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STUDY OF PLASMA DIAGNOSTICS USING LASER INTERFEROMETRY WITH EMPHASIS ON ITS APPLICATION TO CESIUM PLASMAS

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ABSTRACT

A review of the background and current development of gas-laser diagnostics is presented. Included is a résumé of the plasma refractive index and the experimental techniques of the three-mirror laser interferometer and the laser heterodyne system. Sensitivity of the two techniques to electron and neutral-particle densities is covered with emphasis placed on the application of the techniques to a cesium plasma contained in a thermionic diode. The spatial resolution of the techniques as applied to a cesium diode is also presented.

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STUDY OF PLASMA DIAGNOSTICS USING LASER INTERFEROMETRY WITH EMPHASIS ON ITS APPLICATION TO CESIUM PLASMAS

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SUMMARY

A review is presented of the background and current development of gas laser diagnostics of plasmas. The index of refraction of a plasma probed by monochromatic radiation is outlined. The experimental techniques of both the three-mirror laser interferometer which measures phase shifts, and the laser heterodyne system which measures frequency shifts are described in detail. The density range of applicability of the three-mirror system is 10^{19} to 10^{25} electrons per cubic meter, and the heterodyne system can measure electron densities down to 10^{16} per cubic meter. The spatial resolution of either technique is shown to be of the order of 0.25×10^{-3} meter when applied to a plasma of small cross section, and through the use of a perturbation analysis, it appears feasible to reduce that figure by an order of magnitude. Special emphasis is placed on the application of the techniques to a cesium plasma contained in a thermionic diode.

INTRODUCTION

At the present time, there are several popular methods available for probing relatively dense plasmas to determine their intrinsic properties such as electron density and electron temperature. All these methods have their advantages and disadvantages. For a good many years, Langmuir probes have been considered to be more or less the standard for plasma property measurements. These probes have the advantage that their use is straightforward and their measurements are recorded with relative ease. However, they are limited in their application because they perturb the plasma in the region of their placement. Accuracy and resolution, both spatial and time, suffer as a result. There is hope, though, that this drawback can be partially removed through use of miniaturized probes as described in reference 1.

Emission spectroscopy is another method which has been quite successful in probing plasmas. This technique allows the determination of both electron density (in a neutral plasma) and electron temperature with quite good spatial resolution, and of course, it offers no perturbation to the probed plasma. The main parameters which are observed by this technique are Stark broadened transitions (yielding ion and electron densities) and relative line intensities (yielding electron temperatures). The major drawbacks of the technique are that it relies on the existence of local thermodynamic equilibrium, and on the knowledge of other parameters which must be either analytically determined (e.g., oscillator strength and impact parameters) or determined by involved experimental procedure (e.g., absolute line intensities). The method does produce good results, however, and is quite adequately described in reference 2.

Another probing method, which is probably one of the easiest to apply of the methods considered in this paper, is microwave interferometry. It is particularly useful in measurements on transient plasmas. As far as electron-density measurements are concerned, the parameter that is observed in this technique is the phase shift of the transmitted (or reflected) microwave beam. The phase shift is brought about by a change in the index of refraction of the plasma. The major drawback of this method is that it has very poor spatial resolution and has limited range of applicability (10^{16} < electron density < 10^{20} m^{-3}).

The purpose of this paper is to present a review of another diagnostic technique which also measures the index of refraction of the probed plasma. This technique, broadly classified as optical interferometry using gas lasers, can be applied to a plasma in two different ways: In one case, the phase shift of the laser beam is observed (useful for electron densities of 10^{19} to 10^{25} m^{-3}), and in the other, the frequency shift of the laser is detected (useful for electron densities down to 10^{16} m^{-3}). In both cases, however, the parameter which is actually determined is the index of refraction of the plasma. As will be shown, the number densities of the various plasma particles are directly related to the plasma refractive index. Particular emphasis is placed on the application of the technique to a cesium plasma such as that which exists in a thermionic diode. The spatial resolution of the technique as applied to a thermionic diode is shown through preliminary measurements to be equivalent to or better than the best currently used technique (emission spectroscopy).

PLASMA REFRACTIVE INDEX

The relation between the plasma refractive index and the densities of the various plasma constituents has been developed in reference 2 by allowing a monochromatic electromagnetic wave to propagate through a homogeneous medium. The resulting equation, when summed over all states is

$$n(\omega) - 1 = \frac{-e^2}{4\epsilon_o m} \sum_{j=0}^{\infty} \sum_{k=j}^{\infty} \frac{N_j f_{jk}(\omega - \omega_{kj})}{\omega_{jk}\left[(\omega - \omega_{kj})^2 + \left(\frac{\gamma}{2}\right)^2\right]} \tag{1}$$

where $n(\omega) - 1$ is the refractive index at propagating frequency ω, N_j is the density of lower state j, f_{jk} is the absorption oscillator strength between state j and some higher state k, ω_{kj} is the frequency of emitted transition k to j, γ is a damping constant which causes frequency spreading about ω_{kj}, e is the electron charge, m is the electron mass, and ϵ_o is the vacuum dielectric constant.

This equation can be simplified somewhat by adapting it to a particular experiment in which visible or near infrared light is used as the probing frequency. In most cases, the damping term γ can then be neglected because it has a magnitude of the order of 10^8 hertz or less. For this term to be a contributing factor, the difference in wavelength between the probing frequency and the particular transition frequency would have to be approximately 10^{-4} Å (10^{-8} μm). Furthermore, it is quite probable that the difference between these two frequencies is of the order of magnitude of the frequencies themselves; that is,

$$\omega_{kj} \approx \frac{\omega + \omega_{kj}}{2}$$

Therefore, equation (1) becomes,

$$n(\omega) - 1 \approx \frac{-e^2}{2\epsilon_o m} \sum_{j=0}^{\infty} \sum_{k=j}^{\infty} \frac{N_j f_{jk}}{\omega^2 - \omega_{kj}^2} \tag{2}$$

To determine the contribution of free electrons to the refractive index, equation (2) can be simplified further by allowing $\omega_{kj} = 0$. Then

$$n(\omega) - 1\Big|_{\text{Electrons}} = \frac{-e^2}{2\epsilon_o m} \frac{N_e}{\omega^2} = -\frac{1}{2} \frac{\omega_p^2}{\omega^2} \tag{3}$$

where $e^2 N_e/\epsilon_0 m$ is the familiar plasma frequency, ω_p^2. Equation (2) can then be written as

$$n(\omega) - 1 = -\frac{1}{2}\frac{\omega_p^2}{\omega^2} - 2\pi \sum_j^{\infty} N_j \alpha_j \tag{4}$$

where α_j is the electronic polarizability associated with the atoms in state j such that

$$\alpha_j = \frac{e^2}{4\pi\epsilon_0 m} \sum_{k=j}^{\infty} \frac{f_{jk}}{\omega^2 - \omega_{kj}^2} \tag{5}$$

The second term of equation (4) is the contribution to the refractive index of both the unexcited and the excited plasma neutrals. Excited ions could also contribute to this term.

The remaining plasma species that could contribute to the refractive index are the free ions. Because their functional contribution is identical to that of the free electrons and their mass is so much greater than that of the electrons, their contribution is negligible for neutral or near neutral plasmas.

EXPERIMENTAL APPROACH TO REFRACTIVITY MEASUREMENT

As was mentioned previously, there are two different methods of experimentally determining the plasma refractive index using gas lasers. One utilizes the laser-beam phase shift and the other utilizes the laser-beam frequency shift. Both shifts are caused by the change in the plasma refractivity.

Phase-Shift Measurements

This method uses a helium-neon gas laser as both the source and the detector of a Michelson interferometer. The three mirrors that are used in this technique form two resonant cavities. There are two alternate arrangements for these cavities as is shown in figure 1. A cavity containing the laser tube and/or the test plasma is an active cavity, and one containing neither is a passive cavity. The earliest configuration (ref. 3) of this

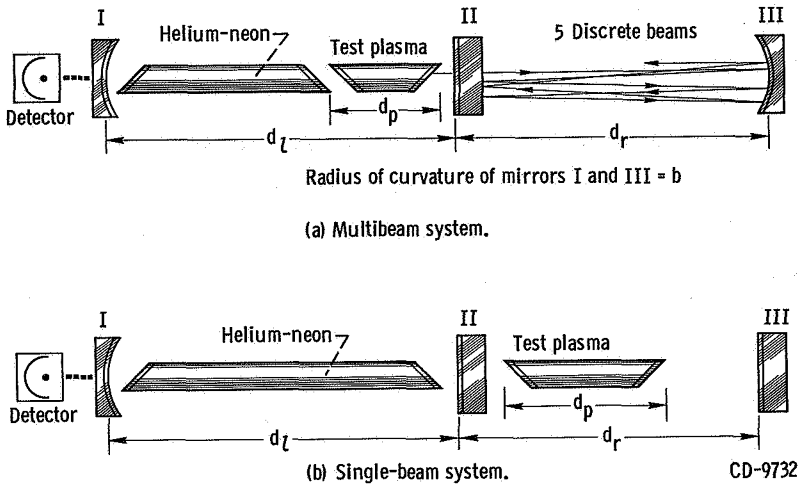

Figure 1. - Three-mirror laser interferometer.

interferometer used a plane mirror (fig. 1(b)) for mirror III, and the test plasma was located between mirrors II and III.

As was shown in reference 4, the sensitivity (i. e., the minimum detectable density) of the technique can be increased by using a spherical mirror for mirror III (fig. 1(a)). Even in this case, the test plasma can be placed between mirrors II and III but the spatial resolution (to be discussed later) of the technique suffers. The radii of curvature of mirrors I and III do not necessarily have to be identical.

The principle of operation is as follows: When the passive cavity is resonant but not in phase with the active cavity, the amplitude of the laser beam is modulated. This occurs when the beam that leaves the active cavity and is reflected a number of times in the passive cavity, finally returns to the active cavity superimposed on itself. The degree of modulation is not particularly important to the experiment, but the cyclid nature of this modulation is important. Reference 5 has shown that, when the optical length of the active cavity is changed (due to the refractivity of the plasma) by an amount equal to $\lambda_o/2s$, one cycle of modulation can be observed on the detected laser output. (λ_o refers to the laser wavelength, and s to the number of discrete beams in the passive cavity.) Therefore, the number of cycles of modulation is a function of the plasma refractivity. This functional relation can be shown by considering the equation (ref. 6) which governs the frequency of a resonant cavity. That equation, for a plane-concave cavity of length d, and concave radius of curvature of b, is

$$\frac{2d}{c}\omega_{m,p,q} = 2\pi\left[q + \frac{1 + m + p}{2\pi}\cos^{-1}\left(1 - \frac{2d}{b}\right)\right] \tag{6a}$$

The symbols m, p, and q refer to the mode indices of the $TEM_{m,p,q}$ mode, with m and p signifying the transverse modes and q, the longitudinal mode. For the experimental arrangement shown in figure 1(a), the transverse modes are excited in the passive cavity by virtue of the third mirror (III) being excited off-axis, and thus s discrete beams are created in the cavity. Reference 4 has shown that $1/2\pi \cos^{-1}[1 - (2d/b)]$ is an angle equal to the ratio of two relative prime integers r/s where the value of an integer r must be such that $0 \leq r/s \leq 1/2$. Therefore, equation (6a) can be written for the passive cavity as

$$\omega_{m,p,q} = \frac{\pi c}{d_r}\left[q + (1 + m + p)\frac{r}{s}\right] \tag{6b}$$

where d_r is now the length of the passive cavity. In the active cavity, only longitudinal modes q' should be excited; that is, m = p = 0. If the transverse modes were excited in this cavity, the modulated signal would be difficult to observe. Hence, the frequency of the active cavity without the test plasma will be given by

$$\omega_{q'} = \frac{c\pi}{d_l}\left[q' + \left(\frac{r}{s}\right)'\right] \tag{6c}$$

where d_l is the overall length of the cavity j, and with the test plasma, the frequency will be

$$\omega_{q'_{plasma}} = \frac{c\pi}{d_l}\,\frac{q' + \left(\frac{r}{s}\right)'}{1 + [n(\omega) - 1]\frac{d_p}{d_l}} \approx \frac{c\pi}{d_l}\left[q' + \left(\frac{r}{s}\right)'\right]\left(1 - \Delta n\frac{d_p}{d_l}\right) \tag{6d}$$

where d_p is the test plasma length and Δn is the plasma refractive index, $n(\omega) - 1$. The approximation $\Delta n\,(d_p/d_l) << 1$ has been made.

Initially, the passive cavity is resonant with the active cavity; that is, $\omega_{m,p,q} = \omega_{q'} = 2\pi c/\lambda_o$. The frequency separation between adjacent resonances in the passive cavity is (ref. 4)

$$\Delta\omega_{m,p,q} = \frac{c\pi}{d_r s} \tag{7}$$

If the resonant frequency of the passive cavity changes by this amount, one cycle of modulation (or one "fringe") will be detected on the output signal of the laser. Now, the change in the plasma refractive index Δn, necessary to produce one cycle of modulation can be obtained by equating equation (6d) to the sum of $\omega_{m,p,q} \pm \Delta\omega_{m,p,q}$ because the two cavities must remain resonant with each other.

$$\frac{2\pi c}{\lambda_o}\left(1 - \Delta n \frac{d_p}{d_l}\right) = \frac{2\pi c}{\lambda_o} - \frac{\pi c}{d_r S} \tag{8}$$

Rearranging terms yields

$$\Delta n = \left(\frac{\lambda_o}{2S}\right)\left(\frac{d_l}{d_r d_p}\right) \tag{9}$$

per cycle of modulation. Thus, by counting the number of cycles of modulation on the output of the laser and using equation (9), the total refractive index of the plasma at λ_o can be determined. It is then necessary to employ equation (4) to separate the contributions of the individual species. Of course, the number of contributing species must be no more than the number of available laser wavelengths.

Frequency-Shift Measurements

This method uses two helium-neon gas lasers and compares the frequency difference between the two to determine the plasma densities. One of these lasers, as in the previous case, contains the test plasma as part of its resonant cavity. The other laser operates in a free-running mode and is considered the local oscillator of the system. They are shown schematically in figure 2. Allowing these lasers to operate only in the mode TEM_{00q}, requires that their frequencies be approximated by the following equations:

$$\omega_A = \frac{q_A c\pi}{d_A\left(1 + \Delta n \frac{d_p}{d_A}\right)} \tag{10}$$

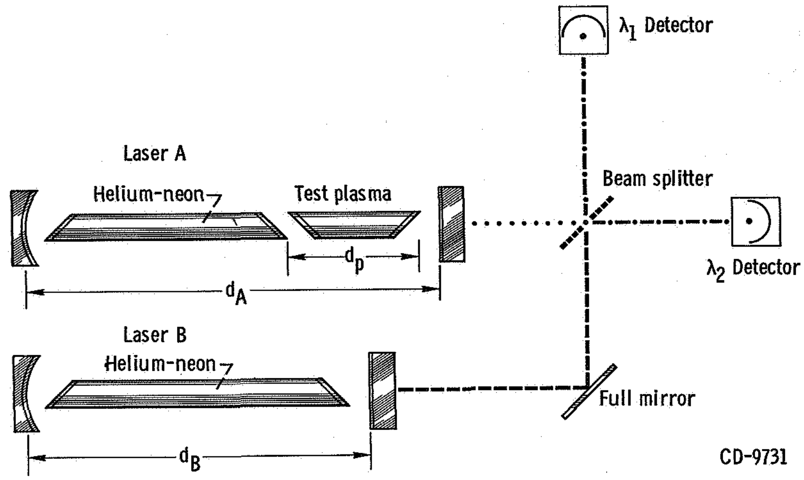

Figure 2. - Laser heterodyne system.

$$\omega_B = \frac{q_B c \pi}{d_B} \tag{11}$$

By mounting all laser windows at their Brewster angles, one is assured of the direction of the polarization vectors of the output laser beams. As long as these two vectors are not perpendicular to each other (parallel orientation is preferred), the two output beams can be mixed together using a beam splitter, and the difference in frequency between the two beams can be detected by a photodetector. This phenomenon can be best understood by considering the signals that induce a current in a photodetector. Most photodetectors are square-law devices; that is, the induced photocurrent is proportional to the square of the field of the incident beam. Hence, $i \propto \overline{E}\,\overline{E}$; however, $\overline{E}$ is the vector sum of the two laser beam field vectors.

$$\overline{E} = \overline{E}_A + \overline{E}_B \tag{12}$$

where

$$\left.\begin{aligned} \overline{E}_A &= \overline{E}_{Ao} \cos (\omega_A t + \varphi_A) \\ &\text{and} \\ \overline{E}_B &= \overline{E}_{Bo} \cos (\omega_B t + \varphi_B) \end{aligned}\right\} \tag{13}$$

where φ_A and φ_B are the respective single-pass phase shifts of the lasers. It follows that, if $\overline{E}_A$ and $\overline{E}_B$ are parallel,

$$\left.\begin{aligned} & i \propto \left(E_A^2 + E_B^2 + 2E_A E_B\right) \\ & \text{or} \\ & i \propto \Big[E_{Ao}^2 \cos^2 (\omega_A t + \varphi_A) + E_{Bo}^2 \cos^2 (\omega_B t + \varphi_B) \\ & \qquad + 2E_{Ao}E_{Bo} \cos (\omega_A t + \varphi_A) \cos (\omega_B t + \varphi_B)\Big] \end{aligned}\right\} \tag{14}$$

Using standard trigonometric identities, the only terms in equation (14) to which any photodetector will respond will be a d-c term and a cosine term involving the frequency and phase differences between the two beams. Therefore,

$$i \propto \left\{\frac{E_{Ao}^2 + E_{Bo}^2}{2} + E_{Ao}E_{Bo} \cos\left[(\omega_A - \omega_B)t + (\varphi_A - \varphi_B)\right]\right\} \tag{15}$$

As long as the phase difference $\varphi_A - \varphi_B$ is time independent, the frequency of the observed signal will be

$$\Delta f = \frac{\omega_A - \omega_B}{2\pi} \tag{16}$$

which includes the initial beat frequency Δf_o of the two lasers in addition to the frequency shift due to the plasma. Substituting equations (10) and (11) into equation (16) yields, for the plasma refractive index,

$$n(\omega) - 1 = \Delta n = (\Delta f_o - \Delta f) \frac{d_A}{d_p} \frac{\lambda_o}{c} \tag{17}$$

Of course, the use of this equation together with equation (4) and the observation of Δf at a different laser transition λ_o for each contributing species in the plasma, will yield the number density of each contributing species.

The unique feature of the helium-neon laser is that three strong transitions oscillate simultaneously, and thus three laser wavelengths (0.6328, 1.15, and 3.39 μm) are available for beat frequency observation.

The development of this technique has been relatively recent, with the first published data obtained using the method appearing in references 7 and 8. A general description of the method also appears in reference 9.

RESOLUTION OF EXPERIMENTAL MEASUREMENTS

Particle Density Sensitivity

In most inert-gas plasmas, the free electrons are the chief contributors to the refractive index at any of the laser transitions considered here. At very low fractional ionizations, though, the unexcited neutrals can have some contributory effect on the index due to the large imbalance between the number densities of the two particles. If this imbalance is not present, the effect of the neutrals is negligible because the first excited states in the inert atoms correspond to energy jumps much greater than the helium-neon laser frequencies; hence, the effect is approximately dependent only on the neutral number density and is independent of the probing frequency (see eq. (1)). For the same reason, the excited neutrals offer negligible contributions under almost any condition existing in an inert-gas plasma. Thus the determination of electron density in this type of plasma is simplified.

However, in an alkali vapor plasma, namely, cesium, a different situation exists. In cesium, there are many allowed transitions which are encompassed by the laser transitions of interest. Therefore, even with a relatively large fractional ionization (easily obtainable in cesium) the effect of the neutrals, whether unexcited or excited, on the refractive index must be considered.

Equation (2) was used to calculate the contributions of the free electrons, the ground-state cesium neutrals, and the first-excited doublet state ($6P_{1/2}$ and $6P_{3/2}$) of cesium for typical conditions that could exist in a cesium-filled thermionic diode (see table I). The oscillator strengths were obtained from reference 10. Although nonexistent in an operating diode, equilibrium conditions were assumed in making these tabulations. The Saha equation was used to determine the fractional ionization for the cesium pressure and electron temperature conditions shown. The equilibrium fractional ionizations represented in this table varied between 1 and 30 percent. A neutral plasma was assumed. Note that the contribution of the neutrals is very sensitive to the particular conditions chosen and that slight variations in the parameters chosen to compute these values could, indeed, change the whole complexion of the table. Also note that a particular species can be the dominant contributor to the refractive index at one wavelength but negligible at another wavelength.

Regardless of the type of plasma (inert or alkali), however, the sensitivity of the experimental measurements is independent of the composition of the plasma; that is, the

TABLE I. - TABULATION OF REFRACTIVE INDEX OF EQUILIBRIUM

CESIUM PLASMA

Species	Cesium pressure, torr	Cesium pressure, N/m^2	Electron temperature, T_e, K	Wave lengths, λ, μm: 0.6328	1.15	3.39
				Refractive index, $n(\omega) - 1$		
Electrons	[a]0.22	29.3	2500 3000	-5.5×10^{-8} -2.4×10^{-6}	-1.8×10^{-7} -8.0×10^{-6}	-1.57×10^{-6} -6.9×10^{-5}
	[b]4.3	573	2500 3000	-1.3×10^{-7} -2.5×10^{-6}	-4.2×10^{-7} -8.3×10^{-6}	-3.6×10^{-6} -7.2×10^{-5}
Unexcited neutrals 6S	0.22	29.3	2500 3000	-1.9×10^{-6} -1.4×10^{-6}	3.6×10^{-6} 2.6×10^{-6}	1.7×10^{-6} 1.2×10^{-6}
	4.3	573	2500 3000	-3.3×10^{-5} -2.7×10^{-5}	6.4×10^{-5} 5.1×10^{-5}	3.0×10^{-5} 2.4×10^{-5}
Neutrals $6P_{1/2}$	0.22	29.3	2500 3000	-4.1×10^{-9} -7.5×10^{-9}	1.4×10^{-9} 2.6×10^{-9}	6.8×10^{-8} 1.2×10^{-7}
	4.3	573	2500 3000	-7.3×10^{-8} -1.8×10^{-7}	2.5×10^{-8} 6.2×10^{-8}	1.2×10^{-6} 3.0×10^{-6}
Neutrals $6P_{3/2}$	0.22	29.3	2500 3000	-5.2×10^{-9} -9.0×10^{-9}	4.1×10^{-9} 7.1×10^{-9}	-3.3×10^{-7} -5.6×10^{-7}
	4.3	573	2500 3000	-9.1×10^{-8} -2.1×10^{-7}	7.2×10^{-8} 1.7×10^{-7}	-5.7×10^{-6} -1.3×10^{-5}

[a]T_{cs} = 500 K.
[b]T_{cs} = 600 K.

limitation on the number density measurements that can be made using either experimental technique discussed herein is not affected by the relative contributions of the plasma particles. Rather, it is a parameter of the technique itself because it is the total refractive index that is experimentally determined (see eqs. (9) and (17)). Therefore, a comparison of the sensitivities of the two techniques can be made by considering a completely ionized plasma.

Reference 4 has shown the practicality of using values of s up to 27 in the three-mirror interferometer method. With this value of s, the corresponding value of d_r (0.843 m for b = 1.0 m), a nominal laser length, d_l, of 1.0 meter, and a laser wavelength of 0.6328 micrometer, one can show from equation (9) that this technique can measure changes of the plasma refractive index of the order of $10^{-6}/d_p$. For a 0.1-meter plasma, this corresponds to a minimum observable electron density, as determined from equation (3), of 5×10^{19} per cubic meter. It is important to note here that one order of magnitude density sensitivity is gained by using a plano-spherical passive cavity (s = 27) instead of a plano-plano passive cavity (s = 1). Results obtained using the various configurations of this interferometer are presented in references 3, 4, and 11 to 17.

Consider now, the same plasma being probed by a heterodyne laser system of similar length and wavelength as before. When it is assumed that the minimum detectable Δf (eq. (17)) is 1.0 kilohertz, the minimum electron density that could be measured would be approximately 1.0×10^{17} per cubic meter; at least two orders of magnitude lower than the three-mirror method.

Even though the heterodyne system offers better density sensitivity than the three-mirror method, care must be exercised when applying the technique using the 3.39-micrometer wavelength. Equation (17) was developed on the basis that there is a time-independent phase relation between the two lasers over the period of observation. This condition may not be fulfilled at the 3.39-micrometer transition, and erroneous density results may be obtained (ref. 7). The error is due to the introduction of the plasma into the laser cavity which changes the gain of the system. The gain change shifts the cavity oscillation frequency such that the net phase shift per pass is unchanged. Hence, equation (10) must be altered as follows:

$$\omega_A = \frac{c\left[q_A \pi + \Delta\varphi(\omega)\right]}{d_A + \Delta n d_p} \tag{18}$$

This phenomenon is called mode-pulling and is analyzed in reference 18. The extent of the error is a function of how rapidly the refractive index of the lasing medium changes near the cavity oscillation frequency. Reference 18 has approximated the correction in the form of

$$\Delta\varphi(\omega) = -\frac{G(\omega_D)(\omega_D - \omega)}{\Delta\omega_D} \tag{19}$$

where $G(\omega_D)$ is the fractional energy gain at line center ω_D, and $\Delta\omega_D$ is the full Doppler width of the particular lasing transition.

Reference 8 has shown that, if

$$\frac{\partial\, \Delta\varphi(\omega)}{\partial\omega} = \frac{G(\omega_D)}{\Delta\omega_D} << \frac{d_A}{c} \tag{20}$$

the error involved in neglecting $\Delta\varphi(\omega)$ in equation (18) is very small for a $d_A = 1$ meter and laser transitions of 0.6328 and 1.15 micrometers. However, for the 3.39-micrometer transition, the correction is significant and, therefore, it should be considered when using that wavelength as a probe.

Spatial Resolution

One of the most important features of using a laser as a diagnostic tool is the very small size of the laser beam. The beam size, of course, determines the spatial resolution of any density measurements made. Because the wave front of the TEM_{00q} beam is Gaussian shaped, the conventional definition of its diameter as given in references 6 and 19, is the width of the Gaussian at 1/e of its axial or peak value. From these references, it can be shown that the diameter or spot size of the beam at the plane mirror of a plano-spherical resonant cavity is given by

$$w = 2\sqrt{\frac{\lambda_o}{\pi}}\left[d(b - d)\right]^{1/4} \tag{21}$$

where λ_o, d, and b, are, as previously defined, the wavelength, mirror separation, and mirror radius of curvature, respectively. At a distance z from the plane mirror, the spot diameter becomes

$$w_z = w\left[1 + \frac{z^2}{d(b - d)}\right]^{1/2} \tag{22}$$

To give an indication of how small the spot size is, again assume a 1.0-meter radius of curvature spherical mirror separated from the plane mirror at a practical distance of 0.995 meter. The spot size at the plane mirror for the 0.6328 micrometer transition would be 0.24×10^{-3} meter. At an arbitrary distance of 0.1 meter from the plane mirror, the spot size would be 0.40×10^{-3} meter. These diameters could, theoretically, at least, be made much smaller by letting d approach b.

Although the spot sizes are small relative to microwave and Langmuir probe standards, it is desirable to make them much smaller in order to probe minute plasma cavities such as exist in thermionic converters. For cesiated thermionic diodes, the interelectrode gap is varied down to less than 0.25×10^{-3} meter. Certain problems arise, however, when a plasma cross section as small as this is placed within an active laser cavity. The basis of these problems is the increased resonant cavity loss brought about not only by the miniature size of the plasma, but also by the associated hardware that is introduced into the cavity by means of the plasma confinement vessel. The second loss is amplified by using cesium vapor as the plasma source. The corrosive effects and high-temperature (approximately 600 K) requirements of the cesium system demand the use of sapphire windows on the diode. However, sapphire is an anisotropic material. Therefore, in order not to destroy the spatial resolution and increase the losses of the laser probe by introducing a double beam at each window, specially oriented, high optical quality sapphire windows must be used. Since any windows must be mounted at their Brewster angle to maintain a specific polarization and minimum losses, the additional requirement for sapphire demands that the C-axis of the material be oriented with respect to the window normal at an angle equivalent to the Brewster angle of sapphire (60^{o}). It has been found in preliminary experiments at this laboratory that the use of unoriented sapphire windows, when placed within a laser cavity, completely quenches the 0.6328-micrometer laser oscillation and severely limits the 1.15-micrometer oscillation.

The introduction of the extremely small plasma cross section into the laser cavity excludes the use of equations (21) and (22) to determine the laser spot size (i.e., the spatial resolution). Those equations were developed for the condition in which the limiting aperture within the cavity is determined by the bore of the laser discharge tube, hence, the major cavity losses are due to laser mirror transmission and Brewster angle window reflection. However, the limiting aperture becomes the interelectrode gap (or part of it) when a thermionic diode is placed within the laser cavity. Diffraction losses

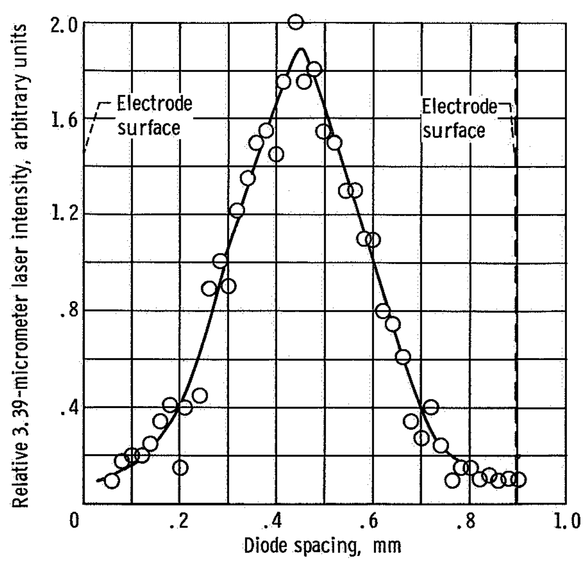

Figure 3. - Relative intensity fo 3.39-micrometer laser containing 0.9-millimeter spaced cesium diode.

due to this aperture now become the controlling cavity loss.

Preliminary experiments have been performed to determine the effect of the increased diffraction losses. A cesium-filled thermionic diode with a 0.9×10^{-3}-meter interelectrode gap and unoriented sapphire windows was placed within a 3.39-micrometer laser cavity. As the diode gap was moved transverse to the cavity axis, laser intensity measurements were made using an indium-antimonide detector with a 4.0×10^{-4} square centimeter sensitive area. A sample of these measurements is shown in figure 3. The most significant information obtainable from these data is that the laser oscillation is maintained to within 0.1×10^{-3} meter of the diode electrodes.

It thus appears feasible to use a technique developed in reference 20 and commonly used with microwave probes (ref. 21) to obtain the desired spatial resolution in the laser-heterodyne-probed diode. That technique is based on a perturbation analysis which relates the change in laser frequency to the electric field distribution in the resonant cavity; that is,

$$\frac{\Delta\omega}{\omega_o} = \frac{1}{2} \frac{\int_V [n(\omega) - 1] E_o^2 \, dV}{\int_V E_o^2 \, dV} \tag{23}$$

where the subscript o pertains to the unperturbed condition (no plasma), and V is the volume of the resonant cavity. Application of the technique requires the additional stipulation that the collision frequency for momentum transfer is much smaller than the probing frequency. This condition, of course, is quite easily met when using a laser as the probe.

In order to extract from equation (23) the refractive index as a function of the radial position within the resonant cavity (i.e., the beam), a functional form of $n(\omega) - 1$, such as a power series expansion, must be assumed. The constants in such an expansion would be obtained by observing $\Delta\omega$ at different positions in the plasma. The validity of such an expansion must be checked to ensure that, in the limit as the plasma cross section becomes much larger than the beam diameter, equation (23) reduces to equation (17). Thus, refractive index profiles may be obtained from a plasma of small cross section even if the ratio of the plasma cross section to the beam diameter is small as long as laser oscillation is not quenched.

System Stability

Both of the techniques discussed in this paper suffer from a common problem. The inherent instability of the laser resonant cavity caused by room vibrations and/or thermal effects place a limitation on either technique as to how long a period the test plasma effects may be observed. When the experimental apparatus shown schematically in figures 1 or 2 is mounted on a heavy steel platform which rests on vibration-reducing inner tubes and lead sheets, the stable period is of the order of 10^{-3} second. Reference 22 describes a method, however, which could, in principle, lengthen the stable period by at least an order of magnitude. That method utilizes piezoelectric transducers mounted on either or both laser mirrors and connected to a stable source through an electronic feedback loop to tune the laser cavity. Reference 9 describes an alternate method of stabilizing the lasers in the heterodyne system. In that method, a tuning tube is placed in the local oscillator cavity. That tube is filled with atmospheric pressure air and has a side arm containing a filament. By using a beat note generated between the two lasers to trigger a feedback signal to the tuning-tube filament, the frequency difference between the two lasers can be kept to within 10 kilohertz for periods of 2 or 3 minutes. This method, of course, has one less frequency with which to determine plasma properties.

The stability problem is not, in general, very severe for the three-mirror system, because a pulsed plasma is required for the correct interpretation of data (i.e., zero phase shift before and after plasma pulse). In the heterodyne system, however, a pulsed plasma is not required, and any frequency stabilizing method is desirable.

CONCLUDING REMARKS

Gas laser interferometry offers a large range of applicability in the diagnostics of plasmas in which electron densities between 10^{16} and 10^{25} per cubic meter are expected. Because of its multifrequency characteristics, it can also probe for unexcited and excited neutral particle densities.

Although in inert gas plasma these particles, in general, contribute little to the plasma refractive index, it has been shown that, in cesium plasmas, they can dominate the refractive index over the effect of the free electrons for certain plasma conditions frequently encountered.

It is a fact that the laser interferometer methods measure only particle densities, however, other parameters, such as electron temperature, can be computed from the density measurements; or the methods can be applied to complement other diagnostic techniques to completely define plasma properties.

In plasmas where spatial resolution is important, such as exist in thermionic diodes, the laser interferometer appears applicable. It offers resolution comparable to the best known spatially resolved technique (emission spectroscopy) but covers a much broader range of electron densities.

Even though its frequency stability problems have yet to be completely solved, it is believed that gas laser interferometry deserves consideration as a primary plasma diagnostic tool.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 15, 1968,
129-02-01-05-22.

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