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Report F-910352-13

Study of Trajectories and Upper Stage Propulsion Requirements for Exploration of the Solar System

Vol. I - Summary

Contract NAS2-2928
Final Report

UNCLASSIFIED

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<u>Requirements for Exploration of the Solar System</u>

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FOREWORD

This document is the Summary Technical Report for the Study of Trajectories and Upper Stage Propulsion Requirements for Exploration of the Solar System. The study effort was sponsored by the Mission Analysis Division of NASA Headquarters, OART, Moffett Field, California, under Contract No. NAS2-2928.

The complete results of the study are contained in the following volumes:

Volume I - Summary
Volume II - Technical Report
Volume III - User's Manual for Power-Limited Trajectory Optimization Computer Program

The current study is an extension to the original one-year contract which began in July 1965. The period of performance for the extension was from August 1966 to September 1967. Interim quarterly reports published under the contract extension are United Aircraft Research Laboratories Report E-910352-10, November 1966, and F-910352-11, February 1967, both entitled "Study of Trajectories and Upper Stage Propulsion Requirements for Exploration of the Solar System", and F-910352-12, "Aids for Analyzing Constant-Thrust, Low-Acceleration Propulsion Systems".

The following personnel contributed to the preparation of this report and to the different phases of the study as indicated:

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R. Gogolewski	High-Low Thrust Planetocentric Studies
C. P. Van Dine	Trajectory Optimization Computer Program
T. N. Edelbaum (Consultant)	Planetocentric-Heliocentric Trajectory Matching

Report F-910352-13

Study of Trajectories and Upper-Stage Propulsion Requirements for Exploration of the Solar System

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SUMMARY

Study Objectives

The basic objective of this research effort is the development of user-oriented computer programs for solving selected trajectory and system optimization problems characteristic of low-acceleration, power-limited, constant-thrust (electrically propelled) interplanetary vehicles. The trajectory optimization is characterized by optimum single or double coast periods with variable or constant power, while the system optimization is concerned with computing the values of exhaust jet velocity (specific impulse) and powerplant fraction which maximize payload fraction. The goal is to optimize both the trajectory and propulsion parameters simultaneously within the programs (system-trajectory optimization).

A secondary objective is the development of a computer program capable of minimizing the initial gross mass of a vehicle using low acceleration solely or in combination with high acceleration. This mass minimization program follows the development of the system-trajectory optimization programs above.

A further objective is the determination of the effects on performance of thrusting (either high-plus-low thrust or low thrust alone) between the planetocentric and heliocentric gravitational fields. The purpose is to employ appropriate equations or assumptions which adequately account for these effects insofar as minimizing the gross mass of the vehicle is concerned.

Study Scope

The research study consists of analytical and computer programming efforts for solving selected trajectory and system optimization problems characteristic

of low-acceleration, power-limited, constant-thrust (electrically propelled) interplanetary vehicle systems. The analytical studies include the variational formulation of several heliocentric, power-limited trajectory problems and the associated payload fraction maximization for a series of interplanetary flight and power modes. These modes include single- or multiple-coast trajectories and consider power either constant or varying with heliocentric position or with time. Missions include flybys of the planets or flight to a heliocentric position and velocity, orbital transfer, planetary rendezvous, and round-trip planetary stopover or flyby. In all of these cases hyperbolic excess speeds on the boundaries are included where appropriate, and the (constant) exhaust velocity and powerplant fraction are optimized to yield maximum payload fraction.

In addition, the analytical studies investigated the problem of the planetocentric low-acceleration spiral and the related problem of thrusting through one gravity field into another. In these investigations, equations are sought which adequately determine system performance with either high-plus-low thrust or low thrust only as the vehicle moves between planetocentric and heliocentric gravitational fields.

The system studies related to the foregoing heliocentric and planetocentric trajectories consist of efforts to employ an expanded rather than simplified payload fraction definition for the electric propulsion stage and to integrate these results with the trajectory optimization computer programs. The expanded payload fraction definition includes the mass of the propellant tanks, thrustors, and miscellaneous structure in addition to the powerplant.

The system study intimately related to the planetocentric and heliocentric trajectories is the minimization of the combined high-low acceleration vehicle initial gross mass for selected flight modes. The combination of hyperbolic excess speeds and the attendant low-acceleration (heliocentric) trajectory requirement is sought which minimizes total vehicle mass for a given payload to be delivered.

The programming effort consists primarily of applying the implicit, finite-difference Newton-Raphson algorithm to solve each system of equations which describes a particular heliocentric trajectory, propulsion system, and control optimization problem. Toward this end, numerical processes and computational techniques are studied which form the basis for the organization and development of the program code. Optimization problems investigated by the algorithm include planet-to-planet rendezvous and one-way planetary flybys, both of which include hyperbolic excess speeds, payload maximization, variable power, and two- or three-dimensional trajectories. Constant-thrust and single- or multiple-coast periods are other important features. This investigation also includes a round-trip stopover mission for the variable-thrust operating mode.

Not within the scope of the study is the computation of numerous trajectory, electric propulsion system, and vehicle mass data for mission and systems analyses. For the computer programs developed, only sufficient run time was accumulated to verify and confirm as thoroughly as possible the basic validity of the deck. Time limitations precluded extensive machine runs to completely determine program operating characteristics, capabilities, and limitations.

Basic Assumptions and Approach

The general approach to the heliocentric trajectory and system optimization problems was to, first, derive the system of differential equations describing each optimization problem by the calculus of variations, and second, solve these systems of equations by the implicit finite-difference Newton-Raphson algorithm. Rather than develop complete individual computer programs for the several special problems, a series of generalized subroutines was prepared which would implement the logical and algebraic aspects of the Newton-Raphson algorithm. These subroutines represent that part of the overall programming task which is common to all the trajectory problems.

The planets are considered to be massless points moving in mutually inclined, elliptic orbits with their heliocentric positions and velocities computed internal to the program based on the latest available astronomical data on the orbital elements. The option for computing two- or three-dimensional heliocentric transfer trajectories is included. During this transfer the motion of the vehicle is governed by the thrust acceleration and the sun's gravitational field; no perturbations due to the planets are included.

The approach to the high-plus-low-acceleration problem was to develop the computer programs for the heliocentric and planetocentric phases independently of each other; the two phases were related in a separate overall mass minimization program which accounts for the condition of the vehicle at the assumed transition between the planetary and heliocentric gravitational fields. The consequent computer programs for each phase provide results for input to the vehicle mass minimization program rather than attempt to integrate each program as a subroutine into a system mass computation program.

The minimization of initial gross mass is accomplished by a search procedure on the hyperbolic excess speeds. These speeds relate the performance of the corresponding planetocentric high-thrust (or atmospheric entry) system to that of the heliocentric electric propulsion system. The high-thrust step mass and entry system computation subroutine is an improved version of that developed in the initial phase of Contract NAS2-2928 (Ref. 1). The subroutine includes gravity losses and optimum thrust-to-weight ratio for minimum step mass. Also, the computation of spacecraft mass for manned round-trip missions employs the procedure developed in the same contract phase.

The problem of accurately describing the vehicle's dynamic condition as it transits the gravitational fields of both the planet and the sun is analyzed both numerically and analytically. Both modes of high thrust combined with low thrust and low thrust alone are considered. From these results it is considered sufficiently accurate for mission and system analysis purposes to assume high-thrust operation within the planet's activity sphere separate from low-thrust operation in heliocentric space.

RESULTS AND ACCOMPLISHMENTS

In general, the research effort produced the following:

1. a set of computer programs which simultaneously optimize both the electric propulsion system parameters and the trajectory thrusting program (system-trajectory optimization),

2. a computer program for minimizing the initial gross mass of a vehicle utilizing a combined high-plus-low-acceleration propulsion system,

3. improvements in the convergence properties of the previously developed constant-thrust, single-coast, system-trajectory optimization computer program,

4. a suggested procedure for optimizing the propulsion parameters of an all-electric vehicle that operates both in planetocentric and heliocentric space, and

5. complete sets of variational equations for a series of system-trajectory optimization problems of present and future interest.

Although attempts were not made to solve all of the formulated problems of the series, those that were successfully programmed represent a considerable achievement in the economical computation of accurate, optimum,constant-thrust, multiple-coast, power-limited trajectories, especially in view of the fact that the propulsion system parameters are simultaneously optimized for given hyperbolic excess speeds and variable power.

Summarized below are specific major results and accomplishments of several programming, numerical, and analytical studies which contributed to the formulation, development, and consequent utilization of the object computer program. Three general areas of effort are presented. These include, first, the computer programs developed for analyzing certain power-limited, heliocentric trajectory and system optimization problems, and for minimizing the mass of vehicles powered by mixed high-plus-low-acceleration propulsion systems. Presented next are the results of the numerical and analytical treatments concerning the problem of thrusting within the planet's sphere of influence (low acceleration solely or in combination with high acceleration) and the associated problem of calculating trajectories which transit the gravitational fields of both the planet and the sun. The third effort consists of the variational formulations for trajectory problems of interest not only to the present study but also of general interest for future programming efforts and subsequent mission mode studies.

Developed Computer Programs

1. Optimization of Heliocentric Power-Limited Trajectories

Planet-to-planet rendezvous is treated with an internal discrimination between one or two coast periods. One-way planetary flybys are included with either one or two coasts allowed. Hyperbolic excess speeds are to be specified at both departure and arrival for the rendezvous, whereas only the departure need be given for the flyby (final hyperbolic speed is open). In both modes the option is given for optimizing either the exhaust velocity or the powerplant fraction or both. Power could be either a function of heliocentric position or a constant. The choice of two- or three-dimensional trajectories is an option.

A round-trip stopover mission can be optimized with respect to the distribution of outbound and inbound legs for fixed total trip time, planetary stay time, and given hyperbolic velocities. The hyperbolic velocities are to be specified at Earth departure, planetary arrival and departure, and either specified or left open for Earth arrival. The variable-thrust operating mode is used.

A user's manual was developed as part of this programming effort. Sufficient information and guidelines are described to reduce the time required to familiarize the user with the general operating characteristics of the program and to expedite the computation of desired trajectories. This manual is given in Volume III of this report and is considered to be an integral part of the heliocentric trajectory optimization program.

An example of a typical result from the program is displayed in Figs. 1 and 2 for a 320-day constant-power, Mars-to-Earth rendezvous in 1980. The first figure illustrates the position-time history of the two-coast trajectory and the times at which the thrust is turned off or on. Although the payload fraction has not been maximized with respect to specific impulse and powerplant fraction, the initial guesses made within the program are very close to the optimum values. The powerplant specific mass of 1 kg/kw was chosen merely to assure convergence for the given example.

Figure 2 shows the magnitude of the primer vector and indicates the regions of thrusting and coasting and their points of occurrence, which are located as anticipated in relation to the sahpe of the curve. Note that, as required for an optimum trajectory, the primer vectors are equal at the initiation and termination of a coast period.

2. Minimization of Hybrid-Thrust Vehicle Mass

The initial mass on Earth parking orbit is minimized for a vehicle employing mixed high- and low-acceleration propulsion. The flight modes are parking-orbit-to parking-orbit, one-way flyby, and round-trip stopover. In the first case, high thrust is used for departure and arrival, while low thrust is employed in between. In the second case, there is no high-thrust propulsion at the arrival point. The third case is a combination of the first two. Actual masses (not dimensionless fractions) are computed for the high-thrust and low-thrust systems once payload mass and hyperbolic speeds are given. A search procedure is used to determine the optimum combination of high plus low thrust which results in minimum vehicle mass for the given payload. Various types of high-thrust propulsion systems are possible through the specification of certain engine parameters.

3. Improved Single-Coast Trajectory Program

An existing single-coast, constant-thrust program was improved by employing closed-form expressions for optimum exhaust velocity and powerplant fraction which are based on a given thrustor efficiency function and a simplified payload fraction definition. These expressions are used in conjunction with the trajectory optimization subroutine to obtain results of interest by themselves or for use as starting guesses for an improved payload fraction definition. This improved definition accounts for propellant tanks, tie-in structure, and thrustor mass and efficiency varying with exhaust velocity. Optimum exhaust velocity and powerplant fraction are computed for rendezvous only (specified hyperbolic velocity at departure and arrival) and for either or both payload definitions.

A closed-form expression is employed for estimating the maximum powerplant specific mass which yields zero payload for a given trajectory. Computations of specific masses greater than this maximum are avoided.

A recently uncovered characteristic of all present versions of the single-coast computer programs is the dependence of computed values of powered time and exhaust velocity upon the input guesses for the ratios of powered time to trip time and the dependence of constant-thrust J on variable-thrust J. Iterations on these ratios using the computed powered time and constant-thrust J are not part of the internal iterations between the exhaust velocity and powerplant fraction in the trajectory optimization subroutine. However, the maximum payload fractions and optimum powerplant fractions so computed do not change significantly with variations in the input time- and J-ratios. In terms of the system mass fractions, the results of the single-coast programs are valid; the powered time and specific impulse are only approximate and may be in error for detailed considerations of time and specific impulse effects on system design and requirements.

The apparent reason for the foregoing problem is that the equations describing the optimum powerplant fraction and exhaust velocity are based on the assumptions that the minimum value of J is invariant with powerplant fraction and that the average thrust acceleration over a minimum-J trajectory is also invariant with powerplant fraction. Therefore, given any constant-thrust minimum-J trajectory and its accompanying powered time, the corresponding average thrust acceleration may be determined and equations derived for the optimum powerplant fraction. No updating of the trajectory which provided the initial J and powered time is necessary insofar as the equations for optimum powerplant fraction are concerned. The mass fractions and exhaust velocity are quite close to exact values although the powered time is not.

Tables I, II, III, and IV summarize the equations used in the program. The closed-form expressions are based on the simplified payload fraction definition which states that the payload is the terminal mass fraction less the powerplant fraction. The expression used for estimating the maximum powerplant specific mass for a given trajectory (represented by an optimum constant-thrust J and powered time, T_p) is given at the bottom of Table I. The thrustor efficiency function employed is illustrated in Fig. 3.

The improved payload fraction definition, sample thrustor specific mass functions, and the optimum system parameters are presented, respectively, in Tables II, III, and IV. The thrustor specific mass functions are analytic fits to the curves presented in Fig. 4 and are intended to be representative of the equations of Table IV. For a given heliocentric transfer (i.e., optimum constant-thrust J and T_p) the equations of Table IV must be solved iteratively to determine the optimum powerplant mass fraction and exhaust velocity (the primes indicate derivatives with respect to exhaust velocity). It should be noted that the suggested thrustor efficiency and specific mass functions given herein need not be used when solving the equations of Table IV. Functions of particular interest could be employed provided they possess continuous first derivatives.

Heliocentric/Planetocentric Trajectory and Systems Analysis

1. Combined High-plus-Low Thrusting Within the Planet's Sphere of Influence

A numerical analysis was performed to determine the effects of neglecting the low-thrust system's operation within the planet's activity sphere immediately after high-thrust burnout. The trajectory problem was analyzed by numerically integrating the planetocentric equations of motion for both high- and low-thrust operation until the sphere of influence is reached. In general, the time in which the low-thrust system has to act is so short that there is negligible difference in performance if the given hyperbolic excess speed is assigned to the high-thrust system and the low-thrust system is assumed to start (heliocentrically) at the center of the massless point planet. Both departure and capture modes were investigated for Jupiter, Mercury, and Earth.

Figures 5, 6, and 7 indicate the increase in hyperbolic excess speed due to low-thrust acceleration within the planet's sphere of influence. For practical thrust-acceleration values of about 10^{-3} to 10^{-4} g's, and in terms of mission and systems analyses, the combining of high-thrust planetocentric and low-thrust heliocentric phases as separate regions related only by the hyperbolic excess velocity is a reasonable assumption.

2. Low-Acceleration Planetocentric Spiral

The low-thrust planetocentric spiral, departure or capture, was studied by using analytic expressions available in the literature. Two aspects were studied; first, the spiral solely within a planet's gravitational field which is assumed to extend to infinity, and second, a spiral that accounts for properly switching the computations from the planet's gravitational field to that of the sun (see Item 3 following). The spiral trajectory requirements were represented by equations representing the final mass ratio as a function of exhaust velocity and powerplant fraction. The study resulted in a procedure (not programmed) for optimizing the exhaust velocity and powerplant fraction of an all-electric vehicle that goes from parking orbit, through a heliocentric transfer, and either captures onto a planetary parking orbit or attains some final heliocentric position or velocity.

3. Heliocentric/Planetocentric Trajectory Matching

A theoretical study of the motion of a low-thrust vehicle as it moves between a planetary gravity field and the solar field was performed to account for the planetary perturbations in the performance calculations. Both spiral and hyperbolic escape trajectories were considered, both assumed to commence (terminate) from (in) a circular parking orbit about a given planet.

For the low-thrust spiral, a point in the spiral is sought at which the computation of vehicle performance for the planetocentric portion of the flight to that point is equivalent to the calculations based upon the actual trajectory profile. The resulting equation for incremental velocity, ΔV, is

$$\Delta V_1 = V_c - 1.84 \left(\frac{T}{m_1} \mu_p\right)^{\frac{1}{4}}$$

where V_c is the circular velocity of the parking orbit, T is thrust, m_1 is the vehicle mass at the transfer point, and μ_p is the gravitational parameter of the planet. The position offset from the center of the planet contributes a change of propellant consumption to the whole trajectory of the order of $\mu^{1/2}$, where μ is the ratio of the planet's mass to that of the sun. For the inner planets at least, this term can be neglected in performance calculations.

Relations for the required velocity and position offsets are derived for the low-thrust hyperbolic trajectories. In both cases it is shown that the error in the approximation is on the order of μ. The effect of assuming, as is done in the analysis, zero rather than a finite periplanet radius is also of the same order.

4. Aids for Analyzing Constant-Thrust Systems

The closed-form expressions for optimum exhaust velocity and powerplant fraction used in the improved single-coast program were plotted to develop a series of graphs for quickly estimating the performance of constant-thrust systems. Given the trajectory requirements in terms of J ($J=\int a^2 dt$) and powered time, the optimum system parameters may be quickly estimated for a given powerplant specific mass, α_w, and thrustor efficiency parameter, d. For the same input values and parameters, a graph is used to estimate the maximum powerplant specific mass which produces zero payload. Although the foregoing graphs are for the simplified payload fraction (defined as the final mass fraction less the powerplant fraction), equations of the optimum system parameters for the improved payload definition were developed along with possible procedures for their solution. These equations were programmed as part of the improved single-coast constant-thrust optimization program.

The corresponding graphs are given in Figs. 8 through 15 for values of $\gamma^2 = \alpha_w J/2000$, the product JT , and the efficiency parameter d = 10, 20, 30, and 40 km/sec where the efficiency η is given by $\eta = 1/(1 + d^2/c^2)$. The powerplant specific mass that yields zero payload fraction is obtained from Fig. 16.

Variational Formulations of Heliocentric Trajectory Problems

Complete sets of differential equations and related transversality conditions for the following problems were developed by use of the calculus of variations. The list is quite extensive, and not all the problems were programmed for solution by the trajectory optimization deck.

Problem 1

This first problem concerns three-dimensional trajectory and control optimization with the thrustor constrained to constant-exhaust-velocity on-off operation. The power available is a given function of position and time corresponding to decaying radioisotope power or solar power. The objective is maximum final mass fraction for given values of powerplant specific mass, powerplant fraction, and exhaust velocity. The boundary conditions correspond to (a) planetary rendezvous, (b) planetary flyby, (c) flyby at a given radius, and (d) orbital transfer.

Problem 2

This problem includes all of problem 1, but in addition, the powerplant fraction μ_w, and the exhaust velocity, C, as well as the trajectory and the associated steering program, are optimized. The objective function is maximum payload fraction which is defined to be everything that is left at the end of the mission except the powerplant, thrustor, and structure.

Problem 3

In this problem, two separate propulsion units are used, one before and one after the coast period. The exhaust velocity and powerplant fraction of each unit are optimized with respect to final payload fraction.

Problem 4

This problem is the same as problem 1 except that the thrust-acceleration vector is constrained to make a constant angle with respect to the radius vector. One constant angle is allowed before coast and another after coast. These two angles are to be separately optimized with respect to maximum final mass.

Problem 5

A round-trip stopover mission is treated for minimizing the mass of the electrically propelled vehicle (after staging of the initial high-thrust Earth departure propulsion) for a given payload back at Earth. High-thrust impulses at Earth departure and planetary arrival and departure are included along with atmospheric braking at Earth return. Two power-limited propulsion systems are employed, one for the inbound and one for the outbound heliocentric transfer; the latter system - including powerplant, thrustor, and tankage - is staged at the planet along with the capture high-thrust stage. The trajectory optimization includes optimizing the distribution of leg times, the launch date for fixed trip time and planetary stay time, and the directions of the hyperbolic excess velocities attributed to high thrust.

The corresponding variable-thrust solution of the round-trip stopover mission is required as a starting approximation. Accordingly, variable-thrust transversality conditions are included corresponding to the constant-thrust case.

Problem 6

A round-trip planetary flyby is considered for the variable-thrust operating mode. The problem is treated in two parts: no constraint on the periradius, and a fixed periradius. The second part is solved if the first produces a periradius lower than the minimum bound imposed by a flight constraint; e.g., radius of the sensible atmosphere. By the use of internal transversality conditions at the planet, both the outbound and inbound legs are solved for simultaneously. The best launch date, best flyby date, and the optimum characteristics of the flyby encounter are computed.

Problem 7

Although not a calculus of variations problem, the problem of substituting analytic solutions for numerical solutions in the coast regions was investigated as a possible approach to reducing the number of mesh points. Analytic solutions for both the trajectory and the primer vector in the coast regions are developed and coupled with the numerical procedure at the switching points. Time did not permit these results to be incorporated into the trajectory optimization programs.

RESULTS FOR GENERAL APPLICABILITY

Because the end product of this research effort consists of computer programs, equations, and graphs, practically all of these results can be used in varying degrees for the evaluation of electric propulsion systems, especially powerplants and thrustors. The following discussion presents particular aspects of the study and their results which are generally applicable to the study of power-limited systems and associated trajectories.

Computer Programs

Of immediate use to the analyst is the computer program for optimizing the trajectory and propulsion system for power-limited interplanetary vehicles. The program consists of the deck itself and the user's manual (Volume III of this report). The program, coded in Fortran IV, was developed and checked using the UNIVAC 1108 and has been run on the IBM 7094 DCS. With the user's manual and a minimum of programming effort, the deck should be a useful tool for analyzing power-limited trajectories and the corresponding electric propulsion system.

Related to the above program is the single-coast, constant-thrust trajectory optimization program previously developed and recently improved to include a simplified optimization procedure and an expanded payload fraction definition.

Although only rendezvous (planet-to-planet) trajectories may be handled, it is of use in those cases where only one coast period is needed and where a better definition of the powerplant, thrustor, and inert structure mass effects is desired.

The hybrid-thrust mass optimization program uses the data from the first two programs to optimize the combination of high and low thrust and is useful in mission and system studies, since an actual mass breakdown of the vehicle is computed. Although particular high-thrust scaling laws are employed along with certain spacecraft inert masses, they are of sufficient detail to provide a reasonable estimate of the system mass requirements. The program should prove useful not only to the evaluation of electric propulsion systems but also to the study of the attendant high-thrust systems and, in general, to the overall capability of hybrid-thrust propulsion.

Analytical and Numerical Information

Possibly the results having the most general applicability are the graphs which give optimum exhaust velocity and powerplant fraction and the resulting maximum payload fraction (Figs. 8 through 16). Even with the simplified payload fraction definition and particular thrustor efficiency curve employed, the prime advantage of these charts is the rapidity with which estimates may be made of electric system performance. Once an optimum constant-thrust J and powered time are known for a particular trajectory, the effects of powerplant specific mass and thrustor efficiency parameter may be seen quickly without recourse to the trajectory optimization program (either multiple-coast or single-coast). In fact, as the multiple-coast program now stands (simplified payload fraction), only two or three different powerplant specific masses need be used to obtain the propulsion parameters as well as the J and powered time. These latter two parameters could then be used to determine the propulsion parameters at any other values of powerplant specific mass (up to the maximum) and thrustor efficiency parameter.

The results of the analytical and numerical studies concerning planetocentric-heliocentric operations provide additional useful information for the analyst although they are not immediately applicable to general mission and system studies. The equation relating the incremental velocity of a low-thrust system undergoing a planetocentric spiral is useful to the analysis of all-electric interplanetary vehicles, since the proper time for switching the performance calculations between planetocentric and heliocentric space is included (see page 8). The corresponding equations for the velocity and position offset which accounts for the planetary perturbation are detailed in Section VI of the Technical Report (Vol. II). Also presented therein are the appropriate equations for high-thrust planetocentric departure or arrival, low-thrust departure or capture on a hyperbolic trajectory, and the planetocentric spiral.

The assumption of separating the high-thrust planetocentric phase from the low-thrust heliocentric trajectory has been shown to be reasonable even though the actual operational procedure would have the different thrusting phases follow each other immediately. For mission and system analyses purposes at least, this assumption could be used in other hybrid-thrust mass optimization programs, thereby providing a measure of simplification.

RECOMMENDATIONS FOR FUTURE STUDIES

The following list of recommended studies is a result of the background and experience obtained in the performance of the study contract. The list is limited to those activities which would directly aid in expanding current capabilities of power-limited flight analysis and in applying such capabilities to the ultimate goal of determining the role of electrically propelled spacecraft in the exploration of the solar system. It should be noted that the first three items listed are essentially study projects, while the third is oriented more toward a survey. The remaining items are basically tasks which contribute to an overall goal of developing valuable study tools for power-limited systems and would therefore contribute significantly to the efforts of the first three recommended studies.

1. A system study should be initiated to determine the implications of high-plus-low-acceleration mission modes on the development of candidate power systems and thrustors and to the identification and, consequently, planning of the role of electrically propelled vehicles in solar system exploration. Such a study should have as its objective the comparison and evaluation of projected power systems and thrustors as related to a range of unamnned and, possibly, manned missions. In addition, the study should determine desirable and feasible characteristics of future primary propulsion power systems and should attempt to combine these characteristics (for different classes of powerplants) into a postulated design which would perform all or most of the missions either singly or by "clustering".

2. To ensure the broadest possible stimulation of new mission and flight mode concepts and to expedite the evaluation of such concepts, a mission/system analysis aids manual would be an invaluable tool. The spirit and philosophy of such an aids manual would parallel that of the NASA Planetary Flight Handbook, SP-35. Because of the coupling between the propulsion system and the power-limited trajectory, it is not possible to merely catalogue tables or graphs of trajectory requirements as is done for impulsive transfers. Therefore, a manual is envisioned which would include not only representative trajectory requirements but also techniques for estimating optimum constant-thrust system parameters, methods of extending payload

definitions and computing the associated parameters, guidelines for determining mixed-thrust trajectory requirements, and general information and background data from past system and mission studies. An additional possibility is the inclusion of a series of computer programs for solving specific trajectory problems.

3. There presently exist several diverse computer programs for solving essentially the same power-limited trajectory problem. A survey should be made of these computer tools to identify their capabilities, limitations, and similarities, such that the possibility of combining some of them could be investigated. The objective here is to develop combined programs which use the best features of each for particular problems. For example, a certain program may be capable of quickly solving the solar probe problem but requires difficult-to-obtain input guesses for certain variables. These may be provided by another program which solves essentially the same problem more slowly but requires only an unsophisticated starting solution. In other cases it may be evident that some particular power-limited trajectory problem is more conveniently and quickly solved by a certain numerical technique than that used in another program.

4. The preliminary procedure developed for optimizing the exhaust velocity and powerplant fraction with respect to payload fraction for a single-stage electric propulsion system should be programmed. This single-stage system is capable of two flight modes: 1) planetary parking orbit departure, heliocentric transfer, and planetary parking orbit capture, and 2) planetary parking orbit departure and heliocentric transfer to a heliocentric position and velocity.

5. The developed multiple-coast trajectory optimization program should be modified to accept the expanded payload fraction definition in a manner similar to that accomplished in the original single-coast program. The capability of allowing for any thrustor efficiency and specific mass variation with exhaust velocity should also be included. This modification is considered to be an add-on item using the approximation techniques employed in the single-coast program modification and is not meant to be a reprogramming effort.

6. Efforts should be made to apply the basic developed computer algorithm to the problem of variable mesh point spacing. An investigation should be initiated to determine the added flexibility and broadened trajectory problem scope that variable mesh spacing produces.

7. The remaining variational problems which were formulated but not solved should be investigated by the basic computer algorithm. Of particular interest here is the constant-attitude, solar-powered trajectory, the round-trip flyby, the orbital transfer, and the staging of one (of two) electric propulsion system before coast.

8. In analyzing the implementation of the finite-difference Newton-Raphson algorithm made to date, two facts stand out very clearly. First it is a lengthy and complex job to complete a computer code for a given problem. Although this difficulty will be eased in the future by the use of generalized subroutines now completed, this advantage will be counteracted by the necessity and desire to attack more difficult problems. Second, once a computer code has been generated to solve a problem by means of this algorithm, solutions can be generated fairly easily and quickly no matter how complicated or nonlinear the problem is. Therefore, recognizing both the difficulties of implementation and the high probability of success, future uses of this algorithm should be made in areas where the resulting data will be extremely useful or in areas where the data are currently essentially unattainable.

In trajectory analysis, three such study areas present themselves. The first is a program to choose simultaneously both the terminal hyperbolic excess speeds and the low-thrust trajectory which minimizes mass on Earth orbit for a given set of vehicle parameters. This area is currently the most time-consuming process in the analysis of hybrid-thrust missions. The approach would be to incorporate the currently used approximations and matching laws into the body of the heliocentric algorithm.

The second is a program to optimize trajectories in a time-varying, n-body, gravitational field. While the usefulness of such a program might be limited to checking out currently used matching criteria, there are very little data available which have been achieved through a unified approach. The questions arising for the case of close approaches to Jupiter are certainly worth answering, and the program would also offer a convenient means to study the guidance problem of low-thrust ascent and descent.

The third is a program for minimum-total-velocity-increment, multiple-impulse, high-thrust trajectories. At present, only a few examples of such transfers are available. It is also extremely likely that once these transfer data become available they would be very useful in demonstrating both the reduction of total energy requirements needed for high-thrust missions and, probably more significantly, the broadening of the launch windows available for these missions.

RELATIONSHIP OF STUDY TO NASA PROGRAMS

In general, the results of this study provide basic tools for an extensive and in-depth evaluation of electric propulsion systems and associated subsystems for unmanned and manned exploration of the solar system. As the unmanned missions become more ambitious, both in size of the scientific payload and intensity of exploration, the payload potential of electric propulsion, especially in combination with high-acceleration propulsion, must be investigated in terms of powerplant and thrustor development feasibility and in relation to the technology improvements of high-thrust systems. One important consequence of such investigations would be the overall effect on Earth-launch booster development in the Saturn V class and beyond.

Past studies (Ref. 1) indicate that, for the same payloads, a combined high-plus-low-acceleration propulsion system requires less total vehicle mass on Earth parking orbit than a corresponding high-thrust system by itself. As the missions become more difficult, e.g., Jupiter and Mercury orbiters, the benefit in mass becomes quite large. Because of this benefit, these more energetic unmanned missions may be accomplished by surface launch boosters in the Saturn IB and Saturn V classes. The major qualifications here, however, are the feasibility of developing powerplants of the required power rating and mass or the mass penalty incurred if a specific type of power system is used in a nonoptimum manner (with a high-thrust system).

The tools developed herein consider realistic thrustor operation (constant-thrust) and account for degrees of electric propulsion stage design sophistication as well as optimum heliocentric trajectory steering and coast periods. With the capability provided by these tools, unmanned orbiters and flybys to planets other than Mars and Venus could be studied as part of NASA's program to evaluate and compare the requirements of advanced power systems and the capabilities of currently projected systems. These types of flights are of the post-Voyager class and could possibly be a logical extension of that mission category.

Of a more fundamental technical nature is the relationship of the present study to the continuing development and extension of computer programs for optimizing power-limited trajectories. Missions corresponding to these trajectories would be out-of-the-ecliptic probes, round-trip planetary flybys, close-in solar orbiters, and constant-attitude solar-powered flights. The present study results include formulations of the systems of equations describing the optimum steering program, coasting periods, and propulsion system parameters for the foregoing missions. These formulations are of use to NASA's activities in developing appropriate study tools and aids for advanced system studies.

REFERENCE

1. Titus, R. R., R. V. Ragsac, R. Gogolewski, and G. Thrasher: Study of Trajectories and Upper Stage Propulsion Requirements for Exploration of the Solar System. United Aircraft Research Laboratories Report E-910352-9, July 1966. Performed under NASA Contract NAS2-2928.

TABLE I

CONSTANT–THRUST OPTIMUM SYSTEM PARAMETERS

SIMPLIFIED PAYLOAD FRACTION DEFINITION

$$C_{OPT} = \left\{ 0.0864 \frac{J T_P}{\gamma^2} \left(1 + \frac{\gamma^2}{0.0864} \frac{d^2}{J T_P} \right) \left[1 - \frac{\gamma}{\left(1 + \frac{\gamma^2}{0.0864} \frac{d^2}{J T_P} \right)^{1/2}} \right] \right\}^{1/2}$$

$$\mu_{W\,OPT} = \gamma \left[\frac{1 + \frac{2\gamma^2}{0.0864} \frac{d^2}{J T_P}}{\left(1 + \frac{\gamma^2}{0.0864} \frac{d^2}{J T_P} \right)^{1/2}} - \gamma \right]$$

$$\mu_{L\,MAX} = 1 - 2\gamma \left(1 + \frac{\gamma^2}{0.0864} \frac{d^2}{J T_P} \right)^{1/2} + \gamma^2$$

WHERE: $\gamma^2 \equiv \frac{\alpha_W J}{2000}$

C, KM / SEC

J, m^2/ SEC^3

T_P, DAYS

α_W, KG / KW

d, KM / SEC

α_W AT ZERO μ_L: $$\alpha_{W\,MAX} = \frac{2000 / J}{1 + \sqrt{\frac{1}{0.0216} \cdot \frac{d^2}{J T_P}}}$$

TABLE II

IMPROVED PAYLOAD FRACTION DEFINITION

$$\mu_L = 1 - \frac{1 + \sigma}{\rho}(1 - \mu_1) - (1 + \sigma)\left(1 + \frac{\alpha_F(C)}{\alpha_W}\right)\mu_W$$

WHERE: $\sigma = \dfrac{m_{STRUCT}}{m_W + m_F + m_{TANKS} + m_{PROP}}$

$\rho = \dfrac{m_{PROP}}{m_{TANKS} + m_{PROP}}$

$\alpha_F(C)$ = THRUSTOR SPECIFIC MASS FUNCTION

α_W = POWERPLANT SPECIFIC MASS

μ_1 = BURNOUT MASS FRACTION

μ_W = POWERPLANT MASS FRACTION

μ_L = PAYLOAD FRACTION

TABLE III

EXPONENTIAL APPROXIMATION TO THRUSTOR SPECIFIC MASS FUNCTION

$$\alpha_F(C) = a_1 e^{-\sigma_1\left(\frac{C}{20}-1\right)} + a_2 e^{-\sigma_2\left(\frac{C}{20}-1\right)}$$

α_F, KG/KW C, KM/SEC

	a_1	σ_1	b_1	σ_2
ELECTRON BOMBARDMENT, 1	1.63542	0.406626	2.46479	1.92452
" " , 2	0.429867	0.403804	1.05991	1.06851
CONTACT, 1	−0.0197516	−0.357073	1.10985	0.342079

<u>CONTACT, 2</u>:

$$\alpha_F(C) = e^{-0.600736\left(\frac{C}{20}-1\right)} \left\{0.590562 \cos\left[\left(13^\circ\ 14.428'\right)\left(\frac{C}{20}-1\right)\right] + 0.275432 \sin\left[\left(13^\circ\ 14.428'\right)\left(\frac{C}{20}-1\right)\right]\right\}$$

TABLE IV

CONSTANT—THRUST OPTIMUM SYSTEM PARAMETERS

IMPROVED PAYLOAD FRACTION DEFINITION

$$\mu_{W\,OPT} = \frac{\mu_1(1-\mu_1)/\rho}{\left(1 + \frac{\alpha_F(C)}{\alpha_W}\right)\left[1 - \frac{1+\mu_1}{2}\left(\frac{\eta' C}{\eta}\right)\right] + \frac{1+\mu_1}{2}\left(\frac{\alpha_F' C}{\alpha_W}\right)}$$

$$\mu_1 = \frac{1}{1 + \frac{\gamma^2}{\eta \mu_W}} \quad ; \qquad \eta = \frac{1}{1 + \left(\frac{d}{c}\right)^2}$$

$$C = \frac{\left[(0.0864)\, J T_P\right]^{1/2}}{1 - \mu_1}$$

WHERE $\gamma^2 = \frac{\alpha_W J}{2000}$

AND
J, m^2/SEC^3
T_P, DAYS
C, KM/SEC
d, KM/SEC
α_F, KG/KW
α_W, KG/KW

FIG. 1

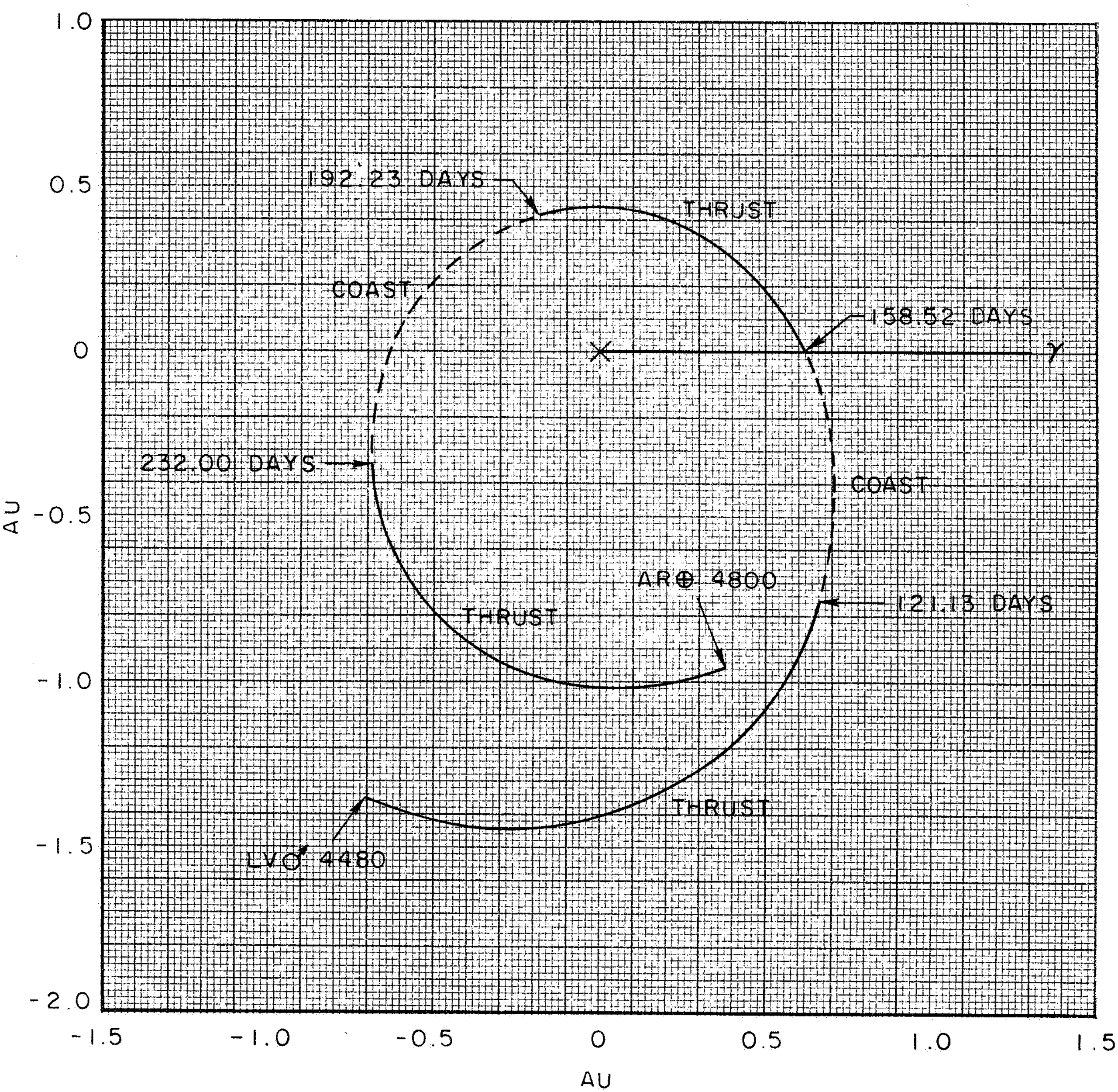
OPTIMUM MULTIPLE - COAST, CONSTANT - THRUST TRAJECTORY

320-DAY 1980 MARS-EARTH TRIP

$J = 20.61\ M^2/SEC^3$ $\mu_L = 0.8063$ $d = 20\ KM/SEC$

$I_{SP} = 19791\ SECS$ $\mu_W = 0.0921$ $\eta = 0.9895$

$V_A = V_B = 0$



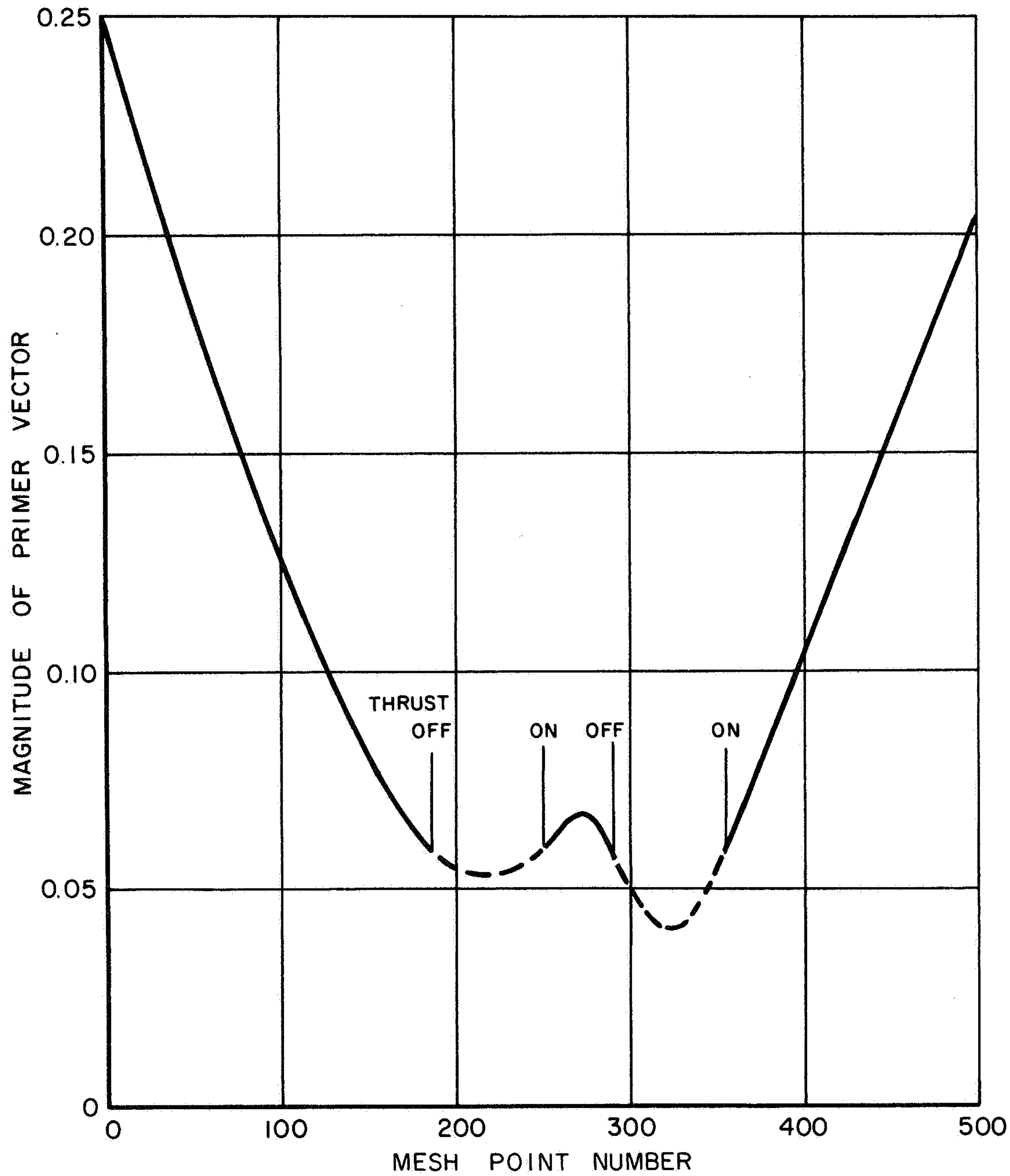
OPTIMUM MULTIPLE-COAST, CONSTANT-THRUST TRAJECTORY

PRIMER VECTOR TIME HISTORY
320-DAY 1980 MARS-EARTH TRIP

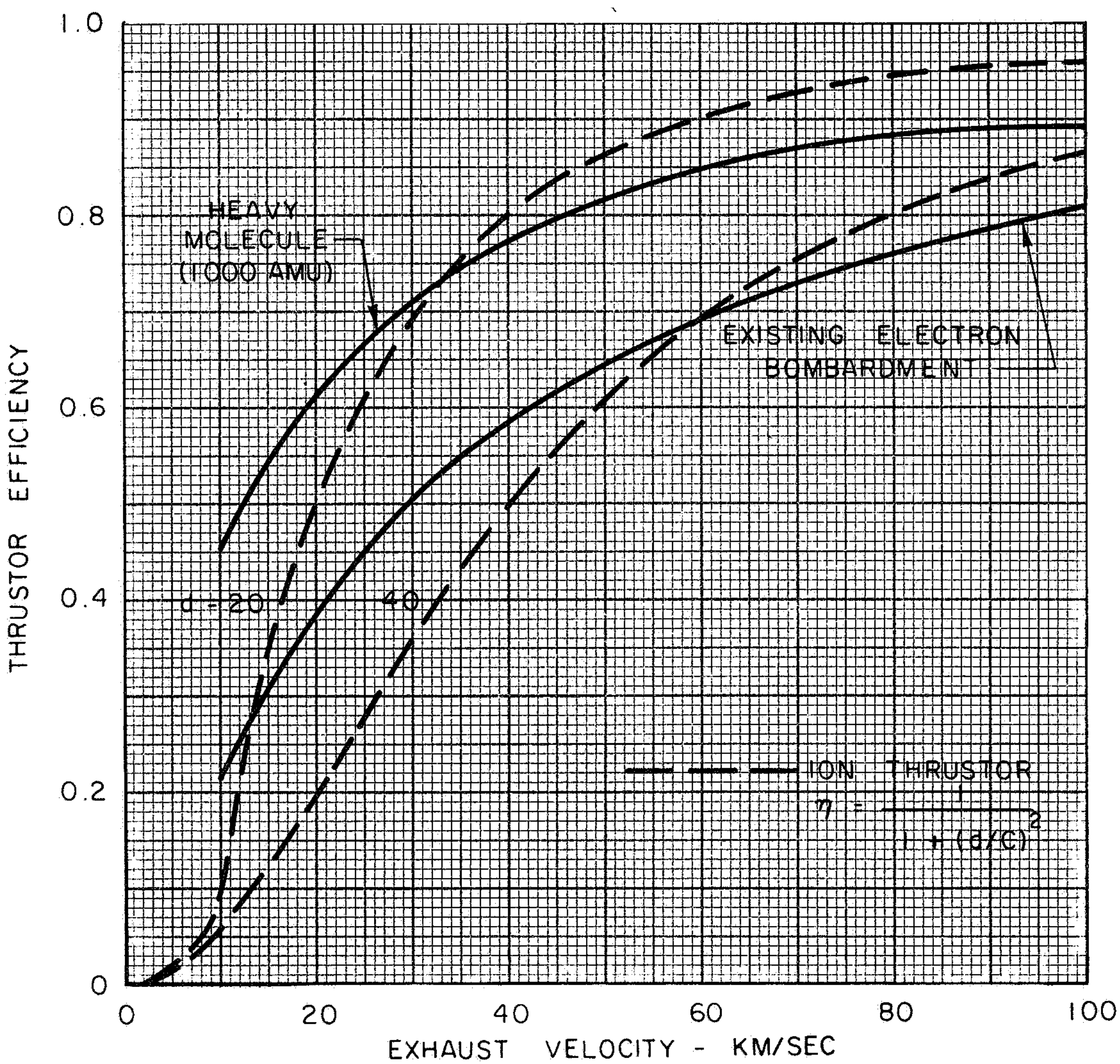
LV♂ 4480 AR⊕ 4800

$J = 20.61 \ M^2/SEC^3$ $\mu_L = 0.8063$

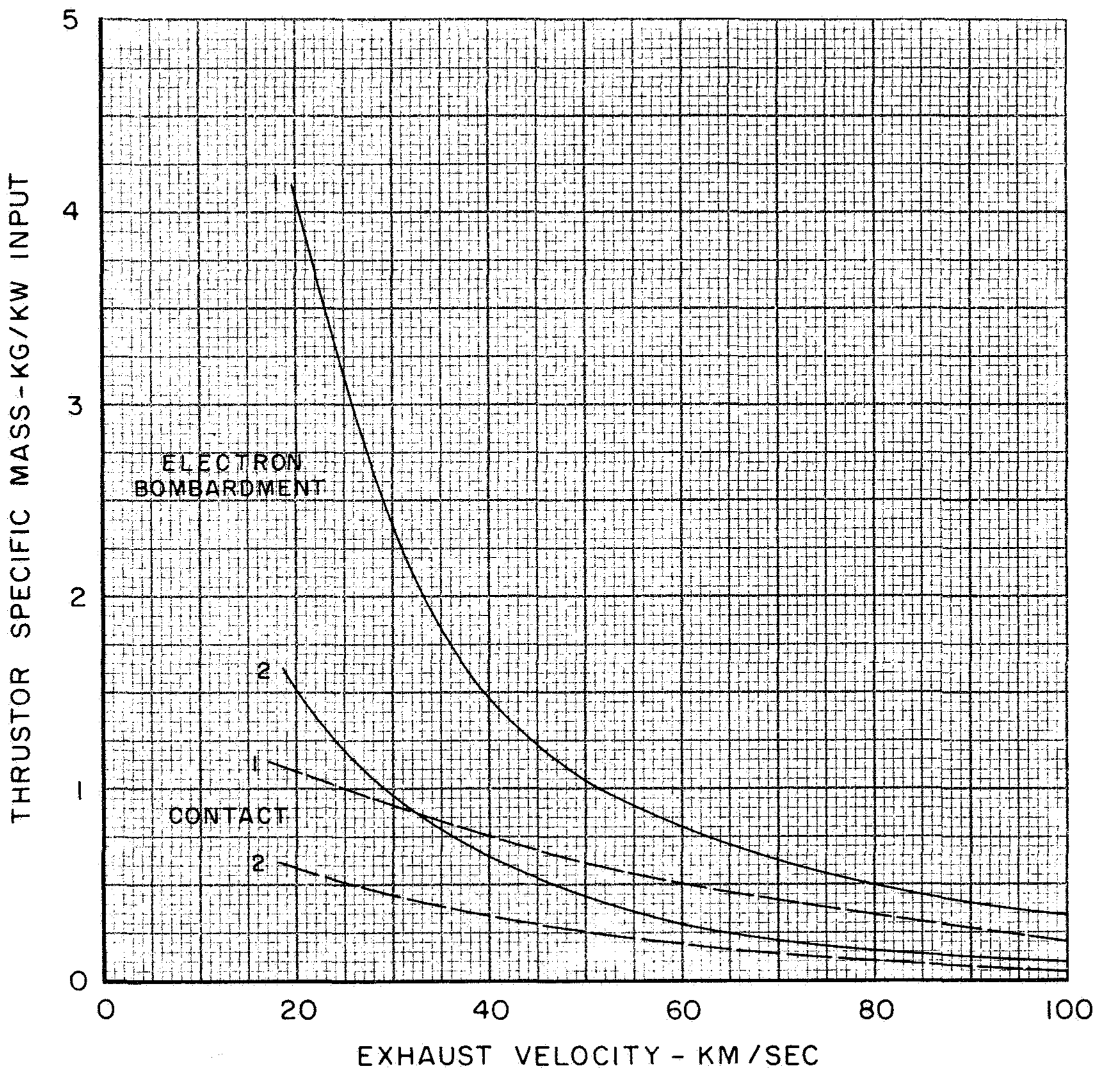
$I_{SP} = 19791$ SEC $\mu_W = 0.0921$



DEPENDENCE OF THRUSTOR EFFICIENCY ON EXHAUST VELOCITY



THRUSTOR SPECIFIC MASS

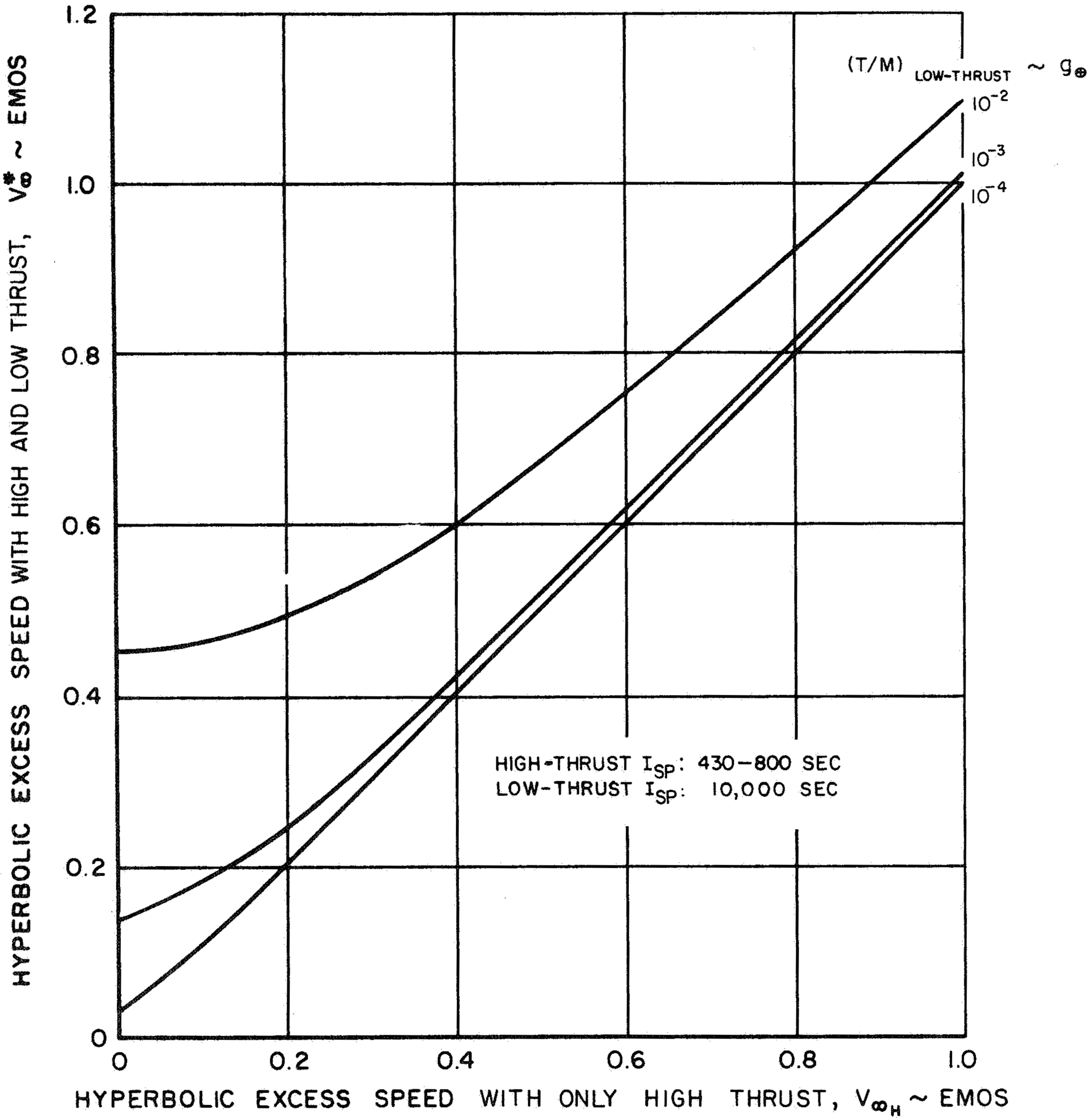


HIGH—LOW THRUST PLANETOCENTRIC OPERATIONS

HYPERBOLIC EXCESS SPEEDS ATTAINED WITH HIGH- AND LOW-THRUST PROPULSION WITHIN SPHERE OF INFLUENCE

EARTH ESCAPE

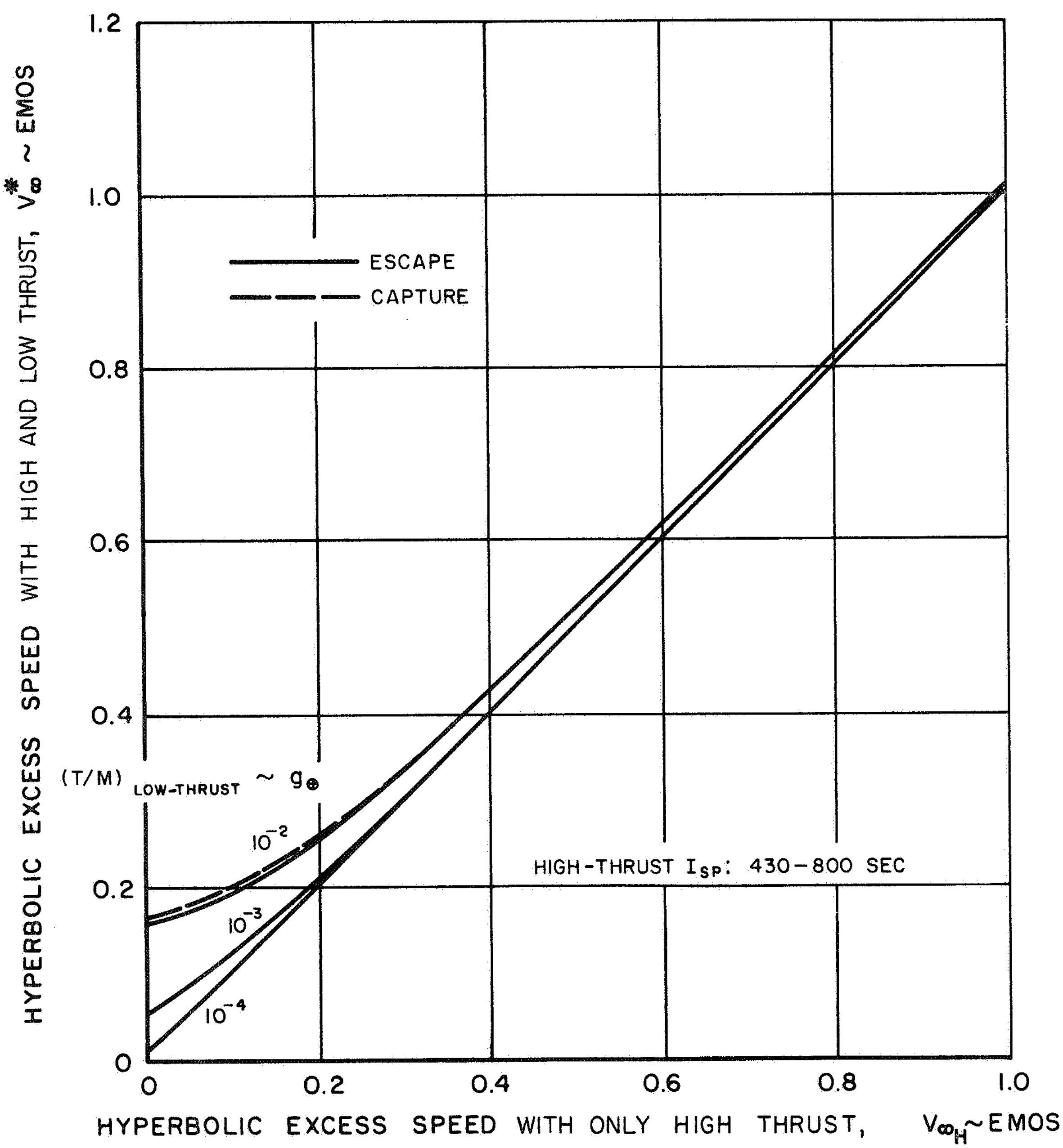
$r_0 = 1.1$



HIGH—LOW THRUST PLANETOCENTRIC OPERATIONS

HYPERBOLIC EXCESS SPEEDS ATTAINED WITH HIGH- AND LOW-THRUST PROPULSION WITHIN SPHERE OF INFLUENCE

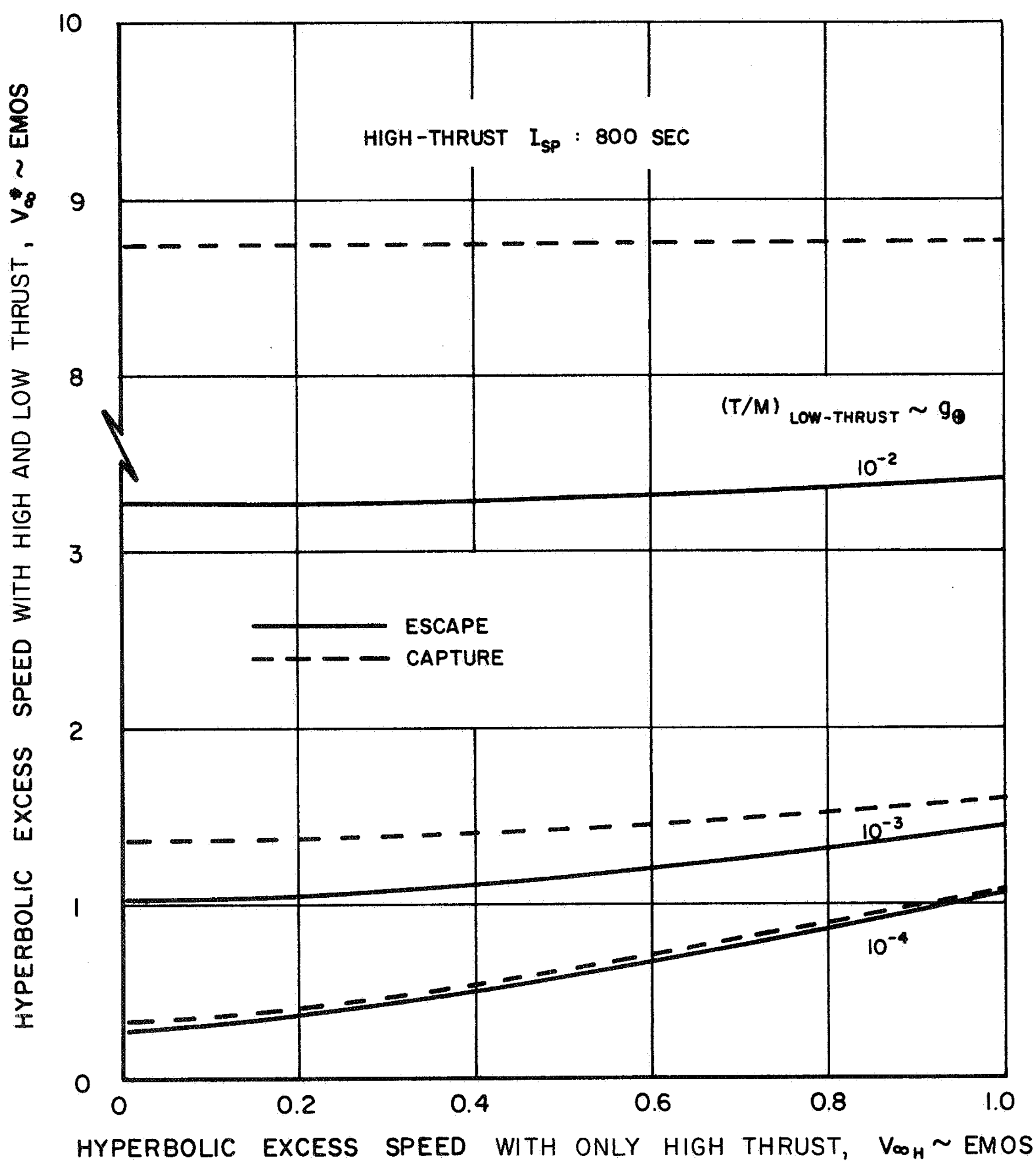
PLANET - MERCURY



HIGH-LOW THRUST PLANETOCENTRIC OPERATIONS

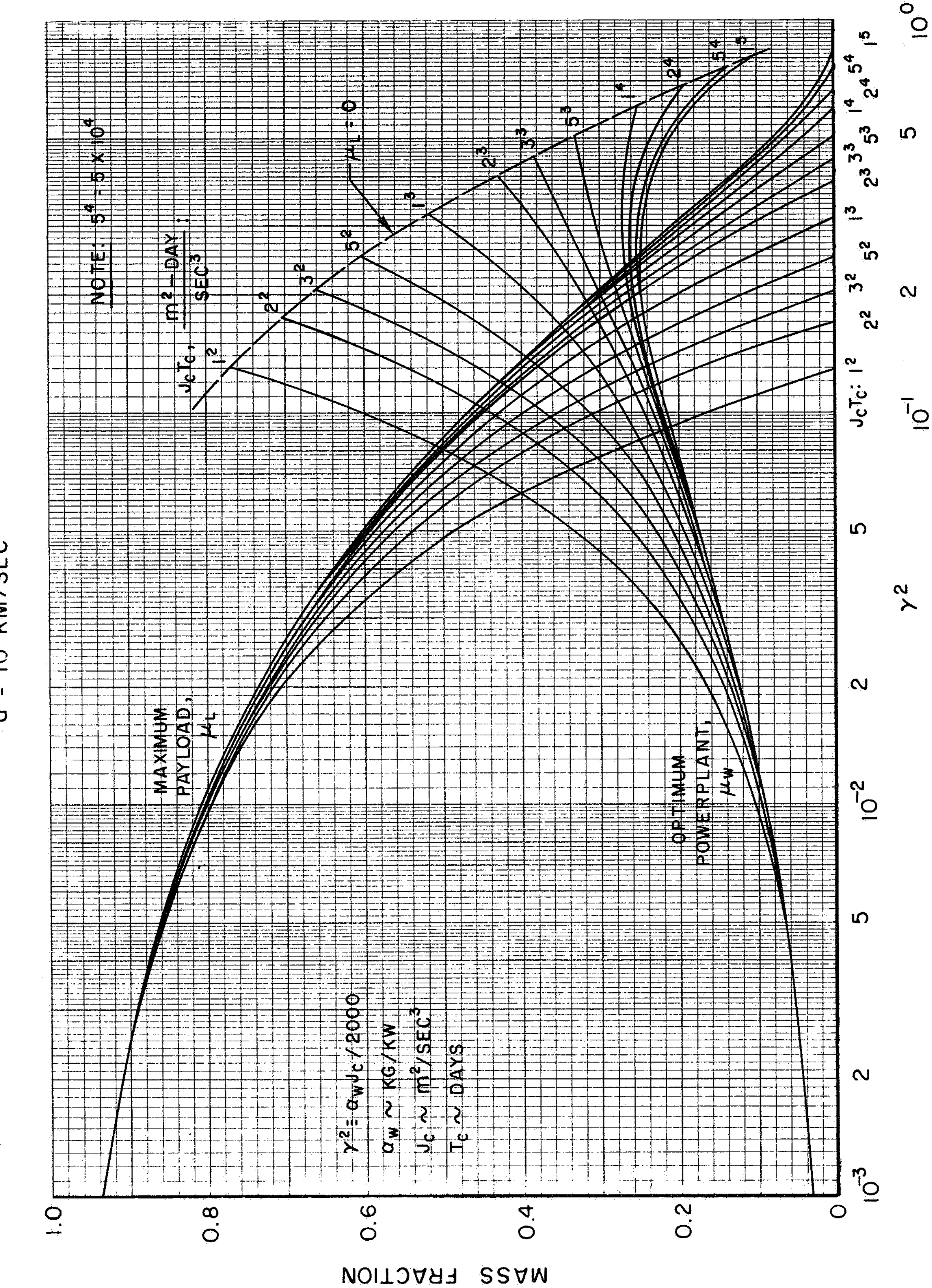
HYPERBOLIC EXCESS SPEEDS ATTAINED WITH HIGH- AND LOW-THRUST PROPULSION WITHIN SPHERE OF INFLUENCE

PLANET – JUPITER

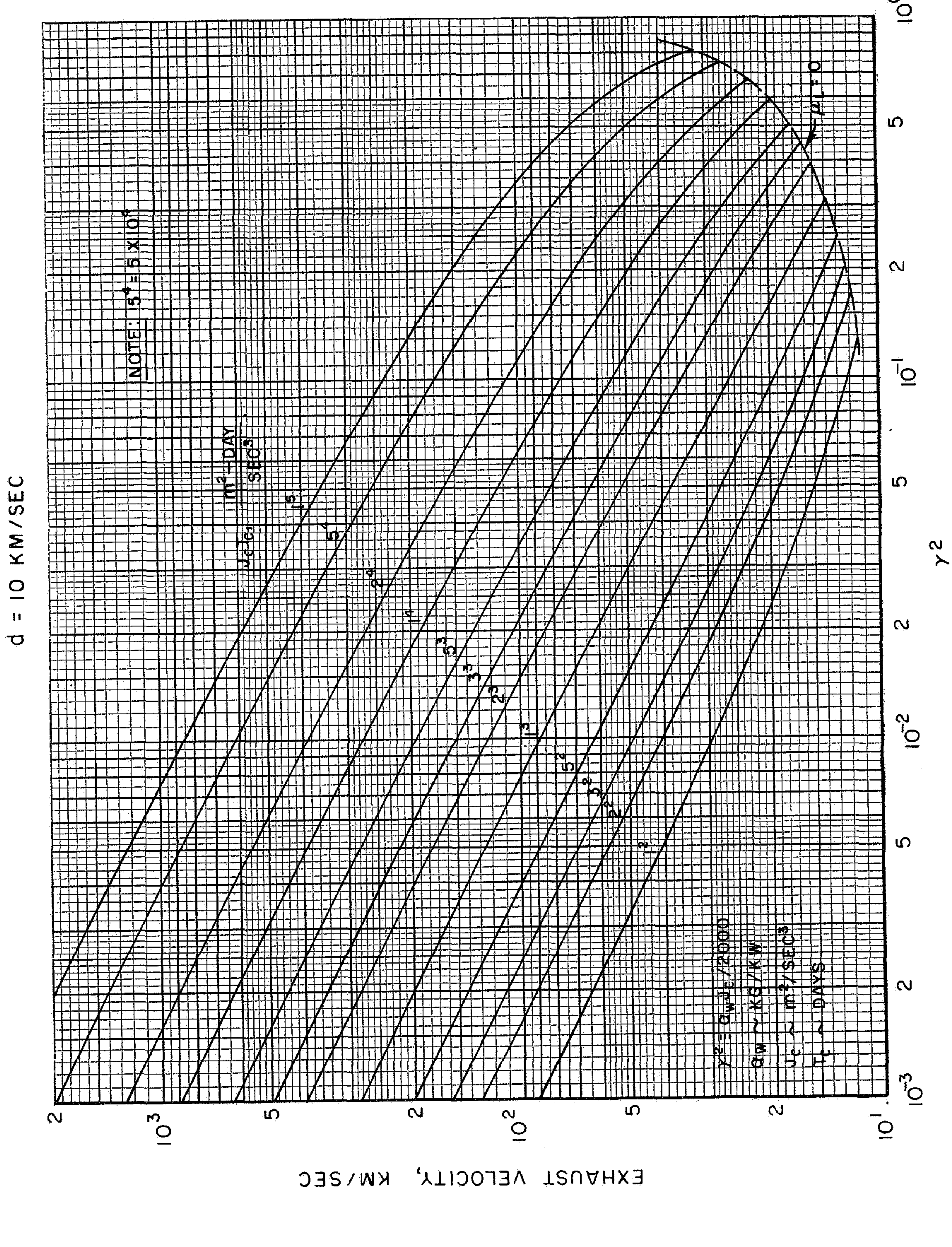


PAYLOAD AND POWERPLANT FRACTIONS FOR CONSTANT—THRUST OPERATION

d = 10 KM/SEC

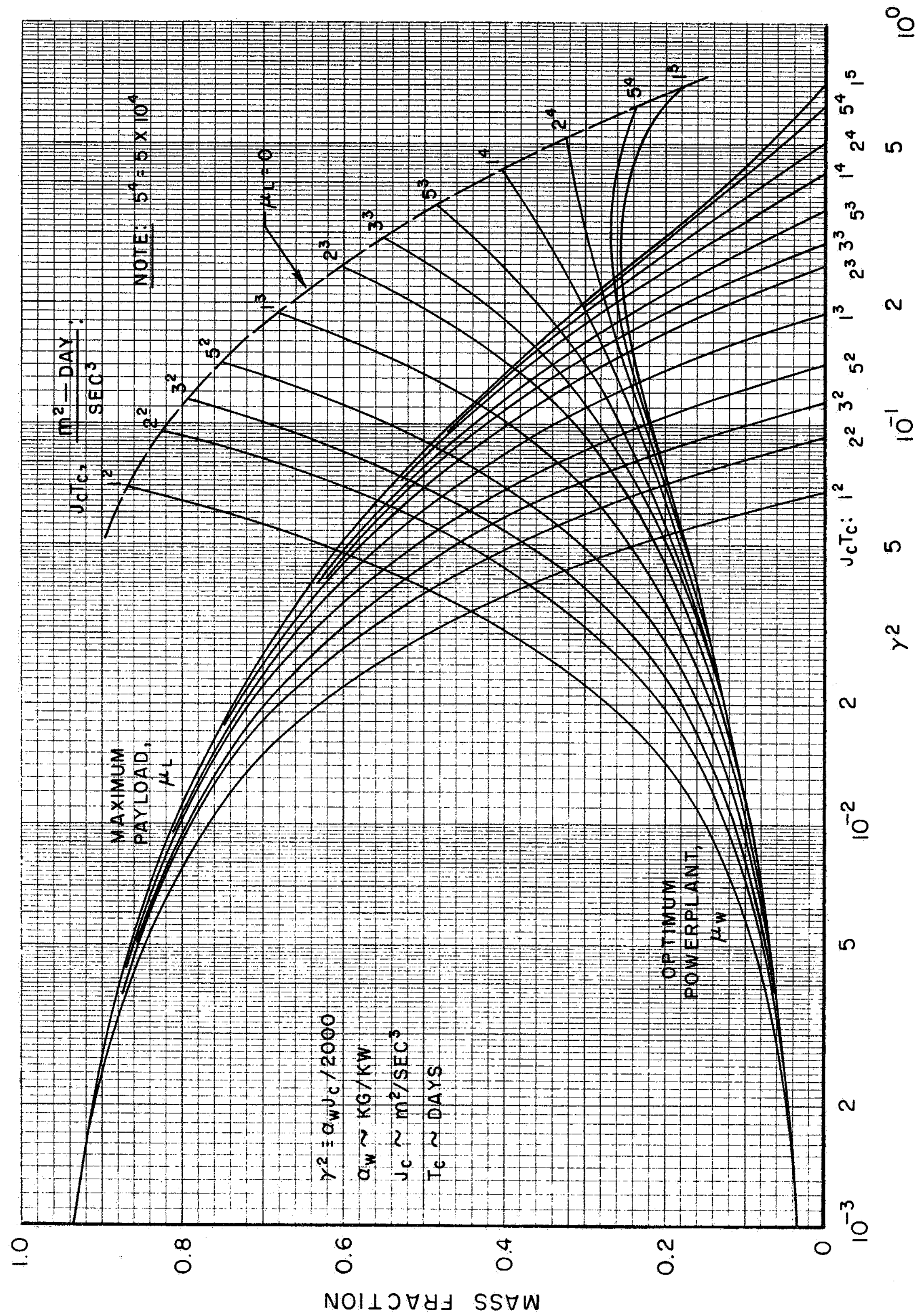


OPTIMUM EXHAUST VELOCITY FOR CONSTANT-THRUST OPERATION

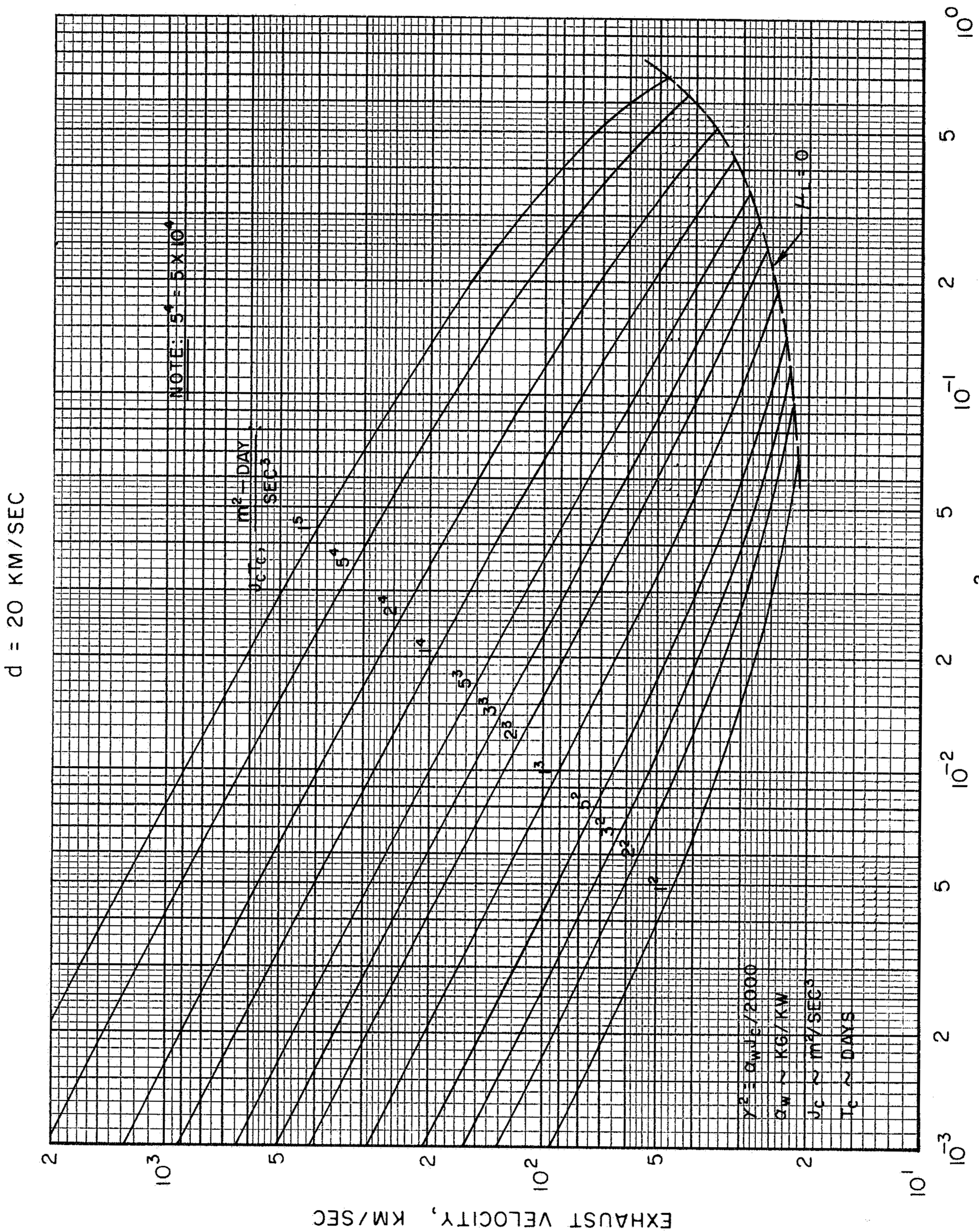


PAYLOAD AND POWERPLANT FRACTIONS FOR CONSTANT-THRUST OPERATION

d = 20 KM/SEC

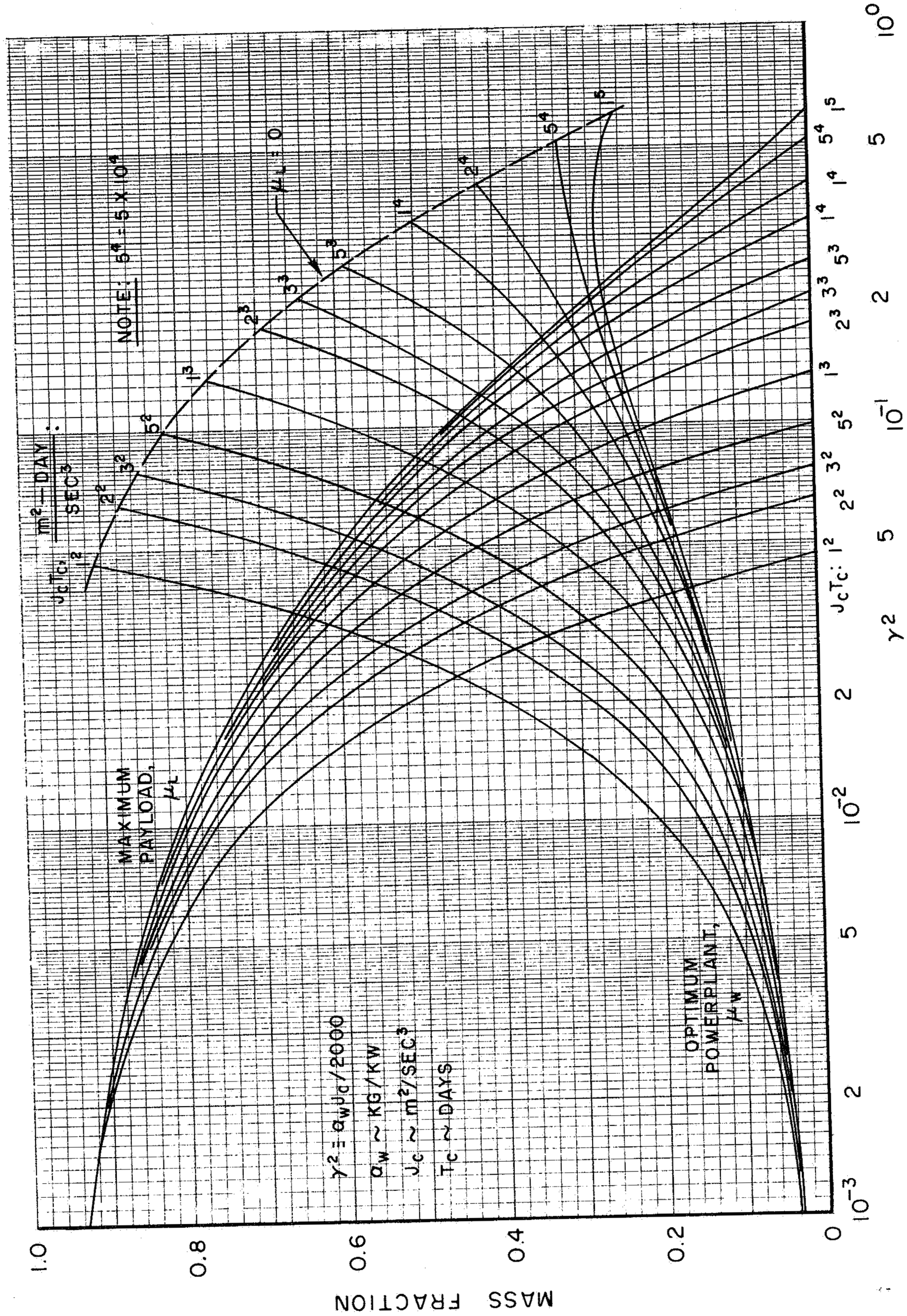


OPTIMUM EXHAUST VELOCITY FOR CONSTANT–THRUST OPERATION



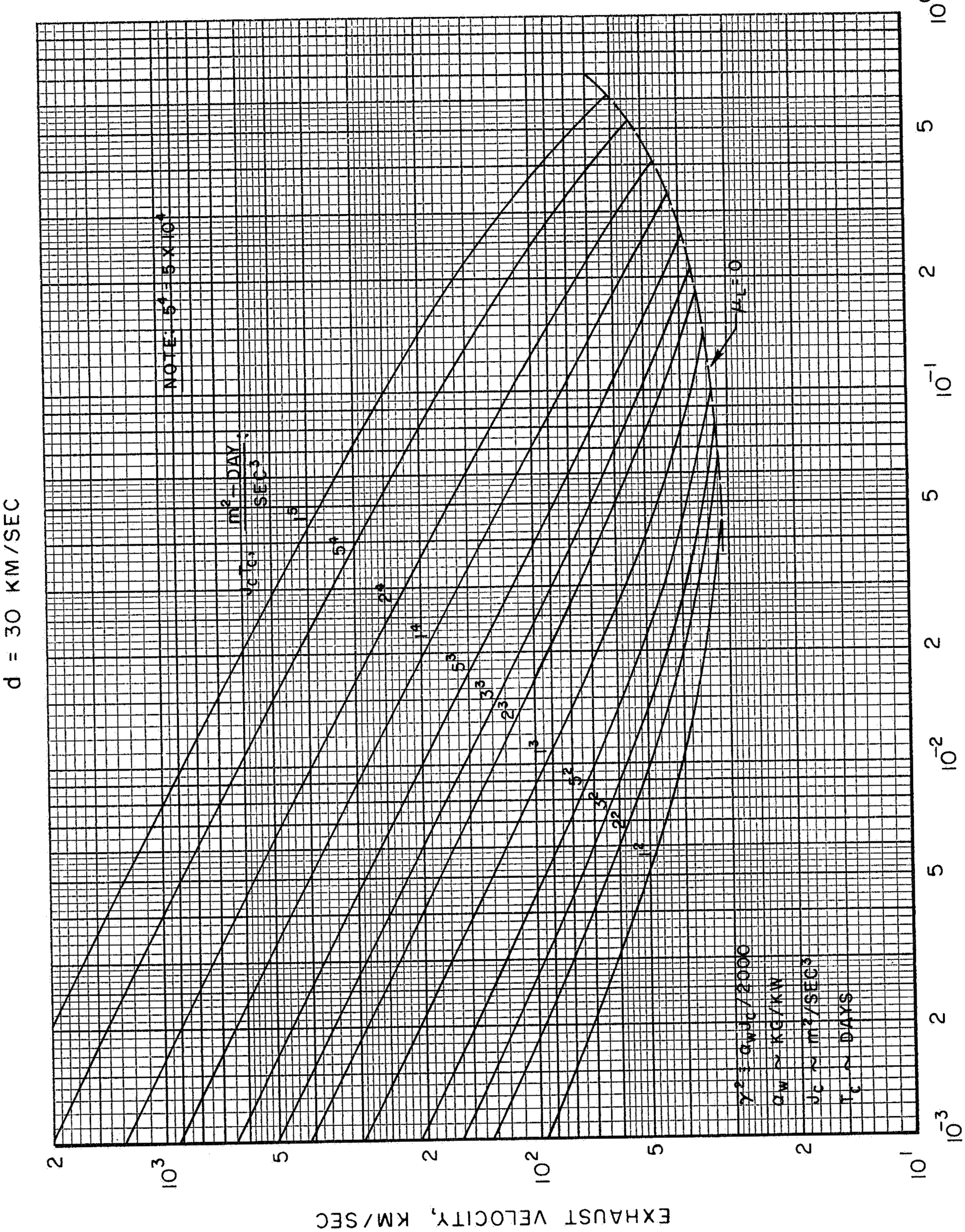
PAYLOAD AND POWERPLANT FRACTIONS FOR CONSTANT-THRUST OPERATION

d = 30 KM/SEC



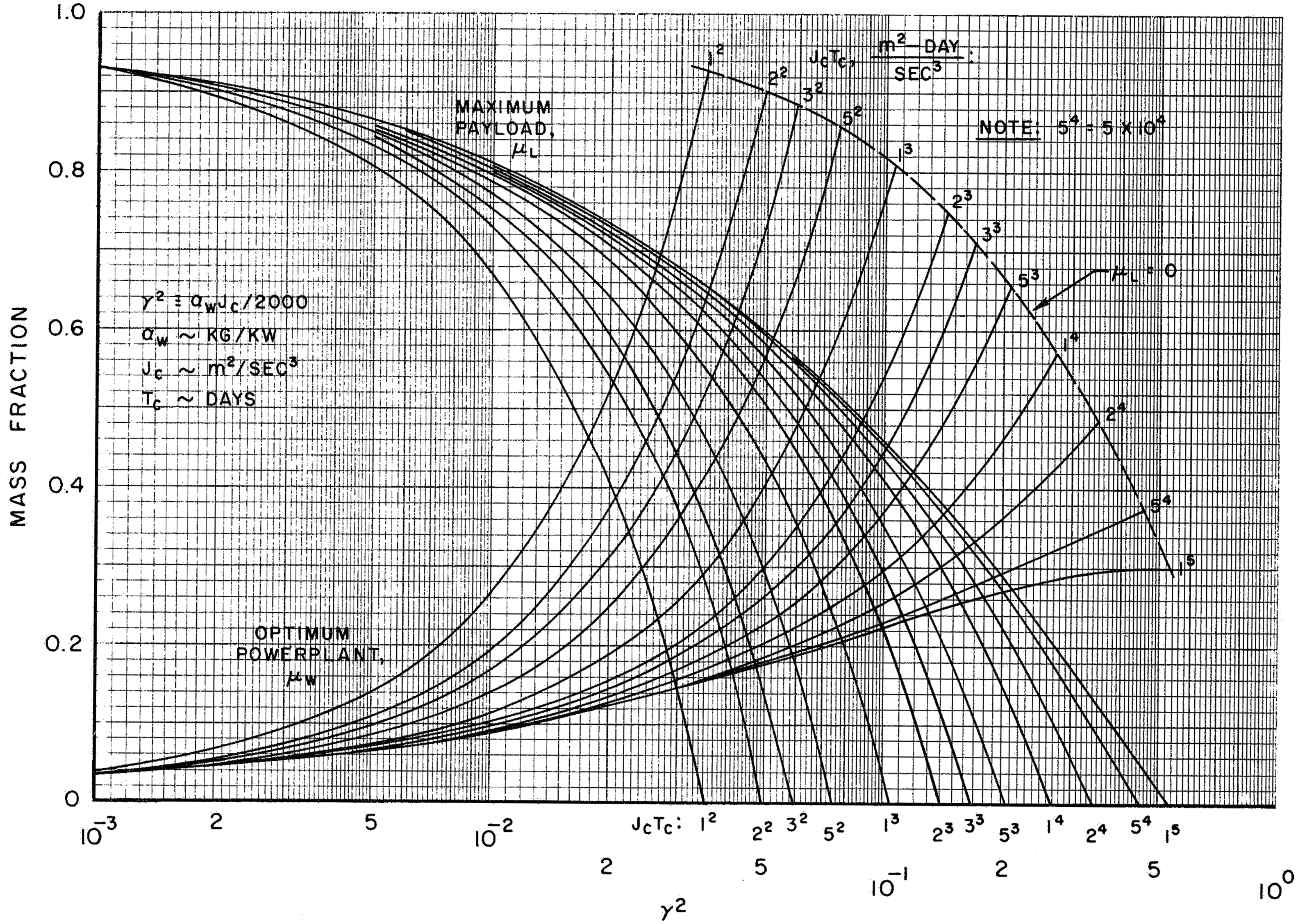
OPTIMUM EXHAUST VELOCITY FOR CONSTANT-THRUST OPERATION

d = 30 KM/SEC



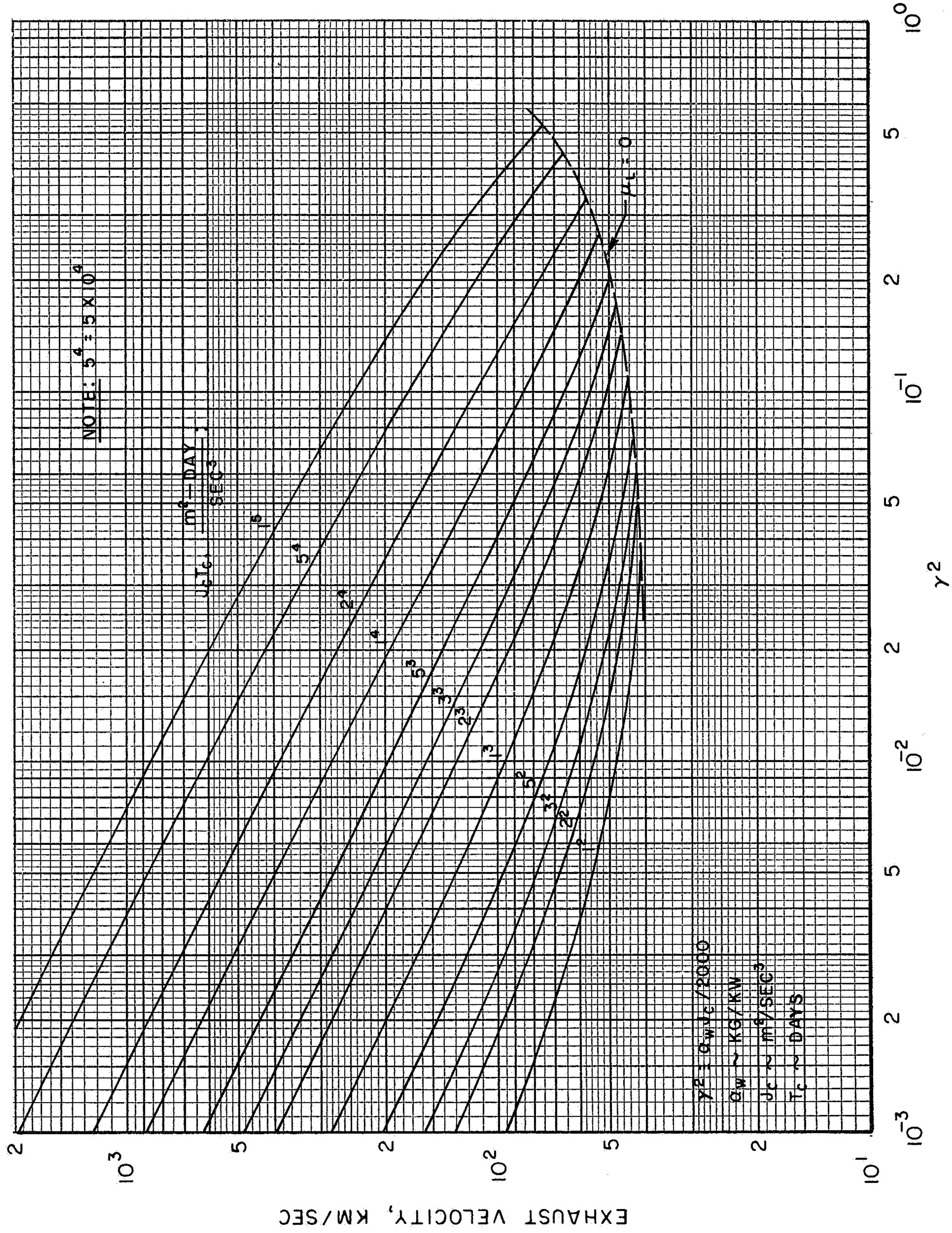
PAYLOAD AND POWERPLANT FRACTIONS FOR CONSTANT-THRUST OPERATION

d = 40 KM/SEC



OPTIMUM EXHAUST VELOCITY FOR CONSTANT-THRUST OPERATION

d = 40 KM/SEC



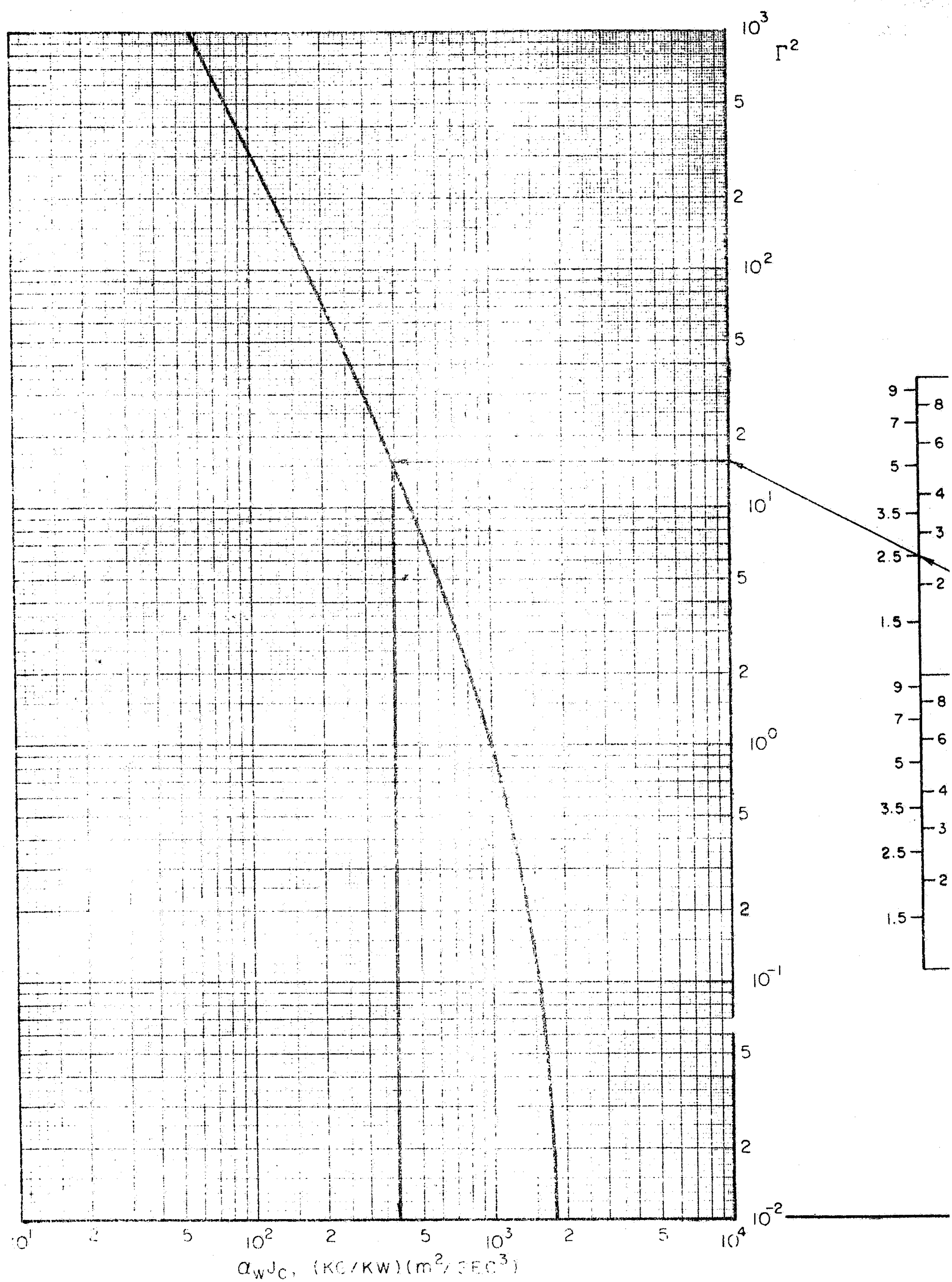
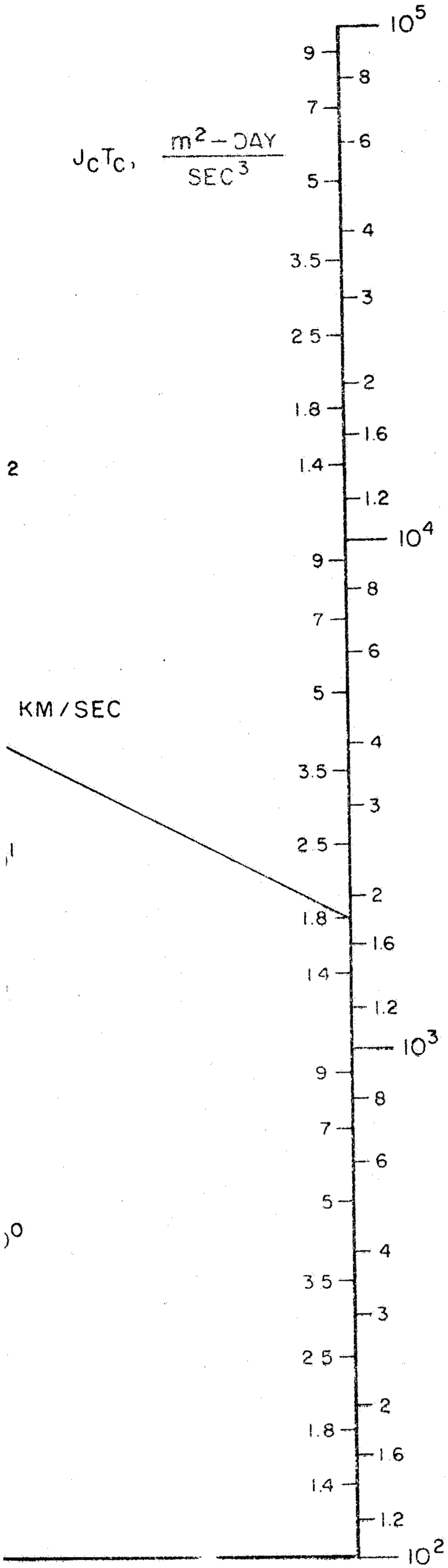

10^3
Γ^2
5
2
10^2
5
2
10^1
5
2
10^0
5
2
10^{-1}
5
2
10^{-2}
9
8
7
6
5
4
3.5
3
2.5
2
1.5
9
8
7
6
5
4
3.5
3
2.5
2
1.5
10^1
2
5
10^2
2
5
10^3
2
5
10^4
$a_w J_c$, (KC/KW)(m^2/SEC^3)

FIG. 16

POWERPLANT SPECIFIC WEIGHT AT ZERO PAYLOAD FRACTION



d = THRUSTOR EFFICIENCY PARAMETER, KM/SEC

J_C = CONSTANT THRUST J, m^2/SEC^3

T_C = POWERED TIME, DAYS

EXAMPLE:

$d = 25, \quad J_C T_C = 1.8 \times 10^3;$

$\therefore \Gamma^2 = 15.7,$

$\& \ \alpha_{WMAX} = \dfrac{3.95 \times 10^2}{J_C}$, KG/KW