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FLOW PARAMETERS OF THE CORONA

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ABSTRACT

The coronal plasma is assumed to be a fully ionized mixture of two species. Nondimensional parameters are obtained from the fundamental equations of plasma flow. Some thermodynamic properties of the coronal plasma are presented in tables which are based on a coronal model computed by F. L. Scarf and L. M. Noble.

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RESEARCH AND DEVELOPMENT OPERATIONS

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LIST OF SYMBOLS

Symbols	Definition
a	Velocity of Sound
a_r	Radiation density constant
α	Friction coefficient
$\vec{B}$	Magnetic induction
c	Velocity of light
C_p	Specific heat at constant pressure
C_V	Specific heat at constant volume
$\vec{D}$	Electric induction
D_d	Coefficient of diffusion
D_r	Diffusion coefficient of radiation
$\vec{E}$	Electric field strength
e	Electron charge
ϵ	Dielectric constant
$\vec{F}$	External force acting on the coronal fluid
$\vec{F}_h$	External heating force
F_r	Froude number
G	Gravitation constant
g	Acceleration of solar gravity

LIST OF SYMBOLS (Continued)

Symbol	Definition
γ	Ratio of specific heats
$\vec{H}$	Magnetic field strength
$\vec{J}$	Electric current density
$\vec{K}$	Flux of matter
k	Boltzmann's constant
k_p	Gas constant of the plasma
κ	Mass absorption coefficient
κ_e	Electron scattering mass absorption coefficient
κ_o	Numerical constant
ℓ	Length
ℓ_o	Characteristic length
ℓ_D	Debye length
ℓ_f	Mean free path
ℓ_ℓ	Larmor radius
$\ell_{\ell e}$	Larmor radius for the electron
$\ell_{\ell p}$	Larmor radius for the proton
λ	Coefficient of thermal conductivity
M	Mach number
$M_\odot$	Solar mass

LIST OF SYMBOLS (Continued)

Symbol	Definition
m	Mean mass of a particle in the plasma
μ	Apparent molecular weight of the plasma
μ_e	Magnetic permeability
μ_v	Coefficient of viscosity
n	Total number density of the plasma
n_e	Number density of the electron gas
ν_h	Magnetic viscosity
$\vec{\Omega}$	Angular velocity
ω_c	Cyclotron angular frequency
ω_{ce}	Cyclotron frequency of the electron
ω_{cp}	Cyclotron frequency of the proton
ω_e	Electrical decay angular frequency
ω_f	Friction angular frequency
ω_p	Plasma angular frequency
P_r	Prandtl number
p	Pressure of the coronal plasma
p_o	Characteristic pressure
p_r	Radiation pressure

Symbol	Definition
p_e	Pressure of electron gas
Π_i	Non-dimensional parameters
q	Concentration
R_e	Reynolds number
R_c	Relativity parameter
R_E	Electrical field parameter
R_h	Magnetic pressure number
R_δ	Magnetic Reynolds number
r	Radius of sphere around center of the Sun
r'	Integration variable
r_o	Fixed value of r
ρ	Density of the coronal plasma
ρ_o	Characteristic density
ρ_e	Electric charge density
S_c	Schmidt number
δ	Electrical conductivity
T	Kinetic temperature of the coronal plasma
T_o	Characteristic temperature
T_r	Radiation temperature of the coronal plasma

LIST OF SYMBOLS (Concluded)

Symbol	Definition
T_e	Effective temperature of the Sun
t	Time
t_o	Characteristic time
$\bar{t}$	Guillotine factor
$\vec{u}$	Flow velocity of the ions
$\vec{u}_o$	Characteristic velocity
V_h	Alfvén velocity
X	Fractional abundance by mass of hydrogen

TECHNICAL MEMORANDUM X-53767

FLOW PARAMETERS OF THE CORONA

SUMMARY

The coronal plasma is assumed to be a fully ionized mixture of two species. Nondimensional parameters are obtained from the fundamental equations of plasma flow. Some thermodynamic properties of the coronal plasma are presented in tables which are based on a coronal model computed by F. L. Scarf and L. M. Noble.

INTRODUCTION

A steady-state, spherically symmetric fluid-dynamic model under quiet-day solar conditions is adopted for the corona. A steady-state quiet solar wind blows continuously with the streaming becoming supersonic at great distances. The local heating energy is transferred primarily by thermal conduction. The corona is assumed to be a fully ionized mixture of two species. Nondimensional parameters are obtained from the following fundamental equations, which describe the plasma flow:

$$p = \frac{k}{m} \rho T$$

$$p_e = k n_e T$$

$$\frac{\partial n}{\partial t} = \nabla \cdot (n \vec{u}) = 0$$

$$nm \frac{\partial \vec{u}}{\partial t} + nm(\vec{u} \nabla) \vec{u} + \nabla (nkT) = \vec{F}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} nkT + \frac{nmu^2}{2}\right) + \nabla \cdot \left(\frac{5}{2} nkT \vec{u} + \frac{nmu^2}{2} \vec{u} - \lambda \nabla T\right) = \vec{F} \vec{u}$$

$$\vec{F} = -\frac{nGM_{\odot} m \vec{r}}{r^3} + \frac{1}{2} \vec{J} \times \vec{B} + \frac{nm}{2} \nabla (|\vec{\Omega} \times \vec{r}|^2)$$

$$+ 2nm \vec{u} \times \vec{\Omega} + \vec{F}_h$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \epsilon \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \mu_e \vec{H}}{\partial t}$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \vec{J} = 0$$

$$\vec{J} = \delta (\vec{E} + \mu_e \vec{u} \times \vec{H}) + \rho_e \vec{u} \quad .$$

In the equations for the coronal plasma, which were given, the variables to be investigated are $\vec{u}$, p, p_e, ρ, T, $\vec{H}$, ρ_e or n_e, $\vec{J}$ or δ. Of these $\vec{u}$, T, and n_e are known from the adopted model. The others are unknown and are derived in this report. In addition some characteristic velocities, frequencies, lengths, and some nondimensional parameters are computed.

The coronal models of the following authors were investigated: Waldmeier, Baumbach, Newkirk, Whang, Liu and Chang, Parker, Chapman, Pottasch, Scarf and Noble. Of all models, the one by Scarf and Noble is most complete and was therefore adopted. The computations of the coronal parameters could not have been carried out with the completeness in which they are presented, using any of the other models.

In the equation for $\vec{F}$, the first term represents the solar gravitational field, the second electromagnetic, the third centrifugal, the fourth Coriolis, and the fifth external heating forces.

In the region between 2 and 15 $R_{\odot}$, where reliable density measurements are available, the solar corona can be described by the Navier-Stokes equations with no external heat source. The coronal flow appears to be stationary, radial, spherically symmetric, and unaffected by electromagnetic forces. The best fit to $n_e(r)$ is obtained for a 90 percent hydrogen, 10 percent helium corona. Numerical values for the solution are contained in Table I.

The streaming speed increases rapidly beyond the sonic transition near $7R_\odot$. The values are not reliable outside the region $2 < R_\odot < 15$.

The properties of the coronal plasma will be presented in tables.

TABLE I. MODEL FOR A 90% H_2-10% He CORONA

$\frac{r}{R_\odot}$	T °K 10^6	n_e m^{-3} 10^6	u $m\,sec^{-1}$ 10^3
1.58	1.500	$6.21 \cdot 10^6$	9.5
1.99	1.470	$2.10 \cdot 10^6$	18.0
2.00	1.470	$2.03 \cdot 10^6$	18.4
4.50	1.030	$1.02 \cdot 10^5$	72.4
4.67	1.020	$9.20 \cdot 10^4$	75.0
6.80	0.889	$3.09 \cdot 10^4$	105.0
7.20	0.873	$2.62 \cdot 10^4$	110.0
7.25	0.870	$2.56 \cdot 10^4$	110.0
9.56	0.810	$1.22 \cdot 10^4$	132.0
10.00	0.792	$1.02 \cdot 10^4$	146.0
12.60	0.755	$4.95 \cdot 10^3$	190.0
13.70	0.735	$5.00 \cdot 10^3$	161.0
15.00	0.735	$2.70 \cdot 10^3$	246.0
214.30	0.270	6.5	354.0

TYPICAL PHYSICAL QUANTITIES

The following quantities, which characterize coronal plasma dynamics, can be introduced into the preceding plasma equations.

1. Length ℓ_o which characterizes the dimension of the flow field. All distances may be expressed in terms of ℓ_o. For the corona, the following distance may be used which is larger than the scale height:

$$\ell_o = 10^9 \, m$$

2. Characteristic velocity u_o. The approximate velocity of expansion at 10^6 °K may be used as characteristic velocity:

$$u_o = 7.5 \cdot 10^4 \, m \, sec^{-1}$$

3. Time t_o which characterizes the interval considered:

$$t_o = \frac{\ell_o}{u_o} = 1.33 \cdot 10^4 \, sec$$

4. Number density of the coronal plasma, n. The values are contained in Table II.

5. Apparent molecular weight of the plasma, μ. With sufficient accuracy, it can be considered as the same as that of the Sun:

$$\mu = 0.62$$

$$m = 1.02908 \cdot 10^{-27} \, kg$$

6. Temperature of the plasma. As shown in Table I, the temperature varies greatly in the corona. The typical temperature may be taken with sufficient accuracy:

$$T_o = 10^6 \, °K$$

TABLE II. THERMAL PARAMETERS OF THE CORONAL PLASMA

$\frac{r}{R_\odot}$	n m^{-3} 10^6	ρ $kg\,m^{-3}$ 10^{-15}	p $kg\,m^{-1}sec^{-2}$ 10^{-5}	T_r °K	μ_v $kg\,m^{-1}sec^{-1}$ 10^{-3}	D_d $m^2\,sec^{-1}$ 10^{13}	λ $kg\,m\,sec^{-3}$ 10^3	D_r deg 10^{19}	δ $ohm^{-1}m^{-1}$ 10^{11}	α $kg\,m^{-3}sec^{-1}$ 10^{-24}
1.58	$11.83 \cdot 10^6$	12.17	24.49	4598	33.06810	0.32715	16.53406	6.69001	12.85983	2.79318
1.99	$4.00 \cdot 10^6$	4.12	8.12	4097	31.43940	0.91876	15.71972	$1.39801 \cdot 10$	12.47597	$3.29161 \cdot 10^{-1}$
2.00	$3.87 \cdot 10^6$	3.98	7.85	4087	31.43940	0.95108	15.71972	$1.43662 \cdot 10$	12.47597	$3.08113 \cdot 10^{-1}$
4.50	$1.94 \cdot 10^5$	$2.00 \cdot 10^{-1}$	$2.76 \cdot 10^{-1}$	2725	12.92040	7.77808	6.46018	$8.47383 \cdot 10$	7.31735	$1.32012\ 10^{-3}$
4.67	$1.75 \cdot 10^5$	$1.80 \cdot 10^{-1}$	$2.46 \cdot 10^{-1}$	2675	12.60900	8.43402	6.30451	$8.90654 \cdot 10$	7.21104	$1.09004 \cdot 10^{-3}$
6.80	$5.89 \cdot 10^4$	$6.06 \cdot 10^{-2}$	$7.23 \cdot 10^{-2}$	2217	8.94203	17.76600	4.47101	$1.50604 \cdot 10^2$	5.86747	$1.51755 \cdot 10^{-4}$
7.20	$4.99 \cdot 10^4$	$5.14 \cdot 10^{-2}$	$6.06 \cdot 10^{-2}$	2154	8.54509	20.01610	4.27255	$1.62849 \cdot 10^2$	5.70978	$1.11930 \cdot 10^{-4}$
7.25	$4.88 \cdot 10^4$	$5.02 \cdot 10^{-2}$	$5.86 \cdot 10^{-2}$	2147	8.47187	20.31900	4.23593	$1.65121 \cdot 10^2$	5.68037	$1.07603 \cdot 10^{-4}$
9.56	$2.32 \cdot 10^4$	$2.39 \cdot 10^{-2}$	$2.60 \cdot 10^{-2}$	1869	7.08588	35.69620	3.54294	$2.28792 \cdot 10^2$	5.10300	$2.70716 \cdot 10^{-5}$
10.00	$1.94 \cdot 10^4$	$2.00 \cdot 10^{-2}$	$2.12 \cdot 10^{-2}$	1828	6.69876	40.32650	3.34938	$2.55805 \cdot 10^2$	4.93385	$1.95786 \cdot 10^{-5}$
12.60	$9.43 \cdot 10^3$	$9.70 \cdot 10^{-3}$	$9.82 \cdot 10^{-3}$	1628	5.94360	73.77420	2.97180	$3.72564 \cdot 10^2$	4.59218	$4.97013 \cdot 10^{-6}$
13.70	$9.52 \cdot 10^3$	$9.80 \cdot 10^{-3}$	$9.66 \cdot 10^{-3}$	1562	5.55776	68.28110	2.77888	$3.25707 \cdot 10^2$	4.41092	$5.27361 \cdot 10^{-6}$
15.00	$5.14 \cdot 10^3$	$5.29 \cdot 10^{-3}$	$5.22 \cdot 10^{-3}$	1492	5.55776	126.49400	2.77888	$5.25848 \cdot 10^2$	4.41092	$1.53731 \cdot 10^{-6}$
214.00	12.38	$1.27 \cdot 10^{-11}$	$4.60 \cdot 10^{-12}$	395	0.45456	$4.30937 \cdot 10^9$	0.22728	$4.06441 \cdot 10^9$	0.98207	$4.00554 \cdot 10^{-11}$

7. The typical pressure p_o may differ for various flow problems. In table II, the densities have been obtained from the equation:

$$\rho = m n$$

and the total number density for the assumed coronal composition from the equation:

$$n_e = 0.525 \, n$$

The pressures are then obtained from the ideal gas equation. The pressure near T_o may be considered as characteristic pressure:

$$p_o = 2.46 \cdot 10^{-6} \text{ kg m}^{-1} \text{ sec}^{-2}$$

8. Boltzmann's constant k which characterizes the gas constant of the plasma:

$$k = 1.38046 \cdot 10^{-23} \text{ joule deg}^{-1}$$

$$k_p = \frac{k}{m} = 1.35456 \cdot 10^{4} \text{ joule kg}^{-1} \text{ deg}^{-1}$$

9. Coefficient of viscosity μ_v, which characterizes the viscous stress of plasma. The values given in Table II have been derived from the equation:

$$\mu_v = 1.2 \cdot 10^{-17} \, T^{5/2} \text{ kg m}^{-1} \text{ sec}^{-1}$$

10. Nonelectric forces, such as the gravitational force. If the coronal flow is assumed to be stationary, radial, spherically symmetric, unaffected by electromagnetic forces, and without external heat sources, the acceleration of solar gravity only appears in the equation for $\vec{F}$:

$$g = 2.7398 \cdot 10^{2} \text{ m sec}^{-2}$$

However, when the coronal gas possesses a sufficiently large density, its self-attraction becomes important. The correct momentum equation for a stationary, spherical, self-gravitating gas in an external field is:

$$\frac{d}{dR}\left[\frac{1}{2}mu^2 - \frac{GM_{\odot}m}{r}\right] + \frac{1}{\rho}\frac{d}{dr}p = -\frac{4\pi Gm}{r^2}\int_{r_o}^{r} r'^2 \rho(r')\,dr'$$

11. Specific heat at constant volume C_V which characterizes the internal energy of the plasma. For a monatomic gas:

$$C_V = \frac{3}{2}\frac{k}{m}$$

After substitution of the previously given values, this equation yields:

$$C_V = 2.03184 \cdot 10^4 \text{ joule kg}^{-1} \text{ deg}^{-1}$$

12. Coefficient of diffusion D_d. Due to the temperature and pressure differences, the molecules in the coronal plasma diffuse from regions of high concentration to regions of low concentration, according to the equation:

$$\vec{K} = -D_d \nabla q \; .$$

In the kinetic theory, the value of D_d is given by the expression

$$D_d = 1.204 \frac{\mu_V}{\rho} \; .$$

The values obtained after the substitutions are contained in Table II.

13. Coefficient of heat conductivity λ, which characterizes the heat conduction of the coronal plasma. It is obtained from the relation:

$$\lambda = 6 \cdot 10^{-12} T^{5/2} \text{ joule m}^{-1} \text{ sec}^{-1} \text{ deg}^{-1} \; .$$

The results are contained in Table II.

14. Radiation constant a_r which characterizes both the radiation pressure and the radiation energy. Its value is

$$a_r = 7.5641 \cdot 10^{-16} \text{ joule m}^{-3} \text{ deg}^{-4} \quad .$$

15. Diffusion coefficient of radiation D_r which characterizes the radiation flux. It is derived from the relation

$$D_r = \frac{4}{3} \frac{a_r \, c \, T_r^3}{\kappa \rho}$$

In this expression, κ reduces to the electron-scattering mass absorption coefficient

$$\kappa_e = 0.019 \, (1 + X) = 0.0361 \text{ m}^2 \text{ kg}^{-1} \quad .$$

Due to the extremely low densities, an expression of the type:

$$\kappa = \kappa_o \rho \frac{T_r^{-3.5}}{\bar{t}}$$

becomes negligible. The radiation temperatures T_r, as evaluated from the effective temperature of the Sun $T_e = 5780°K$ by the square-distance law, are given in Table II. After substitution of the previously given values, the results contained in Table II are obtained.

16. Magnetic induction B . At the photospheric level, the Sun has general induction field of 10^{-4} weber m^{-2} (1 gauss) which leads to a radial field of $2.2 \cdot 10^{-9}$ weber m^{-2} ($2.2 \cdot 10^{-5}$ gauss) at a distance of 1 AU (214 $R_{\odot}$). At the orbit of the Earth, the inclination of the lines of force to the radial direction becomes 56°, approximately; the azimuthal component is $3.8 \cdot 10^{-9}$ weber m^{-2} ($3.8 \cdot 10^{-5}$ gauss). Inside the orbit of the Earth, the quiet-day interplanetary magnetic field is predominately radial, with a tendency to spiral toward the west at the orbit of the Earth. Beyond the orbit of the Earth, the spiral increases so that the field is predominately azimuthal. As the result of small differences in the quiet-day solar wind velocity around the Sun, considerable disordering can be expected.

17. Electric field strength E . Since the electric conductivity is high, neither an impressed nor an induced coronal field can be maintained. The electric field strength for the coronal plasma may be taken as 0.

18. The electron charge e . Its value is

$$e = 1.60207 \cdot 10^{-19} \text{ coulomb} .$$

19. The magnetic permeability μ_e . The characteristic value for the coronal plasma may be taken as the value in free space:

$$\mu_e = 4\pi \cdot 10^{-7} \text{ kg m coul}^{-2}$$

20. Dielectric constant ϵ. The characteristic value for the coronal plasma may be taken as the one in free space:

$$\epsilon = \frac{1}{36\pi} \cdot 10^{-9} \text{ kg}^{-1} \text{ m}^{-3} \text{ sec}^2 \text{ coul}^2$$

21. Electrical conductivity δ. It is obtained from the equation

$$\delta = 7 \cdot 10^2 \, T^{3/2} \text{ ohm}^{-1} \text{ m}^{-1}$$

The values are contained in Table II.

22. Friction coefficient α , which characterizes the effect of the viscous forces on the electromagnetic force. It is given by the relation

$$\alpha = \frac{n^2 e^2}{\delta}$$

Substituting the previously given values, the results contained in Table II are obtained.

These 22 quantities may be used to characterize the coronal plasma. The values in Table II, which belong to $T_o = 1.02 \cdot 10^6$ °K, may be selected as characteristic values.

CHARACTERISTIC VELOCITIES, FREQUENCIES AND LENGTHS

Some velocities, frequencies, and lengths may be derived from the 22 characteristic quantities, which are useful in obtaining nondimensional parameters.

1. Velocity of light: $c = \dfrac{1}{\sqrt{\epsilon \mu_e}} = 2.99793 \cdot 10^8 \text{ m sec}^{-1}$

2. Velocity of sound: $a = \sqrt{\gamma k_p T}$

Substituting

$$\gamma = \frac{5}{3}$$

and the previously given values, the results contained in Table III are obtained.

3. Velocity of the Alfven wave: $V_h = \dfrac{B}{\sqrt{\mu_e \rho}}$

Substituting the previously given values, the results contained in Table III are obtained.

4. The plasma frequency:

$$\omega_p = e\sqrt{\frac{n}{m\,\epsilon}}$$

It is the frequency of oscillation due to the electric field alone. Substituting the previously given values, the results contained in Table III are obtained.

5. The cyclotron frequency:

$$\omega_c = \frac{e\,B}{m}$$

TABLE III. CHARACTERISTIC VELOCITIES, FREQUENCIES, AND LENGTHS

$\frac{r}{R_\odot}$	a m sec^{-1} 10^5	V_h m sec^{-1} 10^5	ω_p sec^{-1} 10^5	ω_{ce} sec^{-1} 10^4	ω_{cp} sec^{-1}	ω_e sec^{-1} 10^{22}	ω_f sec^{-1} 10^{-13}	l_D m	l_f m 10^7	l_{le} m	l_{lp} m
1.58	1.84022	3.23918	57.76650	704.572000	3835.18000	14.54410	2295.14000	0.0245558	2.39231	0.0261183	47.9834
1.99	1.82172	3.50946	33.59030	444.154000	2417.65000	14.11000	798.93400	0.0418053	6.78678	0.0410155	75.3509
2.00	1.82172	3.53503	33.04000	439.723000	2393.54000	14.11000	774.15300	0.0425016	7.02551	0.0414288	76.1099
4.50	1.52490	3.11498	7.39749	86.858900	472.79700	8.27584	66.00600	0.1588980	$6.86393 \cdot 10$	0.1755610	322.5270
4.67	1.51748	3.04879	7.02591	80.650200	439.00200	8.15550	60.55780	0.1664880	$7.47919 \cdot 10$	0.1881560	345.6660
6.80	1.41669	2.47823	4.07607	38.038300	207.05300	6.63596	25.04210	0.2679150	$1.68755 \cdot 10^2$	0.3724380	684.2160
7.20	1.40388	2.40021	3.75174	33.929300	184.68600	6.45762	21.77630	0.2884430	$1.91863 \cdot 10^2$	0.4137660	760.1440
7.25	1.40147	2.39534	3.71017	33.462900	182.14800	6.42436	21.43490	0.2911740	$1.95101 \cdot 10^2$	0.4188130	769.4130
9.56	1.35228	1.99655	2.55817	19.245300	104.75700	5.77136	11.32700	0.4074750	$3.55220 \cdot 10^2$	0.7026550	1290.8700
10.00	1.33717	1.99471	2.33929	17.588900	95.74140	5.58006	9.78930	0.4406190	$4.05832 \cdot 10^2$	0.7602350	1396.6500
12.60	1.30556	1.80415	1.63094	11.078900	60.30570	5.19364	5.12385	0.6170490	$7.60413 \cdot 10^2$	1.1784200	2164.9000
13.70	1.28815	1.51824	1.63869	9.371270	51.01040	4.98864	5.38123	0.6059370	$7.13306 \cdot 10^2$	1.3745700	2525.2700
15.00	1.28815	1.72379	1.20411	7.817300	42.55170	4.98864	2.90607	0.8246260	$1.32144 \cdot 10^3$	1.6478200	3027.2600
214.00	0.78074	172.84900	0.05909	0.038407	0.20906	1.11070	31 539.70000	10.1842000	$7.42762 \cdot 10^8$	203.2810000	373453.00

It is equal to the angular frequency with which particles of mass m and charge e gyrate in a cyclotron. Substituting the previously given values, the results for the electron ω_{ce} and for the proton ω_{cp} contained in Table III are obtained.

6. The electrical decay frequency:

$$\omega_e = \frac{J}{D} = \frac{\delta}{\epsilon}$$

It prescribes the rate of electrical energy converted into joule heat. Substituting the previously given values, the results contained in Table III are obtained.

7. The friction frequency:

$$\omega_f = \frac{\alpha}{m\,n} = \frac{\alpha}{\rho} \quad .$$

It is the frequency of the oscillation if the force of friction is the only external force. Substituting the previously given values, the results contained in Table III are obtained.

8. The Debye length:

$$\ell_D = \sqrt{\frac{k\,T\,\epsilon}{n\,e^2}} \quad .$$

It is a measure of the distance over which an excess electrical charge may be appreciably different from zero. Substituting the previously given values, the results contained in Table III are obtained.

9. The mean free path:

$$\ell_f = 1.255\sqrt{\gamma}\;\frac{\mu_v}{a\rho}$$

Substituting $\gamma = \frac{5}{3}$,

$$\ell_f = 1.6202\;\frac{\mu_v}{a\,\rho} \quad .$$

It is a measure of the distance traveled between collisions of neutral particles. Since the typical length of the flow field is of the order of the mean free path, or smaller, the collision theory should be applied. Substituting the previously given values, the results contained in Table III are obtained.

10. The Larmor radius:

$$\ell_\ell = \frac{a}{\omega_c}$$

It is a measure of the radius of the helical path in a charged particle in a magnetic field. Substituting the previously given values, the results for the electron $\ell_{\ell e}$ and for the proton $\ell_{\ell p}$ are contained in Table III.

NONDIMENSIONAL PARAMETERS OF CORONAL DYNAMICS

The nondimensional parameters which characterize the flow of a fully ionized plasma are obtained by dimensional analysis from the 22 physical quantities given earlier in this report. In the system of units adopted, there are five independent fundamental units: mass, time, length, electrical charge and temperature. Then, by the π-theorem of dimensional analysis, 17 nondimensional parameters may be formed. The following five fundamental units are suggested:

$$\ell_o = 10^9 \text{ m}$$

$$\rho_o = 1.80 \cdot 10^{-16} \text{ kg m}^{-3}$$

$$t_o = 13\,330 \text{ sec}$$

$$e = 1.60207 \cdot 10^{-19} \text{ coulomb}$$

$$T_o = 10^6 \,^\circ\text{K}$$

Nondimensional parameters of all other quantities listed under Typical Physical Quantities are then derived in terms of these fundamental units. They will be denoted as Π_i.

1. The nondimensional parameter for the velocity:

$$\Pi_1 = \frac{u\,t}{\ell}$$

2. The nondimensional parameter for the pressure:

$$\Pi_2 = \frac{p}{\frac{\rho u^2}{2}}$$

3. The ratio of specific heats:

$$\Pi_3 = \gamma = \frac{C_p}{C_V}$$

4. The Mach number:

$$\Pi_4 = M = \frac{u}{a}$$

5. The Reynolds number:

$$\Pi_5 = R_e = \frac{\rho u \ell}{\mu_v}$$

6. The Prandtl number:

$$\Pi_6 = P_r = \frac{\mu_v C_p}{\lambda}$$

7. The Froude number:

$$\Pi_7 = F_r = \frac{u^2}{g\ell}$$

8. The Schmidt number:

$$\Pi_8 = \frac{\mu_v}{\rho D_d}$$

9. The relativity parameter:

$$\Pi_9 = R_c = \frac{u^2}{c^2}$$

10. The electric decay parameter:

$$\Pi_{10} = t\,\omega_e = t\,\frac{\delta}{\epsilon}$$

11. The plasma frequency parameter:

$$\Pi_{11} = t\omega_p = te\sqrt{\frac{n}{m\,\epsilon}}$$

12. The frictional frequency parameter:

$$\Pi_{12} = t\,\omega_f = t\,\frac{\alpha}{m\,n}$$

13. The electrical frequency parameter:

$$\Pi_{13} = R_E = \frac{E}{\mu_e\,u\,H}$$

14. The magnetic pressure number:

$$\Pi_{14} = R_h = \frac{V_h^2}{u^2} = \frac{\mu_e\,H^2}{\rho\,u^2}$$

15. The magnetic Reynolds number:

$$\Pi_{15} = R_\delta = u\,\ell\,\delta\mu_e = \frac{u\,\ell}{\nu_h}$$

$$\nu_h = \frac{1}{\delta\mu_e}$$

16. The radiation pressure parameter:

$$\Pi_{16} = \frac{p_r}{P} = \frac{a_r T_r^4}{3p}$$

17. The radiation flux parameter:

$$\Pi_{17} = \frac{D_r a_r}{\lambda} T_r^3$$

The following seven parameters have considerable influence on the flow problems of the corona:

Π_2, γ, M, R_e, P_r, R_h, and R_δ .

The remaining seven are unimportant parameters which may be neglected:

F_r, S_c, Π_{10}, Π_{11}, Π_{12}, Π_{16}, and Π_{17} .

Substituting the previously given values, the results for the important nondimensional parameters are given in Table IV.

TABLE IV. NONDIMENSIONAL PARAMETERS FOR THE CORONAL PLASMA

$\frac{r}{R_\odot}$	Π_2	$\Pi_4 = M$	$\Pi_5 = R_e$	$\Pi_6 = P_r$ 10^{-2}	$\Pi_{14} = R_h$	$\Pi_{15} = R_\delta$ 10^{19}
1.58	445.9450000	0.0516243	3.496270	6.77280	1162.580000	1.53521
1.99	121.6590000	0.0988077	2.358820	6.77279	380.133000	2.82200
2.00	116.5150000	0.1010030	2.329310	6.77279	369.106000	2.88472
4.50	5.2654100	0.4747850	1.120710	6.77282	18.511100	6.65736
4.67	4.8592600	0.4942400	1.070660	6.77279	16.524700	6.79625
6.80	2.1643000	0.7411640	0.711583	6.77281	5.570630	7.74194
7.20	1.9487400	0.7835430	0.661667	6.77279	4.761170	7.89263
7.25	1.9294700	0.7848900	0.651804	6.77281	4.741860	7.85198
9.56	1.2487000	0.9761290	0.445223	6.77280	2.287770	8.46466
10.00	0.9945580	1.0918600	0.435902	6.77280	1.866610	9.05209
12.60	0.5608700	1.4553100	0.310081	6.77280	0.901651	10.96430
13.70	0.7605530	1.2498500	0.283891	6.77280	0.889260	8.92412
15.00	0.3261180	1.9097200	0.234148	6.77280	0.491019	13.63560
214.00	0.0578067	4.5341600	0.989044	6.77280	2384.120000	4.36873

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APPROVAL

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By Klaus Schocken

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This document has also been reviewed and approved for technical accuracy.

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