AEC-NASA TECH BRIEF

Numerical Inversion of Finite Toeplitz Matrices and Vector Toeplitz Matrices

A numerical technique is described (*1*) that simplifies nth-order Toeplitz matrix inversion; it increases the efficiencies of the numerical methods, involving Toeplitz matrices, by reducing the number of multiplications required by an N-order Toeplitz matrix from N-cubed to N-squared multiplications.

Many problems in mathematical physics, statistics, and algebra lead to the need for the inverse of finite Toeplitz or Hankel matrices. The problems involving convolutions, integral equations with difference kernels, and least-squares approximations by polynomials are well known. Although there is abundant literature on the mathematical properties of Toeplitz matrices, references to the problem of numerical inversion seem to be few. The efficiencies of numerical methods involving Toeplitz or Hankel matrices are often judged on the assumption that the inversion of a Toeplitz matrix of order-n requires roughly n^3 multiplications.

The purpose of this report (*1*) is introduction of a new method by which the exact inversion can be accomplished simply by use of on the order of n^2 multiplications. Some efficient algorithms are given. Extension is made to vector Toeplitz matrices that occurred in the author's work.

Vector Toeplitz matrices are defined as rectangular matrices whose elements are vectors and whose diagonals consist of like elements, except for the vector elements in the last row, which may be obtained by omission of the last components of corresponding full vector elements.

Reference:

1. E. H. Bareiss, *ANL-7440* (Argonne National Laboratory, June 1968)

Notes:

1. This method may interest persons concerned with mathematical physics.
2. Inquiries concerning this innovation may be directed to:

Office of Industrial Cooperation
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439
Reference: B69-10415

Source: E. H. Bareiss
Applied Mathematics Division
(ARG-10445)

Patent status:

Inquiries concerning rights for commercial use of this innovation may be made to:

Mr. George H. Lee, Chief
Chicago Patent Group
U. S. Atomic Energy Commission
Chicago Operations Office
9800 South Cass Avenue
Argonne, Illinois 60439

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