OPTIMUM LINEAR ADAPTIVE DESIGN OF DOMINANT TYPE SYSTEMS WITH LARGE PARAMETER VARIATIONS *

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GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) ______

Microfiche (MF) ______

ff 653 July 65

Semiannual Status Report
NASA Grant NGR 06-003-083

October 1, 1968

OCT 1968
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FACILITY FORM 602
N68-37727 (ACCESSION NUMBER)
(THRU)
10 (PAGES)
1 (CODE)
CR-97312 (NASA CR OR TMX OR AD NUMBER)
07 (CATEGORY)

*The research reported here was supported by the National Aeronautics and Space Administration under Research Grant NGR 06-003-083.

SEMIANNUAL STATUS REPORT

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The following research projects have been undertaken under this grant:

(1) Improved Linear Adaptive Design of Dominant Type Systems with Large Parameter Variations -- use of third-order dominant model (I. Horowitz)

(2) Extension of (1) to a fourth-order dominant model (Don Olson and I. Horowitz)

(3) Computer aided synthesis of third-order linear systems with large parameter variations (John Smay)

(4) Computer programs for synthesis of fourth-order (nondominant) linear systems with large parameter variations (Don Lewis)

(5) Frequency-response approach to design of linear systems with large parameter variations (Ned Oda and I. Horowitz)

(6) Study of feedback systems with simple nonlinearities for the purpose of developing a synthesis approach (Tom McDonald)

(7) The Self-Oscillating-Adaptive-System (SOAS) -- decoupling of the limit-cycle mode and the adaptive loop transmission mode (R. Smisek and I. Horowitz)

(8) Parameter variations in optimal control systems (Fred Taylor)

Descriptions of the research topics and the progress that has been made are given below:

(1) Improved Linear Adaptive Design of Dominant Type Systems with Large Parameter Variations -- use of third-order dominant model

This research project, in common with all the others, except for No. 8, is concerned with the development of optimum synthesis techniques for satisfying time-domain specifications in systems with large parameter variations. One can state without fear of contradiction that very little progress has been made in this area. Certainly, structures have been devised with adaptive properties, but no analytic techniques are available for tailoring their design in a reasonably optimum manner to numerical problems with time-domain specifications. One approach has been the use of a dominant second-order model, permitting easy correlation between the system poles and time-domain performance. Root-locus techniques are then used to control the range of system poles despite the parameter variations. However, the previously available technique is very approximate and restricted to problems in which the time-domain specifications are very rigid. In this research project the specfications need not be so restricted, the technique is far more exact and includes a much superior method for handling the "far-off" region.

The project has been completed for the case of plants with large variations in gain factor and in one or two poles. It has been written up and the manuscript submitted for publication to the IEEE Trans. on Automatic Control. A copy of the manuscript is included in this report.

(2) Improved Linear Adaptive Design of Dominant Type Systems with Large Parameter Variations - use of fourth-order model

In this and the first topic, system response is presumed dominantly second-order. However, minimization of the bandwidth of the adaptive loop is secured by using in an exact manner, a higher-order representation of the loop transmission function. In (2) a fourth-order representation was attempted in contrast to a third-order in research topic (1). The synthesis technique which has been developed relies heavily on use of the computer. The research has been completed for the same kind of time-domain specifications and parameter variations (gain and up to two poles independently varying) as in topic (1). Some work was also done for plants in which a zero also varies, independently of the gain and the poles, but this part will require more work in the future. A research report covering the above (including computer programs) has been written and is now in the production process. Copies will be sent to the sponsors in the near future. It is also planned to condense the report and submit the results for publication.

(3) Computer aided synthesis of third-order linear systems with large parameter variations

An attempt is made here to work with a third-order all-pole system model instead of the simpler second-order model. Given a set of step-response specifications and bounds on the plant parameter ignorance, the objective is to have the computer deliver a design which satisfies the specifications with a minimum loop bandwidth. The approach taken in this project, in contrast to that taken in (4) below, was to first thoroughly analytically master the problem before developing the computer programs. This project has been completed and written up as a research report. The report has been typed and is in its final stages of preparation. It is also planned to submit the results for publication.

(4) Computer programs for synthesis of fourth-order (nondominant) linear systems with large parameter variations

The problem, specifications and objectives are the same as in the first three. However, a more complex model is being used -- four poles and one zero. Also, no attempt is being made to first thoroughly explore the nature of the boundaries, as was done in topic (3). Instead one works entirely with the computer. The analog portion of the Hybrid computer is used as a model of the system, with the digital portion performing the data-analysis, logic and decision-making. Progress has been slowed by some difficulties with the analog portion of the computer. However, these have been overcome as they arose. The situation now is that a gradient technique for achieving a satisfactory design for fixed plant parameters is nearing completion. The next stage is to expand this technique to admit plant parameter variations.

(5) Frequency-response approach to design of linear systems with large parameter variations

The basic problem is the same as in the first four topics. However, a frequency-response formulation is used here instead of the pole-zero formulation. The attractive feature of using frequency-response is that when the performance specifications are in terms of frequency response then there is already available a straightforward optimum design technique to handle systems of any order, no matter how high. However, when the performance specifications are in the time-domain, it becomes necessary to translate them into their equivalent in the frequency domain. The problem is to find systematic means for such translation. The approach taken has been to begin with very simple systems and proceed to the more complex. So far, extensive investigation of first- and second-order systems has been made, including detailed designs and analog computer verification. This is now being written up, before continuing with higher-order systems.

(6) Study of feedback systems with simple nonlinearities for the purpose of developing a synthesis approach

In a linear feedback system, a few key system parameters, (system bandwidth and damping, loop transmission bandwidth and damping) suffice to give the systems engineer an excellent appreciation of the system capabilities and its response to command and disturbance inputs. The general objective is to do the same for nonlinear systems, starting with very simple single nonlinear elements in an otherwise linear system. This study begins with the simple saturating element. The second-order (linear portion) case has been analyzed and simulated on the analog computer.

It is planned to present the results in the form of graphs and then to attempt to generalize for the same non-linearity imbedded in a higher-order linear portion.

(7) <u>The Self-Oscillating-Adaptive-System (SOAS) -- decoupling of the limit-cycle mode and the adaptive loop transmission mode</u>

The SOAS (MH system) has been one of the most successful of the new family of nonlinear adaptive systems. The simplest version consists of the structure in Fig. 1.

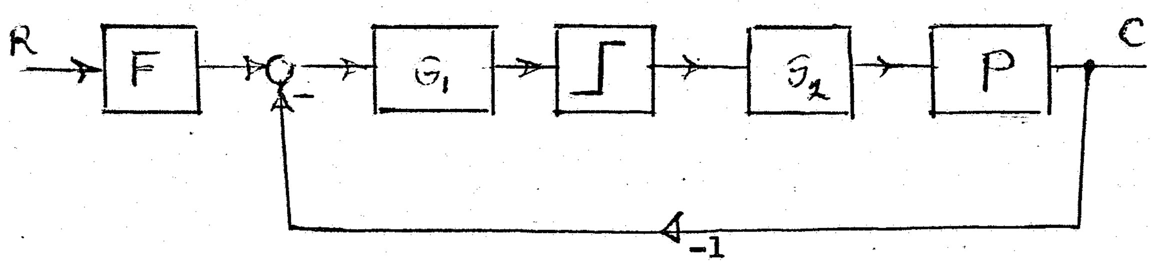

Figure 1

If the forced signal components are sufficiently small and slow relative to the limit cycle component, then quasilinear analysis may be made. In fact, to a significant extent a synthesis procedure[1] exists whereby one can design to specifications. There are two classes of design requirements. One concerns quasilinearity and leads to the minimum value of the oscillating frequency ω_o, which is a function of quantities such as:

the bandwidth and peak levels of the "extreme" signals the plant must process; the maximum tolerable limit

[1] I. Horowitz, Comparison of Linear Feedback Systems with the Self-Oscillating Adaptive System, Trans. IEEE on Automatic Control, Vol. AC-9, pp 386-392, 1967.

cycle amplitude.

The second class of design requirements concerns the adaptive and disturbance attenuation demands being made on the system. The extremely attractive property of the SOAS is that it has zero sensitivity to pure gain changes. However, it is possible in specific problems that the first class of quasilinearity requirements are greater than that of the second class. In such cases the SOAS may be inferior in performance as compared to the purely fixed compensation linear design. Also, in many cases where the SOAS should be decisively superior it may not be so, because of the comparatively rigid relation between ω_o and the adaptive loop. Thus for the ideal relay used in Figure 1, the forced component loop function (L_f) is exactly one half that of the limit cycle loop function (L_D). One can easily

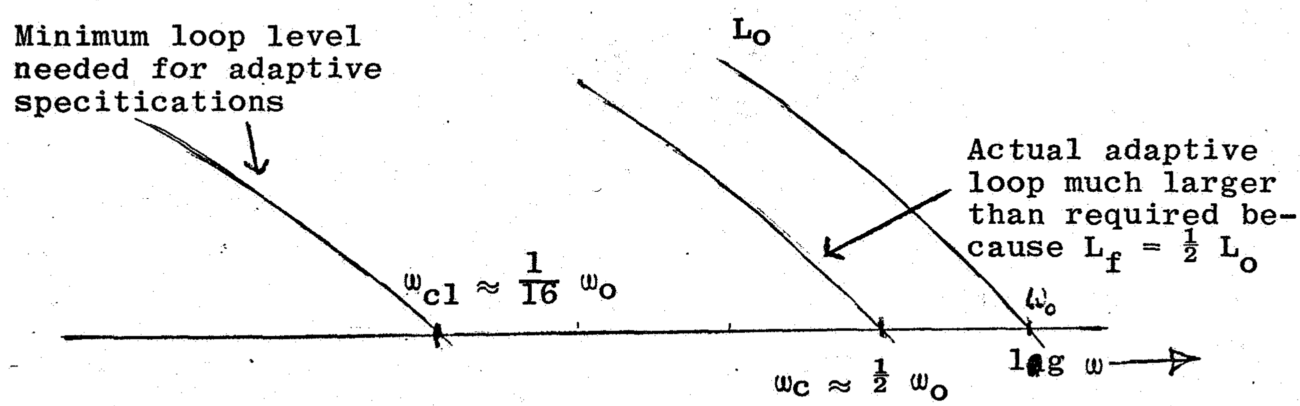

Figure 2

formulate problems where, as in Figure 2, the adaptive loop crossover frequency need only be as large as $\omega_{c1} << \omega_o/2$. However, because willy-nilly $L_f = \frac{1}{2} L_o$, the actual crossover frequency is closely $\frac{1}{2}\omega_o$, so the adaptive loop is much better than need be, which is not good because the system tries to attenuate disturbances much better than need be and is more sensitive to internal noise.

One possible solution appears to use a nonlinear element whose forced-component describing function $N_f = \lambda N_o$, where N_o is the limit cycle describing function of the nonlinear element, and with λ flexible. Thus if the relation between ω_{c1} and ω_o is as shown in Figure 2, then λ could be << 0.5, such that L_f need not be any larger in magnitude and bandwidth than required by the adaptive specifications. Elastic modes near ω_o would then be much less troublesome because the forced component loop is then essentially an open circuit. Actually λ frequency-dependent would be even more valuable, for the reason shown in Fig. 3.

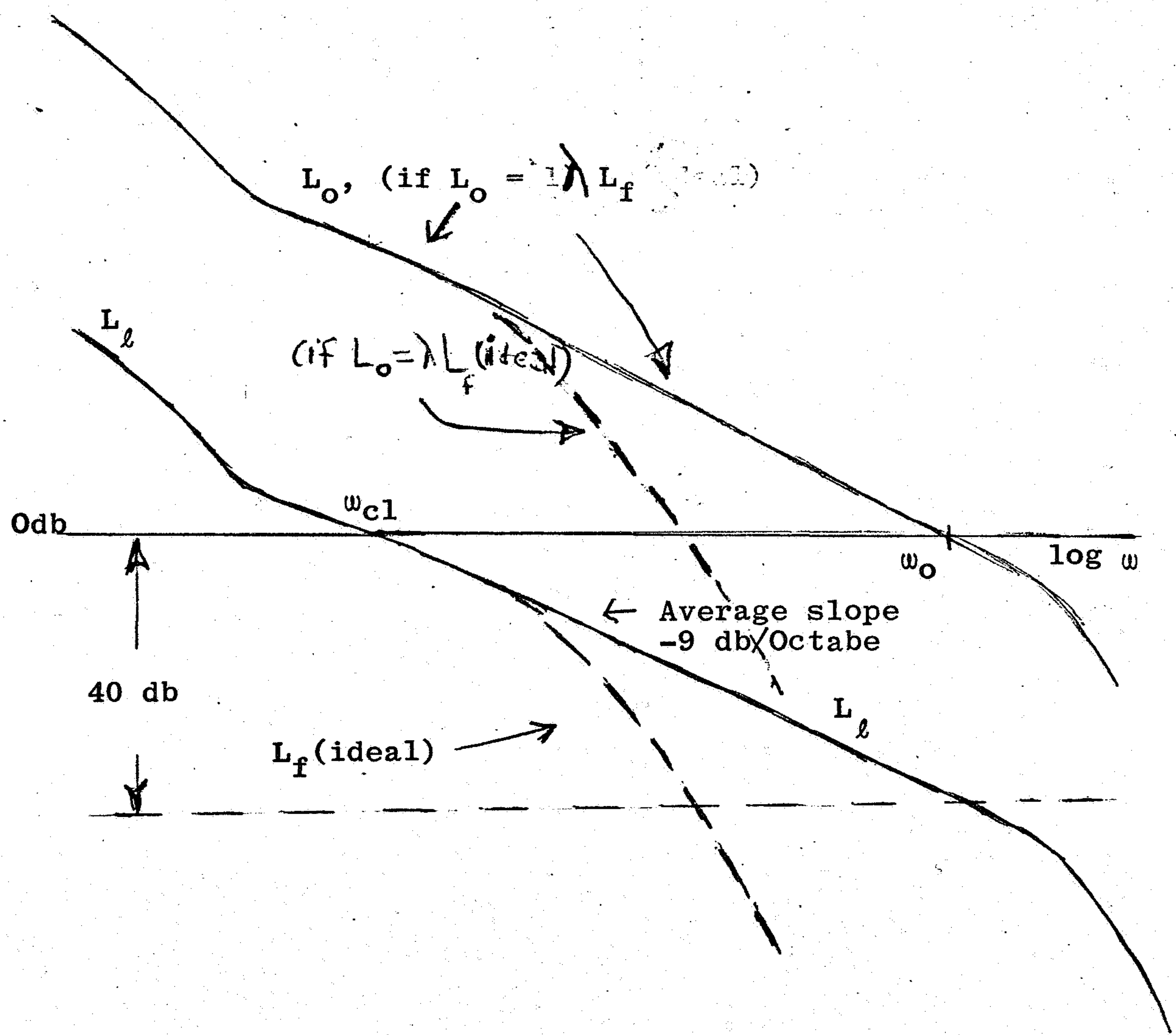

Figure 3

In Figure 3, L_ℓ represents the minimum loop transmission required in a fixed-compensation linear design, in a problem where the plant gain can change by a factor of 100. Hence the gain margin G_M must be more than 40 db, so the average slope for $\omega > \omega_{c1}$ can only be approximately -9 db/octave for about 5 octaves until a gain main margin of say 45 db has been achieved. Only then can L_ℓ be reduced at a much higher rate. Next, consider use of the SOAS structure of Figure 1. Its adaptive loop requirements will be closely similar to those of L_ℓ for $\omega \le \omega_{c1}$. Its potential great advantage is that only a few db gain margin (5 db if the same number as in above is used instead of 45 db as in the fixed compensation linear design) are required. Therefore, it should be possible to reduce L_f very much more rapidly than L_1, as shown by the dashed line in Figure 3. However, even if $L_f = \lambda L_o$ with λ real and small is used, this is not satisfactory because the resulting L_o (also shown in dashed lines in Figure 3) will then have a phase lag much larger than 180^o at $\omega = \omega_o$. Clearly, what is required is a means of generating nonlinearities for the realization of dual input describing functions whose two components can be made independent of each other as a function of frequency. Only then can the full potentialities of the SOAS structure be exploited, for it would then be possible to use L_f(ideal) of Figure 3, and still achieve the value of ω_o required for quasilinear operation.

(8) Parameter variations in optimal control

The usual optimal control law obtained from the use of the maximum principle (or any other approach), applies only at a specific assumed set of fixed or time-varying parameter values. If the parameters have different fixed or time-varying values the results are no longer optimal. Even though a feedback structure is used to implement the control law, the departure from optimum can be just the same as if an open-loop structure is used. This should not be surprising because the feedback is not from the "desired objective" but from the system state variables. One approach to this parameter variation problem has been to include sensitivity parameters in the performance index. However, the resulting performance is then certainly not optimum, but rather some compromise value. Ideally, one wants a control law which adapts itself to the parameter values such that it continues to be optimum as the parameters change. This research project is taking the latter approach.

The state vector is augmented by the inclusion of a parameter set $\dot{\underline{\alpha}} = 0$. For example, if the time varying gain of an amplifier can be expressed by means of $A(t) = a_1 t + a_2 + a_3 e^{a_4 t}$, then the parameter set includes $a_1, \ldots, a_4$. Note that even though the plant parameters are assumed fixed but unknown, it is possible by this means nevertheless to include the time-varying case. A Taylor series expansion of the Hamiltonian is made about the nominal Hamiltonian, i.e., at a nominal value of the parameters, associated with a nominal set of state and control variables. Below y, p, u are the augmented state, costate and control variables respectively, and n refers to the nominal values:

$$H(y,p,u) = H(y,p,u)_n + \left(\frac{\partial H}{\partial y}\right)_n' (y - y^n) - \frac{1}{2} (y - y^n)' \left(\frac{\partial^2 H}{\partial y^2}\right)_n (y - y^n) + \text{---}$$

$$+ \left(\frac{\partial H}{\partial p}\right)_n' (p - p^n) + \text{---} + \left(\frac{\partial H}{\partial u}\right)_n' (u - u^n) + \text{---}$$

The Maximum Principle may be expressed in terms of this series expression for H, but only terms up to and including second-order ones are retained.

The values of $\Delta u \overset{\Delta}{=} u - u^n$ are sought, rather than u. It is found that for their solution a system of ordinary differential equations must be solved even though the original system may be nonlinear and with unrestricted form for the performance index.

Research is being continued in the directions of convenient implementation, bounds on error arising out of Taylor series truncation and computer verification.