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**Design Criteria For And Applications Of The Vortex Generator Mixing Principle**

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## Design Criteria For And Applications Of The Vortex Generator Mixing Principle

### Summary

There are at this date numerable projects, in various Divisions of U.A.C., either in progress or contemplated, which are employing the vortex generator mixing principle. This principle is defined as the method which utilizes tip vortices generated by low aspect ratio airfoils for the purpose of transverse mixing of the flow of a confined fluid medium where the axis of the vortex is parallel to the direction of flow. To date, vortex generator design criteria have not been published. Therefore, for the assistance of those individuals concerned and in the interest of the vortex mixing principle, the following material (subject to modifications) is presented in this memorandum report.

1. The principle of the vortex mixing method.
2. Optimum vortex arrangement with reference to stagger and direction of rotation.
3. Design criteria for:
   a. The vortex generating airfoil
   b. The configurations for various vortex mixing applications

## Introduction

The vortex generator mixing method was first conceived for the purpose of eliminating the flow separation then existing in the U.A.C. 8-ft. tunnel diffuser. The results obtained from this application (see Reference 1) indicated that this mixing method proved to be an effective means of energizing a retarded boundary layer. Initially, the Wind Tunnel Group was concerned with the development of the principle for applications to flow separation problems. During this development, members of the Gas Dynamics Group applied the principle for the purpose of obtaining greater fuel and air mixing in burners. A second application made by the Wind Tunnel Group to increase the efficiency of the root sections of the U.A.C. main tunnel fan (see Reference 2) gave additional experimental verification relative to the effectiveness of this mixing method. Consequently, individuals of United Aircraft Corporation have conceived the following possible applications.

1. Eliminating or delaying flow separation along the surface of a confined fluid medium.
2. Increasing the efficiency of the tip and/or root sections of fan, compressor, or propeller blades.
3. Increasing a burner efficiency by obtaining greater air and fuel mixing.
4. Increasing the rate of transfer of heat from or to the flow of a confined fluid medium.
5. Decreasing an undesirable transverse velocity, temperature, or density gradient of the flow of a confined fluid medium.

Projects are now in progress or are contemplated for the purpose of developing design criteria for the various applications. For the assistance of those individuals concerned, the present report has been written. The report has been subdivided as follows:

Section I — Principles of the vortex mixing method
Section II — Arrangement of the vortices
Section III — Design criteria for:

A. The vortex generating airfoil
B. The configurations for various mixing applications

The design criteria contained in this report are based on information obtained, to date, from flow separation applications being developed by the Wind Tunnel Group. Since this work has not as yet been completed, all design criteria are subject to modifications.

## I. Principles of the Vortex Mixing Method

The vortex generator mixing method is defined as the method which utilizes tip vortices generated by low aspect ratio airfoils for the purpose of transverse mixing of the flow of a confined fluid medium where the axis of the vortex is parallel to the direction of flow.

It is often desirable to have, in the flow of a confined medium containing two or more fluids, a state wherein the fluids are completely mixed. For a single fluid medium, it is sometimes advantageous to eliminate or reduce an existing temperature, velocity or density gradient, or in the case of heat transfer from or to the confined medium, it is desirable to increase the temperature gradient at the confining surface. Thus, a mixing device is indicated having, inherently, low consumption of energy, ease of fabrication, and being effective and efficient in operation.

It is known that a three-dimensional airfoil when immersed in a flowing fluid will release a trailing vortex near its extremities. Such a vortex is defined as a tip vortex. By choosing for a particular application a proper arrangement of these tip vortices, fluid mixing will take place. Thus, a method is suggested for obtaining the desired mixing as mentioned in the previous paragraph. Compared to the existing turbulizers (e.g., screens, etc.), the power consumption for equivalent performance is lower and the effectiveness is higher and better controlled.

II. Arrangement of the Vortices

The vortex generator arrangement, to date, has been the choice of a number of vortex generators located in a plane perpendicular to the axis of flow and so arranged that adjacent generators release oppositely rotating vortices. The following reasons are given for this choice. Refer to Figure I.

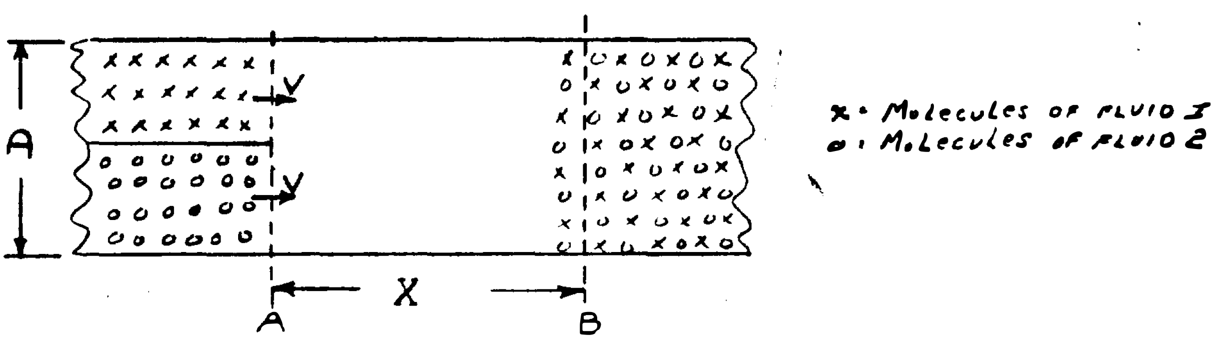


Figure I

Assume that it is desired to obtain by the vortex mixing method complete mixing at Station B between the molecules of Fluid 1 and Fluid 2 entering separately at Station A with equal velocities, V. Obviously, to obtain the desired state at Station B, for a minimum distance, X, the mixing vortices should be located near or at Station A. The desired mixing will be obtained by arranging the vortices so that their axes are parallel to the axis of flow. Inasmuch as the induced velocities due to the mixing vortices are additive (vectorially) in a plane perpendicular to the axis of flow, the optimum arrangement of the vortices is a



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non-staggered arrangement; that is, with all the vortices originating in the same plane. Refer to Figure II.

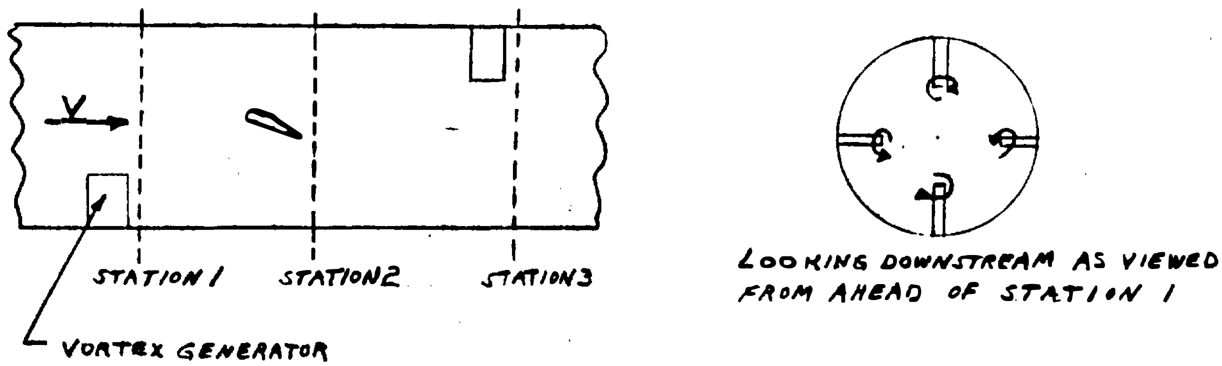



Figure II

If a plane is cut through Station 1 (perpendicular to the direction of the velocity, V), then the induced velocities due to the vortex released by the single airfoil at Station 1 will depend only on the vortex at Station 1 and its image. At Station 2, the induced velocities will depend on the tip vortices generated by the first two airfoils and their images, and so on. Thus, at Station 1, for example, a staggered arrangement of the vortices will not result in an equal amount of mixing compared to a non-staggered arrangement (all vortices located just upstream of Station 1). At some station downstream of all the vortices the amount of mixing obtained by a staggered arrangement will not equal that of a non-staggered arrangement if the first vertex is dissipated by the time it reaches the last vortex station or if the first vortex axis changes direction. It is suggested that a staggered arrangement could be used if:

1. There is no dissipation of the first vortex in the series before it reaches the last vortex.
2. There is no large change of direction of the vortex axis between vortices.
3. The first vortex does not produce unfavorable flow conditions for the succeeding vortex generators.

It has been stated that the mixing vortices have been arranged in a plane perpendicular to the axis of flow with adjacent vortices opposed in direction. It is known that a right-hand airfoil generates a tip vortex whose direction of rotation is clockwise when observed from the leading edge of the generating airfoil. Consequently, a left-hand airfoil generates a counter-clockwise vortex. Thus, a vortex mixing installation could contain all right- (or left-) hand airfoils wherein adjacent vortices would rotate similarly (co-rotating arrangement), or by choosing alternately left- and right-hand airfoils, adjacent airfoils will generate opposed vortices (counter rotating arrangement). To determine which arrangement affects the greater amount of mixing, let us examine the velocity fields set up by four vortices and their images in a duct. These velocity fields were obtained as a result of plotting the streamline patterns for four equal vortices symmetrically placed in a duct of unit radius. Refer to Figures X and XI on Pages 20 and 21. The velocity fields which may be obtained from these plots by the method given on the graphs indicate the amount of mixing which would take place between the fluid in the center and that near the outer portions of the duct. It is noticed that a greater amount of mixing may be obtained from a counter rotating configuration. Experimental confirmation of this fact may be obtained from the results reported in Reference 1.

For the above mentioned reasons, we have been concerned with, and therefore suggest using, the counter rotating configuration to obtain the maximum mixing effect. However, there may be certain installations for which the co-rotating configuration would prove superior. Such an arrangement imparts to the fluid adjacent to the confining surface of the flow a flow direction which may be up to 60 degrees relative to the axis of main flow, depending upon the vortex strength. Therefore, the co-rotating arrangement has possible applications to propeller, fan, or compressor blade root or tip stall problems.

It is pointed out that the aforementioned argument which proved a counter rotating configuration the superior mixer may be applied, as well, to a vortex arrangement consisting of two or more series of generators, i.e., where the succeeding series are located downstream relative to the first series.

## III - A. Design Criteria for the Vortex Generating Airfoil

As has been stated previously, this mixing method utilizes the tip vortex generated by a low aspect ratio airfoil mounted on the confining surface of the flow. Obviously, a well-defined, strong, tip vortex is essential. It is known from aerodynamical theory that the circulation spanwise must, in this case, be essentially constant.

The circulation, $\Gamma$, about an airfoil in terms of the section lift coefficient, $C_L$; the airfoil chord length, C; and the free stream velocity, V; is given by the following theoretical relation.

$$\Gamma = \frac{C_L CV}{2}$$

To maintain constant spanwise circulation when the airfoil spans a velocity gradient, it is necessary to either twist or taper the airfoil an amount determined by the magnitude of the spanwise velocity gradient. Due to the fact that twisting an airfoil to this degree generally would not be feasible, it has been the practice to taper the airfoil an amount determined by the spanwise velocity gradient. It is suggested that a non-tapered airfoil may be used if the spanwise velocity gradient does not appreciably affect the airfoil performance relative to its generating a well-defined tip vortex. To date, experiments conducted with non-tapered airfoils in the field of a spanwise velocity gradient (airfoils spanning a boundary layer) have indicated that, compared to the tapered airfoil, the non-tapered airfoil has a much earlier stall angle, consequently it will not generate as strong a tip vortex.

In addition, the following airfoil design criteria are advanced. These criteria are based on experimental results obtained from experiments with generators mounted in the boundary layer. Optimum airfoil performance has resulted from:

(a) No sweep of the 1/4 chord line.
(b) No spanwise twist (no experimental verification other than 0° twist gave satisfactory results).
(c) An N.A.C.A. $64_1812$ ($a = 0.3$) airfoil section.

By using the suggested N.A.C.A. section, a lift coefficient of approximately 0.85 is obtained at an angle of attack relative to the mean free stream direction of 16°. These airfoils were not observed to stall until an angle of attack of approximately 30° was reached. The critical Mach number for this suggested N.A.C.A. section when operating at a lift coefficient of 0.85 is approximately 0.6. Therefore, when it is desired to use this mixing device in fluid flow having a Mach number greater than 0.6, it is suggested that a wedge-shaped airfoil (triangular in planform) be used inasmuch as the wedge airfoil has better transonic characteristics compared to the conventional airfoil. Further wedge airfoil information is contained in Reference 3.

It has been pointed out that the aforementioned design was developed for the generating airfoil when immersed in the fluid boundary layer. Refer to the following figure.

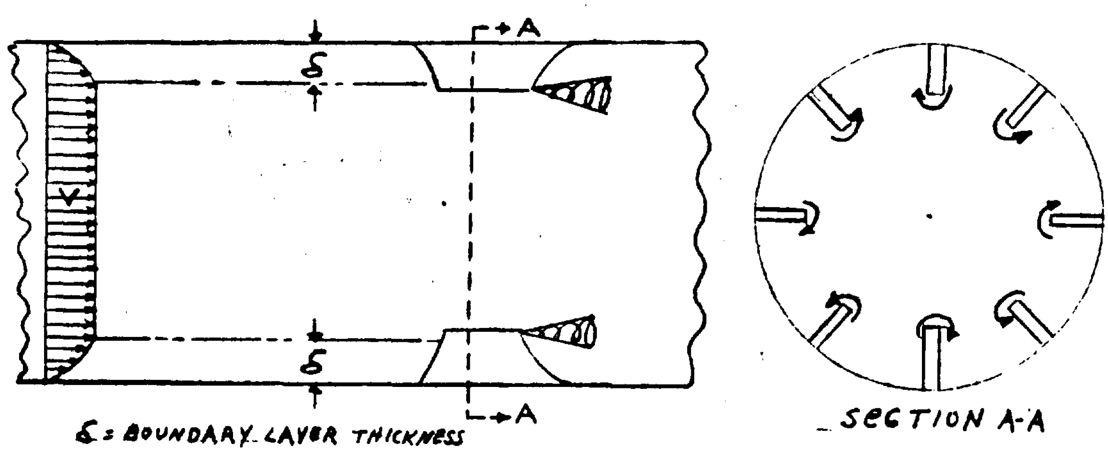
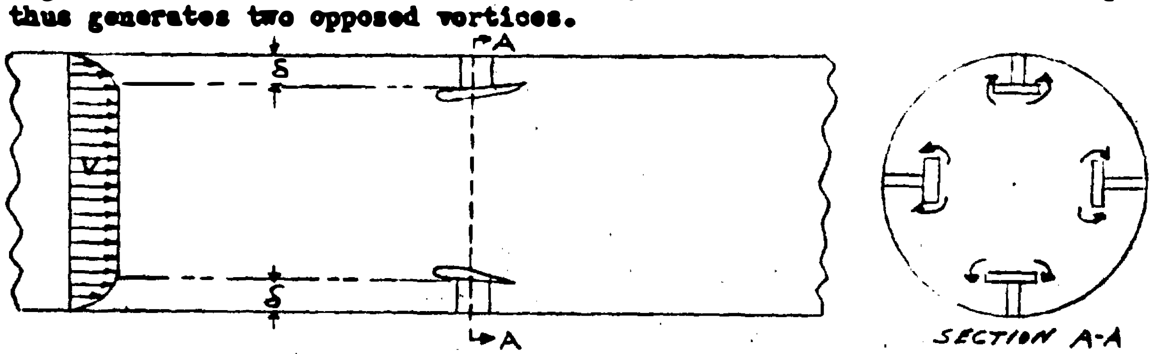



Figure III

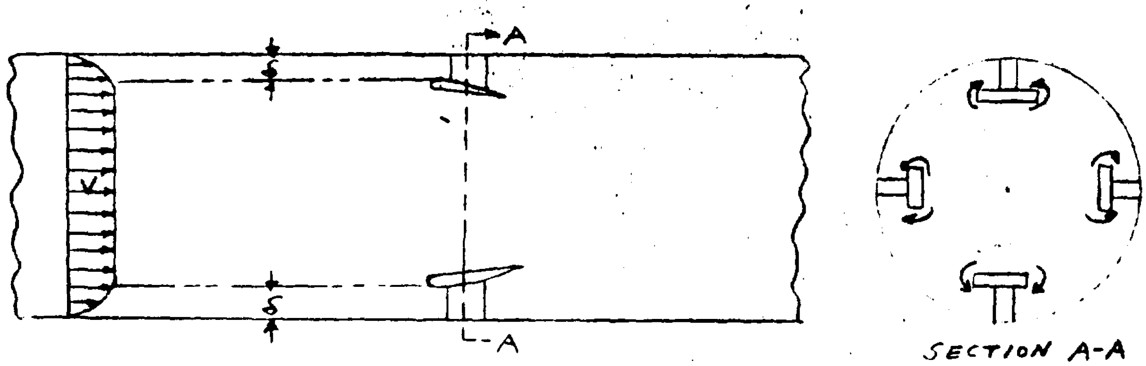
An alternate method of mounting the generating airfoils is indicated in Figures IV and V which follow. This configuration consists of mounting a constant chord airfoil on a strut; the airfoil of such an arrangement thus generates two opposed vortices.




Suspended Airfoils Lifting Away From Confining Surface

Figure IV

An alternate configuration is indicated in Figure V.




Suspended Airfoils Lifting Toward Confining Surface

Figure V

Experiments carried out by the Wind Tunnel Group using the suspended airfoil configuration indicated that the strut design was critical, inasmuch as both configurations had early stall characteristics. Results from this configuration did not duplicate (with the struts tested) the effective results obtained from the configuration indicated in Figure III. However, these tests were not extensive and thus should not be considered as absolute evidence that the suspended configuration is inefficient.

## III - B. Design Criteria For Various Mixing Applications

The following possible applications of the vortex generator mixing principle have been conceived.

1. Eliminating or delaying flow separation along the surface of a confined fluid medium.
2. Increasing the efficiency of the tip and/or root sections of fan, compressor, or propeller blades.
3. Increasing a burner efficiency by obtaining greater air and fuel mixing.
4. Increasing the rate of transfer of heat from or to the flow of a confined fluid medium.
5. Decreasing an undesirable transverse velocity, temperature, or density gradient of the flow of a confined fluid medium.

Design criteria relative to the span and chord lengths, the number, and the location (relative to the initial flow separation point) of the vortex generators for Applications 1 and 2, above, are advanced based on information obtained from results of tests as contained in References 1 and 2 and from tests with a 40° diffuser. For Applications 3, 4 and 5, design criteria are advanced based on information obtained from the same tests. However, the suggested design criteria for Applications 3, 4 and 5 are intended for the purpose of suggesting to the individual concerned a vortex configuration for the initial tests.

It is repeated that these criteria are formulated from information obtained to date and are, therefore, subject to modifications.

### 1. Design Criteria for Application 1 (as stated above)

When fluid flows past a confining surface in viscous motion, it is known that in a thin layer next to the confining surface the fluid velocity is reduced due to friction and shear forces. The thin stratum of fluid so segregated is called the boundary layer. If this fluid must flow against an adverse pressure gradient, the fluid in the boundary layer may have insufficient energy to maintain flow. Thus, this fluid may stop and reverse direction, or, in other words, separation may ensue. In order to prevent or delay the occurrence of separation it is desirable to energize the boundary layer. The flow velocity of fluid in this thin layer may be increased by causing an intermixing between the non-retarded fluid in the main stream and the retarded fluid in the boundary layer. As has been suggested, this intermixing may be affected by a method employing the action of mixing vortices. Experimental confirmation of this application is contained in Reference 1.

To obtain the vortex generator configuration for the purpose of eliminating or delaying fluid separation along a confining surface, it is first necessary to obtain with a reasonable degree of accuracy the point of initial separation and boundary layer velocity profiles and flow directions at various (generally three) stations upstream of the initial separation point. Refer to Figure XII on Page 22 which is an empirical vortex generator design chart. The design areas of this chart were obtained from tests conducted in the U.A.C. 1/12-scale 8-ft. tunnel model with a 7° and a 40° diffuser installed after the test section. The confines of the design areas were obtained from configurations giving, within experimental error, the same diffuser flow or efficiency results. For the 7° diffuser, visual results were plotted because these results were more accurate and informing than were the diffuser efficiency results since this efficiency varied only a few percent with or without separation. However, when separation was delayed in the 40° diffuser, the diffuser efficiency increased by over 100% and thus the efficiency results were used as a figure of merit.

As the ordinates of this design chart, the parameters $\lambda$ and S/d are plotted. The parameter $\lambda$ is defined as (Refer to Figure VI):

$$\lambda = d/\delta = \frac{\text{distance the vortex core lies from the surface}}{\text{the boundary layer thickness}}$$

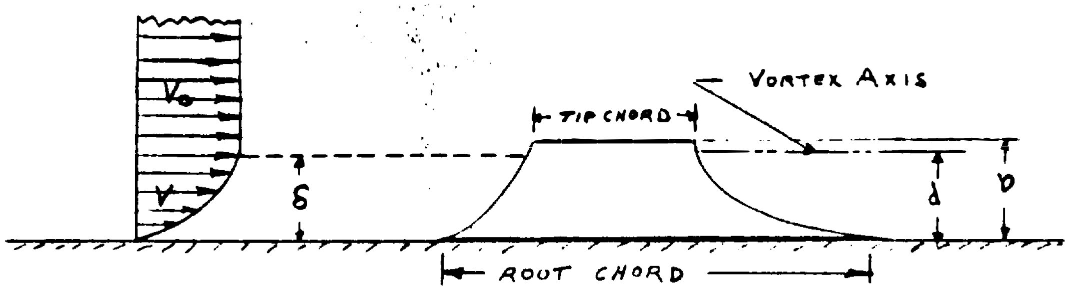



Figure VI $\qquad$ $d \equiv 0.9b$ (by EXPERIMENT)

The parameter S/d is defined as (Refer to the sketch given on the design chart of Figure XII):

$$S/d = \frac{\text{distance between adjacent vortices}}{\text{distance the vortex core lies from the surface}}$$

As the abscissa, the distance ahead of the initial separation point in terms of the local boundary layer thickness* is plotted.

Assume that the initial separation point and boundary layer profiles (and flow directions) upstream of this point, for a flow separation problem, are available. (Refer to Figure VII, following.) By trial and

*This is the thickness at the vortex generator station.



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error calculations, the station is obtained which lies a streamline distance of say 15 $\delta$ ahead of the initial separation point (Station 4).

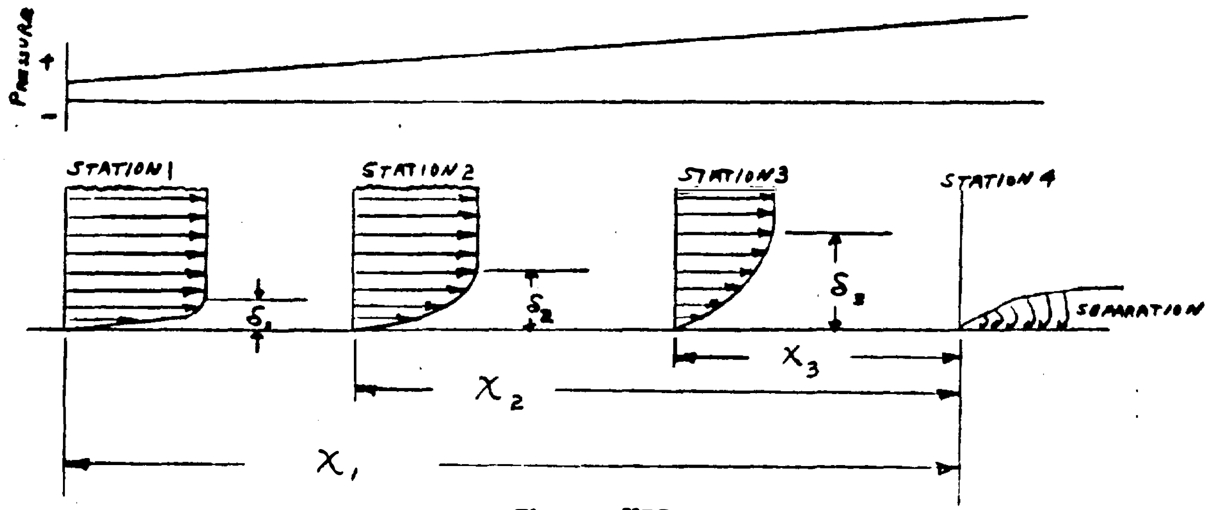


Figure VII

If, for instance, $15\delta_2 = X_2$, then Station 2 is chosen as the vortex generator station. If the boundary layer thicknesses in a plane are not symmetrical, then the maximum thickness is chosen for $\delta_1$, $\delta_2$, or $\delta_3$. By choosing the 12 to 15 $\delta$ station, the generators are placed at that station which results in presenting to Station 4 (the separation point), a full boundary layer; that is, a layer which has been energized by the mixing action and which a positive pressure gradient has not retarded to any extent. This assumes (assumption based on experimental results) that the mixing action is not continuous along the axis; that is, the vortices become diffused. If the 12-15 $\delta$ station is not accessible, physically, then the choice from 5 to 30 $\delta$ is indicated by the design chart*. For values of S/d and $K$, the midpoint of the design areas are chosen. Thus for the 15 $\delta$ station, an S/d of 3 and a $K$ of 1.10 would be chosen.

Assume, for an example, that the 15 $\delta$ station occurred at a duct station where $\delta_{max}$ was measured to be 1.0 in. and the duct diameter, $D$, was 10 in. The number of generators and the span and chord lengths would be determined as follows.

$$K = d/\delta = 1.10 \text{ from chart}$$

$$\text{thus } d = 1.10\ (\delta) = 1.10\ (1.0'') = 1.10''$$

*In general, it has been the practice not to choose a station less than the 12 $\delta$ station inasmuch as it is quite difficult to obtain the initial separation point accurately.

$$\text{Airfoil Span, } b = \frac{d}{0.9} = \frac{1.10}{0.9} = 1.22" \quad (d = 0.9\ b_{s_2} \text{ by experiment})$$

$$S/d = 3.0 \text{ (from chart)}$$

$$\text{Thus } S = 3.0\ (d) = 3(1.10) = 3.30"$$

The number, N, of generators necessary would be:

$$N = \frac{\text{Circumferential distance}}{\text{Distance between generators}} = \frac{\pi(D-d)}{S}$$

$$N = \frac{\pi(10-1.10)}{3.30} = 8.48$$

By rounding off to the nearest even number (to give a symmetrical pattern), the number of generators is determined to be 8. The airfoil tip chord is determined from the following emperical relation (Equation IX-A as derived in the Appendix).

$$C_{tip} = \frac{1.64\ \delta}{C_L}$$

For the example at hand, the tip chord length is (by assuming $C_L = 0.85$ for the N.A.C.A. $64_1$-812 section set at 16°)*:

$$C_{tip} = \frac{1.64(1)}{0.85} = 1.93"$$

The spanwise chord lengths are obtained as follows (using Equation X-A given in the Appendix).

$$C = C_{tip}\left(\frac{V_o}{V}\right)$$

By plotting the spanwise chord length distribution as determined by the boundary layer velocity profile, $V/V_o$, and fairing the curve for ease of construction, the generator planform may be obtained.

The suggested experimental procedure would be to place the 8 generators (counter rotating configuration - see Section II) at the 15 $\delta$ station at an angle of attack of 16°. The angle of attack is then varied positively and negatively until an optimum angle is obtained. If this configuration does not give satisfactory results, the span lengths are cut by approximately 5% and the above experiments are repeated. By cutting the span

*Refer to Sec. III-A, Item (C).

lengths, the optimum span is obtained. This is necessary due to the fact that, generally, the boundary layer thickness is not symmetrical in a plane and that the effective boundary layer thickness at the 15 $\delta$ station may not be as large as the actual measured thickness, as in the case of a steep velocity gradient (from the confining surface) up to a velocity ratio of 0.9 followed by a gradual gradient from a velocity ratio of 0.9 to 1.0. When cutting the span length, enough generators must be added to maintain a S/d factor of approximately 3 $\pm$0.5. It is suggested that, if possible, a tuft survey of the flow about the vortex generator be made. This will indicate if a well-defined tip vortex is being released, if the strength of the vortices are approximately the same, and if the airfoil is or is not stalled. Obviously, if the generating airfoils are not performing efficiently, measures should be taken to correct the design.

<u>Note</u>: It has been stated that the critical Mach number for the N.A.C.A. $64_1812$ section operating at a lift coefficient of 0.85 is approximately 0.6. If the local Mach number at the 15 $\delta$ station is greater than 0.6, it is suggested that a wedge-shaped airfoil be used. Refer to Reference 3.

If one series of generators does not eliminate or sufficiently delay the flow separation, then additional series may possibly be used. To date, there is very little information available relative to the design for the multiple series configurations. It is suggested that a second series is feasible if the first series succeeds in delaying the separation by an amount which allows a second series to be located at least 10 first-series chord lengths downstream. The location of the second series would be determined, presumably, by the same method as that of the first series.

2. <u>Design Criteria for Application 2 (See Page 8)</u>

The sections of propeller, compressor, or fan blades near the confining surface of the flow of a fluid medium may operate at times at an angle of attack greater than the section stall angle. This effect is due, primarily, to the fact that at the confining surface there exists a boundary layer with materially reduced velocities due to friction. A reduction of the stalled blade area and, as a result, an increase in the efficiency of the blade section may be effected by employing the vortex generator mixing principle to energize the boundary layer. By this method, the boundary layer velocities are increased, the effective section blade angle is reduced and the efficiency of the blade section is, therefore, increased. Experimental confirmation for this application is contained in Reference 2.

Generally, when a root or tip blade section stalls "pumping" will be obtained; that is, reversed flow will result over the stalled portions of the blade. This pumping action may or may not cause flow separation ahead of the blade section. If it is determined that such a problem exists, then the vortex generator configuration is arrived at in a similar manner as the method given under the design for Application 1. For the case where no pumping action results, but the blade section near the confining surface is stalled, then the procedure to use for arriving at the generator design is as follows:

(a) Assume that the stalled blade section station is the initial separation point for a flow separation problem (with no fan).
(b) With the assumption of (a), follow the design given for Application 1. This results in presenting to the fan root/or tip sections the fullest (least retarded) boundary layer.

It is possible that the aforementioned design procedure may result in a vortex generator installation located between pre-rotation vanes. If this condition results, it is suggested that the vortex generator configuration be chosen which will generate vortices in such a direction as to sweep unretarded energy into the relatively thick boundary layer existing at the intersection between the pre-rotation vane and the confining surface. Refer to the following figure.

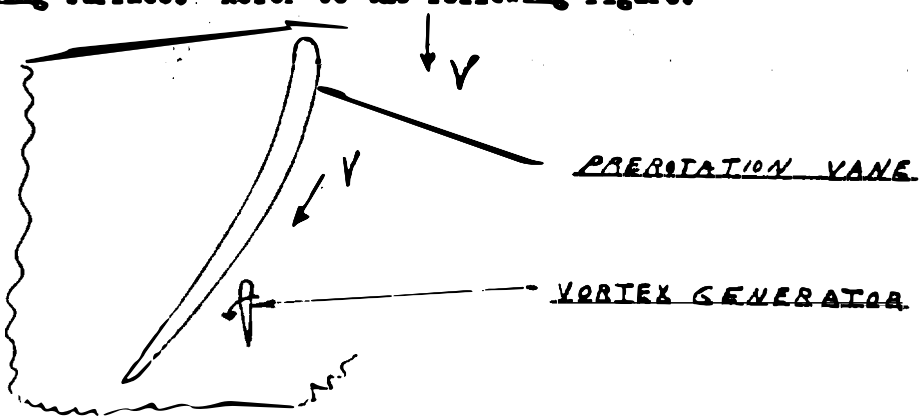


Figure VIII

For this application, it is possible that a co-rotating vortex arrangement might prove superior to the counter rotating arrangement inasmuch as the co-rotating arrangement may result in imparting to the boundary layer a flow angle up to 60° relative to the main flow direction. The result of this directioning coupled with the mixing given by the co-rotating configuration may, in certain such applications, prove superior to the counter rotating configuration. There is, at present, no experimental verification of this point.

3-5. Design Criteria for Applications 3, 4 and 5 (See Page 8)

The design criteria for these applications are based on information obtained from Applications 1 and 2. The main difference between these applications is the amount of fluid to be intermixed.

Assume that the problem exists of obtaining at a predetermined duct station (e. g., at the entrance to a turbine) a uniform temperature distribution. Refer to the following figure.



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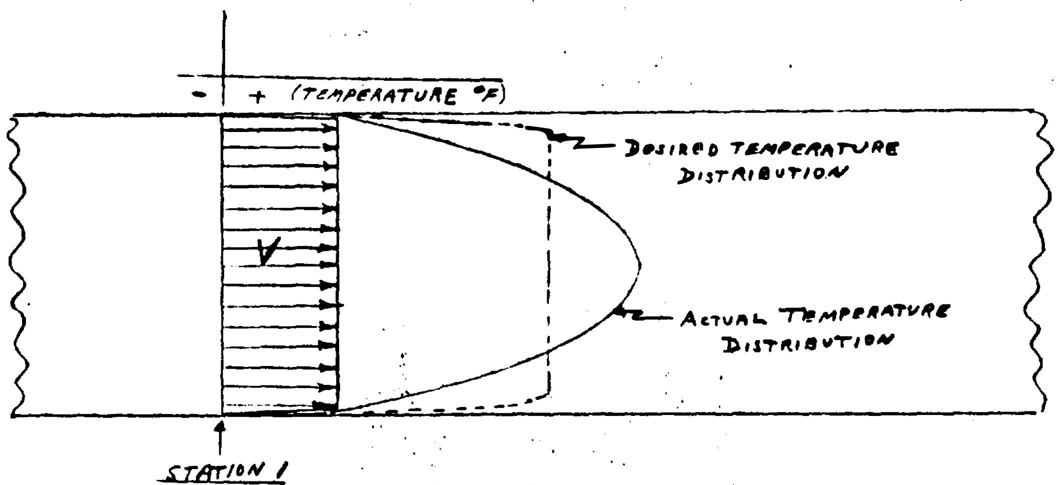



Figure IX

This problem, then, may be treated in a manner similar to the flow separation problem. For this case, Station 1 is analogous to the initial separation point of the flow separation problem. The design chart of Figure XII on Page 22 may be used with the following manipulation for the purpose of obtaining some information relative to the initial choice for the generator station.

Suppose we choose a design $\kappa$ of 1.10, thus

$d/\delta = 1.10$, but since $d = 0.9b$ by experimental observation, we can obtain $\delta$ in terms of the span length, b, as

$$\delta = \frac{0.9b}{1.10} = 0.82b$$

The design chart indicates that sufficient mixing is obtained by placing the generators from 5 to 30 $\delta$ ahead of the initial separation point or, in this case, from approximately 4 to 25 span lengths ahead of the desired Station 1. Since the assumed velocity profile is uniform, a constant chord airfoil may be used. Equation IX-A of the Appendix indicates that the following relation between the tip chord and airfoil span should result in effective mixing.

$$C = \frac{1.34b}{\bar{C}_L}$$

It is recommended that the initial span be no larger than approximately 1/2 the radius of the duct. Larger airfoil spans might easily cause interference between airfoils. By choosing an S/d factor of approximately 3, the number of generators necessary may be determined. The following experimental procedure is suggested.

(a) Choose a generator station between 4 and 25 span lengths upstream of the point where the uniform temperature gradient is desired.
(b) Start with a span length of approximately 1/2 the radius of the ducts.



[redacted]

(c) Set the <u>even</u> number of generators (counter rotating) at an angle of attack of 16°. Vary the angle of attack positively and negatively away from 16° until a maximum effect is obtained.

(d) If the desired mixing effect is not obtained, then cut the span length by 5 to 10 percent. Add sufficient generators to maintain S/d approximately 3 $\pm$.5. Vary the angle of attack as outlined in (c). Repeat the cutting procedure until a <u>maximum</u> effect is obtained.

(e) For a complete program, the value of S/d should also be varied for each span length between 2-6, and finally the axial location of the generators should be investigated for the purpose of checking the application of the design chart to the particular problem.

It is suggested that a probe of the vortex generator be made to determine if a well-defined tip vortex is being obtained. If the section Mach number is above 0.6, then a wedge-shaped airfoil is suggested.

The configurations for other various applications may be obtained by following the procedure outlined above. Since the drag of these low aspect ratio airfoils is almost entirely induced drag, it may be discovered that a greater overall efficiency will be obtained by using a greater number of generators at a relatively low angle of attack to obtain the same amount of mixing as afforded by a few generators at a high angle of attack.

For any vortex generator application, it is emphasized that the aerodynamic conditions at the generator station must be of such a nature as to enable the airfoil to generate a strong tip vortex. The assumption cannot be made that a tip vortex is, in every case, generated by an airfoil.

## References

1. The Elimination of Diffuser Separation by Vortex Generators, by H. D. Taylor. U.A.C. Research Department Report R-4012-3, June 10, 1947.

2. Increasing the Efficiency of the U.A.C. 8-ft. Wind Tunnel Fan by Means of Vortex Generators, by H. D. Taylor. U.A.C. Research Department Report R-4012-4, November 5, 1947.

3. Wind Tunnel Tests on Wedge-Shaped Wings, by R. S. LaValle. U.A.C. Research Department Report R-5503-2, January 22, 1947.

## Appendix

### Development of the Empirical Equations to Determine the Generating Airfoil Chord Lengths

The theoretical relation giving the circulation, $\Gamma$, about an airfoil is as follows:

$$\Gamma = \frac{1}{2} C_L C V \qquad \text{(I-A)}$$

where $C_L$ = section lift coefficient
$C$ = section chord length (ft.)
$V$ = free stream velocity (ft./sec.)

For a vortex generator mixing installation utilizing more than one tip vortex, the vortex strength per airfoil necessary to maintain the required induced velocities may be said to depend principally upon the following.

1. Magnitude of fluid velocity, $V$
2. The vortex generator pattern (ratio of distance between generators to span of generaters, $S/b$).

Thus we may write

$$\Gamma = k_1 V S \qquad \text{(II-A)}$$

where $k_1$ is an empirical constant ($k_1$ varies as the generator pattern is varied). This equation reveals that if the spacing is doubled with constant velocity, then the circulation about the airfoil must be doubled to give the required induced velocities.

Combining equations (I-A) and (II-A) and solving for the chord length, we have

$$C = \frac{2k_1 S}{C_L} \qquad \text{(III-A)}$$

For the generator application to eliminate separation where all but the tip section of the airfoil is immersed in the boundary layer, then equation (III-A) becomes

$$C_{tip} = \frac{2k_1 S}{C_L} \qquad \text{(IV-A)}$$



[redacted]

The parameter $S/d$ is defined as

$$S/d = \frac{\text{distance between adjacent generators}}{\text{distance of the vortex core from the confining surface}}$$

(Refer to the sketch given on Figure XII.)

For optimum performance (based on results to date) we may set

$$S/d = k_2 \qquad \text{(V-A)}$$

where $k_2$ is an empirical constant

The parameter d may also be defined in terms of the span length, b, and the boundary layer thickness, $\delta$, as

$$d = k_3 b = k_4 \delta \qquad \text{(VI-A)}$$

where $k_3$ is an empirical constant based on experimental observation and $k_4$ is the empirical constant which has given optimum results to date. Combine equations (IV-A), (V-A) and (VI-A) and obtain

$$C_{tip} = \frac{2k_1 k_2 k_3 b}{C_L} \qquad \text{(VII-A)}$$

$$\text{or} \quad C_{tip} = \frac{2k_1 k_2 k_4 \delta}{C_L} \qquad \text{(VIII-A)}$$

Based on results to date, the following values have been assigned to the empirical constants.

$k_1 = 0.248$
$k_2 = 3.0$
$k_3 = 0.9$
$k_4 = 1.1$

Substituting the above values into equations (VII-A) and (VIII-A), the following relations are obtained.

$$C_{tip} = \frac{1.34\ b}{C_L} = \frac{1.64\ \delta}{C_L} \qquad \text{(IX-A)}$$

where the second relation of equation (IX-A) is useful in flow separation applications and the first relation is appropriate for entire duct transverse mixing applications, etc. Obviously, when the airfoil is not in the field of a spanwise velocity gradient a constant chord airfoil may



[redacted]

be used, where the chord length is given by equation (IX-A).

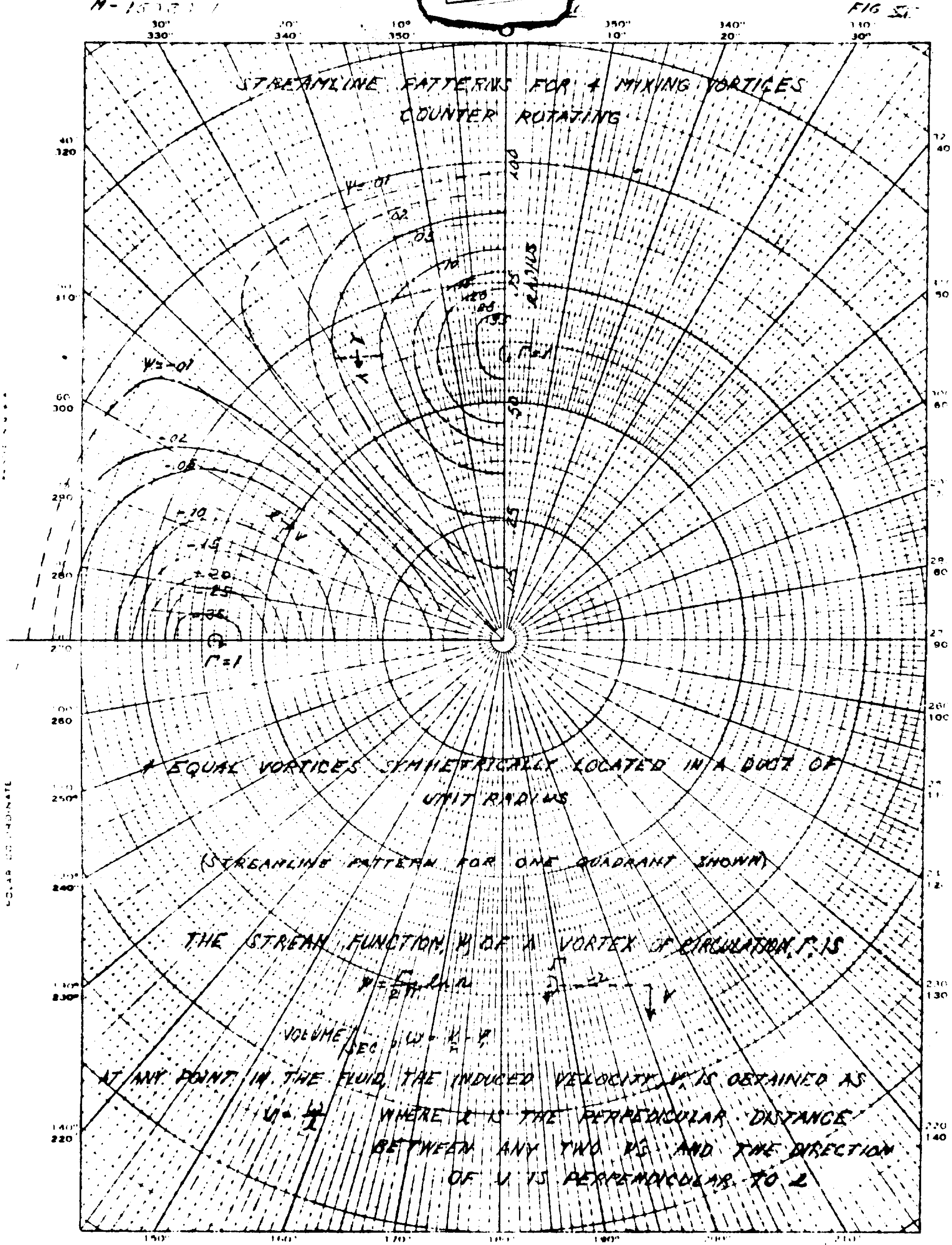
When the generating airfoil is mounted in the field of a spanwise velocity gradient, it is suggested that a tapered airfoil be used (to obtain a well-defined strong tip vortex). For constant circulation spanwise, the chordwise distribution in terms of the tip chord length and the velocity distribution may be obtained by using the relation of equation (I-A) as

$$C = C_{tip} \left(\frac{V_o}{V}\right) \qquad \text{(X-A)}$$

Where $C_{tip}$ is given by equation (IX-A),

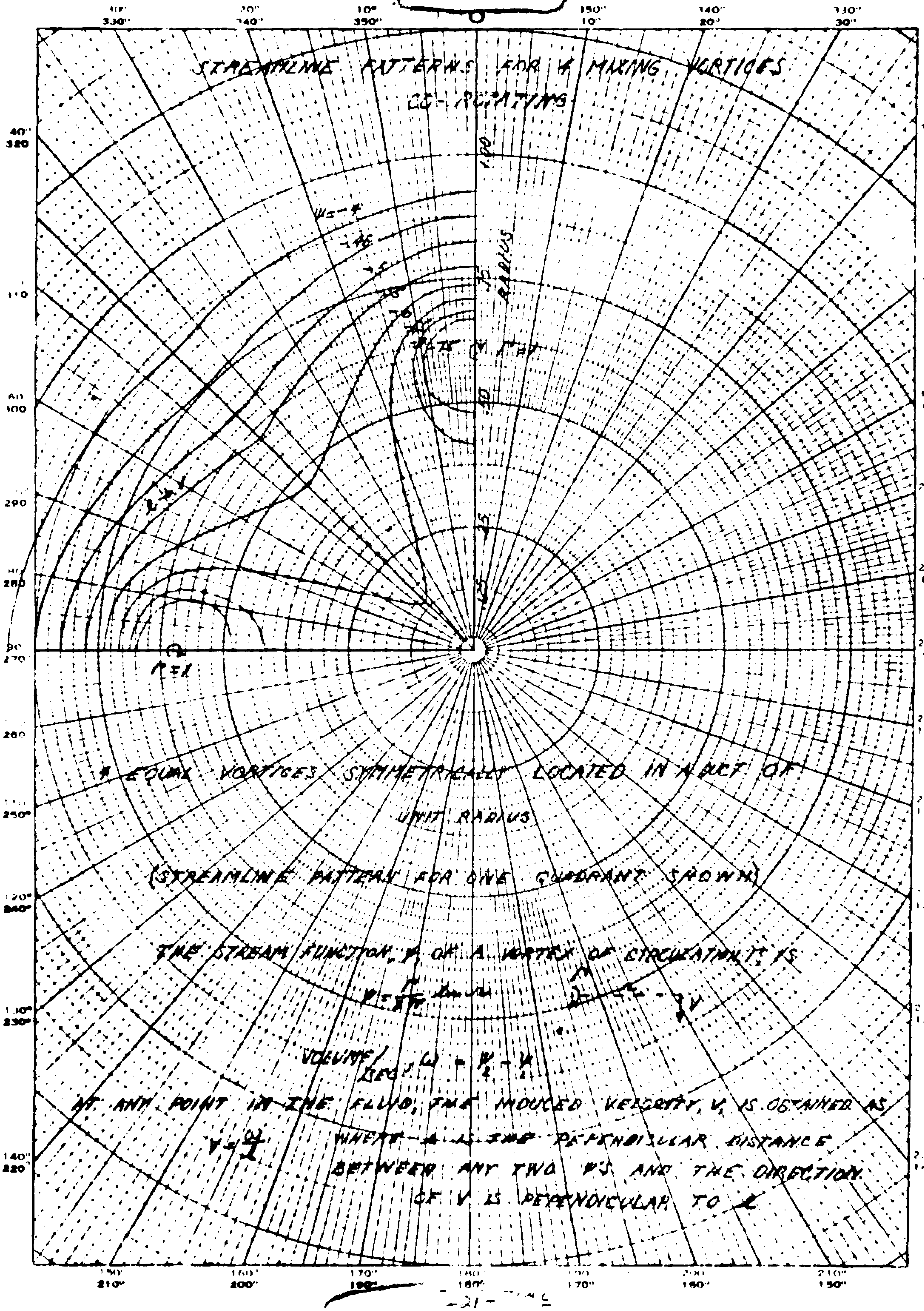
$V_o$ is the local free stream velocity, and
$V$ is the local boundary layer velocity.

At the edge of the boundary layer, $V_o/V$ becomes unity and thus $C = C_{tip}$.


STREAMLINE PATTERNS FOR 4 MIXING VORTICES
COUNTER ROTATING
ψ=.01
.02
.03
.04
.05
.06
RADIUS
100
75
50
25
Γ=1
ψ=-.01
-.02
-.05
-.10
-.15
-.20
-.25
-.35
Γ=1
4 EQUAL VORTICES SYMMETRICALLY LOCATED IN A DUCT OF
UNIT RADIUS
(STREAMLINE PATTERN FOR ONE QUADRANT SHOWN)
THE STREAM FUNCTION, ψ, OF A VORTEX OF CIRCULATION Γ IS
ψ = Γ/2π ln r
VOLUME/SEC
AT ANY POINT IN THE FLUID THE INDUCED VELOCITY, V, IS OBTAINED AS
WHERE ℓ IS THE PERPENDICULAR DISTANCE
BETWEEN ANY TWO ψ'S AND THE DIRECTION
OF V IS PERPENDICULAR TO ℓ

M-15…

FIG X



STREAMLINE PATTERNS FOR 4 MIXING VORTICES
CO-ROTATING
RADIUS
4 EQUAL VORTICES SYMMETRICALLY LOCATED IN A DUCT OF
UNIT RADIUS
(STREAMLINE PATTERN FOR ONE QUADRANT SHOWN)
AT ANY POINT IN THE FLUID, THE INDUCED VELOCITY, V, IS OBTAINED AS
BETWEEN ANY TWO ψ'S AND THE DIRECTION
OF V IS PERPENDICULAR TO

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