THE FLUCTUATION ORIGIN OF COSMIC RADIATION

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ABSTRACT

We have investigated the origin of cosmic radiation in terms of a sudden injection of particles in time, momentum and space. The appropriate boundary conditions for the various regions through which the particles pass were used. With all of the acceleration within a turbulent region, we find that the observed spectrum is explained by a continuous deceleration in which statistical fluctuations dominate. This is contradictory to the usual assumptions in which fluctuation do not play an important part. We find that the exponent of the power law spectrum has a weak momentum dependency as

$$\gamma = 0.057 \ln(P/P_0) + 0.67.$$

Correction Sheet

Abstract: $\bar{\gamma}$ = should read $\gamma - 1$ =

Figure 1: The Intensity Scale should read with a negative sign in front of each division.

Emuision should read Emulsion

Valcano should read Volcano

I. INTRODUCTION

Previously when the problem of the origin of cosmic radiation was considered the influence of fluctuations in the acceleration mechanism was thought to be of small importance [Ginzburg and Syrovatskii, 1964]. Recently, however, it has been shown that when there is continuous deceleration imposed on general acceleration, fluctuations can be an important factor in determining the energy spectrum of cosmic rays [Wayland and Bowen, 1968]. This result leads to the concept of a power index that increases with energy.

In this paper we will investigate the problem in which particles are injected by an explosion into a turbulently expanding region, in which the particles diffuse in both space and momentum. We will treat the momentum process as stochastic. The particle then escapes into interstellar space where they continue to diffuse spatially but are no longer experiencing momentum-changing processes. Thus we must solve a two-region Fokker-Planck diffusion equation with appropriate boundary conditions. In Section II we will give a statement of the problem. Section III contains an outline of the solution of the problem. We will apply our results to the observed cosmic ray spectrum in Section IV.

II. THE MODEL

We will consider the source as a sudden outburst of particles that occurs over such a short period of time that we can write it as a delta function in time. By this we are assuming that the source is active over a period of time that is short in comparison to the time required for the particles to diffuse out of the turbulent region. We will simplify our problem by assuming that this injection of particles is a one-time episode. We will also assume that, in comparison to the momentum range considered, the particles all have the same injection momentum, p_o.

After the particles are injected into the turbulent region, they will diffuse spatially. We can describe this motion by the standard diffusion equation. To a first approximation we will suppose that the diffusion coefficient, D, is independent of the spatial coordinate within a given region. However, we would expect it to have different values in the two regions. At the same time that the particles are undergoing spatial diffusion in the turbulent region, they will undergo diffusion in momentum space. We will assume that at each momentum "scattering", the particle is just at the point of forgetting what has happened in the past. Thus the conditional probability depends only on the value of the momentum at the previous time; i.e., a Markoff process. This will probably be true if the spatial extent of the system, L, is related to the correlation time, T, of the process by $T \gg L/c$. Then momentum diffusion can be described by a Fokker-Planck Eqn. We will use the standard second-order form so that we can take fluctuations into account. We will also make the presupposition that the diffusion coefficient is independent of momentum. In the momentum range in question, this is the same as saying the diffusion mean free path is independent of momentum.

When the particles diffuse to the boundary of the diffusion region we will let them freely escape into interstellar space. The particle density at the boundary and the flux are assumed to be continuous. This corresponds to a very close coupling of the two regions at the boundary. Because we are assuming the diffusion condition, we cannot expect to describe what is happening at the boundary in detail. Only when one is a few mean free paths from the boundary will our solutions be accurate.

We will take the turbulent region to be generally spherical in shape. Although this is never exactly true, normally the departures will be small in comparison to the total extent of the region and we can approximate the surface with a sphere. We also note that there is no reason for the distribution to depend upon any spatial coordinate other than the radial one.

In the interstellar space region we will postulate that the particles are no longer experiencing an acceleration process. However, we note that experimentally they are almost completely isotropic. This, of course, implies that they have also diffused spatially in this region. We will not attempt to give a detailed description of their motion but will assume a macroscopic averaging by writing this in the form of a diffusion equation.

When the particles have diffused to the boundary of the interstellar space we will assume that they "radiate" freely into a region of far less particle density. If the point of observation is far from the boundary with intergalactic space, we would expect that to a good approximation we can treat the interstellar region as spherical. As the particle density approaches this boundary, it should be decreasing and at the boundary assume a small finite value.

We can incorporate all of the above into the following equations:

(1) $$\frac{\partial n_1}{\partial t} - D_1 \nabla^2 n_1 + \frac{\partial}{\partial p}\left(\frac{\langle \Delta p \rangle}{\Delta t} n_1 \right) - \frac{1}{2} \frac{\partial^2}{\partial p^2}\left(\frac{\langle \Delta p^2 \rangle}{\Delta t} n_1 \right) + \frac{n_1}{T_1} = 0,$$

for $t > t_o$ in region I (= turbulent region), and

(2) $$\frac{\partial n_2}{\partial t} - D_2 \nabla^2 n_2 + \frac{n_2}{T_2} = 0,$$

for $t > t_o$ in region II (= interstellar space). At the boundary of region I and II, $r = a_s$, we have

(3) $$n_1 = n_2 ,$$

(4) $$D_1 \frac{\partial n_1}{\partial r} = D_2 \frac{\partial n_2}{\partial r} ,$$

and at the boundary of region II, $r = R$,

(5) $$n_2 \xrightarrow[r \to R]{} O(\epsilon)$$

where ϵ is a small finite number. The initial condition at $t = t_o$ is

(6) $$n = q_o \, \delta(t - t_o) \, \delta(p - p_o) \, \delta(\underset{\sim}{r}).$$

Note that we have assumed the injection source is at the origin of coordinates.

In Eqns. (1) and (2) the first two terms describe the spatial diffusion of the particles. The third and fourth terms account for the diffusion of the particles in momentum space. These terms depict the acceleration and momentum loss in scatterings which produce a continuous change in the cosmic ray particle momentum. The third term arises from the mean statistical momentum change of cosmic ray particles. The fourth term characterizes the statistical fluctuations in the momentum change. The last term accounts for removal interaction processes in which the particles interact with the medium to produce particles other than the particles in question.

III. SOLUTION OF THE BOUNDARY VALUE PROBLEM

As we show in the Appendix, one can often write

$$\frac{\langle \Delta p \rangle}{\Delta t} = a p, \tag{7}$$

and

$$\frac{\langle \Delta p^2 \rangle}{\Delta t} = 2 b p^2. \tag{8}$$

The form of Eqn. (1) suggests a power law solution in p. Thus we will take the Mellin transform w.r.t. p of Eqn. (1), after first inserting the Eqns. (7) and (8), to find

$$\frac{\partial g_1}{\partial t} - D_1 \nabla^2 g_1 - \left[(s-1) a + (s-1)(s-2) b - \frac{1}{T_1} \right] g_1 = 0, \tag{9}$$

where

$$g_1 = \int_0^\infty p^{s-1} n_1(p, \underset{\sim}{r}, t)\, dp.$$

If we write $g_1(s, \underset{\sim}{r}, t) = f(\underset{\sim}{r}, s, t) h(s, t)$ we find that we can write Eqn. (9) in the form

$$\frac{\partial h}{\partial t} - \left[(s-1) a + (s-1)(s-2) b - \frac{1}{T_1} \right] h = 0 \tag{10a}$$

$$\frac{\partial f}{\partial t} - D_1 \nabla^2 f = 0. \tag{10b}$$

If we use the initial condition Eqn. (6)

(11) $$\lim_{\tau \to 0} \int g_1 \, 4\pi r^2 dr = q_0 P_0^{\,s-1},$$

where $\tau = t - t_o$ we find that

(12) $$g_1 = \frac{q_0 P_0^{\,s-1}}{(2\pi D_1 \tau)^{3/2}} \exp\left\{\left[(s-1)a + (s-1)(s-2)b - 1/T_1\right]\tau - \frac{r^2}{4D_1\tau}\right\}$$

Before we can apply the boundary conditions (3) and (4) we must solve Eqn. (2). This leads to the solution

(13) $$g_2 = \frac{A}{r} \exp\left\{-\alpha^2 D_2 \tau - \beta r\right\}$$

where we have used the condition (5) and have set

(14) $$\alpha^2 + \beta^2 = \frac{1}{D_2 T_2}.$$

When we apply the boundary conditions (3) and (4) we obtain

(15) $$g_2 = \frac{a_s q_0}{r P_0 (2\pi D_1 \tau)^{3/2}} \exp A_2 ,$$

where

$$A_2 = s \ln P_0 + [(s-1)a + (s-1)(s-2)b - 1/T_1]\tau + \frac{a_s}{2D_2\tau} - \frac{a_s^2}{4D_1\tau} - 1 - \frac{a_s r}{2D_2\tau} - \frac{r}{a_s}.$$

Then the solution to our problem is obtained by taking the inverse Mellin transform of g_2

(16) $$n_2(P, \underset{\sim}{r}, t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g_2(s, \underset{\sim}{r}, t)\, P^{-s}\, ds.$$

We were able to obtain an asymptotic solution to Eqn. (16) by the method of steepest descent. Under the condition* of $\ln(P/P_o) \gg 1$ we have

(17) $$n_2 = \frac{a_s q_o}{r P_o 2^{5/2} \pi^{3/2} D_1^{3/2} b^{1/2} \tau^2} \exp A_3,$$

where $$A_3 = -s_o \ln(P/P_o) + [(s_o-1)a + (s_o-1)(s_o-2)b - \frac{1}{T_1}]\tau + \frac{a_s^2}{2D_2\tau} - 1 - \frac{a_s r}{2D_2\tau} + \frac{r}{a_s},$$

$$s_o = \frac{1}{2b\tau}\ln(P/P_o) + \frac{3b-a}{2b}.$$

*This requirement can be relaxed to $\ln(P/P_o) > 1$ without a noticeable increase in error.

We know that the intensity of cosmic radiation appears to be constant with time. Then we require the steady state solution for our problem. In the above we have solved for the case of a single source as a function of time. Let us assume that it is a typical source. Then we wish to sum over its contribution to the present cosmic ray density of particles. In a later paper we will investigate the requirements placed upon sources by our solution. For the present we will just find this source's contribution and assume all others behave in a duplicate manner. Thus the momentum spectrum should reflect the observed one. Accordingly we must evaluate the integral

$$\eta(P, \underset{\sim}{r}) = \int_0^\infty n_2(P, \underset{\sim}{r}, \tau)\, d\tau. \tag{18}$$

Laplace integration gives the asympotic solution

$$\eta = \frac{a_s q_0 \exp A_4}{r P_0\, 2^{7/2} \pi D_1^{3/2} b^{1/2} \tilde{\tau}_0} \left(\frac{P_0}{P}\right)^{\gamma}, \tag{19}$$

where $\tilde{\tau}_0 = \dfrac{2b}{a(a-b)}$,

$$\gamma = \frac{a(a-b)}{8b^2} \ln(P/P_0) + \frac{3b-a}{2b}.$$

and $A_4 = -\dfrac{(a-b)}{2b} - \dfrac{a_s r}{2D_1 \tau_0} + \dfrac{r}{a_s} + \dfrac{a_s^2}{2D_1 \tau_0} - 1.$

Note that we can write (19) in the form

$$\zeta = \varphi \left(\frac{\rho_0}{\rho} \right)^{\gamma} \tag{20}$$

Ergo, φ will act as a normalization constant when we attempt to fit Eqn. (20) to the observed spectrum.

IV. COMPARISON WITH EXPERIMENT

The cosmic ray momentum spectrum has been observed to be basically a power law in which the power index increases with increasing momentum. We are interested in the region 10 GeV/c < p < 10^8 GeV/c. Above 10^8 GeV/c the spectrum power index suddenly decreases. This is probably due to a secondary source of cosmic radiation.

We note that only when there is a steady decrease in the mean statistical momentum change, $\langle\Delta p\rangle$, can we fit the observed spectrum. This corresponds to a deceleration. As we show in the Appendix, this could be due to a radial expansion of the turbulent region. This is not the only interpretation; it is, however, one that can fit into many astrophysical phenomena that have been observed. Thus the particles that we see at the very high energies are the result of a series of favorable accelerations. This is then a momentum spectrum in which fluctuations in the acceleration process have a dominating effect. But note that while deceleration prevails, an acceleration process must also be present.

To obtain the integral spectrum we must integrate Eqn. (20) from P to ∞. We will approximate this integral by noting that the result depends mainly on the integrand near the lower limit of integration, P. Thus we may hold γ fixed and equal to its correct value at P. This procedure gives

$$N(P) = \frac{\varphi'}{(\gamma-1)} \left(\frac{P_0}{P}\right)^{\gamma-1} \tag{21}$$

The result of applying Eqn. (21) to the measured integral spectrum is shown in Figs. 1 and 2. We have used the results reported by Bradt, 1965 and Bray, 1965 transposed to the momentum variable. [We have assumed that the composition is mainly protons at low momentum. At the higher momenta

$pc \gtrsim E$, and the true composition is not critical.] We have adopted $a = -.34b$ and $p_o = 3.0$ GeV/c. This choice of p_o was made after noting that the minimum ionization of protons in hydrogen is approximately at p_o.

V. DISCUSSION AND CONCLUSIONS

We have investigated what happens to a source of cosmic rays that is impulsed with a given momentum at a fixed time and position into a turbulent region. Within this region the particles undergo acceleration and deceleration that can result in statistical fluctuations. After diffusing both in space and momentum, the particles escape freely into interstellar space. Here the particles again diffuse spatially until they become isotropic. When the particles reach the boundary with intergalactic space, provided that they do arrive, they escape. Thus we have solved a two-region second-order Fokker-Planck equation with appropriate boundary conditions.

It is shown that the observed spectrum is congruous with the dominance of statistical fluctuations in the case of continuous deceleration. We were able to obtain remarkably good agreement with the observed integral momentum spectrum of primary cosmic rays. The usual integral power law spectrum was found to have an exponent that is weakly dependent upon the momentum:

$$(\gamma - 1) = 0.057 \ln(P/P_0) + 0.67$$

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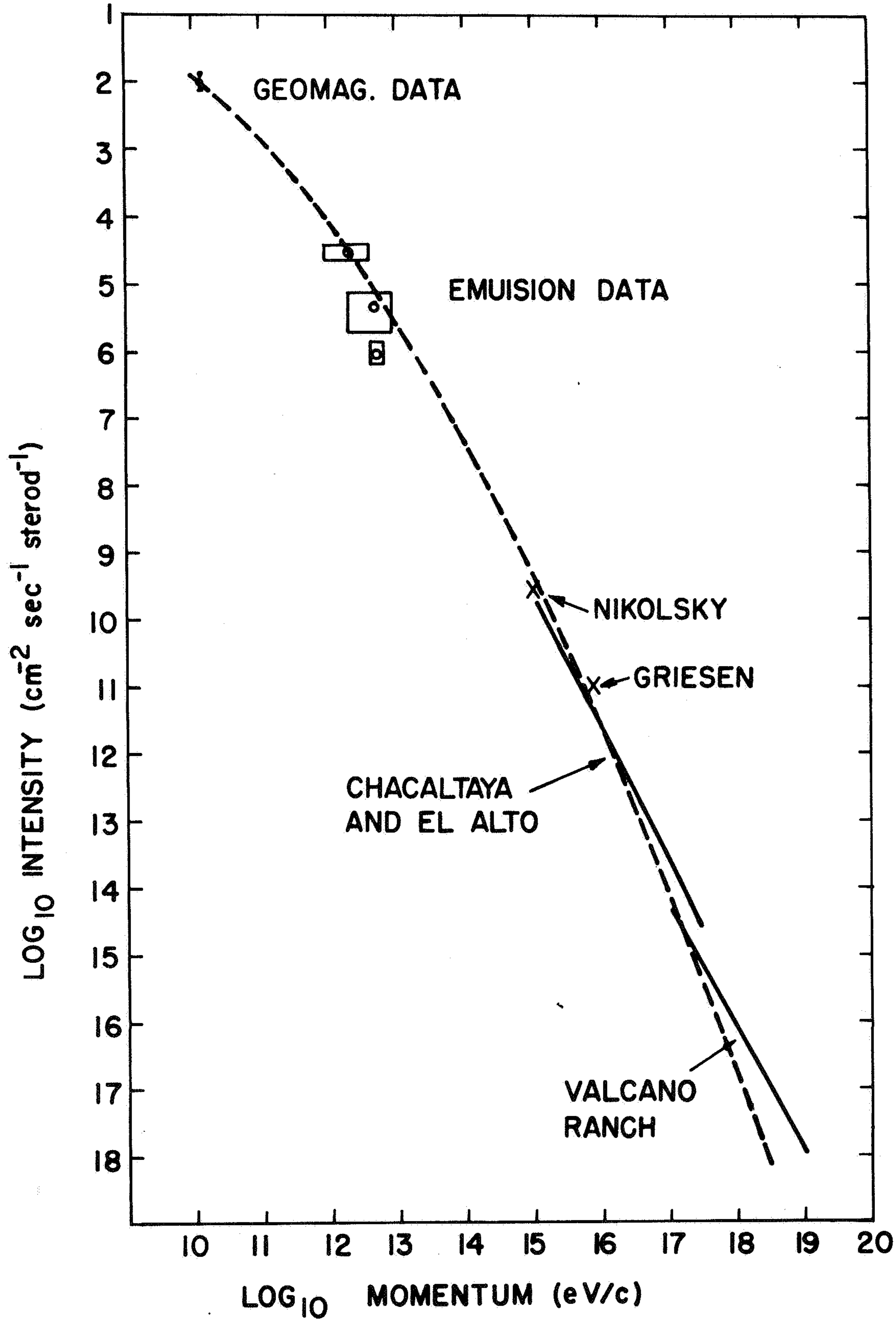
FIGURE CAPTIONS

Fig. 1 The integral momentum spectrum of Cosmic Radiation [Bradt, 1965 and Bray, 1965].

Fig. II The variation of the power law exponent with momentum.



GEOMAG. DATA
EMUISION DATA
NIKOLSKY
GRIESEN
CHACALTAYA
AND EL ALTO
VALCANO
RANCH
LOG10 INTENSITY (cm-2 sec-1 sterod-1)
LOG10 MOMENTUM (eV/c)

Fig. 1

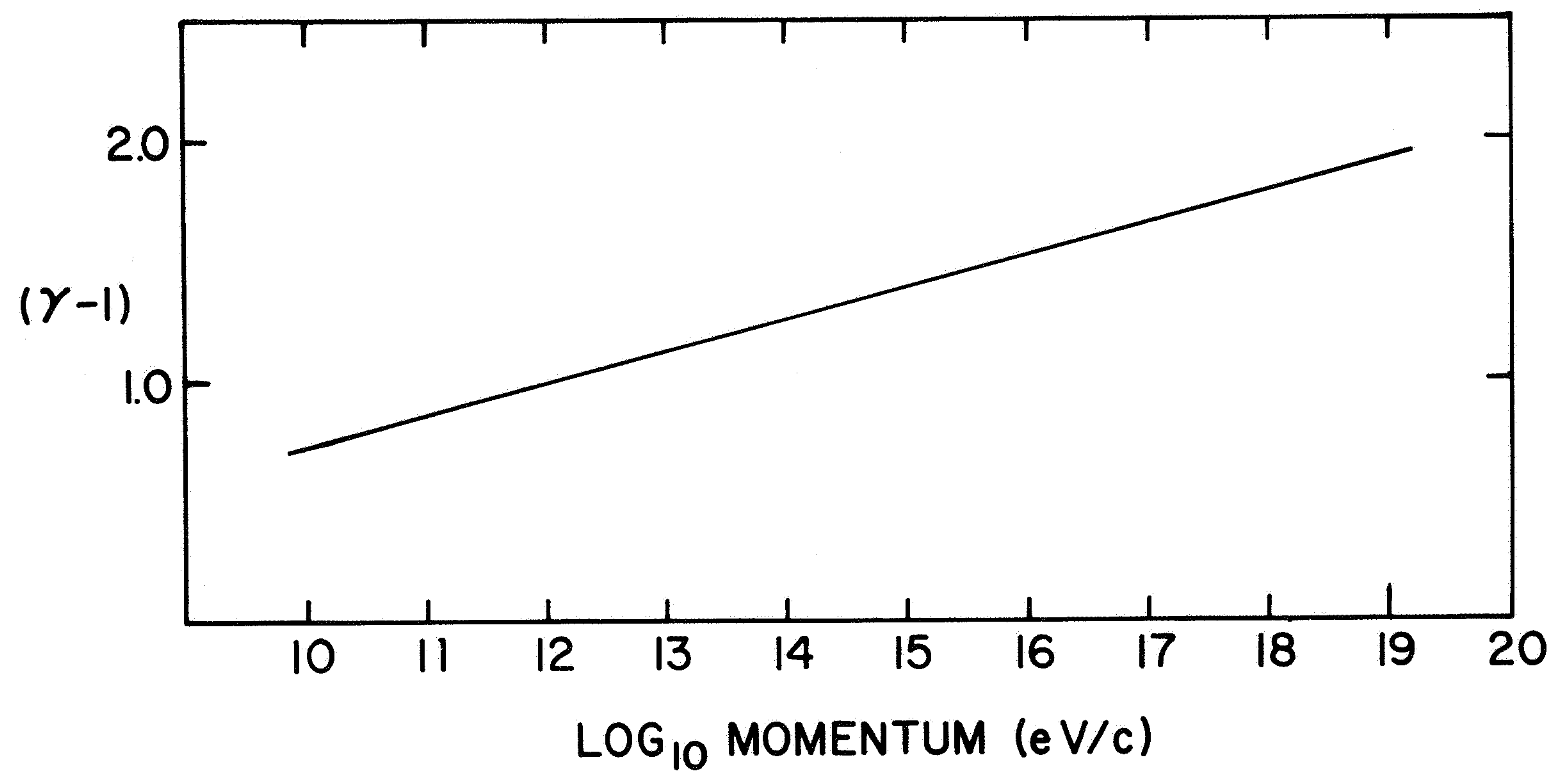


Fig. II

APPENDIX

One can easily show that for a Fermi type of scattering of particles with moving centers, the fractional momentum change is given by

$$\text{(A1)} \qquad \frac{\Delta P}{P} = \left[\Gamma^4 \left\{ \frac{2B}{\beta} + (1+B^2)\cos\alpha \right\}^2 + \sin^2\alpha \right]^{1/2} - 1 ,$$

where $\beta c = v$ = velocity of the particle, $Bc = V$ = velocity of the scattering center and α is the angle between v and V. If we expand this in terms of B and drop anything of order greater than B^4 we find

$$\text{(A2)} \quad \frac{\Delta P}{P} = \frac{2B^2}{\beta^2} + \frac{2B}{\beta}\mu\left(1+3B^2-\frac{2B^2}{\beta^2}\right) + 2B^2\mu^2\left(1-\frac{1}{\beta^2}\right) - \frac{4B^3\mu^3}{\beta}\left(1-\frac{1}{\beta^2}\right) + O(B^4),$$

where $\mu = \cos\alpha$. The collision probability is given in

$$\text{(A3)} \quad \Psi \, d\underset{\sim}{V}\, d\mu = \frac{v_r \, f(\underset{\sim}{V})\, d\underset{\sim}{V}\, d\mu}{\iint v_r \, f(\underset{\sim}{V})\, d\underset{\sim}{V}\, d\mu} ,$$

where

$\frac{v_r}{c}$ = relative velocity in units of the speed of light c,

$$= \left\{\beta^2 + B^2 - 2\beta B\mu - \beta^2 B^2 (1-\mu^2)\right\}^{1/2} \Big/ (1-\beta B\mu),$$

$f(\underset{\sim}{V})$ = velocity distribution of the scattering centers.

If the scattering centers are receding from each other as the result of spherical expansion from a common center, there will be a fractional momentum decrease [Wayland and Bowen, 1968] of

(A4) $$\frac{\Delta \rho}{\rho} = - \frac{B_e \lambda}{R}$$

where $B_e c = V_e$ = velocity of expansion,

R = radius of expansion,

λ = mean free path between scatterings.

We are interested in computing

(A5) $$\langle (\Delta\rho)^n \rangle = \iint (\Delta\rho)^n \, \psi \, d\underset{\sim}{V} \, d\mu .$$

By combining the above and expanding where possible in terms of B we find that in the limit of $\beta \to 1$

$$\Delta t \, a = \frac{8}{3} \bar{B}^2 - \frac{2 B_e \lambda}{R} + \frac{2 B_e \lambda \bar{B}^2}{3R} ,$$

$$\Delta t \, b = \frac{B_e^2 \lambda^2}{R^2} - \frac{4 B_e \lambda \bar{B}^2}{R} + \frac{4}{3} \bar{B}^2 ,$$

where

$$\bar{B}^2 = \int B^2 f(\underset{\sim}{V}) \, d\underset{\sim}{V} .$$

We note that is possibel to have a < 0.