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GASL TECHNICAL REPORT NO. 636

MIXING AND COMBUSTION IN SUPERSONIC FLOW WITH LATERAL
PRESSURE GRADIENT EFFECTS

FINAL REPORT

Prepared Under Contract Number NAS7-590

Prepared for

National Aeronautics and Space Administration
NASA Pasadena Office
Pasadena, California

August 1968

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Prepared by

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August 1968

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LIST OF SYMBOLS

C_f	Skin friction coefficient
C_{p_i}	Specific heat of the i^{th} specie
D_{ij}	Binary diffusion coefficient
H_j	Mixture stagnation enthalpy
h	Mixture static enthalpy
h_i	Specie static enthalpy
$\underline{j_i}$	Diffusion mass flux vector of i^{th} specie
K	Mixture thermal conductivity
Le	Lewis number
M	Mach number
N_s	Step size control factor
n	Coordinate normal to streamline
p	Pressure
Pr	Prandtl number
q	Total velocity
r	Normal coordinate measured from axis
R	Universal gas constant
s	Coordinate along streamline
T	Temperature
$\underline{v}$	Velocity vector
$\dot{w}_i$	Production rate of i^{th} specie
x	Axial coordinate
y	Radial coordinate
α_i	Specie mass fraction
γ	Ratio of specific heats
$\underline{\delta}$	Unit tensor
$\underline{\epsilon}$	Deformation tensor
θ	Flow deflection angle
μ_t or ν	Turbulent viscosity
ρ	Mixture density

σ	Characteristics step size
$\bar{\sigma}$	Shock angle
τ	$\tan \theta$
$\underline{\tau}$	Viscous stress tensor
$\underline{\psi}$	Stream function

MIXING AND COMBUSTION IN SUPERSONIC FLOW WITH LATERAL
PRESSURE GRADIENT EFFECTS

By R. Edelman and G. Weilerstein

I. INTRODUCTION

The design of reliable and efficient propulsion systems for manned and unmanned hypersonic flight has introduced the need for a higher degree of sophistication of analysis than previously required. Problems of aerothermodynamic and structural performance requirements call for detailed investigations of physical phenomena governing the behavior of each of the many components in any particular propulsion system. Of interest here are propulsion systems which derive thrust from the energy that is liberated through the mechanism of exothermic chemical reactions. Systems of interest include (1) pure rockets, (2) pure airbreathing and (3) composites. The latter type of engine system combines the operating characteristics of two or more of the basic engine types of which the ducted fuel rich rocket, and the ejector ramjet are two typical examples. In the case of rocket engines the propellant may be liquid, solid or hybrid where the latter utilizes both solid and liquid fuel and oxidizer combinations. Airbreathing engines differ from rockets in their operation in that only fuel is carried on board and the oxidizer is derived from the air ingested by the engine. A composite engine generally combines both rocket and airbreathing capabilities and as such has the potential for a larger operational flight envelope.

Common to the variety of engines mentioned above are the problems associated with propellant combustion. With the exception of the pure solid rocket, the combustion process is coupled to a fuel/oxidizer mixing process. If a spectrum of pressure, temperatures and flow velocities are considered, the combustion process may be (1) diffusion controlled, i.e., fuel and oxidizer react on contact and the heat release rate is dependent only upon the mixing rate; (2) reaction controlled wherein the kinetics of the oxidation process determine the heat release rate; and (3) an intermediate regime wherein the heat release rate depends upon the coupled mixing and kinetic rates.

The diffusion controlled process is desirable in comparison to the reaction controlled situation because the latter involves a mixing process followed by the combustion process and could lead to excessively long combustion chambers. In addition, for hypersonic flight involving pure airbreathing and composite engines, shock losses, dissociation losses and heat transfer limitations require that the flow remain supersonic during the combustion process. Thus, fuels of interest including hydrocarbons requires consideration of the coupling between the mixing and kinetic processes because flow, mixing and reaction times are generally of equal order of magnitude. Of particular importance in high speed combustion chamber flows is the optimization of the combustion chamber length required to achieve efficient heat release while maintaining reasonable chamber lengths. To achieve an understanding of the relevant parameters and their interrelationships it is necessary to analyze the details of the combustng flow field. A meaningful analysis for such flow configurations requires consideration of turbulent mixing, finite rate combustion and pressure gradients in a model which appropriately accounts for the coupling of these phenomena. It should be noted that the pressure field

...
must be considered in detail because of the sensitivity to pressure of both the chemical kinetics and velocity field. The structure of the pressure field includes the locally induced pressure due to mixing and combustion as well as the pressure impressed on the field due to the interactions with the confining walls.

The purpose of the present work is to develop an analytical and numerical model for the calculation of flows with mixing, finite rate combustion and lateral pressure gradients. This model will extend the prediction capability from flows with negligible lateral pressure gradient such as those considered in References 1, 2 and 3 to flows where lateral pressure gradients become important (supersonic flow throughout). The resulting formulation provides the capability to compute the coupled inviscid-viscous flow field simultaneously without iteration while retaining lateral pressure gradients throughout.

A configuration which characterizes the flows considered in this investigation is shown in Figure 1. Figure 1a represents an under-expanded exhaust plume. Although the complete flow field of the plume involves a complex internal shock structure the present study is limited to the inclusion of only the external shock. The internal shock structure is the subject of a current NASA Contract (NAS8-21264). Figure 1b shows the nozzle, or jet, inside a duct which could represent a number of propulsion systems including a ducted rocket (with afterburning), an ejector scramjet or a gas generator fueled scramjet. In the present study, all shocks are neglected in the ducted jet configuration and therefore the degree of pressure mismatch is limited accordingly. This is not, however, a severe limitation since strong shocks are normally undesirable in propulsion systems; however, their existence is still implied by the present calculation through the occurrence of rather sharp adverse pressure gradients.

The approach commonly adopted for the study of jet type flows is based on boundary layer concepts. The resulting formulation is applicable to viscous flow regions where the pressure normal to the primary flow direction is constant and the pressure along the primary flow direction is specified. Such a treatment is adequate for subsonic and low supersonic Mach number flows and for ducted flows with balanced supersonic primaries and subsonic secondaries. A class of such problems and the adequacy of the "boundary layer" approach is discussed in Reference 3. Methods for computing the effect upon the inviscid outer flow require cumbersome interactions based on boundary layer displacement thickness determinations. The present approach treats the coupled problem by analyzing the entire "disturbed" flow.

II. ANALYSIS

The classical analysis of viscous flow problems involving unequal characteristic length scales in the coordinate directions lead to the boundary layer formulation. The describing equations are parabolic and the pressure is constant across the smaller, or lateral, dimensions of the flow. The pressure variation in the primary flow direction is either specified as a given distribution or determined by approximate methods based on the induced pressure due to the displacement of the inviscid outer flow. In either case the pressure across the flow is assumed constant. In flows with combustible mixtures an accurate prediction of the induced pressure field is crucial in predicting the force on solid surfaces adjacent to the combustion region and moreover the ignition delay and reaction time of the combustion process itself may be critically dependent on the local pressure particularly for pressure levels below sea level. Furthermore, non-uniform transverse pressure distributions are inherent in jets operating either in an underexpanded or overexpanded configuration.

The analysis of this problem requires a study of the full Navier-Stokes equations. The approach will involve the choice of coordinates and a set of physically reasonable assumptions which will provide a tractable formulation and yet preserve the basic character of the flow. Such an analysis was initiated earlier by Moretti and is discussed in Reference 4.

The general describing equations for the steady flow of a reacting mixture of perfect gases are given by:

Global Continuity:

$$\nabla \cdot \rho \underline{V} = 0 \quad (1)$$

Species Continuity:

$$\nabla \cdot \rho \alpha_i \underline{V} = \rho \dot{w}_i - \nabla \cdot \underline{j}_i \quad (2)$$

i = ith specie

where $\underline{j}_i$ is the diffusional mass flux vector:

$$\underline{j}_i = \rho_i (\underline{V}_i - \underline{V})$$

Momentum:

$$\nabla \cdot \rho \underline{VV} = -\nabla P + \nabla \cdot \underline{\tau} \quad (3)$$

where $\underline{\tau}$ is the viscous stress tensor.

Energy:

$$\nabla \cdot \rho \underline{VH} = \nabla \cdot (\underline{\tau} \cdot \underline{V}) - \nabla \cdot \underline{q} - \nabla \cdot \sum_i h_i \underline{j}_i \quad (4)$$

where $H = \sum_i \alpha_i h_i(T) + \frac{\underline{V} \cdot \underline{V}}{2}$ is the stagnation enthalpy, and $\underline{q}$ is the heat conduction vector.

State:

$$\rho = \frac{P}{RT \sum_i \frac{\alpha_i}{W_i}} \quad (5)$$

Transport Processes

In this investigation simple transport will be considered sufficiently accurate for describing the various diffusional processes occurring in the flow.

Viscous Stress

The viscous stress will be represented in the usual way as a linear function of the rate of strain. Thus, for a Newtonian fluid the viscous stress is given by:

$$\underline{\tau} = 2 \mu \underline{\epsilon} - 2/3 \mu \underline{\delta} : \underline{\epsilon} \underline{\delta} \quad (6)$$

where

$$\underline{\epsilon} = \frac{1}{2} (\nabla \underline{v} + \underline{v} * \underline{v}) \quad (7)$$

and $\underline{\delta}$ is the unit tensor

Species Diffusion

The diffusion of each species is assumed to depend only on the gradient of the particular species mass fraction. The assumption requires that the binary diffusion coefficients D_{ij} , for each species are equal and the simple Fick's law is applicable. Under this assumption the diffusional mass flux is given by:

$$\underline{j}_i = -\rho D_{ij} \nabla \alpha_i = -\mu \frac{Le}{Pr} \nabla \alpha_i \quad (8)$$

Heat Conduction

Consistent with the above assumptions the heat conduction may be represented by the Fourier law given by:

$$\underline{q} = -k \nabla T = \frac{\overline{c_p}}{Pr} \mu \nabla T \quad (9)$$

where

$$\overline{c_p} = \sum_i \alpha_i c_{p_i}$$

Transport Coefficients

The flows which are of interest here will, in general, be turbulent. It is assumed, however, that the functional form describing the transport processes discussed above remain unchanged in turbulent flow and the laminar coefficients are replaced by their turbulent counterparts, i.e., $\mu \rightarrow \mu_t$, etc.

Coordinate System

A schematic of the flow field is shown in Figure 2. A coordinate system is sought wherein a minimum of physical assumptions are required to simplify the Navier-Stokes equations and within which a numerical solution is most easily formulated and implemented. It was felt that the implicit coordinate system would provide these desired conditions. Some insight into this choice is obtained by considering the important case of an underexpanded (or overexpanded) jet. The initial flow deflection is governed almost entirely by the adjustment of the pressure. The jet flow is separated from the external flow by the "dividing" streamline and the gradients in velocity, concentrations and temperature will be greatest in the direction normal to the streamline. Of course, as the flow becomes more parallel to the jet axis this condition is still satisfied. It appears then that in this coordinate system certain of the usual boundary layer assumptions may be applied. Furthermore, it is shown later in Section III that a natural consequence of this is a characteristics method of solution which could be implemented with a certain degree of ease.

To demonstrate the above discussion the full equations in intrinsic coordinates are given below followed by an order of magnitude analysis and the assumptions which give rise to the desired set of equations to be solved.

Let s and n be the coordinates along and normal to a streamline in either a plane two-dimensional or an axisymmetric flow. Let q denote the velocity, and θ the streamline deflection relative to the plane or axis of symmetry where for the particular case of axisymmetric flow r is the normal distance from the axis to the point on the streamline in question.

Equations (1) through (4) together with the relations (6), (8), and (9) may be expanded in this coordinate system to give:

Global Continuity:

$$\frac{\partial \rho q}{\partial s} + j \frac{\rho q}{r} \sin \theta + \rho q \theta_n = 0 \quad (11)$$

Momentum:

$$\begin{aligned} s- \quad \rho q \frac{\partial q}{\partial s} = & - \frac{\partial P}{\partial s} + \frac{1}{r^j} \frac{\partial r^j \tau_{sn}}{\partial n} + \frac{1}{r^j} \frac{\partial r^j \tau_{ss}}{\partial s} + \\ & + (\tau_{ss} - \tau_{nn}) \theta_n - 2 \tau_{sn} \theta_s - \tau_{\omega\omega} \frac{j}{r} \sin \theta \end{aligned} \quad (12)$$

$$\begin{aligned} n- \quad \rho q^2 \theta_s = & - \frac{\partial P}{\partial n} + \frac{1}{r^j} \frac{\partial r^j \tau_{nn}}{\partial n} + \frac{1}{r^j} \frac{\partial r^j \tau_{sn}}{\partial s} + 2 \theta_n \tau_{sn} \\ & - \tau_{\omega\omega} \frac{j}{r} \cos \theta \end{aligned} \quad (13)$$

Species Continuity:

$$\begin{aligned} \rho q \frac{\partial \alpha_i}{\partial s} = & \rho \dot{w}_i + \mu \frac{Le}{Pr} \theta_n \frac{\partial \alpha_i}{\partial s} - \mu \frac{Le}{Pr} \theta_s \frac{\partial \alpha_i}{\partial n} + \\ & + \frac{1}{r^j} \frac{\partial (r^j \mu \frac{Le}{Pr} \frac{\partial \alpha_i}{\partial s})}{\partial s} + \frac{1}{r^j} \frac{\partial (r^j \mu \frac{Le}{Pr} \frac{\partial \alpha_i}{\partial n})}{\partial n} \end{aligned} \quad (14)$$

Energy:

$$\begin{aligned} \rho q \frac{\partial H}{\partial s} = & \tau_{ss} q \theta_n + \frac{1}{r^j} \frac{\partial r^j \tau_{ss} q}{\partial s} - \tau_{sn} q \theta_s + \frac{1}{r^j} \frac{\partial r^j \tau_{ns} q}{\partial n} + \\ & + \frac{\mu}{Pr} \left\{ \theta_n \frac{\partial H}{\partial s} - \theta_s \frac{\partial H}{\partial n} \right\} + \frac{1}{r^j} \frac{\partial}{\partial s} \left[r^j \frac{\mu}{Pr} \frac{\partial H}{\partial s} \right] + \\ & + \frac{1}{r^j} \frac{\partial}{\partial n} \left[r^j \frac{\mu}{Pr} \frac{\partial H}{\partial n} \right] + \frac{\mu}{Pr} \theta_n (Le-1) \sum_i h_i \frac{\partial \alpha_i}{\partial s} + \\ & - \frac{\mu}{Pr} \theta_s (Le-1) \sum_i \frac{\partial \alpha_i}{\partial n} + \frac{1}{r^j} \sum_i \frac{\partial}{\partial s} \left[r^j \frac{\mu}{Pr} (Le-1) h_i \frac{\partial \alpha_i}{\partial s} \right] + \end{aligned} \quad (15)$$

$$\begin{aligned}
& + \frac{1}{r^j} \sum_i \frac{\partial}{\partial n} \left[r^j \frac{\mu}{Pr} (Le-1) h_i \frac{\partial \alpha_i}{\partial n} \right] + \frac{\mu}{Pr} \left[\theta_s \frac{\partial q^2/2}{\partial s} - \theta_n \frac{\partial q^2/2}{\partial n} \right] - \\
& - \frac{1}{r^j} \frac{\partial}{\partial s} \left[r^j \frac{\mu}{Pr} \frac{\partial q^2/2}{\partial s} \right] - \frac{1}{r^j} \frac{\partial}{\partial n} \left[r^j \frac{\mu}{Pr} \frac{\partial q^2/2}{\partial n} \right]
\end{aligned}
\tag{15}$$

(Contd)

where the stress components are given by:

$$\begin{aligned}
\tau_{ss} &= 2\mu \frac{\partial q}{\partial s} - 2/3 \mu \left[\frac{\partial q}{\partial s} + \frac{j q}{r} \sin \theta + q \theta_n \right] \\
\tau_{nn} &= 2\mu q \theta_n - 2/3 \mu \left[\frac{\partial q}{\partial s} + \frac{j q}{r} \sin \theta + q \theta_n \right] \\
\tau_{\omega\omega} &= 2\mu \frac{j}{r} q \sin \theta - 2/3 \mu \left[\frac{\partial q}{\partial s} + \frac{j q}{r} \sin \theta + q \theta_n \right] \\
\tau_{ns} &= \tau_{sn} = \mu \left[\frac{\partial q}{\partial n} + q \theta_s \right]
\end{aligned}
\tag{16}$$

and where,

$$j = \begin{cases} 0 & \text{plane two-dimensional} \\ 1 & \text{axisymmetric} \end{cases}$$

Order of Magnitude Analysis

The simplification of the describing Equations (11) through (15) requires that attention be given to a particular type of problem. The problem of present interest involves a class of supersonic jet flows having a principal flow direction which are conditions also found in practical configurations including rocket nozzles and supersonic engine combustion chambers. In a supersonic flow, small deflections can induce a non-uniform pressure field which can have a significant effect upon combustion and pressure forces on surfaces adjacent to the flow. Furthermore, if the flow curvature is small

in regions where viscous effects dominate across the entire field, viz., away from the near region of an underexpanded jet, then boundary layer type approximations are applicable to the diffusive terms in the describing equations. Therefore, it remains to be seen how such approximations may be applied to Equations (11) through (16) to reduce the system to a tractable form retaining the second momentum equation.

Consider the following variables having an order of magnitude of unity:

$$q \sim 01, \quad \rho \sim 01, \quad \alpha_i \sim 01, \quad H \sim 01 \quad (17)$$

In the regions of the flow wherein diffusive effects are fully developed it is assumed that the flow deflection is small and of order δ , i.e.,

$$\sin \theta \sim \theta \sim 0\delta \quad (18)$$

where δ is a measure of the lateral extent of the disturbed region. It follows that the derivatives must satisfy the condition:

$$\frac{\partial}{\partial s} < \frac{\partial}{\partial n} \quad (19)$$

Conditions (18) and (19) provide the following order of magnitudes for the curvatures θ_s and θ_n :

$$\begin{aligned} \theta_s &\sim 0\delta \\ \theta_n &\sim 01 \end{aligned} \quad (20)$$

The relative scale, $\delta/2$, of the flow field is related to one or more parameters of the flow depending on the particular configuration of interest. If the region of interest is dominated by viscous effects and the Reynold's number is very high then the condition that:

$$\left(\frac{L}{\delta} \right)^2 \frac{\mu_o}{\rho_o q_o L} \sim 01 \quad (21)$$

defines the classical boundary layer flow and the second momentum equation yields a zero (higher order) lateral pressure variation. Since a formulation is sought wherein the effect of a lateral pressure variation is retained, relation (21) must be replaced by a somewhat more general condition. The resulting formulation will represent a unified description of regions dominated by viscous effects as well as regions which are principally inviscid. Thus, the flow field is regarded as one in which viscous effects are dominant over a lateral extent of δV whereas the entire field is disturbed over the thickness δ depending on the external flow Mach number. Let,

$$\beta = \left(\frac{\delta V}{\delta} \right)^2 < 1 \quad (22)$$

and consider the condition:

$$\left(\frac{L}{\delta} \right)^2 \left(\frac{\mu_o}{\rho_o q_o L} \right) \sim \beta \quad (23)$$

Substitution of (17) through (20) and (23) into Equation (12) gives:

$$\begin{aligned} & \left[\frac{1}{\rho q} \frac{\partial \bar{q}}{\partial s} \right] = - \frac{L}{\rho_o q_o} \frac{\partial p}{\partial s} + \beta \frac{1}{\bar{r}^j} \frac{\partial \bar{r}^j \bar{\tau}_{sn}}{\partial \bar{n}} + \\ & \left[(\delta/L)^2 \beta \right] \\ & + \left(\frac{\delta}{L} \right)^2 \beta \left\{ \frac{1}{\bar{r}^j} \frac{\partial \bar{r}^j \bar{\tau}_{ss}}{\partial s} + (\bar{\tau}_{ss} - \bar{\tau}_{nn}) \theta_n - 2 \bar{\tau}_{sn} \bar{\theta}_s - \bar{\tau}_{\omega\omega} \frac{j \sin \theta}{\bar{r}} \right\} \end{aligned} \quad (24)$$

where the order of magnitude is shown above each term and the principal contribution of viscosity is due to the second term on the right hand side. Similarly, the second momentum, Equation (13) is given by:

$$\begin{aligned}
 [\delta/L] \quad \overline{\rho q} \quad \overline{\theta}_s \quad \frac{\delta}{L} = & - \frac{L}{\rho_o q_o^2} \frac{\partial P}{\partial n} + \frac{[\delta/L \beta]}{L} \beta \left\{ \frac{1}{r^j} \frac{\partial r^j \tau_{nn}}{\partial n} + \right. \\
 & \left. + \frac{1}{r^j} \frac{\partial r^j \tau_{sn}}{\partial s} + 2 \tau_{sn} - \frac{j \cos \theta}{r} \tau_{\omega\omega} \right\}
 \end{aligned}
 \tag{25}$$

If β is zero the entire flow is inviscid and Equations (24) and (25) are the usual inviscid momentum equations. This is valid inasmuch as (23) would imply a zero viscosity. Furthermore, the retention of the second momentum equation even for $\delta/L < 1$ would not be in error although this flow could be approximated as one dimensional. If $\beta = 1$ relation (23), implies that classical boundary layer theory applies and in Equation (25) it is seen that the inviscid and viscous terms are of the same order of magnitude and are of the order $\delta/L = \delta_v/L < 1$. If $0 < \beta < 1$, then Equation (24) may be approximated by:

$$\rho q \frac{\partial q}{\partial s} = - \frac{\partial P}{\partial s} + \frac{1}{r^j} \frac{\partial r^j \tau_{sn}}{\partial n}
 \tag{26}$$

where the viscous term of highest order is retained and the shear stress reduces to:

$$\tau_{sn} = \mu \frac{\partial q}{\partial n}
 \tag{27}$$

and Equation (25) is approximated by

$$\rho q^2 \theta_s = - \frac{\partial P}{\partial n} \quad (28)$$

where the viscous terms are all of order $\delta/L \beta$ and are neglected. The validity of the resulting Equations (26) and (28) is apparently subject to two principal constraints: (1) the flow should be supersonic so that the disturbed region is bounded by a narrow domain of influence, i.e., $\delta/L < 1$; and (2) the primary effects of viscosity are confined to a region such that $\delta_v/\delta < 1$. The extent of this approximation and the range of validity remain open to further analytical and experimental investigation. The limited results of calculations performed here, however, exhibit certain expected behavior and are discussed in detail later.

The remaining Equations (11), (14) and (15), reduce in a straight forward manner and the resulting system of describing equations become:

Global Continuity

$$(\rho q)_s + \frac{j \rho q}{r} \sin \theta + \rho q \theta_n = 0 \quad (29)$$

S-momentum

$$\rho q q_s + P_s = \frac{1}{r^j} \left[r^j \mu \frac{\partial q}{\partial n} \right]_n \quad (30)$$

n-momentum

$$\rho q^2 \theta_s + P_n = 0 \quad (31)$$

Species Continuity

$$\rho q (\alpha_i)_s = \dot{\rho w}_i + \frac{1}{r^j} \left[r^j \mu \frac{Le}{Pr} (\alpha_i)_n \right]_n \quad (32)$$

Energy:

$$\begin{aligned} \rho q H_s = & \frac{1}{r^j} \left[r^j \frac{\mu}{Pr} H_{r_i} \right]_n + \frac{1}{r^j} \left[\left(1 - \frac{1}{Pr}\right) \mu (q^2/2)_n \right]_n + \\ & + \frac{1}{r^j} \sum_i \left[(Le-1) r^j \frac{\mu}{Pr} h_i (\alpha_i)_n \right]_n \end{aligned} \quad (33)$$

where

$$H = \sum_i \alpha_i h_i(T) + q^2/2 \quad (34)$$

and

$$\rho = \frac{P}{RT \sum_i \frac{\alpha_i}{W_i}} \quad (35)$$

III. SOLUTION TECHNIQUE

The present solution technique is that proposed by Moretti Reference 4, and involves the application of concepts related to both hyperbolic and parabolic systems. Basic to the technique is the assumption that the diffusive effects can be treated as forcing terms in the compatibility equations along the "inviscid" characteristics. The solution is obtained as follows: starting from some line of known data the right and left running characteristics from each mesh point are constructed. The compatibility equations are solved for the flow direction and pressure at the new points in the characteristic grid. Now, a rectangular grid, constructed along and normal to the streamlines, is superposed on the characteristics mesh for the solution of the diffusion equations. The pressure gradient obtained from the solution of the compatibility equations is used in obtaining the solution of the boundary layer equations at the new grid points. This data is used to compute the viscous contributions, i.e., the second derivative terms, at the new points. These terms also appear in the compatibility equations providing a mechanism for an iterative procedure in obtaining the coupled solution. Thus, the average values of the second derivative terms over the interval are substituted into the compatibility equations and a new pressure and a new flow deflection angle are determined. The above cycle of calculations is repeated until the pressure change between successive iterations becomes less than a prescribed tolerance. For the cases which have been considered, generally, a maximum of three iterations was required to converge to within about four significant figures.

As indicated above, the solution technique involves the coupling of both hyperbolic and parabolic equations. It is the purpose of the following discussions to describe the details of the method of

solution which can be described in terms of an "inner" calculation (characteristics) and an "outer" calculation (mixing).

The characteristics calculation is initiated in a standard manner from a "line" of given data as shown in Figure 3.

The characteristic directions are dependent only upon the local state and the left (I) and right (II) running characteristic directions from point K, say, are given by:

$$\tan \epsilon_K = \pm \sqrt{\frac{F}{\rho q^2 \left(\frac{F}{p} - \frac{1}{\rho} - \frac{F}{\rho q^2} \right)}} \quad (36)$$

where

$$F = T \sum_i \alpha_i \frac{dh_i(T)}{dT} \quad (37)$$

and all properties are evaluated at the point K.

The compatibility equations are obtained in the usual way assuming, however, that the viscous terms are part of the forcing function. These equations provide a relation between the flow deflection ($\theta = \tan^{-1} \tau$) and the pressure p, and are given by:

$$\frac{d\tau}{d\ell} \pm b \frac{dp}{d\ell} = \pm g \quad (38)$$

where,

$$b = \frac{1 + \tau^2}{\rho q^2 \tan \epsilon} \quad (39)$$

and,

$$g = \frac{(1+\tau^2)\sin\epsilon}{F} \left[\frac{\mu}{\rho q} (H_{nn} + \frac{\cos\theta}{y} H_n) - \frac{\mu}{\rho} (1 + \frac{F}{q^2}) \{q_{nn} + \frac{q_n \cos\theta}{y}\} - \sum_i h_i D_i + \frac{\rho R T F}{p} \sum_i \frac{D_i}{W_i} - \frac{F \sin\theta}{y} \right] \quad (40)$$

with,

$$D_i = \frac{\mu}{\rho q} \alpha_{inn} + \frac{\mu}{\rho q} \frac{\cos\theta}{y} \alpha_{in} + \frac{\dot{W}_i}{q} \quad (41)$$

The chemical production term, $\dot{W}_i$, represents the production rate of species i through the mechanism of a system of chemical reactions given by:

$$\sum_{j=1}^N \nu'_j A_j \xrightleftharpoons[k_m^b]{k_m^f} \sum_{j=1}^N \nu''_j A_j, \quad m = 1, M \dots \quad (42)$$

where N species are involved in M reactions and the ν_j 's are the stoichiometric coefficients associated with A_j species. The production rate, $\dot{W}_i$, is then given by:

$$\dot{W}_i = \frac{\rho}{W_i} \sum_{j=1}^M (\nu''_{i,j} - \nu'_{i,j}) \left[k_i^f \prod_{\gamma=1}^N y_{\gamma}^{\nu'_{\gamma,j}} k_j^b \prod_{\gamma=1}^N y_{\gamma}^{\nu''_{\gamma,j}} \right] \quad (43)$$

where

$$y_i = \rho \frac{\alpha_i}{W_i} \quad (44)$$

is the molar density of the i^{th} specie.

Characteristics Step

The sequence of calculations is initiated with a characteristics type calculation. Referring to Figure 3 the up-running characteristic from point K-1 and the down-running characteristic from point K are constructed and their point of intersection, C_K , is obtained. This procedure is carried out for all intersections across the field. The minimum forward distance required for an intersection determines the "characteristics" step size which guarantees that all new data computed across the field is within the domain of influence of the initial data.

The next step in the sequence of calculations involves the solution of the compatibility equations along the characteristics. The differenced form of Equations (38) are given by

$$\frac{\tau_C - \tau_{K-1}}{\Delta l_I} + b_{K-1} \frac{p_C - p_{K-1}}{\Delta l_I} = g_{K-1} \quad (45a)$$

and,

$$\frac{\tau_C - \tau_K}{\Delta l_{II}} - b_K \frac{p_C - p_K}{\Delta l_{II}} = -g_K \quad (45b)$$

Where

$$\Delta l_I = \sqrt{(x_{CK} - x_{K-1})^2 + (y_{CK} - y_{K-1})^2} \quad (46a)$$

and,

$$\Delta l_{II} = \sqrt{(x_{CK} - x_K)^2 + (y_{CK} - y_K)^2} \quad (46b)$$

Solving Equations (45a) and (b) for the pressure gives:

$$p_C = \frac{\tau_{K-1} - \tau_K + g_{K-1} \Delta l_I + g_K \Delta l_{II} + b_{K-1} p_{K-1} + b_K p_K}{b_{K-1} + b_K} \quad (47)$$

τ_C is determined by substitution into either Equation (45a) or (45b).

As discussed above, a single forward step size is used and this is done in anticipation of the subsequent mixing calculation discussed below. However, this requires interpolations for the data required at points A, D and finally at E (Figure 3). Thus, the pressure, p , and flow deflection, τ , are determined at A and D by interpolation along the characteristics and then a second interpolation is performed across the field to determine p and τ on the streamline at point E. In this way, the pressure gradient along the streamline is obtained:

$$\frac{dp}{ds} = \frac{p_{EK} - p_K}{\Delta s} \quad (48)$$

This result is utilized in the mixing calculation discussed in the next section.

The explicit bilateral coupling between the pressure field and the viscous mixing process is through the forcing function g in Equations (45a) and (b) and the momentum equation along the streamline, respectively. The former contains the diffusion and chemical reaction terms and the latter contains the pressure gradient. Furthermore, the description given above for the p - τ determination represents a single calculation in an iterative calculation. Thus the forcing function is modified by averaging it over a step which includes the "new" diffusion contributions obtained at the end of the mixing calculation. The p - τ calculation is repeated until the difference in two successively determined pressures is less than a prescribed tolerance.

Mixing Step

The calculation procedure as described so far lends itself uniquely to the application of already existing numerical solutions of the parabolic mixing problem, (see, for example, Ref. 3). The solution is obtained by an explicit finite difference representation of the describing equations along the streamlines. These equations are given by:

Momentum:

$$\rho q \frac{\partial q}{\partial s} + \frac{\partial p}{\partial s} = \mu \frac{\partial^2 q}{\partial n^2} + \frac{\mu \cos \theta}{y} \frac{\partial q}{\partial n} \quad (49)$$

Energy:

$$\rho q \frac{\partial H}{\partial s} = \mu \frac{\partial^2 H}{\partial n^2} + \frac{\mu \cos \theta}{y} \frac{\partial H}{\partial n} \quad (50)$$

Species:

$$\rho q \frac{\partial \alpha_i}{\partial s} = \mu \frac{\partial^2 \alpha_i}{\partial n^2} + \frac{\mu \cos \theta}{y} \frac{\partial \alpha_i}{\partial n} + \rho \dot{w}_i \quad (51)$$

It is important to note the above describing equations have been simplified from their most general form by the assumption that the Prandtl and Schmidt numbers are unity. Furthermore, in the numerical representation of the above equations it is assumed that $n \rightarrow y$. Both of these approximations were made to facilitate the initial stages of the program development but it is believed that the limitations are not critical. As the program continues development these restrictions will be removed.

To illustrate the numerical solution technique, consider the difference form of Equation (49) with reference to Figure 4.

$$\frac{q_{m,n+1} - q_{m,n}}{\Delta} = \left(\frac{\mu}{\rho q} \right)_n \left[\frac{q_{m+1,n} - 2q_{m,n} + q_{m-1,n}}{(\Delta y)^2} + \right. \\ \left. + \frac{\cos \theta_{m,n}}{y_m} \frac{(q_{m+1,n} - q_{m-1,n})}{2\Delta y} \right] - \left(\frac{1}{\rho q} \right)_n \left(\frac{\Delta p}{\sigma} \right) \frac{\sigma}{\Delta} \quad (52)$$

where $\frac{\Delta p}{\sigma}$ is given by the characteristics calculation (see Equation (48)).

Equation (52) represents an explicit finite difference form of the s-momentum equation. The energy and species conservation equations assume a similar form. The forward step size is limited by a stability requirement which for linear parabolic equation is given by:

$$\Delta s \leq \frac{1}{2} \rho q \frac{\Delta y^2}{\mu t} \quad (53)$$

To insure stability of the non-linear system a fraction of the characteristic step, σ , is used:

$$\Delta = \frac{\sigma}{\left(\frac{\sigma}{\Delta s} + 1 \right) N_s} \quad (54)$$

where N_s is an input integer. In cases involving fast reacting systems $N_s = 6$ has been found to be sufficient.

The species conservation equation, Equation (51), is treated in a special way for configurations involving finite rate kinetics. Specifically, two distinct numerical schemes were developed to overcome the difficulties associated with time consuming kinetics calculations and the coupling of kinetics with multidimensional diffusive flow systems. The problem of long computer times for

kinetics calculations was resolved by developing solution techniques for the linearized production terms. The coupling of the kinetics to diffusive flows was accomplished by performing a sequence of calculations wherein a diffusion step is made followed by a number of chemistry steps within the diffusion step. The techniques for the kinetics and the coupling of the kinetics to flows with diffusion have been developed over the past several years and reported upon in References 4 , 5 , and 6 .

The above discussions pertain specifically to the solution technique for the describing partial differential equations. To completely specify a given problem the initial and boundary conditions must be given. In the present work three configurations were investigated: (1) free jet type flows, (2) free jet flows with an external shock, and (3) shockless ducted jet flows.

In all cases initial conditions are specified by giving the distributions of velocity, temperature, pressure and mass fractions in the jet and free stream in the exit plane of the jet:

$$\begin{aligned}
 & @ \quad s = 0 \\
 & \quad y \leq r; \quad q_j(o,y), \quad T_j(o,y), \quad p_j(o,y) \text{ and } \alpha_{ij}(o,y) \\
 & \quad y \geq r \leq y_w; \quad q_e(o,y), \quad T_e(o,y), \quad p_e(o,y) \text{ and } \alpha_{ei}(o,y)
 \end{aligned}
 \tag{56}$$

For the free jet, $y_w \rightarrow \infty$. The boundary conditions depend on the particular configuration and therefore are treated individually.

Free Jet

The free jet without external shock is the simplest configuration and the boundary conditions are given as:

@ x = 0

$$y = y_e \quad q = q_e, \quad T = T_e, \quad p = p_e, \quad \text{and} \quad \alpha_i = \alpha_{ie} \quad (57)$$

Free Jet With External Shock

In this configuration, the domain of integration is between the axis of the jet and the downstream side of the shock. Thus, it is necessary to tie together the specified free stream conditions to the interior flow by considering the Rankine-Hugoniot jump conditions across the shock. To initiate the calculation the shock strength must be assumed. For a pressure balanced jet the initial shock slope is taken equal to the slope of the free stream Mach wave. For an underexpanded jet an iterative calculation is performed wherein a centered expansion is assumed at the jet lip. The calculation proceeds as follows. (See Figure 5).

a) Shock Calculation

1. Assume a shock angle, $\bar{\sigma}$.
2. Compute $\frac{p_{2s}}{p_j} = \left(\frac{p_{2s}}{p_\infty}\right) \frac{p_\infty}{p_j}$ where p_{2s}/p_∞ is obtained from the oblique shock relations using $\gamma = \gamma_\infty (=c_p/c_v)$.
3. Compute flow direction, θ_{2s} .

b) Centered Expansion Calculation

The Prandtl-Meyer equations are used here to determine the flow direction, θ_{2p} , associated with the shock pressure ratio, p_{2s}/p_∞ , determined above. Referring to Figure 5 the flow direction is given by:

$$\theta_{2p} = (\beta_2 - \beta_j) + (\epsilon_{2p} - \epsilon_j) \quad (58)$$

where

$$\beta_2 = \left(\frac{\gamma_j + 1}{\gamma_j - 1} \right)^{\frac{1}{2}} \cos^{-1} \left\{ \left[\left(\frac{p_{2s}}{p_j} \right) \left(\frac{p_j}{p_{j0}} \right) \right]^{\frac{\gamma_j - 1}{\gamma_j}} (\gamma_j + 1)^{-1} \right\} \quad (59)$$

$$\frac{p_j}{p_{j0}} = \left\{ \left(\frac{1}{\gamma_j + 1} \right) \left[1 + \cos \left(2 \left[\frac{\gamma_j - 1}{\gamma_j + 1} \right]^{\frac{1}{2}} \beta_j \right) \right] \right\}^{\frac{\gamma_j}{\gamma_j - 1}} \quad (60)$$

$$\beta_j = \left(\frac{\gamma_j + 1}{\gamma_j - 1} \right)^{\frac{1}{2}} \tan^{-1} \left[\left(\frac{\gamma_j - 1}{\gamma_j + 1} \right)^{\frac{1}{2}} \tan (90 - \epsilon_j) \right] \quad (61)$$

$$\epsilon_{2p} = 90 - \tan^{-1} \left[\left(\frac{\gamma_j + 1}{\gamma_j - 1} \right)^{\frac{1}{2}} \tan \left\{ \left(\frac{\gamma_j - 1}{\gamma_j + 1} \right)^{\frac{1}{2}} \beta_2 \right\} \right] \quad (62)$$

Equations (59) through (62) define the quantities necessary to fix the value of θ_{2p} given by Equation (58).

The desired solution is obtained when:

$$\left| \frac{\theta_{2p} - \theta_{2s}}{\theta_{2s}} \right| \leq \text{tolerance} \quad (63)$$

The remaining state properties at the downstream side of the shock (at the initial station) are then determined by the shock jump conditions:

$$a_s^2 = \frac{\gamma_\infty p_s}{\rho_s} \quad (64)$$

$$q_s = \sqrt{u_{sn}^2 + v_{sn}^2} \quad (65)$$

$$H_s = H_\infty \quad (66)$$

$$h_s = H_s - \frac{q_s^2}{2} \quad (67)$$

$$T_s = \frac{p_s}{p_\infty} \frac{\rho_\infty}{\rho_s} T_\infty \quad (68)$$

The determination of the shock at a generic point downstream of the initial station is made by satisfying the compatibility equations along the uprunning characteristic and the shock jump conditions. Referring to Figure 6, the calculation proceeds as follows:

The location of the shock at the new station, $n+1$, is given by the shock slope at the current station n :

$$y_{s_{n+1}} = y_{KT_n} + \tan \bar{\sigma}_{KT} \Delta x \quad (69)$$

where $\bar{\sigma}$ is the shock angle and KT is the shock point.

A star point is iterated for between points KT and $KT-1$ whose upward running characteristics gives a $y_{s_{n+1}}^* = y_{s_{n-1}}$ within a tolerance of 1×10^{-4} .

$$y_{s_{n+1}}^* = y_{*n} + \tan (\theta + \epsilon)_* \Delta x \quad (70)$$

p_* , q_* , τ_* , ρ_* are interpolated for linearly between points KT and KT-1 and

$$a_*^2 = \gamma p_*/\rho_* \quad (71)$$

$$\beta_* = 1/\tan \epsilon_* = \sqrt{\frac{q_*^2}{a_*^2} - 1} \quad (72)$$

$$\tan(\theta + \epsilon)_* = (\tau_* + 1/\beta_*) / (1 - \tau_*/\beta_*) \quad (73)$$

Once the * point has been found the following parameters are calculated for the iteration of the shock angle.

$$B_* = \beta_*/\rho_* u_{2*} \quad \text{and} \quad (74)$$

$$\tau_{NN} = \tau_* (1 + \Delta x (\tau_* - \tan(\theta + \epsilon)_*) / y_* \quad (75)$$

with

$$u_{2*} = \frac{q_*^2}{1 + \tau_*^2} \quad (76)$$

The iteration for the shock angle is handled in the following manner. A guess of $\bar{\sigma}$ (the value at station n initially) is taken and the following quantities are computed:

$$v_{N_\infty} = v_\infty \sin \bar{\sigma} \quad (77)$$

$$v_{T_\infty} = v_\infty \cos \bar{\sigma} \quad (78)$$

$$M_{N_\infty}^2 = \frac{v_{N_\infty}^2}{\gamma_\infty p_\infty \rho_\infty} \quad (79)$$

$$v_s = \frac{v_{N_\infty} (1 + \frac{\gamma_\infty - 1}{2} M_{N_\infty}^2)}{\frac{\gamma + 1}{2} M_{N_\infty}^2} \quad (80)$$

$$\rho_s = \frac{\rho_\infty v_{N_\infty}}{v_s} \quad (81)$$

$$p_s = p_\infty (1.0 - \frac{\frac{\gamma_\infty + 1}{\gamma_\infty - 1} \rho_s}{\rho_\infty}) / (\frac{\rho_s}{\rho_\infty} - \frac{\gamma + 1}{\gamma - 1}) \quad (82)$$

$$u_{s_N} = v_s \sin \bar{\sigma} + v_{T_\infty} \cos \bar{\sigma} \quad (83)$$

$$v_{s_N} = v_{T_\infty} \sin \bar{\sigma} - v_s \cos \bar{\sigma} \quad (84)$$

$$\tau_s = v_{s_N} / u_{s_N} \quad (85)$$

$$\tau_{s_N} = \tau_{s_{NN}} - B_* (p_s - p_*) \quad (86)$$

Where Equation (86) is the compatibility equation along the up-running characteristic from the * point.

When

$$\left| \frac{\tau_s - \tau_{s_N}}{\tau_s} \right| \leq 1.0 \times 10^{-4} \text{ the iteration for } \bar{\sigma} \text{ is considered converged.}$$

The guess for the second iteration of $\bar{\sigma}$ is given by

$$\tan \bar{\sigma}_{(2)} = \tan \bar{\sigma}_{(1)} + .05 \tan \bar{\sigma}_{(1)} \quad (87)$$

and for all subsequent values of $\bar{\sigma}$ the following equation is used

$$\tan \bar{\sigma}_{i+1} = \tan \bar{\sigma}_i - (\tau_s - \tau_{s_N})_i \frac{(\tan \bar{\sigma}_{i-1} - \tan \sigma_i)}{[(\tau_s - \tau_{s_N})_{i-1} (\tau_s - \tau_{s_N})_i]} \quad (88)$$

With the shock angle converged the properties of the KT shock point at station $n+1$ are computed in the following manner:

$$y_{KT_{n+1}} = y_{s_{n+1}} \quad (89)$$

$$p_{KT_{n+1}} = p_s \quad (90)$$

$$\rho_{KT_{n+1}} = \rho_s \quad (91)$$

$$\tau_{KT_{n+1}} = \tau_s \quad (92)$$

$$H_{KT_{n+1}} = H_{KT_n} \quad (93)$$

$$q_{KT_{n+1}} = \sqrt{u_{s_N}^2 + v_{s_N}^2} \quad (94)$$

$$h_{KT_{n+1}} = H_{KT_{n+1}} - \frac{q_{KT_{n+1}}^2}{2.0} \quad (95)$$

$$T_{KT_{n+1}} = T_\infty \frac{p_s \rho_\infty}{p_\infty \rho_s} \quad (96)$$

$$\alpha_{i_{KT_{n+1}}} = \alpha_{i_{KT_n}} \quad (97)$$

The assumptions made for the shock calculation are (1) chemically frozen constant γ across the shock and (2) the viscous terms in the compatibility equation along the uprunning characteristic from the star, *, point are negligible.

Ducted Flow

The flows considered above pertain to "free" external type flows and as such, interactions with confining walls were not present. To include the effects of wall interactions appropriate boundary conditions must be specified. In the present study the wall is assumed to be impermeable, and adiabatic. The boundary conditions are given by:

$$x > 0, \quad y = y_w(x) \quad \left\{ \begin{array}{l} \frac{\partial \alpha_i}{\partial y} \Big|_w = 0 \\ \frac{\partial H}{\partial y} \Big|_w = 0 \\ \frac{\partial q}{\partial y} = -\frac{c_f}{2} \left(\frac{\rho q^2}{\mu} \right)_{\text{mean}} \end{array} \right. \quad (98)$$

The arbitrary wall contour, $y=y_w(x)$ may be specified in terms of the coefficients of 5th order polynomials covering four adjoining regions:

$$y_w = \sum_0^5 a_n x^n \quad (99)$$

The pressure is determined from the appropriate uprunning characteristic since the flow deflection at the "new" wall point is known from Equation (99).

The calculation proceeds as follows (see Figure 7):

A characteristic step is taken to determine the minimum forward distance, σ , for intersections.

A star, *, point is found by interpolation between the w_n and KT_n points whose uprunning characteristic intersects the wall at the end of the step, $x_{n+1} = x_n + \sigma$. The pressure at the wall is then computed through the compatibility equation:

$$p_{w_{n+1}} = p_n^* + \frac{[\tau_{*n} - \tau_{w_{n+1}} + g_* \sqrt{\sigma^2 + (y_{w_{n+1}} - y_{*n})^2}]}{b_n^*} \quad (100)$$

The remaining properties along the $n+1$ line are determined and the diffusion calculation is performed. The wall boundary conditions for the diffusion step, Equations (98) are imposed by inserting a data point above the wall denoted by $KT+1$. The numerical difference form of Equations (98) are then given by:

$$\begin{aligned} \alpha_{i_{KT+1}} &= \alpha_{i_{KT}} \\ H_{KT+1} &= H_{KT} \\ q_{KT+1} &= q_{KT} - \frac{c_f}{2} \frac{(\rho q)_{\text{mean}}^2}{\mu} (y_{KT+1} - y_{KT}) \end{aligned} \quad (101)$$

where the physical location of the $KT+1$ point is given by:

$$y_{KT+1} = y_w + (y_w - y_{KT}) \quad (102)$$

Finally, the iterative process between the characteristic step and the diffusion step is performed as before. Flow diagrams and sample output are given in Appendix A.

IV. RESULTS OF CALCULATIONS AND DISCUSSIONS

The above analysis and computer program has been applied to a variety of flow configurations including unbounded and bounded flows. Mixing of dissimilar gases and finite rate kinetics are included and comparisons with other, more restricted, analyses have been made.

The mixing problem which forms the basic configuration for all flows considered here is that of a jet in an unbounded stream. Although this type of flow field is basic it has practical application for various nozzle plume and fuel injection problems. The state of development of the treatment of such problems is quite advanced including effects of chemical kinetics and two phase phenomena, References 1 , 2 , and 7 . Therefore, it is important that the M.O.C.V. analysis and computer program be compared to the existing parabolic analysis. Such a comparison has been made for a configuration where initial and boundary conditions minimized lateral pressure variations throughout the flow field. For this purpose an air-into-air pressure balanced jet was chosen with temperature and velocity differences that would not result in significant induced pressures upon mixing. The results are shown in Figures 8a through 8l. Figures 8a through 8d show, in an exaggerated scale, the axial distributions of pressure, velocity, temperature, and Mach number. It is apparent that the pressure disturbance which occurs in the near jet region decays rapidly and because it is small in magnitude it should not influence the development of flow field down stream. Figures 8e through 8h show radial profiles of pressure velocity, temperature, and Mach number at six(6) jet radii downstream. Figure 8e shows the comparison of the static pressure across the viscous region. The M.O.C.V. profile was cut off at a radial point defined by the intersection of the leading Mach line from the jet lip but is not shown because of the radial scale required.

Figures 8f through 8h show that the initial temperature and velocity difference develop into profiles governed by lateral diffusion and the excellent agreement between the M.O.C.V. and parabolic predictions demonstrates the validity of the calculation technique employed in the M.O.C.V. program. This is further demonstrated at 15 radii downstream in Figures 8i through 8l. It is interesting to note that the pressure profile at this station is uniform across the viscous layer but not quite equal to the free stream pressure which is assumed in the parabolic calculation. As indicated above the pressure becomes equal to ambient further out in the inviscid region of the flow.

Three significant conclusions can be drawn from the above comparison; 1) there are no apparent instabilities arising out of the technique employed in the M.O.C.V. calculation, 2) pure diffusion is accurately predicted by the M.O.C.V. program and 3) good resolution is maintained even though the M.O.C.V. domain of integration is substantially greater than that of the parabolic program.

One of the primary objectives of this work is to predict the effects of finite rate combustion in flows where lateral pressure gradients can be important. To demonstrate the significance of lateral pressure gradients in diffusive, reacting flows an axisymmetric jet of hydrogen exhausting into air was considered first with balanced pressures and then with the nozzle weakly under-expanded ($p_e/p_j = .5$) so that shocks were not important. Figure 9 a and b show the pressure and Mach number maps for the balanced jet configuration. Figure 9a represents the pressure field and shows more than a 50% rise in pressure in the neighborhood of the nozzle exit plane. The lateral pressure variations that occur here are particularly pronounced because of the large variance in molecular weights between the nozzle (H_2) and the free stream (air). Thus, upon mixing, large flow deflections develop inducing significant pressure variations through the flow. Upon proceeding downstream

the pressure field becomes more uniform and there is an approach to the "boundary layer" like structure. The effect of combustion is shown more clearly in Figure 9b where the isotherms are given. The combustion field develops smoothly on proceeding downstream and is governed by mixing and finite rate kinetics. The "thermal field" is confined more toward the interior regions than is the pressure field demonstrating that the viscous field is embedded within the overall disturbed flow. Figures 10a and b show the pressure and temperature fields for the same initial Mach numbers and temperatures but in an underexpanded configuration. Localized compressions are essentially eliminated although significant lateral pressure variations exist. Furthermore, combustion is delayed due to the expansion and generally lower pressure levels. In terms of the applicability of the boundary layer theory it is observed that in the balanced pressure configuration lateral pressure variations are localized in a small region relative to the overall domain of interest. Thus, the importance of considering such effects depends upon the objectives of the problem at hand such as the effect of pressure on combustion initiation.

Figures 11 and 12 show some results for an inviscid under-expanded nozzle plume including the external shock where the pressure ratio, p_j/p_e is 10/1. Figures 11a and b show the pressure and Mach number maps including the shock shape. Profiles of pressure and Mach number are shown in Figures 11c and d. The effect of viscosity on the shock shape and the dividing streamline contour are shown in Figure 12a. For the range of constant viscosities used in these calculations the effect is negligible even for a viscosity level two orders of magnitude greater than the corresponding laminar value. There appears to be no experimental data to make a comparison with the theory for underexpanded nozzle flows with other than

quiescent surroundings. Nevertheless, a comparison with other theories is possible and for this case a calculation was made utilizing the "pure" characteristics method of Reference 8 inputting the dividing streamline contour. The comparison is excellent. It is apparent from the above results that the flow field structure, at least for eight nozzle diameters downstream, is controlled by inviscid phenomena associated with the pressure adjustment process. The effect of viscosity is seen more clearly in Figures 12b and c where it is seen that the profiles are modified in a region near the core of the plume for the largest value of the viscosity. It should be noted that levels of turbulent viscosity an order of magnitude larger than the maximum value used here have been observed in flows with combustion. Thus, the effect of viscosity must ultimately be determined for each case by the incorporation of an appropriate eddy viscosity model which will reflect the effects of the initial and boundary conditions on the viscosity level. A review and application of various eddy viscosity models are given in References 2, 3 and 9.

The flow configurations discussed above were primarily concerned with "external" flows involving no interactions with surrounding structures or walls. Since all the engine processes contribute to the plume structure it is necessary to provide the capability to compute, in a consistent manner, the internal flow field of the engine. This problem was initially investigated for configurations where the parabolic assumption was reasonably valid and the flow was analyzed assuming that the bulk mixing and combustion processes were described by "boundary layer" type equations, Reference 3. This description is adequate for the analysis of combustion chamber flows which are subsonic throughout

and for configurations including ejector and augmented rocket systems where the primary (or central) stream is moderately supersonic and the secondary stream is subsonic. In such configurations the lateral pressure gradients will not have an important effect upon wall pressure distribution determinations (thrust). However, for pressure unbalanced primary and secondary streams and in general, for supersonic combustors, pressure wave propagation must be considered. To analyze this more general internal flow process the M.O.C.V. analysis was extended to include wall interactions. A special case of this type of flow is the underexpanded nozzle in a quiescent atmosphere. Neglecting viscous effects, this flow is representative of an internal flow confined by a duct contoured to give a constant pressure along the wall. In addition to the relative simplicity of this configuration experimental data is available for comparison, Reference 10. Figures 13, 14 and 15 show a comparison of the present M.O.C.V. technique (with zero viscosity) with the standard characteristics method of Ref. 7 and with experiments. The good agreement obtained here lends further credence to the basic calculation procedures used in the M.O.C.V. program. Upon completion of this preliminary step the analysis was modified for inclusion of arbitrary wall contours. Figures 16 through 19 show the results of sample calculations for the flow fields associated with ducted nozzles. These calculations were carried out for hydrogen in the primary stream and air in the secondary stream. The duct cross sectional area used in these calculations is constant and the flow is assumed to be chemically frozen. The latter was chosen to isolate the effects of viscosity on the detailed structure of flow field. Finite rate calculations will be performed in connection with the current studies. Figure 16 shows the wall and axis pressure distributions for inviscid flow (no mixing).

Since there is no viscous interaction between the streams each behaves isentropically and the configuration lends itself to a simple check. Here each stream was analyzed utilizing the constant γ , isentropic relations and for the final area of each stream, the velocity, temperature and pressure were determined and compared with the M.O.C.V. output at $x/r_j = 100$. The results compared with each other to within 2%. The effect of viscosity and the under-expansion are shown in Figures 17 thru 19. Figure 17 shows the effect of a constant viscosity in a pressure balanced primary/secondary configuration. Here, as in the case shown in Figure 9a, there is a large pressure rise on the axis in the vicinity of the exit plane. This is due to the large flow deflection generated by the rapid mixing between the hydrogen and the air. The magnitude of the pressure wave is enhanced by the focusing effect on approaching the axis of the flow field. The structure of the flow moving downstream shows the propagation of the pressure field with higher pressures always occurring on the axis due to the focusing effect. Again, a comparison with one dimensional calculations for the fully mixed end state agrees with the mean properties at $x/r_j = 100$. Axial decay and radial profiles of the hydrogen mass fraction, pressure, velocity and temperature are shown in Figures 17b thru 17j.

Figure 18 shows the additional effect of underexpansion on the flow field assuming the same value of viscosity as used in the previous case. Similar behavior is observed with the exception that the initial rise in pressure on the axis is substantially reduced due to the expansion. The axial decay of the hydrogen mass fraction and the additional property profiles are shown in Figures 18b through 18j. Of particular interest in this type of flow configuration is the effect of the mixing rate upon the flow field structure. Figure 19 summarizes the effect of viscosity for a range of viscosity

levels over an axial length of 20 nozzle radii. The previous results for, $\nu = 0$ and $\nu = 4.49 \times 10^{-4}$ lb-sec/ft² are included and, in addition, results for a smaller viscosity level ($\nu = 4.49 \times 10^{-5}$ lb-sec/ft²) and for a "variable" viscosity are presented. The latter has the feature of the shear layer, or two-dimensional diffusion rate which accounts for slower mixing observed in the "potential core" region of jet flows. The comparison shown in Figure 19a and b shows the significant effect of viscosity on the induced pressure level in the vicinity of the nozzle exit plane. This effect is pronounced for the largest viscosity and is much less important for the lower levels including the "inviscid" and variable viscosity cases. The effect of viscous "dispersion" is also noted in Figures 19a and b where the pressure waves for the "lower" viscosity levels intercept the wall and axis at the same points whereas for the largest viscosity level the disturbance is felt at an earlier station. Figure 19 c shows the rate of axial decay of the hydrogen mass fraction as a function of viscosity level. With the exception of the largest viscosity case, this result shows that the width of the pressure field is substantially greater than the spread of the viscous region.

Although the above results serve to indicate the kinds of flows and phenomena that can be analyzed with the M.O.C.V. program the predicted magnitudes of the flow variables are tentative and require experimental verification. Such experiments could serve a dual purpose; 1) to explain the observed phenomena and imply a mixing rate for the particular experiment and 2) to aid in developing an appropriate functional representation for the eddy viscosity which could be used with confidence to predict flow fields for which data has not yet been obtained.

In summary a computer program has been generated which can treat supersonic flows in either free flows with or without external shocks and ducted flows. Finite rate chemistry for the

hydrogen-air system is included in the existing computer program. Current work is underway to extend the capability of the program to include hydrocarbon kinetics with solid carbon and to include the treatment of imbedded subsonic regions within the flow resulting from a local transition due to combustion and/or due to the internal shock structure.

V. REFERENCES

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APPENDIX A

The diagrams shown below give the "overall" computational sequence in the M.O.C.V. program. The various options are given along with the subroutine names and operations performed within each subroutine.

There is also a sample output given showing property profiles at a particular axial station for a nozzle inside a constant area duct.

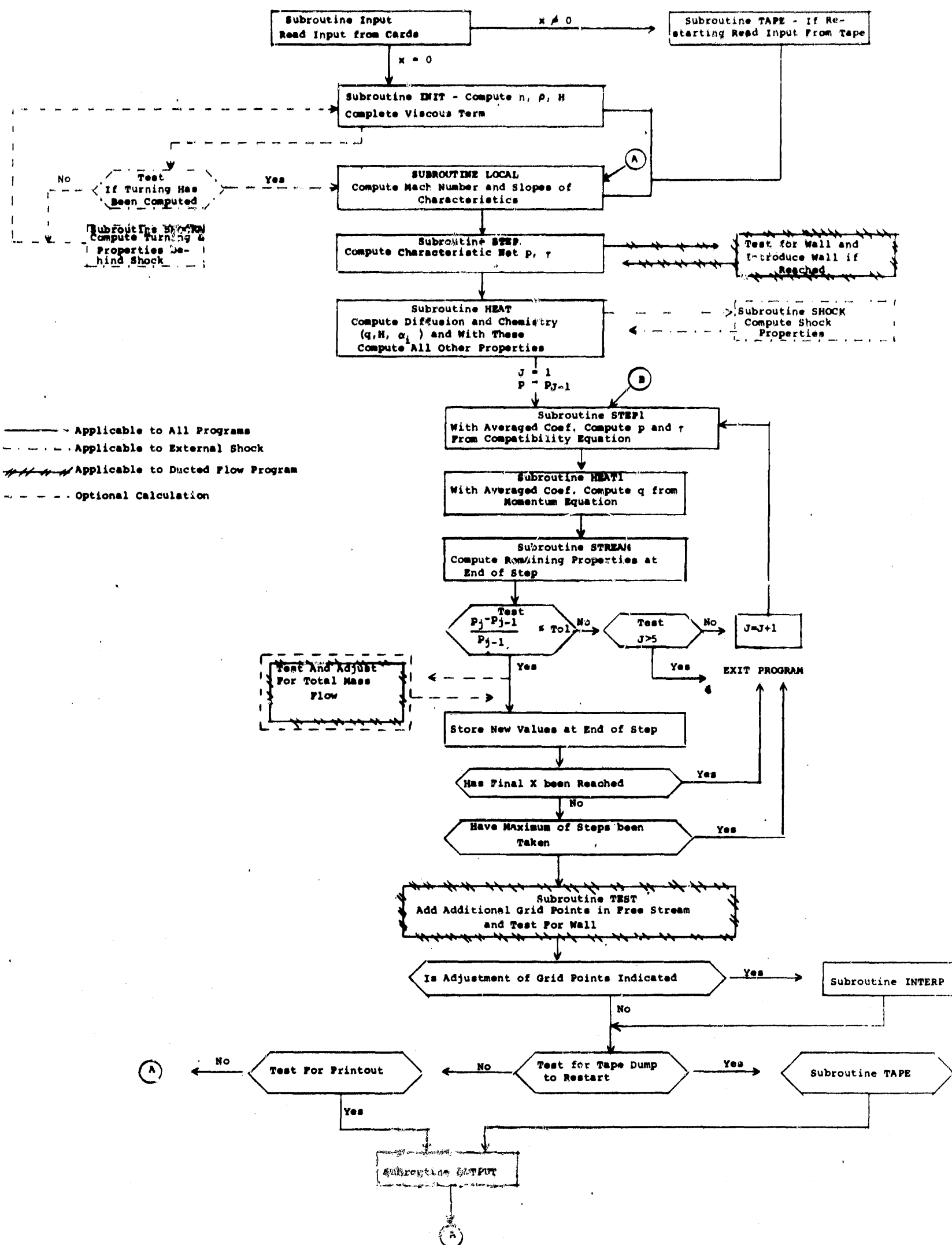


TABLE I - FLOW DIAGRAM FOR M.O.C.V. PROGRAMS

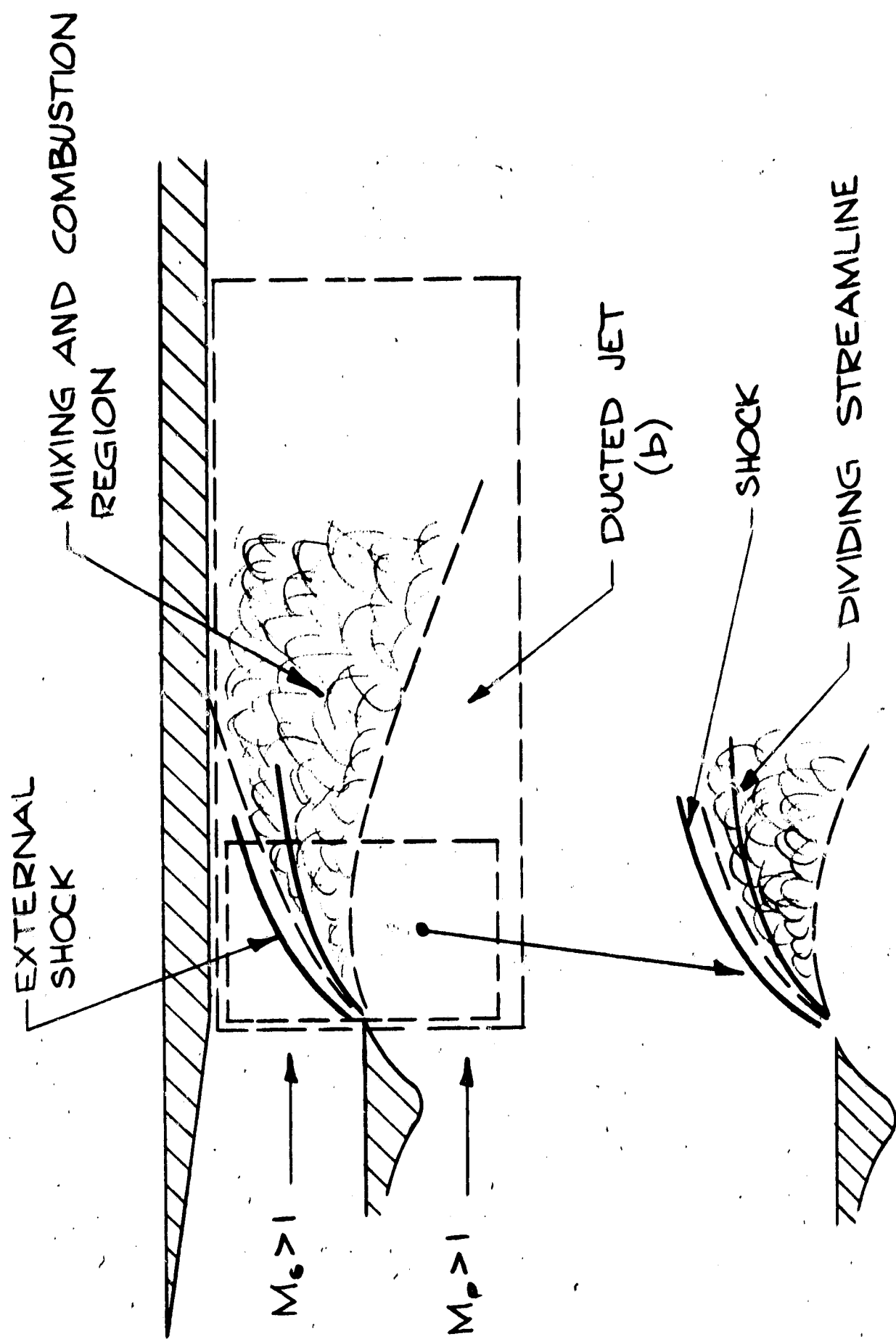
global equivalent ratio
 FROZEN FLOW-HIGH VISC CONST-MALL AT 375 PSI=1 NSTEP6 TO K=1.0 2/19/68
 STATION NUMBER 100
 VISCOSITY (C.G.S.) = 2.1516195E-01 = 4.99 NO → 11.90E
 K (CM.) = 6.692078E-01
 TAU = 0.0
 VELOCITY (CM/SEC) = 2.3751380E 00
 TEMP (°K) = 1.0072273E 03
 CRHU (GM/CM) = 9.0652525E-05
 global equivalent ratio
 FROZEN FLOW-HIGH VISC CONST-MALL AT 375 PSI=1 NSTEP6 TO K=1.0 2/19/68
 STATION NUMBER 100
 VISCOSITY (C.G.S.) = 2.1516195E-01 = 4.99 NO → 11.90E
 K (CM.) = 6.692078E-01
 TAU = 0.0
 VELOCITY (CM/SEC) = 2.3751380E 00
 TEMP (°K) = 1.0072273E 03
 CRHU (GM/CM) = 9.0652525E-05

K	Y	PSI	M	TAU	VELOCITY (CM/SEC)	TEMP (°K)	CRHU (GM/CM)	global equivalent ratio
1	0.	0.	2.363E 00	0.	4.515971E 05	1.0072273E 03	9.0652525E-05	1.6195488E 00
2	1.120E-02	3.372E-02	2.362E 00	-4.752E-03	4.505286E 05	1.0084728E 03	9.1104054E-05	1.6142010E 00
3	2.242E-02	6.757E-02	2.360E 00	-5.568E-03	4.473456E 05	1.0121856E 03	9.2426509E-05	1.5981001E 00
4	3.349E-02	1.016E-01	2.356E 00	-7.445E-03	4.422135E 05	1.0181866E 03	9.4576946E-05	1.5716416E 00
5	4.436E-02	1.358E-01	2.351E 00	-9.427E-03	4.353611E 05	1.0262313E 03	9.7568331E-05	1.5353602E 00
6	5.504E-02	1.703E-01	2.346E 00	-1.133E-02	4.270468E 05	1.0360646E 03	1.0142475E-04	1.4899444E 00
7	6.554E-02	2.051E-01	2.343E 00	-1.304E-02	4.175226E 05	1.0474702E 03	1.0620579E-04	1.4361556E 00
8	7.591E-02	2.404E-01	2.341E 00	-1.445E-02	4.070022E 05	1.0603116E 03	1.1202741E-04	1.3747448E 00
9	8.623E-02	2.764E-01	2.342E 00	-1.544E-02	3.956384E 05	1.0745715E 03	1.1908979E-04	1.3062213E 00
10	9.666E-02	3.133E-01	2.347E 00	-1.587E-02	3.835158E 05	1.0903822E 03	1.2771986E-04	1.2308528E 00
11	1.074E-01	3.514E-01	2.358E 00	-1.554E-02	3.706562E 05	1.1080629E 03	1.3843938E-04	1.1485969E 00
12	1.186E-01	3.912E-01	2.376E 00	-1.418E-02	3.570539E 05	1.1281605E 03	1.5205797E-04	1.0593132E 00
13	1.305E-01	4.333E-01	2.404E 00	-1.141E-02	3.427218E 05	1.1515302E 03	1.6982256E-04	9.6309739E-01
14	1.433E-01	4.785E-01	2.446E 00	-6.764E-02	3.278336E 05	1.1793259E 03	1.9349206E-04	8.6133054E-01
15	1.572E-01	5.281E-01	2.507E 00	1.979E-04	3.127915E 05	1.2126112E 03	2.2546593E-04	7.5729531E-01
16	1.718E-01	5.833E-01	2.590E 00	9.522E-03	2.984960E 05	1.2515015E 03	2.678874E-04	6.5816267E-01
17	1.869E-01	6.454E-01	2.694E 00	2.091E-02	2.856354E 05	1.2973360E 03	3.2165271E-04	5.6973432E-01
18	2.023E-01	7.157E-01	2.811E 00	3.364E-02	2.747452E 05	1.3498183E 03	3.8718383E-04	4.9653589E-01
19	2.172E-01	7.943E-01	2.926E 00	4.621E-02	2.663787E 05	1.4043316E 03	4.5815451E-04	4.4247290E-01
20	2.316E-01	8.803E-01	3.026E 00	5.781E-02	2.602328E 05	1.4568423E 03	5.2869400E-04	4.0517297E-01
21	2.450E-01	9.721E-01	3.100E 00	6.763E-02	2.559884E 05	1.5021294E 03	5.9117427E-04	3.8175295E-01
22	2.575E-01	1.068E 00	3.147E 00	7.550E-02	2.531549E 05	1.5379287E 03	6.4170871E-04	3.6826684E-01
23	2.692E-01	1.165E 00	3.172E 00	8.158E-02	2.486395E 05	1.5646428E 03	6.806296E-04	3.6134153E-01
24	2.797E-01	1.262E 00	3.181E 00	8.586E-02	2.500791E 05	1.5836406E 03	7.0745236E-04	3.5852545E-01
25	2.886E-01	1.357E 00	3.181E 00	8.850E-02	2.492677E 05	1.5973043E 03	7.2686514E-04	3.5798728E-01
26	2.963E-01	1.451E 00	3.176E 00	8.980E-02	2.480485E 05	1.6084798E 03	7.4231805E-04	3.5871825E-01
27	3.035E-01	1.543E 00	3.166E 00	8.980E-02	2.480485E 05	1.6194422E 03	7.5701321E-04	3.6044256E-01
28	3.104E-01	1.633E 00	3.151E 00	8.815E-02	2.473835E 05	1.6321259E 03	7.7361905E-04	3.637696E-01
29	3.172E-01	1.722E 00	3.127E 00	8.410E-02	2.465340E 05	1.6485595E 03	7.9511523E-04	3.676591E-01
30	3.239E-01	1.812E 00	3.095E 00	7.660E-02	2.453877E 05	1.6708249E 03	8.2523351E-04	3.7401673E-01
31	3.305E-01	1.902E 00	3.051E 00	6.471E-02	2.438782E 05	1.7000853E 03	8.6784056E-04	3.8262506E-01
32	3.369E-01	1.994E 00	3.000E 00	4.844E-02	2.420836E 05	1.7346815E 03	9.2422996E-04	3.9252634E-01
33	3.429E-01	2.089E 00	2.949E 00	3.082E-02	2.402768E 05	1.7692731E 03	9.890205E-04	4.0327570E-01
34	3.487E-01	2.186E 00	2.908E 00	1.520E-02	2.387778E 05	1.7977816E 03	1.0508215E-03	4.1182251E-01
35	3.542E-01	2.286E 00	2.880E 00	4.496E-03	2.377276E 05	1.8176514E 03	1.0992341E-03	4.1778538E-01
36	3.596E-01	2.386E 00	2.863E 00	-1.063E-03	2.370723E 05	1.8300053E 03	1.1316028E-03	4.2149461E-01
37	3.650E-01	2.485E 00	2.853E 00	-2.705E-03	2.366969E 05	1.8370680E 03	1.1506040E-03	4.2361618E-01
38	3.704E-01	2.583E 00	2.848E 00	-1.914E-03	2.365077E 05	1.8406248E 03	1.160085E-03	4.2468467E-01
39	3.758E-01	2.680E 00	2.847E 00	0.	2.364509E 05	1.8416914E 03	1.1628707E-03	4.2505514E-01

mass fractions

K	ALPHA(H)	ALPHA(H2O)	ALPHA(H2)	ALPHA(O2)	ALPHA(OH)	ALPHA(N2)	SUM
1	0.	0.	6.1198829E-01	9.0016935E-02	0.	2.9798695E-01	0.999992
2	0.	0.	6.0850610E-01	9.0824803E-02	0.	3.0060122E-01	0.999992
3	0.	0.	5.9812875E-01	9.3232350E-02	0.	3.0863102E-01	0.999992
4	0.	0.	5.8139921E-01	9.7113603E-02	0.	3.2147931E-01	0.999992
5	0.	0.	5.5906748E-01	1.0229457E-01	0.	3.3863009E-01	0.995992
6	0.	0.	5.3199480E-01	1.0857544E-01	0.	3.5942193E-01	0.999992
7	0.	0.	5.0103943E-01	1.1575709E-01	0.	3.8319570E-01	0.999992
8	0.	0.	4.6694990E-01	1.2366588E-01	0.	4.0937652E-01	0.999992
9	0.	0.	4.3029096E-01	1.3217075E-01	0.	4.3753961E-01	0.994992
10	0.	0.	3.9141798E-01	1.4118928E-01	0.	4.6738504E-01	0.999992
11	0.	0.	3.5049792E-01	1.5068275E-01	0.	4.9881158E-01	0.999992
12	0.	0.	3.0763261E-01	1.6062753E-01	0.	5.3173204E-01	0.999992
13	0.	0.	2.6301274E-01	1.7097938E-01	0.	5.660010E-01	0.999992
14	0.	0.	2.1735941E-01	1.8157100E-01	0.	6.0106196E-01	0.994992
15	0.	0.	1.7209360E-01	1.9207273E-01	0.	6.3582618E-01	0.999993
16	0.	0.	1.3005545E-01	2.0182564E-01	0.	6.681105E-01	0.999993
17	0.	0.	9.3254322E-02	2.1036357E-01	0.	6.963757E-01	0.999993
18	0.	0.	6.3077168E-02	2.1736474E-01	0.	7.1955141E-01	0.999993
19	0.	0.	4.0734437E-02	2.2254715E-01	0.	7.3670698E-01	0.999994
20	0.	0.	2.5049188E-02	2.2616735E-01	0.	7.4875731E-01	0.999994
21	0.	0.	1.4805419E-02	2.2856396E-01	0.	7.5662475E-01	0.999994
22	0.	0.	8.4601514E-03	2.3003612E-01	0.	7.6149811E-01	0.994994
23	0.	0.	4.7070109E-03	2.3090689E-01	0.	7.6438071E-01	0.995995
24	0.	0.	2.6287270E-03	2.3138909E-01	0.	7.6597694E-01	0.999995
25	0.	0.	1.5178408E-03	2.3164683E-01	0.	7.6683016E-01	0.994995
26	0.	0.	8.9695190E-04	2.3179089E-01	0.	7.6733706E-01	0.994995
27	0.	0.	5.2884172E-04	2.3187629E-01	0.	7.6758977E-01	0.999995
28	0.	0.	3.0551701E-04	2.3192810E-01	0.	7.6776124E-01	0.999995
29	0.	0.	1.7145346E-04	2.3195919E-01	0.	7.6786414E-01	0.994995
30	0.	0.	9.3162886E-05	2.3197733E-01	0.	7.6792417E-01	0.999995
31	0.	0.	4.894542E-05	2.3198755E-01	0.	7.6795794E-01	0.994994
32	0.	0.	2.4980271E-05	2.3199309E-01	0.	7.6797624E-01	0.999994
33	0.	0.	1.2385415E-05	2.3199595E-01	0.	7.6798573E-01	0.994994
34	0.	0.	5.9854105E-06	2.3199740E-01	0.	7.6799044E-01	0.999994
35	0.	0.	2.8152442E-06	2.3199808E-01	0.	7.6799276E-01	0.994994
36	0.	0.	1.2831619E-06	2.3199839E-01	0.	7.6799377E-01	0.999994
37	0.	0.	5.7147661E-07	2.3199852E-01	0.	7.6799409E-01	0.994994
38	0.	0.	2.7016039E-07	2.3199854E-01	0.	7.6799415E-01	0.999994
39	0.	0.	1.4847689E-07	2.3199855E-01	0.	7.6799420E-01	0.994994

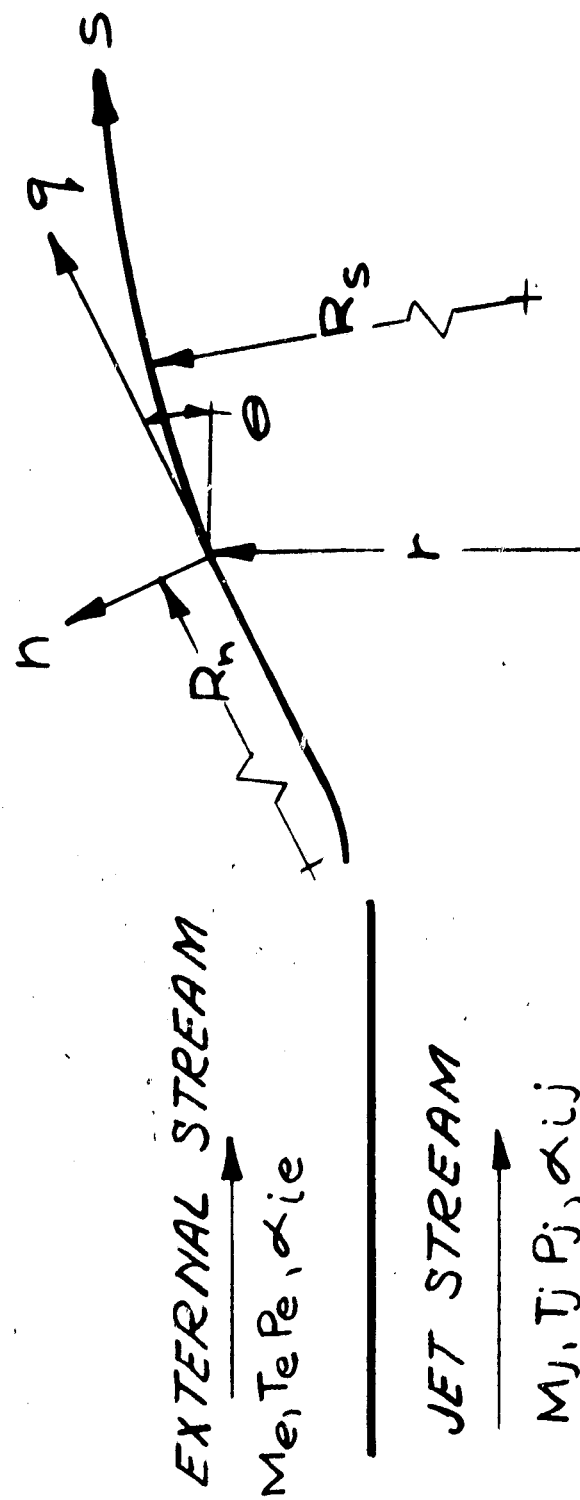
SI05YS
SPAUSE
UPTR. ACTIU. AJSE



FREE JET WITH EXTERNAL SHOCK

FIG. 1 SCHEMATIC OF FLOW FIELD

FIG. 2 SCHEMATIC OF FLOW FIELD WITH LATERAL PRESSURE GRADIENT



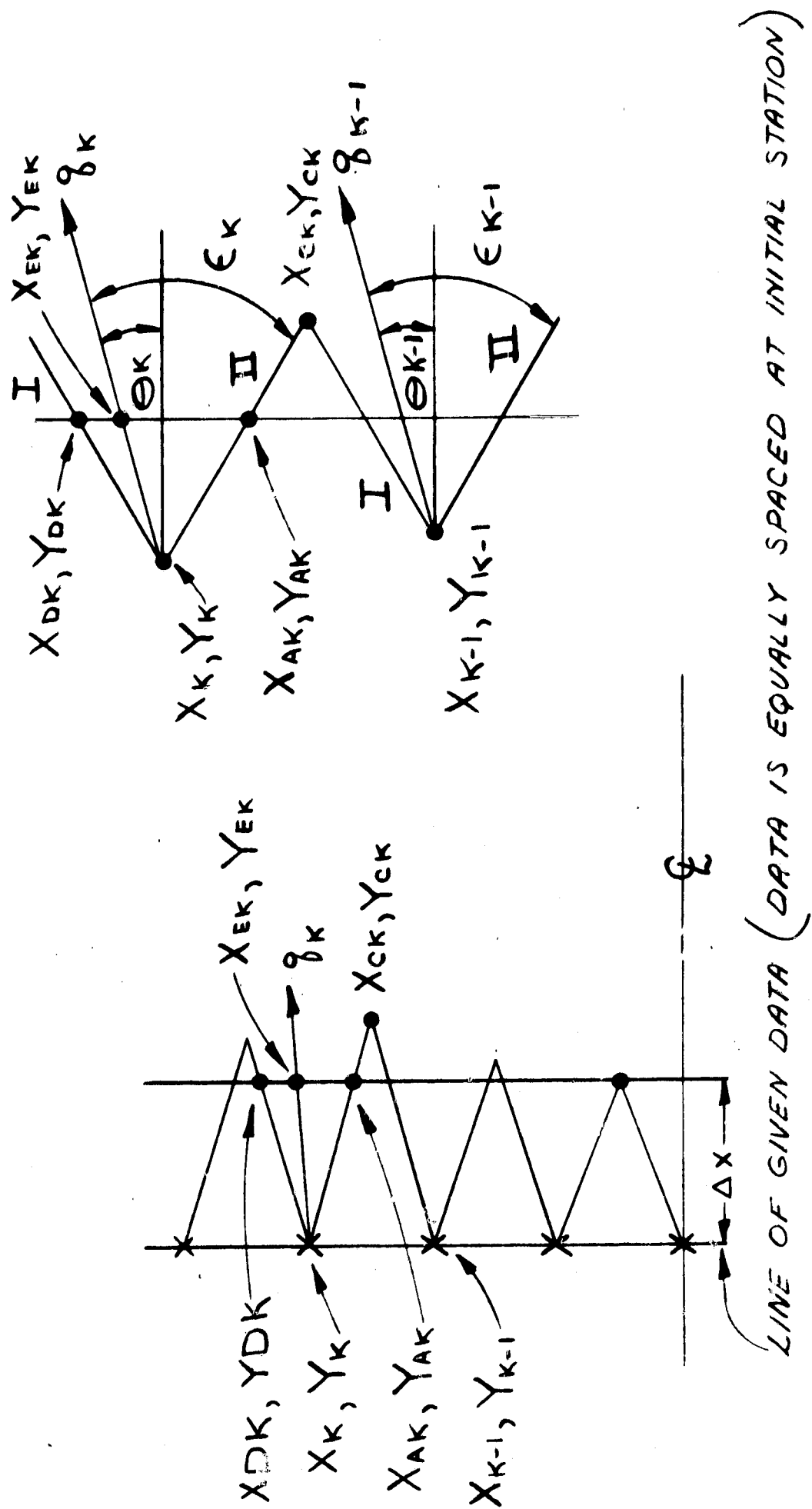


FIG. 3 NUMERICAL NET FOR CHARACTERISTICS STEP

$\sigma \equiv$ CHARACTERISTICS
STEP SIZE

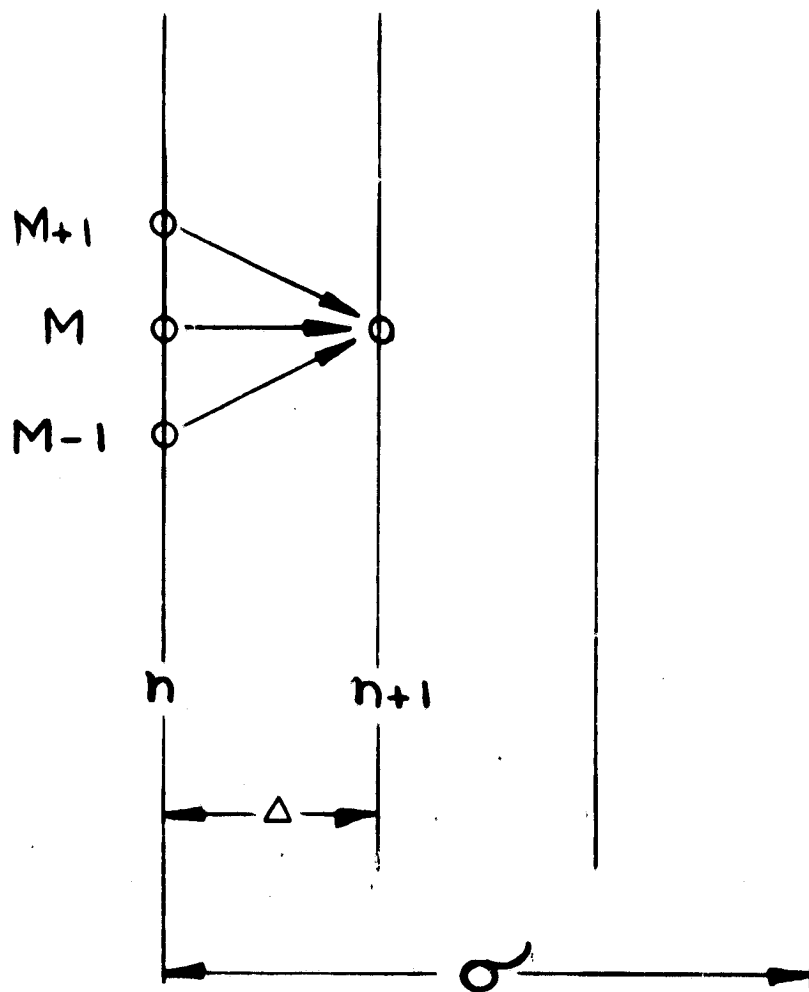


FIG. 4 NUMERICAL GRID FOR MIXING STEP

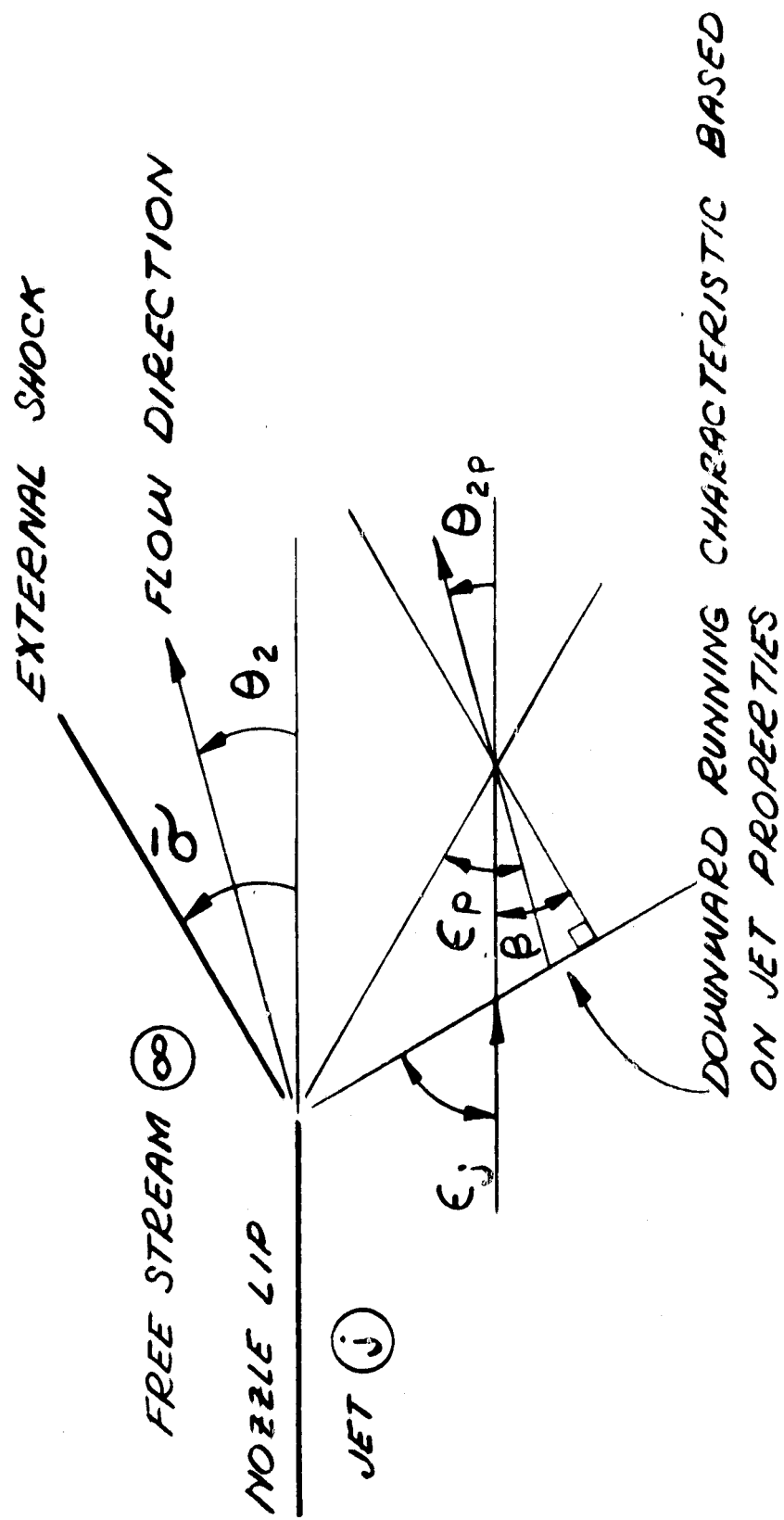


FIG. 5 DETERMINATION OF EXTERNAL SHOCK STRENGTH AT INITIAL STATION

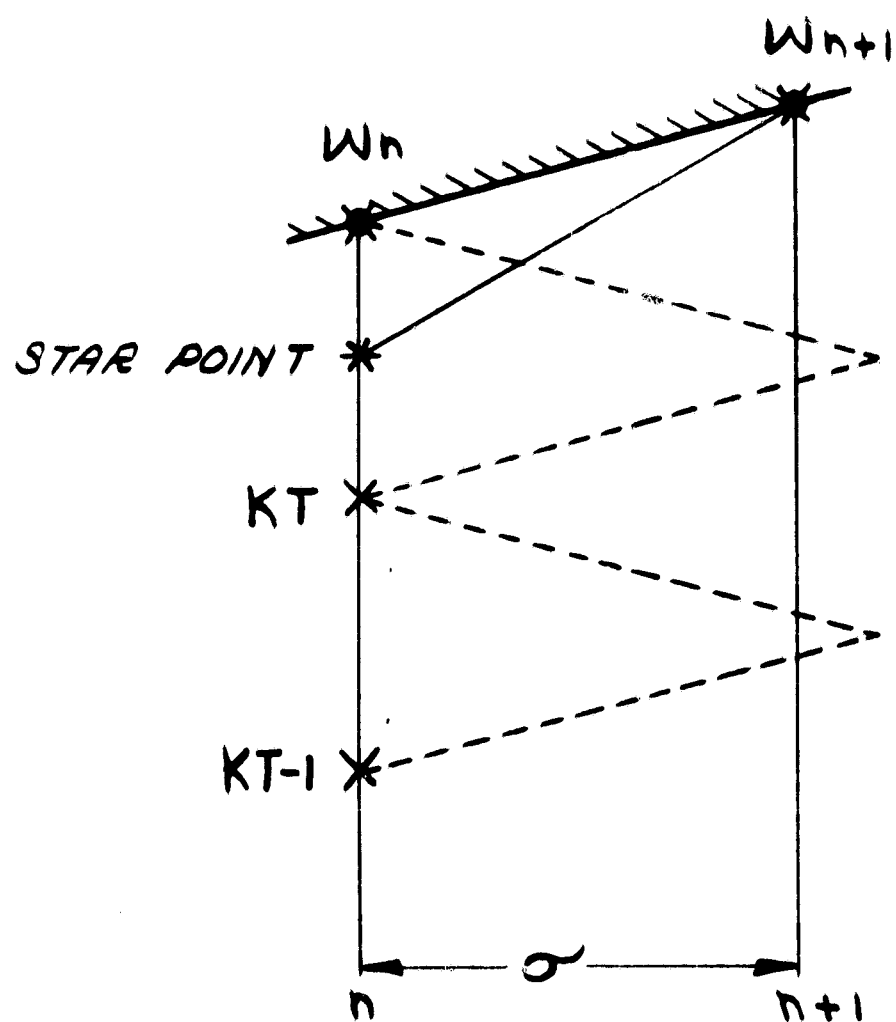
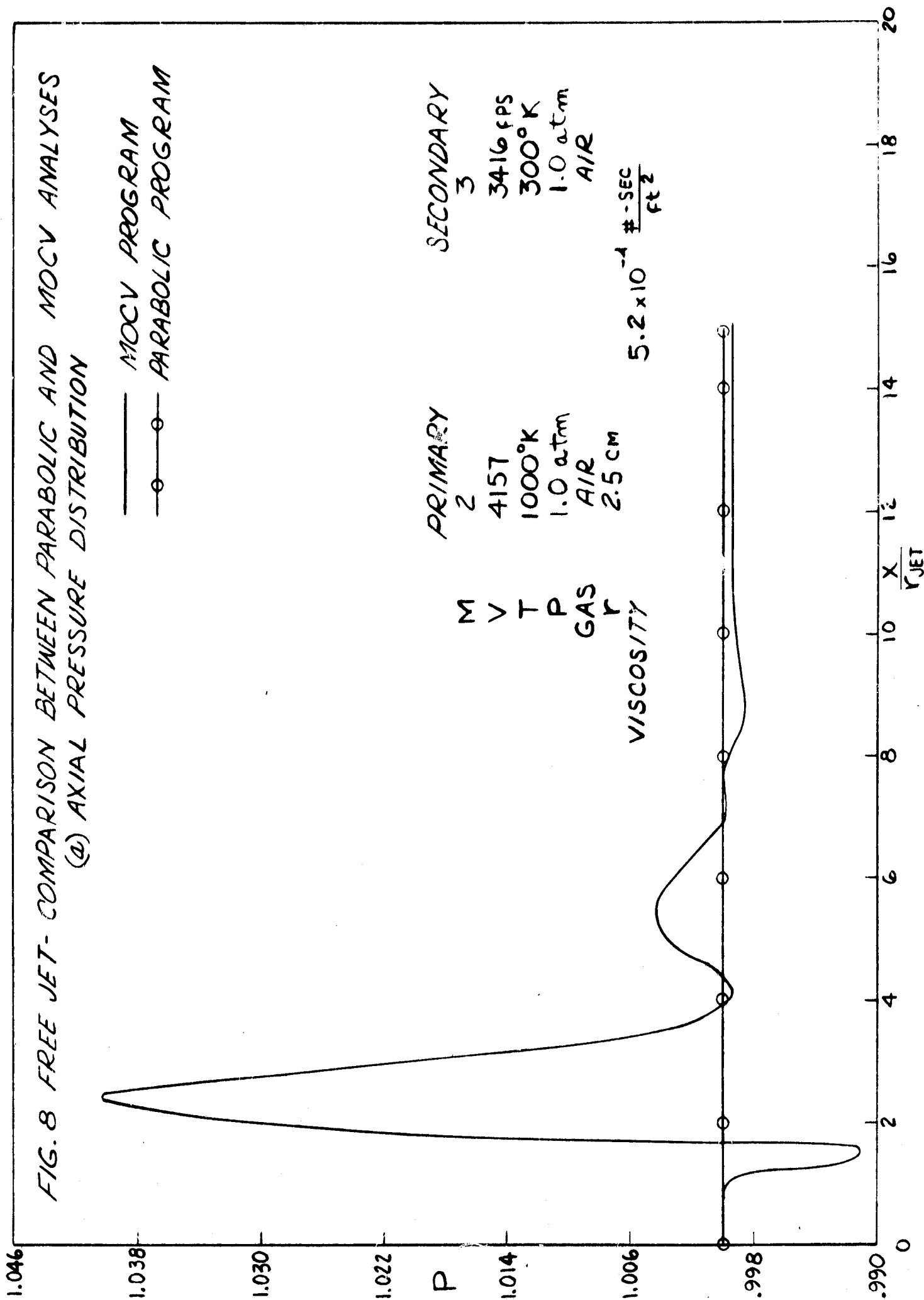
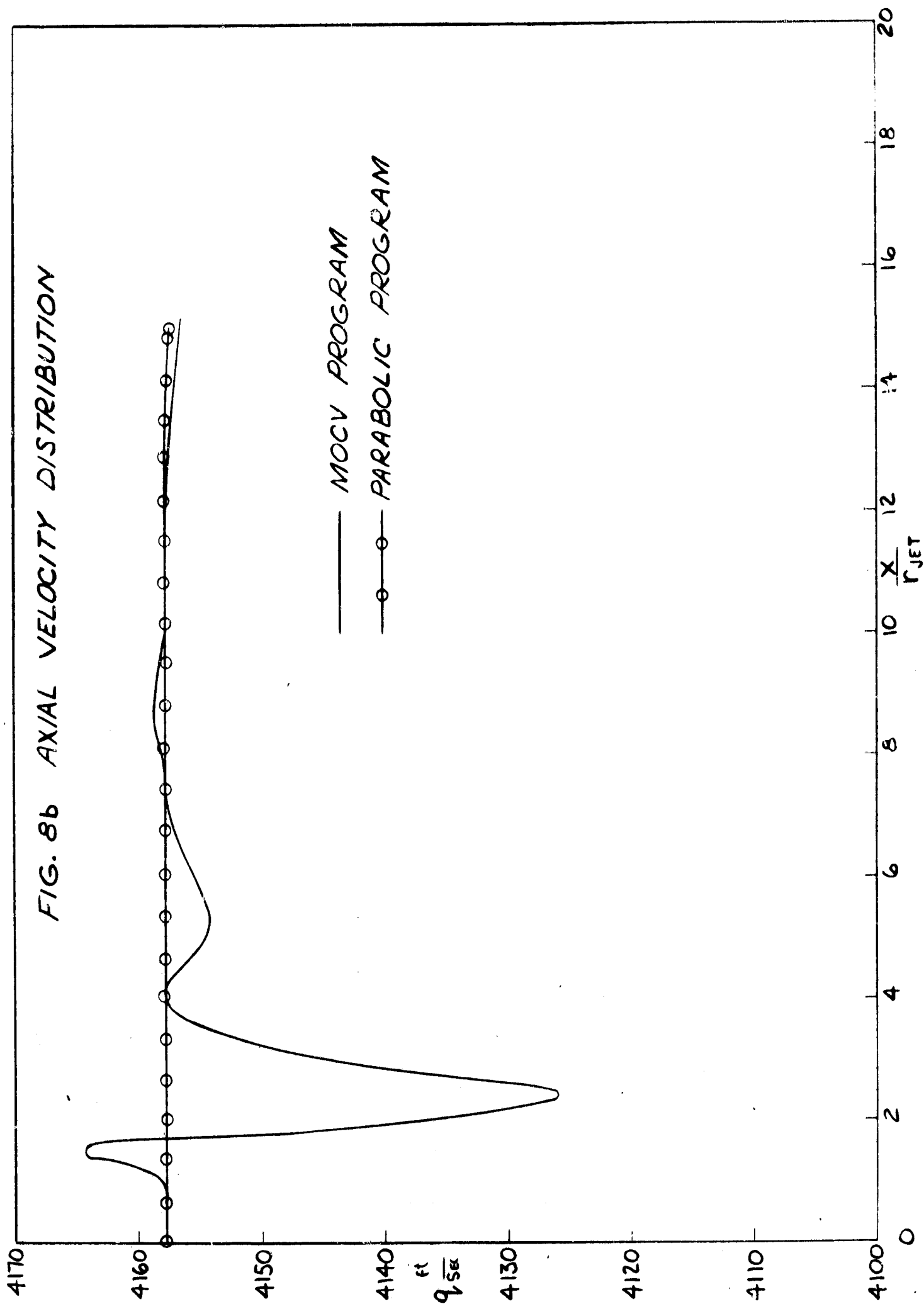
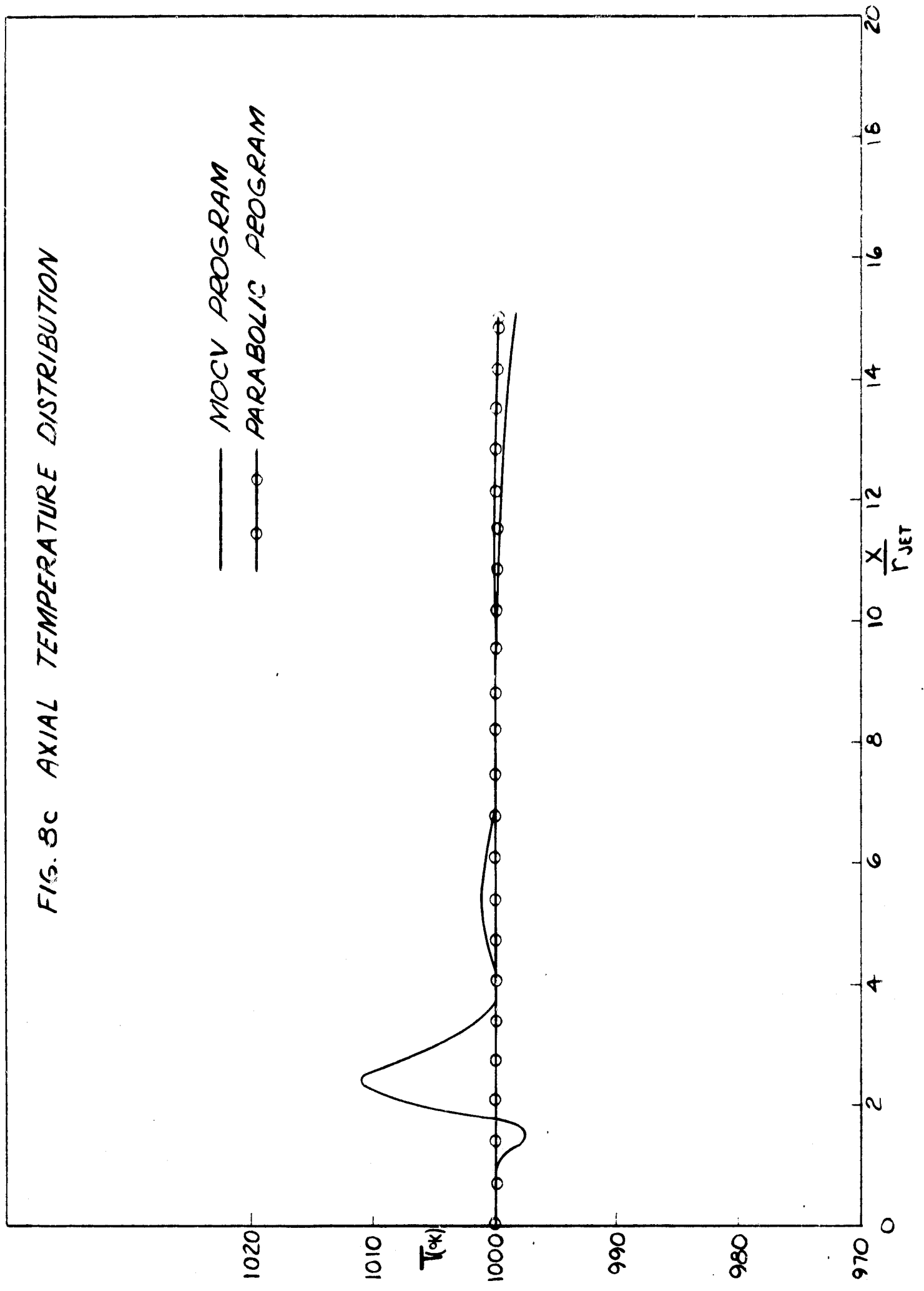


FIG. 7 WALL INTERSECTION







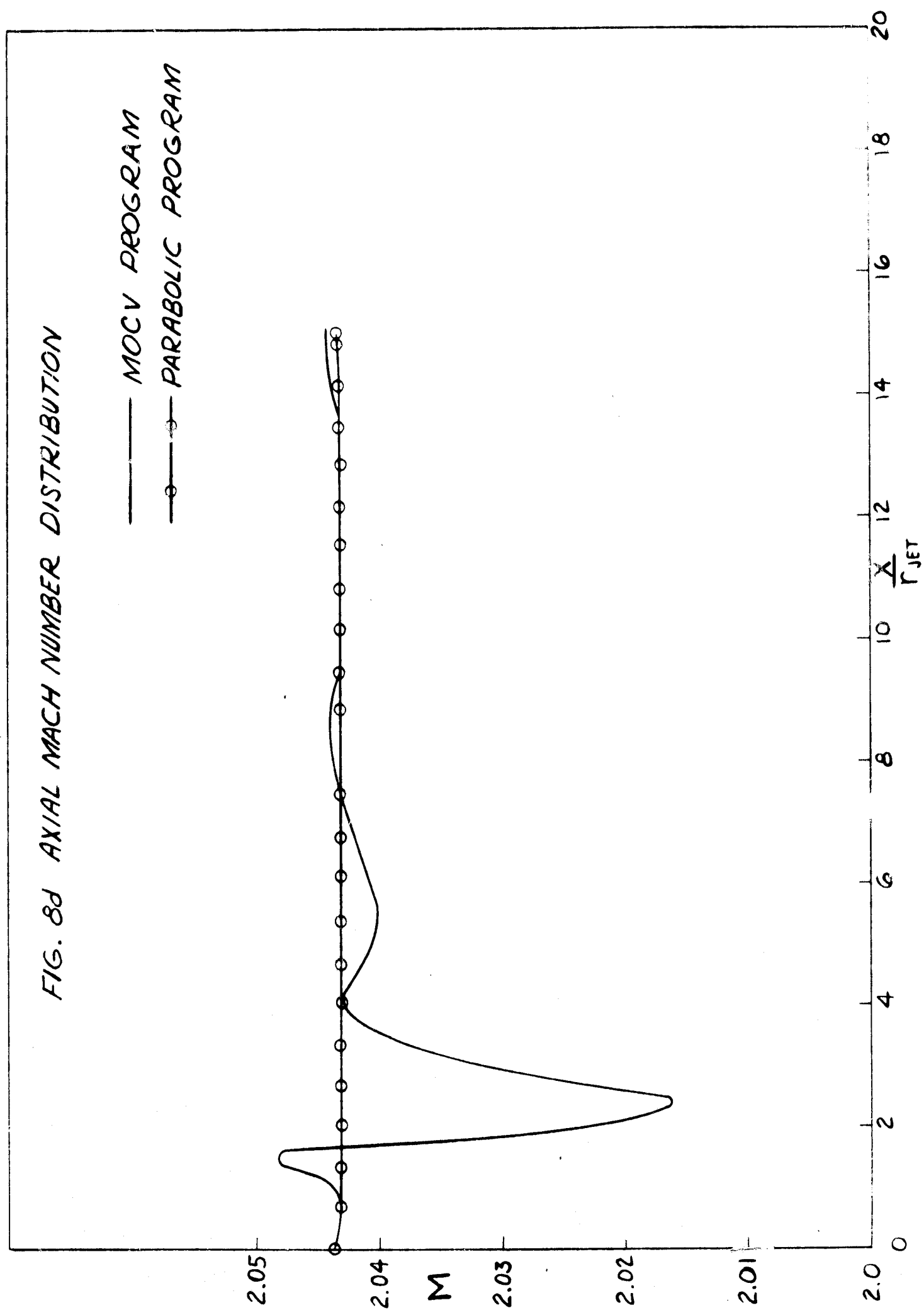
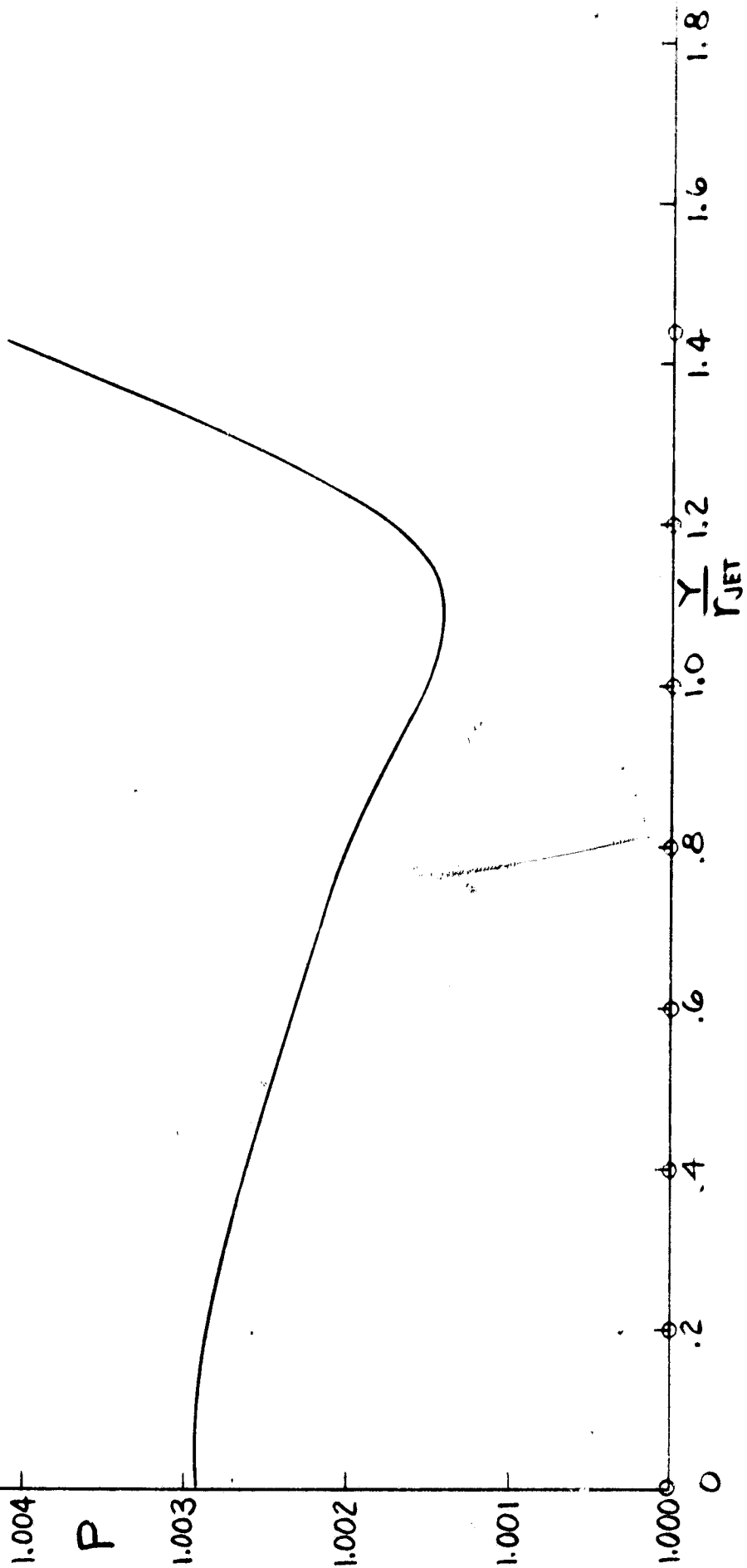


FIG. 8e PRESSURE PROFILE AT $\frac{x}{r_j} = 6$

— MOCV PROGRAM
 —○— PARABOLIC PROGRAM



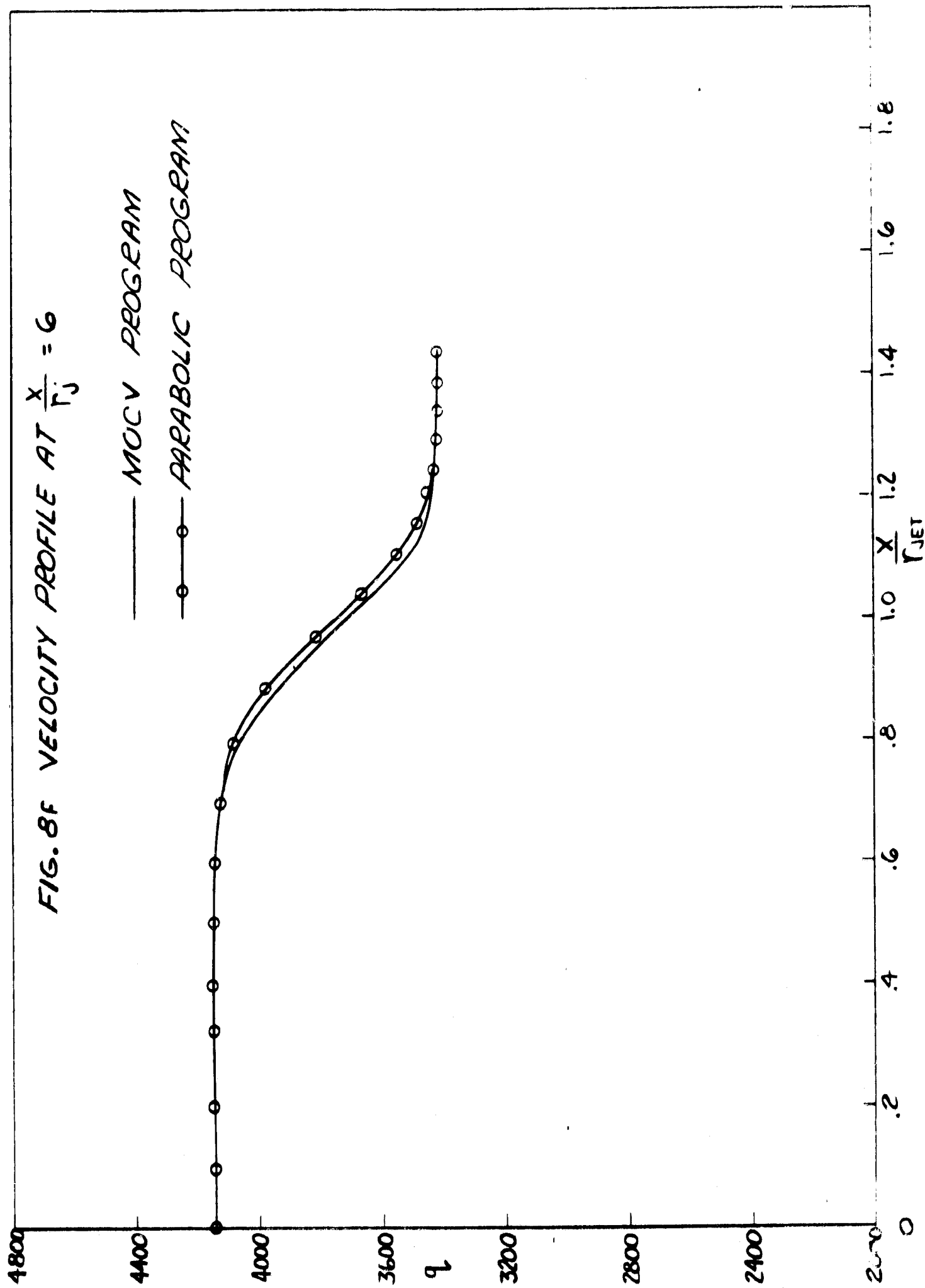
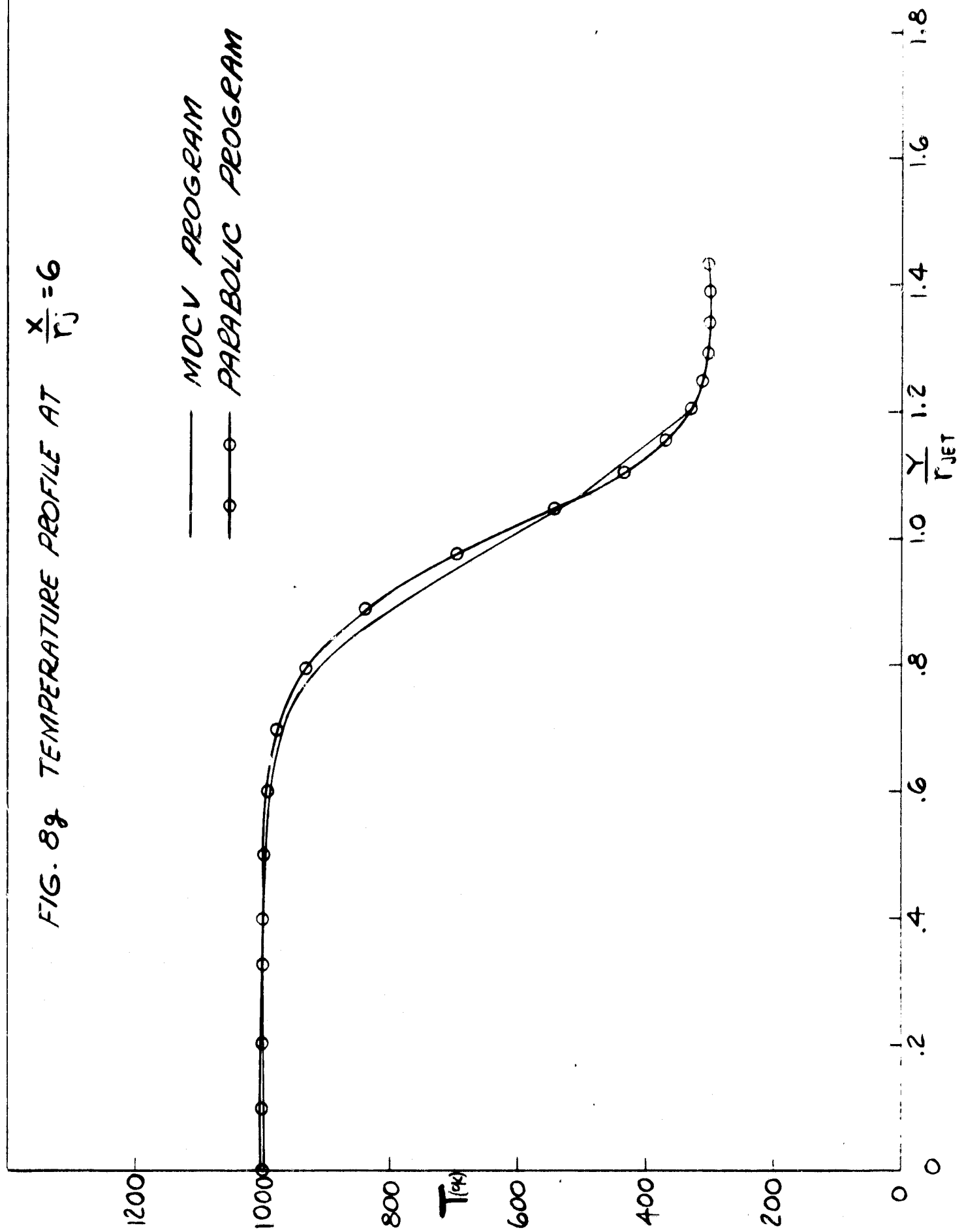
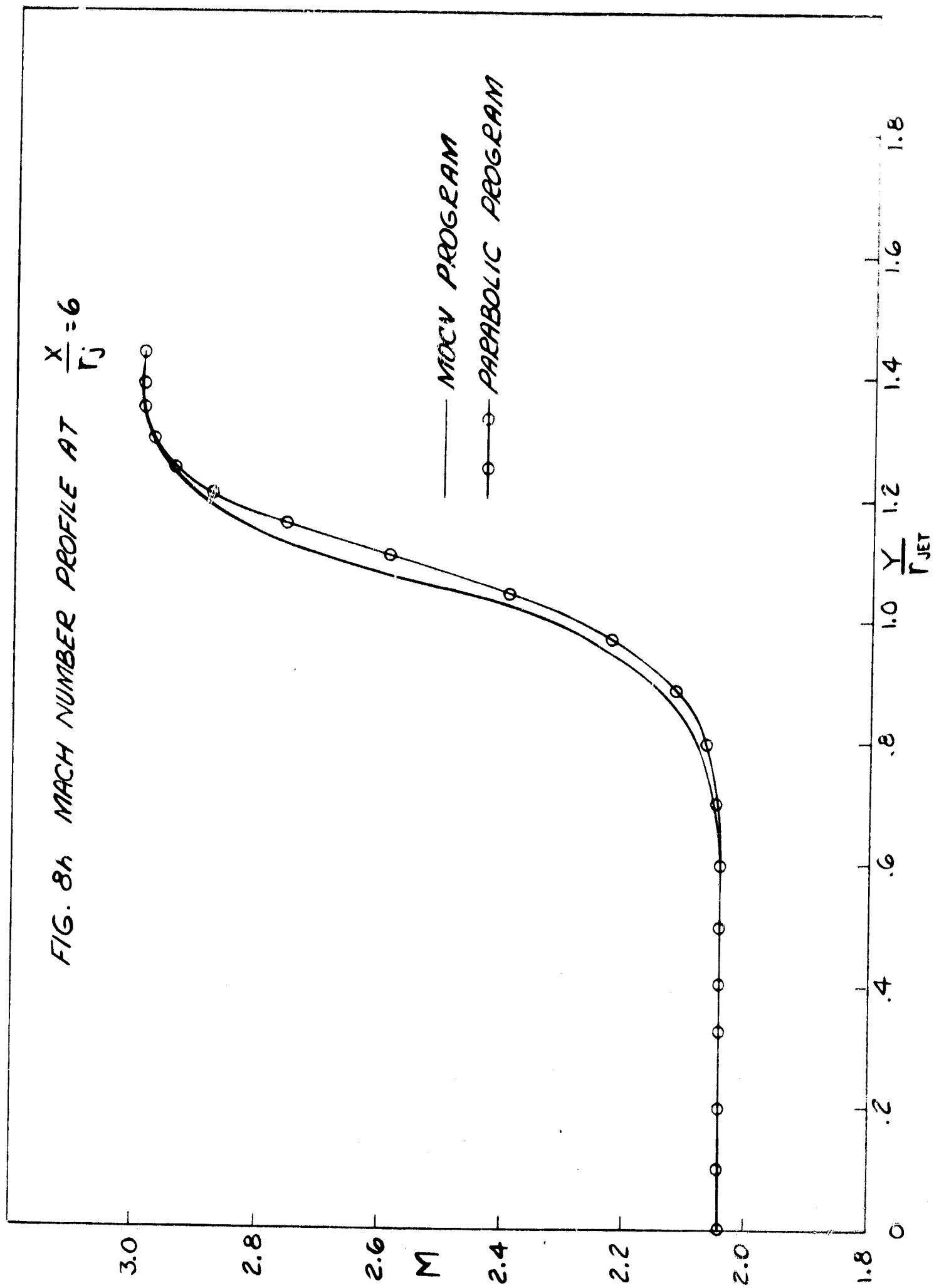
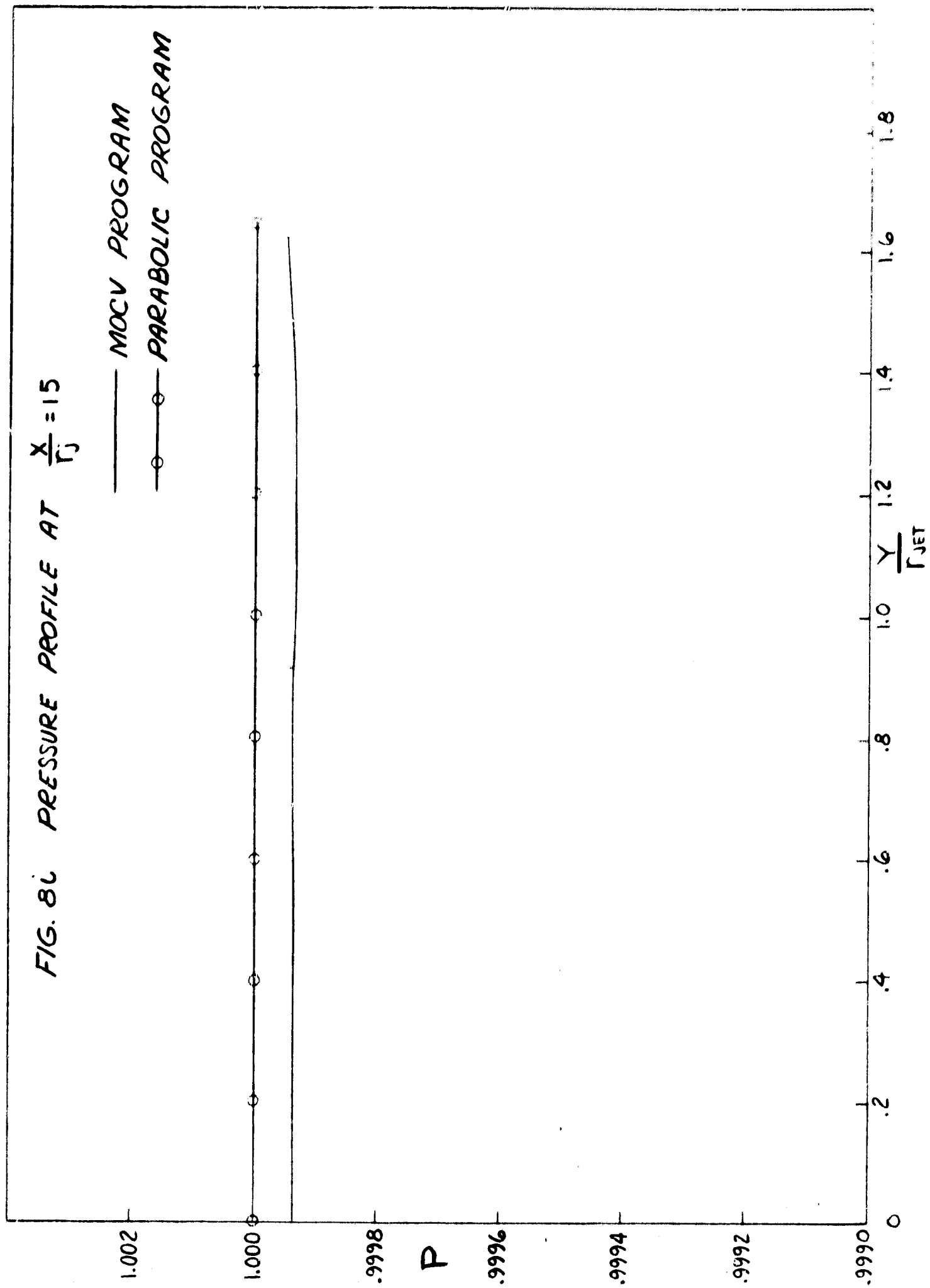


FIG. 8g TEMPERATURE PROFILE AT $\frac{x}{r_j} = 6$







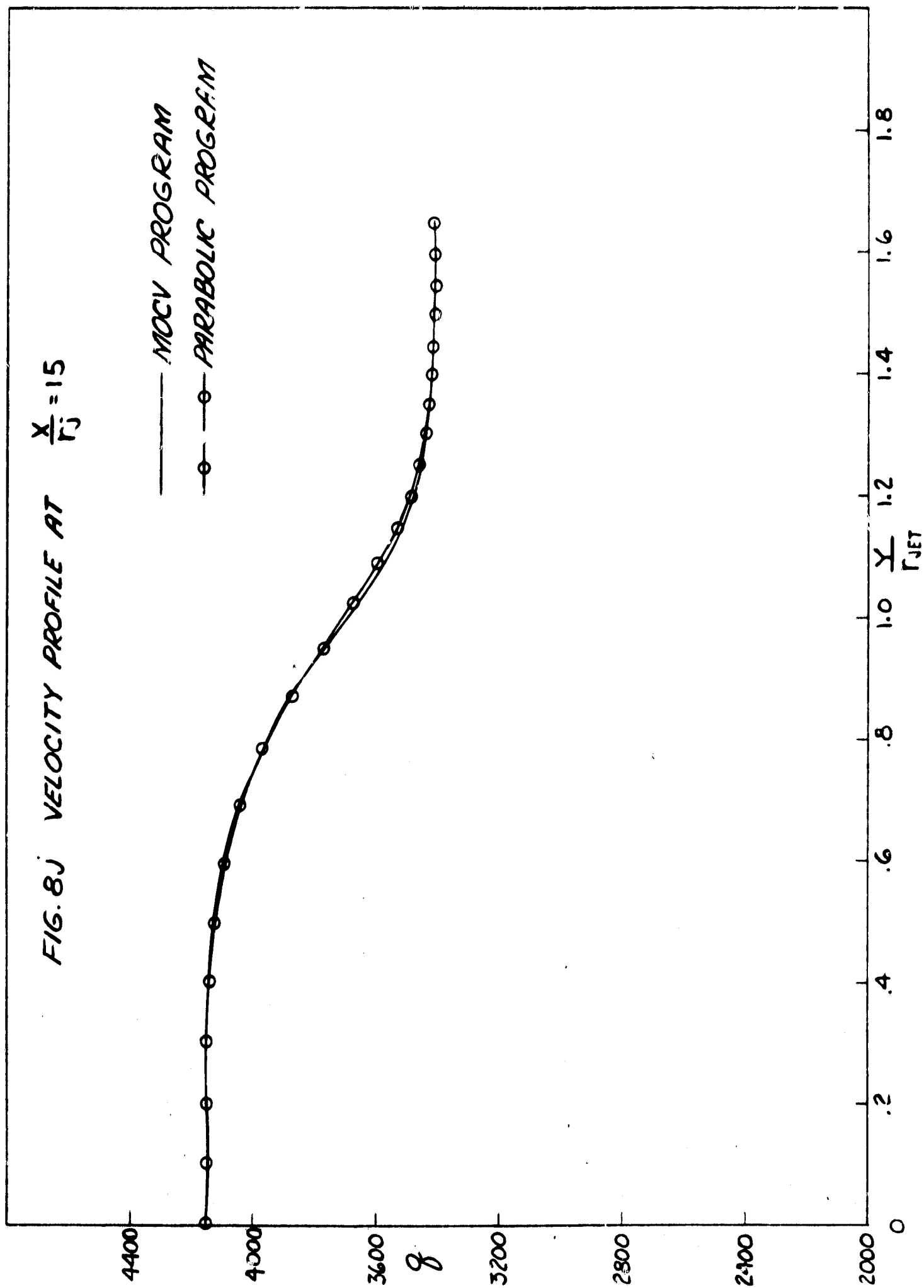
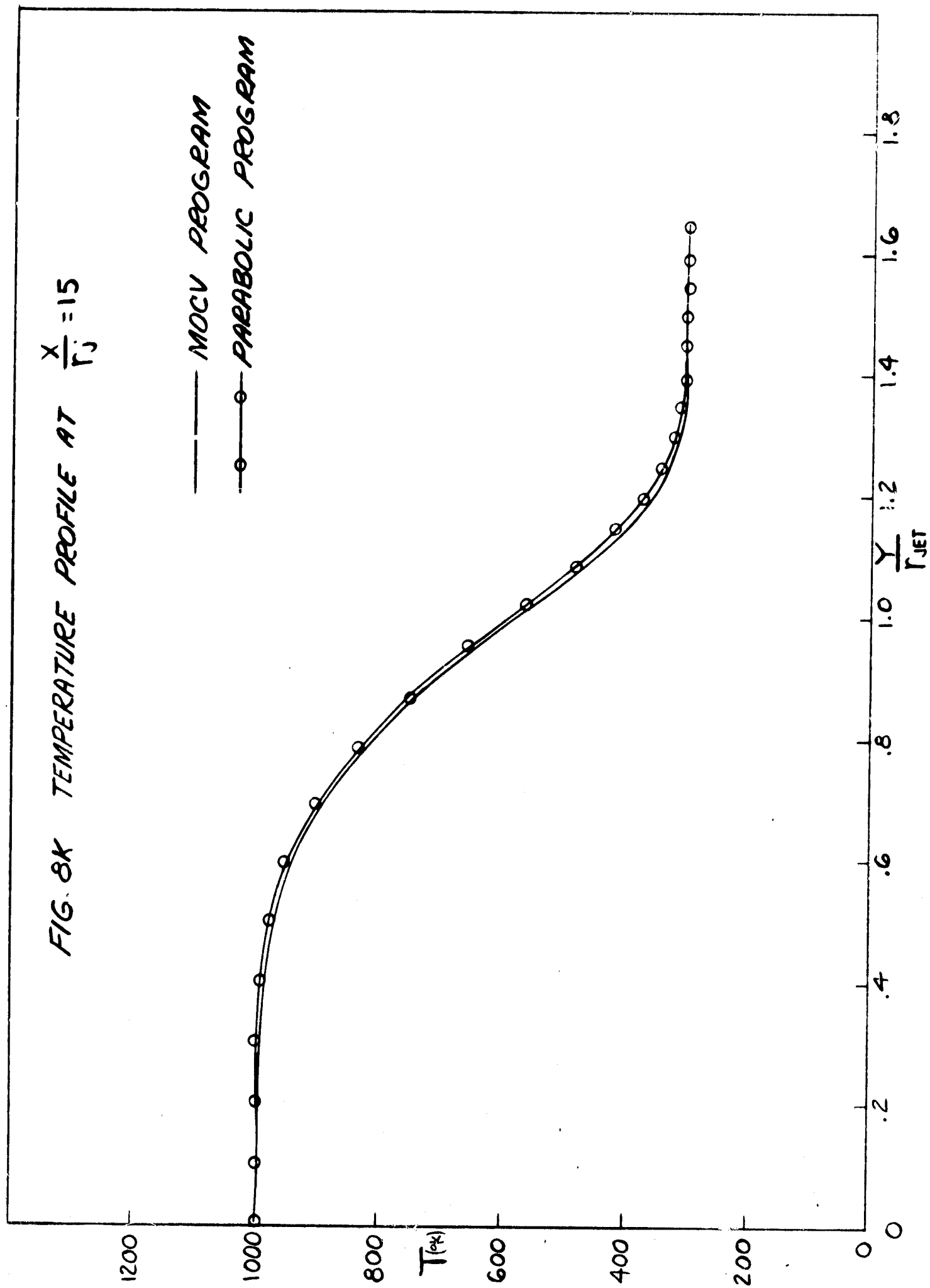


FIG. 8K TEMPERATURE PROFILE AT $\frac{x}{r_j} = 15$



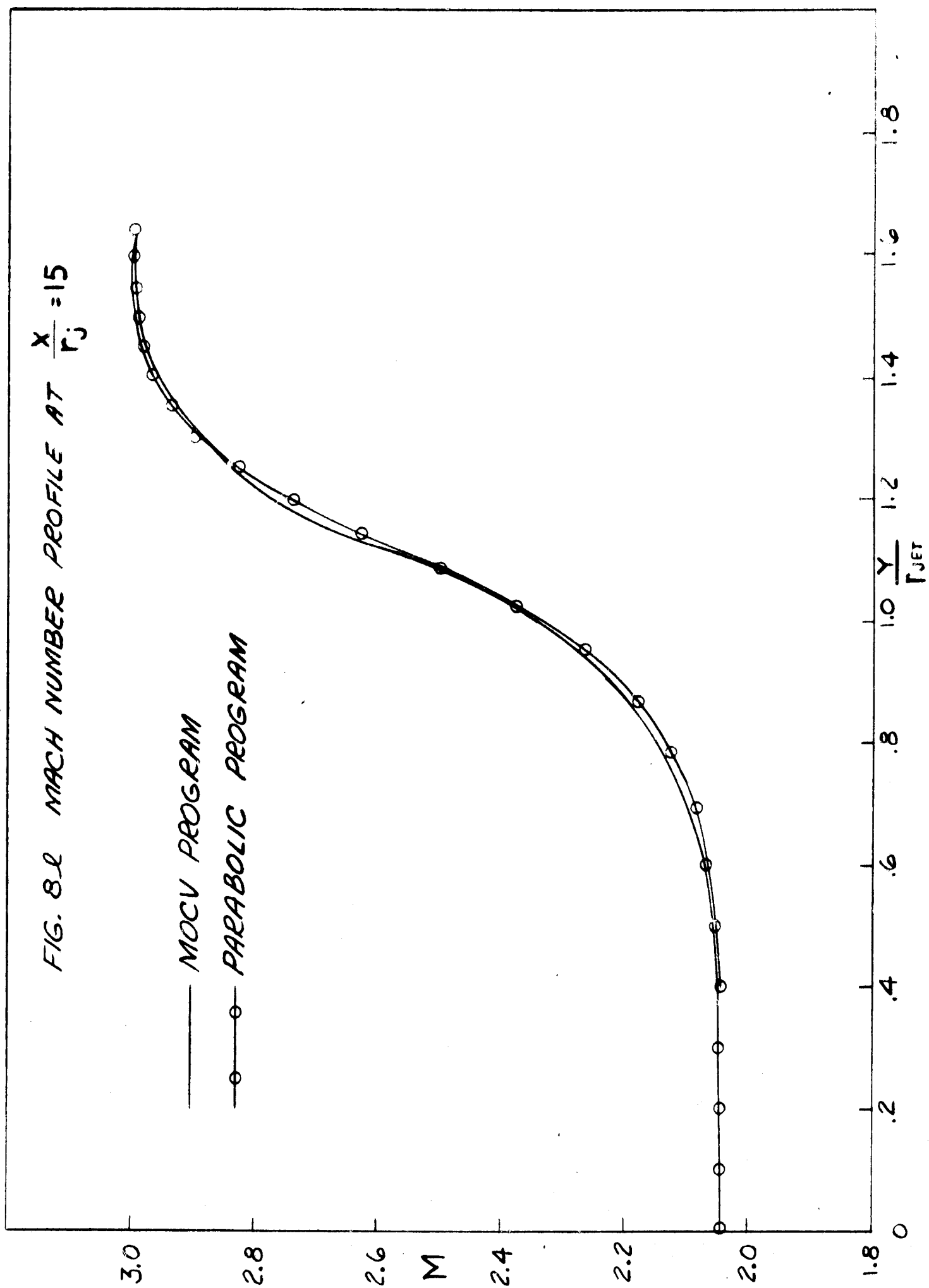
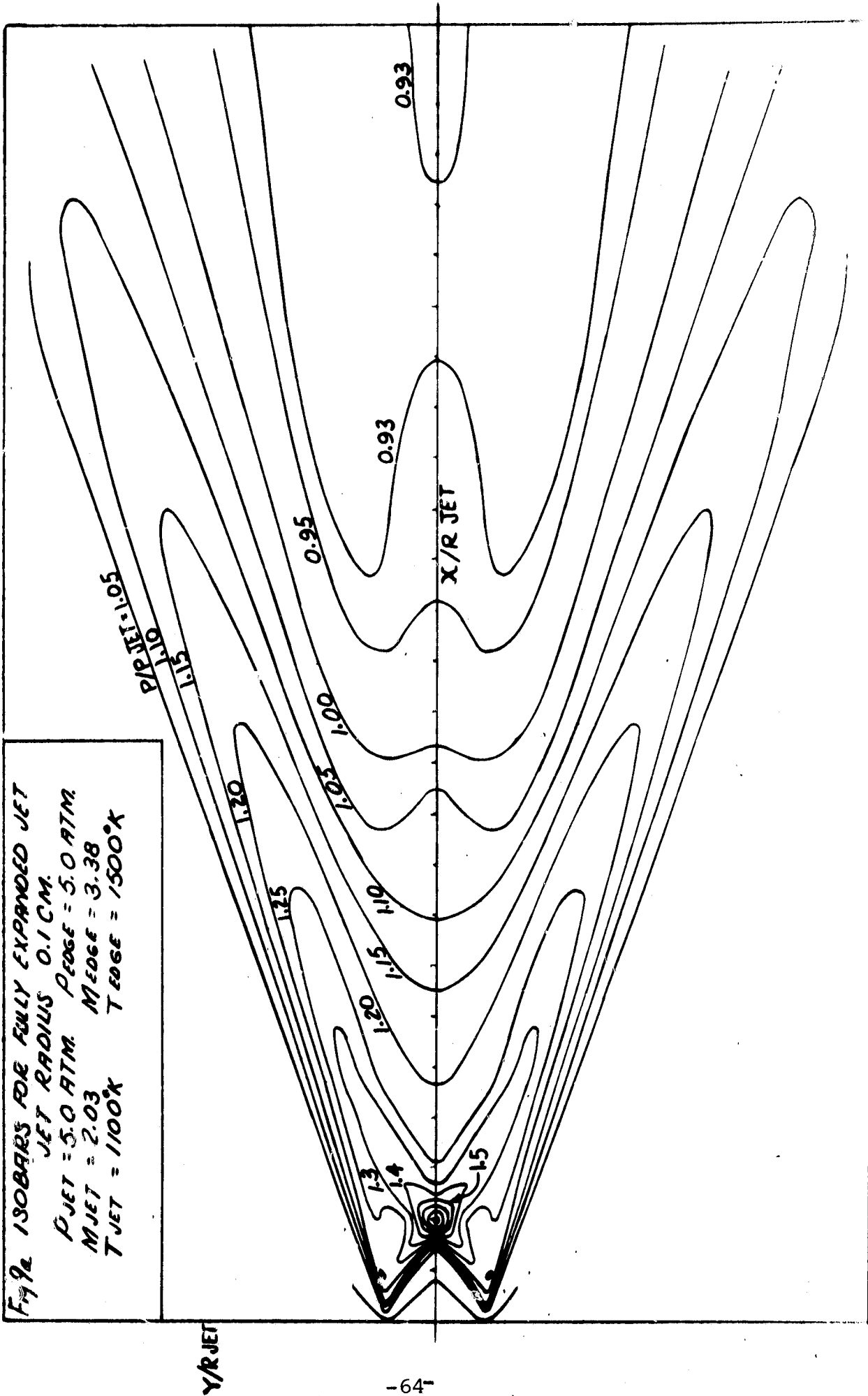
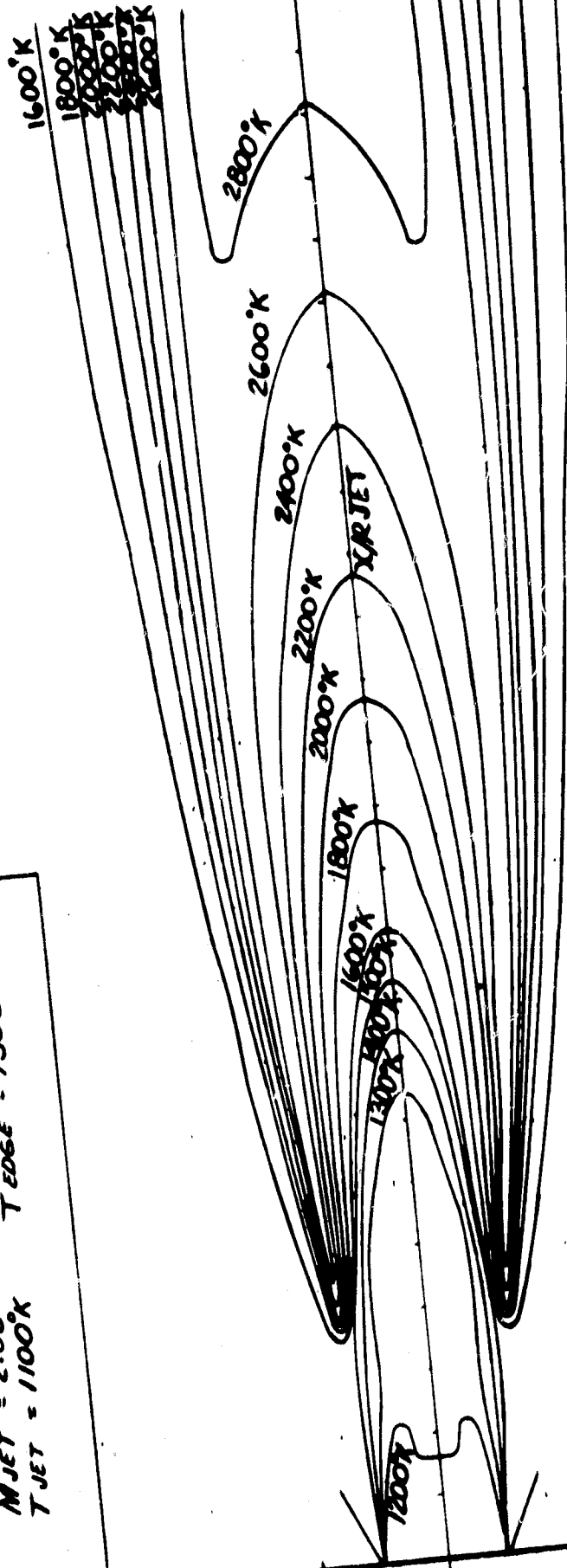


Fig 9a ISOBARS FOR FULLY EXPANDED JET
 JET RADIUS 0.1 CM.
 $P_{JET} = 5.0 \text{ ATM}$ $P_{EDGE} = 5.0 \text{ ATM}$
 $M_{JET} = 2.03$ $M_{EDGE} = 3.38$
 $T_{JET} = 1100^\circ K$ $T_{EDGE} = 1500^\circ K$

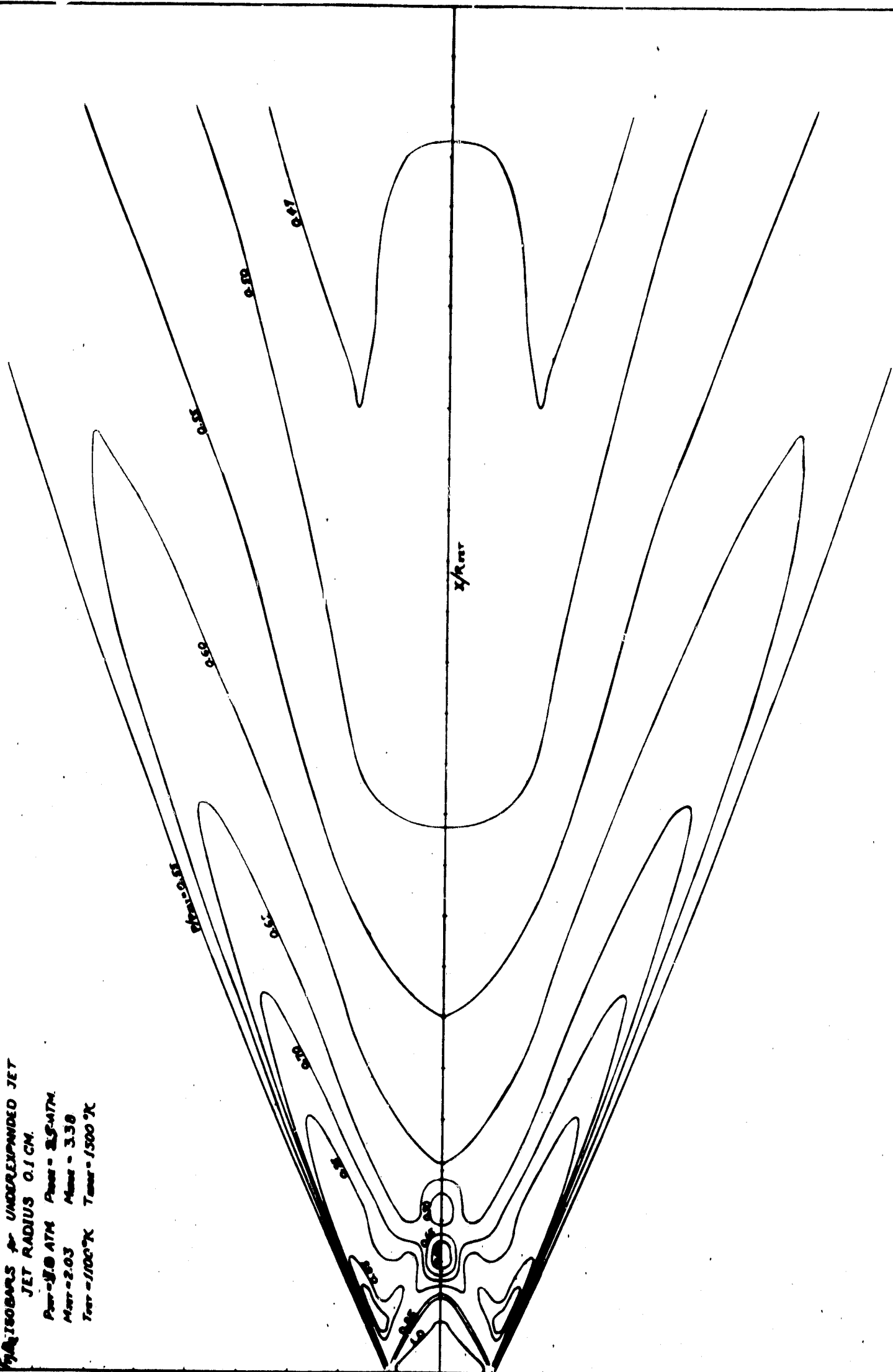


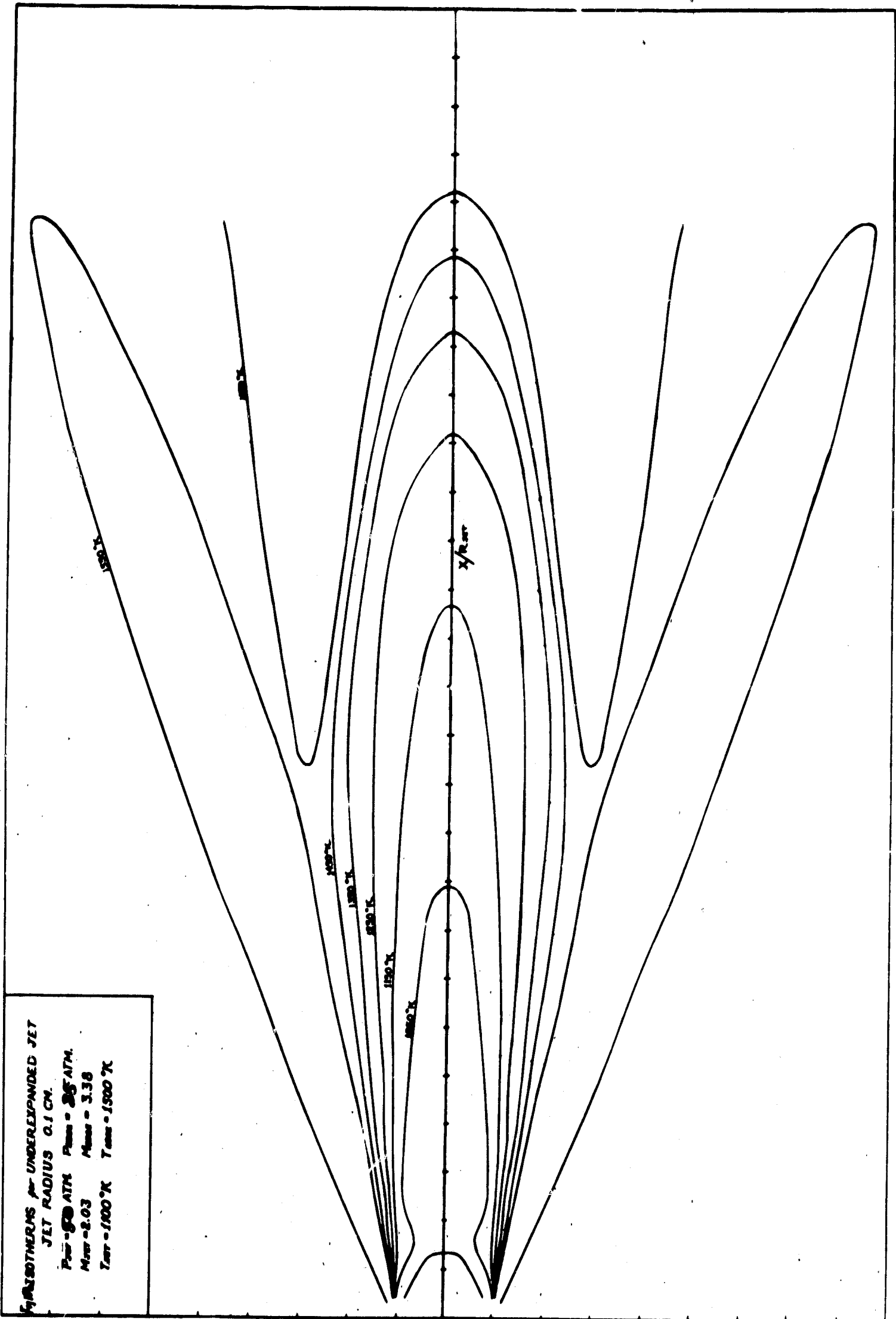
FLOW THERMS FOR FULLY EXPANDED JET
 JET RADIUS 0.1 CM.
 $P_{JET} = 5.0 \text{ ATM.}$ $P_{EDGE} = 5.0 \text{ ATM.}$
 $M_{JET} = 2.03$ $M_{EDGE} = 3.38$
 $T_{JET} = 1100^\circ K$ $T_{EDGE} = 1500^\circ K$



X/RJET

परिचय





ISOTHERMS p^* UNDEREXPANDED JET
 JET RADIUS 0.1 CM.
 $P_{jet} = 50$ ATM $P_{amb} = 25$ ATM.
 $M_{jet} = 2.03$ $M_{amb} = 3.38$
 $T_{jet} = 1000^\circ K$ $T_{amb} = 1500^\circ K$

FIG. 11a PRESSURE MAP FOR UNDEREXPANDED WITH EXTERNAL SHOCK - INVISCID FLOW

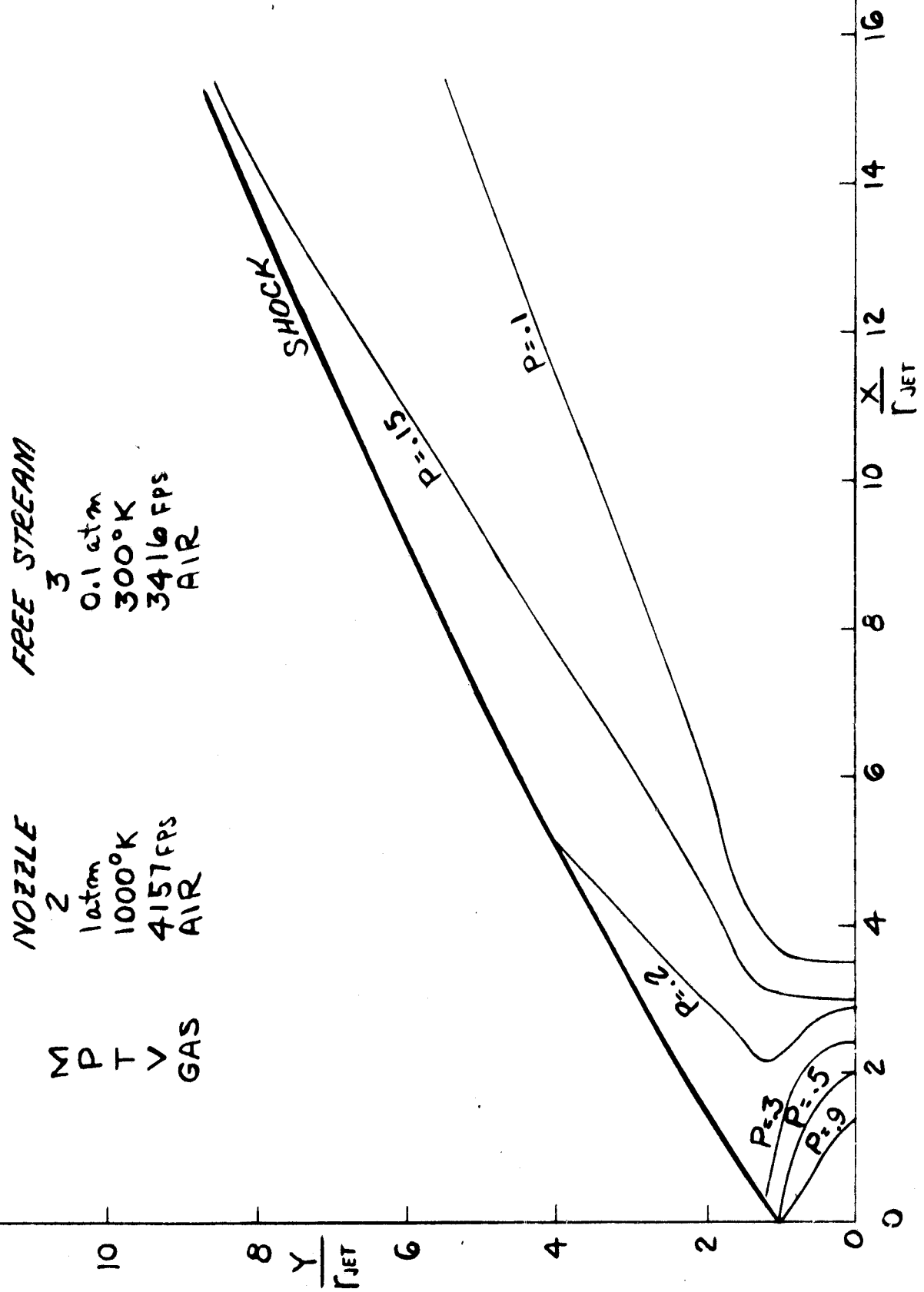


FIG. 116 MACH NUMBER MAP FOR UNDEREXPANDED JET - INVISCID FLOW

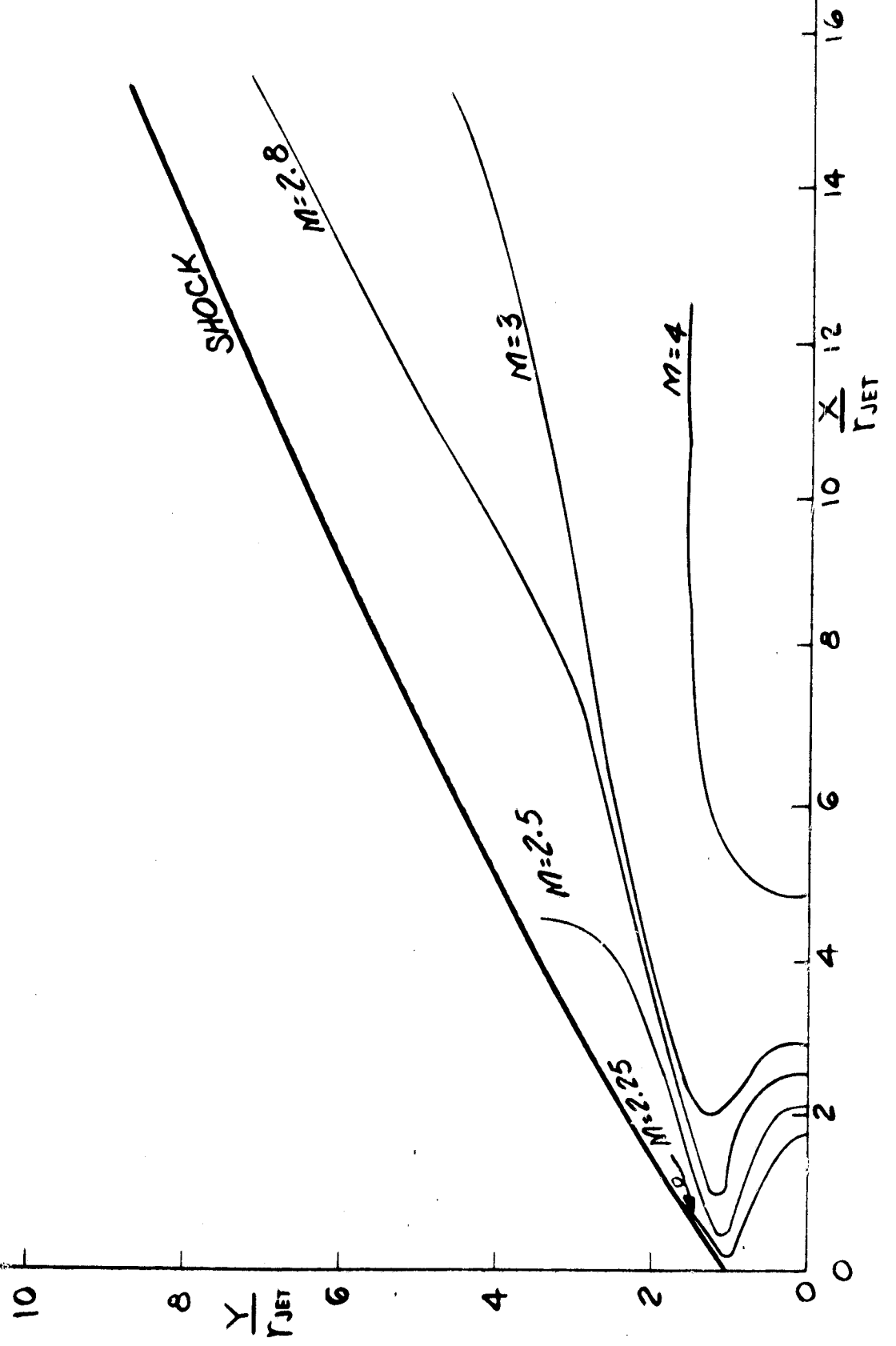


FIG. 11C PRESSURE PROFILES FOR UNDEREXPANDED JET - INVISCID FLOW

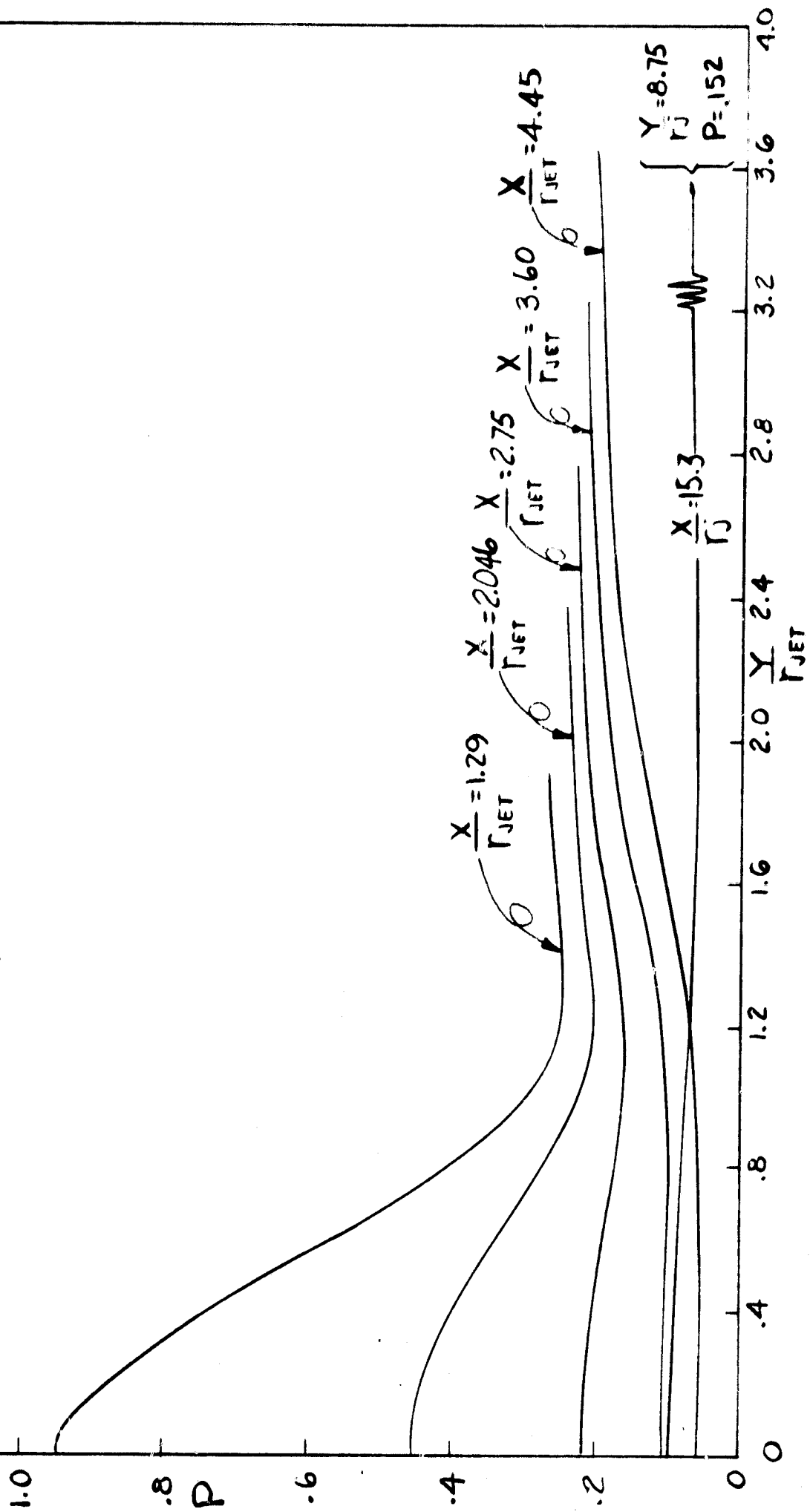


FIG. 11d MACH NUMBER PROFILES FOR UNDEREXPANDED JET - INVISCID FLOW

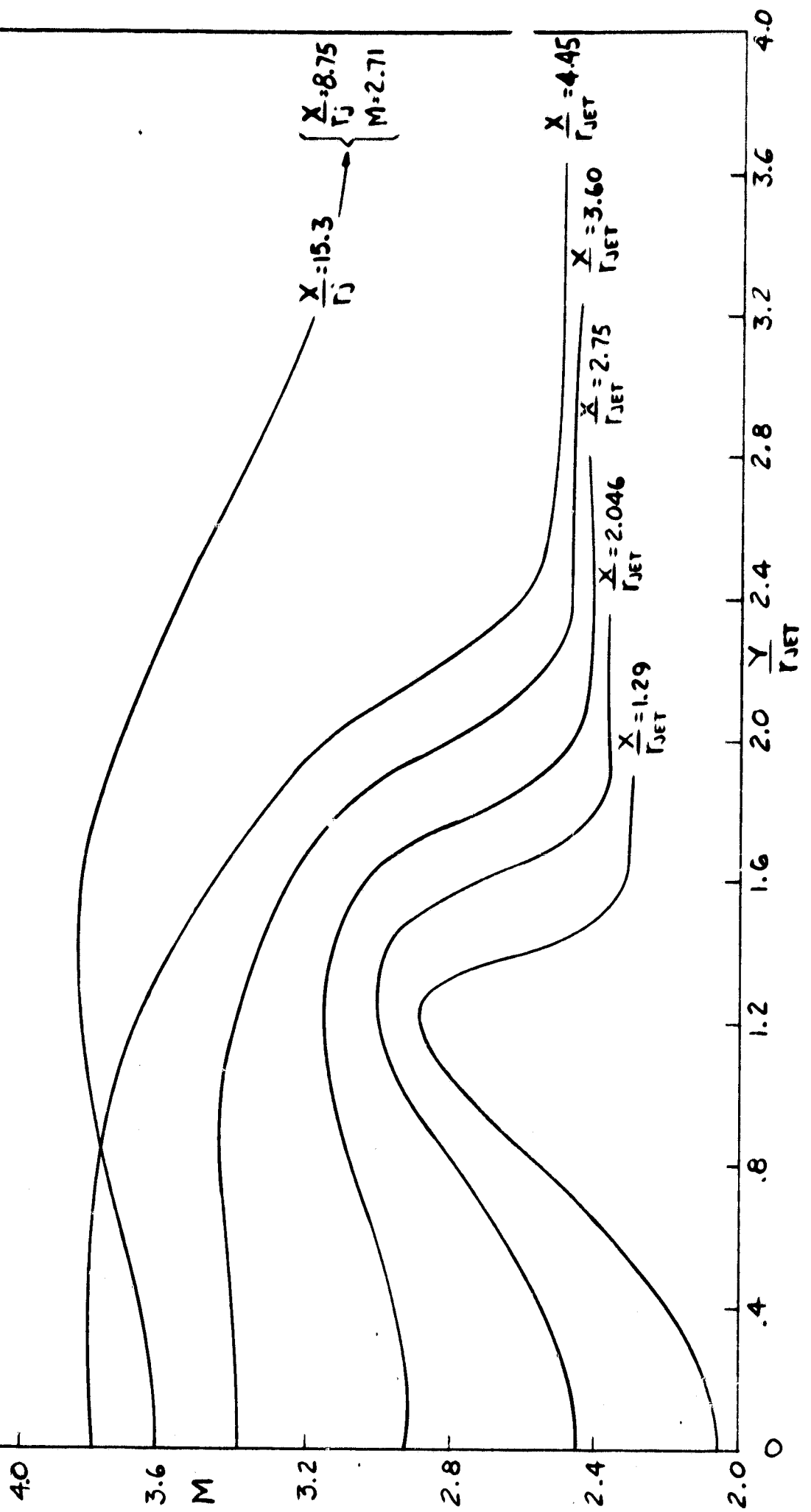


FIG. 12a COMPARISON OF SHOCK AND STREAMLINE SHAPES FOR
VARIOUS VISCOSITY LEVELS

NOZZLE 2 FREESTREAM 3

M	2	3
P	1 atm	0.1 atm
T	1000°K	300°K
V	4157 FPS	3416 FPS
GAS	AIR	AIR

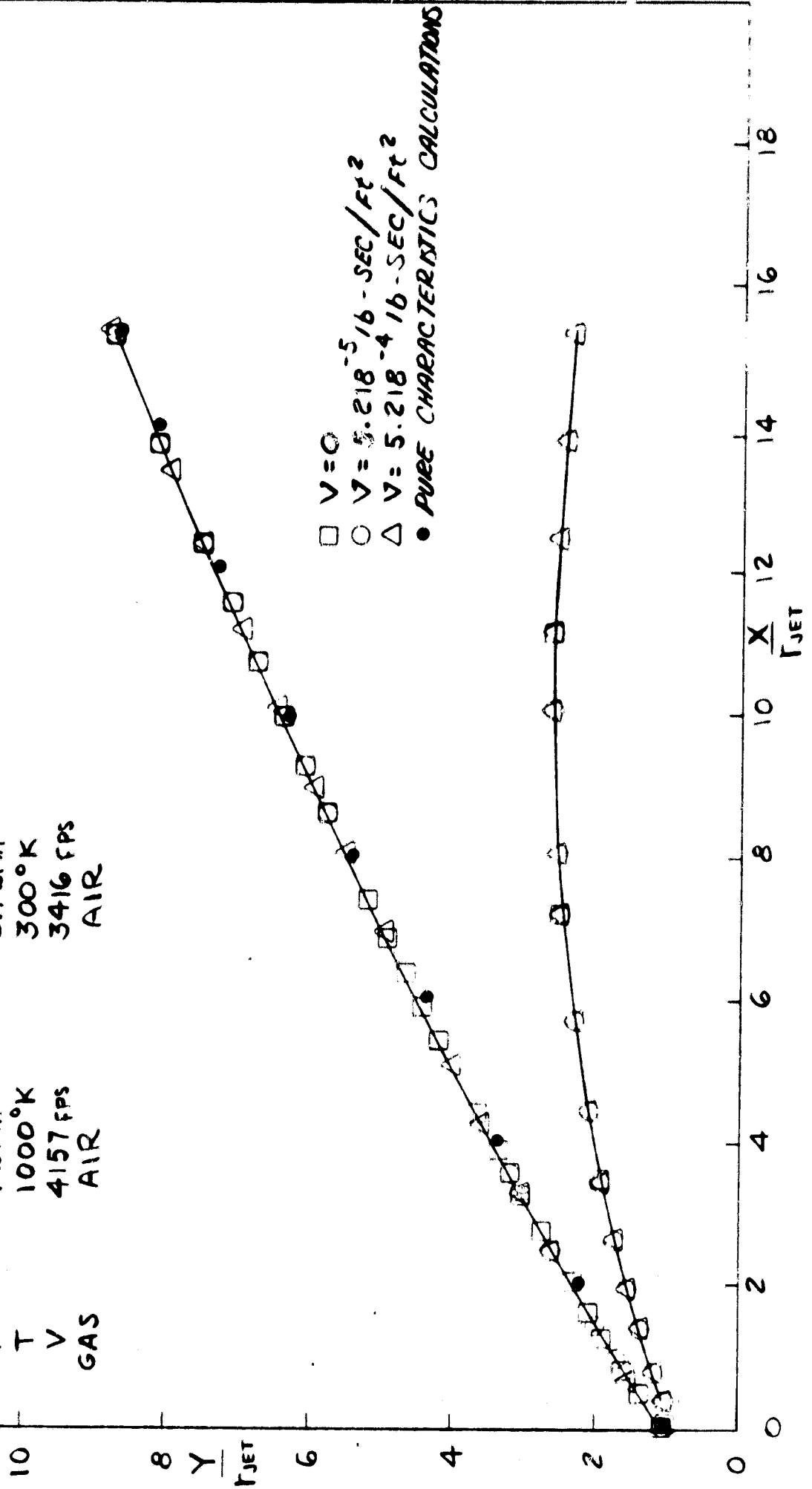


FIG. 126 EFFECT OF VISCOSITY LEVEL ON MACH NUMBER PROFILE

- $\square V=0 \quad \frac{X}{r_{JET}} = 15.25$
 $\circ V=5.218^{-5} \quad \frac{X}{r_{JET}} = 15.25$
 $\triangle V=5.218^{-4} \quad \frac{X}{r_{JET}} = 15.32$

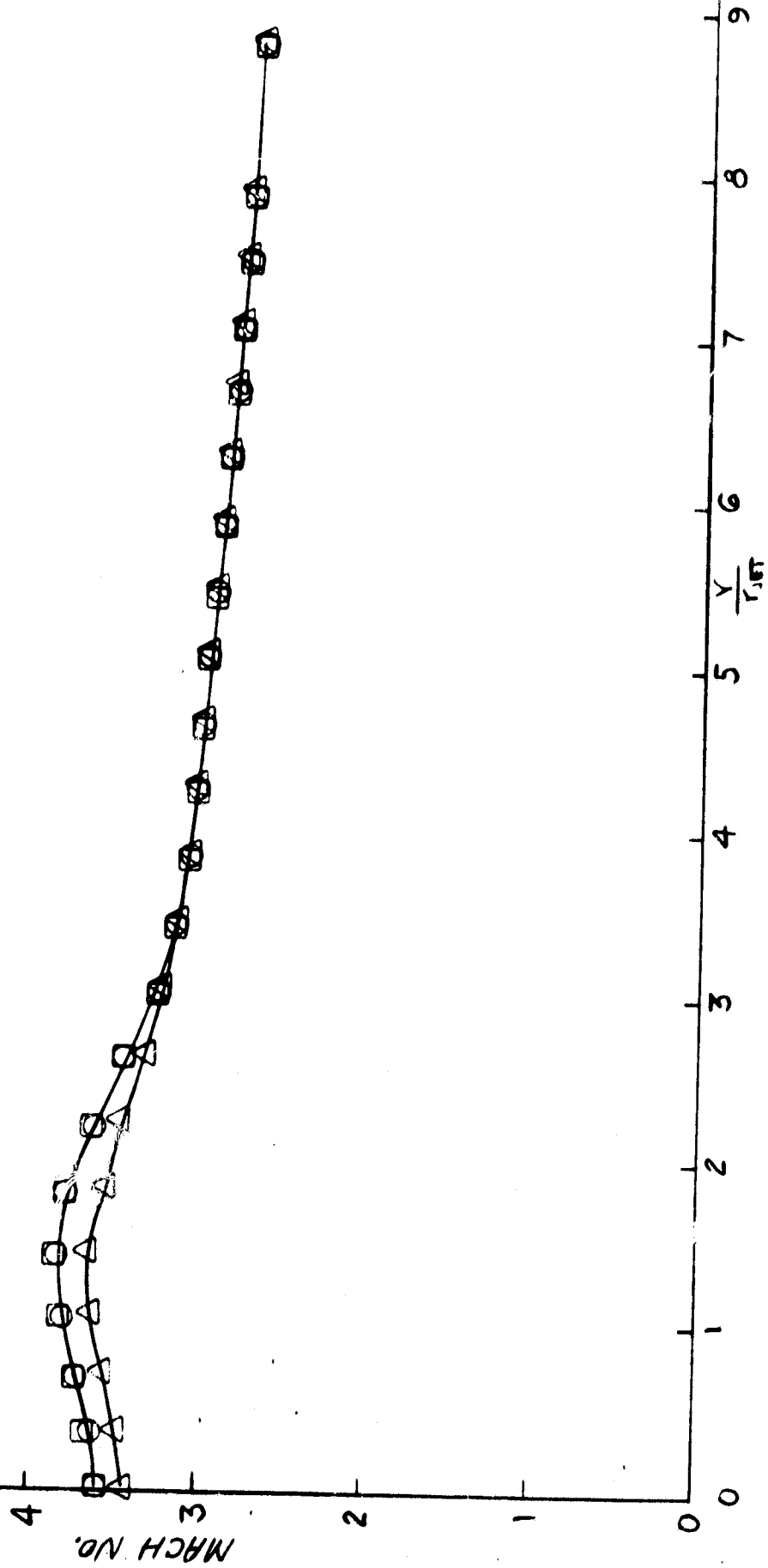
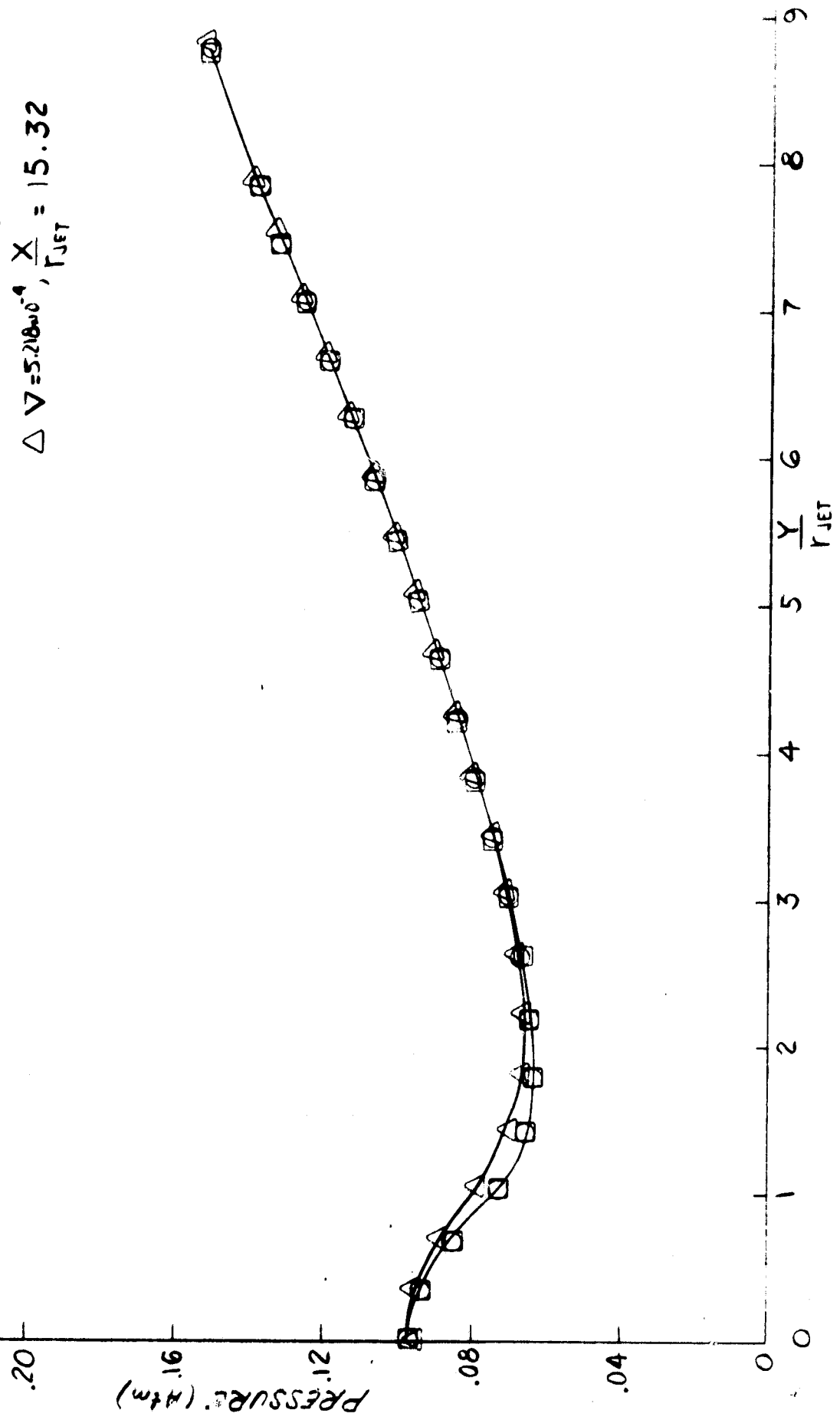
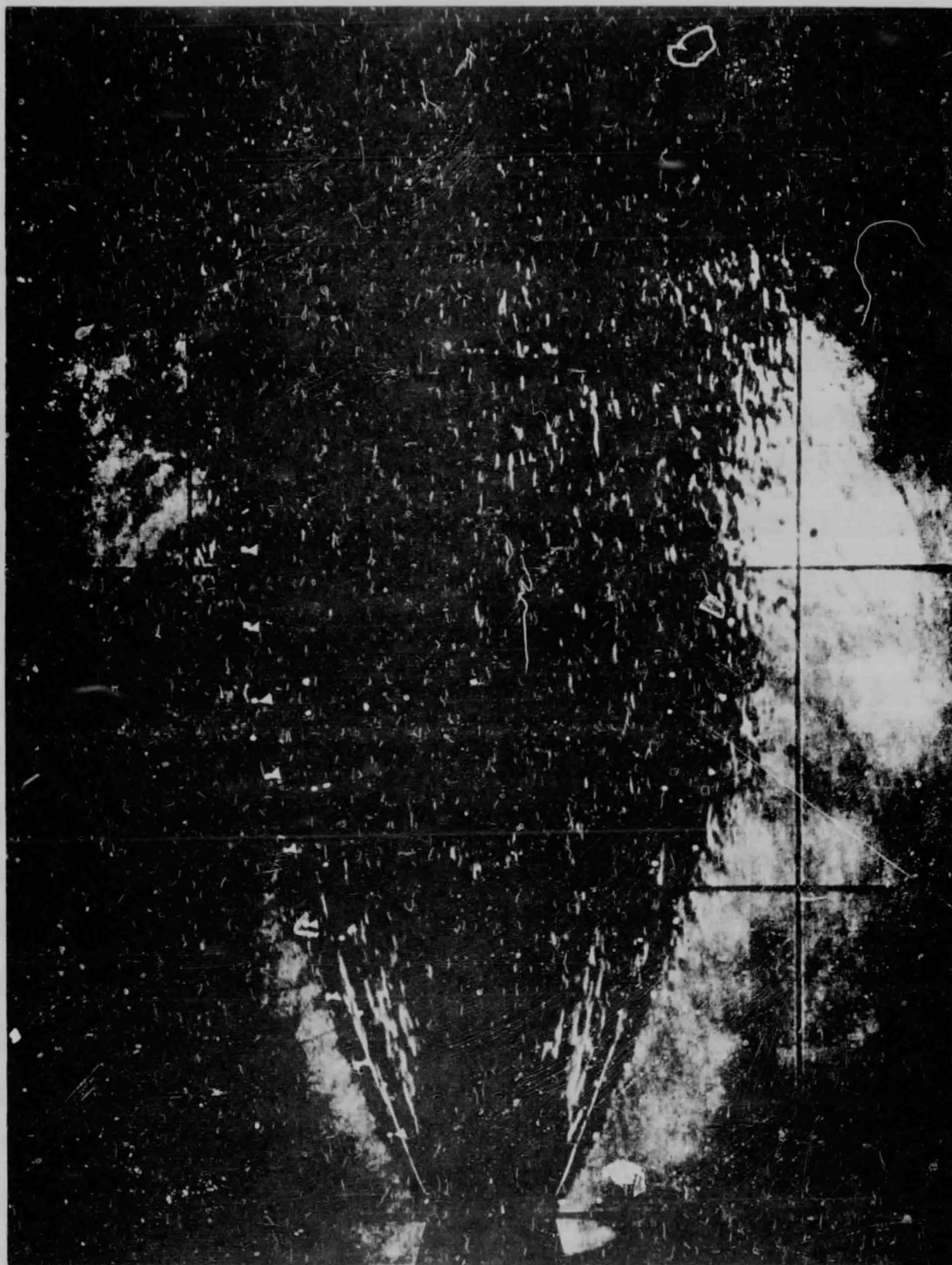


FIG. 12c EFFECT OF VISCOSITY LEVEL ON PRESSURE PROFILE

- $\square V=0, \frac{X}{r_{JET}} = 15.25$
 $\circ V=5.218 \times 10^{-5}, \frac{X}{r_{JET}} = 15.25$
 $\triangle V=5.218 \times 10^{-4}, \frac{X}{r_{JET}} = 15.32$





$\Delta MOCV$

(a) $M_j = 2.22$; $\theta_N = 0^\circ$; $p_j/p_\infty = 14.4$.

FIGURE 13 - Schlieren photographs of jet exhaust plumes with superimposed theoretical boundaries and internal shock



$\Delta MOCV$

(b) $M_j = 2.22$; $\theta_N = 0^\circ$; $P_j/P_\infty = 37.7$.

FIGURE 14



Δ MOCV

(h) $M_J = 1.0$; $\theta_M = 0^\circ$; $P_J/P_\infty = 194.2$.

Figure 15

FIG. 16 UNDEREXPANDED DUCTED JET - INVISCID FLOW

AXIAL PRESSURE DISTRIBUTION			
PRIMARY		SECONDARY	
M	2.03	M	3.37
P	5 atm	P	2.5 atm
T	1100°K	T	1500°K
V	16,639 FPS	V	8,333 FPS
GAS	H ₂	GAS	AIR
r _{JET}	0.1 CM		
r _{WALL}	0.376 CM		

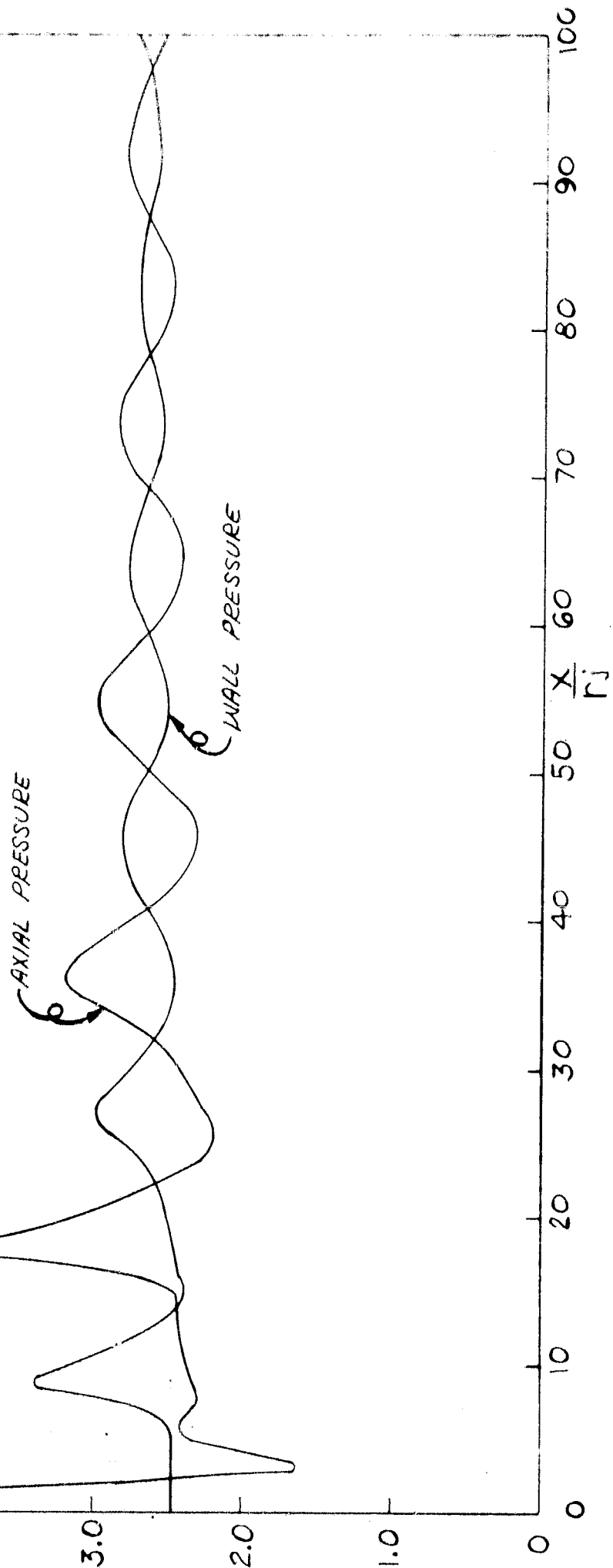


FIG. 17 FULLY EXPANDED DUCTED JET - VISCOUS FLOW
(a) WALL AND CENTERLINE PRESSURE DISTRIBUTION

UNIFORM PRESSURE AT $\frac{x}{r_{JET}} = 0$
CONSTANT VISCOSITY ($4.49 \cdot 10^{-4} \frac{\text{g}}{\text{cm} \cdot \text{sec}}$)
WALL AT $\frac{x}{r_{JET}} = 3.76, \phi = 1$

	PRIMARY	SECONDARY
M	2.03	3.37
P	5 atm	2.5 atm
T	1100°K	1500°K
V	16,639 FPS	8,333 FPS
GAS	H ₂	AIR
r_{JET}	0.1 CM	
r_{WALL}	0.376 CM	

CENTERLINE PRESSURE

WALL PRESSURE

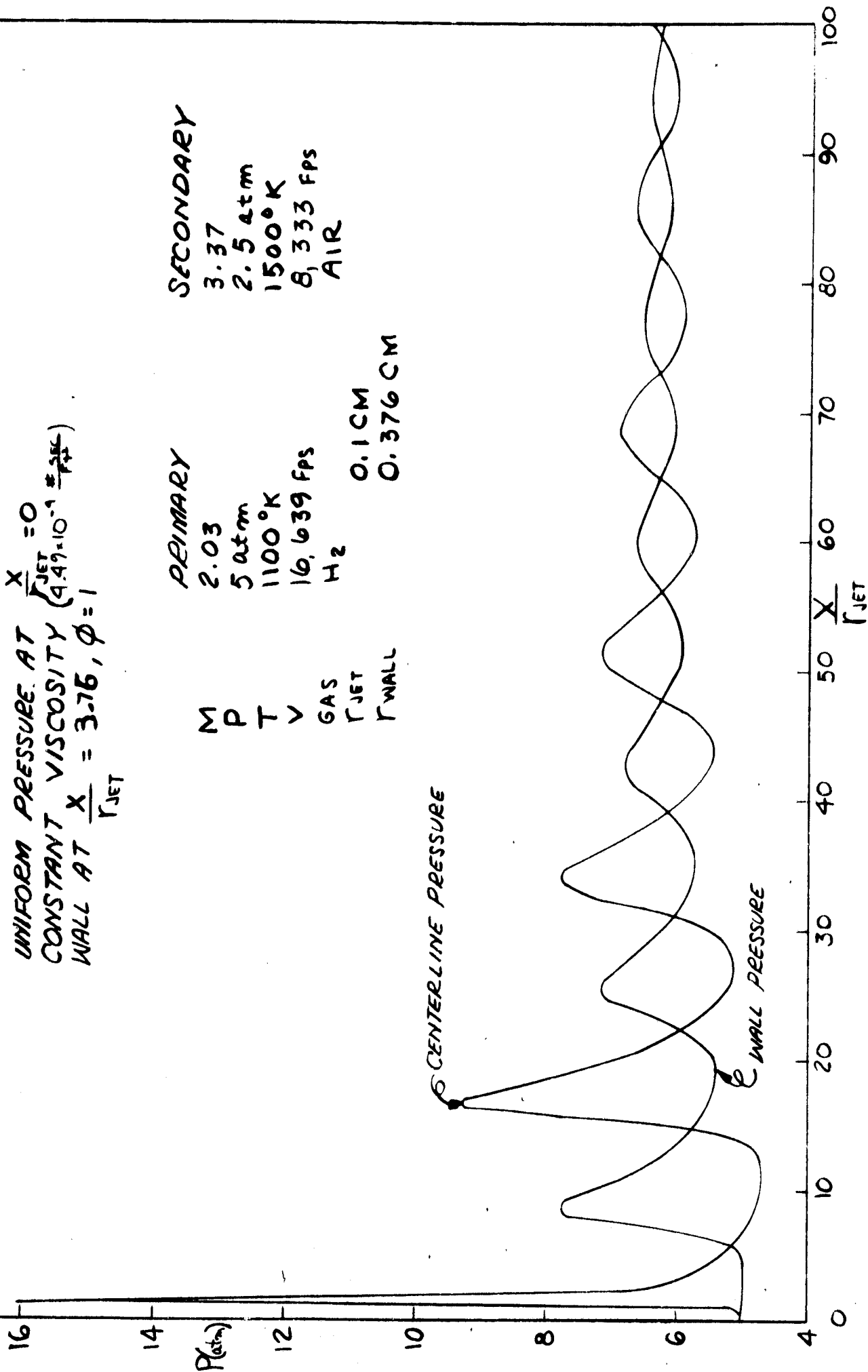


FIG. 176 AXIAL DECAY OF THE HYDEOGEN MASS FRACTION

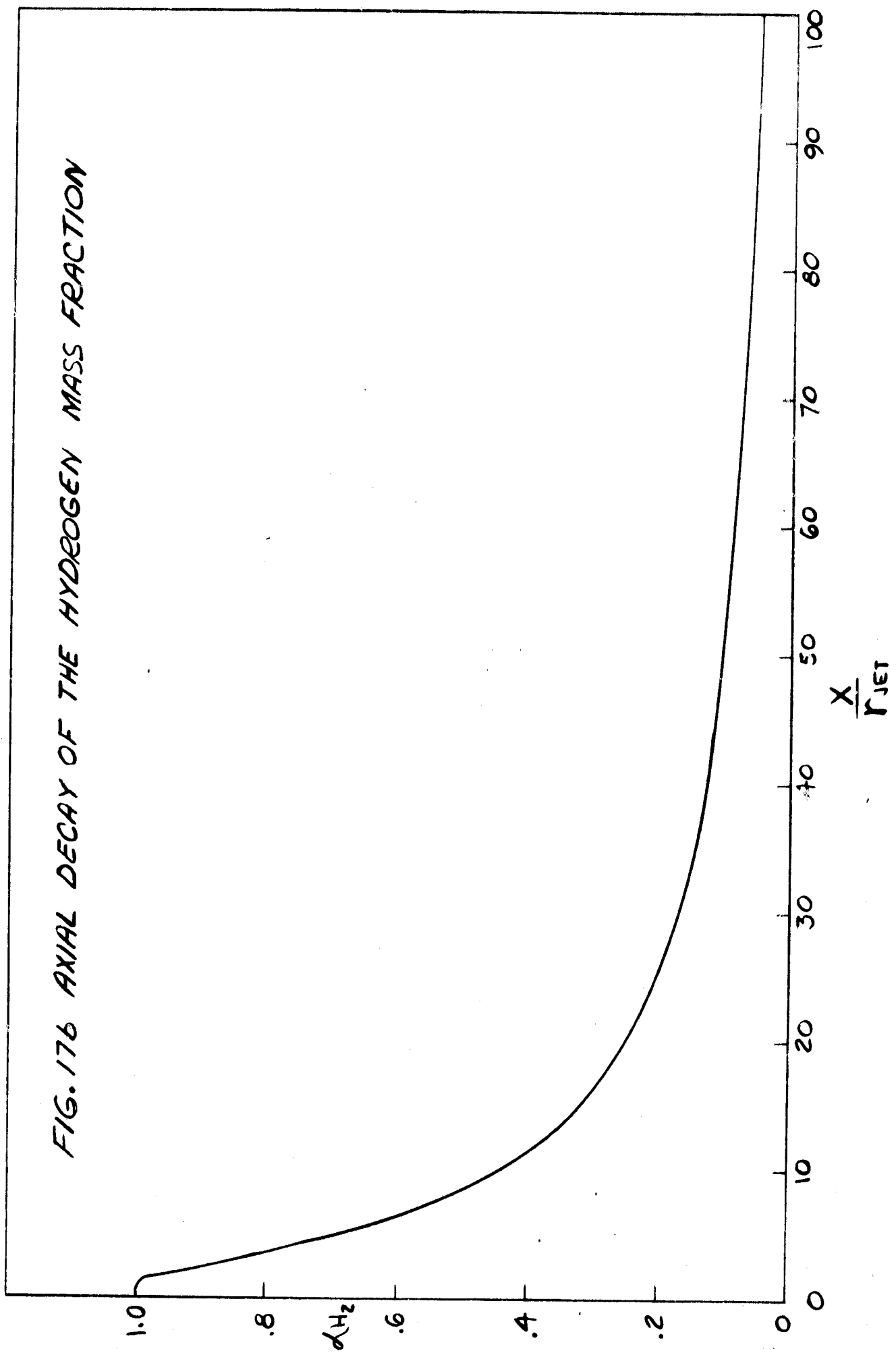


FIG. 17C PRESSURE PROFILE AT $\frac{X}{r_j} = 30$

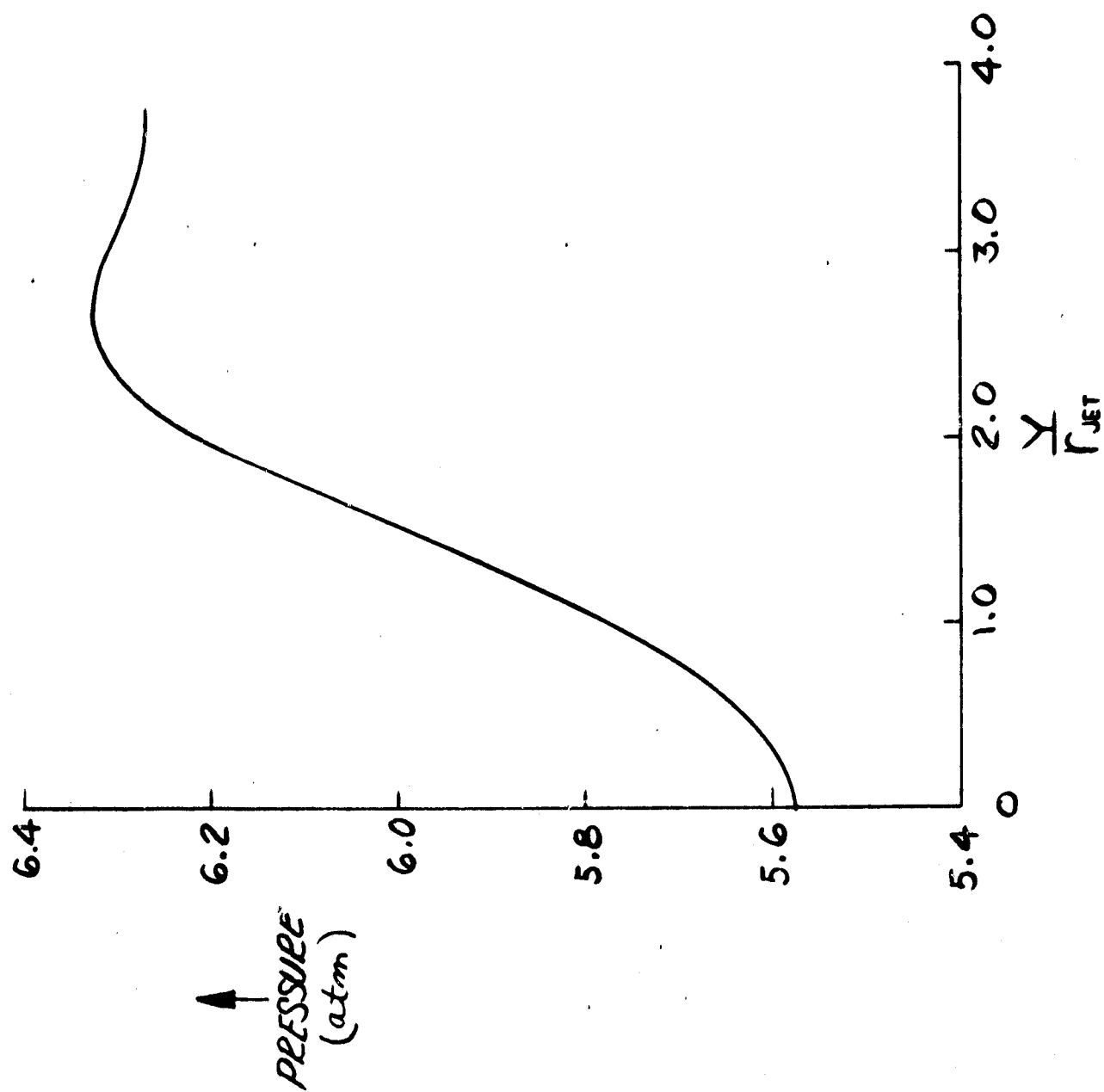


FIG. 17d VELOCITY PROFILE AT $\frac{X}{r_j} = 30$

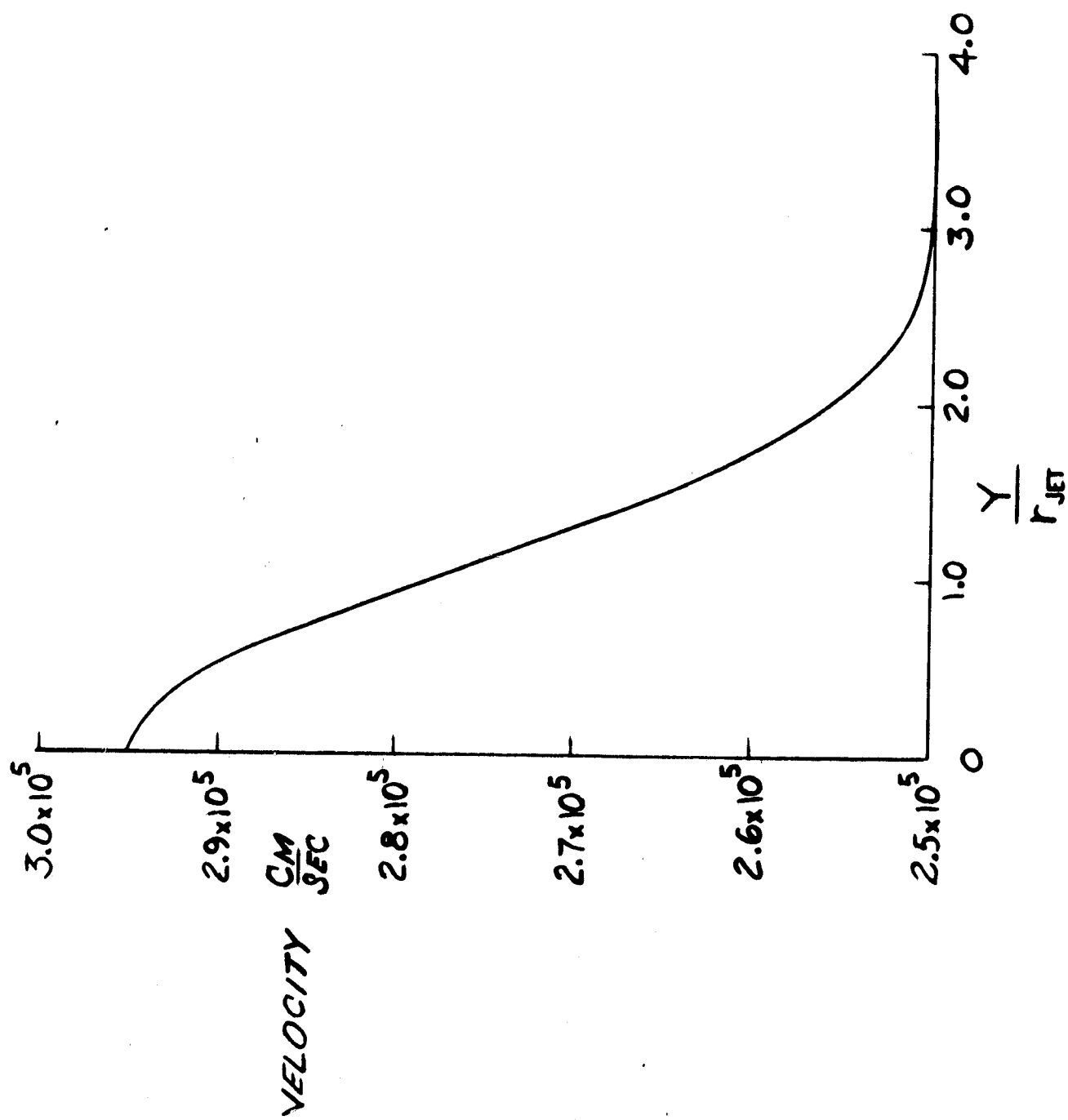
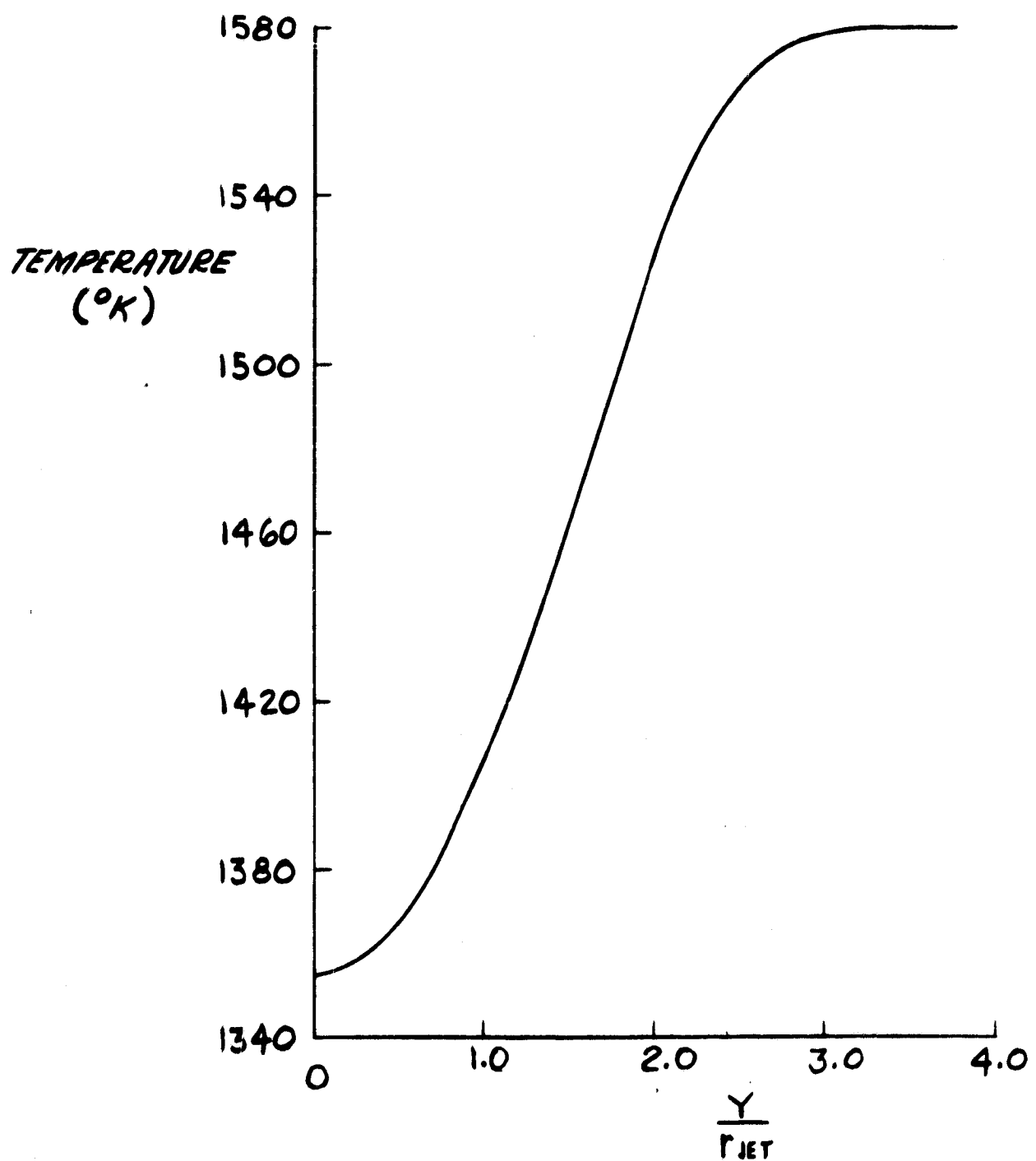


FIG. 17e TEMPERATURE PROFILE AT $\frac{x}{L} = 30$



$$\frac{X}{r_j} = 30$$

FIG. 17F HYDROGEN MASS FRACTION AT

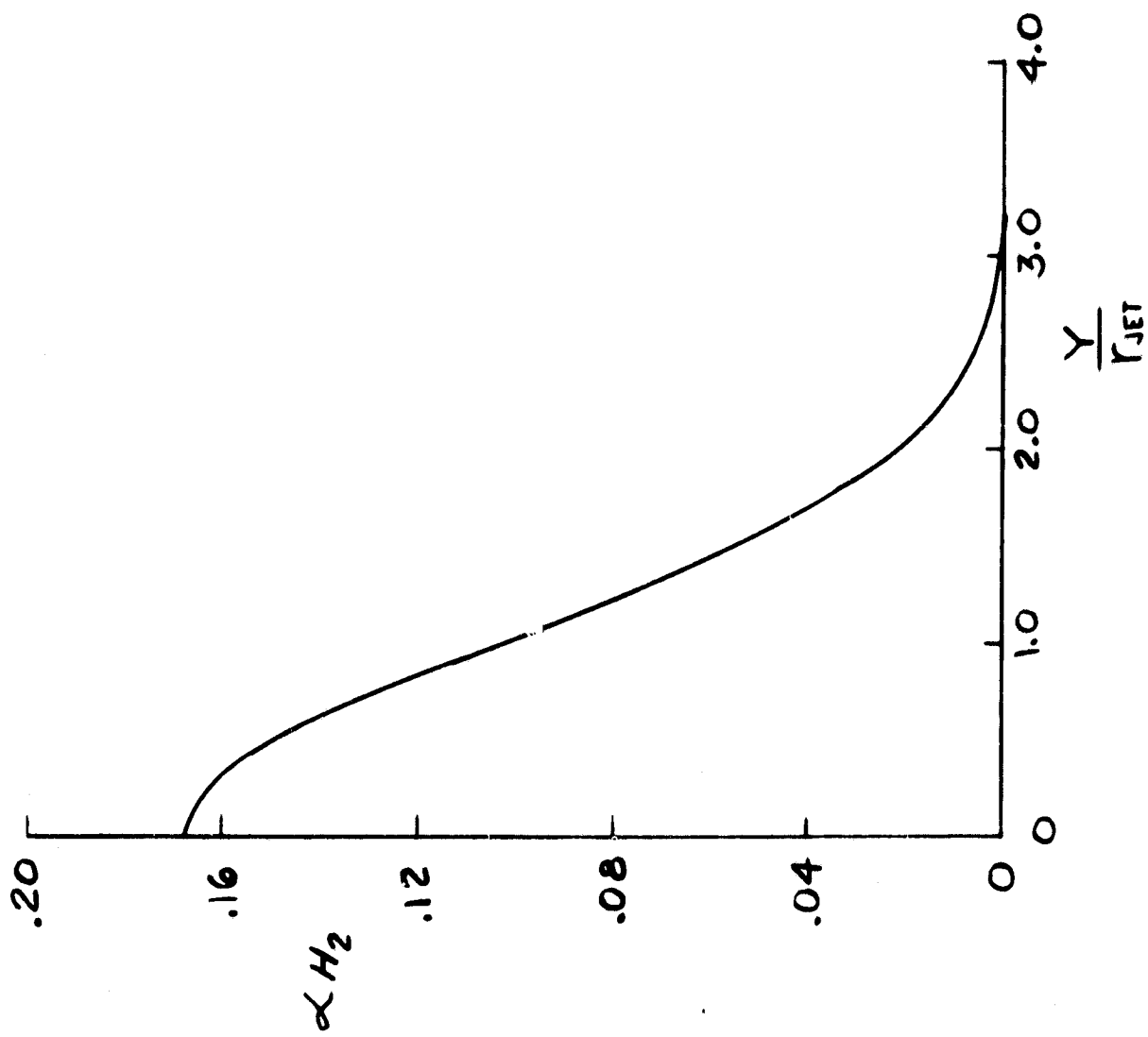


FIG. 17g PRESSURE PROFILE AT $\frac{X}{r_j} = 100$

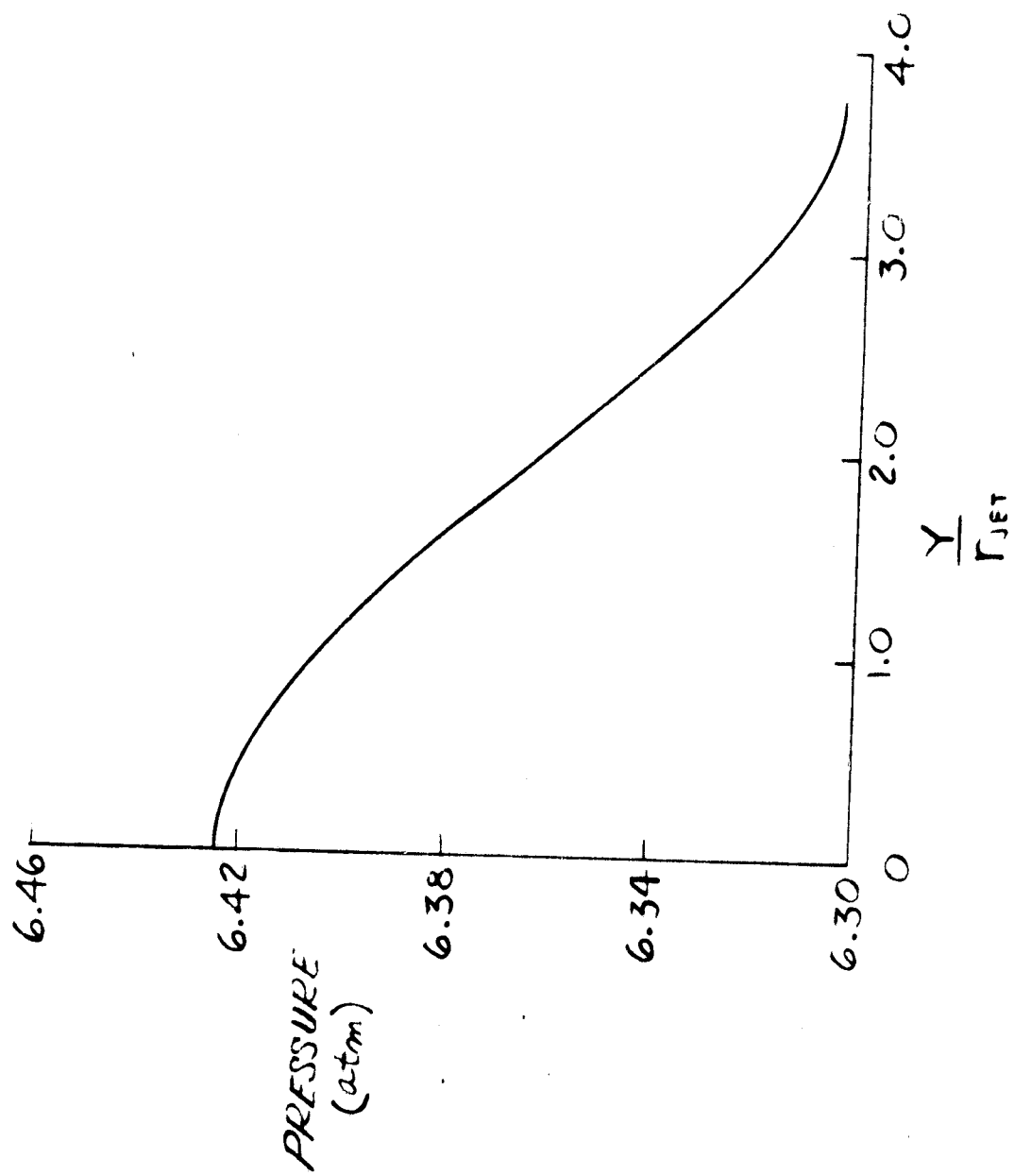


FIG. 17h VELOCITY PROFILE AT $\frac{X}{r_j} = 100$

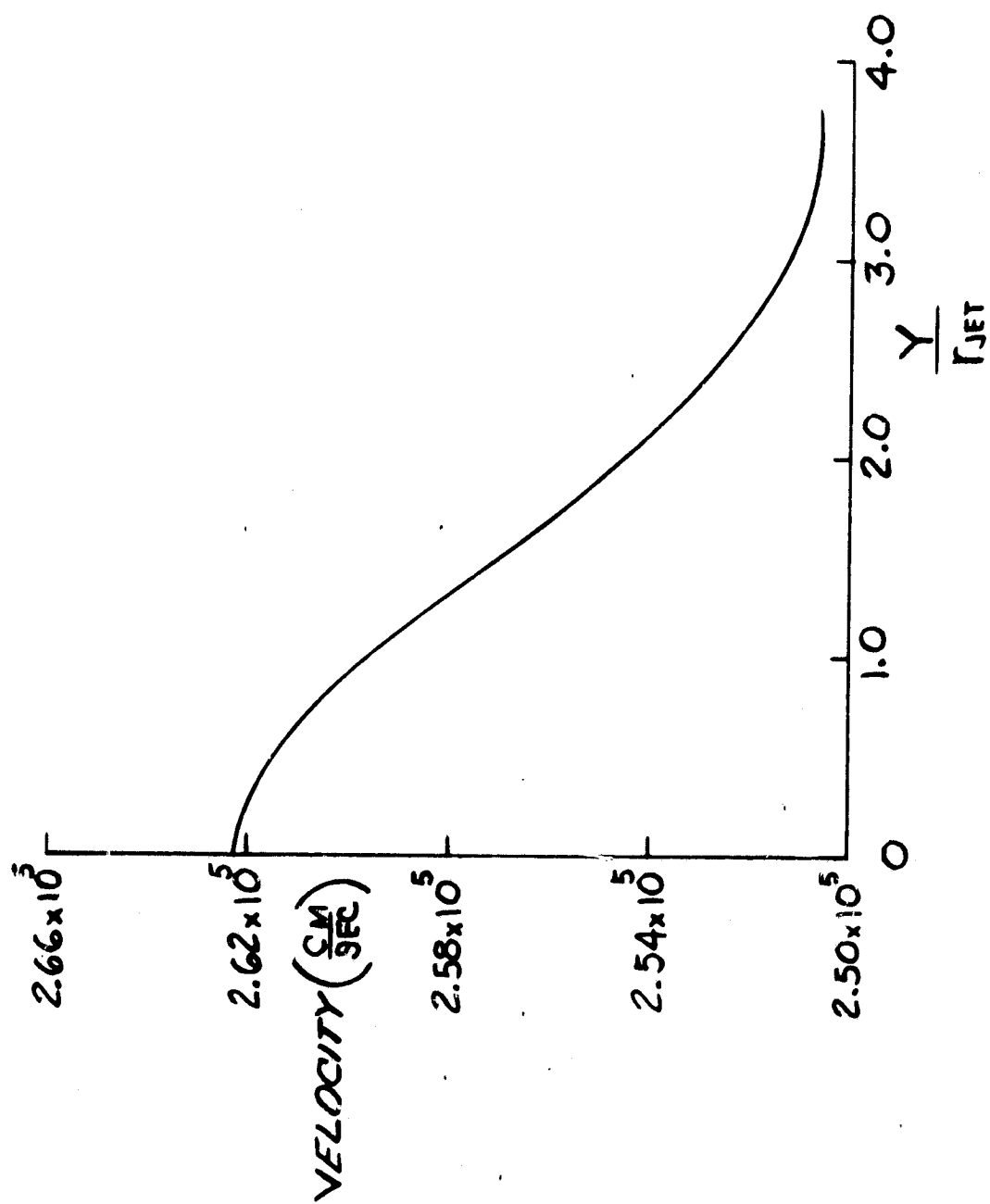


FIG. 17*i* TEMPERATURE PROFILE AT $\frac{X}{r_j} = 100$

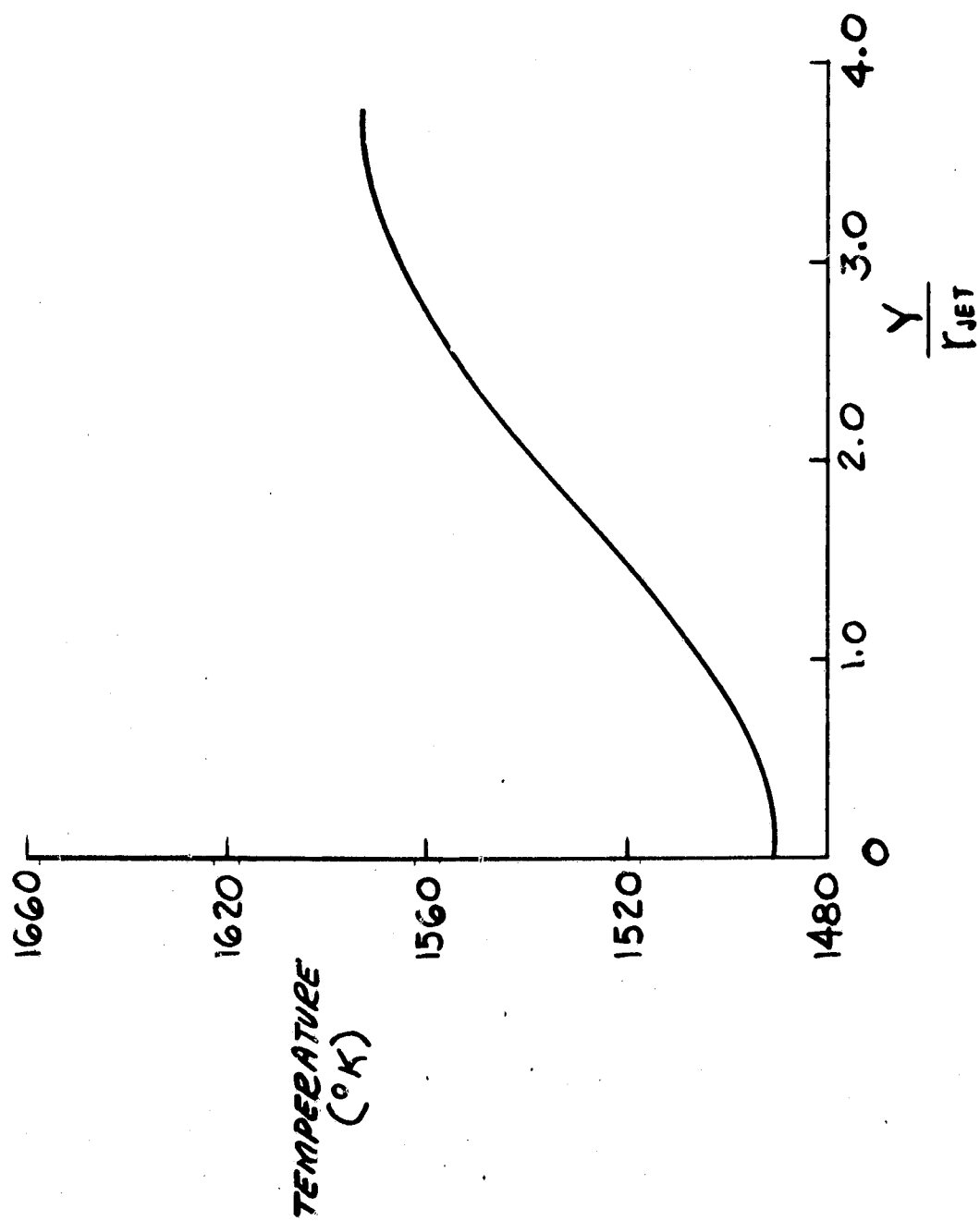


FIG. 17j: HYDROGEN MASS FRACTION AT $\frac{X}{r_j} = 100$

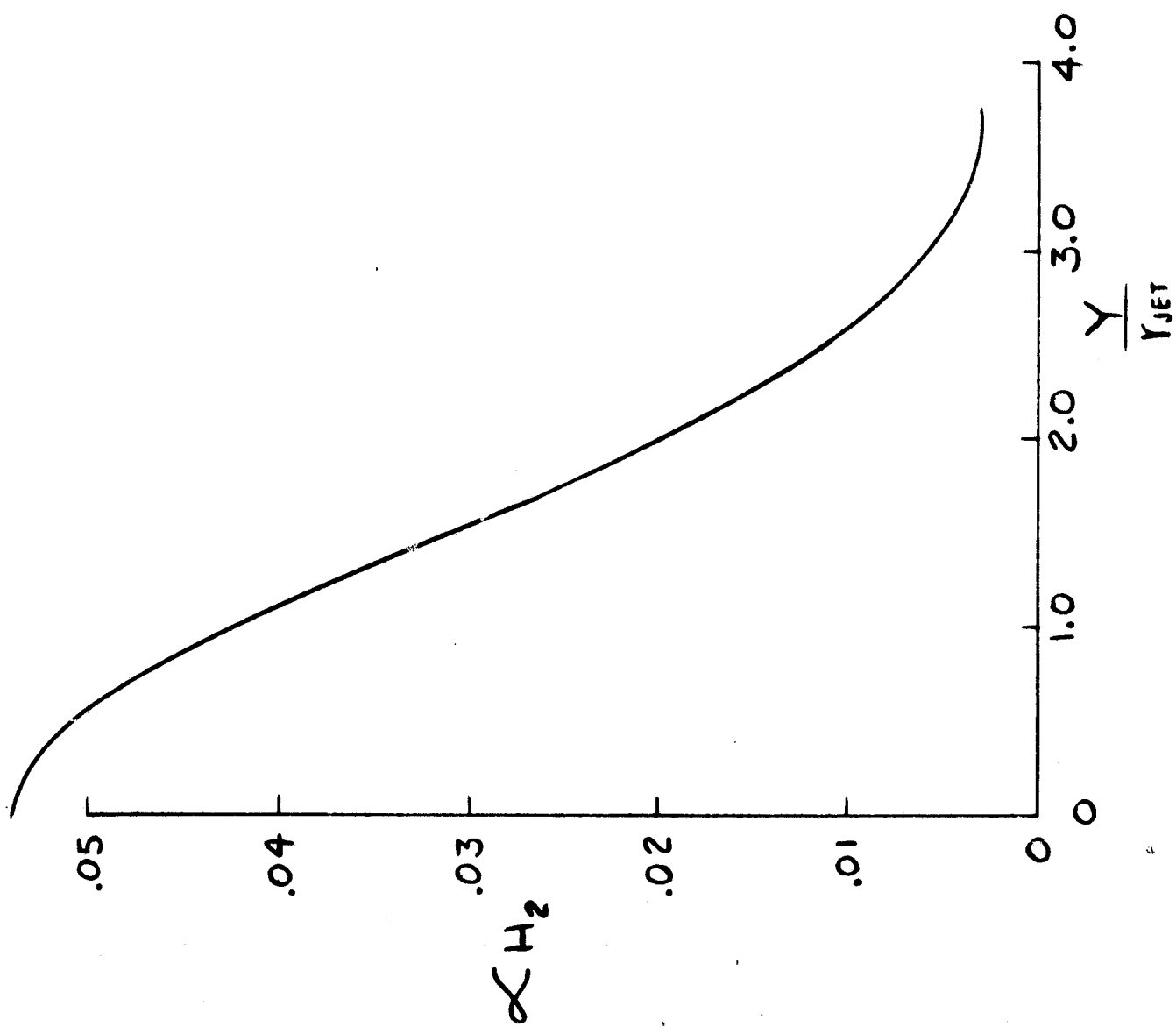


FIG. 18 UNDEREXPANDED DUCTED JET - VISCOUS FLOW
(a) WALL AND CENTERLINE PRESSURE DISTRIBUTIONS

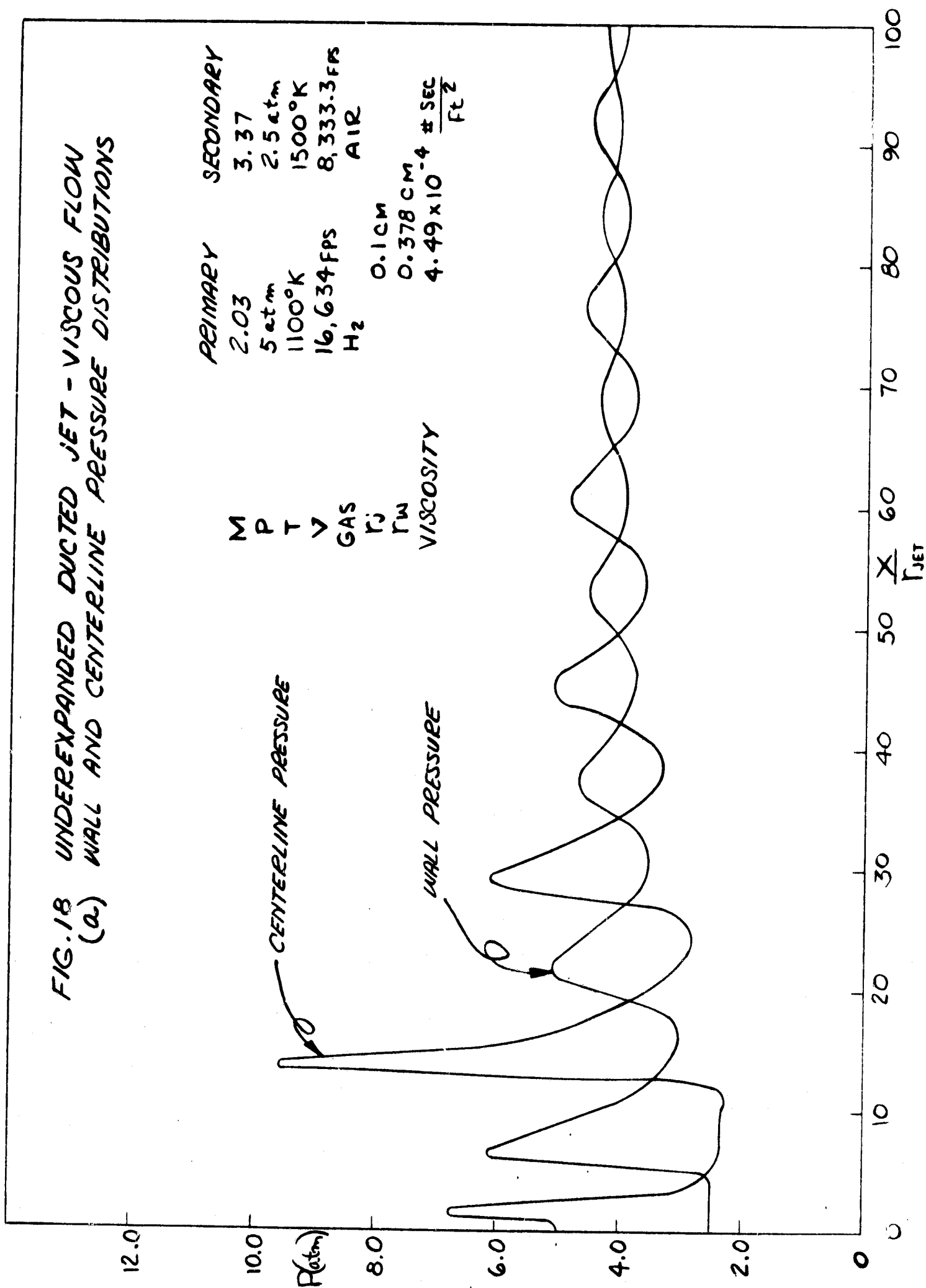


FIG. 186 AXIAL DECAY OF THE HYDROGEN MASS FRACTION

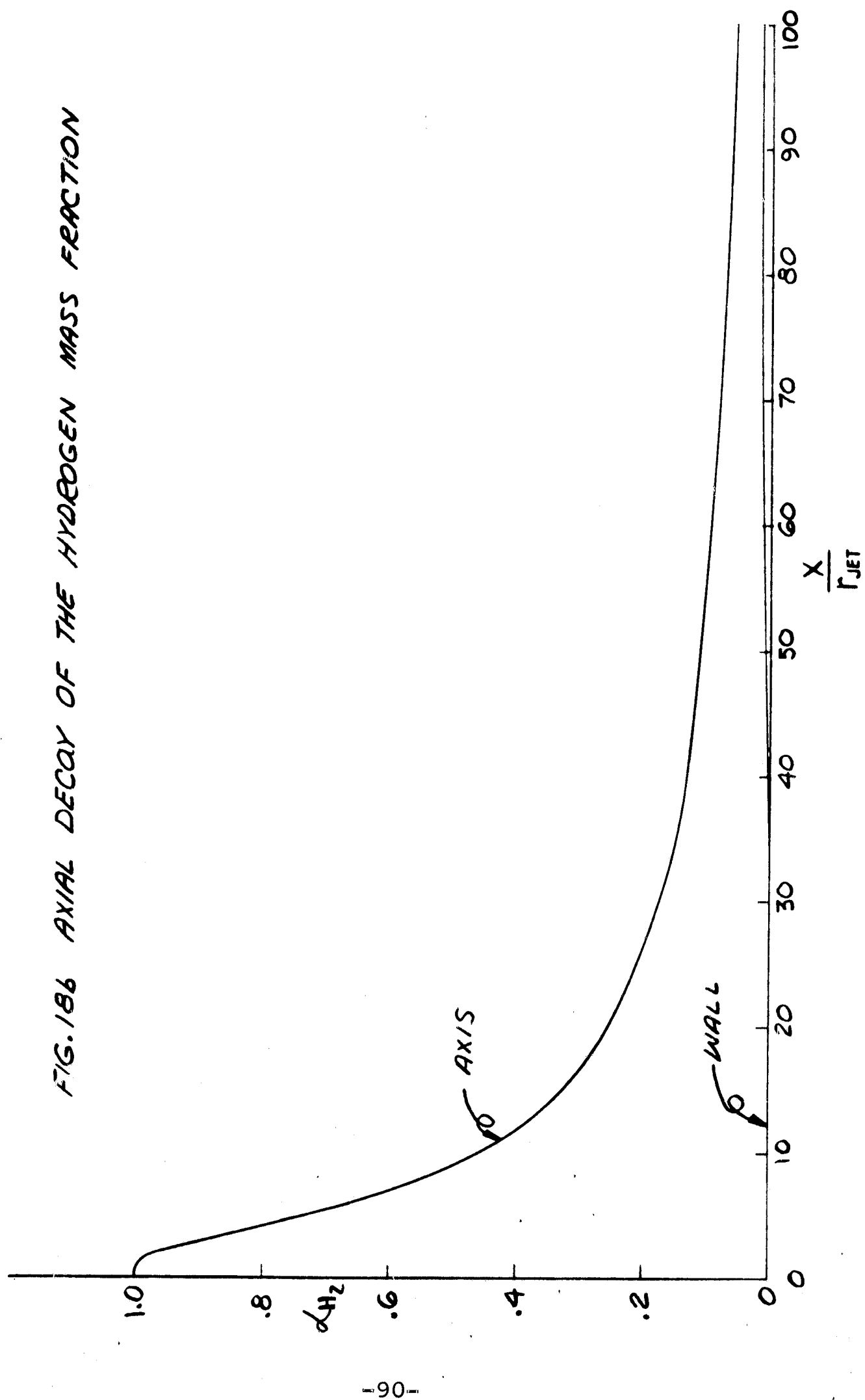


FIG. 18C PRESSURE PROFILE AT $\frac{X}{r_j} = 30$

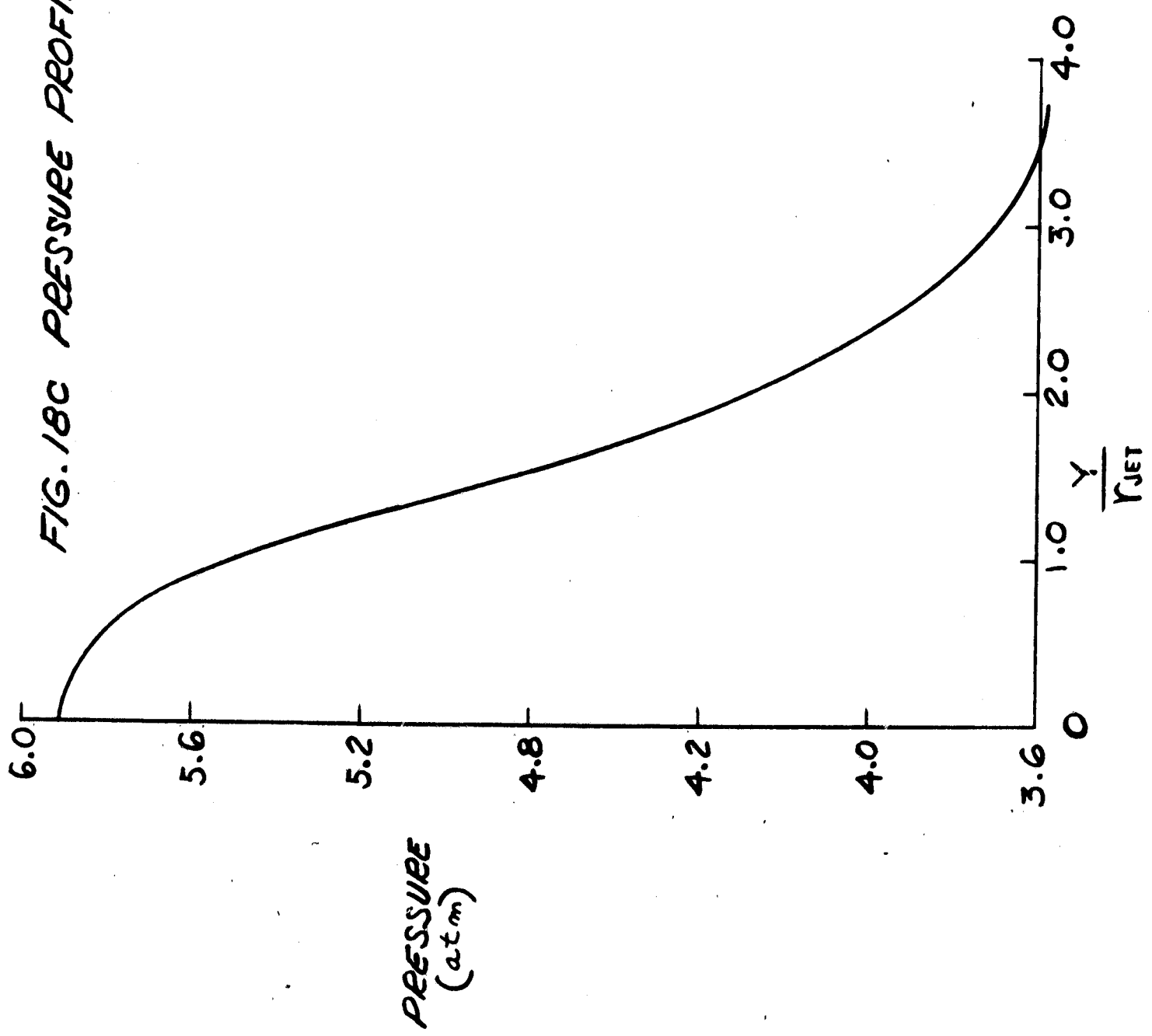


FIG. 18d VELOCITY PROFILE AT $\frac{x}{r_j} = 30$

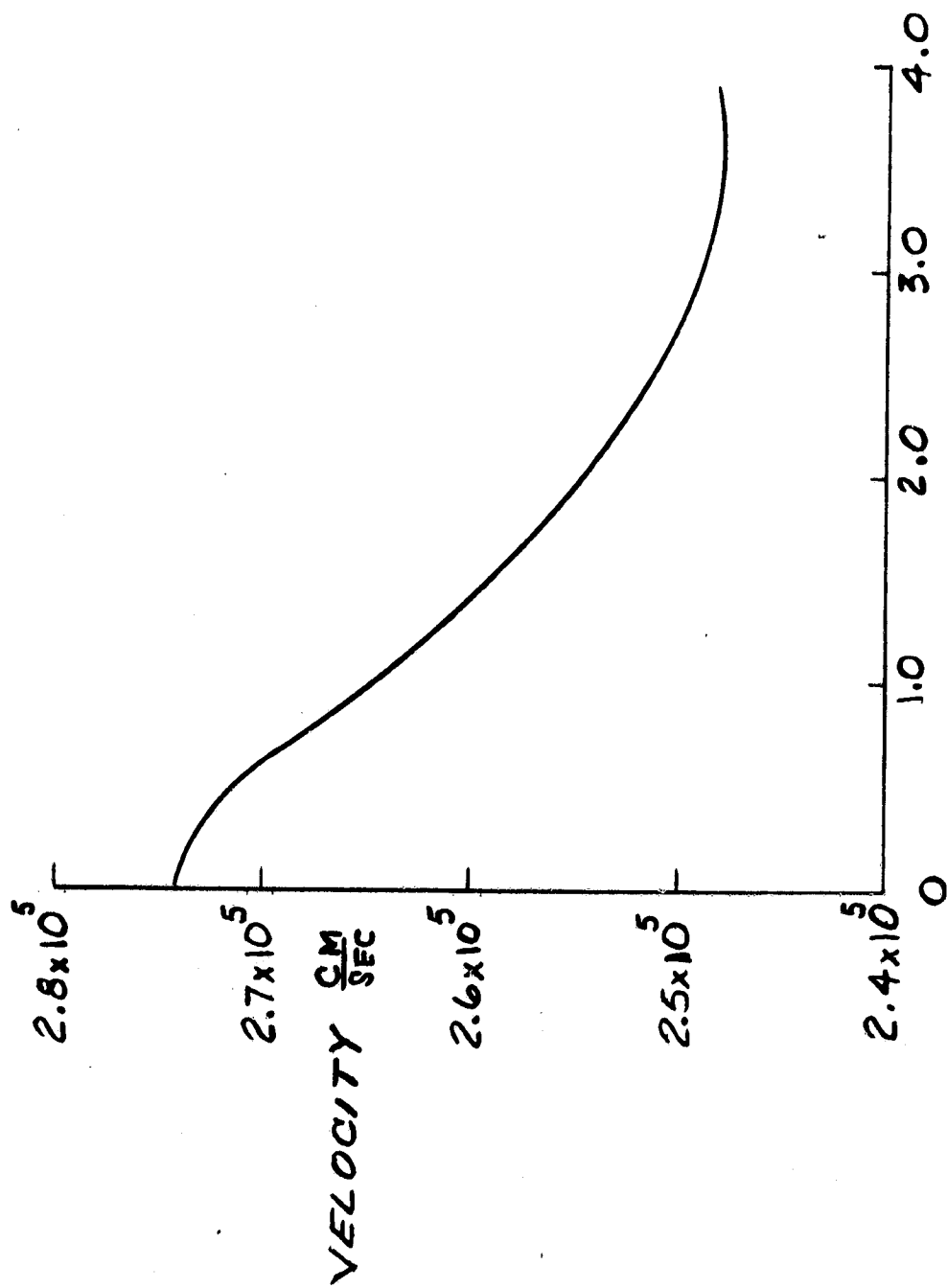


FIG. 18e TEMPERATURE PROFILE AT $\frac{x}{r_j} = 30$

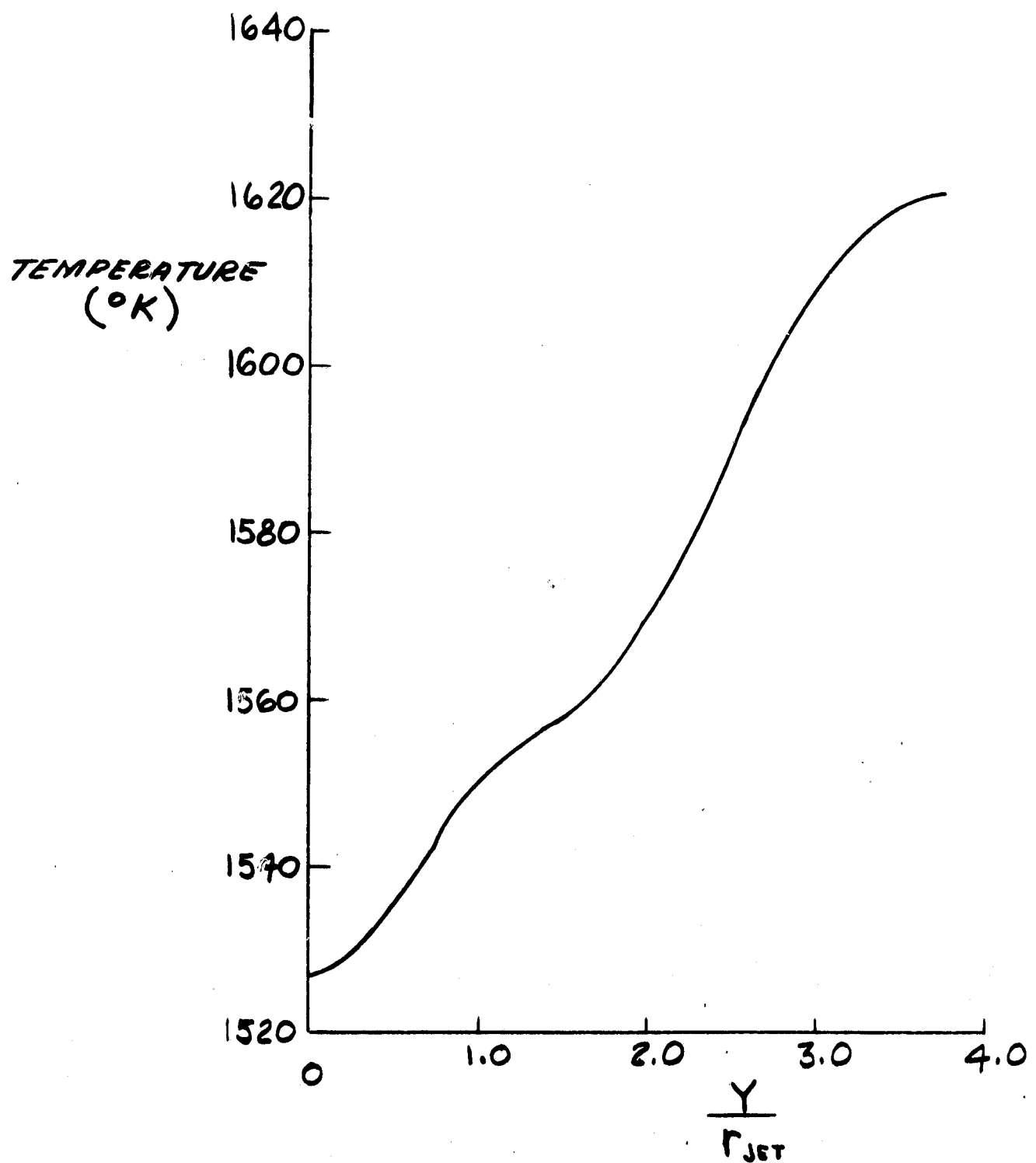


FIG. 18F HYDROGEN MASS FRACTION PROFILE AT $\frac{X}{r_j} = 30$

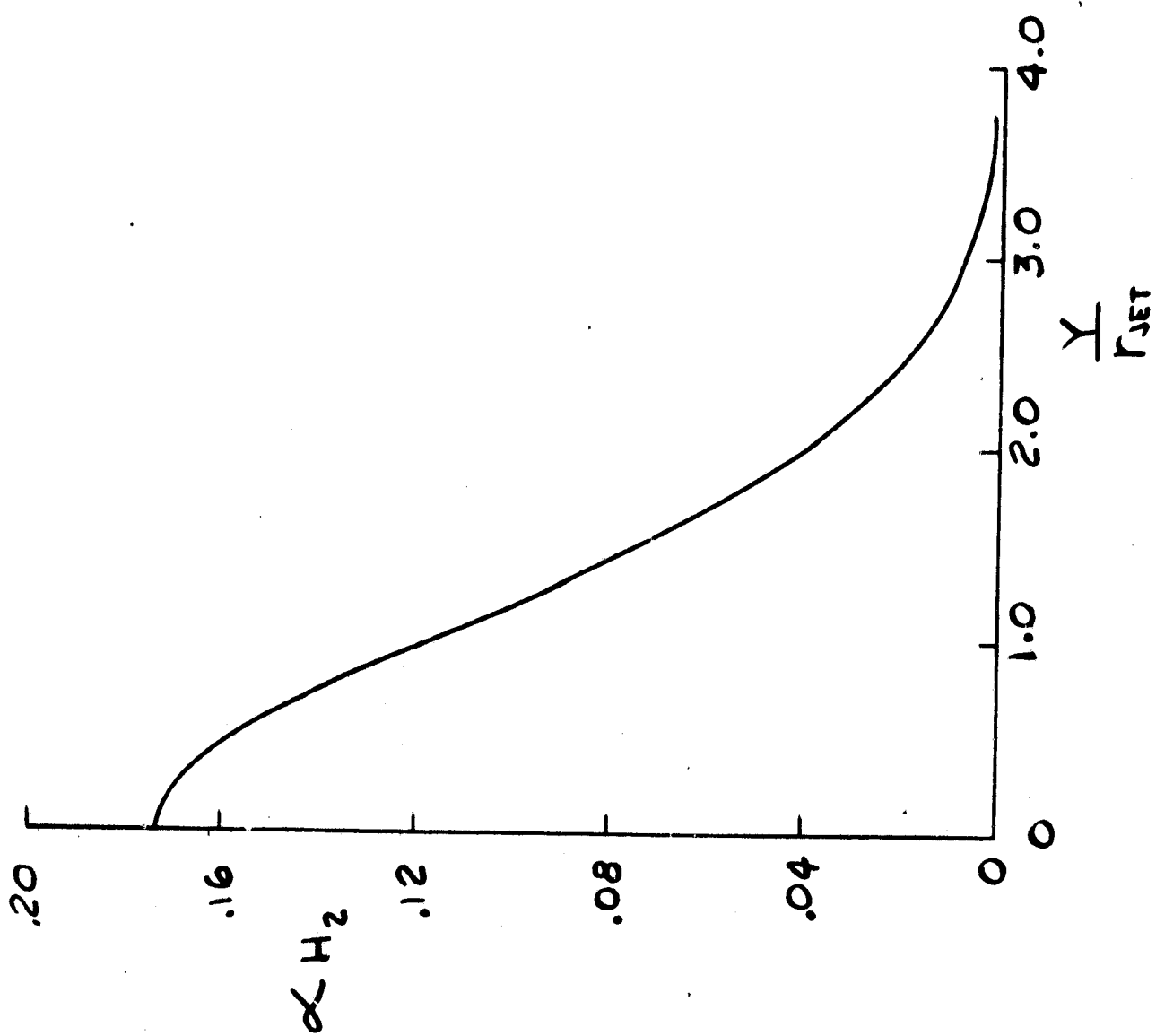


FIG. 18g PRESSURE PROFILE AT $\frac{X}{r_j} = 100$

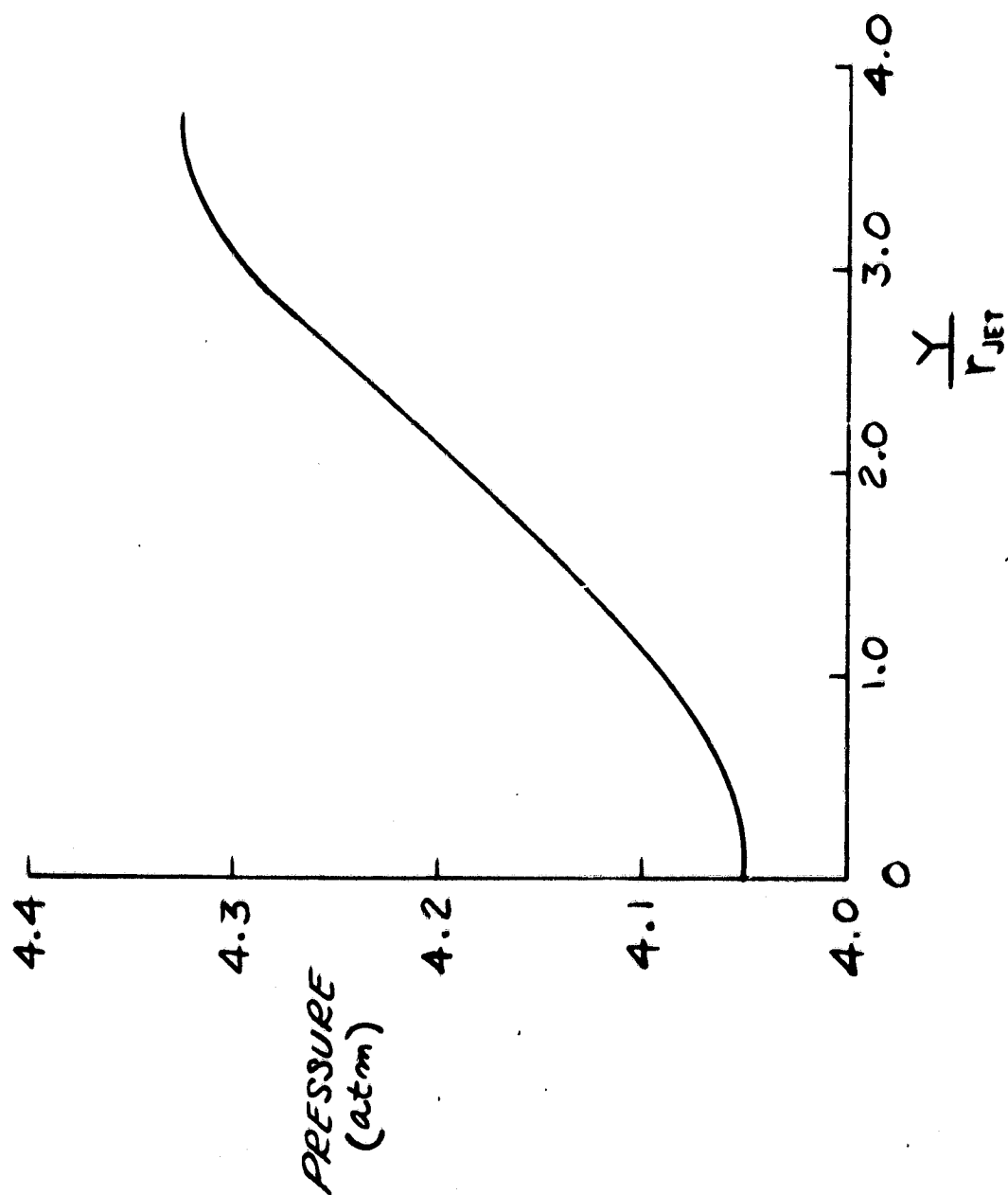


FIG. 18h VELOCITY PROFILE AT $\frac{X}{r_j} = 100$

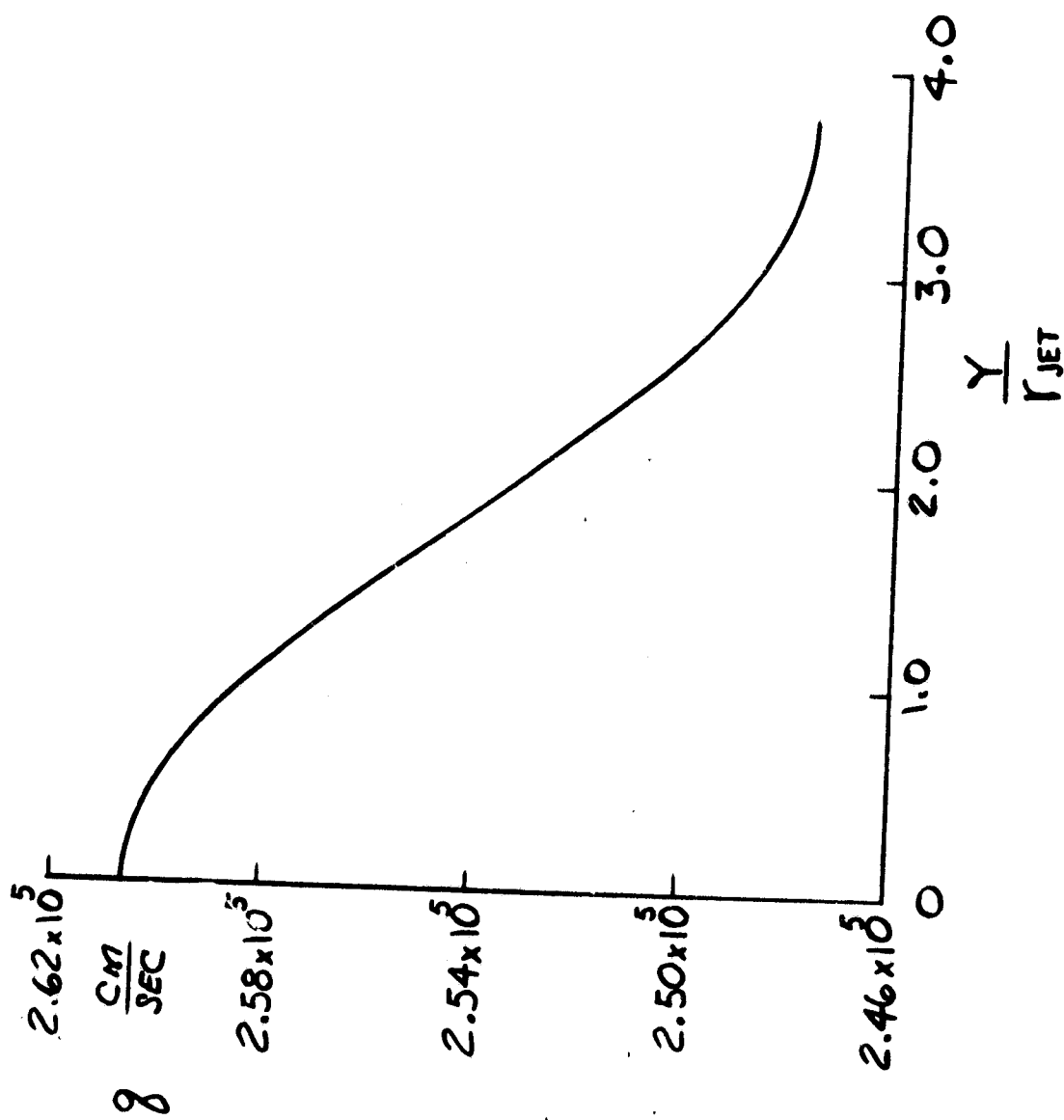


FIG. 18i TEMPERATURE PROFILE AT $\frac{X}{r_j} = 100$

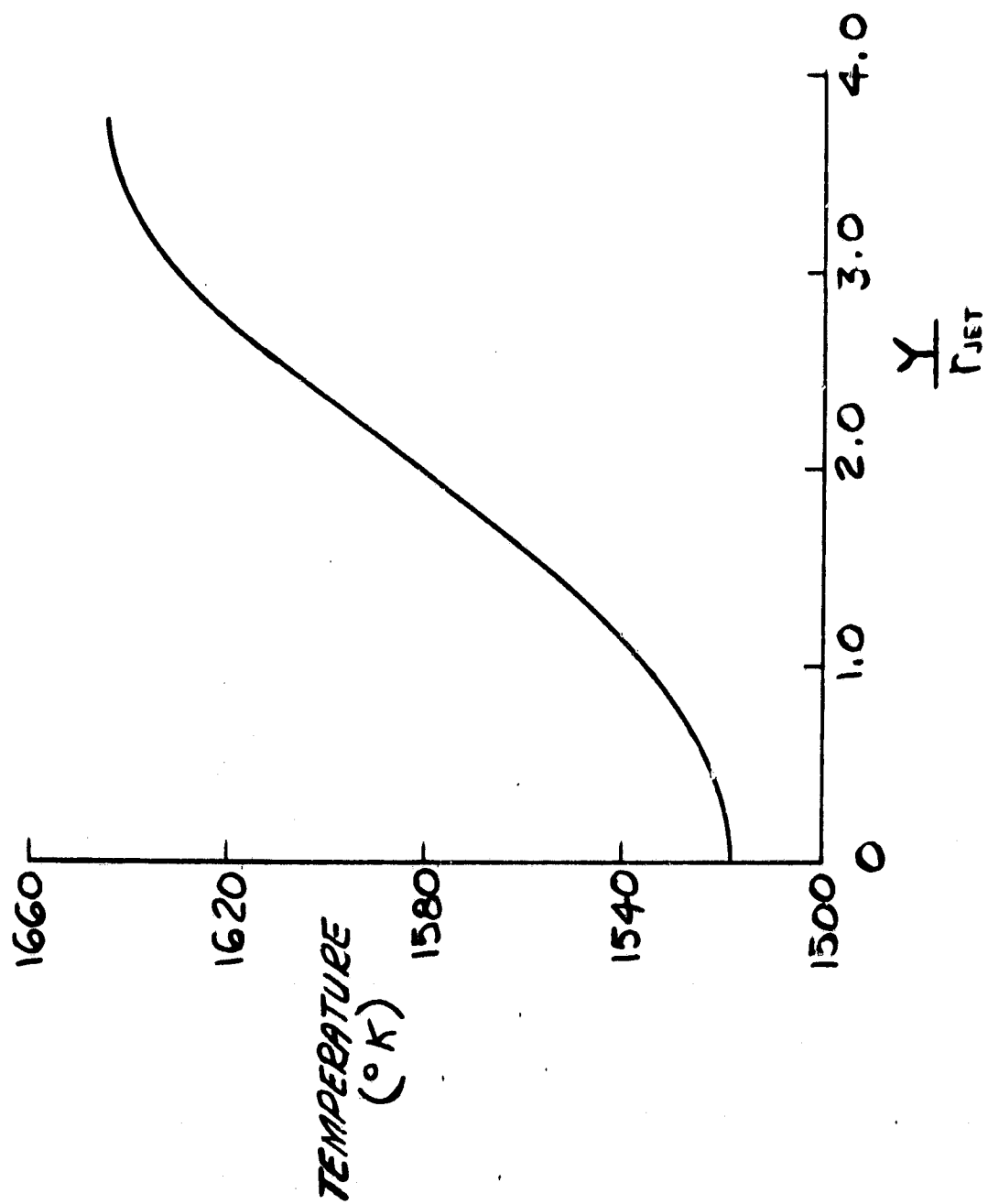


FIG. 18j HYDROGEN MASS FRACTION PROFILE AT $\frac{X}{r_j} = 100$

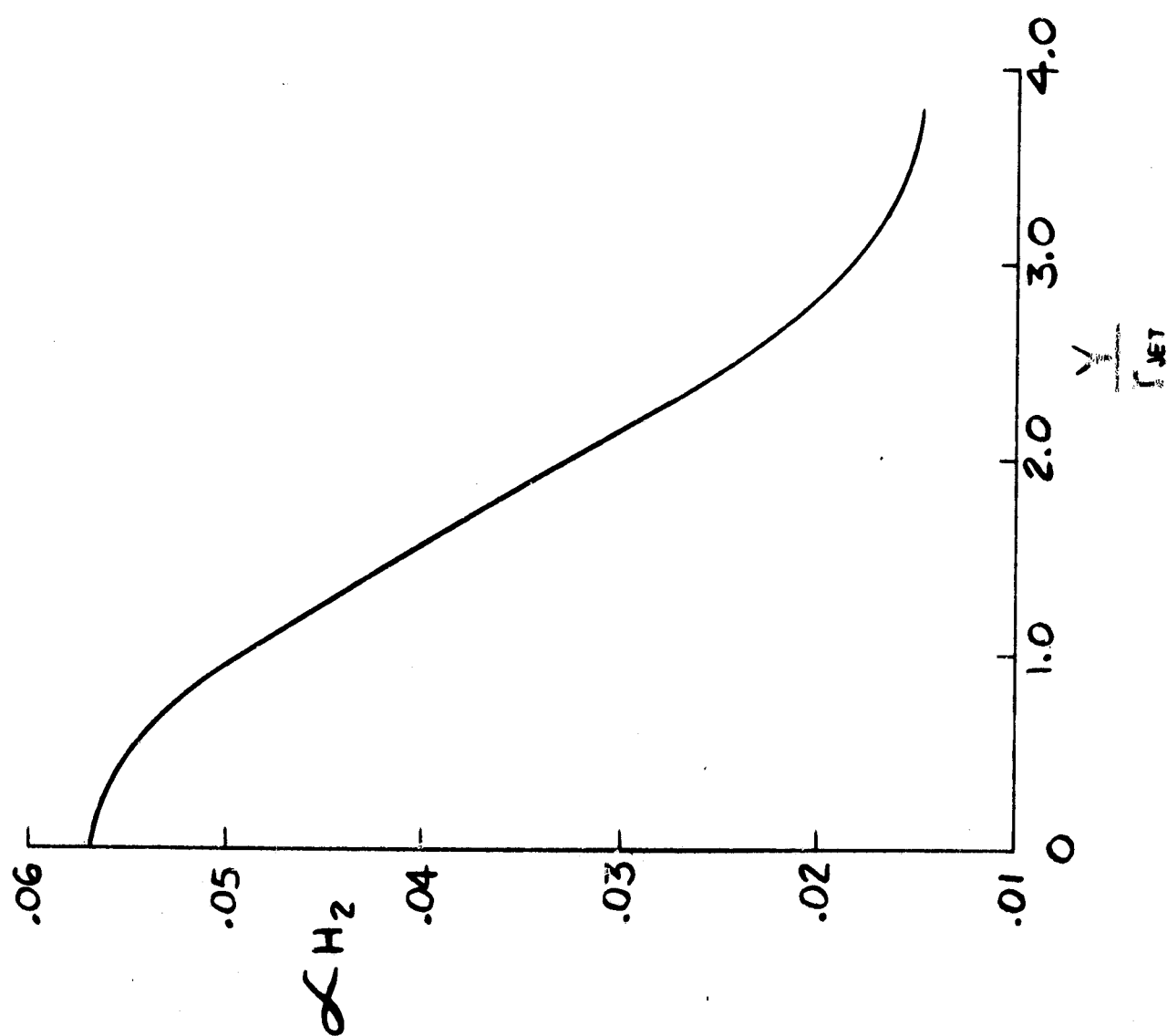


FIG. 19 COMPARISON OF UNDEREXPANDED DUCTED JET FOR VARIOUS VISCOSITY MODELS
(A) CENTERLINE PRESSURE DISTRIBUTION

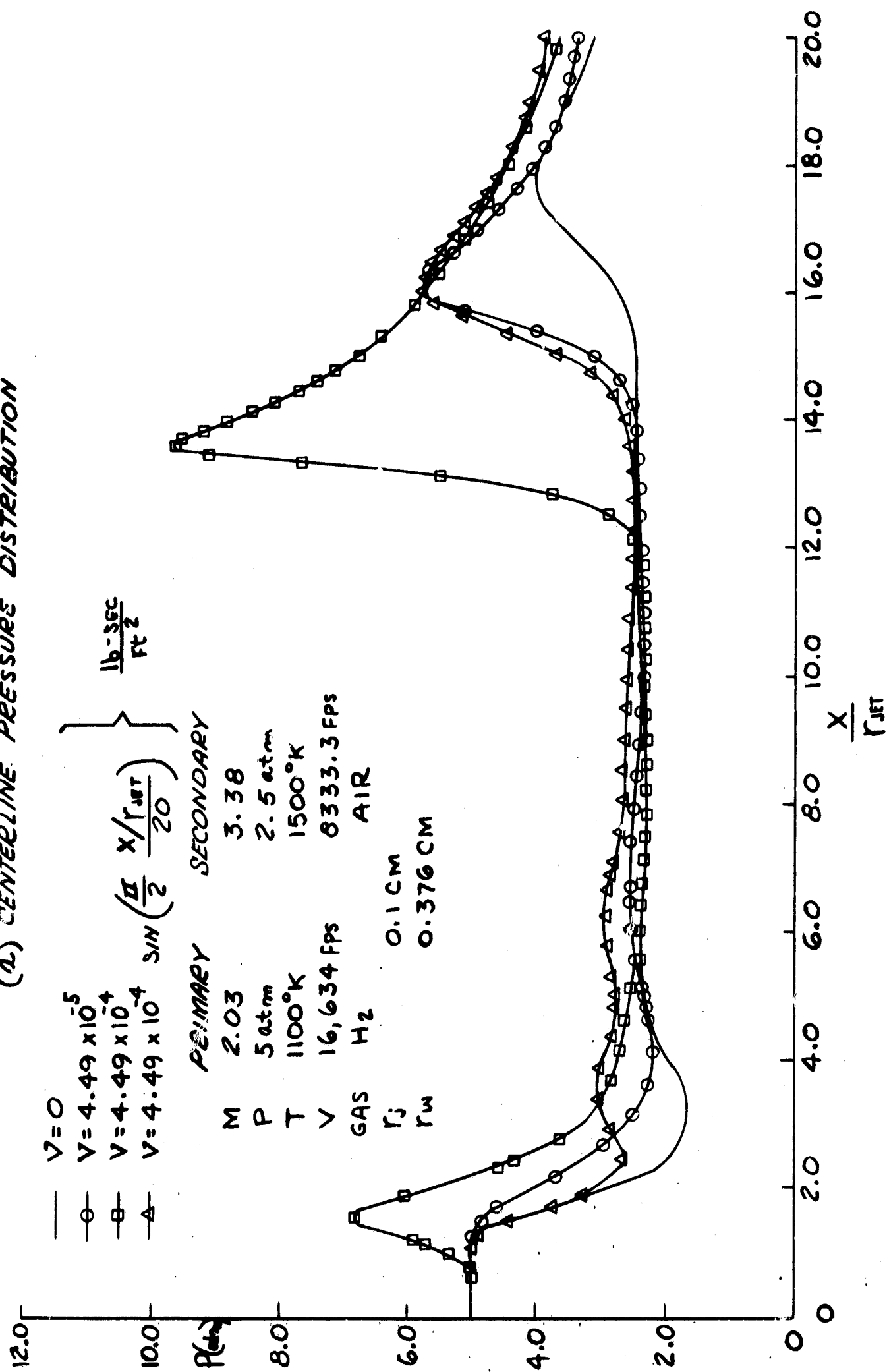


FIG. 196 WALL PRESSURE DISTRIBUTION

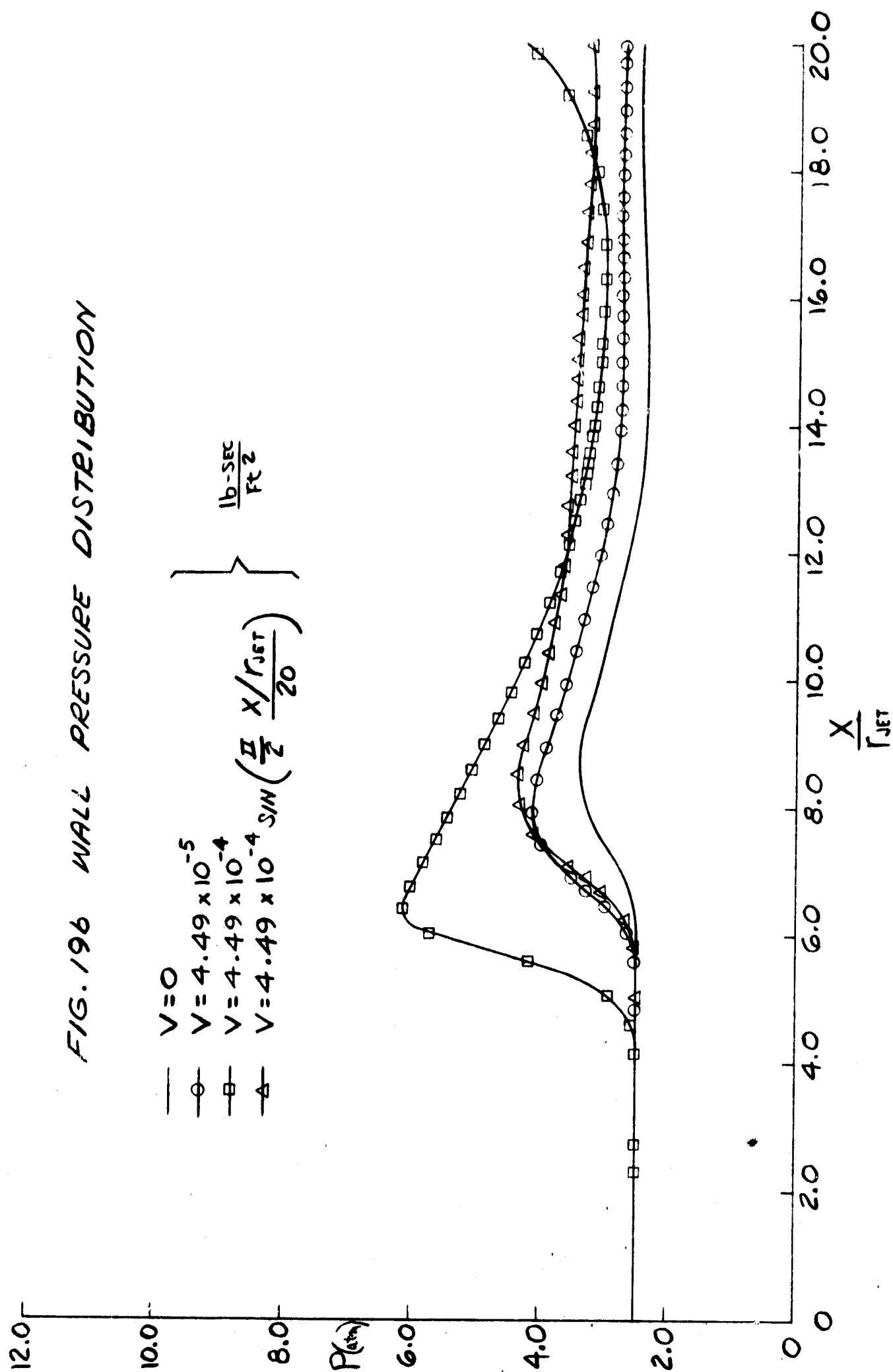


FIG. 19C AXIAL DECAY OF HYDROGEN MASS FRACTION

