

ENGINEERING ESTIMATES FOR SUPERSONIC FLUTTER OF
CURVED SHELL SEGMENTS

William J. Anderson and Kuo-Hsiung Hsu

The University of Michigan

ABSTRACT

Static aerodynamic theory is used to find design curves for the flutter of curved panels. The panels are rectangular segments cut from a circular cylindrical shell. Supersonic flow is directed parallel to the generators of the shell segment. The pressure expression used is general enough to encompass a wide range of physically possible pressure distributions. Design curves are given in the form of a thickness parameter required to prevent flutter as a function of curvature and length-to-width ratio. Upper and lower bounds for the onset of coupled-mode panel flutter are given. Comparisons with other theories and experiments are made. The results are intended to aid in design of wind tunnel models for panel flutter tests.

GPO PRICE \$ _____

CSFTI PRICE(S) \$ _____

Hard copy (HC) 3.00Microfiche (MF) .65

N 68-34201
(ACCESSION NUMBER)
26
(PAGES)
CR-89185
(NASA CR OR TMX OR AD NUMBER)
(THRU) **1**
(CODE) **32**
(CATEGORY)

FACILITY FORM 802

D	$Eh^2/[12(1-\nu^2)]$
F	Airy stress function
H	Thickness parameter, $\left[\frac{\sqrt{M^2-1}}{(1-\nu^2)} \cdot \frac{E}{q} \right]^{1/3} \frac{h}{L}$
h	Panel thickness
L	Length of panel
M	Mach number
m	Axial wave number
N	Number of modes
N_x, N_θ	Stress resultants, see equations (5) and (6)
$p(x, \theta, t)$	Aerodynamic load
q	Integer, also dynamic pressure
R	Radius
t	Time
V	Flow velocity
W	Width of panel
W_{eff}	Effective width of panel, W/n
w	Panel displacement in radial direction
x	Spatial coordinate, flow direction
Z	Curvature parameter, $\frac{L}{R} \frac{L}{h} \sqrt{1-\nu^2}$
δ_{qm}	Kronecker Delta
θ	Angular coordinate
θ_0	Included angle of shell segment
λ	Eigenvalue
ρ	Fluid density

ρ_s	Panel density
ψ	Spatial phase shift
ω	Frequency, rad/sec

LIST OF FIGURES

- Fig. 1. Shell segment.
- Fig. 2. Elastic shell segment imbedded in a cylinder.
- Fig. 3. Thickness requirement for a cylindrical shell segment,
 $\psi = 0^\circ$ (Ackeret theory).
- Fig. 4. Thickness requirement for a cylindrical shell segment,
 $\psi = 90^\circ$ ("slender body" theory).
- Fig. 5. Upper and lower bounds for thickness requirement.
Freely supported edges.
- Fig. 6. Comparison with other theories and experiments.
- Fig. 7. Stability boundaries, $\psi = 0^\circ$ (Ackeret theory), four mode solution.
- Fig. 8. Stability boundaries, $\psi = 90^\circ$ ("slender body" theory).
- Fig. 9. Approximate upper and lower stability boundaries.

1. INTRODUCTION

There is a need for rough estimates of panel flutter boundaries in design work. One specific area which has not been studied extensively involves the flutter of a rectangular panel with curvature (Fig. 1). This panel would in general form a portion of a cylindrical body (Fig. 2), and would be supported at its edges by heavy bulkheads or stringers. Some portions of the outer skin of a missile would correspond to this case. For conventional aircraft, such a panel might represent a window, where the window is relatively weak compared to the surrounding structure. Such windows can be a problem in high speed flight where temperature lowers their rigidity.

The exact mathematical solution to this problem is so difficult, and the results dependent on so many parameters, that there is serious doubt whether it is of any practical value. On the other hand, recent research [1] for cylindrical shells indicates that approximate results can be found by using a steady flow (quasi-static) theory. The approach taken here is to recommend a set of design curves developed by a simple theory with the intent that corrections to these curves are to be made as experimental data are obtained. It is felt that the design parameters used here are somewhat universal and will be the ones which will prove useful in the long run, even after more precise theories are available.

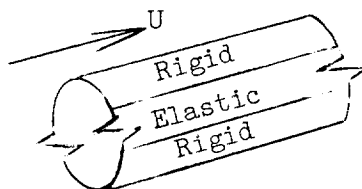
In the mathematical development of the problem, we will remain within the framework of linear shell theory and steady flow theory. A modal

approach will be used, hence the aerodynamic problem resolves to finding pressures on sinusoidally deflected walls. The aerodynamic pressure distribution used in this problem will be generalized in the manner studied in Ref. [2]. A parameter ψ is introduced to typify the spatial pressure distribution. Then ψ is allowed to vary over the range of values which can be expected for such a panel under different physical conditions including boundary layer effects and length-to-width ratio effects. The result is an approximate theory which gives upper and lower limits to the panel thickness requirement to prevent flutter. The upper limit corresponds to the use of Ackeret theory, the lower limit corresponds loosely to a "slender body" type of theory. These two bounds represent extremely different flow situations, yet the dynamics of the system are so insensitive to the details of the pressure distribution as to cause a variation from upper to lower bound of only 35% for most cases.

A series of figures will be presented for design purposes. These should be especially useful in designing models for wind tunnel testing.

Previous work has been done on related problems. Dzygadlo [3] studied the elastic instability of an infinitely long elastic segment of an infinitely long cylinder. The stability boundaries were found for a traveling wave form:

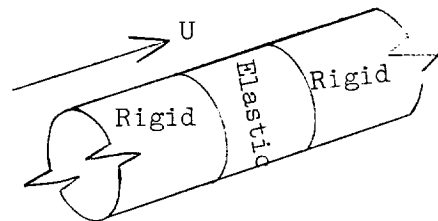
$$w(x, \theta, t) = w(\theta) e^{i\alpha(x - V_t t)}$$



A set of integro-differential equations of motion resulted. These were solved with the aid of a Fourier series in the θ variable. Much effort was placed on a study of the effect of structural (Voigt) damping on the stability boundaries. For moderate amounts of damping, unexpected changes in the panel's stability resulted. The numerical results presented were not extensive. It was concluded that for small damping ratios and for fixed shell thickness and radius, the critical Mach number does not vary greatly for included angles for the segment lying between $\pi/4$ and π .

Another study of interest was by Dowell and Widnell [4]. The case considered was a finite length elastic segment in an infinitely long rigid cylindrical shell. In this case, the generalized aerodynamic forces were found for deflections of the type

$$w(x, \theta, t) = e^{i\omega t} \cos n\theta \sin \frac{m\pi x}{L}$$



Dowell made several comments about the stability of the shell segment merely by looking at the character of the generalized forces. First of all, in the low supersonic Mach number range, a single degree of freedom type of flutter is possible. Secondly, for shell segments with long length-to-width ratios, static divergence takes place. Flutter boundaries for the "coupled-mode" type of flutter were not presented.

Neither of these studies is easy to extend to the current problem. Neither case yields useful design curves (nor were they intended to). The approach used by Dowell would be the more easy to extend to the present case.

The present solution parallels the approach used by McElman [5] to some extent. McElman studied a curved orthotropic panel segment by using a two mode analysis with Ackeret theory. No design curves of the type shown here were presented in McElman's work. (In order to work with lower aspect ratio panels, one needs many modes rather than two.)

2. STATEMENT OF PROBLEM

Consider a cylindrical shell segment as shown in Fig. 1. Supersonic flow passes over the outer surface of the segment, with flow direction parallel to the cylinder axis. The segment is of uniform thickness and of isotropic, homogeneous elastic material. Conventional cylindrical coordinates x, r, θ are used. The shell segment is defined by

$$r = R$$

$$0 \leq x \leq L$$

$$-\frac{\theta_0}{2} \leq \theta \leq \frac{\theta_0}{2}$$

Deflection of the surface of the segment will be given by $w(x, \theta, t)$ measured from the mean radius of the shell. The edges of the shell will be "freely-supported" as defined below. The shell may be internally pressurized. No structural damping will be included.

STRUCTURAL DETAILS

The shell is thin and initially circular. Radial deflections are restricted to be small:

$$\frac{w(x, \theta, t)}{h} \ll 1$$

The in-plane motions of the shell $u(x, \theta, t)$ and $v(x, \theta, t)$ are small compared to $w(x, \theta, t)$ so that inertial effects due to in-plane motion can be neglected (Reissner's assumption). The included angle θ_0 will be less than $\pi/2$ so that Donnell's shallow shell equations can be used.

$$D \nabla^4 w - \bar{N}_x \frac{\partial^2 w}{\partial x^2} - \frac{\bar{N}_\theta}{R^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{1}{R} \frac{\partial^2 F}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} + p(x, \theta, t) = 0 \quad (1)$$

$$\nabla^4 F - \frac{Eh}{R} \frac{\partial^2 w}{\partial x^2} = 0 \quad (2)$$

where D is the bending rigidity of the shell, \bar{N}_x and \bar{N}_θ are constants representing the components of membrane stress due to internal pressurization and $F(x, \theta, t)$ is the stress function defined so that

$$\tilde{N}_x(x, \theta, t) = \frac{1}{R^2} \frac{\partial^2 F}{\partial \theta^2} \quad (3)$$

$$\tilde{N}_\theta(x, \theta, t) = \frac{\partial^2 F}{\partial x^2} \quad (4)$$

Note that \tilde{N}_x and \tilde{N}_θ are the time dependent components of membrane stress due only to panel motion. The total membrane stresses are

$$N_x(x, \theta, t) = \bar{N}_x + \tilde{N}_x(x, \theta, t) \quad (5)$$

$$N_\theta(x, \theta, t) = \bar{N}_\theta + \tilde{N}_\theta(x, \theta, t) \quad (6)$$

Boundary conditions to be applied at $x = 0$, $x = L$ are

$$v = w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 F}{\partial \theta^2} = 0 \quad (7)$$

Boundary conditions at $\theta = \pm \frac{\theta_0}{2}$ are

$$w = \frac{\partial^2 w}{\partial \theta^2} = u = \frac{\partial^2 F}{\partial x^2} = 0 \quad (8)$$

These freely supported boundary conditions have been chosen primarily because they are satisfied (term by term) by the series

$$w(x, \theta, t) = e^{i\omega t} \sum_{m=1}^N a_m \cos \frac{n\pi\theta}{\theta_0} \sin \frac{m\pi x}{L} \left(\begin{array}{l} 0 \leq x \leq L \\ -\frac{\theta_0}{2} \leq \theta \leq \frac{\theta_0}{2} \end{array} \right) \quad (9)$$

These boundary conditions are useful, however, because they result in a dynamically "weak" plate. Hence, the stability boundaries will tend to be conservative for design purposes when applied to damped plates.

At this point, the structural problem has been posed. We need to find the aerodynamic pressures $p(x, \theta, t)$ generated at the panel surface.

AERODYNAMIC DETAILS

A strong assumption on the aerodynamic pressures will be made. The pressure on a panel deflection

$$w(x, \theta, t) = \epsilon e^{i\omega t} \cos \frac{n\pi\theta}{\theta_0} \sin \frac{m\pi x}{L} \quad (10)$$

will be assumed to be

$$p(x, \theta, t) = \epsilon e^{i\omega t} \frac{\rho U^2}{\sqrt{M^2 - 1}} \frac{m\pi}{L} \cos \frac{n\pi\theta}{\theta_0} \cos \left(\frac{m\pi x}{L} + \psi \right) \quad (11)$$

In other words, the pressure will have a magnitude equal to that given by Ackeret theory and a spatial pressure distribution that can vary as desired. (It would be possible to discuss this same assumption later in terms of generalized forces, but this is not as meaningful.)

The above assumption appears to be a strong oversimplification at first glance. One wonders how to choose a proper value of ψ for a cylindrical shell segment. In Reference 6, extensive numerical work was done for pressures on oscillating cylinders exposed to potential flow. The phase angle ψ was found to vary only between 0° and 90° . Hence, we will include values of ψ between 0° and 90° in the present study. In Reference 2, the dynamic results of such an assumption are studied in detail. It is found that the choice of ψ does not drastically affect the thickness requirement for cylindrical shells.*

STABILITY DETAILS

Galerkin's method is used to pose the problem in matrix form. The deflections of the shell segment are

$$w(x, \theta, t) = e^{i\omega t} \cos \frac{n\pi\theta}{\theta_0} \sum_{m=1}^N a_m \sin \frac{m\pi x}{L} \quad (12)$$

Note that this expression allows n half waves in the circumferential direction of the panel. If n takes a value higher than 1, then the effective length-to-width ratio of the panel increases accordingly because there are stationary nodal lines down the length of the panel.

*Footnote: The thickness required to prevent flutter is a continuous function of ψ . For the case studied in Ref. [2], the thickness requirement has a minimum near $\psi = 30^\circ$. This value of the thickness ratio at $\psi = 30^\circ$ is practically identical with that at $\psi = 0^\circ$. Because the calculation for $\psi = 0^\circ$ has more physical meaning (Ackeret theory) it is used as a reference rather than $\psi = 30^\circ$.

The expression for pressure, Eq. (11), is used in conjunction with Eqs. (1) and (2) to yield the set of linear algebraic equations of motion:

$$\sum_{m=1}^N a_m \left[\left\{ m^2 + \left(\frac{L}{W_{\text{eff}}} \right)^2 \right\}^2 + \frac{12 Z^2 m^4}{\pi^4 \left[m^2 + \left(\frac{L}{W_{\text{eff}}} \right)^2 \right]^2} + \frac{N_x L^2}{\pi^2 D} m^2 + \frac{N_0 L^2}{\pi^2 D} \left(\frac{L}{W_{\text{eff}}} \right)^2 \right. \\ \left. - \hat{\lambda} - \frac{24 m \pi}{\pi^4 H^3} \sin \psi \right\} \delta_{qm} + \frac{24}{\pi^4 H^3} \eta_{qm} \cos \psi \Bigg] = 0 \\ (q = 1, 2, \dots, n)$$

where:

$$H = \left[\frac{\sqrt{M^2 - 1} E_1}{(1 - \nu^2)^q} \right]^{1/3} \frac{h}{L}$$

$$Z = \frac{L}{R} \frac{L}{h} \sqrt{1 - \nu^2}$$

$$L/W_{\text{eff}} = \frac{L n}{R Q_0}$$

$$\hat{\lambda} = \frac{\rho_s h \omega^2 L^4}{\pi^4 D}$$

$$\eta_{qm} = \begin{cases} 0 & m+q \text{ even} \\ \frac{4 q m}{q^2 - m^2} & m+q \text{ odd} \end{cases}$$

Thus, a set of linear algebraic equations are obtained. The occurrence of a negative eigenvalue $\hat{\lambda}$ signifies static divergence of the panel and complex $\hat{\lambda}$ signifies flutter.

3. RESULTS

Stability boundaries have been calculated for the aerodynamic loading discussed above. All results will be given for cases with zero membrane stresses N_x and N_0 . This theory would be more inaccurate at positive

values of membrane stress which would cause higher flutter frequencies.

The results are presented using the effective length-to-width ratio L/W_{eff} , a curvature parameter Z and a thickness parameter H . The plots of H versus L/W_{eff} are given as a generalization of the work of Kordes, Tuovila, and Guy [7], and the curvature parameter Z is chosen to correspond to Batdorf's study of cylinder buckling [8].

A four mode solution for $\psi = 0$ (Ackeret theory) is given in Fig. 3. It is easily seen that curvature helps to stiffen the panel and reduce the thickness requirement. An interesting effect is obtained in the regions where H increases with increasing L/W_{eff} . This means that a panel of given physical length and width will flutter in a mode with $n > 1$, giving a higher critical value of L/W_{eff} . As an example, a panel of length 10 inches and width 2 inches has a physical length-to-width ratio of 5. If $Z = 8000$ for this panel then it must have a thickness ratio of $H = 0.065$ to prevent flutter from occurring at an effective length-to-width ratio of 15. This particular panel flutters with $n = 3$, i.e., it has two interior nodal lines extending down its length.

Results for $\psi = 90^\circ$ are given in Fig. 4. These results are somewhat similar to the $\psi = 0$ curves except that the instabilities in the lower left corner are due to static divergence. Again, one must observe the cases where H increases with L/W_{eff} and one must choose the multiple of the geometric length-to-width ratio which gives the critical value of H .

Finally, several of the curves for $\psi = 0^\circ$ and $\psi = 90^\circ$ are combined in Fig. 5. These are approximate bounds for the thickness required to prevent an instability. Note that the difference between the two bounding curves is not great, particularly in certain intermediate regions of L/W_{eff} . This may be an indication as to why Ackeret theory gives relatively good results for the cylinder experiments discussed in Ref. 1. In these tests, a cylinder fluttered in a mode which yields an effective $L/W_{\text{eff}} = 9.21$ and with $Z = 6,950$. Flutter occurred at $H = 0.0624$.

It is felt that figures such as Fig. 5 can be very useful to designers. The curves give rough bounds for the onset of flutter or divergence for freely supported panels. As experiments are carried out, confidence can be obtained for accuracy of such curves.

It is suspected that for L/W_{eff} large, more modes are needed to ensure convergence. Gaspars and Redd [9] studied carefully the number of modes required for convergence on finite aspect ratio flat plates when Ackeret theory is used. They found that as many as 50 modes were needed for flat plates with aspect ratios of 10 or more. The present results are less sensitive to convergence problems because of the presence of curvature and because the flutter parameter H is less sensitive to error in the eigenvalue of the matrix problem.

Other theories and experiments are shown in Fig. 6. Several of the points shown correspond to work for full cylindrical shells. The problem of a shell segment is closely related to that of a full cylinder. Structurally, the major difference is that the full cylinder can flut-

ter in modes with waves travelling in the circumferential direction whereas the segment cannot. Of particular interest in Fig. 6 are the experimental points found for full cylinders by Olson and Fung [1.] and by Stearman, Lock and Fung [10]. It is now suspected that these cylinders did flutter in circumferentially travelling waves [11]. This might explain why the experimental values occurred at slightly higher values of thickness ratio than predicted by the present theory. The experiments of Tuovila and Hess [12] were carried out for a shell segment clamped all around. The tests were done at Mach 1.3, which unfortunately brings in transonic effects into the comparison. In transonic flow the unsteady aerodynamic terms are of importance and their effects are neglected in this theory.

The theories of Voss [13] and Shulman [14] both were done for a complete cylinder with the use of Ackeret theory. These should (and do) correspond with the present calculations and serve as a check.

The theory of Dyzgadlo [15] was carried out for a more exact aerodynamic theory on a finite length cylinder. These were mode calculations. These appear to yield values of H slightly higher than the current work which may reflect the fact that fewer modes were used by Dyzgadlo. (Gasparis and Redd [8] indicate that the thickness requirement decreases with an increase in the number of modes.)

All in all, there are no unclassified experiments known to the authors which furnish the proper comparison with the theory. Such tests would be useful.

Figures 7-9 are cross plots of the same data given in Figs. 3-5.

4. CONCLUSIONS

Design curves have been given for prevention of aeroelastic instability of curved shell segments. The calculations are approximate in the sense that unsteady aerodynamic effects are ignored and because a modal approach was used. These are the very reasons that the results are understandable, however. From a practical standpoint, these design curves, as corrected by experiment, will probably be more useful than exact theories.

The only case illustrated here was the case of freely supported edges. The results should be conservative if applied to panels with clamped edges.

One shortcoming of the current calculations is the limited number of modes used. Only four mode solutions were carried out. If more modes were used, the results presented at higher values of L/W_{eff} would become more accurate.

5. REFERENCES

1. Olson, M. D. and Fung, Y. C., "On Comparing Theory and Experiment for the Supersonic Flutter of Circular Cylindrical Shells," AIAA 5th Aerospace Sciences Meeting, AIAA Paper No. 67-77, New York, January 1967.
2. Anderson, William J., "Aeroelastic Stability of Plates and Cylinders." The University of Michigan Report 08079-2-F, Part A.
3. Dzygadło, Z., "The Problem of Aeroelasticity of a Cylindrical Panel and a Plate Strip Taking into Consideration the Transversal Coupling," Proceedings of Vibration Problems, Warsaw, 2, 5 (1964).
4. Dowell, E. H. and Widnall, S. E., "Generalized Aerodynamic Forces on and Oscillating Cylindrical Shell," Quarterly of Appl. Mech., XXIV, 1, April 1966.
5. McElman, J. A., "Flutter of Curved and Flat Sandwich Panels Subjected to Supersonic Flow," NASA TN D-2192, Langley Research Center, April 1964.
6. Anderson, William J., "Oscillatory Pressures in a Simplified Boundary Layer with Application to Cylinder Flutter," AIAA Journal, Vol. 4, No. 5, May 1966.
7. Kordes, E. E., Tuovila, W. J., and Guy, L. D., "Flutter Research on Skin Panels," NASA TN D-451, Langley Research Center, Sept. 1960.
8. Batdorf, S. B., "A Simplified Method of Elastic-Stability Analysis for Thin Cylindrical Shells," NACA Report 874, Langley Research Center, 1947.
9. Gaspars, P. A. and Redd, B., "A Theoretical Analysis of the Flutter of Orthotropic Panels," NASA TN D-3551, Ames Research Center, August 1966.
10. Stearman, R. O., Lock, M. H., and Fung, Y. C., "Ames Tests on the Flutter of Cylindrical Shells," GALCIT Structural Dynamics Report SM 62-37, December 1962.
11. Private communication with M. Olson and D. Evensen.

12. Tuovila, W. J. and Hess, R. W., "Experimental Investigation of Flutter of Buckled, Curved Panels," NASA Memo 5-18-59L, Langley Research Center, June 1959.
13. Voss, H. M., "The Effect of an External Supersonic Flow on the Vibration Characteristics of Thin Cylindrical Shells," Journal of Aerospace Sciences, Vol. 28, No. 12, December 1961.
14. Shulman, Y., "Vibration and Flutter of Cylindrical and Conical Shells," OSR Technical Report No. 59-776 Massachusetts Institute of Technology, June 1959.
15. Dzygadło, Z., "Self Excited Vibration of a Cylindrical Shell of Finite Length in Supersonic Flow," Proceedings of Vibration Problems, Warsaw, Vol. 3, No. 1, 1962.

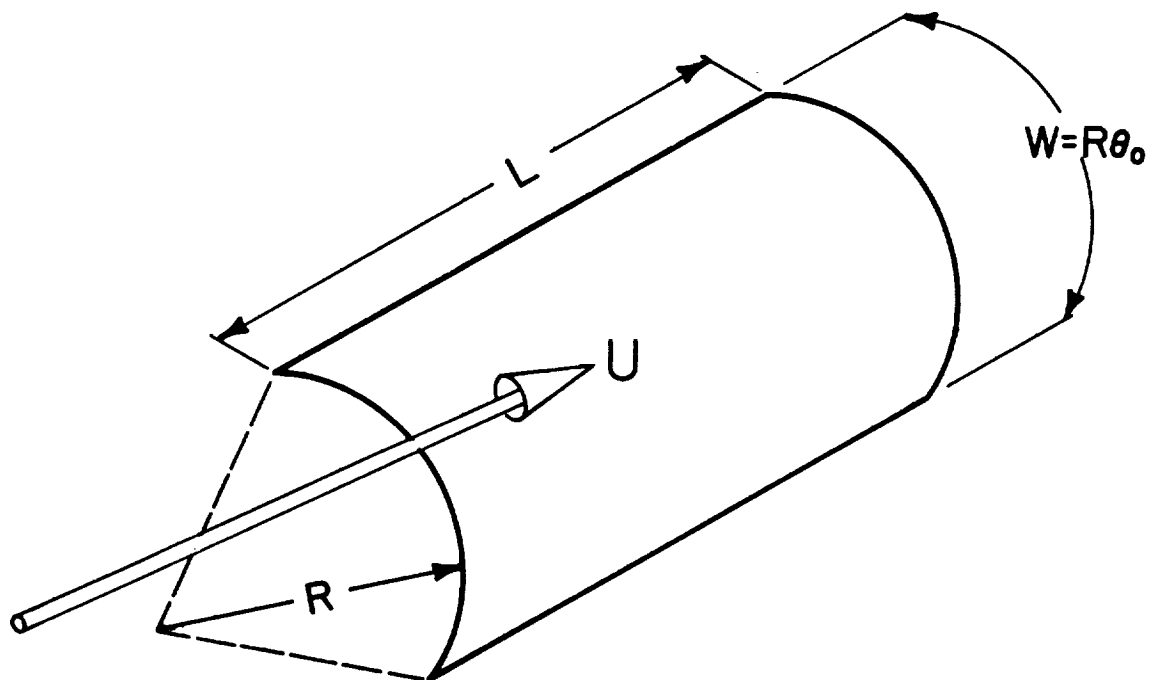


Fig. 1. Shell segment.

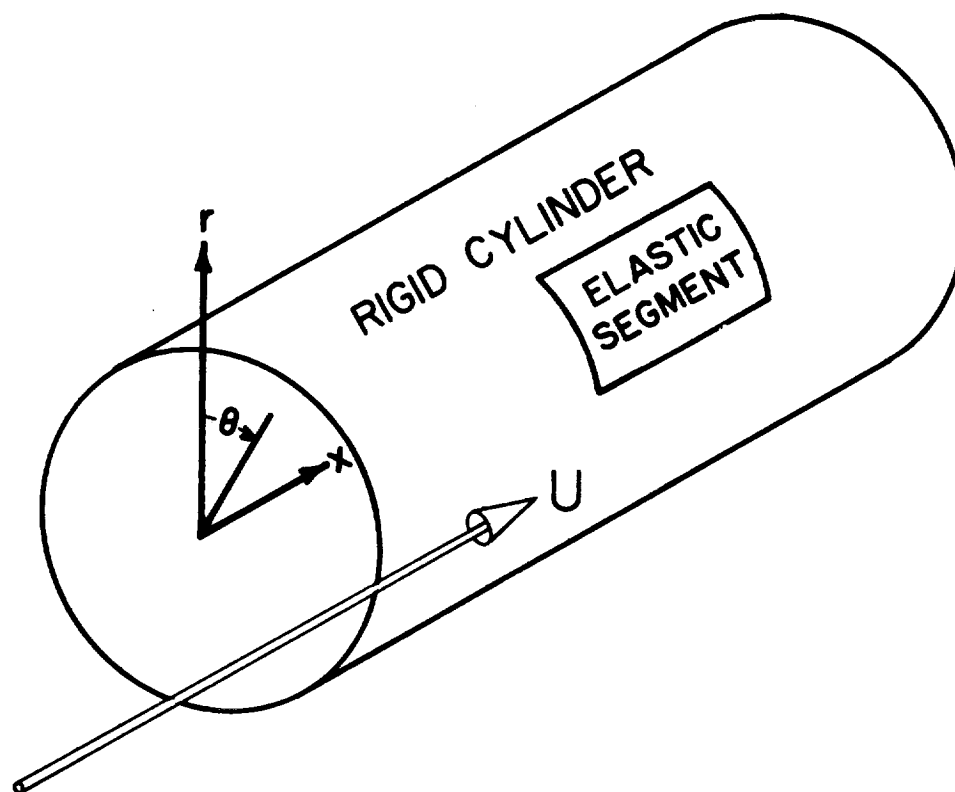


Fig. 2. Elastic shell segment imbedded in a cylinder.

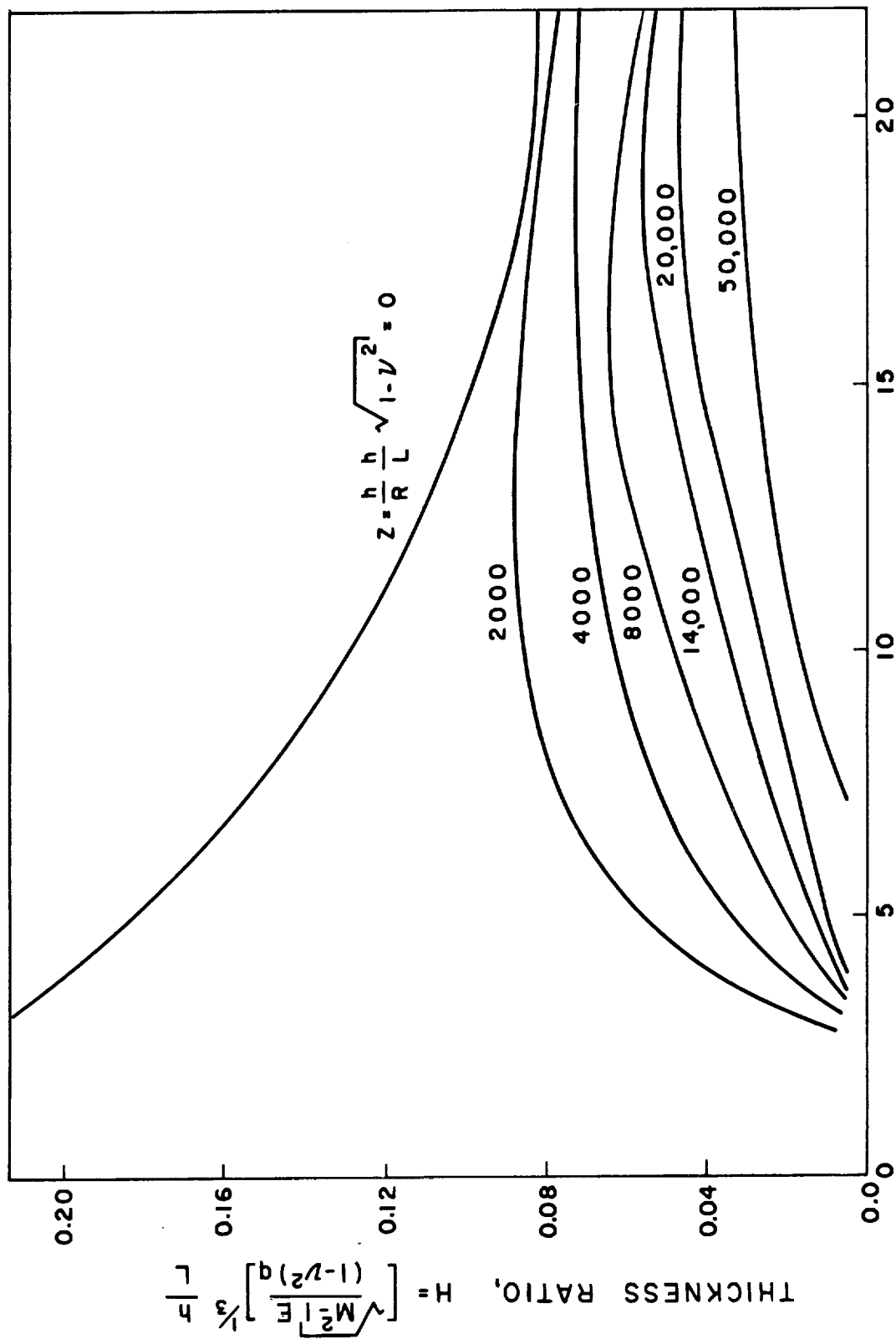


Fig. 3. Thickness requirement for a cylindrical shell segment, $\psi = 0^\circ$ (Ackeret theory).

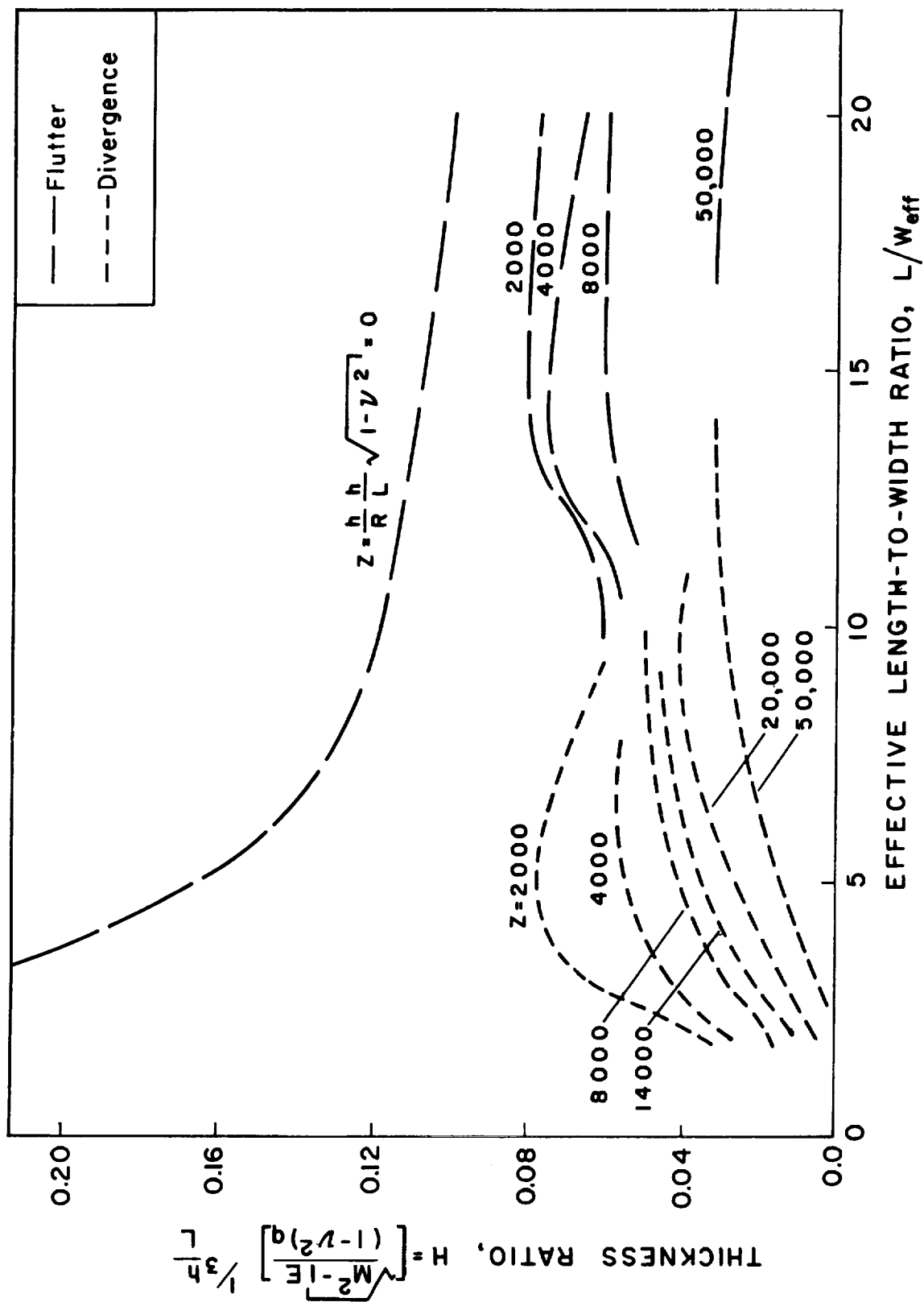


Fig. 4. Thickness requirement for a cylindrical shell segment, $\psi = 90^\circ$ ("slender body" theory).

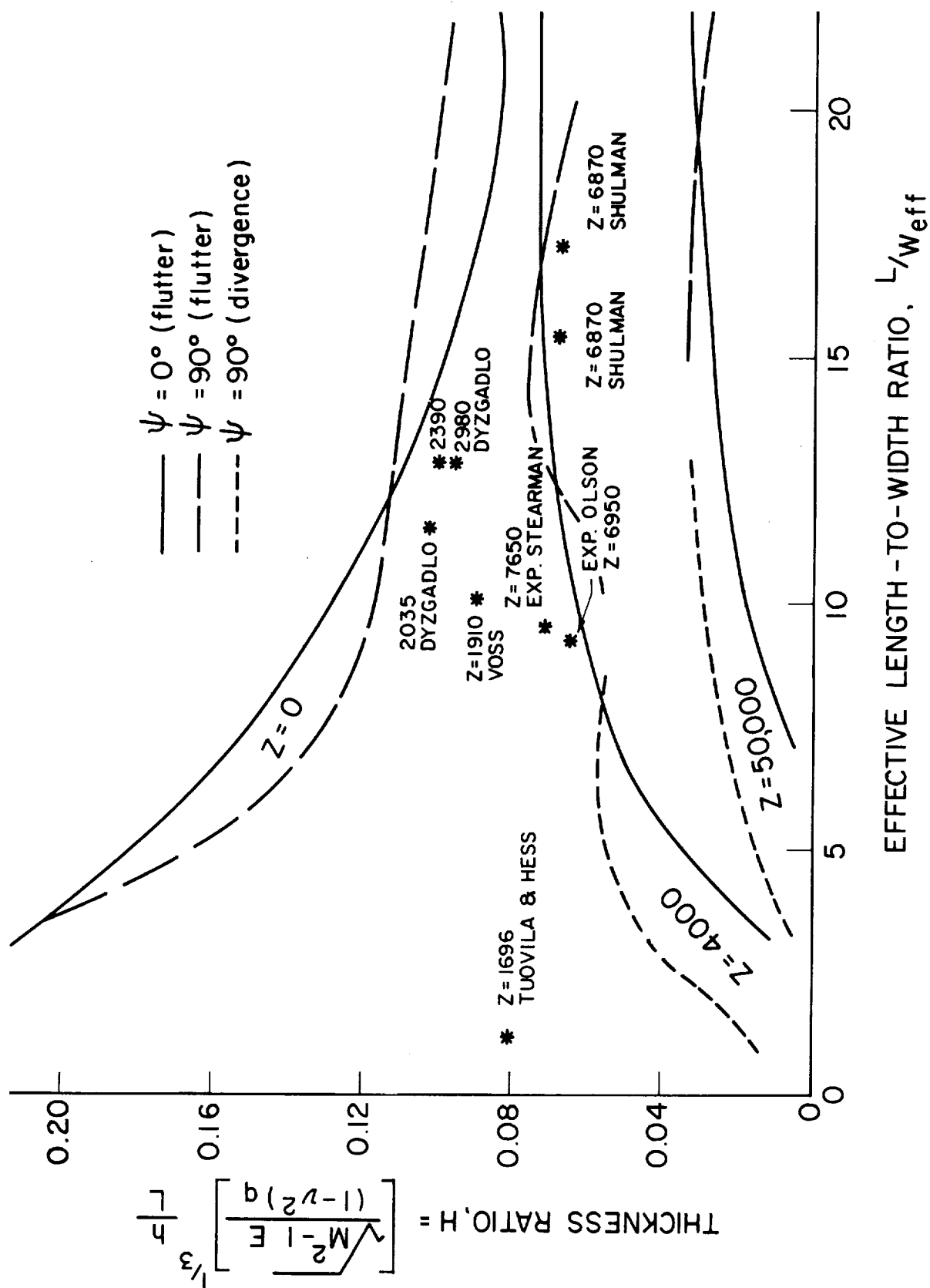


Fig. 5. Upper and lower bounds for thickness requirement. Freely supported edges.

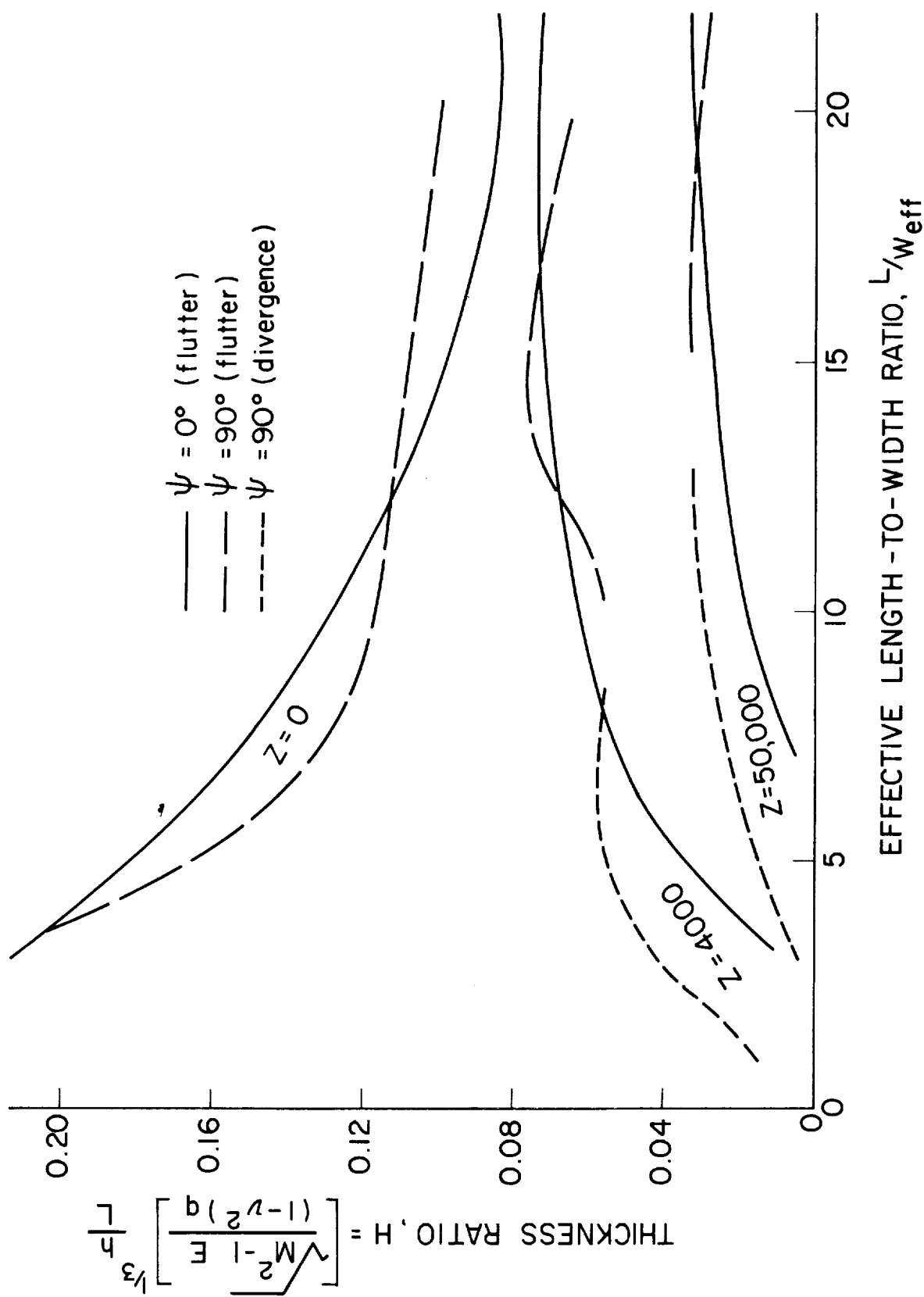


Fig. 6. Comparison with other theories and experiments.

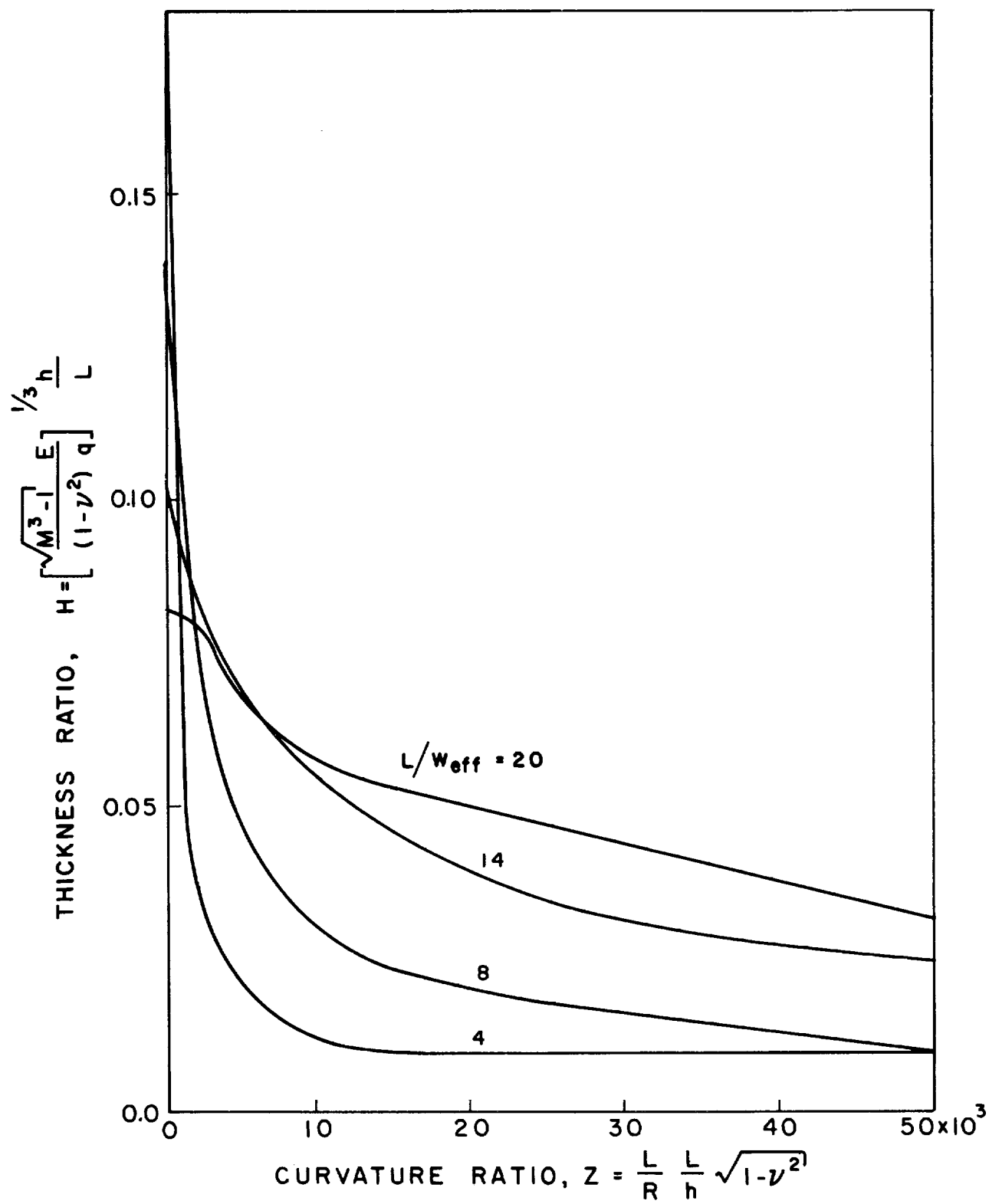


Fig. 7. Stability boundaries, $\psi = 0^\circ$ (Ackeret theory), four mode solution.

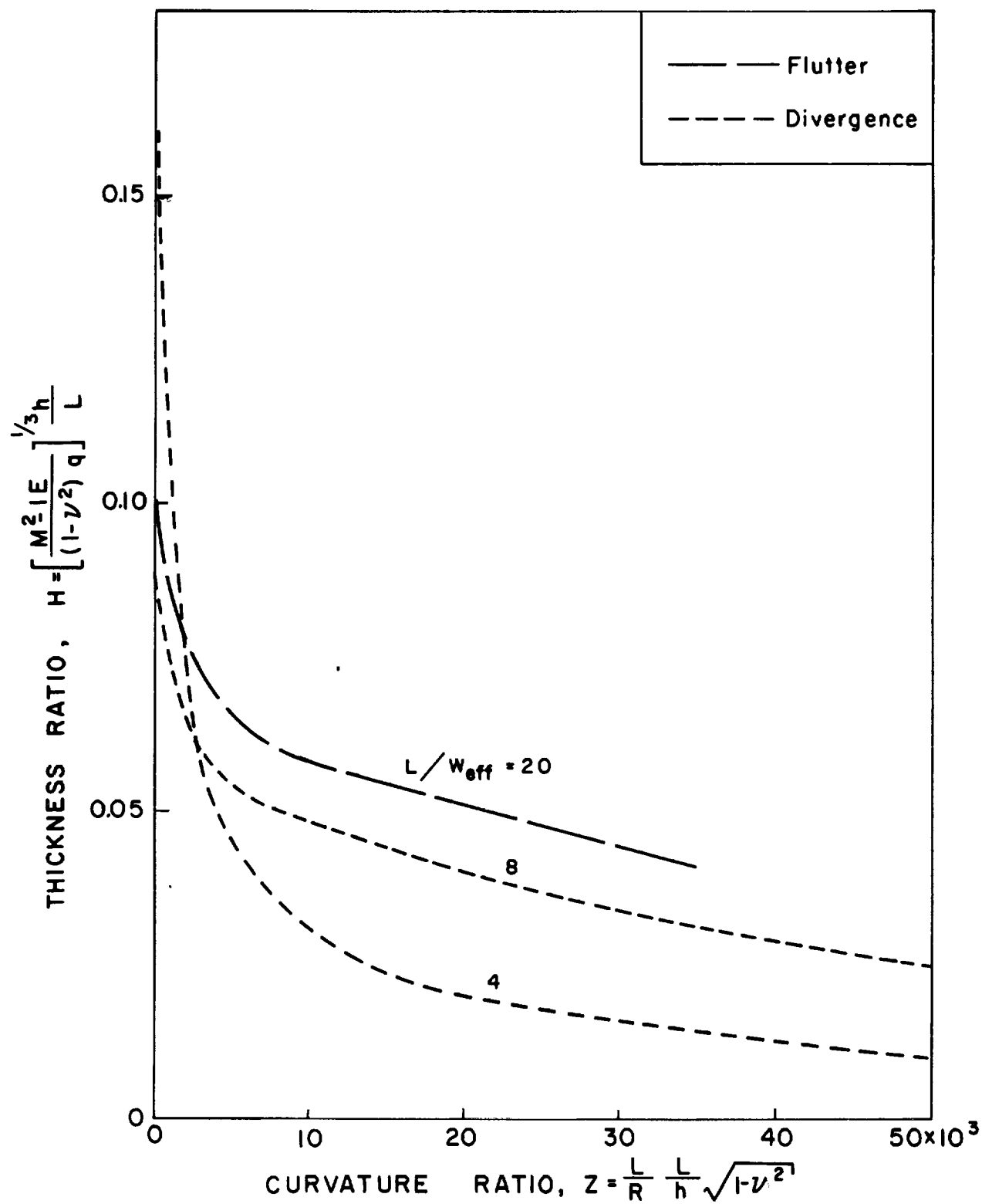


Fig. 8. Stability boundaries, $\psi = 90^\circ$ ("slender body" theory).

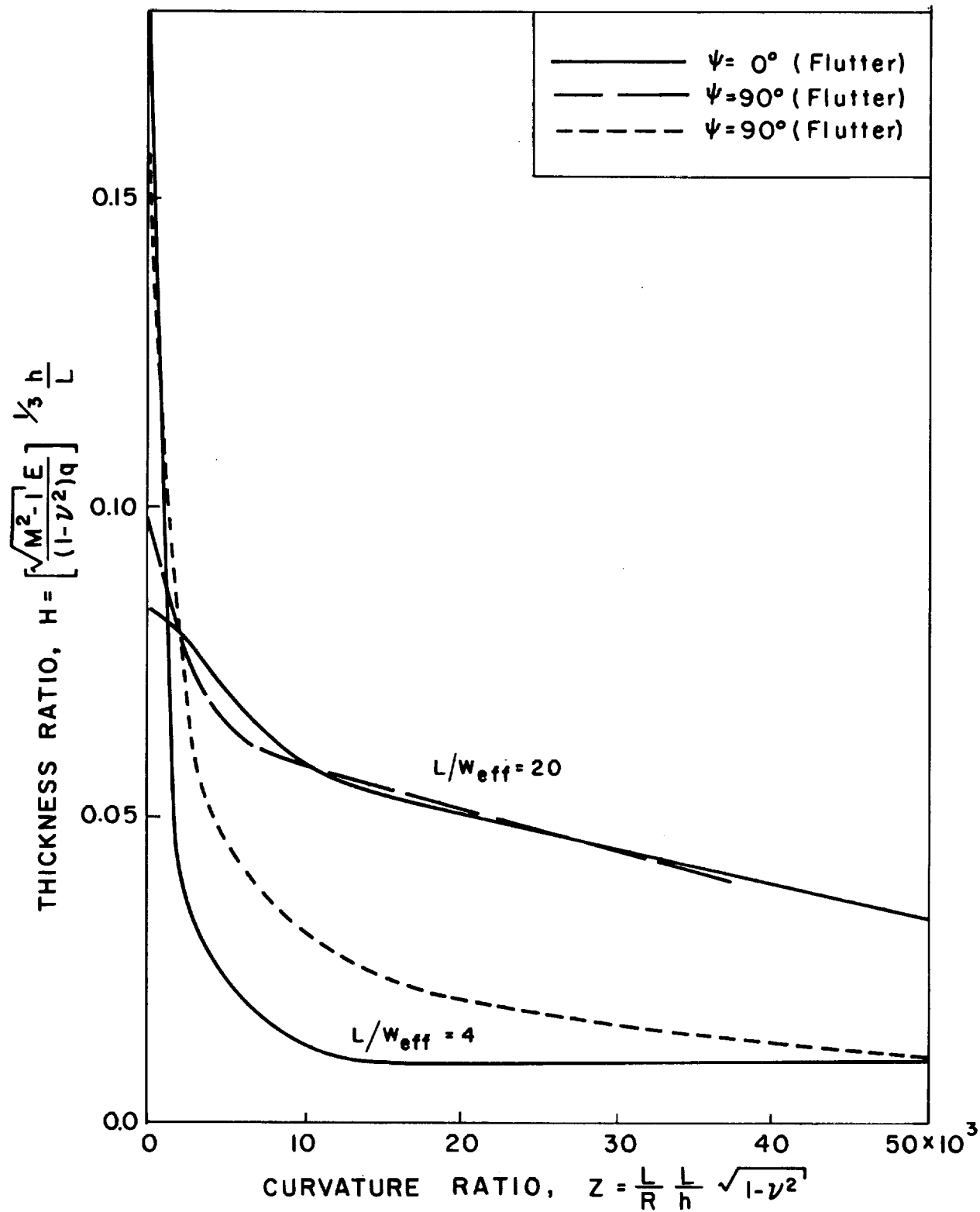


Fig. 9. Approximate upper and lower stability boundaries.