USS-T

Fagurny

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-12487

ST-RWP-AI-10754

N.S.

NUMERICAL METHOD OF CALCULATING THE DOPPLER EFFECT AND THE REDUCED DIFFERENCE OF DOPPLER FREQUENCIES OF RADIOWAVES COHERENTLY EMITTED FROM ARTIFICIAL EARTH'S SATELLITES

by

D.S. Lukin

&

V.A. Shkol'nikov

(USSR)

30 SEPTEMBER 1968

NUMERICAL METHOD OF CALCULATING THE DOPPLER EFFECT AND THE REDUCED DIFFERENCE OF DOPPLER FREQUENCIES OF RADIOWAVES COHERENTLY EMITTED FROM ARTIFICIAL EARTH'S SATELLITES

Kosmicheskiye Issledovaniya
Tom 6, vyp.No.3, pp 389-394
Izd-vo "NAUKA", Moscow 1968.

By
D.S. Lukin and
V.A. Shkol'nikov

SUMMARY

A numerical method is proposed for calculating the Doppler effect and the reduced difference of Doppler frequencies of two coherent radiowaves emitted from artificial Earth's satellites or from geophysical rockets in a three-dimensional inhomogenous magnetoactive medium.

The results of the calculation of the reduced difference of Doppler frequencies of two coherent radiowaves (f_1 = 20 mc, f_2 = 90 mc) along the orbit of a "Kosmos"-type AES are presented for a spherically stratified and horizontally inhomogenous ionosphere for a two-dimensional case. The effect of horizontal gradients of electron concentration on the reduced difference of Doppler frequencies is demonstrated by means of numerical calculations.

*

* *

The method of ionosphere studies based on the investigation of the reduced difference of the Doppler frequencies of two coherent radiowaves emitted from artificial Earth's satellites or from geophysical rockets has become fairly widespread. Both the theoretical premises of this method and a sufficiently large experimental material were the object of numerous studies [1-11]. However, numerous difficulties arising during estimates of the influence of various factors (inhomogeneity and non stationary state of the ionosphere, horizontal gradients, fine structure etc.,) on the magnitude of the Doppler frequency shift fail to provide a clear idea of the vast information on the ionosphere which this value yields. Therefore, it is interesting to make a direct estimate of the influence of various effects by means of a numerical calculation of the reduced difference of the Doppler frequencies of two coherent radiowaves for different models of electron concentration distribution in the ionosphere.

METHOD OF CALCULATING THE DOPPLER EFFECT IN AN INHOMOGENOUS AND MAGNETOACTIVE MEDIUM

Let us write the expression for the electromagnetic field component in the form

$$A(\mathbf{r}) = A_0(\mathbf{r})\,e^{iS(\mathbf{r},\,t)}, \tag{1}$$

where the wave amplitude $A_0(\mathbf{r})$ is a slowly varying function, while the phase S is a large quantity which is function of coordinates and time

$$S(\mathbf{r},\,t) = \psi(\mathbf{r},\,t) - \omega t. \tag{2}$$

By definition the wave frequency is a variation of phase in time; therefore, in a stationary case, the frequency is

$$\omega = -\frac{dS}{dt}, \tag{3}$$

while the phase derivative with respect to the coordinates determines the wave vector $\vec{k}$

$$\nabla\psi(\mathbf{r}) = \mathbf{k}(\mathbf{r},\,\omega). \tag{4}$$

If the medium is nonstationary and the source of radiation is displaced with a velocity $\mathbf{v}_c = \mathbf{v}_c\{v_x = dx/dt,\ v_y = dy/dt,\ v_z = dz/dt\}$, the frequency of the received wave is equal to

$$\omega' = -\frac{dS(\mathbf{r},\,t)}{dt} = -\left[\frac{\partial\psi}{\partial x}\frac{dx}{dt} + \frac{\partial\psi}{\partial y}\frac{dy}{dt} + \frac{\partial\psi}{\partial z}\frac{dz}{dt}\right] + \omega. \tag{5}$$

Taking into account (4) we obtain

$$\omega' = -(\mathbf{k}\mathbf{v}_c) + \omega - \frac{\partial\psi}{\partial t}, \tag{6}$$

wherefrom the frequency variation due to the Doppler effect will be

$$\Delta\omega = \omega - \omega' = (\mathbf{k}\mathbf{v}_c) + \frac{\partial\psi}{\partial t}. \tag{7}$$

For $(\mathbf{k}\mathbf{v}_c) \gg \partial\psi/\partial t$, we have

$$\Delta\omega = (\mathbf{k}\mathbf{v}_c) = k_x v_x + k_y v_y + k_z v_z, \tag{8}$$

where $k_i = \frac{\omega}{c}\,n(\mathbf{r},\,\omega,\,\mathbf{k})\cos\alpha_i$, $n(\mathbf{r},\,\omega,\,\mathbf{k})$ is the refraction index of the medium, while a $\cos\alpha_i$ $(i = 1, 2, 3)$ are the direction cosines of the beam.

Thus

$$\Delta\omega = \frac{\omega}{c} n(\mathbf{r}, \omega, \mathbf{k}) (\cos\alpha_1 v_x + \cos\alpha_2 v_y + \cos\alpha_3 v_z). \tag{8'}$$

To determine k_i (or α_i) we shall use the numerical solution of the first system of differential equations [12]

$$\frac{d\mathbf{k}}{d\tau} = \frac{\partial\omega^2 n^2}{\partial\mathbf{r}} \Big/ \frac{\partial\omega^2 n^2}{\partial\omega}, \qquad \frac{d\mathbf{r}}{d\tau} = \left(2c^2\mathbf{k} - \frac{\partial\omega^2 n^2}{\partial\mathbf{k}}\right) \Big/ \frac{\partial\omega^2 n^2}{\partial\omega}. \tag{9}$$

The first three scalar equations of this system determine the phase trajectory of the beam, while the second three determine the group trajectory.

In the isotropic case the refraction index of the medium is a function which depends solely on coordinates and frequency $n = n(\mathbf{r}, \omega)$, Therefore $\partial\omega^2 n^2 / \partial\mathbf{k} = 0$ * Then, for an isotropic ionosphere with a refraction index $n = \sqrt{1 - \omega_0^2/\omega^2}$ we have

$$\frac{dk_i}{d\tau} = \frac{d\left[\frac{\omega}{c} n(\mathbf{r}, \omega)\cos\alpha_i\right]}{d\tau} = \omega n \frac{\partial n}{\partial x_i}, \tag{10}$$
$$\frac{dx_i}{d\tau} = cn(\mathbf{r}, \omega)\cos\alpha_i = \frac{c^2}{\omega} k_i \qquad (i = 1, 2, 3),$$

while according to [13], in an arbitrary orthogonal system of coordinates the trajectory of the beam is determined by the following system of differential equations

$$\frac{dx_i}{d\tau} = cn\frac{\cos\alpha_i}{H_i},$$
$$\frac{d\alpha_i}{d\tau} = \frac{\operatorname{ctg}\alpha_i}{H_i} c \sum_{j=1}^{3} \left[\frac{\partial(nH_i)}{\partial x_j}\frac{\cos\alpha_j}{H_j}\right] - \frac{c}{H_i \sin\alpha_i}\frac{\partial n}{\partial x_i} - \tag{11}$$
$$- \frac{cn}{H_i \sin\alpha_i} \sum_{j=1}^{3} \left[\frac{\cos^2\alpha_j}{H_j}\frac{\partial H_j}{\partial x_i}\right] \qquad (i = 1\ 2, 3),$$

H_i is the Lamé coefficient.

For the case of radiowave propagation in the troposphere, the refraction index is a function of coordinates $n = n(\mathbf{r})$ only and the trajectory of the beam is determined by the following system of differential equations

$$\frac{dx_i}{d\tau} = \frac{c}{n(\mathbf{r})}\cos\alpha_i,$$
$$\frac{dk_i}{d\tau} = \frac{\omega}{n(\mathbf{r})}\frac{\partial n}{\partial x_i} \qquad (i = 1, 2, 3). \tag{12}$$

---

* The right parts of system (9) were derived by means of the differentiation of implicit function $g(\mathbf{r}, \mathbf{k}, \omega) = c^2\mathbf{k}^2 - \omega^2 n^2(\mathbf{r}, \mathbf{k}, \omega) = 0$ with respect to variables $\mathbf{r}, \mathbf{k}, \omega$, Therefore, in an isotropic case $\omega^2 n^2$ is explicitly independent of $\vec{k}$.

If the analytical or numerical dependence of electron concentration $N(\mathbf{r})$ is known, or the refraction factor of the medium $n(\mathbf{r}, \omega)$, is given, the right hand parts of systems (9)-(12) are known functions and at given initial conditions $\mathbf{k}|_{\tau_0} = \mathbf{k}_0$, $\mathbf{r}|_{\tau_0} = \mathbf{r}_0$, the integration of the systems is obtained in the form of a series or values $k_i(\tau_1)$, $x_i(\tau_1)$; $k_i(\tau_2)$, $x_i(\tau_2)$ etc along the trajectory of the beam over a specific time interval $\Delta\tau$ which is the system's integration step.

Having calculated various $\vec{k}_0$ values (or, which is the same, various outgoing angles of beams α_0 since $k_{0i} = \frac{\omega}{c} n_0 \cos \alpha_{0i}$), at a fixed $\vec{r}_0$, we obtain a family of integral curves, i.e. a family of beam trajectories originating from the point emitter's.

If the trajectory of the artificial Earth satellite is given, then at the points of its intersection with the integral curves (beam trajectories) the components of the wave vector k_i and the actual angles of the direction cosine α_i are known, and the velocity components of object v_i are given.

Consequently, the Doppler frequency shift along the trajectory of the object can be determined at these points by formulas (8) or (8').

METHOD OF CALCULATING THE REDUCED DIFFERENCE OF DOPPLER FREQUENCIES FOR TWO COHERENT RADIOWAVES

Using (8), we shall write the expression for the difference of the Doppler shift of two coherent frequencies ω_1 and ω_2 reduced to the lowest frequency ω_1,

$$\delta\omega_{1,2} = \Delta\omega_1 - \frac{\omega_1}{\omega_2}\Delta\omega_2 = (\mathbf{k}_1\mathbf{v}_c) - \frac{\omega_1}{\omega_2}(\mathbf{k}_2\mathbf{v}_c) \qquad (13)$$

or according to (8')

$$\delta\omega_{1,2} = \frac{\omega_1}{c}\sum_{i=1}^{3}(n_1 \cos\alpha_{1i} - n_2\cos\alpha_{2i})v_i. \qquad (14)$$

Let $\omega_2 \to \infty$, then $n_2 = 1$ (case of rectilinear electromagnetic wave propagation) and the expression (14) will be rewritten in the form

$$\delta\omega_{1,\infty} = \frac{\omega_1}{c}\sum_{i=1}^{3}(n_1\cos\alpha_{1i} - \cos\alpha_i^0)v_i, \qquad (15)$$

where α_i^0 is the angle between the vertical passing through the point at which AES is located and the straight line linking this point with the point of observation.

Formula (15) determines the Doppler shift of frequency ω_1 caused by the inhomogeneity of the medium. On the basis of formula (15) it is possible to determine $\delta\omega_2.\infty$.

It is evident that the reduced difference of two Doppler frequencies ω_1 and ω_2 coherently emitted from an artificial Earth satellite, is

$$\delta\omega_{1,2} = \delta\omega_{1,\infty} - (\omega_1/\omega_2)\,\delta\omega_{2,\infty}. \tag{16}$$

In the following, this will allow us to estimate the error which occurs on account of the fact that, as a rule, the effect of the medium on the higher coherent frequency ω_2 is neglected in the calculations.

CASE OF A HORIZONTALLY INHOMOGENEOUS IONOSPHERE

To investigate the effect of a horizontal inhomogeneity of the ionosphere on the Doppler difference of two coherent frequencies, we shall consider the simplest case when the trajectory of the object coincides with the incidence plane of the electromagnetic wave.

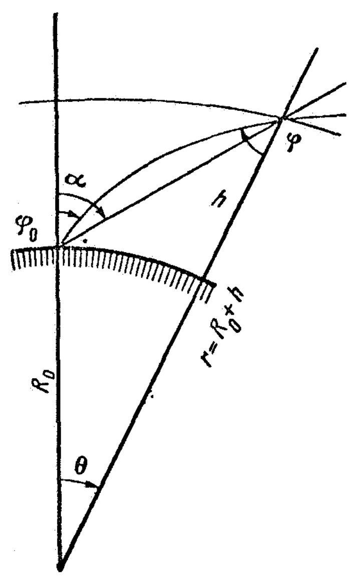

Fig.1

In the polar system of coordinates r, θ (Fig.1) formula (15) has the form

$$\delta\omega_{1,\infty} = \frac{\omega_1}{c}\{[n(r,\theta,\omega_1)\cos\varphi + \cos(\alpha-\theta)]\,v_r + [n(r,\theta,\omega_1)\sin\varphi - \sin(\alpha-\theta)]\,v_\theta\}, \tag{17}$$

where v_r and v_θ are the vertical and horizontal velocity components of the object.

Designating by $\delta\omega_r$ and $\delta\omega_\theta$ the components $\delta\omega_{1,\infty}$, dependent on v_r and v_θ, the expression (17) can be rewritten in the form

$$\delta\omega_{1,\infty} = \delta\omega_r + \delta\omega_\theta = \left(\frac{\delta\omega_r}{v_r}\right)v_r + \left(\frac{\delta\omega_\theta}{v_\theta}\right)v_\theta. \tag{17'}$$

To determine r, θ and ϕ we have a system of differential equations

$$\frac{dr}{d\tau} = n(r,\theta,\omega)\,c\cos\varphi, \qquad \frac{d\theta}{d\tau} = \frac{cn(r,\theta,\omega)}{r}\sin\varphi,$$
$$\frac{d\varphi}{d\tau} = \frac{c}{r^2}\frac{\partial(nr)}{\partial\theta}\cos\varphi - \frac{c}{r}\frac{\partial(nr)}{\partial r}\sin\varphi, \tag{18}$$

which follows from the system (11) at $H_1 = 1$, $H_2 = r$.

On the basis of formula (17) and using the numerical integration of system (18), we have calculated $\delta\omega_r$ and $\delta\omega_\theta$ on an electronic digital computer by the Runge-Kutta method for different models of a spherically laminated and horizontally inhomogeneous ionosphere and different critical frequencies of layer F_2.

In the calculation it was assumed that the dependence of the refraction index on the coordinates is determined by formula $n^2(r, \theta) = 1 - \alpha^2\Phi(r,\Theta)$, where f_0/f, and

$$\Phi(r,\theta)=\begin{cases} 0 & \text{for } r_m - y_m \leqslant r, \\ \left\{1-\left[\dfrac{r_m(1+A\sin\lambda_m\theta)-r}{y_m}\right]^2\right\}^2 & \text{for } r \leqslant r_m(\theta), \\ e^{-b[r-r_m(\theta)]} & \text{for } r > r_m(0). \end{cases} \tag{19}$$

Fig.2 shows the results of calculation of beam trajectories (Fig.2a) and of components $\delta\omega_r/v_r$ and $\delta\omega_\theta/v_\theta$ (Fig.2b,c) of the reduced Doppler frequency shift at various initial angles ϕ_0 (see Fig.1) for the case of a spherically laminated ionosphere (λ_m = 0, r_m = 6670 km, y_m = 200 km, b = $2\cdot10^{-3}$ km^{-1}, a = 0,15; f_1 = 20 Mc). Similar curves can be plotted for any kind of model of a spherically laminated and horizontally inhomogeneous ionosphere.

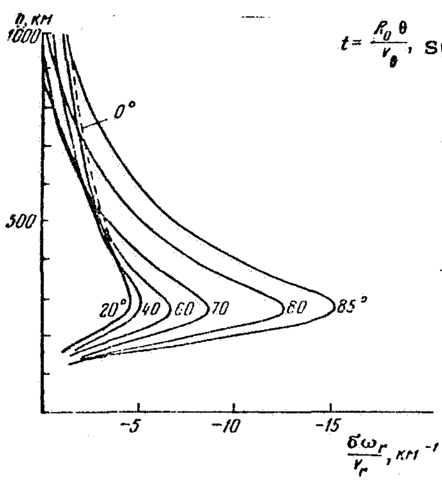

Fig.2-b

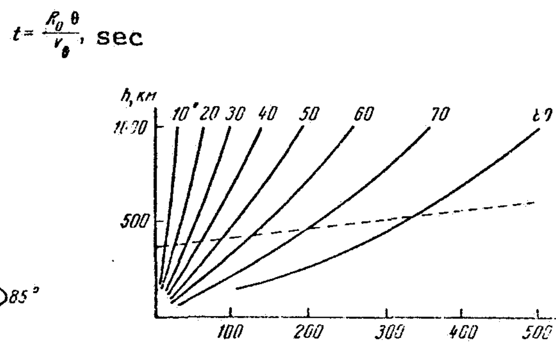

Fig.2-a

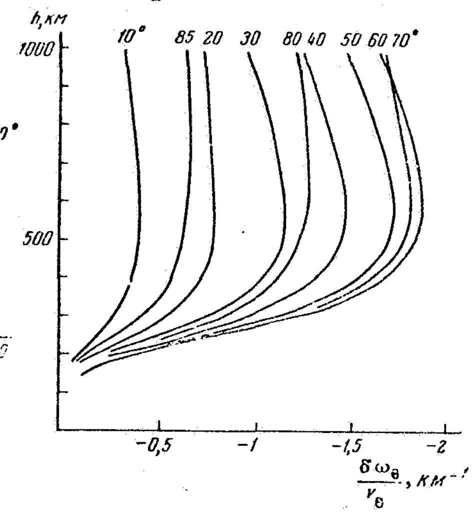

Fig.2-c

Using the curves of Fig.2 it is possible to plot the values of the reduced difference $\delta\omega_{1,\infty}$ for any v_r and v_θ, i.e. for any AES orbit. Fig.2-a shows on the dashed line an AES trajectory under the assumption that v_r = 0.5 km/sec, while v_θ = 5 km/sec.

At these values of the velocity components we have calculated the reduced difference of Doppler frequencies for two coherent radiowaves (Fig.3-a) along the outbound trajectory of an AES from the Earth (Fig.3-b)

On Fig.3-a curve 1 corresponds to the case of a spherically laminated ionosphere (λ_m = 0, r_m = 6670 km, y_m = 200 km, b = $2\cdot10^{-3}$ km^{-1}, a = 0.15; f_1 = 20 Mc) which is the first model, while curve 2 corresponds to the case when the altitude of the electron concentration maximum varies according to the law

$$r(\theta) = r_m(1 + A \sin \lambda_m \theta), \qquad (20)$$

where $\lambda_m = 60$, $A = \pm\, 0.005$ (sign <<+>> corresponds to the right side of Fig.3-a) and $r_m = 6670$ km ($y_m = 200$ km, $b = 2\cdot 10^{-1}$ km, $a = 0.15$; $f_1 = 20$ Mc) which is the second model.

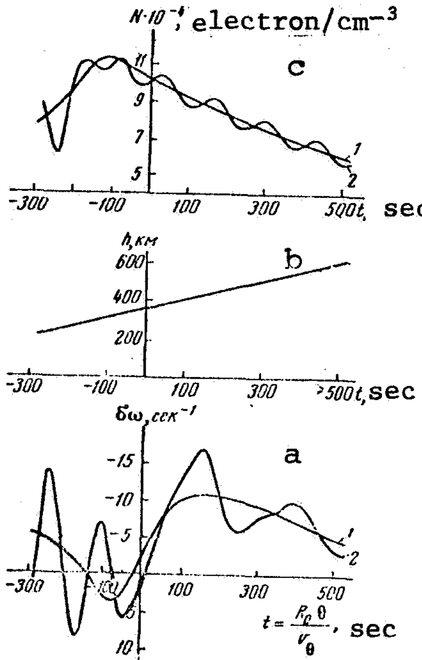

Fig.3

The beginning of time reading was selected at the time of AES passage of the zenith at an altitude of 350 km. At the points where the satellite orbit was intersected by beam trajectories (see Fig.2-a) we have $t = R_0\Theta(\tau)/v_\theta$.

Fig.3-c shows the variation in electron concentration along the selected orbit for, respectively the first (curve 1) and the second (curve 2) model.

The decrease in electron concentration on the left hand side is related to the AES being located below the ionization maximum.

As may be seen from Fig.3-a and c, curves 1 corresponding to the case of spherically laminated ionosphere may be interpreted as an averaging of curves 2 calculated for an horizontally inhomogeneous ionosphere.

Comparison of curves 2 for the Doppler shift difference and the electron concentration along the orbit shows that inhomogeneity dimensions are determined by relations $\rho \approx {}^1/_2\, v_\theta T$ on the right side and $\rho \approx v_\theta T$ on the left side of Fig.3. For the given model these relations are approximately valid every where with the exception of the region near zenith (T is the time interval between the maxima of curve 2, Fig.3-a).

Fig.4 shows the values calculated according to formula (17) of the reduced difference of Doppler frquencies for the first (curve 1) and the second (curve 2) ionosphere model at $f_1 = 20$ Mc (solid curves) and $f_2 = 90$ Mc (dashed curves). The critical frequency $f_0 = 13.5$ cm. Comparison of the curves shows that

$$\delta\omega_{1,2} = \delta\omega_{1,\infty} - \omega_1/\omega_2\delta\omega_{2,\infty} \approx \delta\omega_{1,\infty},$$

i.e. that in calculations with a precision to $\sim 5\%$, the effect of the medium at the higher frequency (90 Mc) may be, for all practical purposes, ignored.

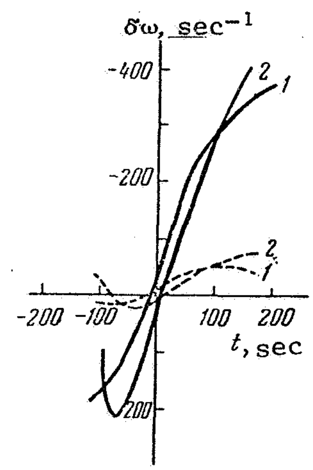

Fig.4

The submitted formulas may be used also in calculating the Doppler frequency shift during vertical launchings of rockets when the point of observation is located at some distance from the take-off point.

It should also be noted that errors in determing satellite and rocket velocity due to the effect of the inhomogeneity of the medium can be calculated on the basis of formula (15).

Manuscript received on
8 December, 1967.

* * * THE END * * *

Contract No.NAS-5-12487
Volt Information Sciences, Inc.,
1145 19th Street, N.W.
Washington, D.C. 20036
Telephone; [202] 223-6700 X-36,37

Translated by
Mr. Daniel Wolkonsky
September 25, 1968
Revised by
Dr. Andre L. Brichant
September 27, 1968

ALB/ldf

R E F E R E N C E S

1. J.C. SEDDON. J. Geophys. Res., 58, No.3, 323, 1953; 59, No.4, 463, 1954.
2. K.I. GRINGAUZ. Dokl. AN SSSR, 120, No.6, 1234, 1958.
3. Ya.L. AL'PERT. Usp. fiz. n., 71, No.3, 369, 1960.
4. K.I. GRINGAUZ, V.A. RUDAKOV, Dokl. AN SSR, 132, No.6, 1314, 1960.
5. K.I. GRINGAUZ, V.A. RUDAKOV, "Iskusstv. Sputniki Zemli", vyp.6, Izd-vo AN SSSR 1961, str.48.
6. Ya.L. AL'PERT. Geomagnetizm i Aeronomiya, 3, No.4, 629, 1963; 4, No.3, 479, 1964.
7. N.A. MITYAKOV, E.Ye. MITYAKOVA, Geomagnetizm i Aeronomiya, 3, No.5, 858, 1963.
8. Ya.L. AL'PERT ET AL. Geomagnetizm i Aeronomiya, 3, No.1, 10, 1963.
9. V.A. MISYURA, G.K. SOLODOVNIKOV, V.M. MIGUNOV, Kosmich. Issled., 3, No.4, 595, 1965.
10. V.A. MISYURA, G.K. SOLODOVNIKOV, V.M. MIGUNOV, Geomagnetizm i Aeronomiya, 6, No.5, 853, 1966.
11. K.I. GRINGAUZ ET AL, Geomagnetizm i Aeronomiya, 6, No.3, 568, 1966.
12. A.N. KAZANTZEV, D.S. LUKIN, Yu.G. SPIRIDONOV, Kosmich. Issled., 5, No.4, 593, 1967.
13. A.N. KAZANTZEV, D.S. LUKIN, S.I. FOMINYKH Idem, p.601.