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STUDY TO DEVELOP METHODS PREDICTING SPACECRAFT MAGNETIC FIELDS

FOR

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

HEAT MAGNETIC

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SYMBOLS

for chapters I - IV

Note: The symbols used in chapters starting with Chapter V are listed in their chapters.

A	without subscript, a dummy constant
A	with subscript, a constant defined by equ. (099-9a)
B	without subscript, a dummy constant
$\vec{B}$	with subscript, magnetic flux-density, weber/m^2
${}^{m}\vec{B}_{n}$	magnetic flux density at point "n" induced by the magnetic dipole at point "m". weber/m^2
C	without subscript, a dummy constant
C	with subscript, a constant, defined by equ. (130-1) through (130-17)
$\vec{D}$	dielectric displacement
E	a dummy vector
F	a dummy vector
$\hat{h}$	unit vector in the direction of the magnetization-vector
$\vec{H}$	magnetomotive force gradient, ampereturn/m
${}^{m}\vec{H}_{n}$	magnetomotive force gradient at point "n" induced by the magnetic dipole at point "m", ampereturn/m
${}^{o}\vec{H}$	mangetomotive force gradient induced only by electric currents, ampereturn/m
$\hat{i}$	unit vector in the x direction
$\hat{j}$	unit vector in the y direction
$\vec{J}$	current density ampere/m^2
$\hat{k}$	unit vector in the z direction
K	constant defined by equ. (109-1 through 35) and (101-4 through 7)
L	constant defined by equ. (102-1 through 5 and 15)

m	magnetic dipole moment, ampereturn $\cdot m^2$
m_m	magnetic dipole moment at point "m", ampereturn $\cdot m^2$
$\vec{M}$	magnetization vector at point "m", identical with the magnetic dipole moment per unit volume at point "m", also identical with the amperian current density on the surface of a small cylinder surrounding point "m", ampereturn/m
p	the last number in a series
R	a distance, m
r	a distance, m
r_{mn}	the distance of point m from the point n ; meters
S_m	a surface spanning a current-loop. m^2
t	time, sec.
V_m	a volume, m^3
x	numerical value of the x coordinate, m
y	numerical value of the y coordinate, m
z	numerical value of the z coordiante, m
θ	angle between the direction of the magnetic dipole moment at point "m" and the direction of the distance r_{mn} between points "m" and "n".
μ_o	permeability of empty space, $4\pi 10^{-7}$ weber.m/ampereturn
μ_r	relative permeability, numeric
Σ	summation
φ	scalar magnetic potential, weber/m

Subscripts

m	at point "m"
n	at point "n"
r or rel	relative
x	component in the x direction

y	component in the y direction
z	component in the z direction

Superscripts

m	induced by the magnetic dipole at point "m"
o	induced by electric current in free space

STUDY TO DEVELOP METHODS OF PREDICTING SPACECRAFT MAGNETIC FIELDS

By Andrew A. Halacsy*

SUMMARY

A procedure to calculate the magnetic field in three dimensions and in the neighborhood of a magnetic body of finite permeability, like a satellite though required, was not known so far.

Such a procedure is presented here, in three sections, as follows.

SectionI. Calculations are presented which define the m.m.f. grad. ${}^{o}\vec{H}_n$ in points of a three dimensional free space, for an arbitrary current system.

Section II. Calculations are presented which determine the total m.m.f. grad., $\vec{H}_n$ at points within magnetic bodies of field dependent permeability. This total m.m.f. grad. results as the sum of the m.m.f. grad. ${}^{o}\vec{H}_n$-s calculated in Section I. and the m.m.f. grad. ${}^{m}\vec{H}_n$-s resulting from dipoles at other points of the magnetic body. The dipoles at each point in their turn are induced by the total m.m.f. grad. This calculation agrees with physics teaching that the magnetic moment of such points arises by the m.m.f. grad. ${}^{o}H_n$ due to the current system and by interaction.

Section III. Calculations are presented which determine the 3 dimensional m.m.f. grad. at any arbitrary point outside the magnetic body or bodies considered in Section II. Values of the total m.m.f. grad. are found by summing the m.m.f. grad. calculated in Section I with the contributions from the points of magnetic bodies considered in Section II.

All calculations are computerized. The computer programs are attached and explained in details.

The calculations are in good agreement with laboratory results.

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I. INTRODUCTION

General

The magnetic field of a spacecraft has its origin in electric currents and magnetic dipoles present in the spacecraft. The magnetic dipoles are not necessarily induced by the electric currents in the spacecraft but can be induced by any other magnetic field present in the space and so can be the current.

Consequently, the magnetic field of the spacecraft does not differ from and can be analyzed as any other magnetic field.

Though several ways are known to analyze magnetic fields in two dimensions there was very little done to develop three-dimensional analysis needed to analyze the three-dimensional field of a spacecraft. The few attempts made for three-dimensional analysis are known to have run into difficulties of great complexities, when field dependent permeability was considered.

An attempt was made to analyze magnetic fields in three dimensions with field dependent permeability of ferromagnetic materials present in the field. This attempt tried to avoid the greatest source of the difficulties which is in the use of a magnetic vector potential. A scalar potential is used in this analysis. This is made possible by performing the analysis only for points where no electric current exists as in the field of a spacecraft.

No such type of analysis is known to the investigator. It is believed that this type of analysis is quite novel in its use of the magnetic scalar potential by which the calculations are reduced to scalars instead of vectors. It turns out that there are two scalar quantities to be calculated at each point considered. They are the magnetic scalar potential and the permeability. They define then the magnitude and the direction of the magnetic field in those points.

This analysis does not require boundary conditions because there is no integration as required if the magnetic vector-potential is used, and so a quite general solution can be reached. Then the geometries can be inserted in a kind of subroutines. This way the computer-program is valid for any geometry except the geometrical subroutine and is less complex than if the vector potential would be used.

Basically, the analysis presented here solved Maxwell's equations directly.

The use of the magnetic scalar potential, φ, leaves only one of Maxwell's equations, $\nabla \cdot \vec{B} = 0$, and this if written in terms of φ and μ, provides one equation for the problem. Another equation for φ and μ is arrived at by writing $H = -\nabla\varphi$ for the permeability curve $\mu = f(\vec{H})$.

The ferromagnetic body is divided into a finite number p of boxes. The magnetic moment of each of which is then concentrated in a single point in its center. The two equations for φ and μ are written for each of these points.

The result is a set of equations, 2p in number for φ and μ, each p in number, that is for 2p unknowns. Originally the equations were partial differential equations, and they are linearized for the numerical solutions. These linear equations are solved, for instance, by matrix inversion.

The calculation was computerized, and the programs sectionalized. Fortram IV language was used for an IBM 360/50 machine or equivalent. This part of the work was done by Professor G. H. Clark, University of Nevada.

Laboratory verification was performed on a limited number of specimens and the fit was 3.16% for the current field, and not so good for the dipoles.

It is realized that a procedure like this one would be useful not only to investigate the magnetic field of satellites but that of any other space with the presence of ferromagnetic bodies and it would be useful to solve several other problems of present day engineering which are yet unsolved, so for instance, for power-flow and short-circuit studies of very large interconnected electric power systems; for analysis and design of elastic structures, etc.

The Physics of the Problem

The electromagnetic field is completely defined by Maxwell's equations

$$\nabla \times \vec{H} = \vec{J} - \frac{\partial \vec{D}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{D} = \rho$$

$$\vec{B} = \mu_r \mu_o \vec{H} \qquad \vec{D} = \epsilon_r \epsilon_o \vec{E}$$

These six equations have six unknowns, $\vec{H}$, $\vec{B}$, $\vec{J}$, $\vec{E}$, $\vec{D}$, t, and so they should be sufficient to calculate these unknowns.

Though the vectorial quantities are fine for theoretical expressions, but how can six equations be solved for six vectors? After all, a vector needs three quantities to define its magnitude and direction; either its three <u>Cartesian components, or its magnitude and two directional cosines</u>.

Though $\vec{H}$ and $\vec{B}$ has the same direction, and so has $\vec{E}$ and $\vec{D}$, and though the $\nabla \times \vec{H}$ equation, and the $\nabla \times \vec{E}$ equation allows to make use of certain orthogonalities between $\vec{H}$, $\vec{E}$, and $\vec{J}$, etc., still great complexities arise.

Fortunately Maxwell's equations simplify drastically when some restraints are permissible.

1.) The most common restraint is to assume no variation in time, the magnetostatic case.

The time derivatives vanish in this case, and the curl of $\vec{H}$ becomes $\nabla \times \vec{H} = \vec{J}$, what is Ampere's law.

2.) The electrostatic field is not required in the discussed case (in the static case). Even if it is, it can be calculated separately because electrostatics and electromagnetics are connected only through the two time derivatives in Maxwell's equations and these time derivatives vanish in the static case. This is equivalent to say that di-electrics and magnetisms are connected only by Faraday's and Henry's law of induction, non-existing in the static case. Of course, the electric currents still produce a magnetic field, and so the remaining Maxwell equations to be solved are then:

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0 \qquad (100\text{-}2)$$

$$\vec{B} = \mu_s \mu_o \vec{H} \qquad (100\text{-}3)$$

3.) Another great simplification occurs when the field is analyzed only outside of real current-carrying conductors, (not dipoles) where $\vec{J} = 0$. This makes $\nabla \times \vec{H} = 0$ with the result that now the vector potential, $\vec{A}$, can be omitted completely, and a magnetic scalar potential, φ, can be used instead of it. The gradient of this magnetic scalar potential, φ is the m.m.f. gradient $\vec{H}$.

$$-\nabla\varphi = \vec{H} \qquad (100\text{-}4)$$

Of course, the curl of a gradient is always zero, and so a scalar potential, φ, chosen this way satisfies the now truncated Maxwell's equation automatically,

$$\nabla \times \vec{H} = 0 \qquad (100\text{-}1)$$

Follows that using a scalar potential, only the equation $\nabla \cdot \vec{B} = 0$ and the magnetizing curve, $B = \mu H$ are to be satisfied.

$$\nabla \cdot \vec{B}_n = \nabla \cdot \mu \vec{H}_n = 0$$

$$\mu = f(H)$$

$H = -\nabla\varphi$ so these are two equations for two unknown scalars φ, μ.

$\nabla\varphi$ can be dissolved into 3 scalar equations for

$$\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z} .$$

Steps Taken To Achieve a Numerical Solution

Several steps were required to make the theory applicable to real problems with numerical dimensions and to arrive at numerical values of H.

A.) The dipoles are induced in the magnetic material by the field. These dipoles have their own field too. The field acting at the location of a dipole is therefore the sum of the field of the current system, ${}^{o}H_n$, and the fields of all other dipoles, $\sum \vec{m}_{H_n}$

$$\vec{H}_n = {}^{o}\vec{H}_n + \sum_{\substack{m \\ m \neq n}} {}^{m}\vec{H}_n$$

B.) The calculation is to be made practical. Therefore the magnetic material is divided into small blocks, then uniformity of H and μ is assumed within each block, and all dipoles of each block are considered lumped into one single dipole located in the center, m of the block.

The strength of the magnetization of m is expressed by the magnetization vector

$$\vec{M}_m = (\mu_{rel} - 1)\vec{H}_m$$

The dipole strength is expressed by the magnetic moment,

$$\vec{m}_m = \vec{M}_m V_m$$

where V_m is the volume of the block.
A diple at m has a potential at another point, say the previously examined n, as follows,

$$\varphi_n = \frac{1}{4\pi} m_m \quad \frac{\cos\theta}{r^2_{mn}}$$

where θ is the angle between the direction of the magnetization vector M_m and the directed distance $\vec{r}_{mn}$ from m to n.

Writing M_m as above, the scalar potential of each dipole at n can be written in terms of the H_m at the location of the other dipoles.

So the H_n at each point can be expressed in terms of the H_m-s at the location of all other dipoles, assumed to be in the center of the subdivisions of the material.

C.) Using this form of H_n, $\therefore$ $B_n = \mu_n H_n$, and the Maxwell equ.

$$\nabla \cdot \vec{B}_n = \nabla \mu \vec{H}_n = \nabla \cdot \mu \left({}^{o}\vec{H}_n + \sum_{\substack{m \\ m \neq n}} {}^{m}\vec{H}_n \right) = 0$$

Then all H-s are expressed as $\vec{H} = -\nabla\varphi$ in this equation and so one has a set of second order partial differential equations, p in number if the material is divided into p subdivisions. These equations are scalar equations.

The unknowns in these equations are the φ-s and the μ-s, each p in number, that is 2p unknowns, twice as much as equations.

However another set of p equations is provided by the permeability curves $\mu = f(H)$, one for each point, in which the H-s could be written again as $H = -\nabla\varphi$.

This makes 2p equ. for the 2p unknown φ-s and μ-s, so they can be solved for these unknowns.

D.) The trouble is that μ cannot be expressed accurately enough as an algebraic function of H.

A solution is used, therefore, as follows:

1.) An arbitrary set of μ-s is plugged into the equations at the start in the form of parametric constants.

2.) The partial differential equations are linearized.

3.) Then the resulting linear equations are solved for the φ-s by any known method, for instance matrix inversion.

4.) The components of H are built as

$$H_x = -\frac{\partial\varphi}{\partial x} \qquad H_y = -\frac{\partial\varphi}{\partial y} \qquad H_z = -\frac{\partial\varphi}{\partial z}$$

according to $H = -\nabla\varphi$

$$H = \sqrt{H_x^2 + H_y^2 + H_z^2}$$

5.) μ is read against H from the permeability curve for each point.

6.) These new μ-s are put into the original equations in place of the originally used μ-s, and the process is iterated until the μ-s become not differing more than allowed from the previous ones.

The calculation was computerized, and the program sectionalized.

Special considerations are given to the air-iron interface but this does require only additional points in the air.

II. THE DETAILED MATHEMATICS OF THE PROBLEM

$\vec{B}$ can be written in terms of φ and μ as shown above

$$\nabla \cdot \vec{B} = \nabla \cdot \mu_o \mu_r \nabla \varphi = 0 \qquad (100\text{-}5)$$

The expanded form of (100-5) is as follows:

$$\mu_r\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) + \frac{\partial \mu_r}{\partial x}\frac{\partial \varphi}{\partial x} + \frac{\partial \mu_r}{\partial y}\frac{\partial \varphi}{\partial y} + \frac{\partial \mu_r}{\partial z}\frac{\partial \varphi}{\partial z} = 0 \qquad (100\text{-}6)$$

The first partial derivatives of φ_n are the directional components of $\vec{H}_n$. These can be written as follows

$$\frac{\partial \varphi_n}{\partial x} = H_{nx} = {}^{o}H_{nx} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m\,(\mu_{rm} - 1)\left[\frac{\partial \varphi_m}{\partial x_m}\frac{\partial}{\partial x_n}\frac{\cos(ir_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \frac{\partial \varphi_m}{\partial y_m}\frac{\partial}{\partial x_n}\frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m}\frac{\partial}{\partial x_n}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] \qquad (100\text{-}7)$$

and similar expressions for $\dfrac{\partial \varphi_n}{\partial y_n} = H_{ny}$ and for $\dfrac{\partial \varphi_n}{\partial z_n} = H_{nz}$.

The derivation of these equations is given below, and the solution is achieved by considerations and steps as follows.

${}^{o}H_{nx}$ appearing in (100-7) is due to an electric current-system and can be calculated independently. It can be considered as a constant in equation (100-7).

Use the first partial derivatives of φ_n in the form of (100-7) in (100-6)

Differentiate (100-7) partially at n in order to have the second partial derivatives for (100-6).

Write (100-6) by using the just described partial derivatives and linearize it by approximating the derivatives by finite differences.

This results in a set of linear equations of the type

$$\varphi_n = f(\mu_{rn}) \tag{100-8}$$

For each n point there is an equation (100-8). n = 1 to p. Use an assumed $\mu_{rn} \neq 1$ for each point n, in (100-8). Solve (100-8) for φ_n - s.

Calculate

$$H_{nx} = \frac{\partial \varphi_n}{\partial x_n} \tag{100-9x}$$

$$H_{ny} = \frac{\partial \varphi_n}{\partial y_n} \tag{100-9y}$$

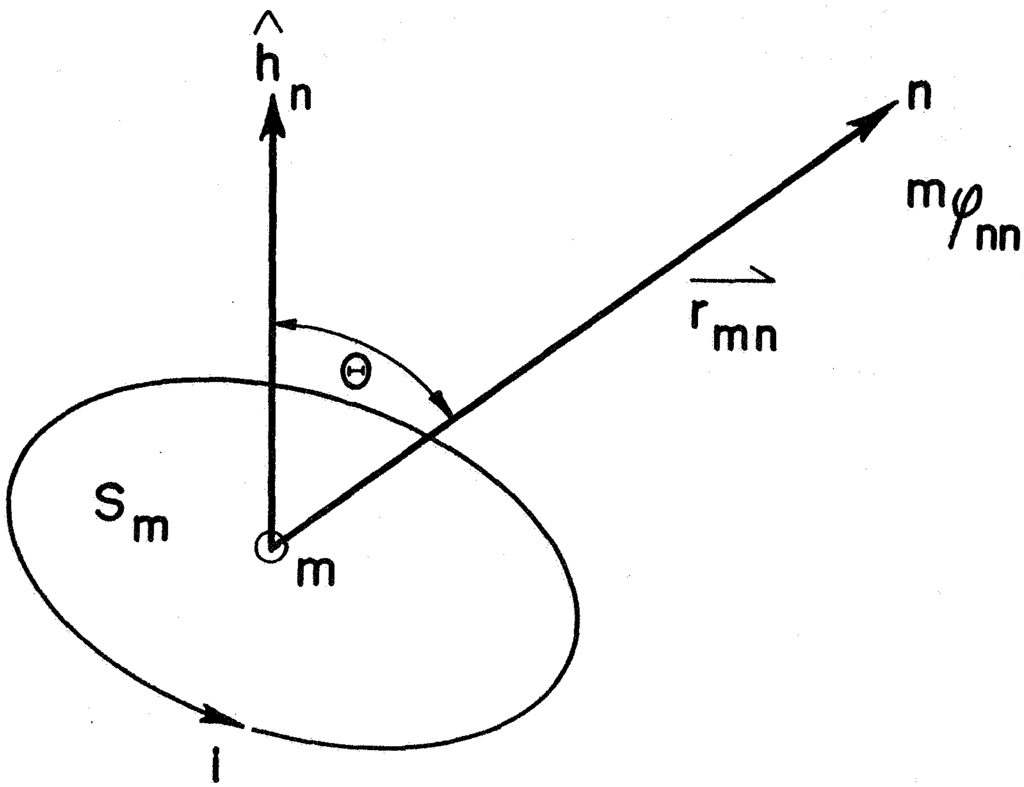
$$H_{nz} = \frac{\partial \varphi_n}{\partial z_n}\text{, numerically.} \tag{100-9z}$$

$$H_n = \sqrt{H_{nx}^2 + H_{ny}^2 + H_{nz}^2} \tag{100-10}$$

Read μ_n from the magnetizing curve, (100-3). Use these μ_n - s in (100-8). Solve (100-8) again for φ_n - s and iterate till the differences in φ-s and μ-s between consecutive steps decrease to acceptable levels.

The Magnetic Scalar Potential, φ

The magnetic scalar potential φ_n, at a point n is usually



THE MAGNETIC DIPOLE-MOMENT AT THE POINT m REPRESENTED BY THE CURRENT I INDUCES A MAGNETIC SCALAR POTENTIAL $^m\varphi_n$ AT THE POINT n

FIGURE 1

written in terms of the magnetic dipole moments, m_m, the dipole being located at a point m, see Fig. 1.

The magnetic dipole moment, m_m, of electric ampereturns, i, enclosing a small surface, S_m, containing the point, m, is

$$m_m = iS_m \text{ Ampereturns·square meter.} \tag{150-1}$$

This is so even if $\lim S_m \longrightarrow 0$. The scalar potential of this magnetic dipole at another point, n, is per ref. 1, p. 59, equ. 4,1.

$$m\varphi_n = \frac{1}{4\pi} m_m \hat{h}_m \cdot \hat{r}_{mn} \cdot \frac{1}{r_{mn}^2} = \frac{1}{4\pi} m_m \frac{\cos\theta}{r_{mn}^2} \tag{150-2}$$

In equ.(150-2) $\hat{h}_m$ is the unit vector normal to the surface S_m at the point m, $\hat{r}_{mn}$ is the unit vector from m to n, see Fig.1(150-1)

Magnetic Dipole-moment and Scalar Potential Of a Current Sheet Enclosing a Volume

Suppose the ampereturns are flowing not in a line like in Fig. 1 , but in a sheet enclosing a cylindrical volume, V_m, containing the point m. This is so even if $\lim V_m \longrightarrow 0$. This is shown in Fig. 2 . Note the height of the cylinder as h_m. The volume of the cylinder is then

$$V_m = h_m S_m \tag{151-1}$$

The ampereturn density, M_m on the surface of the cylinder is

then $M_m = \dfrac{i}{h_m}$ amperturn/meter $\qquad$ (151-2)

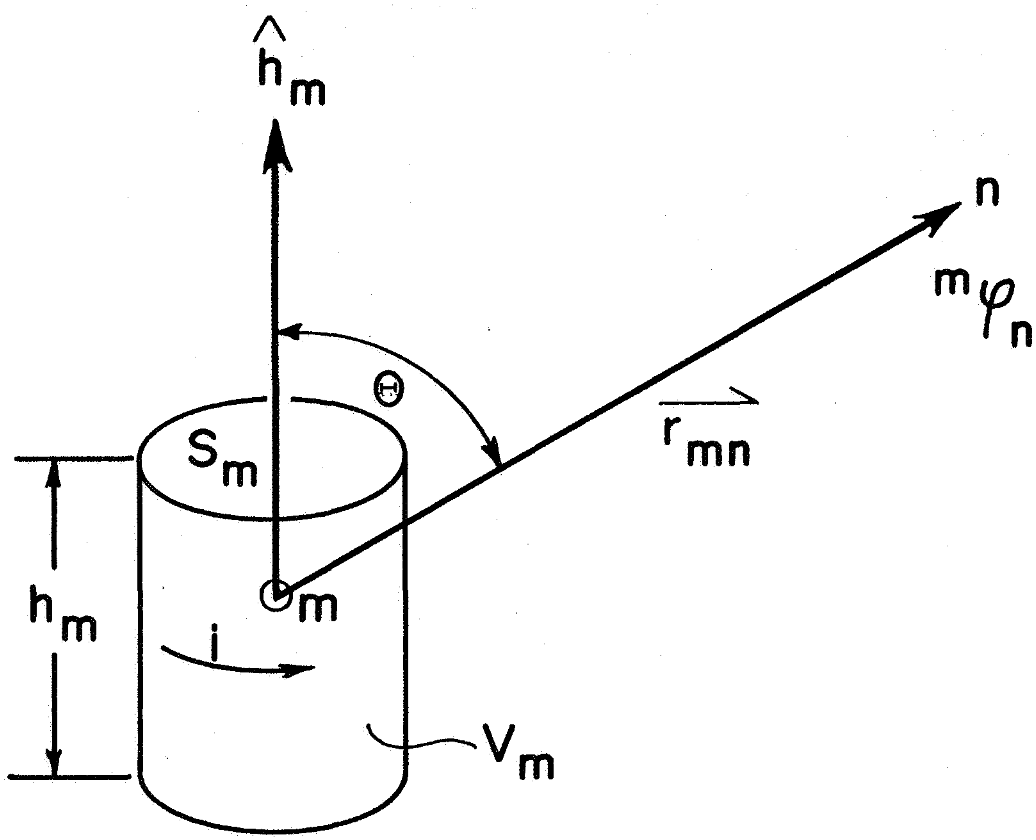
or, in reverse, the ampereturn i can be written as

$$i = M_m h_m \tag{151-3}$$

The magnetic dipole-moment of these ampereturns is, per equ. (150-1) (151-1) (151-3)

$$m_m = i S_m = M_m h_m S_m = M_m V_m \quad \text{Amperturns·square meter} \tag{151-4}$$

Similar to (150-2) the scalar potential of the magnetic dipole m_m at n, is by the combination of (150-2) and (151-4)



THE MAGNETIC DIPOLE-MOMENT AT THE POINT m REPRESENTED BY THE CURRENT SHEET i INDUCES A MAGNETIC SCALAR POTENTIAL ${}^m\varphi_n$ AT THE POINT n

FIGURE 2

$$ {}^{m}\varphi_{n} = \frac{1}{4\pi} M_m V_m \frac{\vec{h}_m}{h_m} \cdot \frac{\vec{r}_{mn}}{r_{mn}} \cdot \frac{1}{r_{mn}^2} \qquad (151\text{-}5) $$

Inspecting equ. (151-5) one recognizes M_m as the magnetization at m, and h as the direction of the magnetization vector.

$$ \vec{M}_m = M_m \hat{H}_m \qquad (152\text{-}1) $$

This is so because M_m is the total of the dipole moments in unit volume, and

$$ M_m = \frac{{}^{m}m}{V_m} \quad \text{from equ. (151-4)} \qquad (152\text{-}1a) $$

The Magnetic Flux Density and the M.M.F. Gradient Induced By Dipole Moments

At n the induction or magnetic flux density ${}^{m}\vec{B}_n$, due to the magnetic dipole moment at m, is the gradient of the potential of that dipole moment, the potential being taken at n, multiplied by the total permeability $\mu_{r\ ,n}\ \mu_o$, taken at the point n.

$$ {}^{m}\vec{B}_n = -\mu_{r\ ,n}\ \mu_o\ \nabla\varphi_n \qquad (152\text{-}2) $$

The m.m.f. gradient at the same point is

$$ {}^{m}\vec{H}_n = \frac{{}^{m}\vec{B}_n}{\mu_{r\ ,n} \cdot \mu_o} = -\nabla\varphi_n \qquad (152\text{-}3) $$

Combine equ. (152-3)(152-2)(151-5)(152-1) and consider V_m a constant.

$$ {}^{m}\vec{H}_n = -\frac{1}{4\pi} V_m \nabla\left(\vec{M}_m \cdot \frac{\vec{r}_{mn}}{r_{mn}^3}\right) \qquad (152\text{-}4) $$

The Total M.M.F. Gradient

The m.m.f. gradient at any point is the total of m.m.f. gradients caused by various sources. The sources are electric currents, and magnetic dipole moments. ${}^{o}\vec{H}_n$ denotes the m.m.f.

gradient component at n, gradient caused by electric currents, ${}^{m}\vec{H}_n$ denotes the m.m.f. component at n, caused by the magnetic dipole moment at m. The total m.m.f. gradient $\vec{H}_n$ at n can be written as follows.

$$\vec{H}_n = {}^{o}\vec{H}_n + \sum_{\substack{m=1 \\ m\neq n}}^{p} {}^{m}\vec{H}_n \qquad (152\text{-}5)$$

Here the summation is for all magnetic dipole moments in points m=1 to p, which have a magnetic potential at n. This, of course, excludes the dipole moment at n from the summation, as indicated.

Note that the summation is a summation of vectors. The induction, or magnetic flux density, $\vec{B}_n$, at n is

$$\vec{B}_n = \mu_{r,n}\mu_o \vec{H}_n \qquad (152\text{-}6)$$

Consider that the magnetization vector $\vec{M}_m$ at m in a material with $\mu_{rel,m}$ is by definition as follows.

$$\vec{M}_m = (\mu_{r\,m}-1)\ \vec{H}_m \ \text{, and } M_m = (\mu_{rm}-1)\ H_m \qquad (153\text{-}1)$$

Combine (152-6), (152-5), (152-4), (153-1)

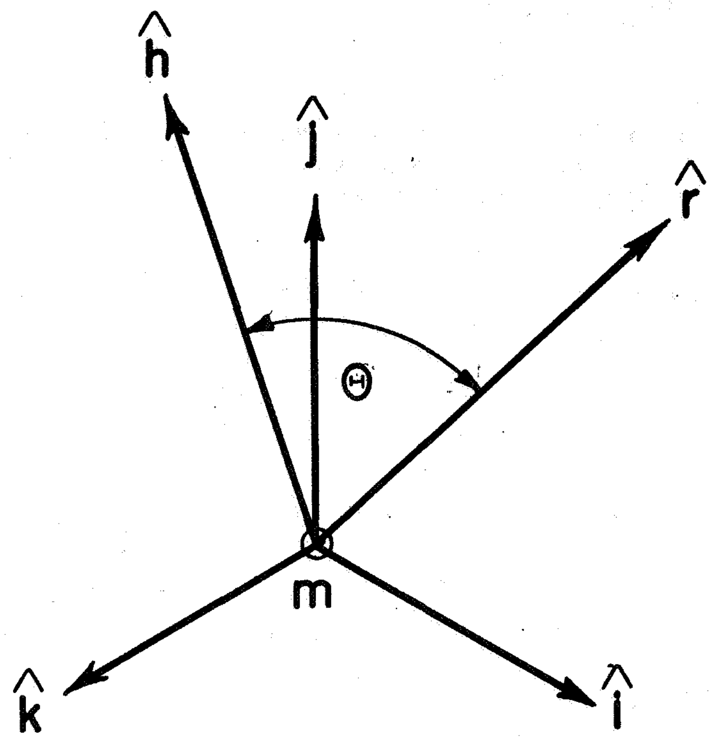
$$\vec{B}_n = \mu_o\mu_{r\,n}\ {}^{o}\vec{H}_n - \sum_{\substack{m=1 \\ m\neq n}}^{p} \frac{1}{4\pi}\mu_o\mu_{r\,n}\ V_m \nabla\left[\vec{M}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right] \qquad (153\text{-}2)$$

and

$$\vec{H}_n = {}^{o}\vec{H}_n - \sum_{\substack{m=1 \\ m\neq n}}^{p} \frac{1}{4\pi} V_m \nabla\left[\vec{M}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right] \qquad (153\text{-}3)$$

Rearrange and combine (153-3), (153-1), (152-1)

$$\vec{H}_n = {}^{o}\vec{H}_n - \frac{1}{4\pi}\sum_{\substack{m=1 \\ m\neq n}}^{p} V_m H_m (\mu_{r\,m}-1)\ \nabla\left(\frac{\hat{h}_m \cdot \hat{r}_{mn}}{r_{mn}^2}\right) \qquad (153\text{-}4)$$



DIRECTIONAL RELATIONS IN A THREE-DIMENSIONAL SYSTEM.

FIGURE 3

A Cartesian coordinate system is used, in which the three axes are x,y,z, and the three unit vectors in the direction of these three axes are $\hat{i}$, $\hat{j}$, $\hat{k}$. The unit vectors $\hat{h}_m$ and $\hat{r}_{mn}$ shown in equ. (155-1) can be expressed in terms of their directional cosines and the unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$, as follows.

$$\hat{h} = \hat{i}\cos(ih) + \hat{j}\cos(jh) + \hat{k}\cos(kh) \qquad (155\text{-}2)$$

$$\hat{r} = \hat{i}\cos(ir) + \hat{j}\cos(jr) + \hat{k}\cos(kr) \qquad (155\text{-}3)$$

and their scalar product is

$$\hat{h} \cdot \hat{r} = \cos(h \cdot r) = \cos\theta \qquad (155\text{-}4)$$

see Fig. 3

$\vec{r}_{mn}$ denotes a vector connecting points m and n and pointing from the point m towards the point n. By comparing (152-5) and (153-4) ${}^m\vec{H}_n$ can be written as follows:

$${}^m\vec{H}_n = -\nabla\, {}^m\varphi_n = -\frac{1}{4\pi} V_m H_m (\mu_{rm} - 1)\, \nabla\left(\frac{\hat{h}_m \cdot \hat{r}_{mn}}{r_{mn}^2}\right) \qquad \begin{matrix}(155\text{-}5\text{-}1)\\(046\text{-}1)\end{matrix}$$

The only variable in this equation is

$$\nabla\left(\frac{\hat{h}_m \cdot \hat{r}_{mn}}{r_{mn}^2}\right) = \nabla\left(\hat{h}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right) \qquad \begin{matrix}(155\text{-}5\text{-}2)\\(046\text{-}2)\end{matrix}$$

This expression can be developed (Appendix III) to the following one:

$$\nabla\left(\hat{h}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right) = \hat{i}\left[\cos(ih_m)\frac{\partial}{\partial x}\frac{\cos(ir_{mn})}{r_{mn}^2} + \cos(jh_m)\frac{\partial}{\partial x}\frac{\cos(jr_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \cos(kh_m)\frac{\partial}{\partial x}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] + \hat{j}\left[\cos(ih_m)\frac{\partial}{\partial y}\frac{\cos(ir_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \cos(jh_m)\frac{\partial}{\partial y}\frac{\cos(jr_{mn})}{r_{mn}^2} + \cos(kh_m)\frac{\partial}{\partial y}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] +$$

$$+ \hat{k}\left[\cos(ih_m)\frac{\partial}{\partial z}\frac{\cos(ir_{mn})}{r_{mn}^2} + \cos(jh_m)\frac{\partial}{\partial z}\frac{\cos(jr_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \cos(kh_m)\frac{\partial}{\partial z}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] \qquad \begin{matrix}(155\text{-}8\text{-}1)\\(049\text{-}2)\end{matrix}$$

Write $^\circ\vec{H}_n$ by its three components in equ. (153-4) and write the second term there by using (049-2)

$$\vec{H}_n = \hat{i}\,^\circ H_{nx} + \hat{j}\,^\circ H_{ny} + \hat{k}\,^\circ H_{nz} - \frac{1}{4\pi}\sum_{\substack{m=1\\m\neq n}}^{p} V_m H_m(\mu_{rm}-1)\cdot$$

$$\cdot\left\{\hat{i}\frac{\partial}{\partial x}(\psi) + \hat{j}\frac{\partial}{\partial y}(\psi) + k\frac{\partial}{\partial z}(\psi)\right\} \qquad (064\text{-}1)$$

where $\psi = \frac{1}{r_{mn}^2}\left[\cos(ih_m)\cos(ir_{mn}) + \cos(jh_m)\cos(jr_{mn}) + \right.$

$$\left. + \cos(kh_{mn})\cos(kr_{mn})\right]$$

The x component of $\vec{H}_n$ is $\vec{H}_{nx}$ and it is as follows:

$$\vec{H}_{nx} = \hat{i}\,^\circ H_{nx} - \frac{1}{4\pi}\sum_{\substack{m=1\\m\neq n}}^{p} V_m H_m(\mu_{rm}-1)\;\hat{i}\frac{\partial}{\partial x}(\psi) \qquad (064\text{-}2)$$

$$\vec{H}_{nx} = \hat{i}\left[{}^{o}H_{nx} - \frac{1}{4\pi}\sum_{\substack{m=1\\m\neq n}}^{p} V_m H_m(\mu_{rm} - 1)\left\{\cos(ih_m)\frac{\partial}{\partial x}\frac{\cos(ir_{mn})}{r_{mn}^2} + \right.\right.$$

$$\left.\left. + \cos(jh_m)\frac{\partial}{\partial x}\frac{\cos(jr_{mn})}{r_{mn}^2} + \cos(kn_m)\frac{\partial}{\partial x}\frac{\cos(kr_{mn})}{r_{mn}^2}\right\}\right] \tag{064-3x}$$

Similarly

$$\vec{H}_{ny} = \hat{j}\left[{}^{o}H_{ny} - \frac{1}{4\pi}\sum_{\substack{m=1\\m\neq n}}^{p} V_m H_m(\mu_{rm}-1)\left\{\cos(ih_m)\frac{\partial}{\partial y}\frac{\cos(ir_{mn})}{r_{mn}^2} + \right.\right.$$

$$\left.\left. + \cos(jh_m)\frac{\partial}{\partial y}\frac{\cos(jr_{mn})}{r_{mn}^2} + \cos(kh_m)\frac{\partial}{\partial y}\frac{\cos(kr_{mn})}{r_{mn}^2}\right\}\right] \tag{064-3y}$$

$$\vec{H}_{nz} = \hat{k}\left[{}^{o}H_{nz} - \frac{1}{4\pi}\sum_{\substack{m=1\\m\neq n}}^{p} V_m H_m(\mu_{rm}-1)\left\{\cos(ih_m)\frac{\partial}{\partial z}\frac{\cos(ir_{mn})}{r_{mn}^2} + \right.\right.$$

$$\left.\left. + \cos(jh_m)\frac{\partial}{\partial z}\frac{\cos(jr_{mn})}{r_{mn}^2} + \cos(kh_m)\frac{\partial}{\partial z}\frac{\cos(kr_{mn})}{r_{mn}^2}\right\}\right] \tag{064-3z}$$

All of the derivatives are taken at the point, n. It is in these equations

$$H_{mx} = H_m \cos(ih_m) \tag{064-4x1}$$

$$H_{my} = H_m \cos(jh_m) \tag{064-4y1}$$

$$H_{mz} = H_m \cos(kh_m) \tag{064-4z1}$$

Combine (064-3x,3y,3z) and (064-4x1,4y1,4z1) and write the absolute values of H_{nx}, H_{ny}, H_{nz}.

$$H_{nx} = {}^{\circ}H_{nx} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m \neq n}}^{p} V_m(\mu_{rm} - 1) \left[H_{mx} \frac{\partial}{\partial x} \frac{\cos(ir_{mn})}{r_{mn}^2} + H_{my} \frac{\partial}{\partial x} \frac{\cos(jr_{mn})}{r_{mn}^2} + H_{mz} \frac{\partial}{\partial x} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \quad (099\text{-}1x)$$

$$H_{ny} = {}^{\circ}H_{ny} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m \neq n}}^{p} V_m(\mu_{rm} - 1) \left[H_{mx} \frac{\partial}{\partial y} \frac{\cos(ir_{mn})}{r_{mn}^2} + H_{my} \frac{\partial}{\partial y} \frac{\cos(jr_{mn})}{r_{mn}^2} + H_{mz} \frac{\partial}{\partial y} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \quad (099\text{-}1y)$$

$$H_{nz} = {}^{\circ}H_{nz} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m \neq n}}^{p} V_m(\mu_{rm} - 1) \left[H_{mx} \frac{\partial}{\partial z} \frac{\cos(ir_{mn})}{r_{mn}^2} + H_{my} \frac{\partial}{\partial z} \frac{\cos(jr_{mn})}{r_{mn}^2} + H_{mz} \frac{\partial}{\partial z} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \quad (099\text{-}1z)$$

The M.M.F. Gradient, $\vec{H}_n$, in Terms of the Magnetic Scalar Potential, φ_n.

$$\vec{H}_n = -\nabla\varphi_n = \hat{i}\left(-\frac{\partial\varphi_n}{\partial x}\right) + \hat{j}\left(-\frac{\partial\varphi_n}{\partial y}\right) + \hat{k}\left(-\frac{\partial\varphi_n}{\partial z}\right) \quad (096\text{-}1)(099\text{-}2)\ (056\text{-}11)$$

$$\vec{H}_{nx} = \hat{i}\left(-\frac{\partial\varphi_n}{\partial x}\right) \quad (096\text{-}2)(099\text{-}3)$$

$$H_{nx} = -\frac{\partial\varphi_n}{\partial x} \quad (096\text{-}3)(099\text{-}4x)$$

and similarly

$$H_{ny} = -\frac{\partial\varphi_n}{\partial y} \quad (099\text{-}4y)$$

$$H_{nz} = -\frac{\partial\varphi_n}{\partial z} \quad (099\text{-}4z)$$

The Divergence of $\vec{B}$ in Terms of φ and μ

Using the directional partial derivatives of φ for the Cartesian components of $\vec{H}$, the equations (099-1x,1y,1z) become equations for the scalar potential, φ, and the permeability, μ.

$$\frac{\partial \varphi_n}{\partial x_n} = - {}^{o}H_{nx} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m \neq n}}^{p} V_m(\mu_{rm} - 1) \left[\frac{\partial \varphi_m}{\partial x_m} \frac{\partial}{\partial x_n} \frac{\cos(ir_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial y_m} \frac{\partial}{\partial x_n} \frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m} \frac{\partial}{\partial x_n} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \quad (099\text{-}5x)$$

$$\frac{\partial \varphi_n}{\partial y_n} = - {}^{o}H_{ny} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m \neq n}}^{p} V_m(\mu_{rm} - 1) \left[\frac{\partial \varphi_m}{\partial x_m} \frac{\partial}{\partial y_n} \frac{\cos(ir_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial y_m} \frac{\partial}{\partial y_n} \frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m} \frac{\partial}{\partial y_n} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \quad (099\text{-}5y)$$

$$\frac{\partial \varphi_n}{\partial z_n} = - {}^{o}H_{nz} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m \neq n}}^{p} V_m(\mu_{rm} - 1) \left[\frac{\partial \varphi_n}{\partial x_m} \frac{\partial}{\partial z_n} \frac{\cos(ir_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial y_m} \frac{\partial}{\partial z_n} \frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m} \frac{\partial}{\partial z_n} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \quad (099\text{-}5z)$$

These expressions for the derivatives of φ are derived in order to use them in the $\nabla.\vec{B} = 0$ equation already written in terms of φ and μ above. An inspection of that equation (100-6) shows that the second partial derivatives of φ are required too. They result from the first derivatives by another differentiation.

Express the second order partial derivative of φ_{n-s}

$\frac{\partial^2 \varphi_n}{\partial x_n^2}$, $\frac{\partial^2 \varphi_n}{\partial y_n^2}$, $\frac{\partial^2 \varphi_n}{\partial z_n^2}$, by differentiating (099-5x,y,z) at n

$$\frac{\partial^2 \varphi_n}{\partial x_n^2} = -\frac{\partial}{\partial x_n}\, {}^{o}H_{nx} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m(\mu_{rm}-1)\left[\frac{\partial \varphi_m}{\partial x_m}\frac{\partial^2}{\partial x_n^2}\frac{\cos(ir_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial y_m}\frac{\partial^2}{\partial x_n^2}\frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m}\frac{\partial^2}{\partial x_n^2}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] \quad \text{(099-10xx)}$$

$$\frac{\partial^2 \varphi_n}{\partial y_n^2} = -\frac{\partial}{\partial y_n}\, {}^{o}H_{ny} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m(\mu_{rm}-1)\left[\frac{\partial \varphi_m}{\partial x_m}\frac{\partial^2}{\partial y_n^2}\frac{\cos(ir_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial y_m}\frac{\partial^2}{\partial y_n^2}\frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m}\frac{\partial^2}{\partial y_n^2}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] \quad \text{(099-10yy)}$$

$$\frac{\partial^2 \varphi_n}{\partial z_n^2} = -\frac{\partial}{\partial z_n}\, {}^{o}H_{nz} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m(\mu_{rm}-1)\left[\frac{\partial \varphi_m}{\partial x_m}\frac{\partial^2}{\partial z_n^2}\frac{\cos(ir_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial y_m}\frac{\partial^2}{\partial z_n^2}\frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m}\frac{\partial^2}{\partial z_n^2}\frac{\cos(kr_{mn})}{r_{mn}^2}\right] \quad \text{(099-10zz)}$$

It must be observed that if the derivatives are taken at the point n, and not at the points m, the quantities at the points m are considered as parametric constants. Such are the first partial derivatives of φ on the right side of the equation but not on the left side. The differentiation must be performed on the components of ${}^{o}\vec{H}_n$, but not on $V_m(\mu_{rm}-1)$.

The differentiation must be performed also on the terms containing the distance r_{mn} between point n and m, because they are functions of the location, n.

The components of $^{o}\vec{H}$ are given by the geometry and the electric currents and can be calculated independently. They can be considered as constants for the φ, μ-equation. The derivatives containing the r_{mn} distance between points m and n are dependent only on the geometry and they too can be considered as constants. Similarly the volumes V_m.

Set of Equations for the Magnetic Scalar Potential and the Permeability

Performing the differentiations and other mathematics and lumping all terms not depending on φ or μ into constants then using the results in the $\nabla.\vec{B} = 0$ equation, transcribed into terms of φ, and μ, (Appendix IV) the $\nabla.\vec{B} = 0$ equation takes a form as follows, after the derivatives are linearized.

$$- K_{nhx}\mu_r(x_{n+1}y_nz_n) + K_{nhx}\mu_r(x_{n-1}y_nz_n) - K_{nhy}\mu_r(x_ny_{n+1}z_n) +$$

$$+ K_{nhy}\mu_r(x_ny_{n-1}z_n) - K_{nhz}\mu_r(x_ny_nz_{n+1}) + K_{nhz}\mu_r(x_ny_nz_{n-1}) +$$

$$- K_{nh}\mu_r(x_ny_nz_n) +$$

$$+ \sum_{\substack{m=1 \\ m\neq n}}^{p} \Big[- \Big\{ M_{mnix}\left[\mu_r(x_my_mz_m)-1\right]\left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)\right] +$$

$$+ M_{mniy}\left[\mu_r(x_my_mz_m)-1\right]\left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)\right] +$$

$$+ M_{mniz}\left[\mu_r(x_my_mz_m)-1\right]\left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})\right] +$$

$$+ L_{mni}\left[\mu_r(x_my_mz_m)-1\right] .\mu_r(x_ny_nz_n)\Big\} \varphi(x_{m+1}y_mz_m) +$$

$$+ \Big\{ M_{mnix}\left[\mu_r(x_my_mz_m)-1\right]\left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)\right] +$$

$$+ M_{mniy}\left[\mu_r(x_my_mz_m)-1\right]\left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)\right] +$$

$$+ M_{mniz}\left[\mu_r(x_my_mz_m)-1\right]\left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})\right] +$$

$$+ L_{mni}\left[\mu_r(x_my_mz_m)-1\right] .\mu_r(x_ny_nz_n)\Big\} \varphi(x_{m-1}y_mz_m) -$$

$$
\begin{aligned}
&-\Big\{ M_{mnjx} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\right] &+\\
&+ M_{mnjy} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right] &+\\
&+ M_{mnjz} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\right] &+\\
&+ L_{mnj} \left[\mu_r(x_m y_m z_m)-1\right] \cdot \mu_r(x_n y_n z_n) \Big\} \; \varphi(x_m y_{m+1} z_m) &+\\
&+\Big\{ M_{mnjx} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\right] &+\\
&+ M_{mnjy} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right] &+\\
&+ M_{mnjz} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\right] &+\\
&+ L_{mnj} \left[\mu_r(x_m y_m z_m)-1\right] \cdot \mu_r(x_n y_n z_n) \Big\} \; \varphi(x_m y_{m-1} z_m) &-\\
&+\Big\{ M_{mnkx} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\right] &+\\
&+ M_{mnky} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right] &+\\
&+ M_{mnkz} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\right] &+\\
&+ L_{mnk} \left[\mu_r(x_m y_m z_m)-1\right] \cdot \mu_r(x_n y_n z_n) \Big\} \; \varphi(x_m y_m z_{m+1}) &+\\
&+\Big\{ M_{mnkx} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\right] &+\\
&+ M_{mnky} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right] &+\\
&+ M_{mnkz} \left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\right] &+\\
&+ L_{mnk} \left[\mu_r(x_m y_m z_m)-1\right] \cdot \mu_r(x_n y_n z_n) \Big\} \; \varphi(x_m y_m z_{m-1}) \Big] &= 0
\end{aligned}
\tag{099-20}
$$

The second equation which is to be satisfied is the magnetizing curve, from which

$$\mu_n = f(\vec{H}_n) \tag{099-22}$$

Combine (099-2) and (099-22)

$$\mu_n = f(-\nabla\varphi_n) \tag{099-23}$$

(099-20) and (099-23) yield two sets of equations, each p in number, for two sets of p unknowns, μ_n, φ_n, each p in number, therefore they can be solved for these two unknowns in theory.

One equation (099-22) the magnetizing curve, is not an algebraic equation. Therefore the practical solution of the two sets of equations (099-22) and (099-23), for the two sets of unknowns μ_n, φ_n where $n = 1$ to p is proposed as follows.

Assume μ_{rn}-s, $\mu_{rn} \neq 1$ in magnetic materials. (099-24)

Use these μ_{rn}-s in (099-20), n=1 to p (099-25)

(099-20) is then a set of p equations for φ_n, unknowns $n = 1$ to p. Solve (099-20) for φ_n. (099-26)

Calculate H_{nx}, H_{ny}, H_{nz} from (099-4x,y,z). (099-27)

Calculate $H_n = \sqrt{H_{nx}^2 + H_{ny}^2 + H_{nz}^2}$. (097-6) (099-28)

Read μ_{rn}-s from the magnetizing curve. (099-22) (099-29)

Use these μ_{rn}-s in (099-20) and iterate

(099-25 → 26 → 27 → 28 → 29 → 25) (099-30)

until $\mu_{rn\,(\text{last reading})} - \mu_{rn\,(\text{last reading}\,-1)} < \Delta\mu_{rn}$ allowed.

Accept the last resulting H_{nx}, H_{ny}, H_{nz} as the directional components of the m.m.f. gradient.

III. SECTIONS OF THE SOLUTION

1.) The above detailed mathematics show that ${}^\circ\vec{H}_n$ can be calculated independently from the induced magnetic dipole moments. This can be done everywhere, including the magnetic bodies in the space. This m.m.f. grad. ${}^\circ\vec{H}_n$ is assumed to be induced by the electric currents and dipoles independent from the magnetizable materials present in the space. Furthermore, the calculation of the ${}^\circ\vec{H}_n$-s assumes constant and unit permeability in all of these points. The points are taken as the geometric centers of parts of the bodies into which these bodies are to be divided, arbitrarily by the analyst.

Solution: The Halacsy Geometric Method,[4,5] the Halacsy-Clark Oxford paper.[3] The MAFCO code,[6] etc.

2.) After the $^{\circ}\vec{H}_n$ is calculated in all points, n, it can be used in the resulting equation (099-20) of the above described calculation. This equation suits to calculate the total m.m.f. gradient, $\vec{H}$, within the magnetizable bodies and not outside them. Points outside the magnetic bodies can be omitted from this calculation because they have no induced dipoles and so no such point influences any other points.

3.) The m.m.f. gradient in points outside the magnetic body, is the total of the m.m.f grad. $^{\circ}\vec{H}_n$ as calculated in #1 above and the m.m.f. grad. induced by the dipole moments of the points of the magnetizable material. These dipole moments are determined by the $\vec{H}_n$-s calculated according to the above #2. The magnetostatic potentials and m.m.f. gradients are then determined from these dipole moments in a way somewhat similar to the one of #1.

These are then three sections into which the calculation can be divided, a very desirable process for the computerization.

IV. SOLUTION OF THE EQUATIONS IN SECTION #2.

Section #2 is the most complex. Equation (099-20) represents a set of linear equations, p in number, for the magnetic scalar potentials, φ_n, at points n, also p in number.

These equations contain a big array of constants discussed in detail in Appendix IV. and tabulated in Appendix V . Of course, these constants depending on the geometry only must be calculated first.

The solution of the set of p linear equations is proposed by known methods. Matrix-inversion was chosen for the present, an available subroutine in computers.

V COMPUTERIZATION

Section I of the computer program

The first section of the computer-program calculates the field induced by electric currents in empty space(air). This part of the program is basically the same as described in "Computerized Calculation of Three Dimensional Magnetic Fields" by A. A. Halacsy, G. Clark, and J. Dunks, in paper #4, presented at the Second International Conference on Magnet Technology, Oxford, England, 1967. That program was slightly modified by applying it to the NASA-Apollo-Helmholtz coil-pairs used in the test of the present work.

The new program is shown on sheets of a main program, called "Helmholtz Field Calculator" into which the geometrical subroutine "MAGFLD" is inserted. These are basically the same as the "MAIN PROGRAM" and Subroutine "COORD" of the above mentioned paper.

The new subroutine MAGFLD includes not only the geometrical subroutine of the above referred paper but also the calculation of the H-field. The reason for this is that the new program calculates the H not only at one point but at a programmed series of points and steps from point to point in the three dimensions, those points being generated by incrementing their x, y, z Cartesian coordinates by DELTAX, DELTAY and DELTAZ respectively. These A, B and C values are the same as the EX, EY, and EZ eccentricities of the original paper.

This program being geared particularly to the Helmholtz-coils, the subroutine specifies the number SEG=K of segments into which each turn of the Helmholtz-coils is segmented. The number of points defining a turn is then KI = 2K+1, because points are taken at the ends and at the middle of each segment. Of course, the radius of the turn with which the program starts RAD = .7299720550 meters and the half distance, AAZ = .35626873 meters of the two coils. with which the program starts are given in the MAGFLD - subroutine. (See also Fig. 4 on P.70) The subroutine then generates the coordinates X(I), Y(I), And Z(I) of the end points of the segments, I, by stepping from I to I+1 until I becomes KI, the number of the last segment in that turn.

Then the MAGFLD - subroutine calculates the values of the Cartesian components UX, UY, and UZ of the H field normalized with respect to inducing current, stepping from segment to segment and adding the contribution of each segment to the total of the H-values induced by the previous segments.

Having calculated the H induced by the first turn, the program steps to the next turn in the axial direction by increasing the index JZ by one, calculates the H field induced by that turn as before, and so on until all 15 turns are considered.

Then the program steps to the next layer of turns in the radial direction, by increasing the JX index by one, calculates the H induced by this turn, and so on until the JX index reaches 16 which is the total number of layers of turns in the radial direction. This completes the calculation of the H field of one coil. The calculation of the H field induced by the second coil of the Helmholtz pair is implemented by switching the index IUP to IUP+1, that is changing AAZ to -.39382017 m. This calculation proceeds through the loop #11 exactly the same way as the one for the first coil.

The main program then switches to the next point in the z-direction in which the H field is required, by increasing the K-index by one, calculates the H-field by using the MAGFLD subroutine, then switches to the next point in the z direction and so on until the last point specified by the index KZ is reached. After this the main program switches to the next point in the y direction by increasing the J index by one, and runs through all J indices similar to the K indices, then and finally does the same in the x direction by running through all points by switching the I indices.

The resulting Cartesian components AHX, AHY, and AHZ of the H-field, are stored on tape, ready to use in Section 2, and are also printed in the A, B and C matrix form.

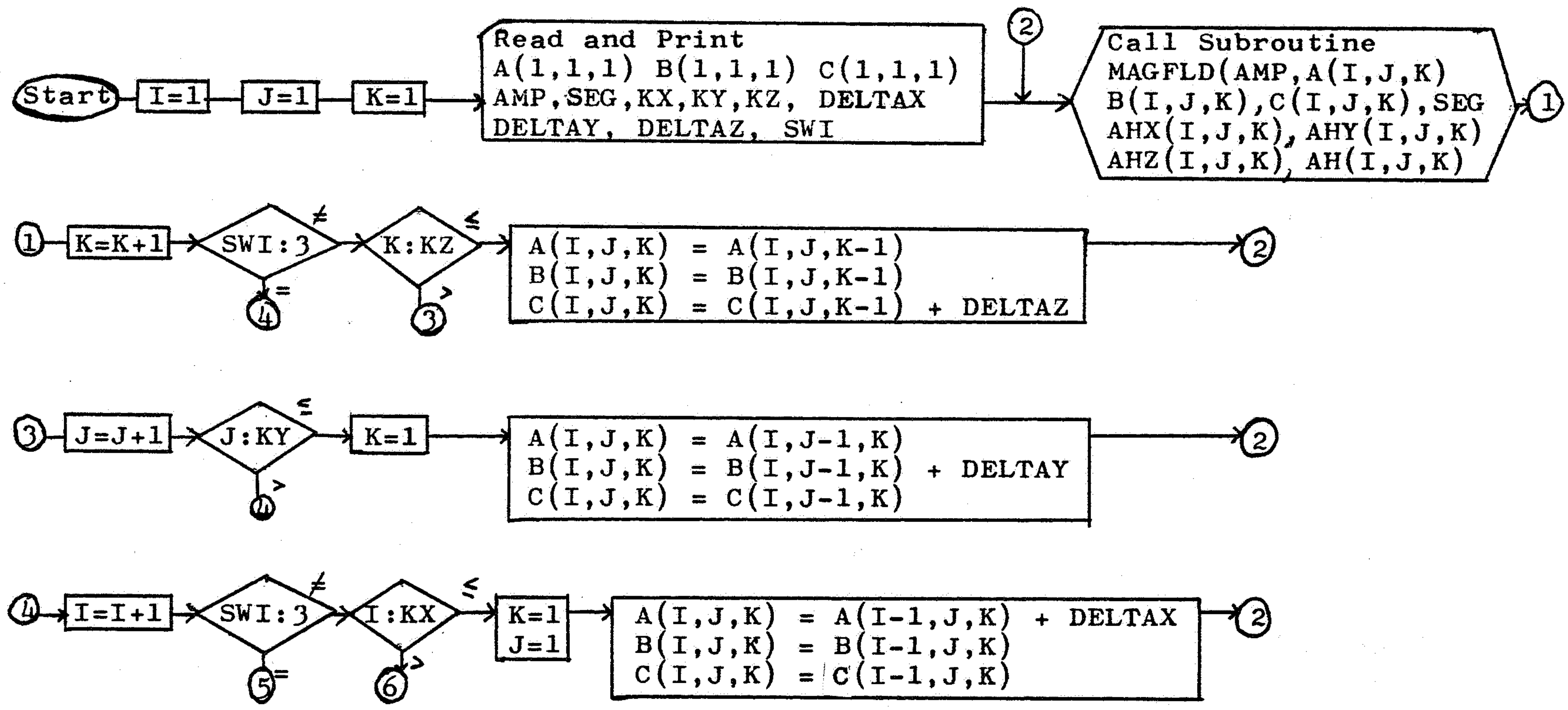
In addition to the above described programs a simplified MAGFLD subroutine was devised.

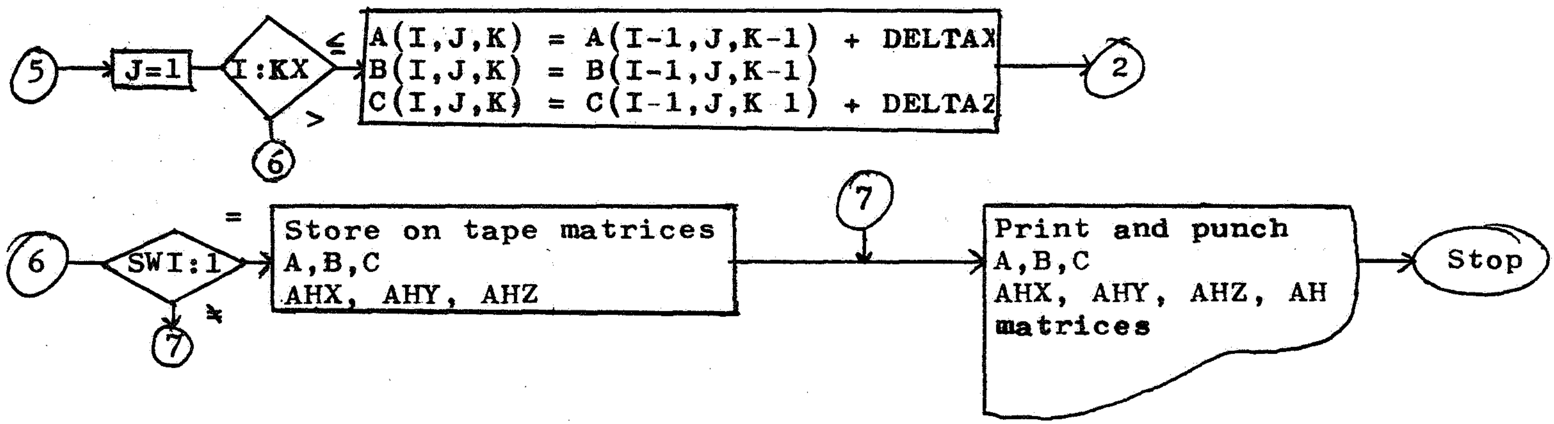
This simplified MAGFLD subroutine lumps three turns in the z axial direction and 4 turns in the x, radial direction into one turn, placed in the geometric center of the lumped 12 turns. Of course, the AMP current value of this imaginary turn is 12 times the AMP value of the original MAGFLD subroutine. Otherwise the simplified MAGFLD subroutine is the same as the original. Due to the lumping of 12 turns into one, the computer time is reduced by a one decade order, and this reduction was the reason for writing the simplified MAGFLD subroutine. It was estimated that the simplified subroutine still will provide sufficient accuracy in the investigated case. Both versions were run, and the results compared. The difference was negligible, therefore, the simplified subroutine was used in this work.

In order to facilitate the understanding of these programs, a List of Symbols is attached.

HELMHOLTZ FIELD CALCULATOR

FLOW SHEET




5
J=1
I:KX
≤
>
6
A(I,J,K) = A(I-1,J,K-1) + DELTAX
B(I,J,K) = B(I-1,J,K-1)
C(I,J,K) = C(I-1,J,K 1) + DELTAZ
2
6
SWI:1
=
≠
7
Store on tape matrices
A,B,C
AHX, AHY, AHZ
7
Print and punch
A,B,C
AHX, AHY, AHZ, AH
matrices
Stop

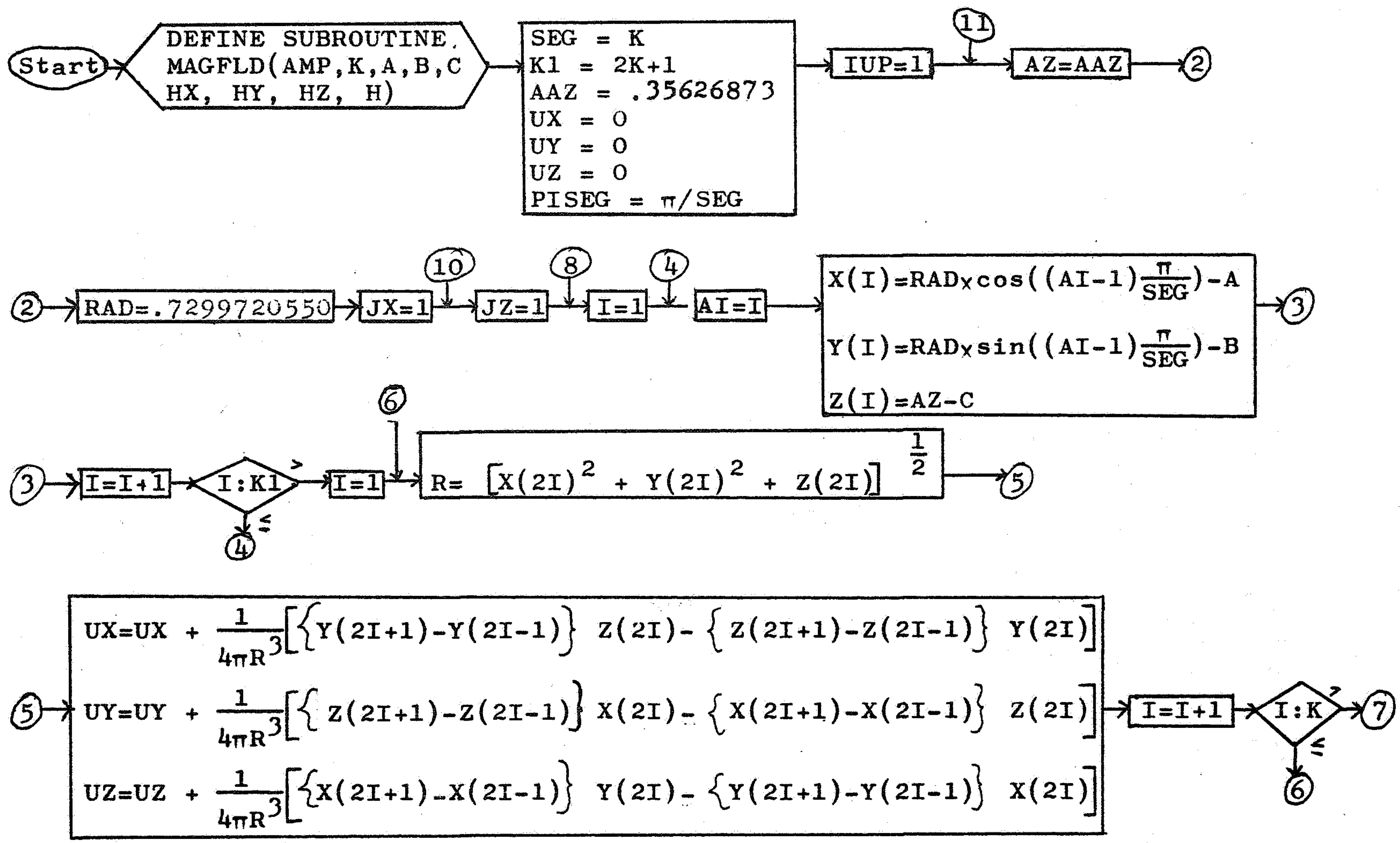
HELMHOLTZ FIELD CALCULATOR

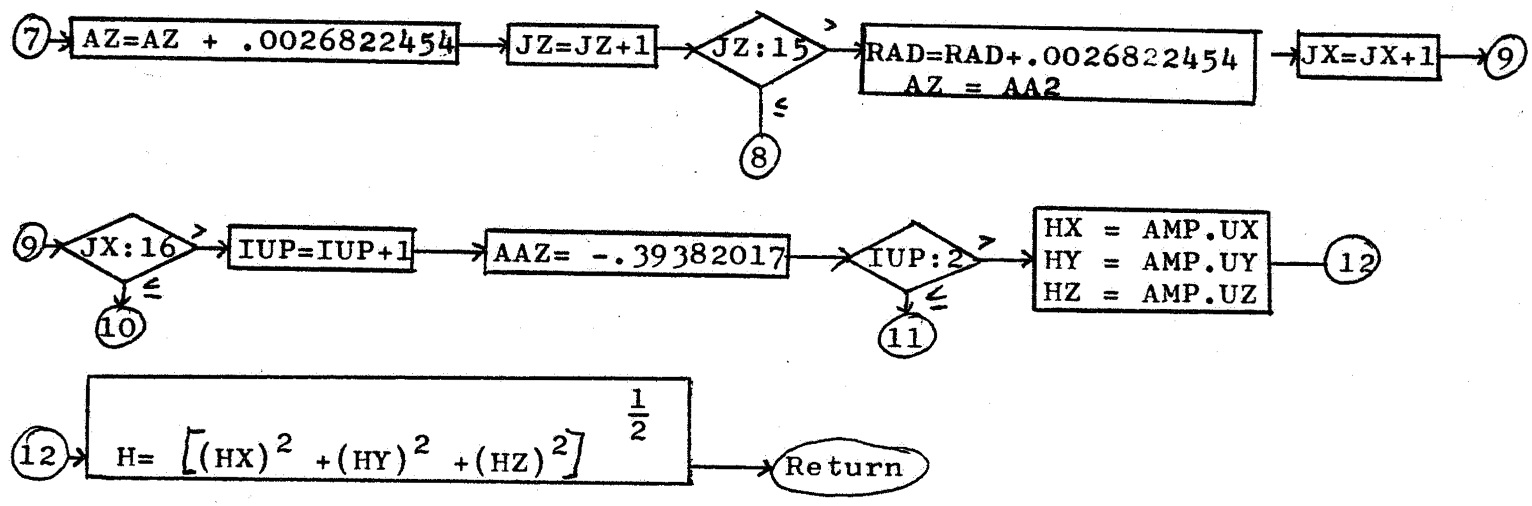
SYMBOLS

A	matrix for the x-coordinates of points where the field is calculated; meter
AH	matrix for the resultant field strength of the desired point; ampereturn/meter
AHX	matrix for the x-component of the resultant field; ampereturn/meter
AHY	matrix for the y-component of the resultant field; ampereturn/meter
AHZ	matrix for the z-component of the resultant field; ampereturn/meter
AMP	current in coil; amperes
B	matrix for y-coordinates of points where field is calculated; meter
C	matrix for z-coordinates of points where field is calculated; meter
DELTAX DELTAY DELTAZ	x, y, z directional increments for generating points; meter
I J K	indices for a three-dimensional index of points; numeric
KX	number of x coordinate values for points generated; numeric
KY	number of y coordinate values for points generated; numeric
KZ	number of z coordinate values for points generated; numeric
SEG	number of segments per turn; numeric
SWI	an index for separating four different configurations of points; numeric

SUBROUTINE MAGFLD

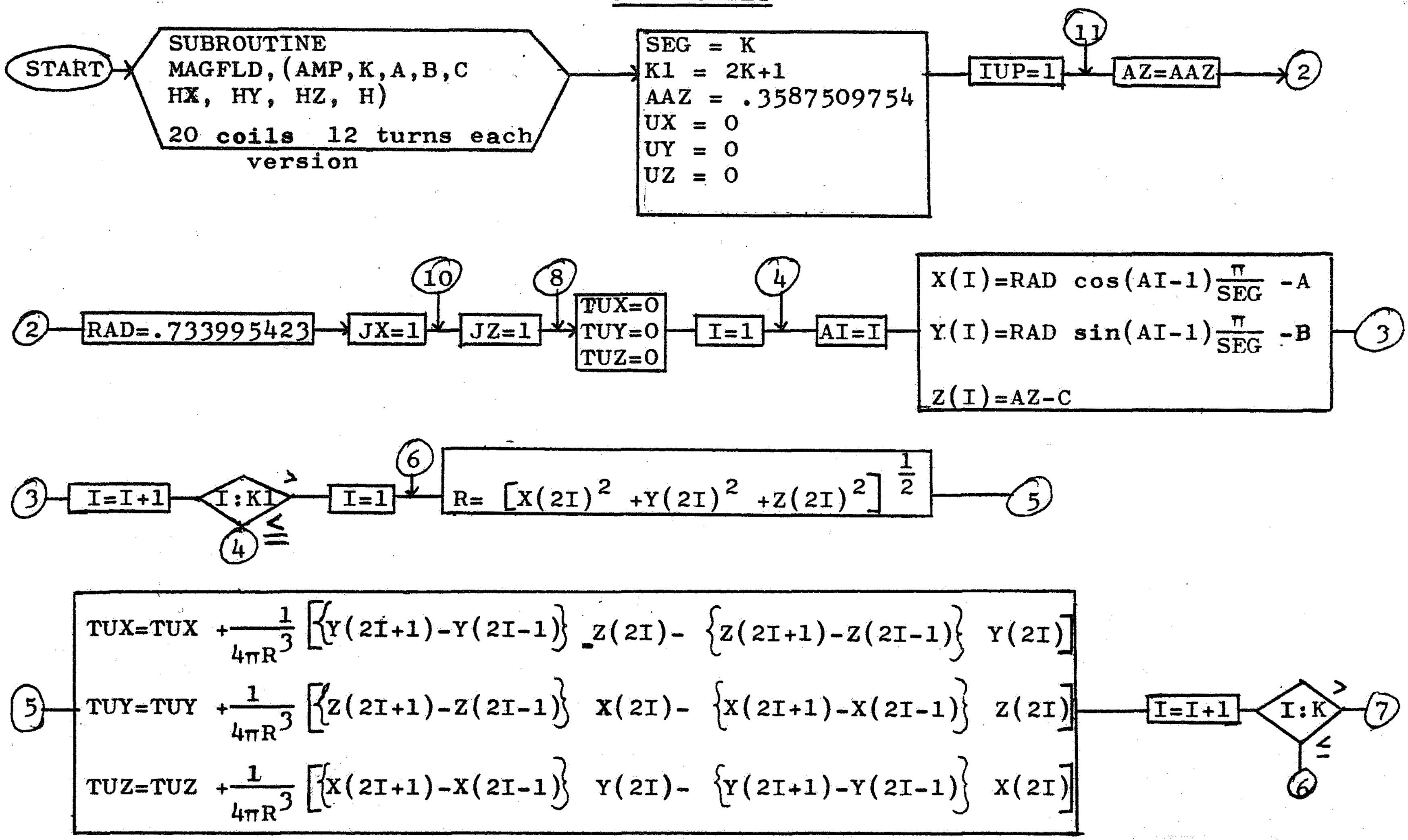
FLOW SHEET

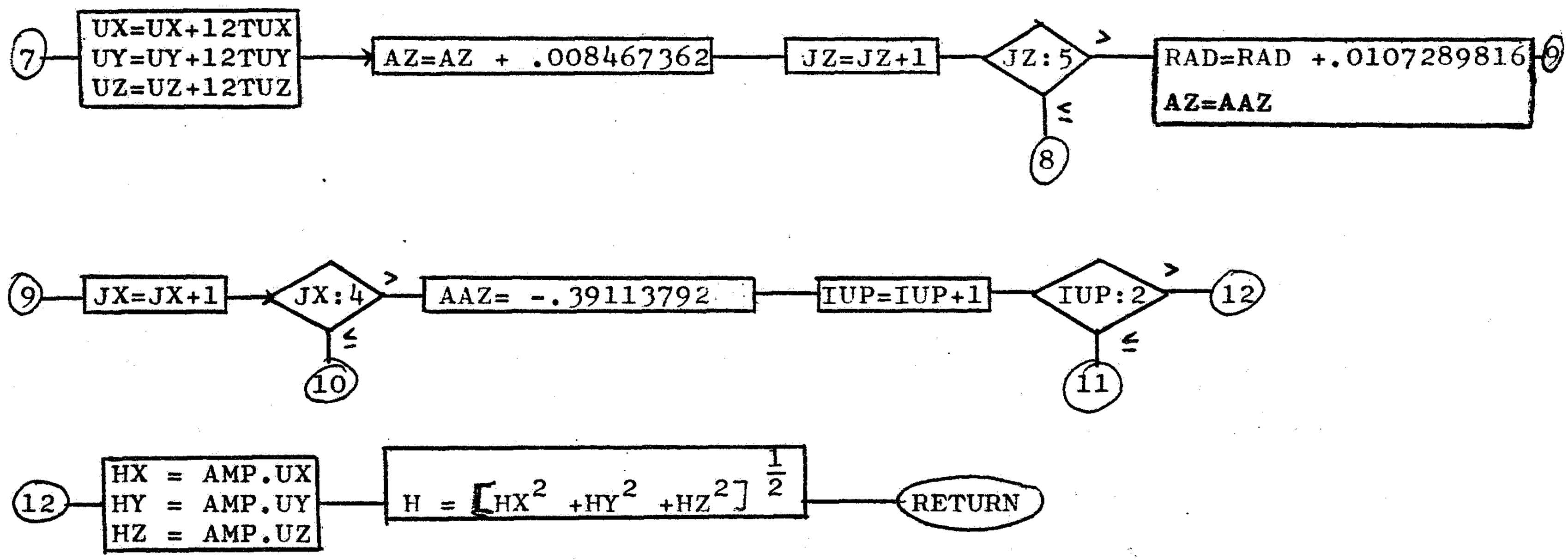



7
AZ=AZ + .0026822454
JZ=JZ+1
JZ:15
>
≤
8
RAD=RAD+.0026822454
AZ = AA2
JX=JX+1
9
9
JX:16
>
≤
10
IUP=IUP+1
AAZ= -.39382017
IUP:2
>
≤
11
HX = AMP.UX
HY = AMP.UY
HZ = AMP.UZ
12
12
H= [(HX)^2 +(HY)^2 +(HZ)^2]^1/2
Return

SIMPLIFIED SUBROUTINE MAGFLD

FLOW SHEET





7
UX=UX+12TUX
UY=UY+12TUY
UZ=UZ+12TUZ
AZ=AZ + .008467362
JZ=JZ+1
JZ:5
>
≤
8
RAD=RAD +.0107289816
AZ=AAZ
9
9
JX=JX+1
JX:4
>
≤
10
AAZ= -.39113792
IUP=IUP+1
IUP:2
>
≤
11
12
12
HX = AMP.UX
HY = AMP.UY
HZ = AMP.UZ
H = [HX² +HY² +HZ²]^(1/2)
RETURN

SUBROUTINE MAGFLD

SYMBOLS

AAZ	half axial distance of the two Helmholtz-coils ; meter
AMP	coil current; amperes
AZ	z direction reference for coils
A, B, C	eccentricities; meter
H	resultant field strength in ampturn/ meter
HX	x component of field strength; ampturn/meter
HY	y component of field strength; ampturn/meter
HZ	z component of field strength; ampturn/meter
I	index for segments
IUP	index for separating calculations for the two coil halves
JX	index for turns in the x direction
JZ	index for turns in the z direction
K	number of segments/turn; numeric
KI	number of points defining a turn; numeric
PISEG	3.1415927 / segments
R	distance from midpoint of segment to the point of which the field is calculated; meter
RAD	radius of a turn of the Helmholtz-coil; meter
SEG	number of segments/turn; numeric
UX, UY, UZ	normalized component values of the computed field strength with respect to the current, ampturn/meter induced by 1 ampere current in the conductors.

$\left.\begin{array}{l}X\\Y\\Z\end{array}\right\}$ arrays for end points of line segments approximating a single turn.

Section II of the Computer Program

The second section of the computer program calculates the scalar magnetic potential, φ, and the relative permeability, μ, in a series of points in the magnetic material which is placed in the space in which the H-field in empty space was calculated in Section I.

The symbol used for φ is PHI and the one used for μ relative is UR.

The main program for Section II is called DIPOLE PROGRAM, (097-140) and there are three subroutines within the main program, subroutine PHICAL calculating the scalar magnetic potential PHI, subroutine HCAL calculating the three Cartesian components of the m.m.f. gradient, H in the points where PHICAL calculated PHI, and the subroutine PERM calculating the relative permeability, UR, in the same points, by reading the values of UR against the values of H, from the permeability-curve of the material and given by test.

The DIPOLE PROGRAM reads and prints the maximum number IP, JP, KP of points in the magnetic material and adjacent to it in which points the H field is to be calculated, reads LIMIT, the maximum number of iterations allowed, EPSI, the accuracy-limit for the relative permeability, ITOT, the number of points on the permeability curve, VOL, the total volume of the ferro-magnetic material, TOL, a small number used to check if the determinant is not zero. Then it reads from the tape-output of Section I the coordinates X(I,J,K), Y(I,J,K), Z(I,J,K) and the Cartesian component values AHX, AHY, and AHZ of the m.m.f. gradient H calculated in the x, y, z points by the HELMHOLTZ FIELD CALCULATOR in Section I, also from the storage-tape resulting from Section I.

Then arbitrary starting values of the permeability are set up in a matrix UR1(I,J,K) and the permeability curve is read in a matrix form, HUR(I,J).

The LIM is set to 1. All required values are read and matrices set up by then, and subroutine PHICAL is called.

Subroutine PHICAL

Subroutine PHICAL is the backbone of the whole program, and it calculates the magnetic scalar potential in specified points in the magnetic material where dipole moments are induced by the electric currents considered in Section I of the program. It does this by setting up the equations (099-20) and solving them by a matrix inversion.

In PHICAL, a point, m, is specified by the indices, I,J,K of its three Cartesian coordinates, and a point, n, is specified by the indices, L,M,N of its three Cartesian coordinates. Before the starting values of these indices are specified, the form of a two-dimensional matrix, AMA(I,J) is set up. This matrix will be the matrix of the set of equations for the magnetic scalar potential. The I's and J's in this matrix are the indices of its rows and columns respectively. Between Steps #1 and #3, both the rows and columns are set up for the total number of points ITP, in the magnetic material.

The computer can handle only finite changes, whereas the change in permeability is step-wise at the boundary between the ferromagnetic material and air. This difficulty is avoided by setting two layers of points in air enveloping the ferromagnetic material. The permeability is set equal to one in these points.

This is done in the computer program by specifying one more point in each of the x,y, and z directions on each side of the material, that is a total of two more points included in IP, JP, KP, along each line of points. This makes the total number of points in the ferromagnetic material AITP = (IP - 2)(JP - 2)(KP-2).

The volume, VM, of an element, that is the volume centered on an m point is either inserted directly or calculated before Step #3. If the magnetic material is in a shape of a parallelepiped, then one volume element is the total volume VOL of the parallelepiped (X(IP-2, JP-2, KP-2) - X(2,2,2))*(Y(IP-2,JP-2,KP-2) - Y(2,2,2))* (Z(IP-2, JP-2,KP-2) -Z(2,2,2)) divided by the total number of volume elements ITPI = (IP - 2)* (JP - 2)* (KP - 2).

The number of points in the three directions can be different IP, JP, KP and so can be their separation, given as DELTAX, DELTAY, DELTAZ in the Helmholtz calculator. Therefore, this program can be used for all type of parallelepipeds. For other geometries, only the generation of points and the calculation of VM is to be changed.

The geometry is shown in Fig. 5, p. 71.

After the base volume VM is calculated, the first point, n, is selected by setting its indices L,M,N to 2. In each direction the first point (L=1, or M=1, or N=1) is in air. The first point in iron is the second point in this direction, and this is the reason why the starting indices, L,M,N, are set to 2.

The value of the relative permeability for air is UR = 1. This is used if the test of the indices between the entry #22 and #5 proves the point being in air.

The partial derivatives

$$\frac{\partial \mu_n}{\partial x_n} = URX, \quad \frac{\partial \mu_n}{\partial y_n} = URY, \quad \frac{\partial \mu_n}{\partial z_n} = URZ$$

of the permeability, UR are calculated in the x,y,z direction next, (after step 5) and also the partial derivatives

$$\frac{\partial H_n}{\partial x_n} = AHXX, \quad \frac{\partial H_n}{\partial y_n} = AHYY, \quad \frac{\partial H_n}{\partial z_n} = AHZZ$$

the x,y,z components of H(before step 6). The proper values of μ_n in air are set in by testing the indices L,M,N and as explained above.

Now the indices I,J,K of a point m are set to their starting value (2,2,2) and compared with the indices L,M,N of the point n, between steps 6 and 7. As shown in the description of the mathematics, m=n must be excluded. This is the case if all three indices I,J,K are the same as L,M,N respectively. In this case the program goes to step 17, increases first K by 1, tests it against the maximum KP and if it is less than KP, returns to between steps 6 and 7, via 18. If the increased value of K is begger than KP, then J is increased and tested, and the program proceeds similar to the result of K. A similar procedure follows for I.

When the indices I,J,K are set and accepted at step #7, then the differences XR, YR, ZR of the coordinates of points n and m are calculated, and from them the distance R between the point n and m.

As soon as R is available, the C constants are calculated between steps 7 and 13.

At this point a transformation of the indices is required. This is necessary because in the matrix of the system of equations for the magnetic scalar potential the dimensionality is two.

Therefore, the subscript for a point (L,M,N) or (I,J,K) must be transformed to a single subscript for the final array.

Recall that L,M,N denote points, n, of the same region as the points, m, denoted by I,J,K. L,M,N corresponding to a point n, denotes a row of the matrix, while I,J,K corresponding to a point m, denotes a column of the matrix.

A simple addition, L+M+N would not yield a single valued subscript because it would result the same subscript for six points, e.g. 1+2+3 = 2+1+3 = 3+1+2 = 3+2+1 = 1+3+2 = 2+3+1.

Therefore, M is multiplied by the greatest N which is the same as the greatest K and is KP. It follows that the second term of the sum is always larger than and never can be the same as the third one.

Similarly, L is multiplied by the greatest N, which is KP and the result is further multiplied by the greatest M, which is JP, the same as the greatest J. It follows that the first term of the sum is always larger than the third and second and never can be the same.

The inversion formula resulting in a new and single subscript is then

$$IA = JP \cdot KP \cdot L + KP \cdot M + N$$

The smallest value of L,M and N is 1, their largest value IP, JP, KP respectively.

This is done before step 10.

At step 10, the calculation of the terms of the matrix for the magnetic scalar potential begins.

The block after step 10 is the starting term B of a row of the matrix. This term is the first part of equ. 097-20 the term independent of PHI.

Then the calculation of the term AM(I,J,K) follows.

Now a part DUM of a matrix term is calculated and temporarily stored. Then another index-transformation is made to have the index JA denoting the column of the matrix. This index-transformation is similar to the transformation resulting in the index IA and described above.

The values of the matrix-terms in a column, JA are then calculated between steps 12 and 17, and stored in the matrix AMA(IA,JA).

The indices JA cannot be zero or negative. Therefore, the program tests the indices JA, and skips the addition of a DUM whenever $JA < 1$ or $JA > ITP$.

The summation by which the multipliers of each PHI are built is performed not first for one multiplier, then for the next and so on, but in a mixed fashion by which computer-time is saved.

After all terms for various indices JA are evaluated, at step 17, the program increases the I, or J, or K index by 1 unless the maximum number KP-1, JP-1, and IP-1 is reached and returns via 18, 19, 20 and 22,23,24 to after step 6, to continue the calculation of other terms of the AMA matrix.

As soon as all terms of the AMA matrix are available, the PHICAL subroutine calls the matrix inversion subroutine MINV. This subroutine solves the system of equations for PHI. The resulting values of the scalar magnetic potential appear in B, indexed by IA. This ends the subroutine PHICAL and the program returns to the main DIPOLE PROGRAM.

The DIPOLE PROGRAM prints the matrix AMA and the value of its determinant DET and tests the latter one against TOL.

Now the value of the magnetic scalar potential, PHI is taken from the solution by the matrix inversion where it appears in the B-matrix and the m.m.f. gradient H is calculated by calling subroutine HCAL.

The values of the magnetic scalar potential are given the index IA in the form of B(IA). They are required with I,J,K indices denoting m points.

Therefore, B(IA) is converted to PHI(I,J,K), at step #2 of the DIPOLE PROGRAM. The conversion is made first for IA=1, then for each consecutive IA = IA + 1.

Subroutine HCAL, 097-137

Subroutine HCAL is rather simple as it calculates the three Cartesian components of H by a linear approximation of the three directional partial derivatives of the static magnetic potential, PHI, between steps #4 and #5.

The indices I,J,K are set to be of points in the ferro-magnetic material. Then the components and the total of H is calculated between steps #4 and #6.

The indices are increased by 1 in sequence and the calculation repeated until H is calculated in each point in the ferro-magnetic material.

Then the program returns to the main DIPOLE PROGRAM again, and the subroutine PERM is called.

Subroutine PERM, 097-138

Subroutine PERM reads the value of the permeability UR2 from the permeability curve of the material against values of H-s at point m, indexed by I,J,K.

The H values resulted in the form HN(I,J,K) from subroutine HCAL.

The permeability curve is given in a tabulated form, HUR, which is a two column matrix.

The first HN(I,J,K) with I = 2, J = 2, K = 2 is tested against the first value HUR(1,1) of H in the HUR matrix. If it is smaller, then the permeability UR2(I,J,K) is taken as the permeability HUR(1,2) corresponding to H = HUR(1,2) and stored. If HN(I,J,K) is bigger then the first μ value HUR(IA,1) in the HUR matrix, then the comparison is carried through with the next H value in the HUR matrix and so on until HN(I,J,K) proves to be smaller or equal to an HUR(IA,1) value. In the latter case, the permeability for HN(I,J,K) is taken as the arithmetic mean of the permeability for this HUR(IA,1) and for the previous HUR(IA-1,1) value of the m.m.f. gradient H, see block between step 3 and 8.

Then the next H value HN(I,J,K = K + 1) is taken, etc. until all K-s are used, then the same follows with the J-s and finally with the I-s.

Then the program returns to the main DIPOLE PROGRAM at step 6, the indices I,J,K are reset to their first value which is 2 for each of them, and the new permeability UR2(I,J,K) is tested against the old one UR1(I,J,K). If their difference is bigger than EPSI units, then the new permeability UR2 is adopted, after step 7. Then or when the difference between the new and old permeability is smaller than EPSI, the next point is taken by increasing the index K by 1, the permeability at this point is tested as it was at the previous point, and so on until all K,J and I indexes are used.

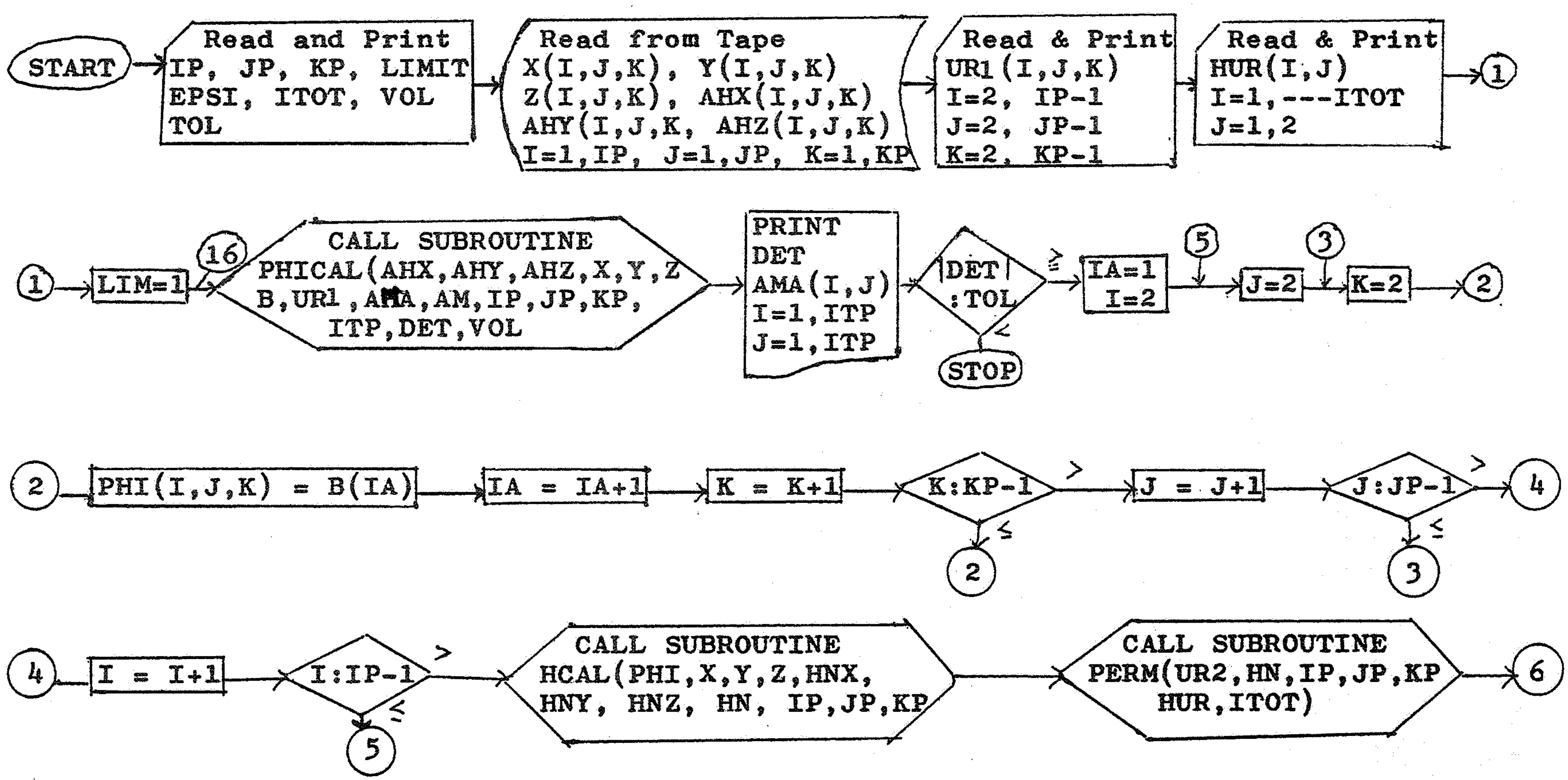
Then the DIPOLE PROGRAM is iterated with the new values of the permeability, UR1 = UR2, until either the difference between consecutive values of permeability becomes less than EPSI or the LIMIT of the number of iterations is reached.

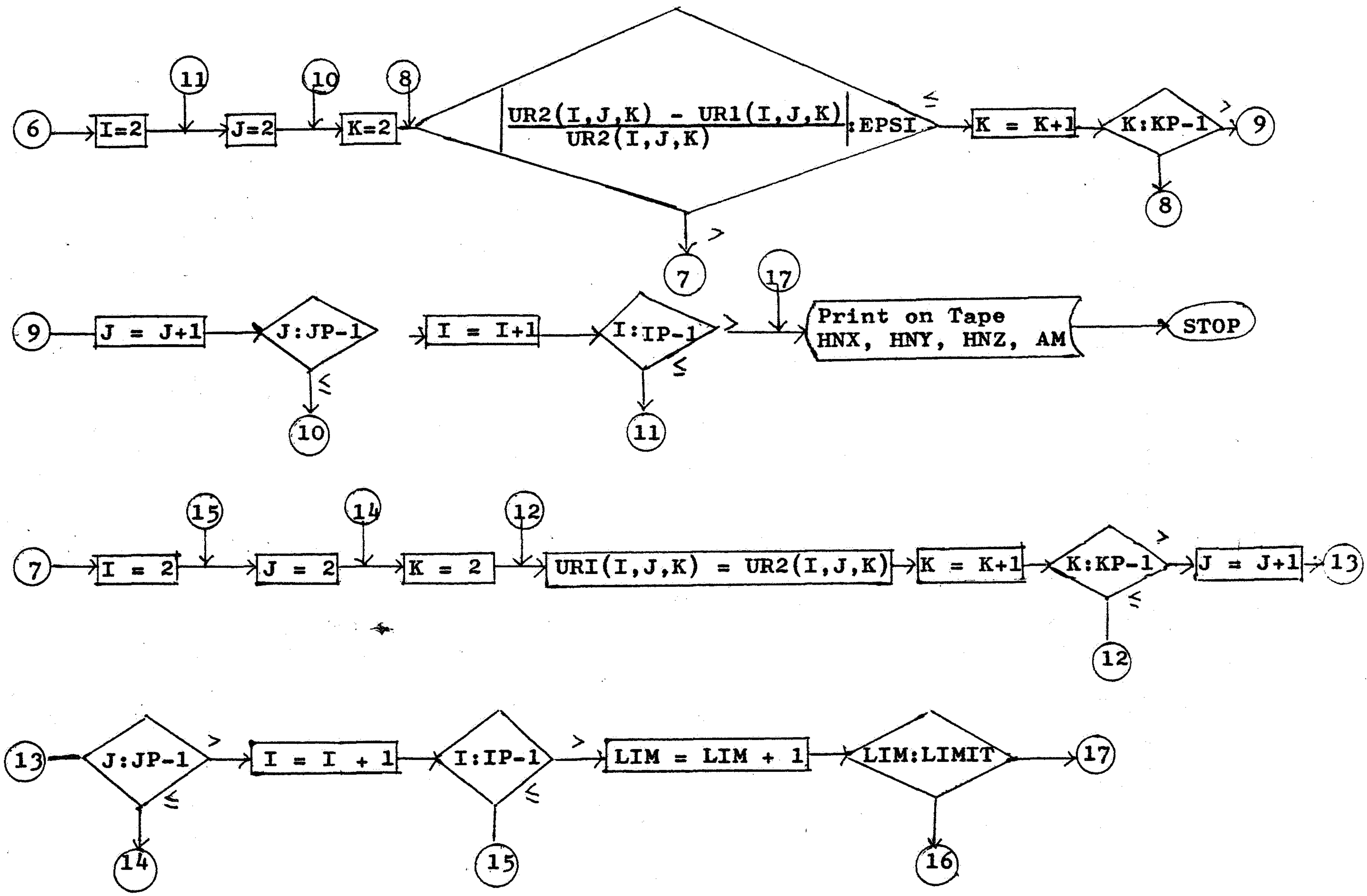
At this point the Cartesian components HNX, HNY, and HNZ of H are accepted and printed.

This completes the calculation of the H-field inside the magnetic body.

DIPOLE PROGRAM

FLOW SHEET





6
I=2
11
J=2
10
K=2
8
|UR2(I,J,K) - UR1(I,J,K) / UR2(I,J,K)| : EPSI
≤
>
7
K = K+1
K:KP-1
>
9
8
9
J = J+1
J:JP-1
≤
10
I = I+1
I:IP-1
>
≤
11
17
Print on Tape
HNX, HNY, HNZ, AM
STOP
7
I = 2
15
J = 2
14
K = 2
12
URI(I,J,K) = UR2(I,J,K)
K = K+1
K:KP-1
>
≤
12
J = J+1
13
13
J:JP-1
>
≤
14
I = I + 1
I:IP-1
>
≤
15
LIM = LIM + 1
LIM:LIMIT
17
16

DIPOLE PROGRAM

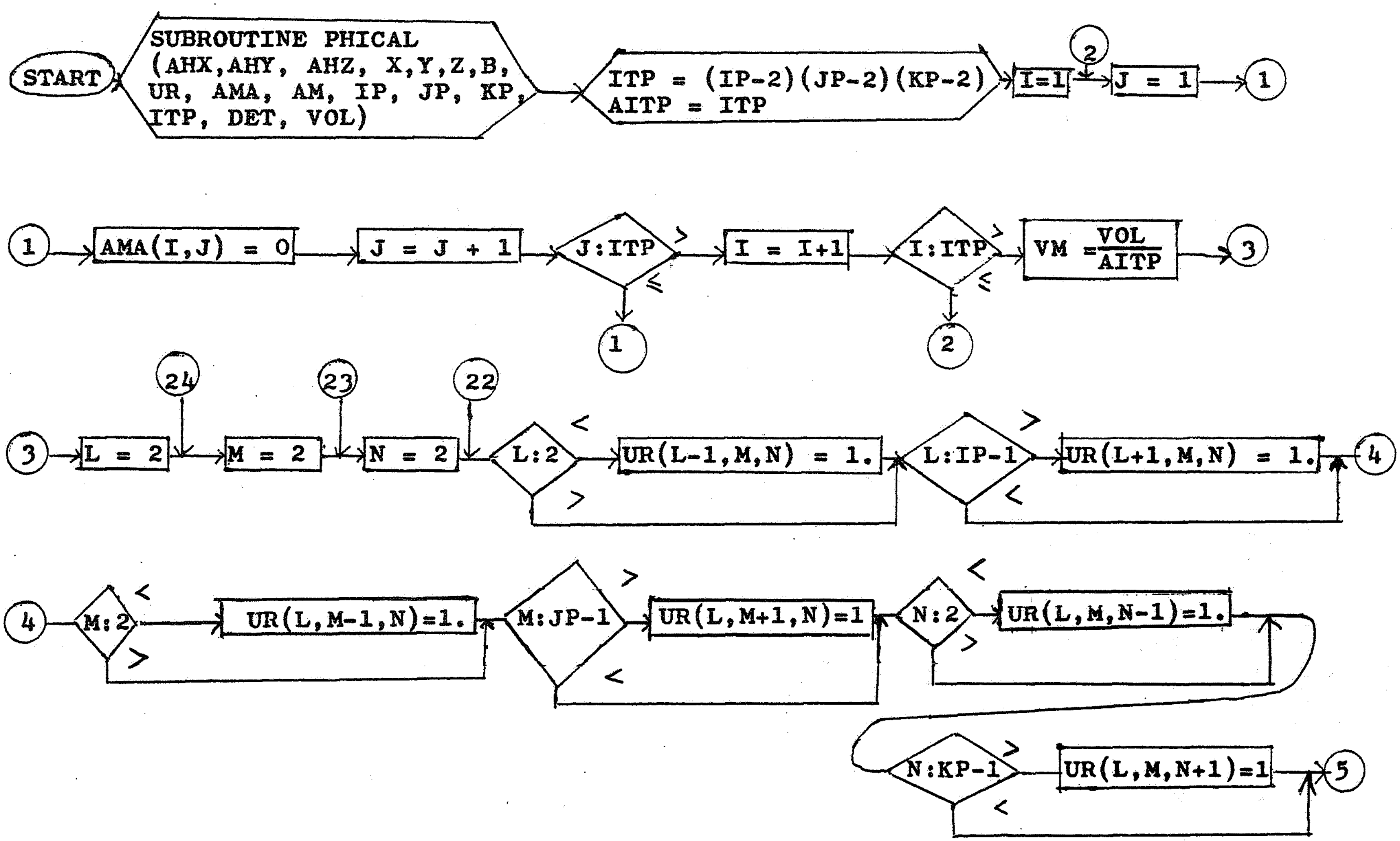
SYMBOLS

AHX AHY AHZ	matrices for x,y,z components of the field strength at points inside the magnetic material assuming relative permeability of unity; ampereturn per meter
AM	matrix of volume constants
AMA	system matrix for the magnetic scalar potentials in magnetic material
B	the terms independent of PHI in the equations. After the matrix inversion, the values of PHI appear as B-s
DET	numeric value of the determinant of the system matrix; numeric
EPSI	limit for permeability accuracy; per unit
HCAL	subroutine for computing the H field from the scalar potentials
HN	matrix of the H field values in the magnetic material; ampereturn per meter
HNX	matrix of the x-component of the HN values; ampereturn per meter
HNY	matrix of the y-component of the HN values; ampereturn per meter
HNZ	matrix of the z-component of the HN values; ampereturn per meter
HUR	matrix for defining the relative permeability curve; numeric
I	index for the x constants; numeric
IP	limit for the x coordinate index of points in the magnetic material; numeric
ITP	number of points in the magnetic material; numeric
ITOT	number of points on the permeability curve; numeric
J	index for the y-coordinates; numeric
JP	limit for the y coordinate index of points in the magnetic material; numeric

K	index for the z-coordinates; numeric
KP	limit for the z-coordinate index of points in the magnetic material; numeric
LIMIT	maximum number of the iterations for testing the permeabilities
PERM	subroutine for computing the new relative permeabilities from the last computed H field
PHI	solution-value for the magnetic scalar potentials at each point in the magnetic material; Ampere-turns
PHICAL	subroutine for computing the magnetic scalar potentials
TOL	a small number to eliminate DET= 0 cases which can not be solved; numeric
UR1	matrix of assumed relative permeabilities; numeric
UR2	matrix of the new computed relative permeabilities; numeric
X, Y, Z	matrices for the x,y, and z coordinates of points in the magnetic material; meter

SUBROUTINE PHICAL

FLOW SHEETS



(5) →

$$URX = \frac{UR(L+1,M,N) - UR(L-1,M,N)}{X(L+1,M,N) - X(L-1,M,N)}$$

$$URY = \frac{UR(L,M+1,N) - UR(L,M-1,N)}{Y(L,M+1,N) - Y(L,M-1,N)}$$

$$URZ = \frac{UR(L,M,N+1) - UR(L,M,N-1)}{Z(L,M,N+1) - Z(L,M,N-1)}$$

→

$$AHXX = \frac{AHX(L+1,M,N) - AHX(L-1,M,N)}{X(L+1,M,N) - X(L-1,M,N)}$$

$$AHYY = \frac{AHY(L,M+1,N) - AHY(L,M-1,N)}{Y(L,M+1,N) - Y(L,M-1,N)}$$

$$AHZZ = \frac{AHZ(L,M,N+1) - AHZ(L,M,N-1)}{Z(L,M,N+1) - Z(L,M,N-1)}$$

→ (6)

(6) → I = 2 → (20) → J = 2 → (19) → K = 2 → (18) → I:L (= → 17; ≠) → J:M (= → 17; ≠) → K:N (= → 17; ≠) → (7)

(7) →

$$XR = X(L,M,N) - X(I,J,K)$$
$$YR = Y(L,M,N) - Y(I,J,K)$$
$$ZR = Z(L,M,N) - Z(I,J,K)$$

→

$$R = \left[XR^2 + YR^2 + ZR^2\right]^{1/2}$$

→

$$CMNIX = \frac{R^2 - 3(XR)^2}{R^5}$$

$$CMNJY = \frac{R^2 - 3(YR)^2}{R^5}$$

$$CMNKZ = \frac{R^2 - 3(ZR)^2}{R^5}$$

→ (8)

(8) →

$$CMNIJ = \frac{-3(XR)(YR)}{R^5}$$

$$CMNJK = \frac{-3(YR)(ZR)}{R^5}$$

$$CMNIK = \frac{-3(XR)(ZR)}{R^5}$$

→

$$CMNIXX = \frac{\left[15\ XR^2 - 9R^2\right] XR}{R^7}$$

$$CMNJYY = \frac{\left[15\ YR^2 - 9R^2\right] YR}{R^7}$$

$$CMNKZZ = \frac{\left[15\ ZR^2 - 9R^2\right] ZR}{R^7}$$

→ (9)

9 →

$$\mathrm{CMNIJJ} = \frac{\left[15\,\mathrm{YR}^2 - 3\mathrm{R}^2\right]\mathrm{XR}}{\mathrm{R}^7}$$

$$\mathrm{CMNIZZ} = \frac{\left[15\,\mathrm{ZR}^2 - 3\mathrm{R}^2\right]\mathrm{XR}}{\mathrm{R}^7}$$

$$\mathrm{CMNJXX} = \frac{\left[15\,\mathrm{XR}^2 - 3\mathrm{R}^2\right]\mathrm{YR}}{\mathrm{R}^7}$$

→

$$\mathrm{CMNJZZ} = \frac{\left[15\,\mathrm{ZR}^2 - 3\mathrm{R}^2\right]\mathrm{YR}}{\mathrm{R}^7}$$

$$\mathrm{CMNKXX} = \frac{\left[15\,\mathrm{XR}^2 - 3\mathrm{R}^2\right]\mathrm{ZR}}{\mathrm{R}^7}$$

$$\mathrm{CMNKYY} = \frac{\left[15\,\mathrm{YR}^2 - 3\mathrm{R}^2\right]\mathrm{ZR}}{\mathrm{R}^7}$$

→

$$\mathrm{IA} = (\mathrm{KP}-2)(\mathrm{JP}-2)(\mathrm{L}-2) + (\mathrm{KP}-2)(\mathrm{M}-2) + \mathrm{N} - 1$$

→ 10

10 →

$$\mathrm{B(IA)} = -\mathrm{AHX(L,M,N)}\cdot\mathrm{URX} - \mathrm{AHY(L,M,N)}\cdot\mathrm{URY} - \mathrm{AHZ(L,M,N)}\cdot\mathrm{URZ} - \mathrm{UR(L,M,N)}\left[\mathrm{AHXX} + \mathrm{AHYY} + \mathrm{AHZZ}\right]$$

→

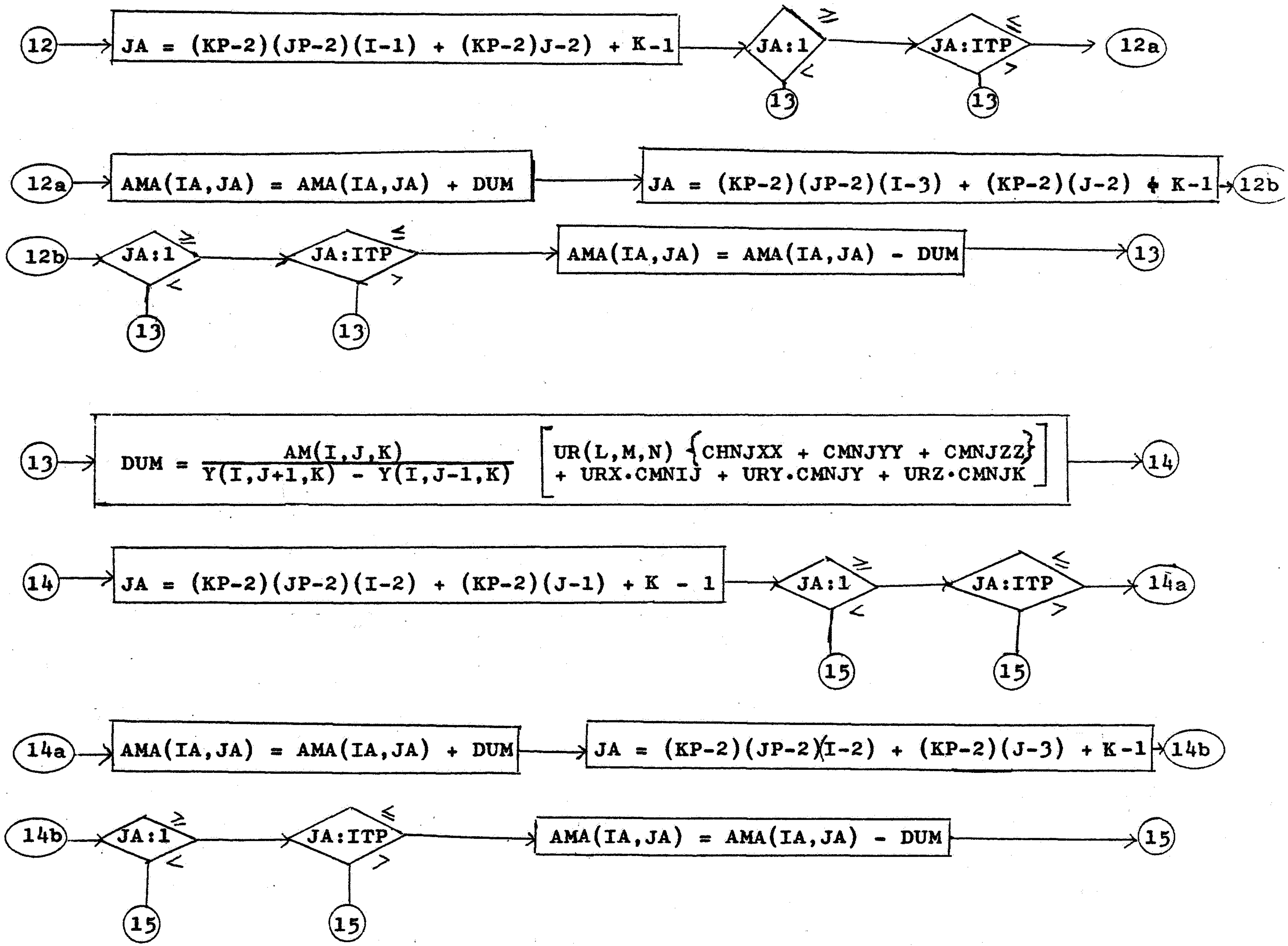
$$\mathrm{AM(I,J,K)} = \frac{1}{4\pi}\cdot\mathrm{VM(UR(I,J,K)} - 1)$$

→ 11

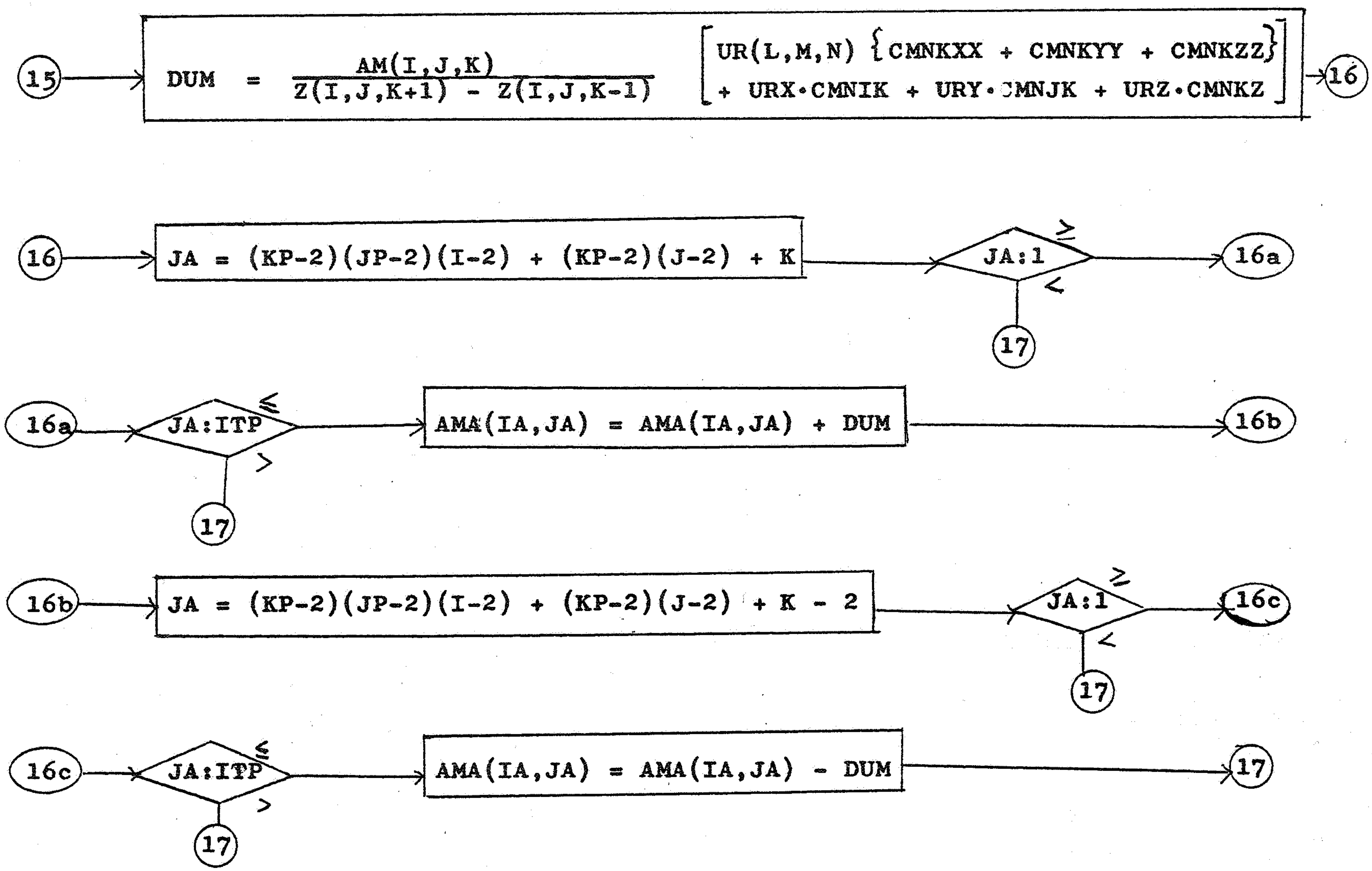
11 →

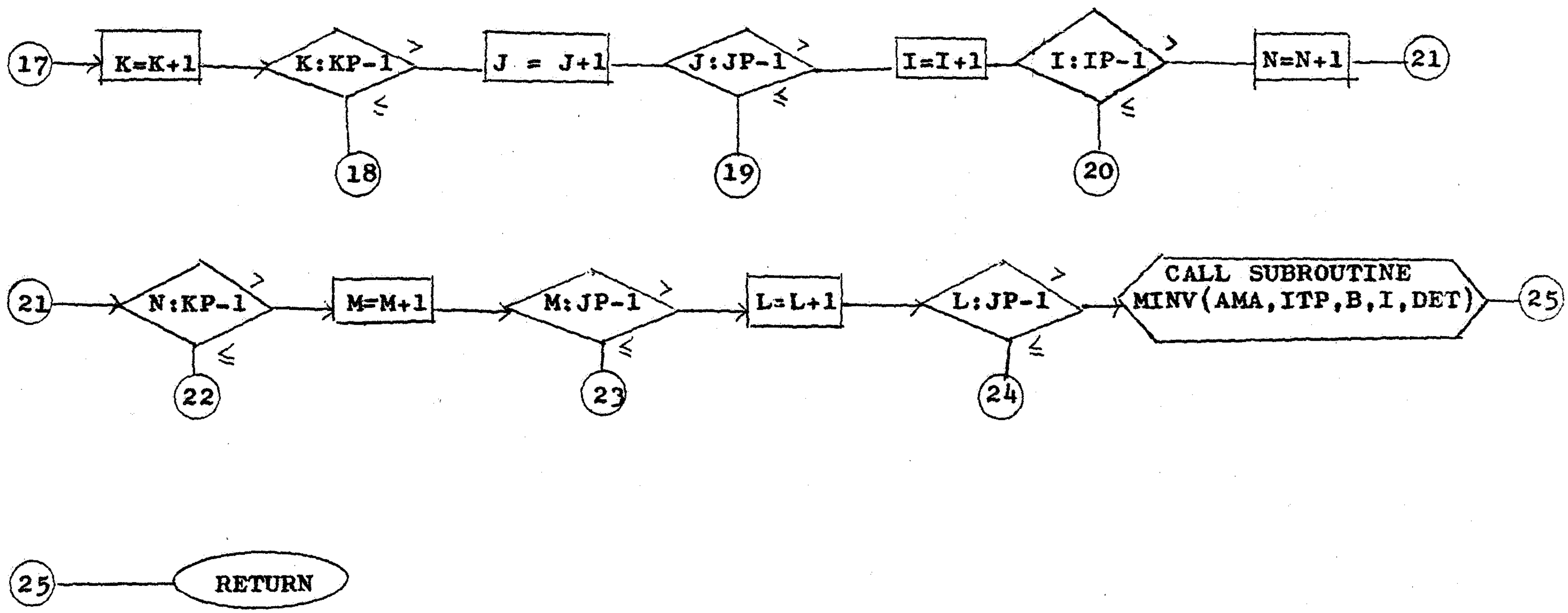
$$\mathrm{DUM} = \frac{\mathrm{AM(I,J,K)}}{\mathrm{X(I+1)J,K)} - \mathrm{X(I-1,J,K)}}\left[\mathrm{UR(L,M,N)}\left\{\mathrm{CMNIXX} + \mathrm{CMNIYY} + \mathrm{CMNIZZ}\right\} + \mathrm{URX}\cdot\mathrm{CMNIX} + \mathrm{URY}\cdot\mathrm{CMNIJ} + \mathrm{URZ}\cdot\mathrm{CMNIK}\right]$$

→ 12



12
JA = (KP-2)(JP-2)(I-1) + (KP-2)J-2) + K-1
JA:1
≥
<
13
JA:ITP
≤
>
13
12a
12a
AMA(IA,JA) = AMA(IA,JA) + DUM
JA = (KP-2)(JP-2)(I-3) + (KP-2)(J-2) + K-1
12b
12b
JA:1
≥
<
13
JA:ITP
≤
>
13
AMA(IA,JA) = AMA(IA,JA) - DUM
13
13
DUM = AM(I,J,K) / Y(I,J+1,K) - Y(I,J-1,K) [UR(L,M,N) {CHNJXX + CMNJYY + CMNJZZ} + URX·CMNIJ + URY·CMNJY + URZ·CMNJK]
14
14
JA = (KP-2)(JP-2)(I-2) + (KP-2)(J-1) + K - 1
JA:1
≥
<
15
JA:ITP
≤
>
15
14a
14a
AMA(IA,JA) = AMA(IA,JA) + DUM
JA = (KP-2)(JP-2)(I-2) + (KP-2)(J-3) + K-1
14b
14b
JA:1
≥
<
15
JA:ITP
≤
>
15
AMA(IA,JA) = AMA(IA,JA) - DUM
15


15
DUM = AM(I,J,K) / (Z(I,J,K+1) - Z(I,J,K-1)) [UR(L,M,N) {CMNKXX + CMNKYY + CMNKZZ} + URX·CMNIK + URY·CMNJK + URZ·CMNKZ]
16
16
JA = (KP-2)(JP-2)(I-2) + (KP-2)(J-2) + K
JA:1
≥
<
16a
17
16a
JA:ITP
≤
>
AMA(IA,JA) = AMA(IA,JA) + DUM
16b
17
16b
JA = (KP-2)(JP-2)(I-2) + (KP-2)(J-2) + K - 2
JA:1
≥
<
16c
17
16c
JA:ITP
≤
>
AMA(IA,JA) = AMA(IA,JA) - DUM
17
17


17
K=K+1
K:KP-1
>
≤
18
J = J+1
J:JP-1
>
≤
19
I=I+1
I:IP-1
>
≤
20
N=N+1
21
21
N:KP-1
>
≤
22
M=M+1
M:JP-1
>
≤
23
L=L+1
L:JP-1
>
≤
24
CALL SUBROUTINE
MINV(AMA,ITP,B,I,DET)
25
25
RETURN

SUBROUTINE PHICAL

SYMBOLS

AHX — matrix for the H field in the magnetic field in the x-direction assuming $\mu_r=1$; ampereturn per meter

AHXX — partial derivative of AHX in the x direction

AHY — matrix for the H field in the magnetic field in the y direction assuming $\mu_r=1$; ampereturn per meter

AHYY — partial derivative of the AHY in the y direction

AHZ — matrix for the H field in the magnetic field in the z direction assuming $\mu_r=1$; ampereturn per meter

AHZZ — partial derivative of the AHZ in the z direction

AM — volume constant matrix

AMA — matrix for the system of magnetic scalar potential equations

B — the terms independent of PHI in the equation. After the matrix inversion the values of PHI appear in B.

CMNIJ $= \text{CMNJX} = \text{CMNIY} = \dfrac{\partial}{\partial y} \dfrac{\cos(ir_{mn})}{r_{mn}^2} = \dfrac{\partial}{\partial x} \dfrac{\cos(jr)}{r^2}$

CMNIK $= \text{CMNKX} = \text{CMNIZ} = \dfrac{\partial}{\partial z} \dfrac{\cos(ir)}{r^2} = \dfrac{\partial}{\partial x} \dfrac{\cos(kr)}{r^2}$

CMNIX $= \dfrac{\partial}{\partial x} \dfrac{\cos(ir)}{r^2}$

CMNIXX $= \dfrac{\partial}{\partial x}$ CMNIX

CMNIY $= \dfrac{\partial}{\partial x} \dfrac{\cos(jr)}{r^2}$

CMNIYY $= \dfrac{\partial}{\partial y}$ CMNIY

CMNIZ $= \frac{\partial}{\partial x} \frac{\cos(kr)}{r^2}$

CMNIZZ $= \frac{\partial}{\partial z}$ CMNIZ

CMNJK $=$ CMNKY $=$ CMNJZ $= \frac{\partial}{\partial z} \frac{\cos(jr)}{r^2} = \frac{\partial}{\partial y} \frac{\cos(kr)}{r^2}$

CMNJY $= \frac{\partial}{\partial y} \frac{\cos(jr)}{r^2}$

CMNJXX $= \frac{\partial}{\partial x}$ CMNJX

CMNJYY $= \frac{\partial}{\partial y}$ CMNJY

CMNJZZ $= \frac{\partial}{\partial z}$ CMNJZ

CMNKZ $= \frac{\partial}{\partial z} \frac{\cos(kr)}{r^2}$

CMNKXX $= \frac{\partial}{\partial x}$ CMNKX

CMNKYY $= \frac{\partial}{\partial y}$ CMNKY

CMNKZZ $= \frac{\partial}{\partial z}$ CMNKZ

DET determinant value; numeric

DUM temporary storage area

IA a single valued index equivalent to a 3 character index. This new index denotes a row of a matrix; numeric

IP number of the x coordinate values of points; numeric

ITP total number of points in magnetic material; numeric

ITP1 number of volume elements; numeric

JA a single valued index equivalent to a 3 character index. This new index denotes a column of a matrix; numeric

JP number of the y coordinate values of points; numeric

KP number of the z coordinate values of points; numeric

I, J, K indices of the x,y,z coordinates of a point m. These indices are used to select the remaining points after removal of a point, n, selected by L,M,N indices; numeric

L, M, N indices of the x,y,z coordinates of a point, m. These indices are used for selecting one point, n, to sum up dipole effects from all other points in the magnetic material; numeric

MINV subroutine for solving sets of simultaneous linear equations

UR relative permeability matrix; numeric

URX $\frac{\partial}{\partial x}$ UR

URY $\frac{\partial}{\partial y}$ UR

URZ $\frac{\partial}{\partial z}$ UR

VM volume of an element associated with a point

X matrix for the x coordinates of points; meter

XR x component of the distance between two points, m and n; meter

Y matrix for the y coordinates of points; meter

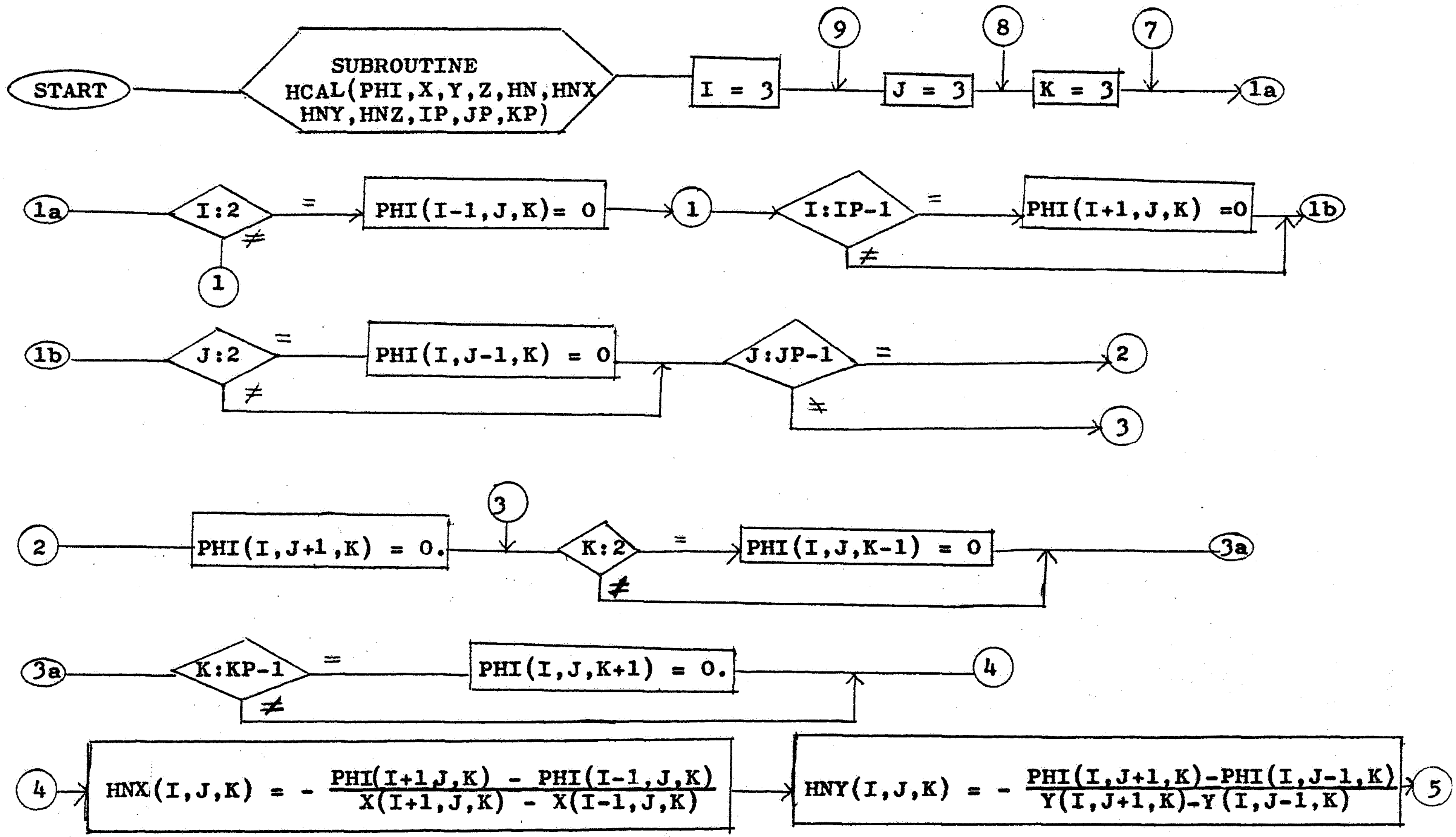
YR y component of the distance between two points, m and n; meter

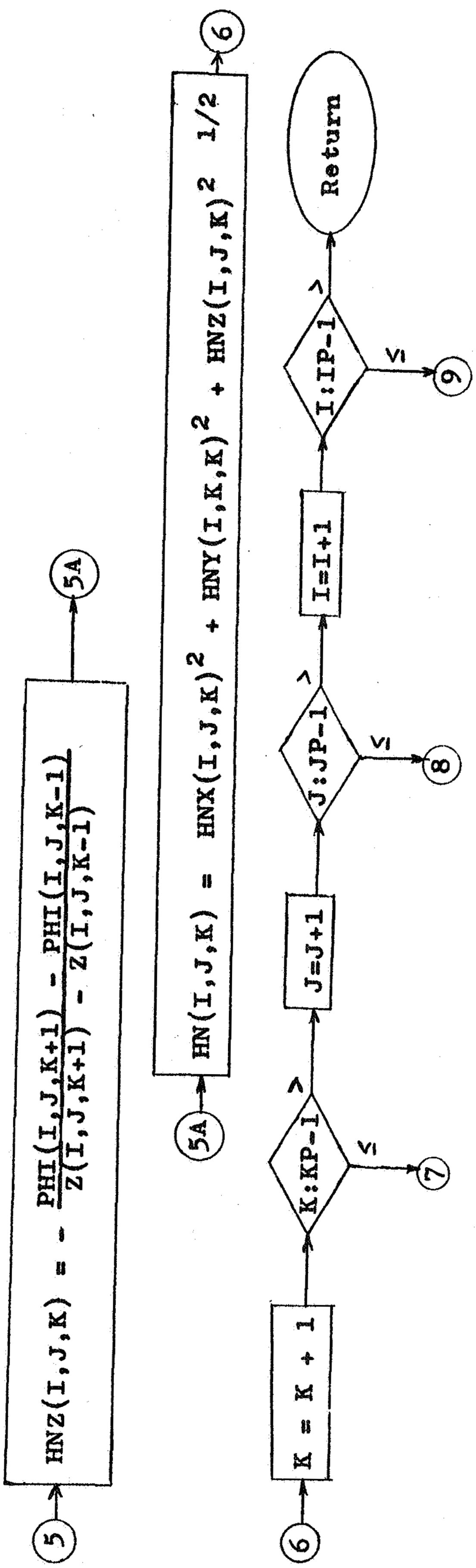
Z matrix for the z coordinates of points; meter

ZR z component of the distance between two points, m and n; meter

SUBROUTINE HCAL

FLOW SHEETS





5
HNZ(I,J,K) = - PHI(I,J,K+1) - PHI(I,J,K-1) / Z(I,J,K+1) - Z(I,J,K-1)
5A
HN(I,J,K) = HNX(I,J,K)^2 + HNY(I,K,K)^2 + HNZ(I,J,K)^2 1/2
6
K = K + 1
K:KP-1
≤
>
7
J=J+1
J:JP-1
8
I=I+1
I:IP-1
9
Return

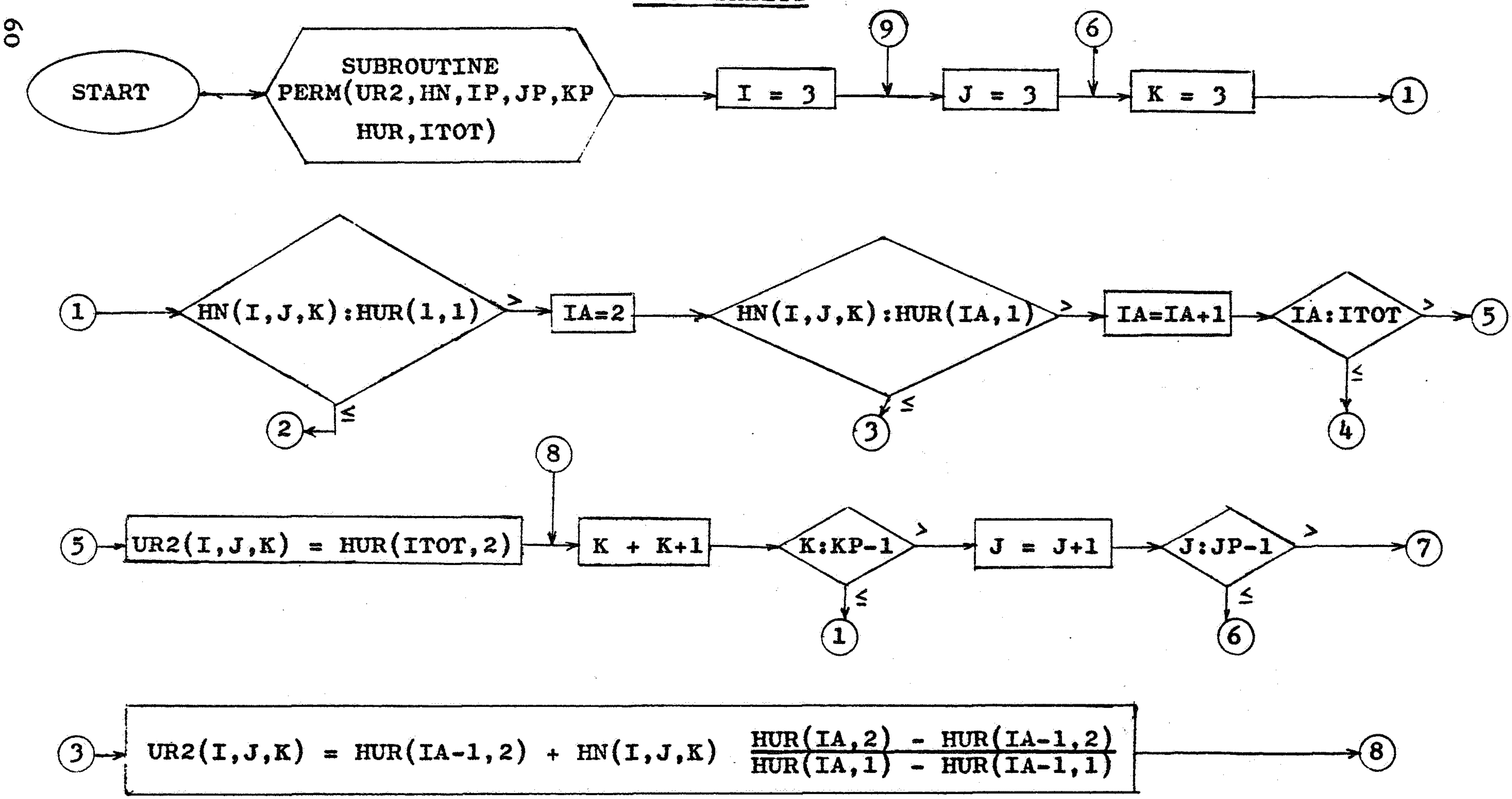
SUBROUTINE HCAL

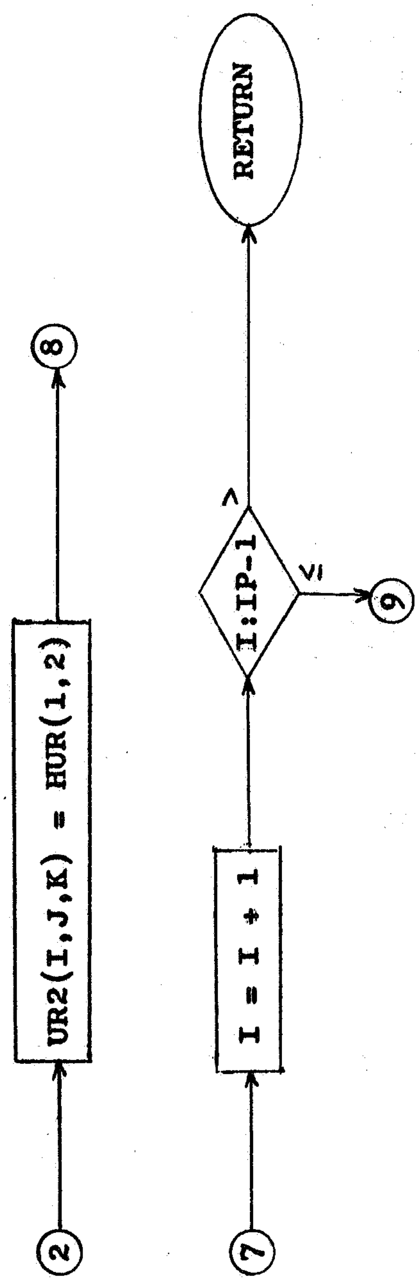
SYMBOLS

HN	matrix for the H field in a magnetic material; ampereturn per meter
HNX HNY HNZ	matrices for the x,y,z components of HN; ampereturn per meter
IP	number of the x coordinates; numeric
JP	number of the y coordinates; numeric
KP	number of the z coordinates; numeric
PHI	matrix of the scalar magnetic potentials; Ampere-turns
X	matrix of the x coordinates of points; meter
Y	matrix of the y coordinates of points; meter
Z	matrix of the z coordinates of points; meter
I1 I2 J1 J2 K1 K2	new indices used to help form finite differences; numeric

SUBROUTINE PERM

FLOW SHEETS





2
UR2(I,J,K) = HUR(1,2)
8
7
I = I + 1
I:IP-1
>
≤
9
RETURN

SUBROUTINE PERM

SYMBOLS

HN	matrix for the H field; ampereturn per meter
HUR	table of the H values; ampereturn per meter VS relative permeabilities defining the magnetizing curve; numeric
IA	index of the HUR matrix
IP	number of the x coordinate values of joints; numeric
ITOT	number of the rows of HUR (points on the permeability curve)
JP	number of the y coordinate values of points; numeric
KP	number of the z coordinate values of points; numeric
UR2	matrix for the new computed relative permeabilities; numeric

Section III of the Computer Program

This section called MAGFIA-PROGRAM calculates the H-field in the space outside the magnetic body, after the field inside the magnetic body was calculated in Section II.

This calculation is quite similar to the calculation in Section I, except that here the field is produced not only by currents but by dipoles as well. However, the complexities of Section II are missing because the relative permeability is unity in this field.

The MAGFIA PROGRAM is nothing else, but the calculation of ${}^{m}H_{n}$ per equation (099-1x,1y,1z)(046-1) of the mathematics, then ~~adding it to the~~ ${}^{o}H_{n}$-s resulting from Section I, the MAGFLD program if this is required. This is done in the block between steps #3 and #4. Of course, one can have only the field of the dipoles if the ${}^{o}H_{n}$-s are not added. This is done by making the values of AHX1 = HAX = 0, AHY1 = HAY = 0, AHZ1 = HAZ = 0, before step #1. Calculating only the dipole field has merit if the dipole field is a very small part of the total. The investigated case of the test samples as described in this report showed a dipole field five decade orders smaller than the field of the current, in some instances. Truncation errors would cloud this field in the field of currents if combined. The same is true for tests and special test methods were used to separate the values of the two fields.

The details of ${}^{m}H_{n}$ are calculated, of course, before that between step #1 and #3.

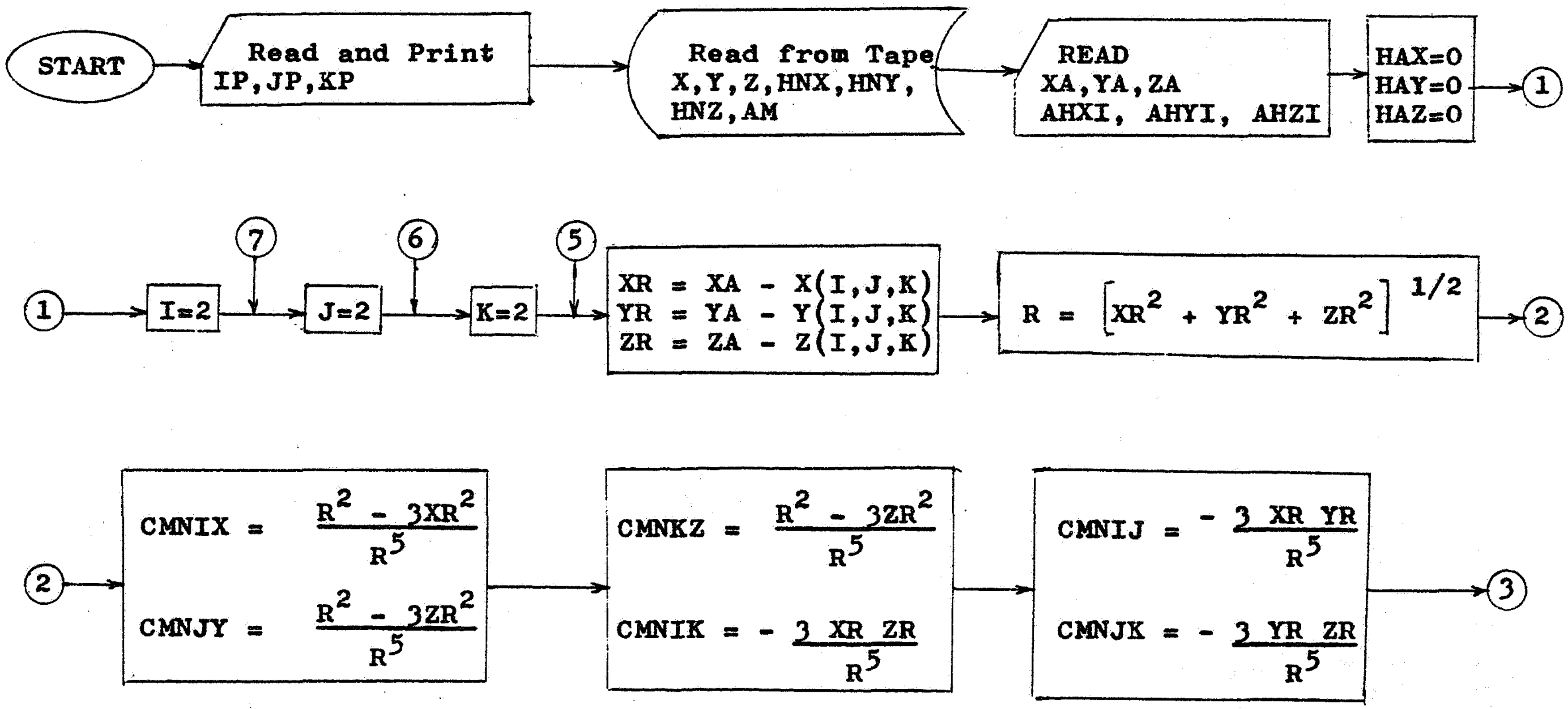
The summation of the ${}^{m}H_{n}$-s is done according to equ. (099-1x,1y,1z) and it is done by subsequent calculation and addition of these terms as controlled between step #4 & #8.

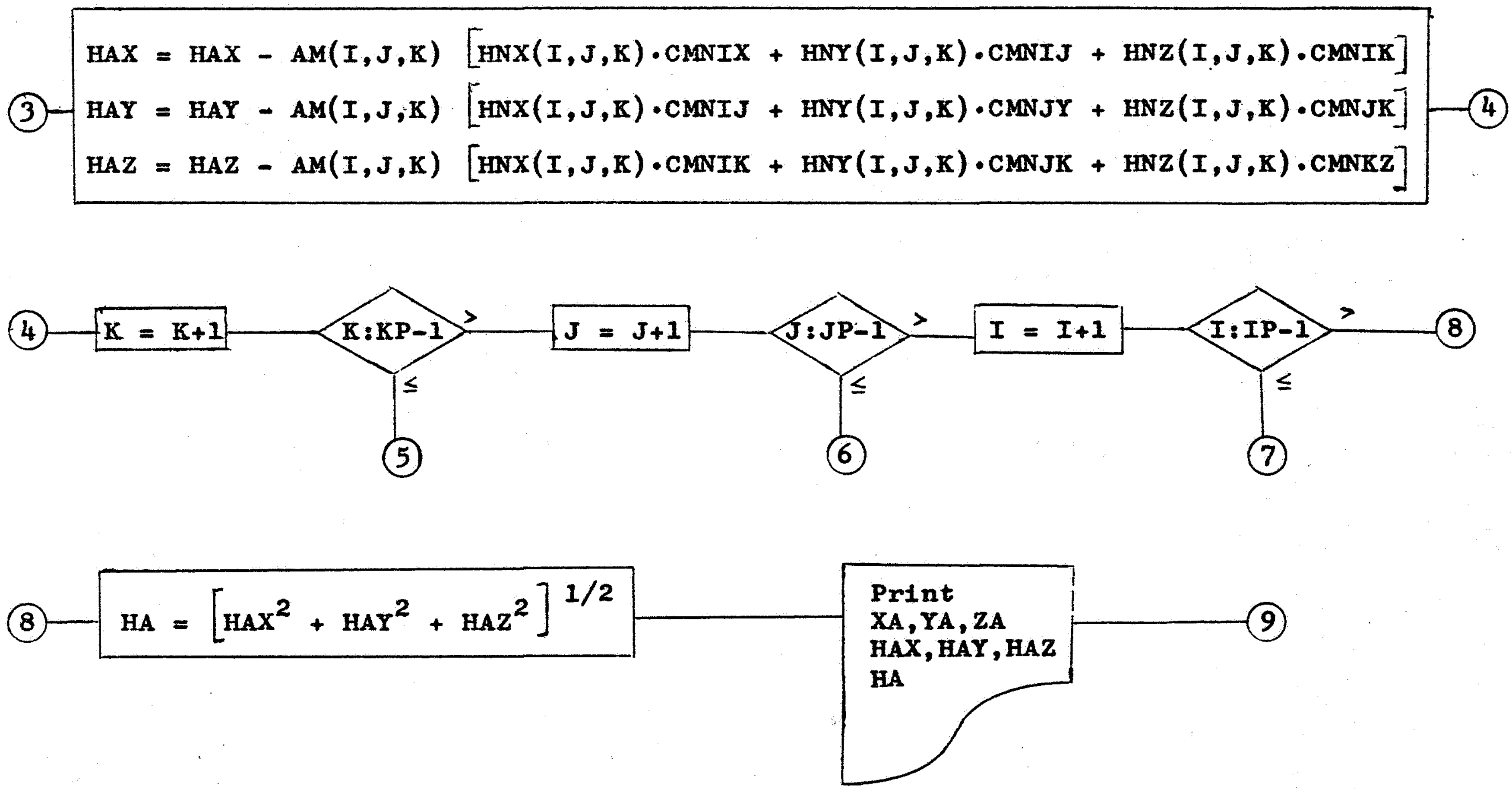
The results are the Cartesian components HAX, HAY, HAZ of the m.m.f. gradient HA.

The total HA of the m.m.f. gradient outside the magnetic body is calculated as the square root of the total of the squares of the three cartesian components HAX, HAY, HAZ at step #8. These components and the totals are printed at the last step before #9.

MAGFIA PROGRAM

FLOW SHEETS




3
HAX = HAX - AM(I,J,K) [HNX(I,J,K)·CMNIX + HNY(I,J,K)·CMNIJ + HNZ(I,J,K)·CMNIK]
HAY = HAY - AM(I,J,K) [HNX(I,J,K)·CMNIJ + HNY(I,J,K)·CMNJY + HNZ(I,J,K)·CMNJK]
HAZ = HAZ - AM(I,J,K) [HNX(I,J,K)·CMNIK + HNY(I,J,K)·CMNJK + HNZ(I,J,K)·CMNKZ]
4
4
K = K+1
K:KP-1
>
≤
5
J = J+1
J:JP-1
>
≤
6
I = I+1
I:IP-1
>
≤
7
8
8
HA = [HAX^2 + HAY^2 + HAZ^2]^1/2
Print
XA,YA,ZA
HAX,HAY,HAZ
HA
9

SUBROUTINE MAGFIA

SYMBOLS

AHX1 AHY1 AHZ1	magnetomotive force gradient in air at points XA, YA, ZA outside the magnetic material induced by the currents of the Helmholtz coil only; ampereturn per meter
AM(I,J,K)	Volume constants of magnetic material, see P. 49
CMNIJ	CMNJX = CMNIY = $\frac{\partial}{\partial x} \frac{\cos(jr)}{r^2}$
CMNIK	CMNKX = CMNIZ = $\frac{\partial}{\partial x} \frac{\cos(kr)}{r^2}$
CMNIX	$\frac{\partial}{\partial x} \frac{\cos(ir)}{r^2}$
CMNJK	= CMNKY = CMNJZ = $\frac{\partial}{\partial y} \frac{\cos(jr)}{r^2}$
CMNJY	$\frac{\partial}{\partial y} \frac{\cos(jr)}{r^2}$
CMNKZ	$\frac{\partial}{\partial z} \frac{\cos(kr)}{r^2}$
HA	total resultant magnetomotive force gradient; ampereturn per meter
HAX HAY HAZ	x, y, z components of resultant magnetomotive force gradient; ampereturn per meter
HNX (I,J,K) HNY (I,J,K) HNZ (I,J,K)	x, y, z components of the magnetomotive force gradient at points inside the magnetic material; ampereturn per meter.
IP	number of points in the magnetic material in the x direction; numeric
JP	number of points in the magnetic material in the y direction; numeric

KP	number of points in the magnetic material in the z direction; numeric
R	distance from a point inside the magnetic material to a point outside the magnetic material; meter
XA YA ZA	coordinate of a point outside the magnetic material; meter
X (I,J,K) Y (I,J,K) Z (I,J,K)	coordinate of a point inside the magnetic material; meter
XR YR ZR	x, y, z components of the distance from a point inside the magnetic material to a point outside the magnetic material; meter

VI TEST

VI Test

Samples of magnetic materials of known properties and geometries were to be placed in a known magnetic field. The magnetic field modified due to the presence of the samples was to be measured at several points at which it was calculated by computer too. The results of tests were to be compared with the computer output.

Apollo-Helmholtz Coil-Pair

The samples were placed in the geometrical center of the Apollo-Helmholtz coil-pair available at Ames Research center.

Each coil of the coil-pair is wound of 240 turns of 22 AWG copper wire, 15 turns in each of the 16 layers per coil.

The dimensions are shown in Figure 4.

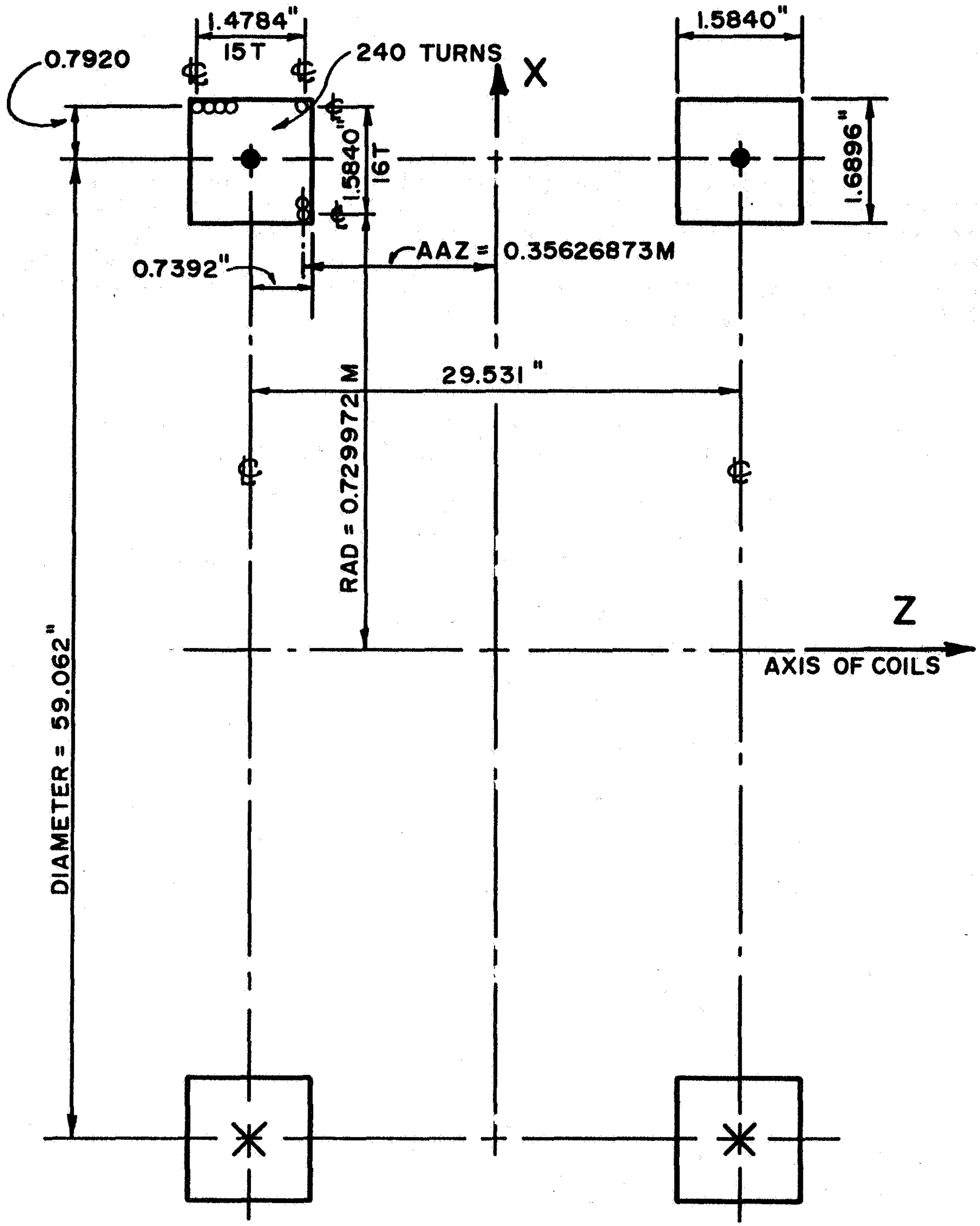
8,688 to 8,72 amperes electric current was specified in the coil conductor.

The geometry of the Samples

The samples to be tested and the sequence of tests were as follows.

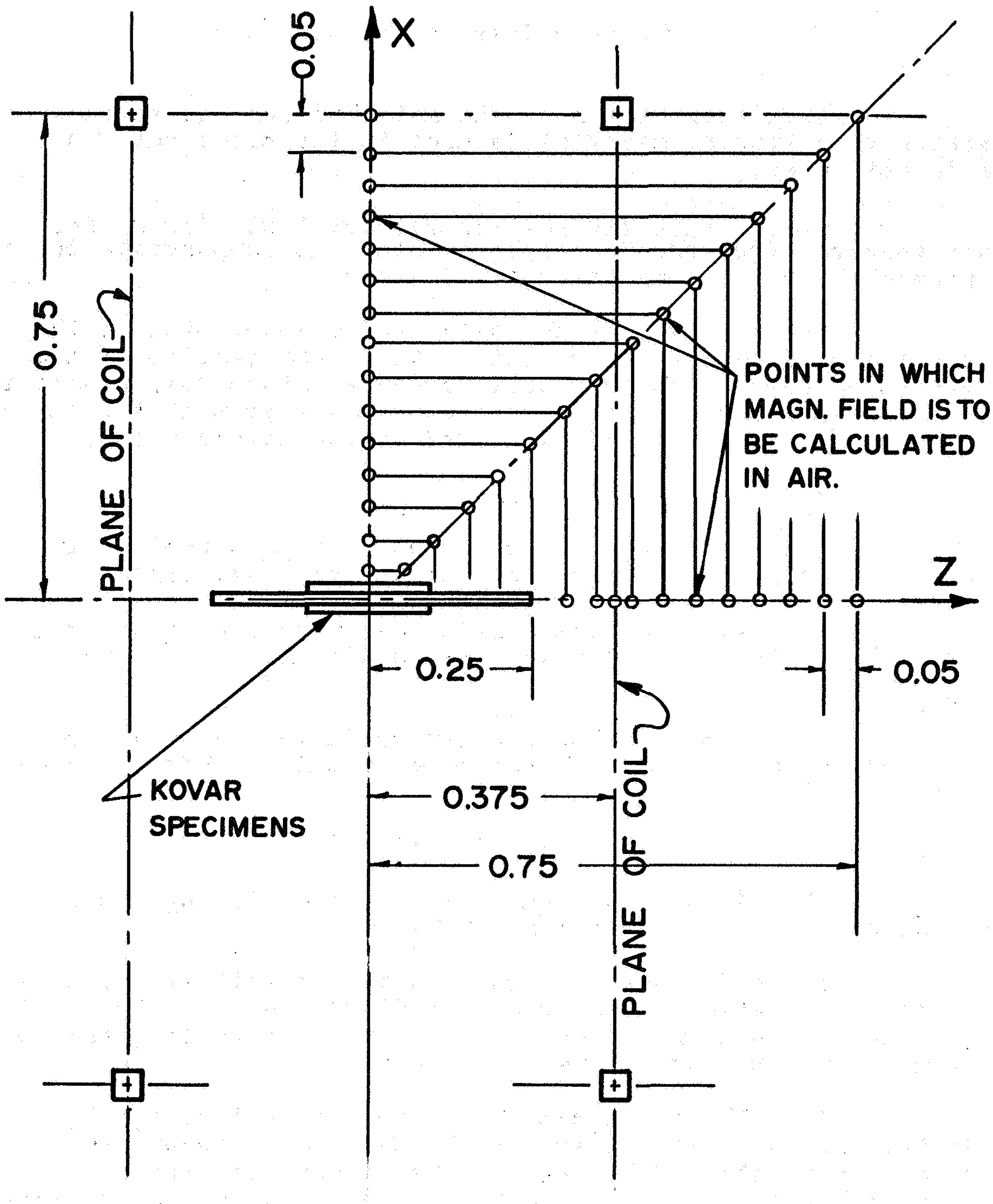
1.)	Cube	0.02541 x 0.02541 x 0.02541
2.)	Parallelepipedon	.02528 x 0.02541 x 0.05081
3.)	" "	0.02542 x 0.02540 x 0.1016
4.)	Square rod	0.02536 x 0.02542
5.)	Round rod	0.02541 x 0.05087
6.)	" "	0.02540 dia x 0.1016
7.)	" "	0.02544 x 0.2032
8.)	" "	0.02536 x 0.3810

The location of the samples relative to the Apollo-Helmholtz coil-pair is shown in Figure 5.



APOLLO - HELMHOLTZ COILS
I = 8.72 AMPS

Figure 4



APOLLO-HELMHOLTZ COIL-PAIR & KOVAR-SPECIMENS

METRIC DIMENSIONS

Figure 5

The materials of the samples

KOVAR was chosen as the material for the samples, because this type of material is used in the spacecraft to be investigated.

Very little information was available for KOVAR. Data received from the Westinghouse Electric corporation indicated a permeability curve as shown in Figure 6.

The scarcity of the data available pointed to the necessity of a permeability test. NASA sent samples of the KOVAR to be used to the National Bureau of Standards, Washington, D. C. for such a test. The test results agreed with the curve of Figure 6, and this curve was used in the calculations.

Test-results

The tests were performed by NASA personnel in the Magnetic Laboratory at Ames Research Center, California.

The magnetic flux density was measured at several points along the z-axis and the x-axis with Sample #1, the Kovar-cube.

The magnetic flux density was measured at several points along the z-axis with the other samples. Fig. 7 shows the results.

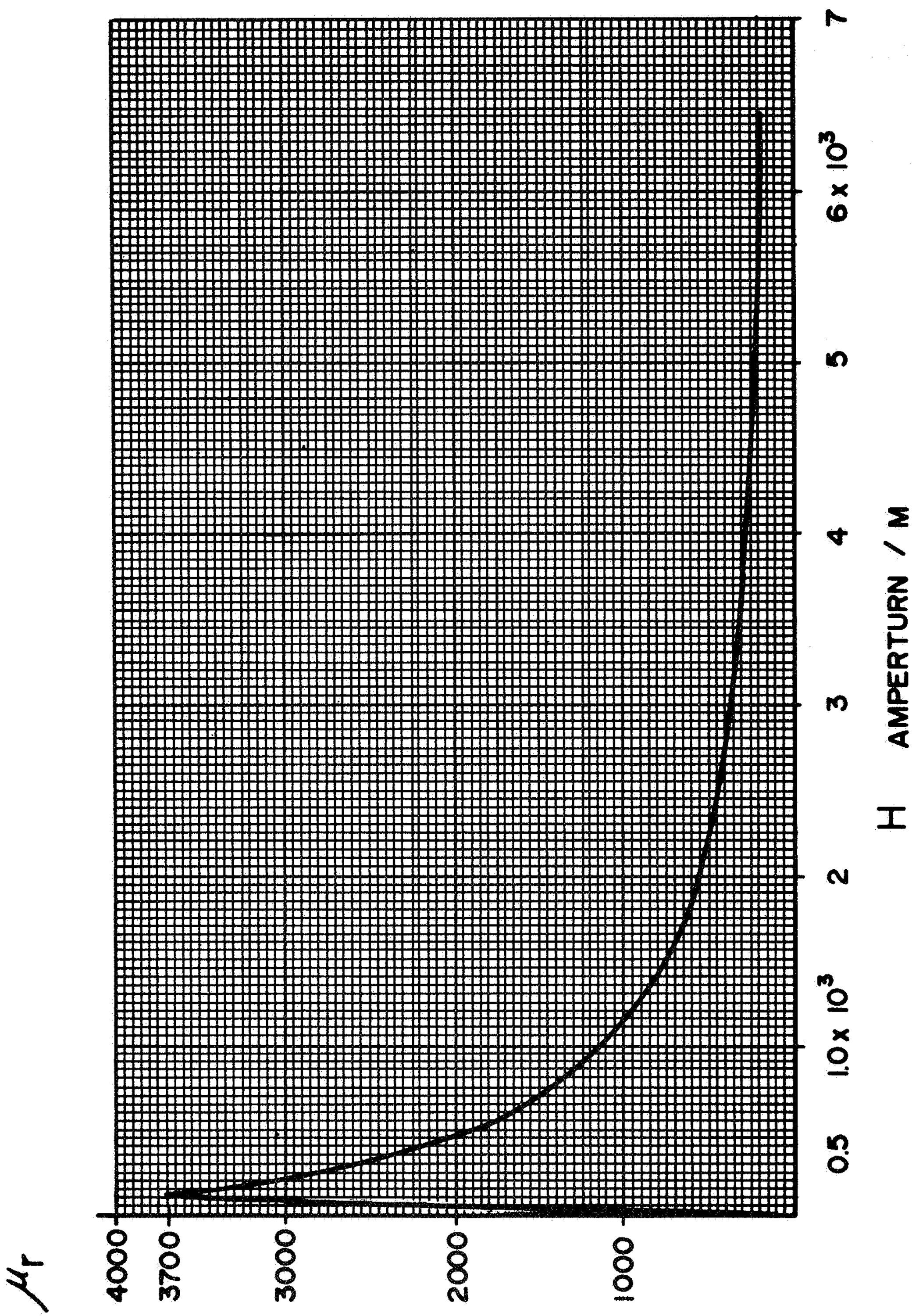
Field in air only

The field in air only was measured during the tests and calculated on an IBM 360/50 computer.

Table I. shows the results and the difference between test and calculation. The greatest difference is 3.29%. The difference increses with the distance from the center, and this phenomenon may be due to two causes.

1.) The coordinates of points were computed by starting from a point and adding the distance between two consecutive points. This process has truncation errors adding with the distance. The net result is an apparent slant of the axis, along which the calculation proceeded, and so an increasing difference of the magnetic field with the distance.

2.) A physical slant of the axis could have been present at the test, because the magnetometer was traveling on rails, suspended as cantilevers in the center. The weight of the magnetometer could have caused a deflection of the cantilever-beam.



RELATIVE PERMEABILITY OF KOVAR

Figure 6

Table I.

Magnetomotive force gradient H, of the Apollo-Helmholtz coil-pair.

1.) Axial scan, along the z-axis
x=0, y=0

z, meters from center	H_z Amperturn/m from test	H_z Amperturn/m from calculation	Difference Ampt/m	Difference %
0	1993	2002	+9	+0.45
0.2	1993	2000	+7	+0.35
0.3	1955	1970	+15	+0.75
0.4	1872	1885	+13	+0.69
0.5	1738	1745	-7	-0.40
0.6	1572	1555	-17	-1.09
0.7	1380	1350	-30	-2.22

2.) Radial scan, along the x-axis
y=0, z=0

x,meters from center	H_x Amperturn/m from test	H_x Amperturn/m from calculation	Difference Ampt/m	Difference %
0	1995	2002	+7	+0.35
0.2	1995	2000.5	+5.5	+0.28
0.3	1972	1978	+6	+0.30
0.4	1935	1905	-30	-1.58
0.5	1746	1735	-11	-0.63
0.55	1605	1595	-10	-0.63
0.6	1421	1415	-6	-0.42
0.65	1223	1192	-31	-2.60
0.7	978	948	-30	-3.16

Test results in gauss were multiplied by 79.579 to have Amperturns/meter.

Field of Kovar-samples

Only a very small distortion of the field in the order of the fourth to sixth decimal occured when the Kovar samples were introduced.

A reasonable accuracy of the tests were achieved by measuring not the absolute field but only the difference, that is the distortion. This was achieved by setting the magnetometer to zero in the full air-field before the Kovar-sample was introduced.

Similarly, the calculation was made only for the difference, by setting HAX, HAY and HAZ to zero in Section

III, the MAGFIA-programm.

Successful test and calculation was performed for a cube, a short cylinder and a sphere. The results are shown on Figure 8, 9 and 10.

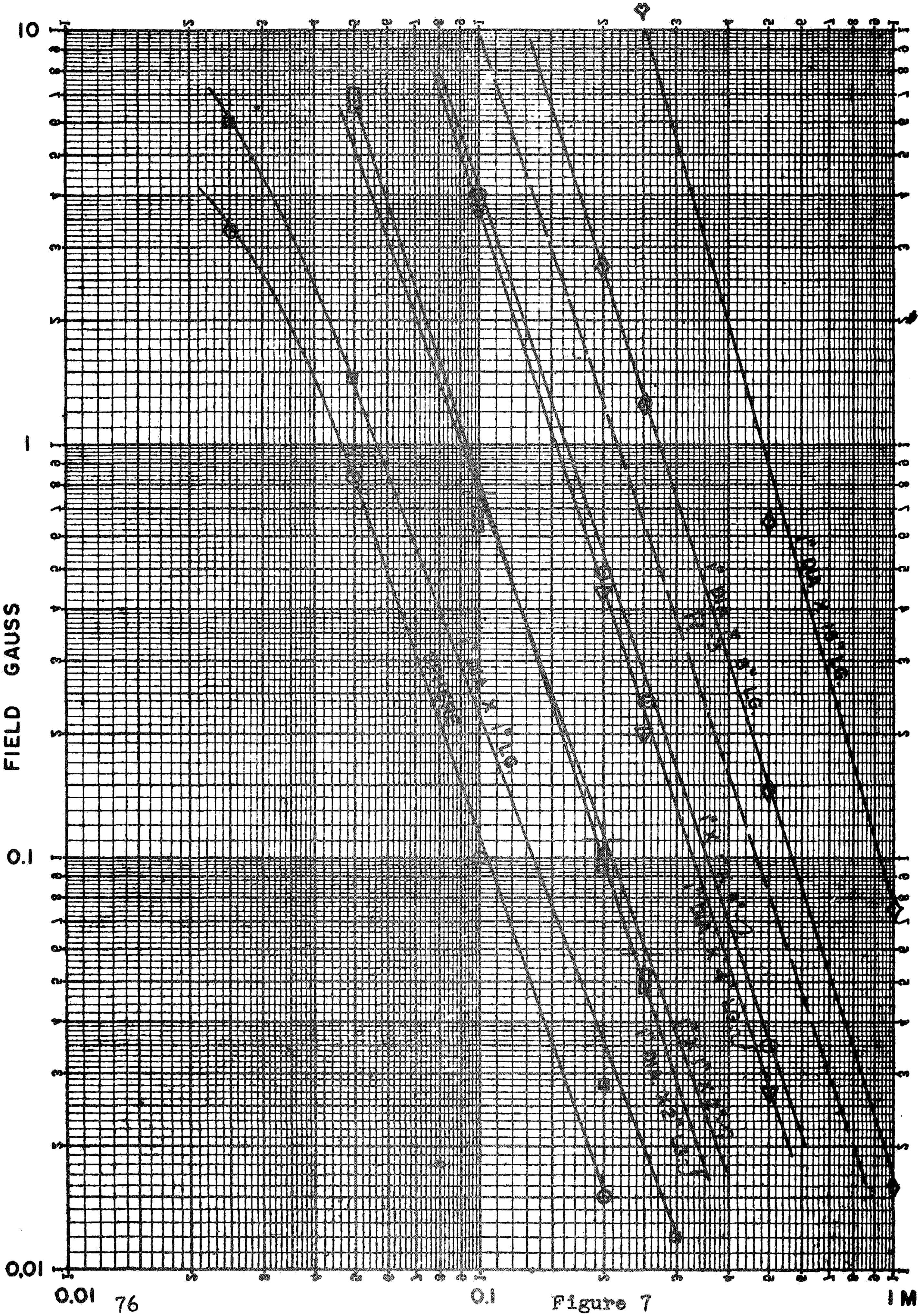
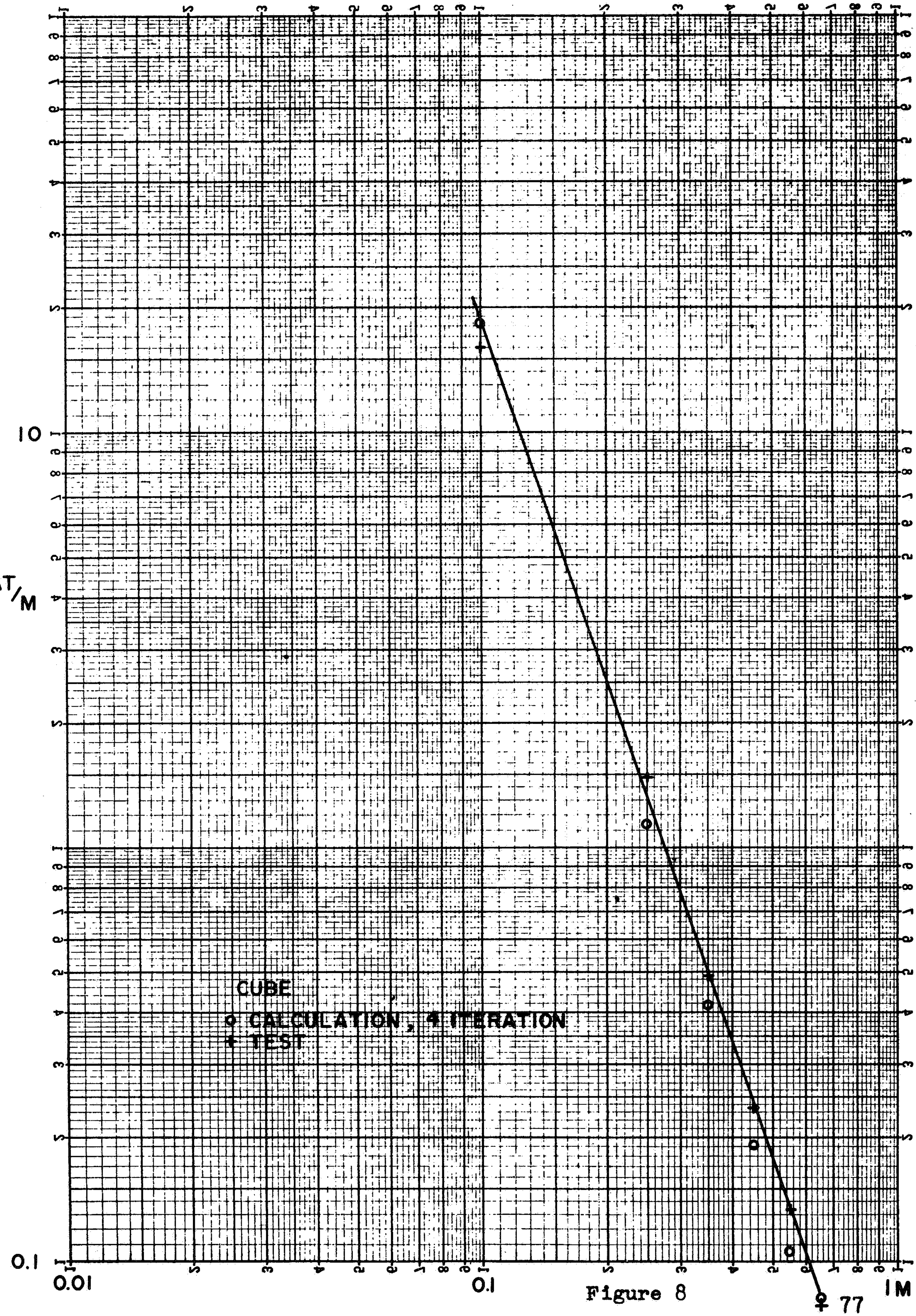


Figure 7



10
0.1
AT/M
0.01
0.1
1M
CUBE
o CALCULATION, 4 ITERATION
+ TEST
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES
LOGARITHMIC 359-115

Figure 8

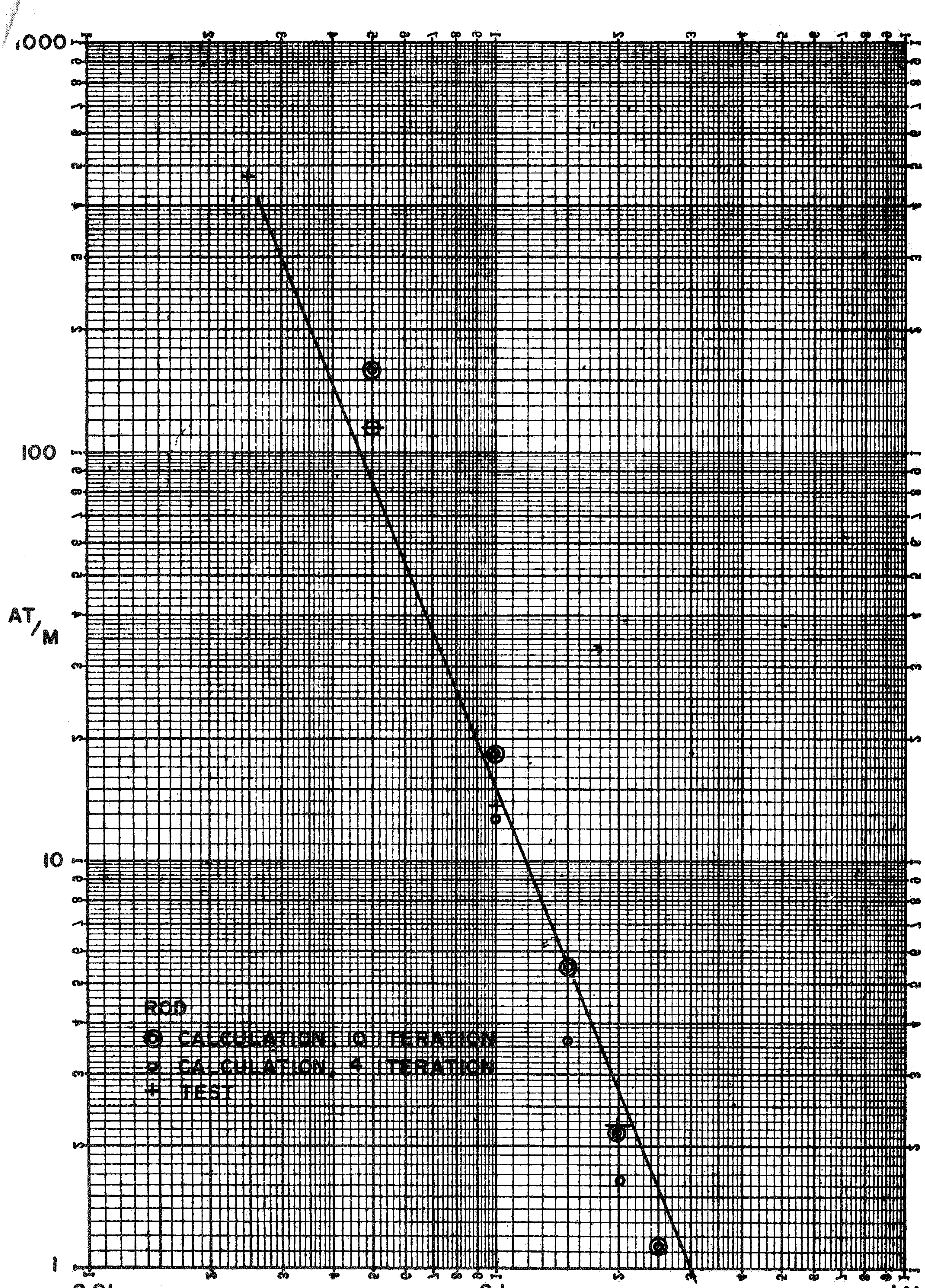

1000
100
10
1
AT/M
0.01
0.1
1 M
ROD
CALCULATION, 10 ITERATION
CALCULATION, 4 ITERATION
TEST

Figure 9

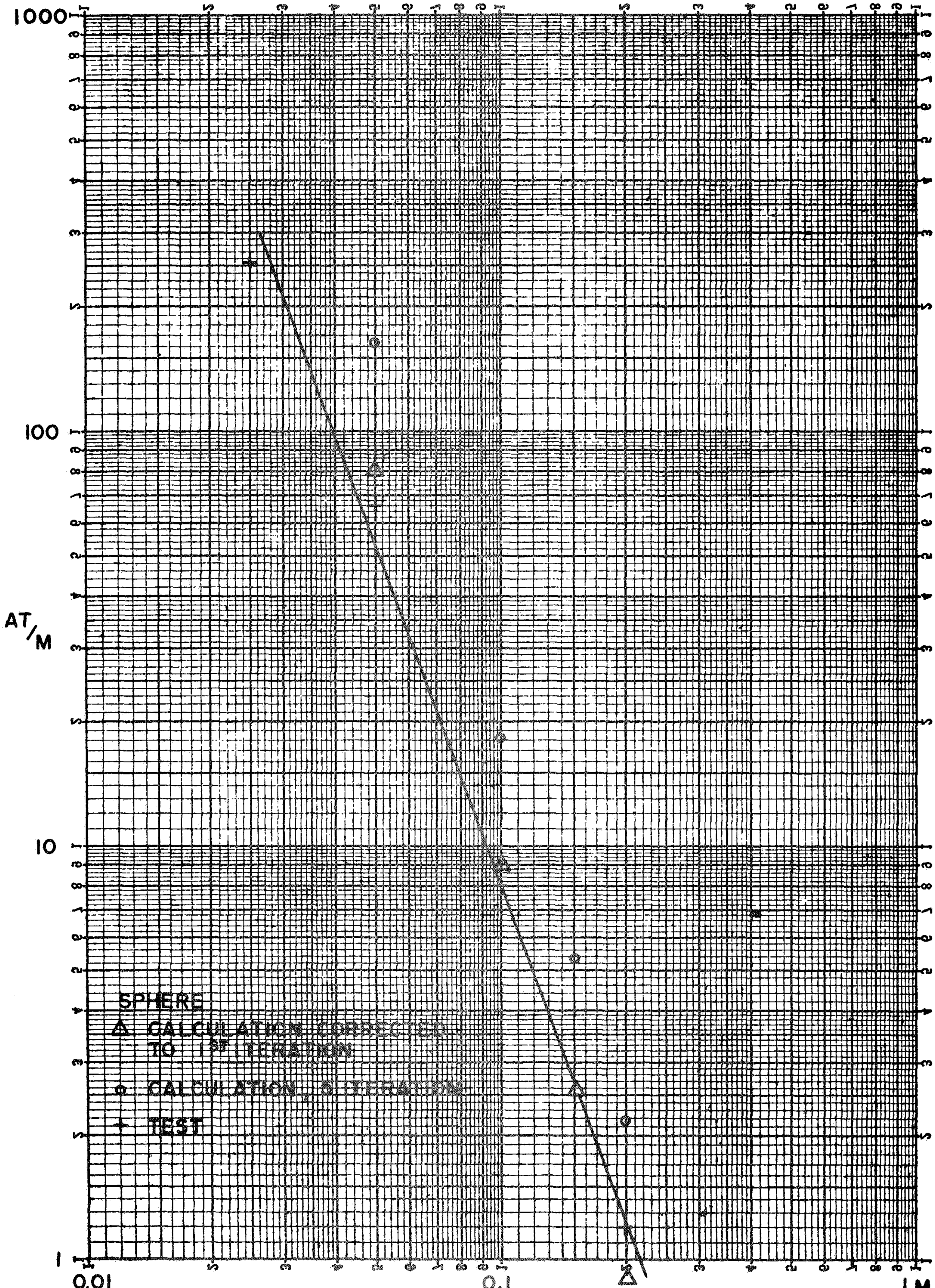

1000
100
10
1
AT/M
0.01
0.1
I M
SPHERE
△ CALCULATION CORRECTED TO 1ST ITERATION
○ CALCULATION, N ITERATION
+ TEST

Figure 10

VII. EVALUATION OF THE COMPUTERIZED CALCULATION

Section I and III

It was established beyond doubt that Section I, and III is valid and works perfectly. This means that the magnetic field in air, induced either by a current system or by a magnetized body can be calculated with a very high accuracy if the current-system and the magnetization of the body is known.

Section II

The same can not be said of Section II yet.

Section II can be evaluated qualitatively and quantitatively.

Qualitatively, good results were achieved.

For instance, the flux-lines of the m.m.f. gradient, H are diverging from the sample towards the air, as they should be.

The m.m.f. gradient shows not only a decreasing trend with increasing distance from the sample, but also the ratio of the decrease is in agreement with the tests. In other words, the trend of curves showing the m.m.f. gradient, H, as a function of the distance from the sample agrees with the test, see Figures 8, 9 and 10.

Qualitatively, Figures 8, 9 and 10 show also a fairly good agreement of calculation and test, but not good enough.

Section II needs corrections also because it seems not to work well if the field of the current system is uniform and parallel. In such a case, and this was the case in all examples considered, the directional partial differentials of the relative permeability, and the field in air tend to go to zero. These are URX, URY, URZ and AHXX, AHYY, AHZZ in the PHICAL-subroutine. The zero value of these quantities makes B(IA), the right side of the equations zero, and this makes the equations homogeneous, with a singular AMA matrix , which can not be solved. This happened fast whenever a solution was attempted by taking more than eight points in the ferromagnetic body, and also when PHI was not assumed zero in the air.

The iteration process in the DIPOLE programm must be investigated further too. It seems that this iteration diverges, and diverges very fast when more than eight points are set up in the ferromagnetic material. This divergence accelerates if PHI is taken not equal zero in the air, which can be explained by the matrix AMA going singular as explained above.

It seems that the best agreement with the test was reached by not using iteration at all, but reading the permeability from the permeability vs. H curve for the H derived from the current-system in air and using this value of the permeability in calculating the dipole strength of the ferromagnetic body. Of course, all cases investigated were close to saturation. It was proposed to investigate cases with less saturation, but time did not permit it. Such cases should be investigated very definitely, and particularly in the neighborhood of the peak of the permeability-curve.

One should also try to use an equation for the permeability vs. H function, and use this function in the main equation, then solve them for the magnetic scalar potential. This would involve rather complex equations, but would dispense with the iteration.

Though Section II does not work reliably and too well in parallel fields it is expected to work well in non-uniform, non-parallel fields, and it should be tried in non-parallel, non-uniform fields. Such an investigation will narrow down the region where corrections are needed.

The machine-times logged are listed in the Table II below.

Table II

Sample	No. of points in the sample	machine-time in minutes for section I for 64 pts.	II for	III for 15 pts.	Total
Rod	2x2x2=8	13.09	4 ITER, 2.80	1.85	17.64
			10 ITER, 2.83	3.62	19.54
Sphere	2x2x2=8	12.31	5 ITER, 2.23	6.45	20.99
Cube	2x2x2=8	12.31	4 ITER, 2.38	1.76	16.45
Cube	5x5x5=125	25.2	—	—	—

VIII. PROPOSITION FOR CONTINUED INVESTIGATION

Reference is made to the end of the previous chapter listing some propositions for the completion of the present work. Further propositions are as follow.

The solution of differential equations by linearizing them and using finite differences-what this work in essence was-has an inherent error due to taking finite differences instead of a continuum.

The error is the less the greater the number of the finite differences is but a great number of finite differences means to have a great number of linear equations with an equally great number of unknowns and the solution of this voluminous set of equations requires excessive computer-time and memory.

One hardly can speak about an optimization in this problem. Optimization would mean to find a balance between tolerable errors and computer-cost. Here the computer-time is increasing so rapidly and the memory-space is outrun already with such a small number of finite differences, that the question is rather whether the magnetic body should be divided into 2 x 2 x 2 or 5 x 5 x 5 blocks each block representing a "finite difference" the number of which is clearly inadequate for any appreciable accuracy for a body beyond the size of very small laboratory samples.

Furthermore, while accuracy can be defined as the per unit difference between the calculated and the real value of the m.m.f. gradient at a point, the value of the accuracy is rather dubious, unless the calculation is repeated several times each time increasing the number of finite differences, and an asymptotic approach by the values of the resulting values of the m.m.f. gradient can be figured out.

In order to make the developed process more useful, -indeed useful at all if the field of a whole satellite is desired- a two-step extension of the work done so far is proposed, as follows.

1.) Investigate ways and means for a better adaptation of the developed method for bodies of sizeable dimensions. One is inclined to investigate not so much various mathematical modifications but rather other approaches, like the exploitation of symmmetries, some kind of a "zooming technique", by which is meant to start with a very small number of "finite differences" then divide and subdivide each of those in always finer parts, etc. Several of such approaches look promising and new ones may emerge during the work as it almost always happens in research.

Mathematical modifications of the developed method are not promising, simply because there are none which could be used. This a common plague of such processes today and a serious limitation to the use of computers.

The propositions made at the end of Chapter VII fall into this category.

2.) Investigate the accuracy of the calculation, and try to develop a process to predict how many of how fine "finite differences" are required to have a predefined accuracy. The prediction may be based on two or three computer runs with very low number of finite differences, and as such rather inexpensive. Preliminary investigation revealed certain regularities of the "accuracy against number of finite differences" curve and this proposal is based on this observation.

Undoubtedly, the step proposed as the first one is more important in view of practical use, but the second can not be neglected either.

APPENDIX I.

THE DEVELOPMENT OF THE ANALYSIS OF THREE-DIMENSIONAL AND STATIC MAGNETIC FIELDS IN THE PRESENCE OF BODIES THE PERMEABILITY OF WHICH IS A FUNCTION OF THE FIELD.

After several futile attempts during the years of 1963-6 it was concluded that the only fundamentally sound approach is to solve Maxwell's equations as they apply. (086-150-225, March, 1966).

The Maxwell-equations for static fields and at a point, n, where no real current is present are

$$\nabla \cdot \vec{B}_n = 0$$

$$\nabla \times \vec{H}_n = 0$$

$$\vec{B}_n = \mu \vec{H}_n$$

It was reasoned that these three equations can be solved for the three unknowns, $\vec{B}_n$, $\vec{H}_n$ and μ.

From $\nabla \cdot \vec{B}_n = 0$ follows that the normal component of B is continuous.

From $\nabla \times \vec{H}_n = 0$ follows that the tangential component of H is continuous. (Slater, Frank; Electromagnetism, p. 71)

1.) Elaborate calculations were performed for the solution of these equations, however an error was made in the mathematics

$$\nabla \cdot \left(\hat{h}_m \frac{\hat{r}_{mn}}{r_{mn}^2}\right) \neq \hat{h}_m \cdot \left(\nabla \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right)$$

2.) The error having been corrected, the equations for the solution looked hopelessly complex, because second partial derivatives appeared now.

The real trouble was however, that $\vec{H}$ was written as the gradient of a scalar potential (086-150-155), fully justified here, but resulting in an identity of the equation $\nabla \times \vec{H}_n = 0$, because the curl of a grad. is always zero. This "equation" is therefore, always satisfied in the analyzed case and it is useless for the required solution. (097-057, Sep 2-24-67) (097-051, equ. 7, Oct. 3, 67)

The original idea, to solve the three Maxwellian equations seemed to be paralyzed, unless some other approach could be found.

3.) In an attempt to go around the difficulty, it was reasoned, that a vector is known, if its div. and curl are known. Then $\vec{B}$ being a vector, it would be necessary only to express its div. and curl by known quantities from the basic three Maxwellian equations. (097-046, -97-058, Sep. 24, 67)

The div. and curl of B were written accordingly.

$\nabla . \vec{B}$ (097-066, Sep. 27, 1966)

$\nabla \times \vec{B}$ (097-051, Oct. 3, 1966)

and it was shown, (097-060, Oct. 3, 1967) how can $\vec{B}$ calculated then.

The reasoning was continued by stating that the div. and curl are known if all components of these quantities are known. These components are the nine first order partial derivatives.

In order to find the nine partial derivatives of $\vec{B}$, the six, figuring in the curl $\vec{B}$, were expressed in terms of μ and of the Cartesian components of $\vec{H}$. (097-061, -062, -063, Oct.3, 1967) and also (097-071, -072, -073, Sept. 7,8, 1967)

These expressions in their turn and of course, required $\vec{H}$ to be expressed by some known values. $\vec{H}$ was expressed by the field of real currents and by the dipole moments induced by this field in the magnetizable material in the original write-up (086-150-153) and could be taken from there.

Of course, this procedure resulted $\vec{B}$ as a function of the independent field, the induced dipoles, and μ-s. The remaining three partial derivatives

$$\frac{\partial B_x}{\partial x} \qquad \frac{\partial B_y}{\partial y} \qquad \frac{\partial B_z}{\partial z}$$

were written from this expression of $\vec{B}$ as functions of the field of the currents, the induced dipoles and μ-s. (097-065, Oct. 3, 1967) (097-074, Sep. 9, 1967)

The resulting equations for B_x, B_y and B_z were, of course, scalar and not vectorial equations. This observance gave the idea that only scalar quantities should be used. This is quite self evident by now, and it is surprising why it was not seen before.

As a byproduct, (097-066, sh. 1, Oct. 3, 1967) it was shown that if $\nabla.\vec{B} = 0$, then $\nabla.\vec{H} = 0$ too. This result gave an expression for $\nabla.\vec{H} = 0$, and as it was proven previously $\nabla\times\vec{H} = 0$ is always satisfied here, it was attempted to use $\nabla.\vec{B} = 0$ (097-066, sh. 2, Oct. 3, 1967) as a function of H_x, H_y, H_z and μ, derived from these equations. Then the $\vec{H}$-components appearing in the expressions of $\vec{B}$ were lumped with real constants into terms parametric in $\vec{H}$, by which manipulation a set of first order partial differential equations was reached for $\vec{H}$ and μ. It was thought to solve them by linearizing them by numerical approximation - an idea maintained in the final solution.

Unfortunately, the set of equations to be solved, turned out to be a set of homogeneous equations - which could be solved only for the ratios of μ- s, and even so only if the determinant of the set and at least one minor was zero — and this is not the case generally. (097-066, sh. 4, Oct. 3, 1967)

Again, the $\vec{H}$-components were not known in the equations for $\vec{B}$, and so these equations were nothing else but the $\vec{B}$-components as functions of $\vec{H}$-components, where both $\vec{B}$ and $\vec{H}$-components were unknown. So, there were now not only three unknowns, but six, the three unknown $\vec{B}$-components and three unknown $\vec{H}$-components, a total of six unknowns, in only three equations.

4.) Several attempts were made to find four independent equations when, during the described work it was realized that the unknowns to be found really are H_x, H_y, H_z, and μ.

One such attempt formulated four equations as follows:

$$\mu\sqrt{H_x^2 + H_y^2 + H_z^2} = \sqrt{B_x^2 + B_y^2 + B_z^2} \qquad (1)$$

and the $\vec{B}$-components on the right side were written in terms of the $\vec{H}$ components derived from $\nabla\times\vec{B} = 0$ and $\nabla\times\vec{H} = 0$. (097-060, October 3, 1967).

$$\nabla.\vec{B} = 0 \qquad (2)$$

the div $\vec{B}$ expressed in terms of partial derivatives, the latter ones again expressed in terms of the $\vec{H}$-components. (097-064, Oct. 3, 1967)

$$\nabla . \nabla \times \vec{B} = 0 \tag{3}$$

was written because the divergence of any curl is zero. (097-076 through -78, Sep. 9, 1967)

$$\mu = f \sqrt{H_x^2 + H_y^2 + H_z^2} \tag{4}$$

The magnetizing curve.

Unfortunately, again, equ. (3) turned out to be an 0 = 0 identity because the curl of the field of currents is identically zero. (097-078 and -079, Sep. 9, 1967). This killed that scheme.

Of course and of same reason a proposition to calculate not $\vec{H}$, but μ, from that equation (097-081, Sep. 11, 1967) did not work either.

At this point it was also observed, that one difficulty of such a proposition was also that $\vec{H}$ is not a single valued function of μ.

Only μ is a single valued function of $\vec{H}$. This observation pointed out that not μ, but $\vec{H}$ must be calculated first; then μ can be read from the magnetization curve of the material.

5.) $\vec{H}$ appeared always in the form of its three Cartesian components. A survey of the work done revealed (097-098, Sep. 15, 1967) that equations between these three Cartesian components and the permeability, μ, were already developed (097-064-3x, 3y, 3z, and 4x1, 4y1, 4z1). So why not try to solve these equations for H_x, H_y, H_z keeping μ as a parameter (097-090, Sep. 15, 1967). This seemed to be possible, because that was a set of 3p equations for 3p unknowns. Then use the resulting $\vec{H}$ in the $\nabla . \vec{B} = 0$ equation and satisfy Maxwell this way, combine this form of $\nabla . \vec{B} = 0$ and the magnetizing curve $\vec{B} = \mu \vec{H}$, where again $\vec{H}$ is the $\vec{H}$ resulting from the above 3p equations and it is in terms of μ, and solve the latter two equations

$$\nabla . \vec{B} = 0$$

$$\vec{B} = \mu \vec{H}$$

for $\vec{H}$ and μ (097-090, -091, -092, Sep. 14, 1967)

When the determinant of the $\vec{H}$-equations was investigated, it was felt better to use the magnetic moment, M, instead of $\vec{H}$ (097-088, -089, Sep. 13, 1967) (097-091, Sep. 14, 1967) because it may yield simpler expressions.

It should be noted that the $\nabla . \vec{B} = 0$ equations are homogeneous, and therefore, the magnetizing curve is not one equation too much, but needed for the solution (097-094, Sep. 24, 1967; 097-095, Sep. 30, 1967).

This reasoning and the algorithm looked to be in order, but from the fact that the magnetizing curve can not be expressed by an algebraic equation difficulties arose. Algebraic approximations like the Fröhlich-equation, though considered (097-095, Sep. 30, 1967) proved neither accurate enough, nor practical.

At this point it seemed that a deadlock was reached.

6.) It seemed that at least there were some useful by-products developed. Such were expressions for the Cartesian components for the partial derivatives, and for the divergence and curl of $\vec{H}$ and $\vec{B}$. Cartesian components of $\vec{H}$ 097-064, sh. 1-2 (in terms of magnetic moments) Cartesian components of $\vec{B}$ 097-064, sh. 2-3.
Partial derivatives of $\vec{B}$ in terms of $\vec{H}$ and μ.

$\nabla . \vec{H}$	097-061, -062, -063, -064, sh 4-5, -065 097-066 sh. 1 097-127
$\nabla . \vec{B}$	097-066 sh. 2-4 097-126
$\nabla \times \vec{H}$	097-057, sh. 9 (also 097-051-7 equ.) 097-127
$\nabla \times \vec{B}$	097-051 through -055 and 097-079,097-126

The calculation of $\vec{B}$ from $\nabla . \vec{B}$ and $\nabla \times \vec{B}$ 097-060.

Many details of these calculations were saved and used directly in the development of the final solution.

7.) The "breakthrough" came with the recognition that the scalar quantities for which a solution can be found are not the three Cartesian components of $\vec{H}$ and the μ, but they are the scalar potential, φ, and μ, that is only <u>two</u> scalar quantities (Sep. 30, 1967), see (097-094 sh. 10, and 097-096).

This was recognized by observing that the scalar potential was everywhere present in the equations though sometimes hidden.

8.) As the next step, and as $\vec{H} = \nabla\varphi$, the Cartesian components of $\vec{H}$ were written as first partial derivatives of φ. For instance $H_x = \frac{\partial\varphi}{\partial x}$ and so on. The result was a partial differential equation for φ and μ. (097-097, Sep. 30, 1967).

9.) Then the partial derivatives of φ and μ, were approximated by a linear approximation (097-098, Oct. 3, 1967).

10.) While first the $\vec{H}$-components on the right side of the equation were maintained and caused difficulties, finally <u>all</u> $\vec{H}$-s were replaced by partial derivatives of φ. In doing this, the following was observed.

a.) The second order partial derivatives of φ could be expressed by the first order partial derivatives of φ and by geometric relations.

b.) The derivatives must be specified carefully as far as their location in the space is concerned (derivative at n or m, see sh. 15, 097-099, Oct. 11, 1967).

By this a mathematical solution was reached and completed.

11.) Quite some more work was required to reduce the theoretical solution to practice.

a.) There are a number of constants to be calculated in the equations. These constants were expressed. (097-099, Oct. 11, 1967, 097-101, -102, Nov. 8, 1967).

b.) Simple examples were sketched to see if the mathematics can be really applied (097-103, Nov. 11, 1967; 097-114, Nov. 25, 1967).

c.) The equations were checked regarding their feasibility for computer-language (097-110).

d.) The algorithm was sketched (097-111, Nov. 25, 1967). This pointed out the necessity of careful indexing.

12.) It was recognized that the only great difficulty remaining is the solution of a large number of simultaneous linear equations. (097-104, -105 Nov. 11, 1967; 097-117, Oct.2 1967).

It is realized that the size of the computer-memory and the length of the computer-time are the limiting factors in the application. Further work is much needed in order to improve the computer-technique. There are some tentative ideas for that purpose,

e.g. a zooming technique (097-106, Nov. 19, and 22, 1967)
exploitation of symmetry (097-112, Nov. 21, 1967)
a block-iterative approach (097-118, Dec. 5, 1967)

Some doubts arose in the correctness and how to calculate in points on the boundary between iron and air. This was clarified (097-115, Nov. 25, 1967).

13.) It was also clarified that the solution dissolves itself into three major steps:

1.) Calculation of $^{\circ}\vec{H}_n$, the m.m.f. gradient due to electric current in all points of the space, and neglecting the presence of iron. This was shown in Paper #4 at the II International Conference on Magnet Technology in Oxford, England.

2.) The calculation of φ and μ at points in the iron (097-099, -100).

3.) The calculation of the m.m.f. gradient, $\vec{H}_n$ in points outside the iron but considering the iron. (097-108)

The really complex part is Step #2. It is the intention to solve it by using a main computer program and geometrical subroutines.

First, simple cases, like a cube, a rod, etc. will be handled by direct solution of the set of linear equations, This will be checked against test-results.

If the first calculation proves to be right and also if the computer-program is debuggeed, then more complex cases will be solved by additional techniques.

Finally, the accuracy of the calculation will be checked.

APPENDIX II

At points where is no current:

$$\vec{J} = 0 \qquad (057\text{-}44)$$

and for static problems

$$\frac{\partial \vec{D}}{\partial t} = 0 \qquad (057\text{-}45)$$

One of Maxwell's equations is

$$\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} \qquad (057\text{-}46)$$

A combination of (057-44, 45, 46) results for static problems and for points without electric current

$$\nabla \times \vec{H} = 0 \qquad (057\text{-}47)$$

APPENDIX III.

The expression in equation 155-5-2, 046-2, can be developed as follows:

$$\nabla\left(\frac{\hat{h}_m \cdot \hat{r}_{mn}}{r_{mn}^2}\right) = \nabla\left(\hat{h}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right) \qquad \begin{matrix}(155\text{-}5\text{-}2)\\(046\text{-}2)\end{matrix}$$

Recall for any two vectors, $\vec{E}$ and $\vec{F}$.

$$\nabla(\vec{E}\cdot\vec{F}) = (\vec{E}\cdot\nabla)\vec{F} + (\vec{F}\cdot\nabla)E + \vec{E}\times(\nabla\times\vec{F}) + \vec{F}\times(\nabla\times\vec{E}) \qquad \begin{matrix}(155\text{-}5\text{-}3)\\(046\text{-}3)\end{matrix}$$

Use in the above equation

$$\vec{E} = \vec{h}_m \; ; \quad \vec{F} = \frac{\hat{r}_{mn}}{r_{mn}^2} \qquad \begin{matrix}(155\text{-}5\text{-}6)\\(046\text{-}6)\\(155\text{-}5\text{-}7)\\(046\text{-}7)\end{matrix}$$

Then $(\vec{F}\cdot\nabla)\vec{E} = 0$ as $\nabla\vec{E} = 0$ (155-5-4,5) (046-4,5)

and $\nabla\times\vec{E} = 0$, because all partial derivatives of $\vec{E}$ at n vanish $\vec{E} = \vec{h}_m$ being a constant at n. With this nomenclature (155-5-2) (046-2) becomes

$$\nabla\left(\hat{h}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^2}\right) = \underbrace{\underbrace{(\hat{h}_m\cdot\nabla)}_{\text{scalar}}\frac{\hat{r}_{mn}}{r_{mn}^2}}_{\text{vector}} + \underbrace{\hat{h}_m\times\underbrace{\left(\nabla\times\frac{\hat{r}_{mn}}{r_{mn}^2}\right)}_{\text{vector}}}_{\text{vector}} \qquad \begin{matrix}(155\text{-}5\text{-}8)\\(046\text{-}8)\end{matrix}$$

The first term on the right hand side of this equation can be developed as follows

$$\frac{\hat{r}_{mn}}{r_{mn}^2} = \hat{i}\,\frac{\cos(ir_{mn})}{r_{mn}^2} + \hat{j}\,\frac{\cos(jr_{mn})}{r_{mn}^2} + \hat{k}\,\frac{\cos(kr_{mn})}{r_{mn}^2} \qquad \begin{matrix}(155\text{-}5\text{-}9)\\(047\text{-}1)\end{matrix}$$

$$\hat{h}_m\cdot\nabla = \cos(ih_m)\,\frac{\partial}{\partial x} + \cos(jh_m)\,\frac{\partial}{\partial y} + \cos(kh_m)\,\frac{\partial}{\partial z} \qquad \begin{matrix}(155\text{-}6\text{-}1)\\(047\text{-}2)\end{matrix}$$

$$(\hat{h}_m\cdot\nabla)\,\frac{\hat{r}_{mn}}{r_{mn}^2} = \hat{i}\left[\cos(ih_m)\,\frac{\partial}{\partial x}\,\frac{\cos(ir_{mn})}{r_{mn}^2} + \cos(jh_m)\,\frac{\partial}{\partial y}\,\frac{\cos(ir_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \cos(kh_m)\,\frac{\partial}{\partial z}\,\frac{\cos(ir_{mn})}{r_{mn}^2}\right] + \hat{j}\left[\cos(ih_m)\,\frac{\partial}{\partial x}\,\frac{\cos(jr_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \cos(jh_m)\,\frac{\partial}{\partial y}\,\frac{\cos(jr_{mn})}{r_{mn}^2} + \cos\,(kh_m)\,\frac{\partial}{\partial z}\,\frac{\cos(jr_{mn})}{r_{mn}^2}\right]$$

$$+ \hat{k}\left[\cos(ih_m)\,\frac{\partial}{\partial x}\,\frac{\cos(kr_{mn})}{r_{mn}^2} + \cos(jh_m)\,\frac{\partial}{\partial y}\,\frac{\cos(kr_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \cos(kh_m)\,\frac{\partial}{\partial z}\,\frac{\cos(kr_{mn})}{r_{mn}^2}\right] \qquad \begin{matrix}(155\text{-}6\text{-}2)\\(047\text{-}3)\end{matrix}$$

and the second term on the right hand side of equ.(046-8) can be developed as follows:

$$\nabla\times\frac{\hat{r}_{mn}}{r_{mn}^2} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\cos(ir_{mn})}{r_{mn}^2} & \frac{\cos(jr_{mn})}{r_{mn}^2} & \frac{\cos(kr_{mn})}{r_{mn}^2} \end{vmatrix} = \qquad \begin{matrix}(155\text{-}7\text{-}3)\\(048\text{-}1)\end{matrix}$$

$$= \hat{i}\underbrace{\left(\frac{\partial}{\partial y}\frac{\cos(kr_{mn})}{r_{mn}^{2}} - \frac{\partial}{\partial z}\frac{\cos(jr_{mn})}{r_{mn}^{2}}\right)}_{A} + \hat{j}\underbrace{\left(\frac{\partial}{\partial z}\frac{\cos(ir_{mn})}{r_{mn}^{2}} - \frac{\partial}{\partial x}\frac{\cos(kr_{mn})}{r_{nm}^{2}}\right)}_{B} +$$

$$+ \hat{k}\underbrace{\left(\frac{\partial}{\partial x}\frac{\cos(jr_{mn})}{r_{mn}^{2}} - \frac{\partial}{\partial y}\frac{\cos(ir_{mn})}{r_{mn}^{2}}\right)}_{C} \qquad \begin{matrix}(155\text{-}7\text{-}4)\\(048\text{-}2)\end{matrix}$$

$$\hat{h}_m \times \left(\nabla \times \frac{\hat{r}_{mn}}{r_{mn}^{2}}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos(ih_m) & \cos(jh_m) & \cos(kh_m) \\ A & B & C \end{vmatrix} = \qquad \begin{matrix}(155\text{-}7\text{-}5)\\(048\text{-}3)\end{matrix}$$

$$= \hat{i}\left[\cos(jh_m)C - \cos(kh_m)B\right] + \hat{j}\left[\cos(kh_m)A - \cos(ih_m)C\right] +$$

$$+ \hat{k}\left[\cos(ih_m)B - \cos(jh_m)A\right] \qquad \begin{matrix}(155\text{-}7\text{-}6)\\(048\text{-}4)\end{matrix}$$

$$= \hat{i}\left[\cos(jh_m)\frac{\partial}{\partial x}\frac{\cos(jr_{mn})}{r_{mn}^{2}} - \cos(jh_m)\frac{\partial}{\partial y}\frac{\cos(ir_{mn})}{r_{mn}^{2}} -\right.$$

$$\left. - \cos(kh_m)\frac{\partial}{\partial z}\frac{\cos(ir_{mn})}{r_{mn}^{2}} + \cos(kh_m)\frac{\partial}{\partial x}\frac{\cos(kr_{mn})}{r_{mn}^{2}}\right] +$$

$$+ \hat{j}\left[\cos(kh_m)\frac{\partial}{\partial y}\frac{\cos(kr_{mn})}{r_{mn}^{2}} - \cos(kh_m)\frac{\partial}{\partial z}\frac{\cos(jr_{mn})}{r_{mn}^{2}} -\right.$$

$$\left. - \cos(ih_m)\frac{\partial}{\partial x}\frac{\cos(jr_{mn})}{r_{mn}^{2}} + \cos(ih_m)\frac{\partial}{\partial y}\frac{\cos(ir_{mn})}{r_{mn}^{2}}\right] +$$

$$+ \hat{k}\left[\cos(ih_m)\frac{\partial}{\partial z}\frac{\cos(ir_{mn})}{r_{mn}^{2}} - \cos(ih_m)\frac{\partial}{\partial x}\frac{\cos(kr_{mn})}{r_{mn}^{2}} -\right.$$

$$\left. - \cos(jh_m)\frac{\partial}{\partial y}\frac{\cos(kr_{mn})}{r_{mn}^{2}} + \cos(jh_m)\frac{\partial}{\partial z}\frac{\cos(jr_{mn})}{r_{mn}^{2}}\right] \qquad \begin{matrix}(155\text{-}7\text{-}7)\\(049\text{-}1)\end{matrix}$$

The equation (155-5-2) (046-2) is then in a more developed form and by combination of (046-8), (047-3), (049-1) as follows. Note that the terms dash-dotted underlined in equ. (047-3) and (049-1) cancel.

$$\nabla\left(\hat{h}_m \cdot \frac{\hat{r}_{mn}}{r_{mn}^{2}}\right) = \hat{i}\left[\cos(ih_m)\frac{\partial}{\partial x}\frac{\cos(ir_{mn})}{r_{mn}^{2}} + \cos(jh_m)\frac{\partial}{\partial x}\frac{\cos(jr_{mn})}{r_{mn}^{2}} + \cos(kh_m)\frac{\partial}{\partial x}\frac{\cos(kr_{mn})}{r_{mn}^{2}}\right] + \hat{j}\left[\cos(ih_m)\frac{\partial}{\partial y}\frac{\cos(ir_{mn})}{r_{mn}^{2}} + \cos(jh_m)\frac{\partial}{\partial y}\frac{\cos(jr_{mn})}{r_{mn}^{2}} + \cos(kh_m)\frac{\partial}{\partial y}\frac{\cos(kr_{mn})}{r_{mn}^{2}}\right] + \hat{k}\left[\cos(ih_m)\frac{\partial}{\partial z}\frac{\cos(ir_{mn})}{r_{mn}^{2}} + \cos(jh_m)\frac{\partial}{\partial z}\frac{\cos(jr_{mn})}{r_{mn}^{2}} + \cos(kh_m)\frac{\partial}{\partial z}\frac{\cos(kr_{mn})}{r_{mn}^{2}}\right]$$

(155-8-1)
(049-2)

APPENDIX IV

The Mathematics of Forming $\nabla \cdot \vec{B} = 0$ Equation in Terms of φ and μ

Use partial derivatives of φ for the components of $\vec{H}$ as shown above. (155-5-1, 046-1)

Linearize, expressing the first partial derivatives of φ_n-s by differences.

$$\frac{\partial \varphi_m}{\partial x_m} = \lim_{\Delta x \to 0} \frac{\varphi(x + \Delta x, y, z) - \varphi(x, y, z)}{\Delta x}$$

$$\simeq \frac{\varphi(x_{m+1} y_m z_m) - \varphi(x_{m-1} y_m z_m)}{2 \dfrac{x_{m+1} - x_{m-1}}{2}} \qquad \begin{matrix}(099\text{-}6x)\\(098\text{-}1x)\end{matrix}$$

$$\frac{\partial \varphi_m}{\partial y_m} = \lim_{\Delta y \to 0} \frac{\varphi(x, y + \Delta y, z) - \varphi(x, y, z)}{\Delta y}$$

$$\simeq \frac{\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m)}{2 \dfrac{y_{m+1} - y_{m-1}}{2}} \qquad \begin{matrix}(099\text{-}6y)\\(098\text{-}1y)\end{matrix}$$

$$\frac{\partial \varphi_m}{\partial z_m} = \lim_{\Delta z \to 0} \frac{\varphi(x, y, z + z) - \varphi(x, y, z)}{\Delta z}$$

$$\simeq \frac{\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1})}{2 \dfrac{z_{m+1} - z_{m-1}}{2}} \qquad \begin{matrix}(099\text{-}6z)\\(098\text{-}1z)\end{matrix}$$

Expressing the first and second partial derivatives of the terms containing the cosines and the r_{mn}-s in equ.(099-5x,y,z) and(099-10xx,yy,zz)

$$C_{mnix} = \frac{\partial}{\partial x_n} \frac{\cos(ir_{mn})}{R^2} = \frac{\partial}{\partial x_n} \frac{x_n - x_m}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{3}{2}}}$$

$$= (x_n - x_m)\left[-\frac{3}{2}\left\{(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right\}^{-\frac{5}{2}} 2(x_n-x_m) + \right.$$

$$+ \frac{1}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{3}{2}}}$$

$$= \frac{-3(x_n-x_m)^2}{\left[R^2\right]^{\frac{5}{2}}} + \frac{(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2}{\left[R^2\right]^{\frac{5}{2}}}$$

$$= \frac{-2(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{R^2 - 3(x_n-x_m)^2}{R^5} \tag{130-1}$$

Similarly

$$C_{mnjy} = \frac{(x_n-x_m)^2 - 2(y_n-y_m)^2 + (z_n-z_m)^2}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{R^2 - 3(y_n-y_m)^2}{R^5} \tag{130-2}$$

and

$$C_{mnkz} = \frac{(x_n-x_m)^2 + (y_n+y_m)^2 - 2(z_n-z_m)^2}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{R^2 - 3(z_n-z_m)^2}{R^5} \tag{130-3}$$

$$C_{mnjx} = \frac{\partial}{\partial x_n}\left(\frac{y_n-y_m}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{3}{2}}}\right)$$

$$= (y_n-y_m)\left\{-\frac{3}{2}\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{-\frac{5}{2}}\right\} 2(x_n-x_m)$$

$$= \frac{-3(x_n-x_m)\ (y_n-y_m)}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(x_n-x_m)(y_n-y_m)}{R^5} \tag{130-4}$$

Similarly

$$C_{mnkx} = \frac{-3(x_n-x_m)(z_n-z_m)}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(x_n-x_m)(z_n-z_m)}{R^5} \tag{130-5}$$

NOTE:

$$C_{mniy} = C_{mnjx} = \frac{\partial}{\partial y_n}\left(\frac{x_n-x_m}{R^{2\,\frac{3}{2}}}\right) = \frac{-3(x_n-x_m)(y_n-y_m)}{R^5} \tag{130-6}$$

Similarly

and $C_{mniz} = C_{mnkx}$ (130-7)

and $C_{mnjz} = C_{mnky} = \dfrac{-3\,(y_n-y_m)(z_n-z_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}}$ (130-8)

Now consider second partials

$$C_{mnixx} = \frac{\partial^2}{\partial x_n^2}\frac{\cos(ir_{mn})}{R^2} = \frac{\partial^2}{\partial x_n^2}\frac{x_n-x_m}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{3}{2}}}$$

$$= \frac{\partial}{\partial x_n}\frac{-2(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}}$$

$$= \left[-2(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]\left\{-\frac{5}{2}\left[(x_n-x_m)^2+(y_n-y_m)^2+\right.\right.$$

$$\left.\left.(z_n-z_m)^2\right]^{-\frac{7}{2}} 2(x_n-x_m)\right]\Bigg\} +$$

$$+ \frac{1}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}}\left[-4(x_n-x_m)\right]$$

$$= \frac{-5(x_n-x_m)\left[-2(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]-4(x_n-x_m)\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{\left[6(x_n-x_m)^2-9(y_n-y_m)^2-9(z_n-z_m)^2\right](x_n-x_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{\left[15(x_n-x_m)^2-9R^2\right](x_n-x_m)}{\left[R^2\right]^{\frac{7}{2}}} \qquad (130\text{-}9)$$

Similarly

$$C_{mnjyy} = \frac{\partial^2}{\partial y_n^2}\frac{\cos(jr_{mn})}{R^2} = \frac{(y_n-y_m)\left[-9(x_n-x_m)^2+6(y_n-y_m)^2-9(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

(130-10)

$$C_{mnkzz} = \frac{\partial^2}{\partial z_n^2}\frac{\cos(kr_{mn})}{R^2} = \frac{(z_n-z_m)\left[-9(x_n-x_m)^2-9(y_n-y_m)^2+6(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

(130-11)

$$C_{mnjxx} = \frac{\partial^2}{\partial x_n^2}\frac{\cos(jr_{mn})}{R^2} = \frac{\partial}{\partial x}\left[\frac{-3(x_n-x_m)(y_n-y_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}}\right]$$

$$= -3(x_n-x_m)(y_n-y_m)\left[-\frac{5}{2}\left\{(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right\}^{-\frac{7}{2}} 2(x_n-x_m)\right]$$

$$+ \frac{-3(y_n-y_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}}$$

$$= \frac{+15(x_n-x_m)^2(y_n-y_m)-3(y_n-y_m)\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(y_n-y_m)\left[12(x_n-x_m)^2-3(y_n-y_m)^2-3(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(y_n-y_m)\left[15(x_n-x_m)^2-3R^2\right]}{R^7} \qquad (130\text{-}12)$$

$$C_{mnkxx} = \frac{\partial^2}{\partial x_n^2}\frac{\cos(kr_{mn})}{R^2} = \frac{\partial}{\partial x_n}\frac{-3(x_n-x_m)(z_n-z_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}}$$

$$= \frac{(z_n-z_m)\left[12(x_n-x_m)^2-3(y_n-y_m)^2-3(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(z_n - z_m)\left[15(x_n - x_m)^2 \quad -3\ R^2\right]}{R^7} \tag{130-13}$$

$$C_{mniyy} = \frac{\partial^2}{\partial y_n^{\ 2}} \cos(ir_{mn}) = \frac{\partial}{\partial y_n} \quad \frac{-3(x_n - x_m)(y_n - y_m)}{R^5}$$

$$= \frac{(x_n - x_m)\left[-3(x_n - x_m)^2 + 12(y_n - y_m)^2 - 3(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(x_n - x_m)\left[15(y_n - y_m)^2 \quad -3\ R^2\right]}{R^7} \tag{130-14}$$

$$C_{mnkyy} = \frac{\partial^2}{\partial y_n^{\ 2}} \frac{\cos(kr_{mn})}{R^2} = \frac{\partial}{\partial y_n} \quad \frac{-3(y_n - y_m)(z_n - z_m)}{R^5}$$

$$= \frac{(z_n - z_m)\left[-3(x_n - x_m)^2 + 12(y_n - y_m)^2 - 3(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(z_n - z_m)\left[15(y_n - y_m)^2 \quad -3\ R^2\right]}{R^7} \tag{130-15}$$

$$C_{mnizz} = \frac{\partial^2}{\partial z_n^{\ 2}} \frac{\cos(ir_{mn})}{R^2} = \frac{\partial}{\partial z_n} \quad \frac{-3(x_n - x_m)(z_n - z_m)}{R^5}$$

$$= \frac{(x_n-x_m)\left[-3(x_n-x_m)^2 -3(y_n-y_m)^2 +12(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2 +(y_n-y_m)^2 +(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(x_n-x_m)\left[15(z_n-z_m)^2 -3R^2\right]}{R^7} \qquad (130\text{-}16)$$

$$C_{mnjzz} = \frac{\partial^2}{\partial z_n^2}\frac{\cos(jr_{mn})}{R^2} = \frac{\partial}{\partial x_n}\frac{-3(y_n-y_m)(z_n-z_m)}{R^5}$$

$$= \frac{(y_n-y_m)\left[-3(x_n-x_m)^2 -3(y_n-y_m)^2 +12(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2 +(y_n-y_m)^2 +(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(y_n-y_m)\left[15(z_n-z_m)^2 -3R^2\right]}{R^7} \qquad (130\text{-}17)$$

All derivatives calculated are numerically defined in a certain problem. They can be considered as constants and denoted by the C-s as shown above.

Now express the first partial derivatives of μ_{rn}

$$\frac{\partial\mu_{rn}}{\partial x_n} = \frac{\mu_{rn}(x_{n+1}y_n z_n) - \mu_{rn}(x_{n-1}y_n z_n)}{2\left(\frac{x_{n+1}-x_{n-1}}{2}\right)} = U_{nx} \qquad \begin{matrix}(098\text{-}3x)\\(099\text{-}8x)\end{matrix}$$

$$\frac{\partial\mu_{rn}}{\partial y_n} = \frac{\mu_{rn}(x_n y_{n+1} z_n) - \mu_{rn}(x_n y_{n-1} z_n)}{2\left(\frac{y_{n+1}-y_{n-1}}{2}\right)} = U_{ny} \qquad \begin{matrix}(098\text{-}3y)\\(099\text{-}8y)\end{matrix}$$

$$\frac{\partial \mu_{rn}}{\partial z_n} = \frac{\mu_{rn}(x_n y_n z_{n+1}) - \mu_{rn}(x_n y_n z_{n-1})}{2\left(\frac{z_{n+1}-z_{n-1}}{2}\right)} = U_{nz} \qquad (098\text{-}3z)\ (099\text{-}8z)$$

Write the here calculated values of the partial derivatives into (099-5x,y,z).

The result is:

$$\frac{\partial \varphi_n}{\partial x_n} = -\,{}^{o}H_{nx} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m(\mu_{rm} - 1)\left[\frac{\varphi(x_{m+1}y_m z_m) - \varphi(x_{m-1}y_m z_m)}{x_{m+1}-x_{m-1}}\,\cdot\right.$$

$$C_{mnix} + \frac{\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m)}{y_{m+1}-y_{m-1}} C_{mnjx} +$$

$$\left. + \frac{\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1})}{z_{m+1}-z_{m-1}} C_{mnkx}\right] \qquad (099\text{-}9x)$$

$$\frac{\partial \varphi_n}{\partial y_n} = -\,{}^{o}H_{ny} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m(\mu_{rm} - 1)\left[\frac{\varphi(x_{m+1}y_m z_m) - \varphi(x_{m-1}y_m z_m)}{x_{m+1}-x_{m-1}} C_{mniy} +\right.$$

$$\left. + \frac{\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m)}{y_{m+1}-y_{m-1}} C_{mnjy} + \frac{\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1})}{z_{m+1}-z_{m-1}} C_{mnky}\right]$$

$$(099\text{-}9y)$$

$$\frac{\partial \varphi_n}{\partial z_n} = -\,{}^{o}H_{nz} - \frac{1}{4\pi}\sum_{\substack{m=1\\ m\neq n}}^{p} V_m(\mu_{rm} - 1)\left[\frac{\varphi(x_{m+1}y_m z_m) - \varphi(x_{m-1}y_m z_m)}{x_{m+1}-x_{m-1}} C_{mniz} +\right.$$

$$\left. + \frac{\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m)}{y_{m+1}-y_{m-1}} C_{mnjz} + \frac{\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1})}{z_{m+1}-z_{m-1}} C_{mnkz}\right]$$

$$(099\text{-}9z)$$

Build the product: $\frac{\partial \mu_n}{\partial x}\ \frac{\partial \varphi_n}{\partial x}$ using (099-8x,y,z) and (099-9x,y,z).

Denote $\frac{1}{4\pi} V_m(\mu_{rm}-1) = A_m$

$$\frac{\partial \mu_n}{\partial x_n}\frac{\partial \varphi_n}{\partial x_n} = -\frac{{}^{\circ}H_x(x_n y_n z_n)}{x_{n+1}-x_{n-1}}\,\mu_r(x_{n+1}y_n z_n) + \frac{{}^{\circ}H_x(x_n y_n z_n)}{x_{n+1}-x_{n-1}}\,\mu_r(x_{n-1}y_n z_n) -$$

$$-\sum_{\substack{m+1\\ m\neq n}}^{p} A_m\left\{\mu_r(x_{n+1}y_n z_n)-\mu_r(x_{n-1}y_n z_n)\right\}\cdot$$

$$\left\{\frac{C_{mnix}}{(x_{n+1}-x_{n-1})(x_{m+1}-x_{m-1})}\left[\varphi(x_{m+1}y_m z_m) - \varphi(x_{m-1}y_m z_m)\right] + \right.$$

$$+ \frac{C_{mnjx}}{(x_{n+1}-x_{n-1})(y_{m+1}-y_{m-1})}\left[\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m)\right] +$$

$$\left. + \frac{C_{mnkx}}{(x_{n+1}-x_{n-1})(z_{m+1}-z_{m-1})}\left[\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1})\right]\right\}$$

(099-9x1)

$$\frac{\partial \mu_n}{\partial y_n}\frac{\partial \varphi_n}{\partial y_n} = -\frac{{}^{\circ}H_y(x_n y_n z_n)}{y_{n+1}-y_{n-1}}\,\mu_r(x_n y_{n+1} z_n) + \frac{{}^{\circ}H_y(x_n y_n z_n)}{y_{n+1}-y_{n-1}}\,\mu_r(x_n y_{n-1} z_n) -$$

$$-\sum_{\substack{m=1\\ m\neq n}}^{p} A_m\left\{\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right\}$$

$$\left\{\frac{C_{mniy}}{(y_{n+1}-y_{n-1})(x_{m+1}-x_{m-1})}\left[\varphi(x_{m+1}y_m z_m) - \varphi(x_{m-1}y_m z_m)\right] + \right.$$

$$+ \frac{C_{mnjy}}{(y_{n+1}-y_{n-1})(y_{m+1}-y_{m-1})}\left[\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m)\right] +$$

$$\left. + \frac{C_{mnky}}{(y_{n+1}-y_{n-1})(z_{m+1}-z_{m-1})}\left[\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1})\right]\right\} \quad (099\text{-}9y1)$$

$$\frac{\partial \mu_n}{\partial z_n}\frac{\partial \varphi_n}{\partial z_n} = -\frac{{}^{\circ}H_z(x_n y_n z_n)}{z_{n+1}-z_{n-1}}\,\mu_r(x_n y_n z_{n+1}) + \frac{{}^{\circ}H_z(x_n y_n z_n)}{z_{n+1}-z_{n-1}}\,\mu_r(x_n y_n z_{n-1}) -$$

$$- \sum_{\substack{m=1 \\ m \neq n}}^{p} A_m \left\{ \mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right\}$$

$$\left\{ \frac{C_{mniz}}{(z_{n+1}-z_{n-1})(x_{m+1}-x_{m-1})} \left[\varphi(x_{m+1} y_m z_m) - \varphi(x_{m-1} y_m z_m) \right] + \right.$$

$$+ \frac{C_{mnjz}}{(z_{n+1}-z_{n-1})(y_{m+1}-y_{m-1})} \left[\varphi(x_m y_{m+1} z_m) - \varphi(x_m y_{m-1} z_m) \right] +$$

$$\left. + \frac{C_{mnkz}}{(z_{n+1}-z_{n-1})(z_{m+1}-z_{m-1})} \left[\varphi(x_m y_m z_{m+1}) - \varphi(x_m y_m z_{m-1}) \right] \right\} \quad (099\text{-}9z1)$$

Note that A_m contains μ_{rm}-1).

Lump the constants in (099-9x1,9y1,9z1) except the μ-s.

$$\frac{\partial \mu_n}{\partial x_n} \frac{\partial \varphi_n}{\partial x_n} = - K_{nhx} \mu_r(x_{n+1} y_n z_n) + K_{nhx} \mu_r(x_{n-1} y_n z_n) -$$

$$- \sum_{\substack{m=1 \\ m \neq n}}^{p} \left\{ K_{mnix} \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n) \right] \varphi(x_{m+1} y_m z_m) - \right.$$

$$- K_{mnix} \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n) \right] \varphi(x_{m-1} y_m z_m) +$$

$$+ K_{mnjx} \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n) \right] \varphi(x_m y_{m+1} z_m) -$$

$$- K_{mnjx} \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n) \right] \varphi(x_m y_{m-1} z_m) +$$

$$+ K_{mnkx} \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n) \right] \varphi(x_m y_m z_{m+1}) -$$

$$\left. - K_{mnkx} \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n) \right] \varphi(x_m y_m z_{m-1}) \right\} \quad (099\text{-}9x2)$$

$$\frac{\partial \mu_n}{\partial y_n} \frac{\partial \varphi_n}{\partial y_n} = - K_{nhy} \mu_r(x_n y_{n+1} z_n) + K_{nhy} \mu_r(x_n y_{n-1} z_n) -$$

$$- \sum_{\substack{m=1 \\ m \neq n}}^{p} \Big\{ K_{mniy} \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n) \right] \varphi\,(x_{m+1} y_m z_m) \quad -$$

$$- K_{mniy} \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n) \right] \varphi(x_{m-1} y_m z_m) \quad +$$

$$+ K_{mnjy} \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n) \right] \varphi(x_m y_{m+1} z_m) \quad -$$

$$- K_{mnjy} \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n) \right] \varphi(x_m y_{m-1} z_m) \quad +$$

$$+ K_{mnky} \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n) \right] \varphi(x_m y_m z_{m+1}) \quad -$$

$$- K_{mnky} \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n) \right] \varphi(x_m y_m z_{m-1}) \Big\} \qquad (099\text{-}9y2)$$

$$\frac{\partial \mu_n}{\partial z_n} \frac{\partial \varphi_n}{\partial z_n} = -K_{nhz} \mu_r(x_n y_n z_{n+1}) + K_{nhz} \mu_r(x_n y_n z_{n-1}) \quad -$$

$$- \sum_{\substack{m=1 \\ m \neq n}}^{p} \Big\{ K_{mniz} \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right] \varphi(x_{m+1} y_m z_m) \quad -$$

$$- K_{mniz} \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right] \varphi(x_{m-1} y_m z_m) \quad +$$

$$+ K_{mnjz} \left[\mu_r(z_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right] \varphi(x_m y_{m+1} z_m) \quad -$$

$$- K_{mnjz} \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right] \varphi(x_m y_{m-1} z_m) \quad +$$

$$+ K_{mnkz} \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right] \varphi(x_m y_m z_{m+1}) \quad -$$

$$- K_{mnkz} \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1}) \right] \varphi(x_m y_m z_{m-1}) \Big\} \qquad (099\text{-}9z2)$$

Total (099-9x2,9y2,9z2)

$$\frac{\partial \mu_n}{\partial x_n} \frac{\partial \varphi_n}{\partial x_n} + \frac{\partial \mu_n}{\partial y_n} \frac{\partial \varphi_n}{\partial y_n} + \frac{\partial \mu_n}{\partial z_n} \frac{\partial \varphi_n}{\partial z_n} = - K_{nhx} \mu_r(x_{n+1} y_n z_n) \quad +$$

$$+ K_{nhx} \mu_r(x_{n-1} y_n z_n) - K_{nhy} \mu_r(x_n y_{n+1} z_n) + K_{nhy}\, \mu_r(x_n y_{n-1} z_n) \quad +$$

$$- K_{nhz} \mu_r(x_n y_n z_{n+1}) + K_{nhz} \mu_r(x_n y_n z_{n-1}) \quad +$$

$$
\begin{aligned}
&+ \sum_{\substack{m=1\\ m\neq n}}^{p} \Big[- \Big\{ K_{mnix} \left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n) \right] + \\
&+ K_{mniy} \left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n) \right] + \\
&+ K_{mniz} \left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1}) \right] \Big\} \varphi(x_{m+1}y_mz_m) + \\
&+ \Big\{ K_{mnix} \left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n) \right] + \\
&+ K_{mniy} \left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n) \right] + \\
&+ K_{mniz} \left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1}) \right] \Big\} \varphi(x_{m-1}y_mz_m) - \\
&- \Big\{ K_{mnjx} \left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n) \right] + \\
&+ Kmnjy \left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n) \right] + \\
&+ K_{mnjz} \left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1}) \right] \Big\} \varphi(x_my_{m+1}z_m) + \\
&+ \Big\{ K_{mnjx} \left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n) \right] + \\
&+ K_{mnjy} \left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n) \right] + \\
&+ K_{mnjz} \left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1}) \right] \Big\} \varphi(x_my_{m-1}z_m) - \\
&- \Big\{ K_{mnkx} \left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n) \right] + \\
&+ K_{mnky} \left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n) \right] + \\
&+ K_{mnkz} \left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1}) \right] \Big\} \varphi(x_my_mz_{m+1}) + \\
&+ \Big\{ K_{mnkx} \left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n) \right] + \\
&+ K_{mnky} \left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n) \right] + \\
&+ K_{mnkz} \left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1}) \right] \Big\} \varphi(x_my_mz_{m-1}) \Big] \qquad (099\text{-}9)
\end{aligned}
$$

Note that the "constants" K_{mnix} through K_{mnkz} contain $\mu_r(x_my_mz_m)$.

Express the second order partial derivative of φ_n - s .

$\frac{\partial^2 \varphi_n}{\partial x_n^2}$, $\frac{\partial^2 \varphi_n}{\partial y_n^2}$, $\frac{\partial^2 \varphi_n}{\partial z_n^2}$, by differentiating (099-5x, 5y, 5z) at n

$$\frac{\partial^2 \varphi_n}{\partial x_n^2} = - \frac{\partial}{\partial x_n} {}^{o}H_{nx} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m\neq n}}^{p} V_m(\mu_{rm}-1) \left[\frac{\partial \varphi_m}{\partial x_m} \frac{\partial^2}{\partial x_n^2} \frac{\cos(ir_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \frac{\partial \varphi_m}{\partial y_m} \frac{\partial^2}{\partial x_n^2} \frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m} \frac{\partial^2}{\partial x_n^2} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \qquad (099\text{-}10xx)$$

$$\frac{\partial^2 \varphi_n}{\partial y_n^2} = - \frac{\partial}{\partial y_n} {}^{o}H_{ny} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m\neq n}}^{p} V_m(\mu_{rm}-1) \left[\frac{\partial \varphi_m}{\partial x_m} \frac{\partial^2}{\partial y_n^2} \frac{\cos(ir_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \frac{\partial \varphi_m}{\partial y_m} \frac{\partial^2}{\partial y_n^2} \frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m} \frac{\partial^2}{\partial y_n^2} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \qquad (099\text{-}10yy)$$

$$\frac{\partial^2 \varphi_n}{\partial z_n^2} = - \frac{\partial}{\partial z_n} {}^{o}H_{nz} - \frac{1}{4\pi} \sum_{\substack{m=1 \\ m\neq n}}^{p} V_m(\mu_{rm} -1) \left[\frac{\partial \varphi_m}{\partial x_m} \frac{\partial^2}{\partial z_n^2} \frac{\cos(ir_{mn})}{r_{mn}^2} + \right.$$

$$\left. + \frac{\partial \varphi_m}{\partial y_m} \frac{\partial^2}{\partial z_n^2} \frac{\cos(jr_{mn})}{r_{mn}^2} + \frac{\partial \varphi_m}{\partial z_m} \frac{\partial^2}{\partial y_n^2} \frac{\cos(kr_{mn})}{r_{mn}^2} \right] \qquad (099\text{-}10zz)$$

Substitute (099-6x,6y,6z) into (099-1oxx,10yy,10zz), and developed from the second partial derivatives of the terms with the iss.

Denote again $\frac{1}{4\pi} V_m(\mu_{rm}-1) = A_m$

The result is

$$\frac{\partial^2 \varphi_n}{\partial x_n^2} = \frac{1}{x_{n+1}-x_{n-1}} \left[-{}^{o}H_x(x_{n+1}y_n z_n) + {}^{o}H_x(x_{n-1}y_n z_n) \right] +$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{p} \left\{ -A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mnixx}\, \varphi(x_{m+1} y_m z_m) \right. +$$

$$+ A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mnixx}\, \varphi(x_{m-1} y_m z_m) -$$

$$- A_m \frac{1}{y_{m+1}-y_{m-1}} C_{mnjxx}\, \varphi(x_m y_{m+1} z_m) +$$

$$+ A_m \frac{1}{y_{m+1}-y_{m-1}} C_{mnjxx}\, \varphi(x_m y_{m-1} z_m) -$$

$$- A_m \frac{1}{z_{m+1}-z_{m-1}} C_{mnkxx}\, \varphi(x_m y_m z_{m+1}) +$$

$$\left. + A_m \frac{1}{z_{m+1}-z_{m-1}} C_{mnkxx}\, \varphi(x_m y_m z_{m-1}) \right\} \qquad (099\text{-}13xx)$$

$$\frac{\partial^2 \varphi_n}{\partial y_n^2} = \frac{1}{y_{n+1}-y_{n-1}} \left[- {}^{\circ}H_y(x_n y_{n+1} z_n) + {}^{\circ}H_y(x_n y_{n-1} z_n) \right] +$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{p} \left\{ -A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mniyy}\, \varphi(x_{m+1} y_m z_m) \right. +$$

$$+ A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mniyy}\, \varphi(x_{m-1} y_m z_m) -$$

$$- A_m \frac{1}{y_{m+1}-y_{m-1}} C_{mnjyy}\, \varphi(x_m y_{m+1} z_m) +$$

$$+ A_m \frac{1}{y_{m+1}-y_{m-1}} C_{mnjyy}\, \varphi(x_m y_{m-1} z_m) -$$

$$- A_m \frac{1}{z_{m+1}-z_{m-1}} C_{mnkyy}\, \varphi(x_m y_m z_{m+1}) +$$

$$\left. + A_m \frac{1}{z_{m+1}-z_{m-1}} C_{mnkyy}\, \varphi(x_m y_m z_{m-1}) \right\} \qquad (099\text{-}13yy)$$

$$\frac{\partial^2 \varphi_n}{\partial z_n^2} = \frac{1}{z_{n+1}-z_{n-1}} \left[- {}^oH_z(x_n y_n z_{n+1}) + {}^oH_z(x_n y_n z_{n-1}) \right] +$$

$$+ \sum_{\substack{m=1 \\ m\neq n}}^{p} \left\{ -A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mnizz}\, \varphi(x_{m+1} y_m z_m) + \right.$$

$$+ A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mnizz}\, \varphi(x_{m-1} y_m z_m) -$$

$$- A_m \frac{1}{x_{m+1}-x_{m-1}} C_{mnjzz}\, \varphi(x_m y_{m+1} z_m) +$$

$$+ A_m \frac{1}{y_{m+1}-y_{m-1}} C_{mnjzz}\, \varphi(x_m y_{m-1} z_m) -$$

$$- A_m \frac{1}{z_{m+1}-z_{m-1}} C_{mnkzz}\, \varphi(x_m y_m z_{m+1}) +$$

$$\left. + A_m \frac{1}{z_{m+1}-z_{m-1}} C_{mnkzz}\, \varphi(x_m y_m z_{m-1}) \right\} \qquad (099\text{-}13zz)$$

Equations (099-13xx,13yy,13zz) are giving the second partial derivatives of φ_n in the form to be used later in the expanded expression of $\nabla \cdot \vec{B}_n$.

A lumping of all constants results in forms of the equations (099-13xx,13yy, 13zz) as follows:

$$\frac{\partial^2 \varphi_n}{\partial x_n^2} = - K_{nhx\pm 1} + \sum_{\substack{m=1 \\ m\neq n}}^{p} \left[- K_{mnixx}\varphi(x_{m+1} y_m z_m) + K_{mnixx}\varphi(x_{m-1} y_m z_m) - \right.$$

$$- K_{mnjxx}\varphi(x_m y_{m+1} z_m) + K_{mnjxx}\varphi(x_m y_{m-1} z_m) -$$

$$\left. - K_{mnkxx}\varphi(x_m y_m z_{m+1}) + K_{mnkxx}\varphi(x_m y_m z_{m-1}) \right] \qquad (099\text{-}13xx1)$$

$$\frac{\partial^2 \varphi_n}{\partial y_n^2} = - K_{nhy\pm 1} + \sum_{\substack{m=1 \\ m\neq n}}^{p} \left[-K_{mniyy}\varphi(x_{m+1} y_m z_m) + K_{mniyy}\varphi(x_{m-1} y_m z_m) - \right.$$

$$- K_{mnjyy}\varphi(x_m y_{m+1} z_m) + K_{mnjyy}\varphi(x_m y_{m-1} z_m) -$$

$$- K_{mnkyy}\varphi(x_m y_m z_{m+1}) + K_{mnkyy}\varphi(x_m y_m z_{m-1}) \qquad (099\text{-}13yy1)$$

$$\frac{\partial^2 \varphi_n}{\partial z_n^2} = - K_{nhz\underline{+}1} + \sum_{\substack{m=1\\m\neq n}}^{p} \Big[- K_{mnizz}\varphi(x_{m+1} y_m z_m) + K_{mnizz}\varphi(x_{m-1} y_m z_m) -$$

$$- K_{mnjzz}\varphi(z_m y_{m+1} z_m) + K_{mnjzz}\varphi(x_m y_{m-1} z_m) -$$

$$- K_{mnkzz}\varphi(x_m y_m z_{m+1}) + K_{mnkzz}\varphi(x_m y_m z_{m-1})\Big] \qquad (099\text{-}13zz1)$$

The sum of the second derivatives will be required. This is written by totaling (099-13xx1, 13yy1, 13zz1)

$$\frac{\partial^2 \varphi_n}{\partial x_n^2} + \frac{\partial^2 \varphi_n}{\partial y_n^2} + \frac{\partial^2 \varphi_n}{\partial z_n^2} = -K_{nhx\underline{+}1} - K_{nhy\underline{+}1} - K_{nhz\underline{+}1} +$$

$$+ \sum_{\substack{m=1\\m\neq n}}^{p} \Big[- (K_{mnixx} + K_{mniyy} + K_{mnizz})\ \varphi\ (x_{m+1} y_m z_m) \quad +$$

$$+ (K_{mnixx} + K_{mniyy} + K_{mnizz})\ \varphi\ (x_{m-1} y_m z_m) \quad -$$

$$- (K_{mnjxx} + K_{mnjyy} + K_{mnjzz})\ \varphi\ (x_m y_{m+1} z_m) \quad +$$

$$+ (K_{mnjxx} + K_{mnjyy} + K_{mnjzz})\ \varphi\ (x_m y_{m-1} z_m) \quad -$$

$$- (K_{mnkxx} + K_{mnkyy} + K_{mnkzz})\ \varphi\ (x_m y_m z_{m+1}) \quad +$$

$$+ (K_{mnkxx} + K_{mnkyy} + K_{mnkzz})\ \varphi\ (x_m y_m z_{m-1}) \qquad (099\text{-}13a)$$

Lumping the constants further

$$\frac{\partial^2 \varphi_n}{\partial x_n^2} + \frac{\partial^2 \varphi_n}{\partial y_n^2} + \frac{\partial^2 \varphi_n}{\partial z_n^2} = - K_{nh} + \sum_{\substack{m=1\\m\neq n}}^{p} \Big[- K_{mni}\varphi(x_{m+1} y_m z_m) +$$

$$+ K_{mni}\varphi(x_{m-1}y_mz_m) - K_{mnj}\varphi(x_my_{m+1}z_m) + K_{mnj}\varphi(x_my_{m-1}z_m) -$$

$$- K_{mnk}\varphi(x_my_mz_{m+1}) + K_{mnk}\varphi(x_my_mz_{m-1})\Big] \qquad (099\text{-}13)$$

Only three different constants, K_{mni}, K_{mnj}, K_{mnk} are used for each mn combination, plus one constant K_{nh} for each n. Note that the constants K_{mni}, K_{mnj}, K_{mnk} contain $\mu_r(x_my_mz_m)$.

$\nabla\times\vec{H}_n = 0$ is automatically satisfied, because (099-14) (100-1)

$\vec{H}_n = -\nabla\varphi_n$ see (099-057 sh. 9) (099-15) (100-4)

$\nabla\cdot\vec{B}_n = 0$ must be satisfied (099-16) (100-2)

$\nabla\cdot\vec{B}_n = \nabla\cdot\left[\mu_{rn}\mu_o(-\nabla\varphi_n)\right] = 0$ (096-7) (099-17) (100-5)

μ_o being a constant, follows

$\nabla\cdot\left[\mu_{rn}\nabla\varphi_n\right] = 0$ (096-8) (099-18)

Expand (099-18)

$$\mu_{rn}\frac{\partial^2\varphi_n}{\partial x_n^2} + \mu_{rn}\frac{\partial^2\varphi_n}{\partial y_n^2} + \mu_{rn}\frac{\partial^2\varphi_n}{\partial z_n^2} + \frac{\partial\mu_{rn}}{\partial x_n}\frac{\partial\varphi_n}{\partial x_n} + \frac{\partial\mu_{rn}}{\partial y_n}\frac{\partial\varphi_n}{\partial y_n} + \frac{\partial\mu_{rn}}{\partial z_n}\frac{\partial\varphi_n}{\partial z_n} = 0$$

(096-9) (099-19) (100-6)

Add (099-9) to μ_{rn} times (099-13) to have (099-19). The result is a linearized equation between φ_n - s and μ_n - s.

$$- K_{nhx}\mu_r(x_{n+1}y_nz_n) + K_{nhx}\mu_r(x_{n-1}y_nz_n) +$$

$$- K_{nhy}\mu_r(x_ny_{n+1}z_n) + K_{nhy}\mu_r(x_ny_{n-1}z_n) +$$

$$- K_{nhz}\mu_r(x_ny_nz_{n+1}) + K_{nhz}\mu_r(x_ny_nz_{n-1}) - K_{nh}\mu_r(x_ny_nz_n) +$$

$$+ \sum_{\substack{m=1\\m\neq n}}^{p}\Bigg[-\Big\{K_{mnix}\left[\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)\right] +$$

$$+ K_{mniy}\left[\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)\right] +$$

$$+ K_{mniz}\left[\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})\right] +$$

$$+ K_{mni}\mu_r(x_n y_n z_n)\Big\}\varphi(x_{m+1} y_m z_m) \quad +$$

$$+ \Big\{K_{mnix}\Big[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\Big] \quad +$$

$$+ K_{mniy}\Big[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\Big] \quad +$$

$$+ K_{mniz}\Big[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\Big] \quad +$$

$$+ K_{mni}\mu_r(x_n y_n z_n)\Big\}\varphi(x_{m-1} y_m z_m) \quad -$$

$$- \Big\{K_{mnjx}\Big[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\Big] \quad +$$

$$+ K_{mnjy}\Big[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\Big] \quad +$$

$$+ K_{mnjz}\Big[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\Big] \quad +$$

$$+ K_{mnj}\mu_r(x_n y_n z_n)\Big\}\varphi(x_m y_{m+1} z_m) \quad +$$

$$+ \Big\{K_{mnjx}\Big[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\Big] \quad +$$

$$+ K_{mnjy}\Big[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\Big] \quad +$$

$$+ K_{mnjz}\Big[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\Big] \quad +$$

$$+ K_{mnj}\cdot\mu_r(x_n y_n z_n)\Big\}\varphi(x_m y_{m-1} z_m) \quad -$$

$$- \Big\{K_{mnkx}\Big[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\Big] \quad +$$

$$+ K_{mnky}\Big[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\Big] \quad +$$

$$+ K_{mnkz}\Big[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\Big] \quad +$$

$$+ K_{mnk}\cdot\mu_r(x_n y_n z_n)\Big\}\varphi(x_m y_m z_{m+1}) \quad +$$

$$+ \Big\{K_{mnkx}\Big[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\Big] \quad +$$

$$+ K_{mnky}\Big[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\Big] \quad +$$

$$= K_{mnkz}\Big[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\Big] \quad +$$

$$+ K_{mnk}\cdot\mu_r(x_n y_n z_n)\Big\}\varphi(x_m y_m z_{m-1})\Big] = 0 \qquad (099\text{-}20)$$

Note that K_{mnix} through K_{mnkz} and K_{mni} through K_{mnk} contain $\mu_r(x_m y_m z_m)$, but K_{nhx} through K_{nhz} and K_{nh} do <u>not</u>.

Expand again the K constants so as to show the μ_r - s.

$$- K_{nhx}\mu_r(x_{n+1}y_nz_n) + K_{mhx}\mu_r(x_{n-1}y_nz_n) \quad +$$

$$- K_{nhy}\mu_r(x_ny_{n+1}z_n) + K_{nhy}\mu_r(x_ny_{n-1}z_n) \quad +$$

$$- K_{nhz}\mu_r(x_ny_nz_{n+1}) + K_{nhz}\mu_r(x_ny_nz_{n-1}) - K_{nh}\mu_r(x_ny_nz_n) \quad +$$

$$+ \sum_{\substack{m=1\\ m\neq n}}^{p} \Big[- \Big\{ M_{mnix} [\mu_r(x_my_mz_m)-1] [\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)] \quad +$$

$$+ M_{mniy} [\mu_r(x_my_mz_m)-1] [\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)] \quad +$$

$$+ M_{mniz} [\mu_r(x_my_mz_m)-1] \quad \mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})] \quad +$$

$$+ L_{mni} [\mu_r(x_my_mz_m)-1] \cdot \mu_r(x_ny_nz_n) \Big\} \quad \varphi(x_{m+1}y_mz_m) \quad +$$

$$+ \Big\{ M_{mnix} [\mu_r(x_my_mz_m)-1] [\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)] \quad +$$

$$+ M_{mniy} [\mu_r(x_my_mz_m)-1] [\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)] \quad +$$

$$+ M_{mniz} [\mu_r(x_my_mz_m)-1] [\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})] \quad +$$

$$+ L_{mni} [\mu_r(x_my_mz_m)-1] \cdot \mu_r(x_ny_nz_n) \Big\} \quad \varphi(x_{m-1}y_mz_m) \quad -$$

$$- \Big\{ M_{mnjx} [\mu_r(x_my_mz_m)-1] [\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)] \quad +$$

$$+ M_{mnjy} [\mu_r(x_my_mz_m)-1] [\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)] \quad +$$

$$+ M_{mnjz} [\mu_r(x_my_mz_m)-1] [\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})] \quad +$$

$$+ L_{mnj} [\mu_r(x_my_mz_m)-1] \cdot \mu_r(x_ny_nz_n) \Big\} \quad \varphi(x_my_{m+1}z_m) \quad +$$

$$+ \Big\{ M_{mnjx} [\mu_r(x_my_mz_m)-1] [\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)] \quad +$$

$$+ M_{mnjy} [\mu_r(x_my_mz_m)-1] [\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)] \quad +$$

$$
\begin{aligned}
&+ M_{mnjz}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\right] + \\
&+ L_{mnj}\left[\mu_r(x_m y_m z_m)-1\right] \cdot\mu_r(x_n y_n z_n)\Big\} \; \varphi(x_m y_{m-1} z_m) - \\
&- \Big\{M_{mnkx}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\right] + \\
&+ M_{mnky}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right] + \\
&+ M_{mnkz}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r)x_n y_n z_{n-1})\right] + \\
&+ L_{mnk}\left[\mu_r(x_m y_m z_m)-1\right] \cdot\mu_r(x_n y_n z_n)\Big\} \; \varphi(x_m y_m z_{m+1}) + \\
&+ \Big\{M_{mnkx}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)\right] + \\
&+ M_{mnky}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)\right] + \\
&+ M_{mnkz}\left[\mu_r(x_m y_m z_m)-1\right] \left[\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})\right] + \\
&+ L_{mnk}\left[\mu_r(x_m y_m z_m)-1\right] \cdot\mu_r(x_n y_n z_n)\Big\} \; \varphi(x_m y_m z_{m-1})\Big] = 0 \qquad (099\text{-}20)
\end{aligned}
$$

All constants evaluated in this Appendix IV are listed in alphabetical order in Appendix V.

APPENDIX V

LIST OF CONSTANTS

$$A_m = \frac{1}{4\pi} V_m(\mu_{rm}-1) \tag{099-9a}$$

$$C_{mnix} = \frac{-2\ (x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{R^2-3(x_n-x_m)^2}{R^5} \tag{130-1}$$

$$C_{mnjx} = \frac{-3\ (x_n-x_m)(y_n-y_m)}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(x_n-x_m)(y_n-y_m)}{R^5} \tag{130-4}$$

$$C_{mnkx} = \frac{-3\ (x_n-x_m)(z_n-z_m)}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(x_n-x_m)(z_n-z_m)}{R^5} \tag{130-5}$$

$$C_{mniy} = \frac{-3\ (x_n-x_m)(y_n-y_m)}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(x_n-x_m)(y_n-y_m)}{R^5} \tag{130-6}$$

$$C_{mnjy} = \frac{(x_n-x_m)^2 - 2(y_n-y_m)^2 + (z_n-z_m)^2}{\left[(x_n-x_m)^2 + (y_n-y_m)^2 + (z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{R^2 - 3(y_n-y_m)^2}{R^5} \tag{130-2}$$

$$C_{mnky} = \frac{-3(y_n-y_m)(z_n-z_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(y_n-y_m)(z_n-z_m)}{R^5} \qquad (130\text{-}8)$$

$$C_{mniz} = \frac{-3(x_n-x_m)(z_n-z_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(x_n-x_m)(z_n-z_m)}{R^5} \qquad (130\text{-}7)$$

$$C_{mnjz} = \frac{-3(y_n-y_m)(z_n-z_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{-3(y_n-y_m)(z_n-z_m)}{R^5} \qquad (130\text{-}8)$$

$$C_{mnkz} = \frac{(x_n-x_m)^2+(y_n-y_m)^2-2(z_n-z_m)^2}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{5}{2}}} = \frac{R^2-3\ (z_n-z_m)^2}{R^5} \qquad (130\text{-}3)$$

$$C_{mnixx} = \frac{\left[6(x_n-x_m)^2-9(y_n-y_m)^2-9(z_n-z_m)^2\right](x_n-x_m)}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{\left[15(x_n-x_m)^2-9\ R^2\right](x_n-x_m)}{R^7} \qquad (130\text{-}9)$$

$$C_{mnjxx} = \frac{(y_n-y_m)\left[12(x_n-x_m)^2-3(y_n-y_m)^2-3(z_n-z_m)^2\right]}{\left[(x_n-x_m)^2+(y_n-y_m)^2+(z_n-z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(y_n - y_m)\left[15(x_n - x_m)^2 \quad -3\ R^2\right]}{R^7} \qquad (130\text{-}12)$$

$$C_{mnkxx} = \frac{(z_n - z_m)\left[12(x_n - x_m)^2 - 3(y_n - y_m)^2 \quad -3(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(z_n - z_m)\left[15(x_n - x_m)^2 \quad -3\ R^2\right]}{R^7} \qquad (130\text{-}13)$$

$$C_{mniyy} = \frac{(x_n - x_m)\left[-3(x_n - x_m)^2 \quad +12(y_n - y_m)^2 \quad -3(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 + (y_n - y_m)^2 \quad +(z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(x_n - x_m)\left[15(y_n - y_m)^2 \quad -3\ R^2\right]}{R^7} \qquad (130\text{-}14)$$

$$C_{mnjyy} = \frac{(y_n - y_m)\left[-9(x_n - x_m)^2 \quad +6(y_n - y_m)^2 \quad -9(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(y_n - y_m)\left[15(y_n - y_m)^2 \quad -9\ R^2\right]}{R^7} \qquad (130\text{-}10)$$

$$C_{mnkyy} = \frac{(z_n - z_m)\left[-3(x_n - x_m)^2 \quad +12(y_n - y_m)^2 \quad -3(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 + (y_n - y_m)^2 + (z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(z_n - z_m)\left[15(y_n - y_m)^2 \quad -3\ R^2\right]}{R^7} \qquad (130\text{-}15)$$

$$C_{mnizz} = \frac{(x_n - x_m)\left[-3(x_n - x_m)^2 \ -3(y_n - y_m)^2 \ +12(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 \ +(y_n - y_m)^2 \ +(z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(x_n - x_m)\left[15(z_n - z_m)^2 \quad -3\ R^2\right]}{R^7} \qquad (130\text{-}16)$$

$$C_{mnjzz} = \frac{(y_n - y_m)\left[-3(x_n - x_m)^2 \ -3(y_n - y_m)^2 \ +12(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 \ +(y_n - y_m)^2 \ +(z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(y_n - y_m)\left[15(z_n - z_m)^2 \quad -3\ R^2\right]}{R^7} \qquad (130\text{-}17)$$

$$C_{mnkzz} = \frac{(z_n - z_m)\left[-9(x_n - x_m)^2 \ -9(y_n - y_m)^2 \ +6(z_n - z_m)^2\right]}{\left[(x_n - x_m)^2 \ +(y_n - y_m)^2 \ +(z_n - z_m)^2\right]^{\frac{7}{2}}}$$

$$= \frac{(z_n - z_m)\left[15(z_n - z_m)^2 \quad -9\ R^3\right]}{R^7} \qquad (130\text{-}11)$$

$$K_{on} = K_{nhx}\left(\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)\right) +$$

$$+ K_{nhy}\left(\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)\right) +$$

$$+ K_{nhz}\left(\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})\right) +$$

$$+ K_{nh} \cdot \mu_r(x_ny_nz_n) \qquad (109\text{-}4)$$

$$K_{nh} = K_{nhx\underline{+}1} + K_{nhy\underline{+}1} + K_{nhz\underline{+}1} \qquad (109\text{-}5)$$

$$K_{nhx} = \frac{{}^\circ H_x(x_ny_nz_n)}{x_{n+1}-x_{n-1}} \qquad (109\text{-}6)$$

$$K_{nhy} = \frac{{}^\circ H_y(x_ny_nz_n)}{y_{n+1}-y_{n-1}} \qquad (109\text{-}7)$$

$$K_{nhz} = \frac{{}^\circ H_z(x_ny_nz_n)}{z_{n+1}-z_{n-1}} \qquad (109\text{-}8)$$

$$K_{nhx\underline{+}1} = \frac{{}^\circ H_x(x_{n+1}y_nz_n) - {}^\circ H_x(x_{n-1}y_nz_n)}{x_{n+1}-x_{n-1}} \qquad (109\text{-}9)$$

$$K_{nhy\underline{+}1} = \frac{{}^\circ H_y(x_ny_{n+1}z_n) - {}^\circ H_y(x_ny_{n-1}z_n)}{y_{n+1}-y_{n-1}} \qquad (109\text{-}10)$$

$$K_{nhz\underline{+}1} = \frac{{}^\circ H_z(x_ny_nz_{n+1}) - {}^\circ H_z(x_ny_nz_{n-1})}{z_{n+1}-z_{n-1}} \qquad (109\text{-}11)$$

$$K_{mni} = K_{mnixx} + K_{mniyy} + K_{mnizz} \qquad (109\text{-}12)$$

$$K_{mnj} = K_{mnjxx} + K_{mnjyy} + K_{mnjzz} \qquad (109\text{-}13)$$

$$K_{mnk} = K_{mnkxx} + K_{mnkyy} + K_{mnkzz} \qquad (109\text{-}14)$$

$$K_{mni} = \frac{A_m}{x_{m+1}-x_{m-1}} \quad (C_{mnixx} + C_{mniyy} + C_{mnizz}) \qquad (109\text{-}1)$$

$$K_{mni} \simeq \frac{1}{4\pi} \; \frac{V_m(\mu_{rm}-1)}{x_{m+1}-x_{m-1}} \Bigg[$$

$$\Bigg[\left\{\frac{4}{(x_{n+1}-x_{n-1})}\,2\right\}\Big\{\left(\cos\left[ir_{(x_m y_m z_m)(x_{n+1} y_n z_n)}\right]/r^2_{(x_m y_m z_m)(x_{n+1} y_n z_n)}\right) -$$

$$- \left(2\cos\left[ir_{(x_m y_m z_m)(x_n y_n z_n)}\right]/r^2_{(x_m y_m z_m)(x_n y_n z_n)}\right) +$$

$$+ \left(\cos\left[ir_{(x_m y_m z_m)(x_{n-1} y_n z_n)}\right]/r^2_{(x_m y_m z_m)(x_{n-1} y_n z_n)}\right)\Big\} +$$

$$+\left\{\frac{4}{(y_{n+1}-y_{n-1})}\,2\right\}\Big\{\left(\cos\left[ir_{(x_m y_m z_m)(x_n y_{n+1} z_n)}\right]/r^2_{(x_m y_m z_m)(x_n y_{n+1} z_n)}\right) -$$

$$- \left(2\cos\left[ir_{(x_m y_m z_m)(x_n y_n z_n)}\right]/r^2_{(x_m y_m z_m)(x_n y_n z_n)}\right) +$$

$$+ \left(\cos\left[ir_{(x_m y_m z_m)(x_n y_{n-1} z_n)}\right]/r^2_{(x_m y_m z_m)(x_n y_{n-1} z_n)}\right) +$$

$$+\left\{\frac{4}{(z_{m+1}-z_{m-1})}\,2\right\}\Big\{\left(\cos\left[ir_{(x_m y_m z_m)(x_n y_n z_{n+1})}\right]/r^2_{(x_m y_m z_m)(x_n y_n z_{n+1})}\right) -$$

$$- \left(2\cos\left[ir_{(x_m y_m z_m)(x_n y_n z_n)}\right]/r^2_{(x_m y_m z_m)(x_n y_n z_n)}\right) +$$

$$+ \left(\cos\left[ir_{(x_m y_m z_m)(x_n y_n z_{n-1})}\right]/r^2_{(x_m y_m z_m)(x_n y_n z_{n-1})}\right)\Big\}\Bigg]$$

$$(101\text{-}4)$$

$$K_{mnj} = \frac{A_m}{y_{m+1}-y_{m-1}} \quad (C_{mnjxx} + C_{mnjyy} + C_{mnjzz}) \qquad (109\text{-}2)$$

$$K_{mnj} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{y_{m+1}-y_{m-1}} \Bigg[$$

$$\Bigg[\left\{\frac{4}{(x_{n+1}-x_{n-1})}\right\}^2 \Big\{\Big(\cos\big[jr(x_m y_m z_m)(x_{n+1} y_n z_n)\big]/r^2(x_m y_m z_m)(x_{n+1} y_n z_n)\Big) -$$

$$- \Big(2\cos\big[jr(x_m y_m z_m)(x_n y_n z_n)\big]/r^2(x_m y_m z_m)(x_n y_n z_n)\Big) +$$

$$+ \Big(\cos\big[jr(x_m y_m z_m)(x_{n-1} y_n z_n)\big]/r^2(x_m y_m z_m)(x_{n-1} y_n z_n)\Big) +$$

$$+\left\{\frac{4}{(y_{n+1}-y_{n-1})}\right\}^2 \Big\{\Big(\cos\big[jr(x_m y_m z_m)(x_n y_{n+1} z_n)\big]/r^2(x_m y_m z_m)(x_n y_{n+1} z_n)\Big) -$$

$$- \Big(2\cos\big[jr(x_m y_m z_m)(x_n y_n z_n)\big]/r^2(x_m y_m z_m)(x_n y_n z_n)\Big) +$$

$$+ \Big(\cos\big[jr(x_m y_m z_m)(x_n y_{n-1} z_n)\big]/r^2(x_m y_m z_m)(x_n y_{n-1} z_n)\Big)\Big\} +$$

$$+\left\{\frac{4}{(z_{m+1}-z_{m-1})}\right\}^2 \Big\{\Big(\cos\big[jr(x_m y_m z_m)(x_n y_n z_{n+1})\big]/r^2(x_m y_m z_m)(x_n y_n z_{n+1})\Big) -$$

$$- \Big(2\cos\big[jr(x_m y_m z_m)(x_n y_n z_n)\big]/r^2(x_m y_m z_m)(x_n y_n z_n)\Big) +$$

$$+ \Big(\cos\big[jr(x_m y_m z_m)(x_n y_n z_{n-1})\big]/r^2(x_m y_m z_m)(x_n y_n z_{n-1})\Big)\Big\}\Bigg]$$

$$(101\text{-}4)$$

$$K_{mnk} = \frac{A_m}{z_{m+1} - z_{m-1}} \quad (C_{mnkxx} + C_{mnkyy} + C_{mnkzz}) \qquad (109\text{-}3)$$

$$K_{mnk} \simeq \frac{1}{4\pi} \quad \frac{V_m(\mu_{rm}-1)}{z_{m+1} - z_{m-1}} \Bigg[$$

$$\Bigg[\left\{\frac{4}{(x_{n+1} - x_{n-1})}\quad 2\right\}\left\{\left(\cos\left[kr(x_m y_m z_m)(x_{n+1} y_n z_n)\right]/r^2(x_m y_m z_m)(x_{n+1} y_n z_n)\right) -\right.$$

$$- \left(2 \cos\left[kr(x_m y_m z_m)(x_n y_n z_n)\right]/r^2(x_m y_m z_m)(x_n y_n z_n)\right) \quad +$$

$$\left.+ \left(\cos\left[kr(x_m y_m z_m)(x_{n-1} y_n z_n)\right]/r^2(x_m y_m z_m)(x_{n-1} y_n z_n)\right)\right\} + \Bigg]$$

$$+\left\{\frac{4}{(y_{n+1} - y_{n-1})}\quad 2\right\}\left\{\left(\cos\left[kr(x_m y_m z_m)(x_n y_{n+1} z_n)\right]/r^2(x_m y_m z_m)(x_n y_{n+1} z_n)\right) -\right.$$

$$- \left(2 \cos\left[kr(x_m y_m z_m)(x_n y_n z_n)\right]/r^2(x_m y_m z_m)(x_n y_n z_n)\right) \quad +$$

$$\left.+ \left(\cos\left[kr(x_m y_m z_m)(x_n y_{n-1} z_n)\right]/r^2(x_m y_m z_m)(x_n y_{n-1} z_n)\right)\right\} \Bigg]$$

$$+\left\{\frac{4}{(z_{m+1} - z_{m-1})}\quad 2\right\}\left\{\left(\cos\left[kr(x_m y_m z_m)(x_n y_n z_{n+1})\right]/r^2(x_m y_m z_m)(x_n y_n z_{n+1})\right) -\right.$$

$$- \left(2 \cos\left[kr(x_m y_m z_m)(x_n y_n z_n)\right]/r^2(x_m y_m z_m)(x_n y_n z_n)\right) \quad +$$

$$\left.+ \left(\cos\left[kr(x_m y_m z_m)(x_n y_n z_{n-1})\right]/r^2(x_m y_m z_m)(x_n y_n z_{n-1})\right)\right\} \Bigg]$$

(101-4)

$$K_{mnix} = \frac{A_m C_{mnix}}{(x_{n+1}-x_{n-1})(x_{m+1}-x_{m-1})} \qquad (109\text{-}15)$$

$$K_{mnix} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{x_{m+1}-x_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(x_{n+1}-x_{n-1})^2} \Bigg\{ \Big(\cos \big[ir(x_m y_m z_m)(x_{n+1} y_n z_n)\big] / r^2 (x_m y_m z_m)(x_{n+1} y_n z_n)\Big) -$$

$$- \Big(\cos \big[ir(x_m y_m z_m)(x_{n-1} y_n z_n)\big] / r^2 (x_m y_m z_m)(x_{n-1} y_n z_n)\Big) \Bigg\} \Bigg] \qquad (101\text{-}7)$$

$$K_{mnjx} = \frac{A_m C_{mnjx}}{(x_{n+1}-x_{n-1})(y_{m+1}-y_{m-1})} \qquad (109\text{-}16)$$

$$K_{mnjx} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{y_{m+1}-y_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(x_{n+1}-x_{n-1})^2} \Bigg\{ \Big(\cos \big[jr(x_m y_m z_m)(x_{n+1} y_n z_n)\big] / r^2 (x_m y_m z_m)(x_{n+1} y_n z_n)\Big) -$$

$$- \Big(\cos \big[jr(x_m y_m z_m)(x_{n-1} y_n z_n)\big] / r^2 (x_m y_m z_m)(x_{n-1} y_n z_n)\Big) \Bigg\} \Bigg] \qquad (101\text{-}7)$$

$$K_{mnkx} = \frac{A_m C_{mnkx}}{(x_{n+1}-x_{n-1})(z_{m+1}-z_{m-1})} \qquad (109\text{-}17)$$

$$K_{mnkx} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{z_{m+1}-z_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(x_{n+1}-x_{n-1})^2} \Bigg\{ \Big(\cos \big[kr(x_m y_m z_m)(x_{n+1} y_n z_n)\big] / r^2 (x_m y_m z_m)(x_{n+1} y_n z_n)\Big) -$$

$$- \left(\cos\left[kr(x_m y_m z_m)(x_{n-1} y_n z_n)\right] / r^2 (x_m y_m z_m)(x_{n-1} y_n z_n)\right)\Big\}\Big]$$

(101-7)

$$K_{mniy} = \frac{A_m \; C_{mniy}}{(y_{n+1} - y_{n-1})(x_{m+1} - x_{m-1})} \qquad (109\text{-}18)$$

$$K_{mniy} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm} - 1)}{x_{m+1} - x_{m-1}} \Bigg[$$

$$\left[\frac{1}{(y_{n+1} - y_{n-1})^2} \Big\{ \left(\cos\left[ir(x_m y_m z_m)(x_n y_{n+1} z_n)\right] / r^2 (x_m y_m z_m)(x_n y_{n+1} z_n)\right) - \right.$$

$$- \left(\cos\left[ir(x_m y_m z_m)(x_n y_{n-1} z_n)\right] / r^2 (x_m y_m z_m)(x_n y_{n-1} z_n)\right)\Big\}\Big]$$

(101-7)

$$K_{mnjy} = \frac{A_m \; C_{mnjy}}{(y_{n+1} - y_{n-1})(y_{m+1} - y_{m-1})} \qquad (109\text{-}19)$$

$$K_{mnjy} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm} - 1)}{y_{m+1} - y_{m-1}} \Bigg[$$

$$\left[\frac{1}{(y_{n+1} - y_{n-1})^2} \Big\{ \left(\cos\left[jr(x_m y_m z_m)(x_n y_{n+1} z_n)\right] / r^2 (x_m y_m z_m)(x_n y_{n+1} z_n)\right) - \right.$$

$$- \cos\left[jr(x_m y_m z_m)(x_n y_{n-1} z_n)\right] / r^2 (x_m y_m z_m)(x_n y_{n-1} z_n)\Big)\Big\}\Big]$$

(101-7)

$$K_{mnky} = \frac{A_m \; C_{mnky}}{(y_{n+1} - y_{n-1})(z_{m+1} - z_{m-1})} \qquad (109\text{-}20)$$

$$K_{mnky} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{z_{m+1}-z_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(y_{n+1}-y_{n-1})^2} \Bigg\{ \left(\cos\left[kr(x_m y_m z_m)(x_n y_{n+1} z_n)\right] / r^2 (x_m y_m z_m)(x_n y_{n+1} z_n)\right) -$$

$$- \left(\cos\left[kr(x_m y_m z_m)(x_n y_{n-1} z_n)\right] / r^2 (x_m y_m z_m)(x_n y_{n-1} z_n)\right) \Bigg\} \Bigg] \quad (101\text{-}7)$$

$$K_{mniz} = \frac{A_m \; C_{mniz}}{(z_{n+1}-z_{n-1})(x_{m+1}-x_{m-1})} \quad (109\text{-}21)$$

$$K_{mniz} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{x_{m+1}-x_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(z_{n+1}-z_{n-1})^2} \Bigg\{ \left(\cos\left[ir(x_m y_m z_m)(x_n y_n z_{n+1})\right] / r^2 (x_m y_m z_m)(x_n y_n z_{n+1})\right) -$$

$$- \left(\cos\left[ir(x_m y_m z_m)(x_n y_n z_{n-1})\right] / r^2 (x_m y_m z_m)(x_n y_n z_{n-1})\right) \Bigg\} \Bigg] \quad (101\text{-}7)$$

$$K_{mnjz} = \frac{A_m \; C_{mnjz}}{(z_{n+1}-z_{n-1})(y_{m+1}-y_{m-1})} \quad (109\text{-}22)$$

$$K_{mnjz} \simeq \frac{1}{4\pi} \frac{V_m(\mu_{rm}-1)}{y_{m+1}-y_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(z_{n+1}-z_{n-1})^2} \Bigg\{ \left(\cos\left[jr(x_m y_m z_m)(x_n y_n z_{n+1})\right] / r^2 (x_m y_m z_m)(x_n y_n z_{n+1})\right) -$$

$$- \left(\cos\left[jr(x_m y_m z_m)(x_n y_n z_{n-1})\right] / r^2 (x_m y_m z_m)(x_n y_n z_{n-1})\right) \Bigg\} \Bigg] \quad (101\text{-}7)$$

$$K_{mnkz} = \frac{A_m \; C_{mnkz}}{(z_{n+1}-z_{n-1})(z_{m+1}-z_{m-1})} \tag{109-23}$$

$$K_{mnkz} \simeq \frac{1}{4\pi} \; \frac{V_m(\mu_{rm}-1)}{z_{m+1}-z_{m-1}} \Bigg[$$

$$\Bigg[\frac{1}{(z_{n+1}-z_{n-1})^2}\Bigg\{\Big(\cos\Big[kr(x_m y_m z_m)(x_n y_n z_{n+1})\Big]/r^2(x_m y_m z_m)(x_n y_n z_{n+1})\Big)-$$

$$-\Big(\cos\Big[kr(x_m y_m z_m)(x_n y_n z_{n-1})\Big]/r^2(x_m y_m z_m)(x_n y_n z_{n-1})\Big)\Bigg\}\Bigg] \tag{101-7}$$

$$K_{mnixx} = \frac{A_m \; C_{mnixx}}{x_{m+1}-x_{m-1}} \tag{109-24}$$

$$K_{mnjxx} = \frac{A_m \; C_{mnjxx}}{y_{m+1}-y_{m-1}} \tag{109-25}$$

$$K_{mnkxx} = \frac{A_m \; C_{mnkxx}}{z_{m+1}-z_{m-1}} \tag{109-26}$$

$$K_{mniyy} = \frac{A_m \; C_{mniyy}}{x_{m+1}-x_{m-1}} \tag{109-27}$$

$$K_{mnjyy} = \frac{A_m \; C_{mnjyy}}{y_{m+1}-y_{m-1}} \tag{109-28}$$

$$K_{mnkyy} = \frac{A_m \; C_{mnkyy}}{z_{m+1}-z_{m-1}} \tag{109-29}$$

$$K_{mnizz} = \frac{A_m \; C_{mnizz}}{x_{m+1}-x_{m-1}} \tag{109-30}$$

$$K_{mnjzz} = \frac{A_m \; C_{mnjzz}}{y_{m+1} - y_{m-1}} \tag{109-31}$$

$$K_{mnkzz} = \frac{A_m \; C_{mnkzz}}{z_{m+1} - z_{m-1}} \tag{109-32}$$

$$K_{x(m+1)} = K_{x(m-1)} =$$

$$= K_{mnix} \; (\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)) \quad +$$

$$+ K_{mniy} \; (\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)) \quad +$$

$$+ K_{mniz} \; (\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})) \tag{109-33}$$

$$+ K_{mni} \quad \mu_r(x_n y_n z_n)$$

$$K_{y(m+1)} = K_{y(m-1)} =$$

$$= K_{mnjx} \; (\mu_r(x_{n+1} y_n z_n) - \mu_r(x_{n-1} y_n z_n)) \quad +$$

$$+ K_{mnjy} \; (\mu_r(x_n y_{n+1} z_n) - \mu_r(x_n y_{n-1} z_n)) \quad +$$

$$+ K_{mnjz} \; (\mu_r(x_n y_n z_{n+1}) - \mu_r(x_n y_n z_{n-1})) \tag{109-34}$$

$$+ K_{mni} \quad \mu_r(x_n y_n z_n)$$

$$K_{z(m+1)} = K_{z(m-1)} =$$

$$= K_{mnkx}\ (\mu_r(x_{n+1}y_nz_n) - \mu_r(x_{n-1}y_nz_n)) \quad +$$

$$+ K_{mnky}\ (\mu_r(x_ny_{n+1}z_n) - \mu_r(x_ny_{n-1}z_n)) \quad +$$

$$+ K_{mnkz}\ (\mu_r(x_ny_nz_{n+1}) - \mu_r(x_ny_nz_{n-1})) \qquad (109\text{-}35)$$

$$+ K_{mnk}\ \mu_r(x_ny_nz_n)$$

$$L_{mni} = \frac{K_{mni}}{(\mu_r(x_my_mz_m)-1)} \qquad \begin{matrix}(102\text{-}1)\\(102\text{-}3)\\(102\text{-}15)\end{matrix}$$

$$L_{mnj} = \frac{K_{mnj}}{(\mu_r(x_my_mz_m)-1)} \qquad \begin{matrix}(102\text{-}1)\\(102\text{-}4)\\(102\text{-}15)\end{matrix}$$

$$L_{mnk} = \frac{K_{mnk}}{(\mu_r(x_my_mz_m)-1)} \qquad \begin{matrix}(102\text{-}1)\\(102\text{-}5)\\(102\text{-}15)\end{matrix}$$

$$M_{mnix} = \frac{K_{mnix}}{(\mu_r(x_my_mz_m)-1)} \qquad \begin{matrix}(102\text{-}6)\\(102\text{-}15)\end{matrix}$$

$$M_{mniy} = \frac{K_{mniy}}{(\mu_r(x_my_mz_m)-1)} \qquad \begin{matrix}(102\text{-}7)\\(102\text{-}15)\end{matrix}$$

$$M_{mniz} = \frac{K_{mniz}}{(\mu_r(x_my_mz_m)-1)} \qquad \begin{matrix}(102\text{-}8)\\(102\text{-}15)\end{matrix}$$

$$M_{mnjx} = \frac{K_{mnjx}}{(\mu_r(x_m y_m z_m)-1)} \qquad \begin{matrix}(102\text{-}9)\\(102\text{-}15)\end{matrix}$$

$$M_{mnjy} = \frac{K_{mnjy}}{(\mu_r(x_m y_m z_m)-1)} \qquad \begin{matrix}(102\text{-}10)\\(102\text{-}15)\end{matrix}$$

$$M_{mnjz} = \frac{K_{mnjz}}{(\mu_r(x_m y_m z_m)-1)} \qquad \begin{matrix}(102\text{-}11)\\(102\text{-}15)\end{matrix}$$

$$M_{mnkx} = \frac{K_{mnkx}}{(\mu_r(x_m y_m z_m)-1)} \qquad \begin{matrix}(102\text{-}12)\\(102\text{-}15)\end{matrix}$$

$$M_{mnky} = \frac{K_{mnky}}{(\mu_r(x_m y_m z_m)-1)} \qquad \begin{matrix}(102\text{-}13)\\(102\text{-}15)\end{matrix}$$

$$M_{mnkz} = \frac{K_{mnkz}}{(\mu_r(x_m y_m z_m)-1)} \qquad \begin{matrix}(102\text{-}14)\\(102\text{-}15)\end{matrix}$$

APPENDIX VI

List of Equations

APPENDIX VII

Computer Program

as output of computer

```
FORTRAN IV G LEVEL 1, MOD 0          MAIN          DATE = 68195          00/43/40          PA

                C     THIS PROGRAM CALCULATES MAGNETIC FIELD INTENSITY IN AMP-TURNS PER  00010
                C     METER FOR A 480 TURN 2 SECTION HELMHOLTZ COIL.IT CALLS SURROUTINE  0002
                C     MAGFLD. THE FIELD AND COORDINATES ARE CALCULATED FOR POINTS IN A    00030
                C     .05 BY .05 BY .3 METER PARALLELPIPED AS WELL AS POINTS OUTSIDE THE 00040
                C     SPECIMEN                                                           00050
 0001                 DIMENSION A(4,4,4),B(4,4,4),C(4,4,4),AHX(4,4,4),AHY(4,4,4),
                     1AHZ(4,4,4),AH(4,4,4)
 0002                 INTEGER SEG,SWI                                                    00080
 0003             100 READ(5,590) SEG,SWI,AMP,A(1,1,1),B(1,1,1),C(1,1,1),KX,KY,KZ,       00090
                     1DELTAX,DELTAY,DELTAZ                                                 100
 0004                 WRITE(6,600)                                                         110
 0005                 WRITE(6,630) SEG,SWI,AMP,A(1,1,1),B(1,1,1),C(1,1,1),KX,KY,KZ,        120
                     1DELTAX,DELTAY,DELTAZ                                                130
 0006                 I = 1                                                                140
 0007                 J = 1                                                              00150
 0008                 K = 1                                                              00255
 0009             180 CALL MAGFLD (AMP,SEG,A(I,J,K),B(I,J,K),C(I,J,K),AHX(I,J,K),          160
                     1AHY(I,J,K),AHZ(I,J,K),AH(I,J,K))                                     170
 0010                 K = K + 1                                                            180
 0011                 IF(SWI.EQ.3) GO TO 340                                               190
 0012                 IF(K.GT.KZ) GO TO 270                                                200
 0013                 A(I,J,K) = A(I,J,K- 1)                                               220
 0014                 B(I,J,K) = B(I,J,K-1)                                                230
 0015                 C(I,J,K) = C(I,J,K-1) + DELTAZ                                       240
 0016                 GO TO 180                                                          00250
 0017             270 J = J + 1                                                            260
 0018                 IF(J.GT.KY) GO TO 340                                                260
 0019                 K = 1                                                                270
 0020                 A(I,J,K) = A(I,J-1,K)                                                280
 0021                 B(I,J,K) = B(I,J-1,K) + DELTAY                                       290
 0022                 C(I,J,K) = C(I,J-1,K)                                              00300
 0023                 GO TO 180                                                            310
 0024             340 I = I + 1                                                            320
 0025                 IF(SWI.EQ.3) GO TO 430                                               330
 0026                 IF(I.GT.KX) GO TO 480                                                340
 0027                 K = 1                                                                350
 0028                 J = 1                                                                360
 0029                 A(I,J,K) = A(I-1,J,K) + DELTAX                                       370
 0030                 B(I,J,K) = B(I-1,J,K)                                                380
 0031                 C(I,J,K) = C(I-1,J,K)                                                390
 0032                 GO TO 180                                                            400
 0033             430 J = 1                                                                410
 0034                 IF(I.GT.KX) GO TO 480
```

```
0035             A(I,J,K) = A(I-1,J,K-1) + DELTAX                         420
0036             B(I,J,K) = B(I-1,J,K-1)                                  430
0037             C(I,J,K) = C(I-1,J,K-1)  + DELTAZ                       0440
0038             GO TO 180                                                450
0039         480 IF(SWI.NE.1) GO TO 520                                   460
0040             REWIND 8
0041             WRITE( 8)(((A(I,J,K),B(I,J,K),C(I,J,K),K=1,KZ),J=1,KY),I=1,KX)   470
0042             WRITE( 8)(((AHX(I,J,K),AHY(I,J,K),AHZ(I,J,K),K=1,KZ),J=1,KY),    480
                1I = 1,KX)                                                490
0043             ENDFILE 8
0044         520 WRITE(6,640)                                             520
0045             DO 560  I = 1,KX                                         530
0046             DO 560  J = 1,KY                                         540
0047             IF(SWI.EQ.3) GO TO 580
0048             K = 1
0049         540 WRITE(6,660) I,J,K,A(I,J,K),B(I,J,K),C(I,J,K),AHX(I,J,K),    560
                1AHY(I,J,K),AHZ(I,J,K),AH(I,J,K)                          570
0050             IF(SWI.EQ.3) GO TO 560
0051             K = K + 1
0052             IF(K.LE.KZ) GO TO 540
0053             GO TO 560
0054         580 K = I
0055             GO TO 540
0056         560 CONTINUE
0057             GO TO 100                                                580
0058         590 FORMAT(2I5,F10.4,3F10.3/3I5,3F10.0)
0059         600 FORMAT(1H1,22HFIELD INTENSITY IN AIR/1H0,3HSEG,2X,3HSWI,2X,3HAMP,   600
                17X,2HXO,8X,2HYO,8X,2HZO,8X,2HKX,3X,2HKY,3X,2HKZ,3X,6HDELTAX,4X,     610
                26HDELTAY,4X,6HDELTAZ)                                    620
0060         630 FORMAT(1H0,2I5,4F10.4,3I5,3F10.4)
0061         640 FORMAT(1H0,3HI  ,3HJ  ,3HK  ,1HX,14X,1HY,14X,1HZ,14X,2HHX,13X,  640
                12HHY,13X,2HHZ13X,1HH)                                    650
0062         660 FORMAT(1H0,3I3,1P7E15.8)                                 660
0063             END                                                      670
```

```
FORTRAN IV G LEVEL 1, MOD 0              MAIN              DATE = 68195         00/43/40          PA

                C      THIS SUBROUTINE APPROXIMATES EACH HELMHOLTZ COIL WITH 20 CONCEN-
                C      TRATED COILS OF 12 TURNS EACH                                      SM0020
  0001                 SUBROUTINE MAGFLD (AMP,K,A,B,C,HX,HY,HZ,H)                         SM0030
  0002                 DIMENSION X(1000),Y(1000), Z(1000)                                 SM0040
  0003                 SEG = K                                                            SM0050
  0004                 K1 = 2 * K + 1                                                     SM0060
  0005                 AAZ = .35895098                                                    SM0070
  0006                 PI = 3.1415927                                                     SM0080
  0007                 UX = 0.0                                                           SM0090
  0008                 UY = 0.0                                                           SM0100
  0009                 UZ = 0.0                                                           SM0110
  0010                 PISEG = PI/SEG                                                     SM0120
  0011                 FRPI = 1./(4. * PI)                                                SM0130
  0012                 DO 450  IUP = 1,2                                                  SM0140
  0013                 AZ = AAZ                                                           SM0150
  0014                 RAD = .73399542                                                    SM0160
  0015                 DO 440  JX = 1,4                                                   SM0170
  0016                 DO 430 JZ = 1,5                                                    SM0180
  0017                 TUX = 0.0                                                          SM0190
  0018                 TUY = 0.0                                                          SM0200
  0019                 TUZ = 0.0                                                          SM0210
  0020                 DO 270  I = 1,K1                                                   SM00220
  0021                 COUNT = I - 1                                                      SM0230
  0022                 ARG = COUNT * PISEG                                                SM0240
  0023                 X(I) = RAD * COS(ARG) - A                                          SM0250
  0024                 Y(I) = RAD * SIN(ARG) - B                                          SM0260
  0025             270 Z(I) = AZ - C                                                      SM0270
  0026                 DO 390  I = 1, K                                                   SM0280
  0027                 ITWO = 2 * I                                                       SM0290
  0028                 INC2 = ITWO + 1                                                    SM0300
  0029                 INC1 = ITWO - 1                                                    SM0310
  0030                 R = SQRT(X(ITWO)**2 + Y(ITWO)**2 + Z(ITWO)**2)                     SM0320
  0031                 XP = X(INC2) - X(INC1)                                             SM0330
  0032                 YP = Y(INC2) - Y(INC1)                                             SM0340
  0033                 ZP = Z(INC2) - Z(INC1)                                             SM0350
  0034                 CON = FRPI /(R**3)                                                 SM0360
  0035                 TUX = TUX + CON * (YP * Z(ITWO) - ZP * Y(ITWO))                    SM0370
  0036                 TUY = TUY + CON * (ZP * X(ITWO) - XP * Z(ITWO))                    SM0380
  0037             390 TUZ = TUZ + CON * (XP * Y(ITWO) - YP * X(ITWO))                    SM0390
  0038                 UX = UX + 12.0 * TUX                                               SM0400
  0039                 UY = UY + 12.0 * TUY                                               SM0410
  0040                 UZ = UZ + 12.0 * TUZ                                               SM0420
  0041             430 AZ = AZ + .8467362E -02                                            SM0430
```

```
0042              AZ = AAZ
0043          440 RAD = RAD + .10728982E-02                          SM0440
0044          450 AAZ = -.39282043
0045              HX = AMP * UX                                      SM0460
0046              HY = AMP * UY                                      SM0470
0047              HZ = AMP * UZ                                      SM0480
0048              H = SQRT(HX**2 + HY**2 + HZ**2)                    SM0490
0049              RETURN                                             SM0500
0050              END                                                SM0510
```

```
              C     THIS PROGRAM COMPUTES THE FIELD INSIDE MAGNETIC MATERIAL DUE TO
              C     AN EXTERNAL FIELD.  IT CALLS SUBROUTINES PHICAL, PERM AND HCAL.
              C     PHICAL CALLS A MATRIX INVERSION ROUTINE
 0001               DIMENSION X(4,4,4),Y(4,4,4),Z(4,4,4),AHX(4,4,4),AHY(4,4,4),
                   1AHZ(4,4,4),UR1(4,4,4),UR2(4,4,4),PHI(4,4,4),AM(4,4,4),HN (4,4,4),
                   2B(8),HNX(4,4,4),HNY(4,4,4),HNZ(4,4,4),HUR(100,2),AMA(8,8)
 0002               COMMON X,Y,Z,AHX,AHY,AHZ,B,UR1
 0003               NUM = 5
 0004               NUM1 = 8
 0005               NUM2 = 6
 0006               NUM3 = 9
 0007               READ(NUM,630) IP,JP,KP,LIMIT,EPSI,ITOT,VOL,TOL
 0008               IP1= IP-1
 0009               JP1= JP-1
 0010               KP1= KP-1
 0011               READ(NUM,640) ((HUR(I,J), J=1,2),I=1,ITOT)
 0012               REWIND NUM1
 0013               READ (NUM1)  (((X(I,J,K),Y(I,J,K),Z(I,J,K),K=1,KP),J=1,JP),I=1,IP)
 0014               READ (NUM1)  (((AHX(I,J,K),AHY(I,J,K),AHZ(I,J,K),K=1,KP),J=1,JP),
                   1I = 1,IP)
 0015               READ (NUM,650) (((UR1(I,J,K),K=2,KP1),J=2,JP1),I=2,IP1)
 0016               WRITE(NUM2,660) IP,JP,KP,LIMIT,EPSI,ITOT,VOL
 0017               WRITE(NUM2,680)
 0018               WRITE(NUM2,690) (((X(I,J,K),AHX(I,J,K),K=1,KP),J=1,JP),I=1,IP)
 0019               WRITE(NUM2,730)
 0020               WRITE(NUM2,690)  (((Y(I,J,K),AHY(I,J,K),K=1,KP),J=1,JP),I=1,IP)
 0021               WRITE(NUM2,750)
 0022               WRITE(NUM2,690) (((Z(I,J,K),AHZ(I,J,K),K=1,KP),J=1,JP),I=1,IP)
 0023               WRITE(NUM2,800)
 0024               WRITE(NUM2,690) ((HUR(I,J),J=1,2),I=1,ITOT)
 0025               DO 490  LIM = 1, LIMIT
 0026               WRITE(NUM2,740)
 0027               WRITE (NUM2,690) (((UR1(I,J,K),K=2,KP1),J=2,JP1),I=2,IP1)
 0028               CALL PHICAL(IP,JP,KP,ITP,AM,DET,VOL,AMA)
 0029               WRITE (NUM2,900) DET
 0030               WRITE(NUM2,1000)
 0031               DO 285 I=1,ITP
 0032           285 WRITE(NUM2,1010) (AMA(I,J),J=1,ITP)
 0033               IF(ABS(DET).LE.TOL) GO TO 620
 0034               IA = 1
 0035               DO 350  I=2,IP1
 0036               DO 350  J=2,IP1
 0037               DO 350  K=2,KP1
```

```
0038             PHI(I,J,K) = B(IA)
0039         350 IA = IA + 1
0040             CALL HCAL(IP,JP,KP,PHI,HNX,HNY,HNZ,HN)
0041             WRITE (NUM2,760)
0042             WRITE(NUM2,690)  (((HNX(I,J,K), K=1,KP),J=1,JP),I=1,IP)
0043             WRITE(NUM2,770)
0044             WRITE(NUM2,690) (((HNY(I,J,K), K=1,KP),J=1,JP),I=1,IP)
0045             WRITE(NUM2,780)
0046             WRITE(NUM2,690) (((HNZ(I,J,K), K=1,KP),J=1,JP),I=1,IP)
0047             CALL PERM(UR2,HN,IP,JP,KP,HUR,ITOT)
0048             DO 420  I=2,IP1
0049             DO 420  J=2,JP1
0050             DO 420  K=2,KP1
0051             IF(ABS((UR2(I,J,K)-UR1(I,J,K)) /UR2(I,J,K)).GT.EPSI) GO TO 430
0052         420 CONTINUE
0053             GO TO 500
0054         430 DO 490  I=2,IP1
0055             DO 490  J=2,JP1
0056             DO 490  K=2,KP1
0057         490 UR1(I,J,K) = UR2(I,J,K)
0058         500 WRITE(NUM2,790)
0059             WRITE(NUM2,690) (((AM(I,J,K),K=2,KP1),J=2,JP1),I=2,IP1)
0060             REWIND NUM3
0061             WRITE(NUM3)(((X(I,J,K),Y(I,J,K),Z(I,J,K),K=1,KP),J=1,JP),I=1,IP)
0062             WRITE(NUM3)(((HNX(I,J,K),HNY(I,J,K),HNZ(I,J,K),K=1,KP),J=1,JP),
                1I=1,IP)
0063             WRITE(NUM3)  (((AM(I,J,K),K=2,KP1),J=2,JP1),I=2,IP1)
0064             ENDFILE NUM3
0065             CALL EXIT
0066         620 WRITE(NUM2,1020)
0067             CALL EXIT
0068         630 FORMAT(4I6,F10.0,I6,E15.8,F10.0)
0069         640 FORMAT(8F10.0)
0070         650 FORMAT(8F10.0)
0071         660 FORMAT(1H0,3HIP=I3,2X,3HJP=I3,2X,3HKP= I3,2X,6HLIMIT=I3,2X,
                15HEPSI=1PE16.8,5HITOT=I6,7HVOLUME=E16.8)
0072         680 FORMAT(1H0,44HFORM IS  X(111),AHX(111),X(111),AHY(112) - -   )
0073         690 FORMAT(1H ,7(1PE15.6))
0074         730 FORMAT(1H0,44HFORM IS Y(111),AHY(111),Y(112),AHY(112) - -  )
0075         740 FORMAT(1H0,8HUR1(IJK))
0076         750 FORMAT(1H0,44HFORM IS Z( 11),AHZ(111),Z(112),AHZ(112) - -  )
0077         760 FORMAT(1H0,8HHNX(IJK))
0078         770 FORMAT(1H0,8HHNY(IJK))
```

```
0079          780 FORMAT(1H0,8HHNZ(IJK))
0080          790 FORMAT(1H0,8H AM(IJK))
0081          800 FORMAT(1H0,7HHUR(IJ))
0082          900 FORMAT(1H0,5HDET =,1PE16.8)
0083         1000 FORMAT(1H0,10HMATRIX AMA)
0084         1010 FORMAT(1H0,(7E15.6))
0085         1020 FORMAT(1H0,15HAMA IS SINGULAR)
0086              END
```

```
                C      THIS SUBROUTINE SETS UP MATRIX AMA TO COMPUTE DIPOLE STRENGHTS. IT
                C      CALLS A SUBROUTINE TO SOLVE SIMULTANEOUS EQUATIONS
0001                   SUBROUTINE PHICAL (IP,JP,KP,ITP,AM,DET,VOL,AMA)
0002                   DIMENSION AM(4,4,4),AHX(4,4,4),AHY(4,4,4),AHZ(4,4,4),X(4,4,4),
                      1Y(4,4,4),Z(4,4,4),UR(4,4,4),B(8),AMA(8,8)
0003                   COMMON X,Y,Z,AHX,AHY,AHZ,B,UR
0004                   ITP = (KP-2)*(JP-2)*(IP-2)
0005                   AITP = ITP
0006                   IP1= IP-1
0007                   JP1= JP-1
0008                   KP1= KP-1
0009                   PI = 3.1415927
0010                   DO 120  I = 1,ITP
0011                   DO 120 J = 1,ITP
0012               120 AMA(I,J) = 0.
0013                   VM = VOL/AITP
0014                   DO 1540 L=2,IP1
0015                   DO 1540 M=2,JP1
0016                   DO 1540 N=2,KP1
0017                   IF(L.NE.2) GO TO  200
0018                   UR(L-1,M,N) = 1.
0019               200 IF(L.NE.IP1) GO TO 210
0020                   UR(L+1,M,N) = 1.
0021               210 IF(M.NE.2) GO TO   220
0022                   UR(L,M-1,N) = 1.
0023               220 IF(M.NE.JP1) GO TO 230
0024                   UR(L,M+1,N) = 1.
0025               230 IF(N.NE.2)   GO TO 240
0026                   UR(L,M,N-1) = 1.
0027               240 IF(N.NE.KP1) GO TO 440
0028                   UR(L,M,N+1) = 1.
0029               440 URX =(UR(L+1,M,N)-UR(L-1,M,N))/(X(L+1,M,N)-X(L-1,M,N))
0030                   URY =(UR(L,M+1,N)-UR(L,M-1,N))/(Y(L,M+1,N)-Y(L,M-1,N))
0031                   URZ =(UR(L,M,N+1)-UR(L,M,N-1))/(Z(L,M,N+1)-Z(L,M,N-1))
0032                   AHXX=(AHX(L+1,M,N)-AHX(L-1,M,N))/(X(L+1,M,N)-X(L-1,M,N))
0033                   AHYY=(AHY(L,M+1,N)-AHY(L,M-1,N))/(Y(L,M+1,N)-Y(L,M-1,N))
0034                   AHZZ=(AHZ(L,M,N+1)-AHZ(L,M,N-1))/(Z(L,M,N+1)-Z(L,M,N-1))
0035                   DO 1540 I=2,IP1
0036                   DO 1540 J=2,JP1
0037                   DO 1540 K=2,KP1
0038                   IF(I.NE.L) GO TO 600
0039                   IF(J.NE.M) GO TO 600
0040                   IF(K.NE.N) GO TO 600
```

```
0041              GO TO 1540
0042          600 XR = X(L,M,N) - X(I,J,K)
0043              YR = Y(L,M,N) - Y(I,J,K)
0044              ZR = Z(L,M,N) - Z(I,J,K)
0045              R2 = XR**2 + YR**2 + ZR**2
0046              R5 = R2**2.5
0047              R7 = R2**3.5
0048              CMNIX = (R2 - 3. * XR**2)/R5
0049              CMNJY = (R2 - 3. * YR**2)/R5
0050              CMNKZ = (R2 - 3. * ZR**2)/R5
0051              CMNIJ = -3. * XR * YR/R5
0052              CMNJK = -3. * YR * ZR/R5
0053              CMNIK = - 3. * XR * ZR/R5
0054              CMNIXX = (15. * XR **2 -9.*R2) * XR/R7
0055              CMNJYY = (15. * YR**2 - 9. * R2) * YR/R7
0056              CMNKZZ = (15. * ZR**2 - 9. * R2) * ZR / R7
0057              CMNJXX = (15. * XR**2 - 3. * R2 )* YR / R7
0058              CMNIYY = (15. * YR**2 - 3. * R2) * XR / R7
0059              CMNIZZ = (15. * ZR**2 - 3. * R2) * XR/R7
0060              CMNKXX = (15.* XR**2 - 3.*R2)*ZR/R7
0061              CMNKYY = (15.* YR**2 - 3.*R2) * ZR/R7
0062              CMNJZZ = (15. * ZR**2 - 3.* R2) * YR/R7
0063              IA =(KP-2)*(JP-2)*(L-2)+(KP-2)*(M-2)+N-1
0064              B(IA) =  -(AHX(L,M,N)*URX + AHY(L,M,N)*URY + AHZ(L,M,N)*URZ
                 1 + UR(L,M,N) * (AHXX + AHYY + AHZZ))
0065         1050 AM(I,J,K) = 1./(4. * PI) * VM * (UR(I,J,K) - 1.)
0066              DUM=AM(I,J,K)/(X(I+1,J,K)-X(I-1,J,K))*(UR(L,M,N)*(CMNIXX+CMNIYY
                 1+ CMNIZZ) + URX*CMNIX + URY*CMNIJ + URZ * CMNIK)
0067              JA =(KP-2)*(JP-2)*(I-1) +(KP-2)*(J-2)+K-1
0068              IF(JA.LT.1)  GO TO 1080
0069              IF(JA.GT.ITP) GO TO 1080
0070              AMA(IA,JA) = AMA(IA,JA) + DUM
0071         1080 JA =(KP-2)*(JP-2)*(I-3) +(KP-2)*(J-2)+K-1
0072              IF(JA.LT.1)  GO TO 1060
0073              IF(JA.GT.ITP)  GO TO 1060
0074              AMA(IA,JA) = AMA(IA,JA) - DUM
0075              DUM=AM(I,J,K)/(Y(I,J+1,K)-Y(I,J-1,K))*(UR(L,M,N)*(CMNJXX+CMNJYY
                 1+ CMNJZZ) + URX*CMNIJ + URY*CMNJY + URZ * CMNJK)
0076         1060 JA =(KP-2)*(JP-2)*(I-2)+(KP-2)*(J-1)+ K-1
0077              IF(JA.GT.ITP)  GO TO 1100
0078              IF (JA.LT.1)  GO TO 1100
0079              AMA(IA,JA) = AMA(IA,JA) + DUM
0080         1100 JA =(KP-2)*(JP-2)*(I-2)+(KP-2)*(J-3)+ K-1
```

```
0081            IF (JA.LT.1)   GO TO 1120
0082            IF(JA.GT.ITP)   GO TO 1120
0083            AMA(IA,JA) = AMA(IA,JA) - DUM
0084            DUM=AM(I,J,K)/(Z(I,J,K+1)-Z(I,J,K-1))*(UR(L,M,N)*(CMNKXX+CMNKYY
               1+ CMNKZZ) + URX*CMNIK + URY*CMNJK + URZ*CMNKZ)
0085       1120 JA =(KP-2)*(JP-2)*(I-2)+(KP-2)*(J-2) + K
0086            IF (JA.LT.1)   GO TO 1140
0087            IF(JA.GT.ITP)   GO TO 1140
0088            AMA(IA,JA) = AMA(IA,JA) + DUM
0089       1140 JA =(KP-2)*(JP-2)*(I-2)+(KP-2)*(J-2) + K - 2
0090            IF (JA.LT.1)   GO TO 1540
0091            IF(JA.GT.ITP)  GO TO 1540
0092            AMA(IA,JA) = AMA(IA,JA) - DUM
0093       1540 CONTINUE
0094            CALL MINV(AMA,ITP,B,1,DET)
0095            RETURN
0096            END
```

COMMON BLOCK /   / MAP SIZE   720

```
             C      THIS SUBROUTINE CONVERTS SCALAR POTENTIAL TO FIELD STRENGTH FOR
             C      POINTS INSIDE THE MATERIAL
  0001              SUBROUTINE HCAL(IP,JP,KP,PHI,HNX,HNY,HNZ,HN)
  0002              DIMENSION PHI(4,4,4),X(4,4,4),Y(4,4,4),Z(4,4,4),HNX(4,4,4) ,
                   1HNY(4,4,4),HNZ(4,4,4),HN(4,4,4)
  0003              COMMON X,Y,Z
  0004              IP1 = IP-1
  0005              JP1 = JP-1
  0006              KP1 = KP-1
  0007              DO 410  I = 2,IP1
  0008              DO 410 J=2,JP1
  0009              DO 410 K=2,KP1
  0010              IF(I.NE.2)   GO TO 140
  0011              PHI(I-1,J,K) = 0.
  0012          140 IF(I.NE.IP1) GO TO 160
  0013              PHI(I+1,J,K) =0.
  0014          160 IF(J.NE.2 )  GO TO 180
  0015              PHI(I,J-1,K) =0.
  0016          180 IF(J.NE.JP1) GO TO 200
  0017              PHI(I,J+1,K) =0.
  0018          200 IF(K.NE.2)   GO TO 220
  0019              PHI(I,J,K-1)  =0.
  0020          220 IF(K.NE.KP1) GO TO 380
  0021              PHI(I,J,K+1) =0.
  0022          380 HNX(I,J,K)=-(PHI(I+1,J,K)-PHI(I-1,J,K))/(X(I+1,J,K)-X(I-1,J,K))
  0023              HNY(I,J,K)=-(PHI(I,J+1,K)-PHI(I,J-1,K))/(Y(I,J+1,K)-Y(I,J-1,K))
  0024              HNZ(I,J,K)=-(PHI(I,J,K+1)-PHI(I,J,K-1))/(Z(I,J,K+1)-Z(I,J,K-1))
  0025          410 HN(I,J,K) = SQRT(HNX(I,J,K)**2 + HNY(I,J,K)**2 + HNZ(I,J,K)**2)
  0026              RETURN
  0027              END
```

```
C     THIS SUBROUTINE ESTIMATES THE NEW VALUES OF PERMEABILITY FROM THE
C     COMPUTED VALUE OF FIELD STRENGTHS
      SUBROUTINE PERM(UR2,HN,IP,JP,KP,HUR,ITOT)
      DIMENSION UR2(4,4,4),HN(4,4,4),HUR(100,2)
      IP1 = IP-1
      JP1 = JP-1
      KP1 = KP-1
      DO 190 I=2,IP1
      DO 190 J=2,JP1
      DO 190 K=2,KP1
      IF(HN(I,J,K).LE.HUR(1,1)) GO TO 150
      IA = 2
  100 IF(HN(I,J,K).LE.HUR(IA,1)) GO TO 170
      IA = IA + 1
      IF(IA.LE.ITOT) GO TO 100
      UR2(I,J,K) = HUR(ITOT,2)
      GO TO 190
  150 UR2(I,J,K) = HUR(1,2)
      GO TO 190
  170 UR2(I,J,K) = HUR(IA-1,2) + HN(I,J,K)*(HUR(IA,2)-HUR(IA-1,2))/
     1(HUR(IA,1)-HUR(IA-1,1))
  190 CONTINUE
      RETURN
      END
```

```
FORTRAN IV G LEVEL 1, MOD 0              MAIN              DATE = 68195          01/24/02          PAGE 0

              C THIS PROGRAM COMPUTES THE FIELD STRENGTH IN AIR OUTSIDE THE SAMPLE
0001                 DIMENSION AM(4,4,4),X(4,4,4),Y(4,4,4),Z(4,4,4),HNX(4,4,4),
                    1HNY(4,4,4),HNZ(4,4,4)
0002                 NUM = 5
0003                 NUM1 = 9
0004                 NUM2 = 6
0005                 REWIND NUM1
0006                 READ(NUM,430) IP,JP,KP
0007                 IP1 = IP-1
0008                 JP1 = JP-1
0009                 KP1 = KP-1
0010                 READ(NUM1)   (((X(I,J,K),Y(I,J,K),Z(I,J,K),K=1,KP),J=1,JP),I=1,IP)
0011                 READ(NUM1) (((HNX(I,J,K),HNY(I,J,K),HNZ(I,J,K),K=1,KP),J=1,JP),
                    1I = 1,IP)
0012                 READ(NUM1)  (((AM(I,J,K) ,K=2,KP1),J=2,JP1),I=2,IP1)
0013             130 READ(NUM,440) XA,YA,ZA,AHX1,AHY1,AHZ1
0014                 HAX = 0.
0015                 HAY = 0.0
0016                 HAZ = 0.
0017                 DO 380 I=2,IP1
0018                 DO 380 J=2,JP1
0019                 DO 380 K=2,KP1
0020                 XR = XA - X(I,J,K)
0021                 YR = YA - Y(I,J,K)
0022                 ZR = ZA - Z(I,J,K)
0023                 R = SQRT(XR**2 + YR**2 + ZR**2)
0024                 CMNIX = (R**2 - 3.*XR**2)/R**5
0025                 CMNJY = (R**2 - 3.*YR**2)/R**5
0026                 CMNKZ = (R**2 - 3.*ZR**2)/R**5
0027                 CMNIJ = (-3.*XR*YR)/R**5
0028                 CMNJK = -3.*YR * ZR/R**5
0029                 CMNIK = -3.*XR * ZR/R**5
0030                 HAX = HAX-AM(I,J,K)*(HNX(I,J,K)*CMNIX + HNY(I,J,K)*CMNIJ +
                    1HNZ(I,J,K) * CMNIK)
0031                 HAY = HAY - AM(I,J,K)*(HNX(I,J,K)*CMNIJ + HNY(I,J,K)* CMNJY +
                    1HNZ(I,J,K)*CMNJK)
0032             380 HAZ = HAZ-AM(I,J,K)*(HNX(I,J,K)*CMNIK + HNY(I,J,K)*CMNJK +
                    1HNZ(I,J,K)*CMNKZ)
0033                 HA = SQRT(HAX**2 + HAY**2 + HAZ**2)
0034                 WRITE(NUM2,450)XA,YA,ZA
0035                 WRITE(NUM2,460) HAX,HAY,HAZ,HA
0036                 GO TO 130
0037             430 FORMAT(3I6)
```

```
0038         440 FORMAT(3E10.0,3E16.7)
0039         450 FORMAT(1H0,3HX =1PE16.6,3HY = E16.6,3HZ = E16.6)
0040         460 FORMAT(1H0,3HHX= 1PE16.6,3HHY= E16.6,3HHZ= E16.6,3HH = E16.6)
0041             END
```

APPENDIX VIII

User Operating Instructions

I. **Programming System.**

360/50 Fortran IV G programming system was used.

II. **Equipment Needed.**

The equipment used and needed was as follows:

IBM 360/50 computer
Disk storage (memory)
Card reader
Printer
Magnetic tapes.

III. **Job Make Up.**

Section I.

Control Cards

```
//
//PES0100 JOB (R4048,TEST,45,5),'CLARK',MSGLEVEL=1
//    EXEC  FORTGCLG,PARM.FORT='LIST,MAP',PARM.LKED='LIST,XREF'
//FORT      EXEC PGM=IEYFORT
//SYSPRINT   DD SYSOUT=A
//SYSPUNCH   DD SYSOUT=B
//SYSLIN     DD DSNAME=SYS1.LOADSET,DISP=OLD,
//              DCB=(RECFM=FB,LRECL=80,BLKSIZE=400)
//FORT.SYSIN DD *
```

Main Programm

See pp. VII-2 and 3.

Subroutine MAGFIA

See pp. VII-4 and 5.

Control Cards

```
//LKED     EXEC PGM=IEWL,PARM=LIST,COND=(0,LT,FORT)
//SYSLIB       DD DSNAME=SYS1.FORTLIB,DISP=OLD
//SYSLMOD      DD DSNAME=&GOSET(MAIN),DISP=(,PASS),SPACE=(CYL,(2,1,1)),
//                UNIT=2311,VOLUME=SER=SYSLB2,DCB=(RECFM=U,BLKSIZE=3625)
//DECKS        DD DSNAME=SYS1.USERLIB,DISP=OLD
//             DD DSNAME=SYS1.USERLIB2,DISP=CLD
//SYSPRINT     DD SYSOUT=A
//SYSUT1       DD DSNAME=SYS1.UT1,DISP=OLD
//SYSLIN       DD DSNAME=SYS1.LOADSET,DISP=OLD
//             DD DDNAME=SYSIN
```

Data Deck

See in "Description of Input".

Section II.

Control Cards

```
//
//PES0100 JOB (R4048,TEST,15,5),'CLARK',MSGLEVEL=1
//   EXEC   FORTGCLG,PARM.FORT='MAP,LIST,EBCDIC',PARM.LKED='MAP,LIST'
//FORT      EXEC PGM=IEYFORT
//SYSPRINT   DD SYSOUT=A
//SYSPUNCH   DD SYSOUT=B
//SYSLIN     DD DSNAME=SYS1.LOADSET,DISP=OLD,
//              DCB=(RECFM=FB,LRECL=80,BLKSIZE=400)
//FORT.SYSIN DD *
```

Main Programm

See pp. VII-6 and 7

Subroutine PHICAL

See pp. VII-9, 10, and 11.

Subroutine HCAL

See pp. VII-12

Subroutine PERM

See pp. VII-13

Control Cards

```
//LKED     EXEC PGM=IEWL,PARM=LIST,COND=(0,LT,FORT)
//SYSLIB     DD DSNAME=SYS1.FORTLIB,DISP=OLD
//SYSLMOD    DD DSNAME=&GOSET(MAIN),DISP=(,PASS),SPACE=(CYL,(2,1,1)),
//              UNIT=2311,VOLUME=SER=SYSLB2,DCB=(RECFM=U,BLKSIZE=3625
//DECKS      DD DSNAME=SYS1.USERLIB,DISP=OLD
//           DD DSNAME=SYS1.USERLIB2,DISP=OLD
//SYSPRINT   DD SYSOUT=A
//SYSUT1     DD DSNAME=SYS1.UT1,DISP=OLD
//SYSLIN     DD DSNAME=SYS1.LOADSET,DISP=OLD
//           DD DDNAME=SYSIN
```

```
//GO         EXEC PGM=*.LKED.SYSLMOD,COND=((0,LT,FORT),(4,LT,LKED))
//FT05F001   DD DDNAME=SYSIN
//FT06F001   DD SYSOUT=A
//GOSET      DD DSNAME=&GOSET,DISP=(OLD,DELETE)
//GO.FT08F001 DD UNIT=TAPE,VOLUME=SER=20039,LABEL=(1,BLP),
//              DISP=(NEW,PASS),
//              DCB=(DEN=2,TRTCH=C,RECFM=V,LRECL=604,BLKSIZE=608)
//GO.FT09F001 DD UNIT=TAPE,VOLUME=SER=20040,LABEL=(1,BLP),
//              DISP=(NEW,PASS),
//              DCB=(DEN=2,TRTCH=C,RECFM=V,LRECL=604,BLKSIZE=608)
//GO.SYSIN DD  *
```

Data Deck

See in "Description of Input"

Section III.

Control Cards

```
//
//PES0100 JOB (R4048,TEST,12,3),'CLARK',MSGLEVEL=1
//  EXEC  FORTGCLG,PARM.FORT='MAP,LIST,EBCDIC',PARM.LKED='MAP,LIST'
//FORT      EXEC PGM=IEYFORT
//SYSPRINT   DD SYSOUT=A
//SYSPUNCH   DD SYSOUT=B
//SYSLIN     DD DSNAME=SYS1.LOADSET,DISP=OLD,
//              DCB=(RECFM=FB,LRECL=80,BLKSIZE=400)
//FORT.SYSIN DD *
IEF236I ALLOC. FOR PES0100  FORT
IEF237I SYSPUNCH ON 00D
IEF237I SYSLIN   ON 191
IEF237I SYSIN    ON 00C
```

Main Programm (MAGFIA)

See pp. VII-14 and 15.

Control Cards

```
//LKED      EXEC PGM=IEWL,PARM=LIST,COND=(0,LT,FORT)
//SYSLIB     DD DSNAME=SYS1.FORTLIB,DISP=OLD
//SYSLMOD    DD DSNAME=&GOSET(MAIN),DISP=(,PASS),SPACE=(CYL,(2,1,1)),
//              UNIT=2311,VOLUME=SER=SYSLB2,DCB=(RECFM=U,BLKSIZE=3625)
//DECKS      DD DSNAME=SYS1.USERLIB,DISP=OLD
//           DD DSNAME=SYS1.USERLIB2,DISP=OLD
//SYSPRINT   DD SYSOUT=A
//SYSUT1     DD DSNAME=SYS1.UT1,DISP=OLD
//SYSLIN     DD DSNAME=SYS1.LOADSET,DISP=OLD
//           DD DDNAME=SYSIN
```

Data Deck

See in "Dexcription of Input"

IV. Description of Input.

The input is given on cards. In addition, scratch-tapes are used to transfer output of Section I and II to input to Section II and III respectively.

Section I.

Main Programm
Read Cards. The data on the Card Deck are shown on line 0003 on p. VII-2. Format is shown on line 0058 on p. VII-3.

Example of Numerical Data is as follows:

See p. VIII-5

SEG	SWI	AMP	XO	YO	ZO	KX	KY	KZ	DELTAX	DELTAY	DELTAZ
100	1	8.7200	-0.0190	-0.0190	-0.0190	4	4	4	0.0127	0.0127	0.0127

The full output of Section I is put on a scratch-tape in binary format and this tape is read in Section II, see lines 0013 and 0014 on p. VII-6.

Section II.

Main Programm

1.) Read Cards for general data. The data on the Card Deck are shown on line 0007 on p. VII-6. Format is shown on line 0068 on p. VII-7.

Example of numerical data is as follows:

```
IP=  4   JP=  4   KP=  4   LIMIT= 10   EPSI=  4.99999970E-02ITOT=   100VOLUME=  1.64063968E-05
```

2.) Read Cards for the permeability matrix. The data on the Card Deck are shown on line 0011 on p. VII-6. Format is shwon on line 0069 on p. VII-7. Example of numerical data is as follows:

HUR(I,J)

4.000000E-02	1.000000E 00	9.999996E-02	2.513200E 00	1.000000E 00	2.513199E 01	2.000000E 0
5.020000E 01	3.000000E 00	7.539999E 01	4.000000E 00	1.005000E 02	5.000000E 00	1.255000E 02
6.000000E 00	1.506000E 02	8.000000E 00	2.008000E 02	1.000000E 01	2.513000E 02	1.200000E 01
3.015999E 02	1.400000E 01	3.517998E 02	1.600000E 01	4.020000E 02	1.800000E 01	4.522998E 02
2.000000E 01	5.025999E 02	2.200000E 01	5.528999E 02	2.500000E 01	6.282998E 02	3.000000E 01
7.538999E 02	3.500000E 01	8.795999E 02	4.000000E 01	1.005300E 03	4.500000E 01	1.130900E 03
5.000000E 01	1.256600E 03	5.500000E 01	1.382200E 03	6.000000E 01	1.507900E 03	6.500000E 01
1.633600E 03	7.000000E 01	1.759200E 03	8.000000E 01	2.018000E 03	8.500000E 01	2.211000E 03
9.000000E 01	2.382000E 03	9.500000E 01	2.536000E 03	1.000000E 02	2.674000E 03	1.040000E 02
2.775000E 03	1.080000E 02	2.868000E 03	1.120000E 02	2.955000E 03	1.160000E 02	3.036000E 03
1.200000E 02	3.111000E 03	1.240000E 02	3.182000E 03	1.280000E 02	3.248000E 03	1.320000E 02
3.311000E 03	1.360000E 02	3.360000E 03	1.380000E 02	3.397000E 03	1.400000E 02	3.424000E 03
1.420000E 02	3.450000E 03	1.440000E 02	3.476000E 03	1.460000E 02	3.501000E 03	1.480000E 02
3.526000E 03	1.500000E 02	3.549000E 03	1.520000E 02	3.572000E 03	1.540000E 02	3.595000E 03
1.560000E 02	3.617000E 03	1.580000E 02	3.638000E 03	1.600000E 02	3.659000E 03	1.620000E 02
3.639000E 03	1.640000E 02	3.618000E 03	1.660000E 02	3.598000E 03	1.680000E 02	3.577000E 03
1.700000E 02	3.557000E 03	1.720000E 02	3.537000E 03	1.740000E 02	3.516000E 03	1.760000E 02
3.496000E 03	1.780000E 02	3.475000E 03	1.800000E 02	3.455000E 03	1.820000E 02	3.435000E 03
1.840000E 02	3.414000E 03	1.860000E 02	3.394000E 03	1.880000E 02	3.373000E 03	1.900000E 02
3.353000E 03	1.920000E 02	3.333000E 03	1.940000E 02	3.312000E 03	1.960000E 02	3.292000E 03
1.980000E 02	3.270000E 03	2.000000E 02	3.250000E 03	2.200000E 02	3.225000E 03	2.400000E 02
3.200000E 03	2.800000E 02	2.960000E 03	3.200000E 02	2.700000E 03	3.600000E 02	2.600000E 03
4.000000E 02	2.340000E 03	4.400000E 02	2.180000E 03	4.800000E 02	2.050000E 03	5.200000E 02
1.930000E 03	5.600000E 02	1.820000E 03	6.000000E 02	1.710000E 03	6.400000E 02	1.640000E 03
6.800000E 02	1.550000E 03	7.200000E 02	1.480000E 03	7.600000E 02	1.410000E 03	8.000000E 02
1.350000E 03	1.200000E 03	9.450000E 02	1.600000E 03	7.350000E 02	2.400000E 03	5.130000E 02
3.200000E 03	4.000000E 02	4.000000E 03	3.260000E 02	4.800000E 03	2.780000E 02	5.600000E 03
2.400000E 02	6.400000E 03	2.130000E 02	8.000000E 03	1.730000E 02	9.600000E 03	1.460000E 02
1.120000E 04	1.260000E 02	1.280000E 04	1.100000E 02			

3.) Read Cards for the starting permeability. The data on the Card deck are shown on line 0015 on p. VII-6. Format is shown on line 0070 on p. VII-7. Example of numerical data is as follows:

```
R1(IJK)
 5.000000E 02   5.000000E 02   5.000000E 02   5.000000E 02   5.000000E 02   5.000000E 02   5.000000E 02
5.000000E 02
```

The full output of Section II is put on a scratch-tape in binary format and this tape is read in Section III, see lines 0010 through 0012 on p. VII-14.

Section III.

Main Programm

1.) Read Cards for maximum value of indices. The data on the Card Deck are shown on line 0006 on p. VII-14. Format is shown on line 0037 on p. VII-14. Example of numerical data is as follows:

IP = 4 JP = 4 KP = 4

2.) Read Cards for coordinates. The data on the Card Deck are shown on line 0013 on p. VII-14. Format is shown on line 0038 on p. VII-15. Example of numerical data is as follows:

See VIII-8

X =	0.0	Y =	0.0	Z =	5.000000E-02
X =	0.0	Y =	0.0	Z =	9.999996E-02
X =	0.0	Y =	0.0	Z =	1.500000E-01
X =	0.0	Y =	0.0	Z =	2.000000E-01
X =	0.0	Y =	0.0	Z =	2.500000E-01
X =	0.0	Y =	0.0	Z =	3.000000E-01
X =	0.0	Y =	0.0	Z =	3.500000E-01
X =	0.0	Y =	0.0	Z =	4.000000E-01
X =	0.0	Y =	0.0	Z =	4.500000E-01
X =	0.0	Y =	0.0	Z =	5.000000E-01
X =	0.0	Y =	0.0	Z =	5.500000E-01
X =	0.0	Y =	0.0	Z =	6.000000E-01
X =	0.0	Y =	0.0	Z =	6.500000E-01
X =	0.0	Y =	0.0	Z =	7.000000E-01
X =	0.0	Y =	0.0	Z =	7.500000E-01

V. Tape Assignments

Only scratch tapes are used. The use of tapes is listed above, under "Description of Input".

VI. Restrictions

See as dimension-statements in the programm, in Appendix VII.

VII. Timing

See p. 81

VIII. Programmed Error Messages are none.

IX. Sample Input

See above in "IV Description of Input"

X. Sample Output

Is given in Appendix IX.

APPENDIX IX

Sample Output

```
FIELD INTENSITY IN AIR
SEG  SWI  AMP      XO        YO        ZO       KX  KY  KZ  DELTAX   DELTAY   DELTAZ
100   1   8.7200  -0.0190   -0.0190   -0.0190    4   4   4   0.0127   0.0127   0.0127
I  J  K  X              Y              Z              HX             HY             HZ             H
 1  1  1-1.90499984E-02-1.90499984E-02-1.90499984E-02 5.50656915E-02 5.48728630E-02-2.00855933E 03 2.00855908E 03
 1  1  2-1.90499984E-02-1.90499984E-02-6.34999946E-03 1.90276168E-02 1.89694054E-02-2.00850708E 03 2.00850684E 03
 1  1  3-1.90499984E-02-1.90499984E-02 6.34999946E-03-1.88581757E-02-1.88545398E-02-2.00850708E 03 2.00850684E 03
 1  1  4-1.90499984E-02-1.90499984E-02 1.90499984E-02-5.49279563E-02-5.47403283E-02-2.00855933E 03 2.00855908E 03
 1  2  1-1.90499984E-02-6.34999946E-03-1.90499984E-02 5.36685996E-02 1.77752711E-02-2.00858203E 03 2.00858179E 03
 1  2  2-1.90499984E-02-6.34999946E-03-6.34999946E-03 1.85541213E-02 6.03604689E-03-2.00853223E 03 2.00853198E 03
 1  2  3-1.90499984E-02-6.34999946E-03 6.34999946E-03-1.84356198E-02-6.01499528E-03-2.00853174E 03 2.00853149E 03
 1  2  4-1.90499984E-02-6.34999946E-03 1.90499984E-02-5.35407402E-02-1.77712440E-02-2.00858154E 03 2.00858130E 03
 1  3  1-1.90499984E-02 6.34999946E-03-1.90499984E-02 5.36633991E-02-1.79788843E-02-2.00858154E 03 2.00858130E 03
 1  3  2-1.90499984E-02 6.34999946E-03-6.34999946E-03 1.85510032E-02-6.28078356E-03-2.00853198E 03 2.00853174E 03
 1  3  3-1.90499984E-02 6.34999946E-03 6.34999946E-03-1.84195042E-02 6.28425926E-03-2.00853174E 03 2.00853149E 03
 1  3  4-1.90499984E-02 6.34999946E-03 1.90499984E-02-5.35360612E-02 1.79641061E-02-2.00858154E 03 2.00858130E 03
 1  4  1-1.90499984E-02 1.90499984E-02-1.90499984E-02 5.50760888E-02-5.52346110E-02-2.00855933E 03 2.00855908E 03
 1  4  2-1.90499984E-02 1.90499984E-02-6.34999946E-03 1.90083869E-02-1.89979896E-02-2.00850708E 03 2.00850684E 03
 1  4  3-1.90499984E-02 1.90499984E-02 6.34999946E-03-1.88789666E-02 1.88483000E-02-2.00850708E 03 2.00850684E 03
 1  4  4-1.90499984E-02 1.90499984E-02 1.90499984E-02-5.49409501E-02 5.50870039E-02-2.00855933E 03 2.00855908E 03
 2  1  1-6.34999946E-03-1.90499984E-02-1.90499984E-02 1.78662613E-02 5.35048768E-02-2.00858130E 03 2.00858105E 03
 2  1  2-6.34999946E-03-1.90499984E-02-6.34999946E-03 6.16861507E-03 1.85104609E-02-2.00853198E 03 2.00853174E 03
 2  1  3-6.34999946E-03-1.90499984E-02 6.34999946E-03-6.15497306E-03-1.83592141E-02-2.00853174E 03 2.00853149E 03
```

Section II

HNX(IJK)

-3.869486E 03	-5.059722E 45	-1.511812E 01	1.301953E 01	-1.714301E 25	-9.293414E 05	1.873299E 3
-2.740567E-59	-8.095556E 46	-9.907814E 05	-8.064772E 05	2.025367E-60	-1.295044E 48	-6.192384E 04
6.784112E 04	-5.995304E 19	-2.701891E 44	-6.733769E 40	-6.832468E 40	7.445178E 28	-1.862700E 43
-3.231329E 01	1.668805E 01	1.745664E-49	-1.014808E 24	-2.141527E 01	4.184029E 01	2.804477E-79
-3.101360E 03	-9.796796E 29	4.529851E-83	5.789575E 18	2.509804E-01	2.509804E-01	-3.498156E 41
2.237528E-72	-1.226569E 01	3.018176E 01	-4.656036E 01	8.573772E-81	-3.498156E 41	3.247771E 01
-3.143675E 01	2.816002E-79	-8.652840E 05	2.509804E-01	3.094850E 26	1.912421E-80	-5.308917E 10
-5.308917E 10	1.873299E 33	5.488614E-79	-5.308917E 10	-5.308918E 10	2.617018E-80	5.517477E 18
-5.308917E 10	-5.308918E 10	2.712032E-79	-1.396263E-58	-5.308917E 10	-5.308918E 10	3.744438E-67
-1.117935E-21						

HNY(IJK)

-6.493510E-59	6.785012E 04	0.0	0.0	5.494726E 18	1.441161E 18	7.142395E-47
8.236092E-84	9.903596E 27	4.529851E-83	1.752470E 26	-0.0	3.187504E 00	-1.606020E 21
3.365267E-80	5.147558E-85	5.783128E 18	4.941655E-83	2.377901E 18	0.0	1.292499E-78
-3.247771E 01	3.143675E 01	8.433758E-81	1.052249E 28	3.018176E 01	-4.656036E 01	-5.309029E 10
3.481370E 18	-1.175178E-57	3.012763E-80	2.633079E-80	2.896489E-79	3.094850E 26	3.530827E 18
0.0	2.816002E-79	-2.141527E 01	4.184029E 01	5.147558E-85	1.052251E 28	3.231329E 01
-1.668805E 01	-5.064925E 04	-3.920299E 19	6.785389E 04	2.549482E-80	4.118046E-83	-6.882863E 11
3.744167E-67	-2.040154E-56	0.0	-1.396263E-58	6.785187E 04	4.529851E-83	3.048897E-03
1.495237E 28	1.441165E 18	-9.979902E-59	2.428922E-78	5.838635E 18	0.0	0.0
0.0						

HNZ(IJK)

5.147558E-85	-0.0	8.124529E 18	2.365112E-04	5.960464E-08	5.147558E-85	-2.384360E-59
1.305078E 01	0.0	5.147558E-85	2.522486E 08	2.712706E-56	5.147558E-85	5.147558E-85
5.413393E-50	3.812615E-59	5.147558E-85	-5.309028E 10	-5.975213E-58	6.784112E 04	0.0
4.656036E 01	3.018176E 01	1.334375E 05	0.0	3.143675E 01	3.247771E 01	-3.253455E-59
2.635549E-82	5.106040E-72	5.122839E-72	-6.447567E-55	5.147558E-85	-5.805725E 04	-1.181007E 23
3.744710E-67	5.147558E-85	1.668805E 01	3.231329E 01	5.130596E 27	0.0	4.184029E 01
2.141527E 01	-7.359004E-59	5.147558E-85	-2.882140E-59	-1.904318E-01	-2.320159E-80	4.118046E-83
1.214461E-78	-5.688161E 36	6.460524E 28	0.0	0.0	5.818695E 18	4.973043E 28
0.0	4.118046E-83	-5.852098E-59	1.290530E-78	2.638020E-80	-1.722330E-01	5.470151E 28
-5.070602E 31						

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Section III

```
X =     0.0            Y =     0.0            Z =    2.500000E-01
HX=     1.019472E-02HY=   -2.128731E-01HZ=    1.136795E 00H =    1.156598E 00
                                                    0.041
X =     0.0            Y =     0.0            Z =    3.000000E-01
HX=     4.586849E-03HY=   -1.207850E-01HZ=    6.573498E-01H =    6.683702E-01
                                                    0.0082
X =     0.0            Y =     0.0            Z =    3.500000E-01
HX=     2.294000E-03HY=   -7.497030E-02HZ=    4.138442E-01H =    4.205862E-01
X =     0.0            Y =     0.0            Z =    4.000000E-01
HX=     1.236480E-03HY=   -4.967163E-02HZ=    2.772297E-01H =    2.816470E-01
X =     0.0            Y =     0.0            Z =    4.500000E-01
HX=     7.036999E-04HY=   -3.458248E-02HZ=    1.947181E-01H =    1.977665E-01
X =     0.0            Y =     0.0            Z =    5.000000E-01
HX=     4.165918E-04HY=   -2.503292E-02HZ=    1.419643E-01H =    1.441550E-01
X =     0.0            Y =     0.0            Z =    5.500000E-01
HX=     2.535656E-04HY=   -1.869800E-02HZ=    1.066731E-01H =    1.082997E-01
X =     0.0            Y =     0.0            Z =    6.000000E-01
HX=     1.570880E-04HY=   -1.433168E-02HZ=    8.217603E-02H =    8.341652E-02
X =     0.0            Y =     0.0            Z =    6.500000E-01
HX=     9.804592E-05HY=   -1.122518E-02HZ=    6.464195E-02H =    6.560934E-02
X =     0.0            Y =     0.0            Z =    7.000000E-01
HX=     6.096810E-05HY=   -8.955158E-03HZ=    5.176237E-02H =    5.253134E-02
X =     0.0            Y =     0.0            Z =    7.500000E-01
HX=     3.719842E-05HY=   -7.258032E-03HZ=    4.208969E-02H =    4.271091E-02
IHC217I
```

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Library Card Abstract

A procedure to calculate the magnetic field in three dimensions in and around a magnetic body of field dependent permeability is presented, written in Fortram IV Computer language for (IBM 7094 machine or equivalent.)
360/50 OS

I. Calculations are presented which define the m.m.f. grad. ${}^{o}\vec{H}_n$ in points of a three dimensional free space, for an arbitrary current system.

II. Calculations are presented which determine the combined m.m.f. grad., $\vec{H}_n$ at points within magnetic bodies of field dependent permeability, by summing the m.m.f. grad. ${}^{o}\vec{H}_n$-s calculated in Section I with the m.m.f. grad. ${}^{m}\vec{H}_n$-s resulting from dipoles at other points which in their turn are induced by the m.m.f. grad. $\vec{H}$, resulting for the external fields of neighboring magnetic bodies or points within the same body. The magnetic moment of such points arises by the m.m.f. grad. ${}^{o}\vec{H}_n$ due to the current system and by interaction.

$$\vec{H}_n = {}^{o}\vec{H}_n + {}^{m}\vec{H}_n$$

III. Calculations are presented which determine the 3 dimensional m.m.f. grad. $\vec{H}_n$, at any arbitrary point outside the magnetic body or bodies. Values of $\vec{H}_n$ are found by summing the m.m.f. grad. ${}^{o}H_n$ calculated in Section I with the contributions ${}^{m}\vec{H}_n$ from the points of magnetic bodies considered in Section II.

Section I can be used independently, Section II requires Section I. Section III requires both Section I and Section II.

Limitations are computer memory-space and machine-time.