AEC-NASA TECH BRIEF

Numerical Integration of Ordinary Differential Equations of Various Orders

A report has been published describing techniques for the numerical integration of differential equations of various orders. Modified multistep predictor–corrector methods for general initial-value problems are discussed and new methods are introduced. Preliminary numerical evidence indicates that this process is faster and more accurate than similar methods applied to equivalent systems of first-order equations.

The investigation was prompted by a desire to deal with higher-order equations directly, rather than converting them to a larger system of first-order equations. It was found that the Nordsieck-type methods (N methods) were the most applicable to higher-order equations since the derivatives are directly available. In pursuing the relation between multistep methods and N methods, a large class of modified multistep methods became apparent. These k-step methods can have degree 2k, be strongly stable, and yet only use one derivative evaluation.

An equivalence relation between the methods was determined. The Nordsieck formulation was found to be best for high-accuracy problems, but for low-accuracy problems either the N methods or versions of the new methods should be used. The conventional Adams multistep methods should almost never be used.

The system of equations studied consisted of s equations in the s unknowns y_i, their first p_i derivatives, and the independent variable x. The most general case for which concrete results are given is when these equations can be solved for the highest-order derivatives of each y_i.

For didactic reasons, the first sections of the report deal with the simplest case, s = p = 1, in order to prepare for discussions of the advantages of various formulations. Later, the N methods are applied directly to the above equation. A numerical comparison of the results of integrating the second-order equation describing $J_{16}(x)$ directly and integrating the related pair of first-order equations is reported. Evidence indicates both a time and accuracy advantage in the direct approach. Some preliminary tests on the integration of singular families are also reported.

Finally, existence and convergence proofs are presented. The matrix formulation used provides a convergence proof equivalent to the usual proof, but considerably more compact and applicable to the system.

Notes:

1. The information is available in "The Numerical Integration of Ordinary Differential Equations of Various Orders," ANL-7126, Argonne National Laboratory, January 1966. This report is available from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Va. 22151; price: $3.00 (microfiche $0.65).
2. Inquiries concerning this report may be directed to:

   Office of Industrial Cooperation
   Argonne National Laboratory
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   Reference: B69-10089

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Patent status:

Inquiries about obtaining rights for commercial use of this innovation may be made to:

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