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SUMMARY REPORT ON CONFIGURATIONS HAVING MAXIMUM LIFT-TO-DRAG RATIO FOR HYPERSONIC FLIGHT

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by

ANGELO MIELE

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Table 1

Optimum Hypersonic Bodies

	Reports	Articles
General considerations	Refs. 1,2,17	Refs. 1,8
Optimum longitudinal contour	Refs. 8,9,10	Refs. 3,4
Optimum transversal contour	Refs. 16,18,19,20	Refs. 9,10,11,12
Similarity laws	Ref. 3	Ref. 2

Table 2

Optimum Hypersonic Wings

	Reports	Articles
General considerations	Refs. 4,5,6	Ref. 5
Two-dimensional wings	Refs. 11,12,13	Ref. 6
Three-dimensional wings	Refs. 14,15	Ref. 7
Similarity laws	Ref. 7	

Summary Report on Configurations Having Maximum Lift-to-Drag Ratio for Hypersonic Flight[1]

NASA Grant No. NGR-44-006-034

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ANGELO MIELE[2]

Abstract. This document summarizes the research performed ~~at Rice University~~ on aerodynamically efficient configurations for hypersonic flight. The methods of the calculus of variations are employed in order to determine two-dimensional wings, three-dimensional wings, and lifting bodies having the maximum lift-to-drag ratio under a variety of geometric and aerodynamic constraints. The geometric constraints are length, thickness, volume, frontal area, and planform area. The aerodynamic constraints are lift and position of the center of pressure. ( )

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[2] Professor of Astronautics and Director of the Aero-Astronautics Group, Department of Mechanical and Aerospace Engineering and Materials Science, Rice University Houston, Texas.

(A) Introduction

Hypersonic flow is a highly supersonic flow, that is, a flow such that the free-stream Mach number is sufficiently large with respect to unity. An important characteristic of this flow is that the shock waves originating at the leading edge of the body lie close to the body. As the Mach number increases, the shock waves move closer to the body. At very high Mach numbers, an approximate picture of hypersonic flow can be provided by assuming that the shock waves are identical with the surface of the body, at least in its front part. Consequently, the pressure coefficient can be determined with the assumption that the particles crossing the shock wave conserve the tangential component of the velocity but lose the normal component. This highly idealized model, called the Newtonian flow model, is remarkable in its simplicity since the local pressure coefficient depends only on the orientation of a body element, that is, it is independent of the geometry of the body portion preceding this element. Hence, the aerodynamic quantities can be simply related to the geometry of the configuration so that a considerable amount of useful analytical work is possible for two-dimensional, axisymmetric, and even three-dimensional shapes.

Within the frame of the Newtonian flow model, a 3-year research program on optimum hypersonic, lifting shapes was developed at Rice University under the sponsorship of the NASA-Langley Research Center. The program was supported with Grants NGR-44-006-034, NGR-44-006-045, and NGR-44-006-063 and was monitored by Mr. Bernard Spencer, Jr. In addition to the principal investigator, the following graduate students participated in the program: Mr. J.C. Heideman, Mr. H.Y. Huang, Mr. D.G. Hull, Mr. A.H. Lusty, Jr., Mr. R.E. Pritchard, and Mr. W.L. Wilson.

While the program involved an idealized fluid mechanics model, its ultimate engineering objective was to develop guidelines for a hypersonic cruise vehicle, a hypersonic glide vehicle, or a maneuverable vehicle reentering the atmosphere from satellite, lunar, and interplanetary missions. With this objective in mind, lifting bodies, two-dimensional wings, and three-dimensional wings were determined so as to maximize the lift-to-drag ratio. In order to attain realistic solutions, constraints were imposed on geometric quantities (length, thickness, volume, frontal area, planform area) and aerodynamic quantities (lift, position of the center of pressure).

Mathematically speaking, two approaches were employed. To achieve a preliminary understanding of the solutions, direct methods were used and configurations were optimized with respect to a limited number of geometric parameters. Then, these restrictions were removed, and the indirect methods of the calculus of variations in either one or two independent variables were employed. For economy of thought, similarity laws were developed. In a typical case, they allow one to deduce an optimum body of arbitrary transversal contour from a known optimum body of semicircular transversal contour.

The results of the above research are described in 20 reports and 12 journal articles. The complete list of reports is given in Section (B) and the list of articles in Section (C). Then, the abstract of each report is presented in Section (D) and the abstract of each article in Section (E). Finally, the reports and articles dealing with bodies are tabulated in Table 1 and those dealing with wings are tabulated in Table 2. Specifically, the research on bodies is subdivided into four groups: general considerations, optimum longitudinal contour, optimum transversal contour, and similarity laws. Analogously, the research on wings is subdivided into four groups: general considerations, two-dimensional wings, three-dimensional wings, and similarity laws.

(B) Reports

1. MIELE, A., Lift-to-Drag Ratios of Slender Bodies at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 9, 1965.

2. MIELE, A., and HULL, D.G., Maximum Lift-to-Drag Ratios of Slender, Flat-Top, Hypersonic Bodies, Part 1, Rice University, Aero-Astronautics Report No. 10, 1965.

3. MIELE, A., Similarity Laws for Lifting Bodies of Minimum Drag at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 11, 1965.

4. MIELE, A., Lift-to-Drag Ratios of Slender Wings at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 13, 1966.

5. MIELE, A., One-Dimensional Approach to the Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Wing, Rice University, Aero-Astronautics Report No. 14, 1966.

6. MIELE, A., Two-Dimensional Approach to the Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Wing, Rice University, Aero-Astronautics Report No. 15, 1966.

7. MIELE, A., Similarity Laws for Lifting Wings of Minimum Drag at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 16, 1966.

8. MIELE, A., and HUANG, H.Y., Power-Law Bodies of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 20, 1966.

9. LUSTY, A.H., Jr., Lifting Bodies of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 21, 1966.

10. LUSTY, A.H., Jr., and MIELE, A., Bodies of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics, Report No. 22, 1966.

11. MIELE, A., and WILSON, W.L., Two-Dimensional, Power-Law Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 23, 1966.

12. HULL, D.G., Two-Dimensional, Lifting Wings of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 24, 1966.

13. HULL, D.G., Two-Dimensional Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 25, 1966.

14. MIELE, A., and HULL, D.G., Three-Dimensional, Lifting Wings of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 26, 1966.

15. MIELE, A., and HULL, D.G., Three-Dimensional Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 27, 1966.

16. MIELE, A., and HUANG, H.Y., Lift-to-Drag Ratios of Lifting Bodies at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 29, 1967.

17. MIELE, A., HULL, D.G., and BROWN, S.L., Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Body, Rice University, Aero-Astronautics Report No. 30, 1967.

18. MIELE, A., HEIDEMAN, J.C., and PRITCHARD, R.E., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 1, Direct Methods, Rice University, Aero-Astronautics Report No. 34, 1967.

19. HUANG, H.Y., Variational Approach to Conical Bodies Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 35, 1967.

20. HUANG, H.Y., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 2, Variational Methods, Rice University, Aero-Astronautics Report No. 36, 1967.

(C) Articles

1. MIELE, A., Lift-to-Drag Ratios of Slender Bodies at Hypersonic Speeds, Journal of the Astronautical Sciences, Vol. 13, No. 1, 1966.

2. MIELE, A., Similarity Laws for Bodies Maximizing the Lift-to-Drag Ratio at Hypersonic Speeds, Journal of the Astronautical Sciences, Vol. 13, No. 3, 1966.

3. LUSTY, A.H., Jr., and MIELE, A., Bodies of Maximum Lift-to-Drag Ratio in Hypersonic Flow, AIAA Journal, Vol. 4, No. 12, 1966.

4. LUSTY, A.H., Jr., Flat-Top Body of Minimum Drag for Given Lift and Length in Hypersonic Flow, Journal of the Astronautical Sciences, Vol. 14, No. 2, 1967.

5. MIELE, A., Maximum Lift-to-Drag Ratio of a Slender Wing at Hypersonic Speeds, ZFW, Vol. 15, No. 7, 1967.

6. HULL, D.G., Two-Dimensional, Hypersonic Wings of Maximum Lift-to-Drag Ratio, Journal of the Astronautical Sciences, Vol. 14, No. 2, 1967.

7. MIELE, A., and HULL, D.G., Three-Dimensional, Hypersonic Wings of Maximum Lift-to-Drag Ratio, Journal of the Astronautical Sciences, Vol. 13, No. 6, 1966.

8. MIELE, A., HULL, D.G., and BROWN, S.L., Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Body, Astronautica Acta, Vol. 13, No. 2, 1967.

9. MIELE, A., and HUANG, H.Y., Engineering Considerations on the Transversal Contour of a Lifting Body, Journal of the Astronautical Sciences, Vol. 14, No. 6, 1967.

10. HUANG, H.Y., Variational Approach to Conical Bodies Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Journal of Optimization Theory and Applications, Vol. 2, No. 5, 1968.

11. MIELE, A., HEIDEMAN, J.C., and PRITCHARD, R.E., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 1, Direct Methods, Journal of the Astronautical Sciences, Vol. 15, No. 2, 1968.

12. HUANG, H.Y., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 2, Variational Methods, Journal of the Astronautical Sciences, Vol. 15, No. 3, 1968.

(D) Abstracts of Reports

1. MIELE, A., Lift-to-Drag Ratios of Slender Bodies at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 9, 1965.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, homothetic body at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. Analytical expressions are derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration.

Since flat-top bodies are naturally suited to produce high lift-to-drag ratios at hypersonic speeds, particular attention is devoted to these bodies. Specifically, the class of flat-top bodies whose longitudinal contour is a power law and whose transversal contour is either semielliptical or triangular is considered, and the effect of the thickness ratio, the exponent of the power law, and the elongation ratio of the cross section on the lift-to-drag ratio is investigated.

It is shown that a set of values of the thickness ratio, the power law exponent, and the elongation ratio exists which yields a maximum value for the lift-to-drag ratio. Specifically, the optimum thickness ratio is such that the friction drag is one-third of the total drag; the optimum power law exponent is one; and the optimum elongation ratio is such that the configuration is bodylike if the cross section is semielliptical and winglike if the cross section is triangular.

For a friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio attainable with a semielliptical cross section is $E = 3.67$; the corresponding value with a triangular cross section is E = 5.29. Thus, bodies of triangular cross section are considerably more efficient than bodies of semielliptical cross section.

2. MIELE, A., and HULL, D.G., Maximum Lift-to-Drag Ratios of Slender, Flat-Top, Hypersonic Bodies, Part 1, Rice University, Aero-Astronautics Report No. 10, 1965.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal and transversal contours is reduced to the extremization of products of powers of integrals related to the lift, the pressure drag, and the friction drag. For the longitudinal contour, the variational approach shows that a conical solution is the best. For the transversal contour, a triangular configuration satisfies the Euler equation for every cross-sectional elongation ratio μ; it satisfies the Legendre condition for $0 \leq \mu \leq 0.65$ but violates it in the neighborhood of the plane of symmetry for $0.65 \leq \mu \leq 1$. The lift-to-drag ratio of the optimum conical body of triangular cross section increases as the elongation ratio of the cross section decreases; for a friction coefficient $C_f = 10^{-3}$, the highest attainable lift-to-drag ratio is E = 5.29.

3. MIELE, A., Similarity Laws for Lifting Bodies of Minimum Drag at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 11, 1965.

Abstract. The problem of determining the longitudinal and transversal contours minimizing the drag of a flat-top body in hypersonic flow is considered under the hypotheses that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. It is also assumed that the body is slender and homothetic and that certain arbitrarily prescribed values are assigned to, at most, the lift, the pitching moment, the planform area, the frontal area, the wetted area, the volume, the length, and the thickness. Two similarity laws are determined.

The Similarity Law for Longitudinal Contours permits one to determine the optimum longitudinal contour of a body of arbitrary transversal contour from the known optimum longitudinal contour of a reference body (a body of semicircular cross section); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed transversal contour.

The Similarity Law for Transversal Contours permits one to determine the optimum transversal contour of a body of arbitrary longitudinal contour from the known optimum transversal contour of a reference body (a conical body); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed longitudinal contour.

4. MIELE, A., Lift-to-Drag Ratios of Slender Wings at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 13, 1966.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, affine wing at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. Analytical expressions are derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration.

The class of flat-top wings whose upper surface is parallel to the free stream is considered. After it is assumed that the chordwise thickness distribution is a power law and the spanwise thickness distribution is proportional to some power of the chord distribution, the effect of the thickness ratio and the power law exponents on the lift-to-drag ratio is investigated.

It is shown that a set of values of the thickness ratio and the power law exponents exists which yields a maximum lift-to-drag ratio. Specifically, the optimum thickness ratio is such that the friction drag is one-third of the total drag; the optimum chordwise power law exponent is one, meaning that a linear thickness distribution is the best in the chordwise sense; and the optimum spanwise power law exponent is one, meaning that a thickness distribution proportional to the chord distribution is the best in the spanwise sense. For a friction coefficient $C_f = 10^{-3}$, the maximum attainable lift-to-drag ratio is 5.29.

5. MIELE, A., One-Dimensional Approach to the Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Wing, Rice University, Aero-Astronautics Report No. 14, 1966.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, flat-top,affine wing at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the chordwise and spanwise contours is reduced to the extremization of products of powers of line integrals related to the lift, the pressure drag, and the friction drag. For the chordwise contour, the variational approach shows that a linear thickness distribution is the best. For the spanwise contour, a thickness distribution proportional to the chord distribution is the best. The lift-to-drag-ratio of the variational solution is independent of the chord distribution and depends on the friction coefficient only. For a friction coefficient $C_f = 10^{-3}$, the maximum attainable lift-to-drag ratio is $E = 5.29$.

6. MIELE, A., Two-Dimensional Approach to the Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Wing, Rice University, Aero-Astronautics Report No. 15, 1966.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, flat-top wing at hypersonic speeds is presented under the assumptions that the planform shape is given, the pressure distribution is modified Newtonian, and

the surface-averaged friction coefficient is constant. The methods of the calculus of variations in two independent variables are employed. It is shown that the optimum wing has a constant chordwise slope so that the chordwise thickness distribution is a linear function of the distance from the leading edge and the trailing edge thickness distribution is proportional to the chord distribution. Also, the friction drag is one-third of the total drag. The lift-to-drag ratio and the thickness ratio of the variational solution are independent of the planform shape and depend on the friction coefficient only. For a friction coefficient $C_f = 10^{-3}$, the maximum attainable lift-to-drag ratio is $E = 5.29$ and the corresponding thickness ratio is $\tau = 0.126$.

7. MIELE, A., Similarity Laws for Lifting Wings of Minimum Drag at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 16, 1966.

Abstract. The problem of determining the chordwise and spanwise contours minimizing the drag of a flat-top wing in hypersonic flow is considered under the hypotheses that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. It is also assumed that the wing is slender and affine and that certain arbitrarily prescribed values are assigned to, at most, the lift, the planform area, the frontal area, the volume, the root chord, the span, and the root thickness. Two similarity laws are determined.

The Similarity Law for Chordwise Contours permits one to determine the optimum chordwise contour of a wing of arbitrary spanwise contour and chord distribution from the known optimum chordwise contour of a reference wing (a wing of constant trailing edge thickness and constant chord); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed spanwise contour and chord distribution.

The Similarity Law for Spanwise Contours permits one to determine the optimum spanwise contour and chord distribution of a wing of arbitrary chordwise contour from the known optimum spanwise contour and chord distribution of a reference wing (a wing with a linear chordwise thickness distribution); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed chordwise contour.

8. MIELE, A., and HUANG, H.Y., Power-Law Bodies of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 20, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, flat-top, hypersonic body is investigated under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. Direct methods are employed, and the analysis is confined to the class of bodies whose transversal contour is semicircular and whose longitudinal contour is a power law.

First, unconstrained configurations are considered, and the combination of power law exponent and thickness ratio maximizing the lift-to-drag ratio is determined.

For a friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 3.6$ and corresponds to a conical configuration of thickness ratio $\tau = 0.118$.

Next, constrained configurations are considered, that is, conditions are imposed on the length, the thickness, the volume, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter.

9. LUSTY, A.H., Jr., Lifting Bodies of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 21, 1966.

Abstract. The problem of minimizing the drag of a slender, flat-top, homothetic body in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. The indirect methods of the calculus of variations are employed, and the necessary conditions to be satisfied by an optimum body are derived for arbitrary conditions imposed on the lift, the wetted area, the volume, the length, and the thickness. The particular cases treated are the following: (a) given lift, (b) given lift and thickness, (c) given lift and wetted area, (d) given lift and length, and (e) given lift and volume. For the first three cases, simple analytical solutions are found; for the last two, parametric solutions of a more complex nature are determined.

10. LUSTY, A.H., Jr., and MIELE, A., Bodies of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 22, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, flat-top body of semicircular cross section in hypersonic flow is investigated with the indirect methods of the calculus of variations. The pressure distribution is assumed to be modified Newtonian and a constant value of the surface-averaged friction coefficient is employed.

First, unconstrained configurations are considered, and it is found that the optimum body is a half-cone whose vertex angle is unique but whose size is undetermined. The scaling factor determining the size of the body can be found providing one geometric constraint is imposed on either the length, the thickness, or the volume.

Next, configurations are considered upon which two geometric constraints are imposed, that is, given thickness and length, given volume and length, and given volume and thickness. For each case, the lift-to-drag ratio parameter, the thickness ratio parameter, and the optimum shape are presented as functions of a single similarity parameter involving the two given quantities, the friction coefficient, and the constant modifying the Newtonian pressure distribution.

11. MIELE, A., and WILSON, W.L., Two-Dimensional, Power-Law Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 23, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, flat-top, hypersonic wing is investigated under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. Direct methods are employed, and the analysis is confined to the class of two-dimensional wings whose chordwise contour is a power law.

First, unconstrained configurations are considered, and the combination of power law exponent and thickness ratio maximizing the lift-to-drag ratio is determined. For a friction coefficient $C_f = 10^{-3}$, the maximum lift-to-drag ratio is $E = 5.29$ and corresponds to a wedge of thickness ratio $\tau = 0.126$.

Next, constrained configurations are considered, that is, conditions are imposed on the length, the thickness, the enclosed area, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter.

12. HULL, D.G., Two-Dimensional, Lifting Wings of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 24, 1966.

Abstract. The problem of minimizing the drag of a slender, flat-top, two-dimensional, lifting wing in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. The indirect methods of the calculus of variations are employed, and the necessary conditions to be satisfied by an optimum airfoil are derived for conditions imposed on the lift, the pitching moment, the profile area, the chord length, and the thickness. Then, the following particular cases are analyzed: (a) given lift, (b) given lift and chord length, (c) given lift and thickness, (d) given lift and profile area, and (e) given lift, pitching moment, and chord length. In all of these cases, analytical expressions are presented for the geometry of the optimum airfoil and the aerodynamic drag.

13. HULL, D.G., Two-Dimensional Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 25, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, two-dimensional, flat-top wing in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. Arbitrary conditions are imposed on the chord, the thickness, and the profile area; and the necessary conditions to be satisfied by an optimum wing are derived with the indirect methods of the calculus of variations. Then, several particular cases are analyzed and, for each case, analytical expressions are determined for the optimum shape and the maximum lift-to-drag ratio.

14. MIELE, A., and HULL, D.G., Three-Dimensional, Lifting Wings of Minimum Drag in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 26, 1966.

Abstract. The problem of minimizing the drag of a three-dimensional, slender, flat-top wing of given span in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. The indirect methods of the calculus of variations in two independent variables are employed, and the minimum drag problem is solved for (a) given lift and (b) given lift and volume.

If the lift is the only given quantity, the optimum wing has a constant chordwise slope and a trailing edge thickness distribution similar to the chord distribution. While the planform area is uniquely determined, the chord distribution is not. In

other words, there exist an infinite number of chord distributions yielding the same maximum value of the lift-to-drag ratio.

If the lift and the volume are given, two solutions are possible depending on the value of the volume-lift parameter, a parameter directly proportional to the volume and inversely proportional to the lift squared. If the volume-lift parameter is greater than a certain critical value, the optimum wing is identical with that of case (a). If the volume-lift parameter is smaller than the critical value, the optimum wing has a constant chord and a constant trailing edge thickness. Also, the chordwise slope is constant in the spanwise sense but not in the chordwise sense. Finally, the maximum lift-to-drag ratio decreases as the volume-lift parameter decreases.

15. MIELE, A., and HULL, D.G., Three-Dimensional Wings of Maximum Lift-to-Drag Ratio in Hypersonic Flow, Rice University, Aero-Astronautics Report No. 27, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, three-dimensional, flat-top wing of given planform in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. The indirect methods of the calculus of variations in two independent variables are employed, and the maximum lift-to-drag ratio problem is solved for (a) unconstrained volume and (b) given volume.

If the volume is unconstrained, the optimum wing surface is unique; it has a constant chordwise slope and a trailing edge thickness distribution similar to the chord distribution.

If the volume is given, the chordwise slope of the optimum wing is constant in the spanwise sense but not in the chordwise sense. A one-parameter family of extremal solutions exists, depending on the value of the volume parameter: this parameter is directly proportional to the volume and inversely proportional to the span and the root chord squared. If the volume parameter exceeds a certain critical value, the optimum wing is convex. If the volume parameter is equal to the critical value, the optimum wing is identical with that of case (a). Finally, if the volume parameter is smaller than the critical value, the optimum wing is slightly concave.

16. MIELE, A., and HUANG, H.Y., Lift-to-Drag Ratios of Lifting Bodies at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 29, 1967.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, homothetic body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal contour is reduced to the extremization of the product of the powers of three integrals related to the lift, the pressure drag, and the friction drag. In this connection, it is proved that a conical solution is the best.

Concerning the transversal contour, a systematic analysis of the effect of the main geometric parameters of a cross section on the lift-to-drag ratio is presented. For a given elongation ratio, the following conclusions are derived: (a) the triangle is aerodynamically superior to the semiellipse and the rectangle, whether flat-top or flat-bottom; (b) the flat-bottom triangle is aerodynamically superior to the flat-top triangle; and (c) the diamond shape is aerodynamically superior to the triangle, whether flat-top or flat-bottom.

17. MIELE, A., HULL, D.G., and BROWN, S.L., Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Body, Rice University, Aero-Astronautics Report No. 30, 1967.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant.

It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal and transversal contours is reduced to the extremization of products of powers of integrals related to the lift, the pressure drag, and the friction drag. With regard to the longitudinal contour, a conical solution is the best. With regard to the transversal contour, the optimum solution is triangular without or with a vertical keel at midsection depending on whether the cross-sectional elongation ratio μ is smaller or larger than the critical value $\mu = 0.206$.

The lift-to-drag ratio of the optimum body increases as the elongation ratio of the cross section decreases; for a Newtonian pressure distribution and a surface-averaged friction coefficient $C_f = 10^{-3}$, the highest attainable lift-to-drag ratio is $E = 5.29$.

18. MIELE, A., HEIDEMAN, J.C., and PRITCHARD, R.E., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 1, Direct Methods, Rice University, Aero-Astronautics Report No. 34, 1967.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, conical body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. The length and the volume are given, and the values of the free-stream dynamic pressure, the factor modifying the Newtonian pressure distribution, and the surface-averaged friction coefficient are known a priori. Direct methods are employed throughout the paper.

First, the following two-parameter families of transversal contours are analyzed: (a) flat-top triangle, (b) flat-top semiellipse, (c) flat-top rectangle, (d) flat-bottom triangle, (e) flat-bottom semiellipse, (f) flat-bottom rectangle. For each of these families, the combination of height and width yielding the maximum lift-to-drag ratio is found. Regardless of the prescribed length and volume, the flat-bottom triangle is aerodynamically superior to the other configurations. An upper bound to the lift-to-drag ratio E exists and is given by $E = 0.529 \sqrt[3]{(n/C_f)}$, where C_f is the surface-averaged friction coefficient and n is the factor modifying the Newtonian pressure law. Therefore, for $C_f = 10^{-3}$ and $n = 1$, the highest attainable lift-to-drag ratio is $E = 5.29$.

Next, the following two-parameter families, endowed with particular aerodynamic properties, are analyzed: (g) bodies having minimum local drag, (h) bodies having maximum local lift, and (i) bodies having maximum local lift-to-drag ratio. It is shown that shapes (g) and (i) are aerodynamically inferior to the flat-bottom triangle. Shape (h) is slightly superior to the flat-bottom triangle for relatively small volumes but inferior for relatively large volumes.

Finally, the following three-parameter families of transversal contours are analyzed: (j) diamond shape and (k) lenticular shape. These shapes are superior to shapes (a) through (i) even though the improvement in the lift-to-drag ratio over that of the flat-bottom triangle is small. The lenticular shape is slightly better than the diamond shape, and its geometry approximates closely that of the variational solution.

19. HUANG, H.Y., Variational Approach to Conical Bodies Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Rice University, Aero-Astronautics Report No. 35, 1967.

Abstract. An investigation of the lift-to-drag ratio E attainable by a slender, conical body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. The length of the body and the elongation ratio of the cross section α are prescribed, and the values of the free-stream dynamic pressure, the factor modifying the Newtonian pressure distribution n, and the surface-averaged friction coefficient C_f are known a priori. The indirect methods of the calculus of variations are employed, and it is found that, for any given value of the length and

the elongation ratio, the optimum transversal contour is a diamond shape. As the elongation ratio increases, the maximum lift-to-drag ratio increases, tending to the limiting value $E = 0.529 \sqrt[3]{(n/C_f)}$ when $\alpha \to \infty$.

20. HUANG, H.Y., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 2, Variational Methods, Rice University, Aero-Astronautics Report No. 36, 1967.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, conical body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. The length and the volume are given, and the values of the free-stream dynamic pressure, the factor modifying the Newtonian pressure distribution, and the surface-averaged friction coefficient are known a priori. The indirect methods of the calculus of variations are employed and numerical solutions are found using an IBM 7040 computer.

It is found that a unique solution exists for each value of the dimensionless parameter $S_* = (3V/\ell^3) \sqrt[3]{(n/C_f)^2}$, where V is the volume, ℓ the length, n the factor modifying the Newtonian pressure distribution, and C_f the surface-averaged friction coefficient. As S_* increases, the maximum lift-to-drag ratio increases, tending to the limiting value $E = 0.529 \sqrt[3]{(n/C_f)}$ when $S_* \to \infty$. The optimum configuration is bodylike for relatively small values of the parameter S_* and winglike for relatively large values of the parameter S_*.

(E) Abstracts of Articles

1. MIELE, A., Lift-to-Drag Ratios of Slender Bodies at Hypersonic Speeds, Journal of the Astronautical Sciences, Vol. 13, No. 1, 1966.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, homothetic body at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. Analytical expressions are derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration.

The class of flat-top bodies whose upper surface is parallel to the free stream is considered. After it is assumed that the longitudinal contour is a power law and the transversal contour is semielliptical, sinusoidal, or triangular, the effects of the thickness ratio, the exponent of the power law, and the elongation ratio of the cross section on the lift-to-drag ratio are investigated.

It is shown that a set of values of the thickness ratio, the power law exponent, and the elongation ratio exists which yields a maximum value for the lift-to-drag ratio. Specifically, the optimum thickness ratio is such that the friction drag is one-third of the total drag; the optimum power law exponent is one; and the optimum elongation ratio is such that the configuration is bodylike if the cross section is semielliptical or sinusoidal and winglike if the cross section is triangular.

For a friction coefficient $C_f = 10^{-3}$, the maximum attainable lift-to-drag ratio is $E = 3.67$ with a semielliptical cross section, $E = 4.05$ with a sinusoidal cross section, and $E = 5.29$ with a triangular cross section. Thus, bodies of triangular

cross section are considerably more efficient than bodies of semielliptical or sinusoidal cross section.

2. MIELE, A., Similarity Laws for Bodies Maximizing the Lift-to-Drag Ratio at Hypersonic Speeds, Journal of the Astronautical Sciences, Vol. 13, No. 3, 1966.

Abstract. The problem of determining the longitudinal and transversal contours maximizing the lift-to-drag ratio of a body in hypersonic flow is considered under the hypotheses that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. It is also assumed that the body is slender and homothetic and that certain arbitrarily prescribed values are imposed on some aerodynamic quantities (drag, lift, pitching moment) and geometric quantities (planform area, frontal area, wetted area, volume, length, thickness). Two similarity laws are determined.

The Similarity Law for Longitudinal Contours permits one to determine the optimum longitudinal contour of a body of arbitrary transversal contour from the known optimum longitudinal contour of a reference body (a flat-top body of semicircular cross section); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed transversal contour.

The Similarity Law for Transversal Contours permits one to determine the optimum transversal contour of a body of arbitrary longitudinal contour from the known optimum transversal contour of a reference body (a conical body); the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed longitudinal contour.

3. LUSTY, A.H., Jr., and MIELE, A., Bodies of Maximum Lift-to-Drag Ratio in Hypersonic Flow, AIAA Journal, Vol. 4, No. 12, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, flat-top body of semicircular cross section in hypersonic flow is investigated with the indirect methods of the calculus of variations. The pressure distribution is assumed to be modified Newtonian, and a constant value of the surface-averaged friction coefficient is employed. First, configurations of given length are considered, and it is found that the optimum body is a half-cone. Next, configurations are considered upon which two geometric constraints are imposed, that is, given thickness and length or given volume and length. For both cases, the lift-to-drag ratio parameter and the optimum shape are presented as functions of a single similarity parameter involving the two given quantities, the friction coefficient, and the constant modifying the Newtonian pressure distribution.

4. LUSTY, A.H., Jr., Flat-Top Body of Minimum Drag for Given Lift and Length in Hypersonic Flow, Journal of the Astronautical Sciences, Vol. 14, No. 2, 1967.

Abstract. The problem of minimizing the drag of a slender, flat-top, body of semicircular cross section in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. The indirect methods of the calculus of variations are employed, and the necessary conditions to be satisfied by the optimum body are derived for the case where the lift and the length are given. These conditions

are then solved in parametric form, and a one parameter family of optimum shapes is determined, the parameter depending on the prescribed values of the lift, the length, the free-stream dynamic pressure, the surface-averaged friction coefficient, and the factor modifying the Newtonian pressure distribution.

5. MIELE, A., Maximum Lift-to-Drag Ratio of a Slender Wing at Hypersonic Speeds, ZFW, Vol. 15, No. 7, 1967.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender wing at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and that the skin-friction drag is porportional to the wetted area. Analytical expressions are derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration. The class of flat-top, affine wings whose upper surface is parallel to the free-stream is considered, and the methods of the calculus of variations are employed in order to determine the configuration which maximizes the lift-to-drag ratio. It is shown that the optimum thickness ratio is such that the friction drag is one-third of the total drag. The subsequent optimization of the chordwise and spanwise contours is reduced to the extremization of products of powers of line integrals related to the lift, the pressure drag, and the skin-friction drag. For the chordwise contour, a linear thickness distribution is the best. For the spanwise contour, a thickness distribution proportional to the chord distribution is the best.

6. HULL, D.G., Two-Dimensional, Hypersonic Wings of Maximum Lift-to-Drag Ratio, Journal of the Astronautical Sciences, Vol. 14, No. 2, 1967.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, two-dimensional, flat-top wing at hypersonic speeds is considered under the hypotheses that the pressure coefficient is modified Newtonian, the surface-averaged friction coefficient is constant, and the base drag is negligible. After the extremal problem is formulated for arbitrary conditions imposed on the chord, the thickness, and the profile area, the indirect methods of the calculus of variations are used to obtain the necessary conditions which must be satisfied by an optimum airfoil. Then, the particular cases of given chord, given chord and thickness, and given chord and profile area are analyzed in detail. Finally, the effects of base drag, leading edge blunting, and inclining the upper surface on the maximum lift-to-drag ratio for a given chord are investigated.

7. MIELE, A., and HULL, D.G., Three-Dimensional, Hypersonic Wings of Maximum Lift-to-Drag Ratio, Journal of the Astronautical Sciences, Vol. 13, No. 6, 1966.

Abstract. The problem of maximizing the lift-to-drag ratio of a slender, three-dimensional, flat-top wing in hypersonic flow is considered under the assumptions that the pressure coefficient is modified Newtonian and the surface-averaged friction coefficient is constant. The indirect methods of the calculus of variations in two independent variables are employed, and the necessary conditions for an extremum are derived for arbitrary conditions imposed on the chord

distribution, the trailing edge thickness distribution, the lift, and the volume. Then, the following particular problems are analyzed under the assumptions that the span is given, the chord distribution is given, and the trailing edge thickness distribution is free: (a) free lift, free volume; (b) given lift, free volume; and (c) free lift, given volume.

8. MIELE, A., HULL, D.G., and BROWN, S.L., Maximum Lift-to-Drag Ratio of a Slender, Flat-Top, Hypersonic Body, Astronautica Acta, Vol. 13, No. 2, 1967.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant.

It is shown that a value of the thickness ratio exists such that the lift-to-drag ratio is a maximum; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the longitudinal and transversal contours is reduced to the extremization of products of powers of integrals related to the lift, the pressure drag, and the friction drag. With regard to the longitudinal contour, a conical solution is the best. With regard to the transversal contour, the optimum solution is triangular without or with a vertical keel at midsection depending on whether the cross-sectional elongation ratio μ is smaller or larger than the critical value $\mu = 0.206$.

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9. MIELE, A., and HUANG, H.Y., Engineering Considerations on the Transversal Contour of a Lifting Body, Journal of the Astronautical Sciences, Vol. 14, No. 6, 1967.

Abstract. An investigation of the lift-to-drag ratios attainable by slender, homothetic bodies flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. The class of bodies optimized with respect to the longitudinal contour is considered: these are conical bodies with the thickness ratio such that the friction drag is one-half of the pressure drag.

A systematic analysis of the main geometric parameters of a cross section on the lift-to-drag ratio is presented. The following shapes are considered: triangle, semiellipse, rectangle, trapezoid, bitrapezoid, triangle with a keel, caret shape, inverted caret shape, and diamond shape. For a given elongation ratio of the cross section, it is shown that the diamond shape exhibits the highest lift-to-drag ratio, closely followed by the flat-bottom triangle.

10. HUANG, H.Y., Variational Approach to Conical Bodies Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Journal of Optimization Theory and Applications, Vol. 2, No. 5, 1968.

Abstract. An investigation of the lift-to-drag ratio E attainable by a slender, conical body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. The length of the body and the elongation ratio of the cross section α are prescribed, and the values of the free-stream dynamic pressure, the factor modifying the Newtonian pressure distribution n, and the surface-averaged friction coefficient C_f are known a priori. The indirect methods of the calculus of variations are employed, and it is found that, for any given value of the length and the elongation ratio, the optimum transversal contour is a diamond shape. As the elongation ratio increases, the maximum lift-to-drag ratio increases, tending to the limiting value $E = 0.529 \sqrt[3]{(n/C_f)}$ when $\alpha \rightarrow \infty$.

11. MIELE, A., HEIDEMAN, J.C., and PRITCHARD, R.E., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 1, Direct Methods, Journal of the Astronautical Sciences, Vol. 15, No. 2, 1968.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, conical body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction

coefficient is constant. The length and the volume are given, and the values of the free-stream dynamic pressure, the factor modifying the Newtonian pressure distribution, and the surface-averaged friction coefficient are known a priori. Direct methods are employed throughout the paper.

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12. HUANG, H.Y., Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds, Part 2, Variational Methods, Journal of the Astronautical Sciences, Vol. 15, No. 3, 1968.

Abstract. An investigation of the lift-to-drag ratio attainable by a slender, conical body flying at hypersonic speeds is presented under the assumptions that the pressure distribution is modified Newtonian and the surface-averaged friction coefficient is constant. The length and the volume are given, and the values of the free-stream dynamic pressure, the factor modifying the Newtonian pressure distribution, and the surface-averaged friction coefficient are known a priori. The indirect methods of the calculus of variations are employed and numerical solutions are found using an IBM 7040 computer.

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