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by

V. G. Fesenkov

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The aim of this paper is the determination of the law according to which the light is reflected from the different parts of the lunar surface under various angles of incidence, reflection and phase.

The photograms of the Moon obtained by Surovtseff in the focus of the normal astrograph of the Tashkent Astronomical Observatory were measured at the Astrophysical Institute of Moscow with the Harmann microphotometer. The photographic densities obtained with this instrument were transformed into brightness of the corresponding points by means of the scale of tubular photometer printed on each plate exactly with the same exposure. The extrafocal images of the Polar star photographed on the same plates with many different exposures were utilized for the determination of the same unit of brightness. The brightness of the different points of the Moon were corrected for effect of halation. It is found that our photometrical determinations for all phases from 0° up to 140° may be represented in a very satisfactory way by the simple expression

$$J = \frac{\Gamma_0 \cos i \cos \epsilon \left(1 + \cos^2 \frac{\alpha}{2}\right)}{\cos i + 0.225 \left(1 + \mathrm{tg}^2 \frac{\alpha}{2}\right) \cos \epsilon},$$

which is a modification of the very well known Lommel-Seeliger's formula.

This paper investigates the law of reflection of the light from lunar surface at different angles of incidence, reflection, and phase. The material for it was provided by the photograms of the Moon taken in the focus of a standard astrograph at Tashkent Astronomical Observatory. The plan of the observations consisted of the following.

The Moon was photographed on the Jougla and Perutz slide plates measuring 8 by 8 in such a manner that the photographs would cover uniformly as far as possible all the lunar phases. In all cases the objective of the astrograph was diaphragmed to an aperture of 16 cm. An exposure of 8 seconds was used. With such a length of the exposure time the constancy of the exposure with accuracy to 0.5 seconds could be provided with an ordinary shutter in front of the objective with the shutter being opened and closed simply by hand. Out-of-focus images of the North Star were plotted on the same plates in order to interconnect them. This was done by moving out the eyepiece end of the tube to 8 cm using the exposures of 8, 12, 18, 27, 40.5, 61, 91, and 136 seconds. However, the first images could not at all serve for tying-in purposes since they stood out extremely faintly against the general background of the plate. Generally speaking, the zenith distances of the Moon and North Star differed a little but their close equality was not set as a necessary condition for the work. An absolute condition was only the presence of a perfectly clear sky.

Next, to convert photographic densities into brightness the scale of a tube photometer was plotted on each plate. The photometer was fabricated at the workshop of the observatory only toward the end of 1924. It consisted of twelve tubes, each of which was 12 cm in length covered on top with diaphragms having the following apertures expressed in mm:

No.	1	2	3	4	5	6	7	8	9	10	11	12
ρ	5.043	4.394	3.907	3.265	2.865	2.523	2.294	2.022	1.753	1.520	1.211	1.089

The above-cited diameters of the apertures are the average result of the measurements of two replicas of the aperture on a photographic plate. Each replica was measured with two screws of the measuring instrument in two directions perpendicular to each other. For all apertures the vertical diameter differs from the horizontal by not more than 0.02 to 0.03 mm.

The light of a perfectly clear sky near the North Pole at noon served as the source of light in plotting the stages of the tube photometer. An exposure of 8 seconds by chronometer was made by shutting and opening the photometer with the hand, i.e., with the same accuracy as in the exposure used for the Moon. The adjustment of the scale intensity was done by means of inserting sheets of tissue paper between two pieces of frosted glass placed in front of the diaphragms of the tubes.

Until the fabrication of the tube photometer the plates were stored without being developed with the exception of three trial plates. The development of the great majority of the plates was done in vertical cuvettes in glycin at the rate of 7 to 9 plates at a time.

All of these photograms taken in Tashkent by V. M. Surovtsev were then sent to the Astrophysical Institute in Moscow for measurements and all necessary processing.

The essence of the processing consisted in the following. The photograms were measured on the Hartmann microphotometer; in doing so, it was decided to measure the photographic densities of the image of the Moon by two coordinates of the instrument's table at equal intervals. In addition to this, around the Moon the background surrounding it and then the stages of the tube photometer were measured, and the out-of-focus images of the North Star along with the nearby background were measured. On the first plate used in the processing, No. 29, the measurements were taken every millimeter along the X- and Y-axes with the total number of the measured points reaching 532 although the phase corresponding to this plate does not exceed 65 degrees. All the remaining plates were measured every 2 mm; nevertheless, this part of the work took the largest amount of time. By necessity different points were measured on different plates with this method of measurements so that the material on the basis of which the final regularity of the reflection of the light from lunar surface was derived was not characterized by uniformity. Of course, it would have been possible to limit ourselves to choosing a certain number of certain details in order that the measurements would relate in all phases to the same points but in that case the number of the measured details would have been rather limited, various troubles would have arisen with their identification under different conditions of illumination and, most important, the regularity derived would not relate in any manner to the lunar surface as a whole and would not characterize its average reflecting properties.

The task set for us is broken down into two separate parts: the photometric part in which the brightness of all of the details examined would be brought out, and the purely geometric part where the angles of incidence, reflection, and phase are to be determined for these same details.

First of all we proceeded to the investigation of the microphotometer wedge. As is known, in this instrument the amount of light which had passed through the photographic plate placed on the instrument's table is balanced by the amount of light passing through the photometric wedge fabricated by the photographic method, with the balancing being done by means of moving the wedge whose position in every given case may be read from the scale with an accuracy to 0. 1 mm. Inasmuch as there already is on the plate the previously plotted scale of the tube photometer which sets the relationship between the blackening and brightness it would seem possible simply to set the relationship between the brightnesses determined on an arbitrary scale by tube photometer and the auxiliary scale of the wedge, and then convert all wedge readings relating to various points on the image of the Moon into the respective brightnesses expressed in the same arbitrary units. However, we will permit such a simplified method only in the case when the entire fog of the plate is completely uniform and there is no need to take it into account. If however, the way it usually occurs, the fog on the plate is nonuniform, then inasmuch as the fog depends on purely photographic factors it is necessary to eliminate it by subtracting the density corresponding to it from the density of the image being analyzed. Indeed, if "I" is the amount of the light falling on the plate and "i" is the amount of the light which had passed through a given image, then according to the definition the photographic density is

$$D = \log \frac{I}{i} .$$

In a similar manner we have for the fog density

$$D_0 = \log \frac{I}{i_0} ,$$

where i_0 is the amount of light which had passed through the plate outside the image.

The difference

$$D - D_0 = \log \frac{i_0}{i}$$

characterizes the degree of blackening produced by the decomposed silver precisely due to the effect of the light when photographing a given object. In this manner the dissimilar transmittance of the glass of the negative, fogging dependent on the developing, and some spoilage of the plate (due to inadvertent exposure) if it had occurred before its exposure may be eliminated. The investigation of the photometric wedge carried out by N. M. Staude was done in the usual manner by means of inserting a system of two Nicol prisms in the path of the rays passing through the wedge. Therefore, it was possible to tie in very simply the readings of the wedge with the readings of the circle of the intensity of Nicol prisms and, consequently, to determine in an arbitrary system of units the transmittance of the wedge in its different parts.

In doing so, the densities measured on the plate are determined up to an arbitrary constant value which drops out in the difference $D - D_0$ so that the densities being sought corresponding to the image as such turn out to be free of any arbitrariness or the luminance of the light source used in the microphotometer.

In this manner all readings of the microphotometer wedge were converted into the respective densities from which the density of the aggregate fogging of the plate was then subtracted. The next stage of the work consisted in the conversion of all these differences into luminances with the help of a tube photometer.

If we denote by ρ the diameter of the aperture and by ℓ the length of the tube, then the amount of the light falling on the plate should be proportional to the magnitude of the solid angle cut out by the aperture in the firmament. It is easy to see that it is equal to

$$i = k\rho^2 \left(1 - \frac{3\,\rho^2}{4\,h^2} + \frac{1\,\rho^4}{8\,h^4} + \ldots\right), \tag{1}$$

where k is the coefficient unknown to us, the same for all tubes.

With the arrangement of the photometer indicated above all terms of the higher order may be discarded and it may be considered that the luminance corresponding to a certain tube is proportional simply to the square of the diameter of the aperture, i.e.,

$$i = k\rho^2. \tag{2}$$

We may take for reference as a unit the luminance of the light sent by the smallest diaphragm No. 12, and consider that

$$\log i = 2\,(\log\rho - \log\rho_{12}). \tag{3}$$

In this manner we find all luminances corresponding to the stages of the tube photometer and tie them in with the values of $D - D_0$ determined in the manner shown above. We construct for each plate the curves of the $D - D_0$ versus log i dependence in a sufficiently large scale. Most of these curves are very similar to each other. These are the usual characteristic curves constructed exclusively for the argument log i and in which the underexposed portion is well represented, which contain the rectilinear portion in its entirety and in which hints of a transition to the overexposed portion are noted. An exception includes only four plates, because they either belonged to a different grade of the plates with smaller sensitivity or else the stages of the tube photometer were printed on them under somewhat anomalous conditions.

Ordinates were read off from these graphs at the intervals of 0.050 $(D - D_0)$; the "i's" which had been found were equalized with respect to the third differences and interpolated for the intermediate values every 0.010 $(D - D_0)$. These tables, compiled for each plate separately, served ultimately for the conversion of all of the densities found in the luminances corresponding to them, both for the Moon and its halo and for the out-of-focus images of the stars.

Even if they are regarded as being expressed in an arbitrary system of units these luminances are not as yet final for the lunar surface. Everybody knows well that the background of the sky considerably increases in luminance in a direct proximity to the bright luminaries. The scattering of the light within the tube itself and the reflection from the rear surface of the plate act in the same direction. For these reasons the background of the plate appears darker near the Moon, and this darkening increases with the approach to the edge of the lunar disc. There is no doubt that this additional luminance continues to increase in accordance with some law on the lunar disc itself distorting the observed luminance and decreasing the degree of the contrast of various details of its surface. Various data compel us to assume that the luminance of this additional halo around an infinitely small luminous element is represented by the expression

$$i = \frac{k}{r^2 + \alpha^2} \tag{4}$$

where r is the distance from the element and α^2 and k are some constant quantities which are to be determined from the observations or derived from the measurements made for each plate. If we have not an infinitely small element but a luminous surface having a finite angular dimension, then knowing the distribution of luminance on this surface, it is possible to find the aggregate effect of the halo and then correct the luminances observed. However, such a statement of the problem is not possible in our case since the luminances on the Moon are distributed in too random a manner. Therefore, it is necessary to proceed from some schematic distribution. We assumed that the visible luminances on the lunar surface are represented in accordance with Lommel-Seeliger's formula:

$$i = \frac{\Gamma \cos i}{\cos i + \lambda \cos \epsilon}, \tag{5}$$

where i and ϵ are the angles of incidence and reflection and λ is a coefficient dependent on the phase. The λ versus phase dependence was taken from the work by E. G. Shoenberg, "Untersuchungen zur Theorie der Beleuchtung des Mondes auf Grund photometrischer Messungen."

The distribution of luminance on the lunar surface was determined in accordance with this formula for most diverse phases and the effect of the halo in the function of rectangular coordinates in relation to the intensity equator was found both for the Moon and for the points outside the Moon located not farther than 0.2 of the radius from the edge of the disc.

Theoretically speaking, the variation of the halo near the edge of the lunar disc is dependent on the constant α^2, and its luminance on the constant k. However, in practice, it turns out that in a fairly wide range the variation of the luminance of the halo is nearly independent of the former constant. Because of this, various attempts of determining α^2 from the observations themselves failed to yield for it a completely satisfactory value. Therefore, it was decided to apply a correction for the halo with $\alpha^2 = 0.06$ which corresponds to the most stable distribution of this effect in the luminance.

The determination of the constant k is done very simply. For this purpose it is sufficient only to know the luminance of the background in direct proximity to the lunar edge.

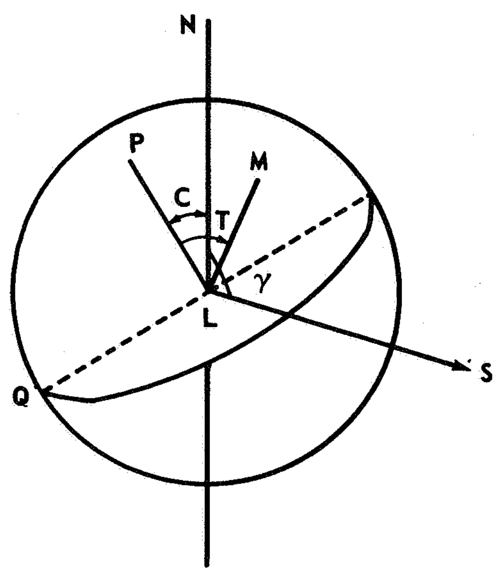

FIGURE 1.

The tables which we have compiled[1] contain one and the same quantity shown in some tabular units in which the luminances of the halo are expressed for all the remaining points. A comparison of these quantities gives the ratio by which all tabular values of the corrections being sought have to be multiplied in order to express them in units used for the expression of luminances on a given plate.

The luminances of the halo are given in the tables contained in the above-mentioned article in relation to the intensity equator, whereas the positions of all points measured on the plates are related to some arbitrary system of rectangular coordinates defined by the scales of the microphotometer table. In order to be able to apply this correction and also for the subsequent determination of the angles of incidence and reflection

[1]Russkiy Astronomicheskiy zhurnal (Russian Astronomical Journal), 3, No. 2.

of light at all the points being investigated, it is necessary to pass on to a new coordinate system in which the origin coincides with the center of the visible lunar disc and the X-axis is directed westward along the intensity equator.

Along with the usual photometric measures which were discussed above, in order to be able to pass on to the new coordinate system, it was necessary to determine from the scales of the microphotometer table the positions of the reference craters, at least two on each plate, whose selenographic coordinates were known in advance.

Knowing the positions of the Earth and the Sun in selenographic coordinates, it is possible to determine without any difficulty the rectangular coordinates of all reference points in relation to the intensity equator which for a terrestrial observer is projected onto the lunar disc in the form of a straight line passing through its center perpendicularly to the line connecting the tips of the horns.

Suppose we have the disc of the Moon with the center at the point L (Figure 1). The declination circle whose northern portion is denoted by N passes through the L. The positional angle of the Moon's pole P is given for each day in various astronomical yearbooks, for example in "American Ephemeris," and is considered to be positive in the eastward direction from the declination circle. We will denote this angle by C. We will indicate the positional angle of the intensity equator by γ and will consider it to be positive in the direction of the illuminated edge of the Moon, and not of the terminator, i.e., in the direction in which the Sun is located.

QQ is the lunar equator. Suppose M is a specified point on the Moon, the selenographic coordinates of which are λ and β and the unknown rectangular coordinates in relation to the intensity equator are x and y. We will consider that both the selenographic coordinates and the rectangular coordinates being sought are positive in the quadrant NW. If s is the angular distance read at the center of the Moon between the points M and L, and L is the angle PLM, then we obviously have

$$x = \sin s \cos (\gamma + c - L)$$

$$y = \sin s \sin (\gamma + c - L) . \qquad (6)$$

To determine the coordinates x and y by these formulas it is necessary to calculate for each plate the angle γ and then for each reference point the angles L and s.

Suppose the selenographic coordinates of the Earth, which may be taken in a similar manner from the "American Ephemeris," will be λ_t and β_t for a given instant. These are, consequently, the coordinates of the center of the visible disc of the Moon.

From the spherical triangle between the pole of the Moon, the center of the lunar disc L and the given point M we find the following:

$$\cos s = \sin\beta \sin\beta_t + \cos\beta \cos\beta_t \cos(\lambda - \lambda_t)$$

$$\sin s \cos L = \sin\beta \cos\beta_t - \cos\beta \sin\beta_t \cos(\lambda - \lambda_t) \tag{7}$$

$$\sin s \sin L = \cos\beta \sin(\lambda - \lambda_t) ,$$

with $\sin s \geq 0$.

Finally, γ is determined as the angle at the visible center of the Moon L between the directions toward the celestial pole N and toward the Sun S. From the spherical triangle NLS we have:

$$\sin\alpha \sin\gamma = \cos\delta_{\odot} \sin(\alpha \ - \alpha_{\odot})$$

$$\sin\alpha \cos\gamma = \cos\delta_{☾} \sin\delta_{\odot} - \cos\delta_{\odot} \sin\delta_{☾} \cos(\alpha_{☾} - \alpha_{\odot}) . \tag{8}$$

From this

$$\mathrm{tg}\,\gamma = \frac{\cos\delta_{\odot} \sin(\alpha_{☾} - \alpha_{\odot})}{\cos\delta_{☾} \sin\delta_{\odot} - \cos\delta_{\odot} \sin\delta_{☾} \cos(\alpha_{☾} - \alpha_{\odot})} . \tag{9}$$

The coordinates x and y may be calculated directly by these formulas inasmuch as we do not take the correction for parallax into account.

Knowing x and y relative to the intensity equator, we can determine immediately the angles of incidence and reflection i and ϵ for the respective points.

First of all the phase of the Moon is calculated by the usual formula:

$$\mathrm{ctg}\,\alpha = \frac{\frac{\pi_{\odot}}{\pi_{☾}} - \cos A}{\sin A} , \tag{10}$$

where

$$\cos A = \cos B_{☾} \cos(\mathcal{L}_{\odot} - \mathcal{L}_{☾}), \tag{11}$$

with $B_{☾}$ being the geocentrical latitude of the Moon, $\mathcal{L}_{☾}$ — its longitude and $\mathcal{L}_{\odot}$ — the longitude of the Sun.

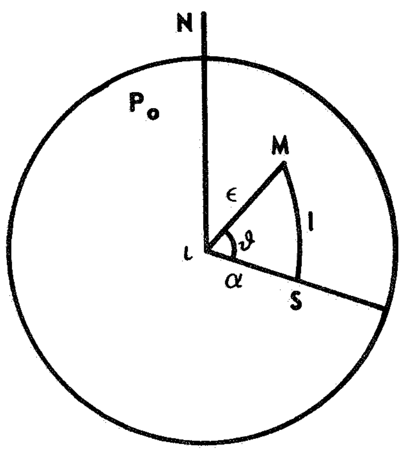

FIGURE 2.

We will now examine the spherical triangle LMS (Figure 2) formed on a sphere described around the center of the Moon by the directions toward the Sun, the Earth and the specified point on the surface of the Moon having the known rectangular coordinates. We will note that in this triangle the angle ϑ between the sides LM and LS is equal to the angle between the intensity equator and the direction from the visible center of the lunar disc toward the specified point. If the rectangular coordinates x and y are expressed in fractions of the lunar radius, then obviously the distance of the specified point on the Moon from the center of the visible disc is in the projection on a plane perpendicular to the ray of vision

$$\sin s = \sqrt{x^2 + y^2} = \sin \epsilon , \tag{12}$$

since $s = \epsilon$.

In addition to this, we have the following from the same triangle:

$$\cos i = \cos \epsilon \cos \alpha + \sin \epsilon \sin \alpha \cos \vartheta, \tag{13}$$

where on the basis of the foregoing

$$\vartheta = \gamma + C - L. \tag{14}$$

Consequently,

$$\cos \epsilon = \sqrt{1 - x^2 - y^2} \tag{15}$$

and

$$\cos i = \cos \epsilon \cos \alpha + x \sin \alpha. \tag{16}$$

It is desirable to have formulas with the help of which it would be possible to calculate, at least for the reference points, the rectangular coordinates in relation to the intensity equator in a manner completely independent of that set forth above. It is easy to do this by introducing into the examination the selenographic coordinates of the Sun, which may be taken from the astronomical ephemerides for any day. Suppose λ_t and β_t are selenographic coordinates of the Earth and $\lambda_\odot$ and $\beta_\odot$ — those of the Sun. Since the phase α is the angle between the direction from the center of the Moon toward the Earth and the Sun, then

$$\cos \alpha = \sin \beta_t \sin \beta_\odot + \cos \beta_t \cos \beta_\odot \cos(\lambda_\odot - \lambda_t). \tag{17}$$

From the spherical triangle LMS which we have just examined we obviously have the following

$$\cos \vartheta = \frac{\cos i - \cos \epsilon \cos \alpha}{\sin \epsilon \sin \alpha} \tag{18}$$

and since

$$\vartheta = \gamma + C - L, \tag{19}$$

then

$$x = \sin \epsilon \cos \vartheta \tag{20}$$

$$y = \sin \epsilon \sin \vartheta,$$

are the rectangular coordinates sought. These expressions contain the as yet undetermined values of the angles i and ϵ. However, we have the following from the respective spherical triangles:

$$\cos i = \sin \beta \;\; \sin \beta + \cos \beta \;\; \cos \beta \cos(\lambda - \lambda \;) \tag{21}$$

$$\cos \epsilon = \sin \beta_t \sin \beta + \cos \beta_t \cos \beta \cos(\lambda - \lambda_t) \;\; ,$$

which fully defines the angles sought since they can vary only within the limits of the first quadrant.

The calculation of x and y for the reference points by both formulas gives a good check of the accuracy of the result.

To clarify the question of the extent to which the parallax of the Moon (which we have not been taking into account up to the present time) affects the results, use may be made of the last formulas shown in which it is necessary to substitute $\lambda_t + \Delta\lambda_t$ and $\beta_t + \Delta\beta_t$ — the selenographic coordinates of the place of observation, instead of the λ_t and β_t — the selenographic coordinates of the center of the Earth. It is easy to show that

$$\Delta\lambda_t \cos \beta_t = \pi \sin z \sin(q + C) \tag{22}$$

$$\Delta\beta_t = \pi \sin z \cos(q + C) ,$$

where q is the parallactic angle of the Moon for the place of observation at a given instant, π is the horizontal parallax and z is the zenith distance of the Moon. We will note that in these formulas both the angles C and q are considered to be positive toward the east, which corresponds to the case when the pole of the Moon is eastward of the declination circle passing through the center of its visible disc and the Moon itself lies to the east of the meridian of the place of observation.

The correction for the parallax appears to be completely immaterial. In nearly all cases it is expressed merely in tenths of a millimeter, whereas the diameter of a lunar image on the plate amounts to not less than 30 mm. Strictly speaking, it can be neglected, the more so because the difference in the calculated and observed positions of the reference points often reaches 0.4 to 0.5 mm in our case. The last is explained first of all by the fact that with the considerable dimensions of the craters the reference points selected in them are not completely identical in the position in different phases and, of course, regularly deviate from the points with a corresponding name shown in the selenographic catalogs. This factor also explains in all probability the discrepancy in the data of the separate catalogs. As an example it may be pointed out that for the crater Menelaus, which in addition is not characterized by considerable dimensions, different authors give the values of the coordinate λ which differ by $0^\circ.7$ from each other.

We took the selenographic coordinates of the reference points from the catalogs compiled by Franz, Neison, Lohrmann, Mädler and even Mayer. In those cases when the coordinates were not in any of the catalogs available to us it was necessary to tie them in to the points with the known coordinates and count them off on the grid. After determining the position of the reference points in relation to the intensity equator it presents no difficulty to pass on to the rectangular coordinates expressed in the same system for all the remaining points at which the luminance on the lunar disc was measured. To accomplish this changeover by a purely calculation method by determining the position of one coordinate system in relation to the other would have been too impractical since this requires the expenditures of considerable labor and time. Therefore, it was decided to resort to the graphic method. We prepared two graphs. On one graph are plotted in a large scale (1 cm on the graph is equal to 1 mm on the scales of the microphotometer) all positions of the reference points and also of the measured points according to the readings from the microphotometer table. On the other graph drawn on semitransparent paper we plot in the same scale the positions of the reference points in relation to the intensity equator and, in addition to this, the boundaries of the illuminated portion of the lunar disc. Then we superimpose the two graphs and cause the reference points on them to coincide. This defines the transition from the coordinate system tied in with the table of the microphotometer, to the intensity equator. It only remains now to count off in the new system the coordinates of all points measured previously. The plates corresponding to a considerable phase, on which the Moon appears in the form of a narrow sickle were specially processed. The coordinates of a considerable number of points located at the edge of the lunar disc were determined for them during the main measurements and, in addition to this, the coordinates of the tips of the sickle were read where this was possible. The position of the center of the visible disc and of the intensity equator was then determined from these points. This assured the feasibility of converting the coordinates by this same graphic method.

The rectangular coordinates obtained in this manner for all measured points on the photograms of the Moon served first of all for the determination of the influence of the halation effect which has already been discussed above. In addition to this, the angles of incidence and reflection, the knowledge of which is necessary to be able to discuss all of the material obtained, were determined by the formulas shown above for these coordinates.

The luminances of different points of the lunar surface are expressed for each plate in an arbitrary system of units in which the unit used is the amount of light falling on the plate from the firmament near the celestial pole at noon after passing through several insertion sheets and through the smallest aperture of the tube photometer.

Thus, there is no doubt that for each plate the luminances are expressed in different units. In order that it be possible to interpret all plates simultaneously it is necessary to express all the luminances obtained in like units. For this purpose, out-of-focus images of the North Star secured as mentioned above upon pulling out the eyepiece tube to 8 cm were plotted on the Tashkent photograms.

Unfortunately, in view of the faintness of the North Star in comparison with the Moon and also because the North Star was photographed with the same aperture of the objective, considerably longer exposures reaching up to 136 seconds had to be used in plotting its out-of-focus images. As a rule, the North Star was photographed with the exposures of 18, 27, 40.5, 61, 91 and 136 seconds, but its prints made with shorter exposures were almost impossible to use since they were characterized by low density and stood out only faintly against the general background of the plate.

Originally it was intended for the densities of the image of the North Star obtained on all plates with the same exposure to be a part of the curves of the tube photometer and to consider certain reference luminance read with the help of these curves as being the same for all plates. This would have been quite correct if the North Star had been photographed with the same exposure as the tube photometer. But in our case this could be correct only when the characteristic curves obtained with different exposures run quite parallel to one another, i.e., when the Schwarzschild exponent is the same for all plates. Indeed, as is generally known, with the same density we have the following relationship between luminance and the time of exposure:

$$I\, t_8 p = I_x t_x p \,, \tag{23}$$

where p is Schwarzschild exponent. With the same p and the same luminance of the North Star I_x the same luminances are obtained for the curves of the tube photometer from this relationship if the same exposure-time ratio is taken.

In order not to introduce an arbitrary assumption regarding the constancy of the p an attempt at its determination was made making use of the circumstance that there are several out-of-focus images of the North Star at our disposal.

From a series of conditional equations of the form

$$\log I = \log I_x + p(\log t - \log t_8) \,, \tag{24}$$

where t, t_8 and I for each plate are known with I being taken from the curve of the tube photometer, we determine the unknown p and I_x.

However, the Schwarzschild constant determined in this manner proves to be very different for different plates — in a range of from 0.5 to 1.0. There is no doubt, however, that in reality this constant cannot vary in such a wide range.

In order to judge the extent to which Schwarzschild exponents are constant for the plates of different kinds and also to judge the error in tying in between the plates in the case of the standard being plotted with an exposure differing from the exposure of the tube photometer, an analysis was made of eight different kinds of Russian and foreign plates, namely, Aerophoto, Ilford Monarch, Ilford Iso Zenith, Agfa Isolar, Agfa Ultra Special, Eastman Ortho, Ilford Special Panchromatic, Ilford Zenith Super Sensitive — altogether 40 samples.

A photometric wedge made up of two cemented wedge-shaped plates, one of a transparent glass and the other of a dark glass neutral in the photographic rays, was printed on a light-sensitive plate with different exposure changing in a geometric progression. A photometric standard whose constancy was thoroughly checked served as the light source.

By plotting the characteristic curves representing, as usual, the photographic density in the function of the wedge readings, i.e., in the function of the logarithm of illumination, and knowing the wedge constant obtained from additional investigations we define Schwarzschild exponent as a quantity proportional to the horizontal distance between the characteristic curves of the same plate that correspond to different exposure. The details of this work are set forth in Astronomicheskiy zhurnal (Astronomical Journal), 5, No. 1 and No. 2.

This investigation leads to the following results:

1. Schwarzschild exponents vary only in a small range, approximately from 0.78 to 0.88 and usually do not vary regularly in passing on from one plate to another.

2. On the average they have a tendency to decrease with an increase in the degree of illumination although in the case of separate plates this tendency is completely hidden owing to local irregularities in the value of the exponent.

3. Schwarzschild exponents are distinguished by the greatest constancy in the center portion of the characteristic curves, i.e., in the region of a normal exposure. In the region of underexposures or overexposures, where the characteristic curve is distinguished by a considerable bend, these values vary considerably more.

4. The error in tying in different plates for which the standard is plotted with an exposure exceeding by ten times the exposure of the scale of the tube photometer amounts approximately to 0.1 zv. [?] of the value for all kinds of the plates investigated.

Because of this, to tie in the plates with each other it was decided to use the out-of-focus images of the North Star obtained with an exposure of at least 60 seconds, since these images have densities falling on the rectilinear portion of the characteristic curve. In this manner, with the density of the out-of-focus image corresponding to a certain exposure for all plates, the luminance which was taken as unity for the plate No. 28 was determined from the curves of the tube photometer. The conversion factors for the remaining plates were taken as the reciprocals of the luminances determined in this manner. The values of these luminances determined by an independent method for three exposures of 60, 91, and 136 seconds are given below.

Plate No.	28	29	30	32	33	34	35	36	37	38	39	51	55	56	147	
K (136^s)	1.00	1.09	0.82	0.86	0.915	0.95	0.93	0.84	0.87	0.75	0.78	0.415	0.81	0.16	-	(25)
K (91^s)	1.00	1.17	0.87	0.835	0.92	0.90	0.93	0.83	0.76	0.78	0.77	0.43	0.70	0.15	2.65	
K (61^s)	1.00	0.96	0.90	0.92	0.92	0.77	1.04	0.86	0.87	0.77	0.76	0.475	0.78	0.14	2.30	

The final value of the conversion factors was derived from the numbers given on these lines assuming the weight of the first two lines to be equal to 3 and that of the last line to 2. Now it only remains to take into account the atmospheric absorption which is dependent on the fact that the zenith distance of the North Star differed somewhat from the zenith distance of the Moon at the time of the observation. The correction for atmospheric absorption was applied simply on the basis of the tables compiled by Wirtz in his "Tafeln und Formeln für Astronomie und Geodäsie." Such a method of tying in the plates with each other is the weakest part of the work being set forth. Because of their nonuniformity, the out-of-focus images are in general hardly suitable for this purpose. It would be much better to plot on all plates under strictly identical conditions the prints of the photometric standard or, still more simply, to print not the stages of the tube photometer, especially from the light of the firmament which differs in its spectral composition from the light of the Moon, but to print the photometric wedge under strictly standard conditions and from a source of light of a suitable color. Thereby the necessity for additional standards would be completely eliminated and the tying in of the plates would be carried out with the greatest possible simplicity. However, it is necessary to have at one's disposal a specially equipped photometric laboratory for this purpose, which was lacking in our case.

After all luminances have been expressed in an arbitrary but identical system of units for all photograms we could pass on to the investigation of the rule in accordance with which the visible luminance of the lunar surface varies in the function of the angles of incidence, reflection and phase.

First of all it should be ascertained to what extent the usually accepted laws of the matte reflection of light are applicable to the Moon. From among these laws it is sufficient to examine only one, namely, the Lommel-Seeliger law, since the two remaining laws — Lambert law and Euler law — clearly do not satisfy the cases observed.

The Lommel-Zeeliger formula has the following form:

$$I = \frac{\Gamma \cos i \cos \epsilon}{\cos i + \lambda \cos \epsilon} , \tag{26}$$

where i is the angle of incidence, ϵ is the angle of reflection, Γ is a constant coefficient dependent on the albedo of a given element of the surface, and λ is a constant coefficient which supposedly varies only with the phase.

This formula representing the amount of light reflected by a unit of area into a unit of a solid angle in a definite direction according to the values of i and ϵ was derived by a theoretical method on the assumption, however, that no reflection of light from a body's surface itself exists and the mechanism of reflection consists in the scattering of the light by the inner elements of a given substance. Inasmuch as we are dealing with such an irregular surface as the lunar surface broken up very possibly by numerous fissures, depressions, etc., which throw shadows and which can be examined in a telescope or taken on a photograph only when they are sufficiently large, there is no reason to believe that this formula has any physical basis in the application to the Moon. According to this formula the visible luminance of a known element of the lunar surface is

$$I = \frac{\Gamma \cos i}{\cos i + \lambda \cos \epsilon} . \tag{27}$$

First we apply this formula for the separate photograms relating to one definite phase. We prepare the images of the separate phases in the form of an orthographic projection on which the outlines of the lunar disc and the positions of all points with a measured brightness are plotted. In addition to this, we plot on these grids the outlines of the lunar seas doing this with the help of original photograms projected directly onto the grid with the help of a slide projector. Smaller details like light rays, etc., were not plotted.

Breaking up the image of the Moon on the grid with the help of concentric circles every 0.1 part of the radius we thereby pick out the points with a constant ϵ but variable i. Indeed, in each ringed zone described around the center of the visible disc of the Moon the angle of reflection remains constant in a narrow range while the angle of incidence varies within all possible limits. The constants Γ and λ may be determined for each one of these zones with the help of the following obvious equation

$$\Gamma - \lambda I \frac{\cos \epsilon}{\cos i} = I. \tag{28}$$

It turns out that for all the zones of a given plate these quantities are approximately constant; at least they do not show a definite trend. Thus for example, for plate No. 29 we have the following values of these constants:

(29)

$\cos \epsilon$	λ	Γ
0.310	0.172	11.0
0.527	0.426	12.9
0.661	0.400	11.9
0.760	0.273	11.2
0.835	0.124	9.6
0.893	0.457	14.4
0.937	0.333	12.3
0.968	0.683	17.1
0.989	0.548	14.3

Therefore, the above-mentioned Seeliger formula may be applied to the entire surface of the Moon as a whole. In order to moderate the nonuniformity of the material the above-cited conditional equation was set up not with respect to each separate point but with respect to a group of neighboring points of one and the same zone. The greater the dispersion of the luminance proved to be, the larger was the number of the separate points used to set up the conditional equation.

The solution of all of these conditional equations by the method of least squares separately for each plate yielded the following results.

No.	α	The Number of Points Used	The Number of Groups	The Sum of the Weights Σg	$\Sigma g:n$	Average Error of One Conditional Equation	λ		Γ		λ_e	Γ_e
28	-76° 15'	88	23	38	1.7	±1.600	0.368 ±	0.046	11.14 ±	0.60	0.37	10.05
29	-64 47	296	36	98	2.7	1.576	0.301	0.030	11.49	0.40	0.31	11.1
30	-53 1	72	18	27	1.5	1.145	0.327	0.057	14.76	0.80	0.28	11.7
32	-31 39	92	24	45	1.9	2.334	0.297	0.113	13.19	1.25	0.24	12.5
33	-20 11	109	15	36	2.4	2.972	0.214	0.137	11.06	1.47	0.23	12.7
36	+25 28	94	13	31	2.4	2.447	0.219	0.106	12.19	1.32	0.235	12.6
37	+35 48	92	21	40	2.0	2.320	0.239	0.040	15.35	0.83	0.24	12.4
38	+47 22	76	20	36	1.8	2.035	0.277	0.058	12.13	0.87	0.265	11.95
39	-96 20	55	23	31	1.3	1.352	0.559	0.062	9.56	0.59	0.52	9.4
46	-65 15	68	20	35	1.8	2.214	0.556	0.082	13.12	1.01	0.32	11.0
51	-141 34	17	3	7	2.3	0.781	1.629	0.322	9.85	1.32	1.95	7.2
55	-126 0	20	4	8	2.0	1.575	0.870	0.194	12.25	1.70	1.06	7.8
56	-115 0	30	5	12	2.4	1.478	0.748	0.374	5.87	1.23	0.79	8.3

Plates No. 34, 147 and 148 were not included in the table shown above since it was not possible to determine the constants λ and Γ for them. Plate No. 34 has no connection with the remaining plates since the focal images of the North Star are plotted on it; the remaining two plates were overexposed to such an extent in comparison with the stages of the tube photometer that a considerable number of points located on the lunar continents which are represented only by the values of λ and Γ given above could not be investigated in the photometric respect.

We see completely clearly that λ has a definite trend with the phase. The same may also be said regarding Γ although here the variance is considerably greater, chiefly because of errors in the determination of the coupling coefficient. These relationships may be represented in a completely satisfactory manner with the help of the following empirical expressions:

$$\lambda = \lambda_0 \left(1 + \mathrm{tg}^2 \frac{\alpha}{2}\right),$$

$$\lambda_0 = 0.225, \tag{30}$$

$$\Gamma = \Gamma_0 \left(1 + \cos^2 \frac{\alpha}{2}\right),$$

with Γ_0 being 6.5 in our arbitrary units.

In this manner the visible luminance of the continents within the range of permissible errors is represented by the following general formula:

$$I = \frac{\Gamma_0 \cos i \left(1 + \cos^2 \frac{\alpha}{2}\right)}{\cos i + 0.225 \left(1 + \mathrm{tg}^2 \frac{\alpha}{2}\right) \cos \epsilon} \tag{31}$$

In the last two columns of the preceding table the values of λ and Γ are given according to their empirical expressions. As may be seen, λ differs from its observed value within the range of the average error in nearly all cases.

The lunar seas could not be investigated like the continents because their luminance is extremely variable. Therefore, each sea was divided into separate regions and luminance in relation to the phase was determined for each one of them. A series of graphs constructed in this manner showed that regardless of where a given region may be located on the lunar surface its luminance reaches the maximum at the time of the full moon. This fact was also repeatedly observed before.

In conclusion the integral luminances of the Moon during different phases were determined on the basis of our material; in doing so, a good agreement was obtained with the Ressel curve of the lunar phases. This indicates that in general the tying in of the plates with each other was done correctly.

Taking personal part in this work were N. M. Shtaude and P. P. Parenago together with whom the processing method was worked out and who carried out a considerable portion of the calculating work. In addition to this, N. M. Shtaude measured the greater part of the plates. Considerable assistance was also given by Ye. M. Vinogradova who carried out all the calculations pertaining to taking the effect of the halo into account.

M. I. Barantseva took part in the measurements and calculations pertaining to different stages of the work. A detailed account of all of the results obtained and also a summary of the luminances determined at all points of the lunar surface is contained in a memoir published in the Works of the State Astrophysical Institute, 4, No. 1.

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13. ABSTRACT

The aim of this paper is the determination of the law according to which the light is reflected from the different parts of the lunar surface under various angles of incidence, reflection and phase.

The photograms of the Moon obtained by Surovtseff in the focus of the normal astrograph of the Tashkent Astronomical Observatory were measured at the Astrophysical Institute of Moscow with the Harmann microphotometer. The photographic densities obtained with this instrument were transformed into brightness of the corresponding points by means of the scale of tubular photometer printed on each plate exactly with the same exposure. The extrafocal images of the Polar star photographed on the same plates with many different exposures were utilized for the determination of the same unit of brightness. The brightness of the different points of the Moon were corrected for effect of halation. It is found that our photometrical determinations for all phases from 0° up to 140° may be represented in a very satisfactory way by the simple expression

$$J = \frac{\Gamma_0 \cos i \cos \epsilon \,(1 + \cos^2 \frac{\alpha}{2})}{\cos i + 0.225\,(1 + \mathrm{tg}^2 \frac{\alpha}{2}) \cos \epsilon},$$

which is a modification of the very well known Lommel-Seeliger's formula.

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