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CONTROL SYSTEM EQUATIONS

Prepared by: J. L. Palmer

Approved by:

P. C. Wheeler

P. C. Wheeler
Project Manager

Approved by:

L. K. Jensen

L. K. Jensen
Manager, Control Systems
Laboratory

Contract No. NAS 5-10189

Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
Greenbelt, Md.

TRW SYSTEMS

ONE SPACE PARK • REDONDO BEACH • CALIFORNIA 90278

ABSTRACT

This document is one in a series covering the development and use of the Generalized Spacecraft Simulation (GSS). The formulation of the basic equations used in programming the simulation is presented in two volumes. Volume I contains the derivation of a set of nonlinear, large-angle dynamics equations for an N-body system, including gravity gradient and disturbance torques and forces. Volume II defines and presents the equations for a generalized active control system.

The basic spacecraft configuration consists of N connected bodies. Body 1 is considered the main spacecraft structure wherein the orbital experiments are housed. Each of the N-1 auxiliary bodies is assumed to be attached to Body 1 at an arbitrary single point, to have up to three degrees of rotational freedom relative to Body 1, and no interconnection with the other auxiliary bodies. The relative rotation of the auxiliary bodies, with respect to the main body, is assumed to take place about a gimbal hinge. For axes with degrees of freedom, the reaction torques may be zero for a frictionless pin, may be proportional to displacement or velocity to represent springs and dampers, or may be control torques for array drives. In addition, the motion about any gimbal axis is limited by a system of elastic gimbal stops.

The sensor routines provide for two rate gyros, two horizon scanners, and two two-axis sun sensors. Sensor dynamics and/or signal processing is provided for each output. The sensors may be patched to the control systems through saturating lead-lag compensation networks. The control systems consist of three gas jet systems, four reaction wheel systems, and two array drive systems, where each control system is provided with six possible control laws. Reaction wheel speed control and unloading loops are provided.

ACKNOWLEDGEMENT

This document covers only a portion of the continuing development and use of the Generalized Spacecraft Simulation (GSS). Additional documentation of the current version C is contained in References 1 and 2. The primary areas of responsibility were assigned as follows:

Spacecraft Configurations and Basic Equations

Mr. J. L. Palmer

Mathematical Analysis Consultant

Dr. I. Tarnove

Digital Programming

Mr. A. W. Jones

In exercising their responsibilities in connection with the present and previous versions of the program, Dr. Tarnove and Mr. Jones have made numerous suggestions and recommendations which have been incorporated in this present version of the program. The basic requirements for the generalized control system defined in this volume were established by Mr. K. J. McKenna of the TRW Technical Staff.

Grateful acknowledgement is given to the technical staff members for the suggestions and comments resulting from their use of the program, in particular Dr. P. C. Wheeler of TRW Systems and Mr. H. R. Freeman of Goddard Space Flight Center, for their many contributions in adapting the current version for the Nimbus Control Systems Design Studies.

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1. INTRODUCTION

This document is one in a series of documents covering the development and use of the Generalized Spacecraft Simulation (GSS). This simulation is a general purpose digital computer program designed to produce a time history of the motion of a generalized multi-body spacecraft. The program input is provided in general form such that the majority of existing and proposed spacecraft may be simulated in an arbitrary orbit about any preselected body. The output is in both tabular and plotted form. Provisions are made to plot up to 48 of up to 256 preselected variables.

The basic document covering the simulation is the User's Handbook, Reference 1, which contains a complete description of the program and its uses. The present document is a reference document to the User's Handbook and consists of two volumes. Volume I, contains the derivation of a set of non-linear large-angle dynamic equations for an N body system including gravity gradient and disturbance torques and forces. The disturbances considered are due to aerodynamics, solar radiation, the magnetic field, thermal bending, eclipse, and boom deployment. The reaction torques between bodies may result from springs, viscous dampers, stops, magnetic hysteresis dampers, and positioning control torques. Volume II of the present document defines and presents the equations for a generalized active control system. The sensor routine provides for two rate gyros, two horizon scanners, and two two-axis sun sensors complete with sensor dynamics. The sensors may be patched to the control systems through saturating lead-lag compensation networks. The control systems consist of three gas jet systems, four reaction wheel systems, and two array drive systems, where each control system is provided with 6 possible control laws. Reaction wheel speed control and unloading loops are provided.

The digital programming of the simulation is covered in a third document, Reference 2, which contains the program listings and flow diagrams.

1.1 CONTROL FORCES AND TORQUES

The basic spacecraft configuration under investigation consists of N connected bodies. Body 1 is considered the main spacecraft structure wherein the orbital experiments are housed. Each of the $N-1$ auxiliary bodies is assumed to be attached to body 1 at a single point, to have up to three degrees of rotational freedom relative to body 1, and no interconnection with the other auxiliary bodies. The relative positions of the mass elements of each of the N bodies are defined with respect to corresponding sets of body coordinates. It is assumed that the mass elements of the bodies may be in motion with respect to each other and that this motion is defined with respect to the body coordinates. In general the allowable relative motion is used to simulate the effects of flexibility (thermal bending), changes in shape (boom deployment), and momentum changes (reaction wheels). These effects are expressed in the dynamic equations as momentum vectors and time rates of change in momentum defined with respect to the corresponding body coordinates. In addition the dynamic equations, presented in Volume I, provide for the application of an external control forces and torques on the main spacecraft body and for internal control torques between the bodies.

The external control force on body 1 is defined as $\bar{F}_{C1}$ applied at the body 1 center of mass, and the external control moment on body 1 is defined as $\bar{T}_{C1}$, Equations (6.1-1) and (6.1-2) of Volume I. They are expressed in the dynamic equations as components $\{F_{C1i}\}$ and $\{T_{C1i}\}$ defined with respect to the body 1 coordinate system with its origin at the body center of mass.

The internal control torques between the main body (1) and auxiliary body i ($2 \leq i \leq N$) are added to the gimbal reaction torques for body i , Equation (7.3-1) of Volume I. They are expressed in the dynamic equations by the 3×1 column matrix $\{T_{Li}\}$ where

$$\{T_{Li}\} = \begin{pmatrix} T_{Lix} \\ T_{Liy} \\ T_{Liz} \end{pmatrix} \quad (1.1-1)$$

The elements of $\{T_{Li}\}$ represent magnitudes of three non-orthogonal torques about the three body i gimbal axes and are not orthogonal components of a resultant torque vector. The elements can be non-zero only for those gimbal axes which have been given a degree of freedom. It should be noted that the central torques $\{T_{Li}\}$ have the same sense as the locked gimbal reaction torques, a positive torque transmitted from body i to the main body. As a result a positive torque T_{Lix} at the x gimbal axis of body i represents a positive gimbal torque on body 1 and a negative gimbal torque on body i .

The reaction wheel momentum components $\{H_{Wii}\}$ and their time rate of change $\{\dot{H}_{Wii}\}$ are added to the corresponding body i momentum, Equations (6.2-1) and (6.2-2) of Volume I. They are expressed in the dynamic equations as components along the body i coordinates ($1 \leq i \leq N$). It should be noted that the components of momentum and rate of change in momentum apply to the reaction wheels. The control torque on body i is the negative of the wheel momentum time rates of change, and the negative sign is included in the dynamic equations.

The control system output parameters transmitted to the dynamic equations are summarized in Table 1-1. In the absence of a control system the $9N+6$ possible control output parameters listed in Table 1-1 are set to zero. The function of the control subroutines is to define expressions for these parameters.

TABLE 1-1
Control Input to Dynamics

<u>Control Output Parameter</u>	<u>Coordinate System</u>	<u>Applicable Body</u>	<u>Description</u>
$\{F_{Cl1}\}$	Body 1	1	Main body external control force components
$\{T_{Cl1}\}$	Body 1	1	Main body external control moment components
$\{T_{Li}\}$	Body i	$(2 \leq i \leq N)$	Gimbal control torques between body i and the main body
$\{H_{Wii}\}$	Body i	$(1 \leq i \leq N)$	Momentum components of body i reaction wheels
$\{\dot{H}_{Wii}\}$	Body i	$(1 \leq i \leq N)$	Acceleration components of body i reaction wheels

2. METHOD

The ultimate objective of this derivation is to provide a set of generalized control system equations to evaluate the control output parameter listed in Table 1-1. The general control system being considered will contain discontinuities, closed loop feedback circuits, lead-lag networks, and reaction wheel unloading circuits. The equations and logic required to represent these effects are sufficiently complexed to warrant an initial discussion of the general approach to be utilized.

2.1 DIGITAL INTEGRATION

In order to discuss the operation of the control system, it will first be necessary to discuss the basic operation of the digital integration package used to solve the dynamic equations. The integration is performed by a Adams-Moulton, Runge-Kutta digital subroutine (AMRK) using a fourth order predictor corrector to control the integration interval. Basically the integration is performed by fitting a curve to the present and past values of each variable and integrating by numerical methods. As a result the effect of continuous varying parameters is obtained using step by step integration. The program normally operates in the Adams-Moulton mode using present and past values to perform the integration and the input error limits to adjust the integration interval, see Section 3.3 of Reference 3. At the start of integration, when past values are not available, the program operates in the Runge-Kutta mode, see Section 3.4 of Reference 3.

The digital integration package (AMRK) has an associated subroutine called the derivative evaluator (VDØT) which must be capable of computing the control output parameters of Table 1-1 as a function of time. As a result, the information supplied to (VDØT) by the control system does not consist of unique values for a given time but must be in the form of expressions for force, torques, and momenta as functions of time.

The important features of the integration system which affect the operation of the control system are summarized below:

1. If h is the integration interval or time step, the program time changes from (t) to $(t + h)$ as the program passes through the integration package (AMRK).
2. In order to prevent past values from being carried across discontinuities, the integration must be restarted in the Runge-Kutta mode at each discontinuity in magnitude. In general the integration is not restarted at discontinuities in the slope of the control parameters. It is assumed that in the Adams-Moulton mode, the integration package can fit a curve through the resulting slope discontinuity in the body acceleration.
3. The function of the control system is to provide, at time t , an expression for the variation in each of the control parameters over the time interval between (t) and $(t + h)$.

2.2 CONTROL SYSTEM OPERATION

The integration package (AMRK) is used only to integrate the body accelerations and velocities, solving the dynamic equations of motion. The integrations required within the control systems are performed by a separate simple method. A cubic polynomial curve fit is maintained for each of the variables in the control system. The present and three past values of each variable are stored and a special curve fit subroutine computes and stores the four polynomial coefficients together with the present time rate of change of the variable. The output for each element of the control system can be computed in closed form in terms of the polynomial coefficients and rate of change of the assumed cubic input signal. As a result, the output of each element is expressed as a function of time and the control system outputs, listed in Table 1-1, are in the correct form to be accepted by the (VDØT) subroutine of the integration package (AMRK).

The integration package (AMRK) is designed to operate only over continuous portions of the dynamic equations and the integration must be restarted at each discontinuity. Therefore, the integration step or interval must be adjusted such that an interval ends at the time of each discontinuity. The time rate of change of each variable, computed at each curve fit, is used to predict the time of each discontinuity. The detailed logic required to predict discontinuities and restart the integration are presented in Reference 2. Two of these requirements are of interest in understanding the operation of the control system and are repeated here for reference. (1) The discontinuities cannot be predicted exactly and an acceptable tolerance must be placed upon the time at which discontinuities occur. This tolerance is an input parameter designated as (HPERR). (2) After a discontinuity has been reached and incorporated into the control parameters and before the integration is restarted, the program must make an extra pass through (VDØT) in order to compute new initial conditions.

A simplified diagram of the program is shown in Figure 2.2-1. Starting at the output of the control system at time t , the control system has determined the control output parameters of Table 1-1 as functions of time. The expressions for these functions are passed to the (VDØT) routine for use in computing the variation in control outputs during the time interval between (t) and $(t + h)$. The program control or flow of information continues through the rest of the program, back to the top and enters the integration package (AMRK) still at time t . (AMRK) determines the integration step (h) and integrates the dynamic equations. When the program control leaves (AMRK) the time is $(t + h)$ and all of the position and velocity data in the program has been updated for time $(t + h)$. In addition the control system output expressions in (VDØT) are used to compute and store the control parameters at time $(t + h)$. Since these values are based upon a projection of a curve fit made at time t , they are defined as the predicted values at time $(t + h)$. After passing through the other subroutines of the program the program control enters the control system at time $(t+h)$. Based upon the updated position and velocity data, the control system computes values of the

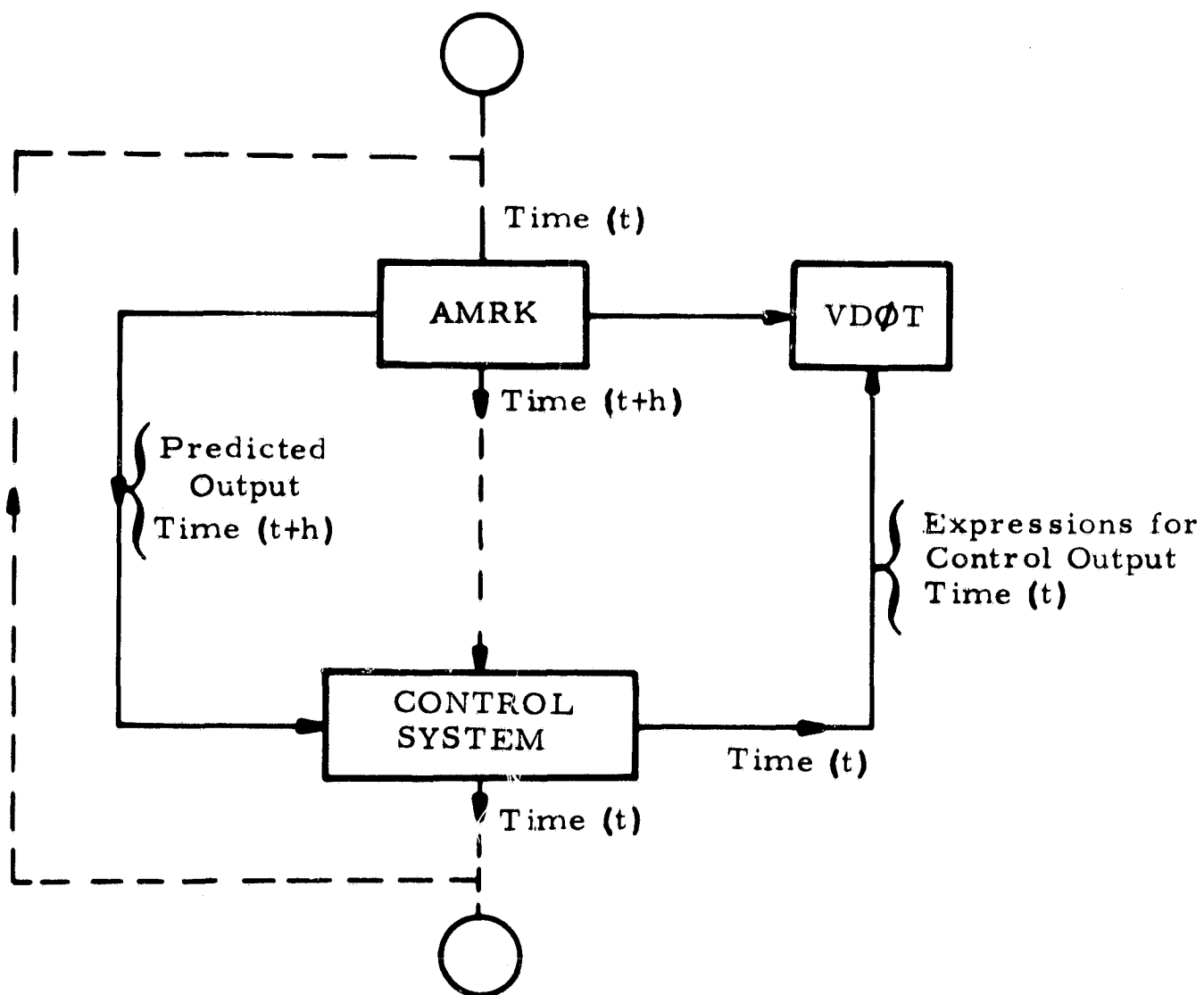
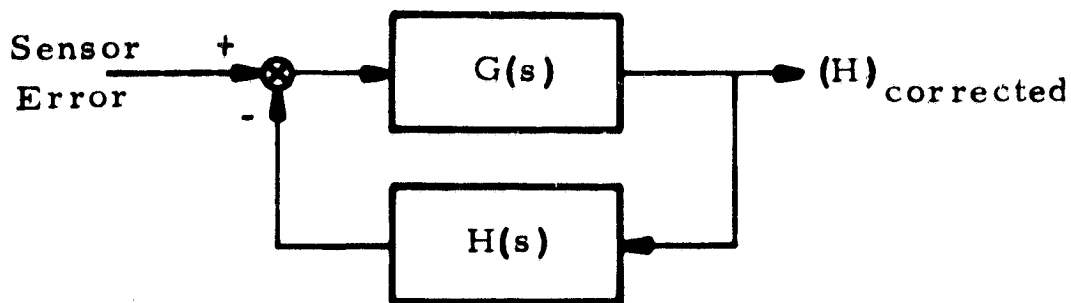


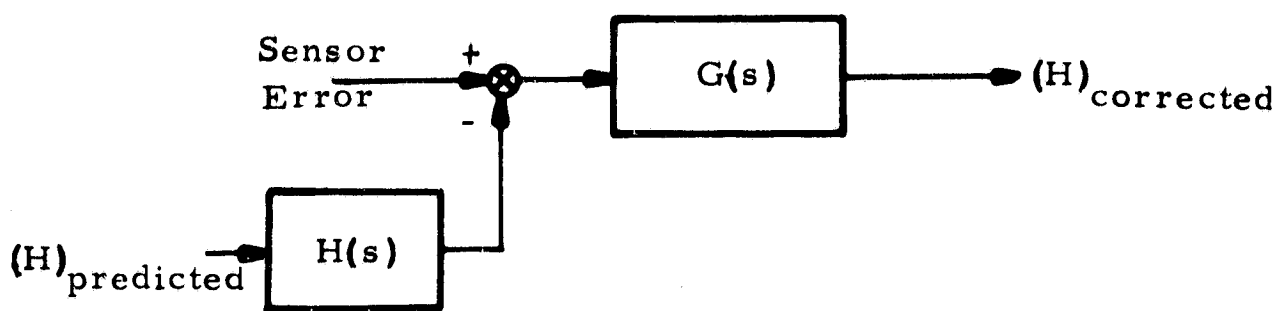
Figure 2.2-1

Control Parameter Time Relationship

control output parameters which are defined as the corrected values. The predicted parameters are then updated by the corrected values and new output expressions based upon a new curve fit at time $(t + h)$ are passed to (VDO T) and the process continues.



(a)



(b)

Figure 2.2-2
Simulation of Feedback Loops

The differences between the predicted and corrected outputs at time $(t + h)$ result from the fact that the predicted values are obtained from a projection of the curve fits at time t , while the corrected values are the actual outputs computed at time $(t + h)$. The differences are then an indication of the accuracy of the curve fits.

The output functions must be in one of two basic forms. For bang-bang type controllers or when linear controllers are saturated or in their deadzone, the output is a linear function (a constant or the integral of a constant). When the linear controller is in the linear region, the output has the same form as the complex input error signal. In the case of linear function the curve fit is exact and the predicted value is equal to the corrected value and need not be updated. For operation in the linear regions of the controllers the curve fit is not exact and the correction must be made. It can be seen that at one point in the program, both the predicted and corrected values are available. Although no use is made of this fact in the program, the difference between the two values is an indication of the accuracy of the curve fits. The important point to be made is the fact that as the program enters the control system a sufficiently accurate prediction of the output is available which greatly simplifies the solution of the closed loop feedback circuits. Consider the system shown in Figure 2.2-2 represented a reaction wheel system with a speed control loop. The system shown in (a) must be solved by iteration while the system shown in (b) is solved in a single pass.

3. DIGITAL SIMULATION OF TRANSFER FUNCTIONS

In order to obtain the output of the control system elements representing lead and/or lag networks, some form of digital integration and differentiation is required. The method which has been selected for this program consists of fitting a polynomial to the input signal and programming the output in terms of the polynomial coefficients, see Section 2.2. The method was verified by writing a special digital program using polynomials from first to fourth order. For the transfer functions being considered, the third degree or cubic polynomial was selected as the best compromise. In general, the linear curve showed the fastest recovery from transients, while the higher degree polynomials produce better steady state accuracy. Typical sets of output curves are shown in Figures (3.0-1) through (3.0-8). The first four figures (3.0-1) through (3.0-4) give the response of a lag network to a step function. As long as the time step or integration interval is less than one half the time constant, there is little difference in the output for the linear, second degree, and third degree polynomials. The output for the fourth degree polynomial tends to oscillate and overshoot. In general, the accuracy improves as the step size h is decreased, however, the required accuracy is a function of the time that the output is used. The second four figures (3.0-5) through (3.0-8) give the response of a lead network to the start of a sine function, where the input is zero before time zero. Figure (3.0-5) shows the smooth starting transient and poor steady state accuracy of the linear curve fit. The remaining figures show the degrading of the transient response and the improved steady state accuracy produced by the higher order curves. In general, the steady state response improves with decreased time step and the transient response improves with increased time step. The transient response is degraded by an increase the ratio of β/P where P is the period of the input signal in seconds. The eight curves included here represent the most common case, the step function into a lag, and the case with the poorest characteristics, the transient into a lead. The curves clearly show the advantages of the second and

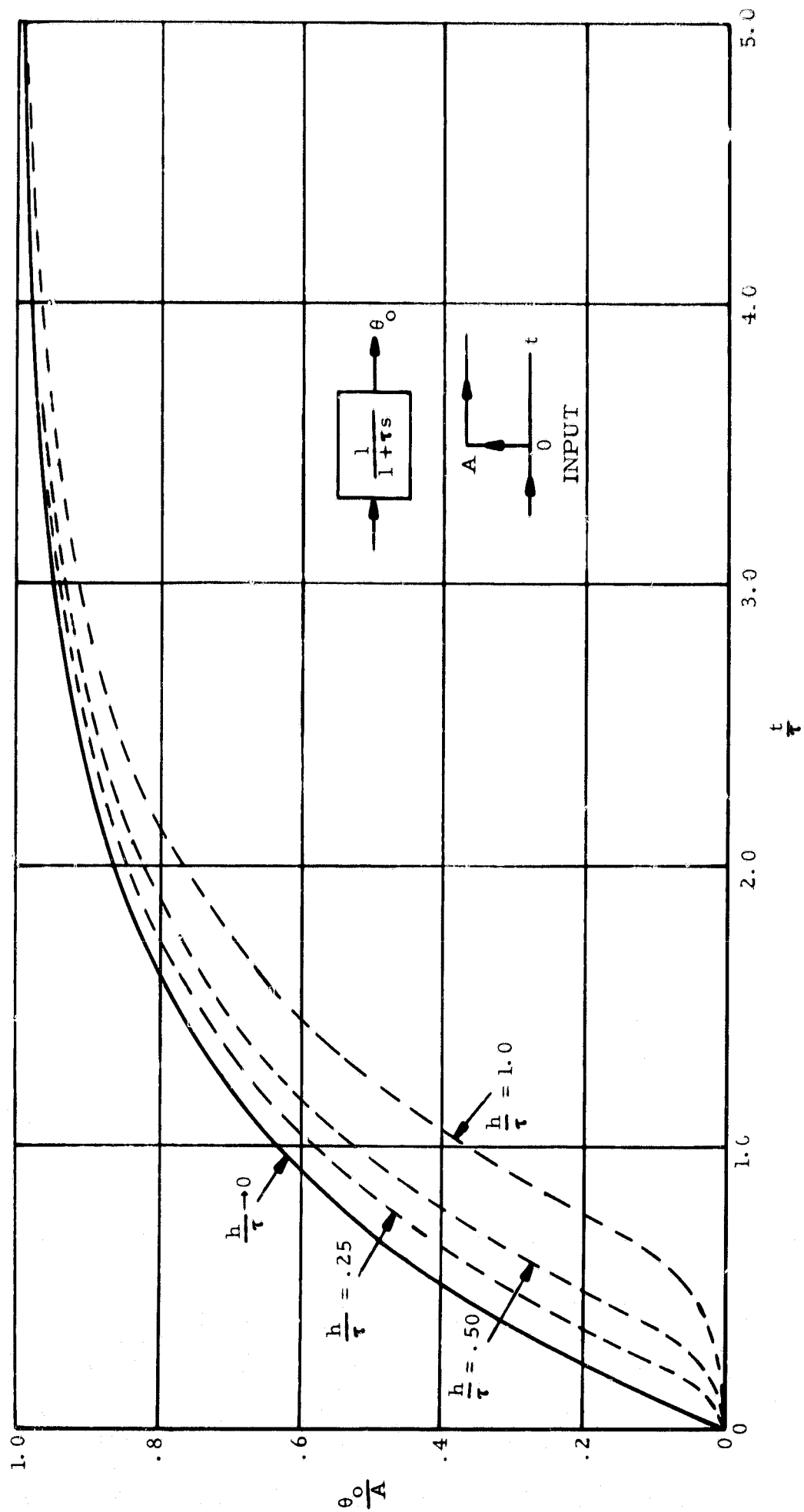


Figure 3.0-1
Response to a Step
Linear Curve Fit

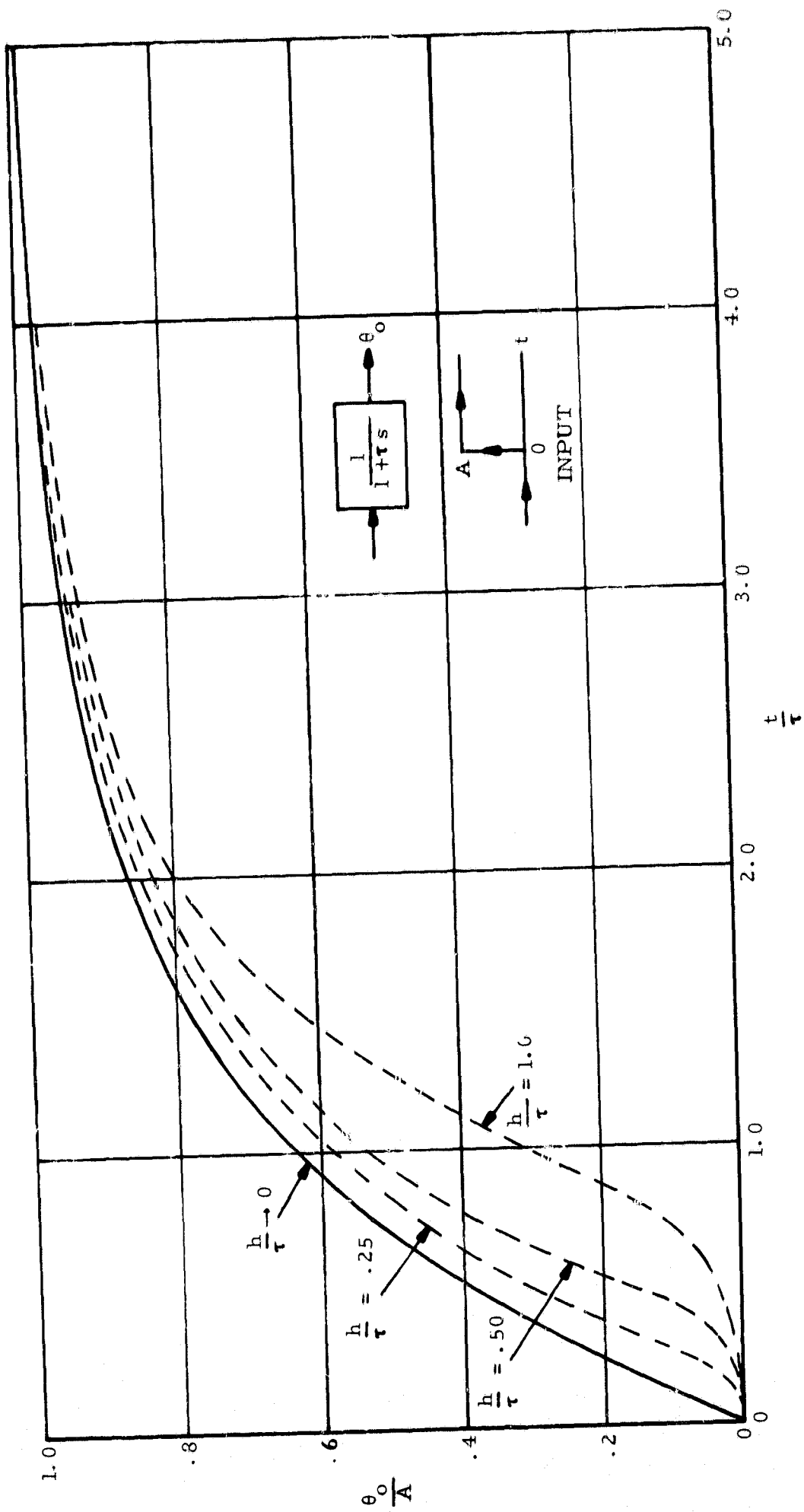


Figure 3.0-2
Response to a Step
Second Order Curve Fit

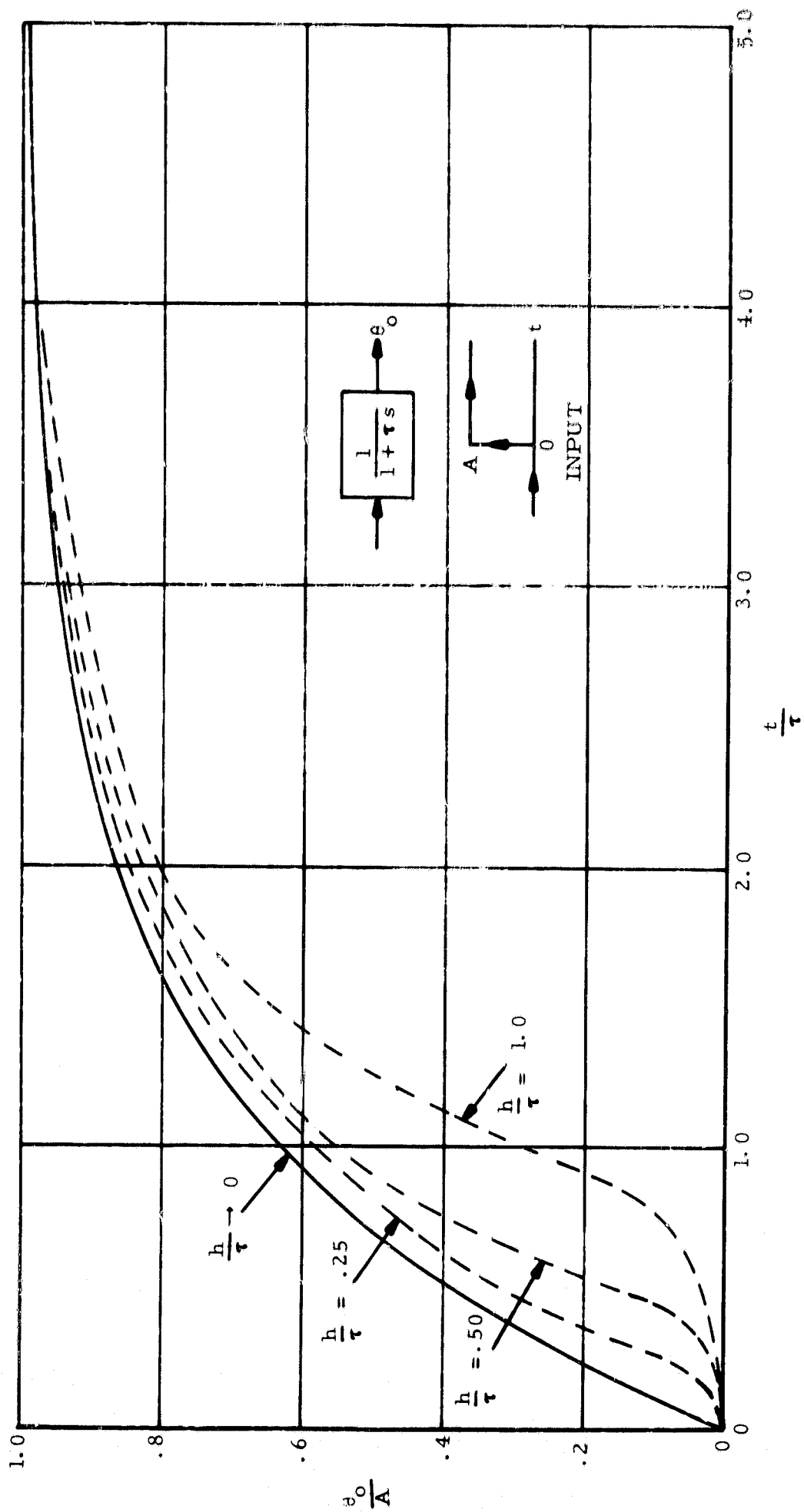


Figure 3.0-3
Response to a Step
Third Order Curve Fit

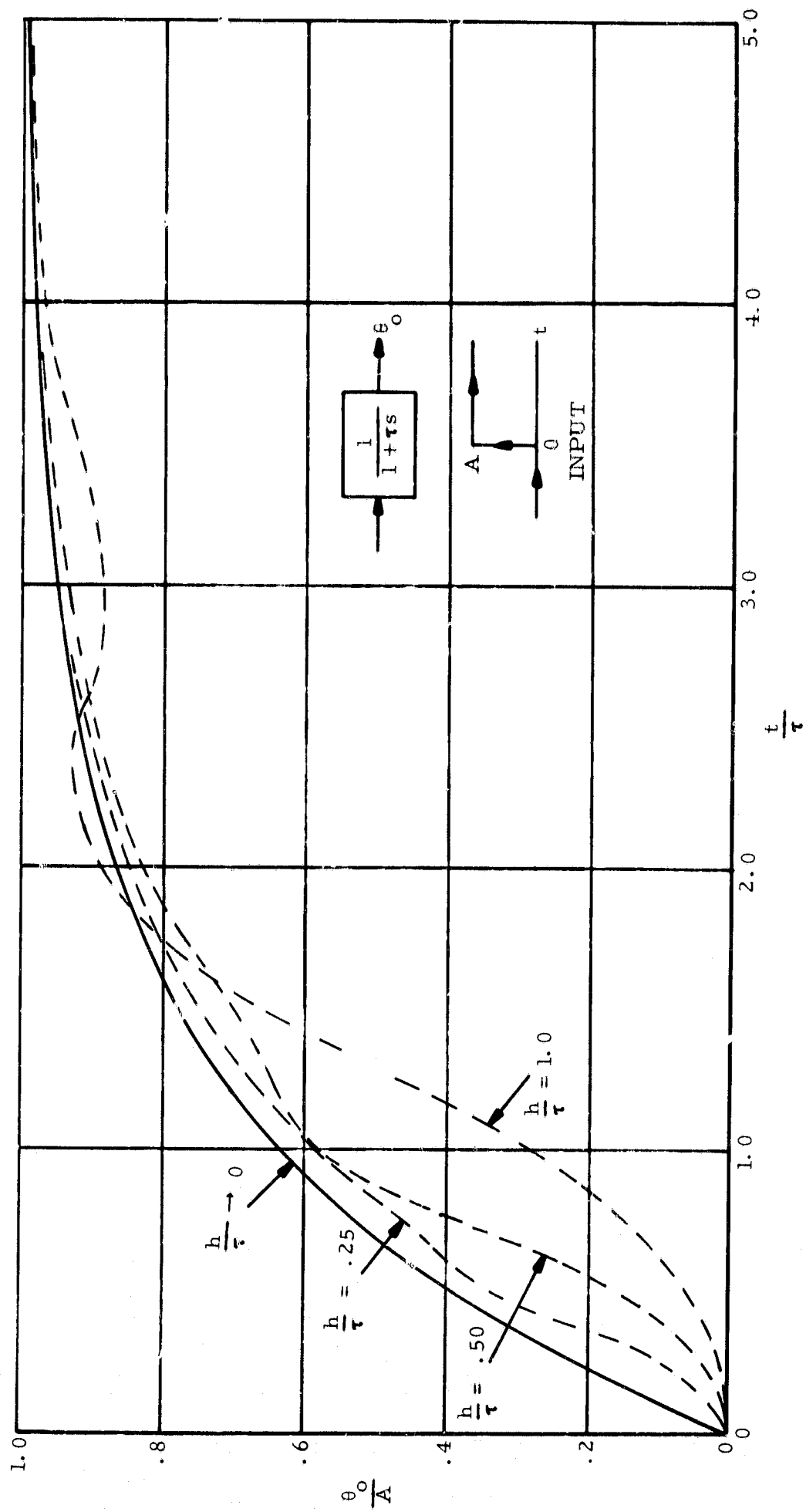


Figure 3.0-4
Response to a Step
Fourth Order Curve Fit

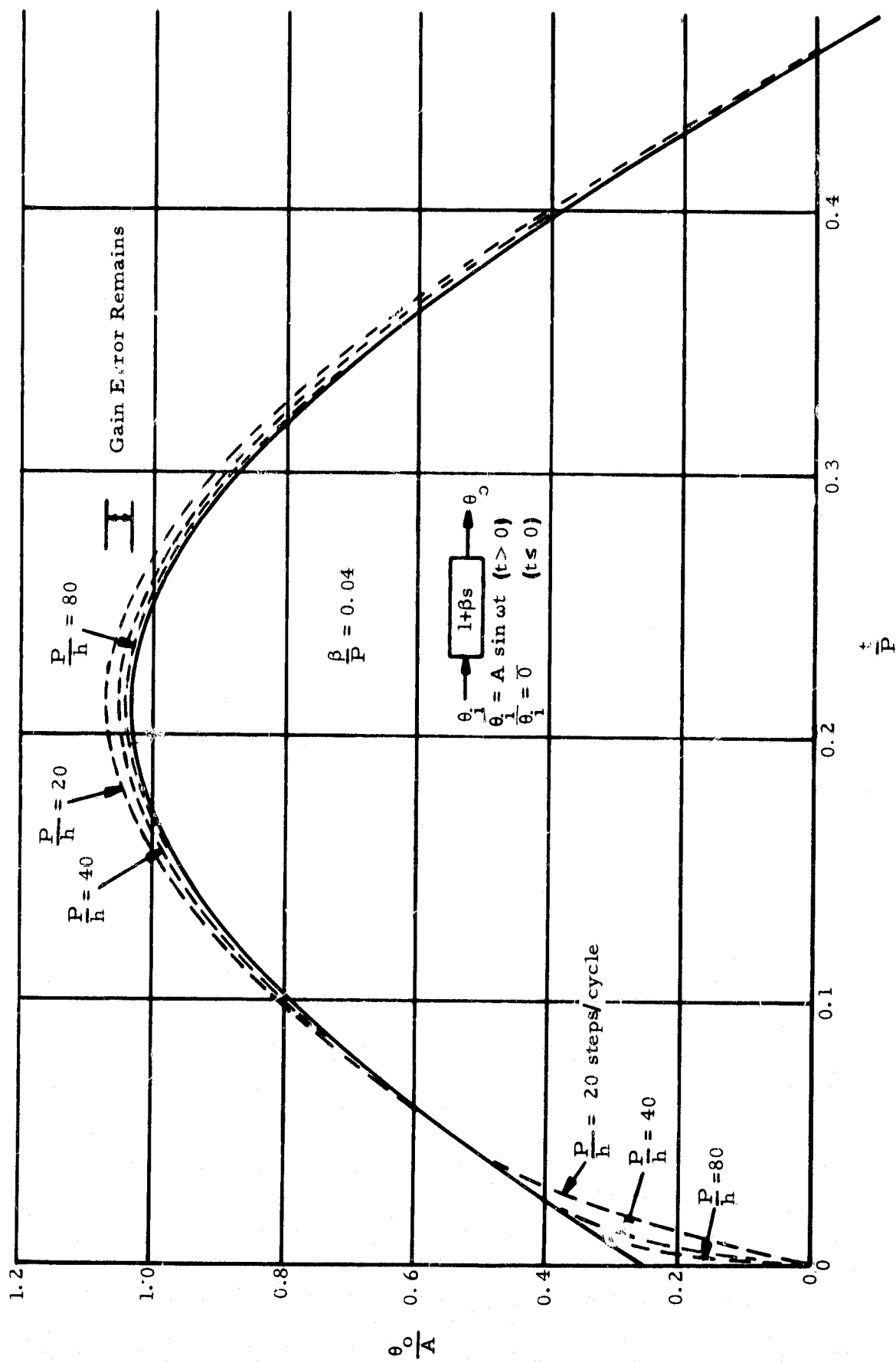


Figure 3.0-5
Response to a Sine Function
Linear Curve Fit

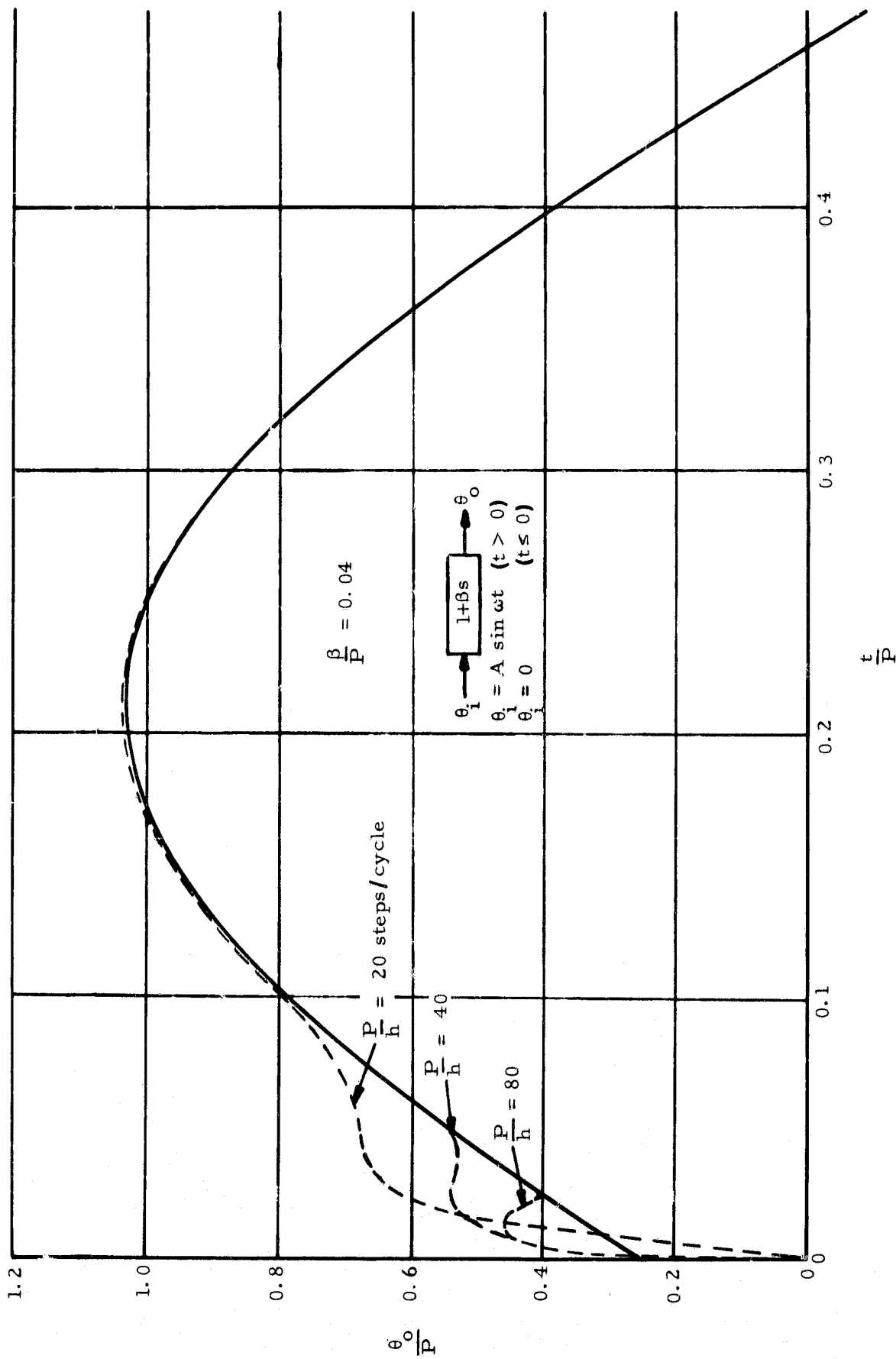


Figure 3.0-6
Response to a Sine Function
Second Order Curve Fit

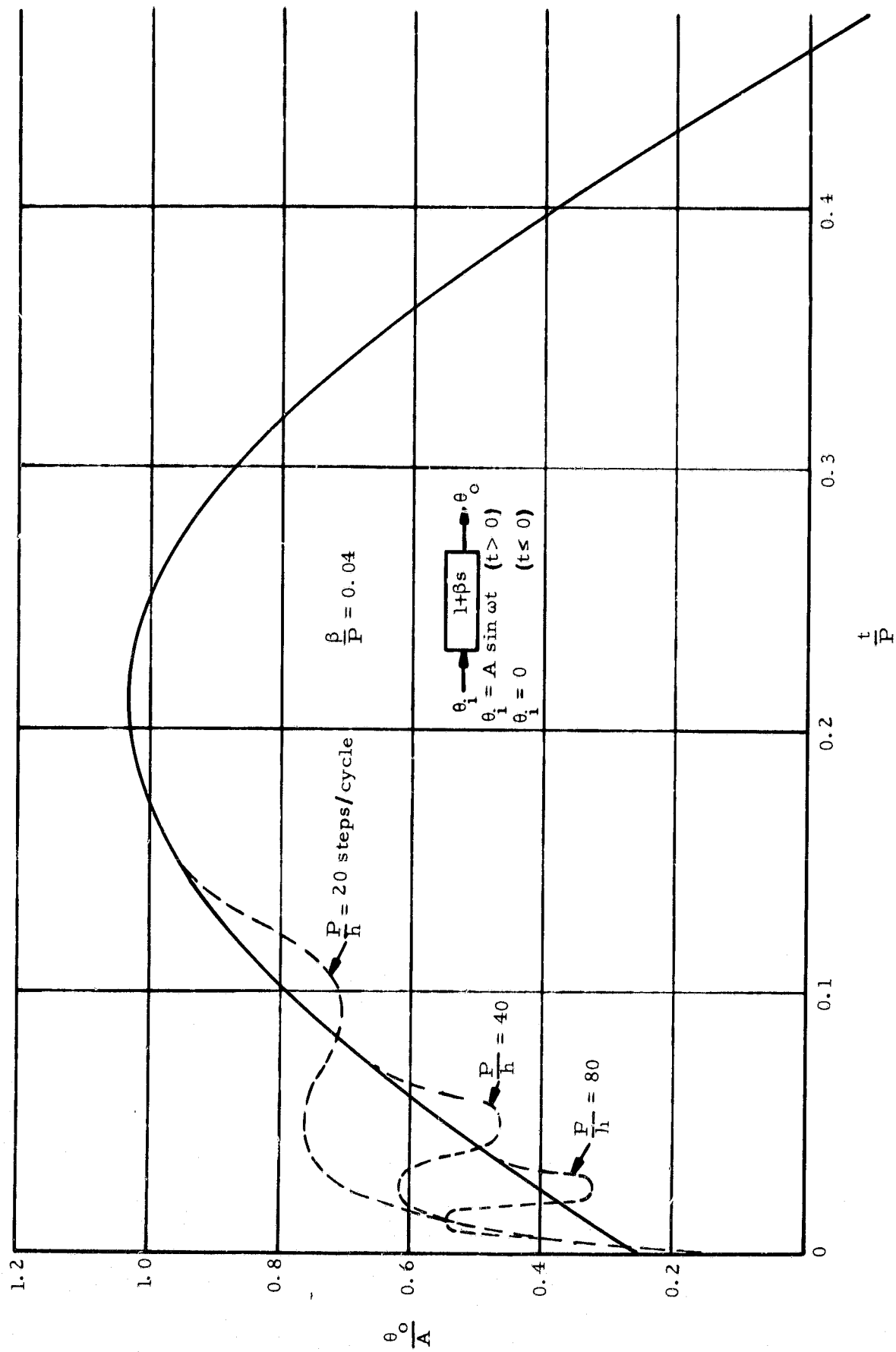


Figure 3.0-7
Response to a Sine Function
Third Order Curve Fit

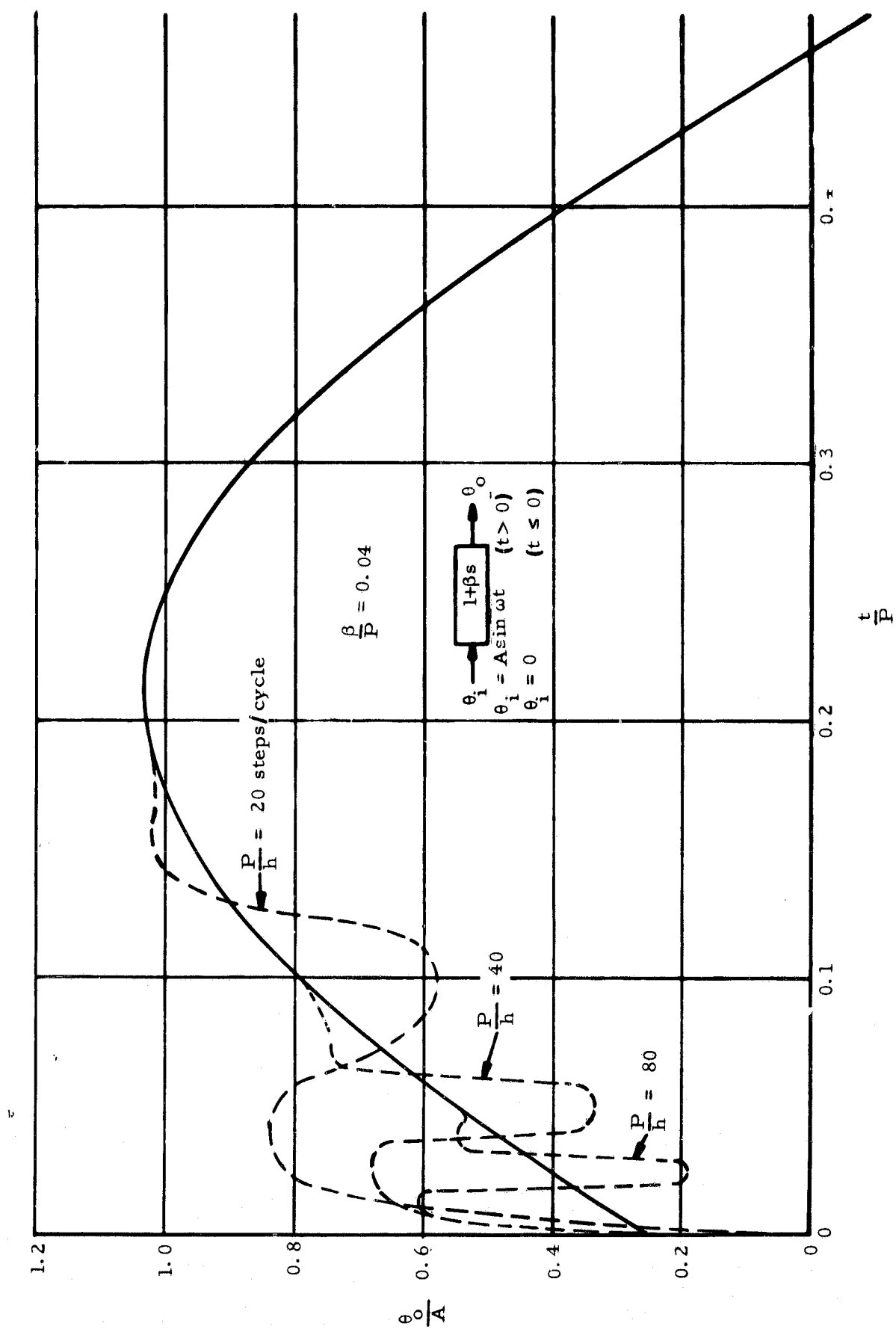


Figure 3.0-8
Response to a Sine Function
Fourth Order Curve Fit

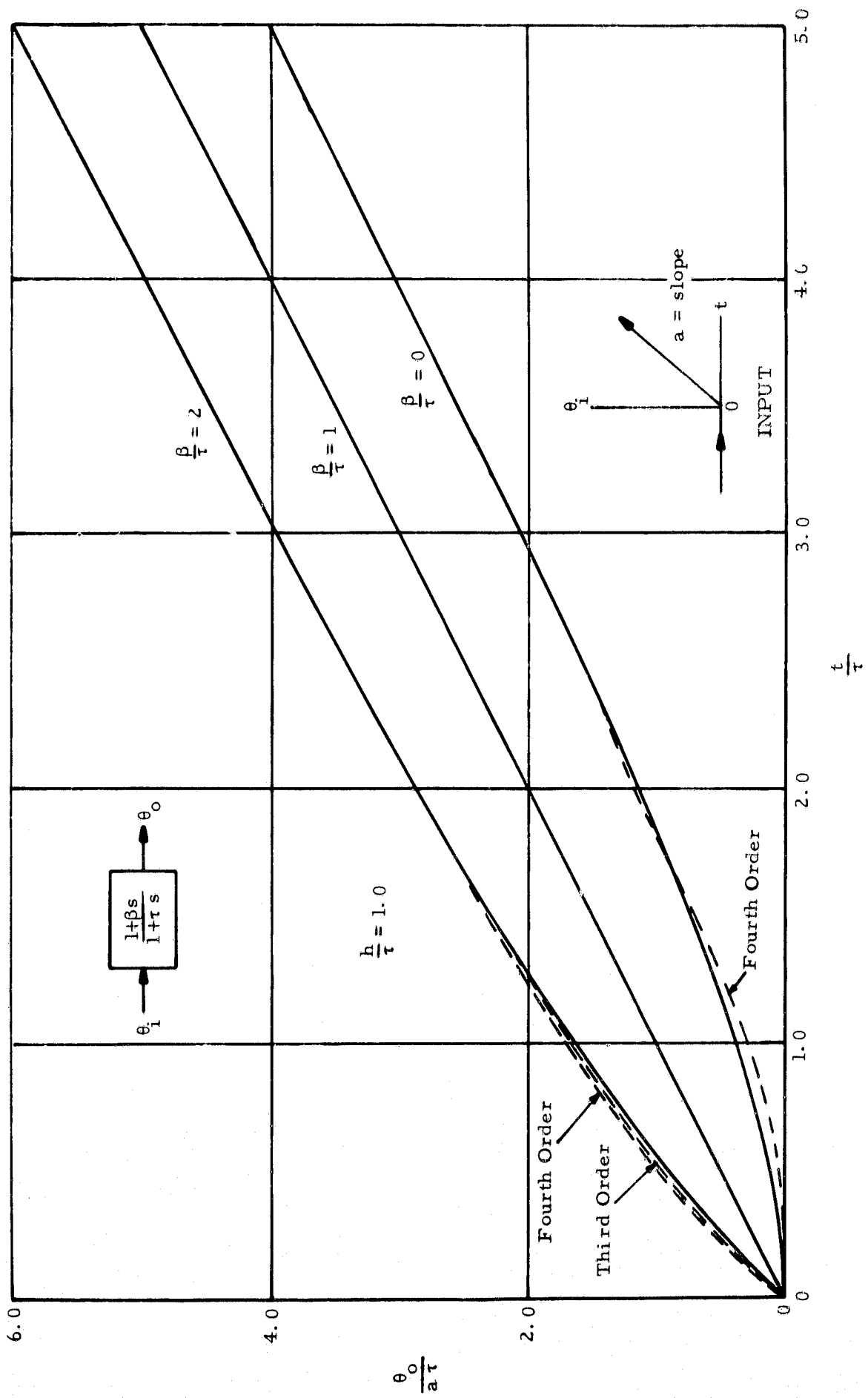


Figure 3.0-9
Response to a Ramp

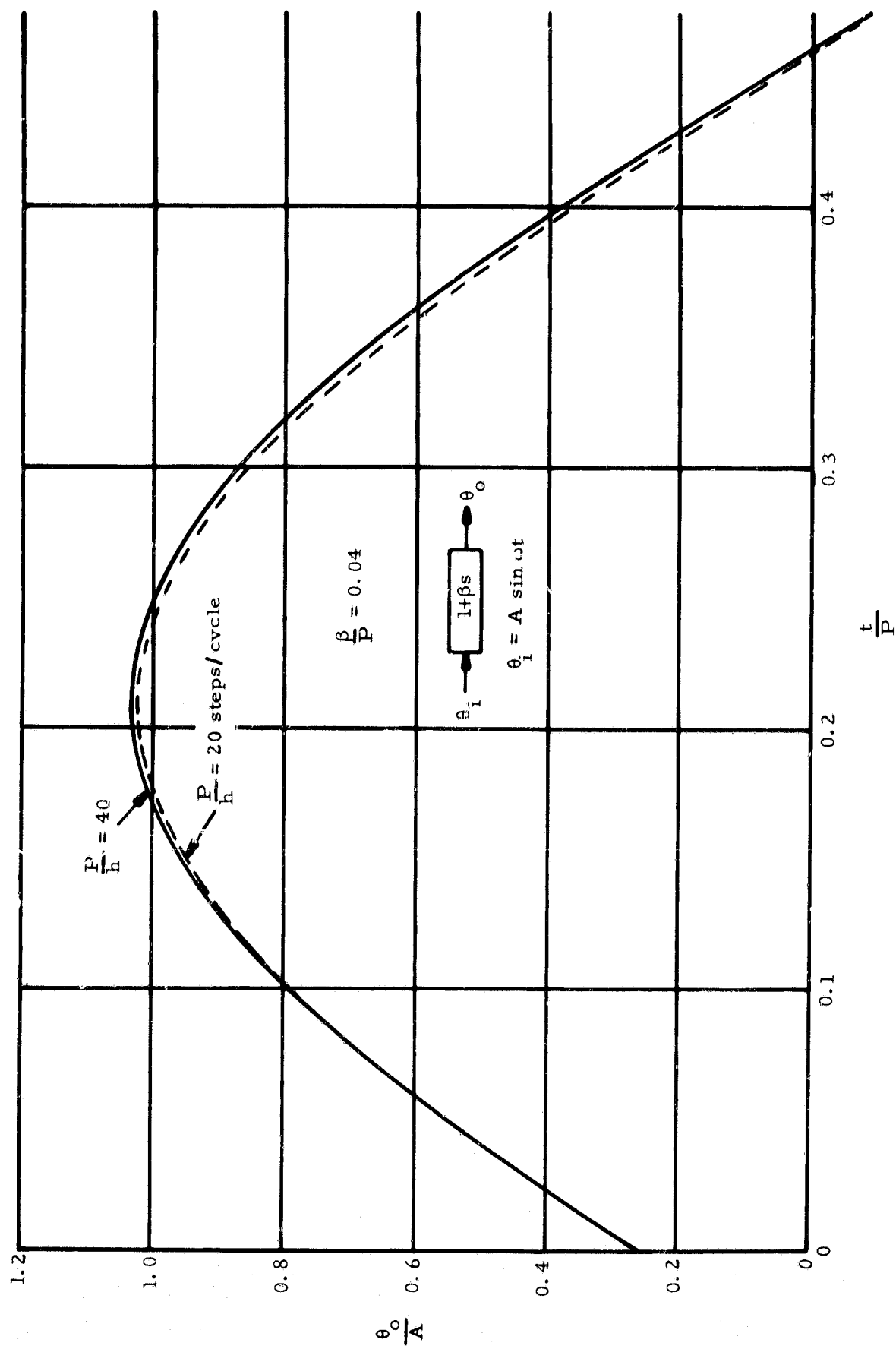


Figure 3.0-10
 Response to a Sine Function
 Third Order Curve Fit

third degree curves over the linear curve for steady state conditions and over the fourth degree curve for transient conditions. The relative merits of the second and third degree curves are not as clearly indicated. Combination lead-lag networks tend to show the characteristics of the pure lag when τ is greater than β and the characteristics of the pure lead when β is greater than τ . The output of a lead-lag network for a ramp input is shown in Figure (3.0-9). As would be expected there is no error for the linear fit. The errors due to the second degree fit are too small to be plotted and the errors due to the cubic fit are detected only for lead networks. The fourth degree fit again shows a tendency to oscillate. The output of a lead network for a continuous sinusoidal input and a cubic curve fit is shown in Figure (3.0-10). The selection of the cubic polynomial for use in the program was based upon the following considerations.

1. With the exception of the start of the program, the principal transients of the system are the on and off switching of the modulations, and the saturation of limiters. The modulator switching represents a step function into a pure lag for which the accuracy is a function of step size and is relatively independent at the type of curve fit, Figure (3.0-1). In the case of limiter saturation the output is set at the limiting value and the curve fit must follow the unlimited signal in order to predict the point at which the limiter becomes unsaturated.
2. The cubic fit provides the greatest steady state accuracies.
3. The majority of the transfer function represent lags in the system. Lead-lag functions are used only in the eight compensation network which receive continuous signals from the sensors.

3.1 CUBIC CURVE FIT PARAMETERS

Let θ_1 be the general input parameter for which a third degree polynomial curve fit is required. The curve is defined with respect to the variable p , equal to zero at the first past value at time t , Figure (3.1-1).

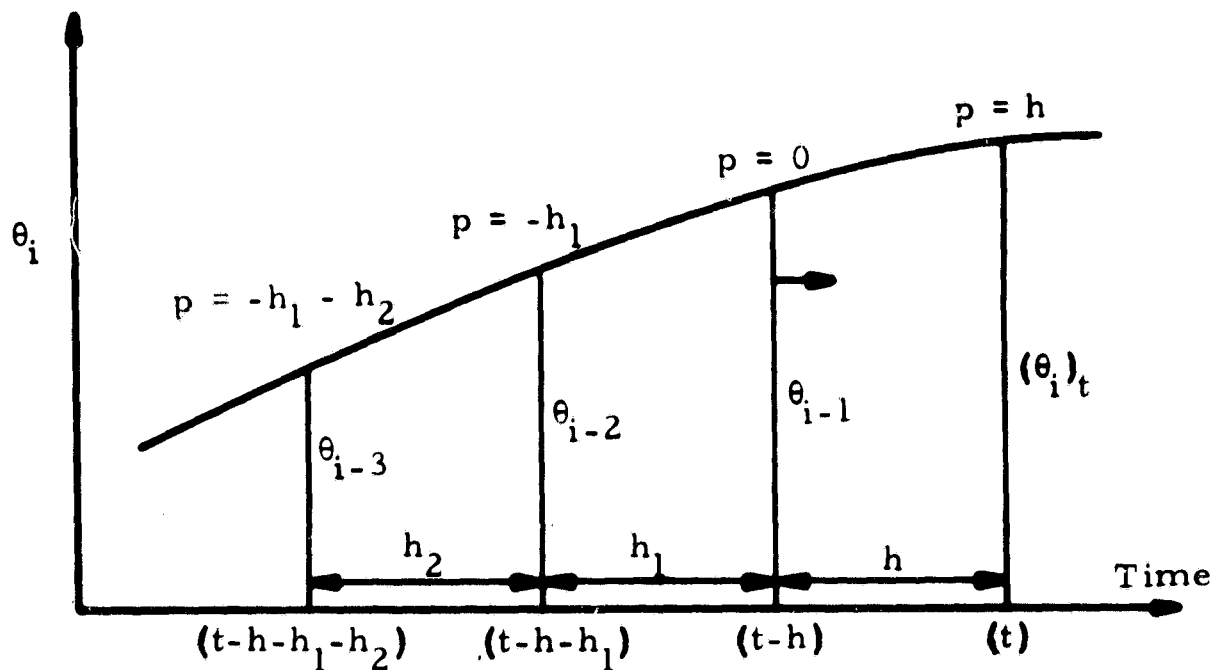


Figure 3.1-1
Cubic Curve Fit Parameters

The input parameters θ_i and the input slope $\dot{\theta}_i$ are given by

$$\theta_i = A_1 + A_2 p + A_3 p^2 + A_4 p^3 \quad (3.1-1)$$

$$\dot{\theta}_i = A_2 + 2A_3 p + 3A_4 p^2 \quad (3.1-2)$$

At time t , let h , h_1 , and h_2 be the three past values of the time step and θ_{i-1} , θ_{i-2} , and θ_{i-3} be the three past values of the input parameter, Figure 3.1-1. Then

$$(\theta_i)_t = A_1 + A_2 h + A_3 h^2 + A_4 h^3 \quad (3.1-3)$$

$$(\theta_{i-1}) = A_1 \quad (3.1-4)$$

$$(\theta_{i-2}) = A_1 - A_2 h_1 + A_3 h_1^2 - A_4 h_1^3 \quad (3.1-5)$$

$$(\theta_{i-3}) = A_1 - A_2 (h_1 + h_2) + A_3 (h_1 + h_2)^2 - A_4 (h_1 + h_2)^3 \quad (3.1-6)$$

Solving Equations (3.1-3) through (3.1-6) for the A's

$$A_1 = \theta_{i-1} \quad (3.1-7)$$

$$A_2 = \frac{h_1^2 h_2 (h_1 + h_2)^2 [\theta_i - \theta_{i-1}] - h^2 (h_1 + h_2)^2 (h + h_1 + h_2) [\theta_{i-2} - \theta_{i-1}] + h^2 h_1^2 (h + h_1) [\theta_{i-3} - \theta_{i-1}]}{h h_1 h_2 (h + h_1) (h_1 + h_2) (h + h_1 + h_2)} \quad (3.1-8)$$

$$A_3 = \frac{h_1 h_2 (h_1 + h_2) (2h_1 + h_2) [\theta_i - \theta_{i-1}] + h (h_1 + h_2) [(h_1 + h_2)^2 - h^2] [\theta_{i-2} - \theta_{i-1}] + h h_1 (h + h_1) (h - h_1) [\theta_{i-3} - \theta_{i-1}]}{h h_1 h_2 (h + h_1) (h_1 + h_2) (h + h_1 + h_2)} \quad (3.1-9)$$

$$A_4 = \frac{h_1 h_2 (h_1 + h_2) [\theta_i - \theta_{i-1}] + h (h_1 + h_2) (h + h_1 + h_2) [\theta_{i-2} - \theta_{i-1}] - h h_1 (h + h_1) [\theta_{i-3} - \theta_{i-1}]}{h h_1 h_2 (h + h_1) (h_1 + h_2) (h + h_1 + h_2)} \quad (3.1-10)$$

The time rate of change of the input parameter at time t is

$$(\theta_i)_t = A_2 + 2A_3 h + 3A_4 h^2 \quad (3.1-11)$$

The function of the curve fit subroutine is to compute and store A_1 , A_2 , A_3 , A_4 , and $(\dot{\theta}_i)_t$ at time t. This process will be referred to as "computing the A's".

If the curve fit is not initialized at the start of the program the present and three past values of each input signal and the three past time steps will all be zero. As a result, the polynomial representation of the input parameters will remain zero until the fourth integration step. The time steps are initialized by storing the initial time step as the three past values. The present value of the input parameter is obtained by passing through the control system before the first time step and computing θ_i present corresponding to the initial input positional data. The critical initial signal would then be a cosine curve for which the initial slope computed from a large present value and past values of zero would be large as shown by curve (1) in Figure (3.1-2).

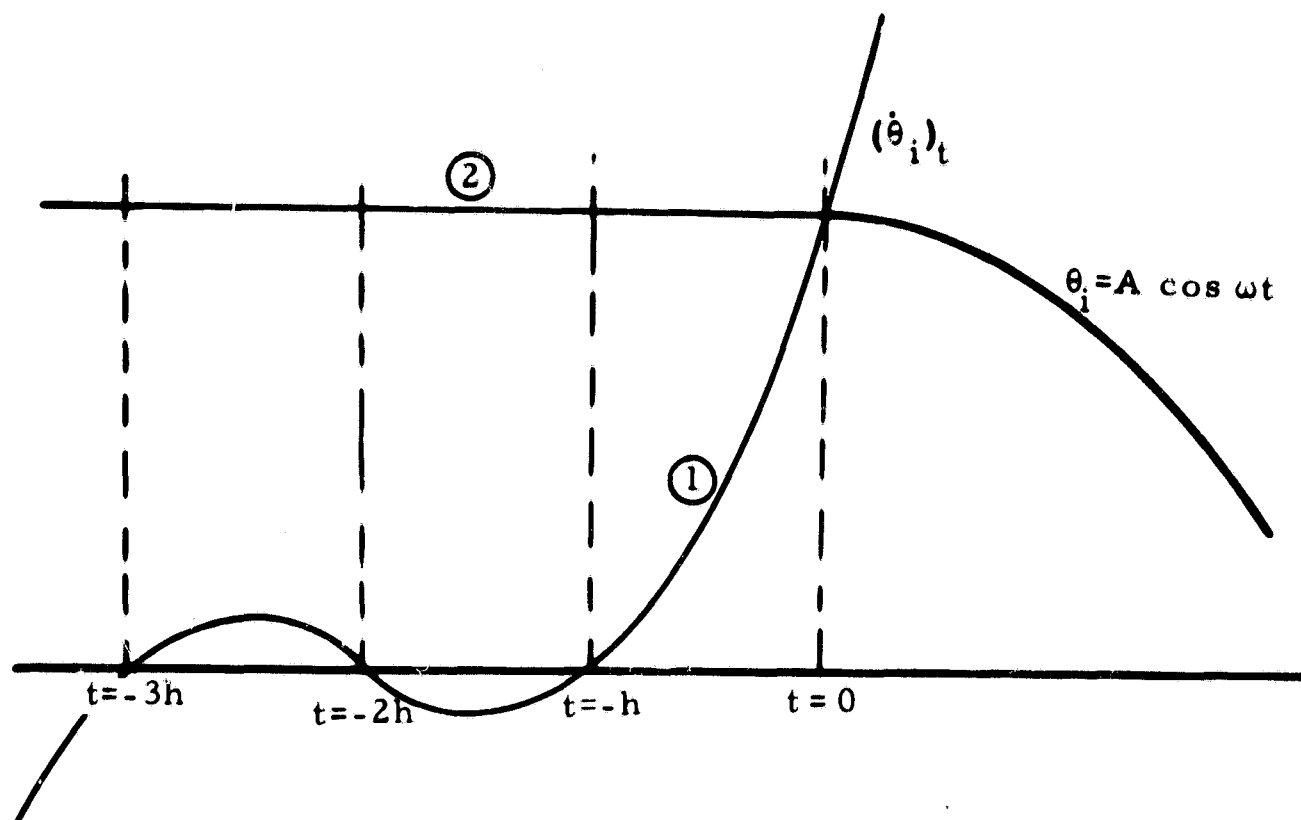


Figure 3.1-2
Initial Conditions for Cosine Function

However, if the initial value of θ_i is stored in all of the past value locations, the initial conditions are represented by curve (2) in Figure (3.1-2) and the initial slope will always be zero. The critical initial signal will then be a sine curve as shown in Figure (3.1-3). The program can more easily recover from the zero initial slope condition and since the initial time steps are always small, the error is assumed to be negligible.

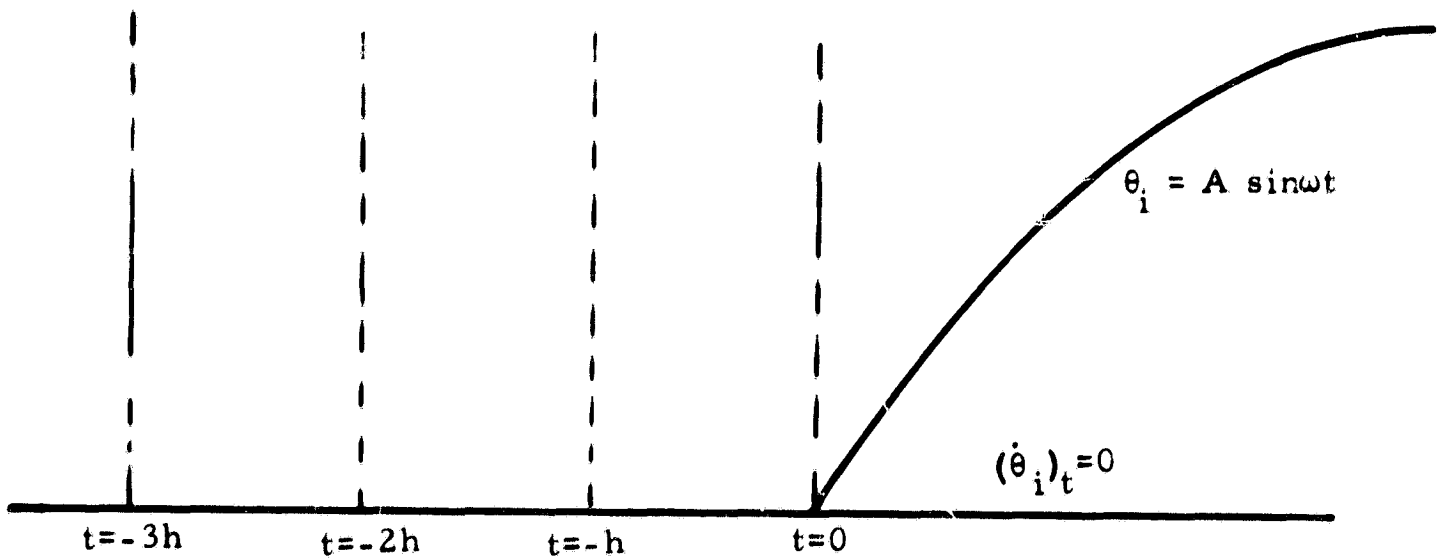


Figure 3.1-3
Initial Conditions for Sine Function

3.2 OUTPUT OF LEAD-LAG NETWORK

The most general transfer function required is the combination lead-lag used in the compensation networks. Let θ_o be the general output parameter resulting from the general input parameter θ_i , Figure (3.2-1).

The output is given by

$$\theta_o = \frac{1 + \beta s}{1 + \tau s} \theta_i \quad (3.2-1)$$

and

$$\theta_o + \tau \dot{\theta}_o = \theta_i + \beta \dot{\theta}_i \quad (3.2-2)$$

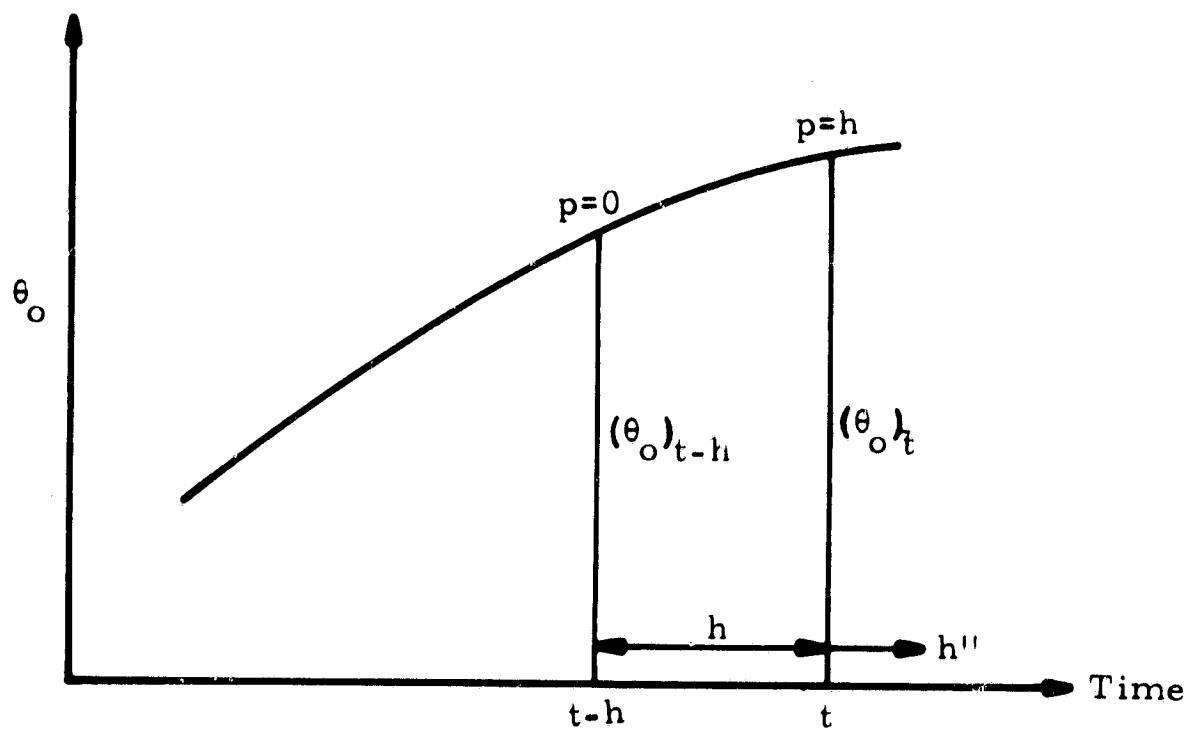


Figure 3.2-1
General Output Expression

Substituting Equations (3.1-1) and (3.1-2)

$$\theta_o + \tau \dot{\theta}_o = (A_1 + \beta A_2) + (A_2 + 2\beta A_3)p + (A_3 + 3\beta A_4)p^2 + A_4 p^3 \quad (3.2-3)$$

Assuming a solution of the form

$$\theta_o = b_1 e^{mp} + b_2 + b_3 p + b_4 p^2 + b_5 p^3 \quad (3.2-4)$$

$$\theta_o + \tau \dot{\theta}_o = b_1(1+m\tau)e^{mp} + (b_2 + b_3\tau) + (b_3 + 2b_4\tau)p + (b_4 + 3b_5\tau)p^2 + b_5 p^3 \quad (3.2-5)$$

Equating coefficients of p in Equations (3.2-3) and (3.2-5) and assuming $b_1 \neq 0$,

$$\begin{aligned} m &= -\frac{1}{\tau} \\ b_2 &= A_1 + A_2 (\beta - \tau) - 2A_3 \tau (\beta - \tau) + 6A_4 \tau^2 (\beta - \tau) \\ b_3 &= A_2 + 2A_3 (\beta - \tau) - 6A_4 (\beta - \tau) \\ b_4 &= A_3 + 3A_4 (\beta - \tau) \\ b_5 &= A_4 \end{aligned} \tag{3.2-6}$$

Equation (3.2-4) may now be written as

$$\theta_o = b_1 e^{-\frac{p}{\tau}} + b_2 + b_3 p + b_4 p^2 + b_5 p^3$$

From Figure (3.2-1) the first past value of the output occurs for $p = 0$ therefore

$$(\theta_o)_{t-h} = b_1 + b_2$$

or

$$b_1 = (\theta_o)_{t-h} - b_2$$

and

$$\theta_o = [(\theta_o)_{t-h} - B_1] e^{-\frac{p}{\tau}} + B_1 + B_2 p + B_3 p^2 + B_4 p^3 \tag{3.2-7}$$

where

$$B_1 = A_1 + A_2 (\beta - \tau) - 2A_3 \tau (\beta - \tau) + 6A_4 \tau^2 (\beta - \tau) \tag{3.2-8}$$

$$B_2 = A_2 + 2A_3 (\beta - \tau) - 6A_4 (\beta - \tau) \tag{3.2-9}$$

$$B_3 = A_3 + 3A_4 (\beta - \tau) \tag{3.2-10}$$

$$B_4 = A_4 \tag{3.2-11}$$

The output of the network consists of the sum of a constant decaying exponentially and a second cubic expression in terms of the B's.

Define a general cubic expression as

$$F_2 [C_i, p] = C_1 + C_2 p + C_3 p^2 + C_4 p^3 \quad (3.2-12)$$

and the time rate of change as

$$\dot{F}_2 [C_i, p] = C_2 + 2C_3 p + 3C_4 p^2 \quad (3.2-13)$$

Then

$$\theta_i = F_2 [A_i, p] \quad (3.2-14)$$

$$\dot{\theta}_i = \dot{F}_2 [A_i, p] \quad (3.2-15)$$

$$\theta_o = [(\theta_o)_{t-h} - B_1] e^{-\frac{p}{\tau}} + F_2 [B_i, p] \quad (3.2-16)$$

$$\dot{\theta}_o = -\frac{1}{\tau} [(\theta_o)_{t-h} - B_1] e^{-\frac{p}{\tau}} + \dot{F}_2 [B_i, p] \quad (3.2-17)$$

Then at time t

$$(\dot{\theta}_i)_t = \dot{F}_2 [A_i, h] \quad (3.2-18)$$

$$(\theta_o)_t = [(\theta_o)_{t-h} - B_1] e^{-\frac{h}{\tau}} + F_2 [B_i, h] \quad (3.2-19)$$

$$(\dot{\theta}_o)_t = -\frac{1}{\tau} [(\theta_o)_{t-h} - B_1] e^{-\frac{h}{\tau}} + \dot{F}_2 [B_i, h] \quad (3.2-20)$$

In order to extend the output past time t, the output must be expressed in terms of the proposed time step h'' in Figure (3.2-1), such that $p = h + h''$. In terms of the first past value $(\theta_o)_{t-h}$

$$(\theta_o)_{t+h''} = [(\theta_o)_{t-h} - B_1] e^{-\frac{h+h''}{\tau}} + F_2 [B_i, (h+h'')] \quad (3.2-21)$$

The value of $(\theta_o)_{t-h}$ is not always available at time t , therefore, from Equation (3.2-19)

$$[(\theta_o)_{t-h} - B_1] e^{-\frac{h+h''}{\tau}} = (\theta_o)_t e^{-\frac{h''}{\tau}} - F_2[B_1, h] e^{-\frac{h''}{\tau}}$$

$$[(\theta_o)_{t-h} - B_1] e^{-\frac{h+h''}{\tau}} = [(\theta_o)_t - F_2(B_1, h)] e^{-\frac{h''}{\tau}}$$

Then in terms of $(\theta_o)_t$ Equation (3.2-21) becomes

$$(\theta_o)_{t+h''} = [(\theta_o)_t - F_2(B_1, h)] e^{-\frac{h''}{\tau}} + F_2[B_1, (h+h'')] \quad (3.2-22)$$

3.3 OUTPUT OF SIMPLE LEAD NETWORK

When τ is set equal to zero the general lead-lag network covered in Section 3.2 reduces to the simple lead network $(1 + \beta s)$. The output expressions derived in Section 3.2 also apply but present special problem in digital programming. The output expressions contain the terms $e^{-\frac{h}{\tau}}$ and $e^{-\frac{h''}{\tau}}$ requiring division by zero. Since the computer defines division by zero as zero, the computed exponents are zero and the exponential functions are set to unity when the correct value is zero. Rather than program around this problem and use the general expressions, a separate and simple set of equations will be used when $\tau = 0$. The output of the network is given by

$$\theta_o = (1 + \beta s) \theta_i = \theta_i + \beta \dot{\theta}_i \quad (3.3-1)$$

and at time t

$$(\theta_o)_t = (\theta_i)_t + \beta (\dot{\theta}_i)_t \quad (3.3-2)$$

Since $(\theta_i)_t$ is known and $(\dot{\theta}_i)_t$ is computed during the curve fit, the output at time t can be obtained directly from Equation (3.3-2). The time rate of change of the output at time t is

$$(\dot{\theta}_o)_t = (\dot{\theta}_i)_t + \beta (\ddot{\theta}_i)_t \quad (3.3-3)$$

where

$$(\ddot{\theta}_i)_t = \ddot{F}_2[A_1, h] = 2A_3 + 6A_4h \quad (3.3-4)$$

The predicted output for $p = h + h''$ is not required for the simple lead network.

3.4 OUTPUT OF SIMPLE LAG NETWORK

When β is set equal to zero the general lead-lag network covered in Section 3.2 reduces to the simple lag network $\frac{1}{1 + \tau s}$. The output expressions derived in Section 3.2 apply and may be used directly. The B's are computed from Equations (3.2-8) through (3.2-11) for $\beta = 0$

$$B_1 = A_1 - A_2\tau + 2A_3\tau^2 - 6A_4\tau^3 \quad (3.4-1)$$

$$B_2 = A_2 - 2A_3\tau + 6A_4\tau^2 \quad (3.4-2)$$

$$B_3 = A_3 - 3A_4\tau \quad (3.4-3)$$

$$B_4 = A_4 \quad (3.4-4)$$

3.4.1 Output of Simple Lag Network for Constant Input

There are a number of points in the control system simulation where the input to a simple lag is constant during any given continuous portion of the signal. As a result the cubic fit to the input is unnecessary and the output can be programmed directly. A_1 is equal to the constant input θ_c and A_2 , A_3 , and A_4 are zero, and the B's are equal to the corresponding A's. The output expressions reduce to

$$(\theta_o)_t = (\theta_o)_{t-h} e^{-\frac{h}{\tau}} + \theta_o (1 - e^{-\frac{h}{\tau}}) \quad (3.4-5)$$

$$(\dot{\theta}_o)_t = \frac{1}{\tau} [\theta_c - (\theta_o)_{t-h}] e^{-\frac{h}{\tau}} \quad (3.4-6)$$

$$(\theta_o)_{t+h''} = (\theta_o)_t e^{-\frac{h''}{\tau}} + \theta_c (1 - e^{-\frac{h''}{\tau}}) \quad (3.4-7)$$

3.5 INTEGRATOR OUTPUT

Consider the output of the function $\frac{1}{\tau s}$ for a cubic input. The output is given by

$$\theta_o = \frac{\theta_i}{\tau s}$$

$$\text{and } \tau \dot{\theta}_o = \theta_i = A_1 + A_2 p + A_3 p^2 + A_4 p^3$$

$$(\theta_o)_t = (\theta_o)_{t-h} + \frac{1}{\tau} A_1 h + \frac{1}{2\tau} A_2 h^2 + \frac{1}{3\tau} A_3 h^3 + \frac{1}{4\tau} A_4 h^4$$

$$(\theta_o)_t = (\theta_o)_{t-h} + h \left(\frac{1}{\tau} A_1 + \frac{1}{2\tau} A_2 h + \frac{1}{3\tau} A_3 h^2 + \frac{1}{4\tau} A_4 h^3 \right)$$

Let

$$B_1 = \frac{1}{\tau} A_1 \quad (3.5-1)$$

$$B_2 = \frac{1}{2\tau} A_2 \quad (3.5-2)$$

$$B_3 = \frac{1}{3\tau} A_3 \quad (3.5-3)$$

$$B_4 = \frac{1}{4\tau} A_4 \quad (3.5-4)$$

then

$$(\theta_o)_t = (\theta_o)_{t-h} + h F_2 [B_i, h] \quad (3.5-5)$$

$$(\dot{\theta}_o)_t = B_1 + 2B_2 h + 3B_3 h^2 + 4B_4 h^3 \quad (3.5-6)$$

4. SENSORS

The output parameters required for nearly any ideal sensor are already available in the digital program. The relative and inertial positions, velocities, and accelerations for all of the spacecraft bodies are computed in the solution of the dynamic equations. The orbital parameters are computed in the orbit routine, and the disturbance routines provide the rotational position of the earth, and the components of the sun vector and magnetic field vector in each body coordinate system. In general the purpose of the sensor routines is to simulate the imperfections and limitations of real sensors.

The program provides for two rate gyros, two horizon scanners, two two-axis sun sensors, and a special "dummy" sensor. The rate gyros may be given any arbitrary orientation on the main spacecraft body while the horizon scanners and sun sensors may have arbitrary orientations on any of the spacecraft bodies. The "dummy" sensor is provided for use in the checkout of new control configurations and to determine the system response to different input functions. The output of this special sensor is the sum of a constant, a ramp, and a sine function and is defined by program input parameters.

4.1 RATE SENSORS

It is assumed that the rate gyros are mounted on the main spacecraft body, each with an arbitrarily oriented input axis. The ideal output of the j^{th} gyro is given by

$$\omega_{gj} = a_{1j} \omega_{lx} + a_{2j} \omega_{ly} + a_{3j} \omega_{lz} + B_{G1} \quad (4.1-1)$$

where a_{1j} , a_{2j} , and a_{3j} are the direction cosines of the j^{th} gyro input axis in main body coordinates and ω_{lx} , ω_{ly} , and ω_{lz} are the components of the inertial angular velocity in body coordinates. B_{G1} is a constant bias specified by a program input parameter. In matrix form

$$\omega_{gj} = \{a_j\}_T \{\omega_{ll}\} + B_{G1} \quad (4.1-2)$$

The simulation of the gyro dynamics is shown in Figure (4.1-1). The general form ($\tau \neq 0$) simulates the gyro dynamic lag, an integrator saturation limit representing the angular stops, and a limiter in the feedback loop representing torquer saturation. Saturation of the integrator is implemented by switch S_g . When the output reaches $\pm K_1$ the switch is opened. The output remains constant until the signal ϵ_g changes sign at which time the switch is closed. It should be noted the K_2 must be less than K_1 in order for K_2 to saturate. For $\tau \neq 0$ the gyro dynamics reduce to a simple limiter representing the gyro stops.

For any value of τ the first step in the solution for the gyro output is a cubic curve fit to the input to the gyro dynamics (ω_{gj}). The curve fit produces the polynomial coefficients A_1 through A_4 and the present time rate of change of ω_{gj} . If $\tau = 0$ only the value of $\dot{\omega}_{gj}$ is needed to predict the time at which $|\omega_{gj}| = K_1$. When $\tau \neq 0$ the gyro dynamics has three modes of operation each requiring separate equations and logic.

In mode one neither the feedback or the integrator is saturated and the magnitude of the output is less than K_2 . The dynamics reduce to a simple lag, Figure 4.1-1. The output and the time rate of change of the output at time t are computed from Equations (3.2-19) and (3.2-20)

$$(\theta_o)_t = [(\theta_o)_{t-h} - B_1] e^{-\frac{h}{\tau}} + F_2 [B_i, h] \quad (4.1-3)$$

$$(\dot{\theta}_o)_t = -\frac{1}{\tau} [(\theta_o)_{t-h} - B_1] e^{-\frac{h}{\tau}} + \dot{F}_2 [B_i, h] \quad (4.1-4)$$

where the B's are computed from the curve fit A's using Equations (3.4-1) through (3.4-4) for a simple lag. The predicted time to the next discontinuity h' is the positive time to saturation level K_2 computed from $(\theta_o)_t$ and $(\dot{\theta}_o)_t$.

In mode two the output is greater than K_2 but less than K_1 . The feedback is saturated and the integrate is not saturated. The dynamics reduces to the integration of the signal ϵ_g , Figure (4.1-1). The input to the integrator (ϵ_g) is

$$\epsilon_g = \theta_i = \omega_{gj} - [\text{sgn}(\theta_o)_{t-h}] K_2 \quad (4.1-5)$$

From Equation (3.5-5) the output at time t is

$$(\theta_o)_t = (\theta_o)_{t-h} + h F_2[B_i, h] - [\text{sgn}(\theta_o)_{t-h}] \frac{K_2}{\tau} h \quad (4.1-6)$$

where the B 's are computed from Equation (3.5-1) through (3.5-4) for an integrator. Using Equation (3.5-6) the time rate of change of the output is

$$(\theta_o)_t = B_1 + 2B_2 h + 3B_3 h^2 + 4B_4 h^3 - [\text{sgn}(\theta_o)_{t-h}] \frac{K_2}{\tau} \quad (4.1-7)$$

The predicted time to the next discontinuity, h' , is the minimum of the positive times to K_2 and K_1 . If the output reaches K_2 the operation of the gyro dynamics changes to mode 1 and if the output reaches K_1 it changes to mode 3.

The instant that the integrator saturates the output is $\pm K_1$ and switch S_g is opened. The gyro dynamics operates in mode 3 with constant output, Figure (4.1-1), until the signal ϵ_g changes sign. The output is given by

$$(\theta_o)_t = [\text{sgn}(\theta_o)_{t-h}] K_1 \quad (4.1-8)$$

The predicted time to the next discontinuity, h' , is the positive time to $\epsilon_g = 0$ where

$$\epsilon_g = \omega_{gj} - [\text{sgn}(\theta_o)_{t-h}] K_2 \quad (4.1-9)$$

at the time $\epsilon_g = 0$ the operation changes to mode 2.

4.2 HORIZON SCANNERS

The horizon scanners may be mounted on any of the spacecraft bodies and have arbitrary orientations with respect to those bodies. A block diagram of the scanners, signal processor, and scanner dynamics is shown in Figure 4.2-1. The equations and logic for the scanners and signal processing methods A and B are given in Reference 4. These data in the form used in the Generalized Spacecraft Simulation are repeated here for reference.

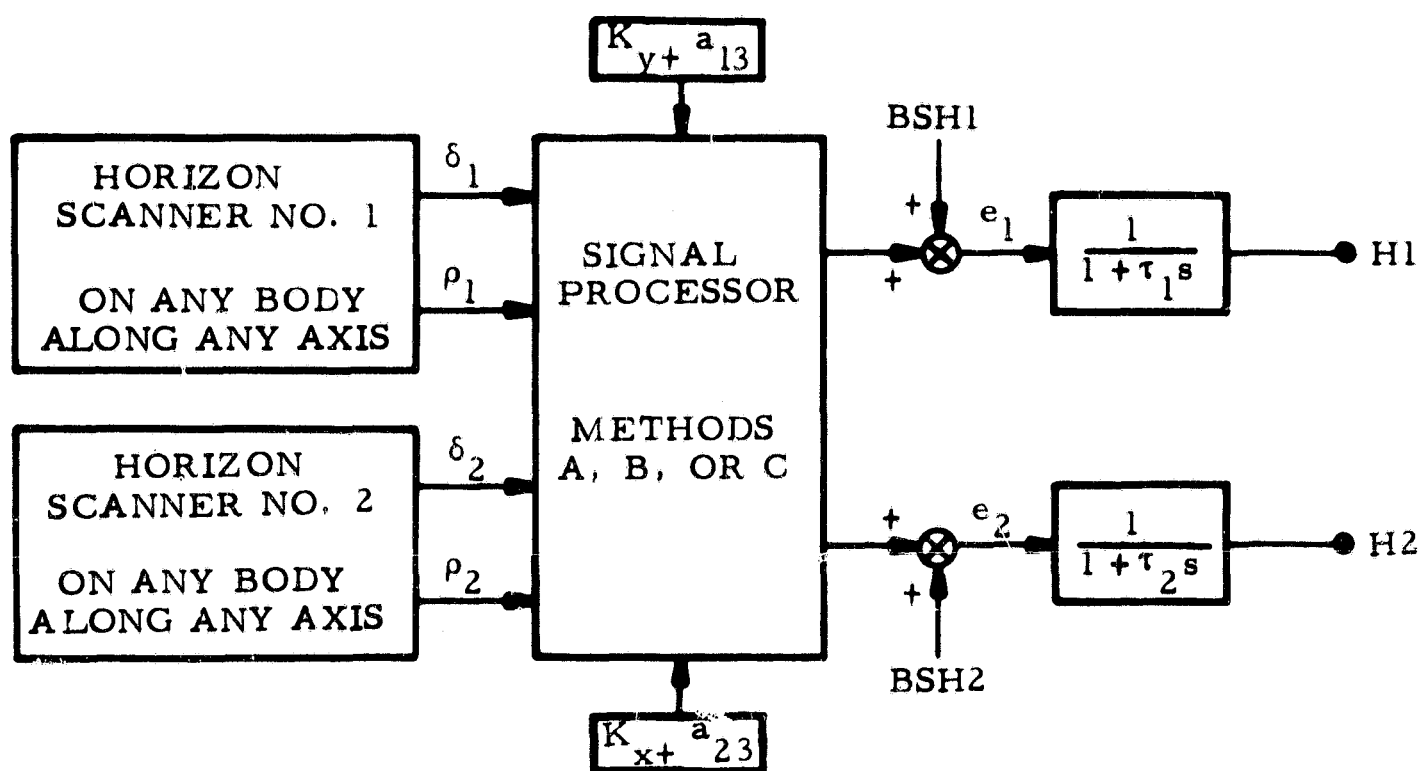


Figure 4.2-1
Horizon Scanner Block Diagram

4.2.1 Single Scanner Output

The scan pattern of a single scanner is a cone. If the scan cone intersects the earth, the sensing element (a bolometer) will emit a pulse. This pulse can be characterized by knowledge of its width (2ρ) and the angular distance (δ) by which it is displaced from a vertical reference marker.

The geometry of a single scan cone relative to the earth and the orbital position of the spacecraft is shown in Figure 4.2-2. The $\bar{x}_R$, $\bar{y}_R$, and $\bar{z}_R$ coordinate frame defines the orbital reference system, Equation (3.2-2) of Volume I.

$$\{\bar{Q}_R\} = \begin{pmatrix} \bar{x}_R \\ \bar{y}_R \\ \bar{z}_R \end{pmatrix} \quad (4.2-1)$$

The transformation from the orbital reference system to the main body coordinates $\{\bar{Q}_1\}$ is $[A]$, Equation (3.3-3) of Volume I

$$\{\bar{Q}_1\} = [A] \{\bar{Q}_R\} \quad (4.2-2)$$

and the transformation from the body 1 system to the body i coordinates is $[D_i]$, Equation (3.5-5) of Volume I

$$\{\bar{Q}_i\} = [D_i] \{\bar{Q}_1\} \quad (4.2-3)$$

Let the arbitrary orientation of the j^{th} horizon scanner on body i be defined by the program input matrix $[S_j]$ such that

$$\{\bar{Q}_{pj}\} = [S_j] \{\bar{Q}_i\} = \begin{pmatrix} \bar{x}_{pj} \\ \bar{y}_{pj} \\ \bar{z}_{pj} \end{pmatrix} \quad (4.2-4)$$

where $\bar{x}_{pj}$ is the scan cone axis, Figure 4.2-2.

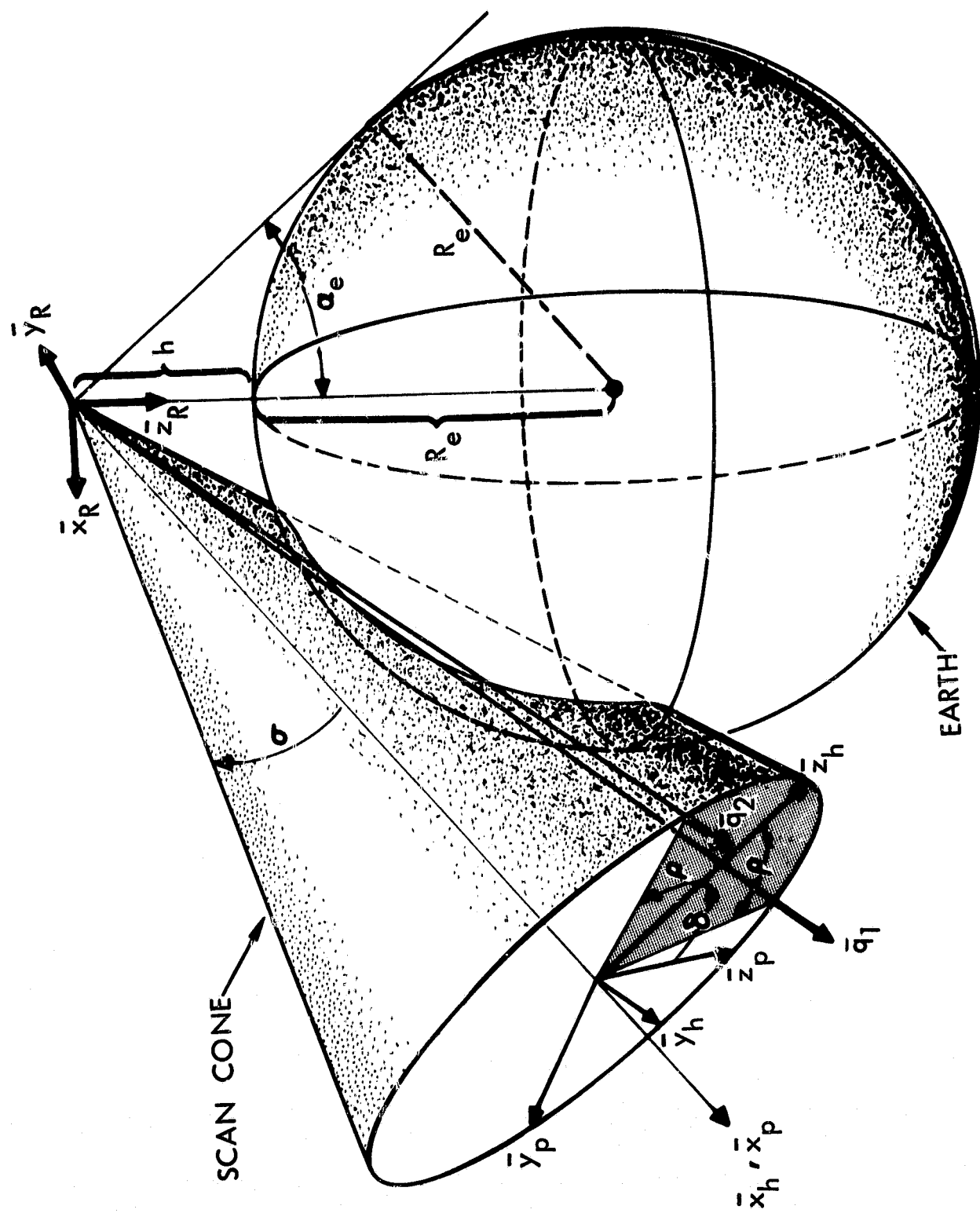


Figure 4.2-2
Scan Geometry

Combining Equations (4.2-2), (4.2-3), and (4.2-4)

$$\{\bar{Q}_{pj}\} = [S_j][D_i][A]\{\bar{Q}_R\} \quad (4.2-5)$$

Let

$$[A_{Sj}] = [S_j][D_i][A] \quad (4.2-6)$$

Then

$$\{\bar{Q}_{pj}\} = [A_{Sj}]\{\bar{Q}_R\} \quad (4.2-7)$$

The matrix $[A_{Sj}]$ defines the j^{th} scanner axes in terms of the orbital reference system. Since $[D_i]$ for $i = 1$ has been defined as the unit matrix $[U]$, the transformation applies for $(1 \leq i \leq N)$.

Define the $\{\bar{Q}_h\}$ coordinate system, Figure 4.2-2, by

$$\{\bar{Q}_h\} = \begin{pmatrix} \bar{x}_h \\ \bar{y}_h \\ \bar{z}_h \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta_j & \sin\delta_j \\ 0 & -\sin\delta_j & \cos\delta_j \end{bmatrix} \begin{pmatrix} \bar{x}_S \\ \bar{y}_S \\ \bar{z}_S \end{pmatrix} \quad (4.2-8)$$

and constrain $\bar{z}_h$ to lie in the plane established by $\bar{x}_R$ and $\bar{z}_R$ with $\bar{z}_h \cdot \bar{z}_R \geq 0$.

Combining Equations (4.2-8) and (4.2-7) for the j^{th} scanner

$$\begin{pmatrix} \bar{x}_h \\ \bar{y}_h \\ \bar{z}_h \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} A_{S11} & A_{S12} & A_{S13} \\ A_{S21} & A_{S22} & A_{S23} \\ A_{S31} & A_{S32} & A_{S33} \end{bmatrix} \begin{pmatrix} \bar{x}_R \\ \bar{y}_R \\ \bar{z}_R \end{pmatrix} \quad (4.2-9)$$

expanding

$$\begin{pmatrix} \bar{x}_h \\ \bar{y}_h \\ \bar{z}_h \end{pmatrix} = \begin{bmatrix} (A_{S11}) & (A_{S12}) & (A_{S13}) \\ (A_{S21} \cos \delta + A_{S31} \sin \delta) & (A_{S22} \cos \delta + A_{S32} \sin \delta) & (A_{S23} \cos \delta + A_{S33} \sin \delta) \\ (-A_{S21} \sin \delta + A_{S31} \cos \delta) & (-A_{S22} \sin \delta + A_{S32} \cos \delta) & (-A_{S23} \sin \delta + A_{S33} \cos \delta) \end{bmatrix} \begin{pmatrix} \bar{x}_R \\ \bar{y}_R \\ \bar{z}_R \end{pmatrix} \quad (4.2-10)$$

In order that $\bar{z}_h$ lie in the plane of $\bar{x}_p$ and $\bar{z}_R$, $(\bar{y}_h \cdot \bar{z}_R)$ must be zero, Figure 4.2-2. This constraint requires that the component of $\bar{y}_h$ along $\bar{z}_R$ must be zero which in turn requires that element 2-3 be zero in the transformation matrix of Equation (4.2-10). Therefore

$$A_{S23} \cos \delta + A_{S33} \sin \delta = 0 \quad (4.2-11)$$

and

$$\frac{\sin \delta}{\cos \delta} = \frac{-(A_{S23})}{(A_{S33})} \quad (4.2-12)$$

To determine the half-width of the earth pulse (ρ) consider the unit vectors $\bar{q}_1$ and $\bar{q}_2$ which define the lines in the cone which are tangent to the earth. From Figure 4.2-2

$$\bar{q}_1 \cdot \bar{z}_R = \bar{q}_2 \cdot \bar{z}_R = \cos \alpha_e \quad (4.2-13)$$

The components of $\bar{q}_1$ in the $\{\bar{Q}_h\}$ coordinate system are

$$\{q_{1h}\} = \begin{pmatrix} \cos \sigma \\ \sin \sigma \sin \rho \\ \sin \sigma \cos \rho \end{pmatrix} \quad (4.2-14)$$

where σ is the cone half-angle.

The components of $\bar{z}_R$ in the $\{\bar{Q}_h\}$ coordinate system are given by Equation (4.2-10) as

$$\{z_{Rh}\} = \begin{pmatrix} A_{S13} \\ A_{S23} \cos \delta + A_{S33} \sin \delta \\ -A_{S23} \sin \delta + A_{S33} \cos \delta \end{pmatrix}$$

Using Equation (4.2-11), the fact that $\bar{z}_R$ is a unit vector and the restriction that $\bar{z}_h \cdot \bar{z}_R \geq 0$

$$\{\bar{z}_{Rh}\} = \begin{pmatrix} A_{S13} \\ 0 \\ + \sqrt{1 - A_{S13}^2} \end{pmatrix} \quad (4.2-15)$$

Substituting Equations (4.2-14) and (4.2-15) into (4.2-13)

$$\cos \rho = \frac{\cos \alpha_e - A_{S13} \cos \sigma}{\sqrt{1 - A_{S13}^2} \sin \sigma} \quad (4.2-16)$$

where

$$\cos \alpha_e = \frac{1}{R_e + h} \sqrt{(R_e + h)^2 - R_e^2} \quad (4.2-17)$$

R_e = radius of earth

h = h spacecraft altitude

In the program $(R_e + h)$ is computed in the orbit routine as the orbital radius S_M . Therefore

$$\cos \alpha_e = \frac{1}{S_M} \sqrt{S_M^2 - R_e^2} \quad (4.2-18)$$

The single scanner outputs are given by Equations (4.2-12) and (4.2-16); however, since the symbols ρ , δ and σ are used elsewhere in the program the subscript f will be added to define horizon scanner parameters and the basic output equations become

$$\frac{\sin \delta_f}{\cos \delta_f} = \frac{-A_{S23}}{A_{S33}}, \quad \delta_f \in [-\pi, \pi] \quad (4.2-19)$$

$$\cos \rho_f = \frac{\cos \alpha_e - A_{S13} \cos \sigma_f}{\sqrt{1 - A_{S13}^2} \sin \sigma_f} \quad (4.2-20)$$

It should be noted that only the third column of the matrix $[A_{Sj}]$ is required for the computations.

The pulse half-width is always in the range between zero and π . If Equation (4.2-20) gives $\cos \rho_f > 1$, ρ_f is then set to zero. If the computed value of $\cos \rho_f$ is less than -1, the scan cone is completely on the earth and ρ_f is set either to π or zero determined by an input option in the program.

4.3 HORIZON SCANNER PROCESSING

Most commonly, pitch and roll errors signals are derived by combining the processed outputs from two scanners. The method of combination generally depends upon the mounting of the scanners. Since the scanner orientation is arbitrary no reference will be made to pitch and roll errors and the scanners are designated as scanners 1 and 2. The scanner outputs from the processor are designated e_1 and e_2 ; however, it should be noted that e_1 is not the output from scanner 1 and e_2 is not the output from scanner 2. Each output may be a combination of the outputs of both scanners. For each scanner two output

parameters q_a and q_b are computed resulting in scanner output parameters q_{a1} , q_{b1} , q_{a2} , and q_{b2} . For two scanner configurations two error signals e_+ as e_- are computed such that

$$e_+ = \frac{90}{\pi} (q_{a1} + q_{a2}) \operatorname{sgn} \delta_{f1} \quad (4.3-1)$$

$$e_- = \frac{90}{\pi} (q_{b1} - q_{b2}) \quad (4.3-2)$$

where e_+ and e_- are in degrees.

The processor output e_1 and e_2 are defined as

$$e_1 = (K_{y+})(e_+) + (K_{y-})(e_-) + B_{SH1} \quad (4.3-3)$$

$$e_2 = (K_{x+})(e_+) + (K_{x-})(e_-) + B_{SH2} \quad (4.3-4)$$

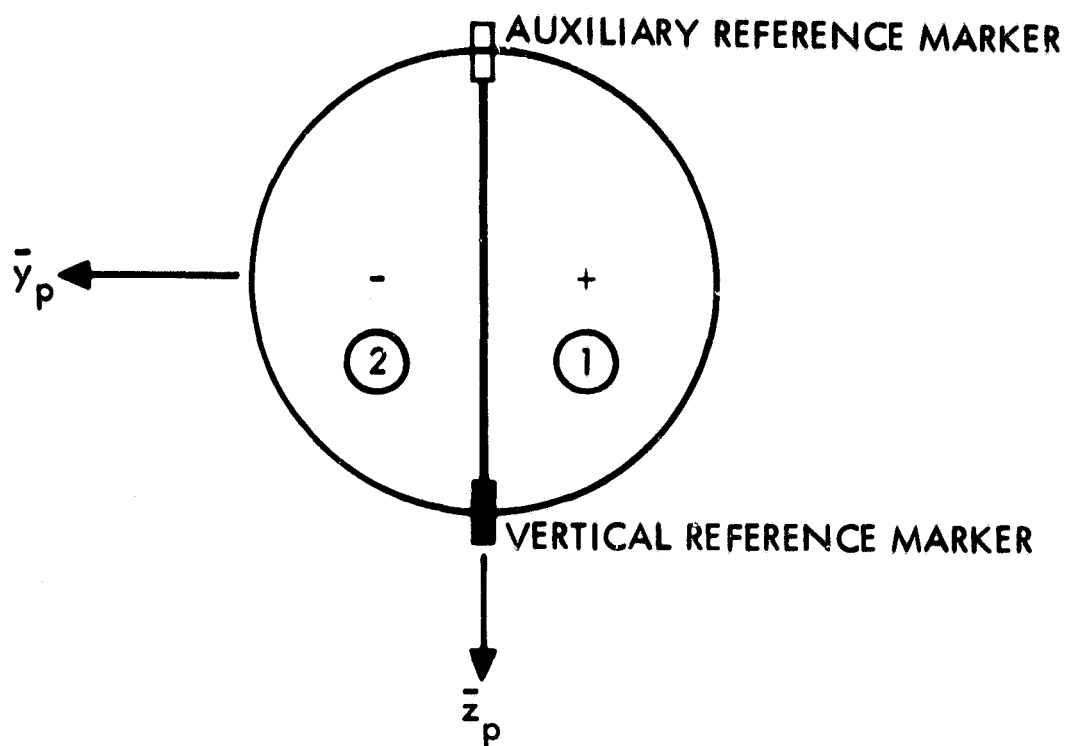
where B_{SH1} and B_{SH2} are constant input bias signals for scanners 1 and 2 in degrees. For single scanner configurations only scanner 1 is used and only e_+ is computed from

$$e_+ = \frac{180}{\pi} q_{a1} \operatorname{sgn} \delta_{f1} \quad (4.3-5)$$

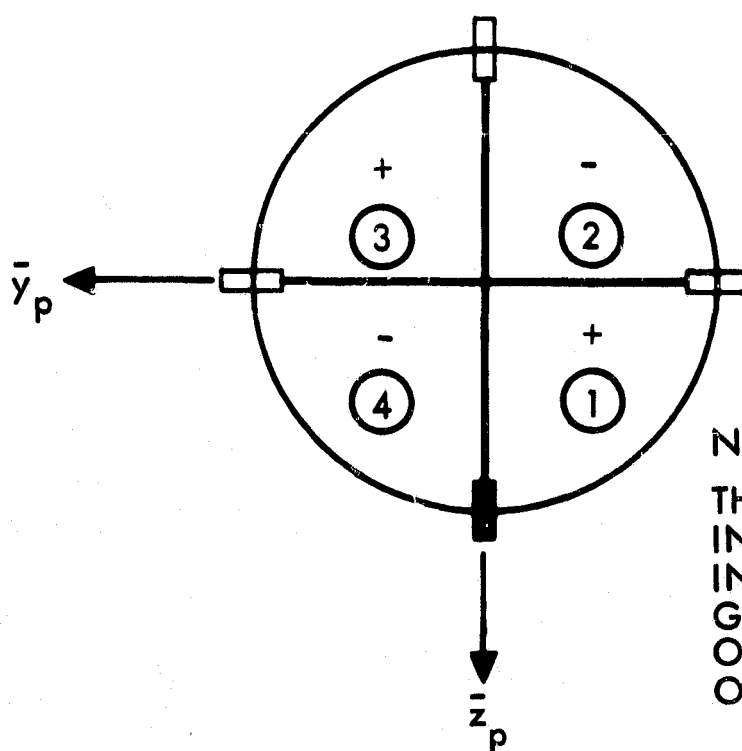
The parameters K_{x+} , K_{x-} , K_{y+} , and K_{y-} are program input gain constants. The parameters q_{a1} , q_{a2} , and q_{b2} are computed in accordance with the type of processing selected (A, B, or C).

4.3.1 Signal Processing Method A

Signal processing method "A" is similar to the "Sin ϕ " processing scheme described in Reference 4. The scan cone is divided into two 180 degree segments with the vertical reference marker as the boundary between the regions. The width of earth pulse on one side of the vertical reference marker ($\bar{z}_p$ axis) is subtracted from that on the other side of the marker to derive the $\bar{x}_p$ axis error for a single scanner, Figure 4.3-1. The results assume a symmetric blanking region of width



(a) "Sin ϕ " PROCESSING:



NOTE
THE SIGN SHOWN
IN EACH REGION
INDICATES THE WEIGHTING
GIVEN TO THE PORTION
OF THE EARTH PULSE
OCCURRING IN THAT REGION

(b) "Sin 2ϕ " PROCESSING:

Figure 4.3-1
Pulse Segment Weighting

$2(\pi - \theta_f)$, centered about the $-\bar{z}_p$ axis (for no blanking $\theta_f = \pi$). The parameters q_a and q_b for a given scanner are functions of δ_f , ρ_f , and θ_f for that scanner and are computed in accordance with Table 4.3-1.

Table 4.3-1
 q_a and q_b for Processing Method "A" with Blanking

Range of $ \delta_f - \rho_f$	Range of $ \delta_f + \rho_f$	q_a	q_b
$-\theta_f, 0$	$0, \theta_f$	$ \delta_f $	ρ_f
$0, \theta_f$	$0, \theta_f$	ρ_f	ρ_f
$-\pi, \theta_f$	$\theta_f, 2\pi - \theta_f$	0	θ_f
$-\theta_f, 0$	$\theta_f, 2\pi - \theta_f$	$1/2(\theta_f + \delta_f - \rho_f)$	$1/2(\theta_f - \delta_f + \rho_f)$
$0, \theta_f$	$\theta_f, 2\pi - \theta_f$	$1/2(\theta_f - \delta_f + \rho_f)$	$1/2(\theta_f - \delta_f + \rho_f)$
θ_f, π	$\theta_f, 2\pi - \theta_f$	0	0
$-\theta_f, \pi$	$2\pi - \theta_f, 2\pi$	$\pi - \rho_f$	$\theta_f + \rho_f - \pi$
$0, \theta_f$	$2\pi - \theta_f, 2\pi$	$\pi - \delta_f $	$\theta_f + \rho_f - \pi$

4.3.2 Signal Processing Method "B"

Signal Processing Method "B" is similar to the "Sin2 ϕ " processing scheme described in Reference 4. The $\bar{x}_p$ axis error signal is derived by considering the algebraic sum of the pulse portions occurring in the various quadrants, Figure 4.3-1. The parameters q_a and q_b for a given scanner without blanking are computed in accordance with Table 4.3-2.

Table 4.3-2
 q_a and q_b for Processing Method "B" (No Blanking)

Range of $ \delta_f - \rho_f$	Range of $ \delta_f + \rho_f$	q_a	q_b
$-\pi/2, 0$	$0, \pi/2$	$ \delta_f $	ρ_f
$-\pi/2, 0$	$\pi/2, \pi$	$\pi/2 - \rho_f$	ρ_f
$-\pi/2, 0$	$\pi, 3\pi/2$	$ \delta_f - \pi/2$	ρ_f
$-\pi/2, 0$	$3\pi/2, 2\pi$	$\pi - \rho_f$	ρ_f
$-\pi, -\pi/2$	$\pi/2, \pi$	$- \delta_f $	ρ_f
$-\pi, -\pi/2$	$\pi, 3\pi/2$	$\rho_f - \pi$	ρ_f
$0, \pi/2$	$0, \pi/2$	ρ_f	ρ_f
$0, \pi/2$	$\pi/2, \pi$	$\pi/2 - \delta_f $	ρ_f
$0, \pi/2$	$\pi, 3\pi/2$	$\rho_f - \pi/2$	ρ_f
$0, \pi/2$	$3\pi/2, 2\pi$	$\pi - \delta_f $	ρ_f
$\pi/2, \pi$	$\pi/2, \pi$	$-\rho_f$	ρ_f
$\pi/2, \pi$	$\pi, 3\pi/2$	$ \delta_f - \pi$	ρ_f

4.3.3 Signal Processing Method "C"

Signal processing method "C" is a representation of ideal small angle pitch and roll sensor. Processor outputs e_1 and e_2 (in radians) are defined by

$$e_1 = K_{y+} a_{13} + B_{SH1} \quad (4.3-1)$$

$$e_2 = K_{x+} a_{23} + B_{SH2} \quad (4.3-2)$$

where K_{y+} and K_{x+} are program input constants and a_{ij} is the ij^{th} element in the transformation matrix $[A]$ which defines the attitude of the main spacecraft body axes with respect to the orbital reference axes. B_{SH1} and B_{SH2} are the constant input bias signals for scanners 1 and 2, in radians.

4.4 SCANNER DYNAMICS

The scanner dynamics are represented by a simple lag for each output signal, Figure 4.2-1. If τ_j for the j^{th} output is zero the scanner output H_j is set equal to the processor output e_j . For τ_j not equal to zero a cubic curve fit is made to the processor output e_j . The resultant A's are used to compute B's for a simple lag using Equations (3.4-1) through (3.4-4). From equations (3.2-19) the scanner output H_j at time t is

$$(H_j)_t = [(H_j)_{t-h} - B_1] e^{-\frac{h}{\tau_j}} + F_2 [B_1, h] \quad (4.4-1)$$

Since the lag is not followed by a discontinuity no prediction is required and therefore $(\dot{H}_j)_t$ is not required.

4.5 SUN SENSORS

The sun sensors may be mounted on any of the spacecraft bodies with arbitrary orientations. Each sensor is defined by the right hand orthogonal sensor axes $\bar{x}_S$, $\bar{y}_S$, and $\bar{z}_S$ where the $\bar{x}_S$ axis is the sensor reference axis, Figure 4.5-1. The orientation of the sensor is defined by the transformation matrix $[M_S]$ such that

$$\{Q_S\} = \begin{Bmatrix} \bar{x}_S \\ \bar{y}_S \\ \bar{z}_S \end{Bmatrix} = [M_S] \{\bar{Q}_i\} \quad (4.5-1)$$

where $[M_S]$ is a constant input matrix.

The components of the unit sun vector $\bar{\delta}_S$ are computed from Equation (10.1-3) of Volume I and are stored for all bodies. The unit sun vector $\bar{\delta}_S$ has been defined as a unit vector from the sun

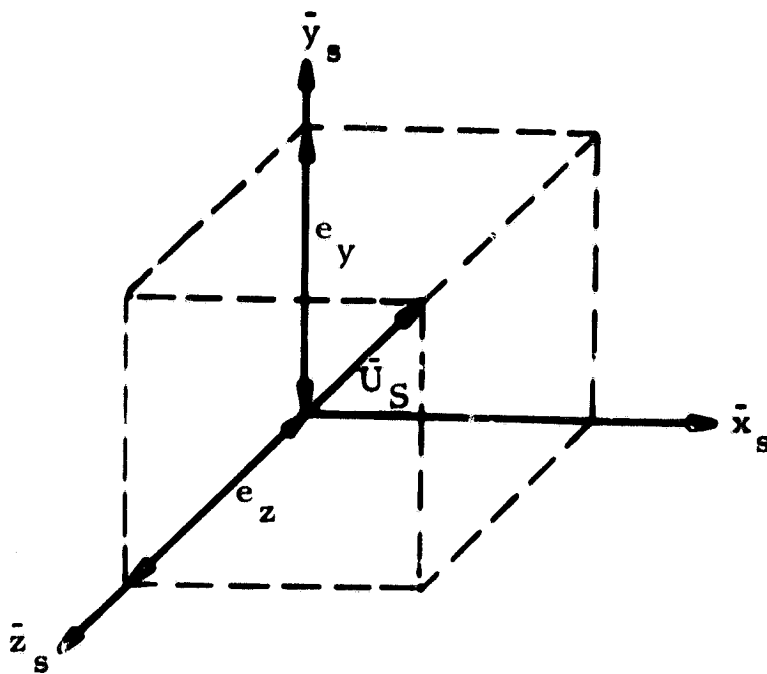


Figure 4.5-1
Sun Sensor Orientation

toward the center of the earth. Let $\bar{U}_S$ be a unit vector from the earth toward the sun. Then

$$\{U_{Si}\} = -\{\delta_{Si}\} \quad (4.5-2)$$

and from Equation (10.1-4) of Volume I

$$\{\dot{U}_{Si}\} = -[\omega_{ii}]\{U_{Si}\} \quad (4.5-3)$$

The y output (e_y) from the ideal sun sensor is the component of $\bar{U}_S$ along the $\bar{y}_S$ axis and the z output (e_z) is the component along the $\bar{z}_S$ axis, Figure 4.5-1. Since the matrix $[M_S]$ is constant the components of $\bar{U}_S$ and $\bar{U}_S$ in sensor axes are

$$\{U_{SS}\} = \begin{pmatrix} e_y \\ e_z \end{pmatrix} = [M_S]\{U_{Si}\} = -[M_S]\{\delta_{Si}\} \quad (4.5-4)$$

$$\{U_{\dot{e}}\} = \begin{Bmatrix} \ddot{e}_x \\ \ddot{e}_y \\ \ddot{e}_z \end{Bmatrix} = -[M_S][J_{\omega ii}]\{U_{S_i}\} = [M_S][J_{\omega ii}]\{\delta_{S_i}\} \quad (4.5-5)$$

4.5.1 Sun Sensor Characteristics

The block diagram for the sun sensors is shown in Figure 4.5-2. The ideal output from each channel is modified by a signal limiter, upper and lower blanking angles, and the addition of a constant bias signal. The output of the j^{th} channel, S_j is given by

$$S_j = e_j + BS_j \quad (4.5-6)$$

where S_j is limited at K_j and set to zero for $|S_j|$ less than D or greater than B , Figure 4.5-2. Outputs S_1 and S_2 are the y and z axis outputs for sun sensor 1 while outputs S_3 and S_4 are the corresponding outputs for sensor 2. It should be noted that the sun sensor outputs are not the angular errors with respect to the reference axis but are components of a unit vector with magnitudes between zero and unity.

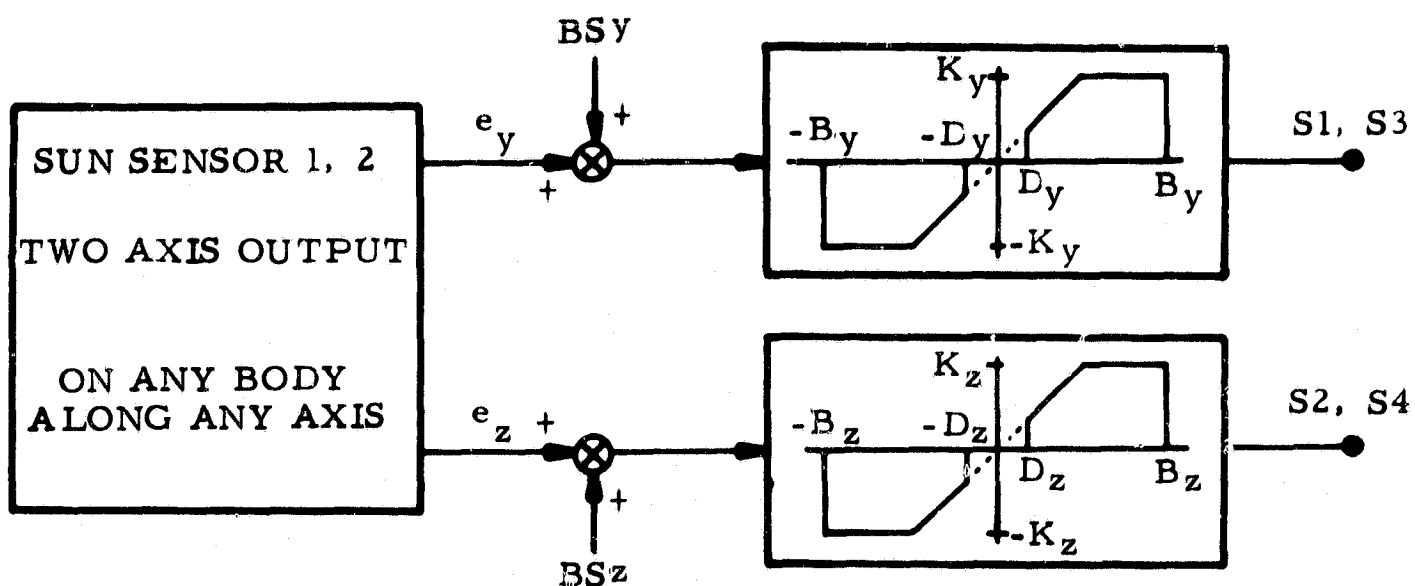


Figure 4.5-2
Sun Sensor Block Diagram

4.6 DUMMY SENSOR

The output of the dummy sensor is given by

$$F = A + B t + C \sin (W t + P) \quad (4.6-1)$$

where A, B, C, W, and P are program input constants.

5. COMPENSATION NETWORKS

The program provides sixteen possible paths through which the sensor outputs may be connected to the control system inputs. These paths are referred to as compensation networks N1 through N16, however, only the first eight, N1 through N8, are true saturating lead-lag compensation networks. The next two, N9 and N10 are signal limiters while the last six N11 through N16 are direct connection paths.

5.1 NETWORK PATCH SYSTEM

The patch system used to connect the sensors and the control systems to the inputs and outputs of the networks is shown in Figure 5.1-1. The method used in the program consists of defining the inputs by specifying the sensor or control system in the location field and the network numbers (1 through 16) in the value field.

The normal sensor outputs (two gyro, two horizon scanner, and four sun sensor) may each be patched to any two network inputs. The "dummy" sensor output may be patched to any four network inputs. The number of sensors which may be patched to a given network is unlimited.

The nine possible control system inputs, three pneumatic systems, four reaction wheel systems, and two array drive systems, are patched to the network outputs in the same manner. In addition, to the capability of patching any control system input to any two network outputs, each possible connection (18) incorporates a constant patch board gain factor.

5.2 LEAD-LAG NETWORKS

The lead-lag network shown in Figure 5.2-1 is defined by the three input parameters, τ , β , and K . By omitting the time constant, τ , the network of form A in Figure 5.2-1 reduces to a simple lead followed

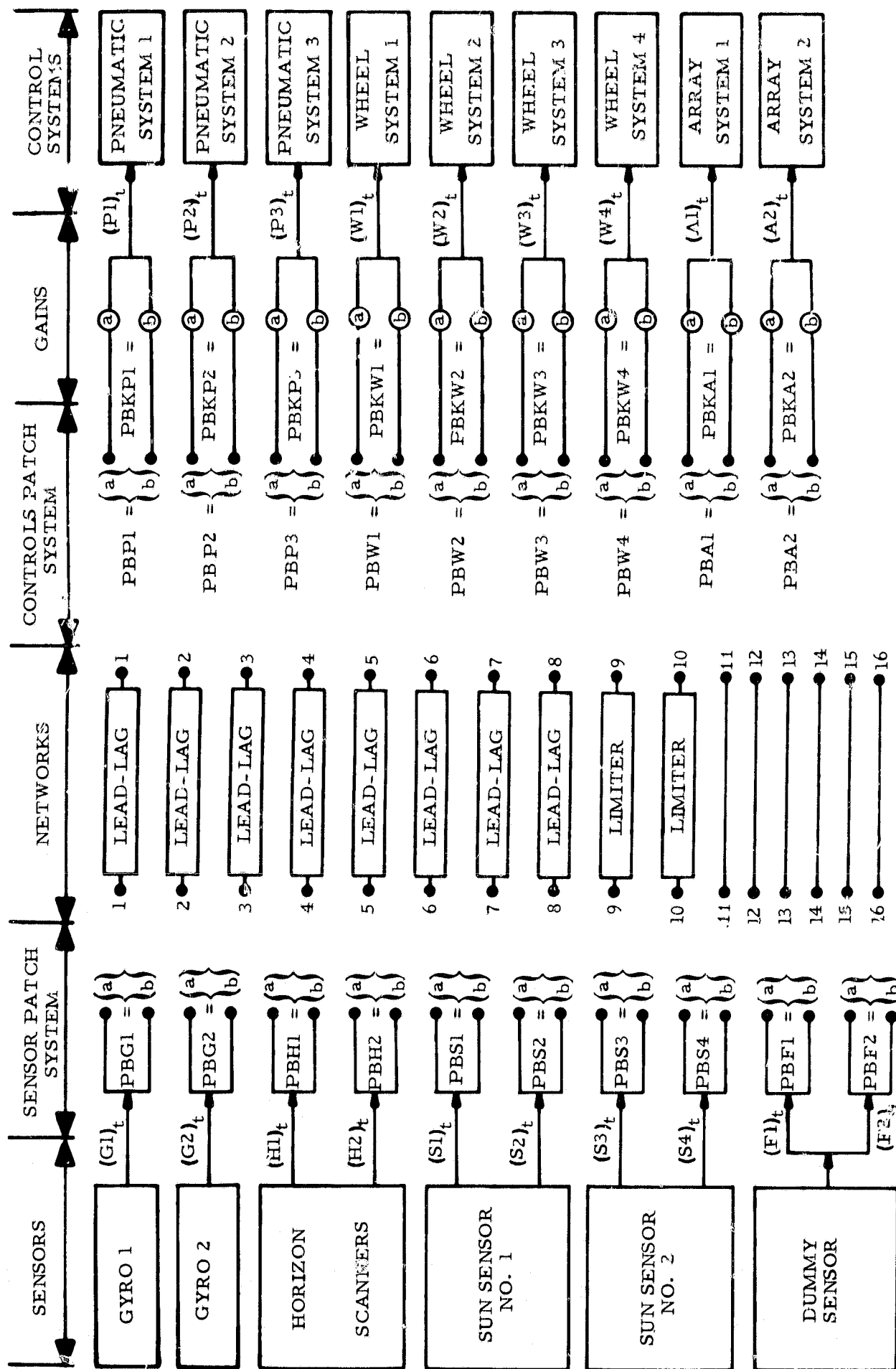
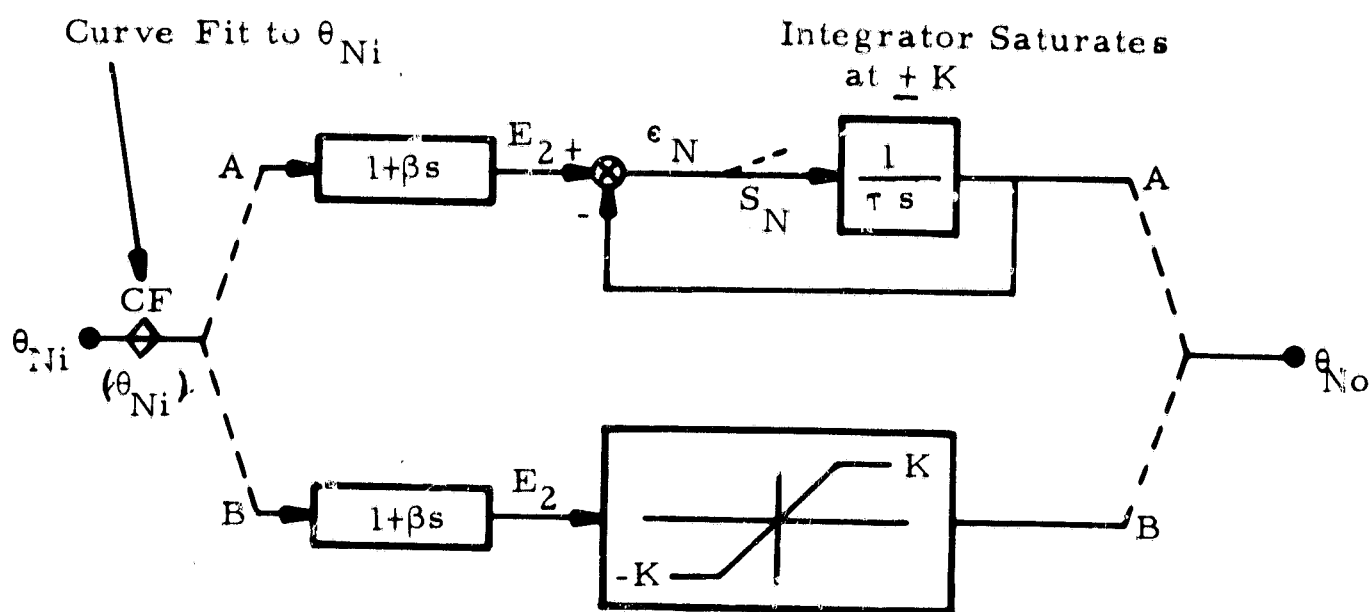
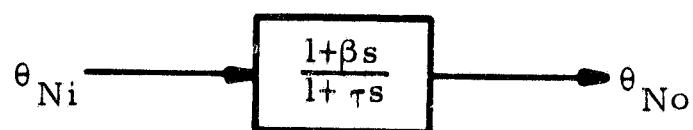


Figure 5.1-1
Patch System



MODE 1 $|\theta_{No}| < K$ (NO SATURATION)



MODE 2 $|\theta_{No}| = K$ (INTEGRATOR SATURATED)

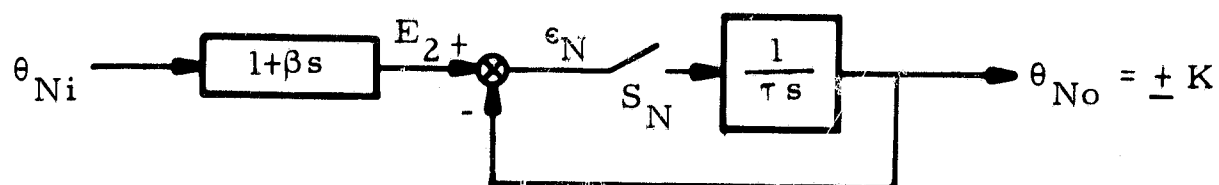


Figure 5.2-1
Typical Compensation Network

by a limiter, form B. By omitting the saturation value K, form A is reduced to a lead-lag network and form B is reduced to a pure lead. In addition, these forms may be further reduced to a lag network and a direct connection, respectively by omitting β .

The general network, form A, is mechanized in a manner similar to that used for the gyro dynamics. However, the general network has only two modes of operation and includes the effects of the lead β . Let θ_{Ni} be the general expression for the input to the j^{th} network where θ_{Ni} is the sum of the sensor outputs patched to network j. The first step in the solution for the output of the first eight networks is a cubic curve fit to θ_{Ni} storing the resulting A's and $\dot{\theta}_{Ni}$.

For operation in mode 1 the integration is not saturated and the magnitude of the network output θ_{No} is less than K. The network may be represented by the function $(1 + \beta s)/(1 + \tau s)$ and from Equations (3.2-19) and (3.2-20)

$$(\theta_{No})_t = [(\theta_{No})_{t-h} - B_1] e^{-\frac{h}{\tau}} + F_2 [B_1, h] \quad (5.2-1)$$

$$(\dot{\theta}_{No})_t = -\frac{1}{\tau} [(\theta_{No})_{t-h} - B_1] e^{-\frac{h}{\tau}} + \dot{F}_2 [B_1, h] \quad (5.2-2)$$

where the B's are computed from Equations (3.2-8) through (3.2-11). The predicted time to the next discontinuity h' is the positive time to saturation level K computed from $(\theta_{Wo})_t$ and $(\dot{\theta}_{No})_t$. At integrator saturation the input signal to the integrator ϵ_N , Figure 5.2-1, is removed and the operation switches to mode 2.

For operation in mode 2 the integrator is saturated and the magnitude of the network output θ_{No} is equal to K. The output remains at $\pm K$ until the signal ϵ_N changes sign. From Figure 5.2-1

$$(\epsilon_N)_t = (E_2)_t - [\text{sgn}(\theta_{No})_{t-h}] K \quad (5.2-3)$$

and $(\epsilon_N)_t$ changes sign when

$$E_2 = [\text{sgn}(\theta_{No})_{t-h}] K \quad (5.2-4)$$

When the output saturates at $\pm K$, the integrator input ϵ_N must have the same sign as $(\pm K)$. Therefore, E_2 must also have the same sign as the output and a magnitude greater than K during saturation. Equation (5.2-4) representing the conditions for the next discontinuity may be written as

$$|E_2| = K \quad (5.2-5)$$

From Equations (3.3-2), (3.3-3), and (3.3-4)

$$(E_2)_t = (\theta_i)_t + \beta (\dot{\theta}_i)_t \quad (5.2-6)$$

$$(\dot{E}_2)_t = (\dot{\theta}_i)_t + \beta (\ddot{\theta}_i)_t \quad (5.2-7)$$

$$(\ddot{\theta}_i)_t = 2A_3 + 6A_4 h \quad (5.2-8)$$

The predicted time to the next discontinuity h' is the positive time until $|E_2| = K$ computed from $(E_2)_t$ and $(\dot{E}_2)_t$. At the discontinuity the operation of the network switches to mode 1.

For the special case when $\tau = 0$, form B in Figure 5.2-1, the input to the limiter is defined by Equations (5.2-4) and (5.2-7) and the discontinuities are defined by Equation (5.2-5) as a result the predicted time to the next discontinuity h' is computed as for mode 2 operation. The network output (θ_{on}) is the minimum of $|E_2|$ and K with the sign of E_2 .

$$(\theta_{on}) = [\text{sgn } E_2] \text{Min}(|E_2|, K) \quad (5.2-9)$$

As noted above the output for mode 2 operation is K with the sign of E_2 . Since $|E_2|$ is always greater than K , Equation (5.2-9) also holds for mode 2 operation and the same equation and logic can be used for mode 2 and $\tau = 0$.

5.3 LIMITING NETWORKS

The limiters used in networks N9 and N10 are simple limiters defined by Equation (5.2-9). It should be noted that the saturation points are not predicated and treated as discontinuities. It is assumed that the integration of the dynamic equations can tolerate the resultant discontinuity in the slope of control torque. It should also be noted that the lead-lag networks N1 through N8 with $\tau = \beta = 0$ reduce to a limiters for which the saturation levels are predicated and treated as discontinuities.

6. TORQUE MODULATORS

Each of the nine control systems incorporates a torque modulator or controller which modulates the control torque as a function of the input error signal. The program provides for six possible types of modulators which are incorporated into a single generalized modulator package, called (PLSMØD) in the digital program. Although all six types are not used in every control system the generalized package is used for all nine systems. The equations and logic required for the generalized modulator routine will be established in this section.

6.1 MODULATOR DESCRIPTIONS

Consider the general torque modulator for use with any of the nine control systems, Figure 6.1-1. The modulator is defined by four

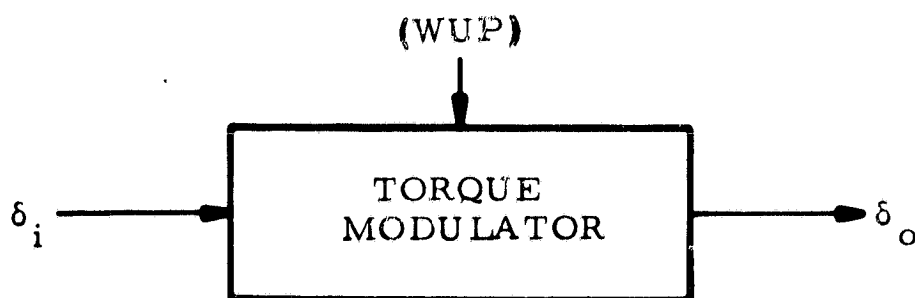
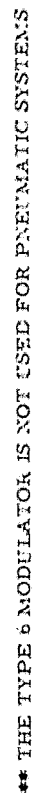


Figure 6.1-1
General Torque Modulator

parameters, type, lag time constant (τ), feedback constant (α), and the hysteresis or slope constant (H). The input signal (δ_i) is the sum of the network signals patched to the control system together with any



6-2

closed loop feedback signals. The input signal (WUP) is the reaction wheel unloading signal and is applied only in the pneumatic control systems. The purpose of the modulator is to determine, at time t , an expression for the normalized control torque (δ_o) over the interval (t) to $(t + h'')$. In addition to the expression for the output over the next integration interval, the program must retain information pertaining to the output over the previous integration interval.

A summary of the equations and logic required for all six modulator types is given in Figure 6.1-2. The first step in the solution for the output is a cubic curve fit to the input signal (δ_i). The polynomial coefficients A_1 through A_4 and the present value of $(\delta_i)_t$ are retained for later use.

6.1.1 The Type 1 Modulator

The type 1 modulator is a saturating linear modulator with a deadzone. The characteristics are shown in Figure 6.1-3, and are defined by the single input parameter H . The deadzone is normalized to plus and minus unity by adjusting the patch board gain at the input of the control system. The output is normalized to the saturation level and $1/H$ is the gain in the linear region.

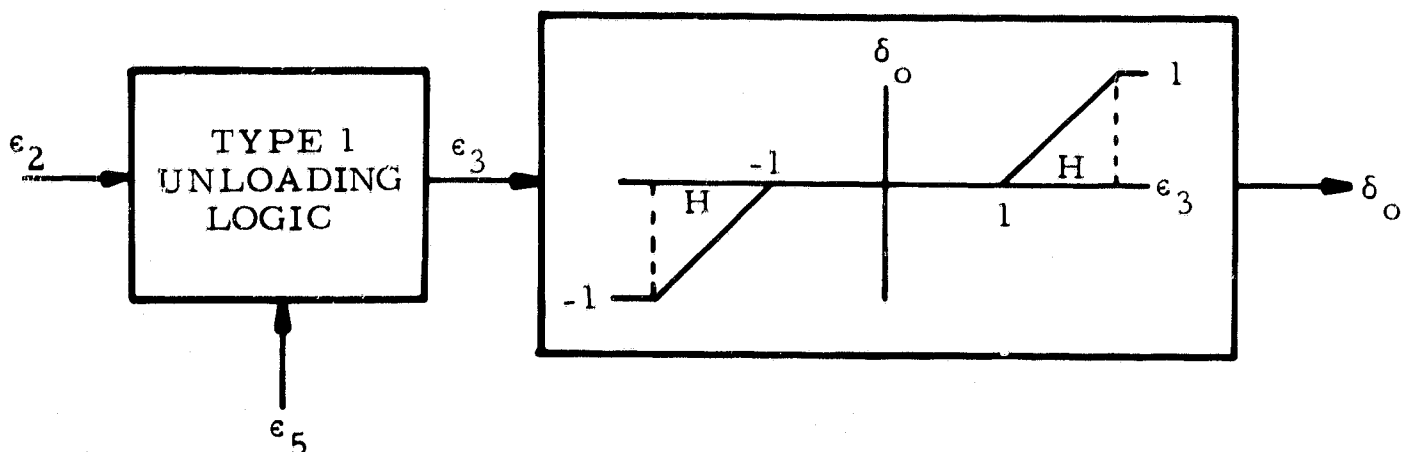


Figure 6.1-3
The Type 1 Torque Modulator

The type 1 modulator is the only modulator combining reaction wheel unloading and a linear switching function. As a result, a special unloading logic is used. Basically a non-zero unloading signal $(\epsilon_5)_t$, at time, is subtracted from the input signal $(\epsilon_2)_t$. If the result is less than unity the unloading signal is ignored and $(\epsilon_3 = \epsilon_2)$. If the resulting signal is greater than unity the output is set at the saturation level. The unloading logic is mechanized as follows:

- 1) If $(\epsilon_5)_t$ is zero set $(\epsilon_3)_t = (\epsilon_2)_t$ and $(\dot{\epsilon}_3)_t = (\dot{\epsilon}_2)_t$. If $(\epsilon_5)_t$ is non-zero compute $(\epsilon_2)_t - (\epsilon_5)_t$.
- 2) If the magnitude of $(\epsilon_2)_t - (\epsilon_5)_t$ is less than unity set $(\epsilon_3)_t = (\epsilon_2)_t$ and $(\dot{\epsilon}_3)_t = (\dot{\epsilon}_2)_t$.
- 3) If the magnitude of $(\epsilon_2)_t - (\epsilon_5)_t$ is greater than unity set $(\epsilon_3)_t$ to a large number with the sign of $(\epsilon_2)_t - (\epsilon_5)_t$ and set $(\dot{\epsilon}_3)_t$ to zero.

The value of $(\epsilon_3)_t$ is used to determine the output for the next time interval and is used in conjunction with $(\dot{\epsilon}_3)_t$ to predict the positive time to the next discontinuity. The discontinuities occur at (ϵ_3) equal to (1.0), (-1.0), $(1 + H)$, and $(1 - H)$, Figures (6.1-2) and (6.1-3).

The modulator output may have five states, positive saturation ($\delta_o = 1$), negative saturation ($\delta_o = -1$), in the deadzone ($\delta_o = 0$), and may be in the positive or negative linear region. When the output is in the linear region an expression for δ_o as a function of time (p) will be required. From Figures (6.1-2) and (6.1-3)

$$\delta_o = \frac{1}{H} [\epsilon_3 - \text{sgn } \delta_o \cdot 1] \text{ (Linear region)} \quad (6.1-1)$$

where

$$\epsilon_3 = A_1 + A_2 p + A_3 p^2 + A_4 p^3$$

6.1.2 The Type 2 Modulator

The type 2 modulator is a bang-bang controller with hysteresis between zero and 100%. The characteristics are shown in Figure 6.1-4, and are defined by the single input parameter H . The deadzone for zero hysteresis is normalized to plus and minus unity by adjusting the patch board gain at the input of the control system. The output is again normalized to unity and may only have values of ± 1.0 and zero. From Figures (6.1-2) and (6.1-4)

$$(\epsilon_3)_t = (\epsilon_2)_t - (\epsilon_5)_t \quad (6.1-2)$$

$$(\dot{\epsilon}_3)_t = (\dot{\epsilon}_2)_t - (\dot{\epsilon}_5)_t \quad (6.1-3)$$

where ϵ_5 is the reaction wheel unloading signal present in pneumatic control systems.

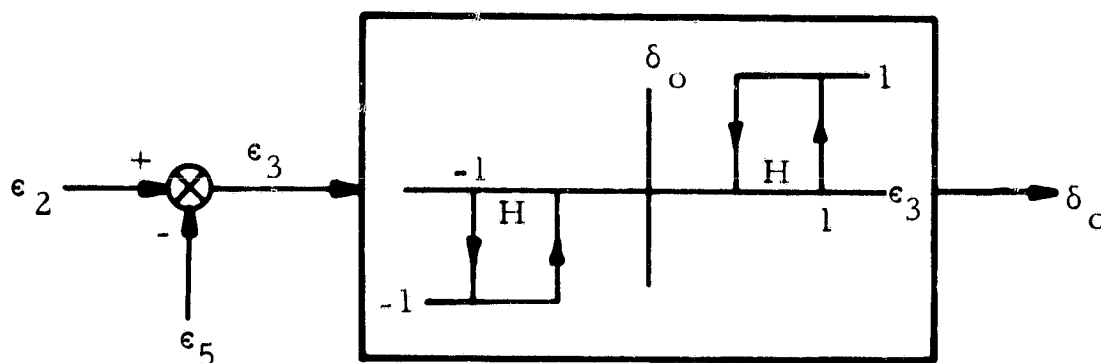


Figure 6.1-4

The Type 2 Modulator

For the type 2 modulator ϵ_2 is equal to the modulator input signal δ_i and

$$(\epsilon_3)_t = (\delta_i)_t - (\epsilon_5)_t \quad (6.1-4)$$

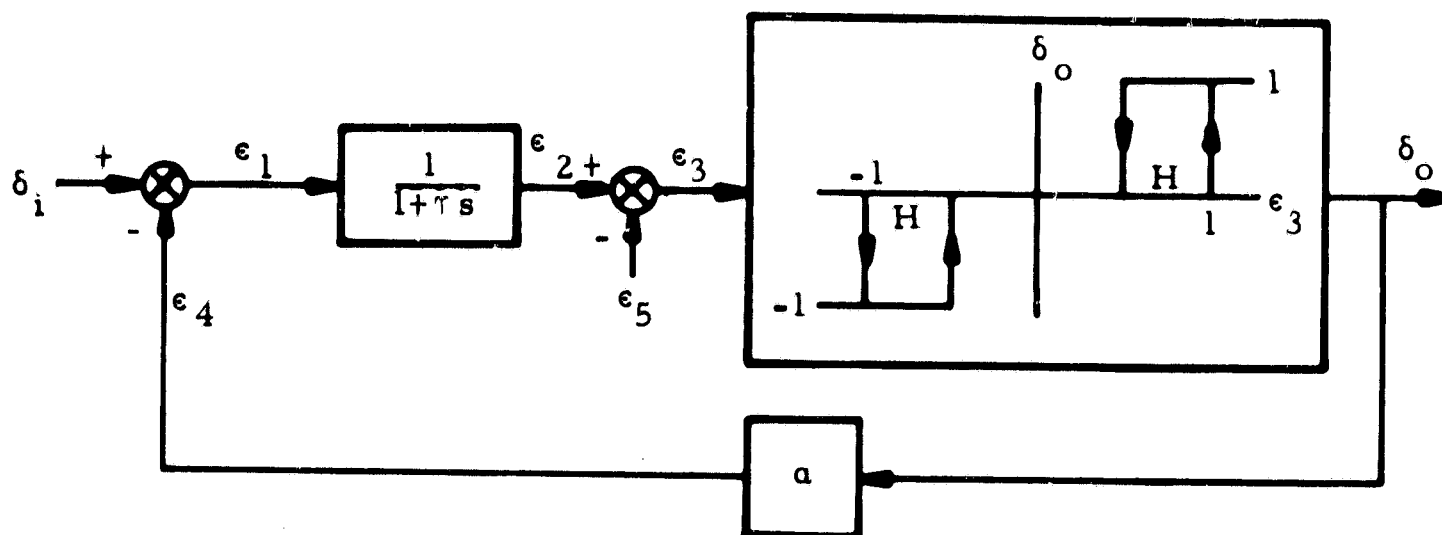
The curve fits to δ_i is only used to determine $(\delta_i)_t$. The value of $(\epsilon_3)_t$ is used to determine the output for the next time interval and is used in conjunction with $(\dot{\epsilon}_3)_t$ to predict the positive time to the next discontinuity. The discontinuities occur at (ϵ_3) equal to (1.0), (-1.0), (1-H), and (H-1), Figures (6.1-2) and (6.1-4). It should be noted that the hysteresis ratio H must be in the range $(0 \leq H \leq 1.0)$.

6.1.3 The Type 3 Modulator

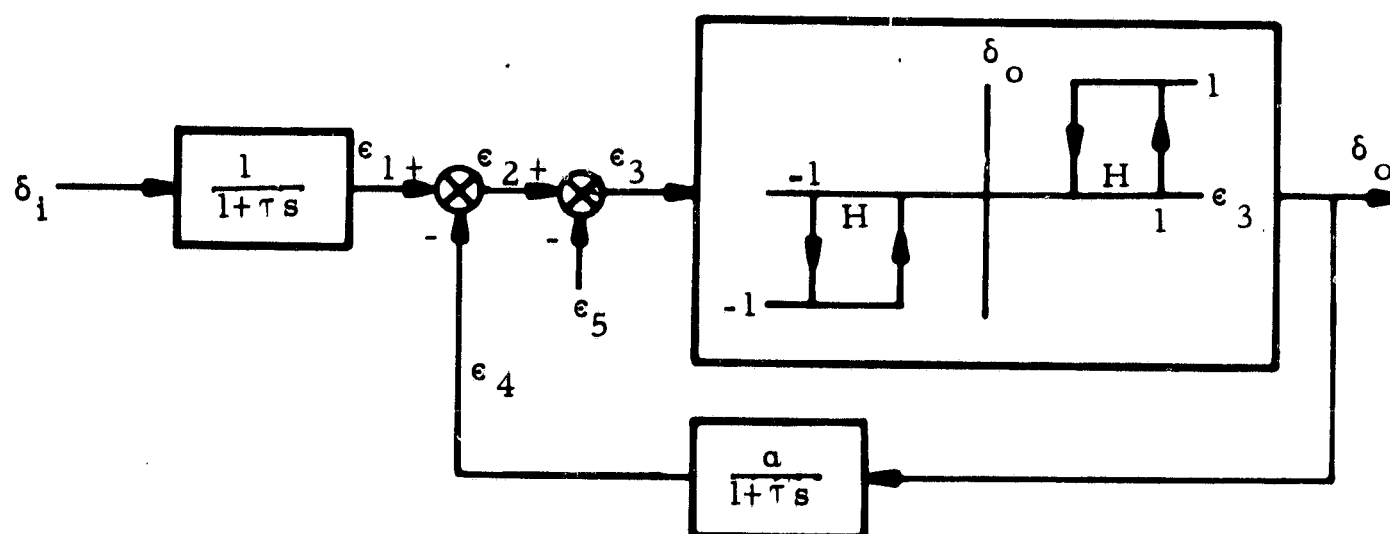
The type 3 modulator is a derived rate pulse modulator with the lag in both the input and feedback paths. The characteristics are shown in Figure 6.1-5(a) and are defined by the input parameters τ , α , and H. Since the integration is stopped and restarted at each discontinuity the modulators shown in Figures (6.1-5a) and (6.1-5b) are equivalent. Considering the system of Figure (6.1-5b), the signal ϵ_1 is obtained by passing the modulator input signal through a simple lag. Using Equations (3.4-1) through (3.4-4) the B's are computed from the A's obtained from the curve fit to δ_i . The value of $(\epsilon_1)_t$ and $(\dot{\epsilon}_1)_t$ are found from Equations (3.2-19) and (3.2-20).

$$(\epsilon_1)_t = [(\epsilon_1)_{t-h} - B_1] e^{-\frac{h}{\tau}} + F_2 [B_i, h] \quad (6.1-5)$$

$$(\dot{\epsilon}_1)_t = -\frac{1}{\tau} [(\epsilon_1)_{t-h} - B_1] e^{-\frac{h}{\tau}} + \dot{F}_2 [B_i, h] \quad (6.1-6)$$



(a)



(b)

Figure 6.1-5

The Type 3 Modulator

The value of $(\epsilon_4)_t$ may be computed using the equations of Section 3.4.1. Assume that ϵ_4 at the start of the last time step, $(\epsilon_4)_{t-h}$, has been retained

$$(\epsilon_4)_t = (\epsilon_4)_{t-h} e^{-\frac{h}{\tau}} + a \delta_{o-1} (1 - e^{-\frac{h}{\tau}}) \quad (6.1-7)$$

$$(\dot{\epsilon}_4)_t = \frac{1}{\tau} [a \delta_{o-1} - (\epsilon_4)_{t-h}] e^{-\frac{h}{\tau}} \quad (6.1-8)$$

where δ_{o-1} is the constant output during the last time step. Then from Figure (6.1-5b)

$$(e_2)_t = (\epsilon_1)_t - (\epsilon_4)_t \quad (6.1-9)$$

$$(\dot{e}_2)_t = (\dot{\epsilon}_1)_t - (\dot{\epsilon}_4)_t \quad (6.1-10)$$

The remaining portion of the modulator is identical to the type 2 modulator. If the values of $(\epsilon_2)_t$ and $(\dot{e}_2)_t$ are substituted into Equations (6.1-2) and (6.1-3) the type 2 modulator equations may be used to obtain the output and to predict the discontinuities for the type 3 modulator, Figure (6.1-2).

6.1.4 The Type 4 Modulator

The type 4 modulator is a derived rate pulse modulator with the lag only in the feedback path. The characteristics are shown in Figure 6.1-6 and are defined by the input parameters τ , α , and H . If the lag in the input path is eliminated the type 3 modulator is identical

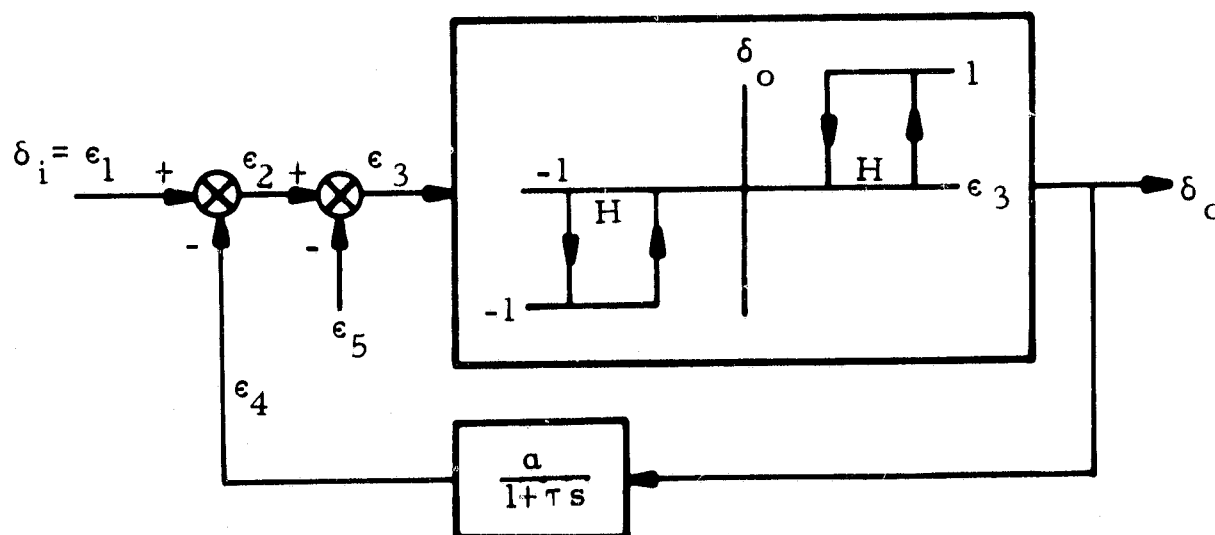


Figure 6.1-6
The Type 4 Modulator

to the type 4. If $(\epsilon_1)_t$ and $(\dot{\epsilon}_1)_t$ are set equal to $(\delta_i)_t$ and $(\dot{\delta}_i)_t$ the type 3 equations and logic can be used for the type 4 modulator, Figure (6.1-2).

6.1.5 The Type 5 Modulator

The type 5 modulator is a simple bang-bang controller without hysteresis. The characteristics are shown in Figure 6.1-7 and are defined simply by type number. The deadzone is normalized to plus and minus unity by the adjusting of the patch board gain at the input of the control system. The output is normalized to unity and may only have

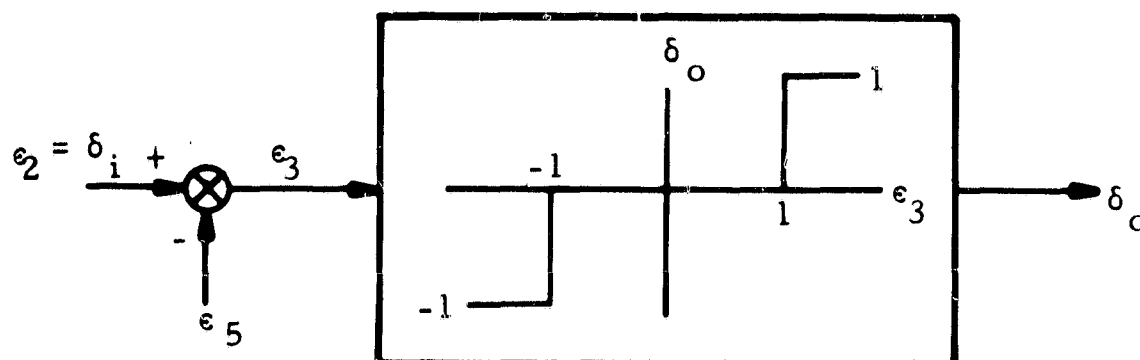


Figure 6.1-7
The Type 5 Modulator

values of ± 1 and zero. The value of ϵ_3 and $\dot{\epsilon}_3$ are required only at time t , therefore,

$$(\epsilon_3)_t = (\delta_i)_t - (\epsilon_5)_t \quad (6.1-11)$$

$$(\dot{\epsilon}_3)_t = (\dot{\delta}_i)_t - (\dot{\epsilon}_5)_t \quad (6.1-12)$$

The value of $(\epsilon_3)_t$ is used to determine the output for the next time interval and is used in conjunction with $(\dot{\epsilon}_3)_t$ to predict the positive time to the next discontinuity. The discontinuities occur at $(\epsilon_3)_t$ equal to ± 1 .

The type 5 modulator is a special controller which has been provided for future expansion of the program. It cannot be used in the present version and selection of a type 5 modulator is equivalent to selecting type zero. The modulator is intended for use in impulsive control systems wherein the control torque, reaction wheel torque, or array torque is applied as an impulse while the integration is stationary at time t . The system momentum is the redistributed and the integration is continued. The system has been proposed as a method of reducing the program running time at some sacrifice in accuracy.

6.1.6 The Type 6 Modulator

The type 6 modulator is a saturating linear controller without a deadzone. It is not used in connection with pneumatic control systems and is available only for a special reaction wheel system and in array drives. The characteristics are shown in Figure 6.1-8 and are defined by the single parameter H . The output is normalized to the saturation level

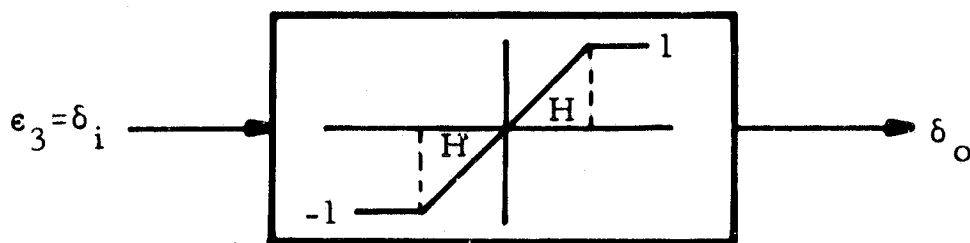


Figure 6.1-8
The Type 6 Modulator

and has three possible states, positive saturation ($\delta_o = 1$) negative saturation ($\delta_o = -1$), and operation in the linear region. Since the modulator will not be used with pneumatic systems there is no reaction wheel unloading signal and

$$(\epsilon_3)_t = (\delta_i)_t \quad (6.1-13)$$

$$(\dot{\epsilon}_3)_t = (\dot{\delta}_i)_t \quad (6.1-14)$$

The value of $(\epsilon_3)_t$ is used to determine the state of the output for the next time interval and is used in conjunction with $(\dot{\epsilon}_3)_t$ to predict the positive time to the next discontinuity. The discontinuities occur at ϵ_3 equal to $\pm H$, Figures (6.1-2) and (6.1-8). For operation in the linear region an expression for the output (δ_o) as a function of time p is required. From Figures (6.1-2) and (6.1-8)

$$\delta_o = \frac{1}{H} \epsilon_3 \quad (\text{Linear region}) \quad (6.1-15)$$

where

$$\epsilon_3 = A_1 + A_2 p + A_3 p^2 + A_4 p^3$$

6.2 TORQUE MODULATOR OPERATION

For a given spacecraft configuration there will be a torque modulator for each control system used. Each of these modulators will operate as a single type (1 through 6) which remains fixed for a given run. As a result each control system will require only one set of polynomial coefficients, one signal rate, one set of input parameters (τ , α , H) and one set of output data. The signal variations δ_i , and ϵ_1 through ϵ_4 are not carried past the modulator switching functions for types 2, 3, 4, and 5. Modulator outputs as a function of time are only required when the type 1 or 6 modulators is in its linear regions of operation. For all other modes of operation the output is constant over the next time interval.

7.0 REACTION WHEEL CONTROL SYSTEMS

Since the reaction wheel unloading signals are required for computations in the pneumatic system, the reaction wheels must be considered first. The generalized reaction wheel system, of which four are provided, is shown in Figure 7.1-1. Each reaction wheel system incorporates a closed loop feedback path through which a function of the wheel momentum is feedback to the input of the torque modulator. This loop is used to control or regulate the wheel speed for a given system. In addition, a single combination feedback loop is provided through which a selected combination of the momenta of the four possible wheels is feed back to the modulator inputs of those wheels used in forming the combination. This loop is used to control or regulate the relative speed of the wheels.

7.1 REACTION WHEEL FEEDBACK LOOPS AND MODULATOR INPUT

As a result of the closed loop speed control paths, it is necessary to use the control system output at time t to compute the input at time t . If the output cannot be expressed as a closed form function of the input, it is usually necessary to solve the system by iteration. However, in the case of the digital program being considered here, a sufficiently accurate predicted value of the output is available for use in the feedback loop computations. As noted in Section 2 the output data passed to the dynamics must be in the form of an expression for wheel momentum as a function of time. The expression for H_w used during the time step from $(t-h)$ to (t) is based upon a cubic curve fit to $(H_w)_{t-h}$ and the previous three values. During integration the curve is extended to time t to give a predicted value of $(H_w)_t$. This predicted value of $(H_w)_t$ will be used to compute the control system inputs due to the feedback loops at time t . The control system equations are then used to compute a corrected value of $(H_w)_t$ based upon the feedback inputs and the positional and velocity

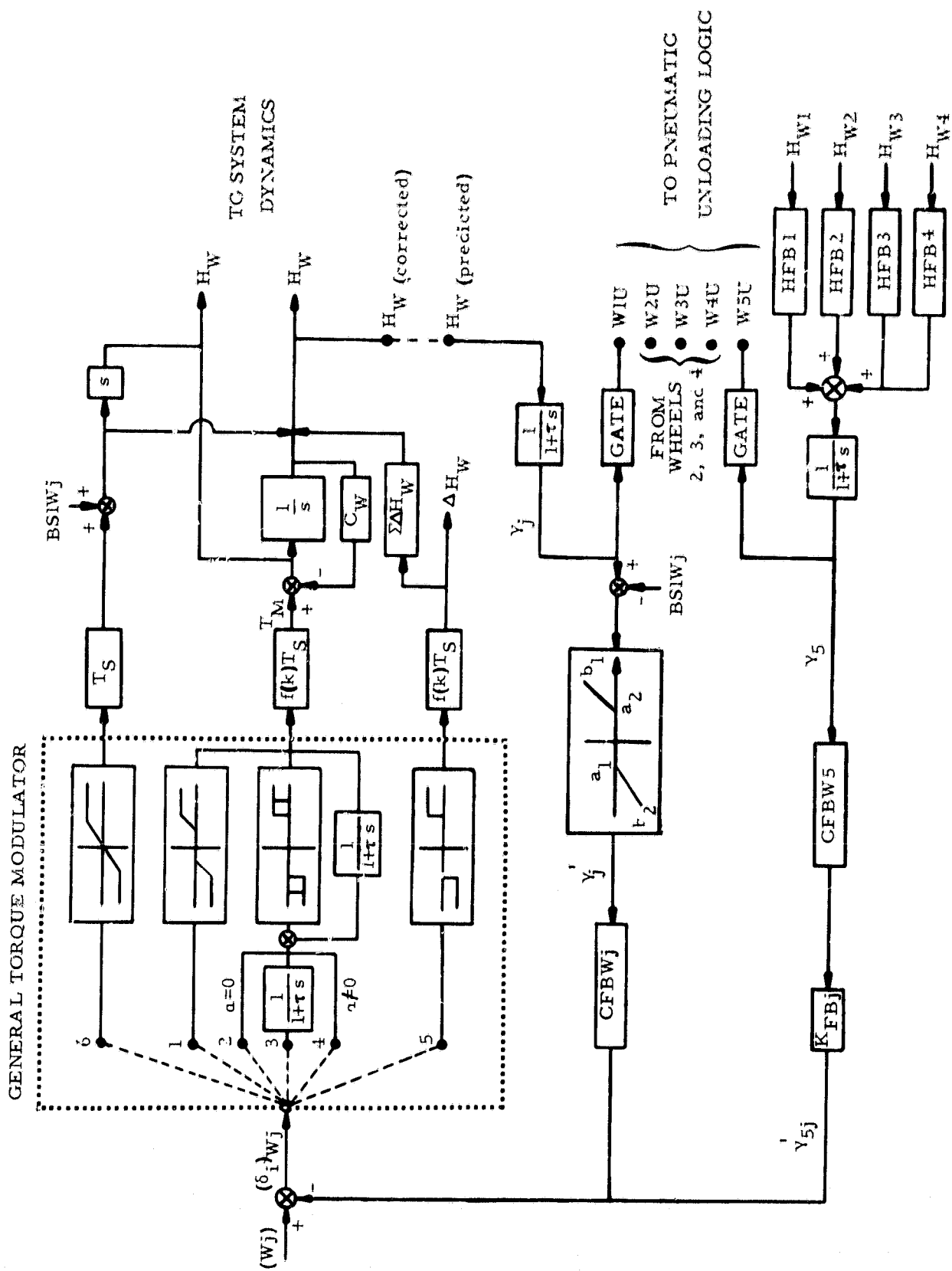


Figure 7.1-1
Reaction Wheel Control Systems

errors at time t . The predicted value is then replaced by the corrected value. Referring to Figure 7.1-1, the momentum of the j^{th} wheel at time t , $(H_{wj})_t$ predicted, passes through a simple lag to form the signal γ_j . This lag is intended to represent the dynamics of the wheel momentum or speed measuring device. Likewise the combined momentum used in the combination feedback loop is passed through a similar lag to form the signal γ_5 . The five lagged feedback signals γ_1 through γ_5 are passed through the sampling gates to form the five wheel unloading signals WIU through W5U.

7.1.1 Speed Control Loops

The first step in the solution of the j^{th} wheel speed control loop is a cubic curve fit to the wheel momentum at time t . It should be noted that the value of $(H_{wj})_t$ used in the curve fit is the predicted value and the three past values used are corrected values. The four polynomial coefficients A_1 through A_4 and the value of $(H'_{wj})_t$ are stored for use in the solution of the tachometer lag. However before proceeding it is necessary to stop and consider the input to the combination feedback loop. Since this loop may also contain a lag a cubic curve fit to the combination signal may also be required. The polynomial coefficients representing the combined signal can best be obtained by forming a combination of the coefficients for the component signals. From Figure (7.1-1) the combination feedback input is given by

$$(H_{wc}) = \sum_j [HFBj] (H_{wj}) \quad (7.1-1)$$

and

$$(H'_{wc}) = \sum_j [HFBj] (H'_{wj}) \quad (7.1-2)$$

where $(HFBj)$ are program input constants.

The polynomial coefficients for the combination path are then

$$(A's)_c = \sum_j [HFB_j] (A's)_j \quad (7.1-3)$$

and the time rate of change of the combination momentum is

$$(H'_{wc})_t = \sum_j [HFB_j] (H'_{wj})_t \quad (7.1-4)$$

The storage locations for the combination path, $(A's)_c$ and $(H'_{wc})_t$, are set to zero at the start of each pass through the control system. As soon as the $(A's)$ and $(H'_{wj})_t$ for the j^{th} wheel have been computed, the lag time constant in the combination path is checked against zero. If the time constant is non-zero, the coefficients for the j^{th} wheel are multiplied by (HFB_j) and added to the coefficient stored for the combination path, Equations (7.1-3) (7.1-4). When the computations for the four possible speed control loops have been completed, the correct polynomial expressions for the combination path will have computed and stored. If the time constant in the combination path is zero, the curve fit to the combination signal is not required and the program can proceed with the computations for the j^{th} speed control loop.

Returning to the j^{th} speed control loop and the computations for the tachometer lag when $\tau \neq 0$, the $(B's)$ are computed from the $(A's)$ using Equations (3.4-1) through (3.4-4). The feedback signal $(\gamma_j)_t$ and the rate $(\dot{\gamma}_j)_t$ are then given by

$$(\gamma_j)_t = [(\gamma_j)_{t-h} - B_1] e^{-\frac{h}{\tau}} + F_2[B_1, h] \quad (\tau \neq 0) \quad (7.1-5)$$

$$(\dot{\gamma}_j)_t = -\frac{1}{\tau} [(\gamma_j)_{t-h} - B_1] e^{-\frac{h}{\tau}} + \dot{F}_2[B_1, h] \quad (\tau \neq 0) \quad (7.1-6)$$

For $\tau = 0$

$$(\gamma_j)_t = (H_{wj})_t \quad (\tau \neq 0) \quad (7.1-7)$$

$$(\dot{\gamma}_j)_t = (H'_{wj})_t \quad (\tau \neq 0) \quad (7.1-8)$$

The values $(\gamma_j)_t$ and $(\dot{\gamma}_j)_t$ are wheel unloading circuit inputs and must be retained for later use.

Again referring to Figure 7.1-1, a constant bias momentum $HBSW_j$ is subtracted from γ_j and the resulting signal is passed through the general feedback processor shown in Figure 7.1-2.

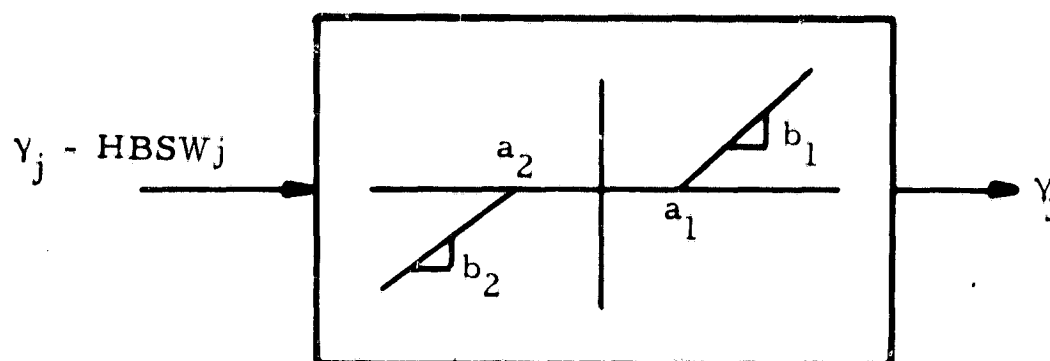


Figure 7.1-2
Reaction Feedback Processor

The breakpoints a_1 and a_2 and the slopes b_1 and b_2 are arbitrary. The output at time t is found from $[(\gamma_j)_t - HBSW_j]$, and $(\dot{\gamma}_j)_t$ is used to predict the time at the breakpoints.

The final feedback signal from the speed control loop at time t is $[CFBW_j](\gamma_j')_t$, where γ_j' is the output of the feedback processor and $(CFBW_j)$ is a program input constant.

7.1.2 Combination Feedback Loop

If the lag time constant in the combination feedback loop is not zero, the combined input momentum will have been computed and stored during the solution of the speed control loops, Equations (7.1-3) and (7.1-4). The data will be in the form of the polynomial coefficients (A's) and the time rate of change of the signal $(H'_{w5})_t$ at time t , and the (B's) for a single lag can be computed from Equations (3.4-1) through (3.4-4). The combination feedback signal $(\gamma_5)_t$ and the rate $(\dot{\gamma}_5)_t$ are

then given by

$$(\gamma_5)_t = [(\gamma_5)_{t-h} - B_1] e^{-\frac{h}{\tau}} + F_2 [B_1, h] \quad (7.1-9)$$

$$(\dot{\gamma}_5)_t = -\frac{1}{\tau} [(\gamma_5)_{t-h} - B_1] e^{-\frac{h}{\tau}} + \dot{F}_2 [B_1, h] \quad (7.1-10)$$

For $\tau = 0$

$$(\gamma_5)_t = \sum_j [HFBj] (H_{wj})_t \quad (7.1-11)$$

$$(\dot{\gamma}_5)_t = \sum_j [HFBj] (\dot{H}_{wj})_t \quad (7.1-12)$$

The values of $(\gamma_5)_t$ and $(\dot{\gamma}_5)_t$ are wheel unloading circuit inputs and must be retained for later use.

The combination feedback signal feed back to the input of the j^{th} reaction wheel system is given by

$$(\gamma'_{5j})_t = K_{FBj} [CFBW5] (\gamma_5)_t \quad (7.1-13)$$

where

$$\begin{aligned} K_{FBj} &= 1.0 \text{ if } HFBWj \neq 0 \\ K_{FBj} &= 0 \text{ if } HFBWj = 0 \end{aligned} \quad (7.1-14)$$

7.1.3 Torque Modulator Input

The net input signal to the j^{th} reaction wheel torque modulator (δ_{iwj}) is the sum of the signal from the compensation networks and the negative feedback signals, Figure 7.1-1. From Figure 5.1.1 the total error signal from the networks at time t is $(Wj)_t$. Then from Figure 7.1-1,

$$(\delta_{iwj})_t = (Wj)_t - (CFBWj) (\gamma'_j)_t - (\gamma'_{5j})_t \quad (7.1-15)$$

$(\delta_{ijw})_t$ is the present value of the input signal to the generalized torque modulator described in Section 6. The first computational step in the solution for the output torque is a curve fit to the present and three past values of (δ_{ijw}) , reference Section 6.1. The use of the curve fit introduces the question of the accuracy of the past values. A portion of the present value $(\delta_{ijw})_t$ results from the feedback signal which is based upon the predicted value of wheel momentum. If this value is stored as the first past value for the next step, all four values used in the curve fit are based on predicted momenta. After the corrected value of wheel momentum has been obtained it could be used to update the present value of (δ_{ijw}) before it is stored as a past value. It is assumed that the predicted momentum is sufficiently accurate and this additional refinement has not been incorporated in the present version of the program.

7.2 MOTOR SPEED- TORQUE CHARACTERISTICS

The relationship between the output torque and the speed of a typical reaction wheel drive motor is shown in Figure 7.2-1. The solid line represents the relationship when the magnitude of the velocity is increasing while the dashed line represents the relationship when the magnitude is decreasing. A normalized relationship is used in the digital program. The motor torque is normalized to the zero speed stall torque and the speed is normalized to the maximum no load speed. The normalized relationship is shown in Figure (7.2-2). Since the reaction wheel inertia is constant the normalized speed k is also a normalized momentum. The normalized torque $f(k)$ is assumed to be

$$f(k) = 1.0 \quad (\delta_o, H_w \text{ opposite sign}) \quad (7.2-1)$$

$$f(k) = 1.0 + C_1 k + C_2 k^2 + C_3 k^3 + C_4 k^4 \quad (\delta_o, H_w \text{ same sign}) \quad (7.2-2)$$

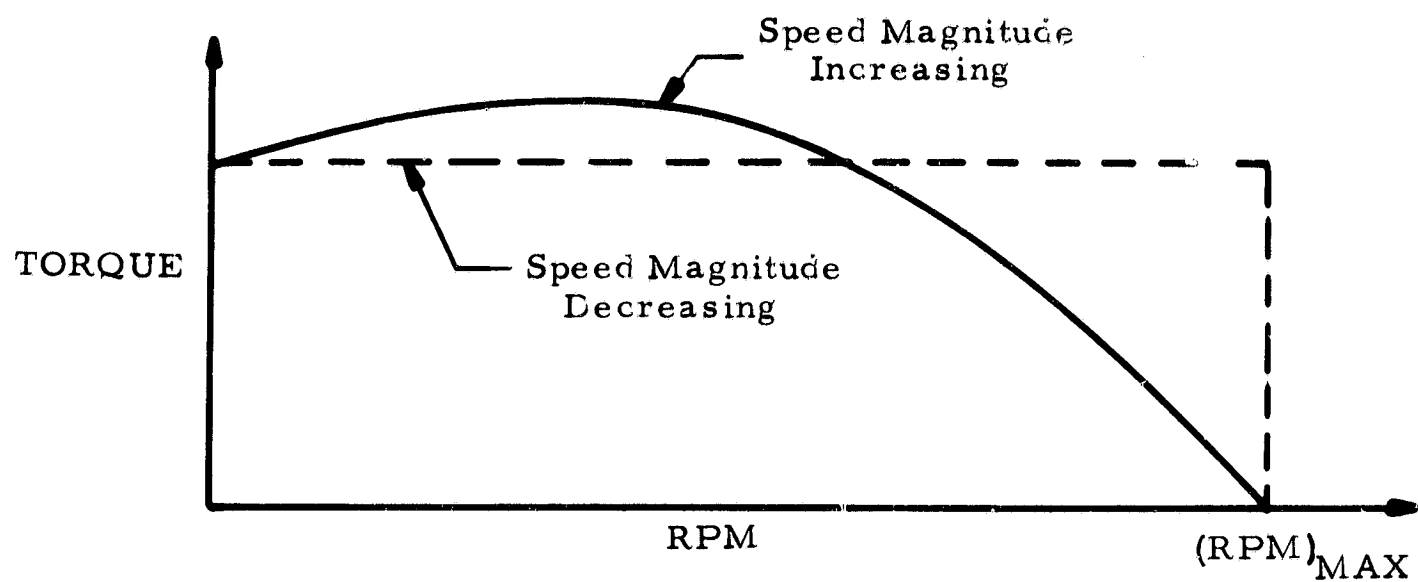


Figure 7.2-1
Typical Speed-Torque Relationship

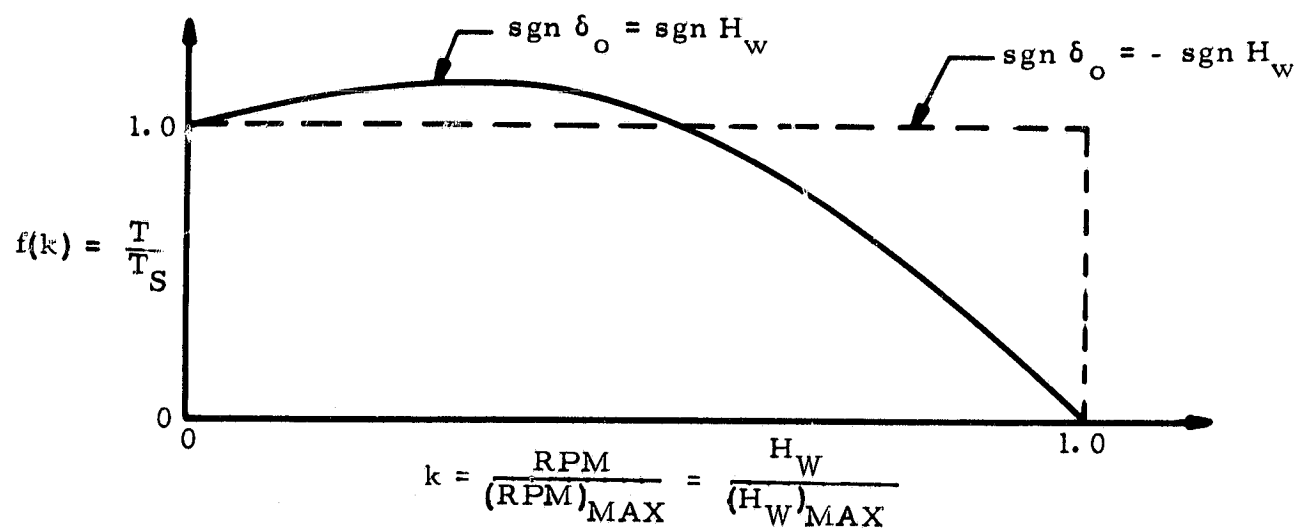


Figure 7.2-2
Normalized Speed-Torque Relationship

where C_1 and C_4 are program input constants. It is assumed that the same coefficients apply for all four reaction wheel motors.

The value of $f(k)$ is assumed to be constant over a given integration interval. Since the predicted value of $(H_w)_t$ is stored at the time k is computed, the value of $f(k)_t$ is based upon predicted values, and the value used for a given interval is the value at the beginning of that interval.

7.3 WHEEL CHARACTERISTICS

For computational purposes the generalized reaction wheel system shown in Figure 7.1-1 may be represented by the block diagram of Figure 7.3-1. The modulator input (δ_i) is covered in Section 7.1.3 and is given by Equation (7.1-15). The generalized torque modulator is covered in Section 6.0 and is shown in Figure (6.1-2). Since the modulator output (δ_o) is normalized with respect to the motor stall torque, it is multiplied by the stall torque (T_S) and the factor $f(k)$ to give the motor torque (T_M) . The speed-torque factor $f(k)$ is covered in Section 7.2. In this section the equations and logic required to transform the motor torque (T_M) into wheel momentum and time rate of change in wheel momentum will be considered.

7.3.1 General Torque Integration

From Figure 7.1-1 the general transformation from modulator output to wheel momentum for modulator types 1, 2, 3, and 4 is shown in Figure 7.3-2. The output (H_w) is given by

$$H_w = \frac{T_M}{C_w + s} = \frac{1}{C_w} \frac{T_M}{(1 + \frac{s}{C_w})} \quad (7.3-1)$$

where C_w is the coefficient of windage and friction.

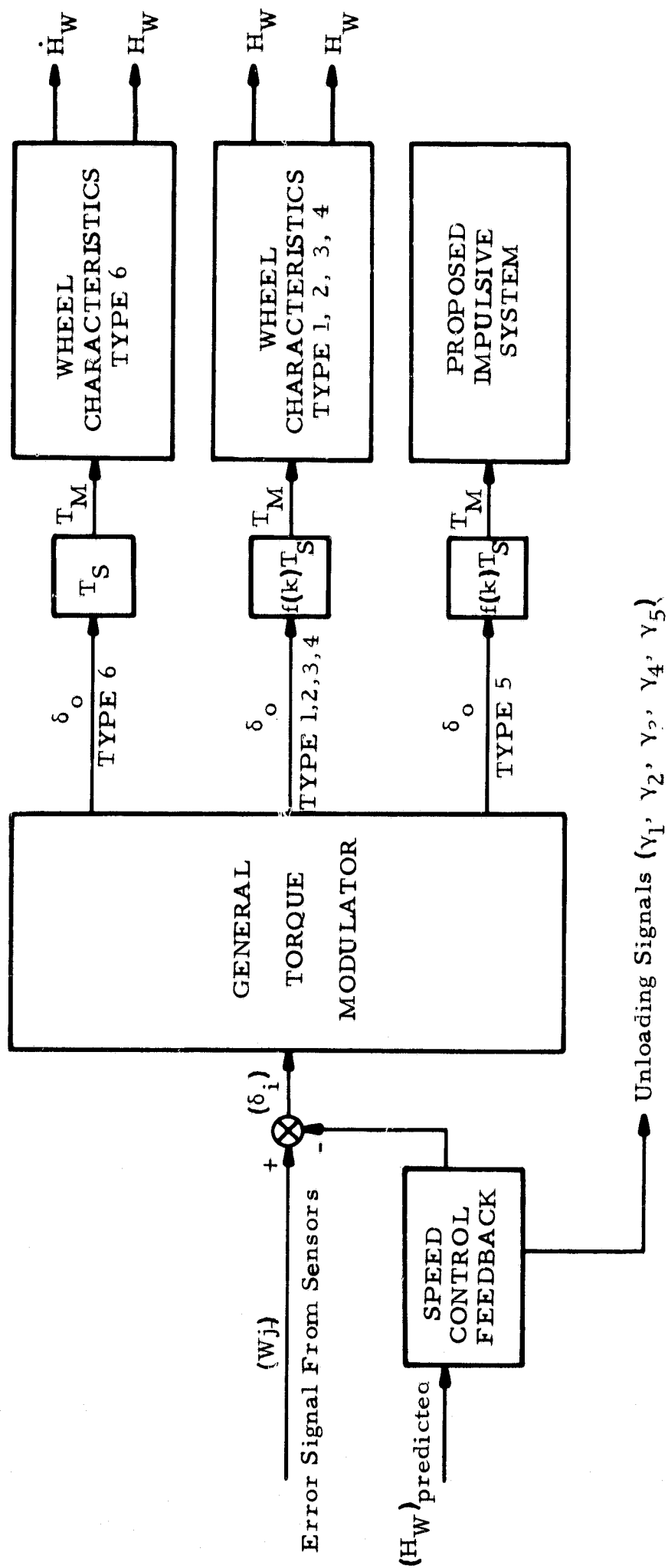


Figure 7.3-1
Block Diagram
Reaction Wheel System

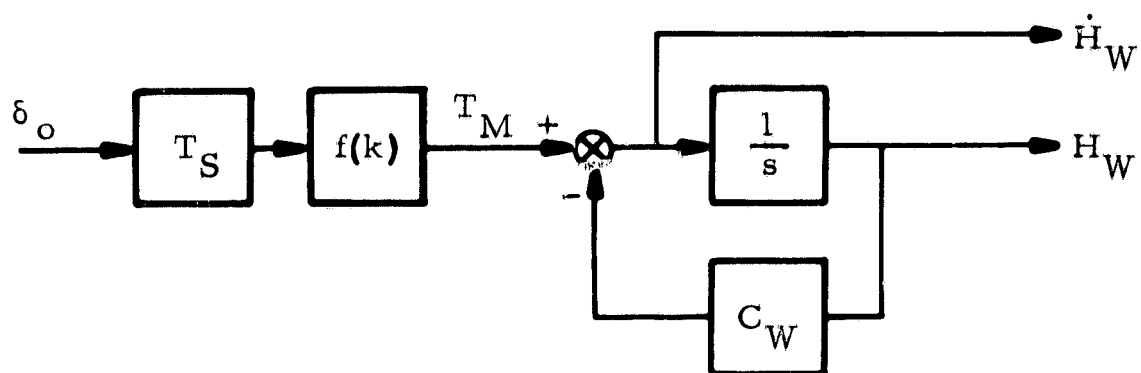


Figure 7.3-2
General Torque Integrator

Let $\tau_w = \frac{1}{C_w}$ then

$$H_w = \frac{\tau_w T_M}{(1 + \tau_w s)} \quad (7.3-2)$$

For $C_w = 0$

$$H_w = \frac{T_M}{s} \quad (7.3-3)$$

For $C_w \neq 0$ the wheel momentum is given by passing $T_M \tau_w$ through a simple lag network. For $C_w = 0$ the output is a single integration of T_M . The equations for H_w and $\dot{H}_w$ for each type of modulator are summarized in Figure (7.3-3). The detailed derivations are given below.



Figure 7.3-3 Reaction Wheel Equations

7.3.2 Type 1 Modulator in the Linear Region

The Type 1 modulator is a saturating linear controller with a deadzone, Figure 6.1-3. When saturated or when operating within its deadzone, it has the same characteristics as the bang-bang controller for types 2, 3, and 4. When operating in its linear region it is the only controller which passes a complex output function to the general torque integrator of Figure 7.3.2.

As noted in Section 2 the purpose of the control system is to provide expressions for H_w and $\dot{H}_w$ as functions of time. In addition it must compute H_w (corrected) and update H_w (predicted) if the last step was in the linear region of the controller. Therefore, the program must first ask the following two questions, Figure 7.3-3.

1. Is δ_{o-1} in the linear region of operation
2. Is δ_o in the linear region of operation

Where δ_{o-1} is the controller output over the last time step and δ_o is the output over the next time step. If the answer to both questions is no, the controller output is either saturated or zero for the last step and for the next step. The present wheel momentum does not need to be updated (see Section 2) and the output for the next step can be computed using the equations for the bang-bang controller. If the answer to either question is yes the expression for δ_o must be passed to the torque integrator. From Equation (6.1-1) the output of the type 1 modulator in the linear region is

$$\begin{aligned}\delta_o &= \frac{1}{H} [\epsilon_3 - \text{sgn } \delta_o \cdot 1] \\ \epsilon_3 &= A_1 + A_2 p + A_3 p^2 + A_4 p^3\end{aligned}$$

Then

$$T_M = \frac{T_S^{f(k)}}{H} [\epsilon_3 - \text{sgn } \delta_o \cdot 1] \quad (7.3-4)$$

$$T_M = f(k) \frac{T_S}{H} (A_1 - \text{sgn } \delta_o \cdot 1) + \frac{T_S}{H} A_2 p + \frac{T_S}{H} A_3 p^2 + \frac{T_S}{H} A_4 p^3$$

$$\text{Let } A'_1 = \frac{T_S}{H} (A_1 - \text{sgn } \delta_o \cdot 1)$$

$$A'_2 = \frac{T_S}{H} A_2$$

$$A'_3 = \frac{T_S}{H} A_3 \quad (7.3-5)$$

$$A'_4 = \frac{T_S}{H} A_4$$

Then

$$T_M = f(k) [A'_1 + A'_2 p + A'_3 p^2 + A'_4 p^3] = f(k) F_2 [A'_i, p] \quad (7.3-6)$$

7.3.2.1 Without Windage and Friction ($C_w = 0$)

For $C_w = 0$ the wheel momentum is given by Equation (7.3-3) and

$$\dot{H}_w = T_M = f(k) F_2 [A'_i, p] \quad (7.3-7)$$

$$\dot{H}_w = f(k) [A'_1 + A'_2 p + A'_3 p^2 + A'_4 p^3]$$

Integrating ($p = 0$ at $t - h$)

$$(H_w) = (H_w)_{t-h} + A'_1 p + \frac{1}{2} A'_2 p^2 + \frac{1}{3} A'_3 p^3 + \frac{1}{4} A'_4 p^4$$

Let the integral of $F_2 [A'_i p] = F_3 [A'_i p]$

$$F_3 [A'_i p] = A'_1 p + \frac{1}{2} A'_2 p^2 + \frac{1}{3} A'_3 p^3 + \frac{1}{4} A'_4 p^4 \quad (7.3-8)$$

$$\text{Then } (H_w) = (H_w)_{t-h} + f(k) F_3 [A'_i p] \quad (7.3-9)$$

If the output during the last step (δ_{o-1}) was in the linear region of the modulator, the momentum at time t must be updated, therefore,

$$(H_w)_t (\text{corrected}) = (H_w)_{t-h} - f(k)_{-1} F_3 [A'_i, h] \quad (7.3-10)$$

where $f(k)_{-1}$ is the torque ratio $f(k)$ for the last step.

If the output during the next step (δ_o) is in the linear region of the modulator, the predicted value of H_w is found from Equation (7.3-9) for $p = h + h''$

$$(H_w)_{t+h''} = (H_w)_{t-h} + f(k) F_3 [A_i', (h + h'')]]$$

and in terms of $(H_w)_t$

$$(H_w)_{t+h''} = (H_w)_t + f(k) \left[F_3 [A_i', (h + h'')]] - F_3 [A_i', h] \right] \quad (7.3-11)$$

and from Equation (7.3-7)

$$(\dot{H}_w)_t = f(k) F_2 [A_i', (h + h'')] \quad (7.3-12)$$

Equations (7.3-11) and (7.3-12) represent the control system output for a type 1 modulator operating in the linear region with $C_w = 0$ ① in Figure 7.3-3. The output expressions, when the type 1 modulator is not in its linear region, ② in Figure 7.3-3, are identical to those for the types 2, 3, and 4 modulator developed in a later section.

7.3.2.2. With Windage and Friction ($C_w \neq 0$)

For $C_w \neq 0$ the wheel momentum is given by Equation (7.3-2) and the (B's) for a simplelag must be computed from the (A' 's), Equations (3.4-1) through (3.4-4). From Equation (3.2-16)

$$H_w = [(H_w)_{t-h} - \tau_w f(k) B_i] e^{-\frac{p}{\tau_w}} + \tau_w f(k) F_2 [B_i, p] \quad (7.3-13)$$

If the output during the last step (δ_{o-1}) was in the linear region of the momentum at time t must be updated. Therefore,

$$(H_w)_t (\text{corrected}) = [(H_w)_{t-h} - \tau_w f(k)_{-1} B_1] e^{-\frac{h}{\tau_w}} + \tau_w f(k)_{-1} F_2 [B_i, h] \quad (7.3-14)$$

If the output during the next step (δ_o) is in the linear region, the predicted value of H_w is found from Equations (7.3-13) for $p = h + h''$

$$(H_w)_{t+h''} = [(H_w)_{t-h} - \tau_w f(k) B_i] e^{-\frac{h+h''}{\tau_w}} + \tau_w f(k) F_2 [B_i, (h+h'')]]$$

and in terms of $(H_w)_t$

$$(H_w)_{t+h''} = \left[(H_w)_t - \tau_w f(k) F_2 [B_i, h] \right] e^{-\frac{h''}{\tau_w}} + \tau_w f(k) F_2 [B_i, (h+h'')]] \quad (7.3-15)$$

differentiating

$$(\dot{H}_w)_{t+h''} = -\frac{1}{\tau_w} \left[(H_w)_t - \tau_w f(k) F_2 [B_i, h] \right] e^{-\frac{h''}{\tau_w}} + \tau_w f(k) \dot{F}_2 [B_i, (h+h'')]] \quad (7.3-16)$$

Equations (7.3-15) and (7.3-16) represent the control system output for a type 1 modulator operating in the linear region with $C_w \neq 0$, (3) in Figure 7.3-3. The output expressions when the type 1 modulator is not in its linear region, (4) in Figure 7.3-3, are identical to those for the type 2, 3, and 4 modulators developed in the next section.

7.3.3 Non-Linear Type 1 and Types 2, 3, and 4

The output of the type 1 modulator when not in its linear region, and the type 2, 3, and 4 modulators can only have values of 1, 0, and -1. The motor torque is

$$T_M = \delta_o T_S f(k)$$

7.3.3.1 Without Windage and Friction ($C_w = 0$)

For $C_w = 0$ the wheel momentum is given by Equation (7.3-3) and

$$\dot{H}_w = \delta_o T_S f(k)$$

integrating

$$H_w = (H_w)_{t-h} + \delta_o T_S f(k) p \quad (7.3-17)$$

Since the past step cannot have been in the linear region of a modulator, the value of $(H_w)_t$ does not need to be updated. The predicted value for the next time step is found from Equation (7.3-17) for $p = h + h''$

$$(H_w)_{t+h} = (H_w)_{t-h} + \delta_o T_S f(k) (h + h'')$$

and in terms of $(H_w)_t$

$$(H_w)_{t+h''} = (H_w)_t + \delta_o T_S f(k) h'' \quad (7.3-18)$$

differentiating

$$\dot{(H_w)}_{t+h''} = \delta_o T_S f(k)$$

Equations (7.3-17) and (7.3-18) represent the control system output for $C_w = 0$, (2) in Figure 7.3-3.

7.3.3.2 With Windage and Friction ($C_w \neq 0$)

For $C_w \neq 0$ the wheel momentum is given by Equation (7.3-2) representing a simple lag network. However, the motor torque (T_M) is constant over any given time interval and the equations of Section 3.4.1 apply. Since the past step cannot have been in the linear region of a modulator, the value of $(H_w)_t$ does not need to be updated. The predicted value for the next time step is given by

$$(H_w)_{t+h''} [(H_w)_t - \tau_w f(k) \delta_o T_S] e^{-\frac{h''}{\tau}} + \tau_w f(k) \delta_o T_S \quad (7.3-19)$$

differentiating

$$\dot{(H_w)}_{t+h''} = -\frac{1}{\tau} [(H_w)_t - \tau_w f(k) \delta_o T_S] e^{-\frac{h''}{\tau}} \quad (7.3-20)$$

Equations (7.3-19) and (7.3-20) represent the control system output for $C_w \neq 0$, (4) in Figure (7.3-3).

7.3.4 Type 5 Modulator

The type 5 modulator is a simple bang-bang controller without hysteresis, Figure 6.1-7. It has been provided for future use in a proposed impulsive control system. The logic required to produce the wheel impulse to be applied at time t is shown in Figure 7.1-1. The required equations are given in Figure 7.3-3. The impulse at time t is

$$(\Delta H_w)_t = f(k) T_S \delta_o \quad (7.3-21)$$

The total wheel momentum at time T is

$$(H_w)_T = (H_w)_o + \sum_0^T (\Delta H_w)_t \quad (7.3-22)$$

Let $\{\Delta M_i\}$ equal the components of the body i momentum change in body i coordinates. Then

$$\{\Delta M_i\} = - (\Delta H_w)_t \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}_H \quad (7.3-23)$$

where i is the body on which the wheel is mounted and the column matrix represents the components of the wheel axis in body i coordinates. The introduction of $\{\Delta M_i\}$ into the dynamic equations has not been incorporated into the present version of the program.

7.3.5 Type 6 Modulator

The type 6 modulator is a saturating linear controller without a deadzone, Figure (6.1-8). It is used in connection with the reaction wheel control systems to provide a simple configuration for which the wheel momentum is proportional to the error signal, Figure 7.1-1.

7.3.5.1 Type 6 In Linear Region

In the linear region of the controller the modulator output (δ_o) is given by Equation (6.1-15)

$$\delta_o = \frac{1}{H} F_2 [A_i, p] \quad (7.3-24)$$

and the wheel momentum is

$$H_w = \frac{T_S}{H} F_2 [A_i, p] + (BSIWj)$$

If the output over the last step (δ_{o-1}) was in the linear region of the modulator, the present value of the wheel momentum must be updated. Letting $p = h$

$$(H_w)_t = \frac{T_S}{H} F_2 [A_i, h] + (BSIWj) \quad (7.3-25)$$

In order to predict the momentum over the next time step let $p = h + h''$

$$(H_w)_{t+h} = \frac{T_S}{H} F_2 [A_i, (h + h'')] + (BSIWj) \quad (7.3-26)$$

differentiating

$$(\dot{H}_w)_{t+h} = \frac{T_S}{H} \dot{F}_2 [A_i, (h + h'')] \quad (7.3-27)$$

Equations (7.3-26) and (7.3-27) represent the control system output for the type 6 modulator in the linear region, (5) in Figure 7.3-3.

7.3.5.2 Type 6 Not In Linear Region

When the type 6 modulator is not in its linear region it is saturated with a constant output of ± 1 . Therefore,

$$(H_w)_{t+h} = \delta_o T_S + (BSIWj) \quad (7.3-28)$$

and

$$(\dot{H}_w)_{t+h''} = 0 \quad (7.3-29)$$

Equations (7.3-28) and (7.2-29) represent the control system output for a saturated type 6 modulator, (6) in Figure 7.3-3.

7.4 SUMMARY OF REACTION WHEEL CONTROL EQUATIONS

The reaction wheel control equations for all six modulator types are summarized in Figure 7.3-3. At time t the control system must provide the integration system with a pair of equations expressing H_w and $\dot{H}_w$ as a function of the length of the next time step. The six possible pairs are designated as (1) through (6) in Figure 7.3-3. H_w and $\dot{H}_w$ are the magnitudes of the wheel momentum and time rate of change in wheel momentum. These magnitudes must be resolved into components along the body i coordinate frame by

$$\{H_{wii}\} = (H_w)_{t+h''} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}_H \quad (7.4-1)$$

$$\{\dot{H}_{wii}\} = (\dot{H}_w)_{t+h''} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}_H \quad (7.4-2)$$

Equations (7.4-1) and (7.4-2) give the reaction wheel parameters in the form specified in Table 1-1. The subscript i designates the body upon which the wheel is mounted and the 3×1 column matrix specifies the orientation of the wheel in body i axes.

8. REACTION WHEEL UNLOADING SYSTEM

Provisions have been made to fire gas pulses from the three pneumatic systems as a function of the wheel momenta in the four reaction wheel systems. In section 6, Figure 6.1-2 a wheel unloading pulse $(\epsilon_5)_t$ was introduced at the input of each controller used in pneumatic systems. Since there are three possible pneumatic systems, there will be these possible unloading signals $(\epsilon_{51})_t$, $(\epsilon_{52})_t$, and $(\epsilon_{53})_t$ together with three time derivatives $(\dot{\epsilon}_{51})_t$, $(\dot{\epsilon}_{52})_t$, and $(\dot{\epsilon}_{53})_t$. These unloading signals are linear combinations of the five sampled momentum feedback signals γ_j covered in Section 7.1 and shown on Figure 7.1-1.

8.1 SAMPLING GATES

The feedback signals γ_1 through γ_4 represent the momenta of the four reaction wheels after passing through the tachometer lags, Figure 7.1-1. $(\gamma_j)_t$ and $(\dot{\gamma}_j)_t$ are computed from Equations (7.1-5) and (7.1-6) and stored during the reaction wheel computations. The feedback signal γ_5 represents the momentum in the combination loop after passing through the tachometer lag. $(\gamma_5)_t$ and $(\dot{\gamma}_5)_t$ are computed from Equations (7.1-11) and (7.1-12).

Each of the feedback signals (γ_j) is passed through a sampling gate to produce the unloading signal WjU , Figure 7.1-1. Basically the sample gate is a switch which is closed at equal intervals of time (sample time, $HSTWj$) and remains closed for a short period of time (pulse time, $HPTWj$). The gate is mechanized by setting a constant K_G equal to zero if the gate is opened and equal to unity if the gate is closed. Then

$$(WjU)_t = K_{Gj} (\gamma_j)_t \quad (8.1-1)$$

$$(WjU)_t = K_{Gj} (\dot{\gamma}_j)_t \quad (8.1-2)$$

In general the sample time (HSTWj) is large, and therefore, is not predicted. A sample period starts at the first time step after the elapsed time since the last sample period exceeds HSTWj. The pulse time on the other hand is general short and the time step is adjusted such that the total on time is equal to HPTWj. With not further input data the initial sample time will occur at the start of the program. Since a sample period at time zero is not always desirable and addition input is provided to specify the initial sampling time.

8.2 COMBINED UNLOADING SIGNAL

The pneumatic unloading signals $(e_{51})_t$, $(e_{52})_t$, and $(e_{53})_t$ are linear combinations of the feedback unloading signals $(W_j U)_t$ ($1 \leq j \leq 5$). Since the orientation of the three pneumatic jets on body 1 and the four reaction wheels on bodies i ($1 \leq i \leq N$) is arbitrary, a perfectly general combination of signals must be available. Let $\{AWP1\}$ be a five element column matrix defined by input constants such that

$$\{AWP1\} = \begin{pmatrix} AWP11 \\ AWP12 \\ AWP13 \\ AWP14 \\ AWP15 \end{pmatrix} \quad (8.2-1)$$

then

$$(e_{51})_t = \sum_{j=1}^5 (AWP1j) (W_j U)_t \quad (8.2-2)$$

$$(\dot{e}_{51})_t = \sum_{j=1}^5 (AWP1j) (\dot{W}_j U)_t \quad (8.2-3)$$

Likewise for pneumatic systems 2 and 3

$$(e_{52})_t = \sum_{j=1}^5 (AWP2j) (WjU)_t \quad (8.2-4)$$

$$(\dot{e}_{52})_t = \sum_{j=1}^5 (AWP2j) (\dot{WjU})_t \quad (8.2-5)$$

$$(e_{53})_t = \sum_{j=1}^5 (AWP3j) (WjU)_t \quad (8.2-6)$$

$$(\dot{e}_{53})_t = \sum_{j=1}^5 (AWP3j) (\dot{WjU})_t \quad (8.2-7)$$

9. PNEUMATIC CONTROL SYSTEMS

The generalized pneumatic system, of which three are provided, is shown in Figure 9.0-1. Since there are no feedback loops, the net input to the j^{th} pneumatic torque modulator is the total error signal from the networks (P_j), Figure 5.1-1. Only modulator types 1, 2, 3, and 4 are used in the present version of the program. As in the case of the reaction wheel systems, a type 5 modulator is provided for future expansion of the program.

The nominal control torque (T_N) is found by simply multiplying the modulator output by the saturation torque. This torque signal can be modified by a simple lag network provided to represent nozzle characteristics. The total jet impulse is computed as an addition output parameter for use in gas consumption studies. The net torque magnitude (T_P) can be resolved as the sum of three moment components about the main body axes and three forces along the main body axes. The equations for the net control torque (T_P) for each type of modulator are summarized in Figure 9.0-2. The detailed derivations are given below.

9.1 TYPE 1 MODULATOR IN THE LINEAR REGION

The type 1 modulator is a saturating linear controller with a deadzone, Figure 6.1-3. When saturated or when operating within its deadzone, it has the same characteristics as the bang-bang controller for types 2, 3, and 4. When operating in its linear region it is the only controller which passes a complex function to the nozzle dynamics, Figure 9.0-1. As noted in Section 2 the purpose of the control system is to provide an expression for T_P as a function of the next time step. In addition it must compute T_P (corrected) and update T_P (predicted) if the last step was in the linear region of the controller. Therefore, the program must first ask the following two questions, Figure 9.0-2.

1. Is δ_{o-1} in the linear region of operation
2. Is δ_o in the linear region of operation

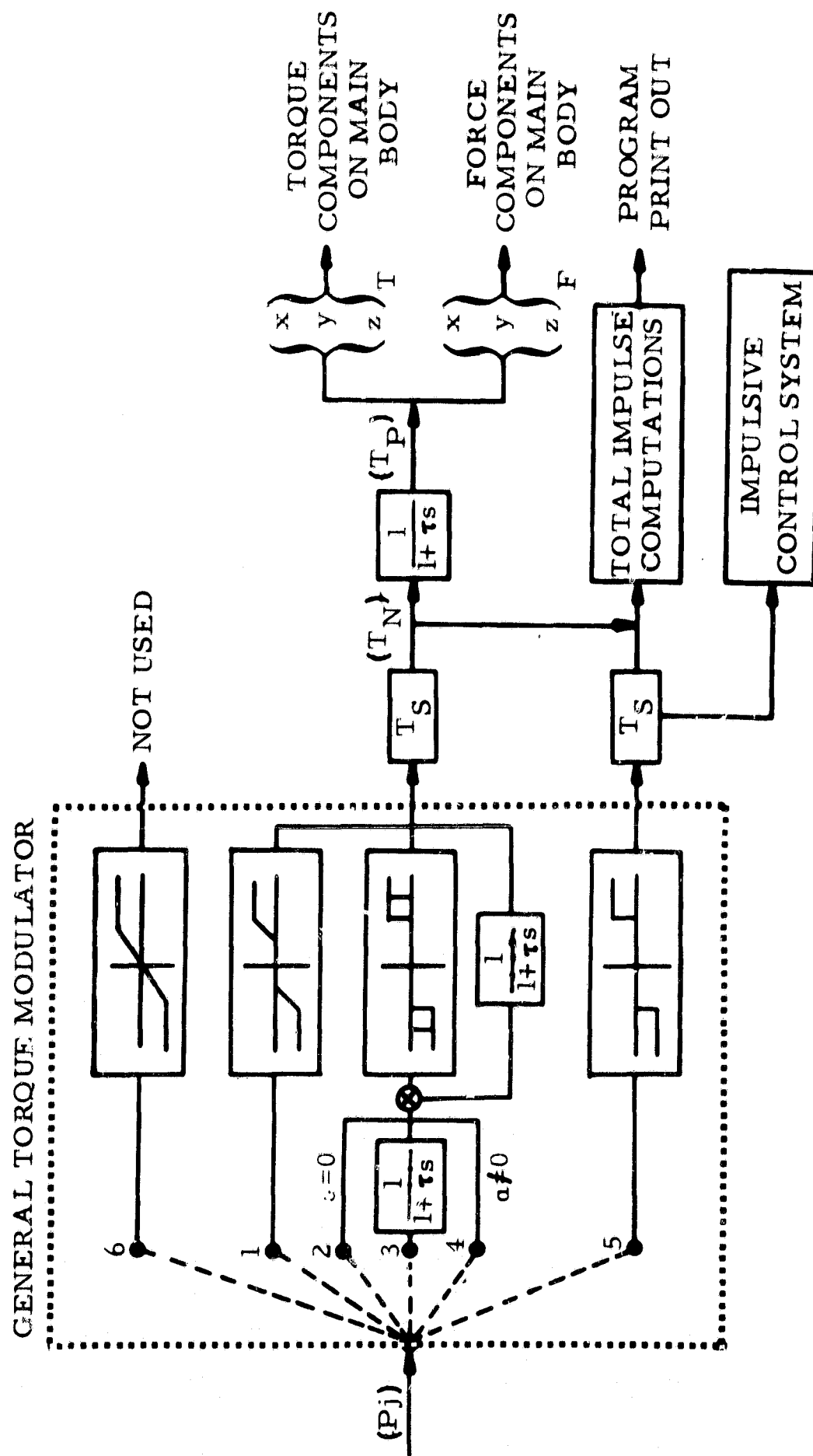


Figure 9.0-1
Pneumatic Control Systems

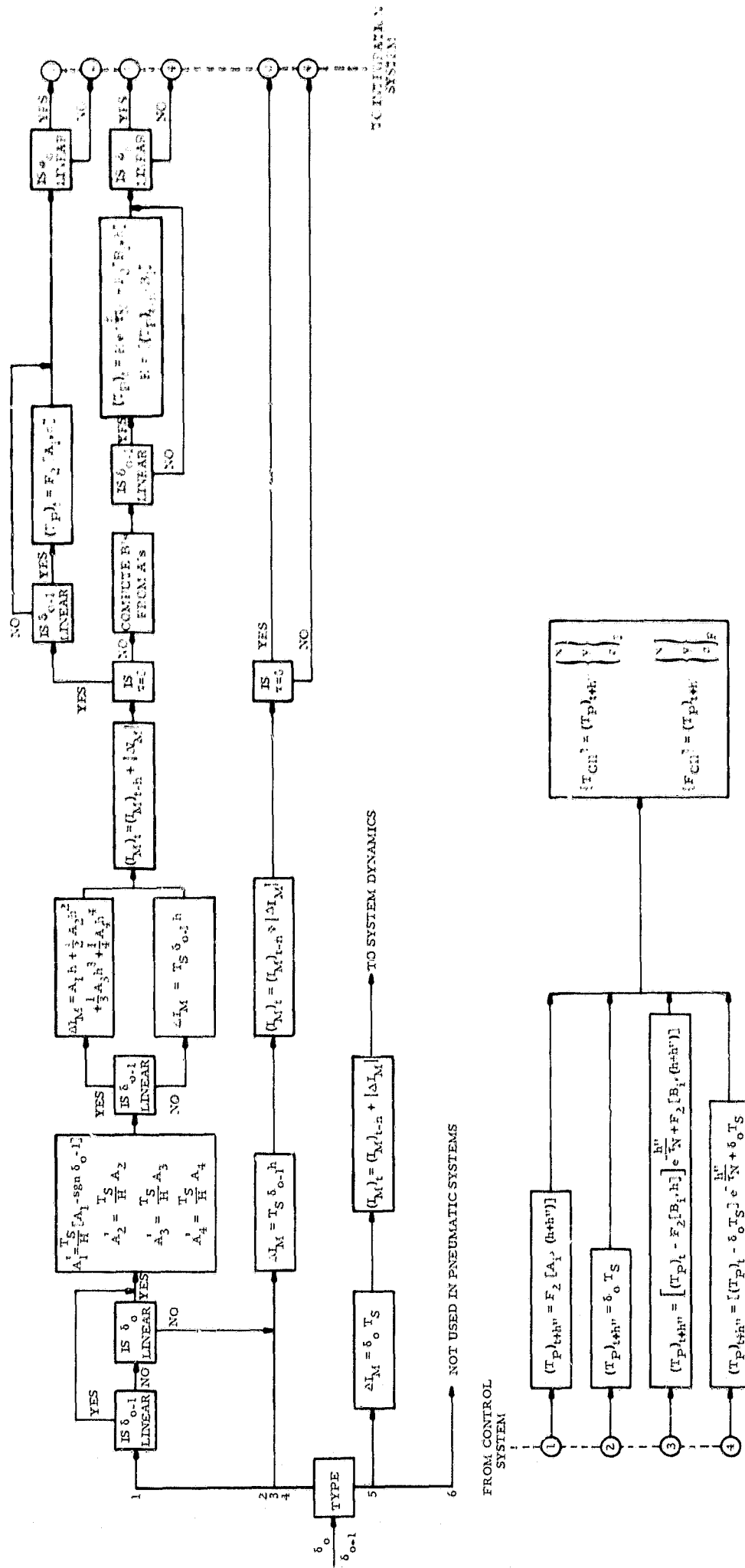


Figure 9.0-2
Pneumatic Control Equations

Where δ_{o-1} is the controller output over the last time step and δ_o is the output over the next time step. If the answer to both questions is no, the controller output is either saturated or zero for the last step and for the next step. The present control torque does not need to be updated (see Section 2) and the output for the next step can be computed using the equations for the bang-bang controller. If the answer to either questions is yes the expression for δ_o must be passed to the nozzle dynamics. From Equation (6.1-1) the output of the type 1 modulator in the linear region is

$$\delta_o = \frac{1}{H} [e_3 - \text{sgn } \delta_o \cdot 1]$$

$$e_3 = A_1 + A_2 p + A_3 p^2 + A_4 p^3$$

Then

$$T_N = \frac{T_S}{H} (A_1 - \text{sgn } \delta_o \cdot 1) + \frac{T_S}{H} A_2 p + \frac{T_S}{H} A_3 p^2 + \frac{T_S}{H} A_4 p^3$$

Let

$$A'_1 = \frac{T_S}{H} (A_1 - \text{sgn } \delta_o \cdot 1) \quad (9.1-1)$$

$$A'_2 = \frac{T_S}{H} A_2 \quad (9.1-2)$$

$$A'_3 = \frac{T_S}{H} A_3 \quad (9.1-3)$$

$$A'_4 = \frac{T_S}{H} A_4 \quad (9.1-4)$$

Then

$$T_N = F_2 [A'_i, p] \quad (9.1-5)$$

9.1.1 Total Impulse For Type 1 Modulator

The total impulse at time t is given by

$$(I_M)_t = (I_M)_{t-h} + |\Delta I_M| \quad (9.1-6)$$

where $(I_M)_{t-h}$ is the total impulse at time $(t-h)$ and ΔI_M is the impulse over the last step of length h seconds.

If the last step was not in the linear region of the controller, the output (δ_{o-1}) was constant and the impulse is

$$\Delta I_M = T_S \delta_{o-1} h \quad (9.1-7)$$

If the last step was in the linear region the nominal torque is given by Equation (9.1-5) and

$$\Delta I_M = \int_0^h F_2 [A_i', p] dp$$

From Equation (7.3-8)

$$\Delta I_M = F_3 [A_i', h] \quad (9.1-8)$$

9.1.2 Without Nozzle Dynamics

If the lag time constant (τ_N) representing nozzle dynamics is zero the control torque (T_P) is equal to the nominal torque (T_N)

$$T_P = F_2 [A_i', p] \quad (9.1-9)$$

If the output during the last step (δ_{o-1}) was in the linear region of the controller, the torque at time t must be updated, therefore,

$$(T_P)_t = F_2 [A_i', h] \quad (9.1-10)$$

If the output during the next step (δ_o) is in the linear region of the controller, the predicted value of T_P is found from Equation (9.1-9) for $p = h + h''$

$$(T_P)_{t+h} = F_2 [A_1', (h + h'')] \quad (9.1-11)$$

Equation (9.1-11) is the control system output for a type 1 modulator operating in the linear region without nozzle dynamics, (1) in Figure 9.0-2. The output expression, when the type 1 modulator is not in its linear region, (2) in Figure 9.0-2, is identical to the output for the types 2, 3, and 4 modulator developed in a later section.

9.1.3 With Nozzle Dynamics

If the lag time constant (τ_N) representing nozzle dynamics is not zero the control torque (T_P) is given by

$$T_P = \frac{T_N}{1 + \tau_N s}$$

and the (B's) for a simple lag must be computed from Equations (3.4-1) through (3.4-4). Then from Equation 3.2-16)

$$T_P = [(T_P)_{t-h} - B_1] e^{-\frac{p}{\tau_N}} + F_2 [B_1, p] \quad (9.1-12)$$

If the output during the last step (δ_{o-1}) was in the linear region of the controller, the torque at time t must be updated. Therefore,

$$(T_P)_t = [(T_P)_{t-h} - B_1] e^{-\frac{h}{\tau_N}} + F_2 [B_1, h] \quad (9.1-13)$$

If the output during the next step (δ_o) is in the linear region of the controller, the predicted value of T_P is found from Equation (9.1-12) for $p = h + h''$.

$$(T_P)_{t+h''} = [(T_P)_{t-h} - B_1] e^{-\frac{h+h''}{\tau} N} + F_2 [B_i, (h + h'')]]$$

Using Equation (9.1-12) to eliminate $(T_P)_{t-h}$

$$(T_P)_{t+h''} = \left[(T_P)_t - F_2 [B_i, h] \right] e^{-\frac{h''}{\tau} N} + F_2 [B_i, (h + h'')]] \quad (9.1-14)$$

Equation (9.1-14) is the control system output for a type 1 modulator operating in the linear region with nozzle dynamics, (3) in Figure 9.0-2. The output expression, when the type 1 modulator is not in its linear region, (4) in Figure 9.0-2, is identical to the output for the type 2, 3, and 4 modulator developed in the next section.

9.2 Non-Linear Type 1 and Types 2, 3, and 4

The output of the type 1 modulator when not in its linear region, and the type 2, 3, and 4 modulators can only have values of 1, 0 and -1. The constant nominal control torque is

$$T_N = \delta_o T_S \quad (9.2-1)$$

Since the past step cannot have been in the linear region of the modulator, the predicted value of the control torque does not need to be updated and the impulse over the last step is given by Equation (9.1-7).

9.2.1 Without Nozzle Dynamics

If the lag time constant (τ_N) representing nozzle dynamics is zero the control torque (T_P) is equal to the nominal torque (T_N).

$$T_P = \delta_o T_S \quad (9.2-2)$$

Since the torque is independent of time

$$(T_P)_{t+h''} = \delta_o T_S \quad (9.2-3)$$

Equation (9.2-3) is the control system output without nozzle dynamics expressed as a function of time step, (2) in Figure 9.0-2.

9.2.2 With Nozzle Dynamics

If the lag time constant (τ_N) representing nozzle dynamics is not zero the control torque (T_P) is given by

$$T_P = \frac{T_N}{1 + \tau_N s} = \frac{\delta_o T_S}{1 + \tau_N s} \quad (9.2-4)$$

Since the nominal torque (T_N) is constant over any given time interval, the equations of Section 3.4.1 apply, and the predicted value over the next time step is given by

$$(T_P)_{t+h''} = [(T_P)_t - \delta_o T_S] e^{-\frac{h''}{\tau_N}} + \delta_o T_S \quad (9.2-5)$$

Equation (9.2-5) is the control system output with nozzle dynamics expressed as a function of time step, (4) in Figure 9.0-2.

9.3 TYPE 5 MODULATOR

The type 5 modulator is a simple bang-bang controller without hysteresis, Figure 6.1-7. It has been provided for future use in a proposed impulsive control system. The logic required to produce the main body impulse at time t is shown in Figure 9.0-1. The required equations are given in Figure 9.0-2.

9.4 SUMMARY OF PNEUMATIC CONTROL FUNCTIONS

The pneumatic control equations for all 5 modulator types are summarized in Figure 9.0-2. At time t the control system must provide the integration system with an equation expressing (T_P) as a

function of the length of the next time step. The four possible equations are designated as 1 through 4 in Figure 9.0-2. (T_P) is the magnitude of the pneumatic control torques which may be resolved into force and torque components along the main body axes by

$$\{T_{Cl1}\} = (T_P)_{t+h''} \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix}_T \quad (9.3-1)$$

$$\{F_{Cl1}\} = (T_P)_{t+h''} \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix}_F \quad (9.3-2)$$

Equations (9.3-1) and (9.3-2) give the pneumatic control parameters in the form specified in Table 1-1.

10. ARRAY DRIVE SYSTEMS

The generalized array drive system, of which two are provided, is shown in Figure (10.0-1). Associated with each array drive is an auxiliary body i representing the body being controlled. The first array drive (A1) will always control its associated body about its x gimbal axis. Since the location of the gimbal axes are arbitrary, there is no loss in the generality of the program. If the second array drive (A2) is on a different body than (A1) it will also control its associated body about the x gimbal axis. If, however, both array drives are given the same body designation, system A1 will control the x gimbal axis and system A2 will control the y gimbal axis.

In the process of solving the dynamic equations, the integration system will provide the displacement, velocity, and acceleration about each gimbal axis which has been given a degree of freedom. When the program enters the array drive systems at time t the value of $(\theta)_t$, $(\dot{\theta})_t$, and $(\ddot{\theta})_t$ for each gimbal axis will have been computed and stored.

10.1 ARRAY DRIVE FEEDBACK LOOP AND MODULATOR INPUT

Each array drive system is provided with a speed control loop which feeds a signal (γ_{Aj}) proportional to the gimbal velocity back to the modulator input. From Figure 10.0-1

$$(\gamma_{Aj})_t = [(BS1Aj) - (\dot{\theta})_t] (CECAj) \quad (10.1-1)$$

The net input signal to the j^{th} array drive torque modulator (δ_{iAj}) is the sum of the signal from the compensation networks and the feedback signal, Figure 10.0-1. From Figure 5.1-1 the total error signal from the networks at time t is $(Aj)_t$. Then

$$(\delta_{iAj})_t = (Aj)_t + (\gamma_{Aj})_t \quad (10.1-2)$$

Note that the signal is feed back with the positive sense.

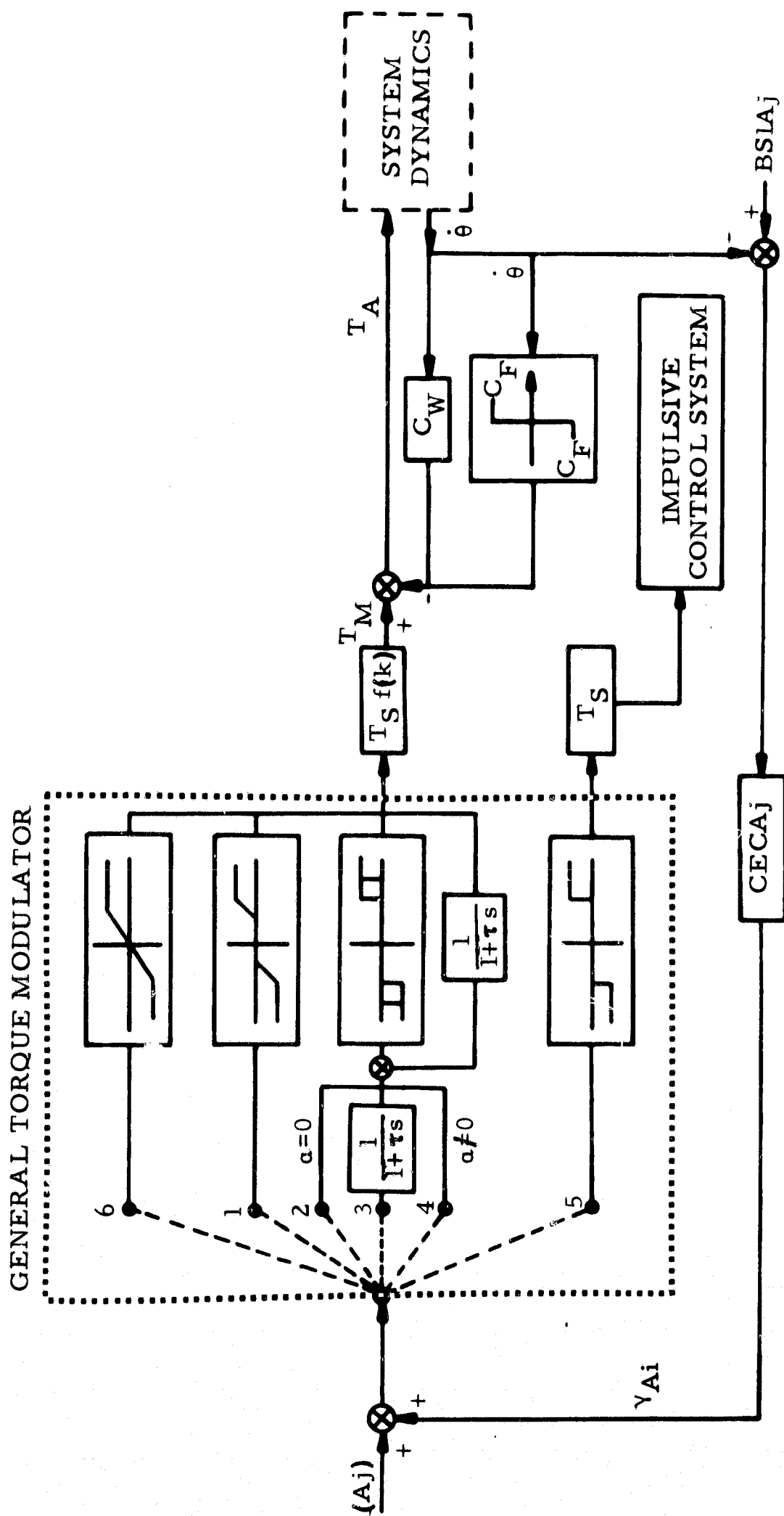


Figure 10.0-1
Array Control Systems

10.2 MOTOR SPEED-TORQUE CHARACTERISTICS

The motor speed-torque characteristics used for the array drive are identical in form to those used for the reaction wheel motors, Section 7.2. A second set of polynomial coefficient for Equation (7.2-2) are provided to establish the characteristics for both array drives. The parameter k , Figure 7.2-2 is computed as the ratio of array velocity to maximum velocity at time t .

10.3 ARRAY FRICTION

The magnitude of the control torque transmitted to the integration system is designated as the shaft torque, Figure 10.0-1. The motor torque (T_M) is modified by a viscous windage and friction term and a coulomb friction term to produce the shaft torque (T_A). From Figure 10.0-1

$$T_A = T_M - C_w \dot{\theta} - \text{sgn } \dot{\theta} C_F \quad (10.3-1)$$

It has been noted in connection with the other control parameters that the control system must transmit to the integration system an expression for torque in terms of the next time step h'' . A more general statement of this requirement is that the expression must be in terms of time and the independent variables of the system. The system already has expressions for the other variables, including $\dot{\theta}$, as functions of time. Therefore, the array drive torques will be expressed in terms of h'' and $\dot{\theta}$ including $\text{sgn } \dot{\theta}$. As a result it is only necessary to express T_M in Equation (10.3-1) as a function of h'' .

10.3.1 Coulomb Friction Effects

The use of coulomb friction introduces a number of complex programming problems. From Equation (10.3-1) and Figure 10.0-1 it may be seen that the expression for the coulomb friction torque is not only discontinuous but is undefined for $\dot{\theta}$ equal to zero. The correct

expression for the coulomb friction at $\dot{\theta}$ equal to zero, must allow C_F' to assume the magnitude and sign required to maintain zero velocity. When the magnitude of C_F' is greater than C_F motion begins and Equation (10.3-1) will apply. This process is mechanized by establishing two states or modes of operation for the drive system. In the locked gimbal mode the degree of freedom is removed and $\dot{\theta}$ is set to zero. The program computes the locked gimbal reaction torque (T_A') and applies it to the main body. In the free gimbal mode the auxiliary body is in motion relative to the main body, $\dot{\theta}$ is not zero, and Equation (10.3-1) applies. In the locked gimbal mode $\dot{\theta}$ is zero, C_P' opposes T_M , and Equation (10.3-1) can be written as

$$T_A' = T_M - C_P' \quad (10.3-2)$$

where T_A' = the locked gimbal reaction torque

C_F' = coulomb friction required for $\dot{\theta} = 0$

then

$$C_F' = T_M - T_A'$$

and at time t

$$(C_F')_t = (T_M)_t - (T_A')_t \quad (10.3-3)$$

The criteria for unlocking the gimbal is

$$|C_F'| > C_F$$

or

$$|(T_M)_t - (T_A')_t| > C_F \quad (10.3-4)$$

Due to the fact that $(T_M)_t$ is computed in the control routine while $(T_A')_t$ is computed during integration, there is a one time step delay in unlocking the gimbal in accordance with Equation (10.3-4). This

delay is not objectionable in the case of the linear modulators (types 1 and 6) since the unlocking friction or static friction is always higher than the sliding friction C_F . In the case of the bang-bang modulators (types 2, 3, and 4), the large motor torque which occurs when the controller switches on should be applied during the initial on step. Therefore, an additional mode switching criteria is provided which unlocks the gimbal when the present modulator output (δ_o) exceeds 0.99.

The gimbal locking criteria is based upon the magnitude of $\dot{\theta}$. Since $\dot{\theta} = 0$ represents a system discontinuity the time at which θ reaches zero is predicted on the bases of $\dot{\theta}$ and $\ddot{\theta}$. Therefore, if the predicted time to $\dot{\theta} = 0$ is less than (HPERR) or if the magnitude of $\dot{\theta}$ is less than 2.0×10^{-6} radians per second, the gimbal is locked.

10.3.2 Viscous Friction Effects

It was noted in connection with Equation (10.3-1) that the viscous damping term $C_w \dot{\theta}$ is the only control torque term which is not a direct function of time. In order to correctly simulate viscous damping the value of $\dot{\theta}$ as a function of time should be used in computing $C_w \dot{\theta}$ in the integration system. In order to simplify the digital programming a constant value of $C_w (\dot{\theta})_t$ was used, where $(\dot{\theta})_t$ is the velocity at the beginning of the integration interval. This assumption is probably a poor approximation for a large number of configurations. However, it should be noted the viscous damping term is a redundant input. For each gimbal axis with a degree of freedom, the basic dynamics provides for a spring restraint, a correctly simulated viscous damper, a magnetic hysteresis damper, and elastic stops, see Equation (7.3-1) of Volume I. Each of these features is available for array drive systems and viscous damping should be introduced as an input to the basic dynamic system. The approximate simulation in the control routine is only used when it is desired that the viscous damping term be included in the printed and plotted parameter T_A .

10.4 MOTOR TORQUE

The array drive system output transmitted to the integration system consists of an expression for motor torque (T_M) as a function of the length of the next time step. Since no curve fits are required after the torque modulators, the predicted output at time t does not require updating. All six modulator types are used. However, as in the case of the reaction wheel and pneumatic systems, the type 5 is provided only for future expansion of the program.

The drive motor torque (T_M) is found by simply multiplying the modulator output by the saturation torque and the speed-torque factor.

$$T_M = \delta_o T_S f(k) \quad (10.4-1)$$

Except for operation in the linear regions of the types 1 and 6 modulators, the motor torque is constant over any given integration interval. The equations for each modulator type are summarized in Figure 10.4-1. The detailed derivations are given below.

10.4.1 The Type 1 and 6 Modulators In the Linear Region

The type 1 and 6 modulators are saturating linear controllers differing only in that the type 1 incorporates a deadzone. When saturated or in the deadzone of the type 1, they have the same characteristics as the bang-bang controller for the type 2, 3, and 4 modulators. When operating in the linear region they are the only modulators that produce a time varying motor torque over a given integration interval. From Equations (6.1-1) and (6.1-15) the outputs in the linear regions are

$$\delta_o = \frac{1}{H} [\epsilon_3 - \text{sgn } \delta_o \cdot 1] \quad (\text{type 1}) \quad (10.4-2)$$

$$\delta_o = \frac{1}{H} \epsilon_3 \quad (\text{type 6}) \quad (10.4-3)$$

where

$$\epsilon_3 = A_1 + A_2 p + A_3 p^2 + A_4 p^3$$

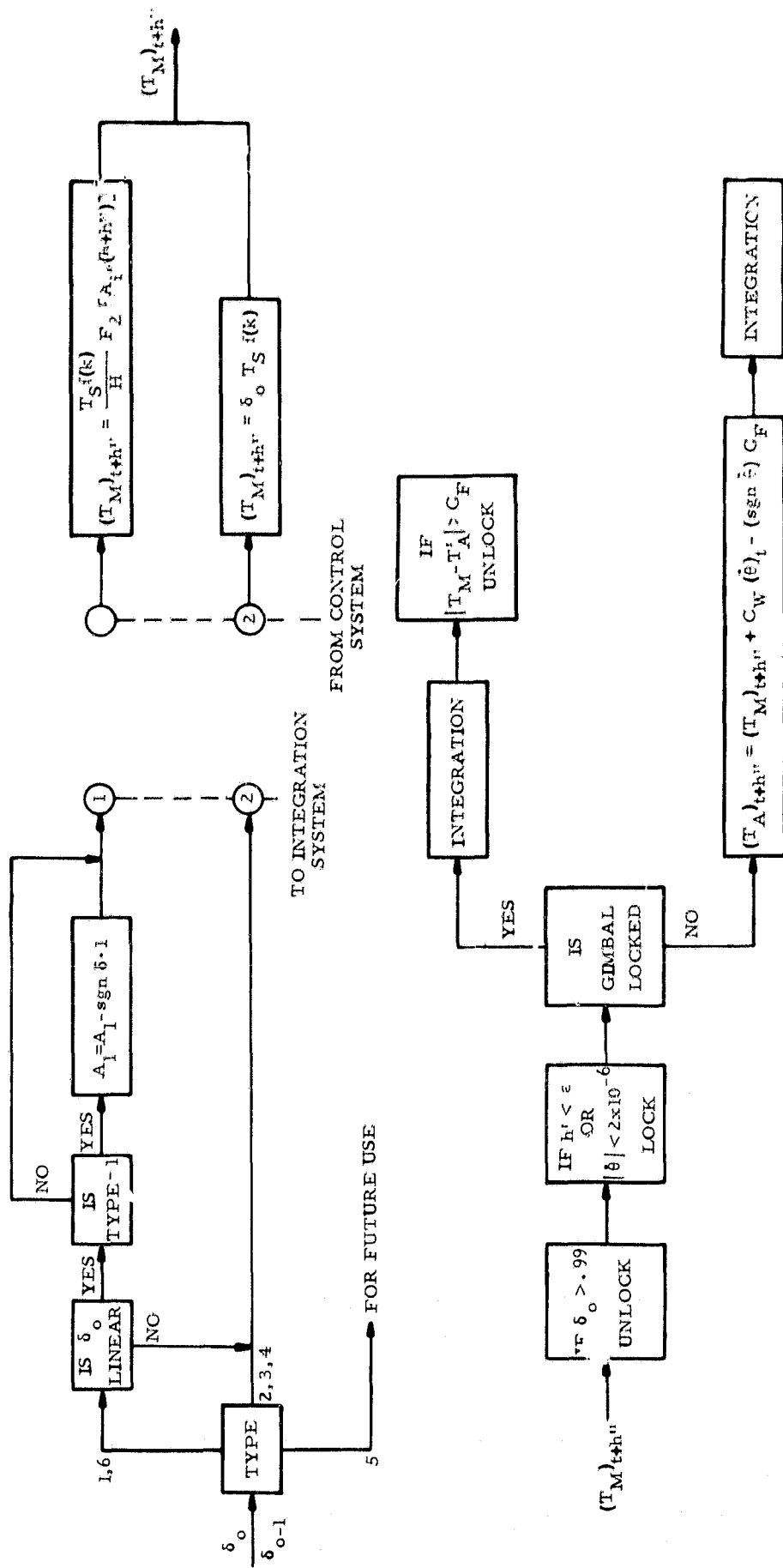


Figure 10.4-1
Array Control Equations

If in the case of the type 1 modulator A_1 is replaced by $(A_1 - \text{sgn } \delta_o \cdot 1)$ both output expressions become

$$\delta_o = \frac{1}{H} F_2 [A_1, p] \quad (10.4-4)$$

and from Equation (10.4-1)

$$T_M = \frac{T_S f(k)}{H} F_2 [A_1, p] \quad (10.4-5)$$

That output over the next time step is given by

$$(T_M)_{t+h''} = \frac{T_S f(k)}{H} F_2 [A_1, (h + h'')] \quad (10.4-6)$$

The motor torque at time t is found from Equation (10.4-6) by letting $h'' = 0$. Equation (10.4-6) is the motor torque expression transmitted to the integration system when the type 1 or 6 modulator is in the linear region.

10.4.2 Non-Linear Types 1 and 6 and Types 2, 3, and 4

The output of the type 1 or 6 modulator when not in the linear region and the output of the type 2, 3, or 4 modulators can only have values of 1, 0, or -1. The constant motor torque is given by Equation (10.4-1). The output over the next time step is given by

$$(T_M)_{t+h''} = \delta_o T_S f(k) \quad (10.4-7)$$

and at time t

$$(T_M)_t = \delta_o T_S f(k) \quad (10.4-8)$$

10.5 INTEGRATION LOGIC

If the modulator output (δ_o) for the next step is greater than .99 the gimbal flag must be set to the unlocked designation, Figure 10.4-1. If the predicted time to $\dot{\theta}$ equal to zero is less than (HPERR) or if $|\dot{\theta}|$ is less than 2×10^{-6} set the gimbal flag to the locked designation. The gimbal flag is now properly set and can be sampled for the next step.

If the gimbal is locked, the integration step h'' is taken and the absolute value of $(T_M - T_A')$ at $t + h''$ is compared with C_F . If greater, set the flag to the unlocked position.

If the gimbal is unlocked, the torque expresion to be used during the integration step is

$$(T_A)_{t+h''} = (T_M)_{t+h''} + C_w (\dot{\theta})_t - \text{sgn} (\dot{\theta})_t \cdot C_F \quad (10.5-1)$$

If the array drive is the first array on body i

$$\{T_{Li}\} = (T_A)_{t+h''} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \quad (10.5-2)$$

If the array is the second array on body i

$$\{T_{Li}\} = \begin{Bmatrix} (T_{A1})_{t+h''} \\ (T_{A2})_{t+h''} \\ 0 \end{Bmatrix} \quad (10.5-3)$$

Equations (10.5-2) and (10.5-3) give the array drive control parameters in the form specified by Table 1-1.

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