AEC-NASA TECH BRIEF

Storage of Electric and Magnetic Energy in Passive Nonreciprocal Networks

The effect of nonreciprocal network elements on stored electric and magnetic energy within a system is discussed (ref. 1). This examination of the relation of storage of electric and magnetic energy to the terminal behavior of nonreciprocal passive networks shows both similarities and important differences between wholly reciprocal systems and systems containing nonreciprocal elements. The presence of a dispersive medium necessitates further modification.

Most expositions of linear passive-network synthesis show how the terminal parameters of a network are related to the average energy functions. For real frequencies

$$\underset{\sim}{I}^{+}\underset{\sim}{V} = P + 2jw(T - V)$$

where T and V have the physical significance of average magnetic and electric energies, and P is the power dissipation. For example, the functions of a one-port for unit current excitation define the impedance Z by

$$Z = P + 2jw(T - V)$$

so that $R = P$ and $X = 2w(T - V)$. Thus any one-port having the same reactance, at any real frequency, necessarily has the same difference of electric and magnetic energies.

These results are old and so widely used in reciprocal-circuit theory that they are often assumed to hold for nonreciprocal networks as well. This, however, is not the case: recent workers in nonreciprocal-network synthesis have shown that one-port synthesis is possible with use of only one type of reactive element, with consequent energy-storage of only one type. Thus a network that stores only magnetic energy may be synthesized by one storing only electric energy.

To examine the relation between the energy functions and the terminal behavior, consider a passive network described by the mesh equation

$$\underset{\sim}{V} = jw\underset{\sim}{(L)}\underset{\sim}{I} + \underset{\sim}{(R)}\underset{\sim}{I} + (1/jw)\underset{\sim}{(D)}\underset{\sim}{I} + \underset{\sim}{(G)}\underset{\sim}{I}$$

where (L), (R), and (D) are the usual frequency-independent symmetric matrices associated with the inductive, resistive, and capacitive elements, and (G) represents intermesh gyrator coupling: (G) is a constant, real, antisymmetric matrix, although it need only be anti-hermitian.

By forming the product $\underset{\sim}{I}^{+}\underset{\sim}{V}$ and identifying the terms with the sums of average energies of the individual components of the network, we obtain (as a replacement for the initial equation)

$$\underset{\sim}{I}^{+}\underset{\sim}{V} + P + 2jw(T - V) + S$$

where $S = \underset{\sim}{I}^{+}\underset{\sim}{(G)}\underset{\sim}{I}$. Thus the nonreciprocity introduces an additional term, the gyrator term S, which invalidates the initial equation for nonreciprocal systems. This observation is independent of whether the terminal parameters behave reciprocally or not.

Even in reciprocal sytems the presence of dispersion in the polarization process complicates the equation, for then T or V must be interpreted as the total kinetic or potential energy, including the motion of internal degrees of freedom associated with the polarization process.

The total energy-storage of both reciprocal and nonreciprocal networks is also discussed in the reference.

Reference:

1. Smith, W. E.: Electric and Magnetic Energy Storage in Passive Nonreciprocal Networks. Electron. Letters, vol. 3, No. 8, Aug. 1967, pp. 389–391.

Notes:

1. This information may interest academic theorists and educators.
2. Inquiries concerning this information may be directed to:

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Patent status:

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