

AD 672792

AMPEX

FACILITY FORM 602

N 68 37198	
(ACCESSION NUMBER)	(THRU)
25	
(PAGES)	(CODE)
CR-97260	12
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

ON FLUID INERTIA EFFECTS IN  
INFINITELY WIDE FOIL BEARINGS

RR 68-15

July 15, 1968

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) \_\_\_\_\_

Microfiche (MF) \_\_\_\_\_

ff 653 July 65

This document has been approved  
for public release and sale; its  
distribution is unlimited

Ampex Corporation  
Research and Advanced Technology Division

Prepared under

Contract No. Nonr-3815(00)(X)  
Task NR 062-297

Supported jointly by

Department of Defense  
Atomic Energy Commission  
National Aeronautics and  
Space Administration

Administered by

S. Doroff  
Fluid Dynamics Branch  
Office of Naval Research  
Department of the Navy

ON FLUID INERTIA EFFECTS IN INFINITELY  
WIDE FOIL BEARINGS

Prepared by: \_\_\_\_\_

*A. Eshel*  
A. Eshel

Member of the Research Staff

Approved by: \_\_\_\_\_

*M. Wildmann*  
M. Wildmann

Manager, Mechanics Section

Approved by: \_\_\_\_\_

*W. A. Gross*  
W. A. Gross

Vice President, Research &  
Advanced Technology,  
Director of Research

REPRODUCTION IN PART OR IN WHOLE IS PERMITTED FOR ANY PUPOSE  
OF THE UNITED STATES GOVERNMENT.

**Ampex Corporation**

**Research and Advanced Technology Division**





5  
PRECEDING PAGE BLANK NOT FILMED

#### ACKNOWLEDGEMENTS

This work was sponsored by the Office of Naval Research,  
Fluid Dynamics Branch, under Contract Nonr 3815(00)(X) administered by  
Mr. S. Doroff.

Thanks are due to Miss T. Woo for assistance in programming.

### ABSTRACT

Equations for a foil over a lubricating film in which the effects of fluid inertia are taken into account are derived. Approximate solutions showing the effect of inertia and fluid compressibility are obtained. The effect of inertia is to increase the fluid film thickness.

## NOMENCLATURE

C	Compresibility Parameter $\frac{p_a r_o}{T}$
C*	Normalized Compressibility Parameter $(1 + C)H^*$
f	Shape Factor for Velocity Profile
h	Radial Clearance
H	Dimensionless Clearance = $\frac{h}{r_o} \epsilon^{-2/3}$
$\bar{H}$	Normalized Clearance = $H/H^*$
I	Inertia Parameter $1/2 \rho_a U^2 / (T/U_o)$
I*	Normalized Inertia Parameter $IH^*$
k, m, n	Exponential Measures
L	Distance of Guide from Point of Tangency
p	Pressure under the Foil
$p_a$	Atmospheric Pressure
r	Polar Angle
$r_o$	Spindle Radius
R	Local Radius of Curvature
t	Temperature
$t_a$	Ambient Temperature
T	Tension per Unit Width of Foil
$v_e$	Velocity Component in the Angular Direction
$v_r$	Velocity Component in the Radial Direction
$\hat{v}$	Dimensionless Radial Component of Velocity

## NOMENCLATURE (Cont)

$\hat{u}$	Dimensionless Angular Component of Velocity
$U$	Speed of Foil or Shaft
$\epsilon$	Dimensionless Parameter $\frac{6\mu U}{T}$
$\phi$	Small Perturbation of Clearance
$\mu$	Air Viscosity
$\rho$	Air Density at $(p, t_a)$
$\rho_a$	Air Density at Ambient Conditions $(p_a, t_a)$
$\pi$	Dimensionless Pressure = $(p - p_a)/(T/r_o)$
$\theta$	Polar Angle
$\Theta$	Wrap Angle
$\xi$	Dimensionless Polar Angle
$\bar{\xi}$	Normalized Polar Angle
$\eta$	Dimensionless Radial Distance from Spindle



CONTENTS

	Page
1.0 INTRODUCTION	1
2.0 ANALYSIS	1
3.0 ASYMPTOTIC BEHAVIOR	10
REFERENCES	17

## LIST OF FIGURES

1. Schematic View of Problem under Consideration
2. Roots of Characteristic Equation
3. Bounds of Uncertainty in the Evaluation of  $H^*$  ( $C^* = \infty$ )
4. Nominal Clearance  $H^*$  as a Function of Compressibility (C) and Inertia (I) Parameters
5. Nominal Clearance  $H^*$  as a Function of Compressibility (C) and the Normalized Inertia ( $I^*$ ) Parameters

## ON FLUID INERTIA EFFECTS IN INFINITELY WIDE FOIL BEARINGS

### 1.0 INTRODUCTION

Recent applications of foil bearings (Fig. 1) in high speed tape transports and in high speed rotor supports necessitate the study of fluid inertia effects. This report is devoted to an approximate treatment of these effects.

### 2.0 ANALYSIS

We start with the steady, two-dimensional, compressible Navier-Stokes equations in polar coordinates:

$$\begin{aligned} \rho \left[ v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} \right] = & - \frac{\partial p}{\partial r} + \mu \left[ 2 \frac{\partial^2 v_r}{\partial r^2} - \frac{2}{3} \left\{ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial r v_r}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right) \right\} + \right. \\ & \left. + \frac{1}{r} \frac{\partial}{\partial \theta} \left\{ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right\} + \frac{2}{r} \left( \frac{\partial v_r}{\partial r} - \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r} \right) \right] \quad (1) \end{aligned}$$

$$\begin{aligned} \rho \left[ v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} \right] = & - \frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left[ \frac{2}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right) - \frac{2}{3r} \frac{\partial}{\partial \theta} \left\{ \frac{1}{r} \frac{\partial r v_r}{\partial r} + \right. \right. \\ & \left. \left. + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right\} + \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right\} + \frac{2}{r} \left\{ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right\} \right] \quad (2) \end{aligned}$$

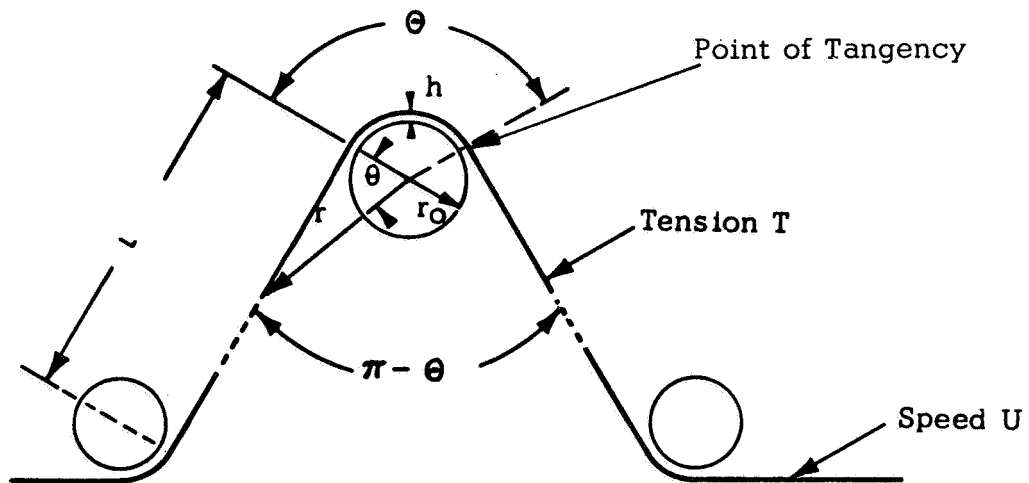


Fig. 1 Schematic View of Problem under Consideration

The continuity equation is

$$\frac{\partial(\rho v_r r)}{\partial r} + \frac{\partial(\rho v_\theta)}{\partial \theta} = 0 \quad (3)$$

The perfectly flexible foil equation is

$$p - p_a = \frac{T}{R} \quad (4)$$

The isothermal equation of state is

$$\frac{p}{p_a} = \frac{p}{p_a} \quad (5)$$

The curvature in polar coordinates is

$$\frac{1}{R} = \frac{2\left(\frac{dr}{d\theta}\right)^2 - r \frac{d^2 r}{d\theta^2} + r^2}{\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]^{3/2}} \quad (6)$$

and the clearance

$$h = r - r_0 \quad (7)$$

In previous analyses of foil bearings [ 1], a small parameter expansion was used to formally discard terms of higher order. In the following, the same expansion is used for the Navier-Stokes equations. This will yield a consistent set of equations, including the effect of inertia. This reduction may not be correct in the large gap inlet and exit regions of the flow since the expansion is incomplete in terms of boundary conditions. It is expected, however, that for small inertia parameters, the effect of the boundary conditions will not penetrate into the small gap regions. Due to this rather heuristic character of the derivation, the results must be viewed as estimates only.

We introduce the dimensionless parameters

$$E = \frac{6\mu U}{T} \quad \text{foil bearing number} \quad (8)$$

$$C = \frac{p_a}{T/r_o} \quad \text{compressibility parameter} \quad (9)$$

$$I = \frac{\frac{1}{2} p_a U^2}{T/r_o} \quad \text{inertia parameter} \quad (10)$$

and the dimensionless variables

$$\hat{\psi} = \frac{v_r}{U} E^{-k} \quad (11)$$

$$\hat{u} = \frac{U_0}{U} \quad (12)$$

$$\pi = \frac{p - p_a}{T/r_0} \quad (13)$$

$$\xi = \theta e^{-m} \quad (14)$$

$$\eta = \frac{y}{r_0} e^{-n} \quad (15)$$

$$H = \frac{h}{r_0} e^{-n} \quad (16)$$

where  $k$ ,  $m$ ,  $n$  are constants to be determined.

In accordance with conventional foil bearing theory, the following requirements are imposed on the equations for the purpose of the determination of  $k$ ,  $m$ ,  $n$ : (a) The expansion of equation (6) has to retain the capability of satisfying the condition that the tape becomes flat at infinity. (b) The purpose of this work is to study cases in which inertia effects are significant. It will be required, therefore, that at least some inertia term will be of the order of some viscous term. (c) Finally, it is required that the lateral and longitudinal terms in the continuity equations will be of the same order. These three requirements result in the conditions

$$n = 2/3 \quad ; \quad m = 1/3 \quad ; \quad k = 1/3 \quad (17)$$

With these results, the equations can be shown to degenerate, to the zeroth approximation, into Prandtl's boundary layer equations.

$$I \left( 1 + \frac{\pi}{c} \right) \left[ \hat{u} \frac{\partial \hat{u}}{\partial \xi} + \hat{v} \frac{\partial \hat{u}}{\partial \eta} \right] = - \frac{1}{2} \frac{d\pi}{d\xi} + \frac{1}{12} \frac{\partial^2 \hat{u}}{\partial \eta^2} \quad (18)$$

$$\frac{\partial}{\partial \eta} \{ (c + \pi) \hat{v} \} + \frac{\partial}{\partial \xi} \{ (c + \pi) \hat{u} \} = 0 \quad (19)$$

$$\pi = 1 - H'' \quad (20)$$

We assume that the velocity profile is parabolic. The profile which satisfies the boundary conditions

$$\hat{u} = 1 \quad \eta = 0 \quad (21)$$

$$\hat{u} = 0 \quad \eta = H \quad (22)$$

is

$$\hat{u} = -f \left( \frac{\eta}{H} \right)^2 + (f-1) \left( \frac{\eta}{H} \right) + 1 \quad (23)$$



where  $f$  is to be determined.

The three unknown functions  $\pi$ ,  $H$ ,  $f$  will be determined from the momentum and continuity integrals between  $\eta = 0$  and  $\eta = H$  and from the elastic equation.

The momentum integral becomes

$$I \left\{ \left( f + \frac{5}{2} \right) \frac{df}{d\xi} - \frac{1}{(\pi+c)H} \frac{d[(\pi+c)H]}{d\xi} \left( \frac{f-5}{2} \right) \right\} = -15 \frac{d\pi}{d\xi} + \frac{5f}{H^2} \quad (24)$$

The continuity integral is

$$\frac{d}{d\xi} \left[ (\pi+c) H \left( \frac{1}{2} + \frac{f}{6} \right) \right] = 0 \quad (25)$$

Equations (20), (24), and (25) constitute the approximate formulation of the problem. The first integral of Eq. (25) is

$$(\pi+c) H \left( 1 + \frac{f}{3} \right) = (1+c) H^* \quad (26)$$

where  $(1+c) H^*$  denotes the integration constant. The physical interpretation of  $H^*$  is the value of  $H$  in the central or uniformity region of the bearing. It is also a measure of the flow rate through the bearing.

From previous analyses [ 1], the boundary conditions representing the straight asymptotic behavior of the foil far from the spindle are:

$$H \sim \xi^2/2 + \dots \quad (27)$$

$$H' \sim \xi + \dots \quad (28)$$

as  $\xi \rightarrow \pm \infty$

The formulation is, thus, a function of three parameters: I, C, H\*. It is convenient to eliminate H\* from Eq. (26) For this purpose, the transformation

$$\frac{H}{H^*} = \bar{H} \quad ; \quad \frac{\xi}{H^*} = \bar{\xi} \quad ; \quad (1+C)H^* = C^* \quad ; \quad IH^* = I^* \quad (29)$$

is used. The equations become

$$f = \frac{3C^*}{(C^* - \bar{H}'')\bar{H}} - 3 \quad (30)$$

$$\bar{H}''' = \frac{\frac{5f}{\bar{H}^2} - I^* \frac{f^2 + 6f + 5}{\bar{H}} \bar{H}'}{15 - I^* \frac{(f^2 + 6f + 5)(f + 3)}{3C^*}} \quad (31)$$

with

$$\begin{aligned} \bar{H}' &\sim H^* \bar{S} \\ \bar{H}'' &\sim H^* \end{aligned} \quad (32)$$

as  $\bar{S} \rightarrow \infty$

Following the procedure of [1], the solution is started from the central region towards the exit with a matched  $H^*$  at the central region. This is done by a linearized approach

$$\begin{aligned} \bar{H} &= 1 + \varphi \\ \bar{H}' &= \varphi' \\ \bar{H}'' &= \varphi'' \end{aligned} \quad (33)$$

where

$$\varphi \ll 1$$

The linearized continuity and momentum equations become

$$f = 3 \left( \frac{\varphi''}{c^*} - \varphi \right) \quad (34)$$

$$\varphi''' \left( 1 - \frac{I^*}{3c^*} \right) - \frac{\varphi''}{c^*} + \frac{I^*}{3} \varphi' + \varphi = 0 \quad (35)$$

The roots of the characteristic equation are graphically shown in Fig. 2.

### 3. ASYMPTOTIC BEHAVIOR

It can be observed that as

$$\begin{aligned} \bar{H} &\rightarrow \infty & f &\rightarrow -3 \\ \bar{H}''' &\sim -\frac{4}{15} \frac{H'}{H} = -\frac{4}{15} \frac{d(\ln \bar{H})}{d\xi} \end{aligned} \quad (36)$$

Thus, the boundary conditions cannot be satisfied since  $\bar{H}''$  does not approach a constant as required by Eq. (32). It must be observed, however, that in the region of interest  $\bar{H}'''$  is fairly small.  $\bar{H}$  is, therefore, nearly parabolic, until finally the logarithmic contribution becomes overwhelming. Thus, one may consider that the boundary condition of asymptotically straight tape is approximately satisfied. \*

---

\*This interpretation of the results is due to Prof. H. G. Elrod Jr.

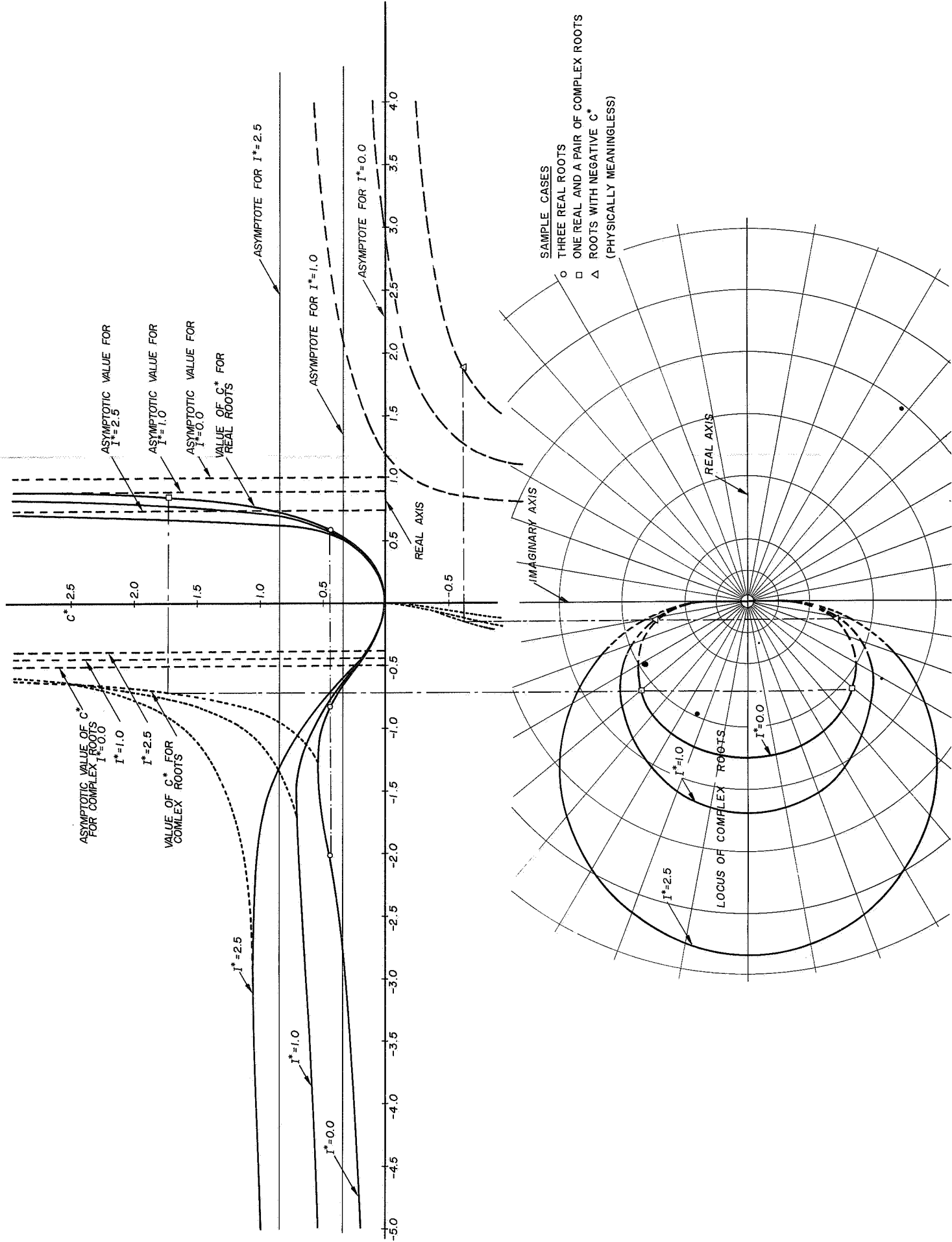


Fig. 2 Roots of Characteristic Equation

Due to the imperfect satisfaction of the boundary conditions, a degree of uncertainty is involved which increases with the inertia parameter  $I$  and vanishes for  $I \rightarrow 0$ . In the actual computation work the point at which the logarithmic behavior takes over was assumed to lie between  $\bar{H} = 20$  and  $\bar{H} = 1000$ . The value of  $H^*$  taken as "correct" was chosen as that corresponding to  $\bar{H} = 100$ .

Figure 3 shows the corresponding range of uncertainty in  $H^*$ . The main result of this work is given in Fig. 4. Here, the values of  $H^*$  as a function of the compressibility and inertia parameters is given. A similar chart, where  $H^*$  is expressed in terms of  $C$  and  $I^*$  which naturally appear in the equations, is given in Fig. 5.

For the purpose of practical calculations, an analytical formula was fitted to the results of Fig. 4:

$$H^* = 0.643 + 0.286 I + 1.905 I^2 - \frac{0.183}{C}$$

The formula approximates the results well in the region

$$0 < \frac{1}{C} < 1$$

$$0 < I < 0.5$$

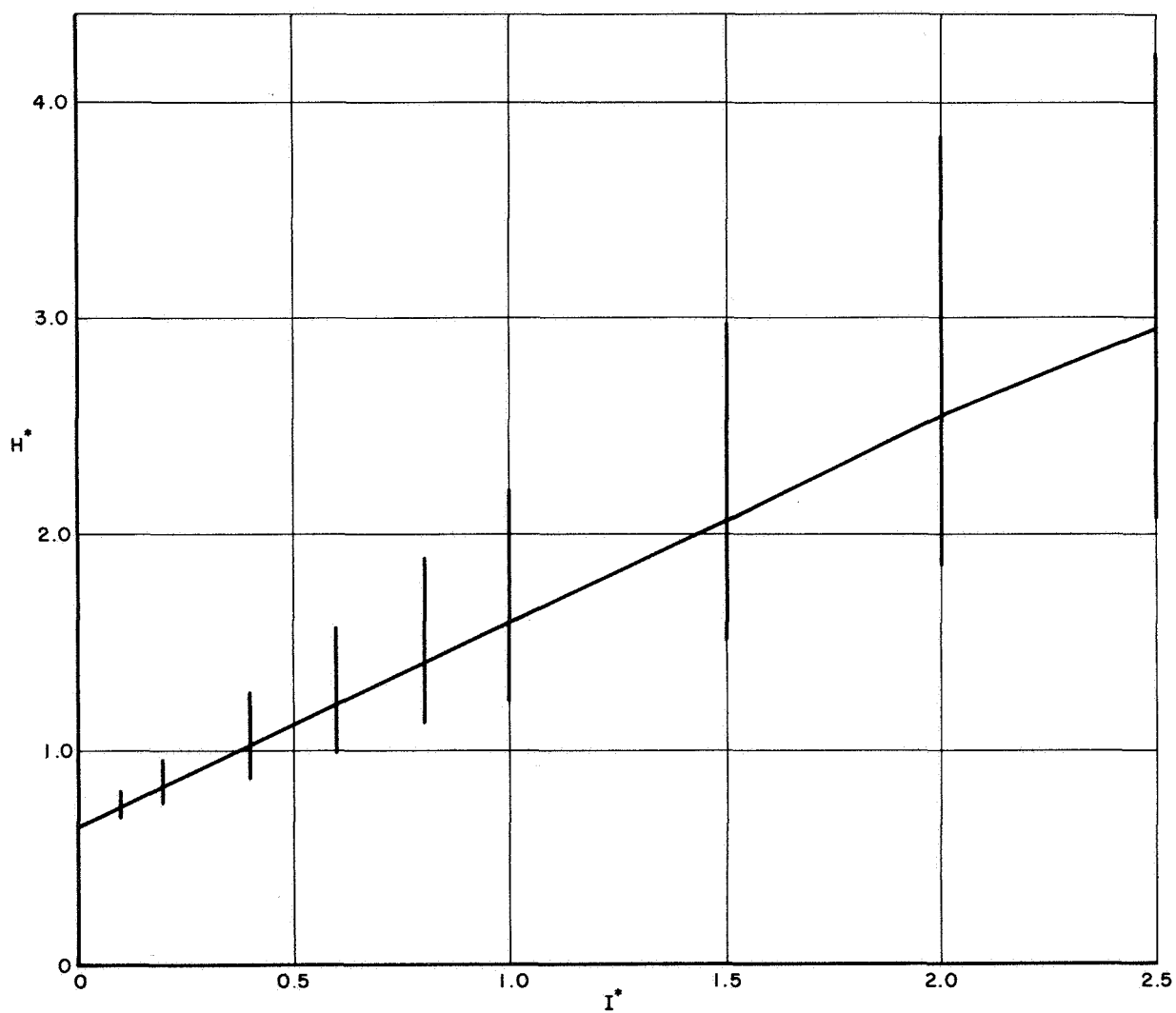


Fig. 3. Bounds of Uncertainty in the Evaluation of  $H^*$   
( $C^* = \infty$ )

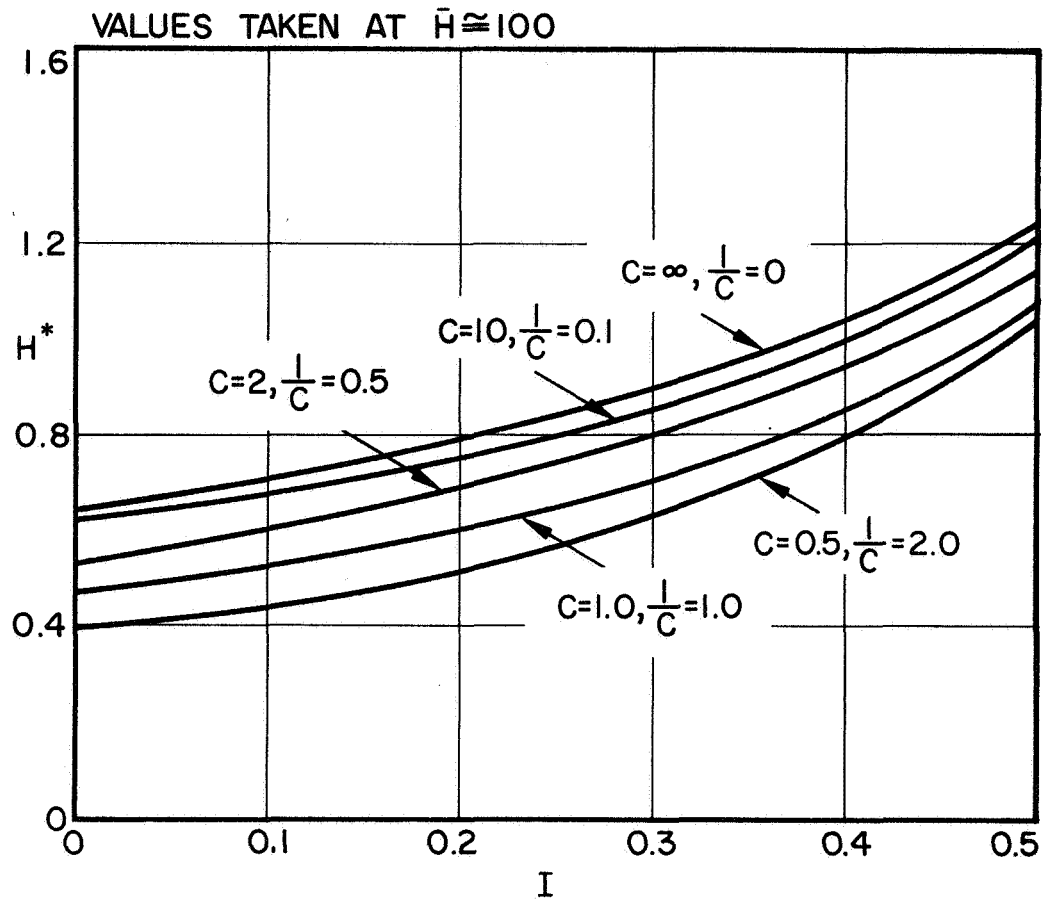


Fig. 4 Nominal Clearance  $H^*$  as a Function of Compressibility (C) and Inertia (I) Parameters



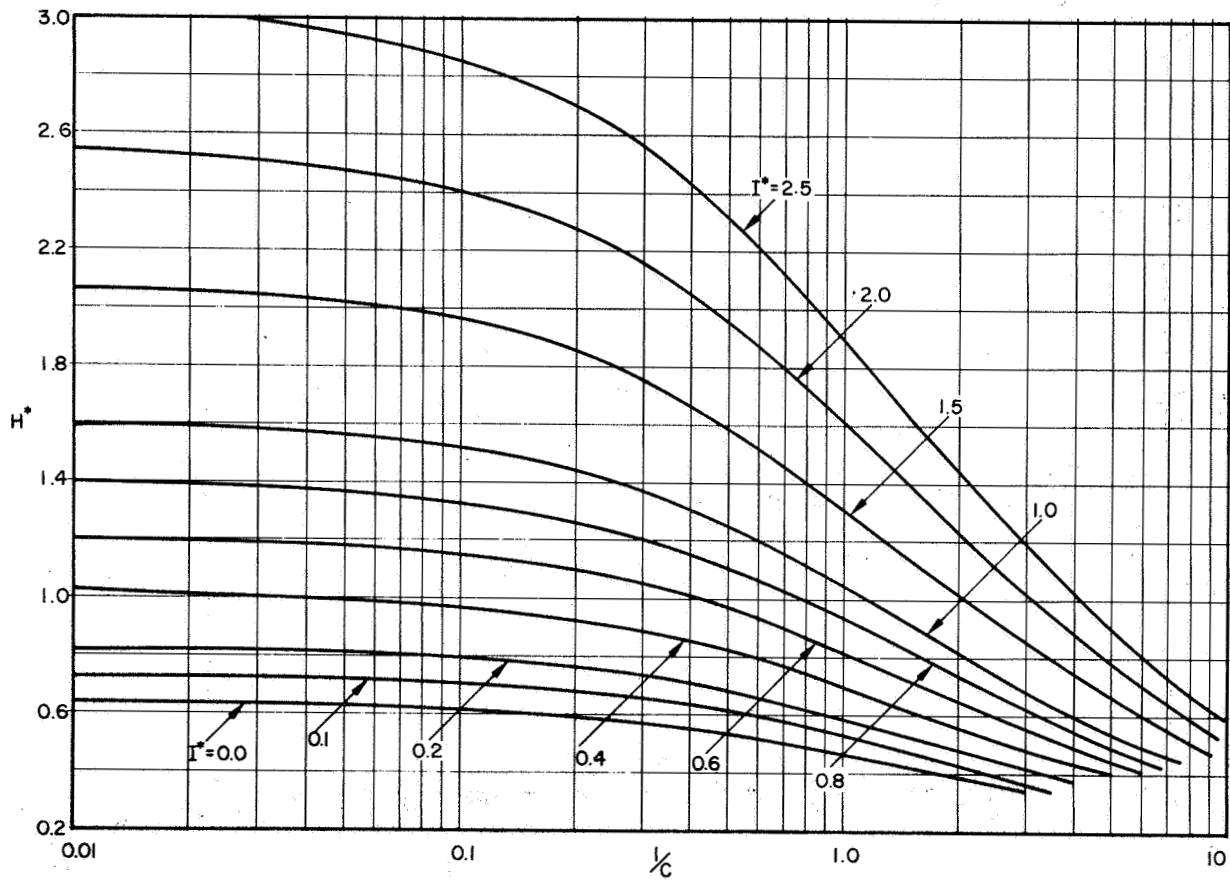


Fig. 5 Nominal Clearance  $H^*$  as a Function of Compressibility ( $C$ ) and the Normalized Inertia ( $I^*$ ) Parameters

## REFERENCES

1. A. Eshel and H. G. Elrod Jr. , "The Theory of the Infinitely Wide, Perfectly flexible Self-Acting Foil Bearing, " Journal of Basic Engineering, Trans. ASME, Series D, Vol. 87, Dec. 1965, pp. 831 - 836.



APPROVED DISTRIBUTION LIST FOR UNCLASSIFIED  
TECHNICAL REPORTS ISSUED UNDER CONTRACT

NONR 3815(00), NR 062-297

Defense Documentation Center Cameron Station Alexandria, Virginia	(20)	Cryogenic Data Center National Bureau of Standards Boulder, Colorado 80302
Mr. Stanley L. Zedaker Department 244-2, Building 71 Autonetics P. O. Box 4181 Anaheim, California 92803		Massachusetts Institute of Technology Instrumentation Laboratory Attn: Library 68 Albany Street Cambridge, Massachusetts 02139
Officer in Charge Annapolis Division Naval Ship Research and Development Center Attn: Mr. Watt V. Smith Code 800 Annapolis, Maryland 21402	(1) (1)	Professor Herbert H. Richardson Room 3-461 Massachusetts Institute of Technology Cambridge, Massachusetts 02139
Superintendent Naval Academy Attn: Library Annapolis, Maryland 21402		Mr. Robert S. Siegler Rocketdyne Nucleonics Subdivision 6633 Canoga Avenue Canoga Park, California 91304
Air Force Office of Scientific Research Attn: Code SREM 1400 Wilson Boulevard Arlington, Virginia 22209		Commander Charleston Naval Shipyard Naval Base Charleston, South Carolina 29408
Mr. David Craig, Jr. Mechanical Design Section Grumman Aircraft Engineering Corporation Bethpage, New York 11714		Dr. Edgar J. Gunter, Jr. University of Virginia School of Engineering and Applied Science Charlottesville, Virginia 22903
Director Office of Naval Research Branch Office 495 Summer Street Boston, Massachusetts 02210		Director Office of Naval Research Branch Office 219 South Dearborn Street Chicago, Illinois 60604
Commander Boston Naval Shipyard Boston, Massachusetts 02129		Professor L. N. Tao Illinois Institute of Technology Chicago, Illinois 60616
		Commander Naval Weapons Center Attn: Code 753 China Lake, California 93555

Mr. Robert H. Josephson, Manager  
Clevite Corporation  
Mechanical Research Division  
540 East 105th Street  
Cleveland, Ohio 44108

Director, Lewis Research Center  
National Aeronautics and Space  
Administration  
Attn: Mr. E. E. Bisson  
Cleveland, Ohio 44871

NASA Scientific and Technical  
Information Facility  
P. O. Box 33  
Attn: Acquisitions Branch (S-AK/DL)  
College Park, Maryland 20740

Battelle Memorial Institute  
505 King Avenue  
Attn: Dr. Russell Dayton (1)  
Mr. J. W. Kannel (1)  
Columbus, Ohio 43201

Dr. John E. Mayer, Jr.  
Research & Engineering Center  
Ford Motor Company  
P. O. Box 2053  
Dearborn, Michigan 48123

Mr. Henry J. Elwertowski  
Admiralty Compass Observatory  
Ditton Park, Slough, Bucks, England

Commanding Officer  
Army Research Office  
Attn: ESD-AROD  
Box 6M, Duke Station  
Durham, North Carolina 27706

Professor A. Charnes  
The Technological Institute  
Northwestern University  
Evanston, Illinois 60201

C. C. Moore, Manager  
Advance Bearing & Seal Technology  
General Electric Company  
Flight Propulsion Division  
Mail Drop H25  
Evendale, Ohio 45215

Mr. L. R. Manoni  
Manager, Advanced Programs  
United Aircraft Corporation Systems Center  
1690 New Britain Avenue  
Farmington, Connecticut 06032

Commanding General  
Army Engineering Research and  
Development Laboratories  
Attn: Mr. W. M. Crim (1)  
Technical Documents Center (1)  
Fort Belvoir, Virginia 22060

Mrs. Hisaw, Engineering Librarian  
The LaFLEUR Corporation  
16659 South Gramercy Place  
Gardena, California 90247

Mr. W. G. Wing  
Sperry-Gyroscope Company  
2T120  
Great Neck, New York 11020

Mr. H. Walter  
Vice President, Research & Development  
Worthington Corporation  
Harrison, New Jersey 07029

Dr. Charles C. W. Ng  
Pratt and Whitney Aircraft Corporation  
E. Hartford, Connecticut 06108

Mr. Stanley Abramovitz, Director  
Industrial Tectonics, Inc.  
Fluid Film Bearing Division  
New South Road and Commercial Street  
Hicksville, New York 11803

Mr. P. R. Broussard, Jr.  
Guidance and Control Division  
National Aeronautics and Space Administration  
George C. Marshall Space Flight Center  
Huntsville, Alabama 35812

Professor J. F. Kennedy, Director  
Iowa Institute of Hydraulic Research  
University of Iowa  
Iowa City, Iowa 52240

Commanding Officer  
Naval Avionics Facility  
21st and Arlington Avenue  
Attn: Mr. J. G. Weir RDM-4  
Indianapolis, Indiana 46218

Dr. H. N. Linhardt  
Director of Engineering  
Cryogenic Division  
Airedution Company  
1900 Lane Road  
Irvine, California 92664

Captain L. S. McCready  
Merchant Marine Academy  
Kings Point, New York 11024

Professor J. Modrey  
School of Mechanical Engineering  
Purdue University  
Lafayette, Indiana 47907

Dr. B. Sternlicht  
Mechanical Technology Incorporated  
968 Albany-Shaker Road  
Latham, New York 12110

Mr. P. Eisenberg  
Hydronautics, Incorporated  
Pindell School Road  
Howard County, Laurel, Maryland 20810

Mr. Walter Carow  
Kearfott Division  
General Precision Incorporated  
1150 McBride Avenue  
Little Falls, New Jersey 07425

Mr. J. Levine  
Ford Instrument Company  
31-1 Thomson Avenue  
Long Island City, New York 11101

Commander  
Long Beach Naval Shipyard  
Long Beach, California 90802

Mr. Jerry Glaser  
Senior Project Engineer (Dept. 37)  
The Garrett Corporation  
9851 S. Sepulveda Boulevard  
Los Angeles, California 90009

Mr. Howard F. Traeder  
Instruments Engineering Department  
AC Electronics Division  
General Motors Corporation  
Milwaukee, Wisconsin 53201

Mr. J. W. Lower, Chief  
Engineer-Inertial Components  
Honeywell Aero Division  
2600 Ridgway Road  
Minneapolis, Minnesota 55455

Superintendent  
Naval Postgraduate School  
Attn: Library  
Monterey, California 93940

Professor Paul F. Pucci  
Mechanical Engineering Department  
Naval Postgraduate School  
Monterey, California 93940

Research Committee in Information  
The American Society of Mechanical  
Engineers  
345 East 47th Street  
New York, New York 10017

Engineering Societies Library  
345 East 47th Street  
New York, New York 10017

• Office of Naval Research  
New York Area Office  
207 W. 24th Street  
New York, New York 10011

Commanding Officer  
Office of Naval Research Branch Office  
Box 39 FPO New York 09510 (25)

Columbia University  
Department of Mechanical Engineering  
Attn: Professor V. Castelli (1)  
Professor H. Elrod (1)  
New York, New York 10027

Mr. Gerald E. Speen, Division Manager  
Conductron  
P. O. Box 844  
Northridge, California 91324

Ralph F. De Angelias  
Norden Division of United  
Aircraft Corporation  
Helen Street  
Norwalk, Connecticut 06852

Mr. E. L. Swainson  
Precision Products Department  
Nortronics  
100 Morse Street  
Norwood, Massachusetts 02062

United States Atomic Energy Commission  
Division of Technical Information  
Extension  
P. O. Box 62  
Oak Ridge, Tennessee 37830

Mr. R. E. MacPherson  
Oak Ridge National Laboratory  
P. O. Box Y  
Oak Ridge, Tennessee 37831

Mr. R. G. Jordan  
Oak Ridge Gaseous Diffusion Pl.  
Union Carbide Corporation - Nuclear  
Division  
P. O. Box P  
Oak Ridge, Tennessee 37830

National Research Council  
Aeronautical Library  
Attn: Miss O.M. Leach, Librarian  
Montreal Road  
Ottawa 7, Canada

Dr. Calus G. Goetzl, Orgn 52-30  
Bldg. 201, Plant 2  
Lockheed Missiles and Space Company  
3251 Hanover Street  
Palo Alto, California 94304

Director  
Office of Naval Research Branch Office  
1030 E. Green Street  
Pasadena, California 91101

Dr. E. E. Sechler  
Executive Officer for Aero  
California Institute of Technology  
Pasadena, California 91109

Mr. Carl F. Graesser, Jr.  
Director of Research  
New Hampshire Ball Bearings, Inc.  
Peterborough, New Hampshire 03458

Professor D. D. Fuller  
Laboratories for Research & Development  
The Franklin Institute  
Philadelphia, Pennsylvania 19104

Mr. Otto Decker, Manager  
Friction and Lubrication Laboratory  
Franklin Institute  
20th and Parkway  
Philadelphia, Pennsylvania 19103

Professor P. R. Trumpler  
Towne School of Civil and Mechanical  
Engineering  
University of Pennsylvania  
Philadelphia, Pennsylvania 19104

Commander  
Philadelphia Naval Shipyard  
Attn: Technical Library (Code 249b)  
Philadelphia, Pennsylvania 19112

Mr. John Boyd  
Westinghouse Research Incorporated  
Churchill Boro  
Pittsburgh, Pennsylvania 15235

Professor M. C. Shaw, Head  
Department of Mechanical Engineering  
Carnegie Institute of Technology  
Pittsburgh, Pennsylvania 15213

Dr. F. Osterle  
Department of Mechanical Engineering  
Carnegie Institute of Technology  
Pittsburgh, Pennsylvania 15213

Commander  
Naval Missile Center  
Attn: Technical Library  
Point Mugu, California 93041

Commander  
Portsmouth Naval Shipyard  
Portsmouth, New Hampshire 03801

Commander  
Norfolk Naval Shipyard  
Portsmouth, Virginia 23709

Mr. Phillip J. Mullan  
Staff Engineer-Advanced Tapes  
IBM Data Systems Division  
Development Laboratory- Box 390  
Poughkeepsie, New York 12602

Redstone Scientific Information Center  
Army Missile Command  
Attn: Chief, Document Section  
Redstone, Arsenal, Alabama 35809

Dr. W. A. Gross (1)  
Dr. J. Licht (1)  
Ampex Corporation  
401 Broadway  
Redwood City, California 94063

Editor  
Applied Mechanics Reviews  
Southwest Research Institute  
8500 Culebra Road  
San Antonio, Texas 78206

Office of Naval Research  
San Francisco Area Office  
1076 Mission Street  
San Francisco, California 94103

Technical Library, Code H245C-3  
Hunters Point Division  
San Francisco Bay Naval Shipyard  
San Francisco, California 94135

Mr. G. R. Fox  
Research and Development Center  
General Electric Company  
P. O. Box 8  
Schenectady, New York 12301

Fenton Kennedy Document Library  
The Johns Hopkins University  
Applied Physics Laboratory  
8621 Georgia Avenue  
Silver Spring, Maryland 20910

Commander  
Naval Ordnance Laboratory  
Attn: Librarian  
Silver Spring, White Oak, Maryland 20910

Dr. H. S. Grassam  
University of Southampton  
Department of Mechanical Engineering  
Southampton, England

Professor J. Foa  
Department of Aeronautical Engineering  
Rensselaer Polytechnic Institute  
Troy, New York 12180

Professor R. C. DiPrima  
Department of Mathematics  
Rensselaer Polytechnic Institute  
Troy, New York 12180

Walt Tucker  
Nuclear Engineering Department  
Brookhaven National Laboratory  
Upton, Long Island, New York 11973

Shipyard Technical Library  
Code 130L7 Bldg 746  
San Francisco Bay Naval Shipyard  
Vallejo, California 94592

Mr. E. Roland Maki  
Mechanical Development Department  
General Motors Corporation  
12 Mile and Mound Roads  
Warren, Michigan 48090

Chief of Naval Research  
Department of the Navy  
Attn: Code 438 (3)  
Code 473 (1)  
Code 463 (1)  
Washington, D. C. 20360

Director  
Naval Research Laboratory  
Attn: Code 2027 (6)  
Code 5230 (1)  
Washington, D. C. 20390



Director  
Special Projects Office  
Department of the Navy  
Washington, D. C. 20360

Director, Special Projects Office  
Department of the Navy  
Attn: Code SP230  
Washington, D. C. 20360

Commander  
Naval Ship Systems Command  
Department of the Navy  
Attn: Code 6340 (1)  
Code 6345 (1)  
Code 6644C (1)  
Washington, D. C. 20360

Commanding Officer and Director  
Naval Ship Research and Development  
Center  
Attn: Code 042 (1)  
Code 513 (1)  
Washington, D. C. 20007

Commander  
Naval Air Systems Command  
Attn: AIR 52034 (1)  
AIR 53651 (1)  
Washington, D. C. 20360

Commander  
Naval Ordnance Systems Command  
Attn: ORD 03 (1)  
ORD 913 (1)  
Washington, D. C. 20360

Division of Engineering  
Maritime Administration  
441 G Street, N.W.  
Washington, D. C. 20235

Librarian Station 5-2  
Coast Guard Headquarters  
1300 E Street, N.W.  
Washington, D. C. 20226

Chief of Research and Development  
Office of Chief of Staff  
Department of the Army  
The Pentagon, Washington, D. C. 20310

Chief of Staff  
U.S. Air Force  
The Pentagon  
Attn: AFDRD-AS/M  
Washington, D. C. 20330

Commander  
Air Force Office of Scientific Research  
Attn: SRHM  
Washington, D. C. 20333

Science and Technology Division  
Library of Congress  
Washington, D. C. 20540

National Science Foundation  
Engineering Division  
1800 G Street, N.W.  
Washington, D. C. 20550

Headquarters Library  
Atomic Energy Commission  
Washington, D. C. 20545

Mr. N. Grossman, Chief  
Engineering Development Branch  
Reactor Development Division  
Atomic Energy Commission  
Washington, D. C. 20545

Mr. Clarence E. Miller, Jr.  
Division of Reactor Development and  
Technology  
U.S. Atomic Energy Commission  
Washington, D. C. 20545

Defence Research and Development Attache  
Australian Embassy  
1735 Eye Street, N. W.  
Washington, D. C. 20006

Dr. W. F. Sawyer  
Department of Mechanical Engineering  
Catholic University  
Washington, D. C. 20017

Mr. Don Moors (1)  
Dr. J. S. Ausman (1)  
Litton Systems, Inc.  
5500 Canoga Avenue  
Woodland Hills, California 91364

Mr. George H. Pedersen  
Curtiss-Wright Corporation  
Wright Aeronautical Division  
Woodridge, New Jersey 07075

AF Flight Dynamics Laboratory  
Attn: Mr. W.C. Buzzard (FDFM)  
Wright-Patterson AFB, Ohio 45433

AF Aero Propulsion Laboratory  
Attn: Mr. M.R. Chasman (APFL)  
Wright-Patterson AFB, Ohio 45433

Commander, Research & Technology Division  
Air Force Systems Command  
Attn: L.M. Medgepeth (APIP-1) (1)  
R.W. McAdory (AVNE) (1)  
Wright-Patterson AFB, Ohio 45433

Hamilton Standard Systems Center  
1690 New Britain Avenue  
Attn: Librarian  
Farmington, Connecticut 06032

Commanding Officer and Director  
Naval Ship Research and  
Development Center, Code 800  
Washington, D.C. 20007



UNCLASSIFIED

## Security Classification

DOCUMENT CONTROL DATA - R&D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1. ORIGINATING ACTIVITY (Corporate author) Ampex Corporation 401 Broadway Redwood City, Calif.		2a. REPORT SECURITY CLASSIFICATION Unclassified
3. REPORT TITLE  On Fluid Inertia Effects in Infinitely Wide Foil Bearings		2b. GROUP
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report		
5. AUTHOR(S) (Last name, first name, initial) Eshel, A.		
6. REPORT DATE July 15, 1968	7a. TOTAL NO. OF PAGES 25	7b. NO. OF REFS 1
8a. CONTRACT OR GRANT NO. Contract No. Nonr-3815(00)(X)	9a. ORIGINATOR'S REPORT NUMBER(S) RR 68-15	
b. PROJECT NO.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
c.		
d.		
10. AVAILABILITY/LIMITATION NOTICES  Qualified requesters may obtain copies of this report from DDC.		
11. SUPPLEMENTARY NOTES  None	12. SPONSORING MILITARY ACTIVITY Department of the Navy Office of Naval Research Washington, D. C. 20360	
13. ABSTRACT  Equations for a foil over a lubricating film in which the effects of fluid inertia are taken into account are derived. Approximate solutions showing the effect of inertia and fluid compressibility are obtained. The effect of inertia is to increase the fluid film thickness.		

## Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Foil Bearings						
Fluid Mechanics						
Reynolds Equation						
Fluid Films						
Mathematical Analysis						
Inertia Effects						

## INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through \_\_\_\_\_."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through \_\_\_\_\_."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through \_\_\_\_\_."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.