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A CRITICAL EXAMINATION OF DISCRETE LATTICE AND DISPERSED BARRIER HARDENING

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ABSTRACT

This paper examines two theories of thermally activated deformation of metals; i.e., the lattice (Peierls-Nabarro) and dispersed barrier hardening. The predictions of two hardening theories show considerable similarities. Only forecasts with respect to the impurity content dependence of the yield stress and activation volume are significantly different. Available data on low temperature deformation in $\alpha$-Ti and bcc metals have been considered. The presently available theories of lattice hardening cannot explain strong dependence of the yield stress of $\alpha$-Ti and Mo on the interstitial content. In the case of other bcc metals, it is not possible to distinguish between these two types of hardening on an experimental basis. In fact, except in extremely pure materials, it is suggested that the two types of hardening may cooperate to control plastic flow.
INTRODUCTION

In recent years, a great deal of effort has been expended to identify thermally activated plastic deformation mechanisms which control dislocation dynamics in various materials at low temperatures. However, a great deal of controversy still exists with respect to the rate controlling mechanism in bcc metals and in such hcp metals as Ti at ambient temperature and below. While one group of investigators is confident that this mechanism of plastic flow essentially involves discrete lattice hardening, i.e., dislocations overcoming the Peierls-Nabarro barrier, others are equally convinced that dispersed barrier hardening due to interstitial impurity atoms is responsible for low temperature strengthening. In the latter mechanism, the barriers are the interaction between a moving dislocation and those interstitial impurity atoms in solution that possess a highly asymmetric strain field.

The purpose of this paper is to re-examine the theoretical frameworks, which were employed in interpreting thermally activated dislocation motion in bcc and hcp metals at low temperatures. We shall particularly query the validity of the assertion that plastic deformation in these metals is singly controlled by either dispersed barrier hardening or discrete lattice hardening, since the two hardening mechanisms show considerable similarities. It will be shown that the data presently available in these bcc and hcp materials cannot often differentiate the two hardening mechanisms under question. In the light of these discussions, results of recent as well as old experiments will be critically examined.

Because of extensive studies in Fe and Ti, the data on these metals will receive careful attention. As clarification of conflicting issues are sought, it will become apparent that critical experiments are indeed scarce.
We shall also discuss the possibility of a hardening mechanism in which two major types of hardening act cooperatively to control the overall plastic flow.

THEORETICAL BACKGROUND

We shall begin by summarizing the salient predictions of the dispersed barrier and lattice hardening theories. Detailed presentations of these theories, however, will not be made as review articles can be found elsewhere on the subjects.

1) Dispersed Barrier Hardening

In this mechanism, randomly distributed obstacles impede the motion of glide dislocations on slip planes. These obstacles include point defects and their aggregates, impurity atoms either singly or in cluster forms, precipitates, forest dislocations, dislocation loops and dipoles, etc. This mechanism has been known for a long time. Cottrell probably was the first to suggest a theory based on this mechanism, and Seeger developed the theory of the yield stress in terms of cutting of forest dislocations. However, Fleischer was the first to propose that atomic sized obstacles, such as self-interstitials and impurity interstitials, could impede dislocation motion and thus increase the yield stress significantly via dispersed barrier hardening. This mechanism predicts the following:

(D1) Increase in the thermally activated component of the yield stress, \( \tau^* \), is proportional to the reciprocal of the average distance between defects on slip planes. In terms of defect concentration, \( C \), this is equivalent to a linear relation between \( \tau^* \) and \( C^{1/2} \); namely,

\[ \tau^* \propto C^{1/2} \]  

(1)
(D2) Below the critical temperature $T^*$ is strongly temperature dependent. Relations between $T^*$ and $T$ are typically concave upward with a particular shape of the $T^*$ vs $T$ curve being dependent on how the dislocation-obstacle interaction potential is approximated. When normalized with respect to $T^*$ at $0^\circ$K ($T_0$) and $T_c$, most of $T^*$ vs $T$ curves fall in the shaded area in Fig. 1a. Ono previously discussed details of the interaction potentials. Note that these normalized $T^*$ vs $T$ curves are concentration independent.

(D3) Activation volume, $V^*$, is dependent on the defect concentration. At constant temperature (equivalent to a constant applied force on the obstacles), $V^*$ is proportional to the reciprocal of $C^{1/2}$, i.e.,

$$V^* \propto C^{-1/2}$$

Thus, $V^*$ can become very large for small values of $C$. On the other hand, $V^*$ can be small at large $C$ and at high stresses where the activation distance $d^*$ decreases.

(D4) $V^*$ decreases with increasing $\tau^*$ (or decreasing $T$). For a constant defect concentration, a $V^*$ vs $\tau^*$ relation can be obtained from specific model of dispersed barrier hardening. Fig. 2 shows 8 such relations, which are normalized at $\tau^*/\tau_0 = 0.1$ (See ref. 14 for details of these 8 relations.) Six of these curves are very similar to each other and the shaded area in Fig. 1b represents them. Two remaining curves behave somewhat differently although the main feature is preserved, i.e., $V^*$ decreases with increasing $\tau^*$.

(D5) Little change in $V^*$ is expected during the early stage of plastic deformation.
(D6) The total interaction energy between a dislocation and an obstacle is always proportional to the elastic constants.

It has been established unequivocally in several cases that the dispersed barrier hardening is applicable. These are (a) hardening of ionic crystals via divalent ion impurities and irradiation induced defects\(^\text{18}\), and (b) electron irradiation hardening of fcc metals near 20°K.\(^\text{19}\)

(2) Lattice Hardening

The Peierls-Nabarro stress has been known from the early days of dislocation theory, but its application to the yield stress was not advocated until about 1960. Seeger\(^\text{20}\) originally proposed a Peierls-Nabarro mechanism, in which dislocation motion is simply controlled by the attractive force between a pair of kinks\(^\text{21}\). The most sophisticated theory of this type applicable to low temperature plastic deformation processes of metals is that due to Dorn and Rajnak\(^\text{5}\). In this mechanism, stress-aided nucleation of kink pairs controls the overcoming of the Peierls hills. The shape of the Peierls hill is not known at present and is approximated by various models\(^\text{22}\). The one employed most often is a sinusoidal hill. The lattice hardening mechanism predicts the following:

(L1) \(\tau^*\) is independent of the defect concentration.

(L2) \(\tau^*\) strongly depends on \(T\) below \(T_c\), the details depending on models for the Peierls hill. Three of \(\tau^*\) vs \(T\) curves given by Dorn and Rajnak are shown in Fig. 1a.

(L3) \(V^*\) is independent of the defect concentration. Since \(V^*\) is primarily determined by the critical length of kink pairs, and upper limit of \(V^*\) (i.e., 40 to 100 \(b^3\)) exists.
V* depends on $\tau^*$ (or $i'$). A particular shape of $V^*$ vs $\tau^*$ curves is dependent on the model of the Peierls hill. Fig. 1b shows three such curves, taken from the work of Dorn and Rajnak.

V* changes little with plastic deformation, as kink nucleation processes are unaltered.

Activation energy for kink nucleation is proportional to elastic constants.

The lattice hardening mechanism definitely appears to control plastic deformation of covalent materials like germanium and silicon, where bond energies are directionally sensitive and high Peierls stresses are expected.

Comparison Between Dispersed Barrier and Lattice Hardening Mechanisms.

Before one can decide which of the two hardening mechanisms is responsible for a deformation process, one should recognize various limitations that are inherent in the theories and in experimental results. The most fundamental limitation is the lack of uniqueness.

A theory is said to be valid when all of its predictions are observed in experiments. This apparent agreement, however, does not preclude other theories from achieving an even better agreement with the experiments. That is, uniqueness of the agreement between a theory and experiments is not established simply by comparison of data with a single theory.

Upon critical scrutiny, it can be seen that some of the predictions of the two hardening mechanisms usually employed to discriminate between these models on the basis of experimental results cannot be used for this purpose. Clearly, predictions (L5) and (D5) are identical, so as are predictions (L6) and (D6). Close examination of Fig. 1a indicates that
the $\tau^*$ vs $T$ curves predicted by the two hardening mechanisms are very similar to each other. This is also true in the case of $V^*$ vs $\tau^*$ curves shown in Fig. 1b. As discussed below, accuracy of experimentally determined relations among $\tau^*$, $V^*$ and $T$ is poor. Thus, predictions (D2) and (L2) also cannot be used to distinguish the two hardening mechanisms. Similarly, (D4) and (L4) cannot be employed. At this point, it is worthwhile to remind the fact that the predictions (L2), (L4), (D2) and (D4) were derived assuming a particular shape of the dislocation-obstacle interaction and of the Peierls hill. These assumptions were made largely because of convenience. Therefore, the model employed does not necessarily represent any physical process. This is perhaps, a sufficient reason why these predictions cannot be employed to distinguish the two hardening mechanisms.

In the present context of low temperature deformation processes, the following experimental limitations are normally expected:

(1) The magnitude of the yield stress at 0°K, $\tau_o$, is often estimated by an extrapolation of data obtained above 77°K or higher. (Limited ductility and twinning are often responsible.) This introduces an error in $\tau_o$ amounting easily to ± 10%.

(2) Accurate determination of $T_c$, above which all the thermal component of the yield stress vanishes, is very difficult to attain due to the interference of such phenomena as strain aging, recovery, etc. Different investigators give estimates of $T_c$ which may differ over 100°K with $T_c$ typically ranging from 300°K to 600°K. $T_c$ can be estimated to ± 10% at best. This produces large uncertainties in the values of $\tau^*$ near $T_c$ and those of the activation energy at zero effective stress (this corresponds to either the total interaction energy or the critical kink energy).

(3) Scatter of yield stress data is relatively large (commonly over ± 5%).
Consequently, experimental $\tau^*$ vs $T$ curves following a normalization procedure may be accurate to only $\pm$ 20%.

Finally, purification of most bcc and hcp metals is quite difficult. Moreover, analysis of minute impurities is also a complicated task by itself.

It is now apparent that only two pairs of the predictions can be utilized to eliminate one hardening mechanism or the other, although these are not enough to positively identify one or the other, as pointed out previously by Fleischer. Those are predictions (D1) and (L1) and also (D3) and (L3). The dispersed barrier hardening predicts a strongly concentration dependent $\tau^*$ and $V^*$, while in the lattice hardening mechanism $\tau^*$ and $V^*$ are supposed to be concentration independent. These are the only means to distinguish the two hardening mechanisms.
DISCUSSIONS

(1) Low Temperature Hardening in Ti.

Let us begin our review by considering the situation as it exists in \( \alpha \)-Ti. A number of investigators have observed the effect of temperature on the yield and flow stress of this metal containing various levels of total interstitial content. Conrad\(^{12}\) summarized these works and suggested that the dispersed barrier hardening is responsible for Ti, since \( \tau^* \) extrapolated to 0\(^\circ\)K increases linearly with the square root of the total effective interstitial concentration that is \( \Sigma(\mathit{O} + 2\mathit{N} + 3/4\mathit{C} + \mathit{H}) \).

The range of the total (but not effective) interstitial concentration employed was from 100 ppm to 4,000 ppm. Even though a large scatter of data exists in the \( \tau_0 \) vs \( c^{1/2} \) plot and the use of an effective interstitial concentration is not well founded, it is very clear that \( \tau_0 \) strongly depends on the interstitial concentration of the range studied. This observation is consistent with the first prediction of the dispersed barrier hardening (D1), but definitely contradicts that of the lattice hardening (L1).

Levine\(^{7}\) studied the yield stress of zone refined Ti single crystals and concluded that the lattice hardening primarily controls the plastic flow. One of his arguments is the fact that the yield stress observed in his experiment, in which a starting material containing 100 ppm of total interstitials is used, is similar to that observed by Spangler and Herman.\(^{26}\) The total interstitial concentration of Ti in the latter study is said to be less than 100 ppm. This argument does not make sense, unless the total interstitial concentration of Ti used by Spangler and Herman is substantially less than 100 ppm, say 10 ppm. This is highly unlikely, in view of extreme difficulties of purifying this metal. This argument of Levine cannot rule out the dispersed barrier hardening, which predicts hardening of correct magnitude. No other data is available below 100 ppm and therefore, conclusions cannot be drawn for purer Ti in regard to the predictions (D1) and (L1).
The observed activation volume $V^*$ also depends strongly on interstitial concentration between 100 ppm and 4,000 ppm. For example, $V^* = 7 \, b^3$ for Ti with interstitial of 4,000 ppm is found at 77°K, while $V^* = 50 \, b^3$ is obtained for zone refined Ti containing 100 ppm at the same temperature. Thus, in these materials, prediction (D3) is tenable but (L3) must be eliminated from contention. Again, no data is available below 100 ppm, and conclusions cannot be drawn at present with regard to purer material.

Another observation warrants further comment. The values of $V^*$ do not appear to vanish as test temperature is decreased; e.g., in the zone-refined Ti, $V^* = 25 \, b^3$ is found at 4°K. At first sight, this might appear to conflict with the dispersed barrier hardening theories. However, this apparent difficulty is solely due to the nature of approximated force-distance curves. However, there is actually no physical requirement that $V^*$ must vanish at 0°K. That is, the activation distance $d^*$ may remain finite at the maximum applied force on the obstacle, as in the case of the square potential.

As pointed out in the previous section, four pairs of predictions, that is (D2) & (L2), (D4) & (L4), (D5) & (L5), (D6) & (L6), cannot distinguish between the dispersed barrier and lattice hardening mechanisms. The last two predictions have not received much attention in Ti and no more comment is made here. On the other hand, the prediction (L2) and (L4) have been employed as the main arguments by Levine for the lattice hardening. Conrad cited the prediction (D2) as a supporting argument. We shall demonstrate below that the data employed by these authors can support both mechanisms just as well and cannot establish one of the two mechanisms as the rate controlling process.

In Fig. 3a, the normalized $\tau^*$ vs $T$ data are plotted, which were taken from Levine's and Conrad's work. Although the interstitial concentration
differs vastly, one can hardly separate the two sets of data. The solid line
in Fig. 3a represents a $\tau^*$ vs $T$ curve as predicted by the Dorn-Rajnak theory
of the lattice hardening. A reasonable agreement between the data points and
the curve indicates that the Conrad's data can be represented in a Dorn-Rajnak plot.
Recalling the closeness of $\tau^*$ vs $T$ curves predicted by the two mechanisms,
Fig. 3a also represents a good agreement between the Levine's data and the theoreti-
cal curves of the dispersed barrier hardening. (If $T_c$ is increased by only
20°C, the agreement between theory and experiment becomes indeed excellent.)
The data for $V^*$ vs $\tau^*$ from these two experiments are shown in Fig. 3b, where
the solid curve represents one of the Dorn-Rajnak prediction. Again, it is not
at all difficult to claim reasonable match between theory and experiment,
regardless of the hardening mechanism employed. From these discussions, it has
become apparent that either of the hardening mechanisms can account for the
observed data. The predictions (D2) and (L2), and (D4) and (L4) should not
be used to justify or eliminate one of the two mechanisms in question.

In the preceding discussion we have been comparing experimental observations
in $\alpha$-Ti with the Dorn-Rajnak approach to the Peierls-Nabarro mechanism. This
however is not the only approach available. Another approach which has been
used is due to Seeger. Guyot and Dorn have considered the Seeger theory
from an analytical standpoint and have concluded that it is improper. On the
other hand, Conrad has calculated the activation energy (at $T_c$) for a number
of bcc metals using both approaches and found the results to be similar. In
Table I, we have estimated the double kink nucleation energy from both view-points
in the case of $\alpha$-Ti. Inspection of this table indicates that the values of
$2U_k$ obtained by Dorn-Rajnak approach for the case of a screw dislocation
(method 2) are similar to the quantities, $H$, determined by experiment (method 1 or 3),
irrespective of the level of interstitial content. Note, however, that this
agreement results from the use of different values of $v_p$ (or $\tau_0$). In contrast, the Seeger approach only yields a reasonable value for $2\mu$ if one considers Ti of commercial purity. If the Peierls mechanism is to operate in this material, one would certainly expect better agreement with experiment and theory in a higher purity metal than in that of commercial purity. These results indicate that the Seeger approach is not appropriate in this problem.

On the basis of the available data in $\alpha$-Ti, if one must choose between a dispersed barrier hardening model and the presently constructed lattice hardening approaches, one has little choice but to conclude that the low temperature deformation of this metal is, at least within the interstitial concentration range studied so far, not controlled by the latter mechanism $^{29}$. Extension of experimental work using Ti of lower interstitial concentration is highly desirable. Further study of dislocation dynamics must be sought at different levels of controlled interstitial addition to high purity starting stock.

(2) Iron and Other BCC Metals.

A number of investigators have considered the low temperature deformation data presently available for bcc metals. These deliberations have resulted in the formation of two distinct schools of thought with regard to the rate controlling mechanism. Conrad and others who favor the lattice hardening mechanism generally cite the following arguments: $^{1-9,30}$

(1) The apparent insensitivity of the thermally activated component of the yield stress and the activation volume to overall impurity interstitial content variation.

(2) The substantial agreement between experiment and the Dorn-Rajnak approach to the Peierls-Nabarro theory with regard to the $\tau^*$ dependencies of $T$ and $V^*$. 
(3) The insensitivity of the thermally activated plastic flow process in these metals at low temperatures to extensive neutron irradiation.

(4) $V^*$ is insensitive to plastic deformation.

(5) The existence of certain universal relationships between the activation parameters observed for this process in the various bcc metals (i.e., $V^*$, $\tau_0$, $H_0$ etc.) and the elastic properties of these metals; e.g. the maximum activation energy $H_0$ for this type of low temperature hardening in $T_o$ was found to vary linearly with Young's modulus which in turn was varied by additions of large amounts of substitutional solutes.

The arguments (2), (4), and (5) are essentially the same as the predictions (L2), (L4), (L5) and (L6). These arguments are untenable for the purpose of distinguishing the two hardening mechanisms as noted earlier. The arguments (1) and (3) are equivalent to the prediction (L1) and (L3) and need a closer examination.

Argument (3) is not decisive at all. This argument is based on the assumption that defects produced via neutron irradiation can influence the yield stress only through the dispersed barrier hardening. This assumption has not been well established yet. In terms of the dispersed barrier hardening, two alternative interpretations are readily available. Firstly, since the starting materials contain sufficiently large number of interstitial obstacles, any addition by neutron irradiation will produce only insignificant change in the yield stress. Second, the small dislocation loop type obstacles that are produced by neutron irradiation interact so strongly with glide dislocations that only the athermal component is affected as a result.³¹

Fleischer and others who favor the dispersed barrier hardening mechanism emphasize the following arguments:¹⁰,¹¹,¹³

(i) $\tau^*$ depends on the interstitial impurity content in solution.
A few examples of $\tau^* \propto C^{1/2}$ have been cited.

(ii) Relations between $\tau^*$, $V^*$ and $T$ agree with those predicted by theory.

(iii) The activation volume at small $\tau^*$ is generally too large to be consistent with the lattice hardening scheme. The dispersed barrier hardening encounters no such difficulty.

The argument (i) corresponds to the prediction (D1). The argument (ii) corresponds to the predictions (D2) and (D4) and cannot be regarded as evidence for dispersed barrier hardening, as discussed earlier. The argument (iii) is consistent with a part of the prediction (D3). However, it cannot be regarded as strong evidence, because the main idea of the prediction (D3), namely $\tau^* \propto V^{*-1}$, has not been established. Moreover, a number of difficulties is present in measurement of $V^*$ via strain-rate cycling that has been employed most commonly to obtain the data under consideration. One of such difficulties is the change in mobile dislocation density upon strain-rate cycling. Perhaps the most important one, especially at low values of $\tau^*$, is dynamic strain-aging due to the interstitial diffusion to dislocations. This would reduce strain rate sensitivity of the flow stress, raising the apparent values of $V^*$ erroneously. In fact, strain-aging has been previously cited as a problem in determining $T_c$.

Thus, only the argument (i) is useful in order to distinguish the two hardening mechanisms.

Effects of impurity interstitial on the yield stress of iron and other bcc metals have been studied extensively. Nonetheless, consensus has not been achieved. For instance, reviewing earlier works, Aleen collected those yield stress data in iron which indicate strong dependence on the carbon content. On the other hand, Conrad concluded that the yield stress extrapolated to $0^\circ K$ is independent of the impurity content in the bcc metals.
In considering various observations, it should be noted that the solubility of interstitials in bcc metals is rather limited. For example, the solubility of carbon in iron decreases to less than 10 ppm at 350°C. The precaution to keep the interstitials in solution has not necessarily been employed. In some experiments, it is probable that precipitation of carbide or clustering of carbon atoms in iron have occurred, keeping the interstitial content in solution relatively constant. The situation in other bcc metals is similar. The claim of impurity insensitivity of the yield stress is not accompanied by a proof that the actual impurity interstitial content in solid solution is varied significantly below the solubility limit. In regard to the saturation of carbon in iron, McMahon has shown that, when 50 ppm of carbon is uniformly distributed in iron by quenching it from above 500°C, intergranular brittleness develops. Only when some carbides are allowed to form, is the ductility restored in the room temperature testing. It appears, therefore, that much of the earlier data on the interstitial effects of the yield stress of polycrystalline iron must have been obtained in the presence of carbide particles; that is, interstitial concentration in solution was always that of saturation.

Several recent experiments have taken advantage of a ZrH₂ treated hydrogen purification system, which was developed by Stein et al, for the reduction of carbon content in iron. Stein and Low showed that the yield stress (at 0.01% offset) of iron single crystals, when measured at 77°K, decreases from 15 kg/mm² to 11 kg/mm² after the ZrH₂-hydrogen purification. The carbon content of the purified iron was reduced to 0.005 ppm from the initial value of 44 ppm. While their rather limited observations can be fitted to a \( \tau^* \sim C^{1/2} \) relation, a careful examination of these results indicates that more data will be necessary to make the fit significant. By varying the nitrogen concentration in iron single crystals, Keh and Nakada studied its effect on the
yield stress of iron. They found that the upper yield stress and the proportional limit increase with the nitrogen content in the range of 3 to 360 ppm, in agreement with Stein and Low.11

On the other hand, Keh and Nakada37 observed that the lower yield stress (or the flow stress at about 10% shear strain) is insensitive to the variation in the nitrogen content. They also showed that the proportional limit measured at 113°K can be reduced from 17 kg/mm² to 8 kg/mm² by prestraining at room temperature. They concluded that the lower yield stress should correspond to the lattice friction while the concentration dependence of the proportional limit (or the upper yield stress) is due to changes in the initial mobile dislocation density. Others have also reported that the yield stress of iron and its dilute alloys is insensitive to the interstitial content.38,39

These results do not lead us to an unambiguous conclusion on the interstitial effect in iron. This may be due to the presence of residual oxygen interstitials of about 20 ppm in even the purest iron specimens.40 Mechanical twinning also obscures the yield stress measurement at lower temperatures. Further investigations are needed to clarify the role of various variables.

In the case of molybdenum single crystals, investigations of Lawley et al.41 and Stein13 clearly show that the low temperature yield stress decreases with greater purification. Lawley et al.41 determined the critical resolved shear stress (CRSS) of zone refined Mo to be 53-62 kg/mm² at 4.2°K and 26-28 kg/mm² at 77°K. Stein13 treated zone refined Mo with the ZrH₂-hydrogen purification system. The CRSS of ZrH₂-treated Mo single crystals of five different orientations was 25-55 kg/mm² at 20°K and 11-39 kg/mm² at 77°K. It can be concluded that \( \tau_0 \) for Mo definitely increases with higher impurity concentration. The dispersed barrier hardening is favored for this part of \( \tau_0 \) which depends on the impurity content, since the prediction(L1) is
obviously not obeyed. It should be noted that $\gamma_o$ is quite large even for the purest molybdenum. Further investigation is necessary to ascertain whether this remaining $\gamma_o$ is concentration dependent or not.

In other bcc metals, critical experiments are still inadequate. More effort is needed before we can identify hardening mechanism.

(3) Other Possible Hardening Mechanisms

During preceding discussions of this paper, only two hardening mechanisms are emphasized; i.e., the lattice and dispersed barrier hardening. Recalling the comment on "uniqueness", we have to continue to search for a "best possible" mechanism. Besides the two hardening theories, several others have been considered. Although we will not discuss them in detail, they are listed below.

1. Breaking away from an interstitial atmosphere, 42
2. Non-conservative motion of jogs, 43
3. Overcoming interstitial precipitates, 44
4. Cross-slip 45 and
5. Sessile-glissile splitting. 46, 47 The last process is a promising mechanism. However, its quantification appears difficult, since dislocation-dislocation interaction in the core region must be taken into account.

Finally, we wish to point out another possible hardening scheme. The presently available lattice hardening theory considers the nucleation of a double kink on an infinitely long dislocation segment. However, when the total impurity concentration is more than 10 ppm, the length of a dislocation segment may no longer be regarded as infinite. The double kink could, then, be nucleating on an impurity limited loop length. This concept was initially proposed by Mason in 1955. 48 He discussed the details of such a Peierls mechanism in an attempt to rationalize the Bordoni internal friction peak in some fcc metals. Conrad and Hayes 49 also suggested in passing that thermally activated plastic flow in bcc metals could obey this model. Several investigators 4, 8 also considered this problem, but no satisfactory treatment of this concept has
been given. Equilibrium kink energy, $U_k$, increases with decreasing loop length. This suggests that energy of double kink nucleation is also a function of the loop length, which in turn is related to the impurity concentration. Thus, the thermally controlled part of the yield stress should increase with higher impurity content. It remains to be seen whether rigorous formulation would predict the $C^{1/2}$ dependence of $\tau^*$. 

It appears feasible to develop a cooperative hardening mechanism, combining the lattice and dispersed barrier hardening. Figure 4 illustrates the details of one of such models. When several kinks have been nucleated ($F^2$; 4B) a dislocation segment reaches a critical configuration at A (Fig. 4C). With or without the aid of thermal activation, an obstacle at A is overcome in the manner of the dispersed barrier hardening (Fig. 4D). If an analytical model can be constructed for this model, this would represent a unification of the two hardening mechanisms and close the controversy now existing. Although Fleischer$^{24}$ raised objections to this type of rate controlling process, there is no fundamental reason why this should not be operative. One should recall that the yield stress of even the purest iron and molybdenum retains large temperature dependence. This remaining temperature dependence may arise from the overcoming of the Peierls-Nabarro hills.$^{50}$ The concentration dependent part of the yield stress could then be due to either dispersed barrier hardening or one of the cooperative mechanisms.

Recent observations of dislocation structures of deformed $\alpha$-Ti and its alloys render strong support to the cooperative hardening scheme. Both Cass$^{51}$ and Pittinato and Fredrick$^{52}$ found numerous parallel screw dislocations and zigzag dislocations of edge character, indicating a large Peierls energy. Since the yield stress of $\alpha$-Ti is strongly dependent on the impurity content, the lattice hardening approach must incorporate effects of impurity limited loop length.
CONCLUSIONS

(1) The lattice and dispersed barrier hardening theories of thermally activated deformation of metals are examined. Both theories predict similar relationships among $\tau^*$, $V^*$ and $T$. Neither $\tau^*$ vs. $T$ nor $V^*$ vs. $T$ relations can be employed as evidence in order to distinguish the two hardening mechanisms. The lattice hardening, as is now formulated, is independent of the interstitial concentration, while the dispersed barrier hardening predict the $\tau^* \propto C^{1/2}$ and $V^* \propto C^{-1/2}$ dependencies. Only these forecasts are significantly different.

(2) Available experimental data on low temperature deformation in $\alpha$-Ti and bcc metals are considered. The yield stress of $\alpha$-Ti and Mo depends on the interstitial concentration, disfavoring the lattice hardening. On the other hand, definitive evidence for such dependence is not available for iron and other bcc metals. The $V^*-C$ relationship has not been adequately studied. Thus, it is not possible to distinguish between the two types of hardening on an experimental basis.

(3) An apparent agreement between the predictions of a theory and experimental observations does not establish uniqueness of the agreement. One must continue to search for a best possible theory and to strive for refinement of experimental data. In fact, except in extremely pure materials, it is suggested that two types of hardening may cooperate to control plastic deformation.
Figure Captions

Fig. 1 The stress dependencies of the activation energy and activation volume, which are predicted by dispersed barrier hardening and by Dorn-Rajnak theory of lattice hardening. Shaded areas represent predictions of most of dispersed barrier hardening theories as reviewed by Ono. The activation volume for dispersed barrier hardening is normalized at $\gamma^*/\gamma_o = 0.1$.

Fig. 2 The stress dependencies of the activation volume for eight dispersed barrier interaction potentials (for details of the potentials, see reference 14).

Fig. 3 Comparison of experimental data on prismatic slip in $\alpha$-Ti with theoretical predictions of Dorn-Rajnak theory. Levine's data are shown by triangles and Conrad's data by circles. Solid lines are the Dorn-Rajnak curves with $\alpha = 1$.

Fig. 4 A cooperative mechanism of thermally activated dislocation movement.
Table Caption

Table I. Estimation of double kink nucleation energy for the Peierls barrier in α-Ti.
REFERENCES

20. A. Seeger, Phil. Mag. 1, 651 (1956).
21. Guyot and Dorn criticized this approach. Later, we will show that the Seeger mechanism leads to a quite spurious conclusion when applied to the case of γ-Ti.
22. An equivalent approximation procedure in the dispersed barrier hardening theories is to adopt an interaction potential for a dislocation-obstacle interaction. Most models used for the interaction potential, as well as for the Peierls hill, have no justification at all with regard to physical reality.

23. Recently, Alexander and Haasen observed dislocation arrangements in deformed germanium to be essentially identical to that in soft fcc metals (Cand. J. Phys. 45, 1209 (1967)). Thus, the word "definitely" is perhaps too strong.


25. We use ppm in weight throughout this paper.


29. This does not imply, however, that the dispersed barrier hardening is the only rate controlling mechanism. As will be discussed later, there is still a possibility of other processes being responsible.


40. Tomalin and Stein, (Trans. AIME 233, 2056 (1965)) reduced the oxygen content of iron single crystals to 6 ppm by zone refining. After the ZrH₂-hydrogen treatment, the yield stress at 77°K was 16-22 kg/mm². This value is higher than that obtained by Stein and Low 11, suggesting that interaction between various interstitials must also be considered.
44. B. L. Mordike and P. Haasen, Phil. Mag. 7, 459 (1962).
52. G. F. Pittinato and S. F. Frederick (to be published).
### Table I

**ESTIMATION OF DOUBLE KINK NUCLEATION ENERGY**

**FOR THE PEIERLS'S BARRIER IN $\alpha$Ti**

<table>
<thead>
<tr>
<th>DORN-RAINAK APPROACH</th>
<th>TOTAL INTERSTITIAL CONTENT</th>
<th>TOTAL INTERSTITIAL CONTENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $H = \frac{1}{2}k + 2u_k$</td>
<td>0.9 ev</td>
<td>1.2 - 1.6 ev</td>
</tr>
<tr>
<td>(2) $2a_k + 2L_0\sqrt{\frac{\left(\frac{2.67}{L_0}\right)^2 - 1}{2}}$</td>
<td>0.3 ev ($a \cdot b$ FOR EDGE DISLOCATIONS)</td>
<td>0.9 ev ($a \cdot b$ FOR EDGE DISLOCATIONS)</td>
</tr>
<tr>
<td>WHERE $L_0 = \Omega / 6$, AND FOR $a = 1$, THE ABOVE REDUCES TO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2\pi a_k - 5.7(\frac{p_d b}{m_L})^{1/2}$</td>
<td>0.6 ev ($a = 1.59b$ FOR SCREW DISLOCATIONS)</td>
<td>1.8 ev ($a = 1.59b$ FOR SCREW DISLOCATIONS)</td>
</tr>
<tr>
<td>$\tau_p = \tau^*$ ($t = 0^1$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) $2a_k = \frac{kx_0}{m_a - m_b} \ln \frac{\frac{y_2}{y_1}}{\frac{x_2}{x_1}}$ (I.E., DORN-RAINAK $\pi^*$ VS T EXTRAPOLATION TECHNIQUE)</td>
<td>0.7 ev</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SEEGER APPROACH</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $2H = \frac{1}{2}k \left(1 + \frac{1}{16} \ln \frac{p_d}{m_L}\right)$ OR $\frac{\Delta H}{\Delta \Phi} = \frac{1}{8} - \frac{2u_k}{\Phi^*}$</td>
</tr>
</tbody>
</table>

$+ \text{ ASSUME THIS VALUE TO BE } 2u_k$
Figure 2.
Figure 3.
Figure 4.

A) Dislocation bowed out by applied stress so that it lies in part along Peierls valley with active loop lengths in these valleys determined by interstitial distribution in the slip plane. (i.e., single kink motion is assumed to be athermal.)

B) Thermally activated nucleation of a double kink in length L (interstitial pinning is assumed to be of the same order of magnitude but slightly more energetic than overcoming of Peierls barrier).

C) Dislocation of length L moves forward to the next Peierls valley after successful double kink nucleation.

D) After some time, T, depinning will occur at A. If aPeierls is very much less than the deforming energy then the dislocation length L may actually move forward one more time by double kinking prior to this event.
This paper examines two theories of thermally activated deformation of metals; i.e., the lattice (Peierls-Nabarro) and dispersed barrier hardening. The predictions of two hardening theories show considerable similarities. Only forecasts with respect to the impurity content dependence of the yield stress and activation volume are significantly different. Available data on low temperature deformation in α-Ti and bcc metals have been considered. The presently available theories of lattice hardening cannot explain strong dependence of the yield stress of α-Ti and Mo on the interstitial content. In the case of other bcc metals, it is not possible to distinguish between these two types of hardening on an experimental basis. In fact, except in extremely pure materials, it is suggested that the two types of hardening may cooperate to control plastic flow.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
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<tr>
<td></td>
<td>ROLE</td>
<td>ROLE</td>
<td>ROLE</td>
</tr>
<tr>
<td></td>
<td>wt</td>
<td>wt</td>
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<tr>
<td>1. Dislocations</td>
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<td>2. Thermally Activated Plastic Deformation</td>
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<td>3. Dispersel Barriers</td>
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<td>4. Peierls-Nabarro Stress</td>
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<td>5. Titanium</td>
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<td></td>
<td></td>
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<tr>
<td>6. Iron</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>7. Molybdenum</td>
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