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HELICOPTER NOISE — BLADE SLAP

Part 1: Review and Theoretical Study

by John W. Leverton

Prepared by
UNIVERSITY OF SOUTHAMPTON
England
for Langley Research Center

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PREFACE

This report reviews the blade slap investigation carried out in connection with the N.A.S.A. Helicopter Noise Contract. Although it is appreciated that many of the problems discussed in this report are unanswered, this account of the investigation attempts to outline the present state of the art as regards the understanding and interpretation of the various aspects of blade slap.

The experimental results obtained during the investigation are not included in this report since they will be the subject of a separate report to be issued in the near future.

Similar reports on the other aspects of helicopter noise are being prepared and these will be issued during the next six months.

HELICOPTER NOISE - BLADE SLAP

PART 1: REVIEW AND THEORETICAL STUDY

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ABSTRACT

This report reviews previous work on the topic of blade slap and the various mechanisms for its generation. It is concluded that blade/tip vortex interaction is the most likely noise producing mechanism. A theoretical approach to the problem leading to the development of a blade slap factor (BSF), which will give an indication of the severity of blade slap likely on any helicopter, is also included. A comparison between some subjective assessments of the blade slap noise and the BSF is also given.

1 INTRODUCTION

Blade slap is the colloquialism which has been applied to the sharp cracking sound associated with helicopter rotors. Although the term 'blade slap' denotes this one type of noise, it varies considerably in intensity and quality with the type of helicopter and flight condition.

When blade slap occurs, not only is it predominant over any other rotor-craft noise, but because of its characteristics it is most objectionable. The degree of annoyance from it has grown with the increase in size and all up weight (A.U.W.) of single rotor helicopters and the development of large tandem rotor helicopters. The importance and need for an understanding of this phenomenon has been further increased by the introduction into service of a number of helicopters which produce blade slap in practically all flight regimes. Also in the present generation of helicopters the noisy piston engines have been replaced by quieter turbine engines, with the result that not only is blade slap the first noise heard, but it positively identifies the approaching aircraft as a helicopter. This noise problem has manifested itself internally as well as externally, and is of importance both militarily and commercially. In military operations it rules out the possibility of a surprise attack and causes mental fatigue to the crew and combat troops. Commercially, inter-city operators have been forced to modify their flight paths to avoid subjecting residential areas to these high noise levels (1). In addition it adds to passenger apprehension and annoyance.

SYMBOLS

a_o	lift curve slope
b	half chord of blade (C/2)
B	number of blades
BSF(E)	blade slap factor based on energy of impulse
BSF(P)	blade slap factor based on power of impulse
C	blade chord
c_o	speed of sound
E_B	'bang' energy
f	frequency
F_i	fluctuating force
GW	gross weight of helicopter
h	dipole separation
I	acoustic radiated intensity
k	wave number
L	lift per blade
Ls	loading per unit span
L(s)	blade loading at any point s
L_m	loading at any point s due to m^{th} harmonic of gust
m	harmonic order of gust (i.e. 1, 2, 3 etc.)
$(p - p_o)$	acoustic pressure
r	distance of observation point from acoustic source
r_c	radius of vortex core
r_p	radius at which motion in a vortex is considered to change from a rigid body to potential flow

r_v	radial distance from vortex centre
$(r_o - r_1)$	span length over which gust acts
R	radius of blades
s	any point measured from beginning of gust
t	time
V	velocity of rotor blade
V_t	tangential component of the induced velocity in a vortex
V_T	blade tip velocity
V_M	maximum tangential velocity in a vortex
w	rotor disc loading
W	peak velocity amplitude of gust
W_B	'bang' power
W_m	velocity amplitude of m^{th} harmonic of gust
x	non-dimensional length of gust (= Y/b)
$\overline{x_n}$	normal coordinate of observation point from mid-point of blade area under consideration.
x_i	coordinates of any point
x_n	distance measured in direction normal to blade chord
Y	actual length of gust
ρ	density
Γ	strength of circulation
ν	kinematic viscosity
Ω	angular frequency
θ	pitch angle

ABBREVIATIONS

$k_m \;=\; \dfrac{2\Pi b\ m}{Y}$

$\phi \;=\; \tan^{-1} k_m/0.13$

$\alpha \;=\; \tan^{-1} k_m$

2 REVIEW OF PREVIOUS WORK

Although a large number of papers have been published on helicopter and rotor noise, until recently there was only one paper (2) which treated the subject in any detail. Many authors (3,4,5,6) agree, however, that "when it occurs" blade slap is the loudest type of rotor noise.

A large section of a report of Sikorsky's work, issued in October 1966 by USAAVLABS (10) was concentrated on blade slap, and more recently H. Sternfeld of Boeing/Vertol presented a paper (11) solely connected with the practical aspects of the blade slap. To date, however, the only attempt at a quantitative study of the problem appears to have been the papers published by F.W. Taylor and the author (12,13). Bell (2) showed, using their two bladed HU-1A helicopter, that on a narrow band analysis blade slap caused an increase in pressure spectrum level over a frequency range of 20-1000 Hz, with the maximum intensity around 200 Hz. They also showed that when blade slap occurred there was an increase in the modulation depth of the noise in the audio range at the blade passing frequency. The paper dealt with both single and tandem rotor helicopters and the authors expressed the view that there were two mechanisms for its generation. At low speeds, they thought that the noise was caused by the rapid change in angle of attack of the blade as it encountered the wake of a previous blade; while at high speed it was considered that it was more likely produced by local shock waves on the advancing blade. Bell,however, freely admitted that there were still many unanswered questions and a lack of a basic understanding of the origins of the noise. Their comments agree in general with those obtained during the investigation conducted by the author.

A paper by Davidson and Hargest (5) stated that blade slap did not add to the disturbance or annoyance value of a helicopter. This is in direct conflict with the views expressed in the majority of other papers and the author's observations. It can only be assumed that these comments were based on information derived from British built helicopters such as the Whirlwind and without reference to the latest generation of helicopters. Blade slap is defined by Davidson and Hargest as a "random noise amplitude and freuqency modulation", but it is difficult to interpret the meaning of this statement or to tie it up with information derived during this investigation.

Bell (7) have reiterated their ideas on the generation of blade slap outlined in reference 2, in connection with their UH-ID IROQUOIS helicopter, which apparently produces two types of blade slap. At low forward speeds they suggest it is due to blade/vortex interaction, while at high forward speeds they feel it results from local supersonic air flow which can occur near the tip of the advancing blade. Vertol (7) originally postulated that blade slap was due to separated flow and that another type of noise which they termed 'blade bang' resulted from shock wave formation. They contended that this 'blade bang' was the major noise source and the predominant mechanism on their tandem helicopters. In the light of this it would appear that Vertol have termed 'blade bang' the noise mechanism mormally referred to as blade slap. Since then, Vertol has made a full investigation into blade slap (or bang) on their helicopters (11) and have shown that in fact it arises from the interaction of a rotor blade and the tip vortex shed by some other blade.

Schlegl (9) has associated blade slap with the retreating blade at the 270° position, this being the region where the angle of attack is greatest and the occurrence of stall most likely. This mechanism is reiterated and discussed in detail in Sikorsky's recent report (10). The report also discussed the phenomenon of compressible drag divergence and the resulting formation of shock waves, and suggested that these can contribute to the blade slap noise. Except for putting forward the stall hypothesis, this paper is in general agreement with that of Bell (2) and Vertol (11).

3 BLADE SLAP MECHANISMS

The three main mechanisms postulated for blade slap in the literature are summarized below:-

1. Fluctuating forces caused by blade/vortex interaction.

2. Fluctuating forces resulting from stalling and un-stalling of the blade.

3. Shock wave formation due to local supersonic flow: it is suggested that this is either (a) a direct result of operating a blade at a high tip speed or (b) caused by a blade/vortex interaction.

At the present time detailed information on the above mechanisms is still limited and, therefore, it is practically impossible to state from a scientific basis which is the most likely mechanism. Certain deductions can, however, be made by considering the possible occurrence of the various mechanisms on actual helicopters and correlating them with test results and flight observations. The best approach appears to be to study the idealised cases in the first instance.

(1) Blade/vortex interaction.- Mechanisms 1 and 3(b) are illustrated in figure 1, which is reproduced from the M.Sc. thesis of F.W. Taylor(14). This figure shows possible blade/vortex interactions when the core is parallel to the span of the blade. If an interaction as given by path 'A' occurs, mechanism '1' is clearly produced. In this case the blade is subjected first to a 'down velocity' change and then to an 'up velocity' change, which produces a rapid change in angle of attack and subsequent impulsive loading. If path 'B' is followed a similar loading fluctuation occurs, but, of course, at a much smaller magnitude. In addition to this, however, it is possible that the velocity of the blade is such that when combined with the tangential velocity of the tip vortex, it exceeds the sonic velocity and produces a local shock wave. Path 'C' would produce a similar fluctuating force variation to that experienced by a blade travelling along path 'B', but in this case there is very little chance of sonic flow being produced. Thus mechanism '1' could result from interactions of the form indicated by paths 'A','B', and 'C', and mechanism '3b' from a blade/vortex intersection of the type indicated by path 'B' only.

In the type of intersection described above, large changes in the angles of attack occur and it is suggested that the blade would stall. This fluctuation takes place, however, very quickly and it is unlikely that stall would occur in this instance (see discussion below on stall).

(2) Blade stall.- The 'stall' hypothesis is much more difficult to visualise. It is well known from fan and propellor studies that there is an increase in the broadband type of noise when a blade section is stalled. Even if the blade could be stalled and un-stalled to produce a burst of broadband noise, this would not have the impulsive nature of blade slap. The impulse is of a short duration and must therefore be a direct results of rapid load (lift) or force fluctuation. Since typical impulses on a single rotor helicopter are of the order of 1-5 ms, the stall sequence or change in lift must therefore occur in an azimuthal blade movement of a few degrees. A study of recent papers concerned with stall (15,16, 17,18) illustrates the complexity of the problem and suggests that it is impossible at the present time to obtain the necessary details required on stall to make even an elementary estimate of the impulsive type of noise (if any) associated with occurrence of stall. There is, however, a general feeling that the occurrence of stall is a relatively slow process. Ham (15), for example, showed that when a blade is taken rapidly above the 'stall angle' the high values of lift are substained for a time equivalent to something of the order of 1/8th blade revolution. The delay in occurrence of stall is thought to be a function of the rate of change of angle of attack and the sustained upper surface suction associated with the chordwise passage of vorticity shed during the stall process. It is, of course, a well known fact that the onset of dynamic stall occurs at much higher angles of attack than associated with the static stall. Thus there is very little chance of a blade stalling as it passes through, or close to, a tip vortex.

Work at Southampton (19) showed that induced angles of $\pm 25^{\circ}$ could be tolerated without any apparent stall, provided the time scale was small.

On an actual helicopter it is, however, true that blade slap is often associated with flight conditions in which stall can easily occur, i.e. high speed and large A.U.W. cases. It appears that this is the main reason for explaining blade slap as a result of stall. A closer study shows that stall can occur without blade slap being detected; and often blade slap occurs when stall is unlikely. It was shown using a WESSEX (19) that a $45/50^{\circ}$ bank turn to port and starboard produced identical blade slap, both in amplitdue and characteristic. It is difficult to explain this in terms of the 'stall theory' since a steady bank turn would not be expected to effect the stall characteristics to any great extent. It also appears that the 'stall' conditions of the majority of single rotor helicopter configurations are approximately the same, yet their blade slap characteristics are completely different varying from no slap to extremely loud slap. Also during the WESSEX tests (19) it was found that in level flight,blade slap could be induced by decreasing as well as increasing collective pitch: it is difficult to see how this can be explained in terms of the stall phenomenon.

A typical angle-of-attack contour for a single-rotor helicopter is shown in figure 2. If stall produces the 'bang', then as the blade rotated it would be expected that a continuous series of impulses would be produced as more of the blade became stalled. Experimental results however, indicated that the main 'bang' on a single rotor helicopter is a single impulse of short duration. A typical bang duration in terms of blade azimuth movement is also shown on this figure. It will be noted that there is very little correlation between this and the stall duration.

A further point against the stall idea is the observation of the effect of very low wind on a hovering helicopter (20) and a whirl tower (11); although stall is very unlikely in these cases, blade slap was produced in both. Distortion of the vortex filament path and blade/vortex interaction, is of course, much more likely in these circumstances.

The above discussion has been based on the traditional and classical concepts of stall. Recent studies, however, suggest that this form of stall does not occur on rotor blades and that compressibility is the important parameter (21).

To summarize, it would appear that although the 'stall blade slap' hypothesis cannot be completely disproved, the above points indicate the unlikelihood of blade slap being the result of such a mechanism.

(3) Shock wave formation.- The formation of shock waves on any aerofoil is very complex. It is suggested that on a single rotor helicopter a shock is formed when the local flow becomes supersonic, while on a tandem

helicopter it is postulated that shock formation is a result of blade/vortex interaction as already outlined (Mechanism 3b). Information of the details of shock wave formation is very sparse and there is practically no work on rotating systems such as the helicopter rotor.

Before considering the possible production of a shock wave on a rotor, it is worth noting that a blade travelling at subsonic speed with a small shock wave on it will itself not produce any noise. There is no comparison between this case and the sonic boom produced on supersonic aircraft. The noise source would, if it occurred, be a result of the flúctuation, or change, in lift caused by the formation of a discrete shock wave.

Although shock wave formation is discussed in detail in references, 22, 23,24, there still appears to be a general lack of understanding of the topic.

The present position appears to be that the actual Mach number at which shock waves occur can be found accurately only by experiment; but this is complicated by the fact that weak shock waves which form owing to localised sonic flow are extrmely difficult to detect. In fact on many aerofoils the local regions can become supersonic without the formation of extensive shock waves.

Since the local flow is dependent on the conditions of the flow across the aerofoil section, the formation of the shock wave would be expected to be more random than indicated by the measurements, which suggest that it occurs (within the measuring accuracy) at blade passing frequency.

Thus although local shock wave formation may contribute to the severity of blade slap, it is not thought to be a predominant mechanism. On tandem helicopters the relative angle between the 'vortex' and blade direction is such that in forward flight the mechanism illustrated in figure 1 is unlikely to occur.

(4) Summary.- From the preceding discussions it would appear, therefore, that the blade/vortex interaction mechanism is still the most likely. This fact appears to be supported by the recent full scale Boeing-Vertol investigation (11). Thus unless specifically stated, in the remainder of the report blade slap will be assumed to be a direct result of the fluctuation in the lift caused by the interation of a blade and a vortex filament. This can be either an actual intersection when a blade cuts a vortex filament or the effect of a blade passing very close to a vortex filament.

4 BLADE/VORTEX INTERACTION ON HELICOPTERS

Although it is easy to imagine a blade and a tip vortex intersecting, it is extremely difficult to visualise the details of such an encounter and practically impossible to describe it mathematically.

Consider in the first instance the single rotor case. It was initially thought that the tip vortex took a consdierable time after it was shed to form into a discrete filament, and that it moved away rapidly from the rotor plane after it had been shed. Experimental evidence has shown that the converse is often true. The photograph (plate 1) of the Westland Westminister taken in high humidity condition shows the tip vortex filaments as condensation trails. The vortex trailed behind the blade in the port quarter is shown very clearly. Also shown, but much more faintly is the vortex trailed by the previous blade (at approximately the 300^{o} position) which can be traced to the forward position ; it will be noted that this appears to go above the following blade. Although the blades are not cutting any vortex filament in this particular case, they are obviously passing very close to the vortex filament. Smoke tests on a model two bladed rotor rig situated in the wind tunnel at Southampton (25) showed that at the front of the disc there is a strong upwash with the result that the tip vortex after formation rises above the rotor disc. A typical trajectory of the cross-section of a vortex at the 180^{o} position (i.e. at the front of the rotor) is reproduced in figure 3. This is in general agreement with calculations and blade pressure measurements made by Ham (26). In addition to this upwash at the front of the rotor considerable distortion in the flow pattern occurs around the disc, particularly in the 45^{o} and 315^{o} regions. This is shown clearly in Taraine's study (27) of the local flow around a rotor disc and was observed during the Southampton tests (25). Computations by Crimi (28) and White (29) for a two bladed rotor, and by Scully (30) have all demonstrated this trend.

The investigation mentioned above has also confirmed that the basic tip vortex forms very rapidly. This is substantiated in the propeller research by Adams (3) and the study of the flow behind an aircraft wing by McCormick and Tangler (31). Simons et al (24) concluded that the rolling-up is completed in about the time taken to travel a distance equivalent to one blade radius, which is in general agreement with that found for the vortex rolling-up process on wings. Thus the tip vortex would be practically in its final form by the time any blade vortex interaction could occur.

Flight tests are limited, although differential blade pressure measurements at the Bell Helicopter Company (2) have shown clearly that rapid changes of pressure of the type expected if a blade/vortex interaction occurs are present when blade slap is being produced. If it is accepted that vortices can pass above or near the rotor disc, then it is clear that blade vortex interaction can occur and it is very likely that a blade can actually cut a vortex filament. For the practical case the situation will be more complicated than the model tests indicate since turbulence, the helicopter fuselage and the general non-symmetrical nature of blade loading will cause even further distortion of the flow patterns.

On a tandem rotor helicopter, particularly if the blade overlap is large, it is easy to imagine a blade cutting a vortex filament since one rotor will be passing through the downwash of the other. This obviously accounts for the fact that blade slap is much more severe on tandem rotor helicopters. The Boeing Vertol Company (11) have recently carried out a detailed experimental programme in which smoke was generated at the blade tips of a tandem helicopter. This showed clearly blade vortex intersection and that the position at which it occured could be computed using a relatively simple analytical model. It is worth noting that although on a tandem the rear rotor is above the front rotor in hover, the position can be reversed in forward flight due to the tilting of the rotor discs. Thus, as shown by Vertol (11) the rear rotor blades can cut the tip vortex filament shed by the front rotor system.

The flow visualisation technique reported in reference 25, was applied by Simons to a preliminary study of the 'tandem rotor' flow patterns. To date this work has not been published, but a selection of the photographs taken are reproduced in plate 2.

The photographs show clearly how the isolated tip vortex passes through the lower rotor and since the vortices are in the form of a continuous filament, it is obvious that an intersection can occur under certain conditions. The bottom photograph illustrates the 'unwinding' of the vortex filament which appears to take place after the interaction.

The details of any interaction are extremely complicated and estimation of actual fluctuation in load is not possible at the present time. An insight into the problem can, however, be obtained by considering the types of idealised interactions likely on helicopters. Figure 4 show some typical cases where the vortex filament is represented by a rotating cylinder. Even an elementary study of this simplified situation will reveal the complex nature of determining the appropriate velocity profiles 'seen' by the blade. In practice the position will be further complicated since the circulation of the blade and the vortex will affect oneanother and cause severe distortion of the filament when the blade and vortex are close together. On a single rotor helicopter the blade will most likely either pass close to the vortex filament (a), or cut through the filament (b). On a tandem helicopter it is more likely that one rotor will cut the vortex filament from the other rotor as illustrated in figure (c). Although the details are not known it is clear that the velocity profile in the direction of motion will be similar in each case, and takes the form given by the interaction of a blade and an isolated vortex with its axis parallel to the span. The fact that large fluctuations in lift occur when a blade passes close to a vortex filament are obvious, as illustrated by Simons (33).

The 'peak' velocity amplitude encountered by the blade will be practically independent of the type of interaction and thus noise from any interaction, to a first approximation, will only be dependent on the vortex size and blade parameter. The theoretical development (section 7) is based on this assumption and thus will predict the less favourable result, since any of the type of interaction illustrated in figure 4 will tend to reduce slightly the peak amplitude and, more important, the rate of change of loading.

.5 TIP VORTICES

Any aerofoil or lifting surface produces a system of trailing vortices as a result of the lift or circulation variation across the span. This trailing sheet of free vortices, which represents a surface of discontinuity, is unstable and cannot persist in this form. Instead the sheet tends to roll up rapidly behind the wing to form a pair of discrete vortex filaments. Thus the trailing wake some distance behind the wing or rotor blade will consist only of a root and tip vortex, in opposite sense to each other. Since the tip vortex is very concentrated and the root vortex very diffuse, it is usual to consider only the tip vortex.

Theoretically it is very difficult to estimate the tangential velocity of a vortex. For two-dimensional potential flow the circumferential velocity varies inversely with the radius according to the relationship

$$V_t = \frac{\Gamma}{2\Pi r_v} \tag{5.1}$$

where V_t is the tangential component of induced velocity

r_v is the radius from the vortex centre

Γ is the circulation.

This results in a distribution as illustrated by the "dotted" line in figure 5., with the velocity at the centre being infinite. In a real fluid this could not occur and V_t would take the form shown by the "solid" line. In this case the velocity departs from the potential theory as the vortex is traversed, and reaches a maximum before decreasing to zero at the vortex centre. This centre region is known as the core, within which the fluid motion approaches that of a solid rotation body. If in fact the core is simplified to a rigid body, then the velocity distribution in the core would take the form indicated by the "dashed" line in figure 5, i.e.

$$V_t = \frac{\Gamma}{2\pi r_p^2} \cdot (r_v) \tag{5.2}$$

where r_p is the radius at which the motion is considered to change from that of rigid body to potential flow(see figure).

As already mentioned, a real fluid has a somewhat different profile, a good approximation (33) to which is given by Lamb's solution for a viscous fluid (34):-

$$V_t = \frac{\Gamma}{2\pi r_v} \left(1 - e^{-r_v^2/4\nu t}\right) \tag{5.3}$$

where ν is the Kinematic Viscosity

t is the time (decay interval).

This illustrates another property of a vortex, namely that the core diffuses with time due to viscous effects. It also follows that the maximum tangential velocity, V_M is dependent on the lift produced by the blade since Γ is directly related to the bound circulation on the aerofoil and the vortex structure.

In addition to the maximum tangential velocity, another important parameter is the core diameter. This is not a well defined dimension since the actual velocity distribution approaches asymptotically the potential distribution. For convenience of this work, the core diameter has been taken as the diameter of a circle bounded by the maximum tangential velocity.

Since Γ, the circulation can be calculated for a helicopter blade, it is only necessary to know the core size to obtain V_M and an estimation of the velocity distribution. As far as could be determined by the author, there has not been any such estimation of vortex core sizes for helicopter blades. A number of investigators have, however, made estimations for the tip vortices shed by wings. Piercy's (35) results, for example, indicate that the core radius is approximately 1/12 of the span while Spreiter and Sacks (36) gave a core radius equivalent to .155 times the semi-span for an elliptically loaded wing. If the rotor in question is considered to be a wing with an aspect ratio of 20, then these results suggest that the core diameter is about 3 chords (i.e. 3C)

Although only a few investigators have actually attempted to measure the profile of a tip vortex, their results indicate a much smaller core. An early study by Piercy (37) suggested a core diameter of 0.2C, with the maximum tangential velocity, being equivalent to an induced angle of $\pm 17\frac{1}{2}^{\circ}$. In a more recent paper which presented a study on the V107 helicopter rotor, (38) McCormick approximated the outer portion of the rotor blade by a wing and measured some velocity profiles about 6 chords behind it. This gave core diamters of the order of 0.25C and maximum induced angles of $\pm 18^{\circ}$.

Simons, Pacifico and Jones (25) carried out some experiments in a wind tunnel using a model helicopter rotor and measured the vortex profile with a 'Hot-wire anemometer'. This gave a core diameter of the order of 0.1C with a 4 in. chord blade. McCormick and Tangler (32) studied the vortex sheet behind a wing of an actual aircraft (U.S. Army Cessna L-19) and compared the results with a one-twelfth scale semi-wing in the wind tunnel. These wind tunnel results also suggested a core diameter of 0.1c, while the flight results indicated a value of half this value. Maximum induced angle in both cases were of the order of $\pm$ 22°.

Although the 'core diameter' has the major effect of the noise produced by any vortex/blade interaction the remainder of the vortex cannot be neglected when estimating the noise. The information on overall size of a vortex is even more limited the details on the vortex size given above. An examination of the available information tends to suggest that the complete profile takes the form shown in figure 6, with an "overall width"(see figure) of 0.75 C. Although the 'peak' amplitdue V_m is a function of blade speed and C, the experimental results already discussed indicate that the maximum induced angles are of the order of $\pm$ 20° as presented in the figure.

Equation 5.3, Lamb's solution, defines the radial distribution of the velocity through the vortex as a function of time. From this it follows that the maximum velocity is at a radius given by:-

$$r = \sqrt{1.26\,(4\nu t)} \tag{5.4}$$

Thus the maximum velocity V_M is

$$V_M = 0.638 \quad \frac{\Gamma}{2\pi}\sqrt{\frac{1}{4\nu t}} \tag{5.5}$$

These equations suggest that the core diameter increases as a function of $t^{\frac{1}{2}}$, while the maximum velocity decays as a function of $t^{-\frac{1}{2}}$.

Simons et al (25) suggested a modified relationship for the core radius where time 't' was replaced by ($t + t_o$). This implies that at t = 0, when the vortex is assumed to leave the blade, the vortex core is of finite size. This illustrates clearly the problem encountered, namely what to consider as zero time (i.e. t = 0). Since there are indications of a well formed vortex core soon after the vortex leaves the blade, it would appear that the time must be assumed to start from a datum in advance of this time. Alternately a solution of the form outlined by Simon et al (25) must be used. Thus to generalise, it would appear that although Lamb's solution can be used to predict the shape of the velocity distribution through an isolated vortex core and indications of changes in vortex structure likely with time, it cannot be used to predict absolute values since 'zero time' cannot be defined. Equations 5.4 and 5.5 are, therefore, not general working solutions. For the particular case under consideration, however, the time between the vortex leaving the blade and a likely blade vortex interaction is fairly small and thus, if it is assumed that change of dimensions is of the form indicated by Lamb's solution, the change in vortex core size and peak velocity would not be expected to be large.

It would be reasonable, therefore to assume that

$$V_M \propto \Gamma \qquad \text{(the circulation strength of the vortex)} \qquad (5.6)$$

The circulation strength of a vortex Γ , is equal to the maximum bound circulation on the rotor blades. Consider the case of helicopter in ideal hovering conditions with constant circulation.

The lift on an element dr at radius r is

$dL = V\rho\Gamma dr$ and $V = \omega r$; thus

$dL = \rho\Gamma r\omega\, dr$

Thus total lift for 1 blade, $L = \int_0^R \frac{dL}{dr}.dr = \frac{\rho\Gamma\omega R^2}{2}$

Now total lift = Gross Weight of Helicopter:

$B.L = (G.W.)$

where B = No. of blades.

Thus the ideal vortex strength is given by

$$\Gamma = \frac{2(GW)}{V_T.B.R} \qquad (5.7)$$

Hence from equation 5.6

$$V_M \propto \frac{(GW)}{V_T.B.R} \qquad (5.8)$$

It should be remembered that the above is based on two dimensional analysis for a perfect fluid containing isolated vortices.

When the blade cuts a vortex filament the blade can be considered to pass through a cylinder of vorticity as illustrated in figure 4. In estimating the noise from any interaction (section 7) it is necessary to know the span width affected by the vortex and the details of the 'gust length' in the direction of blade motion. These, as shown in figure 4 are obviously functions of the width of the vortex filament D.

The experimental work already discussed on the vortex size appears to suggest that the vortex width, or more precisely the vortex core, is a function of the blade chord. Correlation between various experimental results is poor and it is likely that measurements of the vortex size are given in terms of the blade chord simply because this is a convenient method of quoting the results. The theory on the other hand, suggests that the vortex size is independent of all parameters except vicosity and time.

It is far more likely, however, that the core size is directly related to the span loading of the blade and in particular to the loading near the tip (21). If this is assumed to be correct then, since the blade loading characteristics of the majority of helicopters are similar it would appear reasonable to assume (at least until further information is available) that the spanwidth effected by the vortex is a constant. Since the overall vortex filament is considered to be of the same order as the blade chord, the 'gust' or vortex filament width in the direction of blade motion can also be assumed constant for any of the isolated interactions illustrated in figure 4.

The difficulties of predicting vortex paths have already been outlined. If however, the tip vortex path just after leaving the blade is determined by simply assuming it to follow the blade tip path and estimation of where the interaction is likely to occur can be obtained. There is some justification in this since relative to the shew-helical wake configuration, the wake distortion appears to occur in the vertical plane (27,28,30). Also since any interaction is likely to be caused by the proceeding blade the distortion in the rotor disc plane can be expected to be minimum since the time involved is small. The Boeing Vertol Company (11), used this type of approach successfully on their tandem rotor helicopter. From this type of study it is clear that for the majority of helicopters the interaction is likely to occur over the outer portion of the blade. In estimating the noise (section 7), the velocity of the blade at the intersection point is required. Unless a particular rotor configuration and flight condition is being investigated it would appear reasonable to take the less favourable case and assume the intersection velocity V, to be the same as the blade tip velocity V_T.

6 THEORETICAL MODEL

The two extreme profile distributions for an isolated blade/tip vortex interaction are illustrated in figures 7.1 and 7.2. Figure 7.1(a) and (b) shows the velocity profiles along the span and in the direction of motion respectively for a blade passing through a vortex with the core axis parallel to the span.

Figure 7.2 shows similar results when the axis of the vortex core is parallel to the direction of motion. The theory (section 7) has been developed for the case shown in figure 7.1. The vortex is treated as a gust with a velocity profile equivalent to the velocity distribution that would be experienced by a blade if it passed through the centre of the vortex. The blade is assumed to be moving as a wing at a velocity equal to that of the blade section at the centre of the gust.

Since it is the blade loading fluctuation in the direction of blade motion which is important, the theory can also be modified to the case illustrated in figure 7.2. For this the gust is treated as two separate gusts, one acting upwards and one downwards as shown in figure. 7.3. The 'power' from each of these is numerically equal and it is therefore, only necessary to calculate it for one of these and modify the solution to give the total power. If it is assumed that the two parts of the gust act as separate and isolated dipole sources then there is no cancellation effect and the total power is just double that for one part of the gust. It has been shown (39) by considering two point dipoles that the ratio of the total power, W_2, to the power in the far field for the single dipole W_1, takes the following form:-

$$\text{As } kh \to 0 \qquad W_2/W_1 \to \frac{(kh)^2}{5} \tag{6.1}$$

$$\text{As } kh \to \infty \qquad W_2/W_1 \to 2 \quad \text{(isolated dipole case)} \tag{6.2}$$

Where 'h' = separation of point dipoles.

$$k = \frac{2\pi f}{C} \qquad \text{and } f = \text{frequency.} \tag{6.3}$$

W_2/W_1 versus 'kh' is shown in figure A1, thus, if details of interaction were known the total power could be easily found. It would also be necessary to modify the equations developed for the case shown in figure 7.1., to take account of the different spanwise loading: this effect is, however, small and can for all practical purposes be neglected. As discussed in section 4 the 'peak amplitude' and the rate of change of the 'velocity' profile will be approximately the same for both the cases illustrated. Thus power and energy predicted by equations 7.10 and 7.11 (section 7) respectively will be equally applicable to either type of interaction, within the accuracy limits already outlined.

Although the treatment of the source as two separate dipoles is not exact, the theory based on two point dipoles can be used to give an estimate of the accuracy of the approach. If it is assumed that:-

1. the extremes of the vortex core diameter are the dipole centres (i.e. h = core diameter);

2. the core diameter is in the order of 0.2 of the blade chord (see section 5 on experimental results);

3. the typical frequency of the sound is given by $f = \frac{V'}{C}$ where V' = flow speed relative to blade.

Then:-

$$kh = \frac{2\pi fh}{c_o} = \frac{2\pi Mh}{C} \qquad (M = \frac{V}{c_o}\text{ , Mach no. of blade})$$

and taking a typical Mach number of 0.75 at the blade tip gives kh $\doteq$ 1.2 in which case W_2/W_1 is approximately 0.3 as shown in figure A1. Thus the final solution using the method outlined above is likely to be of the order of 8 dB below the exact solution.

7 THEORY

7.1 General

Although the loudness of single impulsive sound such as sonic booms have been evaluated (40,41), there is practically no information available on estimating the loudness of repeating impulses, except in connection with recommendations for damage risk criteria (42,43). For the single impulse, the loudness is determined by considering the sound energy in the impulse and applying appropriate weighting functions. For continuous sounds it is more usual to work in terms of the source power since this is directly related to the loudness and sound pressure level measurements. At the present time it is not clear which method of evaluation is best suited in estimating the loudness of repetitive type of noises like blade slap. Before new material becomes available, however, it would appear that the evaluation of the power of the impulse is the most applicable (44), but since it is not definite both forms are given in the following theoretical development.

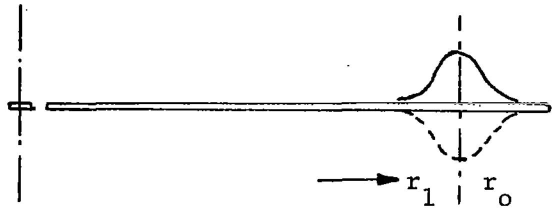
7.2 Acoustic Theory

It was shown in the earlier papers (12,13) that when a blade is subjected to a fluctuating load L_s per unit span, the total sound energy per unit time **radiated** into the far field is given by

$$W_s = \frac{1}{12\pi\rho c_o 3}\cdot\left[\frac{\partial L_s}{\partial t}\right]^2 \cdot \quad (r_o - r_1)^2 \qquad (7.1)$$

where $(r_0 - r_1)$ is the span width subjected to the disturbance. This derivation which is reproduced in Appendix 1, assumes that both the blade chord and the region affected by the fluctuating load are small compared with the acoustic wavelength.

It was also assumed in developing equation (7.1) that the blade loading is constant over the span width $(r_o - r_1)$. A more representative spanwise loading for the type of blade/gust interaction under consideration is illustrated below:-



A good approximation to this is a half sine wave. If this is used then equation (7.1) is modified to

$$W_s = \frac{1}{3\pi^3 \rho c_o^3} \cdot \left[\frac{\partial L_s}{\partial t}\right]^2 \cdot (r_o - r_1)^2 \qquad (7.2)$$

In the original papers a frequency analysis of the measured sound was made in terms of the blade passing frequency as fundamental, and for this reason the subsequent theory was developed in terms of harmonics of this frequency. This form does not, however, lend itself readily to working relationships for use with actual helicopters. The following theory therefore treats the blade loading impulse as an isolated event; the total radiated energy from impulse or 'bang' will be given, rather than the average power over a complete blade passing cycle. Recent work on impulsive noise and the subjective assessment of blade slap on actual helicopters seems to suggest that this approach is more appropriate for loudness prediction.

Equation (7.2) can be used to calculate the acoustic power or energy radiated from the blade following a single gust or impulse. This simply involves an integration over the duration of the impulse, and leads to the following expression for 'bang power' W_B and 'bang energy' E_B respectively:

$$W_B = \frac{(r_o - r_1)^2}{3\pi^3 \rho c_o^3} \cdot \frac{1}{\text{bang duration}} \cdot \int_{\text{bang}} \left[\frac{\partial L_s}{\partial t}\right]^2 dt \qquad (7.3)$$

$$E_B = \frac{(r_o - r_1)^2}{3\pi^3 \rho c_o^3} \int_{\text{bang}} \frac{\partial L_s}{\partial t}^2 dt \qquad (7.4)$$

since E_B = (bang duration) x W_B.

7.3 Blade loading

To calculate W_B and E_B accurately it would of course, be necessary to know the variation of blade loading as it passes through or over the tip vortex filament. Thus an exact gust velocity profile in the direction of blade motion would be required. The time history of the blade loading is very important, since W_B and E_B are dependent on the rate of change of loading and not just the amplitude. The spanwise distribution normal to the direction of motion is not so important, although for an absolute estimate of noise it would be needed.

A study of the tip vortex paths associated with even a simple rotor system shows that it is practically impossible to estimate the gust profile experienced by the blade: this is discussed in section 4. It is therefore impossible at the present time to develop the theory in terms of absolute values for an actual helicopter.

It is possible, however, by considering an ideal blade/vortex interaction and making various simplifications, to obtain a relationship which shows both the important parameters for an actual helicopter, and indicates the severity of blade slap likely on any helicopter.

When blade/vortex interaction occurs, the blade effectively passes through a gust of known dimensions. The resulting lift can be calculated using Kussner's function (45). It is convenient to represent the gust as a series of harmonics based on the gust width as the fundamental length.

Using this approach, the loading is given by two separate expressions, one when the blade is experiencing the gust(equation 7.5) and the other when the loading decays as the blade passes out of its effect (equation 7.6)

$$L_m = \tfrac{1}{2}VCa_oW_m\left[\frac{0.065}{\sqrt{k_m^2+(0.13)^2}}\left(\sin(k_m s-\Phi)+e^{-0.13s}\frac{k_m}{\sqrt{k_m^2+(0.13)^2}}\right)+\frac{0.5}{\sqrt{k_m^2+1}}\left(\sin(k_m s-\alpha)+e^{-s}\frac{k_m}{\sqrt{k_m^2+1}}\right)\right] \tag{7.5}$$

$$L_m = \tfrac{1}{2}\rho VCa_oW_m\left[\frac{0.065e^{-0.13s}}{\sqrt{k_m^2+(0.13)^2}}\left(e^{0.13x}\sin(k_m x-\Phi)+\frac{k_m}{\sqrt{k_m^2+(0.13)^2}}\right)+\frac{0.5e^{-s}}{\sqrt{k_m^2+1}}\left(e^{x}\sin(k_m x-\alpha)+\frac{k_m}{\sqrt{k_m^2+1}}\right)\right] \tag{7.6}$$

In the above relationships

m = Harmonic order of gust

$$K_m = \frac{2\Pi b.m}{Y}$$

s = non-dimensional distance measured from beginning of gust given by $S = \frac{d}{b}$

where d is actual distance, b = half chord = C/2

Y = total length of gust

x = non-dimensional length of gust= Y/b,

$\alpha = \tan^{-1} k_m$, $\phi = \tan^{-1} (k_m/0.13)$

These expressions could be evaluated numerically for each case. The effort involved is not, however, justified unless the details of the gust profiles are accurately known. As already discussed this is not the case, and it is more useful to obtain a simplified solution which gives an estimate of W_B in terms of overall features of the gust.

Since the harmonics of the typical gust shown in figure 8.1 fall off rapidly (figure 8.2) it is not necessary to consider more than, say, the first three harmonics. The total loading (based on the first three gust harmonics) determined using equations 7.5 and .6 is shown by the continuous line in figure 9.4 If the calculation is limited to the first harmonic alone, then the blade loading shown by the broken line on figure 10 is obtained. The two curves are very similar in shape, suggesting that it is the first harmonic that largely determines the shape of the loading curve. It should be noted that equation 7.3 and 7.4 depend on the rate of change of loading ($\partial L/\partial t$) and the shape is important as well as the amplitude.

Thus a good estimate of W_B and E_B can be expected if the calculation is based on the first harmonic. The loading for the first harmonic is given by equations 7.5 and 7.6 with m = 1. It will be noted that this is made up of two terms, with an amplitude ratio of approximately 1:8. These two terms are shown on figure 10. The smaller term is of a similar shape to the larger, with the result that although it affects the amplitude it has very little effect on the shape. Since this term is small compared with the other it seems reasonable to neglect it in which case the loading is given by a much simplified equation.

It is still theoretically necessary to harmonically analyse the gust. If however, the calculation is made using the peak gust amplitude W instead of W_1, then the resulting loading is increased in amplitude and now approximates closely the overall amplitude, as well as the shape, originally obtained using the first three harmonics. This is, of course, due to the fact that the gust profile approximates a sine wave and that the ratio of gust width to blade chord approaches unity. The agreement is shown in figure 11, which compares the loading based on the first three harmonics (continuous line) with that obtained for only the first harmonic (m = 1) when W_1 is taken as the peak amplitude W of the gust and the first term is omitted as described above (broken line).

Since it is the rate of change of loading which is important, the main contribution to the 'bang' power and energy is expected to come while the blade leading edge is within the gust (equation 7.5) rather than from the exponential decay of loading after the leading edge emerges from the gust (equation 7.6). Part of equation 7.5 has been discarded already; the remaining terms of a sine term and an exponential term are compared in figure 12.

If the exponential decay term can be neglected, the loading is given by the following simplified relationship:

$$L_s = \tfrac{1}{4}\rho VCWa_o \cdot \frac{1}{\sqrt{k_1^2+1}} \cdot \sin(k_1 s-\alpha), \quad (0\leq s<x) \tag{7.7}$$

This is valid, within the limits already outlined, provided the gust lengths is of the same order as the blade chord. It may appear that the simplifications made to obtain equation 7.6 are extremely severe. To obtain an indication of the difference between the exact solution (based on the first three harmonics of the gust) and equation 7.7 the time variation of $(\partial L/\partial t)$ has been computed using both methods. The results are compared in figure 13, and from this it can be seen that the 'peak levels' are under-estimated by a factor of 2.

W_B and E_B depend on the time integral of the square of the curves given in figure 4.7; this has been computed and shows that the value calculated using the exact solutions is 1.7 times as great as that given by the simplified solution. The comparison is illustrated in figure 14.

Differentiation of equation 7.7 with respect to $t = Cs/2V$ gives

$$\left[\frac{\partial L_s}{\partial t}\right]^2 = \tfrac{1}{4}a_o^2\rho^2V^4W^2 \cdot \frac{k_1^2}{k_1^2+1} \cdot \cos^2(k_1 s-\alpha); \tag{7.8}$$

This expression is to be integrated over the duration of the impulse, which in the present approximation is from $k_1 s = 0$ to 2. Approximating $k_1^2/(k_1^2+1)$ by 1 (on the grounds that $Y\simeq C$, so $k_1 \simeq \Pi$) leads to the result

$$\int_{\text{bang}} \left[\frac{\partial L_s}{\partial t}\right]^2 dt \simeq \tfrac{1}{8}a_o^2\rho^2YV^3W^2 \tag{7.9}$$

Substitution in equation 7.3 and 7.4 give the bang power and energy as

$$W_B \doteq \frac{a_o^{\,2}}{24\pi^3 C_o^{\,3}} \quad \rho V^4 W^2 \, (r_o - r_1)^2 \tag{7.10}$$

$$E_B \doteq \frac{a_o^{\,2}}{24\pi^3 C^3_{\;o}} \quad \rho Y V^3 W^2 \, (r_o - r_1)^2 \tag{7.11}$$

7.4 Bang Power and Energy in Terms of Helicopter Parameters

$(r_o - r_1)$ is the span length subjected to the gust, or in the case of a real helicopter the blade length affected by the tip vortex filament. This is discussed in section 5 and it would appear reasonable to assume that $(r_o - r_1)$ is a constant.

The study of possible tip vortex paths has already shown that blade/vortex interaction is most likely to occur near the blade tip. Thus V can be replaced in the above equations by the blade tip velocity V_T.

It has also been shown in section 5 that although an exact solution for the maximum tangential velocity of a tip vortex is not possible V_M is connected to the parameters of a helicopter by the following proportionality:-

$$V_M \propto \frac{(GW)}{V_T BR} \qquad \text{(equation 5.8)}$$

In this particular application $W \equiv V_M$ and hence,

$$W_B \propto \frac{V_T{}^2 (GW)^2}{B^2 R^2} \qquad (7.12)$$

$$E_B \propto \frac{Y.V_T.(GW)^2}{B^2 R^2} \qquad (7.13)$$

Since it was assumed in the derivation of the blade loading that the gust length is of the same order as the blade chord and since any blade/vortex interaction is likely to be a function of a blade chord (section 4), Y in equation 7.13 can be replaced by C, whence

$$E_B \propto \frac{V_T (GW)^2 C}{B^2 R^2} \qquad (7.14)$$

7.5 Blade Slap Factor

Equations 7.12 and 7.14 which give the 'bang power' and 'bang energy' respectively can be used to compare the relative levels of blade slap likely from any helicopter. For convenience these equations have been termed the

BLADE SLAP FACTORS and are referred to as 'BSF(P) and BSF(E) for the estimation based on power and energy respectively

$$BSF(P) = \left(V_T \cdot \frac{(GW)}{RB}\right)^2 \quad ; \quad BSF(E) = \left(\frac{GW}{BR}\right)^2 CV_T \tag{7.15}$$

or in terms of the disc loading, w

$$BSF(P) = \left(\frac{V_T\, wR}{B}\right)^2 \quad ; \quad BSF(E) = \left(\frac{wR}{B}\right)^2 CV_T \tag{7.16}$$

The above equation can be used to illustrate the most important parameter on an actual helicopter. Consider in the first instance one particular helicopter, then C, R and B will be fixed, in which case

$$BSF(P) \propto (V_T . GW)^2 \quad ; \quad BSF(E) \propto V_T (GW)^2 \tag{7.17}$$

If now the 'pitch' is assumed constant, it follows that the vortex size or strength is directly proportional to the velocity of the blade, hence $GW \alpha V_T^2$ and

$$BSF(P) \propto V_T^6 ; \quad BSF(E) \propto V_T^5 \tag{7.18}$$

This assumes of course, that all other conditions and parameters are constant and is the typical law for the dipole type of radiation.

Consider next the case when the tip velocity is fixed, then:-

$(GW) \doteq K.\theta$ where θ is the pitch angle.

Thus BSF(P) and BSF(E) $\propto \theta^2$ (7.19)

The Blade Slap Factors have been computed for a number of helicopters and the results are shown in table 1 together with some subjective assessments.

8 DISCUSSION ON THE BLADE SLAP FACTORS

The blade slap theory has been developed on the assumption that the blade chord and the spanwidth effect of the vortex are small (see Appendix 1). The

B.S.F. is therefore more likely to be applicable to the helicopters with relatively small chord blades. For large chord blades it would be necessary to treat the source as an array of dipoles and obtain the overall effect by a summation process. Although this would be relatively simple, it does not appear justified since the details of blade/vortex intersection likely on any helicopter are so vague.

A comparison between the subjective assessments and the values of B.S.F.(P) and B.S.F.(E) given in table 1 is shown in figures 15 and 16 respectively. The B.S.F. has been plotted on a log scale, which is, of course, equivalent to using a dB scale. In addition to the helicopters referred to in the survey (14) the values for the Sikorsky S65 and Milhail (USSR) Mil 10 are shown. It will be observed that there is fairly good correlation between the B.S.F.'s and the subjective observation, particularly for the power solution (B.S.F.(P) Figure 15).

The values for the Mil 10 are extremely large, while observations of the helicopter suggest it is very quiet and without blade slap(20). At the time of this assessment of the Mil 10 the gross weight was only 85000 lbs in which case the B.S.F.(P) and B.S.F.(E) are reduced to 4540×10^7 and 20.8×10^7 respectively. These are, however, still well above those of the other helicopters considered.

With the information at present available it is not possible to explain why the Mil 10 has such a large blade factor while not having any blade slap.

Information on the S65 is also sparse, but there are indications that blade slap is not a significant problem on this helicopter, even although the value of the B.S.F. is relatively large.

As already mentioned the theory is not really suited to helicopters with large chord blades. The chord width of the Mil 10 is very large (39 inches), which could account for the fact that the B.S.F. does not appear to agree with the subjective assessment in this case. The chord of the S65 blade is also relatively large (26 inches), but this is only slightly greater than the chord width of the UH-1D blade(21 inches). The UH-1D result, as shown in the figure, appears to agree well with the subjective assessments of the blade slap noise

An examination of the limited number of results suggests that although the blade slap factor is appropriate for single rotor helicopters with a low number of blade (2 or 3) and tandem rotor helicopters it is not applicable to multi-bladed (5 or more) single rotor helicopters. It should be remembered that the factor assumes not only that blade/vortex interaction occurs, but that it occurs in the less favourable form. It could be that the wake distortion on single rotor helicopters with a large number of blades is considerably less than on those with, say, two blades, with the results that blade/vortex intersection is less likely to occur. Until further data becomes available it is not, however, possible to draw any definite conclusions.

9 EXPERIMENTAL RESULTS AND FUTURE REPORTS ON BLADE SLAP

A second report on blade slap is being prepared. This will report the full scale flighttests and model tests carried out to date. In addition it is hoped to include some of the subjective evaluations of various impulse shapes which are being computed at present under the direction of Mr. C.G.Rice of the Audiology Group, as well as the results of some initial studies made at the I.S.V.R. of the subjective aspects of blade slap.

A review of the possible methods of reducing blade slap on helicopters will also be included.

APPENDIX A

ACOUSTIC THEORY

A theory has been developed (19) for the noise produced when a blade passes through an impulse or gust of known form and profile. The rotor blade is assumed to be rigid and not to deflect when passing through the gust. For the noise calculations it is treated as a thin plate.

Lighthill (45) has shown that the pressure at a point whose position relative to a point dipole has co-ordinates x_i $(i = 1,2,3)$ takes the form

$$p - p_o = \frac{x_i}{4\pi r^2}\left[\frac{1}{c_o}\cdot\frac{\partial F_i}{\partial t}\left(t - \frac{r}{c_o}\right) + \frac{1}{r}\cdot F_i\left(t - \frac{r}{c_o}\right)\right] \tag{A1}$$

where F_i is the fluctuating force and r the distance of the observation point from the source.

The second term in the prior equation becomes small when the distance from the point to the source r is much larger than a typical wavelength of the sound radiated. Thus, for the "far field",

$$p - p_o = \frac{x_i}{4\pi r^2}\cdot\frac{1}{c_o}\cdot\frac{\partial F_i}{\partial t}\left(t - \frac{r}{c_o}\right) . \tag{A2}$$

The loading per unit span L_s, on each small area of blade can be assumed to act as a point dipole acoustic source, provided the chord is small compared with a wavelength. Thus for a small element of span

$$(p - p_o) = \frac{x_n}{4\pi r^2}\cdot\frac{1}{c_o}\cdot\frac{\partial L_s}{\partial t}\left(t - \frac{r}{c_o}\right) \quad dx \tag{A3}$$

so that the total radiated noise is

$$p - p_o = \int_{span} \frac{1}{4\pi r}\cdot\frac{x_n}{r}\cdot\frac{1}{c_o}\cdot\frac{\partial L_s}{\partial t}\left(t - \frac{r}{c_o}\right) \quad dx \tag{A4}$$

In addition, if the span length over which the gust acts, $(r_o - r_1)$, is small compared with the acoustic wavelength, and if L_s is assumed constant over $(r_o - r_1)$, then

$$\int_{\text{span}} \frac{\partial L_s}{\partial t}\left(t - \frac{r}{c_o}\right) d_x = \left[\frac{\partial L_s}{\partial t}\right] (r_o - r_o) \qquad \text{(A5)}$$

and the factor $(1/4\pi r)$ (x_n/r) in equation (A) can be replaced by $(1/4\pi\bar{r})$ $(\bar{x}_n/\bar{r})$ where r is the distance of the observation point from the mid-point of the span length and x_n is the co-ordinate normal to the span at this mid-point.

The mean radiated intensity, I in the "far field" is

$$I = \frac{\overline{(p - p_o)^2}}{\rho_o c_o} \qquad \text{(A6)}$$

and the total radiated energy W_s is the surface integral of the intensity over the surface of a large sphere. It therefore follows that

$$W_s = \frac{1}{12\pi\rho_o c_o^3} \overline{\left[\frac{\partial L_s}{\partial t}\right]^2} (r_o - r_1)^2 \qquad \text{(A7)}$$

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Table 1

Helicopter	No. of Blades	Rotor Radius	Blade Chord Width	A.U.W.	Rotor R.P.M.	Tip Speed	Blade Slap Factor		Subjective Assessment
							B.S.F.(P)	B.S.F.(E)	
		ft.	ins.	lbs.		ft/sec	(x 10^7)	(x 10^7)	
Bell: UH-1B	2	22	21	8500	324	745	2050	5.05	Very loud
UH-1D	2	24	21	9500	324	745	2310	5.25	Very loud
Sikorsky: S58	4	28	16.4	13000	195	570	440	1.0	*
S61	5	31	18.3	19000	203	660	650	1.48	*
S65	6	36.1	26	35000	185	700	1290	4.05	+
Westland: Wasp/Scout	4	16	10.5[3]	5500	400	670	330	0.44	Little slap at high altitude.
Wessex 2 & 5	4	28	16.4	12600	221	650	625	1.25	Slight banging
Wessex 3	4	28	16.4	13500	228	670	655	1.27	Slight banging
Whirlwind	3	26.5	16.4	8000	218	600	370	0.80	No slap
Belvedere	2 x 4	24.5	15.5	19000	250	640	385	0.78	No slap
Boeing Vertol: V107	2 x 3	25	18	19650	264	690	820	1.78	Loud
Chinook	2 x 3	30	25.3	33000	230	720	1740	5.02	Very loud
Hiller: SL 4	2	17.7	14	3100	370	690	365	0.64	No slap
FH - 1100	2	17.7	10.2	2750	368	680	285	0.36	*
Hughes: OH - 6A	4	13.2	6.8	2400	470	650	87	0.08	*
269 A	3	12.7	6.8	1600	483	650	78	0.069	*
Kaman: K 20	4	22	21.6	8637	277	640	395	1.12	*
K 600 - 3	4	23.5	15.7	5969	248	610	150	0.32	*

Table 1 contd.

Helicopter	No. of Blades	Rotor Radius	Blade Chord Width	A.U.W.	Rotor R.P.M.	Tip Speed	Blade Slap Factor B.S.F.(P)	Blade Slap Factor B.S.F.(E)	Subjective Assessment
		ft.	ins.	lbs.		ft/sec	$(\times 10^7)$	$(\times 10^7)$	
Lockheed: XH-51A	4	17.5	12	4700	355	660	195	0.30	*
Milhail: Mil 10	5	57.4	39	95790	120	720	5760	26.4	+

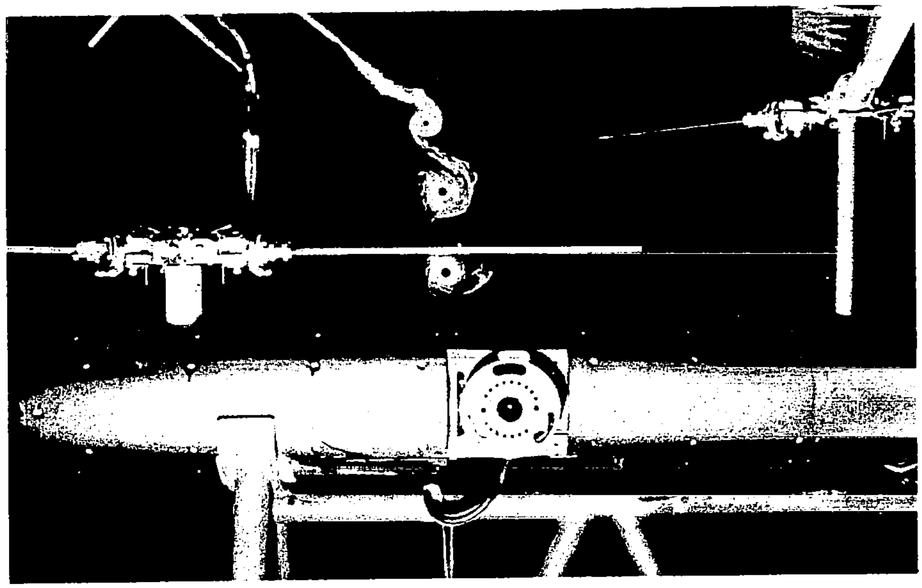
Notes: 1. Details of rotor parameters, speed and A.U.W. for all the helicopters except the Mil 10 were obtained from VERTICAL WORLD June 1967. Details of the Mil 10 were obtained from FLIGHT 23.3.67

2. Information from F.W. Taylor's questionnaire/survey given in reference (14). * indicates that there was no specific mention to these helicopters, + indicates that the helicopter was not in service at the time of the survey.

3. Blade Chord Width at tip.

Plate 1. Photograph of Westland Westminster showing trailed vortices.

PHOTO: WESTLAND HELICOPTERS LTD



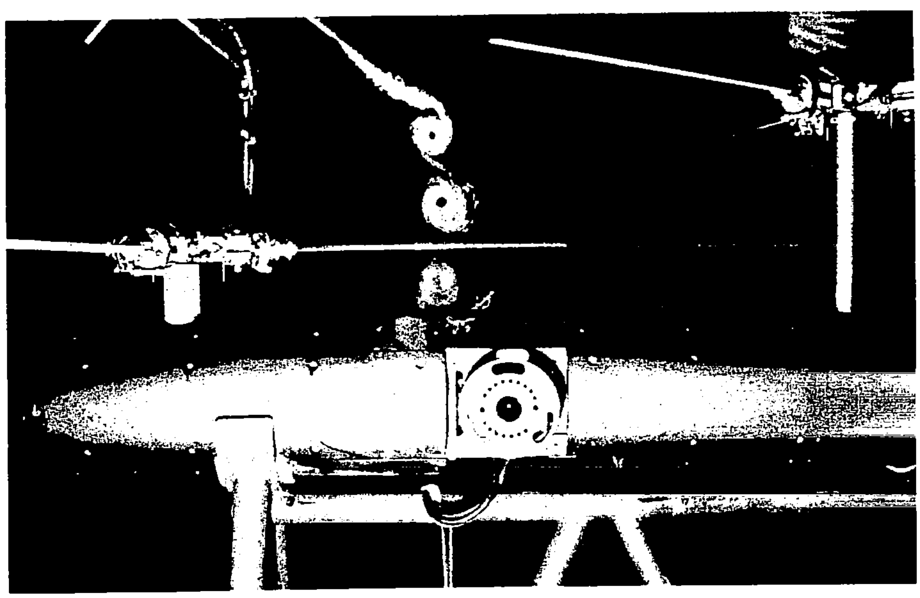


Plate 2. Tip vortices on tandem rotor rig. (Collective pitch – 10° Tip speed – 400 f. p.s.)

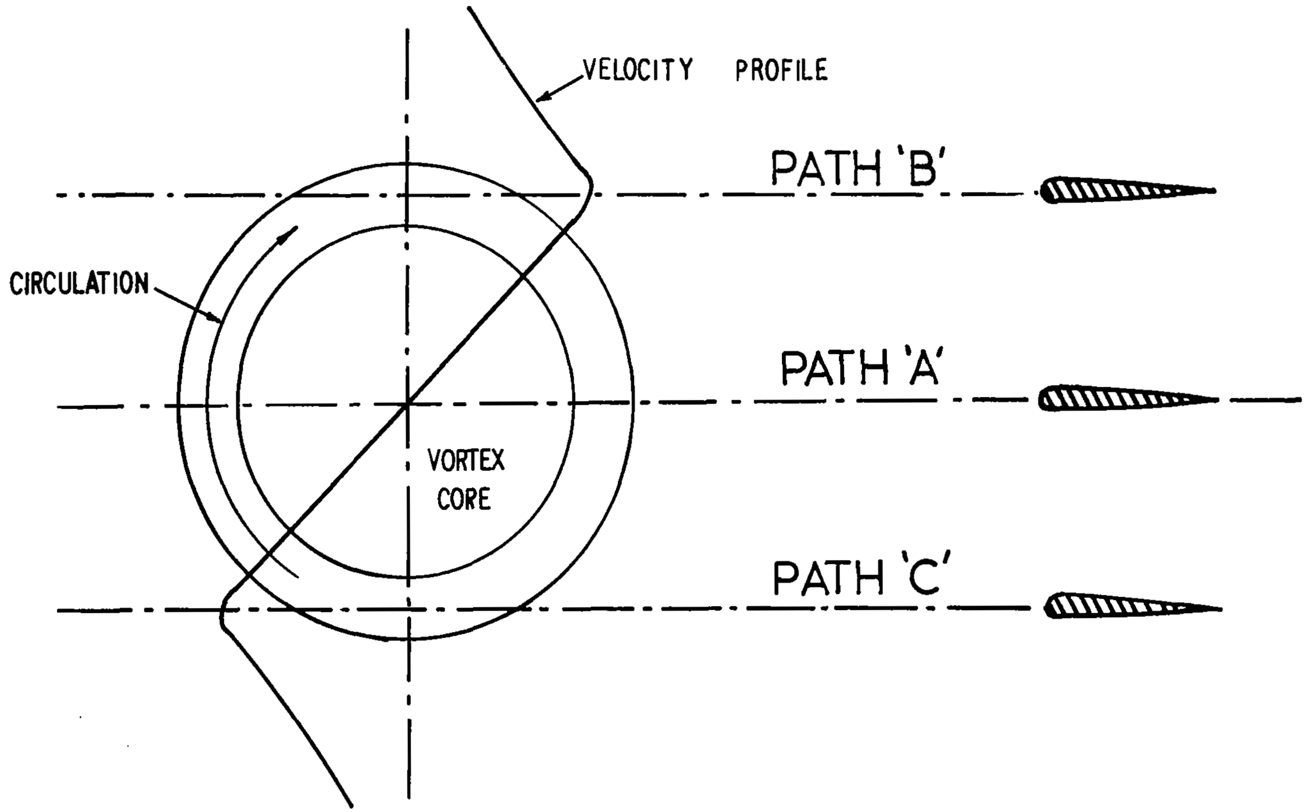


FIG. 1. IDEALIZED BLADE VORTEX INTERSECTION. (REFERENCE 14)

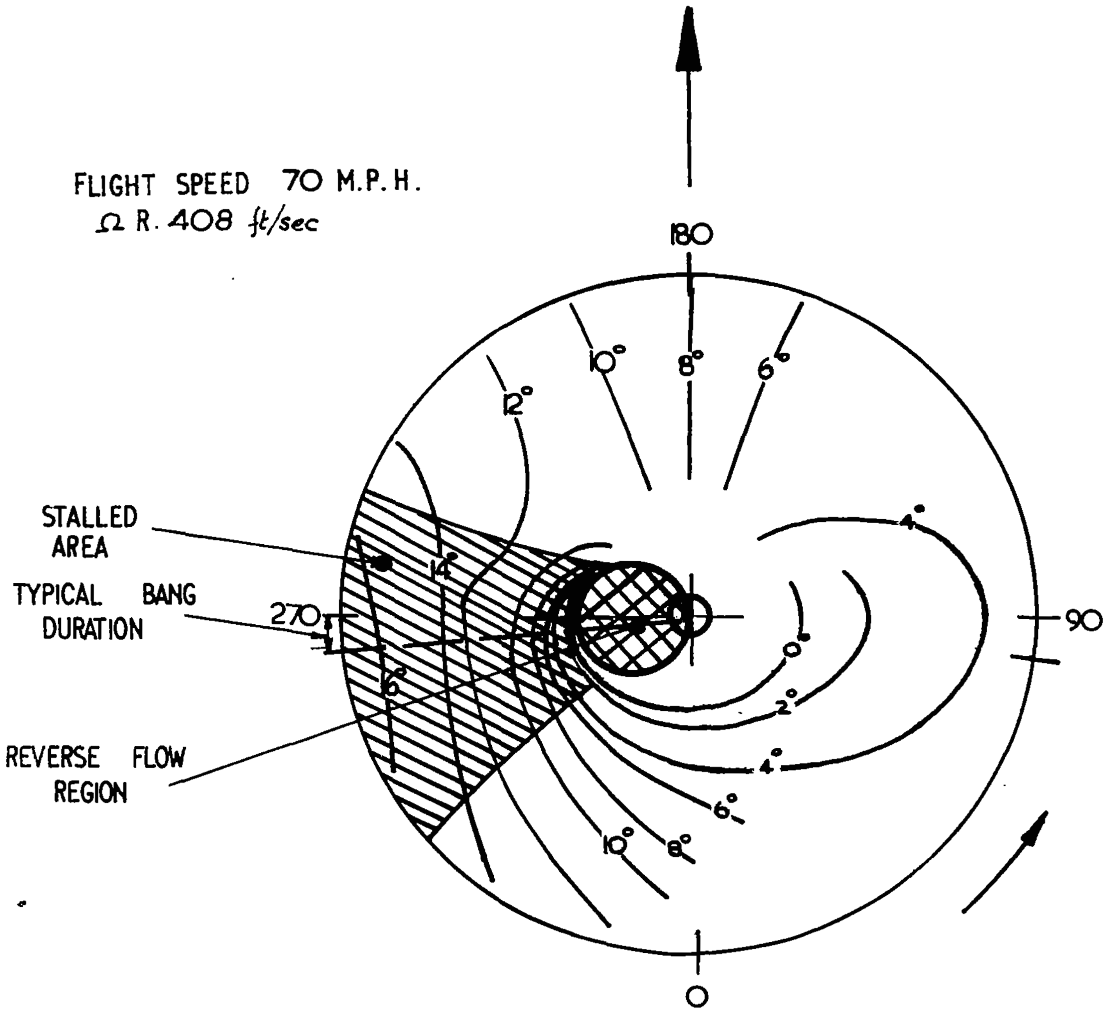


FIG.2 TYPICAL ANGLE-OF-ATTACK-CONTOUR FOR SINGLE ROTOR HELICOPTER.

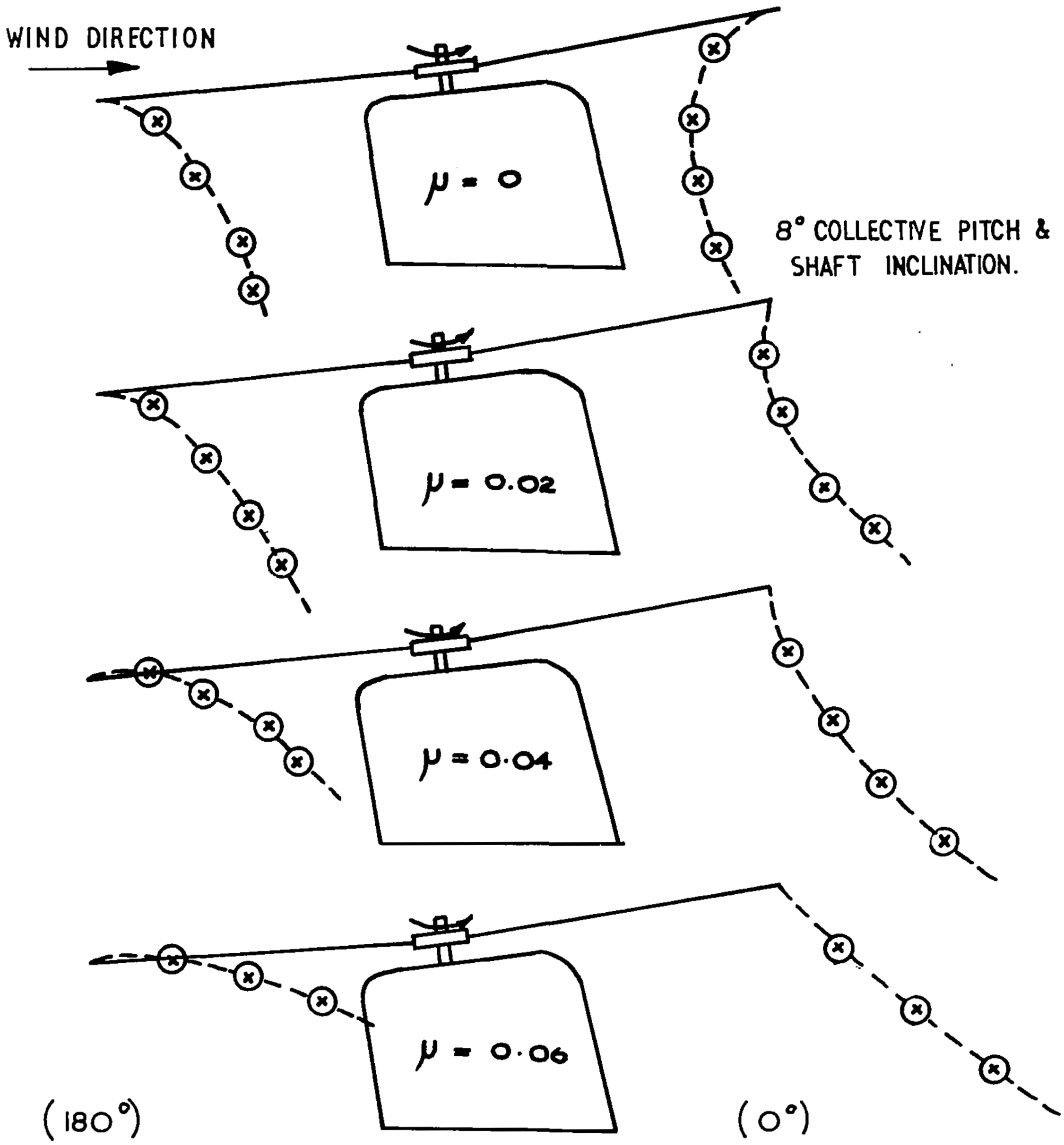


FIG.3. VORTEX PATHS IN THE FORE-AND-AFT PLANE. (REFERENCE 25)

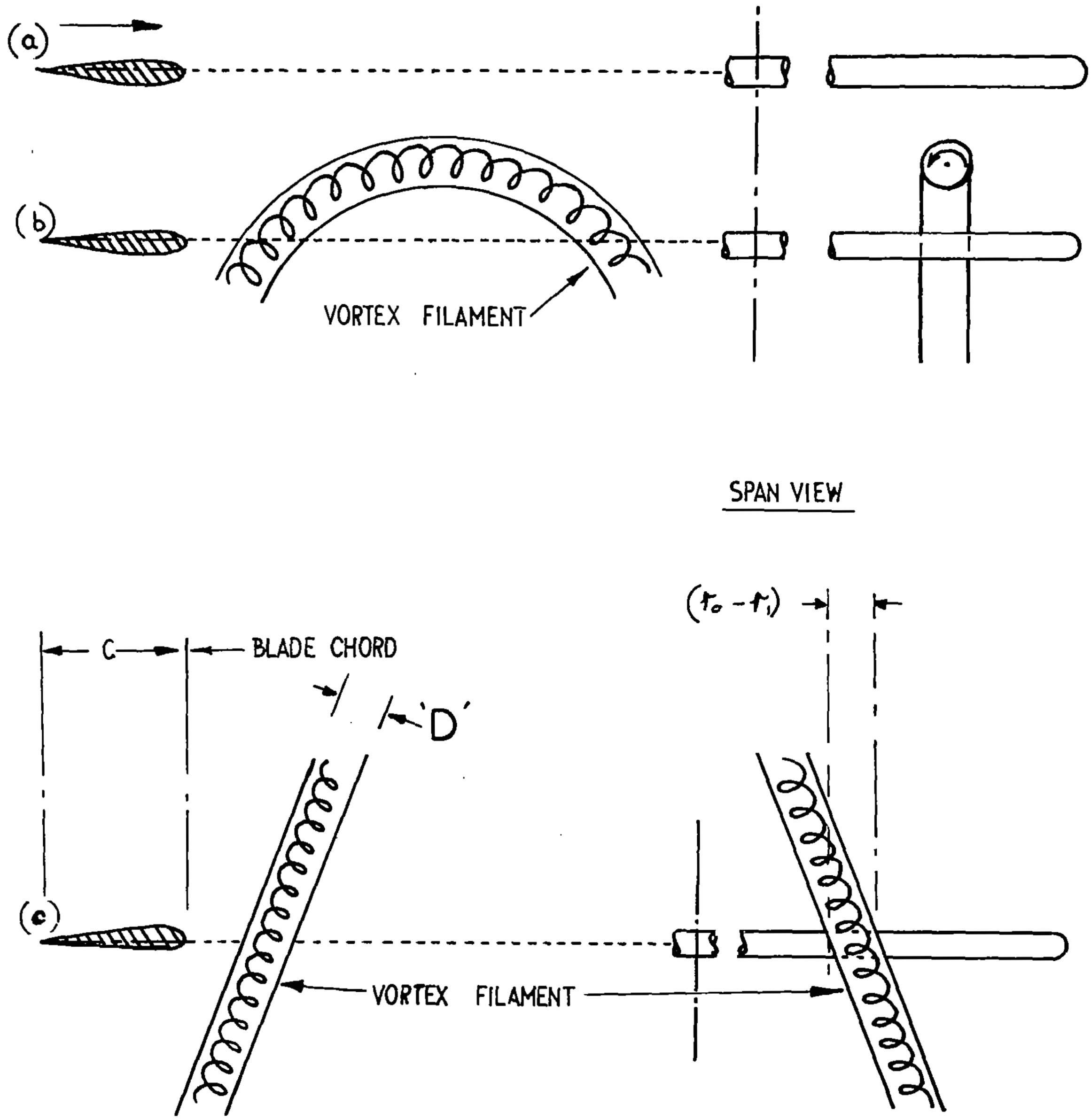


FIG.4. IDEALIZED BLADE VORTEX FILAMENT INTERSECTION

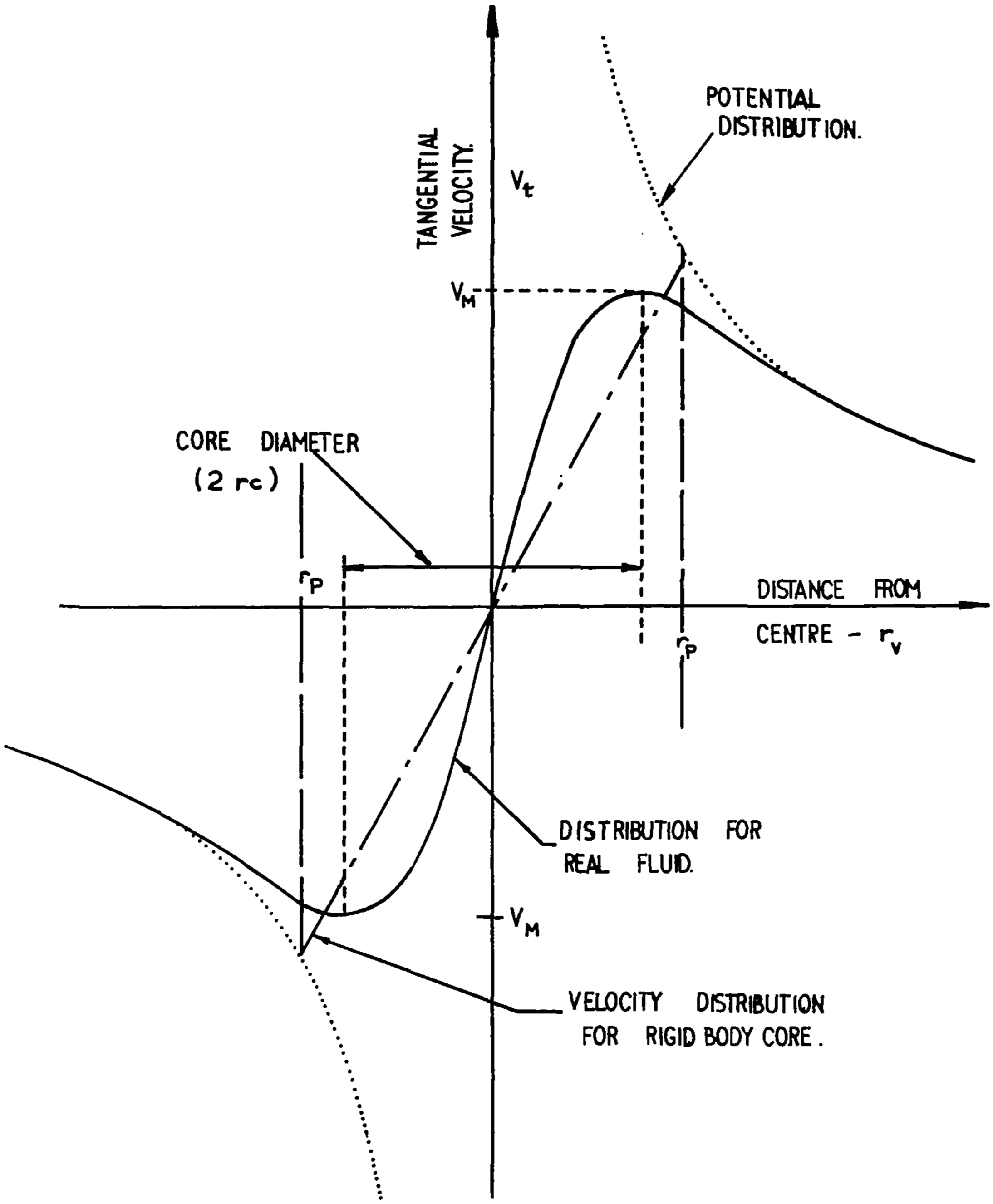


FIG. 5. VELOCITY DISTRIBUTION THROUGH A VORTEX CORE.

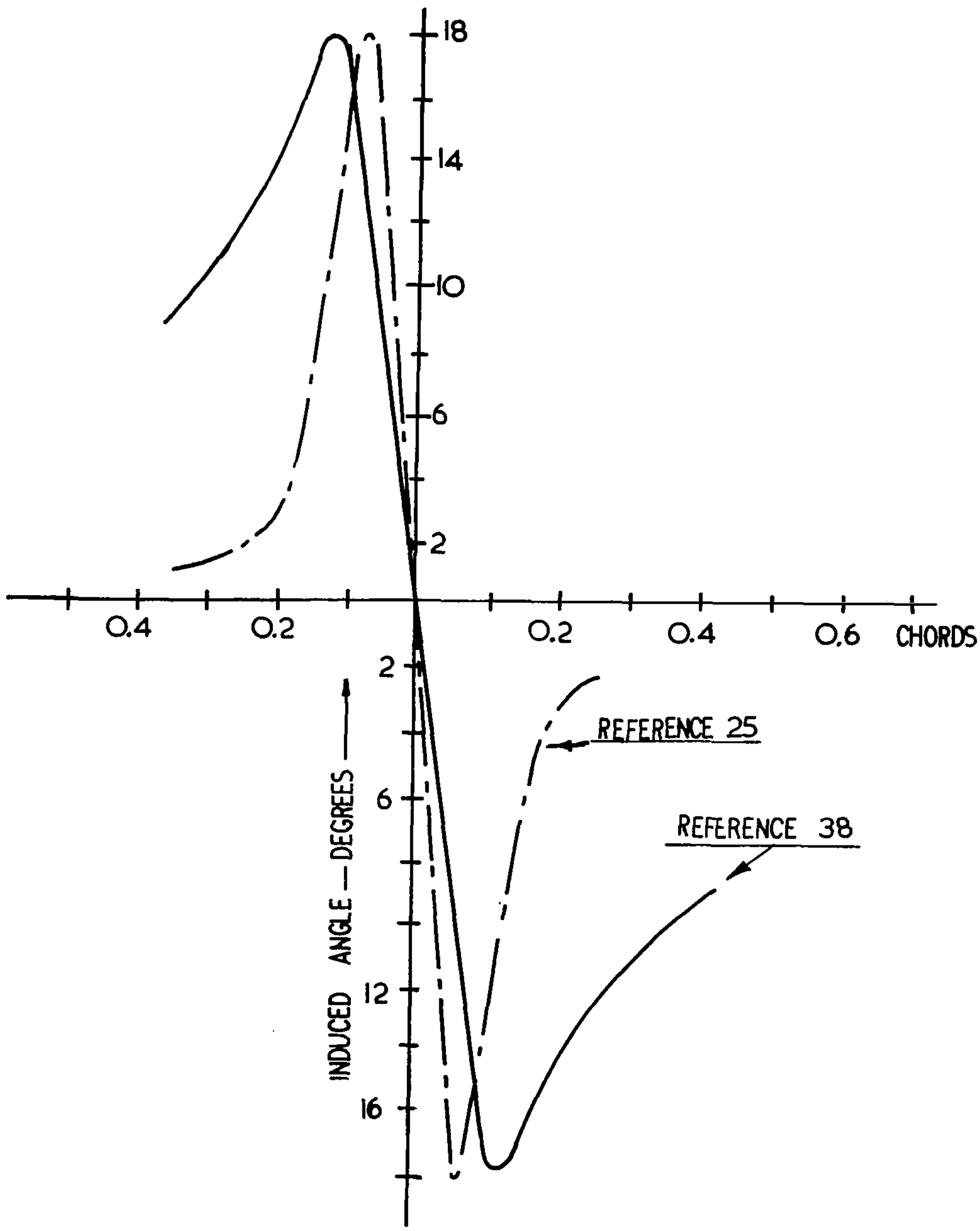


FIG. 6. MEASURED VORTEX PROFILES.

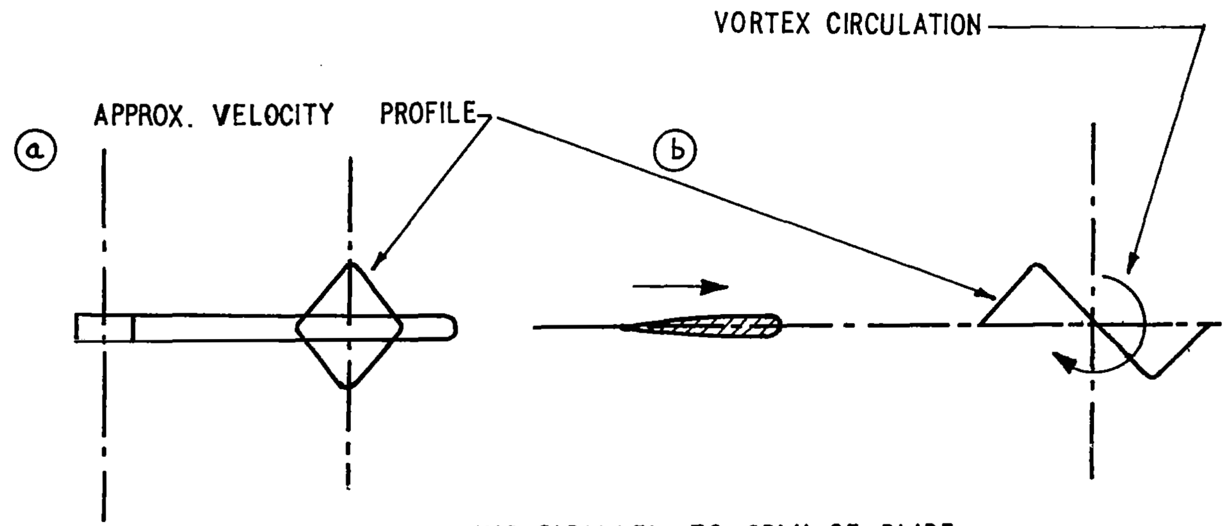


FIG.7·1 VORTEX CORE AXIS PARALLEL TO SPAN OF BLADE.

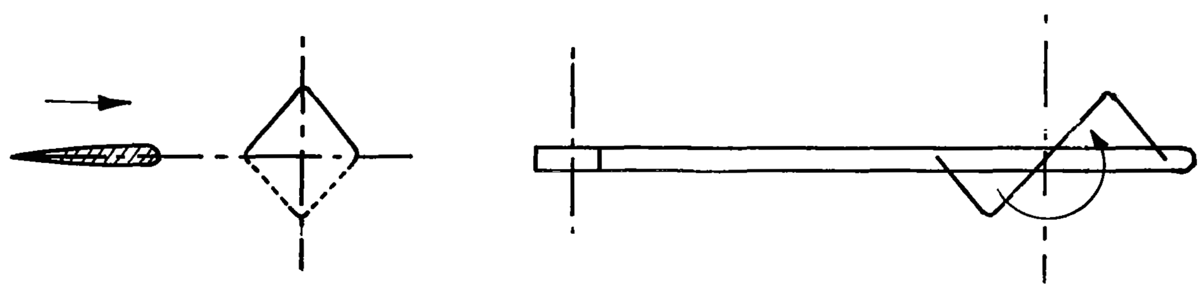


FIG.7·2 VORTEX CORE AXIS PARALLEL TO DIRECTION OF BLADE MOTION.

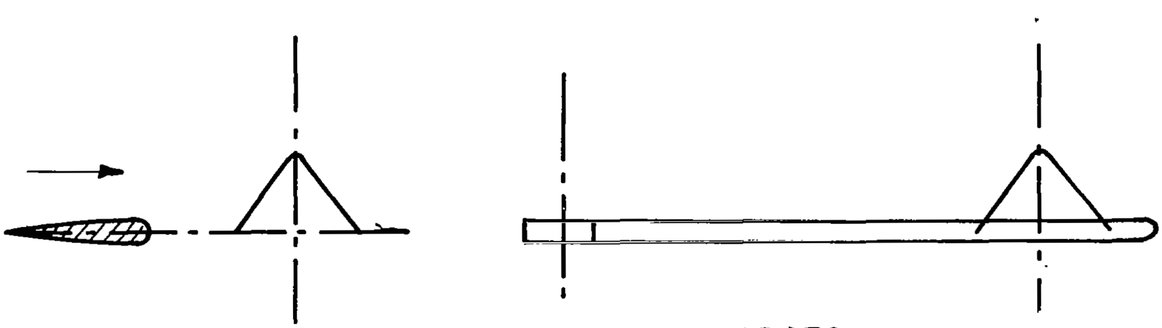
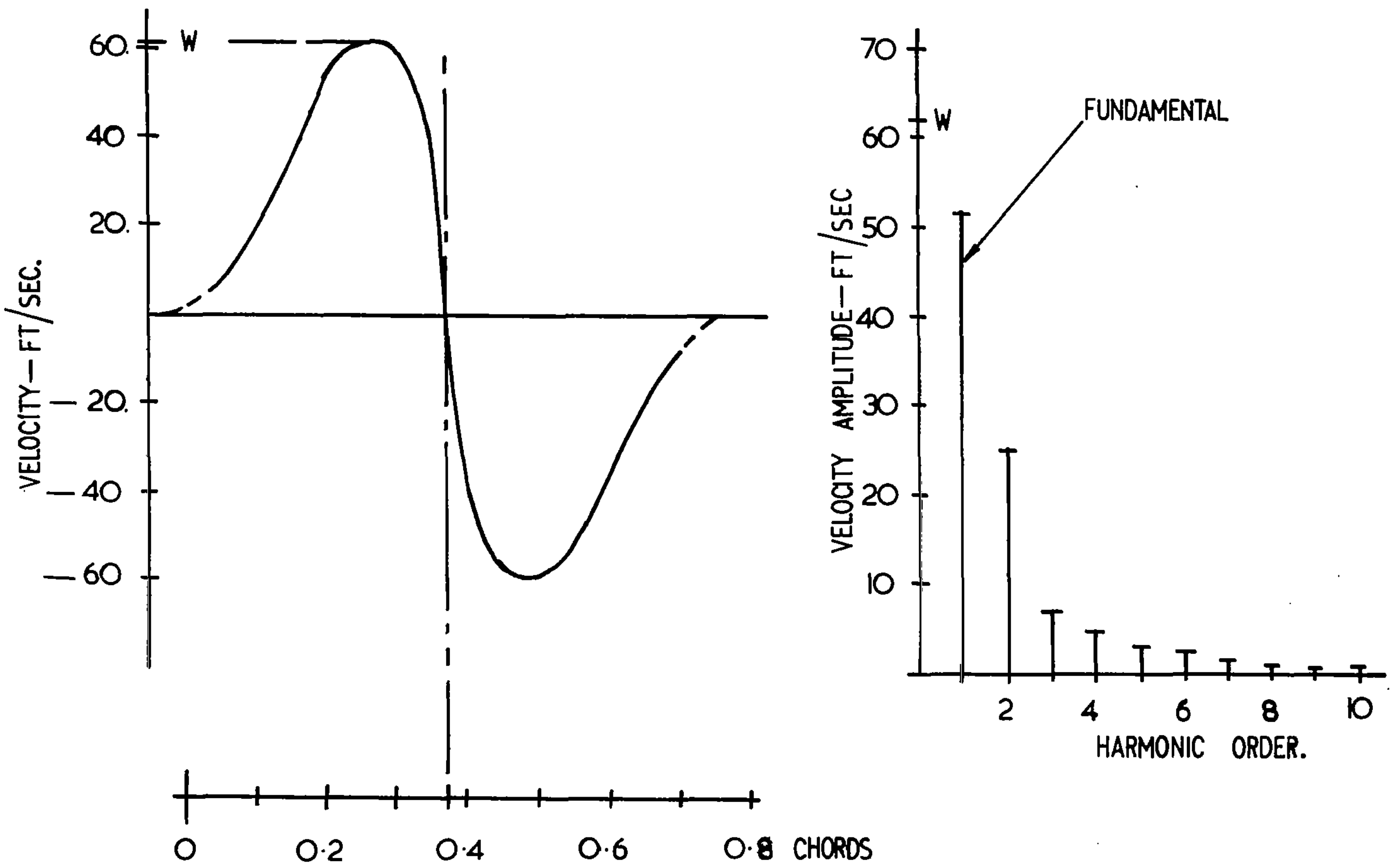


FIG.7·3. THEORETICAL MODEL FOR CASE SHOWN IN FIG.7·2

FIG.7. BLADE VORTEX INTERSECTIONS.



8·1 – PROFILE.

8·2 – HARMONIC CONTENT.

FIG. 8. GUST CHARACTERISTICS.

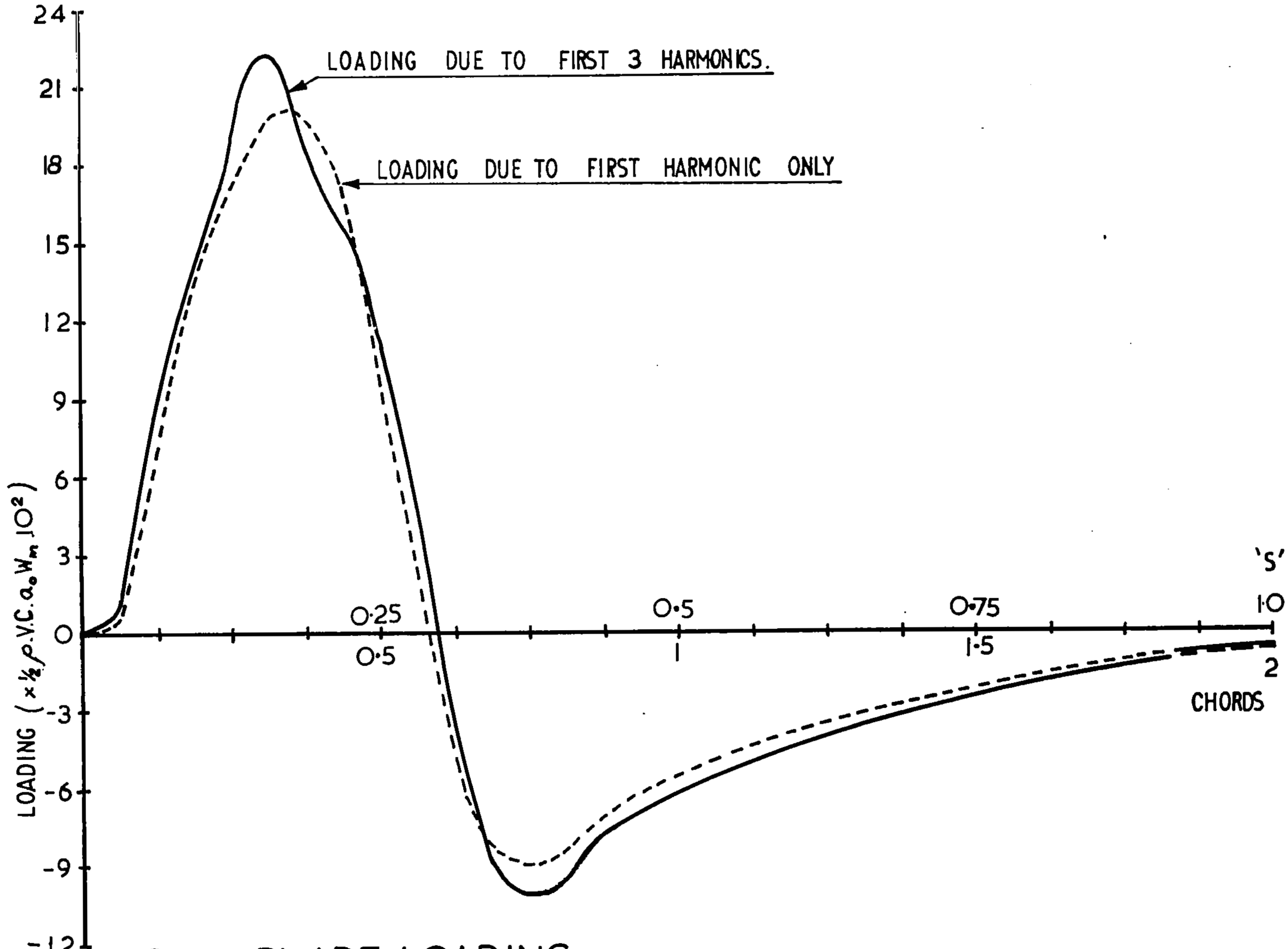


FIG.9 BLADE LOADING

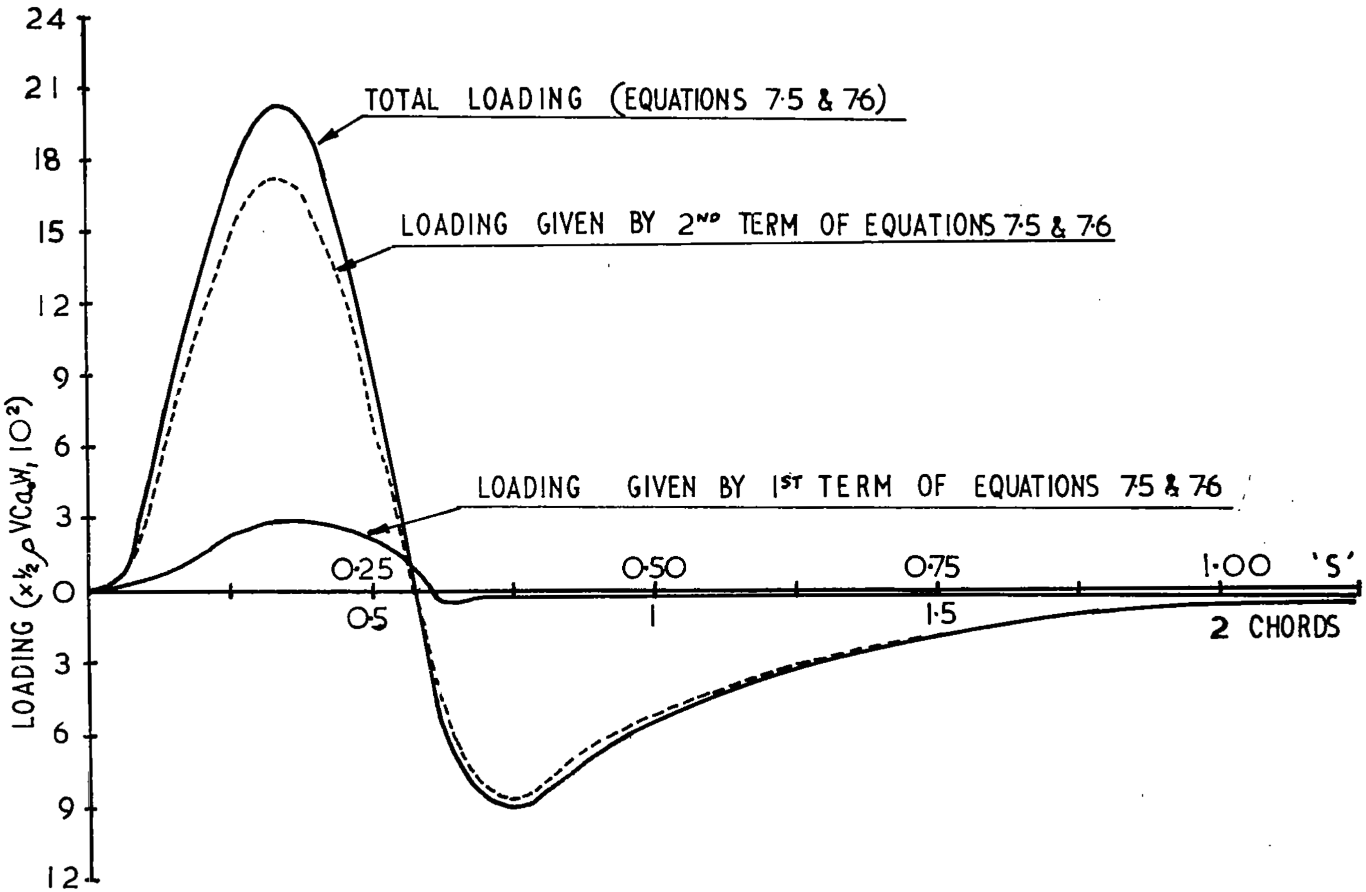


FIG.10. BLADE LOADING FOR FIRST HARMONIC

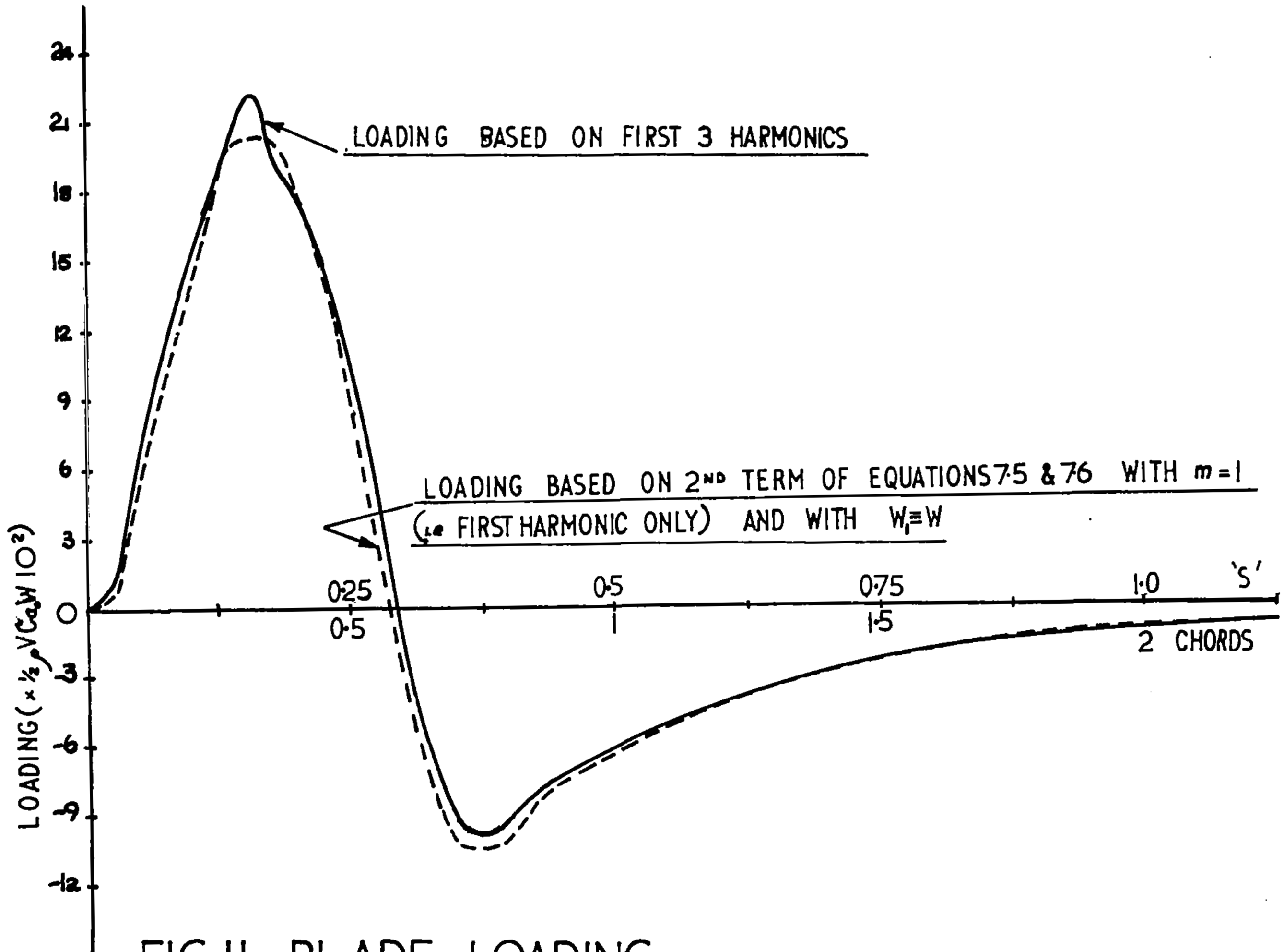


FIG. 11. BLADE LOADING.

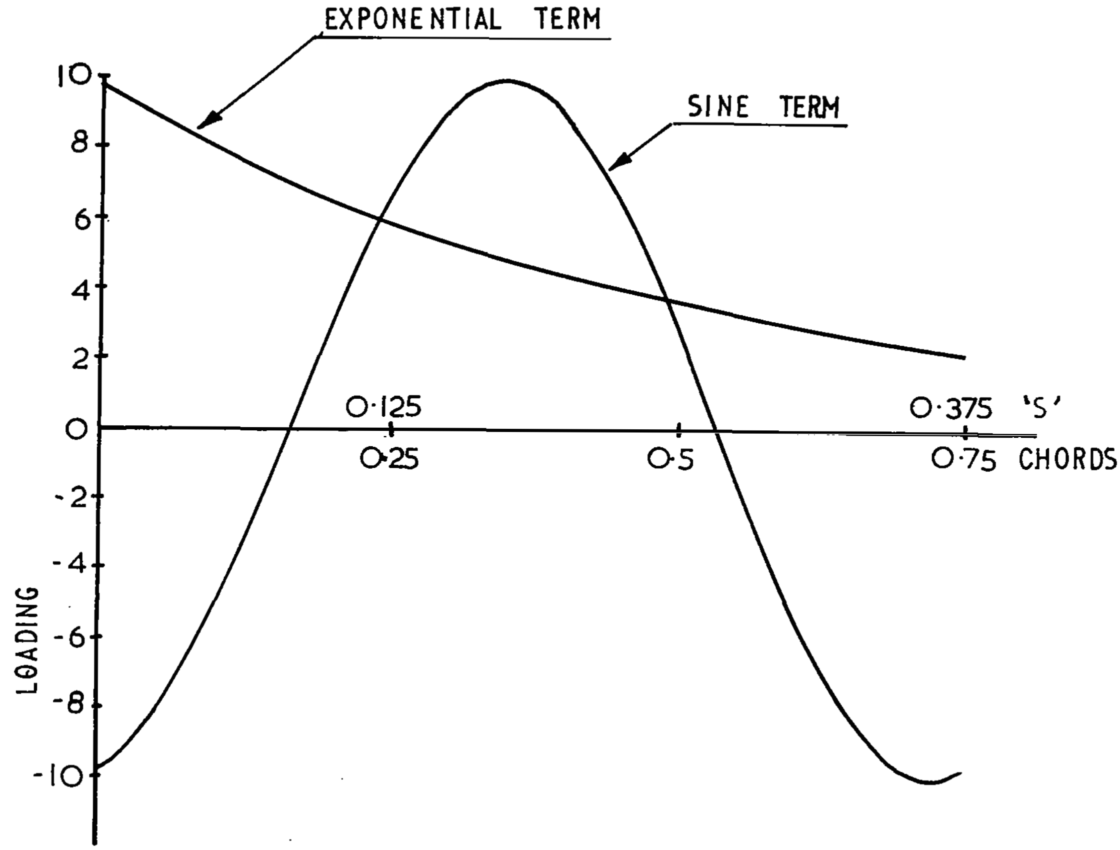


FIG. 12. BLADE LOADING - COMPARISON OF EXPONENTIAL & SINE COMPONENTS OF 2ND. TERM OF EQUATION 7·5

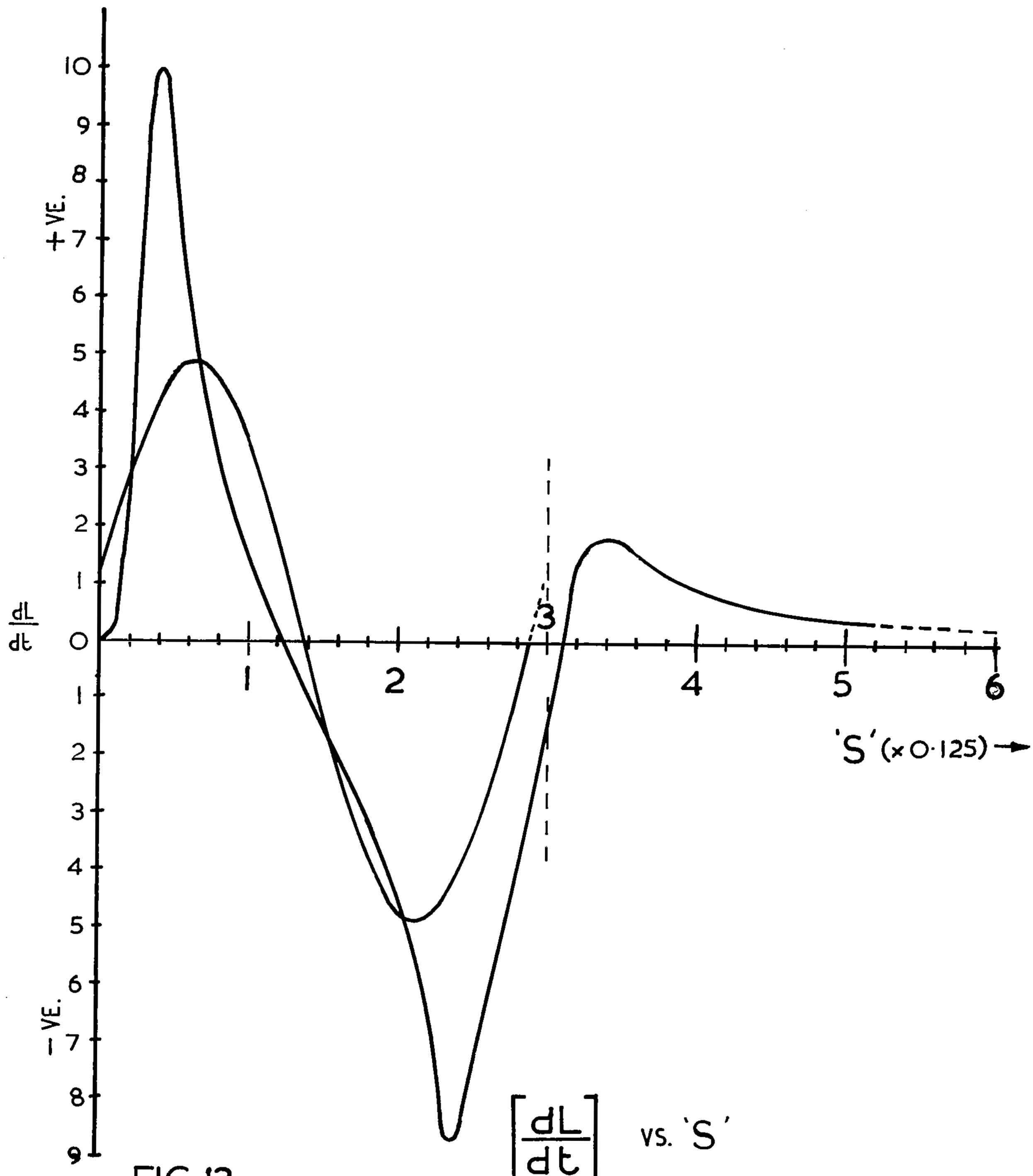


FIG. 13.

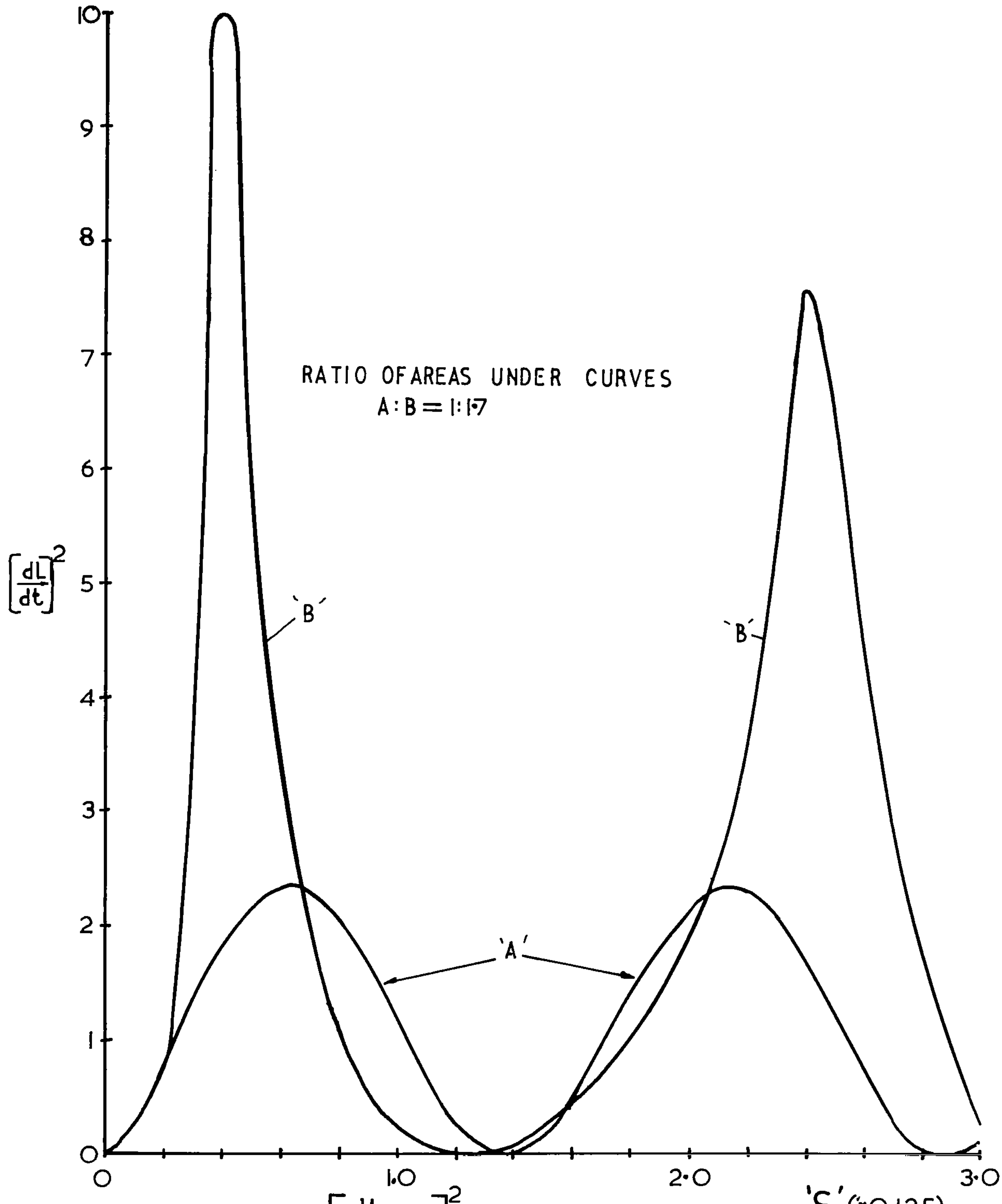


FIG. 14. $\left[{dL}/{dt}\right]^2$ VS. 'S'

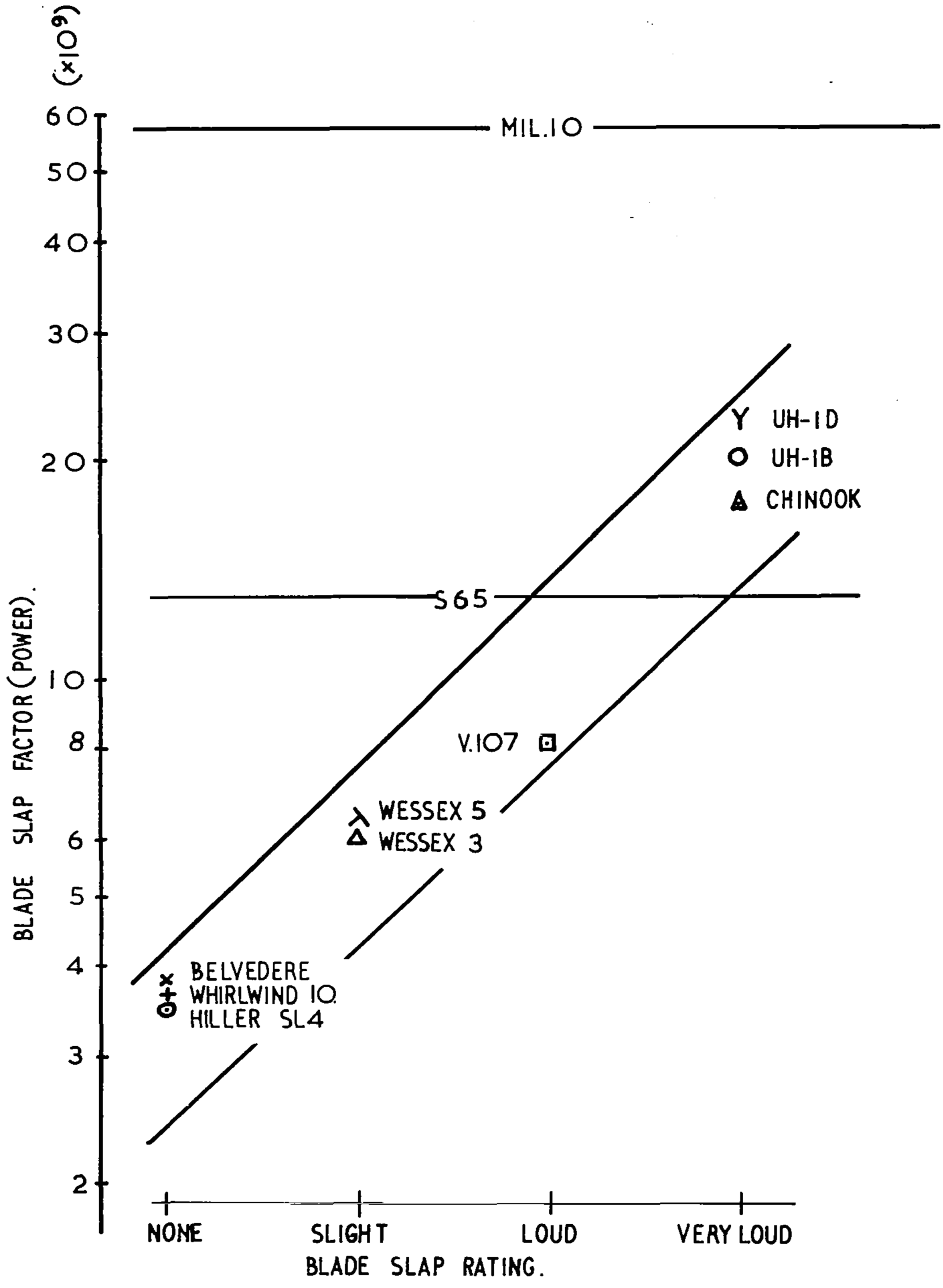


FIG. 15. BLADE SLAP FACTOR (POWER) VS. SUBJECTIVE ASSESSMENT.

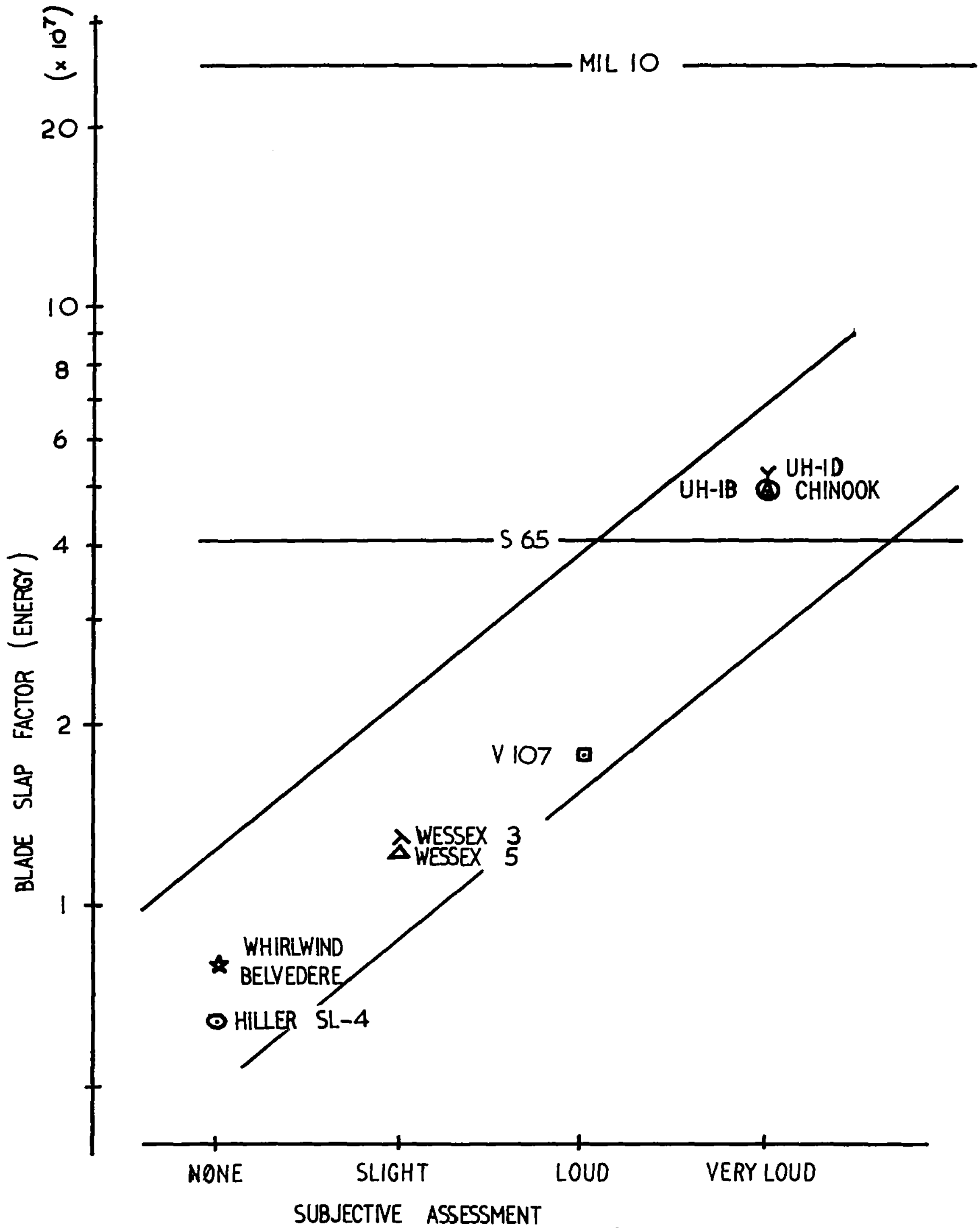


FIG. 16. BLADE SLAP FACTOR (ENERGY) VS. SUBJECTIVE ASSESSMENT.

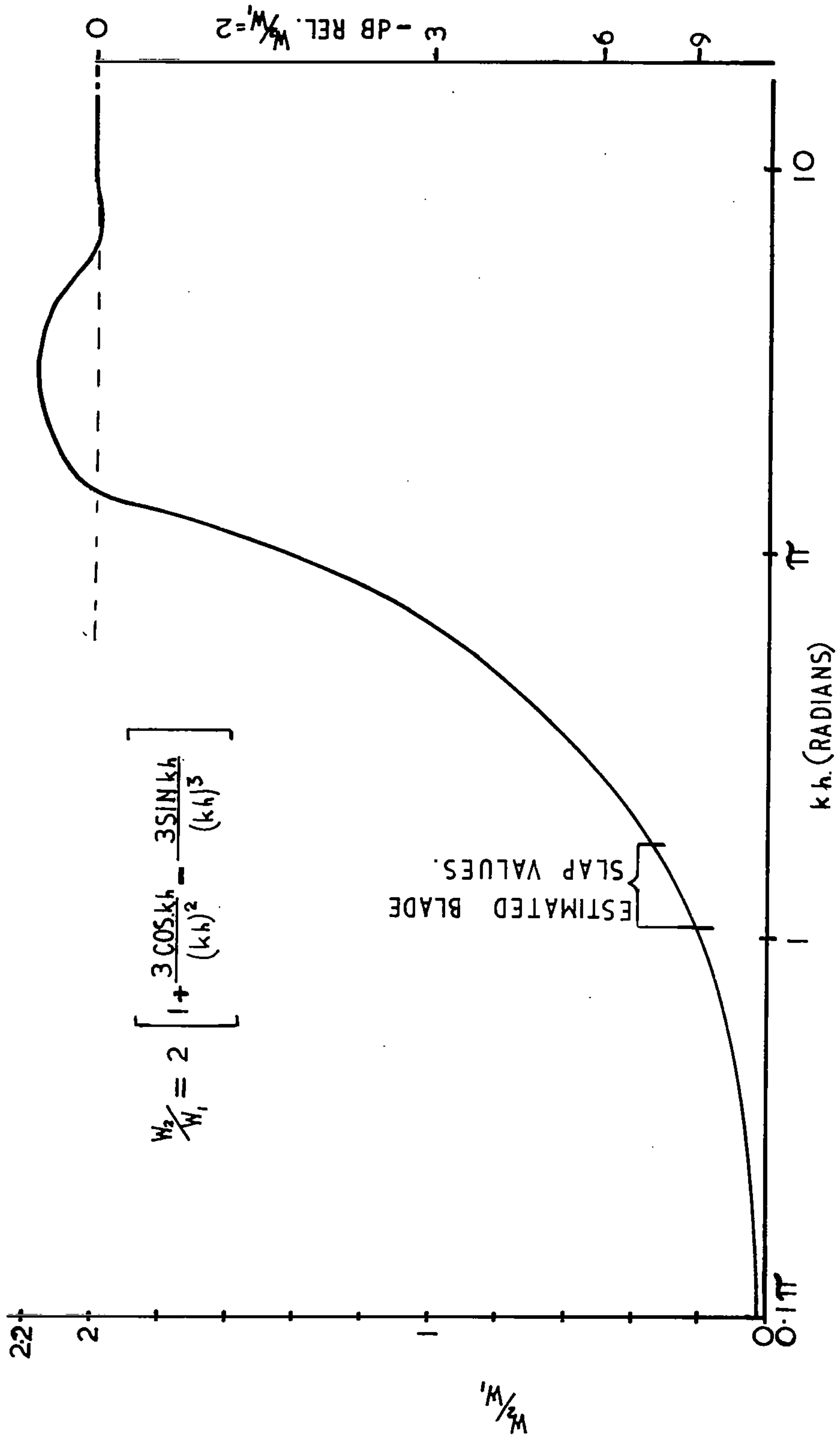


FIG. A1. W_2/W_1 vs. kh

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