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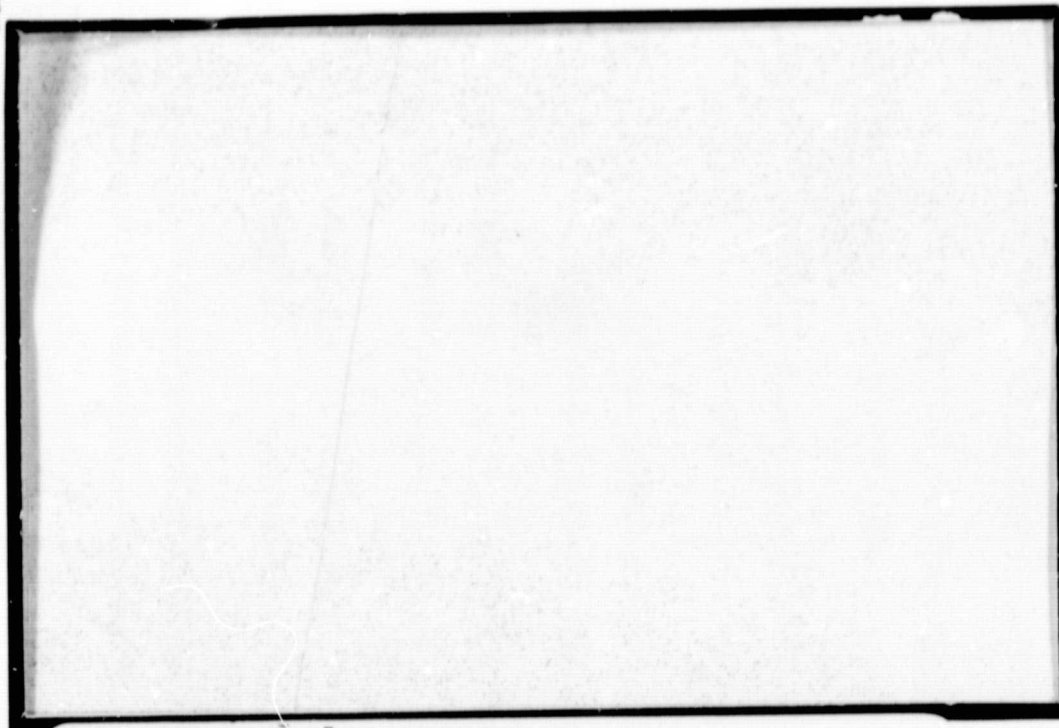
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DIGITAL COMPENSATION OF THE THRUST
VECTOR CONTROL SYSTEM

PREPARED BY

SAMPLED-DATA CONTROL SYSTEMS GROUP

AUBURN UNIVERSITY

C. L. Phillips, Project Leader

SEVENTH TECHNICAL REPORT

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LIST OF SYMBOLS

- C_1 = Aerodynamic moment coefficient
 C_2 = Control moment coefficient
 F = Total thrust of vehicle
 I_E = Engine moment of inertia about gimbal point
 I_{xx} = Pitch plane moment of inertia about vehicle c.g.
 l_{cg} = Distance from vehicle c.g. to engine gimbal
 l_{sj} = Slosh moment arm
 M, m = Total mass of vehicle
 M_{sj} = Slosh mass
 m_i = Generalized mass
 N' = Aerodynamic force
 R' = Thrust of control engines
 $\bar{S}_E$ = First moment of swivel about gimbal point for one engine
 $\bar{V}$ = Velocity of vehicle
 X = Drag force
 $k_3 = (F - X)/m$
 $k_4 = R'/m$
 $k_7 = N'/m$
 X_D = Station of displacement gyro
 X_β = Station of engine gimbal
 $\eta_i(X_\beta)$ = Normalized displacement at station X_β of the i^{th} bending mode
 $\eta'_i(X_\beta)$ = Normalized slope at station X_β due to i^{th} bending mode

$Y'_i(X_D)$ = Normalized slope at station X_D due to the i^{th} bending mode.

Z = Displacement normal to reference

β = Total engine deflection

β_c = Input to controller

η_i = Generalized displacement of the i^{th} mode

ϕ = Attitude angle

ϕ_D = Platform angle

ζ_{sj} = Damping factor for j^{th} sloshing mode

ξ_i, ζ_i = Bending mode damping of i^{th} mode

ω_{sj} = Frequency of the j^{th} sloshing mode

ω_i = Frequency of the i^{th} bending mode

$q(kT)$ = Round off error due to quantization at k^{th} sampling instant

Δ = Quantizer granularity

$\Delta\phi(kT)$ = System error at k^{th} sampling instant due to quantization

$\Delta\phi(z)$ = z-transform of the sequence, $\Delta\phi(kT)$

$q(z)$ = z-transform of the sequence, $q(kT)$

$\underline{x}(kT), \underline{y}(kT)$ = State vectors for quantized system.

A, g, h = Matrices used in the state representation of the quantized system.

λ = Symbol representing a matrix eigenvalue

$V(\underline{x}(kT))$ = The Liapunov function evaluated at system state, $\underline{x}(kT)$

$\Delta V(\underline{x}(kT))$ = The first difference of the Liapunov function at $\underline{x}(kT)$

B, C = Positive definite matrices used in the construction of the Liapunov function

ρ, ρ' = Scalar quantities representing the radius of a spherical ball in E^n

$\|\underline{x}(kT)\|$ = The norm of the state vector, $\underline{x}(kT)$

FOREWORD

This document is a technical summary of the progress made since September 28, 1966, by the Auburn University Electrical Engineering Department toward fulfillment of Phase A of Contract No. NAS8-11274. A technical report covering the progress made on Phase B is forthcoming. This contract was awarded to Auburn Research Foundation, Auburn, Alabama, May 28, 1964, and was extended September 28, 1966, by the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama.

SUMMARY

A new approximate technique is given for determining the gain margin in unsampled loops of sampled-data control systems. The limitations of this method are discussed and an example is given which compares results obtained using the new technique with those obtained from existing techniques.

Some of the problems associated with digitally stabilizing the rate-derived Thrust Vector Control System are discussed, and several alternative solutions of these problems are given. A set of design specifications is established and, by the use of the Nyquist criterion, a digital compensation function is evolved. The performance of the compensation is evaluated analytically at several times throughout the S-IC flight.

The effects of quantization in the Thrust Vector Control System are investigated using a simplified system model and a technique based on Liapunov's Direct Method. The practicality of the technique is discussed and an assessment of its merits is made.

A digital computer program is developed to determine the sampled frequency response of the Saturn V. This program includes the effects of both sloshing and bending dynamics.

This report summarizes the progress made to date on Phase A of Contract No. NAS8-11274. A technical report covering the progress of Phase B is forthcoming in the near future.

LIST OF PERSONNEL

The following named staff members of Auburn University have actively participated on this project:

- C. H. Holmes - Head Professor of Electrical Engineering
- G. T. Nichols - Associate Professor of Electrical Engineering
- C. L. Phillips - Professor of Electrical Engineering
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DIGITAL COMPENSATION OF THE THRUST VECTOR CONTROL SYSTEM

I. INTRODUCTION

Gain margin is a design criterion frequently used in the analysis of multiloop sampled-data control system. The gain margin in a given loop can be determined by inspection of the Nyquist diagram of the z-transfer function for the system opened in that loop. However, if a loop is composed completely of continuous elements, no z-transfer function can be written. In Chapter II a new approximate technique is presented which circumvents this problem and provides extremely accurate gain margin approximations for a certain class of systems.

The problem of digitally compensating the rate-derived Thrust Vector Control System is considered in Chapter III. In order to stabilize the Saturn V launch vehicle, it is shown that additional phase lead in the rigid body frequency region must be introduced by the control system. Further shaping of the compensation is necessary to account for the effects of body bending, fuel depletion, and varying dynamic pressure.

In Chapter IV, the technique advanced in the Sixth Technical Report [3] for determining an upper bound on quantization error is applied to the Thrust Vector Control System. The example, although highly simplified, gives a realistic view of the applicability of the method to design problems.

In Chapter V a digital computer program is developed which computes sampled frequency response data from the transfer function relating the engine command to attitude error. The program includes the effects of bending and sloshing, although the coupling effects between the two are neglected.

II. DETERMINATION OF GAIN MARGINS IN UNSAMPLED LOOPS

Summary

A new approximate technique for determining the gain margin in unsampled loops of sampled-data control systems is developed, and the limitations of the technique are presented. An example is given which compares results obtained using the new technique with those obtained using existing techniques.

Introduction

At the present time, the attitude control system of space vehicles of the Saturn class is designed by conventional, or classical, procedures. A design criterion of primary importance is the gain margin. Generally a minimum gain margin is specified in each loop of a multiloop system. For a continuous-data system, i.e., a system without sampling, the gain margin of a loop can be determined by developing the transfer function for the system opened in that loop. The gain margin is then found by inspection from the Nyquist diagram or the Bode plot of the open-loop transfer function.

The same procedure can be applied to a multiloop sampled-data control system, provided that a sampler appears in the loop under consideration. However, if loops are present in the system that are completely comprised of continuous-data elements, no transfer function can be written for the system opened in that loop.

In this paper, two known approximate techniques for obtaining open-loop characteristics in an unsampled loop are reviewed. Then a new approximate technique is developed. For comparative purposes, the three approximate techniques are utilized to determine the gain margins in the unsampled loops of a Saturn V attitude control system. The accuracy of the new technique is shown to be considerably greater than that of the existing techniques, for the system under consideration. The limitations of the new technique are discussed.

Known Approximate Techniques

The developments in this paper will be illustrated using a particular sampled-data control system. The system considered is the linearized attitude control system of a Saturn V vehicle, and is shown in Figure 1. In this control system, ϕ is the attitude of the vehicle, $\dot{\phi}$ is the attitude rate, β_c is the command signal to the engine actuator, and β_e is the engine angle. W_{ss} is the engine actuator transfer function, and G_1 is the transfer from β_e to ϕ , and represents the vehicle dynamics. Then $G_2 = sG_1$.

A transfer function can be written for the system of Figure 1 opened at the point $\dot{\phi}$. This transfer function is

$$M_{\dot{\phi}}(z) = \left(\frac{H_o G_2 W_{ss}}{1 - G_1 W_{ss}} \right)(z) \quad (\text{II-1})$$

Transfer functions cannot be written for the system opened at either ϕ or β_e . Two techniques for obtaining approximate open-loop transfer functions at both ϕ and β_e will now be reviewed.

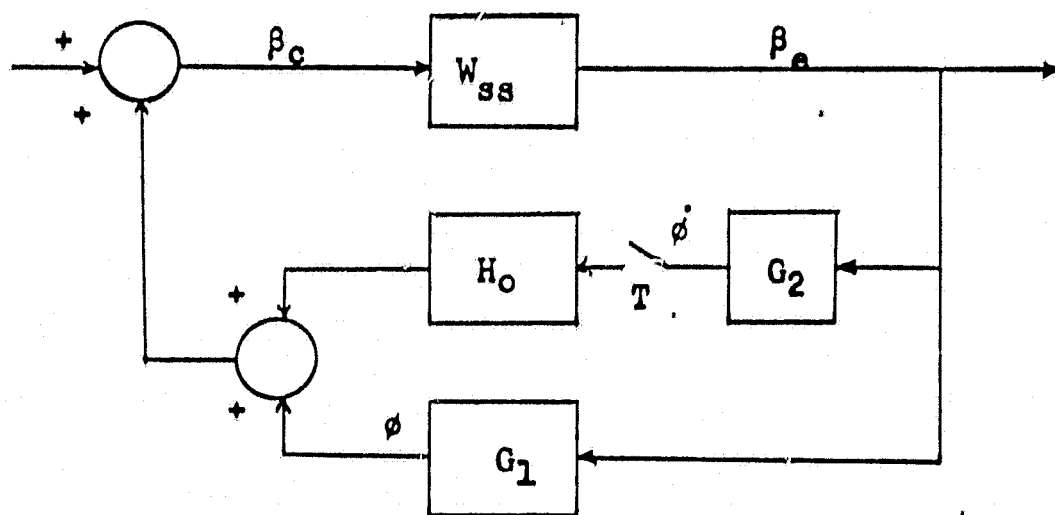


Figure 1. Attitude Control System

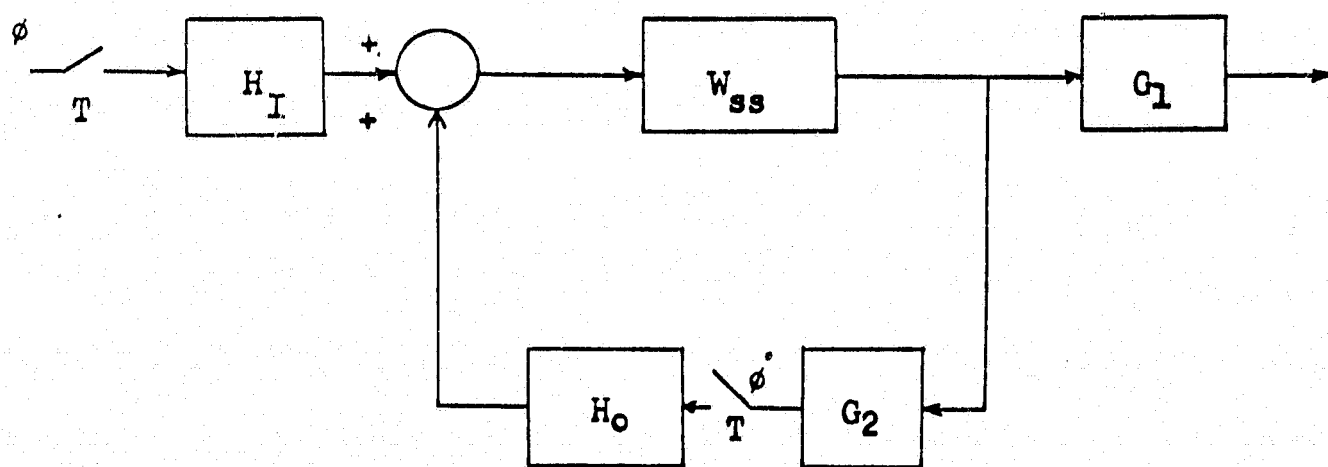


Figure 2. Attitude control system opened in ϕ channel.

The first method [1], called the one-term approximation or the continuous-system approximation, can be developed from the starred expression of (II-1).

$$M_{\phi}^*(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \frac{H_o G_2 W_{ss}}{1 - G_1 W_{ss}} (s + jn\omega_s) \quad (\text{II-2})$$

where ω_s is the sampling frequency. If the system is sufficiently low-pass with respect to $\omega_s/2$, (II-2) can be approximated, for $s = j\omega$, by

$$M_{\phi}^*(j\omega) = \frac{1}{T} \frac{H_o G_2 W_{ss}(j\omega)}{1 - G_1 W_{ss}(j\omega)} \quad (\text{II-3})$$

This approximation applies only for $\omega < \omega_s/2$. An approximate characteristic equation, as a function of ω , is, from (II-3),

$$1 - G_1 W_{ss}(j\omega) - \frac{1}{T} H_o G_2 W_{ss}(j\omega) = 0 \quad (\text{II-4})$$

From (II-4), an approximate open-loop transfer function for the system opened at ϕ is

$$M_{\phi}^*(j\omega) = \frac{G_1 W_{ss}(j\omega)}{1 - \frac{1}{T} H_o G_2 W_{ss}(j\omega)}, \quad (\text{II-5})$$

and an approximate open-loop transfer function for the system opened at β_e is

$$M_{\beta}^*(j\omega) = G_1 W_{ss}(j\omega) + \frac{1}{T} H_o G_2 W_{ss}(j\omega) \quad (\text{II-6})$$

Equations (II-5) and (II-6) apply only for $\omega < \omega_s/2$, and the accuracy of the approximations depends upon the low-pass nature of the system.

A second approximate technique, the fictitious-sampler method [2], can be illustrated by the system of Figure 2. Figure 2 is the attitude control system of Figure 1, opened at ϕ , with a fictitious sampler and ideal hold added as shown. The transfer function for the system of Figure 2 is

$$M_{\phi}^*(j\omega) = \frac{(H_I G_1 W_{ss})^*(j\omega)}{1 - (H_o G_2 W_{ss})^*(j\omega)} \quad (\text{II-7})$$

From the characteristics of the ideal hold, which is an ideal low-pass filter that has a pass band for $|\omega| < \omega_s/2$, (II-7) becomes

$$M_{\phi}^*(j\omega) = \frac{G_1 W_{ss}(j\omega)}{1 - \frac{1}{T} \sum_{n=-\infty}^{\infty} H_o G_2 W_{ss}(j\omega + jn\omega_s)} \quad (\text{II-8})$$

If the system of Figure 1 is opened at β_e and a fictitious sampler and ideal hold added, the open-loop transfer function is found to be

$$M_{\beta}^*(j\omega) = G_1 W_{ss}(j\omega) + G_2(j\omega) \frac{1}{T} \sum_{n=-\infty}^{\infty} H_o W_{ss}(j\omega + jn\omega_s) \quad (\text{II-9})$$

Both (II-8) and (II-9) apply only for $|\omega| < \omega_s/2$. The accuracy of (II-8) and (II-9) depend upon the low-pass nature of the system. This limitation

is the same as for the one-term approximation of (II-5) and (II-6). However, (II-8) and (II-9) are seen to contain correction terms for the harmonics generated by the sampler.

An exact method exists for finding the gain margins in both the ϕ and the β channels. Consider (II-1), the exact transfer function for the system opened at ϕ . If the gain of G_1 is varied in (II-1) until the Nyquist diagram of the system, obtained from (II-1), indicates marginal stability, then the factor by which the gain of G_1 was changed is the gain margin of the ϕ channel. The gain margin of the β channel can be found in the same manner by varying the gain of W_{ss} in (II-1). It is evident that this method is time consuming and inefficient, since a Nyquist diagram must be generated for each value of gain considered.

The New Technique

The new approximate technique for determining the open-loop gain margin in a continuous-data loop of a sampled-data system will now be developed. Consider the exact characteristic equation of the system, obtained from (II-2).

$$1 - \frac{1}{T} \sum_{n=-\infty}^{\infty} \frac{H_o G_2 W_{ss}(j\omega + jn\omega_s)}{1 - G_1 W_{ss}(j\omega + jn\omega_s)} = 0 \quad (\text{II-10})$$

To simplify notation in the following development, let

$$H_o G_2 W_{ss}(j\omega + jn\omega_s) = A_n \quad (\text{II-11})$$

and

$$G_1 W_{ss}(j\omega + jn\omega_s) = B_n \quad (\text{II-12})$$

Then (II-10) can be expressed as

$$1 - \frac{1}{T} \left[\frac{A_0}{1-B_0} + \frac{A_1}{1-B_1} + \frac{A_{-1}}{1-B_{-1}} + \frac{A_2}{1-B_2} + \dots \right] = 0 \quad (\text{II-13})$$

Also, to simplify notation, let

$$\prod_{\infty} (1-B_n) = (1-B_0)(1-B_1)(1-B_{-1})\dots \quad (\text{II-14})$$

and

$$\frac{k}{\prod} (1-B_n) = \frac{\prod_{\infty} (1-B_n)}{(1-B_k)} \quad (\text{II-15})$$

Equation (II-13) can be expressed as

$$1 - \frac{1}{T} \left[\frac{A_0 \prod_{\infty} (1-B_n) + A_1 \prod_{\infty} (1-B_n) + A_{-1} \prod_{\infty} (1-B_n) + \dots}{\prod_{\infty} (1-B_n)} \right] = 0 \quad (\text{II-16})$$

now

$$\begin{aligned} \frac{k}{\prod} (1-B_n) &= 1 - B_0 - B_1 - B_{-1} - \dots - B_{k-1} - B_{k+1} - \dots \\ &\quad + B_0 B_1 + B_0 B_{-1} + \dots \end{aligned} \quad (\text{II-17})$$

Assume that all product terms in (II-17) are negligible. Note that the term $B_0 B_{-1}$ is negligible compared to B_0 if

$$|B_0| \gg |B_0 B_{-1}|, \quad (\text{II-18})$$

for $\omega < \omega_s/2$. Or, (II-18) may be written as

$$1 \gg |B_{-1}| \quad (\text{II-19})$$

In a like manner, $B_i B_j$ can be neglected compared to B_i , $|i| < |j|$, provided that

$$1 \gg |B_j| \quad (\text{II-20})$$

For a low-pass system, this is generally true provided that (II-19) is satisfied. And if $B_i B_j$ can be neglected compared to B_i , $|i| < |j|$, then so can $B_i B_j B_k$, $|i| < |j| < |k|$.

The substitution of (II-12) into (II-19) gives

$$1 \gg |G_1 W_{ss}(j\omega - j\omega_s)| \quad (\text{II-21})$$

for $\omega < \omega_s/2$. It is then seen that, for $|\omega| > \omega_s/2$, (II-17) can be approximated by

$$\prod_{n=0}^k (1 - B_n) = 1 - B_0 - B_1 - B_{-1} - \dots - B_{k-1} - B_{k+1} - \dots, \quad (\text{II-22})$$

provided that (II-21) is satisfied. Then (II-16) becomes

$$\begin{aligned} (1 - B_0 - B_1 - B_{-1} - \dots) - \frac{1}{T} \left[A_0 (B_1 + B_{-1} + \dots) + A_1 (B_0 + B_{-1} + B_2 + \dots) \right. \\ \left. + A_{-1} (B_0 + B_1 + B_2 + \dots) \right] - \frac{1}{T} (A_0 + A_1 + A_{-1} + \dots) = 0 \end{aligned} \quad (\text{II-23})$$

By the addition and subtraction of certain terms, (II-23) can be expressed as

$$\begin{aligned} 1 - \frac{1}{T} (A_0 + A_1 + A_{-1} + \dots) - (B_0 + B_1 + B_{-1} + \dots) - \frac{1}{T} \left[A_0 (B_0 + B_1 + B_{-1} + \dots) \right. \\ \left. + A_1 (B_0 + B_1 + B_{-1} + \dots) - A_0 B_0 - A_1 B_1 - A_{-1} B_{-1} + \dots \right] = 0 \end{aligned} \quad (\text{II-24})$$

By dividing (II-24) by all terms not containing B_i , $i=0, \pm 1, \pm 2, \dots$, the following expression is obtained.

$$1 - \frac{\sum_{n=-\infty}^{\infty} B_n + \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n \sum_{n=-\infty}^{\infty} B_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n B_n}{1 - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n} = 0 \quad (\text{II-25})$$

A_n and B_n are defined by (II-11) and (II-12), respectively. The second term of (II-25) is seen to be an approximate transfer function for the system of Figure 1 opened at ϕ , and applies only for $\omega < \omega_s/2$.

$$M_{\phi}^*(j\omega) = \frac{\sum_{n=-\infty}^{\infty} B_n + \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n \sum_{n=-\infty}^{\infty} B_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n B_n}{1 - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n} \quad (\text{II-26})$$

If, in (II-26), only $n=0$ is used in the numerator, the approximate transfer function reduces to (II-8), the transfer function obtained using the fictitious sampler. If, in addition, only $n=0$ is used in the denominator of (II-26), the approximate transfer function reduces to (II-5), the one-term approximate transfer function.

To determine the approximate transfer function for the system opened at β_e , define, in (II-10),

$$C_n = H_o G_2(j\omega + jn\omega_s)$$

$$D_n = G_1(j\omega + jn\omega_s) \quad (\text{II-27})$$

$$E_n = W_{ss}(j\omega + jn\omega_s)$$

Using the same method as above, the approximate characteristic equation can be expressed as

$$1 - \sum_{n=-\infty}^{\infty} D_n E_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} C_n E_n = 0 \quad (\text{II-28})$$

The assumption necessary for the derivation of (II-28) is

$$1 \gg |G_2 W_{ss}(j\omega + jn\omega_s)|$$

The approximate open-loop transfer function for the system opened at β_e is then seen to be, from (II-28),

$$M_{\beta}^*(j\omega) = \sum_{n=-\infty}^{\infty} G_1 W_{ss}(j\omega + jn\omega_s) + \frac{1}{T} \sum_{n=-\infty}^{\infty} H_o G_2 W_{ss}(j\omega + jn\omega_s) \quad (\text{II-29})$$

This approximate transfer function is valid only for $|\omega| < \omega_s/2$, and is based on the assumption that

$$1 \gg |G_2 W_{ss}(j\omega - j\omega_s)| \quad (\text{II-30})$$

for $\omega < \omega_s/2$.

An Example

The approximate transfer functions (II-26) and (II-29) were used to analyze the system of Figure 1 with the system transfer functions as given in Table 1. A sampling frequency of five hz. was used. A plot of (II-3), shown in Figure 3, indicates that the system is low-pass with respect to $\omega_s/2$, except in the vicinity of 2.9 hz., which is the third bending-mode frequency. Since $f_s/2$ is 2.5 hz., then 2.9 hz. is reflected back into the primary band of frequencies at 2.1 hz. Thus inaccuracies in the vicinity of 2.1 hz. are to be expected by any method that is dependent upon the system being low pass.

The gain margin of the ϕ channel was calculated first. The Nyquist diagram of the system opened in the ϕ channel, obtained using the one-term approximation of (II-5), is shown in Figure 4, and indicates that a reduction in gain by a factor of 0.40 will force the system to marginal stability at a frequency of 1.95 hz. The Nyquist diagram obtained using the fictitious-sampler approximate transfer function of (II-8) is also plotted in Figure 4, and indicates that an increase in gain by a factor

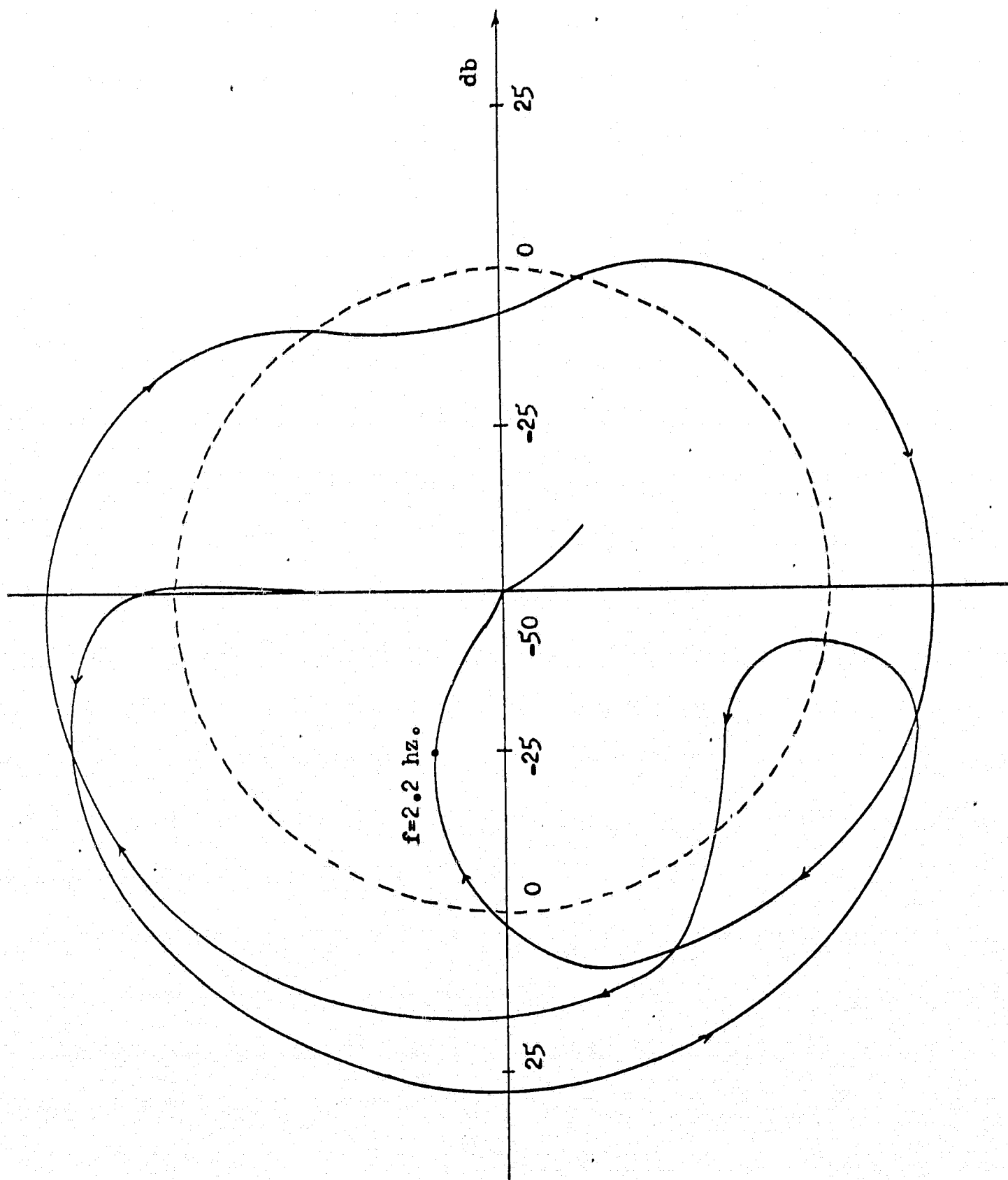


Figure 3. Nyquist diagram using one-term approximation for system opened in ϕ channel.

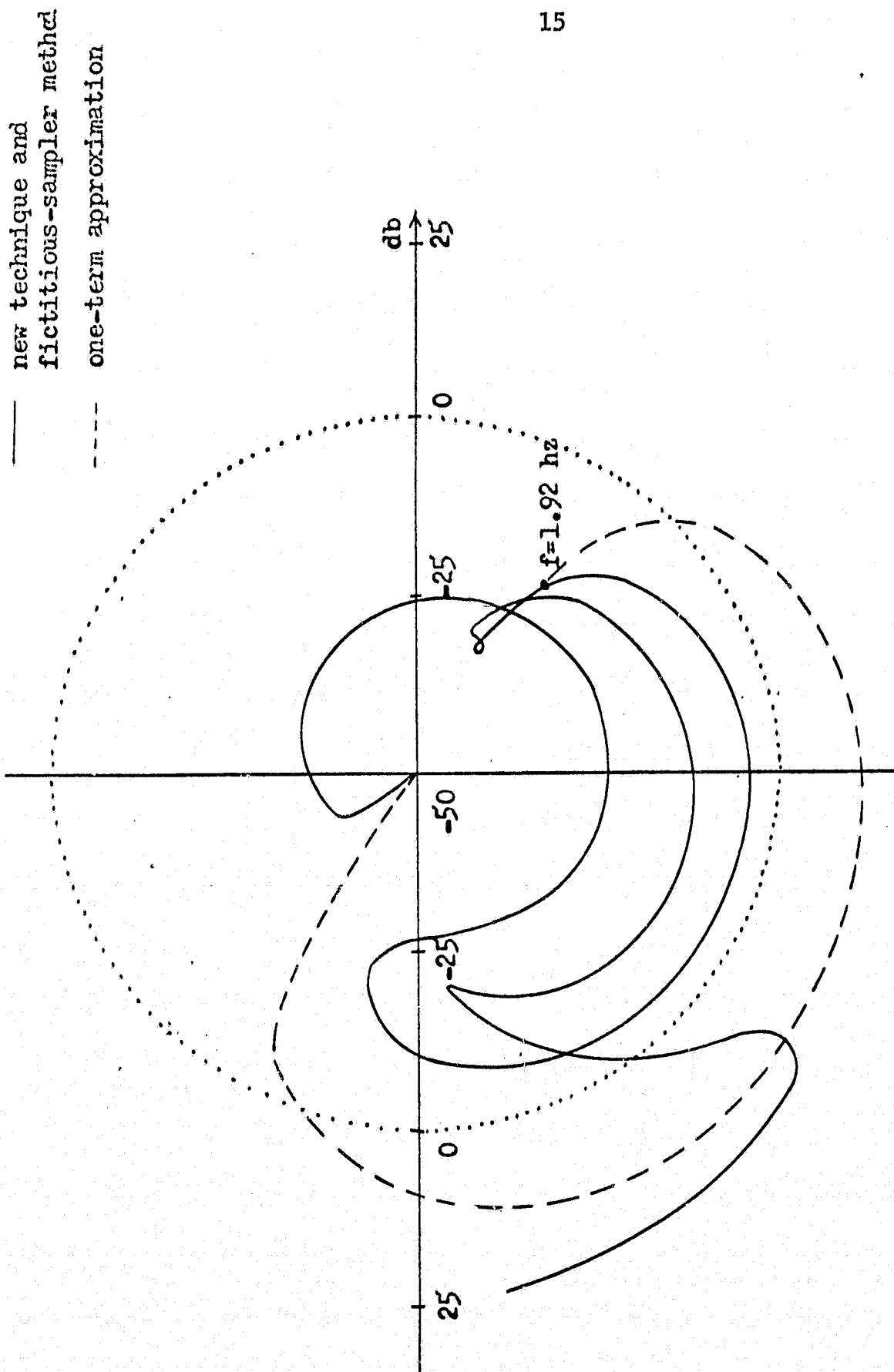


Figure 4. Nyquist diagrams using approximate methods for system opened in ϕ channel.

of 3.84 will force the system to marginal stability at a frequency of 1.96hz. The Nyquist diagram of the system opened at ϕ was also calculated using the new approximate transfer function of (II-26). It was approximately the same as that obtained using (II-8), and could not be plotted as a separate curve. However, the new method indicated that an increase in gain by a factor of 3.06 will force the system to marginal stability at a frequency of 1.96hz. A check by the exact method showed that an increase in gain by a factor of 2.95 forces the system to marginal stability at a frequency 1.96hz.

Next the gain margin of the β channel was calculated. The one-term approximation of (II-6) indicated a decrease in gain by a factor of 0.870 forces marginal stability at a frequency of 1.95hz. The fictitious-sampler approximation of (II-9) indicated a decrease in gain by a factor of 0.213 forces marginal stability at a frequency of 1.92hz. The new approximation of (II-29) indicated a decrease in gain by a factor of 0.709 forces marginal stability at a frequency of 2.07hz. A check by the exact method showed that a decrease in gain by a factor of 0.719 forces marginal stability at a frequency of 2.07hz. These results are summarized in Table 2.

As a further check on the accuracy of the new approximate technique, the approximate open-loop transfer functions were reflected back to the ϕ loop and compared with the exact transfer function of the system. Consider first the approximate transfer function of the ϕ channel. The approximate characteristic equation, obtained from (II-25), is

$$1 - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n - \sum_{n=-\infty}^{\infty} B_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n \sum_{n=-\infty}^{\infty} B_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n B_n = 0 \quad (\text{II-31})$$

Equation (II-31) can then be expressed as

$$1 - \frac{\frac{1}{T} \sum_{n=-\infty}^{\infty} A_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n \sum_{n=-\infty}^{\infty} B_n - \frac{1}{T} \sum_{n=-\infty}^{\infty} A_n B_n}{1 - \sum_{n=-\infty}^{\infty} B_n} = 0 \quad (\text{II-32})$$

where A_n and B_n are defined by (II-11) and (II-12), respectively. The second term of (II-32) is seen to be the approximate transfer function of system opened in the ϕ channel, using the approximations of the new techniques. The exact transfer function for the system opened in the ϕ channel is given in (II-2). Nyquist diagrams obtained from both (II-2) and (II-32) are shown in Figure 5, and could not be plotted as separate curves. Also the Nyquist diagram of (II-3), the one-term approximation, is shown in Figure 5. In addition, the method used to obtain (II-32) was used to obtain the approximate transfer function from the fictitious sampler method. The Nyquist diagram obtained from this transfer function could not be plotted as a separate curve from the exact Nyquist diagram.

The approximate transfer functions for the β channel were also referred back to the ϕ channel by the method used above. The approximate transfer function for the system opened at ϕ , obtained from (II-29) is

$$M_{\phi\beta}^* = \frac{\frac{1}{T} \sum_{n=-\infty}^{\infty} H_o G_2 W_{ss}(j\omega + jn\omega_s)}{1 - \sum_{n=-\infty}^{\infty} G_1 W_{ss}(j\omega + jn\omega_s)} \quad (\text{II-33})$$

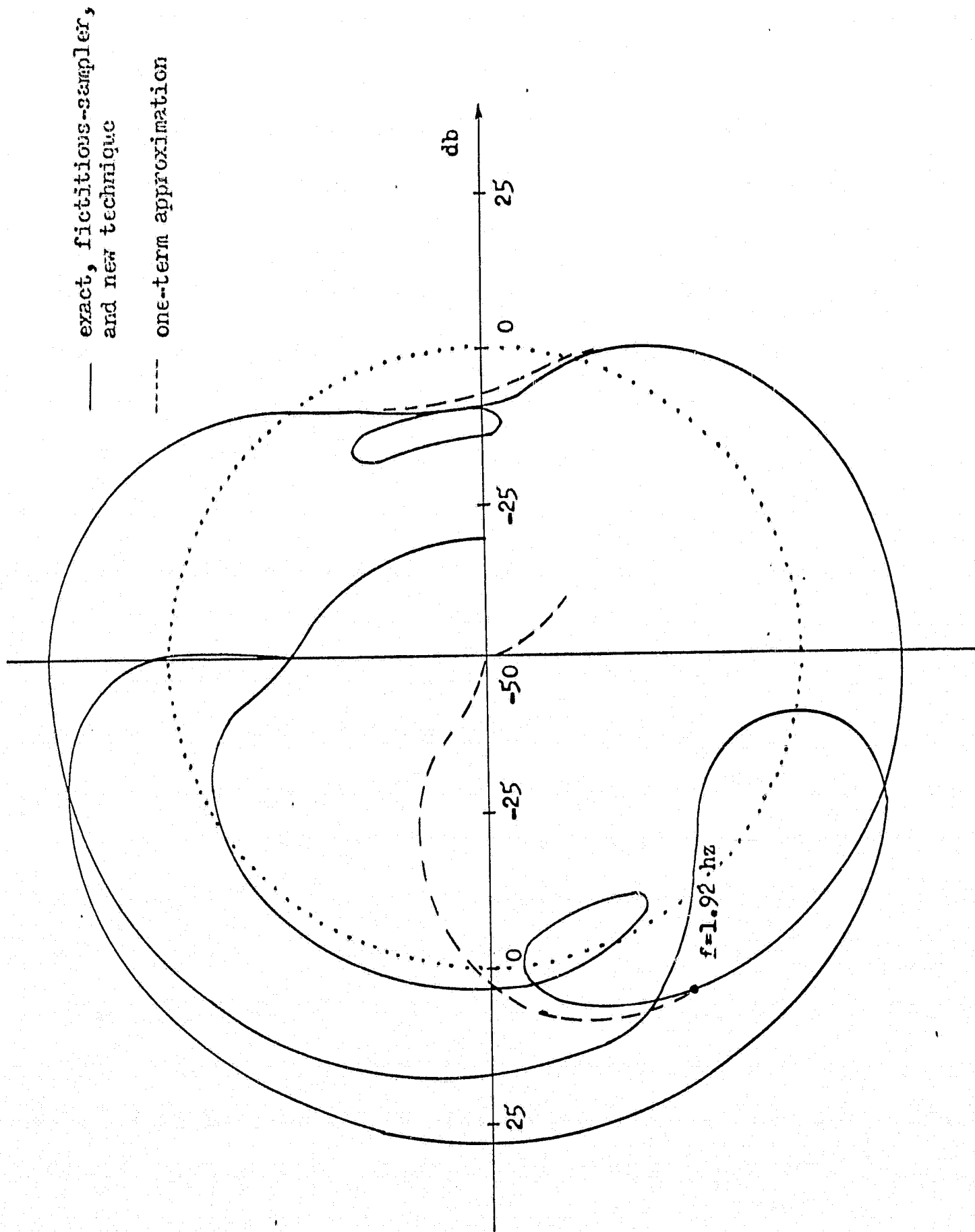


Figure 5. Nyquist diagram by exact and approximate methods in ϕ channel for system opened in ϕ channel.

There was no appreciable difference between the Nyquist diagram obtained from (II-33) and that obtained from (II-2), the exact transfer function. These Nyquist diagrams are shown in Figure 6, as one curve. However, the Nyquist diagrams obtained from both the one-term approximation and the fictitious-sampler method, referred from the β channel back to the ϕ channel, show considerable variation from the exact Nyquist diagram, as is seen in Figure 6.

Conclusion

An approximate technique for determining the gain margins in unsampled loops of sampled-data control systems is developed. The limitations of the technique are presented, and the technique is applied to the analysis of the attitude control system of a Saturn V vehicle. The results obtained from the new technique are compared with those obtained using known approximate methods, and are seen to be significantly more accurate.

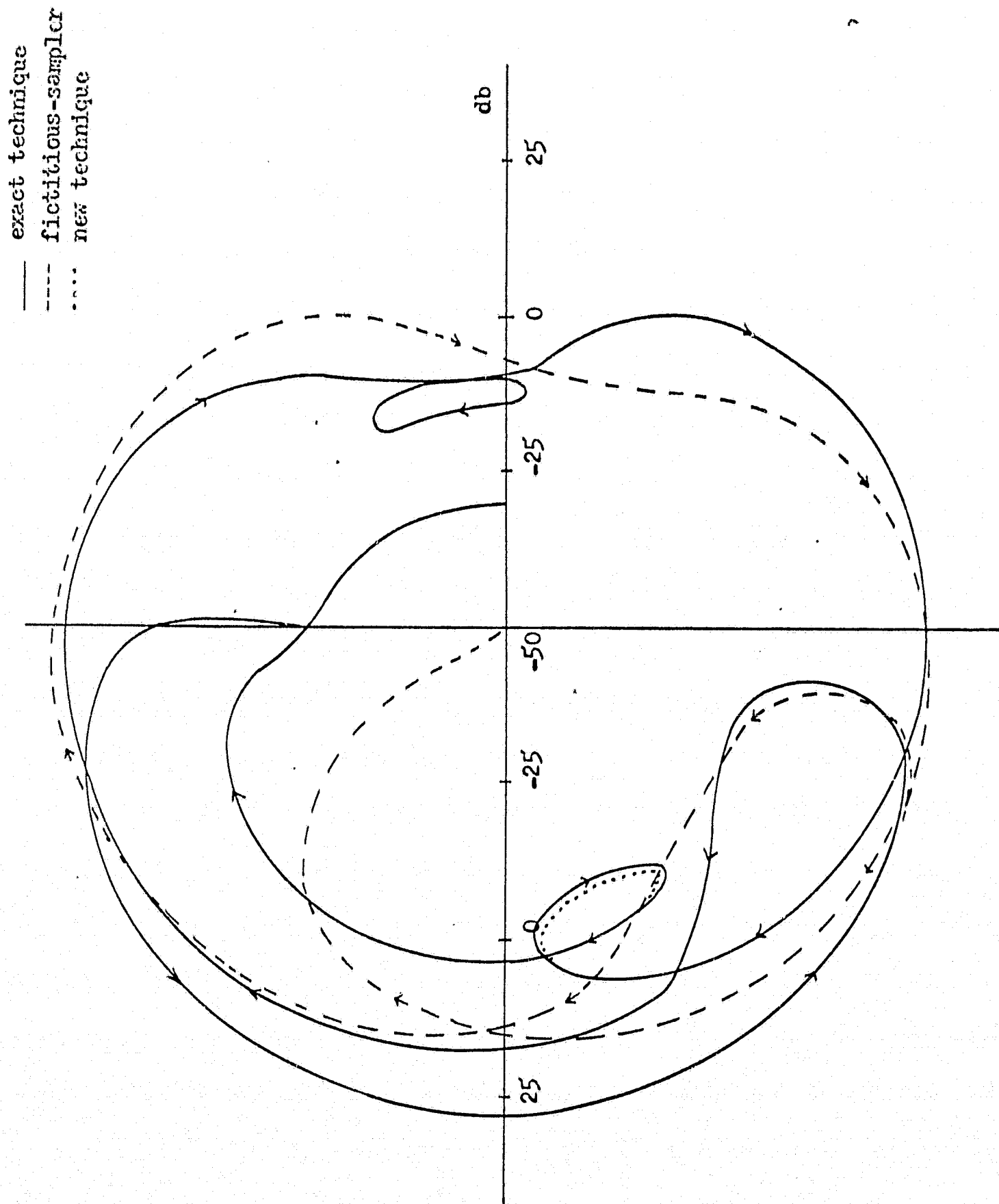


Figure 6. Nyquist diagram by exact and approximate methods in β channel for system opened in ϕ channel.

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1. B. C. Kuo, "Analysis and Synthesis of Sampled-Data Control Systems," Prentice-Hall, Inc., Englewood Cliffs, N. J., pp. 220-223.
2. C. L. Phillips, et. al., "Stability Analysis Methods for Hybrid Control Systems", Paper No. 75, Southeastern Symposium on Missiles and Aerospace Vehicles Sciences, Huntsville, Ala.; December, 1966

TABLE 1

Transfer Functions of Attitude Control System

$$G_{1R} = \frac{-0.94068468}{s^2 - 0.02972784}$$

$$G_{1B1} = \frac{0.65323138 \times 10^{-2}(s^2 + 498.59362)}{s^2 + 0.64905305 \times 10^{-1}s + 42.126986}$$

$$G_{1B2} = \frac{-0.40378959 \times 10^{-2}(s^2 + 485.48033)}{s^2 + 0.1201345s + 144.32299}$$

$$G_{1B3} = \frac{-0.53896739 \times 10^{-2}(s^2 + 470.36052)}{s^2 + 0.18378317s + 144.32299}$$

$$G_{1B4} = \frac{-0.58368238 \times 10^{-2}(s^2 + 469.03256)}{s^2 + 0.22481237s + 505.40603}$$

$$G_2 = s(G_{1R} + G_{1B1} + G_{1B2} + G_{1B3} + G_{1B4}) = s G_1$$

$$W_{ss} = \frac{625.0}{s^2 + 25.0s + 625.0}$$

$$H_0 = \frac{1 - e^{-Ts}}{s}$$

TABLE 2
RESULTS OF EXAMPLE

Technique	ϕ Channel		β Channel	
	Gain-Factor Change	Frequency hz.	Gain-Factor Change	Frequency hz.
One-Term Approximation	0.40	1.95	0.870	1.95
Fictitious- Sampler	3.84	1.96	0.213	1.92
New Technique	3.06	1.96	0.709	2.07
Exact Technique	2.95	1.96	0.719	2.07

III. DIGITAL COMPENSATION OF THE THRUST VECTOR CONTROL SYSTEM

1. Introduction

In this chapter, the results are presented of a preliminary effort in digitally compensating the pitch attitude control system of the Saturn V AS-504 launch vehicle during S-IC flight.

The system was stabilized in the "rate-derived" configuration, which is shown in Figure 7. Stability studies were performed using w-plane frequency response data and the Nyquist criterion, and relative stability was investigated on the basis of phase and gain margins.

2. Simplifications

In order to facilitate the system analysis, certain simplifying assumptions were made. Although these assumptions do not allow the compensation function to be determined in its final form, the general form of the compensation function can be established. The simplifications made in the analysis are as follows:

- (1) Sloshing dynamics are neglected.
- (2) Digital computer transport delay is neglected.
- (3) Lateral drift of the vehicle center of gravity is assumed negligible.

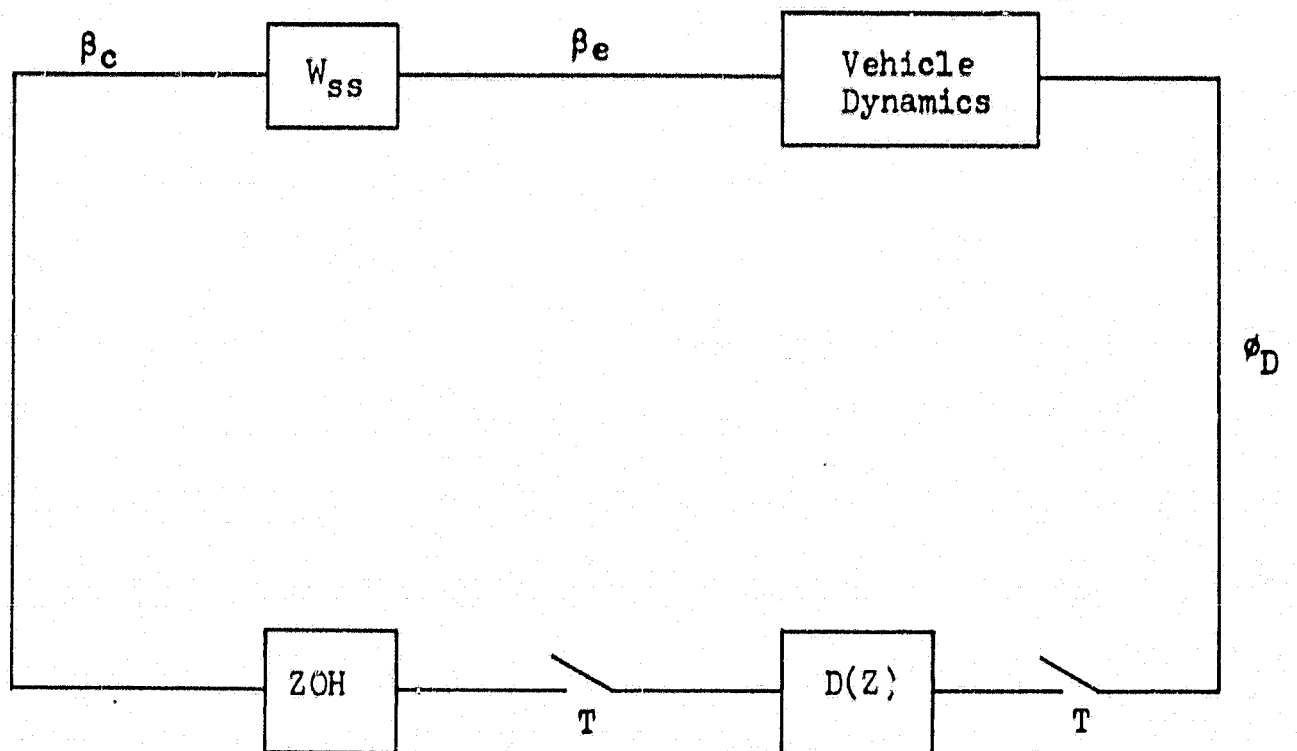


Figure 7. Rate-derived Configuration.

3. Design Criteria

The primary design objective used in determining the compensation was the achievement of adequate stability margins with a minimum sacrifice of system performance.

It was found that the simplified system model could be stabilized by introducing phase lead in the w-plane region of 0.01 to 0.3 Hz, and by gain stabilizing all bending modes except the first. The first bending mode was phase stabilized. Acceptability bounds on gain and phase margins were arbitrarily set as follows: 6db and 30 degrees for rigid body stability margins, 60 degrees for phase stabilized bending mode phase margins, and 6db for gain stabilized bending mode gain margins. The minimum acceptable control gain was set at 0.30 in the w-plane.

4. Compensation Design

The form of the necessary compensation function was determined by an analysis of the vehicle motion equations, which appear in simplified form in Appendix B. From these perturbation equations, the transfer function from B_c , the controller input, to ϕ_D , the platform angle, was derived. From this transfer function the sampled frequency response was generated. The ϕ_D/B_c (s) transfer function derivation and a digital computer program for computing the sampled frequency response were discussed previously in the Sixth Technical Report [3].

The vehicle dynamics are influenced greatly by the effects of fuel depletion and changing dynamic pressure throughout the course of the S-IC flight. Therefore, to stabilize adequately the control

system during the entire S-IC flight, it was necessary to perform stability studies for several representative times in the flight trajectory. Thus, the system transfer function was determined for each of these points. The flight times chosen for the point studies were 20, 40, 60, 80, 100, 120, and 140 seconds.

In each of the point studies, the s-plane sampled frequency response was computed from the system transfer function, using the series approximation method with three terms [3]. Then, to facilitate the use of continuous system compensation techniques, the s-plane frequencies were converted to the corresponding w-plane frequencies through the transformation

$$\omega_w = \frac{\sin \omega T}{1 + \cos \omega T}, \quad (\text{III-1})$$

where T is the sampling period, ω is the s-plane frequency, and ω_w is the w-plane frequency. A sampling period of 0.04 seconds was used in the compensation studies discussed in this chapter.

The open-loop vehicle transfer function indicates the existence of one unstable pole (due to negative C_1) throughout most of the S-IC flight. Hence, for system stability, the Nyquist criterion requires one negative encirclement of the -1 point of the system Nyquist diagram. The problems arising in obtaining the necessary encirclement become apparent upon an examination of the uncompensated system w-plane Nyquist diagrams, which appear in Appendix A. The introduction of phase lead in the 0.01 to 0.3 Hz. frequency region

provides the required encirclement; however, it also produces the undesired effect of increasing the bending mode peak amplitudes. Three alternative solutions to this problem were investigated. They are discussed as follows:

A. The control gain might be decreased sufficiently to attenuate the bending mode peaks to an acceptable level. However, this cannot be achieved without greatly sacrificing system response.

B. One or more lightly damped second order filters might be utilized to attenuate the bending modes. However, the variation in bending resonances as a function of the other system parameters presents a limitation to the use of this approach. That is, the filter band widths must be sufficiently large to account for wide variations in bending frequencies, and this is difficult to achieve without loss in rejection capabilities.

C. The insertion of a first order lag-lead term in the compensation function with very low break frequencies has the effect of attenuation without phase shift in the bending frequency region. However, the breaks must be chosen sufficiently low as to minimize the effects of the additional phase lag in the rigid body frequency range. This scheme is especially attractive in that it provides good rigid-body phase margin without sacrificing control gain and steady-state accuracy. One generally undesirable side effect of this technique, however, is the accompanying decrease in system bandwidth.

It was found that a satisfactory compensation function could be developed using a combination of these three schemes. The w-plane

compensation $D(w)$ resulting from this effort is given below:

$$D(W) = \frac{a_w (w/.004+1) (w^2/.135^2+2(.1)w/.135+1) (w^2/.330^2+2(.1)w/.330+1) (w/.0018+1)}{(w/.1+1) (w/.1+1) (w/.15+1) (w/.226+1) (w/.330+1) (w/.00072+1.)}, \quad (\text{III-2})$$

where a_w is used to denote the w -plane control gain. Phase and gain plots of $D(j\omega_w)$ are shown in Figure 8. The corresponding z -plane compensation was obtained from the transformation

$$w = \frac{z - 1}{z + 1}. \quad (\text{III-3})$$

Then,

$$D(Z) = \frac{a_z (1-.992z^{-1}) (1-1.88z^{-1}+.948z^{-2}) (1-.517z^{-1}) (1-.9964z^{-1})}{(1-.82z^{-1}) (1-.82z^{-1}) (1-.739z^{-1}) (1-.633z^{-1}) (1-.504z^{-1}) (1-.99856z^{-1})}, \quad (\text{III-4})$$

where a_z denotes the z -plane control gain.

A physical implementation of $D(z)$ with digital elements would require the retention of five significant decimal digits in certain computations. This, as might be expected, adds considerably to the complexity of the implementation problem. This problem will be considered later.

It was found that a control gain change was necessary near the middle of the S-IC flight to obtain sufficient rigid body stability

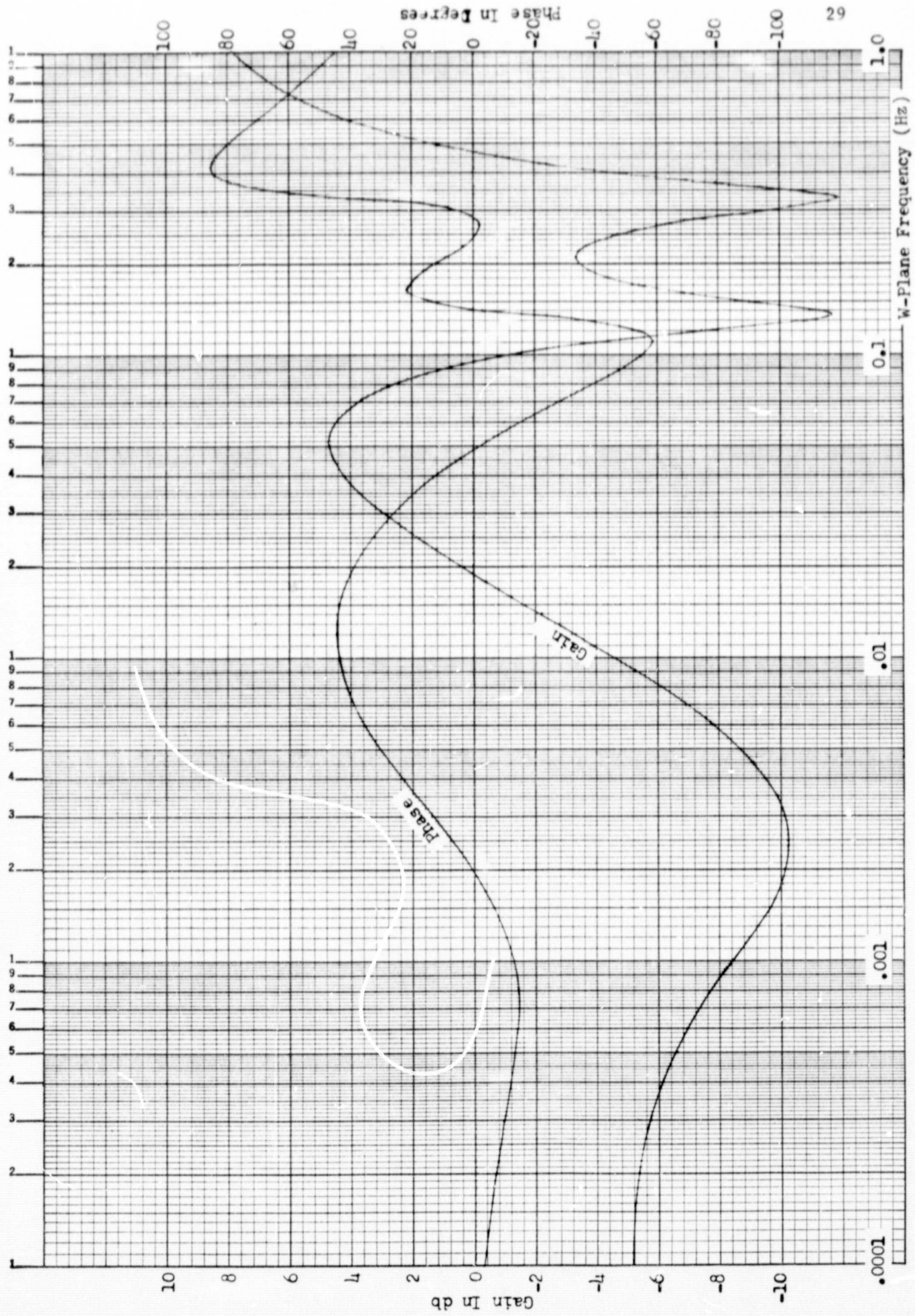


Figure 8. Gain and Phase Plots of $D(j\omega)$.

margins. The attitude control gain schedule corresponding to the compensation function $D(w)$ is shown in Table 3.

The effects of the compensation function upon system performance can be seen in terms of the compensated w -plane Nyquist diagrams appearing in Appendix C. The rigid body and bending mode stability margins have been obtained from these diagrams and are shown in Table 4 for each of the flight times investigated.

5. Hybrid Simulation of the Pitch Attitude Control System

A hybrid simulation of the Saturn V S-1C pitch attitude control system is necessary for the development and verification of a proposed digital compensation function. This simulation is being implemented on an Electronics Associates Incorporated, model 16-31R, analog computer at Auburn University.

Three simplifying assumptions have been made in the simulation. It has been assumed that: (1) The transport delay in the flight computer is negligible; (2) $\phi = \alpha$; i.e., the lateral drift of the vehicle is negligible; and, (3) that the actuator dynamics can be described accurately by a fourth order system.

Preliminary scaling and patching has been accomplished for the actuator, the four bending modes, and the rigid body at 40 seconds of flight time. Presently, scaling factors and associated gain and potentiometer settings are being computed for selected times throughout the S-1C flight.

In order to gain an insight into the performance of the digitally-compensated system, an approximate continuous model of the digital compensation function $D(z)$ will be used to stabilize the system. Further, the continuously compensated system will be used to verify the scaling of system variables.

TABLE 3

CONTROL GAIN SCHEDULE

Flight Time	Control Gain a_w
0 - 100 sec.	0.55
100 - 159 sec.	0.34

TABLE 4

SATURN V RIGID BODY AND BENDING MODE STABILITY MARGINS

Flight Time (sec.)	Rigid Body G.M. (db)	Rigid Body P.M. (deg.)	1 st Bending Mode P.M. (deg.)	2 nd Bending Mode G.M. (db)	3 rd Bending Mode G.M. (db)	4 th Bending Mode G.M. (db)
20	7.0	35.0	95.0	4.3	17.6	30.7
40	7.0	35.0	95.0	5.5	16.5	33.1
60	6.0	33.0	90.0	7.7	15.0	46.3
80	5.9	32.0	86.0	11.7	12.4	28.8
100	5.0/6.4	29.0/41.0	76.0/97.0	20.0/24.2	35.8/40.0	9.6/13.8
120	7.7	36.0	95.0	31.0	13.0	24.0
140	5.5	28.0	86.0	22.8	15.9	20.0

IV. QUANTIZATION ERRORS

1. Introduction

In this chapter, the technique for determining bounds on quantization errors, which was outlined in Chapter 3 of Reference [3], is illustrated by example. The system which is studied is an elementary model of the Thrust Vector Control System. The departures from a more realistic representation of this system are:

- (1) All bending and sloshing effects are neglected.
- (2) A perfect control engine actuator is assumed.
- (3) A simplified compensation function is assumed.

2. Application of the Method

The system which will be investigated can be represented by the block diagram shown in Figure 9.

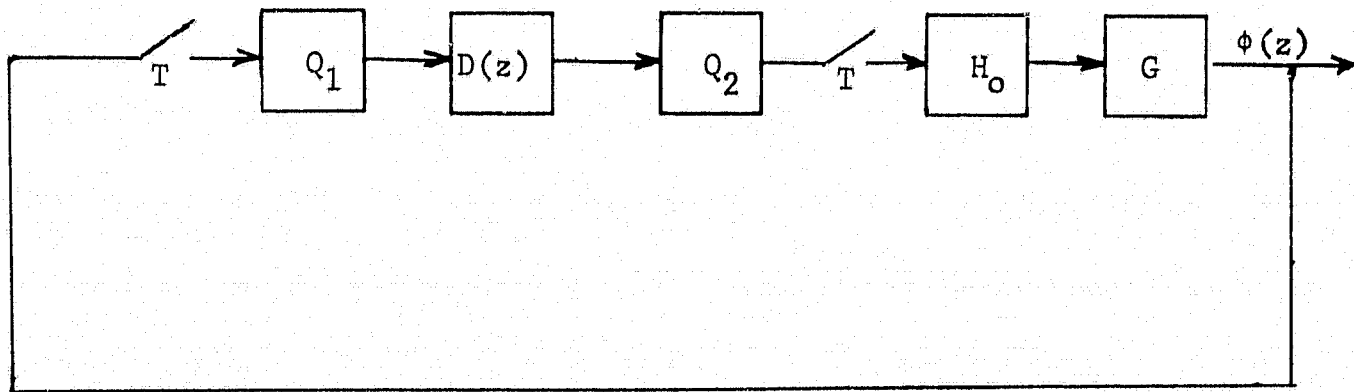


Fig. 9--Block diagram of simplified Thrust Vector Control System.

The transfer functions which are used in the example are

$$G(s) = \frac{0.9166}{s^2 - 0.024435}, \quad (\text{IV-1})$$

$$H_o = \frac{1 - e^{-Ts}}{s}, \quad (\text{IV-2})$$

$$D(z) = \frac{12.5(1.004z - 0.996)}{(1.1z - 0.90)}, \quad (\text{IV-3})$$

where $T = 0.04$ seconds.

The z -transform of the transfer functions which represent the continuous portion of the system is

$$[G \cdot H_o](z) = \frac{P[1 - \cosh \omega_o T][z + 1]}{z^2 - 2z \cosh \omega_o T + 1}, \quad (\text{IV-4})$$

where $P = 0.9166/0.024435$, and $\omega_o = \sqrt{0.024435}$.

Consequently, the discrete equivalent of the system of Figure 9 is presented in Figure 10. Note that each quantizer is replaced by a unity gain element and a summing junction at which a noise sequence representing quantizer round off errors is added.

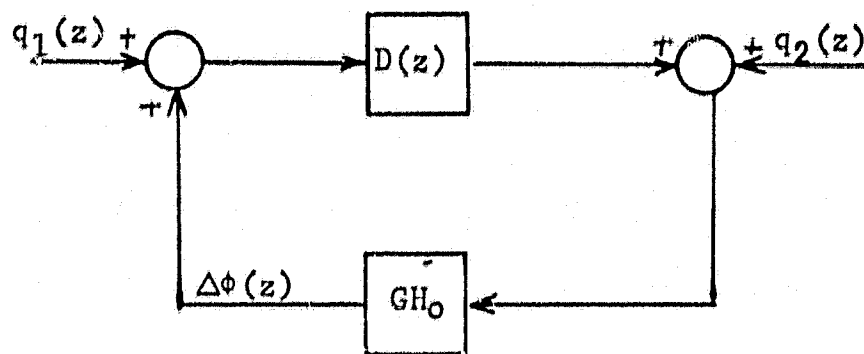


Fig. 10--Discrete diagram showing quantization error input sequences.

It is apparent from Figure 10 that the system response, $\Delta\phi(z)$, resulting from the input error sequences is given by,

$$\Delta\phi_2(z) = \frac{GH_O(z)}{1 - D(z) GH_O(z)} q_2(z) , \quad (\text{IV-5})$$

$$\Delta\phi_1(z) = \frac{D(z) GH_O(z)}{1 - D(z) GH_O(z)} q_1(z) , \quad (\text{IV-6})$$

and

$$\Delta\phi(z) = \Delta\phi_1(z) + \Delta\phi_2(z) . \quad (\text{IV-7})$$

The transfer function from $q_2(z)$ to $\Delta\phi_2(z)$ was found to be

$$\frac{\Delta\phi_2(z)}{q_2(z)} = \frac{D(1.1z^2 + 0.2z - 0.9)}{(d_3z^3 + d_2z^2 + d_1z + d_0)} , \quad (\text{IV-8})$$

where $D = P(1 - \cosh \omega_0 T)$,

$$d_3 = 1.1,$$

$$d_2 = -(2.2 \cosh \omega_0 T + 0.9 + (12.5)(1.004)D),$$

$$d_1 = (1.1 + 1.8 \cosh \omega_0 T - (0.008)(12.5)D), \text{ and}$$

$$d_0 = .996(12.5)D - 0.9 .$$

In a similar manner, $\Delta\phi_1(z)/q_1(z)$ is

$$\frac{\phi_1(z)}{q_1(z)} = \frac{12.5 D(z+1)(1.004z - 0.9961)}{(d_3 z^3 + d_2 z^2 + d_1 z + d_0)} . \quad ((V-9))$$

In order to apply the error bound evaluation technique, it is necessary to develop a state variable representation for the system. This can be done by manipulation of (IV-8) or (IV-9). The state equations may be written in the following forms by using (IV-8) as a starting point.

$$\underline{x}(k+1) = A \underline{x}(k) + gq_2(k) , \quad (IV-10)$$

and

$$\Delta\phi_2(k) = h_2^T \underline{x}(k) , \quad (IV-11)$$

Matrices A and g are described by (IV-12) and (IV-13) respectively. The total attitude error which results from these two quantization error sequences is

$$\Delta\phi(k) = \Delta\phi_1(k) + \Delta\phi_2(k) = h_1^T y(k) + h_2^T x(k). \quad (IV-18)$$

By applying norm inequalities, (IV-18) may be written as

$$|\Delta\phi(k)| \leq \|h_1\| \cdot \|y(k)\| + \|h_2\| \cdot \|x(k)\|. \quad (IV-19)$$

The quantities $\|h_1\|$ and $\|h_2\|$ may be computed by evaluating the square root of the sum of the squares of the components of h_1 and h_2 respectively.

An upper bound for $\Delta\phi(k)$ can be determined if bounds on $\|y(k)\|$ and $\|x(k)\|$ are known. The bounds for $\|y(k)\|$ and $\|x(k)\|$ may be computed by making use of the results given in Chapter 3 of Reference [3].

These bounds are

$$\rho_1 = \left[\frac{\|g^T B A\|}{\min_i \lambda_i(C)} + \left[\frac{\|g^T B A\|^2}{\min_i \lambda_i^2(C)} + \frac{g^T B g}{\min_i \lambda_i(C)} \right]^{\frac{1}{2}} \right] \cdot \left[\frac{\max_i \lambda_i(B)}{\min_i \lambda_i(B)} \right]^{\frac{1}{2}} \cdot \left[\frac{q_1 \max}{2} \right], \quad (IV-20)$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}, \quad (\text{IV-12})$$

$$g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (\text{IV-13})$$

$$h_2 = [-0.9D, 0.2D, 1.1D]^T, \quad (\text{IV-14})$$

and $a_0 = d_0/d_4$, $a_1 = d_1/d_4$, $a_2 = d_2/d_4$.

If the same operations are performed on transfer function (IV-9), similar results are obtained.

$$\underline{y}(k+1) = A \underline{y}(k) + gq_1(k), \quad (\text{IV-15})$$

and

$$\Delta\phi_1(k) = h_1^T \underline{y}(k). \quad (\text{IV-16})$$

where,

$$h_1 = [-12.5D(.996), 12.5D(.004), 12.5D(1.004)]^T. \quad (\text{IV-17})$$

and

$$\rho_2 = \left[\frac{||g^T BA||}{\min_i \lambda_i(C)} + \left[\frac{||g^T BA||^2}{\min_i \lambda_i^2(C)} + \frac{g^T Bg}{\min_i \lambda_i(C)} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \cdot \left[\frac{\max_i \lambda_i(B)}{\min_i \lambda_i(B)} \right]^{\frac{1}{2}} \cdot \left[\frac{q_2 \max}{2} \right] \quad (IV-21)$$

The elements of matrices B and C are determined by using the matrix equation

$$A^T BA - B = -C \quad (IV-22)$$

Normally an arbitrary positive definite matrix is assumed for the C matrix, and the $\frac{1}{2}(n^2 + n)$ equations in $\frac{1}{2}(n^2 + n)$ unknowns are solved for the elements of the B matrix. The most common choice for the C matrix is the identity matrix. When B and C are known, and the maximum values of quantization error are specified, then numerical solutions for ρ_1 and ρ_2 may be advanced. It is worthwhile to note that $\rho_2 = (q_2 \max / q_1 \max) \rho_1$. Consequently, only one of the complicated expressions, (IV-20) and (IV-21), needs to be evaluated.

In this preliminary analysis, $\phi(\max)$ will be assumed to be 15. degrees and $\beta_c(\max)$ to be 7.5 degrees. If an eight bit word length plus one sign bit is used in the digital representation of these variables, the maximum round off error may be determined by using the following expression:

$$q_1(\max) = \frac{\phi(\max)}{2^8 \times 57.3} = \frac{15}{256 \times 57.3} = 1.02 \times 10^{-3} \text{ radians,}$$

and similarly,

$$q_2(\max) = .51 \times 10^{-3} \text{ radians.}$$

The numerical values, which have been calculated for the coefficients of the characteristic equation of the system under consideration, are

$$a_0 = -0.82648124E + 00,$$

$$a_1 = 0.26364623E + 01, \text{ and}$$

$$a_2 = -0.28098548E + 01 .$$

If these values are used to construct the system A matrix, and if C is assumed to be the identity matrix, then by solving matrix equations (IV-22), the B matrix can be found to be:

$$B = \begin{bmatrix} 0.51068571E + 06 , & -0.11185781E + 07 , & 0.61767665E + 06 \\ -0.11185781E + 07 , & 0.24504522E + 07 , & -0.13533753E + 07 \\ 0.61767665E + 06 , & -0.13533753E + 07 , & 0.74763022E + 06 \end{bmatrix}$$

The leading principal minors of B are positive; therefore, it may be asserted that B is positive definite.

Let

$$\rho_2 = \rho_2' \left[\frac{\max_i \lambda_i(B)}{\min_i \lambda_i(B)} \right]^{\frac{1}{2}} \quad (\text{IV-23})$$

Since all of the eigenvalues of a positive definite matrix are positive, it follows that $\rho_2 \leq \rho_2'$. It was found that

$$\|h_2\| \rho_2' = .815 \text{ radians}$$

Thus, the computed error bound of $\Delta\phi(k)$, resulting from the $q_2(k)$ sequence alone, is greater than or equal to .815 radians. This result is so unreasonable that there is little need to compute the contribution because of the $r_1(k)$ sequence, which will only further increase the error bound.

3. Conclusions

It is not possible to evaluate completely the error bound method used in this chapter because there is only one application. The numerical results, which have been obtained, have been carefully checked, and they appear to be free of errors.

Although no concrete analytical evidence is presently available to support this conclusion, it is intuitively evident that the bound predicted by this method is unreasonably large. Other methods [4] will be applied to the system described in this chapter in order to obtain a comparative estimate of quantization error bounds.

There are several possible causes for the extreme conservatism exhibited by this Liapunov-based technique. For example, the norm inequalities, upon which (IV-20) and (IV-21) are based, are likely to cause the generation of a pessimistic bound. Furthermore, the

selection of the C matrix as the identity matrix is entirely arbitrary, and it is possible that a different positive definite C matrix would provide a tighter bound. If this is true, then a systematic procedure for the selection of the C matrix is needed.

V. ANALYSIS OF THE DIGITALLY COMPENSATED PITCH ATTITUDE CONTROL SYSTEM WITH SLOSHING DYNAMICS INCLUDED

1. Introduction

In order to ascertain the acceptability of the compensation function synthesized in Chapter III of this report, it is essential that sloshing dynamics be considered. The computer program described in this report represents a first step toward evaluation of the effect of sloshing on the digitally compensated system.

2. The Program.

Because of the complexity of the vehicle dynamics during first stage flight, the development of the transfer function from B_c to ϕ_D is extremely difficult. Thus, in order to make the problem more amenable to signal flow graph analysis, the coupling between the bending and sloshing differential equations was neglected. More explicitly, the bending variables were removed from the sloshing equations and the sloshing variables were removed from the bending equations (see Appendix B). The resulting signal flow graph is depicted in Figure 11. The symbols listed on the graph are defined in Appendix D. The transfer function from B_c to ϕ_D may be determined by application of Mason's gain formula. The result is written

$$\frac{\phi_D}{B_c} = H_0 W_{ss} \left[\sum_{i=1}^4 W_i + TF \right], \quad (V-1)$$

where

$$TF = \left[\frac{1}{s^2} \left(\sum_{i=1}^4 T_i + A_2 s^2 + A_1 \right) \left(1 - \sum_{i=1}^4 T_{2i} T_{4i} \right) + \right. \\ \left. \frac{\left(\sum_{i=1}^4 U_i + k_4 + \frac{s}{M} s^2 \right) \left(\sum_{i=1}^6 T_{2i} T_{3i} \right)}{\Delta} \right], \quad (V-2)$$

and

$$\Delta = 1 - \left[\frac{-C_1}{s^2} + \sum_{i=1}^6 T_{1i} T_{3i} + \sum_{i=1}^6 T_{2i} T_{4i} + A_3 \sum_{i=1}^6 T_{2i} T_{3i} \right. \\ + (T_{21} T_{31}) \sum_{i=2}^6 T_{1i} T_{4i} + (T_{22} T_{32}) \sum_{\substack{i=1 \\ i \neq 2}}^6 T_{1i} T_{4i} \\ + (T_{23} T_{33}) \sum_{\substack{i=1 \\ i \neq 3}}^6 T_{1i} T_{4i} + (T_{24} T_{34}) \sum_{\substack{i=1 \\ i \neq 4}}^6 T_{1i} T_{4i} \\ + (T_{25} T_{35}) \sum_{\substack{i=1 \\ i \neq 5}}^6 T_{1i} T_{4i} + (T_{26} T_{36}) \sum_{i=1}^5 T_{1i} T_{4i} \left. \right] \\ - \frac{C_1}{s^2} \left(\sum_{i=1}^6 T_{4i} T_{2i} \right)^* \quad (V-3)$$

*Where H_{0Wss} , ω_i , T_i , U_i , T_{1i} , T_{2i} , T_{3i} and T_{4i} are functions of s .

It is evident that (V-1) is an extremely complicated function of products of transfer functions. The problem of manipulating (V-1) into the form of a ratio of polynomials in s is a monumental one with a high likelihood of error. Consequently, it was decided to compute the frequency response from the unreduced form given by (V-1), (V-2) and (V-3). It is important to note that before any conclusions can be reached concerning the stability of the closed loop system, the pole locations of $\Delta(s)$ must be known. Consequently, it is necessary that $\Delta(s)$ be reduced to a polynomial in s . This has not yet been accomplished.

A computer program, with which to generate the required frequency response data, has been written and is now operable. This program is given in Appendix E. The data obtained from this program for forty seconds of flight is shown in polar form in Figure 12. These results should be regarded as preliminary since the program has not been completely verified.

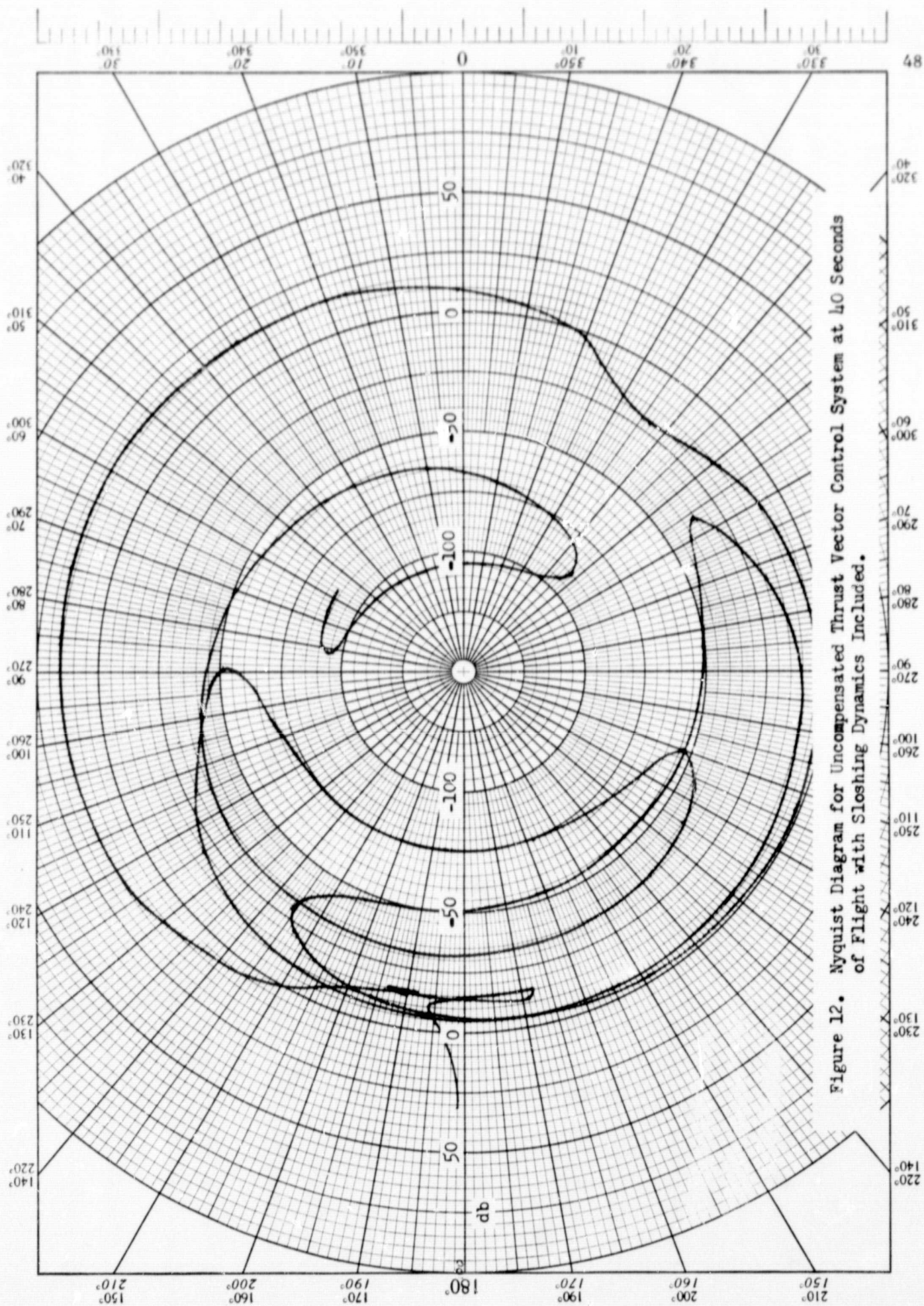


Figure 12. Nyquist Diagram for Uncompensated Thrust Vector Control System at 40 Seconds of Flight with Sloshing Dynamics Included.

VI. CONCLUSIONS

A problem of considerable importance in the analysis of hybrid control systems is that of determining continuous channel stability margins. A new approximate technique was developed in Chapter II and the limits on its applicability were discussed. The new method was used to analyze a model of the Thrust Vector Control System and the results of the analysis were compared with results which were developed by application of two conventional techniques. The new method provided substantially more accurate results than was obtained from the other two techniques.

The problem of digitally compensating the pitch attitude control system of the Saturn V AS-504 launch vehicle during S-IC flight was considered in Chapter III. The system was simplified to the extent that sloshing dynamics were neglected, computer transport delay was assumed to be zero and lateral drift of the vehicle center of gravity was assumed negligible. A discussion of possible approaches to the compensation problem was given and design goals were established for the system. A z-domain transfer function of order six over six, which enables the system to meet these design goals, was presented. Nyquist diagrams were included to provide graphical evidence of the suitability of the proposed compensation function at different times of flight. A summary of the status of the digitally compensated Thrust Vector Control System analog simulation was also included.

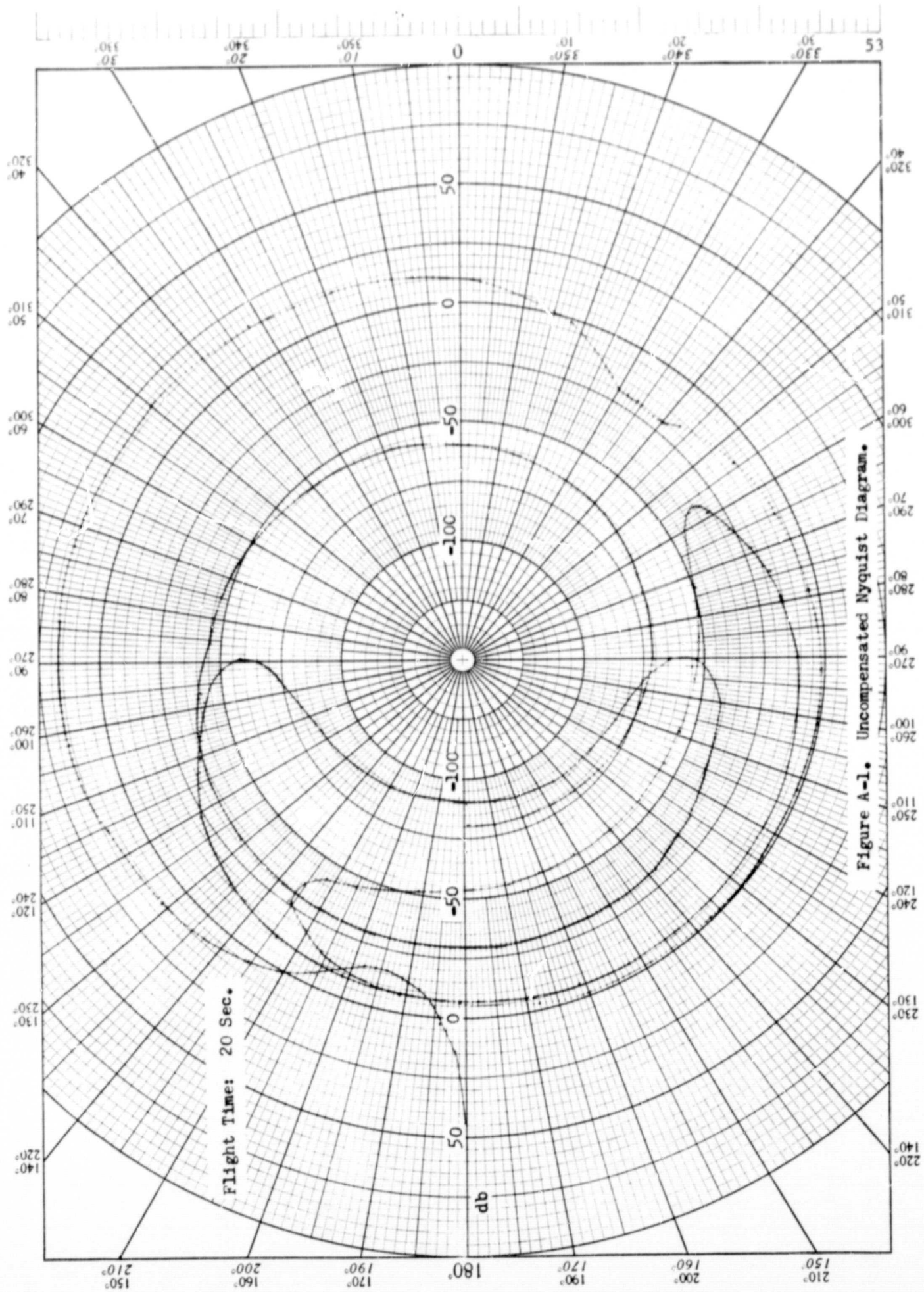
The problem of determining the effect of digital computer quantization on the performance of the Thrust Vector Control System was considered in Chapter IV. A method, which is based on the Second Method of Liapunov, was applied in an effort to establish a bound on system error due to quantization. The results which were obtained by application of the method appear to be excessively conservative and therefore they were of little practical value to the designer. Further studies are required in order to more conclusively evaluate the technique.

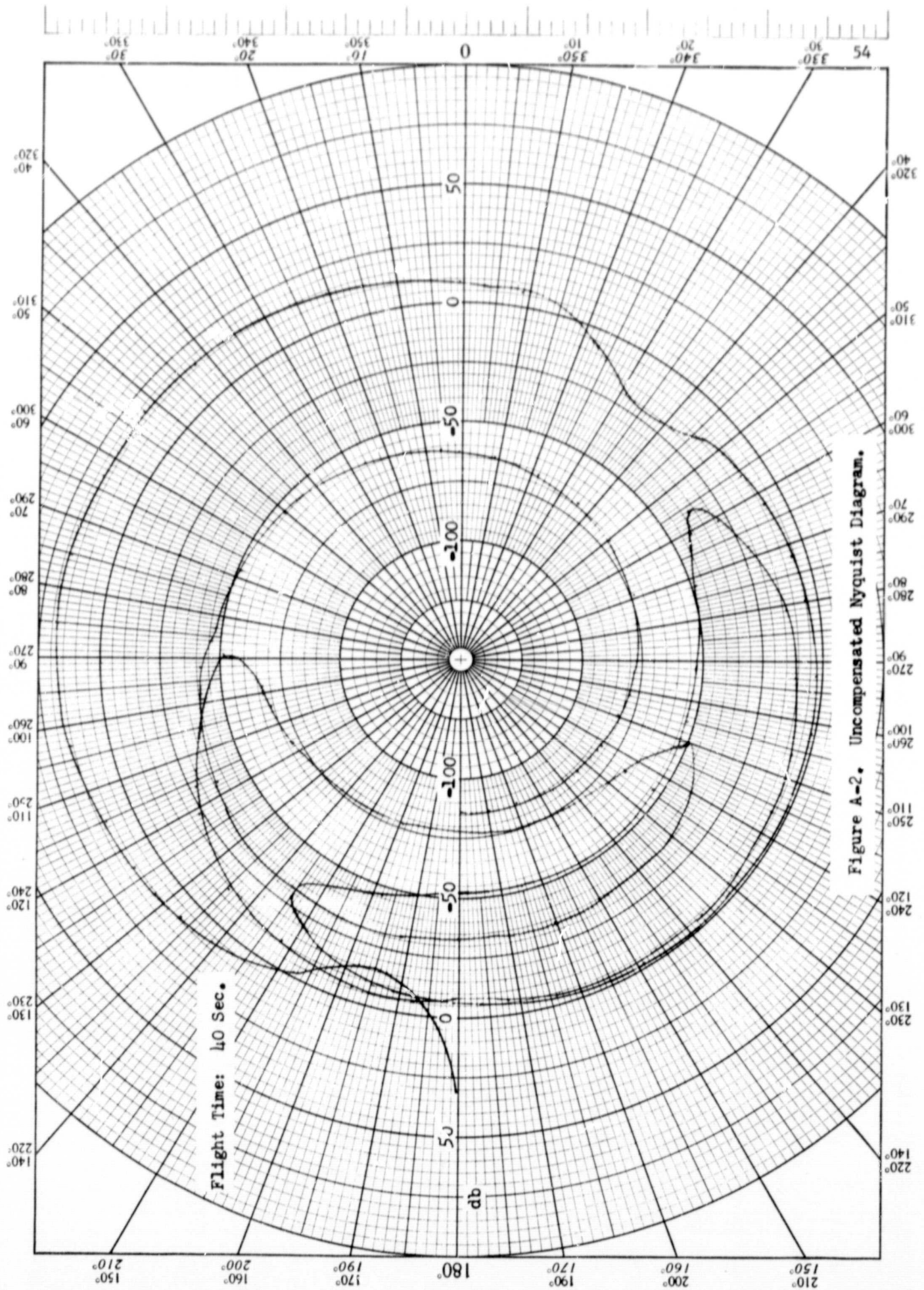
A description of a digital computer program which was developed to determine system frequency response with sloshing dynamics included was given in Chapter V. In order to expedite the development of this program, it was necessary to neglect intercoupling between the sloshing and bending differential equations. An example of the program output was provided in the form of a Nyquist diagram for the uncompensated system at 40 seconds of flight.

REFERENCES

1. B. C. Kuo, "Analysis and Synthesis of Sampled-Data Control Systems," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, pp. 220-223.
2. C. L. Phillips, et. al., "Stability Analysis Methods for Hybrid Control Systems," Paper No. 75, Southeastern Symposium on Missiles and Aerospace Vehicles Sciences, Huntsville, Alabama; December, 1966.
3. C. L. Phillips, "Digital Compensation of the Thrust Vector Control System," Sixth Technical Report, Auburn Research Foundation, Auburn, Alabama, September, 1966.
4. J. E. Bertram, "The Effect of Quantization in Sampled-Feedback Systems," Trans. AIEE on Applications and Industry, pt-2, vol. 77, pp. 177-181, September, 1958.

APPENDIX A





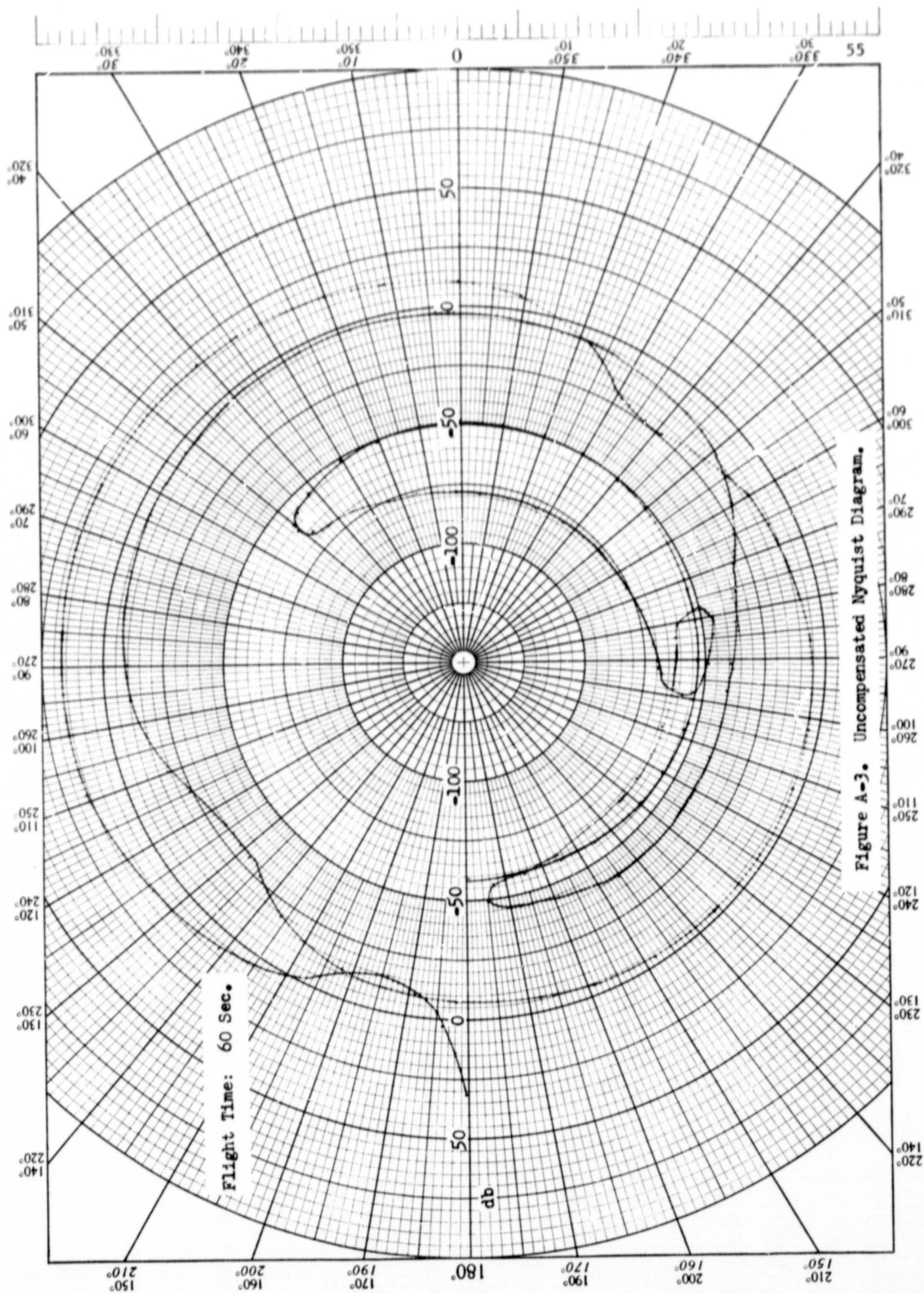


Figure A-3. Uncompensated Nyquist Diagram.

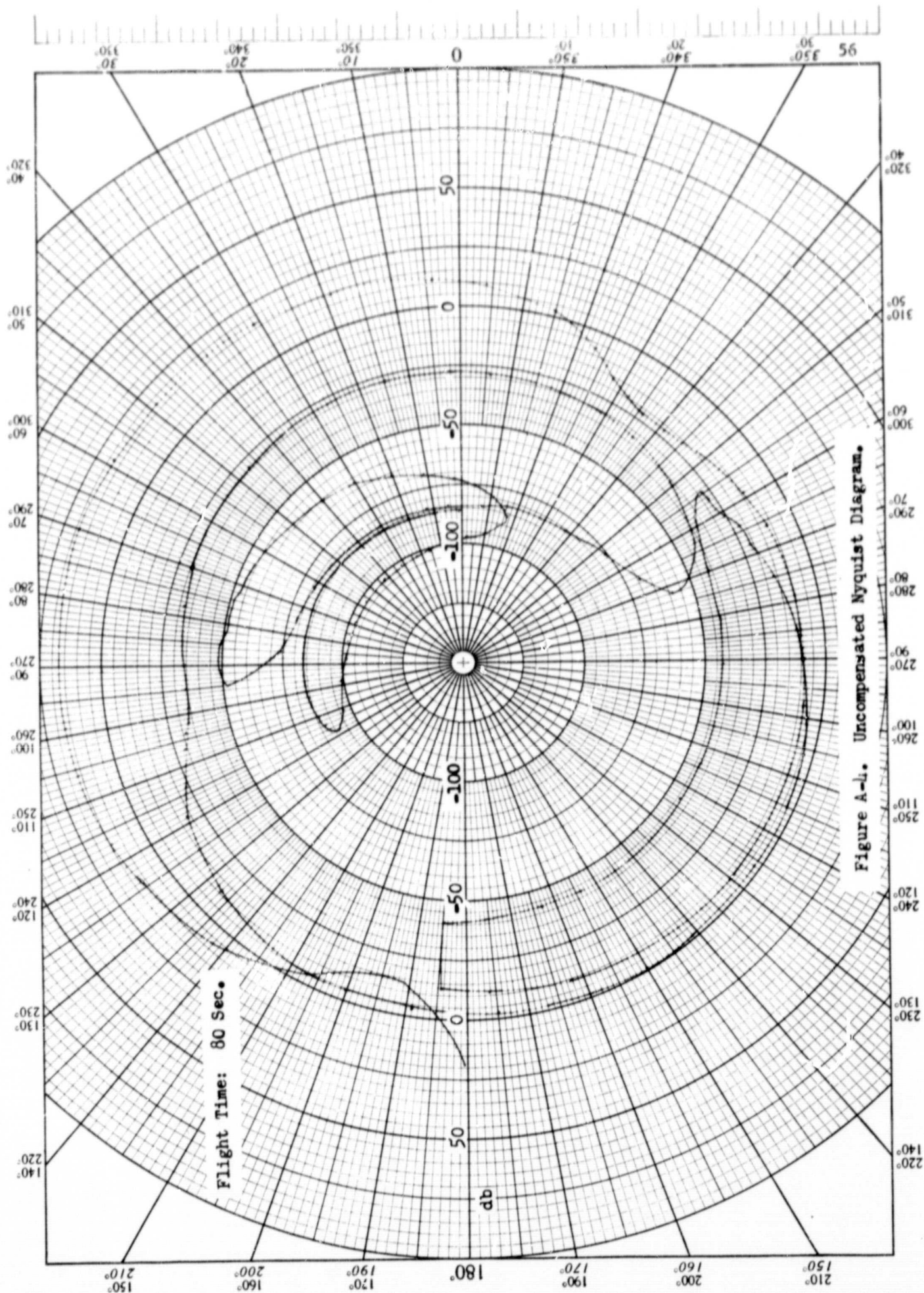
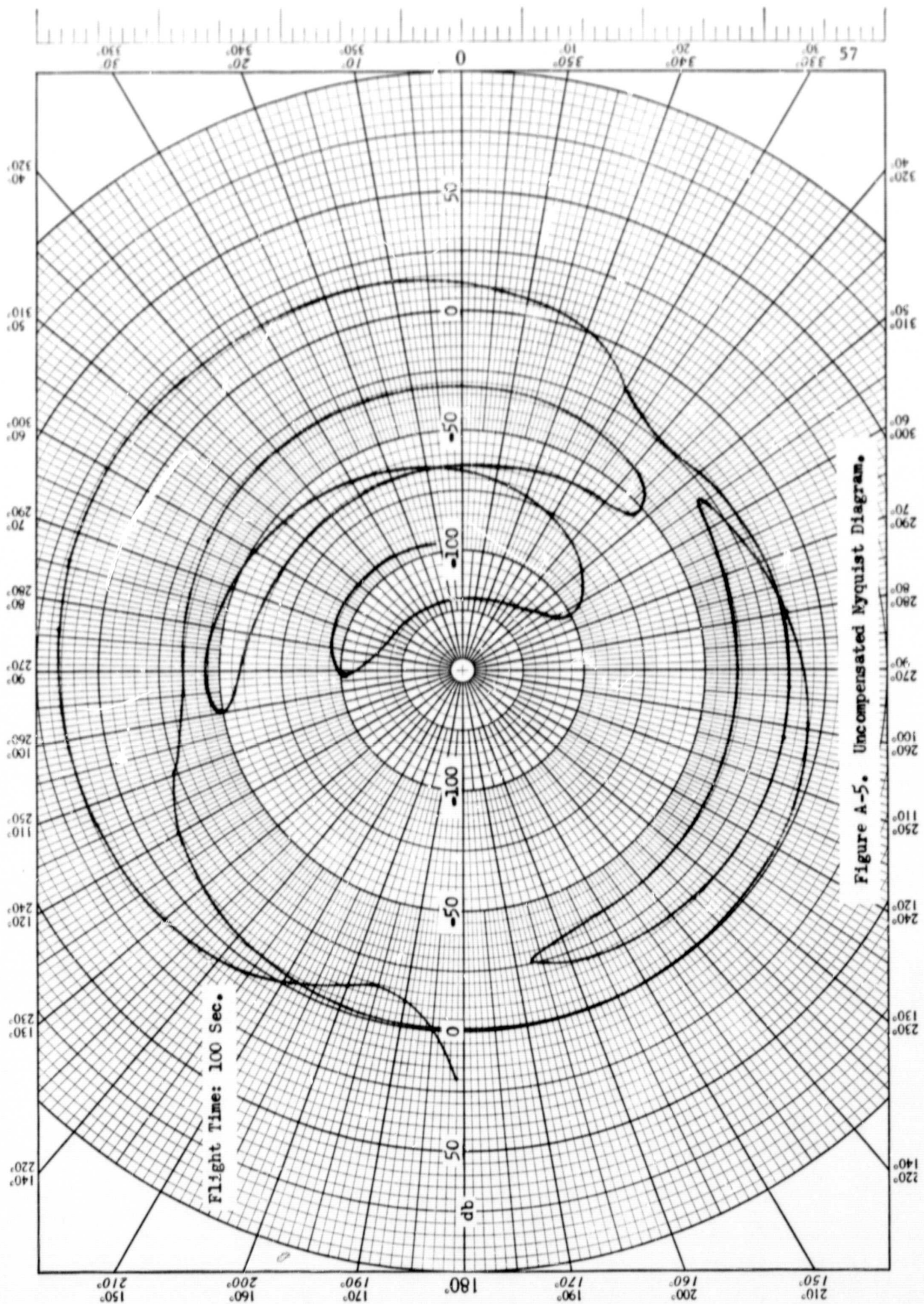
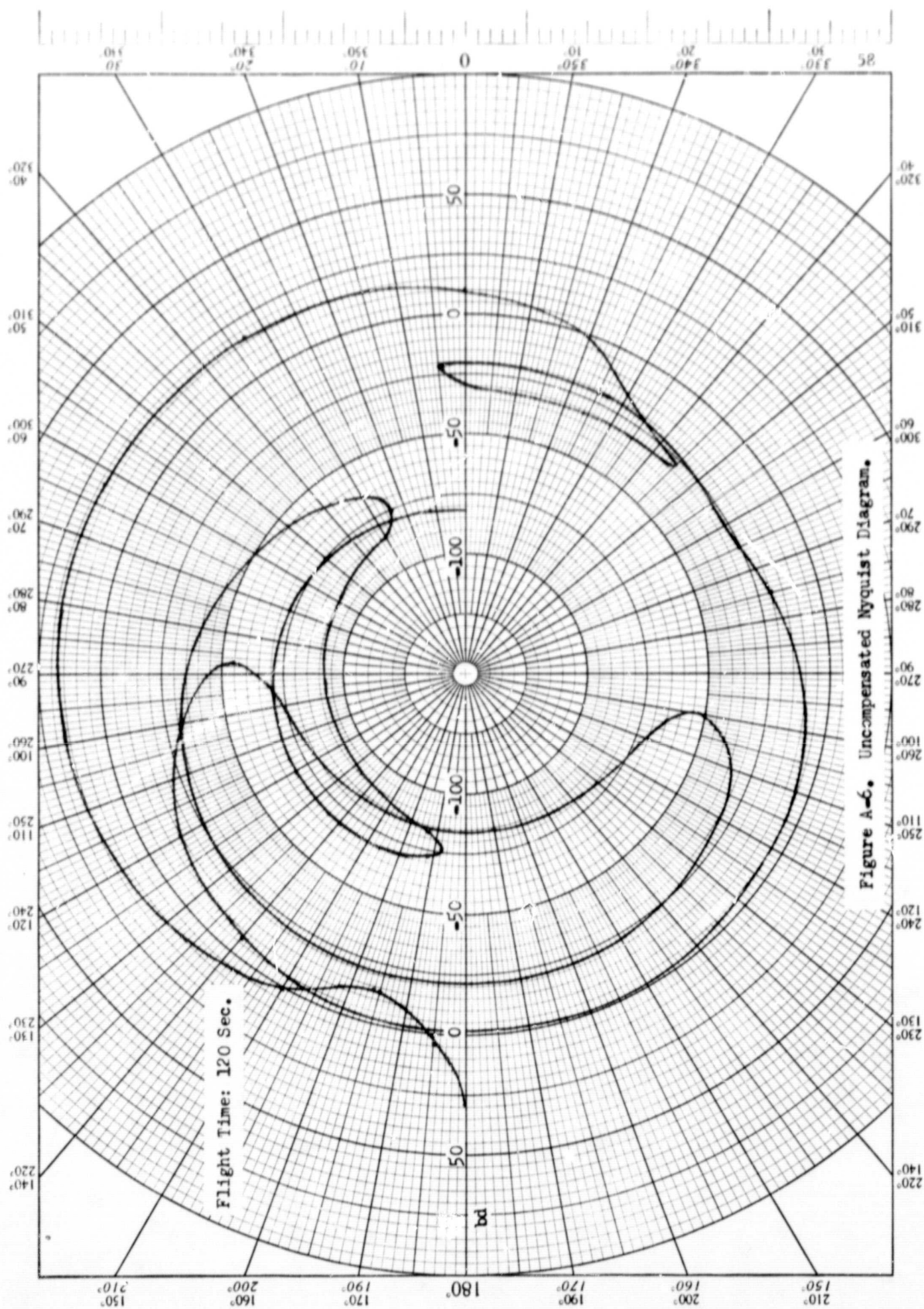
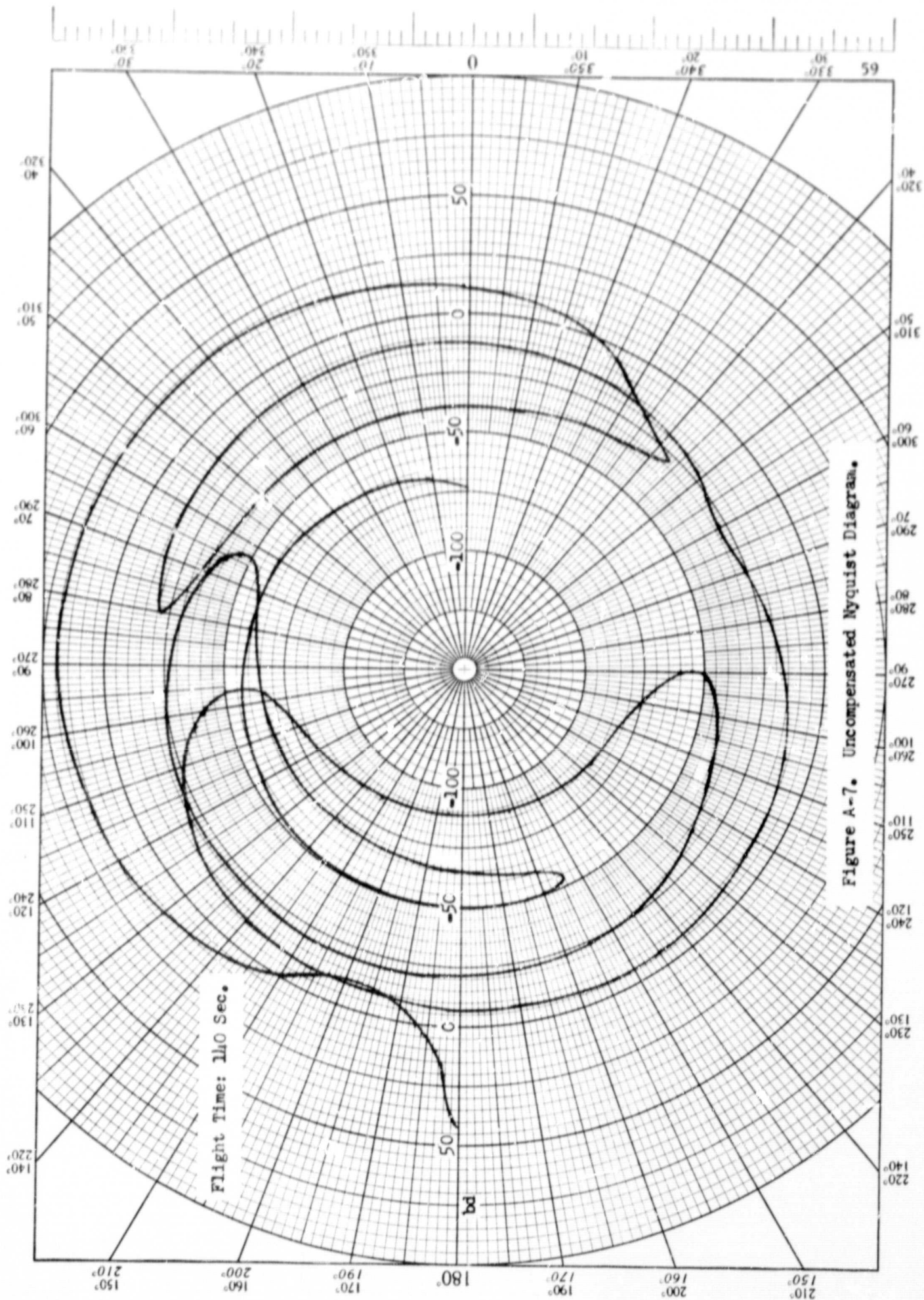


Figure A-4. Uncompensated Nyquist Diagram.







APPENDIX B

LIST OF SIMPLIFIED SATURN V EQUATIONS OF MOTION

1. Moment Equation

$$\begin{aligned} \phi = & -C_1 \phi - C_2 \beta - \frac{F l_{cg}}{I_{xx}} \sum_{i=1}^4 Y'_i(X_\beta) \eta_i \\ & - \frac{F}{I_{xx}} \sum_{i=1}^4 Y_i(X_\beta) \eta_i - \left(\frac{l_{cg} S_E}{I_{xx}} + \frac{I_E}{I_{xx}} \right) \ddot{\beta} - \frac{k_3 S_E}{I_{xx}} \beta \end{aligned} \quad (B-1)$$

2. Normal Acceleration of Vehicle c.g.

$$\ddot{z} = k_3 \phi + k_4 \beta + k_7 \alpha + \sum_{i=1}^4 \frac{F}{m} Y'_i(X_\beta) \eta_i + \frac{S_E}{m} \ddot{\beta} \quad (B-2)$$

3. Bending Mode Equations

$$\begin{aligned} \ddot{\eta}_i + 2\zeta_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = & \frac{1}{m_i} \left\{ R' Y(X_\beta) \beta + \left[S_E Y_i(X_\beta) \right. \right. \\ & \left. \left. - I_E Y'_i(X_\beta) \right] \beta \right\} \\ i = & 1, 2, 3, 4 \end{aligned} \quad (B-3)$$

4. Fuel Slosh Equations

$$\begin{aligned} \ddot{z}_{sj} + 2\zeta_{sj} \omega_{sj} \dot{z}_{sj} + \omega_{sj}^2 z_{sj} = & \ell_{sj} \ddot{\phi} + \left[\frac{F - X}{m} \right] \phi - \ddot{z} \\ j = & 1, 2, 3, 4, 5, 6 \end{aligned} \quad (B-4)$$

5. Miscellaneous Equations

$$\phi_D = \phi + \sum_{i=1}^4 Y'_i (X_D) \eta_i \quad (B5)$$

APPENDIX C

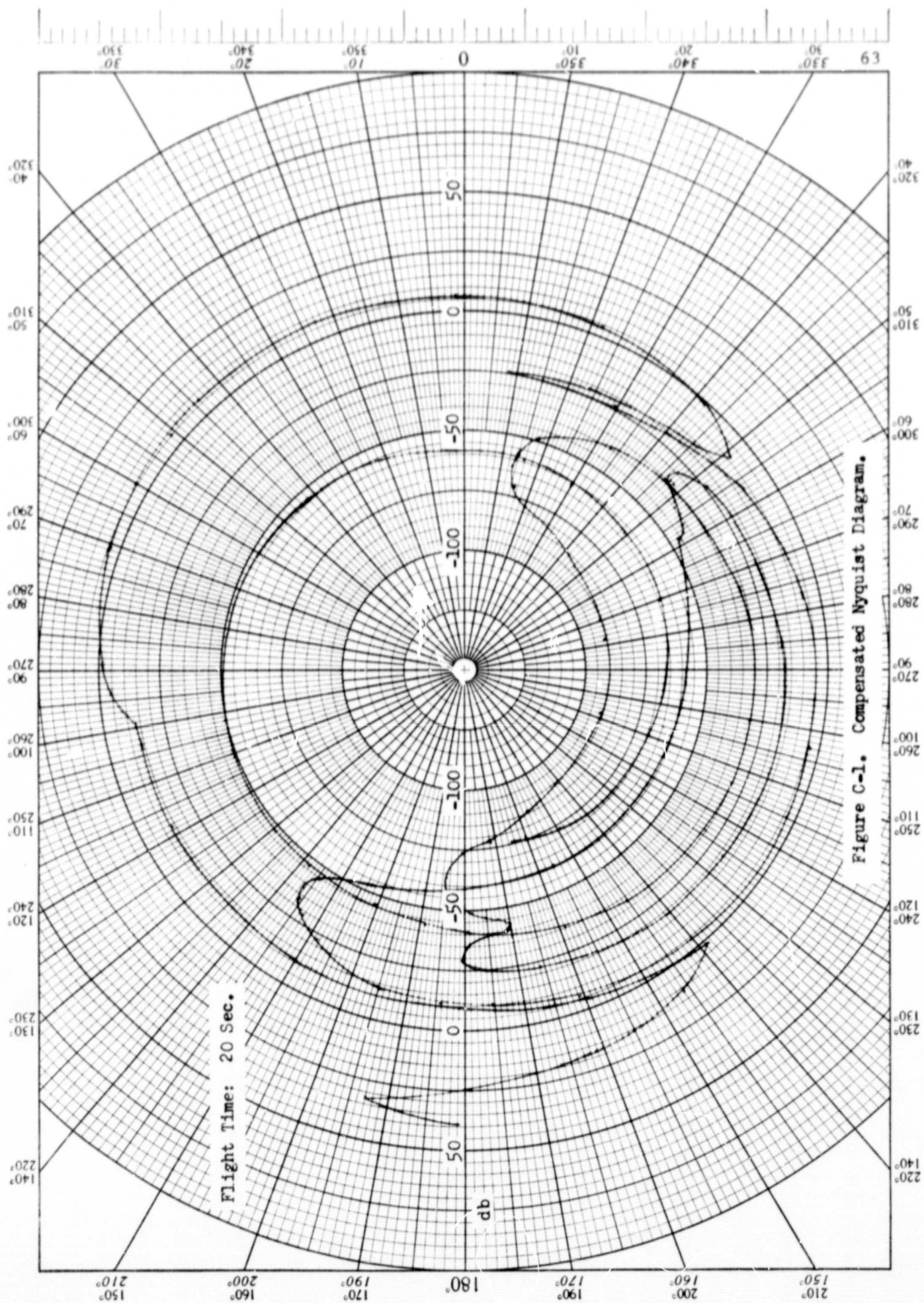


Figure C-1. Compensated Nyquist Diagram.

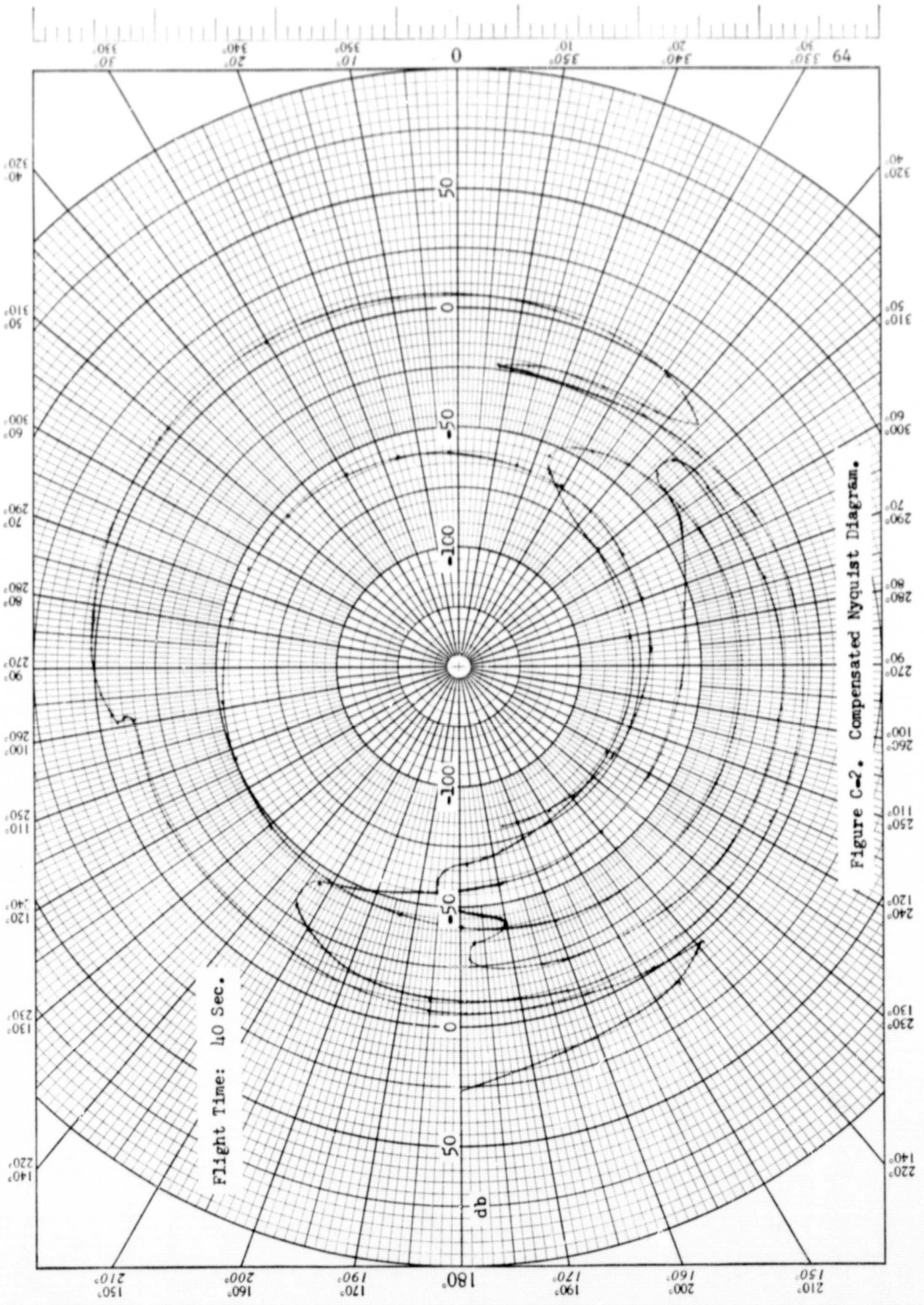


Figure C-2. Compensated Nyquist Diagram.

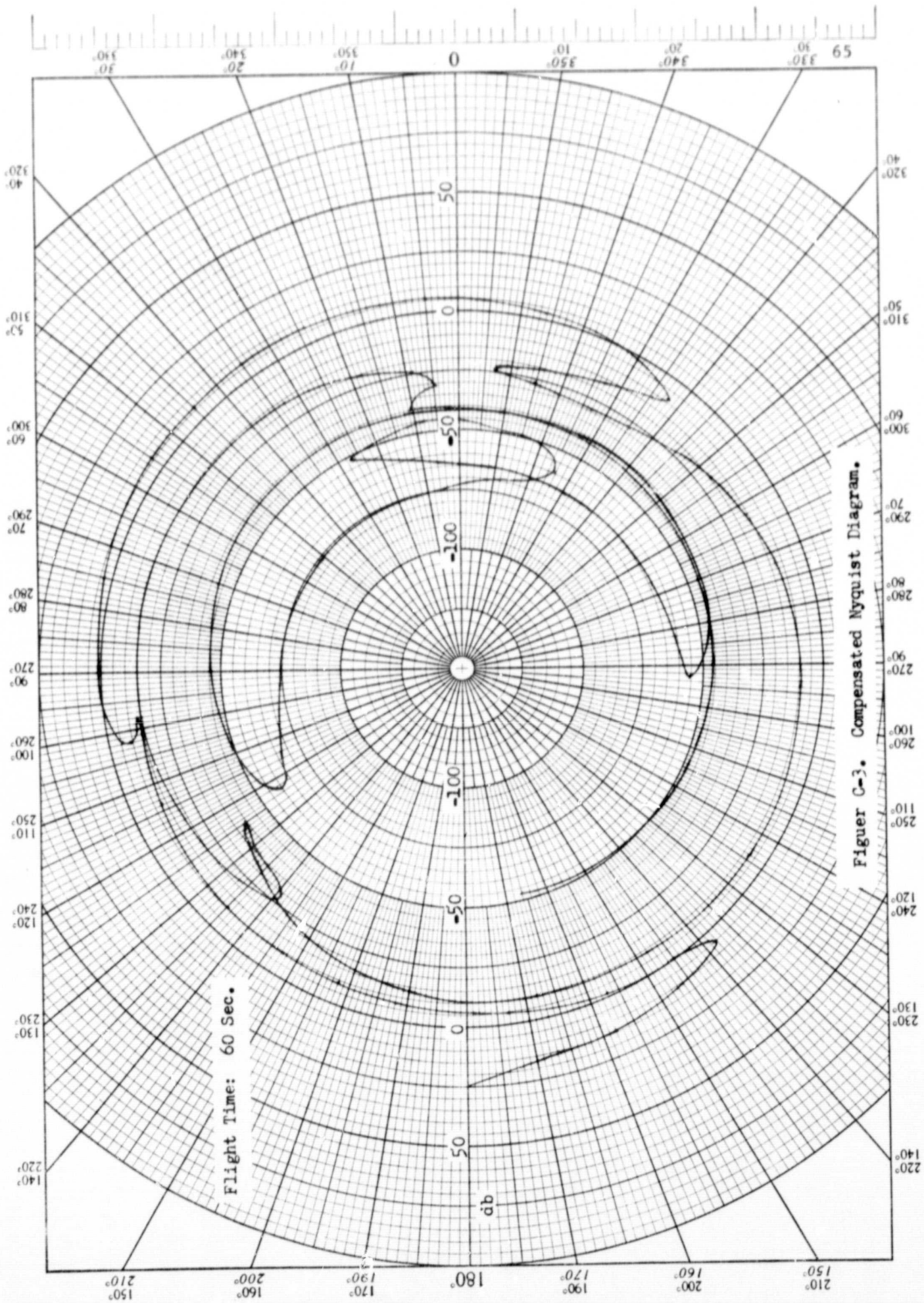


Figure C-3. Compensated Nyquist Diagram.

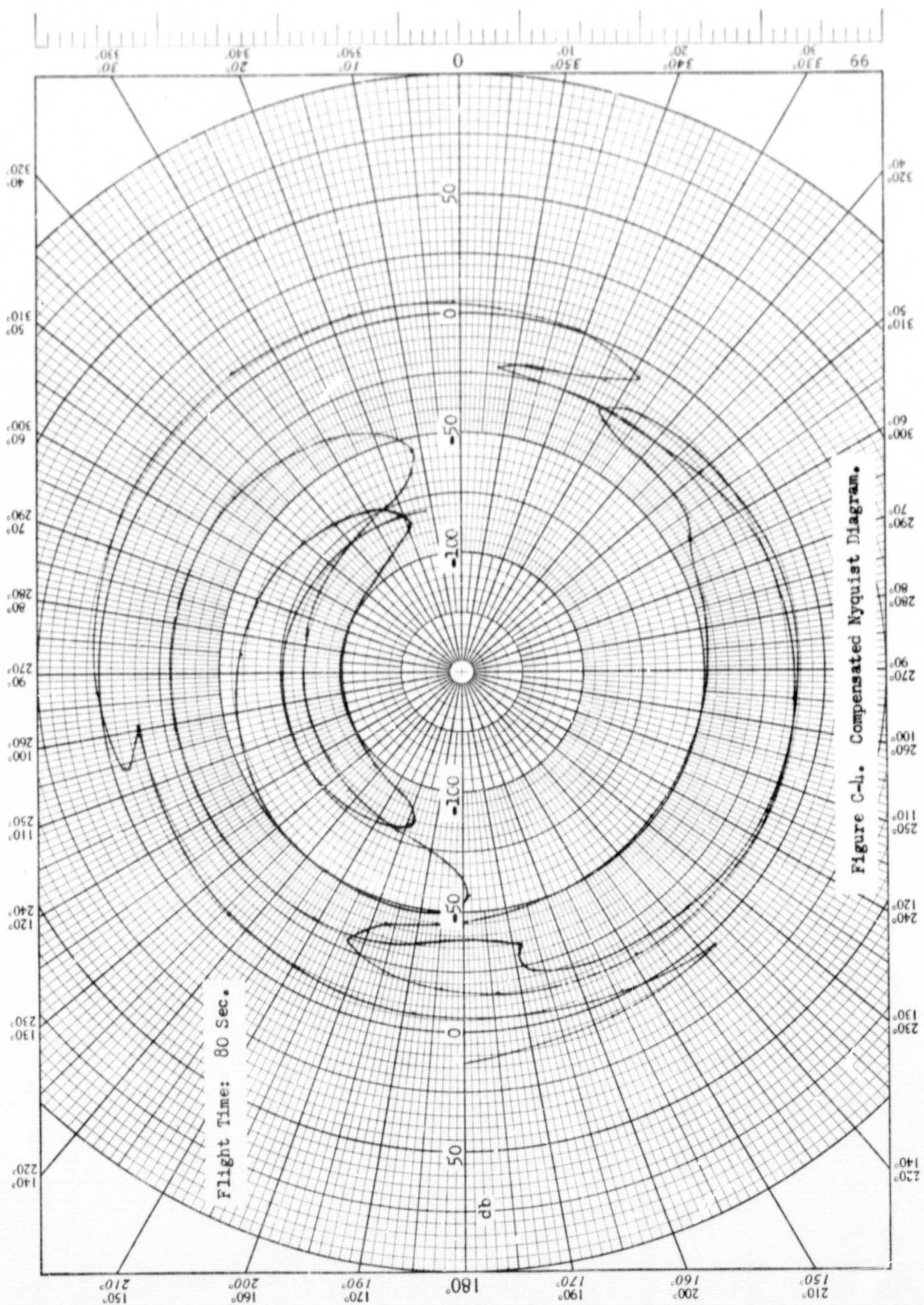
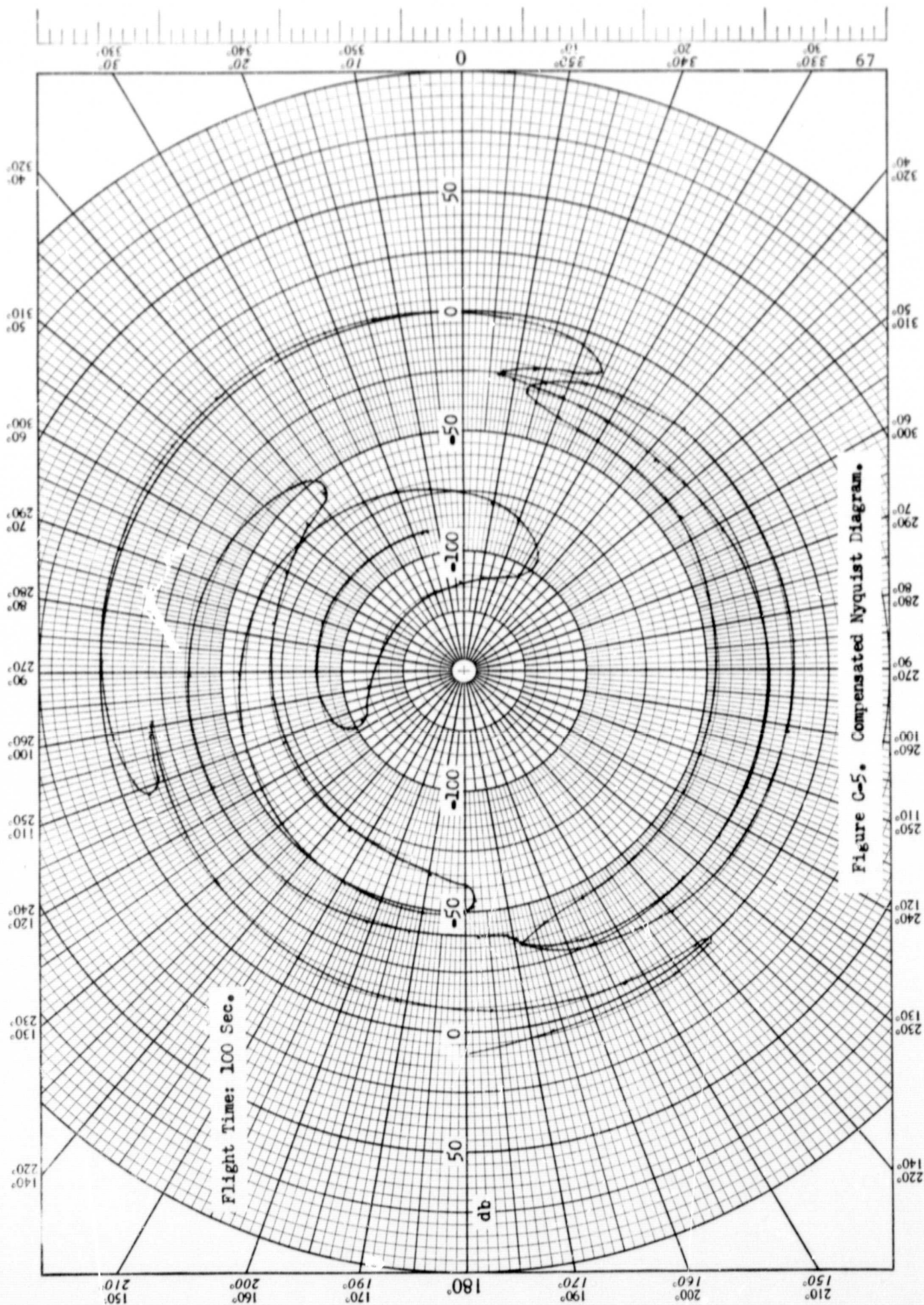


Figure C-4. Compensated Nyquist Diagram.



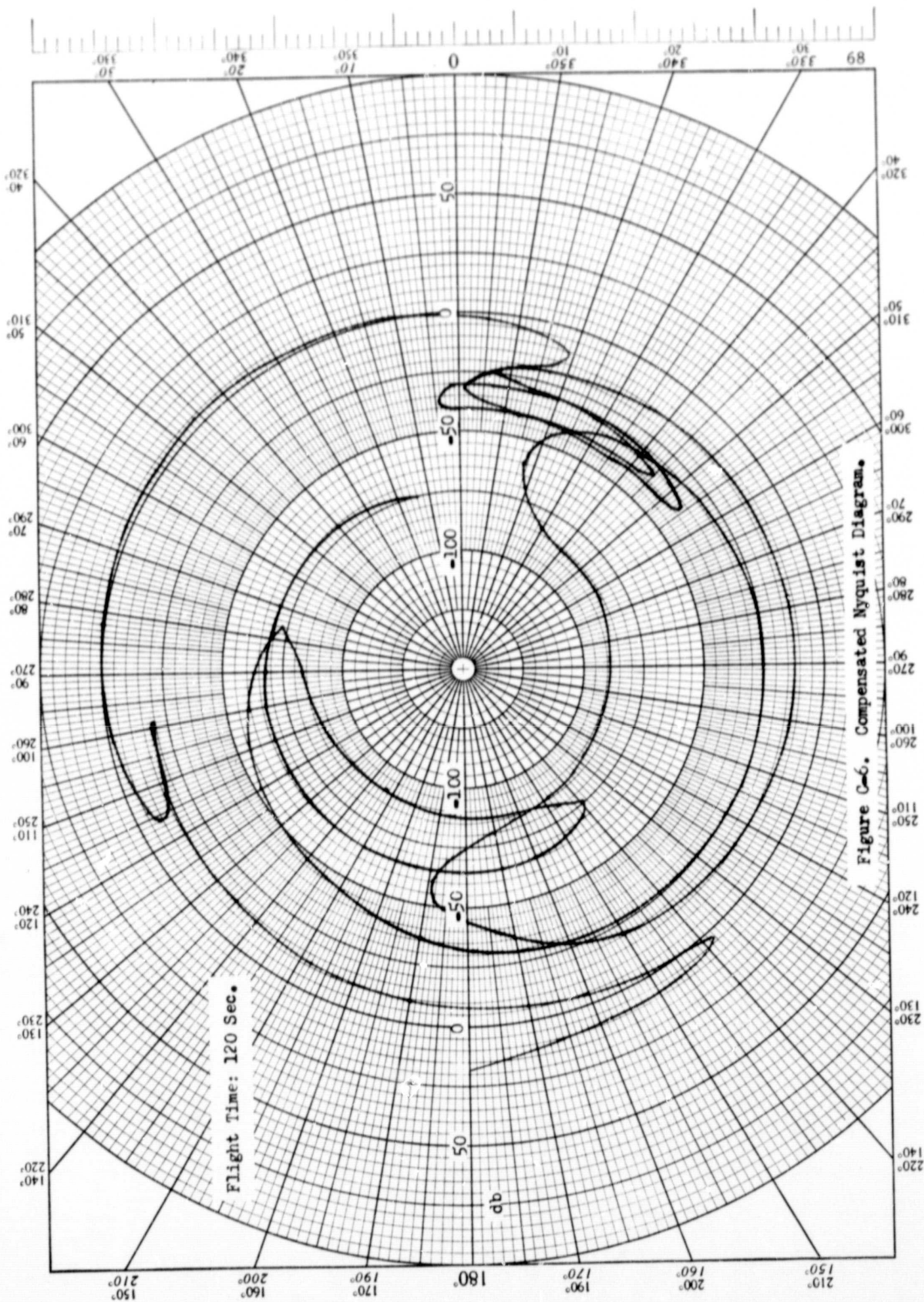


Figure C-6. Compensated Nyquist Diagram.

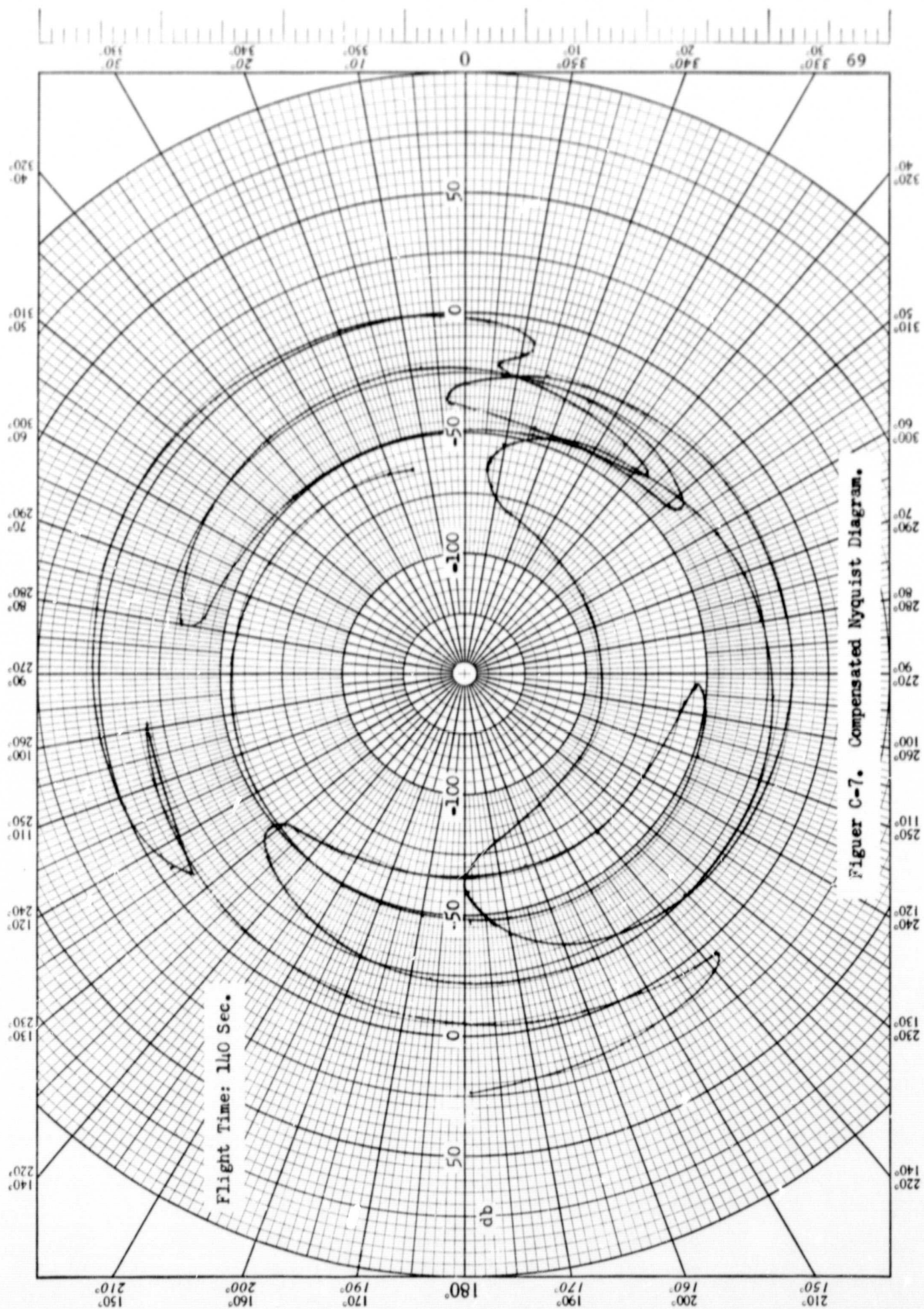


Figure C-7. Compensated Nyquist Diagram.

APPENDIX D

LIST OF SYMBOLS AND TRANSFER FUNCTIONS USED IN THE DEVELOPMENT OF $[\phi_D/B](s)$ WITH SLOSHING DYNAMICS INCLUDED

Bending Equations

$$\omega_i(s) = \left[\frac{R'Y_i(X_\beta)}{M_i} + \frac{1}{M_i} \left(S_E Y_i(X_\beta) - I_E Y_i'(X_\beta) s^2 \right) \right] \frac{Y_i'(X_D)}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \quad (D-1)$$

$$U_i(s) = \left[\frac{R'Y_i(X_\beta)}{M_i} + \frac{1}{M_i} \left(S_E Y_i(X_\beta) - I_E Y_i'(X_\beta) s^2 \right) \right] \frac{\left[\frac{F}{M} Y_i'(X_\beta) \right]}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \quad (D-2)$$

$$T_i(s) = \left[\frac{R'Y_i(X_\beta)}{M_i} + \frac{1}{M_i} \left(S_E Y_i(X_\beta) \right) s^2 \right] \cdot \left[\frac{-F l_{cg} Y_i'}{I_{xx}} \quad \frac{F Y_i(X_\beta)}{I_{xx}} \right] \frac{1}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \quad (D-3)$$

$$i = 1, \dots, 4$$

Constants

$$A_1 = - \left[C_2 + \frac{K_3 S_E}{I_{xx}} \right] \quad (D-5)$$

$$A_2 = - \left[\frac{l_{cg}s_E}{I_{xx}} + \frac{I_E}{I_{xx}} \right] \quad (D-6)$$

$$A_3 = k_3 + k_7 \quad (D-7)$$

Sloshing Equations

$$T_{1j}(s) = \frac{l_{sj}s^2 + k_3}{s^2 + 2\zeta_{sj}\omega_{sj}s + \omega_{sj}^2} \quad (D-8)$$

$$T_{2j}(s) = \frac{-1}{s^2 + 2\zeta_{sj}\omega_{sj}s + \omega_{sj}^2} \quad (D-9)$$

$$T_{3j}(s) = \frac{M_{sj}}{I_{xx}} \frac{(l_{sj}s^2 + k_3)}{s^2} \quad (D-10)$$

$$T_{4j}(s) = \frac{-M_{sj}s^2}{M} \quad (D-11)$$

$$j = 1, \dots, 6$$

Actuator Equation

$$W_{ss}(s) = \frac{1.0 + .25s}{(1.0 + 0.292s + 0.0124s^2 + 0.402 \times 10^{-3}s^3 + 0.564 \times 10^{-5}s^4 + 0.498 \times 10^{-7}s^5 + 0.16 \times 10^{-9}s^6 + 0.176 \times 10^{-12}s^7)} \quad (D-12)$$

Data Hold

$$H_O(s) = \frac{1 - e^{-Ts}}{s}$$

APPENDIX E

COMPUTATION OF PHILADELPHIA SAMPLED FREQUENCY RESPONSE WITH
SIX SLOSHING MODES INCLUDED AND WITH SLOSH-BENDING UNCOUPLED

```

C
C
C
C
    DIMENSION W(4),ZFIA(4),GM(4),YR(4),YPR(4),YPD(4),SM(6),SL(6),
1  SZFIA(6),SW(6),Q1(4),Q2(4),Q3(4),Q4(4),P1(4)
    COMPLEX CMPLX,CFXP,S,SUMAW,SUMAT,SUMU,T1(6),T2(6),T3(6),T4(6),
1  AWP(4),AW(4),AT(4),U(4),FR,CHAR,TOTAL,WSS,HOL,TF,FRP,FRQ,H(6),
2  ADDT, NC, DC, GCOMP, WP
10  FORMAT(3(5X,F15.8))
11  FORMAT(4(5X,F15.8))
    READ(5,10) C1,C2,F
    READ(5,11)ALCG, AIXX, AIF, T
    READ(5,10) ASF,AK3,AK4
    READ(5,11)AK7, AM, RP, V
    READ(5,11)W(1), W(2), W(3), W(4)
    READ(5,11)ZETA(1), ZETA(2), ZETA(3), ZETA(4)
    READ(5,11)GM(1), GM(2), GM(3), GM(4)
    READ(5,11)YR(1), YR(2), YR(3), YR(4)
    READ(5,11)YPR(1), YPR(2), YPR(3), YPR(4)
    READ(5,11)YPD(1), YPD(2), YPD(3), YPD(4)
    READ(5,10) SM(1),SM(2),SM(3)
    READ(5,10) SM(4), SM(5), SM(6)
    READ(5,10) SL(1),SL(2),SL(3)
    READ(5,10)SL(4), SL(5), SL(6)
    READ(5,10) SW(1),SW(2),SW(3)
    READ(5,10)SW(4), SW(5), SW(6)
33  FORMAT(1H1,4X,10HINPUT DATA,/)
    PRINT 33
    PRINT 10, C1,C2,F
    PRINT 11, ALCG, AIXX, AIF, T
    PRINT 10,ASF,AK3,AK4
    PRINT 11, AK7, AM, RP, V
    PRINT 11, W(1), W(2), W(3), W(4)
    PRINT 11, ZETA(1), ZETA(2), ZETA(3), ZETA(4)
    PRINT 11, GM(1), GM(2), GM(3), GM(4)
    PRINT 11, YR(1), YR(2), YR(3), YR(4)
    PRINT 11, YPR(1), YPR(2), YPR(3), YPR(4)
    PRINT 11, YPD(1), YPD(2), YPD(3), YPD(4)
    PRINT 10, SM(1), SM(2), SM(3)
    PRINT 10, SM(4), SM(5), SM(6)
    PRINT 10, SL(1), SL(2), SL(3)
    PRINT 10,SL(4), SL(5), SL(6)
    PRINT 10, SZFIA(1), SZFIA(2), SZFIA(3)
    READ(5,10) SZFIA(1),SZFIA(2),SZFIA(3)
    READ(5,10) SZFIA(4), SZFIA(5), SZFIA(6)

```

```

PRINT 10, SZETA(4), SZETA(5), SZETA(6)
PRINT 10, SW(1), SW(2), SW(3)
PRINT 10, SW(4), SW(5), SW(6)
C CALCULATE COEFFICIENTS
AK3 = AK3*57.29578
AK4 = AK4*57.29578
AK7 = AK7*57.29578
A1 = -(C2+AK3*ASF/AIXX)
A2 = -(ALCG*ASE/AIXX+AIE/AIXX)
A3 = AK3 + AK7
DO 1 I = 1,4
W(I) = W(I)*2.0*3.1415927
Q1(I) = RP*YB(I)/GM(I)
Q2(I)=F*YPB(I)/AM
Q3(I)=-F*ALCG/AIXX*YPB(I)-F*YB(I)/AIXX
Q4(I) = YPD(I)
1 P1(I) = (ASE*YB(I)-AIE*YPB(I))/GM(I)
C CALCULATE FREQUENCY RESPONSE
DFL = 0.2
XK = 1.0
66 FORMAT(1H1,4X,30HFREQUENCY RESPONSE OUTPUT DATA ,//)
PRINT 66
OMEG = .005
GO TO 4
77 DEL = DEL/2.0
OMEG = OMEG-DEL
4 TF = CMPLX(0.0, 0.0)
TS = 0.04
OMEGS = 2.0*3.1415927/TS
NTILT = 1
NX = 2*NTILT+1
DO 24 J = 1, NX
XJ = -NTILT+J-1
OMEG1 = OMEG+XJ*OMEGS
S = CMPLX(0.0, OMEG1)
DO 3 I = 1,6
T1(I) = (SL(I)*S**2 +AK3)/(S**2 +2.*SZETA(I)*SW(I)*S +SW(I)**2)
T2(I) = -1./((S**2+2.*SZETA(I)*SW(I)*S + SW(I)**2)
T3(I) = SM(I)/AIXX*(SL(I)*S**2 +AK3)/S**2
3 T4(I) = -SM(I)/AM*S**2
DO 5 K=1,4
AWP(K) = (P1(K)*S**2 + Q1(K))/(S**2 +2.*ZETA(K)*W(K)*S+W(K)**2)
AW(K) = Q4(K)*AWP(K)
AT(K) = Q3(K)*AWP(K)
5 U(K) = Q2(K)*AWP(K)
SUMAW = CMPLX(0.0,0.0)
SUMAT = CMPLX(0.0,0.0)
SUMU = CMPLX(0.0,0.0)
DO 6 L = 1,4
SUMAW = SUMAW + AW(L)

```

```

SUMAT = SUMAT + AT(L)
6 SUMU = SUMU + U(L)
  ADDT = CMPLX(0.0,0.0)
  DO 100 I = 1,6
100 ADDT = ADDT + T1(I)*T4(I)
  DO200 I=1,6
200 H(I) = T2(I)*T3(I)*(ADDT-T1(I)*T4(I))
  FR = CMPLX(0.0,0.0)
  DO300 I=1,6
300 FR = FR + T4(I)*T2(I)
  FRP = CMPLX(0.0,0.0)
  DO 400 I = 1,6
400 FRP = FRP + T2(I)*T3(I)
  FRQ = CMPLX(0.0,0.0)
  DO 500 I= 1,6
500 FRQ = FRQ + T1(I) *T3(I)
  CHAR = 1.0 -(-C1/S**2 +FR +FRQ + A3*FRP+ H(1)+H(2)+H(3)+H(4)+
1H(5)+H(6))-C1/S**2*FR
  WSS = (1.0 + 0.25*S)/(1. + 0.292*S .0124*S**2 + 0.402E-03*S**3 +
10.564E-05*S**4 + 0.498E-07*S**5 + .16E-09*S**6 +.176E-12*S**7)
  HOL = (1.0 - CEXP(-TS*S))/S
  TOTAL = -1./TS*HOL*WSS*(SUMAW+((A1 A2*S**2+SUMAT)*(1.-FR)/S**2
1+(SUMU+AK4+ASE/AM*S**2)*FRP)/CHAR)
24 TF =TF + TOTAL
  WR = SIN (TS*OMEG)/(1.0 + COS (TS*OMEG))
  WP = CMPLX (0.0, WR)
  NC=.5*(WP/.004+1.)*(WP**2/.135**2+2.*.10*WP/.135+1.)*(WP/.002+1.)
1*(WP**2/.330**2+2.*.10*WP/.330+1.)
  DC = (WP/.1+1.)*(WP/.1+1.)*(WP/.15 1.)*(WP/.001+1.)
1*(WP/.226+1.)*(WP/.330+1.)
  GCOMP = NC/DC
  TF = TF*GCOMP
  ABSVAL = CABS(TF)
  DB = 20.0*ALOG10 (ABSVAL)
  PHASE = 57.29578*ATAN2(AIMAG(TF), REAL(TF))
  IF(PHASE) 30,30,31
30 PHASE = PHASE+360.0
31 CONTINUE
  IF (XK-1.0) 32,32,39
39 CONTINUE
  DELT1 = ABS (DELT-PHASE)
  DALT1 = ABS (DALT-DB)
  IF (DALT1-10.0) 44,44,77
44 IF (DELT1-15.0) 32,32,777
777 IF (DELT1-350.0) 77,77,32
32 DELT = PHASE
  DALT = DB
  XK = XK + 1.0
25 FORMAT(5X,6HOMEGA=F7.3,5X,3HDB=F9.3,5X,6HPHASE=F9.3,5X,2HW=F7.3)
  PRINT 25,OMEG,DB,PHASE,WR

```

```
FXIT = 3.1415927/TS  
DEL = 0.2  
OMEG = OMFG + DEL  
55 IF(OMEG-EXIT) 4,4,7  
7 CONTINUE  
STOP  
END
```

```
$ENTRY  
$IBSYS
```