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ON THE COMPUTATION OF SOLAR ELEVATION ANGLES AND THE DETERMINATION OF SUNRISE AND SUNSET TIMES

by Harold M. Woolf

National Meteorological Center, ESSA
Hillcrest Heights, Md.

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SUMMARY

The complete procedure for precise computation of solar elevation angle as a function of latitude, longitude, date, and time is given. Construction of a graphical aid for determining times of sunrise and sunset, with a precision of one minute, as functions of latitude, longitude, date, and altitude is described. This Sunrise-Sunset Finder is contained in the Appendix.

CONTENTS

ON THE COMPUTATION OF SOLAR ELEVATION ANGLES AND THE DETERMINATION OF SUNRISE AND SUNSET TIMES

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Environmental Science Services Administration
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INTRODUCTION

Upper-atmosphere research workers often require solar elevation (or zenith) angles as input for various problems. Examples include the determination of solar-radiation corrections* for stratospheric radiosonde data and the calculation of photo-dissociation and ionization rates in the mesosphere and thermosphere [2]. Solar elevation as a function of declination, local time, and latitude is presented in convenient graphical form in the Smithsonian Meteorological Tables [3]. However, for many purposes the charts do not permit solar elevation to be determined with sufficient precision. In addition, many of the calculations in which these angles are employed are performed by electronic computer. Thus it seems worthwhile to summarize the steps required for precise determination of solar elevation. The procedure is quite readily programmed for computer operation; one such program has been in daily use since the beginning of 1964 [4].

On the other hand, there are applications of earth-sun relationships in which direct calculation is not the optimum approach. For example, it is often necessary in connection with the planning of such experiments as chemical trail releases from rockets, to know the time of sunrise and/or sunset at a particular location and altitude. While tables of sunrise/sunset times at the earth's surface are readily available [5], adjustment for altitude entails tedious exercises in earth-sun geometry and computations with trigonometric functions as in the case of solar elevation. However, with proper manipulation of the relevant astronomical parameters and formulas, it is possible to perform the

*These corrections are based on the average differences between daytime and nighttime observations, which have been found [1] to depend strongly on the solar elevation angle of the daylight sounding. They are needed to achieve compatibility of data, obtained at various local times and with many different instruments, for purposes of synoptic analysis [4].

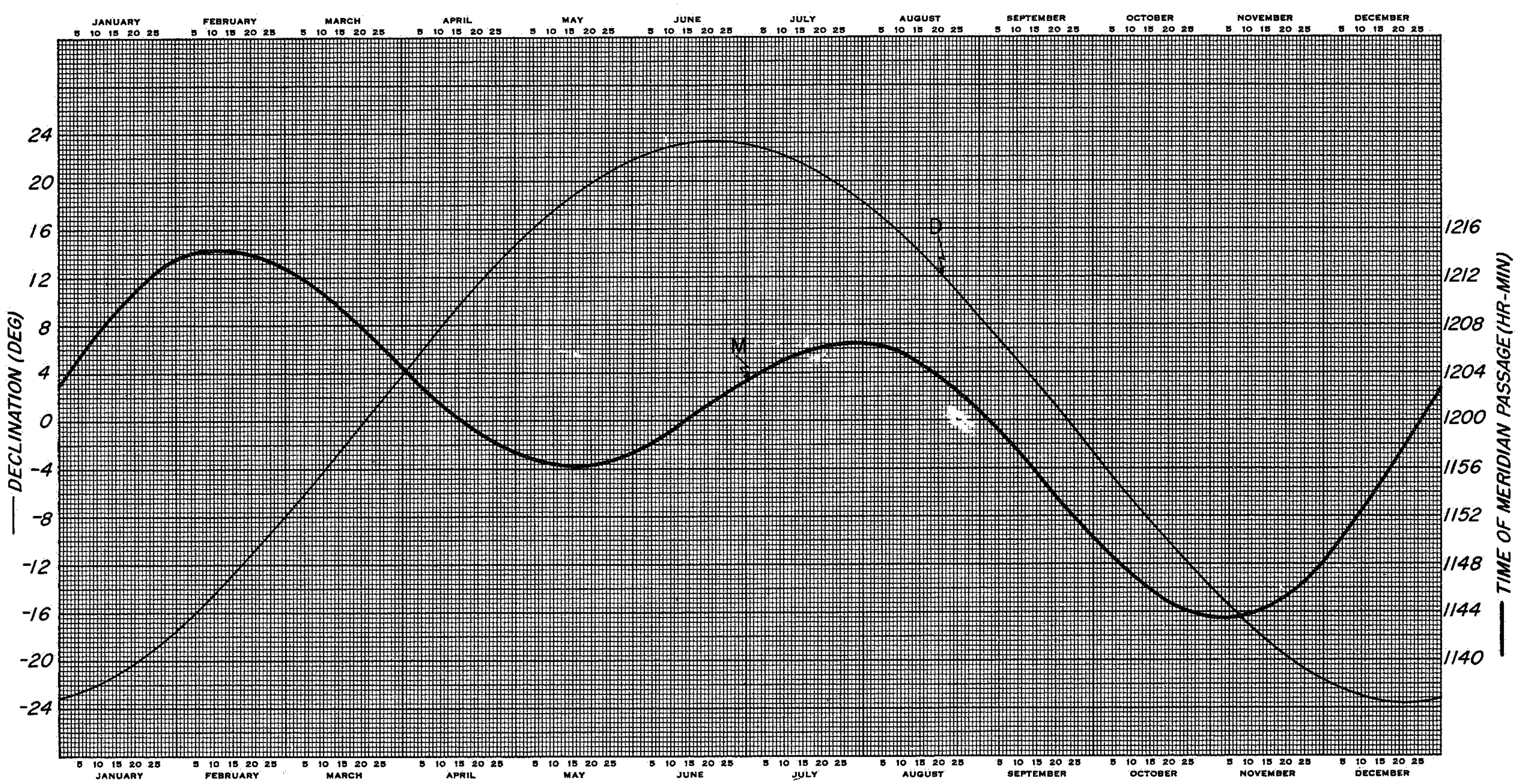

Figure 1. Solar declination (D) and time of meridian passage (M) versus day of the year.

tedious calculations once and for all, and construct tables or graphs to expedite and simplify the determination of sunrise/sunset times for any altitude and location.

CALCULATION OF SOLAR ELEVATION ANGLE

The solar elevation angle α is determined by the relation

$$\sin \alpha = \sin \phi \sin D + \cos \phi \cos D \cos h \tag{1.1}$$

where ϕ is the latitude, D the solar declination, and h the solar hour angle. At first glance, declination (Fig. 1) is a simple sinusoidal function of date, with maximum and minimum at the summer and winter solstices, respectively, and nodes at the equinoxes. Careful inspection of the curve, however, reveals a slight asymmetry, due to the ellipticity of the earth's orbit, which is accounted for in the following procedure for computing declination*:

$$\sin D = (\sin 23°26'37.8") \sin \sigma \tag{1.2}$$

where $\sigma(\text{deg}) = \ell + 0.4087\sin\ell + 1.8724\cos\ell$

$$- 0.0182\sin 2\ell + 0.0083\cos 2\ell \tag{1.3a}$$

$$\ell(\text{deg}) = 279.9348 + d, \tag{1.3b}$$

and d is the angular fraction of a year represented by a particular date, given by

$\dfrac{[(\text{number of day in year})-1]\times 360}{365.242}$ deg. For example, on 1 January, d=0 and on 21 March, $d = 79(0.98565) = 77.86635° = 77°52'$.

It is convenient to combine equations (1.3a) and (1.3b) to yield

$$\sigma = 279.9348 + d + 1.914827\sin d - 0.079525\cos d$$

$$+ 0.019938\sin 2d - 0.001620\cos 2d \tag{1.4}$$

The solar hour angle h, a measure of the longitudinal distance to the sun from the point for which the calculation is being made, is given by

$$h(\text{deg}) = 15(T-M)-L \tag{1.5}$$

*Equations (1.2) and (1.3) and the definition of d used here were obtained from the Nautical Almanac Office, U.S. Naval Observatory, Washington, D.C. Year-to-year variations in the relationships involving σ, ℓ, and d are negligible for atmospheric applications.

where T(hr) is the time (GMT) of the calculation; M(hr) is the time of meridian passage, or true solar noon; and L(deg) is longitude, counted positive west of Greenwich. M, which is shown along with D in Fig. 1, is given by

$$M(\mathrm{hr}) = 12 + 0.123570\sin d - 0.004289\cos d + 0.153809\sin 2d + 0.060783\cos 2d \qquad (1.6)$$

where d is as defined earlier.

In the operational program for applying radiation corrections to high-altitude radiosonde data [4], T in equation (1.5) is replaced by $H_o + c$, where H_o is a standard synoptic observation time such as 0000 or 1200 GMT, and c accounts for the time elapsed between release of the balloon and its estimated time of arrival at successive standard pressure levels. Since actual release time and ascent rate are not included in upper-air reports transmitted over the meteorological telecommunications network, representative values of these parameters along with standard-atmosphere [6] heights have been employed to construct a table of c-values for use in the program (Table 1).

TABLE 1. Corrections to nominal observation time based on estimated time of radiosonde arrival at selected constant-pressure surfaces using ascent rate of 305 meters/minute (after [4]).

P(mb)	Z(m)	c_e(hr)	c_ℓ(hr)
100	16180	0.134	0.551
70	18442	0.258	0.675
50	20576	0.374	0.791
30	23849	0.553	0.970
20	26481	0.697	1.114
10	31055	0.947	1.364
7	33453	1.078	1.495
5	35791	1.206	1.623

c_e: for "early release," nominally 45 min. prior to observation time.

c_ℓ: for "late release," nominally 20 min. prior to observation time.

A GRAPHICAL AID FOR DETERMINING SUNRISE AND SUNSET TIMES

Derivation

The solar day extends from M-H to M+H, where M is the time of meridian passage and H is the solar hour angle corresponding to the time of the sun's appearance or disappearance on the horizon. The latter phenomena are described uniquely by the sunrise/sunset solar elevation angle A, which is measured with respect to the earth's surface and therefore decreases (increases in the negative sense) with height. Once the variation of A with altitude is known, H can be found for any height, latitude, and solar declination.

From the earth-sun geometry, values of A were determined at 5-km intervals from the surface to 120 km, and corrected for refraction with an assumed solar angular radius (limb to center of disk) of 0.25 deg. The variation of A(deg) with geometric height Z (km), shown in Fig. 2, is described almost exactly (mean error -0.005 deg, r.m.s. error 0.057 deg) by the relation

$$A = -1.76459Z^{0.40795} \qquad (Z \geqslant 1), \tag{2.1}$$

obtained by least squares.

Equation (1.1) may be rewritten, for the special case of sunrise/sunset, as

$$\sin A = \sin\phi \sin D + \cos\phi \cos D \cos H \tag{2.2a}$$

$$\text{or} \quad H = \arccos\left\{\frac{\sin A - \sin\phi \sin D}{\cos\phi \cos D}\right\} \tag{2.2b}$$

Equation (2.2b) was solved by high-speed computer with A varying from -0.9° (surface) to -12.5° (120 km) in 0.1-deg increments, ϕ from 0 to 85° in 1-deg increments, and D from -24° to 24° in 1-deg increments. The resulting values of H were printed out in units of hours and minutes.

Presentation

After considerable experimentation, the following method was chosen as optimum for presenting the results of the above computations in a convenient and readily usable form. For each of 10 latitudes (0, 10, 20, 30, 40, 45, 50, 55, 60, and 65°) a chart was constructed on which H is a function of height for a given declination. The resolution in declination on individual charts, and in latitude between charts, is such that values of H obtained by linear interpolation are accurate to one minute. Graphical display is not practical for latitudes above 65°, where H changes very rapidly with declination, especially near the summer solstice.

The complete Sunrise-Sunset Finder is contained in the Appendix. In order to avoid excessive photo-reduction and consequent loss of readability, the charts for 55, 60, and 65° are each presented in two sections.

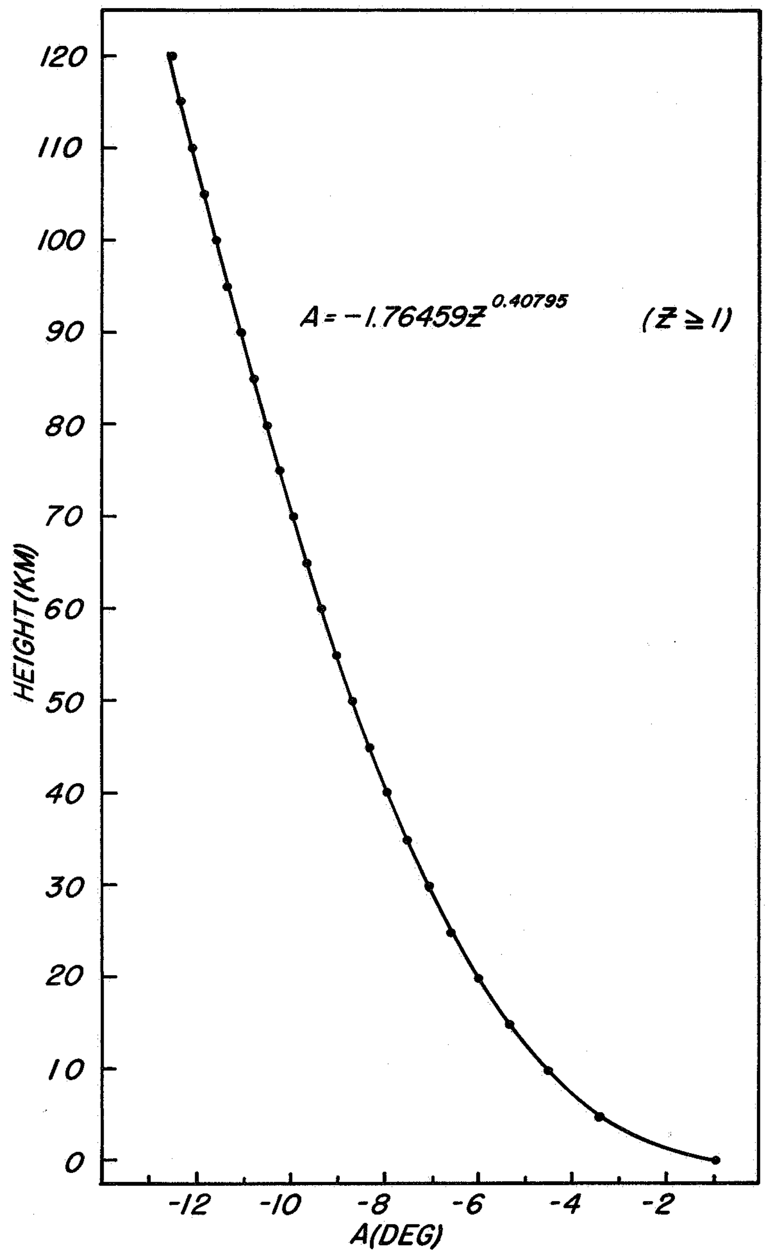

Figure 2. Variation with height of solar elevation angle, relative to the earth's surface, at sunrise and sunset. Equation was obtained by least-squares fit to points at 5-km intervals.

Once H has been found, times of sunrise and sunset are given in Greenwich Mean Time (GMT) by the relations

$$T_{SR} = \frac{L}{15} + M - H \tag{2.3a}$$

$$T_{SS} = \frac{L}{15} + M + H \tag{2.3b}$$

where L is longitude in degrees, counted positive west of Greenwich. All quantities in equations (2.3a, b) are in units of hours and minutes. If the time zone of the point is known, T_{SR} and T_{SS} may readily be converted to Local Standard Time if desired.

ACKNOWLEDGMENTS

This work was supported by the National Aeronautics and Space Administration, under its Experimental Inter-American Meteorological Rocket Network (EXAMETNET) program. The computer program used in the preparation of the Sunrise-Sunset Finder was written by W. P. Townshend, and the charts were expertly drafted by J. A. Booth.

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1. Teweles, S. and F.G. Finger, 1960: Reduction of diurnal variation in the reported temperatures and heights of stratospheric constant-pressure surfaces. J. Meteor., 17, 177-194.

2. Craig, R.A., 1965: The Upper Atmosphere-Meteorology and Physics. New York, Academic Press.

3. List, R.J., ed., 1958: Smithsonian Meteorological Tables. Washington, D.C., Smithsonian Institution.

4. Finger, F.G., and H.M. Woolf, and C.E. Anderson, 1965: A method for objective analysis of stratospheric constant-pressure charts. Mon. Wea. Rev., 93, 619-638.

5. U.S. Naval Observatory, 1945: Tables of sunrise, sunset, and twilight (Supplement to the American Ephemeris, 1946). Washington, D.C., Government Printing Office.

APPENDIX

SUNRISE-SUNSET FINDER

INSTRUCTIONS

The Finder permits the determination of times of sunrise and sunset for any altitude from sea level to 120 km, any latitude up to 65°, and any longitude and date.

Example:

Determine times of sunrise and sunset at 55 km, at 39°N 77°W, on January 1.

(1) From Fig. 1 of the main text, extract solar declination D (degrees) and time of meridian passage M(hours and minutes) for the desired date.

$$D = -23°, \; M = 12 \text{ hr } 03 \text{ min.}$$

(2) Charts in this Appendix are plots of hour angle H at sunrise and sunset versus height at selected latitudes, with declination as the defining parameter for each curve. Select the chart for the latitude closest to that desired, and extract the hour angle for the desired height using linear interpolation in declination. Repeat for the chart closest in latitude on the other side of that desired, and obtain the required hour angle by linear interpolation in latitude.

latitude 40°: H = 05 hr 29 min.

30°: H = 05 hr 49 min.

39°: H = 05 hr 31 min.

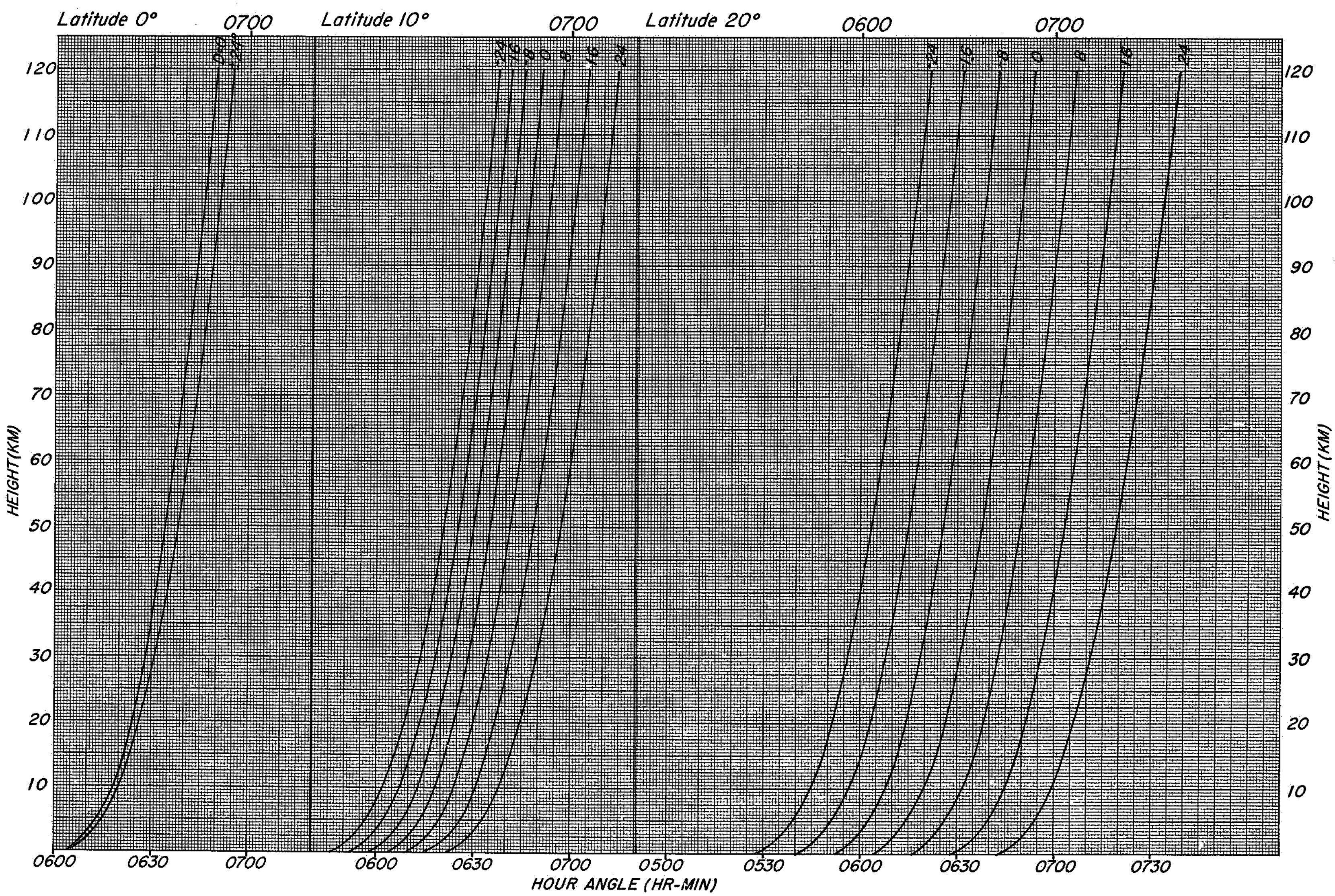
(3) Divide the longitude L (degrees, positive west of Greenwich) by 15. The integral part of the quotient is then in hours; multiply the remainder by 60 to convert it to minutes.

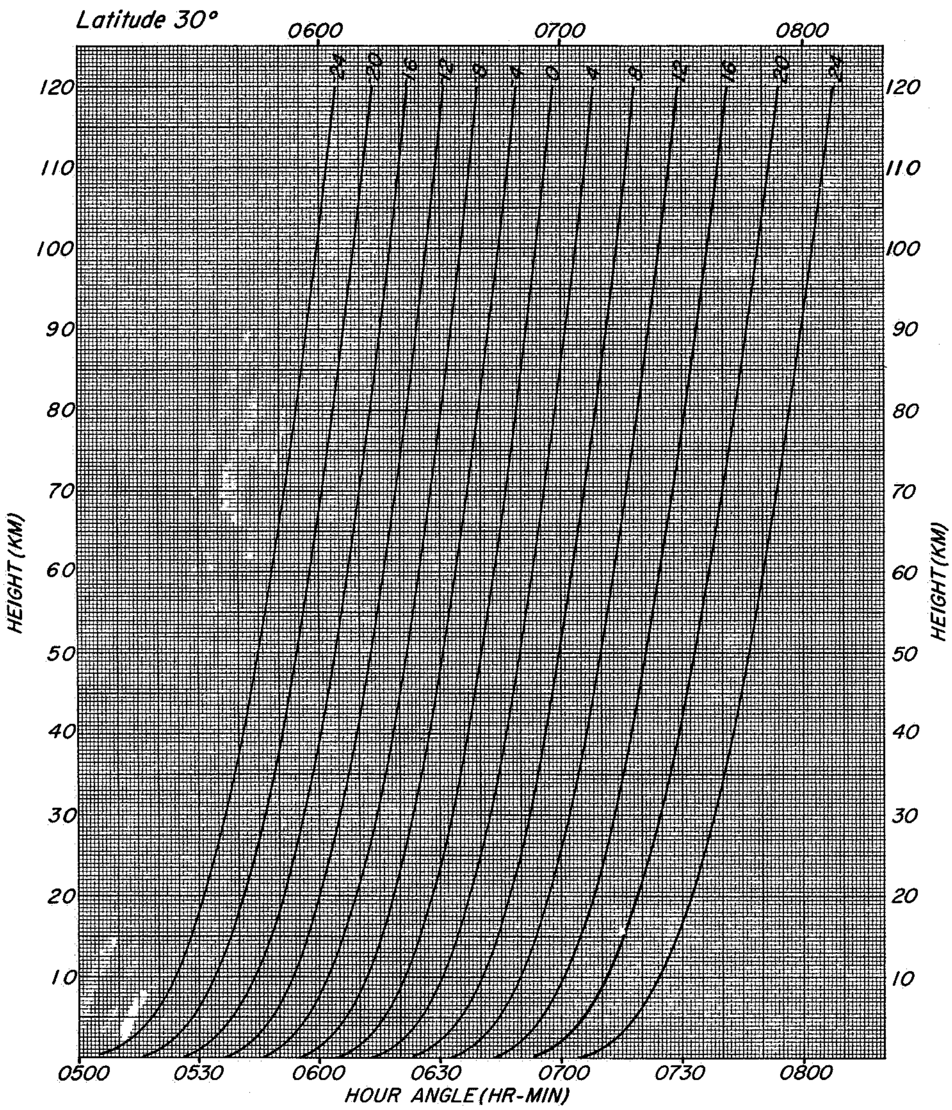
$$\frac{L}{15} = \frac{77}{15} = 5 + \frac{2}{15}(60) = 05 \text{ hr } 08 \text{ min.}$$

(4) Determine the times (GMT) of sunrise and sunset in the following manner (all quantities are in hours and minutes):

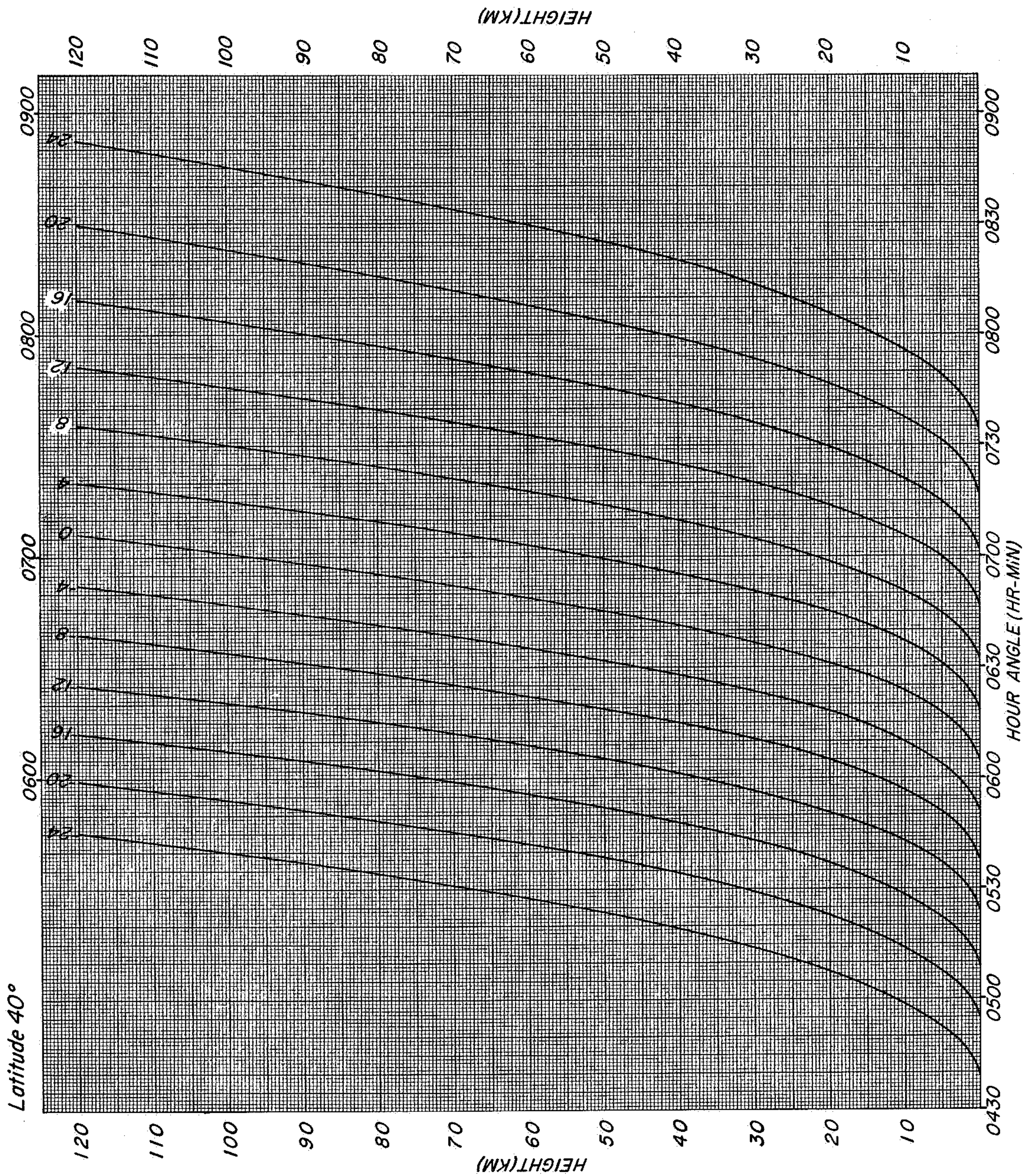
(a) Sunrise: $T_{SR} = \frac{L}{15} + M - H = 0508 + 1203 - 0531 = 1140$ GMT

(b) Sunset: $T_{SS} = \frac{L}{15} + M + H = 0508 + 1203 + 0531 = 2242$ GMT

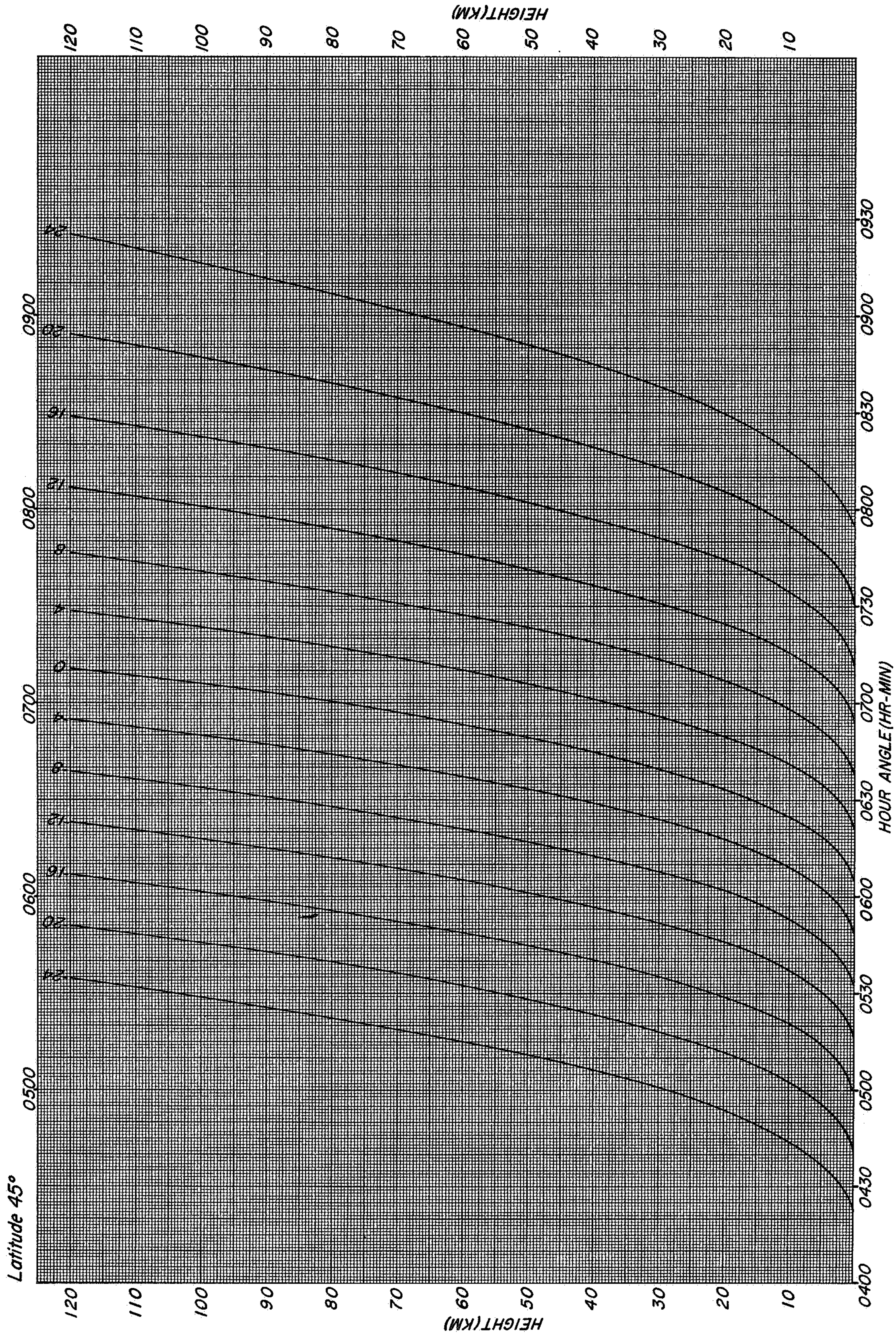

Latitude 0°
Latitude 10°
Latitude 20°
HEIGHT (KM)
HOUR ANGLE (HR-MIN)
0600
0630
0700
0730
0500
0530
10
20
30
40
50
60
70
80
90
100
110
120
-24
-16
-8
0
8
16
24

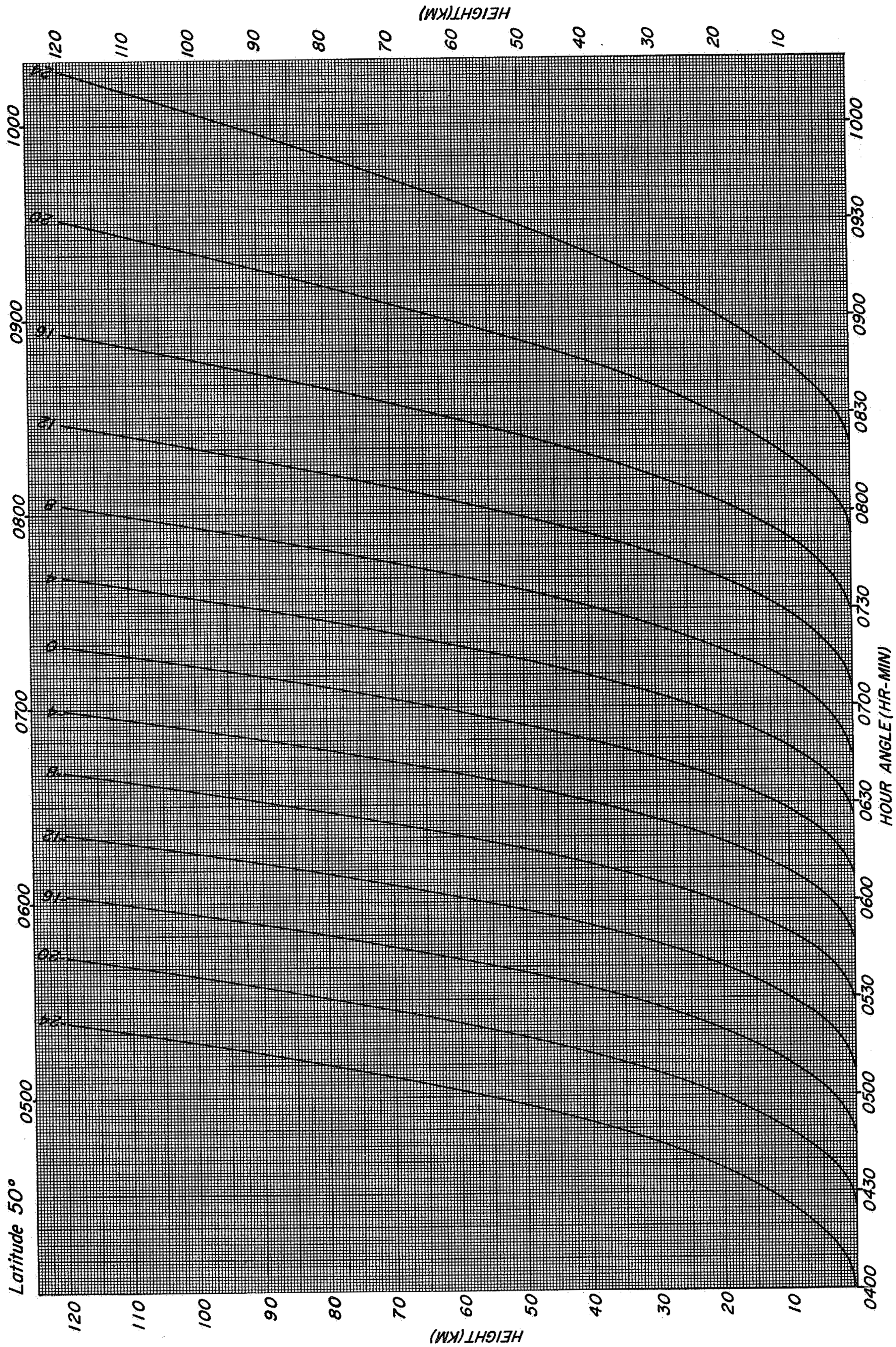


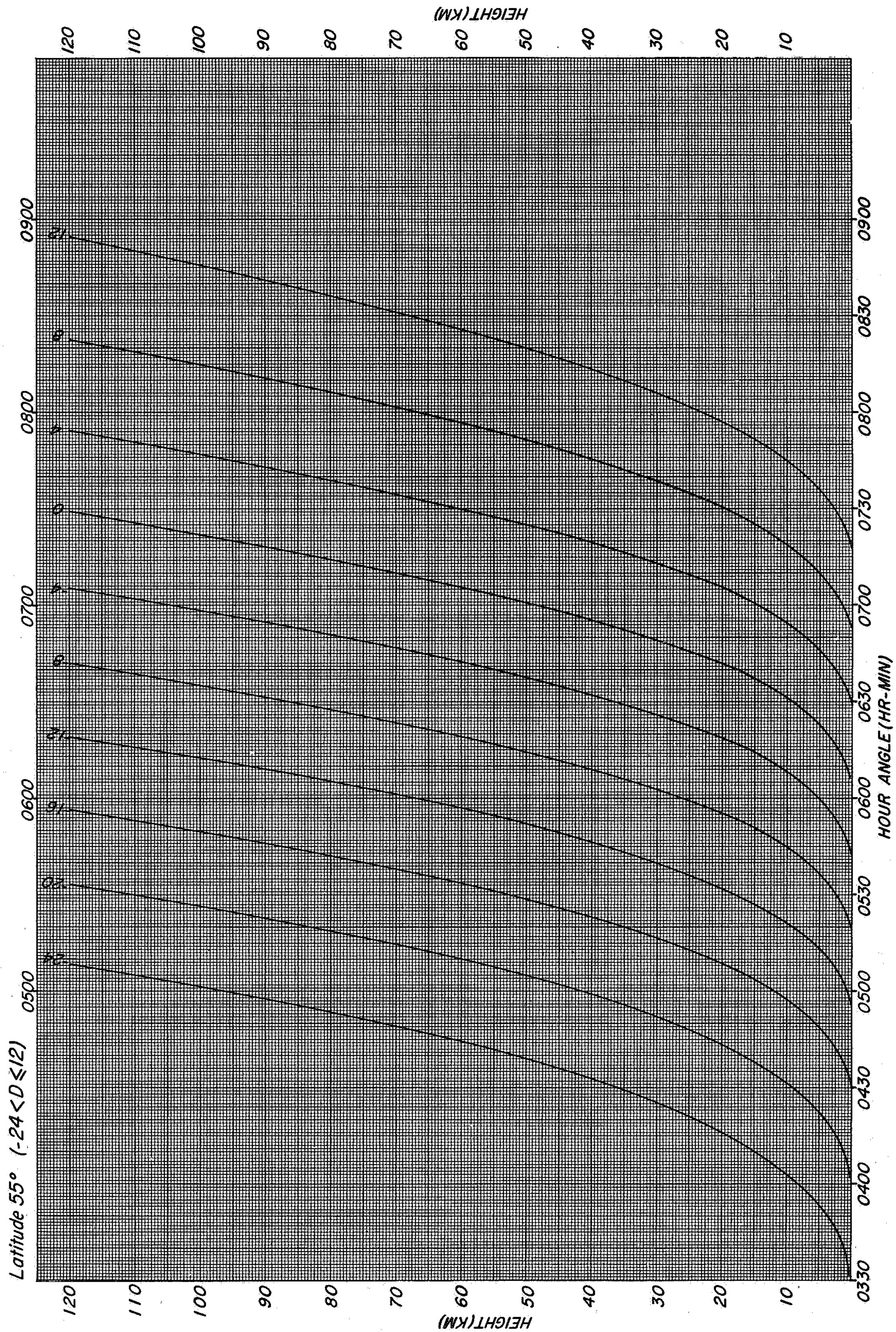
Latitude 30°
0600
0700
0800
24
20
16
12
8
4
0
4
8
12
16
20
24
120
110
100
90
80
70
60
50
40
30
20
10
HEIGHT (KM)
0500
0530
0600
0630
0700
0730
0800
HOUR ANGLE (HR-MIN)



Latitude 40°
HEIGHT(KM)
120
110
100
90
80
70
60
50
40
30
20
10
HOUR ANGLE (HR-MIN)
0430
0500
0530
0600
0630
0700
0730
0800
0830
0900
24
20
16
12
8
4
0
4
8
12
16
20
24

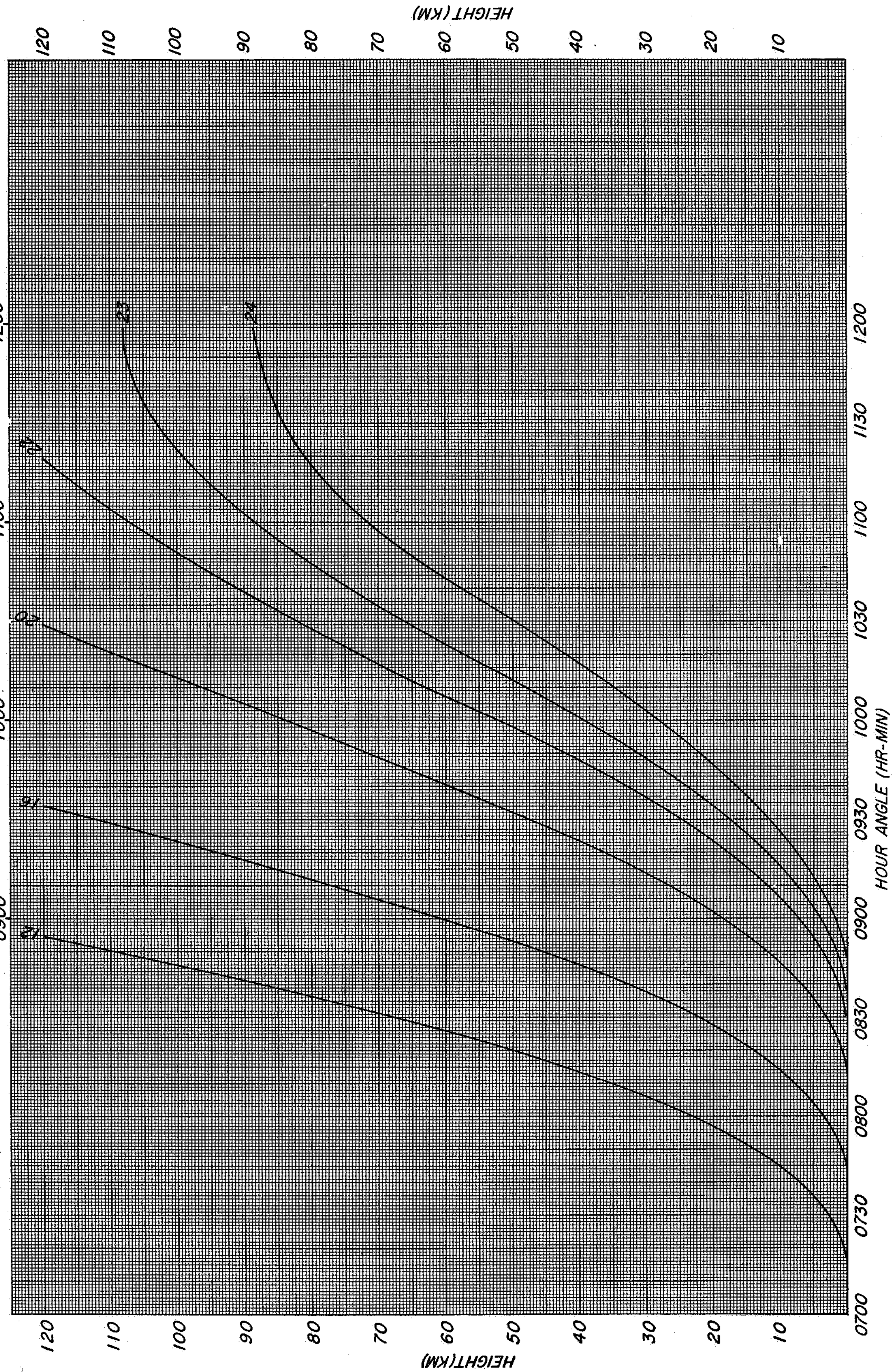

Latitude 45°
HEIGHT(KM)
120 110 100 90 80 70 60 50 40 30 20 10
HOUR ANGLE (HR-MIN)
0400 0430 0500 0530 0600 0630 0700 0730 0800 0830 0900 0930
24 20 16 12 8 4 0 4 8 12 16 20 24

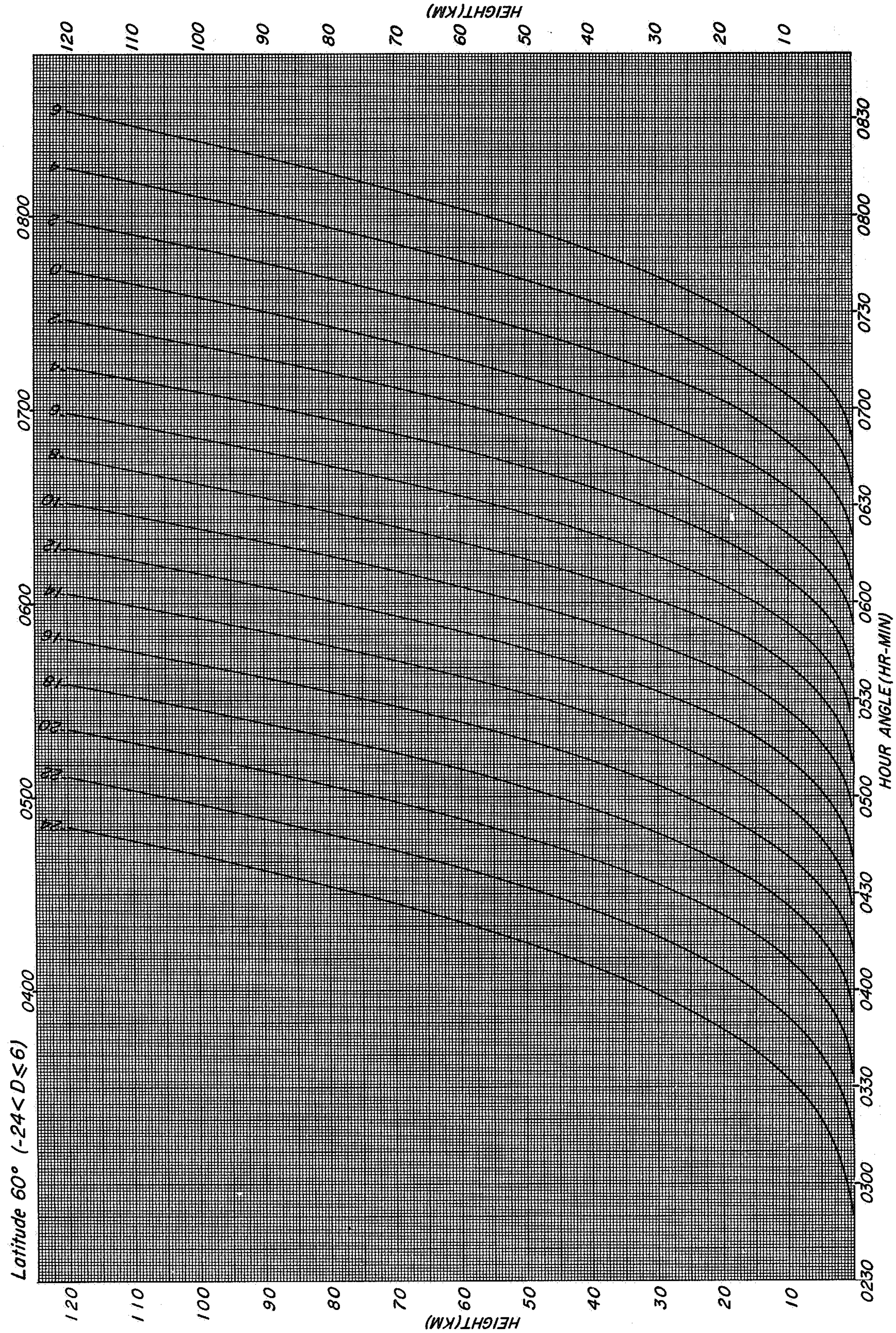

Latitude 50°
HEIGHT(KM)
HOUR ANGLE (HR-MIN)
0400
0430
0500
0530
0600
0630
0700
0730
0800
0830
0900
0930
1000
10
20
30
40
50
60
70
80
90
100
110
120
24
20
16
12
8
4
0
-4
-8
-12
-16
-20
-24

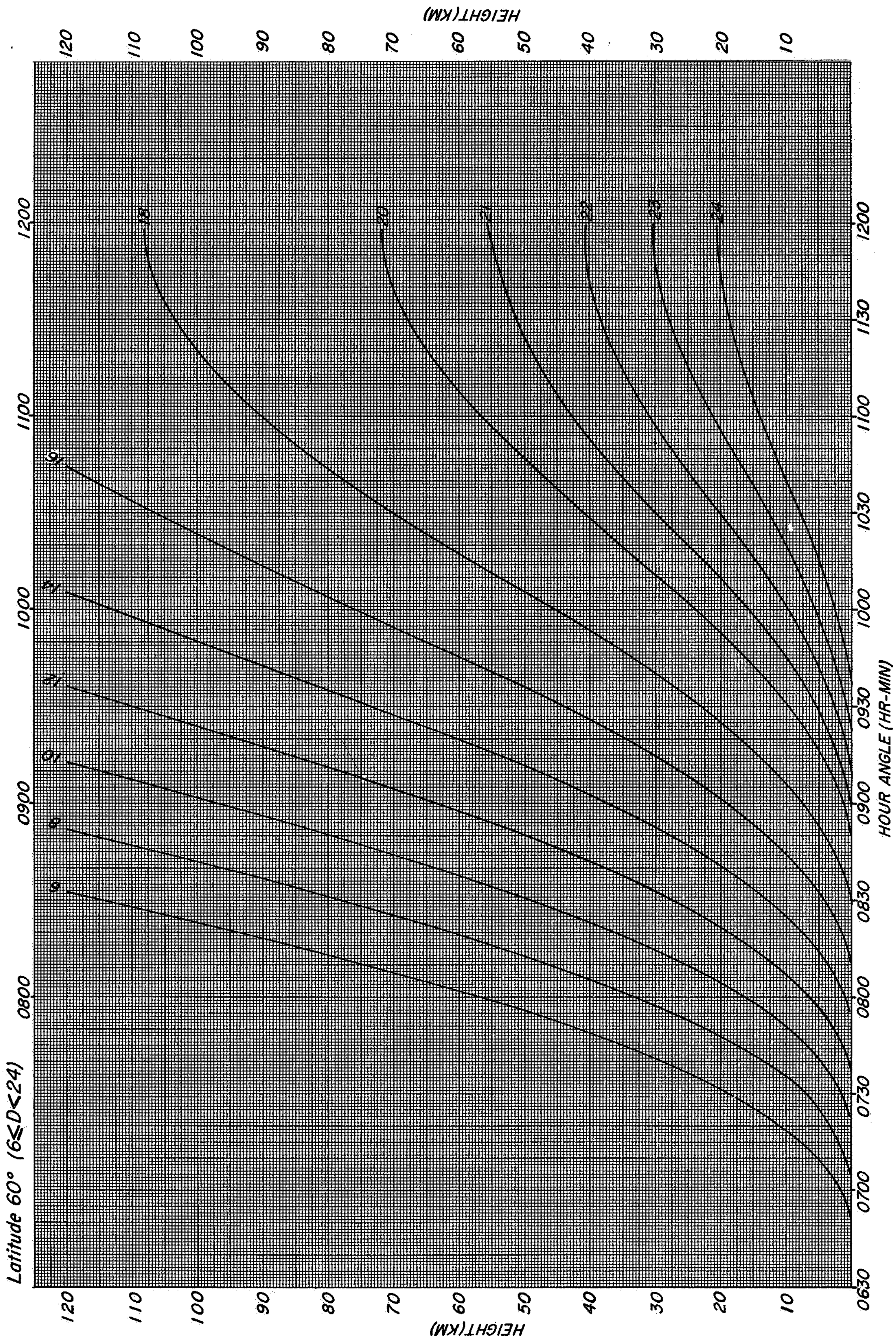


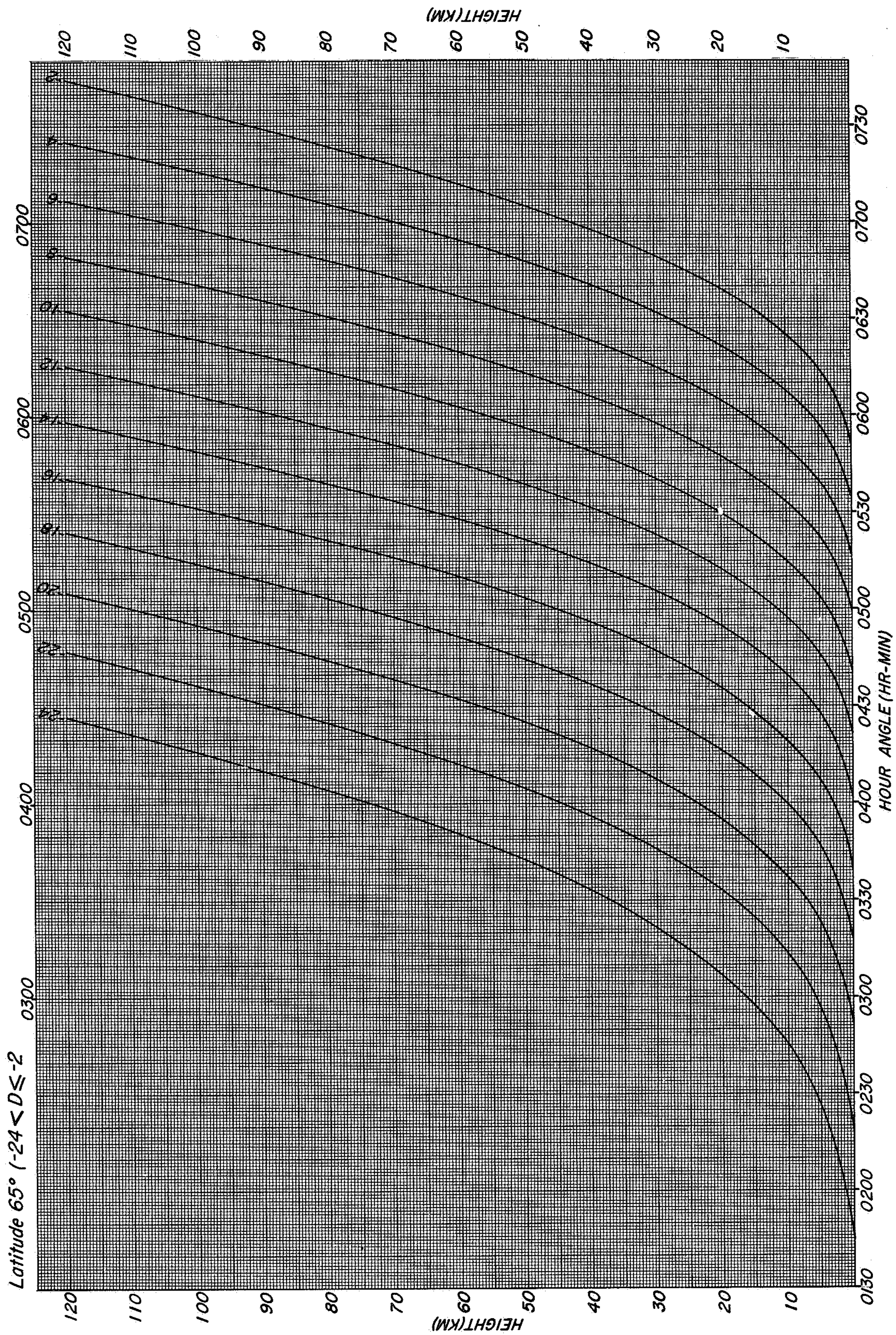
Latitude 55° (-24 < D ≤ 12)
HEIGHT (KM)
120
110
100
90
80
70
60
50
40
30
20
10
HOUR ANGLE (HR-MIN)
0330
0400
0430
0500
0530
0600
0630
0700
0730
0800
0830
0900
12
8
4
0
-4
-8
-12
-16
-20
-24

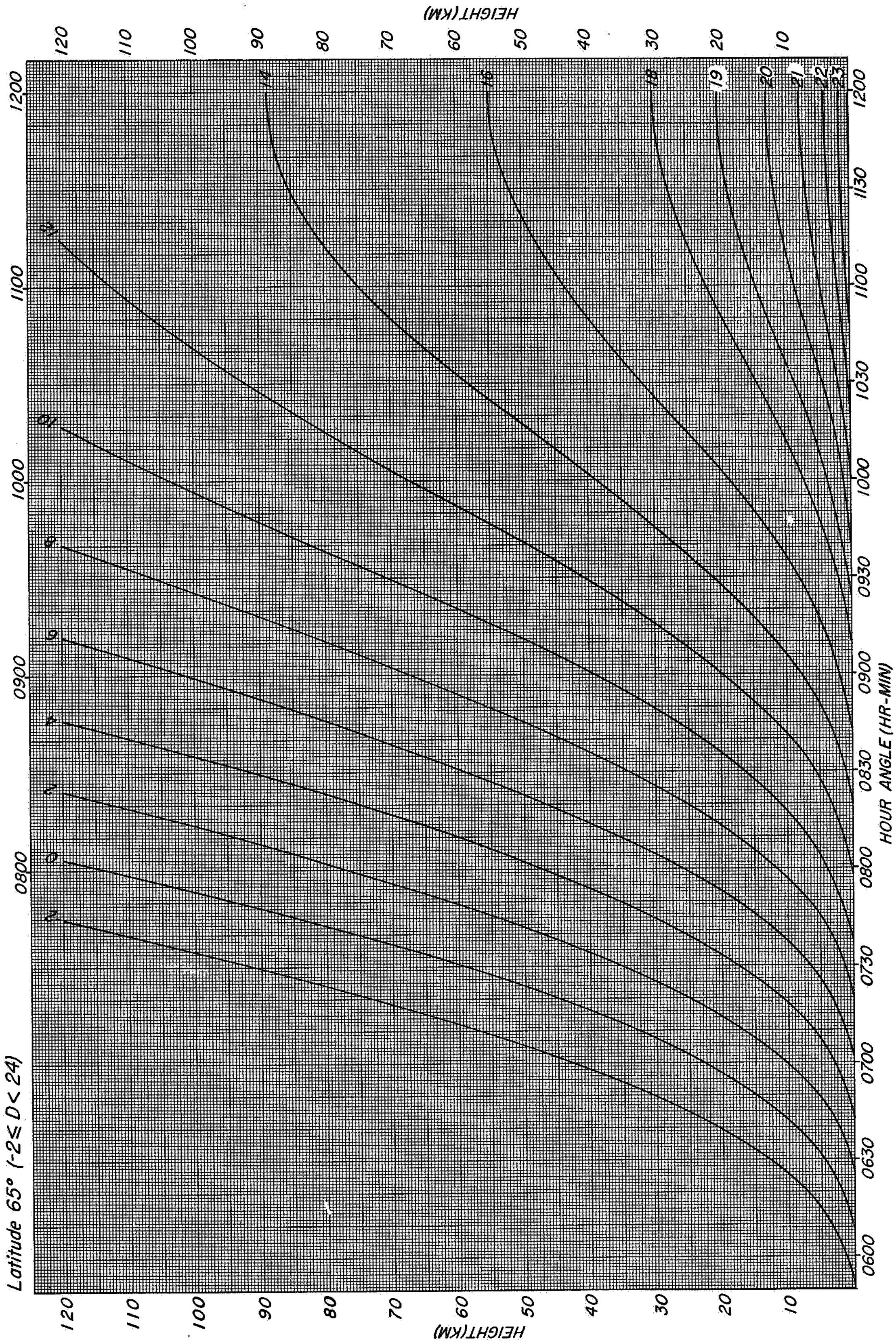
Latitude 55° (12≤D<24)




Latitude 60° (-24<D≤6)
HEIGHT(KM)
HOUR ANGLE (HR-MIN)
0230
0300
0330
0400
0430
0500
0530
0600
0630
0700
0730
0800
0830


Latitude 60° (6≤D<24)
HEIGHT(KM)
120
110
100
90
80
70
60
50
40
30
20
10
HOUR ANGLE (HR-MIN)
0630
0700
0730
0800
0830
0900
0930
1000
1030
1100
1130
1200
6
8
10
12
14
16
18
20
21
22
23
24


Latitude 65° (-24 < D ≤ -2)
HEIGHT (KM)
HOUR ANGLE (HR-MIN)
0130
0200
0230
0300
0330
0400
0430
0500
0530
0600
0630
0700
0730
10
20
30
40
50
60
70
80
90
100
110
120
2
4
6
8
10
12
14
16
18
20
22
24


Latitude 65° (-2 ≤ D < 24)
HEIGHT (KM)
HOUR ANGLE (HR-MIN)

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