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Report 68-9

THE CURRENT DISTRIBUTION ON RESISTIVE LINEAR ANTENNAS

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The Current Distribution on Resistive Linear Antennas

By W.H. Schoendorf

Introduction

The purpose of this work is to determine the effect of resistive losses on the current distribution of long, thin cylindrical antennas. The first section of this report describes a conputational technique for obtaining the current on a thin antenna.

Section II presents the results obtained for a 1 1/2 and a 3 wavelength antenna. These calculations were performed for a variety of source positions and for perfectly conducting and uniform lossy wires. The resistance per unit length of these imperfectly conducting wires was taken as 0.08565 ohms/meter. This choice was made in order to accentuate the effects of antenna losses and is larger than the value which will be encountered in the KWOT structure.

The final section describes an approximation to the current distribution which allows the resistive losses to be estimated and then extended to antennas longer than those considered in this work.

I. Computational Technique

The current distribution, $I(x)$, on a thin perfectly conducting or resistive antenna is described by the Fredholm equation shown below

$$\int_{L_1}^{L_2} I(y)\ K(x,y)\ dy = g(x). \tag{1}$$

The kernal, $K(x,y)$ is dependent on the antenna geometry, $g(x)$ is the appropriate forcing function, and the antenna extends from $x = L_1$ to $x = L_2$. In the case of a thin cylindrical antenna of circular cross section the kernal may be written as

$$K(x,y) = \frac{1}{i\omega 4\pi\epsilon_o}\left[\frac{\partial^2}{\partial x^2} + k^2\right]\frac{e^{ik\sqrt{(x-y)^2 + a^2}}}{\sqrt{(x-y)^2 + a^2}} + \delta(x-y)\ Z_{int} \tag{2}$$

where a is the antenna radius, Z_{int} is the internal impedance per unit length of the antenna and an $e^{-i\omega t}$ time dependence has been assumed.

The numerical solution of equation (1) is accomplished by transforming the kernel $K(x,y)$ into one, $\hat{K}(x_o,y)$, which closely approximates a delta function. This is performed by multiplying both sides of (1) by $T(x_o,x)$ and integrating the resulting expression over the variable x. The transformed current equation then appears as

$$\int_{L_1}^{L_2} I(y)\, \hat{K}(x_o,y)\, dy = \hat{g}(x_o) \tag{3}$$

where

$$\hat{K}(x_o,y) = \int_{L_1}^{L_2} T(x_o,x)\, K(x,y)\, dx \tag{4}$$

and

$$\hat{g}(x_o) = \int_{L_1}^{L_2} T(x_o,x)\, g(x)\, dx \;. \tag{5}$$

If the transformed kernel, $\hat{K}(x_o,y)$, is a good approximation to a delta function, $\delta(x_o-y)$, then equation (3) is solved by inspection. The problem reduces to determining the function $T(x_o,x)$ which when introduced into (4), will give the desired result.[1] This may be accomplished by writing $T(x_o,x)$ as

$$T(x_o,x) = \sum_{n=1}^{N} A_n(x_o)\, W_n(x), \tag{6}$$

where the $W_n(x)$ are a chosen set of functions and the coefficients $A_n(x_o)$ are determined by constraining the integral of $\hat{K}(x_o,y)$ to be unity over a small region surrounding the point $y = x_o$, and by minimizing the mean square value of $\hat{K}(x_o,y)$ outside this region.

The computations presented below were obtained by choosing the $W_n(x)$ to be a set of adjacent rectangular pulses extending over the entire antenna from $x = L_1$ to $x = L_2$

$$\begin{aligned} W_n(x) &= 1 \qquad x_n < x < x_{n+1} \\ W_n(x) &= 0 \qquad \text{otherwise} \end{aligned} \tag{7}$$

and

$$x_n = \frac{(n-1)\ (L_2 - L_1)}{N}, \tag{8}$$

where N is the total number of pulses employed. The source is a one volt, one megacycle slice generator, positioned as indicated in the figures.

II. Numerical Results

Figures 1 through 3 display the current distribution and phase on a perfectly conducting 1 1/2 wavelength antenna with feed points positioned at L/2, L/4 and L/8 respectively. These results are similar to those obtained by a conventional point matching technique, (2) and were used to evaluate the accuracy of the present method. The traveling wave behavior of the antenna becomes evident in figure 3 as the souce is shifted toward one end of the antenna. An investigation of the effect of internal antenna resistance is the major objective of this work. Figure 4 shows the current and phase distribution on a resistive 1 1/2 wavelength antenna with the source located at $x = L/8$. The internal resistance, R_{int} is taken as 0.08565 ohms/meter for reasons described earlier. It is seen that the current phase distribution is the same as in the perfectly conducting case and that the locations of the maxima and minima of the current amplitude remain the same. A comparison of the magnitude of the current distribution in both the resistive and perfectly conducting case is shown in figure 5. The current minima are not affected by the addition of uniform losses on the antenna, while the maxima are reduced.

Computations were performed on a 3 wavelength antenna in order to determine if this behavior varies when the antenna length is increased. Figures 6 and 7 show this structure excited at $x = L/4$ for the perfectly conducting and the resistive case. The traveling wave behavior of the antenna is again in evidence in these two plots. The phase distribution is unaffected by the addition of uniform losses on the antenna and figure 8 indicates that the minima and maxima of the current amplitude behave in the same manner as in the 1 1/2 wavelength case.

III. Approximation of Uniform Losses on Thin Cylindrical Antennas

Since the primary objective of this work is to investigate the effect of resistive losses on thin wire antennas, a study of the reduction of the maxima of the current amplitudes was performed. It was found that an expression for the current distribution of the form

$$I(x) = Ae^{iw(x-x_s)} + Be^{-iw(x-x_s)} + Ce^{iw\left|x-x_s\right|} , \tag{9}$$

where x_s is the source position and w is given by

$$w = k + i\,\alpha \tag{10}$$

gave a good approximation for the positions of the current minima and maxima. The coefficients A, B and C were determined by requiring that the current be zero at the endpoints of the antenna and that there be one volt across the source. This latter condition was applied by prescribing the input impedance of the antenna. It was found that the real part of the input resistance had to be slightly higher than the computed values in order to match the current maxima. This is due to the fact that equation (9) does not account for the decrease in current due to radiation losses, as the attenuation constant, α, is used solely to represent losses due to antenna resistance and was chosen from uniform transmission line theory to be

$$\alpha = \frac{R_{int}}{2Z_o} , \tag{11}$$

Z_o being the free space impedance.

Figures 9 and 10 show the current magnitude predicted by equation (9) for the 1 1/2 and 3 wavelength antennas. It is seen from these figures that this value of α results in a good approximation of the current reduction due to resistive losses. The maxima and minima of the current distribution occur in the correct locations on the antenna and the current maxima are very close to the computed values. The value of the current maxima for the resistive antenna in this approximation, may be closely determined by multiplying the current maxima in the perfectly conducting case by $e^{-\alpha L}$. This operation must be performed in conjunction with increasing the real part of the input impedance by a factor of $e^{\alpha L}$. It was found that the ratio of the two maxima was very insensitive to the original input impedance chosen for the perfectly conducting case. Thus, the reduction of the current peaks due to resistive losses could be determined without an accurate description of the current in the perfectly conducting case.

The reduction of the current peaks in the case of the computed currents shown in figures 5 and 8 was also given very accurately by the $e^{-\alpha L}$ factor. The largest deviation from the computed value was 5.8%. This agreement indicates the possibility of using equations (9) and (11) to estimate the effect of uniform resistive losses on antennas longer than those treated in this work.

This was done in the case of a 12 wavelength antenna and the behavior noted above was again in evidence. The current peaks were decreased by a factor of $e^{-\alpha L} = 0.81$, and the positions peaks and nulls, and magnitude of the nulls were unaffected by the addition of losses.

Since the resistance of 0.08565 ohms/meter is much larger than the value which will be encountered in the KWOT antenna, it appears that the problem of resistive losses will not seriously affect the antenna operation.

WHS:rdc

REFERENCES

1. C.E. Schensted, "Filtering to Improve Resolution in the Presence of Noise", University of Michigan Radio Astronomy Observatory Report Number 64-2, March 1964.

2. R.F. Harrington, J.R. Mautz, I.E.E.E. Transactions on Antennas and Propagation, Vol. AP-15, pp 502-515, July 1967.

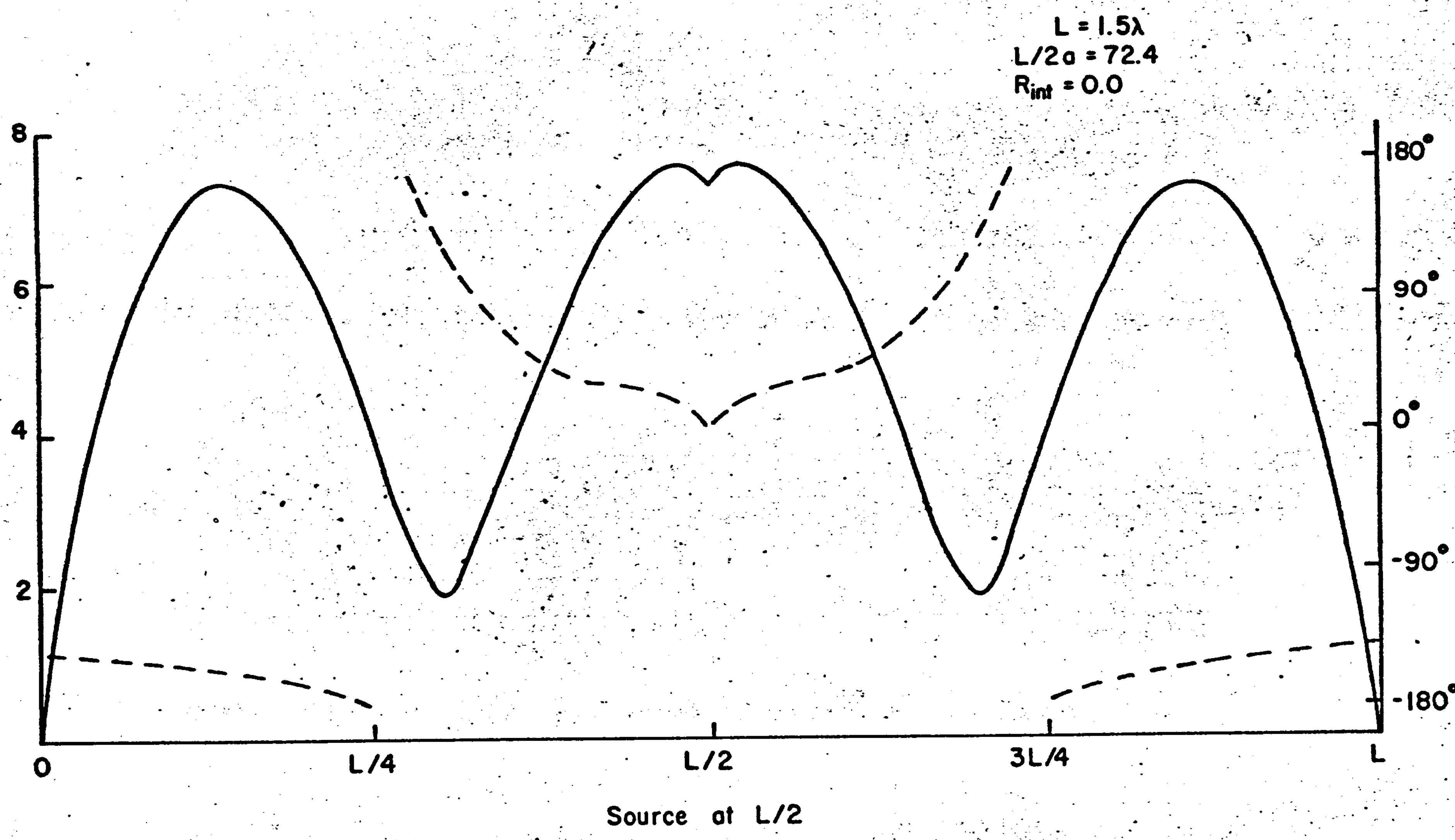
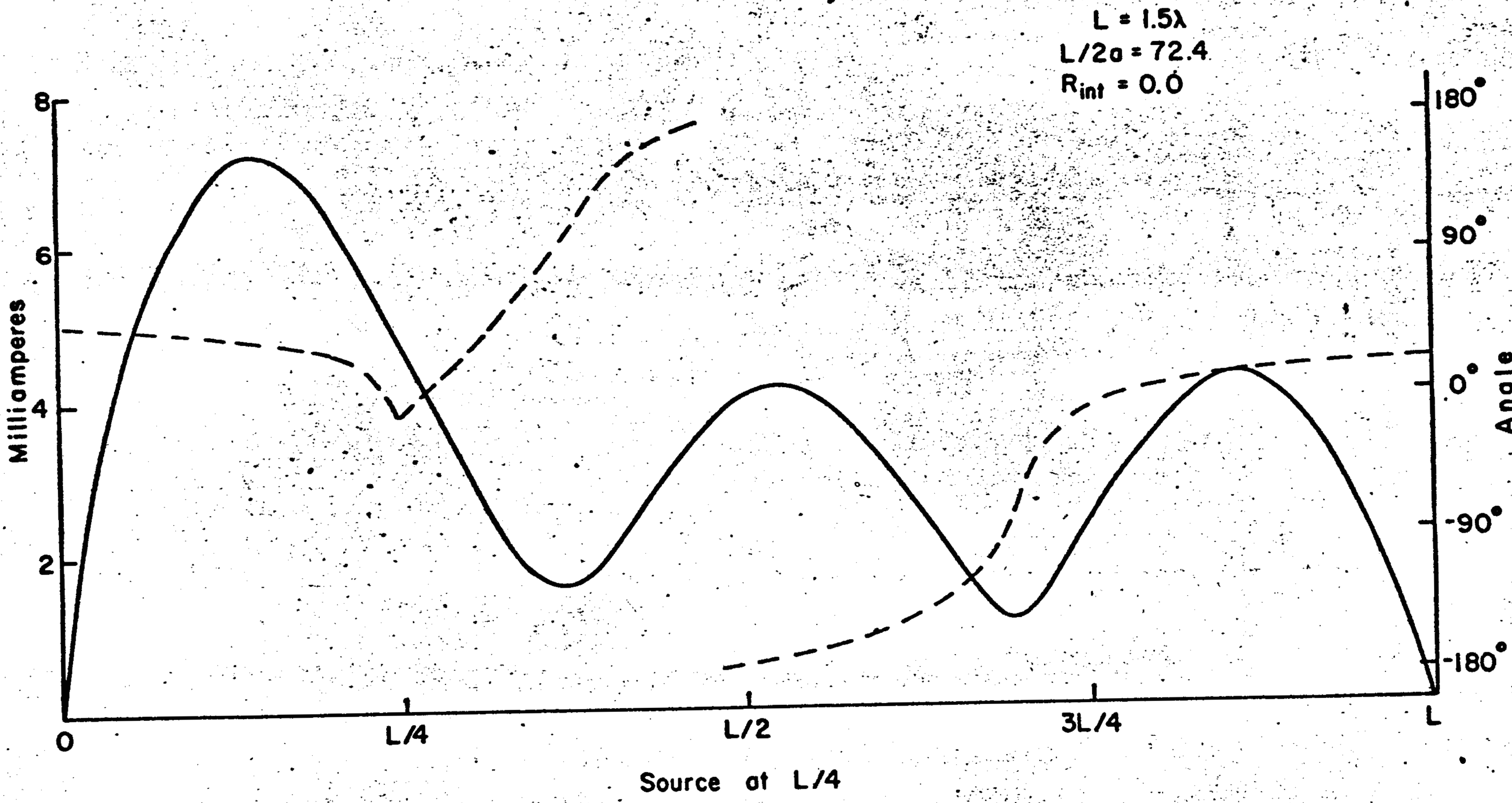
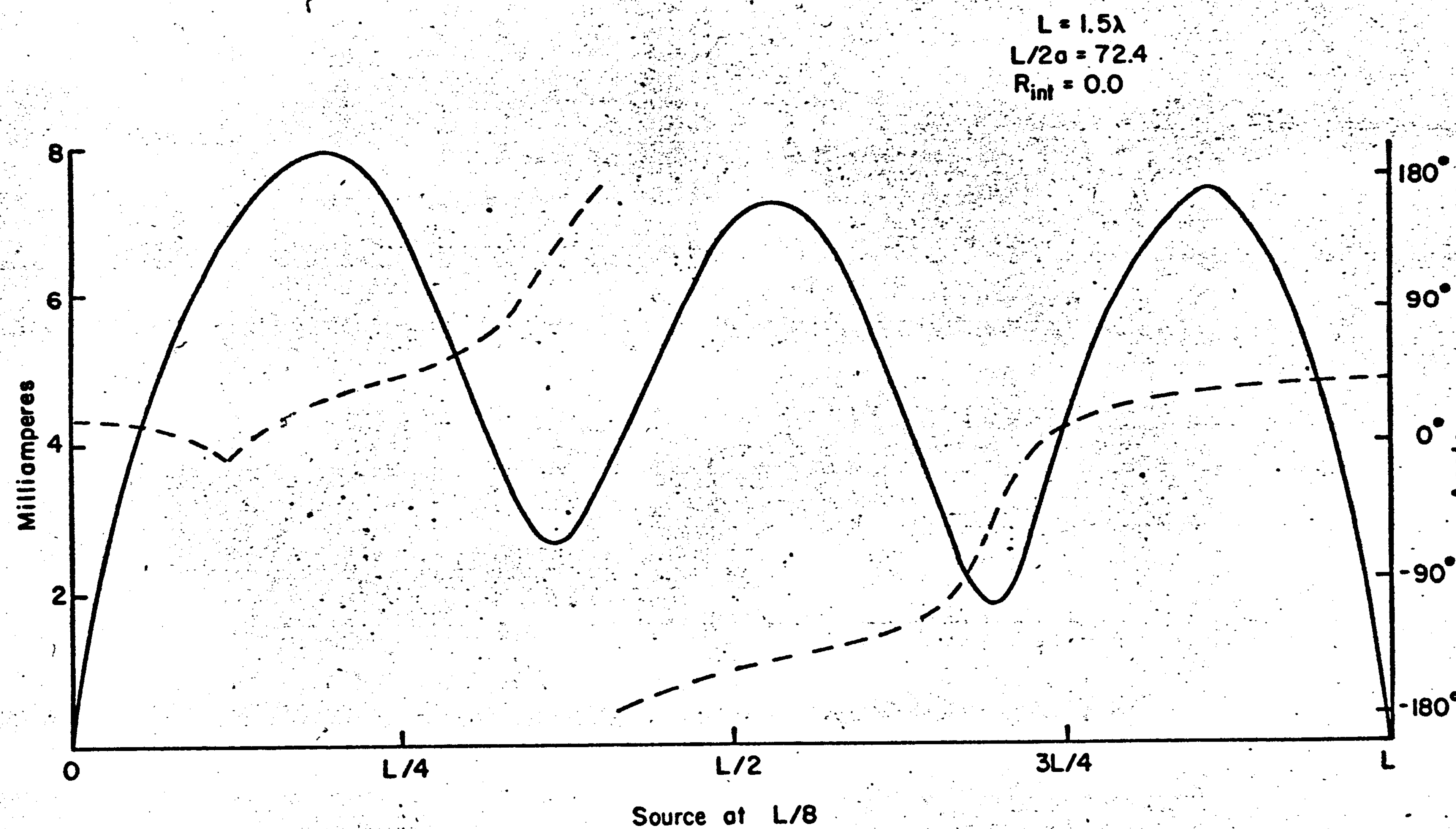

L = 1.5λ
L/2a = 72.4
R_int = 0.0
8
6
4
2
0
L/4
L/2
3L/4
L
180°
90°
0°
-90°
-180°
Source at L/2

Figure 1



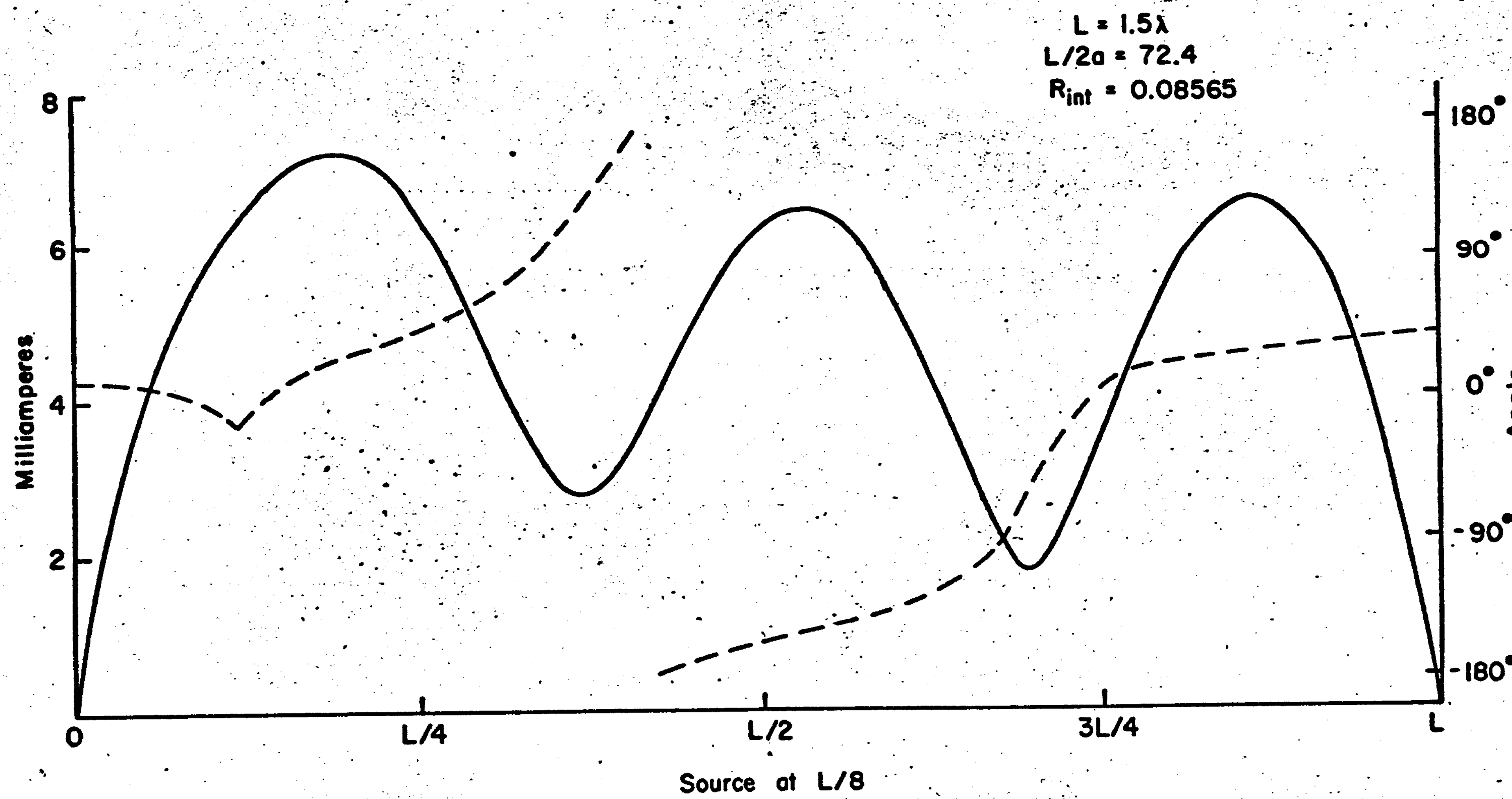
L = 1.5λ
L/2a = 72.4
R_int = 0.0
Milliamperes
8
6
4
2
0
L/4
L/2
3L/4
L
180°
90°
0°
-90°
-180°
Angle
Source at L/4

Figure 2



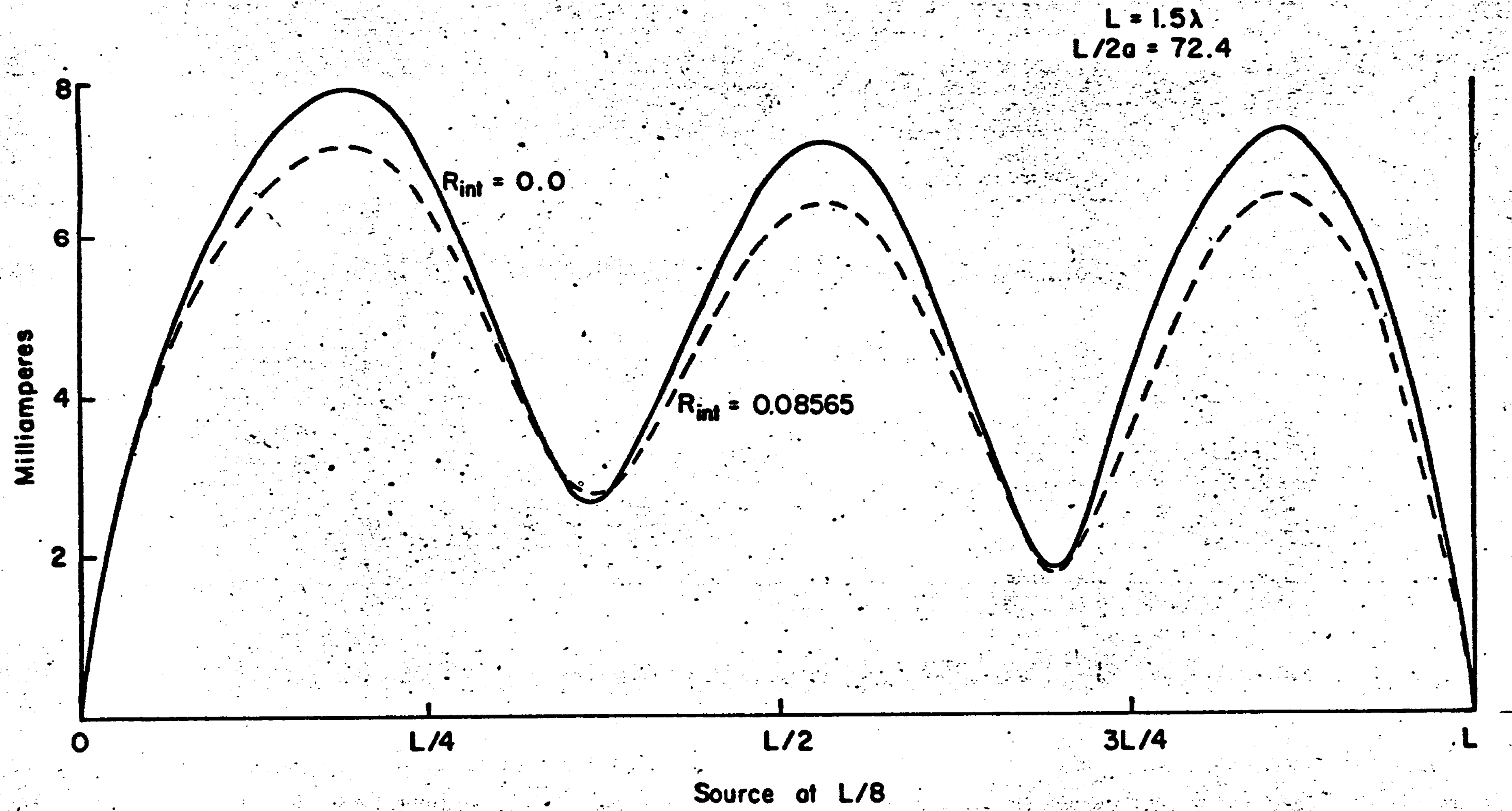
L = 1.5λ
L/2a = 72.4
R_int = 0.0
8
6
4
2
Milliamperes
180°
90°
0°
-90°
-180°
Angle
0
L/4
L/2
3L/4
L
Source at L/8

Figure 3



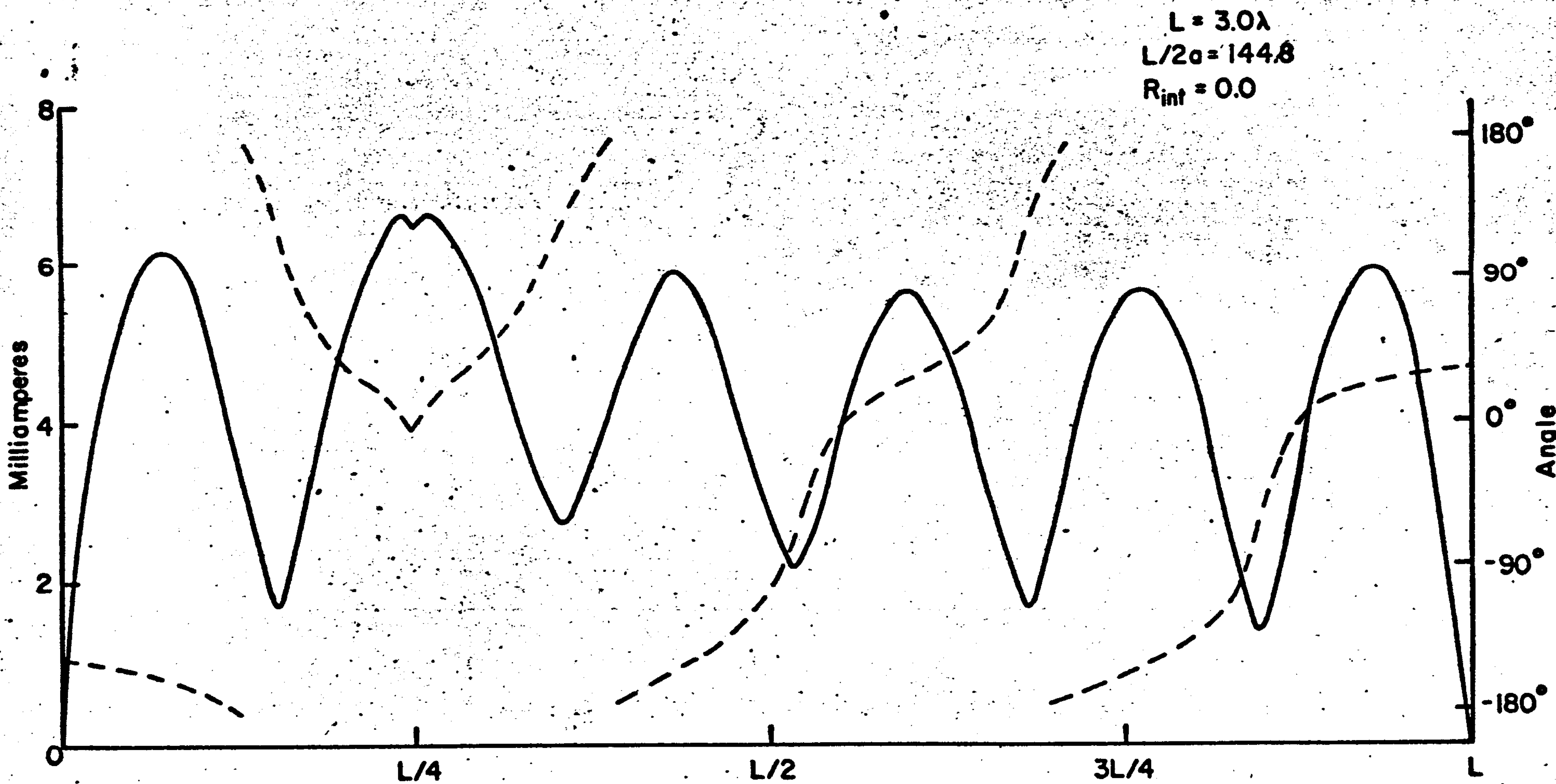
L = 1.5λ
L/2a = 72.4
R_int = 0.08565
8
6
4
2
Milliamperes
180°
90°
0°
-90°
-180°
Angle
0
L/4
L/2
3L/4
L
Source at L/8

Figure 4



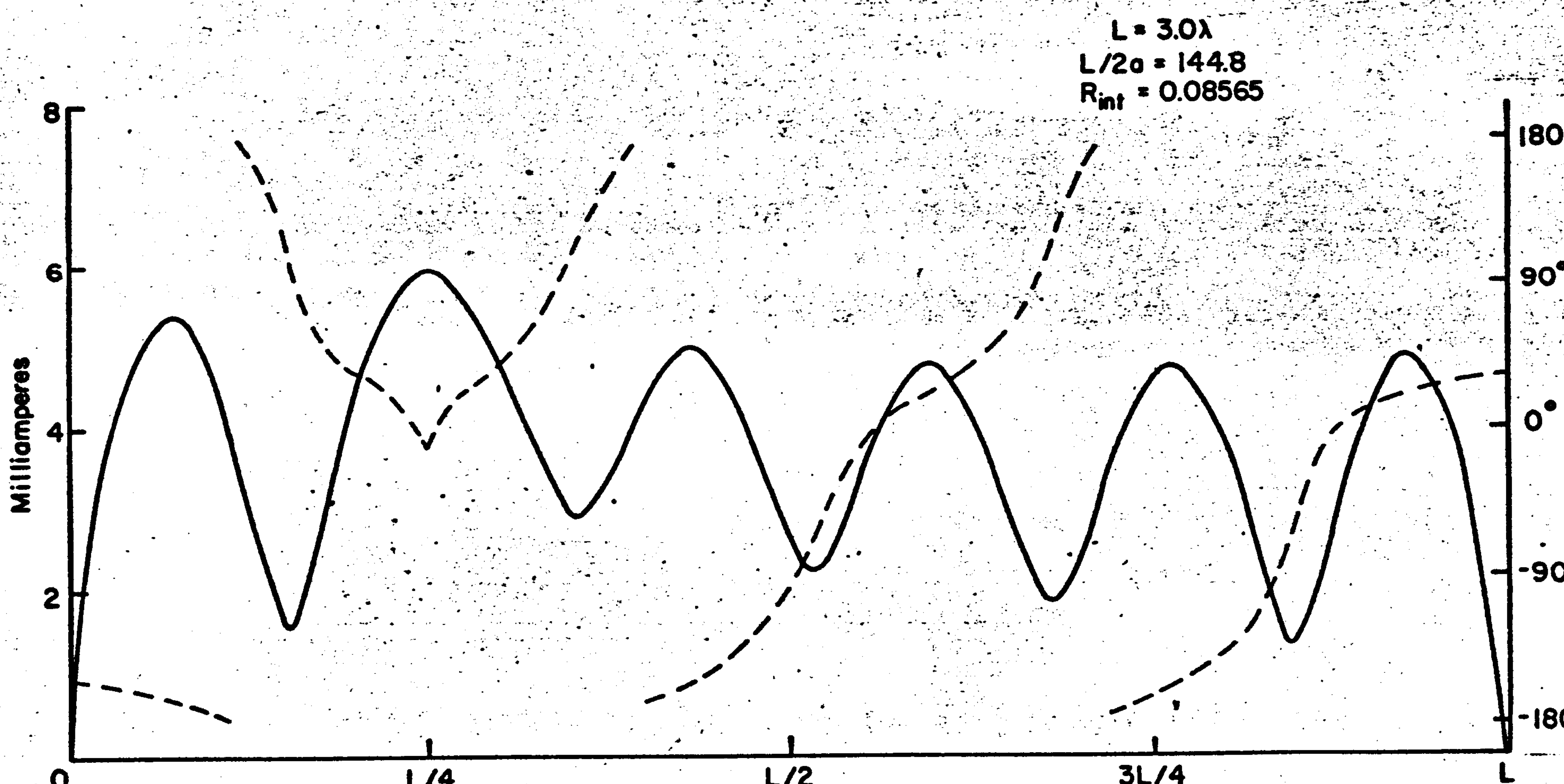
L = 1.5λ
L/2a = 72.4
R_{int} = 0.0
R_{int} = 0.08565
Milliamperes
8
6
4
2
0
L/4
L/2
3L/4
L
Source at L/8

Figure 5



Source at L/4

Figure 6



Source at L/4

Figure 7

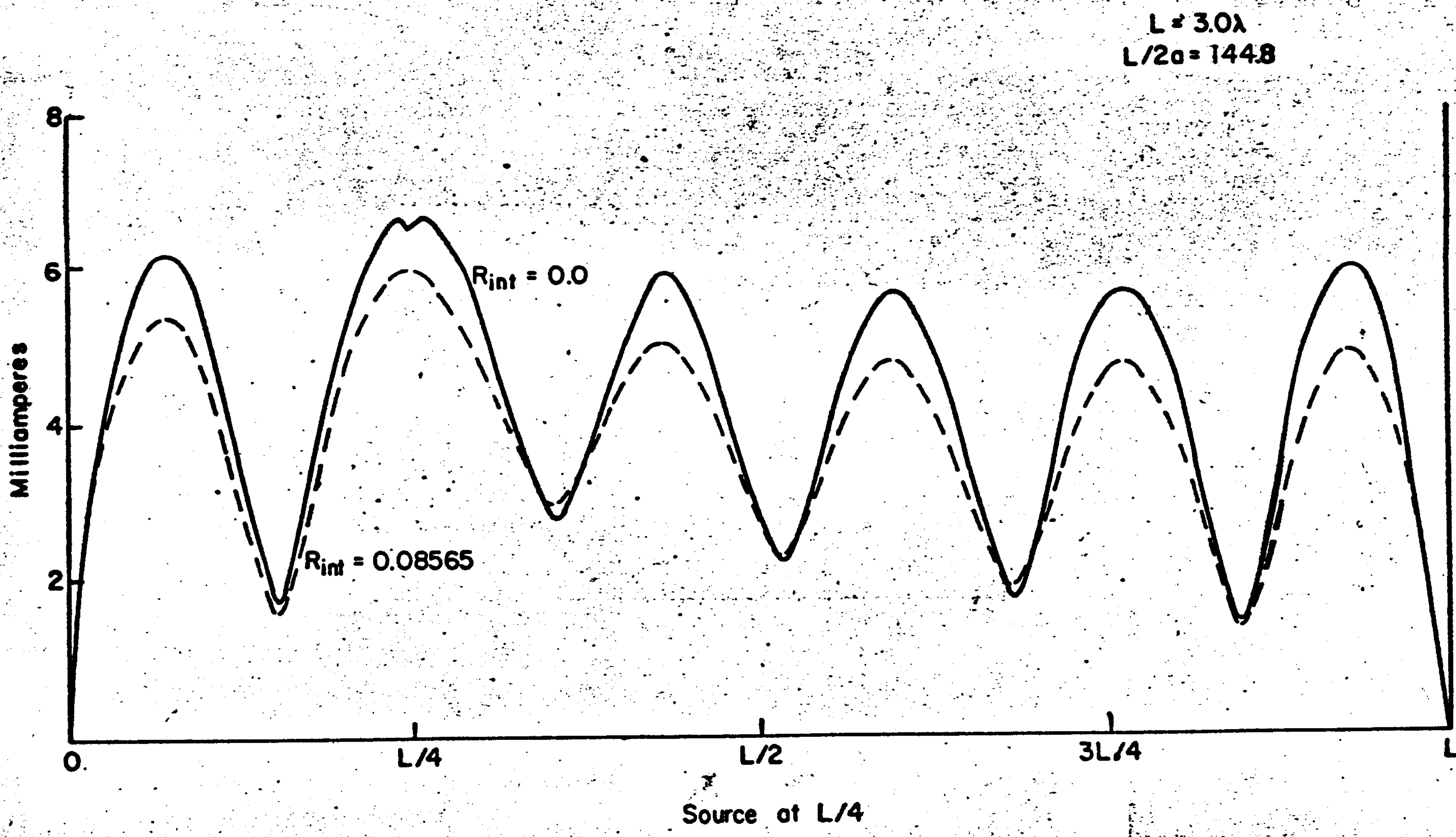
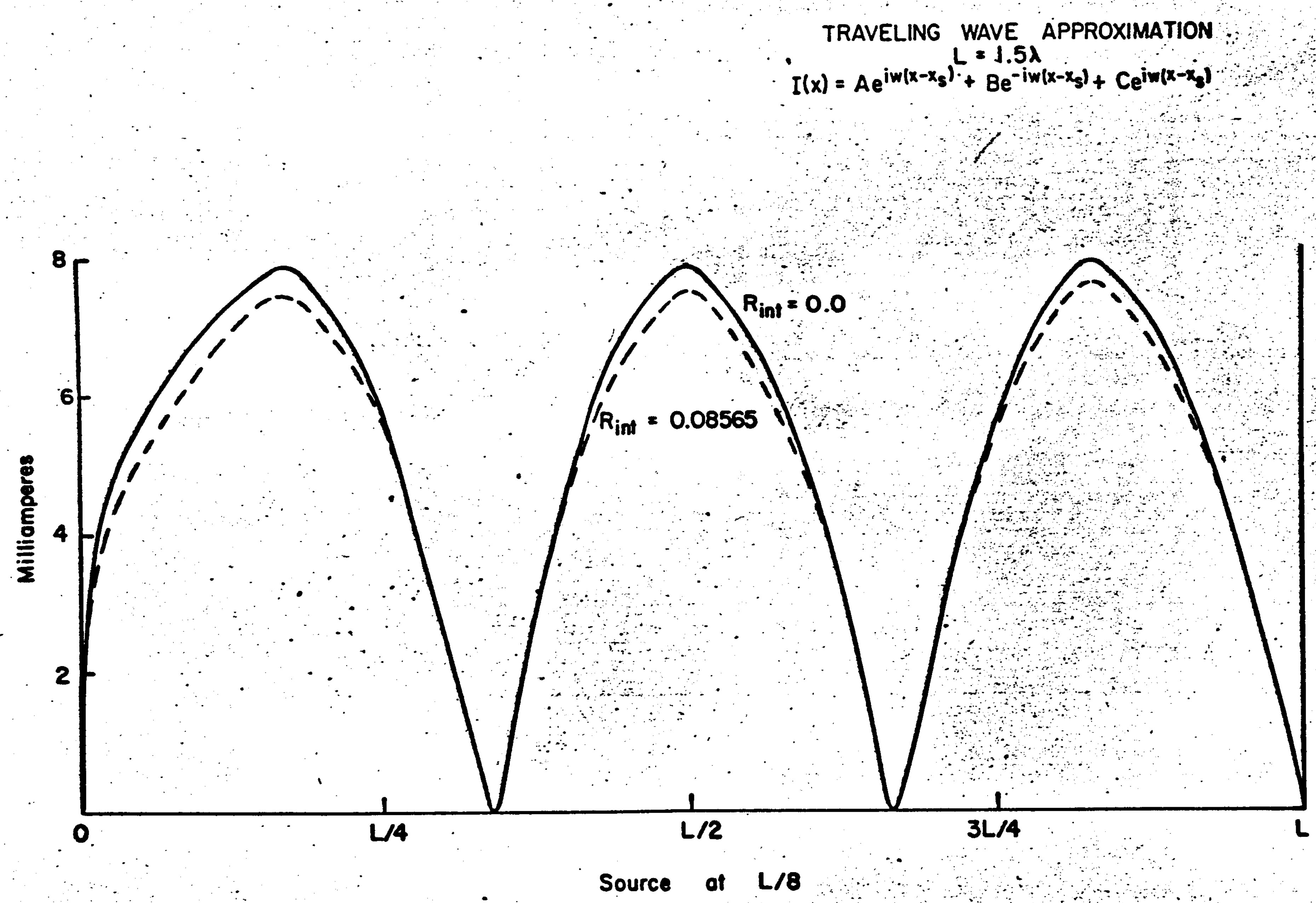

L = 3.0λ
L/2a = 144.8
8
6
4
2
Milliamperes
R_int = 0.0
R_int = 0.08565
0
L/4
L/2
3L/4
L
x
Source at L/4

Figure 8



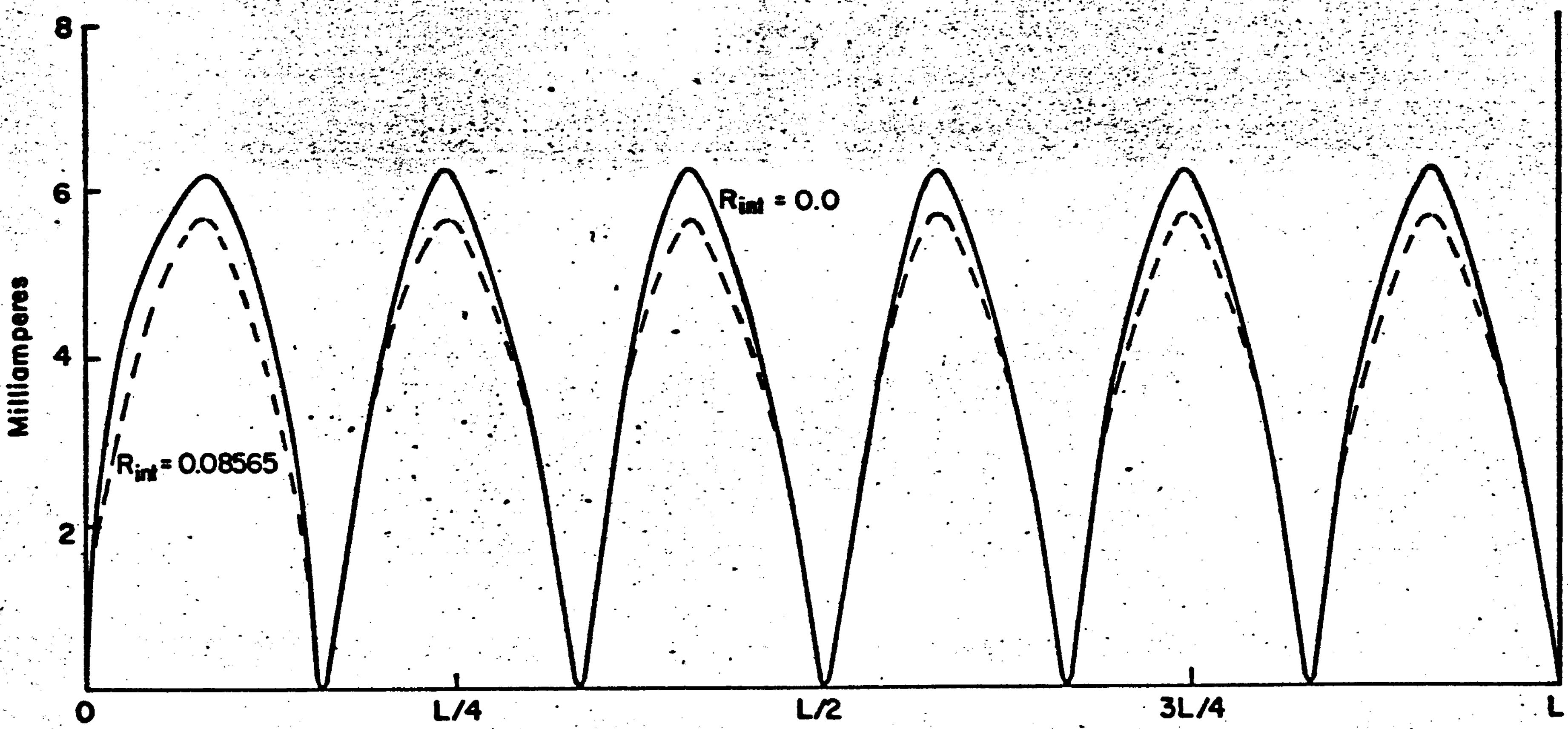
TRAVELING WAVE APPROXIMATION
L = 1.5λ
I(x) = Ae^{iw(x-x_s)} + Be^{-iw(x-x_s)} + Ce^{iw(x-x_s)}
8
6
4
2
Milliamperes
R_int = 0.0
R_int = 0.08565
0
L/4
L/2
3L/4
L
Source at L/8

Figure 9

TRAVELING WAVE APPROXIMATION

$L = 3.0\lambda$

$I(x) = Ae^{iw(x-x_s)} + Be^{-iw(x-x_s)} + Ce^{iw(x-x_s)}$



Source at L/4

Figure 10