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WAKE AND INTERREFLECTION EFFECTS IN THE CALCULATION OF FREE MOLECULAR FLOW DRAG COEFFICIENTS

By James O. Ballance
Aero-Astrodynamics Laboratory

NASA

George C. Marshall
Space Flight Center
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ABSTRACT

A Monte Carlo technique has been applied to the problem of calculating drag coefficients in free molecule flow where one body shields a second body from the incoming flow and where multiple collisions with the bodies are allowed. First, the applicability of the Monte Carlo approach is demonstrated showing that it produces compatible results with other theories considering various parameters such as the accommodation coefficient, the mode of reflection, the angle of attack, etc. Then, a system of two coaxial circular discs are studied showing the effects of shielding of one body by another, and the effects of multiple collision. Although the interaction effects of the system always tended to decrease the drag coefficients, for practical purposes, they could be ignored. It was also shown (1) that hyperthermal flow-type calculations can be modified slightly to assume some small divergence in the flow which is dependent on the speed ratio and (2) that modifications will provide very good agreement to the wake effects observed in the Monte Carlo solution.

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AEROPHYSICS DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

DEFINITION OF SYMBOLS

Symbol	Definition
X	separation distance = d/R_2
a	angle of attack
C	aerodynamic coefficient
C_D	drag coefficient
ρ	mass density
U	velocity of free stream
A	projected area
F	force
ΔV	change in velocity
N	number of molecules
n	number density
$\bar{v}$	average velocity, $(8kT/\pi m)^{1/2}$
v_m	most probable velocity, $(2kT/m)^{1/2}$
S	speed ratio, v_m/U
$\chi(S)$	$e^{-S^2} + S\sqrt{\pi}\,[1 + ERF(S)]$
α	accommodation coefficient
E	average kinetic energy

SUBSCRIPTS

T	total
∞	free stream

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
SUBSCRIPTS (Cont'd)	
m	molecule
i	incident
r	reflected
s	surface
w	wall
g	gas
ℓ	leading
t	trailing
int	interaction

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WAKE AND INTERREFLECTION EFFECTS IN THE CALCULATION OF FREE MOLECULAR FLOW DRAG COEFFICIENTS

SUMMARY

A Monte Carlo technique has been applied to the problem of calculating drag coefficients in free molecule flow where one body shields a second body from the incoming flow and where multiple collisions with the bodies are allowed. First, the applicability of the Monte Carlo approach is demonstrated showing that it produces compatible results with other theories considering various parameters such as the accommodation coefficient, the mode of reflection, the angle of attack, etc. Then, a system of two coaxial circular discs are studied showing the effects of the above parameters on the drag coefficients, the effects of shielding of one body by another, and the effects of multiple collision. Although the interaction effects of the system always tended to decrease the drag coefficients, for practical purposes, they could be ignored. It was also shown (1) that hyperthermal flow-type calculations can be modified slightly to assume some small divergence in the flow which is dependent on the speed ratio and (2) that modifications will provide very good agreement to the wake effects observed in the Monte Carlo solution.

I. INTRODUCTION

The calculation of aerodynamic forces in free molecule flow has only recently been extended to bodies where the incoming molecules could be re-emitted for the body and collide with the body again [1,2,3,4]. These solutions, derived initially by M. T. Chahine, however, are limited to rather special geometries (i.e., concave hemispheres, concave semi-cylinders, etc.). Another problem in free molecule flow calculations which has not been solved explicitly is the determination of the effect of the one body in the flow field on another downstream. In hyperthermal flow (i.e., flow where the relative motion of the body to the molecule is much greater than the most probable thermal velocity of the molecules), the currently accepted procedure is to project downstream the frontal outline presented to the flow by the leading body parallel to the flow vector (see figure 1), thus removing any of the "shadowed" area from consideration. The additional problem of interreflections of one body with a second body (similar to concave body problems) has not been considered. The principal reason for neglecting these areas in free molecule flow calculations is that the mathematical formulation is quite complicated and, even in the most simple cases, is rather difficult to solve.

For several years the author has been analyzing free molecule flow in ducts where there was relative motion between the duct and the stream in order to determine the molecular kinetic characteristics and the overall response of various geometries used to couple density and mass spectrometer gauges to the atmosphere when used on sounding rockets and satellites. The apparent success of the techniques for these problems and the relative ease in the formulation of the analysis suggested a general approach to the family of problems outlined above. This report presents the results of a portion of the study. Specifically, this report presents the free molecule drag coefficients for a system consisting of two coaxial, circular, flat discs which can be separated from each other. The parameters considered are (1) the speed ratio (ratio of the speed of the system relative to the gas to the most probable thermal speed of the molecules), (2) the energy accommodation coefficient, (3) the ratio of the temperature of the surface to the temperature of the gas, (4) the ratio of the area of the forward disc to the area of the trailing disc, (5) the angle of attack, (6) the type of reflection, (7) the separation distance between the discs, and (8) the interreflection of the molecules between the discs.

II. APPROACH

The application of the Monte Carlo method to free molecule flow has been demonstrated in several studies. This approach can be easily extended to aerodynamic coefficient calculation, which is usually defined as

$$C = \frac{F_T}{\frac{1}{2}\rho_\infty U^2 A} ,$$

where

F_T = force

ρ_∞ = mass density of the freestream

U = velocity of the body relative to the free stream

A = projected area of the body.

The force from a single molecule is determined from the velocity exchange

$$F_m = m\ \Delta V_m,$$

when

m = molecule mass

ΔV_m = change in velocity.

The total force F_T is given by summing over all the molecules. Thus,

$$F_T = \sum NF_m = \sum Nm\ \Delta V_m$$

when

N = number of molecules.

The number of molecules striking a unit area in free molecule flow is given by [6]

$$N = n_i \frac{\bar{v}}{4} \left[e^{-S^2} + S\sqrt{\pi}\ [1 + ERF(S)] \right],$$

where

n_i = number density of the freestream

$\bar{v}$ = average velocity of the molecule

$= [8kT/\pi m]^{1/2}$

$S = v_m/U$

v_m = most probable velocity of the molecules

$= (2kT/m)^{1/2}$.

Normally, $\chi(S)$ is defined as

$$\chi(S) = e^{-S^2} + S\sqrt{\pi}\,[1 + ERF(S)]$$

so that

$$N = n_i \frac{\bar{v}}{4} \chi(S).$$

From reference 7,

$$n_i \frac{\bar{v}}{4} = \frac{1}{2\sqrt{\pi}} \frac{\rho_\infty}{m} v_m,$$

so that, when the above equations are combined,

$$C = \frac{1}{\sqrt{\pi}} v_m \frac{\chi(S)}{u^2} \sum \Delta V_m$$

for a unit area. The computer program calculates a speed which is given by

$$V_m = \gamma v_m$$

where γ is determined randomly from the velocity distribution function such that

$$\sum \Delta V_m = \frac{1}{N_{sam}} \sum_{i=1}^{N_{sam}} \gamma_i v_m$$

$$= \frac{v_m}{N_{sam}} \sum_{i=1}^{N_{sam}} \gamma_i$$

where N_{sam} is the number of molecules followed. The computer program then calculates a drag coefficient given by

$$C = \frac{1}{\sqrt{\pi}} \frac{\chi(S)}{S^2 N_{sam}} \sum_{i=1}^{N_{sam}} \gamma_i .$$

The trailing plate (disc 2) in the system was used at the starting point. After choosing (1) a random location (from a uniform distribution) on disc 2, (2) the total velocity of the molecule including the thermal motion and the relative motion, and (3) the direction cosines of the molecule's trajectory, the incoming path was projected back toward the leading disc (disc 1) to see if it would have been intercepted by that disc. If the discs had equal radii or disc 1 had the larger radius and they were not separated, all the molecules would strike disc 1. If the radius of disc 1 were less than disc 2 and the discs were not separated, the number of molecules striking disc 1 and disc 2 were proportional to the ratio of the area of disc 1 and exposed area of disc 2. When the discs were separated, the flux of each disc was dependent on the distance of separation, the speed ratio and the angle of attack. Using this flux information, the wake effects can be investigated.

When the molecule collided with the surface, the amount of energy lost to or gained from the surface was determined by the energy accommodation coefficient being used. For this study there was no consideration given to any angle-of-incidence dependence of the accommodation coefficients. The energy accommodation coefficient is defined as

$$\alpha = \frac{E_i - E_r}{E_i - E_s} ,$$

where

E_i is the average kinetic energy of the incoming molecule,

E_r is the average kinetic energy of the reflected molecules,

E_s is the average kinetic energy of the molecules leaving at the surface temperature.

The speed of the reflected molecule can then be related to the speed of the incoming molecule as

$$\frac{V_r}{V_i} = [E_r/E_i]^{1/2}$$

$$= \left[1 - \alpha \left(1 - \frac{E_s}{E_i} \right) \right]^{1/2} .$$

In the Monte Carlo calculation, the thermal motion of the free stream molecule is arbitrary so that the surface temperature (and thus E_s) is expressed in terms of the free stream molecules. This expression then allows one to examine the effects of the accommodation coefficient and the temperature ratio on the aerodynamic coefficients. Next, the reflection parameters, which could be either diffuse or specular, were determined. The cosines of the new direction were calculated and the molecule emitted from the surface. The trajectory of the molecule was tested to see if it would strike the rear of disc 1. If it did, the collision procedure was again followed, the molecule re-emitted, and the trajectory followed. Any number of interreflections between the disc could be followed; however, except for very small separation distances (i.e., less than the radius of disc 2), few molecules would make more than 30 collisions with the discs.

III. RESULTS

A. Monte Carlo Calculation of Drag Coefficients on a Flat Plate

To illustrate the usefulness of the method to calculate drag coefficients, figures 2 through 5 show typical results for a flat plate. Figure 2 presents the drag coefficient of a flat plate for various speed ratios, considering diffuse reflections, accommodation coefficients of 0, 0.25, 0.5, 0.75, and 1.0 and a wall-temperature-to-gas temperature ratio of 0.25. The angle of attack in this report is defined as the angle between the incoming flow vector and the normal to the plate. In figure 2 the angle of attack is 0° and the flow is normal to the surface. Also shown is the hyperthermal value for these conditions as calculated by Schamberg [5].

Figures 3 and 4 present the drag coefficients based on the area projected normal to the flow vector for a speed ratio of 10, T_w/T_g ratio of 0.25, and various accommodation coefficients as a function of the angle of attack for diffuse and specular reflections. Again, the solid curve is the value from Schamberg's model.

Figure 5 shows the drag coefficient for a speed ratio of 10, angle of attack of 0°, and various accommodation coefficients as a function of the wall temperature-to-gas-temperature ratio, T_w/T_g. The results are quite consistent with other approaches, showing that the wall temperature contributes little to the drag coefficient values where the accommodation coefficient is less than 1. When $\alpha = 1.0$, the dependence on the ratio is obvious.

From these results, then, it is seen that the Monte Carlo approach works quite well and provides information consistent with other solutions.

B. Wake Effects

As stated earlier, the standard method of calculating drag coefficients for bodies in free molecular flow where one portion of the body shields another portion from the incoming stream of molecules is to project the leading profile normal to the flow onto the trailing body. Any contribution to the drag due to molecules from the trailing body striking the back of the leading body is ignored. A convenient way to express this for a system might be

$$C_{D_{total}} = A_\ell \, C_{D_\ell} + A_t \, C_{D_t} \, \delta(X,S,a,\gamma) + C_{D_{int}}(X,S,a,\gamma) \qquad (1)$$

where

$C_{D_{total}}$ = total drag coefficient for the system

A_ℓ, A_t = area projected normal to the flow for the leading body and the trailing body, respectively

C_{D_ℓ}, C_{D_t} = drag coefficient for the leading body and trailing body, respectively

$\delta(X,S,a,\gamma)$ = correction factor to relate reference area A_t to that predicted by hyperthermal flow (equals 1 in hyperthermal flow)

$C_{D_{int}}(X,S,a,\gamma)$ = drag contribution due to molecules rebounding between the leading and trailing bodies

X = separation distance of the bodies, d/R_2

S = speed ratio

a = angle of attack

γ = variable to indicate the dependence of the parameters on the geometry of the bodies.

In this section only the first two terms of the above equation will be examined.

For the simple geometry under study, the leading body is disc 1 and the trailing body is disc 2. The area A_t can easily be determined for the standard method of calculation. Table I shows the values for the case when $A_1/A_2 = 1.0$, at various angles of attack as a function of the separation distance X. With these area calculations the drag coefficients are easily determined in hyperthermal flow from the first two terms of equation (1).

In the Monte Carlo calculations, the wake effects are observable through the percentage of particles which collide with the trailing disc. Since the disc is uniformly covered with molecules, the percentage of molecules which strike the trailing disc should be proportional to the areas exposed. Thus, this percentage is the value of the term δ in the above equation.

Figures 6 and 7 compare the area, A_t, as determined by using hyperthermal approach to that determined by the Monte Carlo method for various angles of attack and speed ratios of 5 and 10, respectively.

C. Interactions

The third term in equation (1) results from the molecules reflecting from the trailing disc and striking the rear of the leading disc. In this study, any number of reflections between the disc could be permitted. It was found, however, that except for the very small separation distance, i.e., X = 0.25, or total specular reflection, few molecules made more than 30 reflections between the discs. Also, little contribution to the magnitude of the coefficients after several (~ 5) reflections was noticed as long as the accommodation coefficient was fairly large.

While, in principle, the reflection mode is highly dependent on the energy transfer as expressed by energy accommodation coefficient, it is convenient to consider this mode to be independent and to be purely diffuse, purely specular or some linear combination of these two. The interaction effects for diffuse reflections were examined for different speed ratios. To better present this information, the results are expressed in the following convention. The contribution to the total drag coefficient as expressed in equation (1) above has been determined from the Monte Carlo results and is expressed in terms of fraction of the total drag coefficient when the discs are not separated (X = 0). This fraction is graphically shown in figures 8 through 14 for diffuse reflections. Figures 8 through 11 show the contribution as a function of the separation distance for flow incident on the discs at zero angle of attack at speed ratios of 3, 5, and 10, for area ratios (area of the leading disc to the area of the trailing disc) of 0.25, 0.5, 0.75, and 1.0. Figures 12 through 14 present the contribution as a function of the separation distance for an area ratio of 1.0, a speed ratio of 10, temperature ratio of 0.25, and accommodation coefficients of 0.75, at different angles of attack. In all cases it is seen that the contribution as so expressed is negative.

The assumption of diffuse reflection may not be physically correct for actual interactions of gas molecules with surface molecules at high velocity impact. The dependence of the reflection coefficients on the accommodation coefficients has been shown by several investigators (e.g., Schamberg), but some insight into the effects of varying reflection coefficients can be obtained in a rather simple manner with the Monte Carlo program. This is done by a simple procedure of randomly choosing a certain fraction of the molecules to have a specular reflection rather than a diffuse one. This was done for discs with an area ratio of 1.0, speed ratio of 10, accommodation coefficient of 0.75, and an angle of attack of 15°. The molecules were allowed to undergo total specular reflections for some arbitrary number of collisions with all subsequent reflections being diffuse. It was also possible to choose only a fraction of the molecules to have a specular reflection. The results of these calculations are shown in figure 15. Here the total drag coefficient for the system is shown as a function of the specular reflections undergone.

D. Total Drag Coefficient

The total drag coefficients for the system of two discs as expressed by equation (1) are shown in figures 16 through 22 and have been normalized to the value for no separation of disc. The first series of data, figures 16 through 19, shows the relationship of the ratio of the area of the leading disc to the area of the trailing disc and of the speed ratio at a fixed angle of attack, T_W/T_g ratio, and energy accommodation coefficient. The second series of data (figures 20 through 22) shows the effects of angle of attack and of energy accommodation coefficient on the system with a fixed area ratio and temperature ratio. In all cases the total drag coefficient is normalized to the drag coefficients for the system when the separation distance is zero. The reference area is πR_2^2 where R_2 (the radius of the trailing disc) was 1.0 for this study. Since the figures show only the drag coefficient normalized to the zero separation value, tables II through VII contain the actual coefficient as calculated for equation (1).

E. Discussion of Results

From this study, several obvious conclusions may be drawn. First, the Monte Carlo method can be applied to the calculation of aerodynamic characteristics of bodies in free molecular flow and gives excellent agreement (i.e., 1 to 2 percent) with more conventional solutions. Second, the Monte Carlo method, because of its inherent characteristics, can be used to study conveniently some aspects of free molecular flow which cannot be studied or are quite difficult to study with more conventional methods. These aspects include the problems of wake effects, multiple reflection, varying geometrical shapes, etc.

To examine the wake effects and interreflection effects in free molecular flow, a system consisting of two coaxial circular discs was studied. By restraining the system to the configuration where the discs were not separated, the problem of calculating drag on a flat plate could be examined, along with parameters such as the energy accommodation coefficient, the angle of attack, the mode of reflection, and the ratio of the wall temperature to the gas temperature. These results, shown in figures 2 through 5, demonstrate several interesting features. Figure 2 shows that the concept of hyperthermal flow is easily satisfied by speed ratios of approximately 5 or larger when the energy accommodation coefficient is less than 1.0. For $\alpha = 1.0$, a ten percent error in drag coefficient is evident at $S = 5$. Figure 5 shows that, unless $\alpha = 1.0$, the temperature ratio, T_W/T_g, has little effect on the drag coefficient at high speed ratios.

By separating the discs, the wake effects from the leading disc on the trailing disc and the interreflection between the discs could be examined (see figures 6-15). Consider first the wake effects as shown in figures 6 and 7. In each figure we can see that, except for the case of the angle of attack equals zero, both the S = 5 and S = 10 results agree quite well with the hyperthermal prediction for shading. For angle of attack equals zero, however, very significant differences appear even at small separations (e.g., 5 radii). Of course, the hyperthermal flow approach does not allow any flux on the trailing disc at this angle of attack while, for X = 5, the S = 5 case shows about 37 percent of the disc actually being hit by molecules, and the S = 10 case, 19 percent. In these instances, these percentages both indicate an error of that amount to the total drag of the system as calculated by hyperthermal theory. It is interesting to consider why these effects are so noticeable for zero angle of attack and not other angles of attack. The reason is the symmetry of the system being studied. The true trajectory of the molecules in free molecular flow is not a beam where each molecular path is parallel to the others but where each path diverges slightly from the parallel path due to its thermal energy. For the case where the two coaxial discs are separated from each other at zero angle of attack, this slight divergence results in the trailing disc being struck along its entire outer edge (for $A_1/A_2 = 1.0$). At angles of attack, however, only a small portion is affected. Thus, for this particular system, very good agreement to hyperthermal theory is noticed at angles of attack, but in general, this may not be true for other configurations (i.e., a long rectangular plate shielding another long plate).

The effects of interreflections are shown in figures 8 through 15. In general, the interreflection effects are of little consequence (i.e., less than 10 percent) to the drag coefficient for both diffuse or specular reflections. The larger effects were observed with the smaller values of the accommodation coefficient. In all cases, though, the interreflection resulted in a decrease in the total drag on the system.

Using the Monte Carlo data, it is possible to see how the hyperthermal flow approach can be modified to consider the small divergence from parallel beams of molecules. Figure 23 presents the total flux which would be intercepted by the system of two discs if the flow were not purely hyperthermal and normal to the disc but had a small divergence angle due to the thermal motion of the molecules. This figure shows that a good approximation for flow S = 3 is a divergence angle of 4°, for S = 5, 2°, and for S = 10, 1°.

The Monte Carlo approach to the investigation of wake and interreflection effects for aerodynamic force coefficient calculations has been demonstrated. The continuing program of study has been extended to concave bodies such as spheres, cones, cylinders, wedges, and to skewed and parallel plates. Although only drag coefficients have been presented in this report, other coefficients are easily obtained.

TABLE I

Ratio of the Flux on the Trailing Disc to the Flux on the Leading Disc Using Hyperthermal Theory

$A_1/A_2 = 1.0, \quad X = d/R_2$

Angle of Attack (degs)	X: .25	.5	.75	1.0	1.5	2	3	4	5
0	0	0	0	0	0	0	0	0	0
15	.043	.085	.128	.170	.254	.337	.498	.648	.784
30	.092	.183	.273	.362	.533	.692	.942	1.00	1.00
45	.159	.315	.466	.609	.856	1.00	1.00	1.00	1.00
60	.274	.533	.764	.942	1.00	1.00	1.00	1.00	1.00
75	.571	.979	1.00	1.00	1.00	1.00	1.00	1.00	1.00
90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE II

Drag Coefficients for a System of Two Coaxial Circular Discs

$A_1/A_2 = 0.25$, $T_w/T_g = 0.5$, $\alpha = 0.5$, Angle of Attack = 0°

X	S = 3	S = 5	S = 10	Diffuse Reflection
0	3.06	2.99	2.96	
.25	3.02	2.94	2.90	
.5	3.00	2.91	2.87	
.75	3.00	2.92	2.88	
1	2.98	2.89	2.86	
1.5	3.05	2.92	2.89	
2	3.10	2.98	2.92	
3	3.24	3.04	2.95	
4	3.35	3.11	2.95	
5	3.42	3.17	2.97	
10	3.59	3.36	3.12	
20	3.68	3.52	3.34	
30	3.74	3.58	3.43	
40	3.76	3.62	3.48	
50	3.78	3.66	3.54	
60	3.76	3.63	3.54	
70	3.81	3.68	3.58	
80	3.80	3.68	3.59	
90	3.80	3.69	3.60	
100	3.81	3.68	3.61	
140	3.81	3.71	3.64	
200	3.82	3.71	3.65	

TABLE III

Drag Coefficients for a System of Two Coaxial Circular Discs

$A_1/A_2 = 0.5$, $T_w/T_g = 0.5$, $\alpha = 0.5$, Angle of Attack = 0°

X	S = 3	S = 5	S = 10	Diffuse Reflection
0	3.05	2.98	2.95	
.25	2.96	2.92	2.88	
.5	2.96	2.87	2.85	
.75	2.98	2.88	2.83	
1	2.99	2.87	2.85	
1.5	3.11	2.91	2.87	
2	3.24	3.00	2.88	
3	3.50	3.16	2.98	
4	3.67	3.27	3.00	
5	3.78	3.36	3.04	
10	4.13	3.78	3.35	
20	4.32	4.07	3.73	
30	4.42	4.19	3.93	
40	4.46	4.26	4.03	
50	4.50	4.32	4.12	
60	4.48	4.31	4.15	
70	4.54	4.36	4.21	
80	4.55	4.37	4.23	
90	4.54	4.39	4.27	
100	4.55	4.40	4.28	
140	4.56	4.42	4.32	
200	4.57	4.43	4.35	

TABLE IV

Drag Coefficient for a System of Two Coaxial Circular Discs

$A_1/A_2 = 0.75$, $T_w/T_g = 0.5$, $\alpha = 0.5$, Angle of attack = 0°

X	S = 3	S = 5	S = 10	Diffuse Reflection
0	3.07	3.00	2.99	
.25	3.00	2.92	2.88	
.5	3.00	2.91	2.87	
.75	3.09	2.90	2.86	
1	3.15	2.94	2.86	
1.5	3.29	3.01	2.89	
2	3.52	3.15	2.95	
3	3.79	3.33	3.03	
4	4.00	3.51	3.12	
5	4.20	3.66	3.19	
10	4.68	4.21	3.61	
20	5.00	4.63	4.19	
30	5.13	4.86	4.47	
40	5.19	4.94	4.63	
50	5.23	5.00	4.73	
60	5.22	5.03	4.79	
70	5.28	5.07	4.85	
80	5.28	5.05	4.86	
90	5.29	5.10	4.91	
100	5.30	5.10	4.93	
140	5.30	5.13	4.99	
200	5.32	5.16	5.04	

TABLE V

Drag Coefficient for a System of Two Coaxial Circular Discs

$A_1/A_2 = 1.0$, $T_w/T_g = 0.5$, Angle of attack = 0°, $\alpha = 0.5$

X	S = 3	S = 5	S = 10	Diffuse Reflection
0	3.07	2.98	2.95	
.25	3.18	3.04	2.97	
.5	3.25	3.10	3.00	
.75	3.35	3.14	3.02	
1	3.45	3.18	3.04	
1.5	3.63	3.28	3.12	
2	3.87	3.44	3.17	
3	4.22	3.66	3.27	
4	4.47	3.85	3.39	
5	4.69	4.07	3.48	
10	5.26	4.69	4.00	
20	5.68	5.26	4.65	
30	5.83	5.48	4.50	
40	5.90	5.59	5.21	
50	5.96	5.68	5.34	
60	5.95	5.70	5.42	
70	6.02	5.77	5.50	
80	6.02	5.78	5.54	
90	6.04	5.81	5.60	
100	6.05	5.83	5.62	
140	6.08	5.88	5.68	
200	6.09	5.89	5.76	

TABLE VI

Drag Coefficients for a System of Two Coaxial Circular Discs

$A_1/A_2 = 1.0$, $T_w/T_g = 0.25$, $\alpha = 0.5$, $S = 10$

X	a = 0°	a = 15°	a = 30°	a = 45°	a = 60°	a = 75°	Diffuse Reflection
0	2.95	2.91	2.82	2.66	2.47	2.24	
.25	2.97	3.03	3.03	3.07	3.11	3.11	
.5	3.01	3.14	3.28	3.43	3.72	4.12	
.75	3.01	3.26	3.49	3.80	4.18	4.31	
1	3.04	3.37	3.74	4.15	4.50	4.35	
1.5	3.05	3.56	4.32	4.72	4.75	4.39	
2	3.16	3.84	4.66	5.08	4.81	4.42	
3	3.28	4.31	5.30	5.24	4.87	4.45	
4	3.40	4.73	5.54	5.28	4.90	4.46	
5	3.49	5.10	5.58	5.29	4.91	4.46	
10	4.00	5.74	5.62	5.32	4.93	4.47	
20	4.64	5.81	5.63	5.34	4.93	4.48	
30	4.99	5.82	5.63	5.33	4.94	4.48	
40	5.20	5.83	5.63	5.33	4.95	4.49	
50	5.34	5.83	5.63	5.34	4.94	4.48	
60	5.41	5.83	5.63	5.34	4.94	4.48	
70	5.48	5.83	5.63	5.34	4.94	4.48	
80	5.54	5.83	5.63	5.34	4.94	4.48	
90	5.57	5.83	5.63	5.34	4.94	4.48	
100	5.62	5.83	5.63	5.34	4.94	4.48	
140	5.69	5.83	5.63	5.34	4.94	4.48	
200	5.74	5.83	5.63	5.34	4.94	4.48	

TABLE VII

Drag Coefficients for a System of Two Coaxial Circular Discs

$A_1/A_2 = 1$, $T_w/T_g = 0.25$, $\alpha = 0.75$, $S = 10$

X	a = 0°	a = 15°	a = 30°	a = 45°	a = 60°	a = 75°	Diffuse Reflection
0	2.67	2.65	2.58	2.48	2.34	2.18	
.25	2.69	2.75	2.80	2.85	3.53	3.46	
.5	2.72	2.86	3.01	3.21	4.01	4.04	
.75	2.74	2.96	3.23	3.55	4.30	4.22	
1	2.77	3.06	3.46	3.83	4.55	4.26	
1.5	2.82	3.27	3.88	4.46	4.59	4.29	
2	2.87	3.50	4.29	4.77	4.64	4.31	
3	2.95	3.94	4.88	4.91	4.64	4.33	
4	3.06	4.34	5.10	4.93	4.66	4.33	
5	3.17	4.63	5.13	4.95	4.67	4.34	
10	3.64	5.22	5.16	4.95	4.67	4.35	
20	4.23	5.29	5.16	4.95	4.67	4.35	
30	4.53	5.30	5.17	4.95	4.67	4.35	
40	4.71	5.30	5.17	4.95	4.67	4.35	
50	4.85	5.31	5.17	4.95	4.67	4.35	
60	4.94	5.31	5.17	4.95	4.67	4.35	
70	4.99	5.31	5.17	4.95	4.67	4.35	
80	5.03	5.31	5.17	4.95	4.67	4.35	
90	5.05	5.31	5.17	4.95	4.67	4.35	
100	5.07	5.31	5.17	4.95	4.67	4.35	
140	5.17	5.31	5.17	4.95	4.67	4.35	
200	5.21	5.31	5.17	4.95	4.67	4.35	

TABLE VIII

Drag Coefficients for a System of Two Coaxial Circular Discs

$A_1/A_2 = 1.0$, $T_w/T_g = 0.25$, $\alpha = 1.0$, $S = 10$

X	a = 0°	a = 15°	a = 30°	a = 45°	a = 60°	a = 75°	Diffuse Reflection
0	2.10	2.10	2.09	2.07	2.05	2.02	
.25	2.12	2.19	2.28	2.40	2.62	3.27	
.5	2.14	2.27	2.46	2.73	3.40	3.84	
.75	2.16	2.37	2.66	3.01	3.61	4.01	
1	2.18	2.45	2.84	3.32	3.91	4.03	
1.5	2.21	2.63	3.19	3.82	4.07	4.04	
2	2.27	2.80	3.52	4.07	4.08	4.04	
3	2.35	3.15	3.98	4.13	4.09	4.05	
4	2.41	3.43	4.14	4.13	4.09	4.05	
5	2.51	3.68	4.16	4.13	4.09	4.05	
10	2.85	4.12	4.17	4.14	4.09	4.04	
20	3.32	4.19	4.17	4.14	4.10	4.05	
30	3.58	4.19	4.17	4.14	4.10	4.05	
40	3.72	4.19	4.17	4.14	4.10	4.05	
50	3.81	4.19	4.17	4.14	4.10	4.05	
60	3.87	4.19	4.17	4.14	4.10	4.05	
70	3.91	4.19	4.17	4.14	4.10	4.05	
80	3.94	4.19	4.17	4.14	4.10	4.05	
90	3.97	4.19	4.17	4.14	4.10	4.05	
100	3.99	4.19	4.17	4.14	4.10	4.05	
140	4.05	4.19	4.17	4.14	4.10	4.05	
200	4.09	4.19	4.17	4.14	4.10	4.05	

GEOMETRICAL MODEL

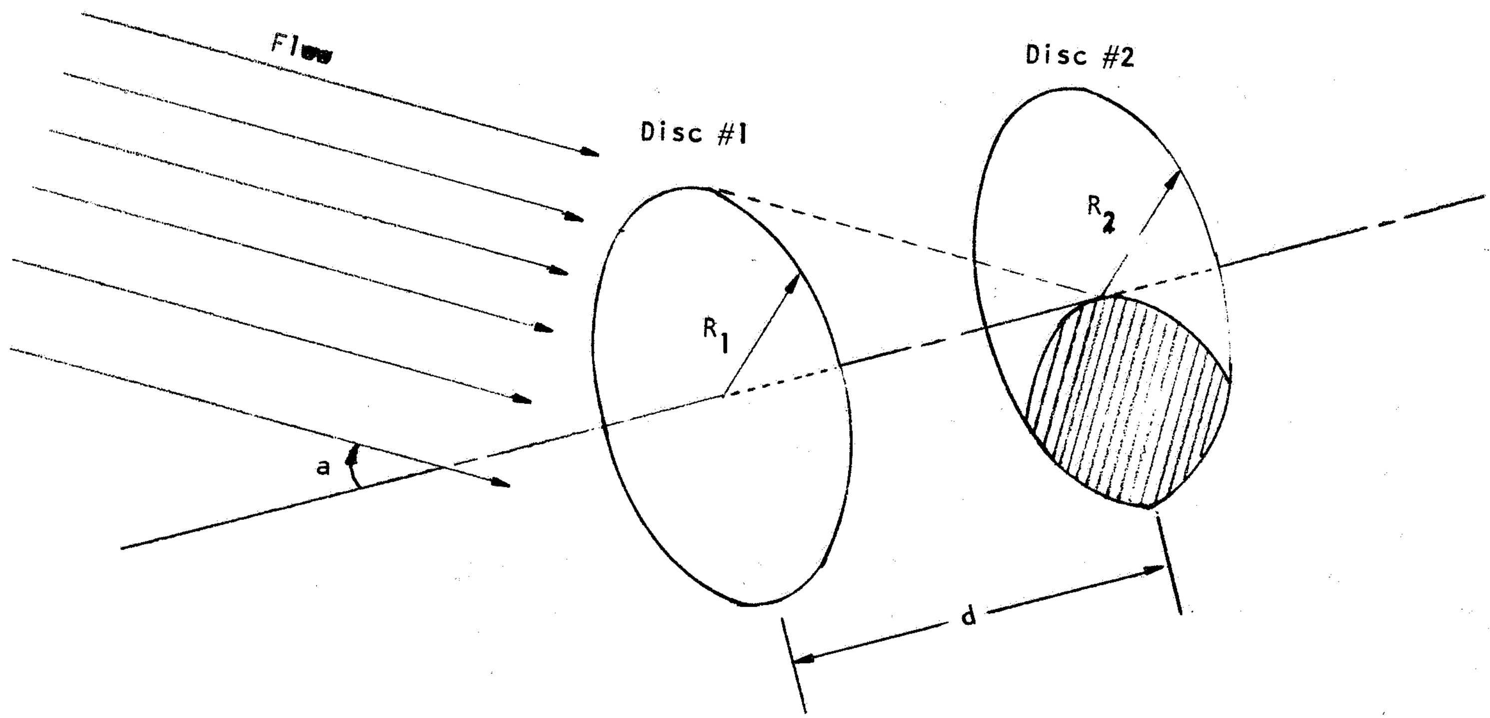

FIGURE 1.

DRAG COEFFICIENTS FOR A FLAT PLATE NORMAL TO THE FLOW DIRECTION
DIFFUSE REFLECTIONS $T_w/T_g = 0.25$

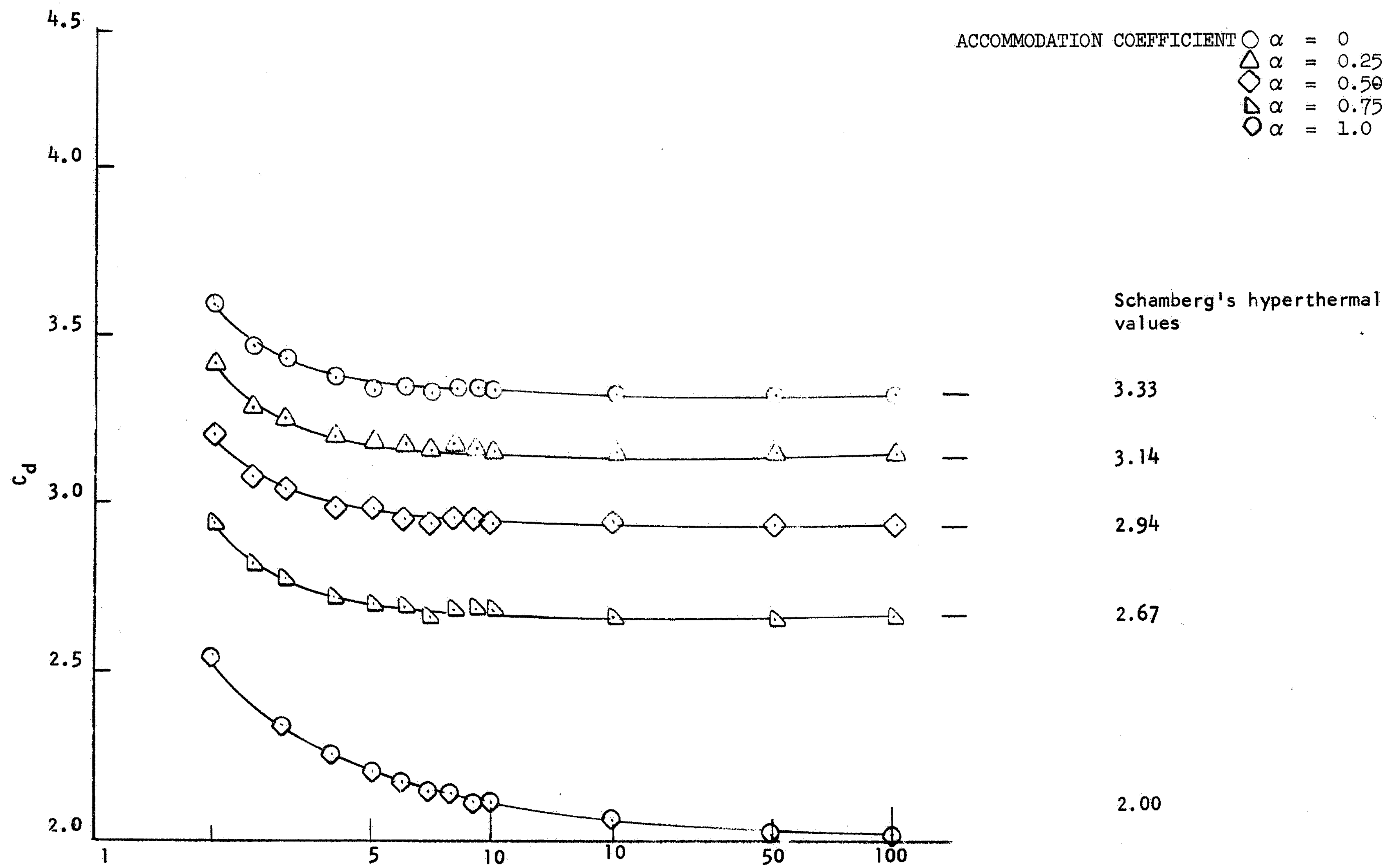

FIGURE 2

DRAG COEFFICIENTS FOR A FLAT PLATE AT ANGLES OF ATTACK
SPEED RATIO = 10 T_w/T_g = 0.25 DIFFUSE REFLECTIONS

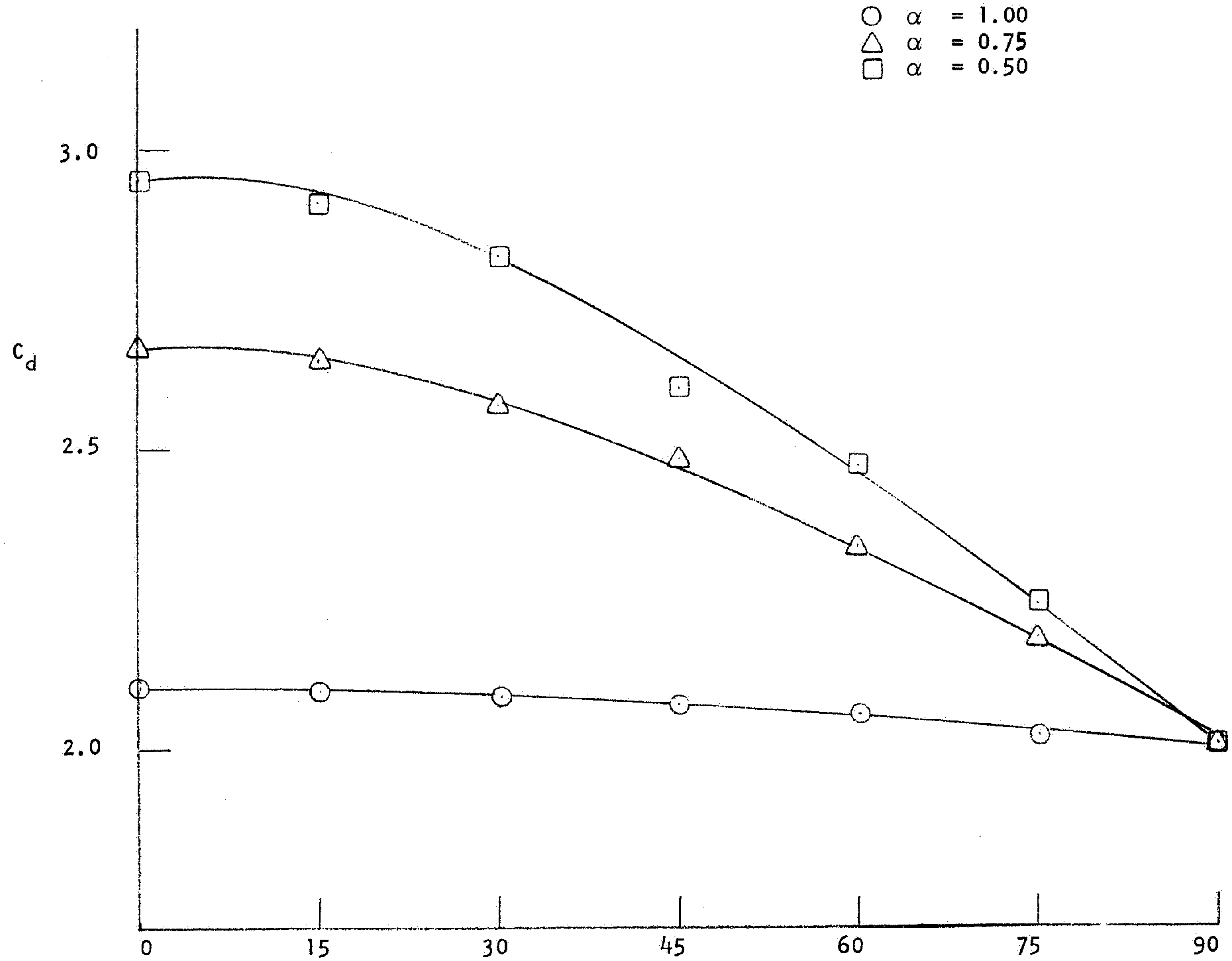

FIGURE 3

ANGLE OF ATTACK (DEGREES)

DRAG COEFFICIENTS FOR A FLAT PLATE AT ANGLES OF ATTACK
SPEED RATIO = 10 T_w/T_g = 0.25   SPECULAR REFLECTIONS

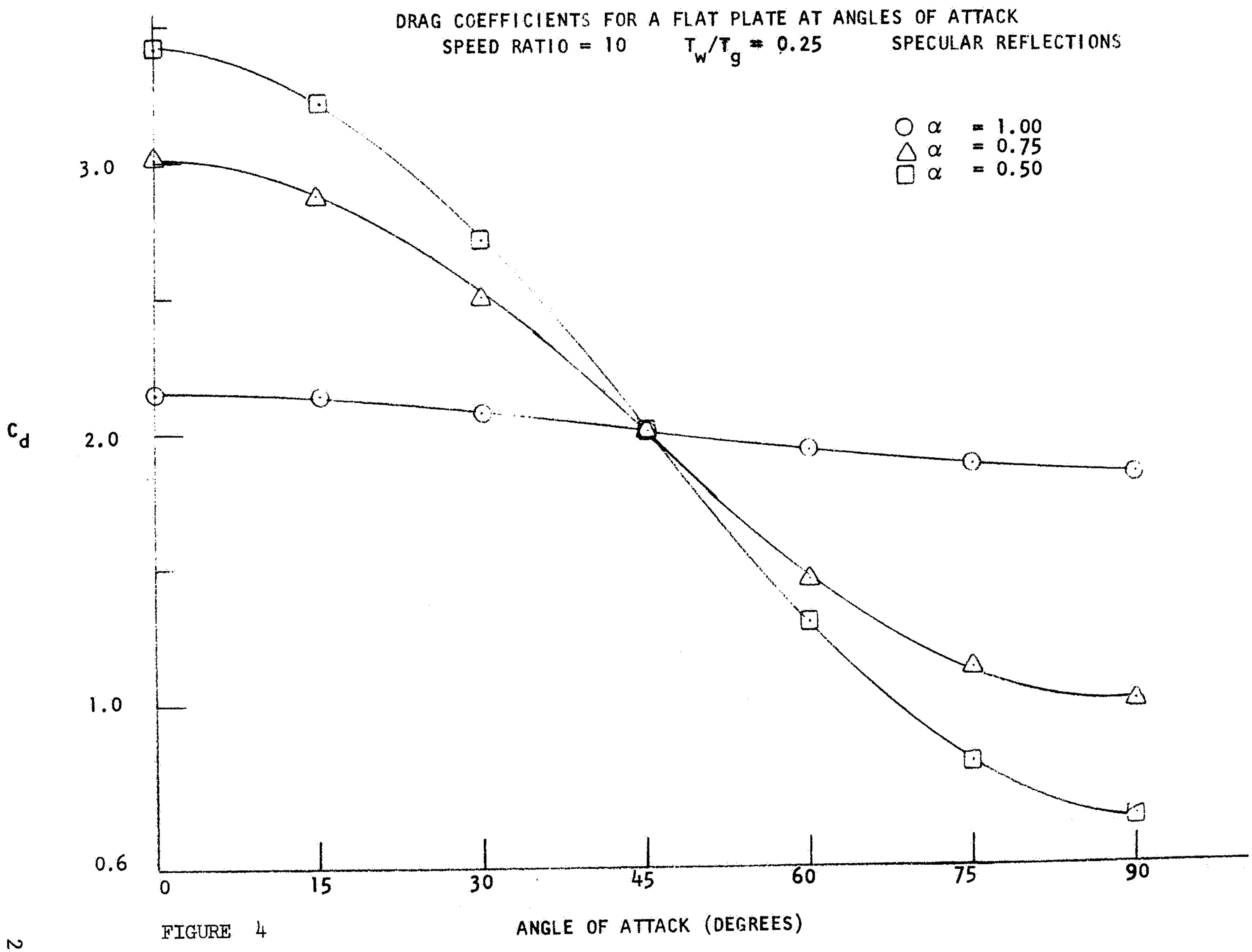

FIGURE 4

DRAG COEFFICIENTS FOR A FLAT PLATE NORMAL TO THE FLOW AT A SPEED RATIO OF 10 FOR DIFFERENT TEMPERATURE RATIOS AND DIFFUSE REFLECTIONS

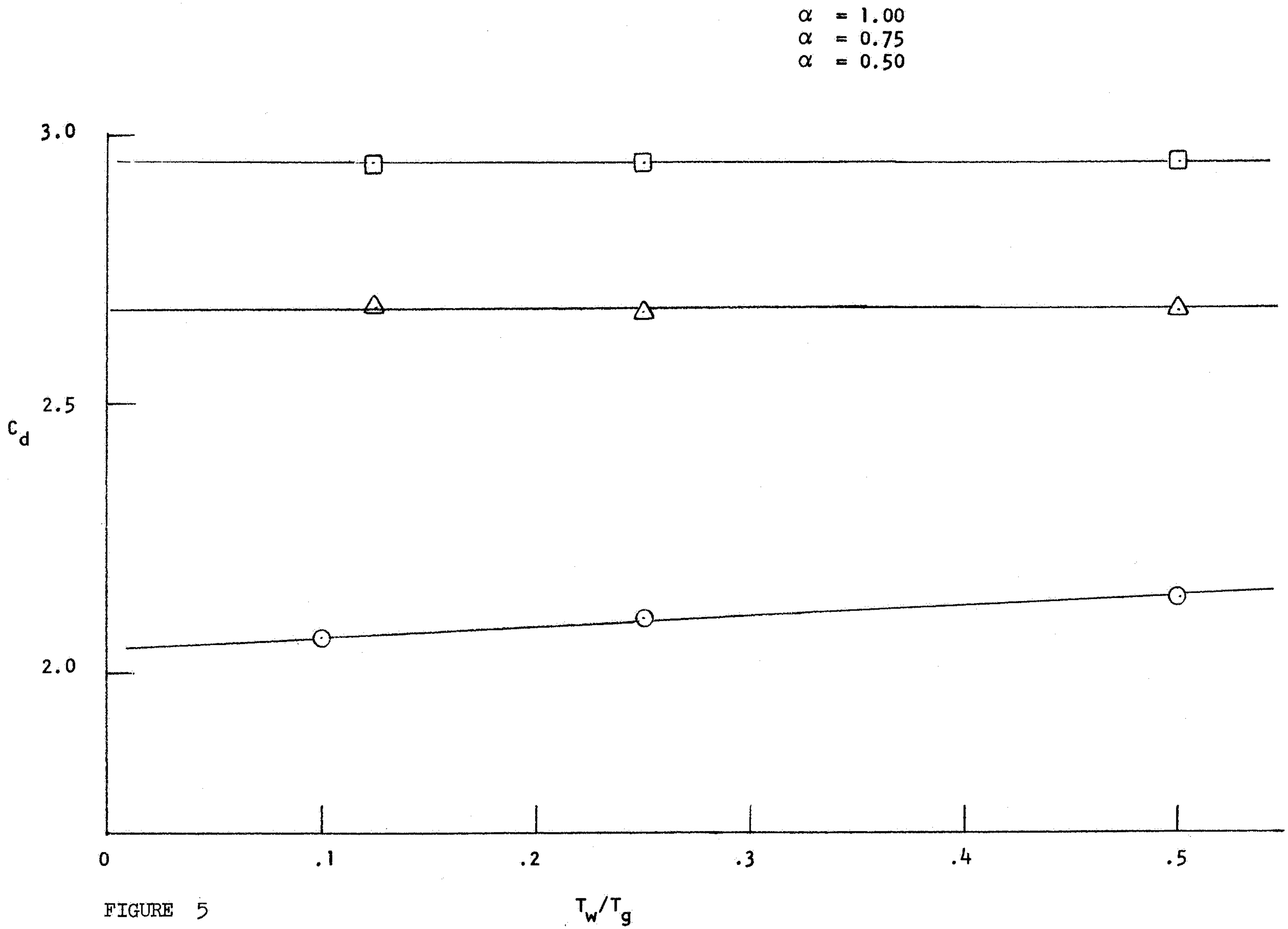

FIGURE 5

RATIO OF MOLECULAR FLUX ON TRAILING DISC TO MOLECULAR FLUX ON LEADING DISC FOR DIFFERENT SEPARATION DISTANCES X

$A_1/A_2 = 1.0 \quad S = 5.0$

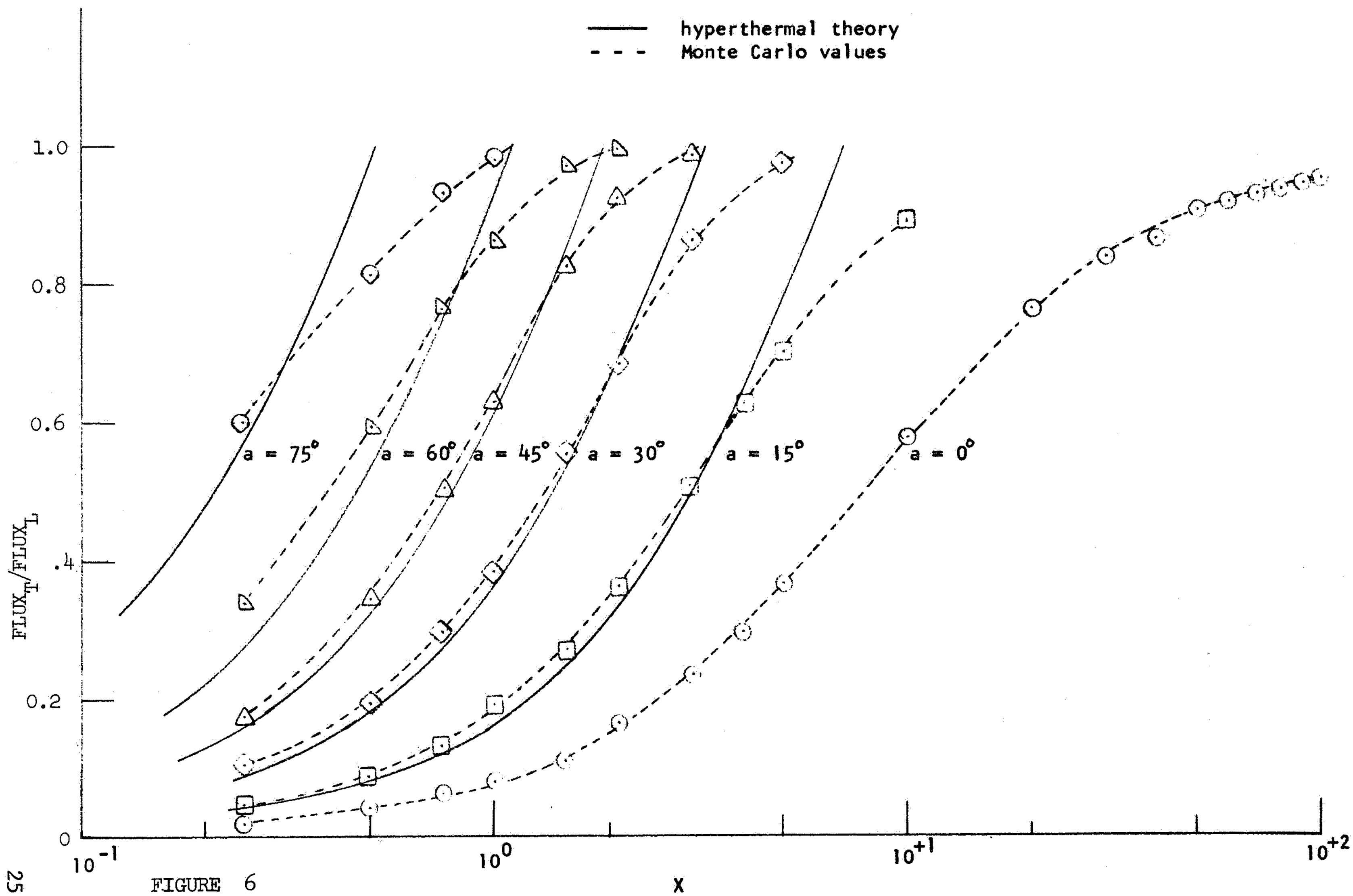

FIGURE 6

RATIO OF MOLECULAR FLUX ON TRAILING DISC TO MOLECULAR FLUX ON LEADING DISC FOR DIFFERENT SEPARATION DISTANCES X

$A_1/A_2 = 1.0$ $S = 10.0$

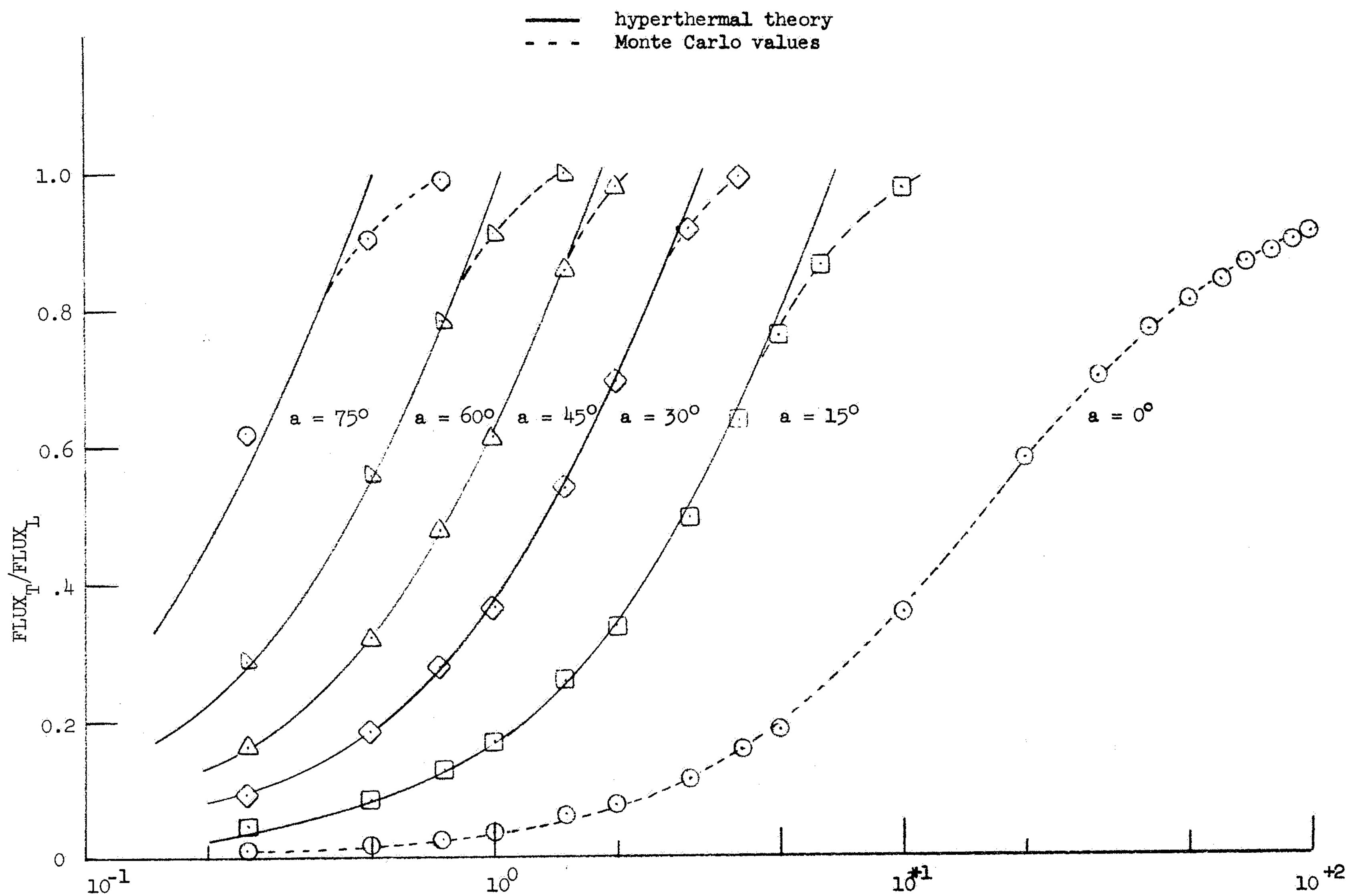

FIGURE 7

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 0.25$ $T_w/T_g = 0.5$ $\alpha = 0.5$

DIFFUSE REFLECTION

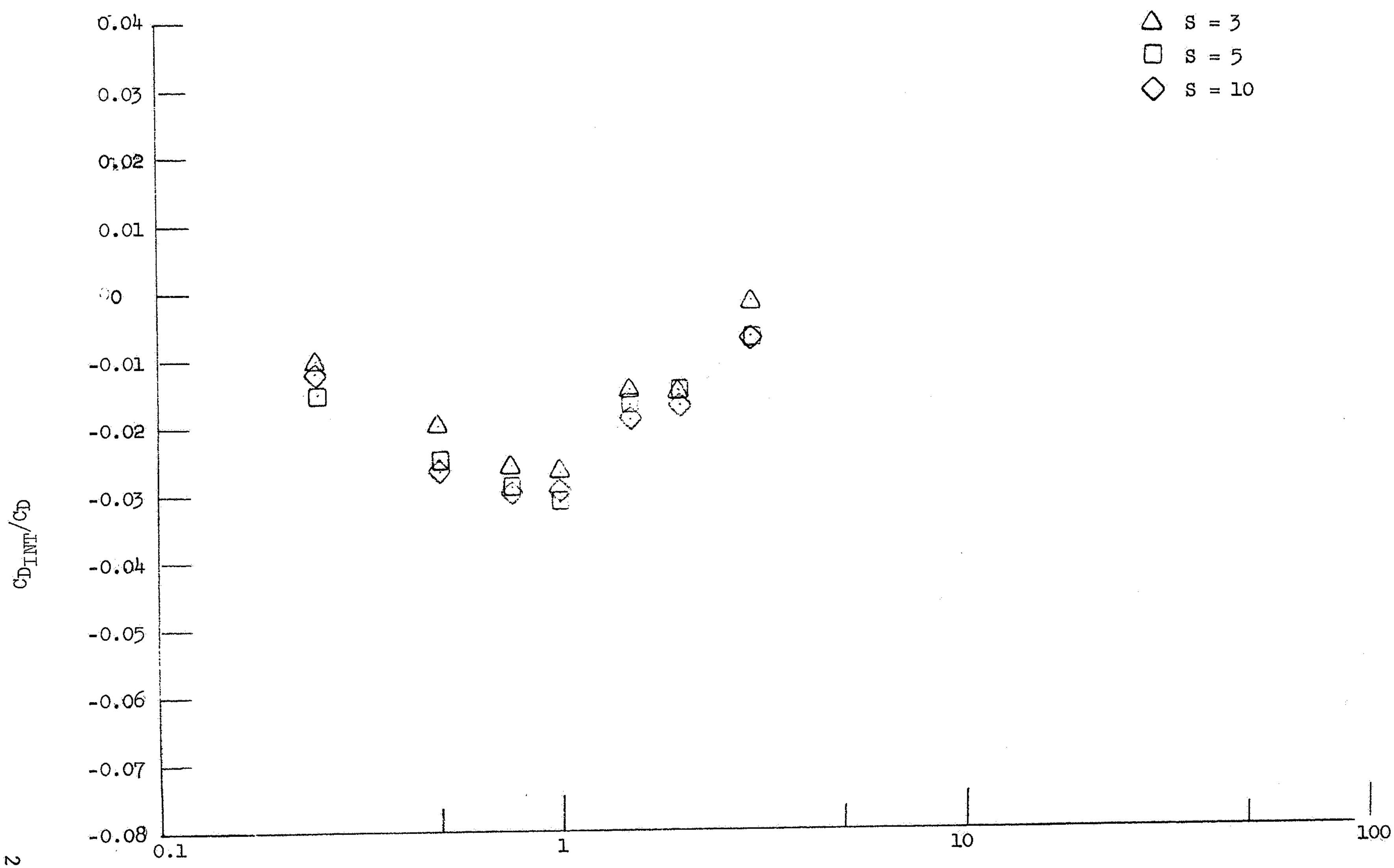

FIGURE 8

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 0.5$ $T_w/T_g = 0.5$ $\alpha = 0.5$

DIFFUSE REFLECTION

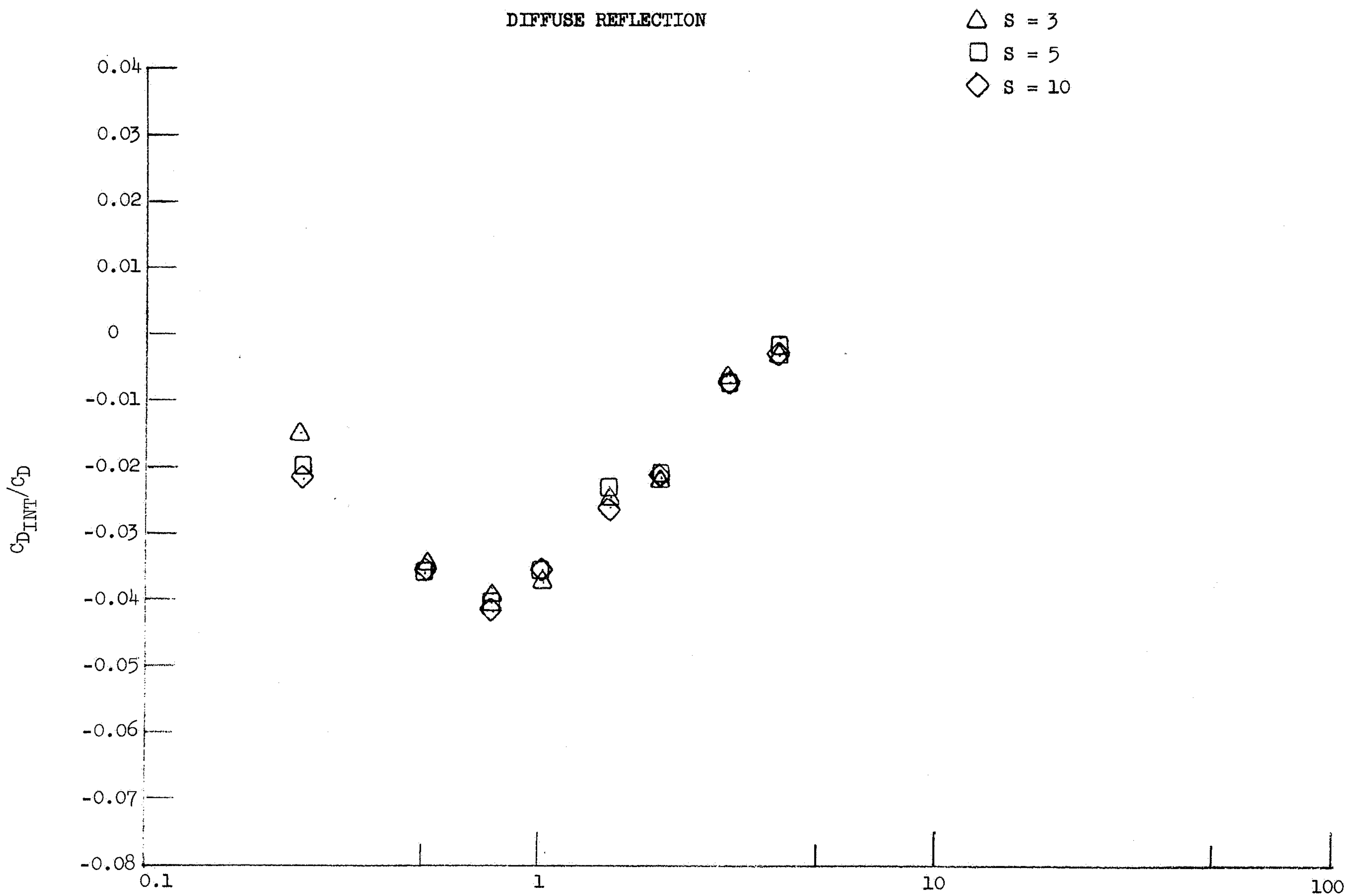

FIGURE 9

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 0.75$ $T_w/T_g = 0.5$ $\alpha = 0.5$

DIFFUSE REFLECTION

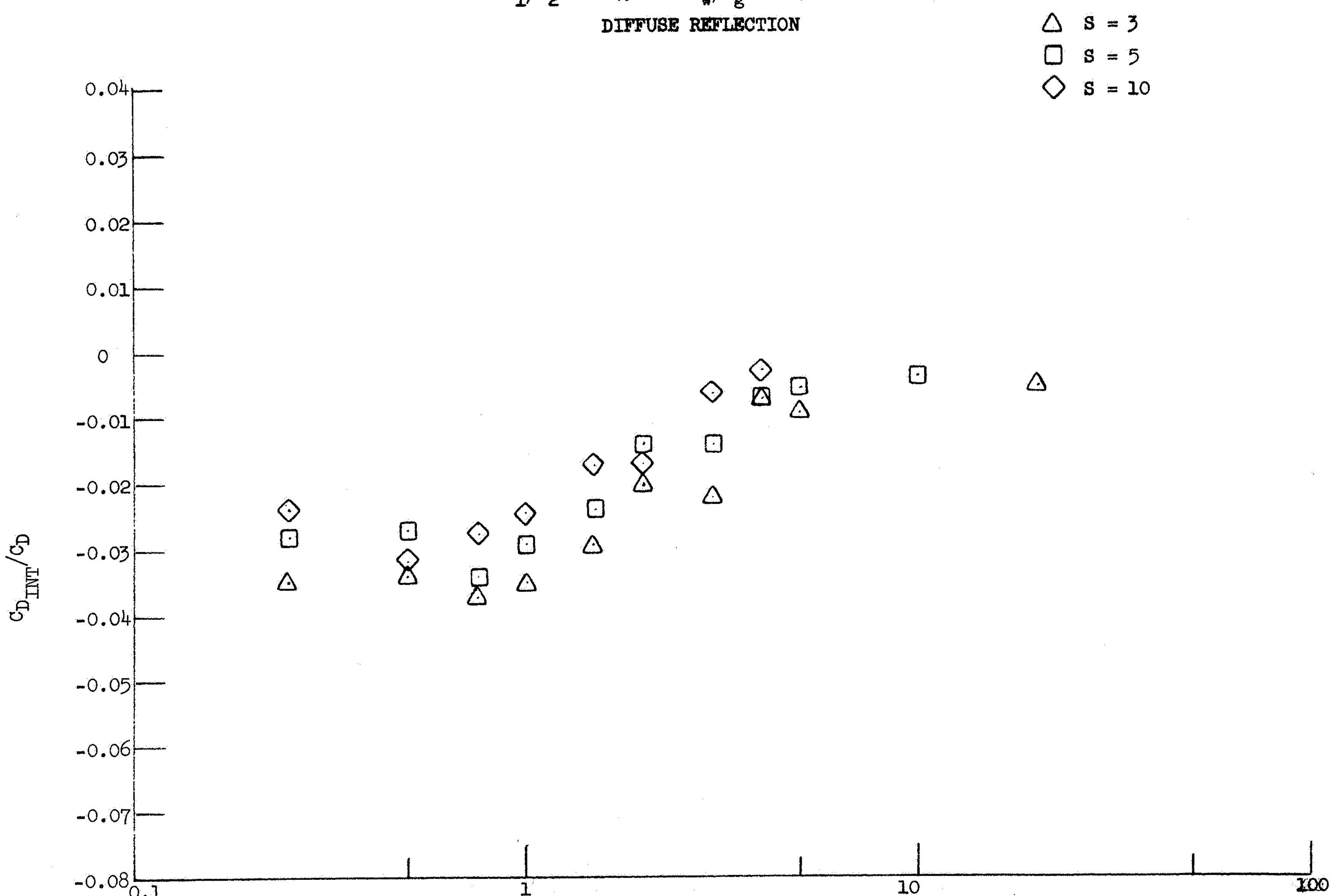

FIGURE 10

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 1.0$ $T_w/T_g = 0.5$ $\phi = 0.5$

DIFFUSE REFLECTION

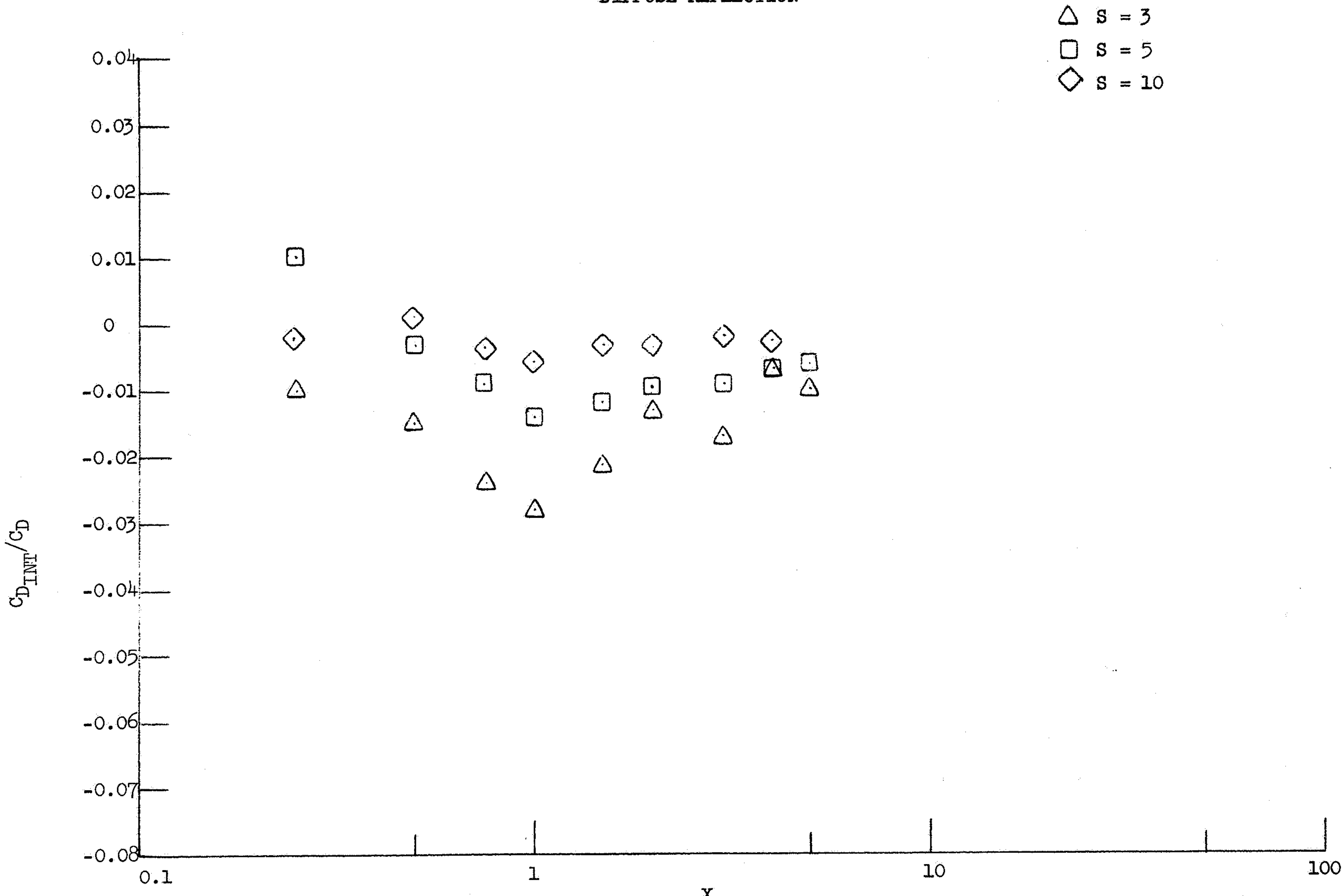

FIGURE 11

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 1.0$ $S = 10$ $T_w/T_g = 0.25$ $\alpha = 0.5$

DIFFUSE REFLECTION

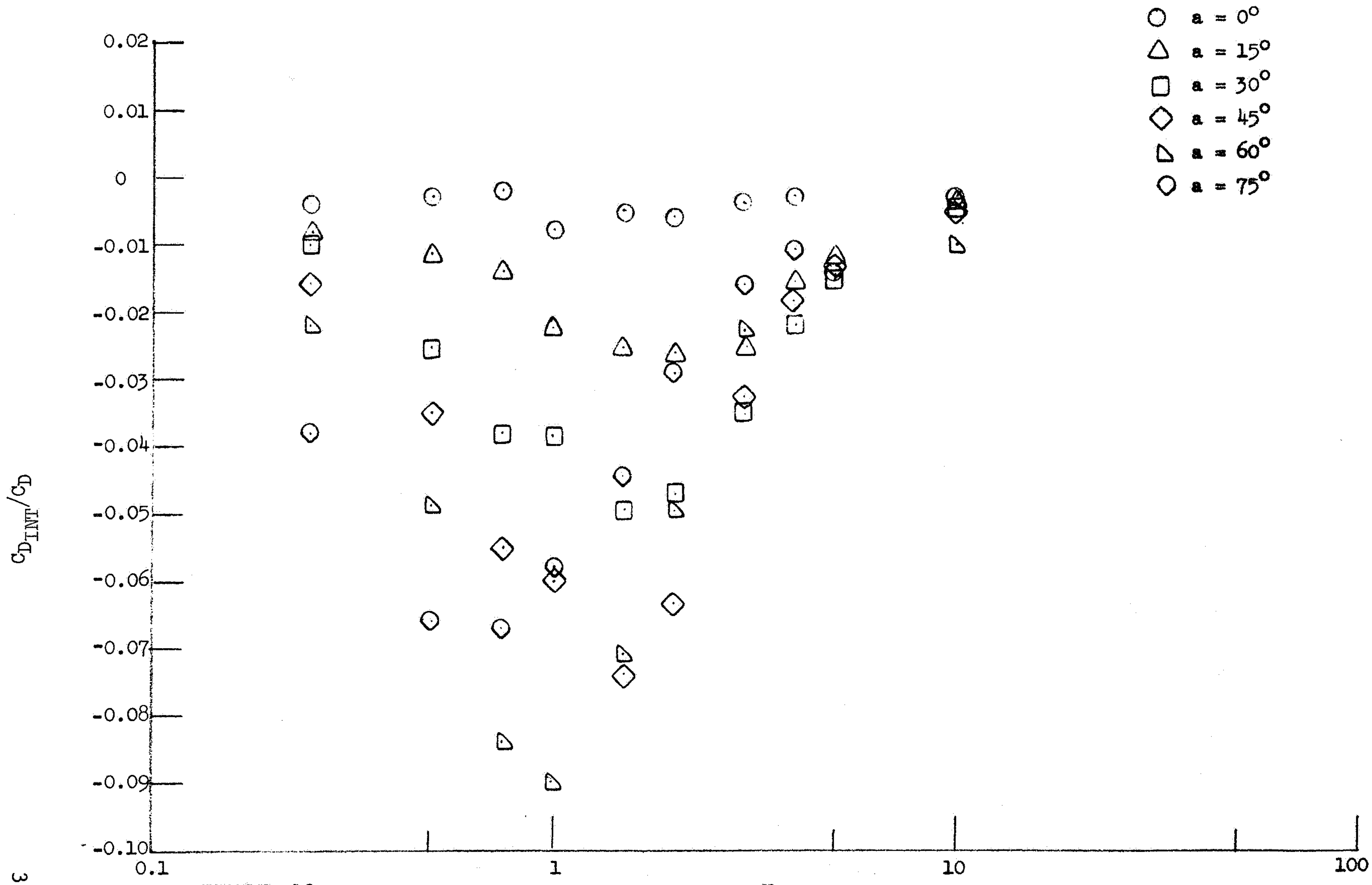

FIGURE 12

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 1.0$ $S = 10$ $T_w/T_g = 0.25$ $\alpha = 0.75$

DIFFUSE REFLECTION

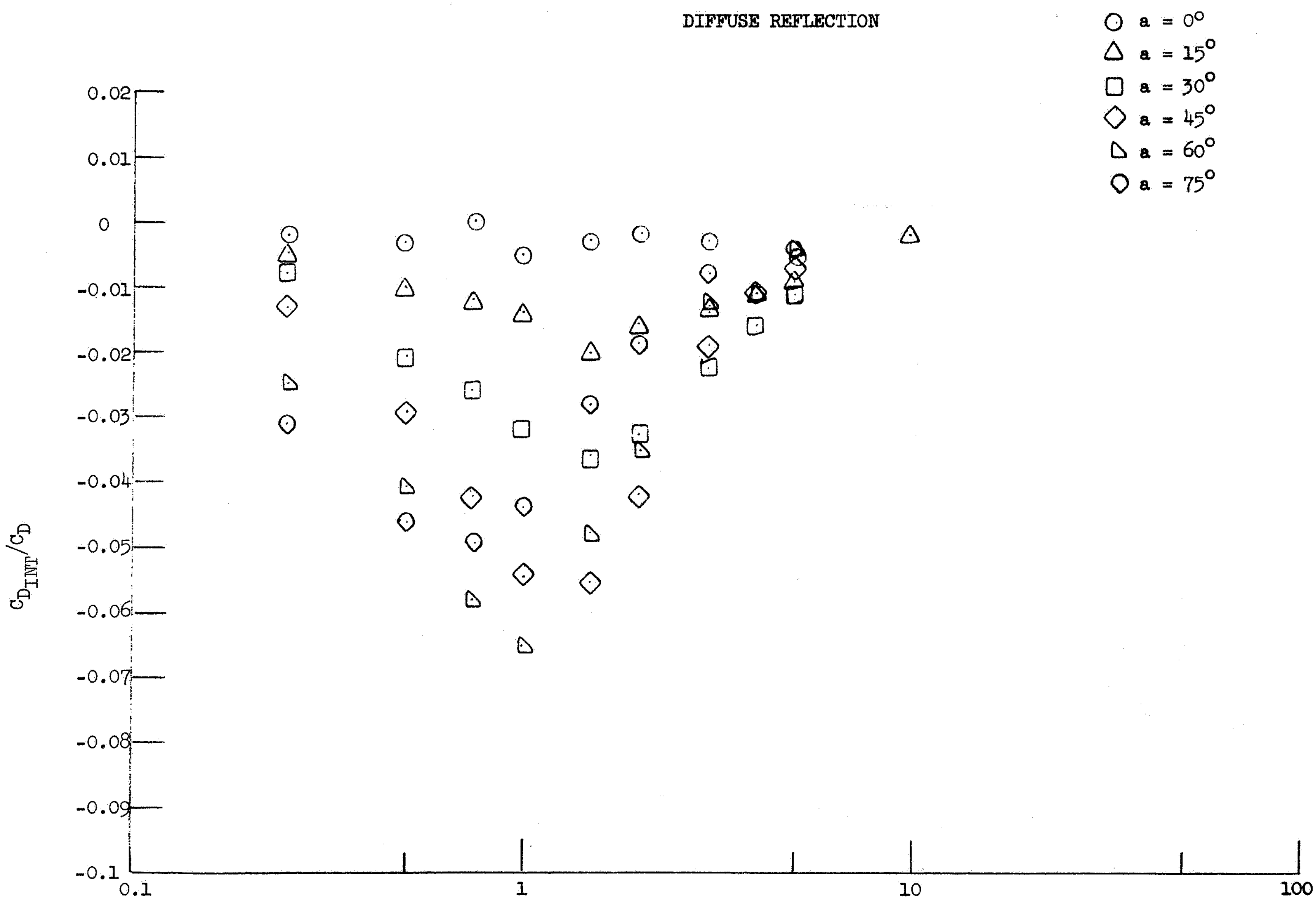

FIGURE 13

INTERACTION DRAG CONTRIBUTION AT VARIOUS SEPARATION DISTANCES

$A_1/A_2 = 1.0$ $S = 10$ $T_w/T_g = 0.25$ $\alpha = 1.0$

DIFFUSE REFLECTION

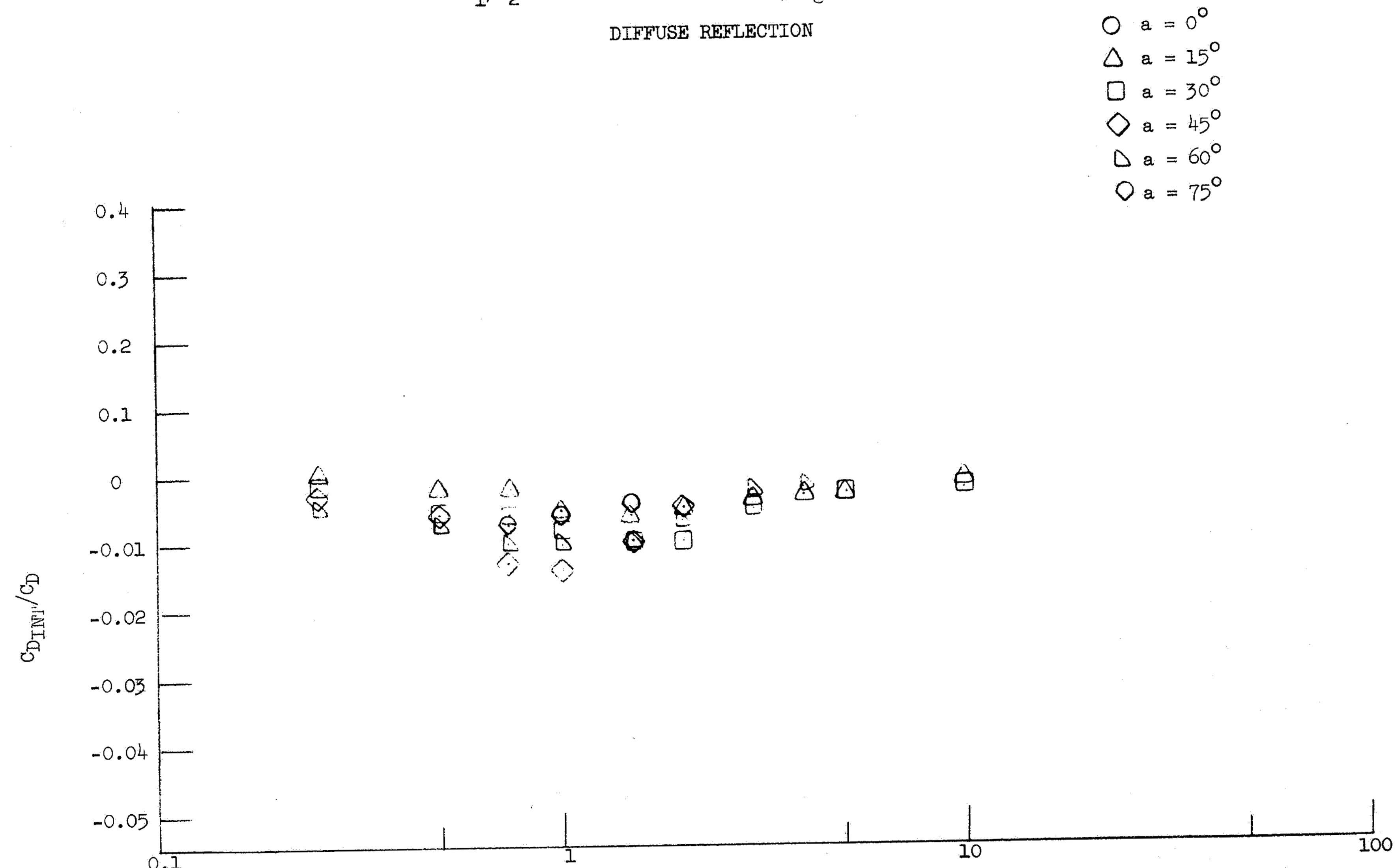

FIGURE 14

TOTAL DRAG COEFFICIENTS FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISC AT VARIOUS SEPARATION DISTANCES AS A FUNCTION OF THE NUMBER OF SPECULAR REFLECTIONS MADE BY THE MOLECULES

$A_1/A_2 = 1.0$ $\alpha = 0.75$ $T_w/T_g = 0.25$ $S = 10$ ANGLE OF ATTACK = 15°

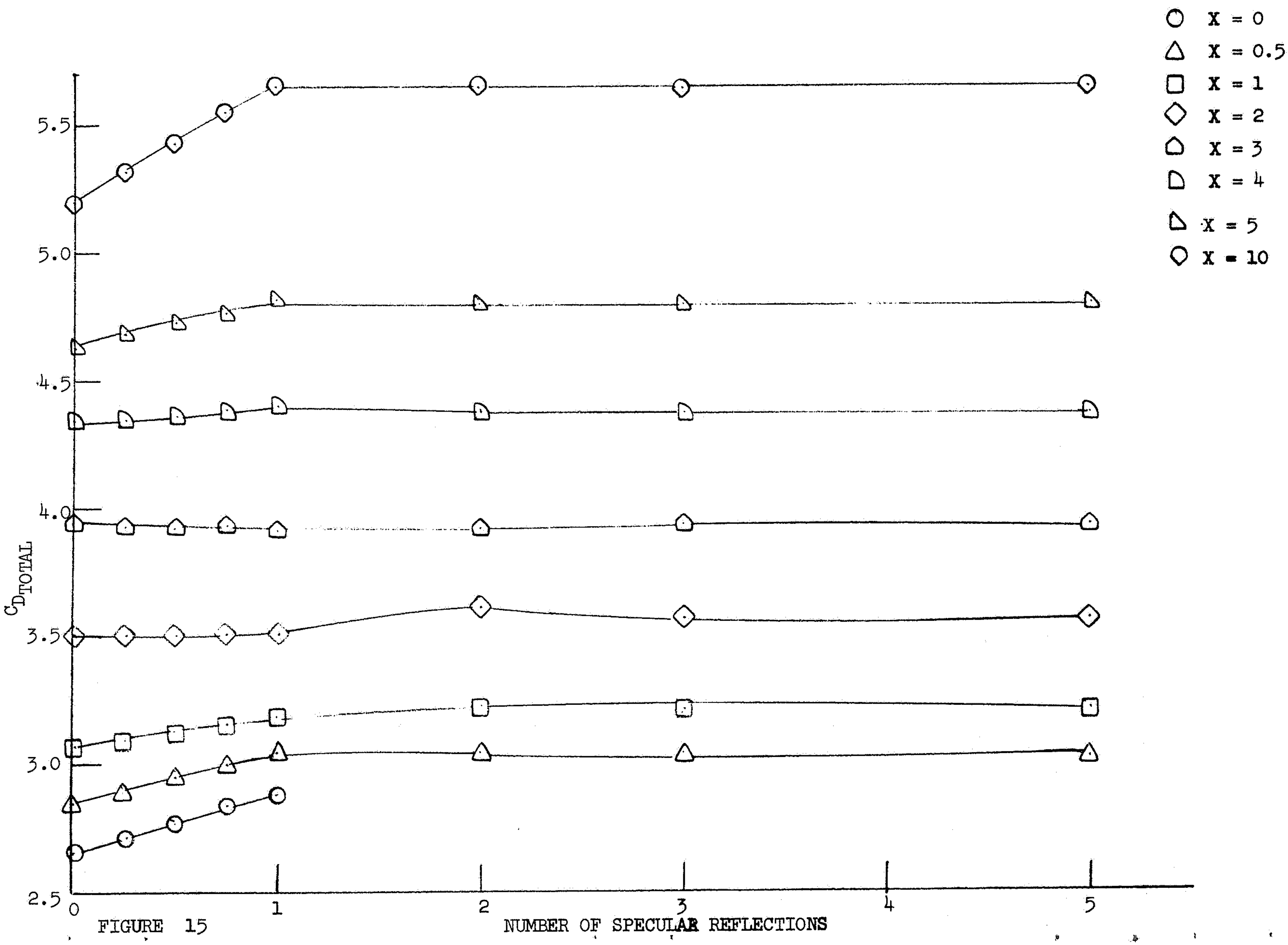

FIGURE 15

NORMALIZED DRAG COEFFICIENTS FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF THE SEPARATION DISTANCE

ZERO ANGLE OF ATTACK $A_1/A_2 = 0.25$ $\alpha = 0.5$ $T_w/T_g = 0.25$

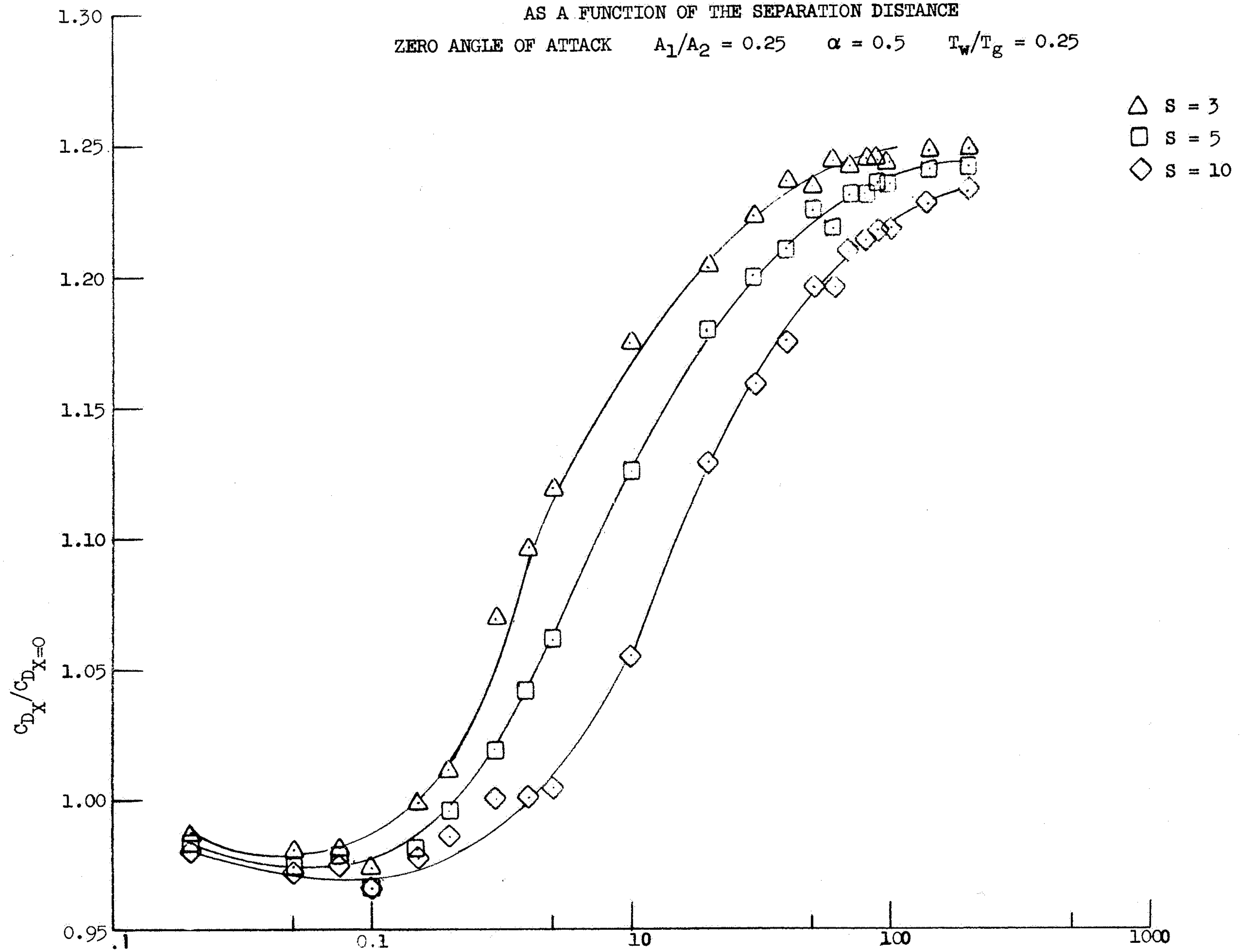

FIGURE 16

NORMALIZED DRAG COEFFICIENTS FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF THE SEPARATION DISTANCE

ZERO ANGLE OF ATTACK $A_1/A_2 = 0.5$ $\alpha = 0.5$ $T_w/T_g = 0.25$

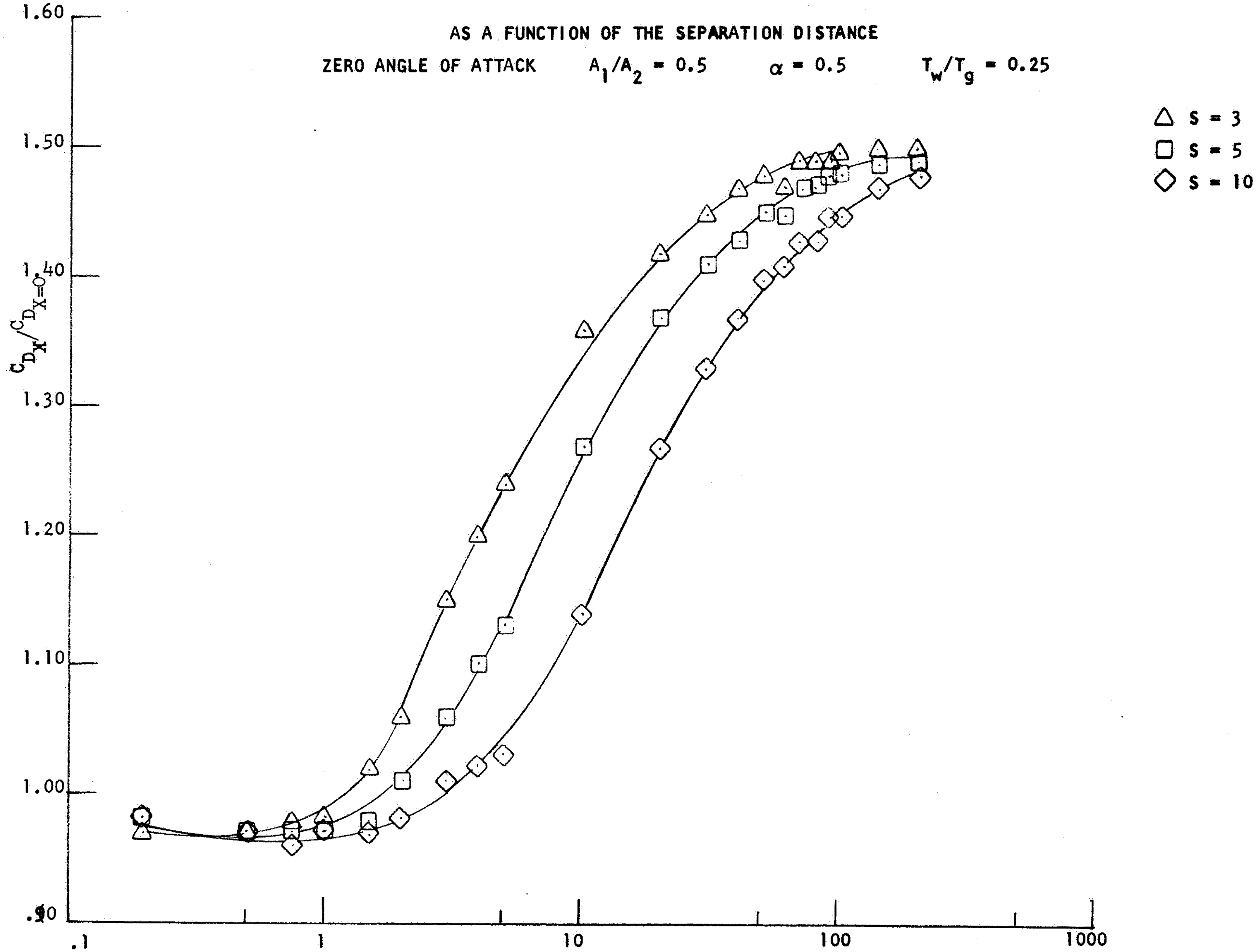

FIGURE 17

NORMALIZED DRAG COEFFICIENTS FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF THE SEPARATION DISTANCE

ZERO ANGLE OF ATTACK $A_1/A_2 = 0.75$ $\alpha = 0.5$ $T_w/T_g = 0.25$

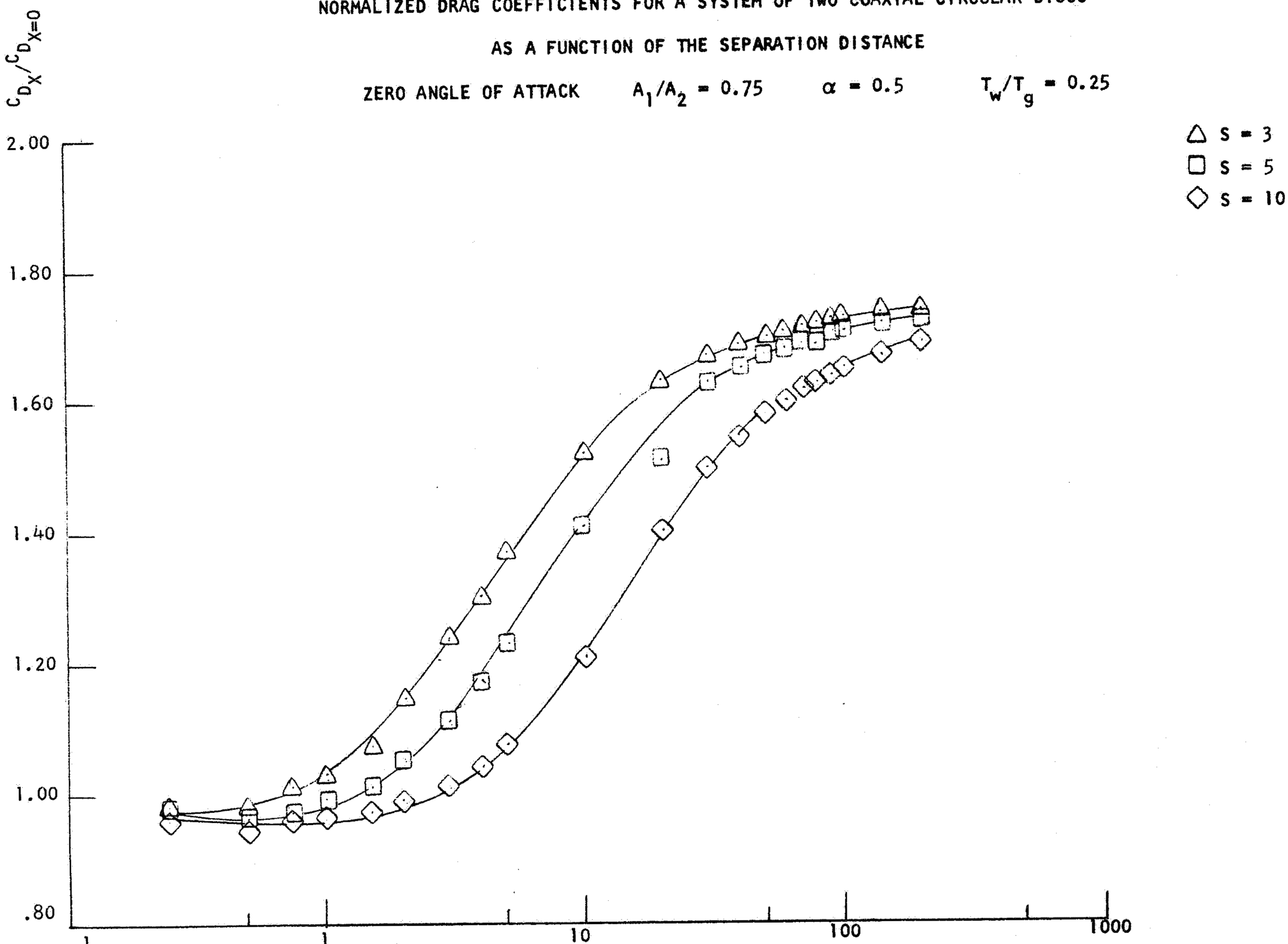

FIGURE 18

NORMALIZED DRAG COEFFICIENTS FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF THE SEPARATION DISTANCE

ZERO ANGLE OF ATTACK $A_1/A_2 = 1.0$ $\alpha = 0.5$ $T_w/T_g = 0.25$

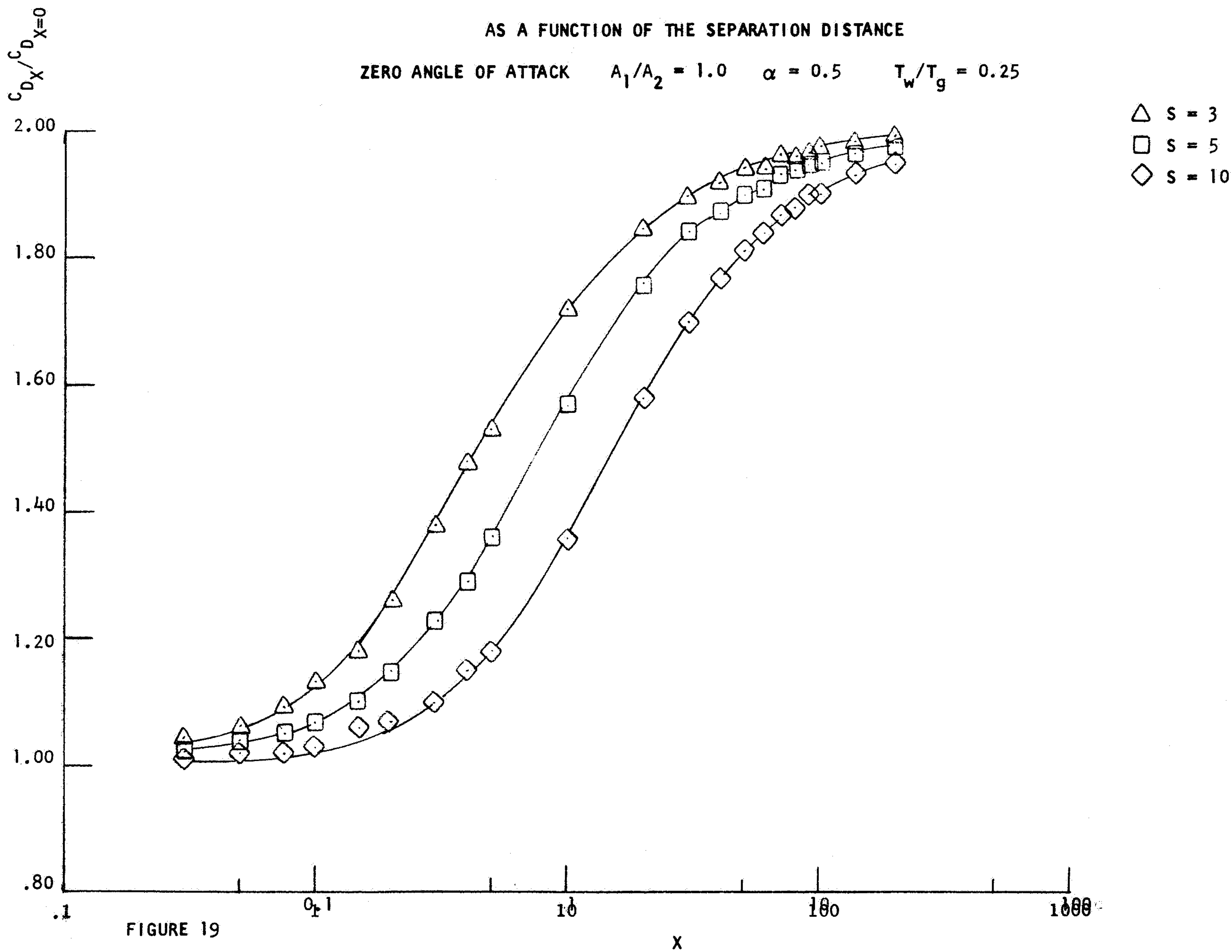

FIGURE 19

NORMALIZED DRAG COEFFICIENT FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF SEPARATION DISTANCE FOR VARIOUS ANGLES OF ATTACK

S = 10 T_w/T_g = 0.25 A_1/A_2 = 1.0   a = 0.5

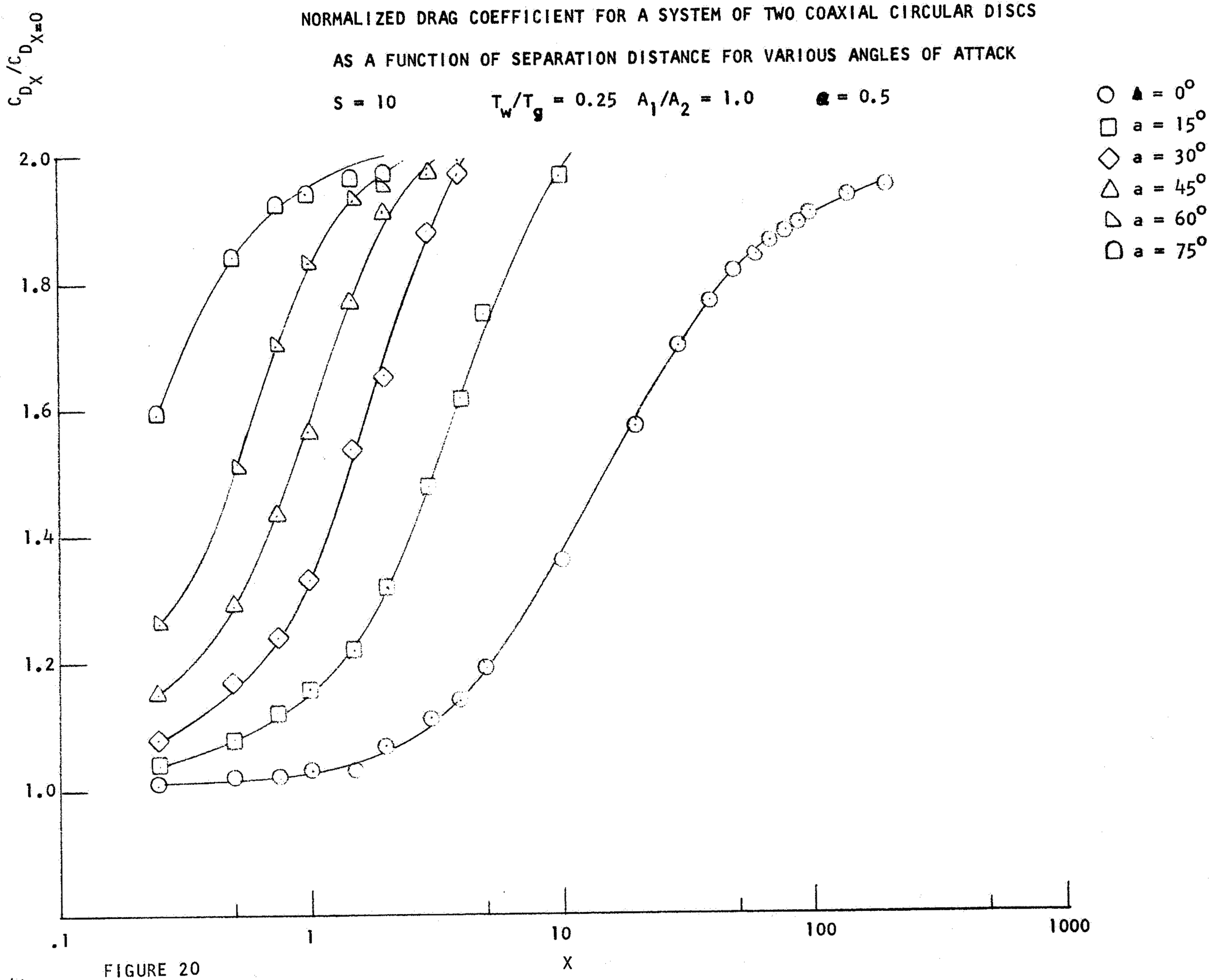

FIGURE 20

NORMALIZED DRAG COEFFICIENT FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF SEPARATION DISTANCE FOR VARIOUS ANGLES OF ATTACK

$S = 10$ $T_w/T_g = 0.25$ $A_1/A_2 = 1.0$ $\alpha = 0.75$

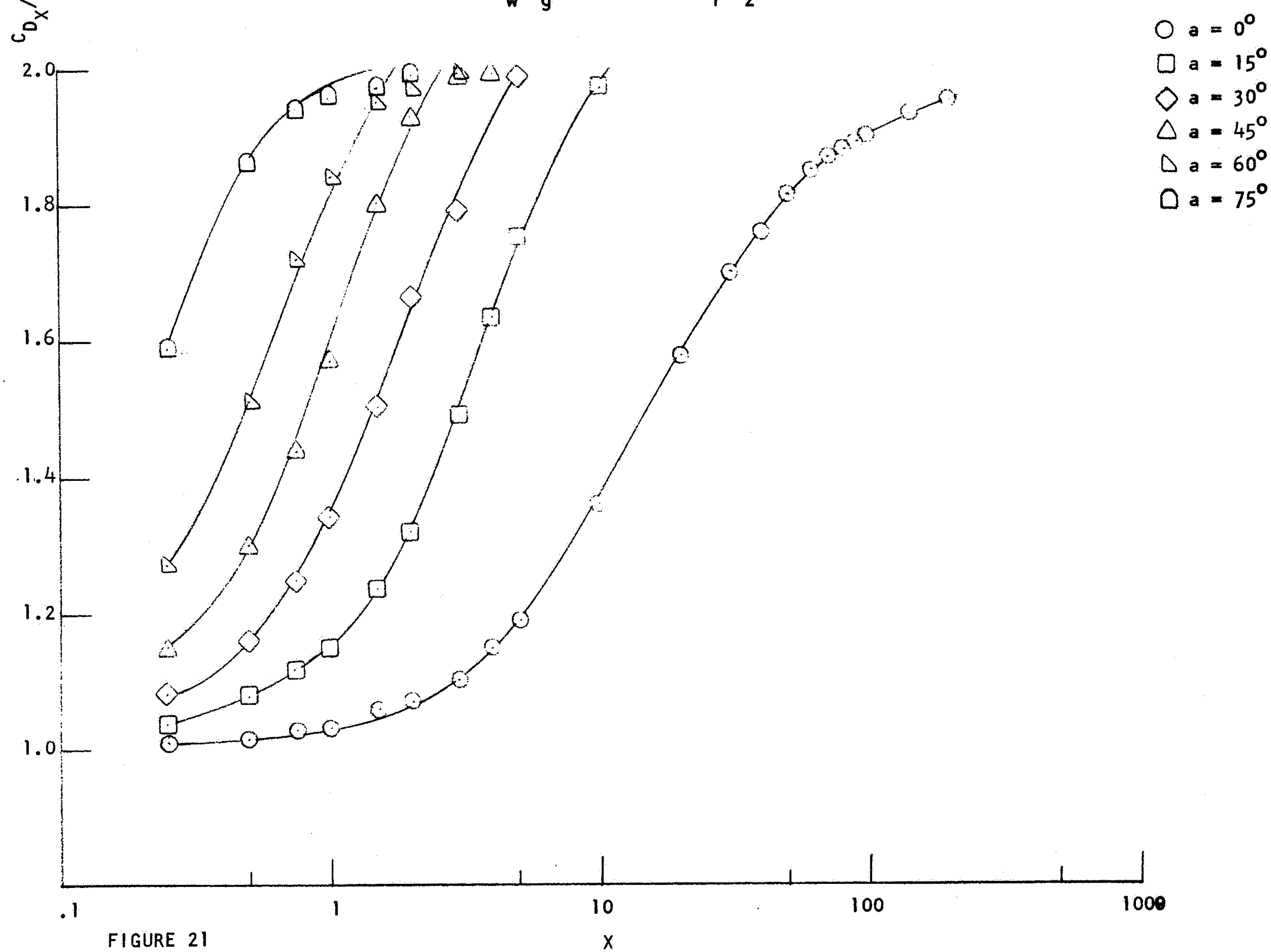

FIGURE 21

NORMALIZED DRAG COEFFICIENT FOR A SYSTEM OF TWO COAXIAL CIRCULAR DISCS

AS A FUNCTION OF SEPARATION DISTANCE FOR VARIOUS ANGLES OF ATTACK

$S = 10 \qquad T_w/T_g = 0.25 \qquad A_1/A_2 = 1.0 \qquad \alpha = 1.0$

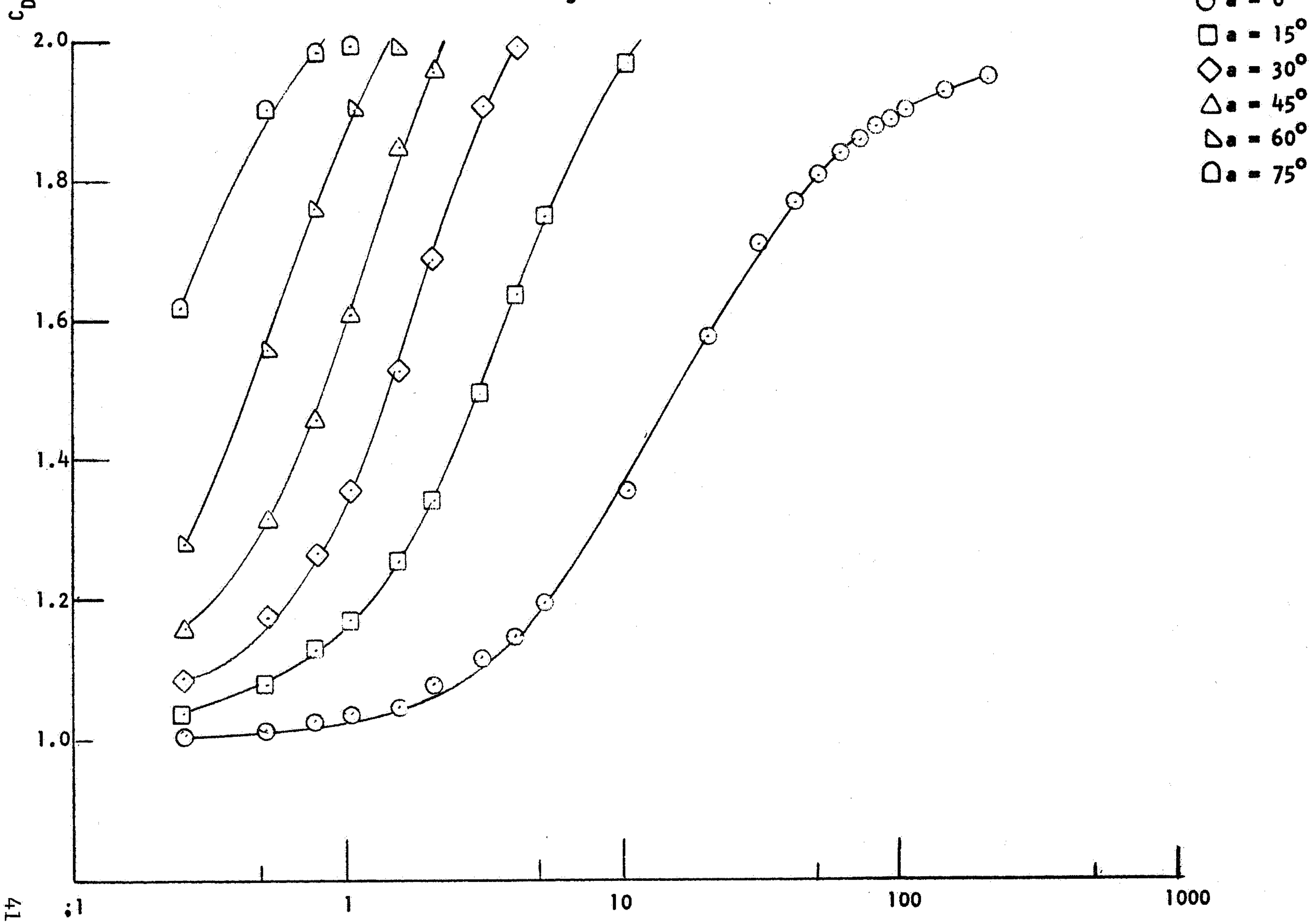

FIGURE 22

TOTAL MOLECULAR FLUX ON A SYSTEM OF TWO COAXIAL CIRCULAR DISCS AS A FUNCTION OF SEPARATION DISTANCE

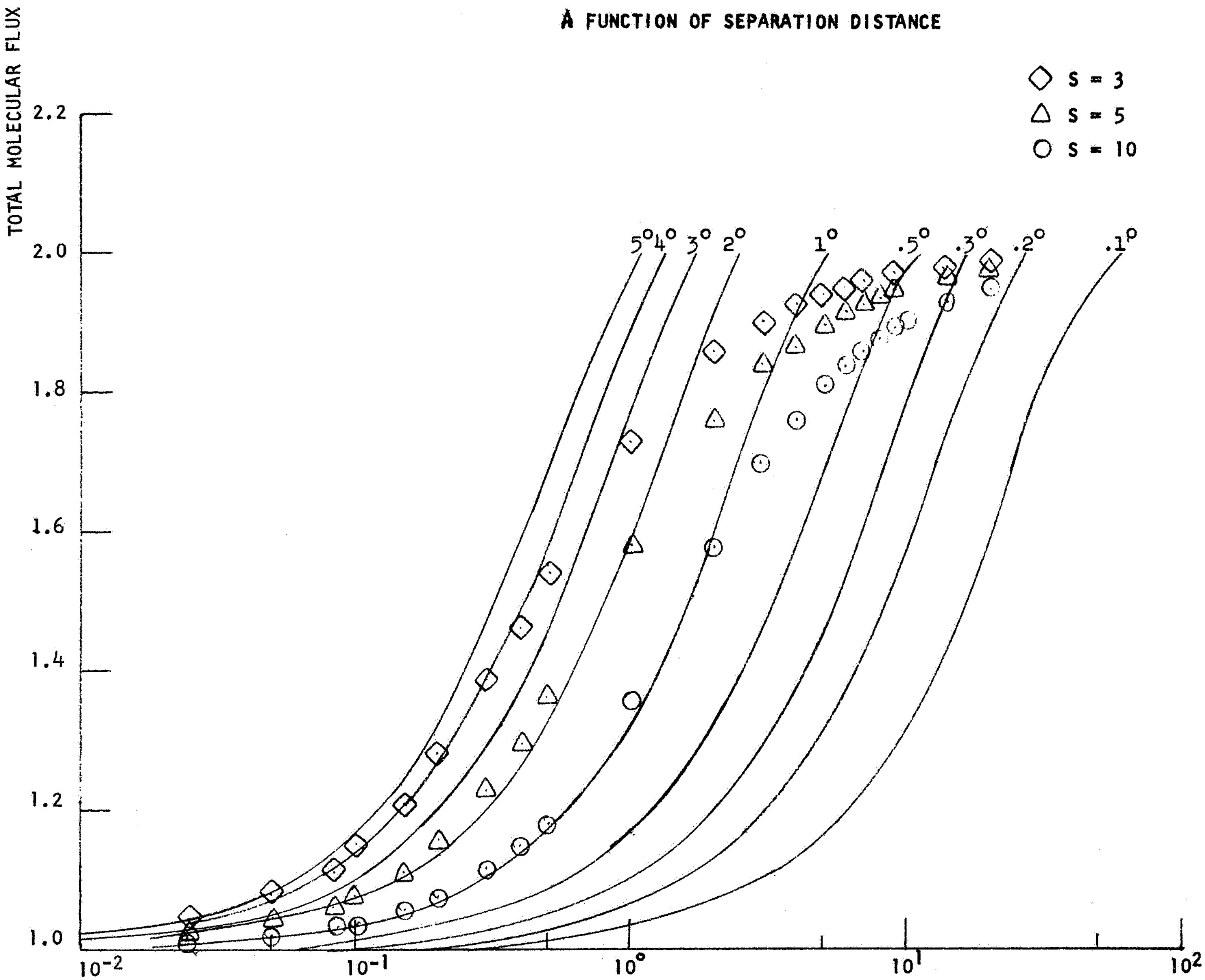

FIGURE 23

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WAKE AND INTERREFLECTION EFFECTS IN THE CALCULATION OF FREE MOLECULAR FLOW DRAG COEFFICIENTS

by James O. Ballance

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W. Dahm

W. K. Dahm
Chief, Aerophysics Division

E. D. Geissler
Director, Aero-Astrodynamics Laboratory

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