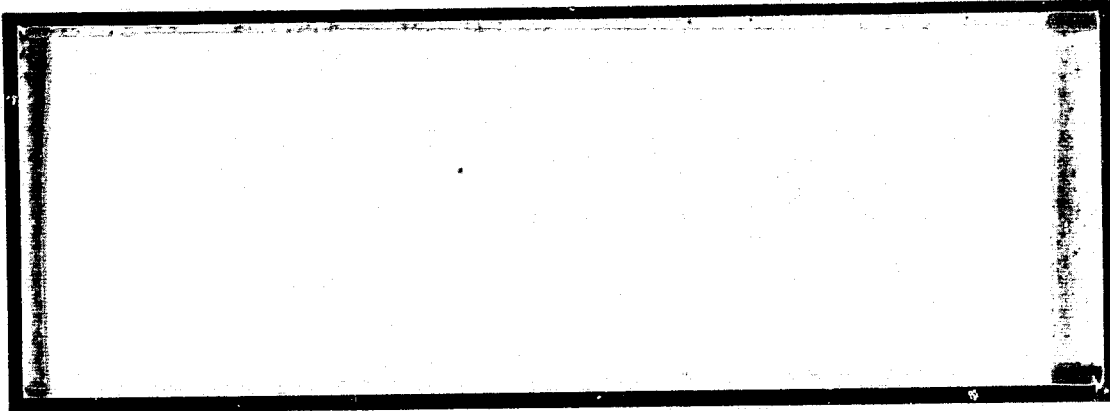


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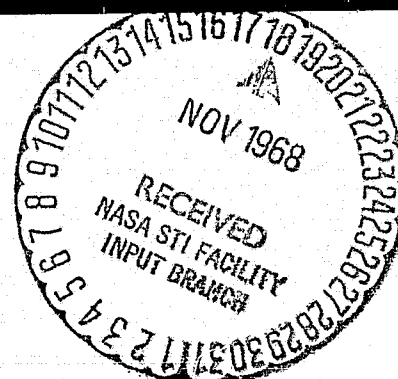
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HOUSTON, TEXAS



NASA CR 92368

LUNAR TERRAIN AND REFLECTIVITY STUDY*

Final Report

Contract NAS 9-6760

By

H. S. Hayre

TR-68-17

Wave Propagation Laboratories
Department of Electrical Engineering

UNIVERSITY OF HOUSTON
3801 Cullen Boulevard
Houston, Texas 77004

October 1968

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ABSTRACT

A lunar landing trajectory, landing module electronics and radar doppler and altitude beams, and the associated probable landing sites and along-the-trajectory lunar surface were modeled for ultrasonic simulation. The model time was selected to be the real time in this simulation and the model doppler velocity and altitude beam widths were identical to those of the landing module. This objective was achieved by appropriate masking of the 1.0 megacycle (corresponding to 10 GHz radar frequencies) and 0.9 megacycle lead zirconate piston transducers of 1.0 inch diameter.

The entire trajectory was divided into three sections: A (altitude 25,000 - 17,950 feet); B (17,950 - 3,400 feet); and C (3,400 to 200 feet). The doppler distance and velocity scale factors for these sections were 300, 300, and 78; 6850, 6850 and 1080; and 6×10^3 , 6×10^3 , 1.56×10^3 , respectively. At thirty points along the trajectory radar cross-section, data was taken. Dynamic data was also taken and tapes supplied to NASA Manned Spacecraft Center. Statistical analysis of this data was also carried out.

Theoretical work was also performed and verified by simulation results in the area of LM radar return bias by lunar surface and probable errors in the LM altitude as a function of the lunar surface perturbations.

CHAPTER I

INTRODUCTION

A Lunar Module (LM) trajectory was supplied by NASA Manned Spacecraft Center to the University of Houston, and a number of probable LM landing sites were pre-selected from Orbitor II data by NASA, MSC. The basic objective was to simulate the probable lunar surface bias on the LM radar signals so that the resulting data could be coordinated with work performed by other NASA MSC contractors on LM project.

The following is a list of major goals of this program:

1. Surface Models
2. LM Radar Simulation
3. LM Electronic System Simulation
4. Data Recording System
5. Trajectory Simulation
6. Lunar Surface Bias - RCS Cross-Section at Points
Along Trajectory
7. Dynamic Data
8. Theoretical Work
9. Analysis

Each of these objectives was met and is reported in detail in the following chapters.

NASA MSC supplied the parallel-to-serial converter as well as the magnetic tape recorder. The delay in the formal report was caused by various equipment failures not within the control of either the University of Houston or NASA MSC and its other contractors.

CHAPTER II

SIMULATED LUNAR MODULE SYSTEMS

2.1 Ultrasonic Simulation of Lunar Module Guidance and Control Signals

The lunar surface features affect lunar module altimeter and doppler velocity detector signals and it is desired to simulate both the lunar surface effects and the landing trajectory on a computer or in a laboratory. Various factors such as long distances and very fast velocities in portions of LM descent suggest ultrasonic simulation to be the only realistic controlled laboratory method to study these effects. In this work the said trajectory, the altitude and velocity sensors and the signal waveforms and beam shapes, etc., were simulated using 1.0 and 0.9 megacycle source frequencies. Lead zirconate cylindrical transducers were employed with special lenses to appropriately shape their beams. The lunar surface was modeled using the latest NASA Surveyor, Ranger, and Orbiter data obtained from high and low resolution cameras. The simulated trajectory position can be located within plus-minus one hundredth of an inch in x and y planes, the pitch angle of the transducer package is adjustable to within ± 0.75 mm/sec over an integration period of 0.1 second, and the altitude variations of the order of ± 1.524 mm can be detected. The surface features and their return have also been successfully simulated.

2.1.1 Electronic System

The NASA radar LM system was modeled by an electronics systems shown below, where the modeled radar beam widths were exactly the same as those of the LM doppler velocity and altitude radar systems.

2.1.2 Doppler Velocity Channel

Each doppler velocity beam information is fed into the counter every second at intervals of 1/3 second in a cyclic fashion, thus sampling all three doppler inputs.

A typical doppler velocity channel consists of 1 MHZ crystal oscillator which feeds a power amplifier, a line driver and subsequently the doppler transmit transducer. The doppler receiving transducer signal is RF amplified and fed into the line driver for input into the squaring amplifier and level converter for the counter. The RF amplifier output is also available for the mixer, whose reference input consists of signal from the 1 MHZ crystal oscillator for mixer output. Furthermore, the RF amplifier output is passed through the log-linear amplifier and detector and level converter for recording of the signal envelope.

2.1.3 Altimeter Channel

A sawtooth generator is used to generate the LM sawtooth in real time to drive the voltage controlled oscillator which in turn feeds a line driver and a power amplifier (x-ducer driver)

for altimeter transmitter-transducer as well as the reference signal to the mixer. The altimeter receiver transducer signal at 0.9 MHz is amplified by RF amplifier and fed into line driver and limited by a limiter and its output goes into the mixer for generating the difference and sum. The mixer output is filtered by low pass filter and converted to dc level corresponding to the input frequency and tape recorded.

2.1.4 Motor Controls

Both the x- and y-drive motor velocities are monitored at their control tachometer readouts in volts and reduced in level for dc signal recording.

2.1.5 Position Controls

Both x- and y- positions of the carriage are referenced to a given point. For instance, the target face is located at $y = 25 \frac{7}{16}$ " and the effective (for us) target length is located between $x = 25$ " and $x = 150$ ".

Each of the positions are monitored by shaft encoders whose output is digital form for tape recording and decimal form for display on nixie tubes is available in the control room.

The above is also true of the pitch angle of the transducer package.

2.1.6 Transducer Package (LM Radar)

The relative position of all the three doppler velocity beams and the altimeter beam is exactly the same as for the LM radar.

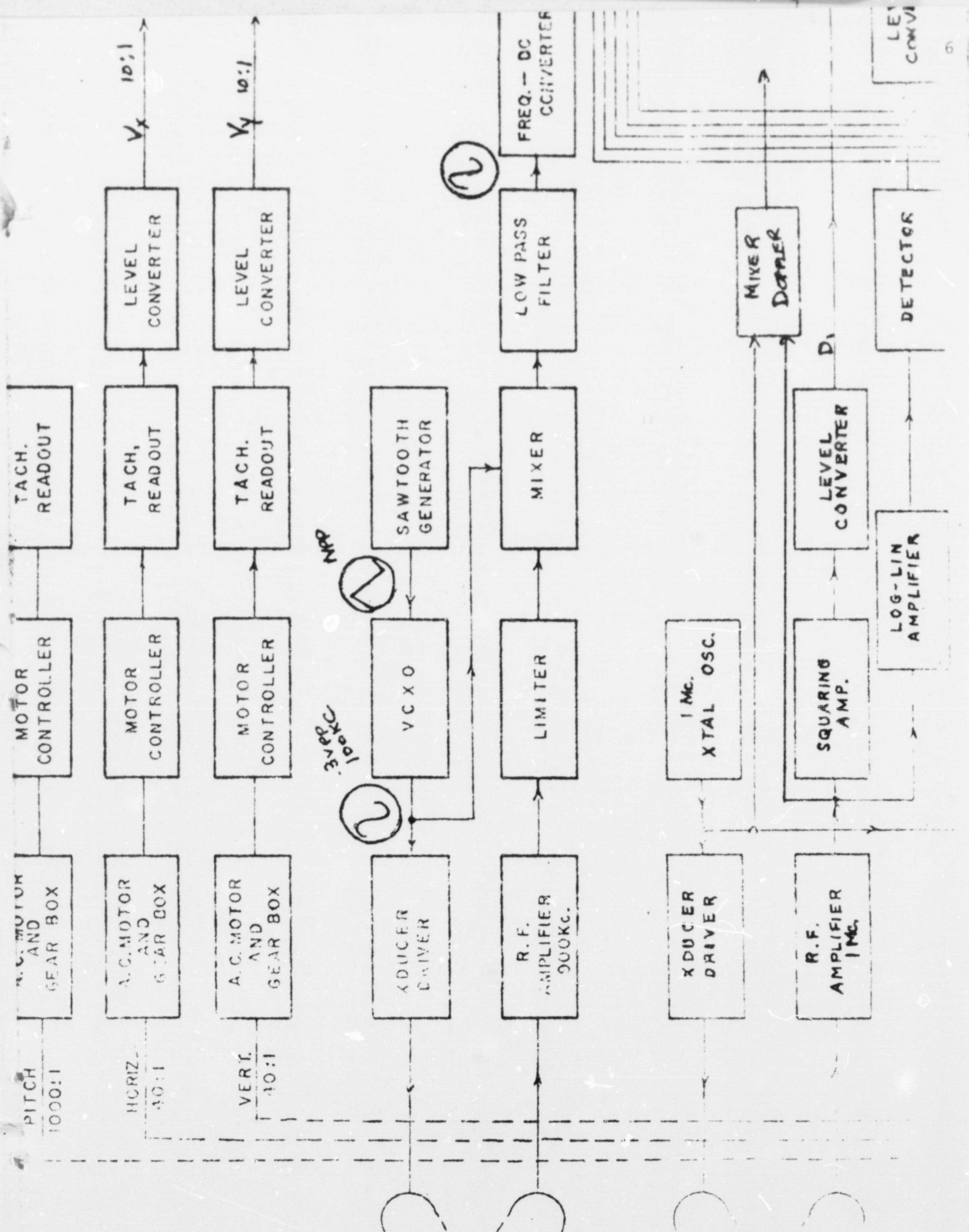


Figure 2-1.

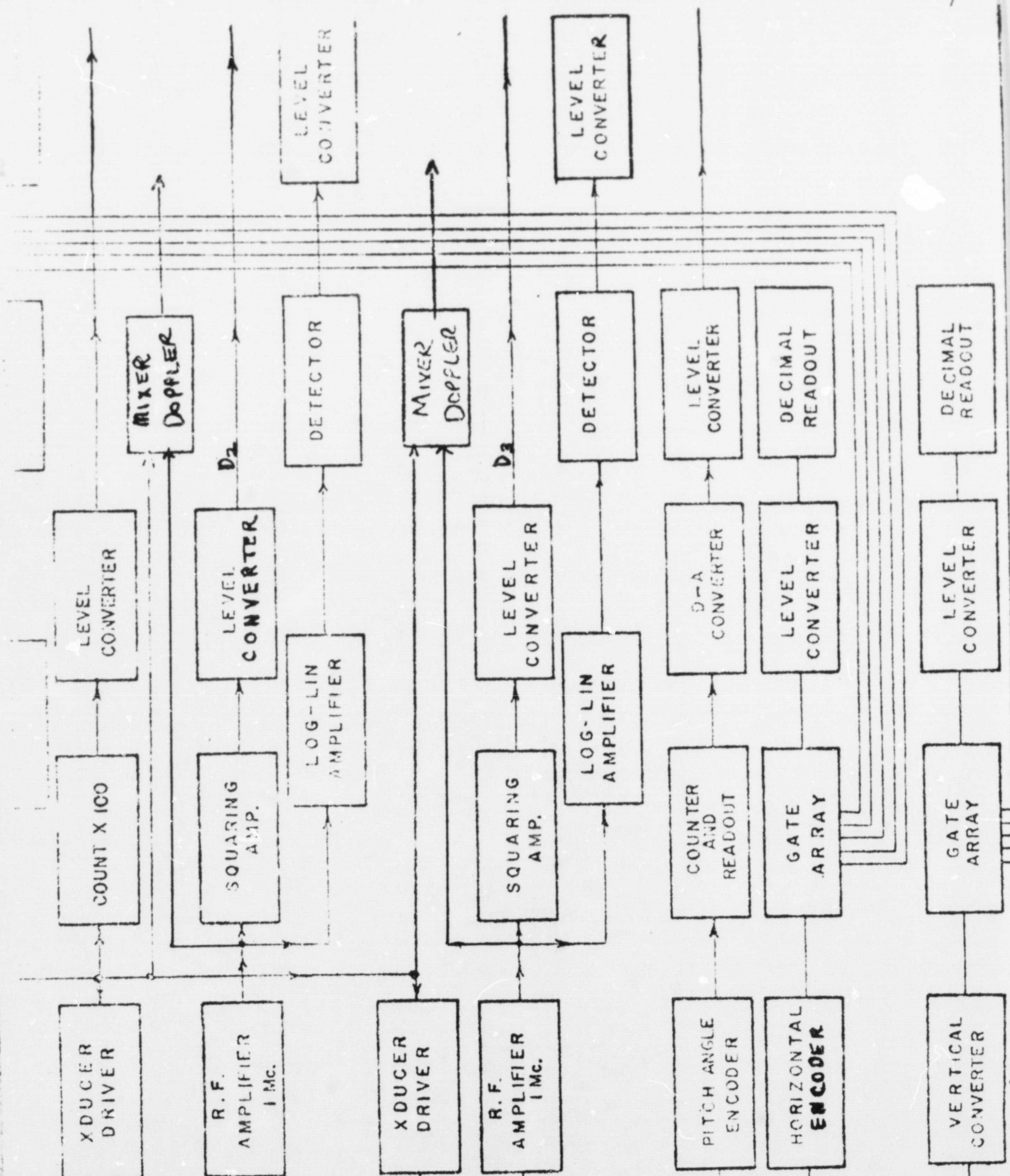


Figure 2-2.

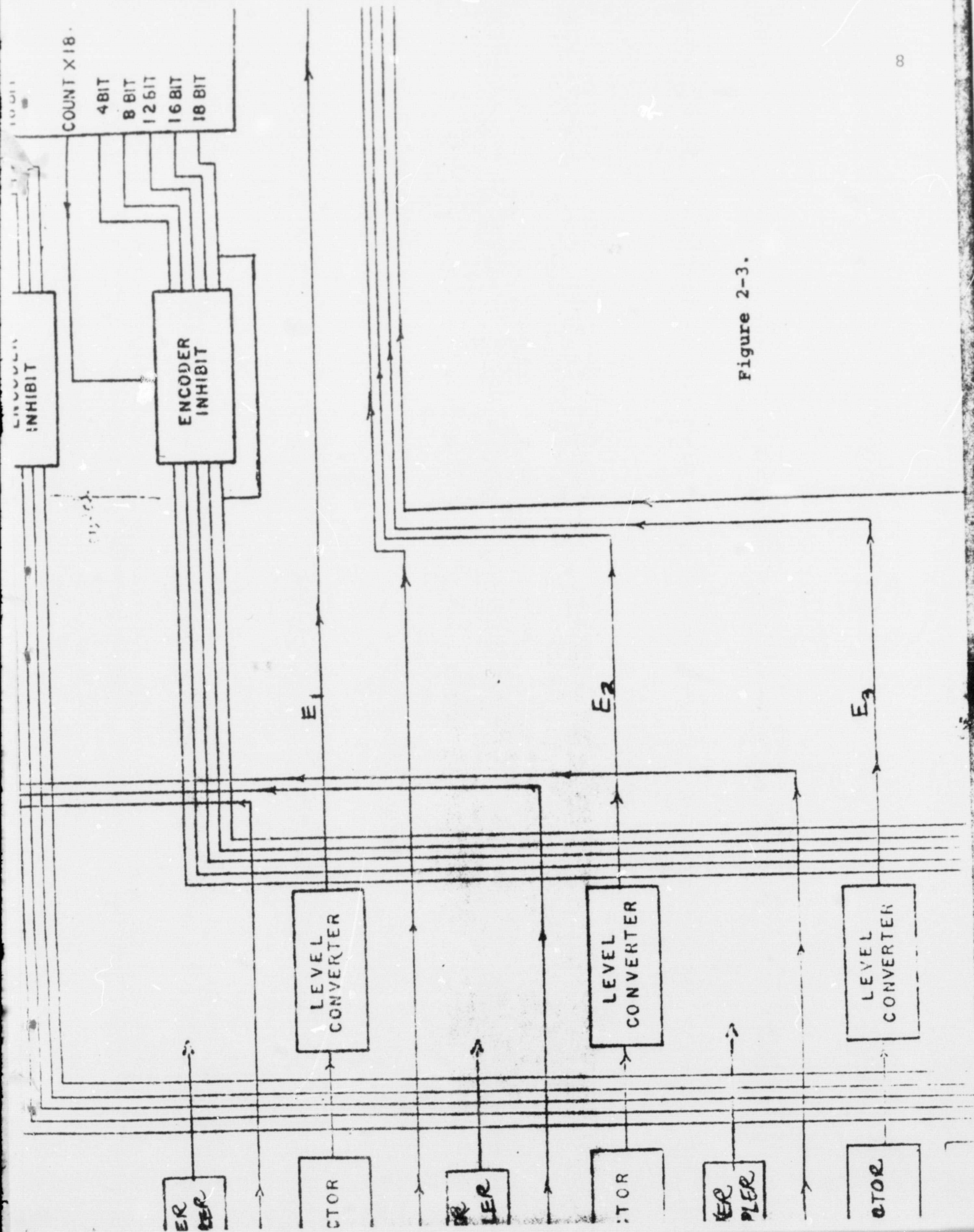
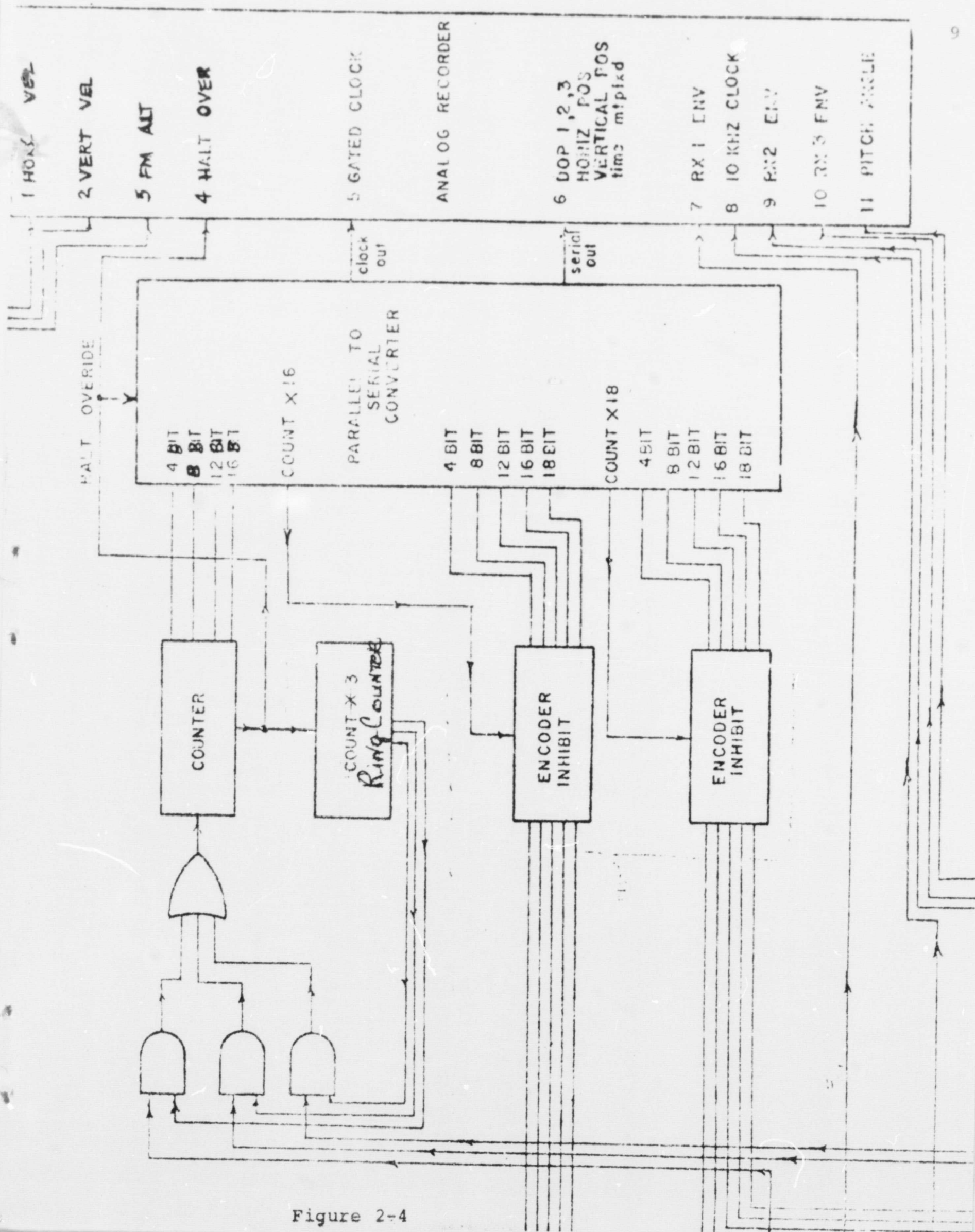


Figure 2-3.

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VELOCITY KADAR SIMULATION:

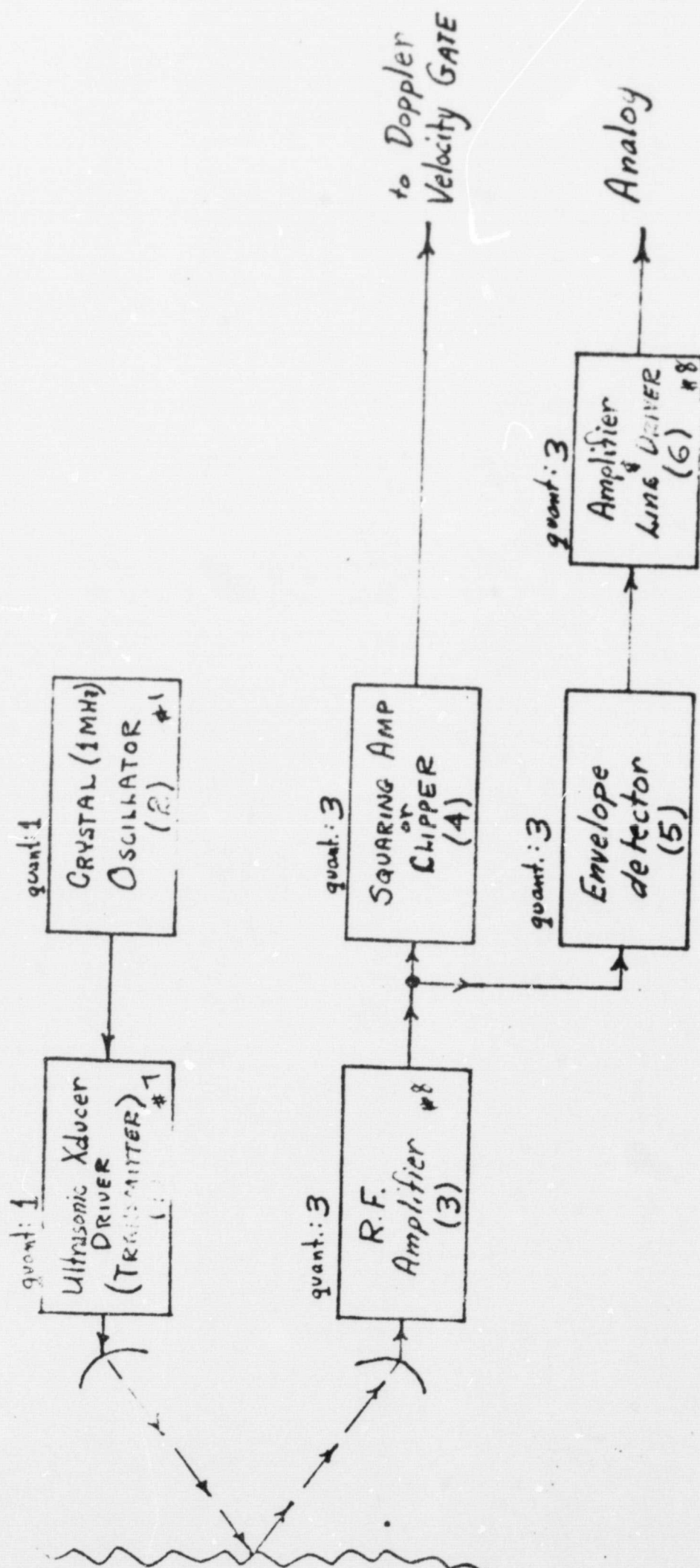


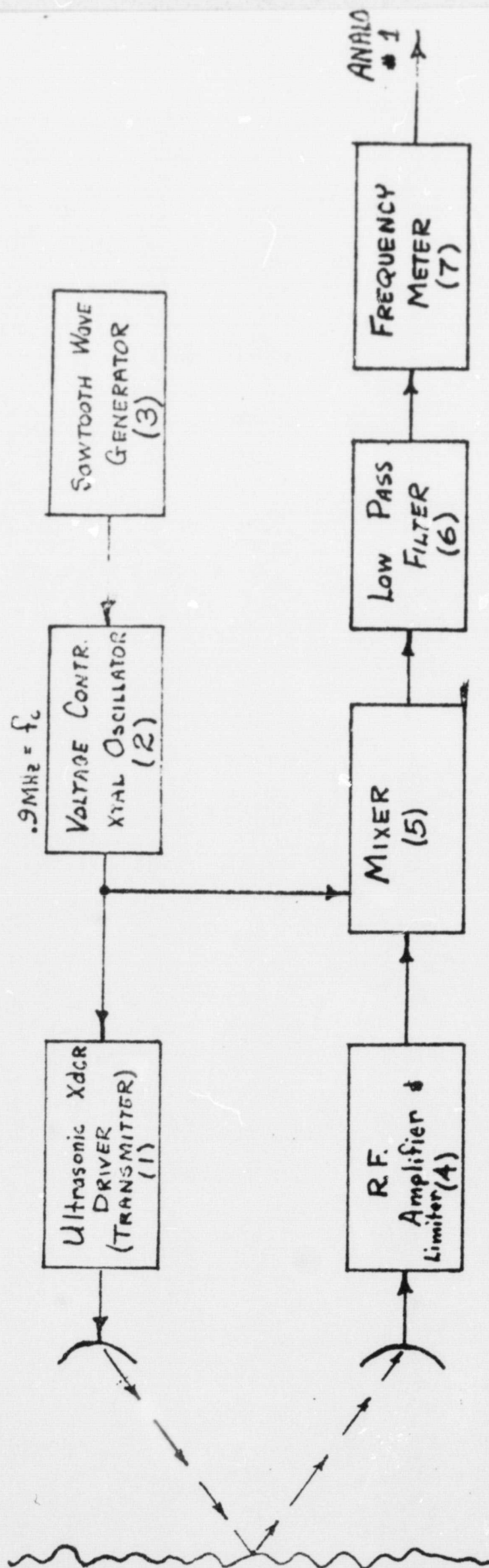
Figure 2-5

This system is duplicated for the three velocity radars. The transmitter and crystal oscillator drive three transducers in parallel. Thus, only modules (3) R.F. Amp., (4) Clipper, (5) Envelope Det., and (6) Amplifier & line driver need to be duplicated.

TECHNICAL SPECS.:

- 1) TRANSMITTER: Capable of 100v p.p across three parallel xducers @ 1.MHz.
- 2) CRYSTAL OSCILLATOR: 1 MHz, $\pm 3p$ in 10^9 /day stability out of CMC 800A counter.
- 3) R.F. Amplifier:
- 4) Clipper:
- 5) Envelope Detector:
- 6) Amplifier & Line Driver:

Altimeter Radar Simulation

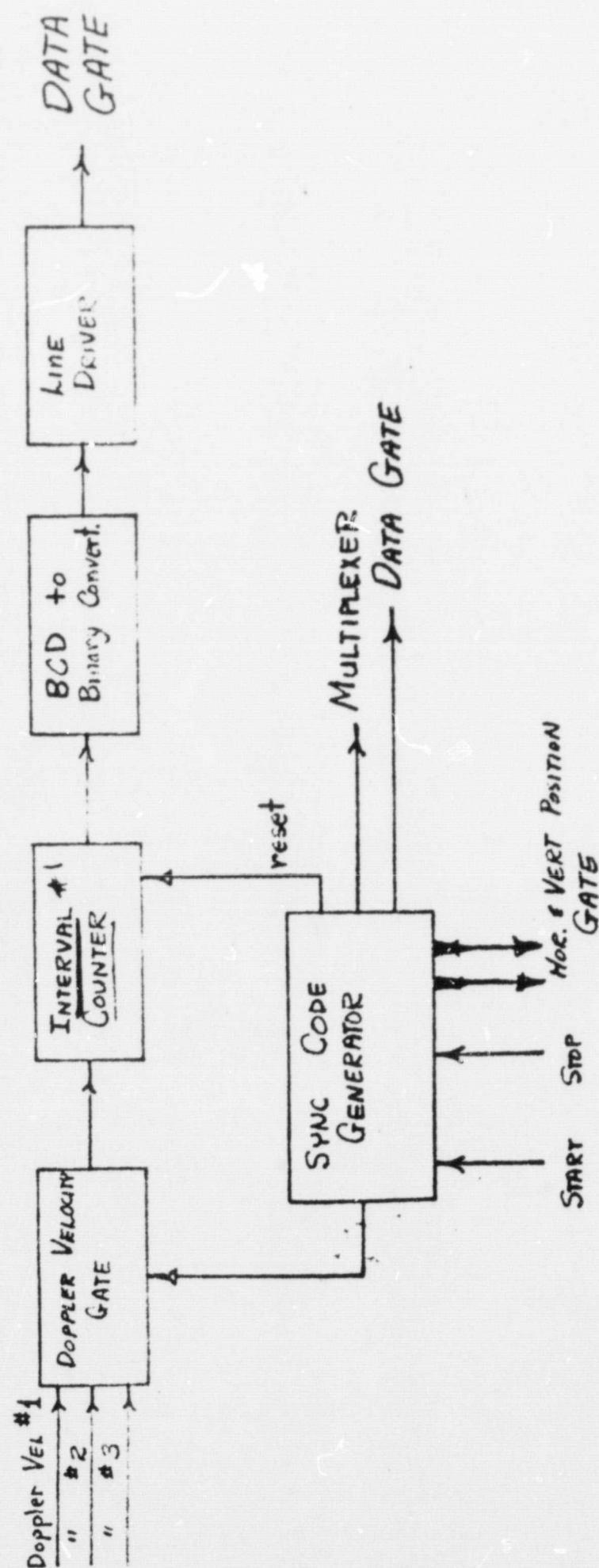


Technical Specs:

- 1) TRANSMITTER: Capable of 100 v. P-P output @ .9 MHz across transducer.
- 2) VCXO: $f_c = .9 \text{ MHz}$, $\pm 0.5\%$ f_c deviation, $2.5 \text{ p}/10^5 f_c$ stability, $\pm 5\%$ linearity
- 3) SAWTOOTH GENERATOR: (dual function capability), hi: f_o
- 4) R.F. Amplifier: (including limiter)
- 5) MIXER: inputs: $(.9 \text{ MHz} \pm 10 \text{ KHz})$ $(.9 \text{ MHz} \pm 10 \text{ KHz nominal} \pm \text{range} \pm \text{doppler})$; output: D.C. to 10 KHz
- 6) Low Pass Filter: Bandpass: D.C. to 10 KHz $\pm 1 \text{ db}$.
- 7) FREQUENCY METER: Linearity $\pm 0.5\%$, 0-100 Hz, 0-1 KHz, 0-10 KHz scales, 0-5 v.d.c. out

Figure 2-6

DOPPLER VELOCITY MEASURING and DIGITIZING SYSTEM



Counter Accuracy: ± 1 count $\pm$ osc. stability

$f_d = \pm 1$ Hz measurement in 0.1 sec.

Figure 2-7

REFERENCE CLOCK GENERATOR

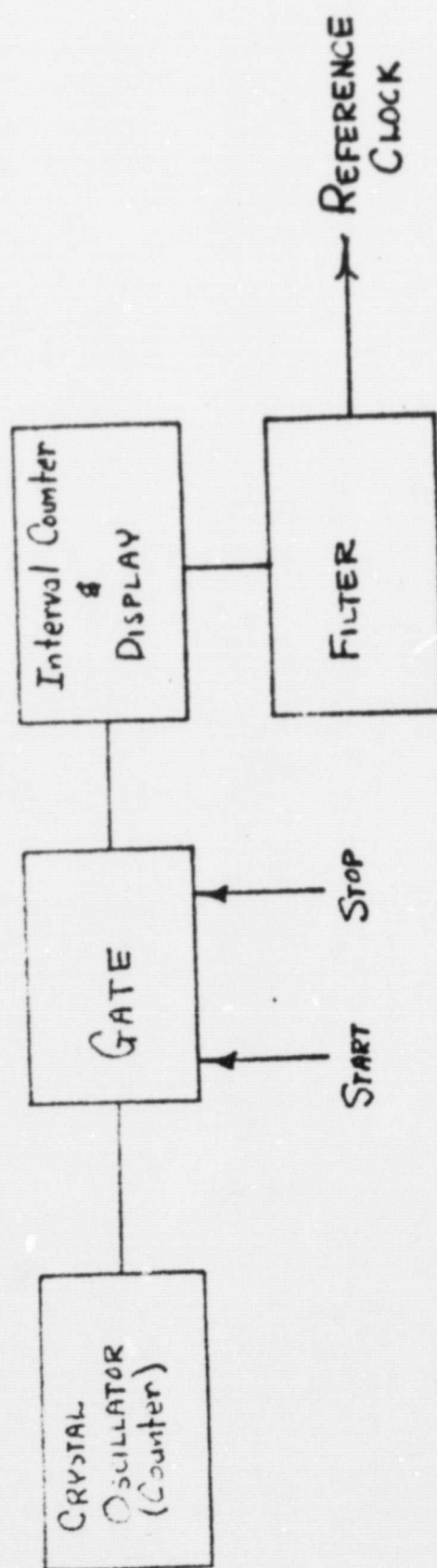


Figure 2-8

TRAJECTORY POSITION READOUT and DIGITAL DATA SYSTEM

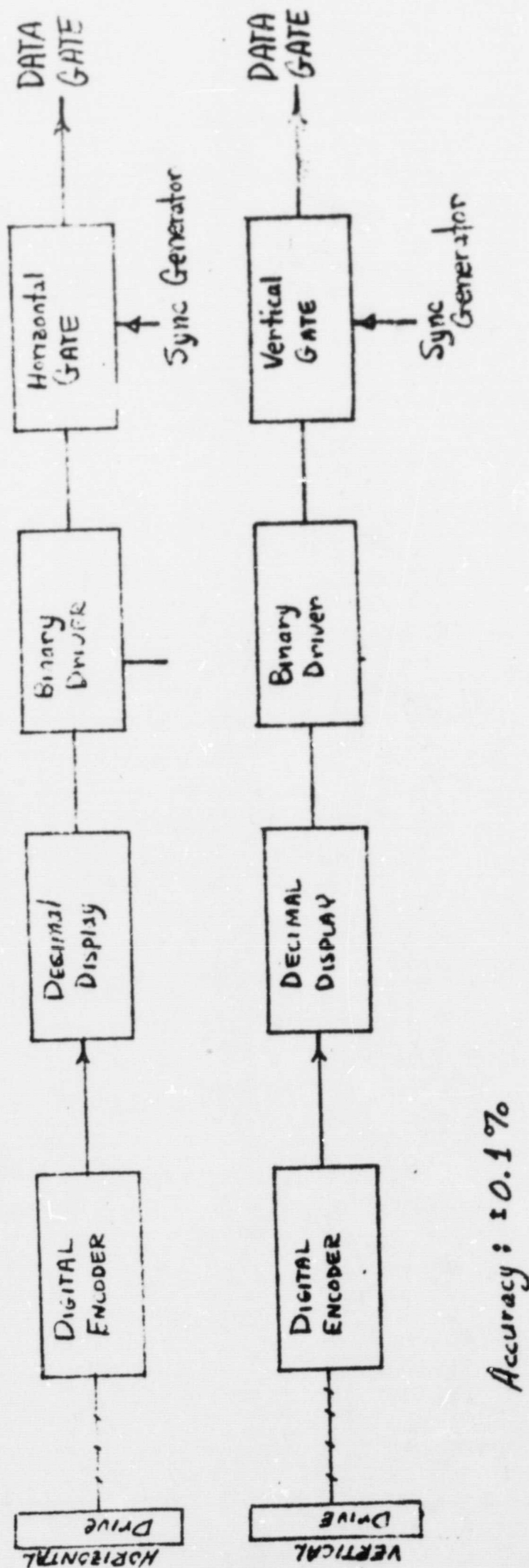
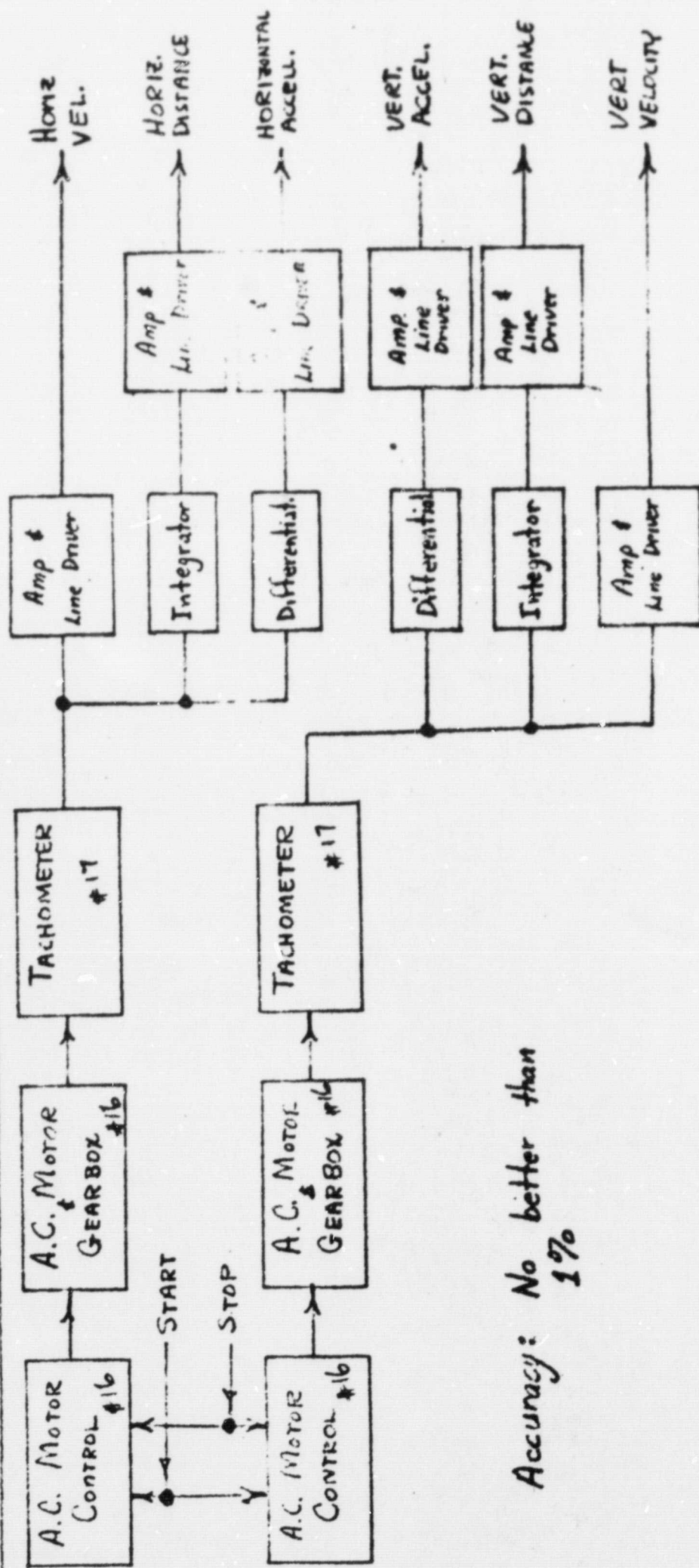


Figure 2-9

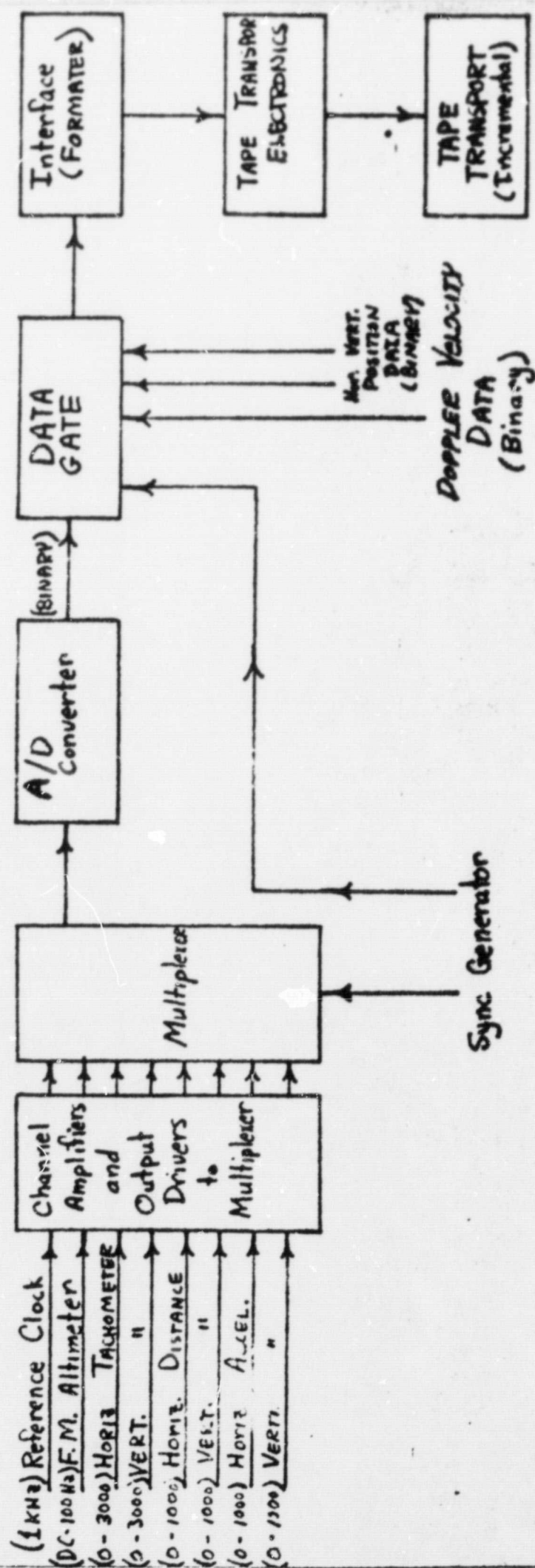
MOTOR CONTROL and TRAJECTORY DATA (analog) SYSTEM



Accuracy: No better than 1%

Figure 2-10

ANALOG - DIGITAL RECORDING SYSTEM

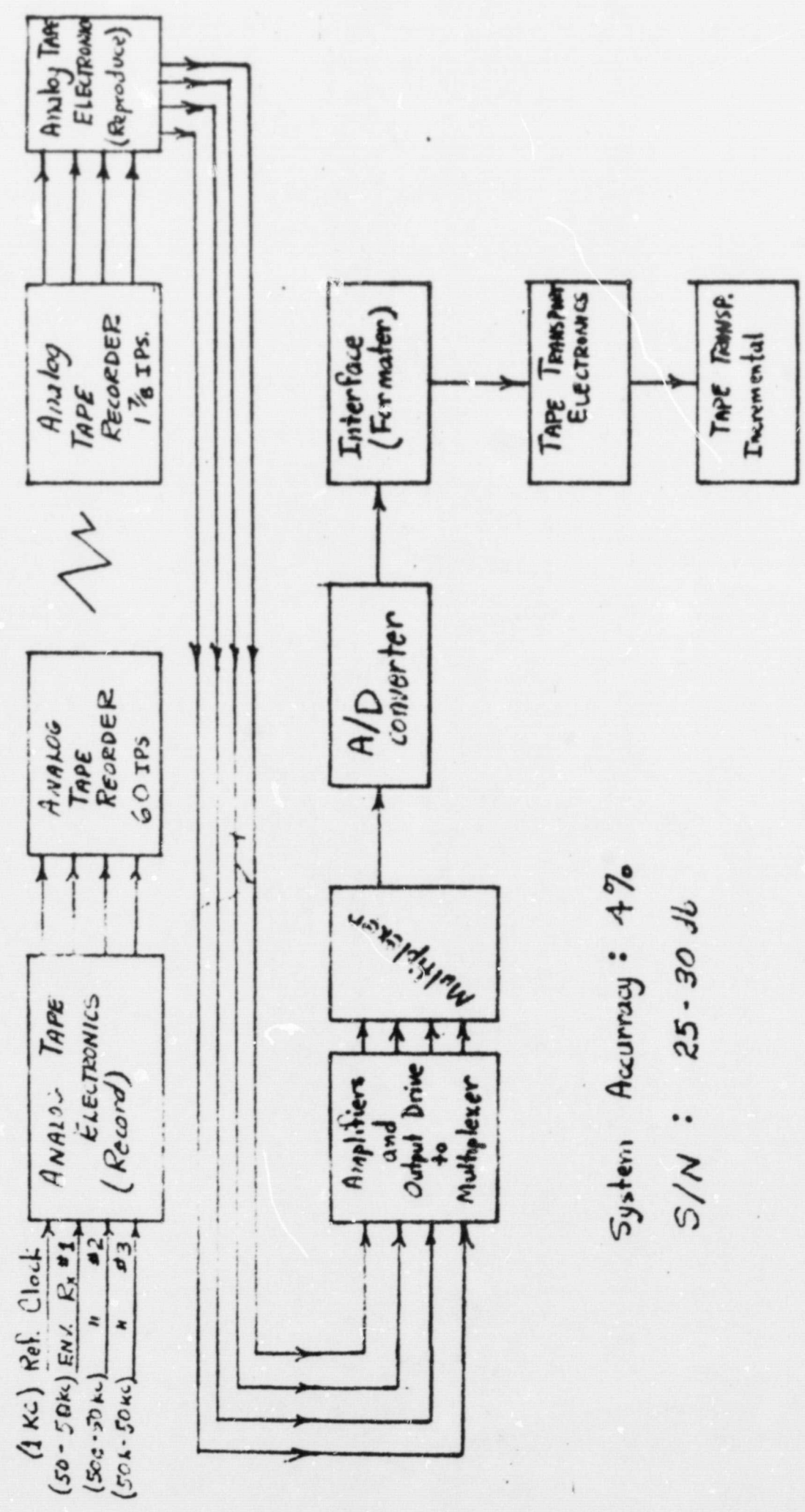


Accuracy : 0.2%

S/N : 50-60db.

Figure 2-11

ENVELOPE RECORDING SYSTEM



System Accuracy: 4%

S/N : 25-30 dB

Figure 2-12

SUGGESTED FORMAT FOR A SAMPLE TEST TAPE - UNIVERSITY OF HOUSTON DATA

60"/per second

ANALOG TRACK NO.	TAPE CALIB. TIME	CALIBRATED SIGNAL-TIME		TIME		TIME	
		00:00 - 00:40	00:40 - 01:40	01:40 - 02:00	02:00 - 03:00	03:00 - 04:00	04:00 - 05:00
1	Calibration	Not Used			Constant Level-dc volts (V _x)	Not Used	Typical Static Data (M ₁)
2	Calibration	Not Used			Halt Override	Not Used	Typical Static Data
3	Calibration	Not Used			Constant Level-dc volts (V _y)	Not Used	Typical Static Data (M ₂)
4	Calibration	Not Used			Gated Clock	Not Used	Typical Static Data
5	Calibration	Not Used			Constant Level-dc volts (alt.)	Not Used	Typical Static Data
6	Calibration	Not Used			Serial Output for known inputs	Not Used	Typical Static Data
7	Calibration	Not Used			Constant Level-dc volts (E ₁)	Not Used	Typical Static Data
8	Calibration	Not Used			100 kHz Sine Wave	Not Used	100 kHz Sine Wave
9	Calibration	Not Used			Constant Level-dc volts (E ₂)	Not Used	Typical Static Data
10	Calibration	Not Used			10 kHz Clock	Not Used	10 kHz Clock
11	Calibration	Not Used			Constant Level-dc volts (E ₃)	Not Used	Typical Static Data
12	Voice	Voice			Voice	Voice	Voice
13	Not Used	Not Used			Not Used	Not Used	Not Used (M ₃)
14	Not Used	Not Used			Not Used	Not Used	Not Used

Figure 2-13

2.1.7 Signals Recorded

The final maximum output of this system consists of the following signals:

1. Horizontal Velocity (dc or slowly varying dc)
2. Vertical Velocity (dc or slowly varying dc)
3. FM Alt. (dc or slowly varying dc)
4. Halt Over-ride - Digital
5. Gated Clock - Digital
6. Serial Output - Digital Data Doppler 1, 2, 3, Horizontal Position, Vertical Position (time multiplexed)
7. Envelope Doppler Channel 1
8. 10 KHZ Clock
9. Envelope Doppler Channel 2
10. Envelope Doppler Channel 3
11. Pitch Angle

2.2 Modification of Ultrasonic Beamwidths by Aperture Control

The narrow pencil beam of an ultrasonic transducer can be shaped into a fan beam by appropriate masking of the transducer aperture. The mask described herein was elliptical in shape with semi-minor axis one half the semi-major axis. As expected, the far field pattern parallel to the semi-minor axis was found to be approximately twice that parallel to the semi-major axis.

Summary of Beam Width--LM and Simulated

Doppler - Transmitter $3^{\circ} 37' \times 8^{\circ} 04'$
 - Receiver $7^{\circ} 39' \times 12^{\circ} 15'$
 Altimeter - Transmitter $8^{\circ} 8' \times 4^{\circ} 4'$
 - Receiver $11^{\circ} \times 7^{\circ} 10'$

The far field patterns of various shapes of apertures have been derived by many authors. Several methods of shaping beams from such apertures have also been described. These include lenses, reflectors, aperture control, preformed transducer elements and a combination of sources. The method of beam forming discussed here is that of controlling the aperture to shape the beam as decided.

2.2.1 Theoretical Discussion

Consider an ellipse lying in the x-y plane as shown in Fig. 2-14.

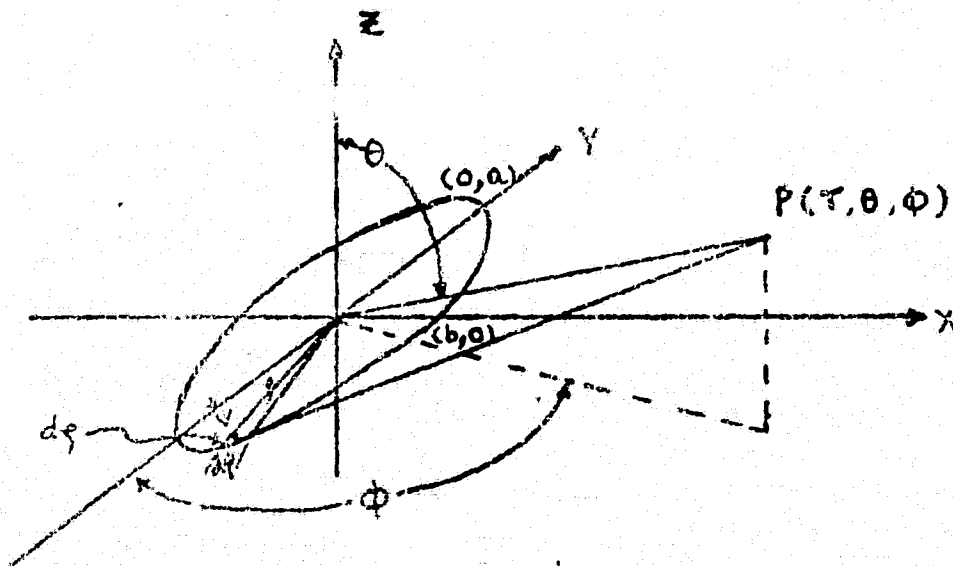


Figure 2-14

The change in pressure at a point P due to an infinitesimal area of the aperture is given by:

21

$$dp = i \frac{p_0 c k}{2\pi r'} u ds e^{i(\omega t - kr')}$$

2-1

where

p = Equilibrium pressure

k = Wave number

u = Particle velocity

c = Velocity of propagation

ds = Infinitesimal area of aperture

By methods of analytic geometry,

$$r' = [r^2 + \rho^2 - 2r\rho \cos(\phi - \psi) \sin \theta]^{\frac{1}{2}}$$

2-2

Expanding Eq. 2-2 by the binomial theorem

$$r' = (r^2 + \rho^2)^{\frac{1}{2}} + \frac{1}{2} (r^2 + \rho^2)^{-\frac{1}{2}} [-2r\rho \cos(\phi - \psi) \sin \theta] + \dots$$

$$= r + \frac{\rho^2}{2r} + \dots - r\rho \cos(\phi - \psi) \sin \theta.$$

2-3

$$\cdot \left[\frac{1}{r} - \frac{\rho^2}{2r^3} + \dots \right]$$

Since $r \gg \rho$

$$r' \simeq r - \rho \cos(\phi - \psi) \sin \theta$$

2-4

And

22

$$\frac{1}{r'} \approx \frac{1}{r}$$

2-5

Upon substitution of Eqs. 2-4 and 2-5 into Eq. 2-1,

$$dp = i \frac{P_0 c k}{2\pi r} u ds e^{i(\omega t - kr)} e^{ik\rho \cos(\phi - \psi) \sin\theta}$$

2-6

Integrating gives

$$p = i \frac{P_0 c k}{2\pi r} u e^{i(\omega t - kr)} \int_s e^{ik\rho \cos(\phi - \psi) \sin\theta} ds$$

2-7

But $ds = \rho d\theta d\psi$. Therefore

$$p = K e^{i(\omega t - kr)} \int_0^{2\pi} \int_0^{\theta^*} \rho e^{ik\rho \cos(\phi - \psi) \sin\theta} d\theta d\psi$$

2-9

where

$$K = i \frac{P_0 c k u}{2\pi r} \quad \text{and}$$

$$\theta^* = \frac{ab}{\sqrt{a^2 \sin^2 \phi + b^2 \cos^2 \phi}}$$

The integral in brackets becomes

$$\frac{1}{k^2 \sin^2 \theta \cos^2(\phi - \psi)} \left[e^{\frac{i k a b \sin \theta \cos(\phi - \psi)}{\sqrt{a^2 \sin^2 \psi + b^2 \cos^2 \psi}}} \left(1 - \frac{i k a b \sin \theta \cos(\phi - \psi)}{\sqrt{a^2 \sin^2 \psi + b^2 \cos^2 \psi}} \right) + 1 \right]$$

Then

$$P = \frac{k e^{i(\omega t - k r)}}{k^2 \sin^2 \theta} \int_0^{2\pi} \frac{1}{\cos^2(\phi - \psi)} \left[e^{\frac{i k a b \sin \theta \cos(\phi - \psi)}{\sqrt{a^2 \sin^2 \psi + b^2 \cos^2 \psi}}} \left(1 - \frac{i k a b \sin \theta \cos(\phi - \psi)}{\sqrt{a^2 \sin^2 \psi + b^2 \cos^2 \psi}} \right) + 1 \right] d\psi \quad 2-9$$

This reduces to

$$P = \frac{k e^{i(\omega t - k r)}}{k^2 \sin^2 \theta} \int_0^{2\pi} \frac{e^{\frac{i k a b \sin \theta \cos(\phi - \psi)}{\sqrt{a^2 \sin^2 \psi + b^2 \cos^2 \psi}}} \left(1 - \frac{i k a b \sin \theta \cos(\phi - \psi)}{\sqrt{a^2 \sin^2 \psi + b^2 \cos^2 \psi}} \right) d\psi}{\cos^2(\phi - \psi)} \quad 2-10$$

since

$$\int_0^{2\pi} \frac{d\psi}{\cos^2(\phi - \psi)} = 0.$$

Suppose now that $b = a$. Eq. 2-10 reduces to the well known expression (1) resulting from a circular aperture. Let the 3 db beamwidth of the circular aperture be α degrees. If $b = a/2$ is now used in Eq. 2-10, the resulting expression

applies to an elliptical aperture with semi-minor axis one-half the semi-major axis. The pattern parallel to the semi-minor axis will then have a beamwidth 2α degrees while the pattern parallel to the semi-major axis will be α degrees in width.

2.2.2 Experimental Results

The experimental arrangement used is depicted in Fig. 2-15.

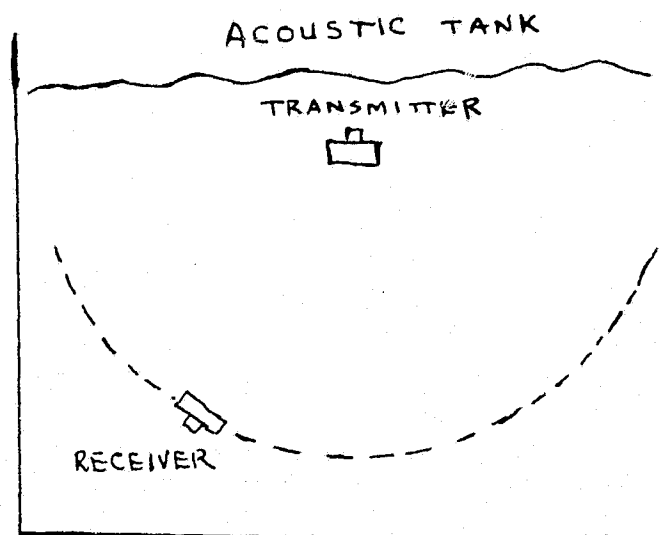


Fig. 2-15

The apparatus consisted of a stationary transmitter with masked aperture and a movable receiver. Receiver position readout was provided by an indicator arm whose motion across a circular scale was synchronized with the receiver motion. After the pattern in one plane was obtained, the mask on the transmitter was rotated 90° for the pattern in the other plane. The patterns for the two orientations of the mask are shown in Figs. 2-16 and 2-17.

12-187

210°
150°

200°
160°

190°
170°

180°

170°
190°

160°
200°

150°
210°

25

Beam Pattern Along the Major Axis

220°
140°

230°
130°

240°
120°

250°
110°

260°
100°

270°
90°

280°
80°

290°
70°

300°
60°

310°
50°

320°
40°

140°
220°

130°
230°

120°
240°

110°
250°

100°
260°

90°
270°

80°
280°

70°
290°

60°
300°

50°
310°

40°
320°

330°
30°

340°
20°

350°
10°

0

10°
350°

20°
340°

30°
330°

Polar

12-187

210°
150°

200°
160°

190°
170°

180°

170°
190°

160°
200°

150°
210°

26

Beam Pattern Along the Minor Axis

140°
220°

130°
230°

120°
240°

110°
250°

100°
260°

90°
270°

80°
280°

70°
290°

60°
300°

50°
310°

40°
320°

220°
140°

230°
130°

240°
120°

250°
110°

260°
100°

270°
90°

280°
80°

290°
70°

300°
60°

310°
50°

320°
40°

330°
30°

340°
20°

350°
10°

0

10°
350°

20°
340°

30°
330°

Polar
Co-ordinate

Fig 2-17

2.2.3 Conclusions

The transducers used in this application were 1 MHz Piezoelectric type. Without the mask, the beam was circular with a beamwidth of 3.5° . Reference to the 3db points on the curves of Figs.2-16 and 2-17 indicate a width of approximately 8° parallel to the semi-minor axis and 4° parallel to the semi-major axis when the aperture is masked. This is in very close agreement to the theoretical conclusion in that if $\alpha = 3.5^\circ$, $2\alpha = 7^\circ$, whereas the experimental values are 4° and 8° .

CHAPTER III

SURFACE MODELS

3.1 Surface Models

A surface height $\zeta(x,y)$ may be considered a spatial random process, and its special inhomogeneous features may be incorporated in the model in a manner similar to that used for LM (Tong and Hayre, 1967). The statistical parameters of a surface can be modeled reasonably well by assuming $\zeta(x,y)$ to be the superposition of a random component $\zeta_R(x,y)$ and one or more quasi-periodic components $\zeta_{P_i}(x,y)$, in keeping with the limitations of the validity of the Kirchhoff-Huygens integral.

The random component $\zeta_R(x,y)$ is the height function representing the general and relatively uniform roughness. It has been shown (Hayre, 1961) that a large number of naturally occurring rough surfaces may be said to have their heights normally distributed above and below their zero mean value and the probability density function of heights is given as

$$p(h) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-h^2}{2\sigma^2}\right) \quad 3-1$$

where

h = height, and

σ = standard deviation of heights.

In order to obtain a complete description of a rough surface, the knowledge of the spatial height distance correlation function is necessary as it, among other things, indicates

whether the protrusions are crowded close together or are widely separated. A widely used correlation function $c(\tau)$ is

$$C(\tau) = \sigma^2 \exp\left(\frac{-\tau^2}{B^2}\right) \quad 3-2$$

where

τ = distance between two points on the reference plane,

and

B = decorrelation distance, where $c(\tau)$ is equal to $\sigma^2 e^{-1}$.

A contour map study by Hayre and Moore (1961) showed that an exponential form of the spatial correlation function is often a realistic form for many surfaces as

$$C(\tau) = \sigma^2 \exp\left(\frac{-\tau^2}{B^2}\right) \quad 3-3$$

The variance σ and the decorrelation distance B are the intrinsic parameters of the surface. A description of surface roughness described by σ and B (Hayre, 1962) is used to depict various types of roughness in Figures 3-1a, 3-1b, and 3-1c.

The quasi-periodic component $\xi_p(x,y)$ represent a special surface feature. $\xi_p(x,y)$ is assumed to be divided into intervals of T , each interval containing one such special feature as shown in Fig.3-2. Since $\xi_p(x,y)$ may not be located periodically from one interval to the next, it is therefore a quasi-periodic random process. The distance x from the reference

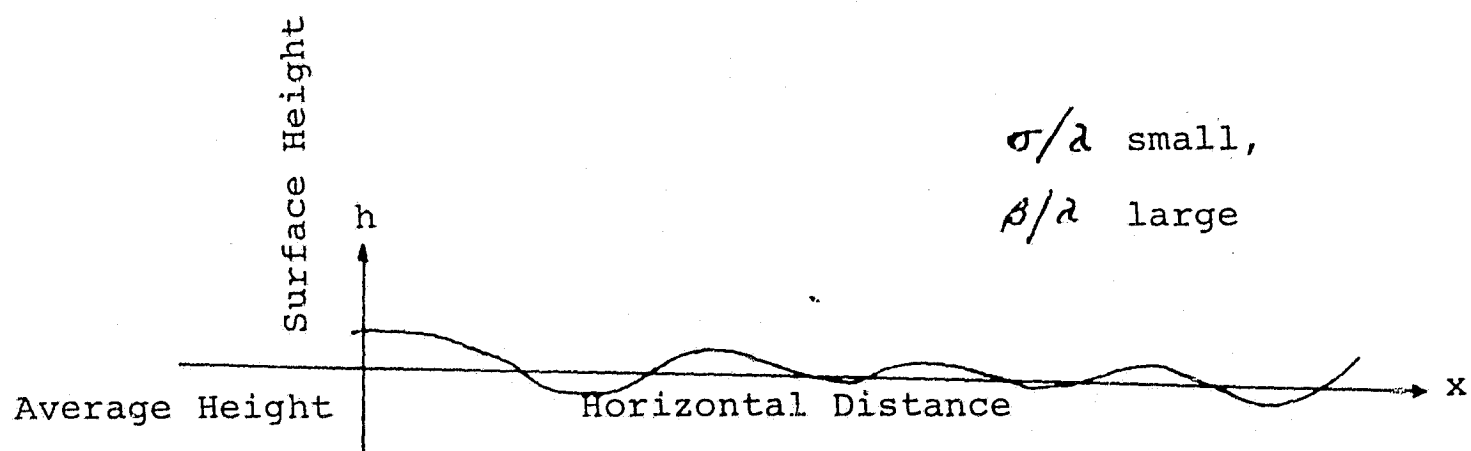


Fig. 3-1a. Relatively Smooth Surface

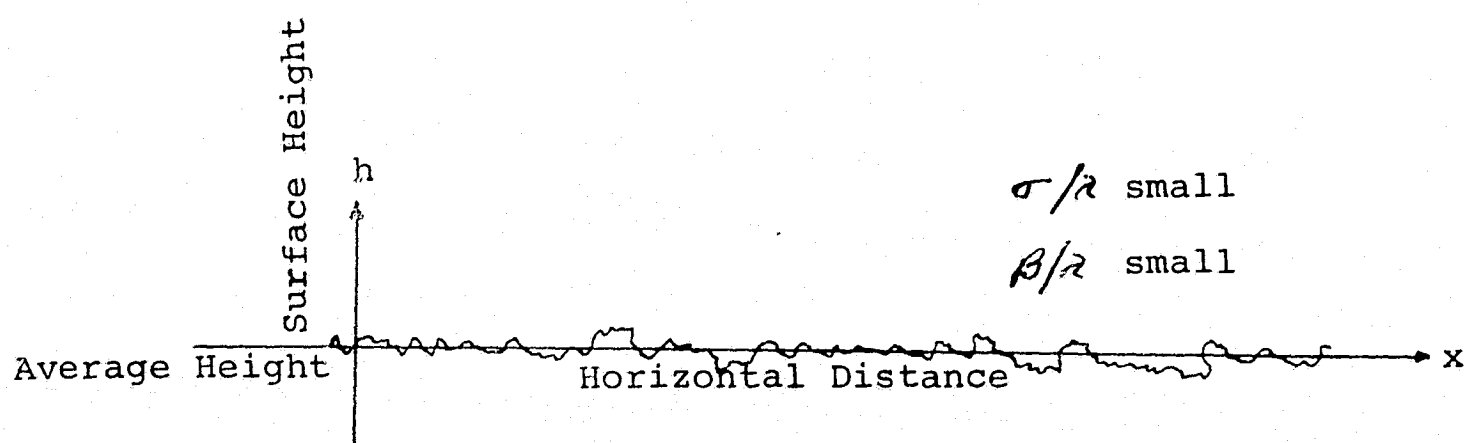


Fig. 3-1b. Very Rough Surface

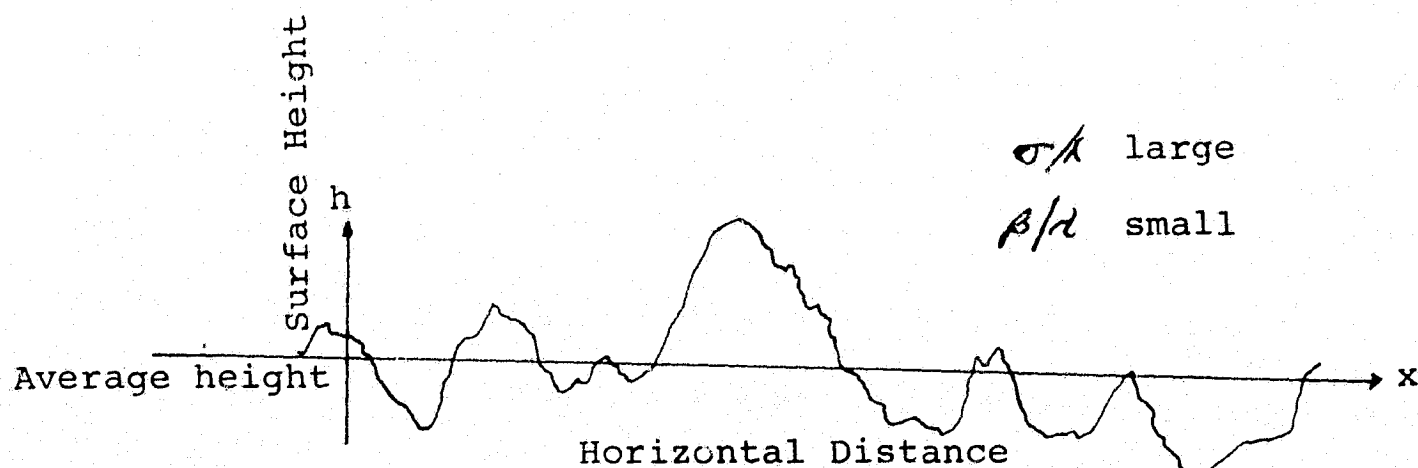


Fig. 3-1c. Rough Surface

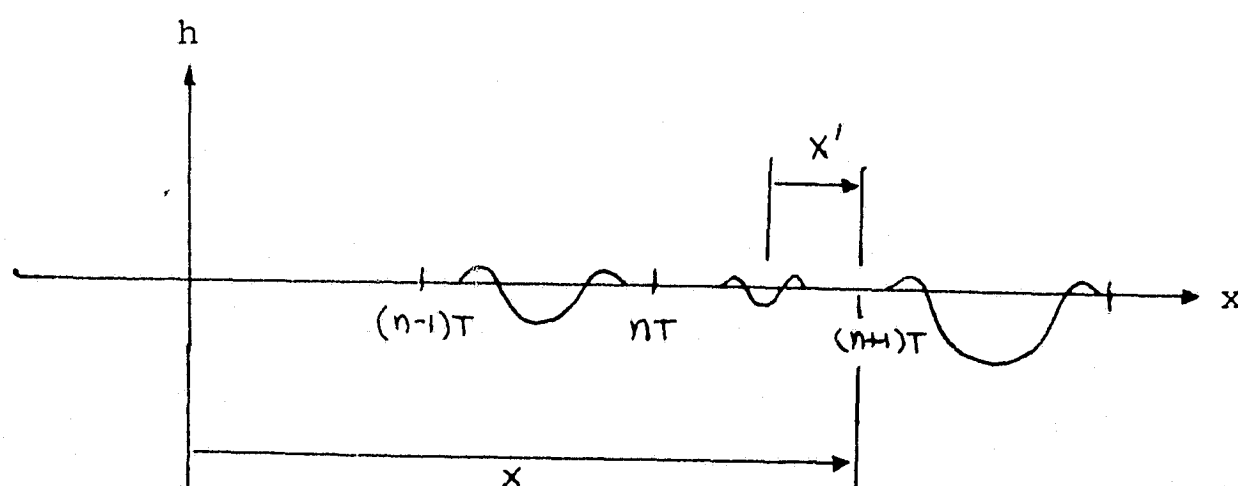


Fig. 3-2 Quasi-Periodic Surface

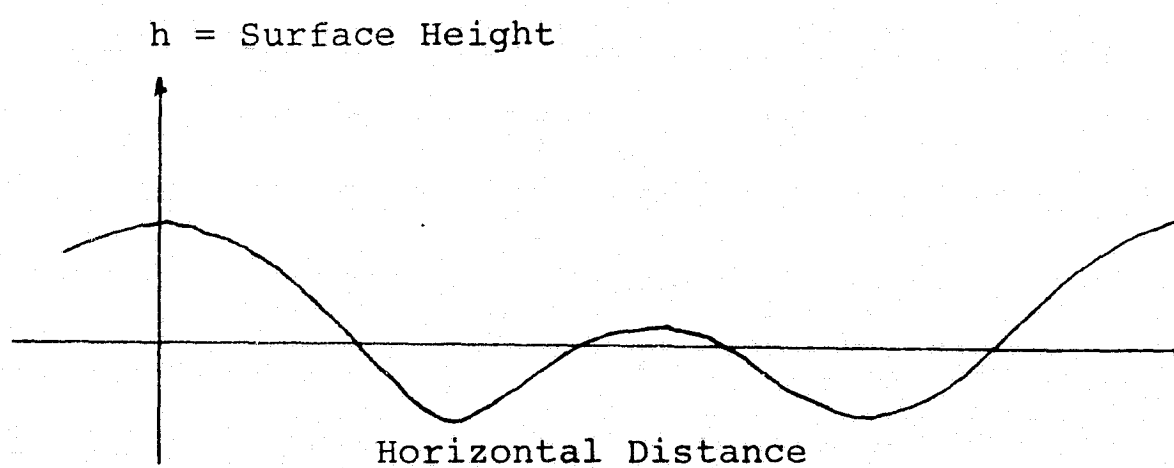


Fig. 3-3 Sandy Desert, Large-Scale Roughness

point is defined as

$$x = (n + \frac{1}{2})T + x' \quad 3-4$$

The functions $\xi_R(x,y)$ and $\xi_P(x,y)$ may be assumed to be statistically independent in some cases. Whereas for other rough terrains, a realistic description would require a specification of cross correlation between $\xi_R(x,y)$ and $\xi_P(x,y)$. However, the added complication in the calculation of the radar return may be difficult in such cases. This application of such models of a few rough terrains occurring in nature is discussed below.

- A. Sandy Desert. The sandy desert surface roughness depends on the natural process such as wind and rain. In general, a desert profile is mainly shaped by wind which usually does not blow in certain preferred directions so that the surface periodicity may not exactly apply in general and the quasi-periodic surface ξ_P may therefore be set to zero. The surface function may then be represented only by the random portion ξ_R , which for this type of sandy desert is a large-scale-rough-feature having $\frac{\sigma}{\lambda} \gg 1$ and $\frac{b}{\lambda} \gg 1$ as shown in Figure 3-3. However, a directional wind may shape the sand hills into a quasi-periodic surface, thus the surface function ξ is the combination of a random component

ξ_r and a periodic component ξ_p . The dimension of a sand grain is usually small compared to the normal radar wavelength (centimeters). Therefore, $\frac{\sigma}{\lambda}$ and $\frac{b}{\lambda}$ is very small, and the uniform random roughness may be the ineffective surface feature insofar as nonspecular radar return is concerned. Thus the surface function has only the quasi-periodic component as in Figure 4 and such a desert surface model may be described by a sinusoid in one-dimension as

$$\xi(x) = h \sin \frac{2\pi}{T} x \quad 3-5$$

where h or T or both may be random with a certain probability density function.

- B. Hills. If the vertical and horizontal dimensions of the hills are large compared to those of the radar beam cross-section, and it essentially slopes in one direction, the calculation of the surface reflectivity may be approximated by a one-dimensional surface representation. The surface ξ is the combination of the uniform general roughness ξ_r and the quasi-periodic component. The general roughness has the usual statistical parameters, such as the autocovariance function $c(\tau)$ of the form $\exp(-\frac{|\tau|}{B})$ and the standard deviation σ . The component ξ_p is triangular in nature and one can assign a probability

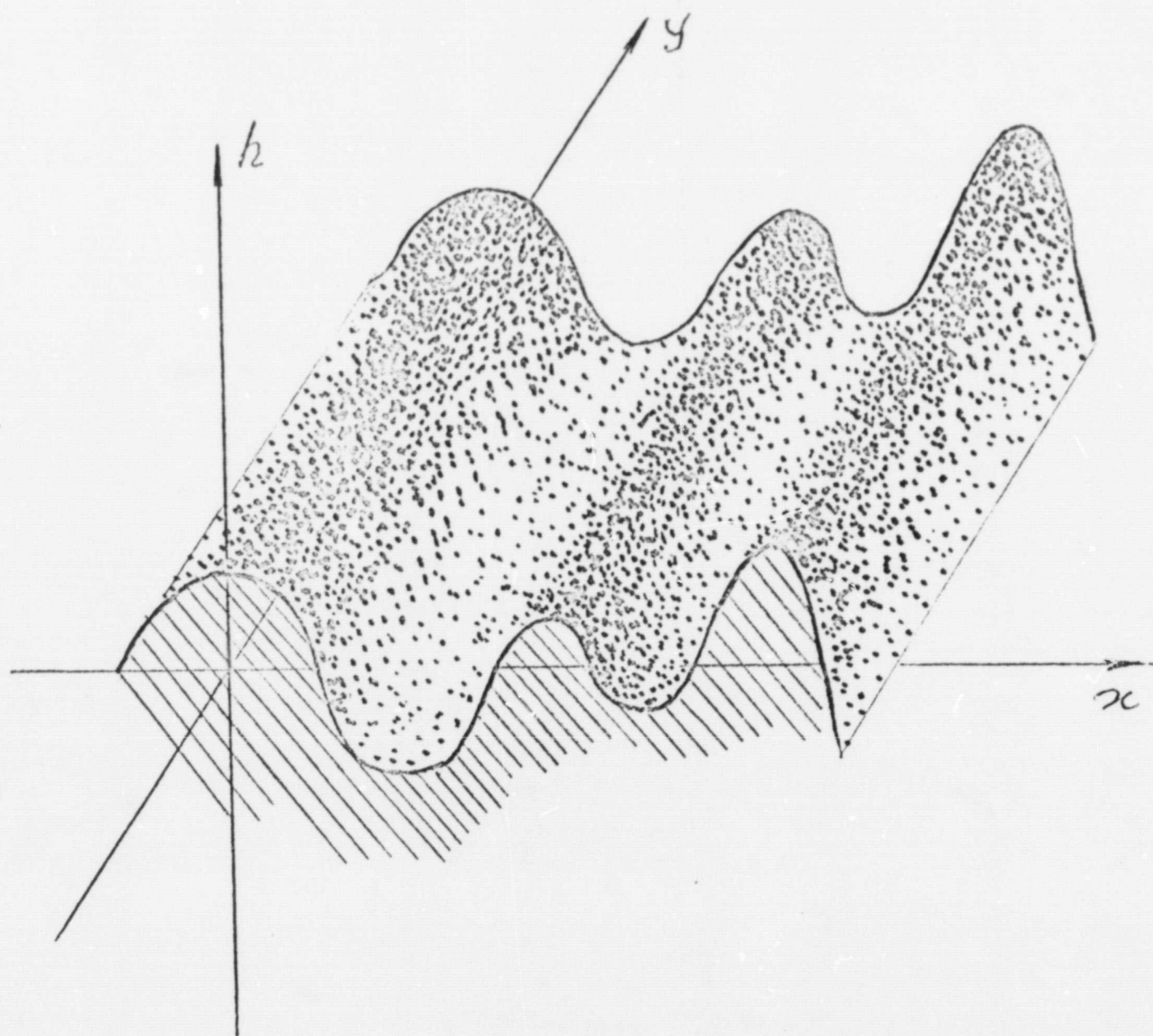


Fig. 3-4 Periodic Desert Profile

density function to the height h as shown in Fig. 3-3 whereas h and d are linearly related.

In case of low hilly terrains, where the radar beam cross-section is large compared to the sizes of the hills, the one-dimensional model is no longer applicable, and a two-dimensional surface description is necessary. The general random component is the same in all cases, but the quasi-periodic component ξ_p may be taken as an array of cones. In a way similar to the lunar model with its crater-array, the cones are assumed to occur once in each sample interval and the axes of the cones coincide with the centers of the intervals. One particular cone is shown in Fig. 3-6, where h and d are related. The density distribution of d may be assumed to be of the one-sided gaussian form.

$$p(d) = \frac{2}{\sqrt{2\pi}\sigma^2} \exp\left(\frac{-d^2}{2\sigma^2}\right) U(d) \quad 3-6$$

or Rayleigh

$$p(d) = \frac{d}{\sigma^2} \exp\left(\frac{-d^2}{2\sigma^2}\right) U(d) \quad 3-7$$

- C. Forest. The description of forest for the calculation of the radar reflectivity can also be appropriately modeled by the use of the superposition method. The random component ξ_r is applicable in

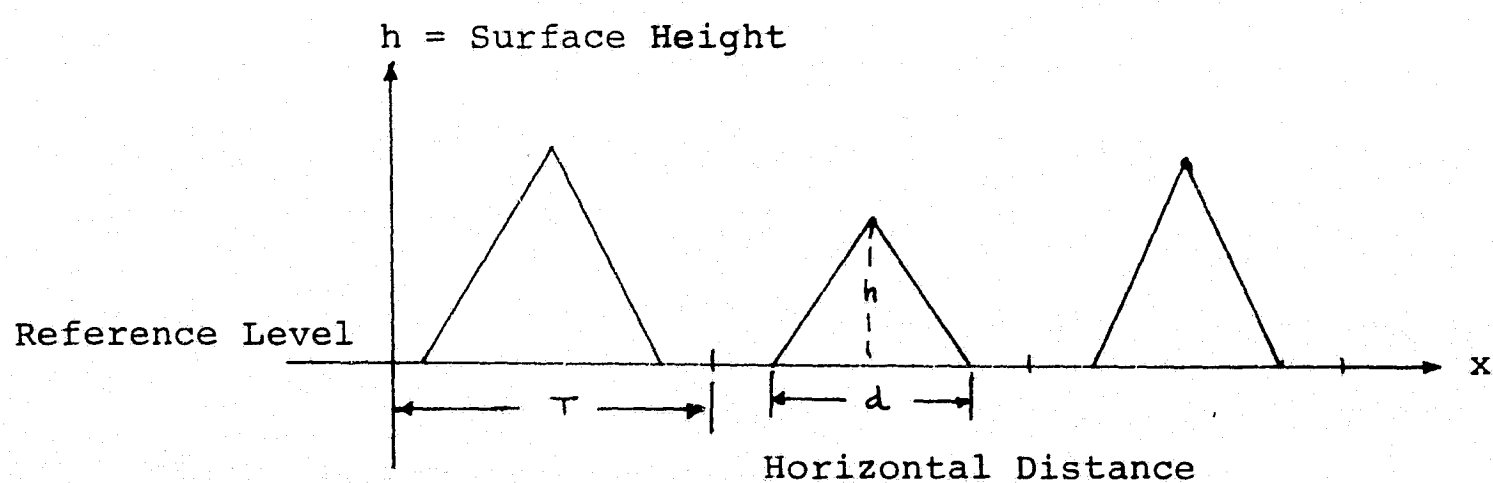
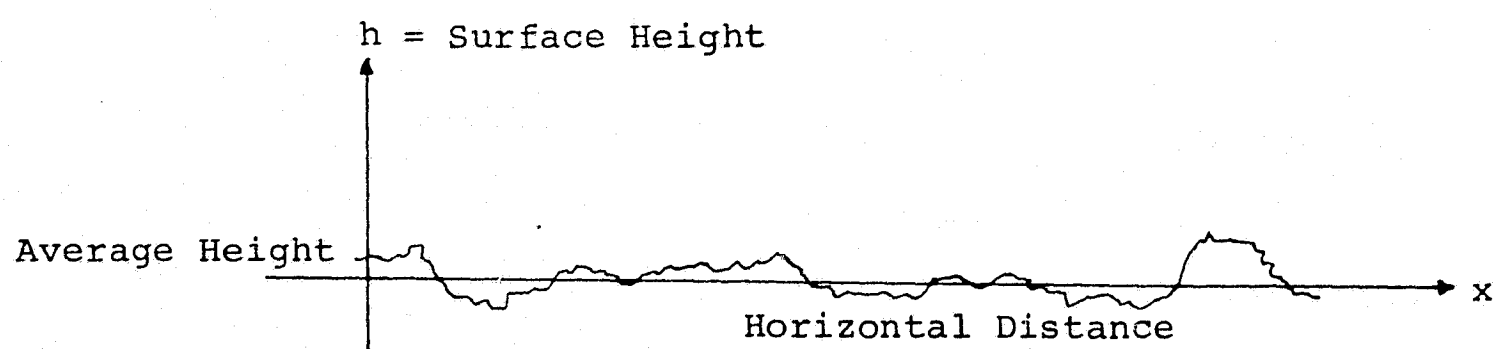
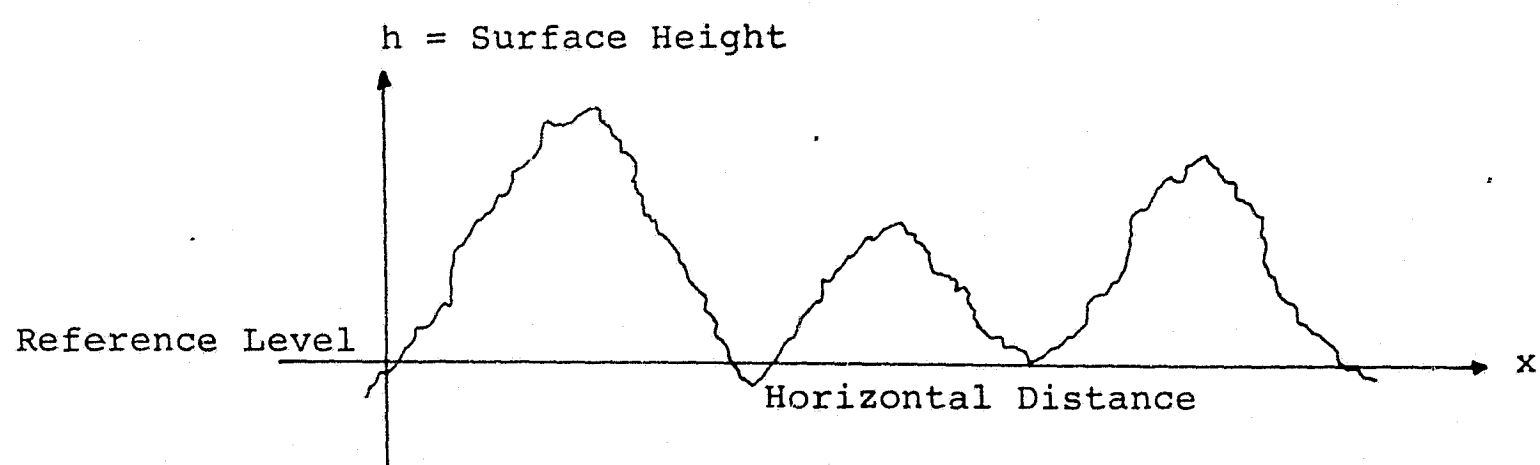


Fig. 3-5 One-Dimensional Hill Model

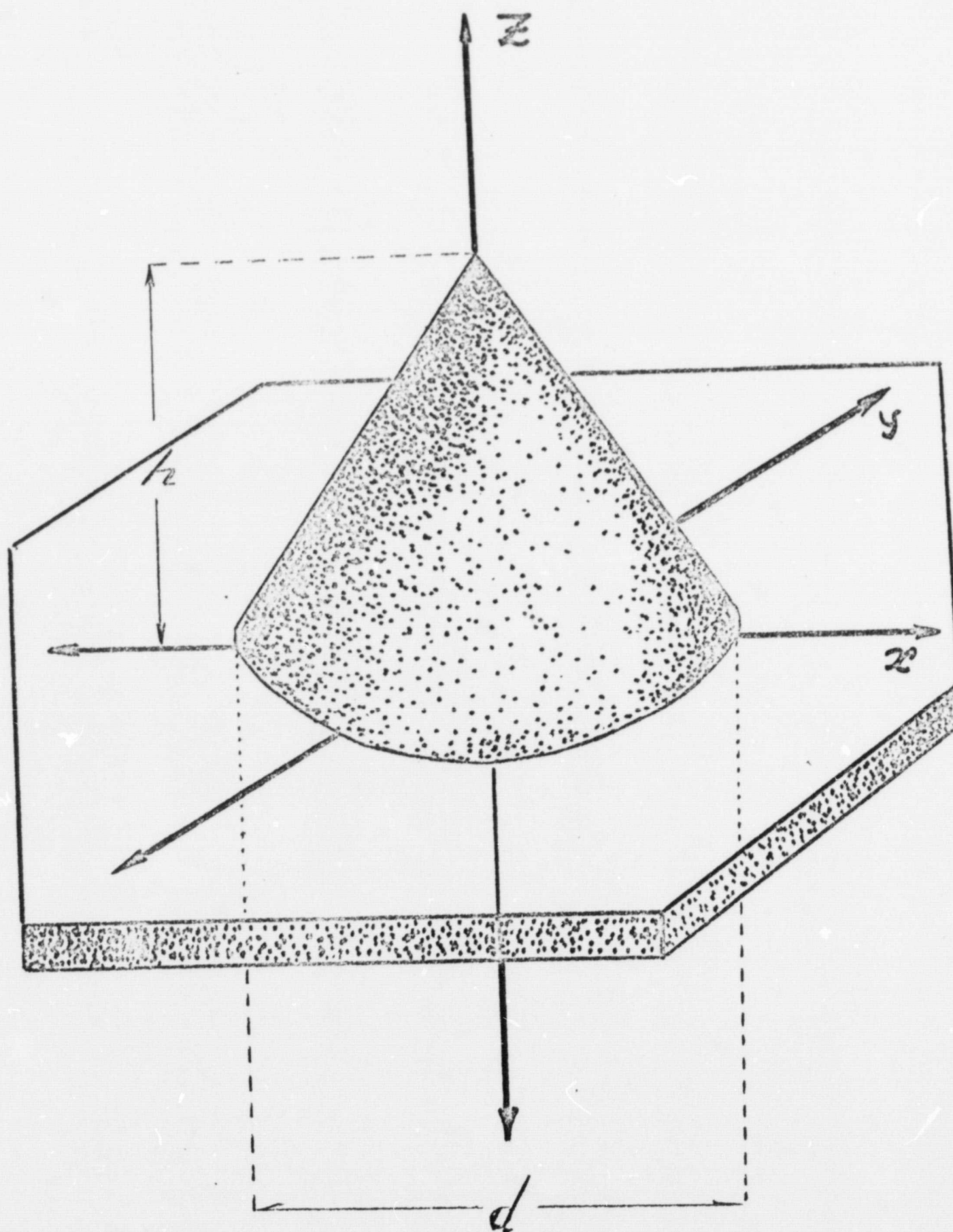


Fig. 3-6 The Periodic Component of the Two-Dimensional Hill Model

all cases. Its characteristics are the standard deviation and the decorrelation distance B . The periodic component, however, may have different descriptions for various kinds of trees. The Arctic forest may best be treated as the superposition of a random roughness and a triangular cone array, while the tropical forest is the combination of a random surface and a global array. In all assumptions, the array parameters may be linearly related to one random variable, which has a definite probability distribution.

The superposition model of a random and a periodic component seems to describe most rough surfaces for the calculation of this radar reflectivity. The random and the periodic component each has its own flexible but mutually independent probability densities and this enables one to obtain a solution which although complicated is still mathematically manageable.

3.2 Ultrasonic Modeling of Lunar Approach of the Lunar Excursion Module

This section covers the ultrasonic system modeling of the LM system and the lunar surface for three possible landing areas, P-6, P-8 and P-11, selected from Orbitor II data, as agreed to between NASA - Manned Spacecraft Center, and the University of Houston.

3.2.1 Modeling Factors for Ultrasonic Simulation of LM Landing Trajectory

The modeling scheme presented in this section is based on complex nonlinear ultrasonic scaling of radar system parameters. In short, this means distances, antenna velocities and frequencies may be selected independently in accordance with the size of the ultrasonic simulation facility, whereas other parameters such as doppler frequency, wavelength, time delay, etc., are then specified in terms of the previously specified parameters.

The transducers to be used in the simulation are piezo-electric type and are designed to operate around a center frequency on 1 MHZ. The velocity beams of the LM radar operate at 10.5 GHZ. The ratio of radar frequency to the ultrasonic frequency of one megacycle/second gives the frequency scale factor, f_x . This and other factors are listed in Table 3-1.

In order to simulate the large lunar distances properly, two or three model surfaces are being prepared for each site

TABLE 3-1
SCALE FACTORS USED IN LUNAR SIMULATION

PARAMETER	IN AIR	IN WATER	SCALE FACTOR
Frequency	10.5 GHZ	1 MHZ	$f = 10.5 \times 10^3$
Distance	12950 Ft. 164050 Ft.	12 Ft. 24 Ft.	$rs_1 = 1080$ $rs_2 = 6850$
Velocity of Propagation	3×10^8 m/sec.	1.5×10^3 m/sec.	$C_s = 2 \times 10^5$

as discussed in the later sections. Each model surface will be 10-12 feet in length and 4 feet in width. Each model surface represents the area traversed by radar beams during different sections of the LM trajectory. One of the models represents the lunar surface within half of the probability ellipse for that site while the others will represent the lunar surface along the approach to the ellipse. The probability ellipses are 7.9 km along the East-West direction. Therefore, the distance scale factor r_s , for this portion of the lunar surface is the ratio of $7.9 \text{ km}/2 = 12,950 \text{ feet}$ to 12 feet, listed in Table 3-1.

When the LM is at an altitude of 25,000 feet above the lunar surface, the distance to the landing site is 177,000 feet. After scaling the portion of the surface within the landing ellipse, there remains 164,050 feet of the total approach path. This remaining distance is scaled to 24 feet which is equivalent to two 12 foot models or one 12 foot model with double strip each to be traversed in the two consecutive runs. The distance scale factor along the approach is the ratio of 164,050 to 24. This produces the scale factor r_{s2} .

In addition to the frequency and distance scale factors, the scale factor for the velocity of propagation C_s is defined as the ratio of the velocity of light in air to the velocity of sound in water. Table 3-1 shows this and all other scale factors used in this study.

The primary consideration in selecting simulated antenna velocities are the maximum and minimum velocities which may be obtained by the primary (longitudinal) and secondary (cross) carriages of the acoustic facility used to position the transducers with respect to the simulated lunar surface. Simulation velocities are also limited by the size of the water tank.

The velocity scale factors were so chosen as to limit all simulated velocity variations to lie between the two extremes dictated by such physical limitations. Because three different component surface models are expected to be used for each site, three velocity scale factors, $V_{s1} = 1.56 \times 10^3$ for the simulated ellipse area, $V_{s2} = 6 \times 10^3$ for velocities at upper altitudes of the approach as shown in Table 3-1.

These scale factors and the coordinate system shown in the diagram below were used to scale different LM radar parameters such as distance, velocity, acceleration and (approximate) time measured from the starting point used as a reference are given in Table 3-2. All unprimed parameters correspond to the landing radar while the primed ones indicate the scaled parameters.

Diagram of coordinates for parameters in Table 3-2.

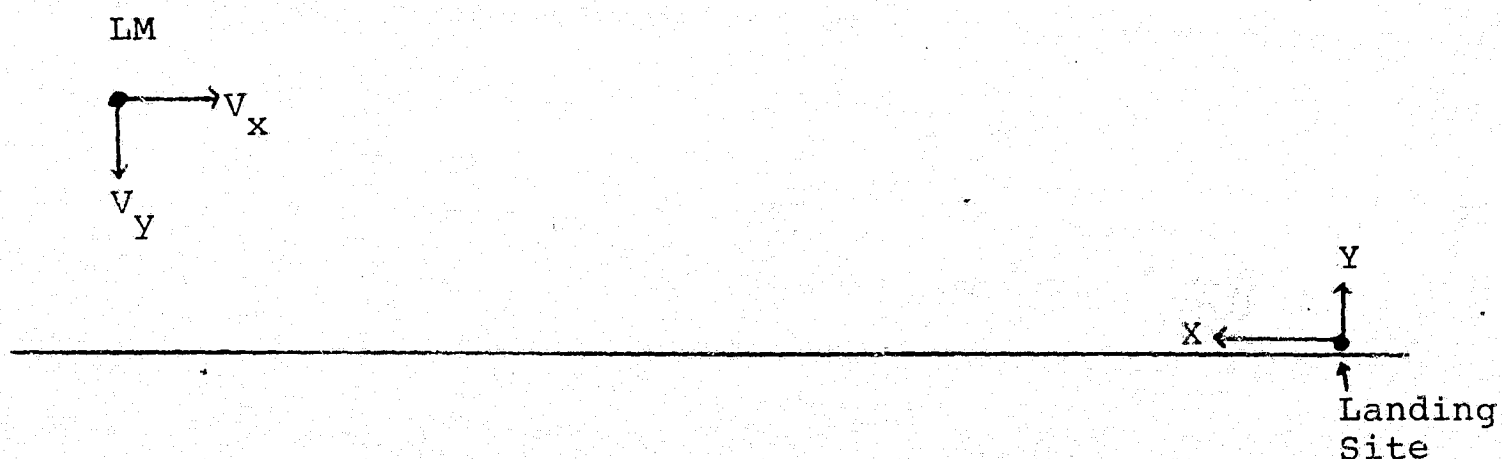


TABLE 3-2

X FT.	X' FT.	Y FT.	Y' FT.	V _X FT/SEC	V _X FT/SEC	V _Y FT/SEC	V _Y FT/SEC	A _X FT/SEC ²	A _Y FT/SEC ²	t' SEC
195	.181	200	.185	26.3	.0168	10.6	.0068	0.96x10 ⁻³	3.06x10 ⁻⁴	2.61
246	.228	220	.204	30.2	.0193	11.8	.0076	1.15x10 ⁻³	3.17x10 ⁻⁴	14.2
670	.620	373	.345	55.6	.0356	18.9	.0121	1.3x10 ⁻³	2.83x10 ⁻⁴	1.7
740	.685	400	.370	59.1	.0379	19.7	.0126	1.34x10 ⁻³	3.31x10 ⁻⁴	12.7
1380	1.280	500	.555	86.9	.0556	26.3	.0168	1.61x10 ⁻³	3.54x10 ⁻⁴	5.1
1710	1.584	700	.647	99.5	.0638	29.1	.0186	1.67x10 ⁻³	3.66x10 ⁻⁴	9.3
2430	2.250	900	.832	123.6	.0792	34.3	.0220	1.88x10 ⁻³	3.82x10 ⁻⁴	14.4
3870	3.580	1290	1.19	165.1	.106	42.9	.0275	2.02x10 ⁻³	4.1x10 ⁻⁴	14.4
5740	5.310	1760	1.63	210.8	.135	52.2	.0384	2.21x10 ⁻³	4.6x10 ⁻⁴	14.5
8100	7.500	2300	2.13	260.2	.167	62.6	.0401	2.28x10 ⁻³	5.0x10 ⁻⁴	14.7
11000	10.70	3000	2.78	312.7	.200	74.2	.0475	2.47x10 ⁻³	5.3x10 ⁻⁴	8.5
12950	12.00	3400	3.15	345.0	.221	81.0	.0520			

K FT. K FT.

12.95	0	3.4K	.496	345	.057	81.0	.0135	1.11x10 ⁻³	2.78x10 ⁻⁴	3.6
14.4	.212	3.8K	.555	367.7	.061	87.2	.0145	1.14x10 ⁻³	2.84x10 ⁻⁴	8.8
18.3	.782	4.8	.702	424.6	.071	101.9	.0170	1.07x10 ⁻³	3.22x10 ⁻⁴	3.4
22.8	1.44	5.87	.858	483.0	.080	118.5	.0197	1.28x10 ⁻³	2.82x10 ⁻⁴	3.9
25.0	1.76	6.5	.950	512.0	.085	125.0	.0208	.7x10 ⁻³	2.78x10 ⁻⁴	7.3
27.9	2.18	7.15	1.04	542.0	.090	137.3	.0228	1.14x10 ⁻³	4.0x10 ⁻⁴	7.0
32.5	2.86	8.31	1.21	590.0	.098	154.0	.0256	1.1x10 ⁻³	3.3x10 ⁻⁴	2.82
33.7	3.04	8.622	1.26	602.0	.10	157.0	.0262	1.9x10 ⁻³	.63x10 ⁻⁴	1.58
34.9	3.20	8.938	1.30	617.0	.103	158.0	.0263	1.04x10 ⁻³	-.52x10 ⁻⁴	1.92
36.2	3.40	9.250	1.35	633.0	.105	155.6	.0362	1.6x10 ⁻³	-1.5x10 ⁻⁴	21.3
53.9	6.00	12.8	1.87	832.0	.139	138.0	.023	1.4x10 ⁻³	-1.5x10 ⁻⁴	20.0
75	9.07	15.8	2.31	1000	.167	120.0	.020	2.37x10 ⁻³	-3.96x10 ⁻⁴	2.53
78.3	9.50	16.2	2.37	1037	.173	114.0	.0190	2.77x10 ⁻³	1.93x10 ⁻⁴	8.30
88.5	11.04	17.3	2.53	1175	.196	124.0	.0206	1.67x10 ⁻³	.42x10 ⁻⁴	4.8
95	12.00	17.95	2.62	1225	.204	125.0	.0208			<u>101</u>

MODEL 2 - RUN 2

X	X'	Y	Y'	V _x	V' _x	V _y	V' _y
95K	0	17.95K	2.62	1225	.079	125	.008
100K	.70	18.4K	2.69	1300	.083	130	.0083
107.5K	1.80	19.2	2.80	1396	.089	134	.0086
125K	4.40	20.8	3.04	1650	.106	141	.0090
150K	8.00	23.0	3.36	1820	.117	145	.0093
177K	12.00	25.0	3.65	2020	.15	147	.0094

A' _x	A' _y	ω_x	ω_y	α_x	α_y	t
1.36x10 ⁻³	.91x10 ⁻⁴	14.0	1.43	16.3x10 ⁻³	1.09x10 ⁻³	5.5
1.23x10 ⁻³	.49x10 ⁻⁴	14.9	1.49	14.8x10 ⁻³	.59x10 ⁻³	8.1
1.47x10 ⁻³	.41x10 ⁻⁴	16.1	1.54	17.6x10 ⁻³	.49x10 ⁻³	17.0
.82x10 ⁻³	.19x10 ⁻⁴	18.9	1.62	9.8x10 ⁻³	.23x10 ⁻³	20.8
.96x10 ⁻³	.096x10 ⁻⁴	20.9	1.66	11.5x10 ⁻³	.115x10 ⁻³	20.8
		23.2	1.69			72

The doppler frequencies at all points have not as yet been determined. If V_1 , V_2 and V_3 represent the velocities along the velocity beams and V'_x , V'_y the horizontal and vertical components of velocity, then

$$V_3 = \Gamma_1 V'_x - \Gamma_2 V'_y$$

$$V_1 = \Gamma_3 V'_x + \Gamma_4 V'_y$$

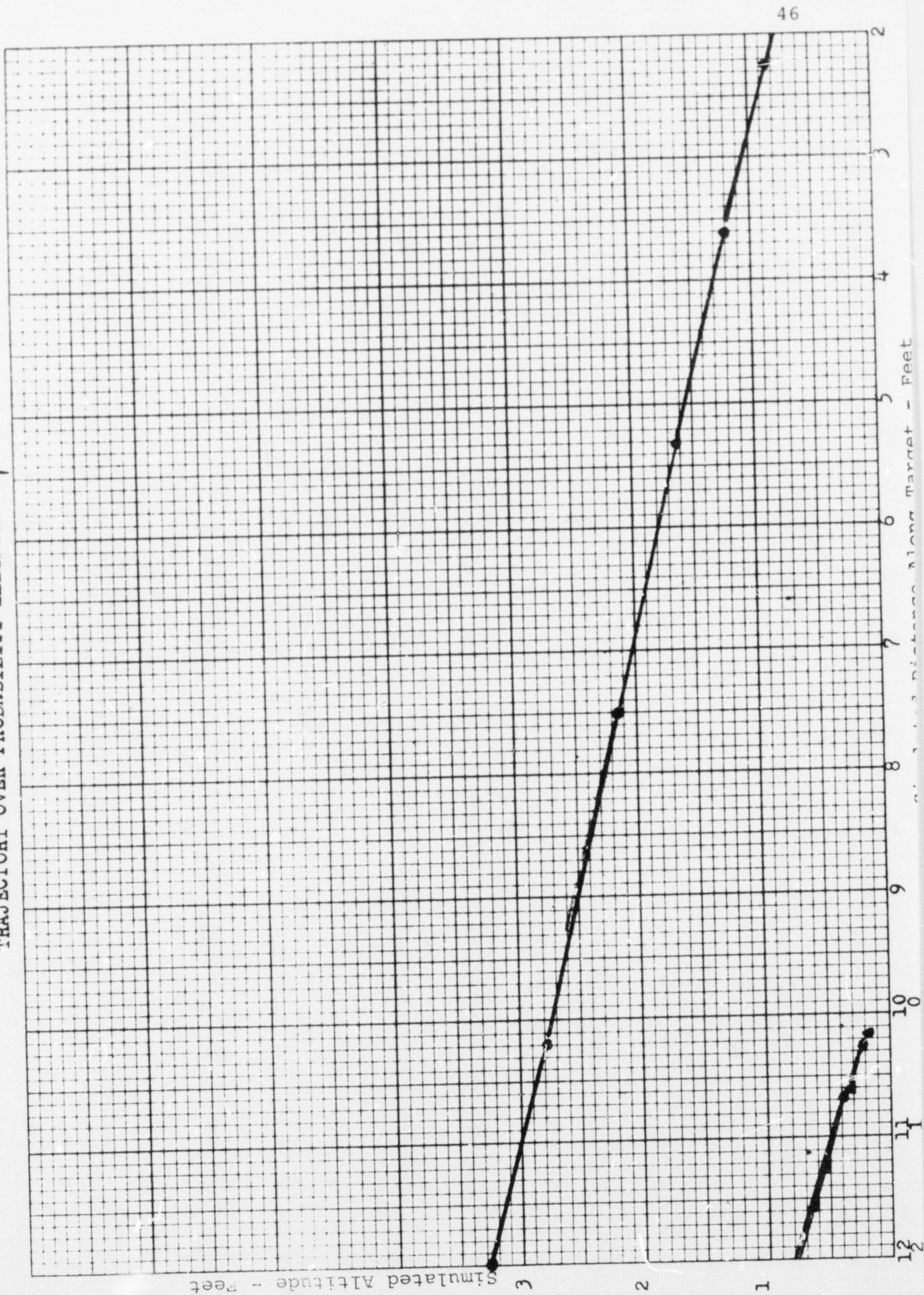
where the Γ 's are constants for a given pitch angle. The doppler frequencies along the beams are related to these velocities by

$$f_{D1} = f_{D2} = \frac{2f'}{c} [\Gamma_3 V'_x + \Gamma_4 V'_y]$$

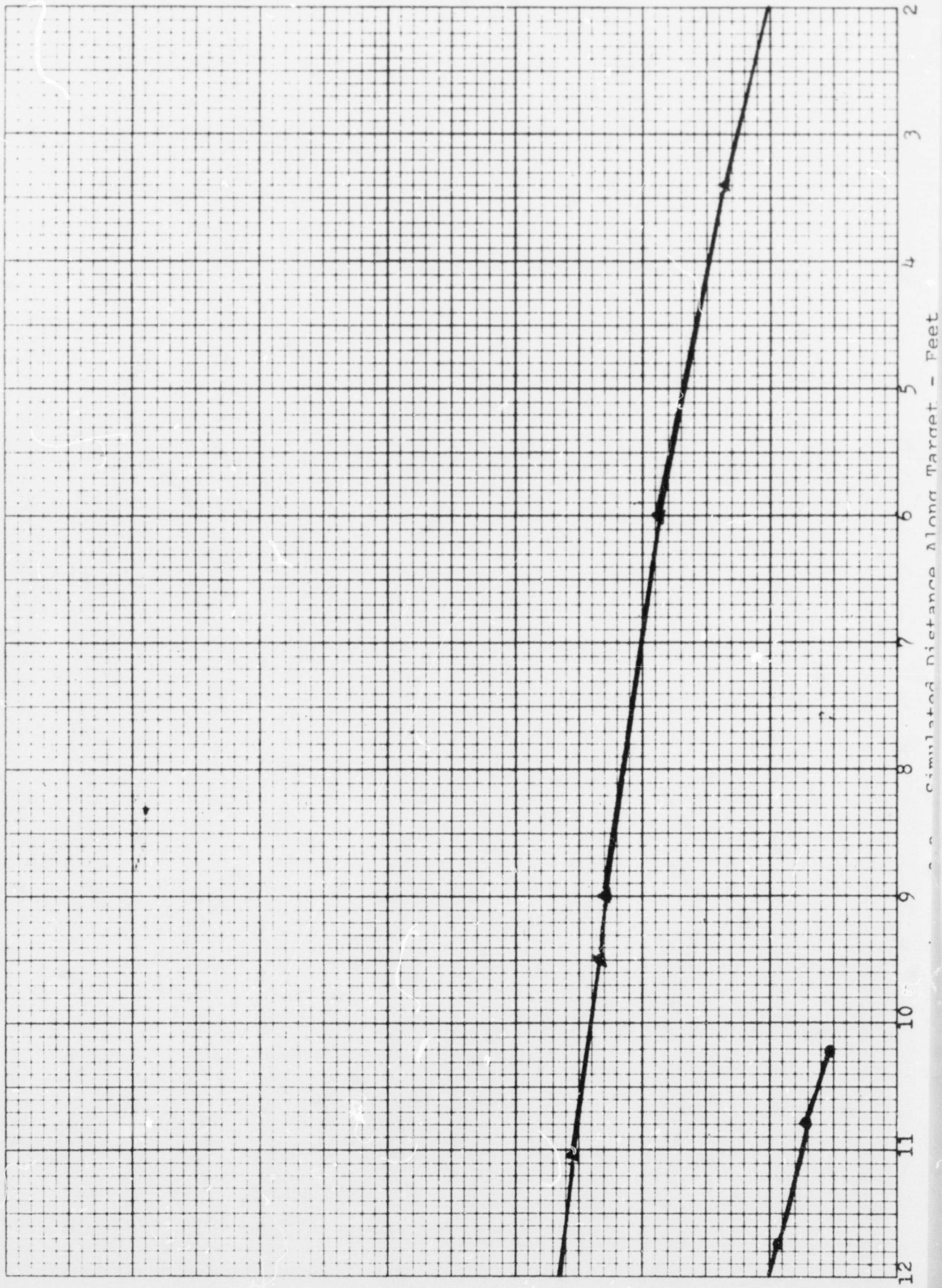
$$f_{D3} = \frac{2f'}{c} [\Gamma_1 V'_x - \Gamma_2 V'_y]$$

Preliminary calculations indicate these frequencies will lie between 0 and 150 Hz for all points along the simulated trajectory.

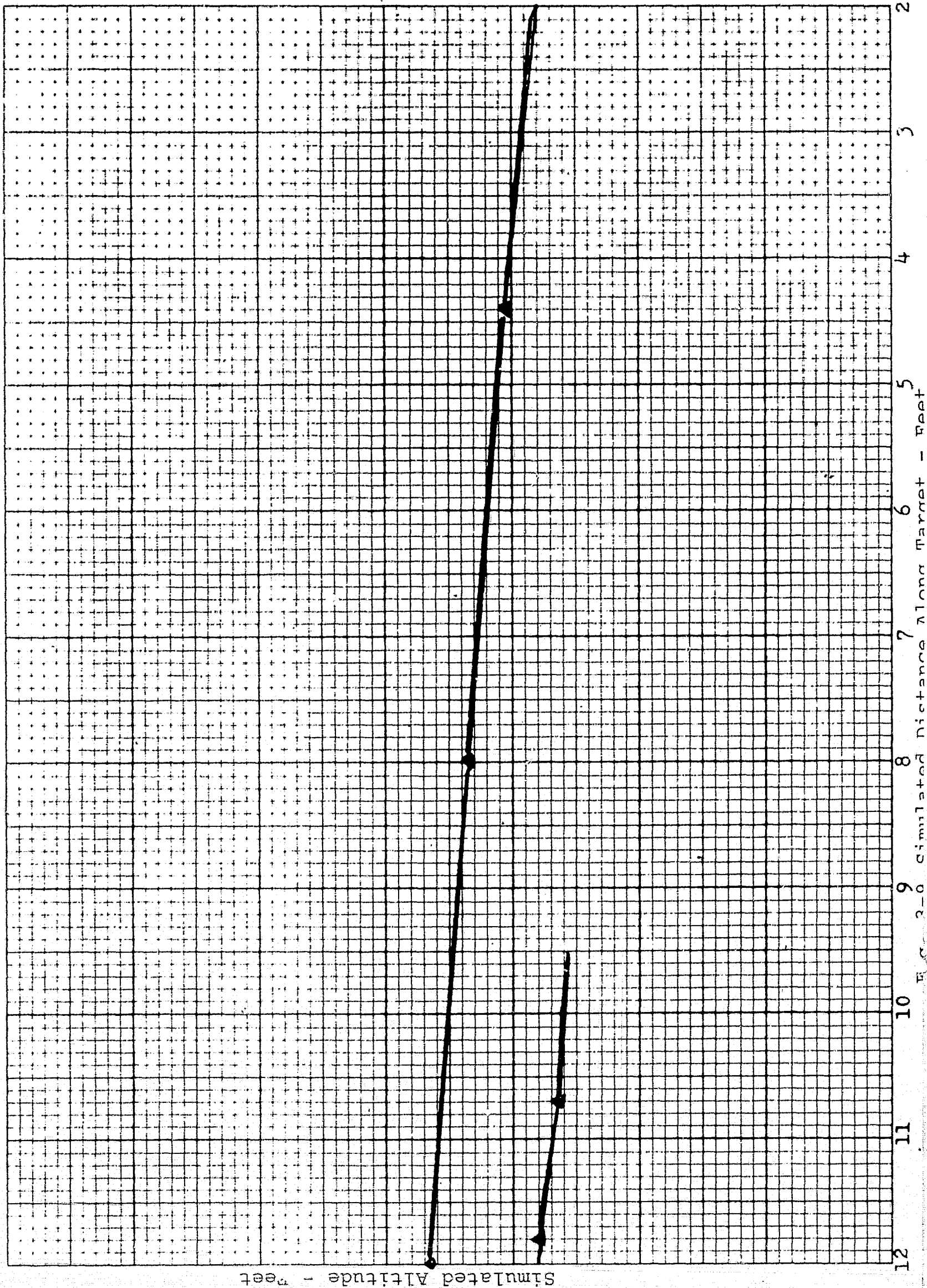
TRAJECTORY OVER PROBABILITY ELLIPSE $K_1 = 1080$



TRAJECTORY #1 OVER TARGET #2 $V_{S_2} = 6850$

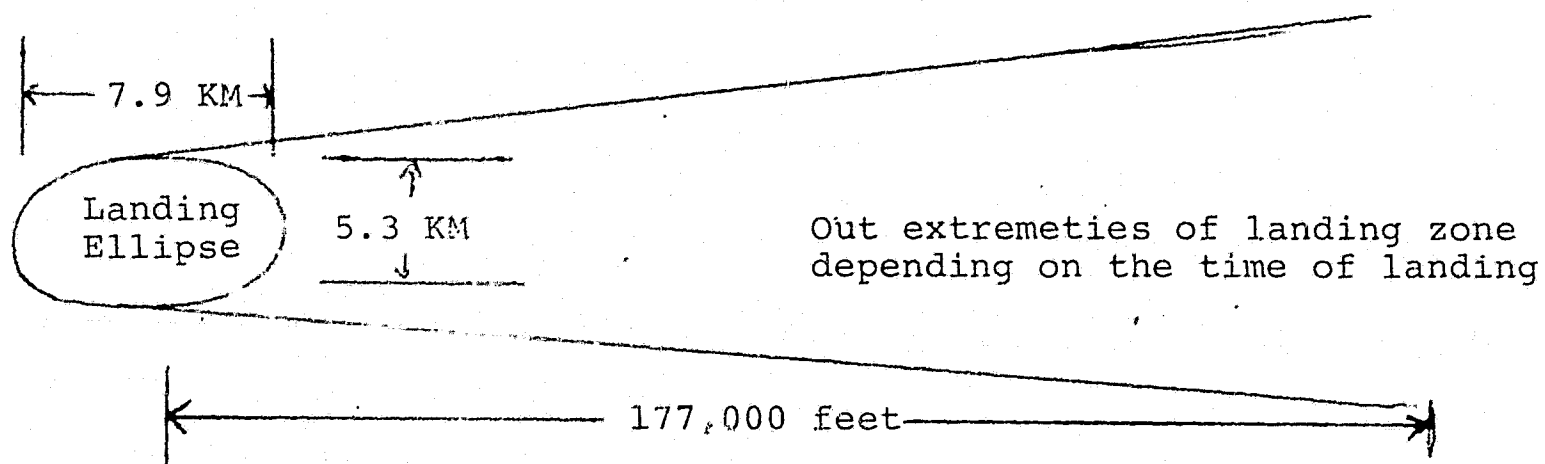


TRAJECTORY #2 OVER TARGET #2 $V_s = 6850$



3.2.2 Lunar Surface Modeling

The LM radar beam covers a distance of 177,000 feet on the lunar surface during its final journey. It approaches from the east and is expected to land on a pre-selected probability ellipse with major and minor axes of 7.9 and 5.3 km respectively, as shown below:



It is assumed that the vehicle is going to land near the center of the landing ellipse and that more detailed data is necessary for this area. Therefore, scale models are designed for the ellipses, while the rest of the approaching track is simulated at smaller scale.

The lunar surface roughness is assumed to be composed of a random component ξ_R and a more or less recurrent component ξ_p .

Random roughness :

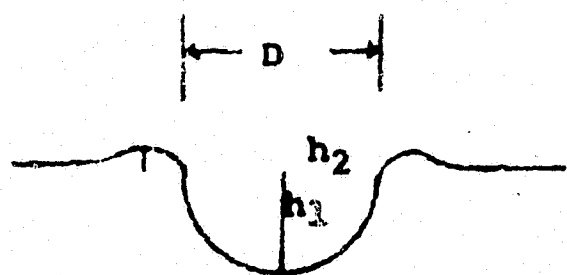
$$\langle \xi_R \rangle = 0$$

The criterion for roughness for vertical incidence is

$$\frac{\sigma}{\lambda} = .125 \quad \text{for vertical incidence} \quad [\text{Hayre 1962}]$$

The recurrent component of surface roughness ξ_p is made up of craters, crater rims, and hills as shown below by one typical segment of it

Crater

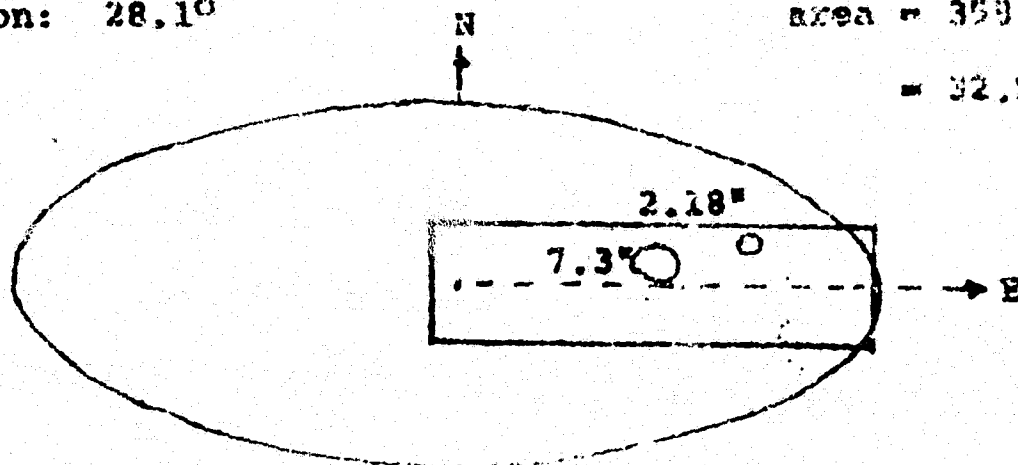


$$D = ah_1 = bh_2$$

where $\langle a \rangle \geq 3$ and $\langle b \rangle = 30$

Sun elevation: 28.1°

$$\begin{aligned} \text{area} &= 359 \text{ K}^2 \text{ ft}^2 \\ &= 32.9 \text{ K}^2 \text{ m}^2 \end{aligned}$$



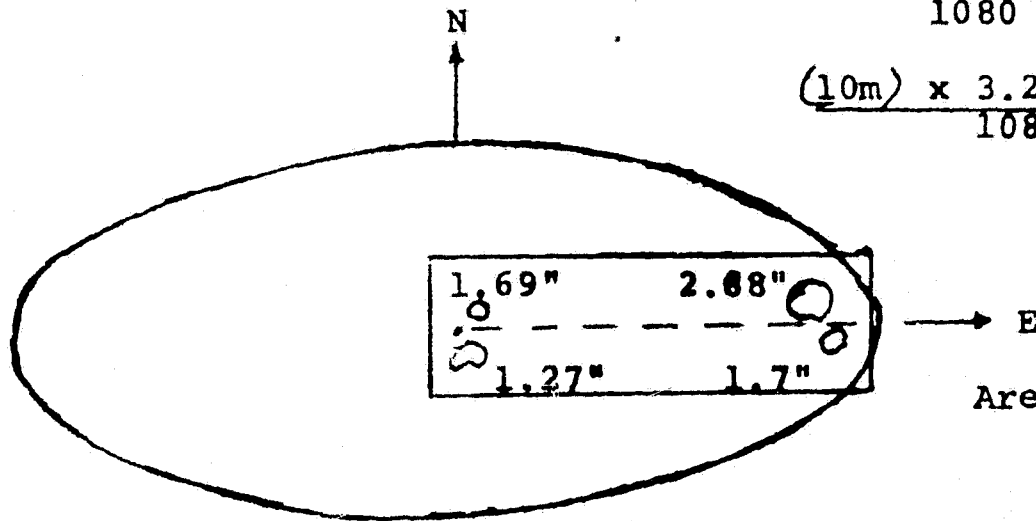
Site II P-8

Scale: 1080:1

Sun elevation: 27.8°

$$\frac{(\text{KM}) \times 3281}{1080} = 3.03 \text{ ft/Km}$$

$$\frac{(10\text{m}) \times 3.281 \times 12}{1080} = .363 \text{ in/10m}$$



$$\begin{aligned} \text{Area} &= 358\text{K}^2\text{ft}^2 \\ &= 32.9\text{K}^2\text{m}^2 \end{aligned}$$

Significant Hazards

Depth	Diameter	Mean	h'	h ₂	Number	Area	% Area
19m	51m		17m	1.7m	1		
19m	1.85"		.62"	.062"	1		
35m	95m		31.7m	3.17m	1		
	3.44"		1.14"	.114"			
47m	118m		39m	3.9m	1		
	4.25"		1.4"	.14"			
74m	200m		67m	67m	1		
	7.3"		2.43"	.243"			
<u>Others</u>							
11.4m → 22.8m		17.1	.12" → .275"	.012" → .0275"	11,632	2.56K ² m ²	7.8
.41" → .825"		.62"					
22.8m → 34.2m		28.5	.275" → .41"	.0275" → .041"	610	.423K ²	1.3%
.825" → 1.24"		1.03"					
34.2m → 45.6m		39.9	.41" → .55"	.041" → .055"	215	.266K ²	.8%
1.24" → 1.66"		1.45"					

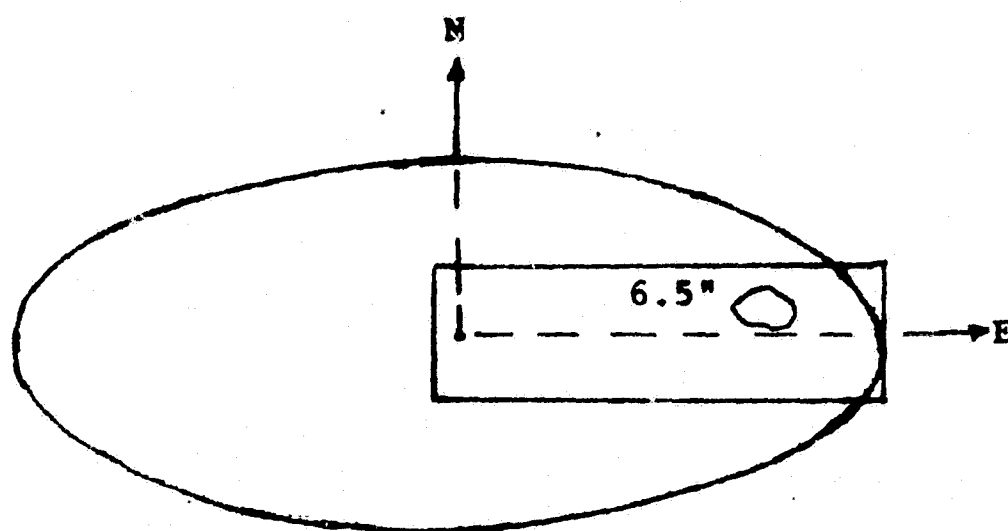
Site II P-11

Sun Elevation 28° Area = $388K^2ft^2$ = $32.9K^2m^2$

Scale 1080:1

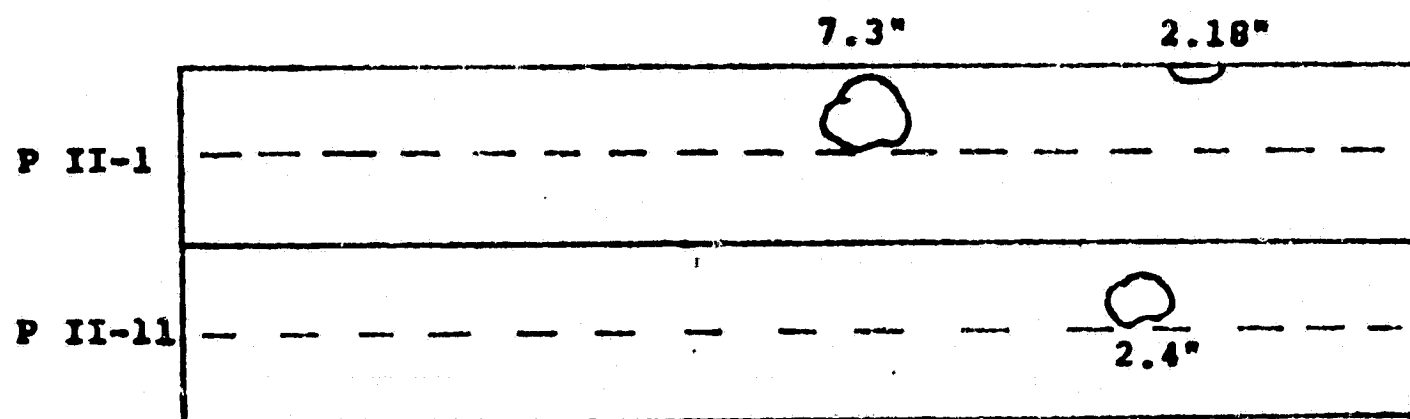
$$\frac{(KM) \times 3281}{1080} = 3.03 \text{ ft/Km}$$

$$\frac{(10m) \times 3.281 \times 12}{1080} = .363 \text{ in/10m}$$

Significant Hazards

Depth	Diameter	Mean	h_1	h_2	Number	Area	% Area
66m 2.4"	180m 6.5"		60m 2.18"	6m .218"	1		
<u>Others</u>							
	11.4m → 22.8m .41" → .825"	17.1m .62"	.12" → .275"	.012" → .0275"	8840	$2K^2m^2$	6.1%
	22.8m → 34.2m .825" → 1.24"	28.5m 1.03"	.275" → .41"	.0275" → .041"	324	$.21K^2$	.64%
	34.2m → 45.6m 1.24" → 1.66"	39.9m 1.45"	.41" → .55"	.041" → .055"	139	$.18K^2$	.55%

Further study of the elliptical landing zone and its modeling suggests that Sites P II-6 and P II-11 approach areas be modeled on separate portions on the same 4' x 12' frame without their respective features overlapping throughout the entire radar beam coverage.

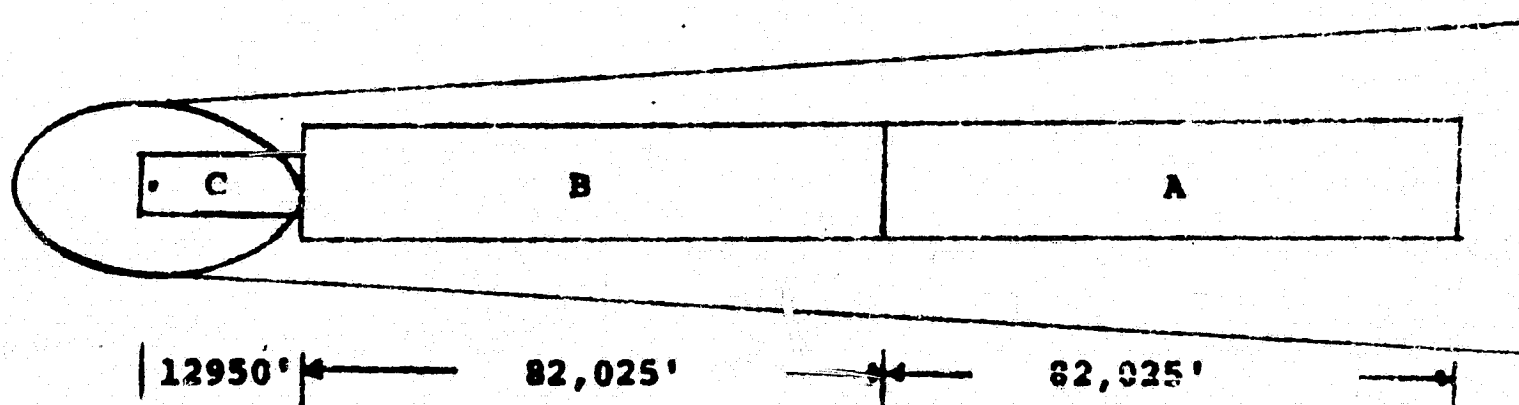


The significant hazards within the elliptical sites will be modeled using linear scale factor techniques whereas the other smaller craters with their size and location randomly distributed over the entire model plane will be modeled with similar distribution of protuberances in a random fashion.

3.2.3 Approaching Radar Track

The approach zones which extend approximately 164,050 feet are simulated by two sections, each 82,025 feet in length, and modeled by 4' x 12' surface each, as shown below.

Scale: 6850:1



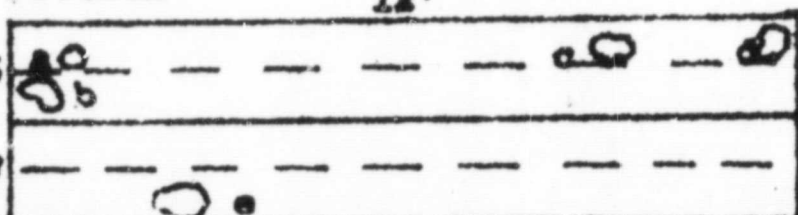
A close study of the medium resolution pictures of the three selected sites indicates that Sites P II-6 and P II-8 should be modeled on the same plane. Since the roughness of the sections A and B of these two sites are dotted sections by craters while site P II-11 is hilly. Therefore Site P II-11 has to be modeled separately.

A & B Section

12'

Scale: 6850:1

P II-6



P II-8

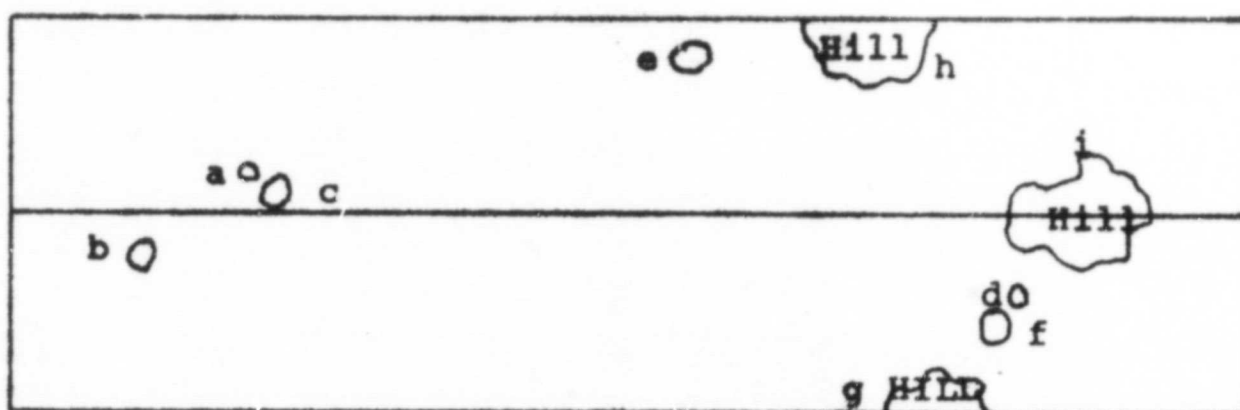
$$\frac{(KM) \times 3281}{6850} = .48 \text{ ft/Km}$$

$$\frac{(10m) \times 3.281 \times 12}{6850} = .057 \text{ in/10}$$

P II-6 Hazards

	Depth	Diameter	h_1	h_2
A.	41m	112m	37m	3.7m
	235"	.64"	.21"	.021"
B.	54m	148m	49m	4.9m
	.31"	.85"	.28"	.028"
C.	106m	280m	96.5m	9.65m
	.61"	1.6"	.55"	.055"
D.	178m	375m	125m	12.5m
	.79"	2.15"	.72"	.072"
<u>P II-8 Hazards</u>				
E.	105m	285m	95m	9.5m
	.6"	1.63"	.54"	.054"

B Section of P II-11

Site P II-11 Hazards

Scale 6850:1

.48 ft/Km
.057 in/10m

	Depth	Diameter	h_1	h_2
A.	47m	128m	42.5m	4.25
	.21"	.738"	.24"	.024"
B.	49m	134m	45m	4.5m
	.28"	.77"	.26"	.026"
C.	69m	188m	62.5m	6.25m
	.385"	1.07"	.356"	.0367
D.	78m	213m	71m	7.1m
	.45"	1.22"	.4"	.04"
E.	94m	256m	85m	8.5m
	.54"	1.47"	.49"	.05"
F.	133m	360m	120m	12m
	.76"	2.06"	.69"	.07"

Hills

G. 261m 1.5"
H. 286m 1.64"
I. 320m 1.83"

SIGNIFICANT HAZARDS

Depth	Diameter	h_1	h_2	Number
24m	65.5m	21.7m	2.17m	1
.87"	2.18"	.73"	.073"	
74m	200m	67m	6.7m	1
2.68"	7.3"	2.43"	.243"	

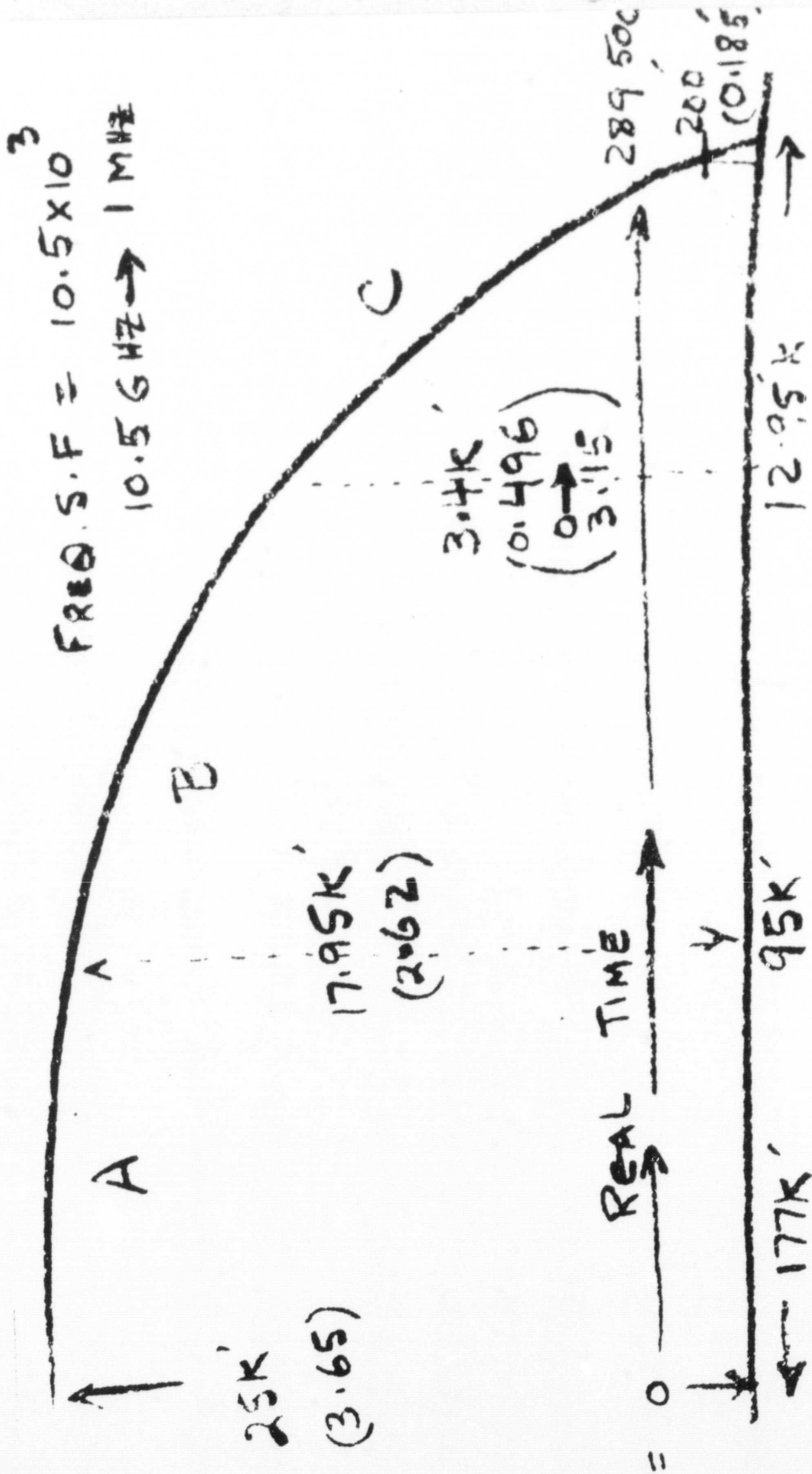
OTHER CRATERS

Diameter	Mean	h_1	h_2	Number	Area	% Area
11.4m → 22.8m	17.1m			5345	$1.23K^2m^2$	3.75%
.41" → .825"	.62"	.12" → .275"	.012" → .0275"			
22.8m → 34.2m	28.5m			289	$-2K^2m^2$	.6%
.825" → 1.24"	1.03"	.275" → .41"	.0275" → .041"			
34.2m → 45.6m	39.9m			92	$.115K^2m^2$	.35%
1.24" → 1.66"	1.45"	.41" → .55"	.041" → .055"			

The following is a complete summary of sites and their major characteristics and scale factors, etc.

ALTITUDE		DISTANCE		SCALE FACTORS		
AIR (FT)	MODEL (IN)	AIR (FT)	MODEL (IN)	DISTANCE	VELOCITY	DOPPLER
25K	43.79	177K	144.74	6850	6×10^3	300 A
		136				
17.95K	31.45	95K	0			
		54	144.74	6850	6×10^3	300 B
3.4k	5.95	12.95k	0			
3.4k	37.7	6.47k	143.89	1080	1.56×10^3	78 C
200	2.22	195	2.17			

TRAJECTORY



d.s.f 6850 1080
 f.s.f 300 78
 6×10^3 1.56×10^3

DOPPLER SCALE FACTS

$$f_{da} = f_{ca} \frac{2 v_a}{c_a} \cos \theta$$

$$f_{dw} = f_{cw} \frac{2 v_w}{c_w} \cos \theta$$

$$f_{da} / f_{dw} = (f_{ca} / f_{cw}) (v_a / v_w) \left(\frac{c_w}{c_a} \right)$$

$$= (10^{10} / 10^6) (3 \times 10^8 / 1.5 \times 10^3)^{-1} (v_a / v_w)$$

$$= 10^4 \times (2 \times 10^5)^{-1} (v_a / v_w)$$

$$= 5 \times 10^{-2} (v_a / v_w)$$

Zone:

$$C = 5 \times 10^{-2} \cdot 1.56 \times 10^3 = 78$$

$$A, B = 5 \times 10^{-2} \cdot 6 \times 10^3 = 300$$

MODELS

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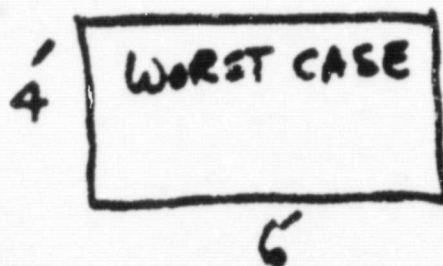
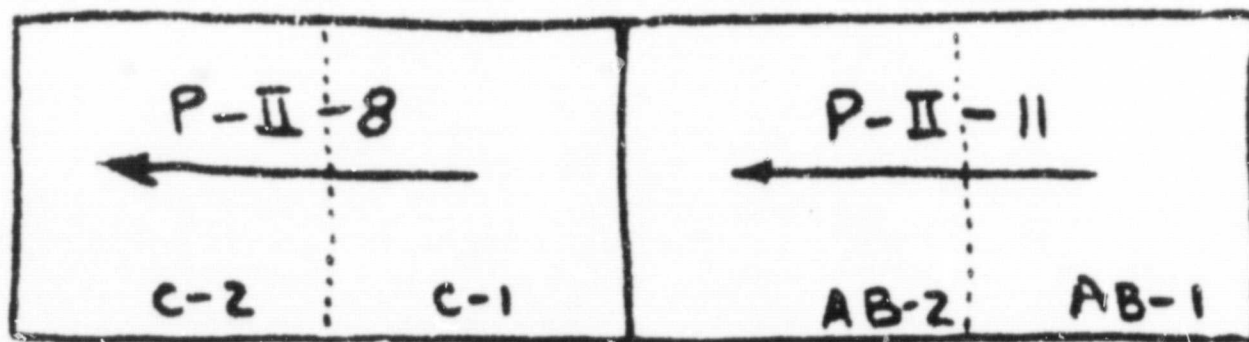
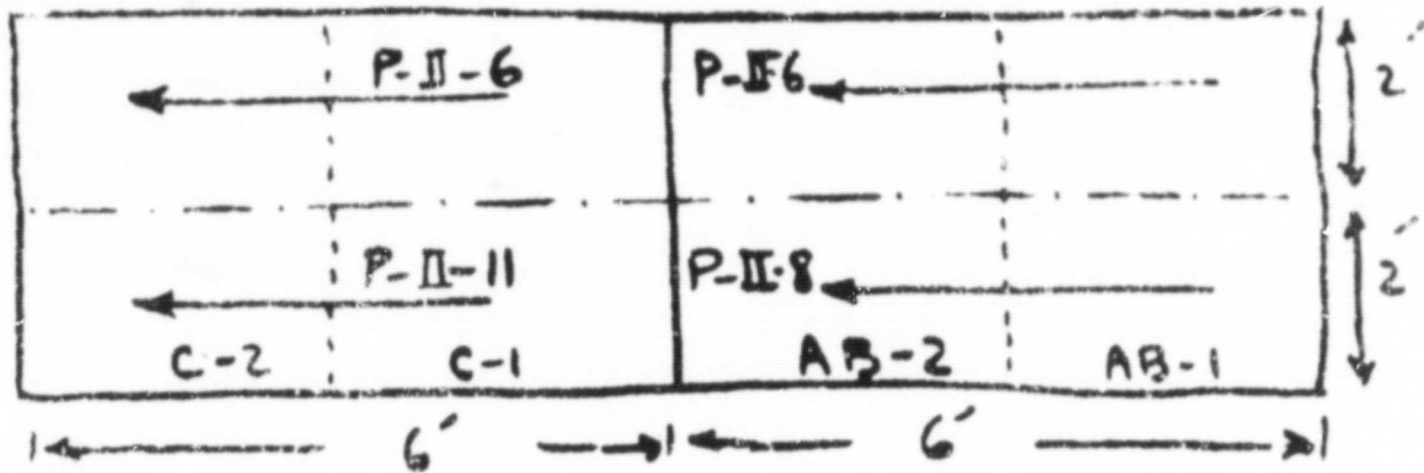


Figure 3-12

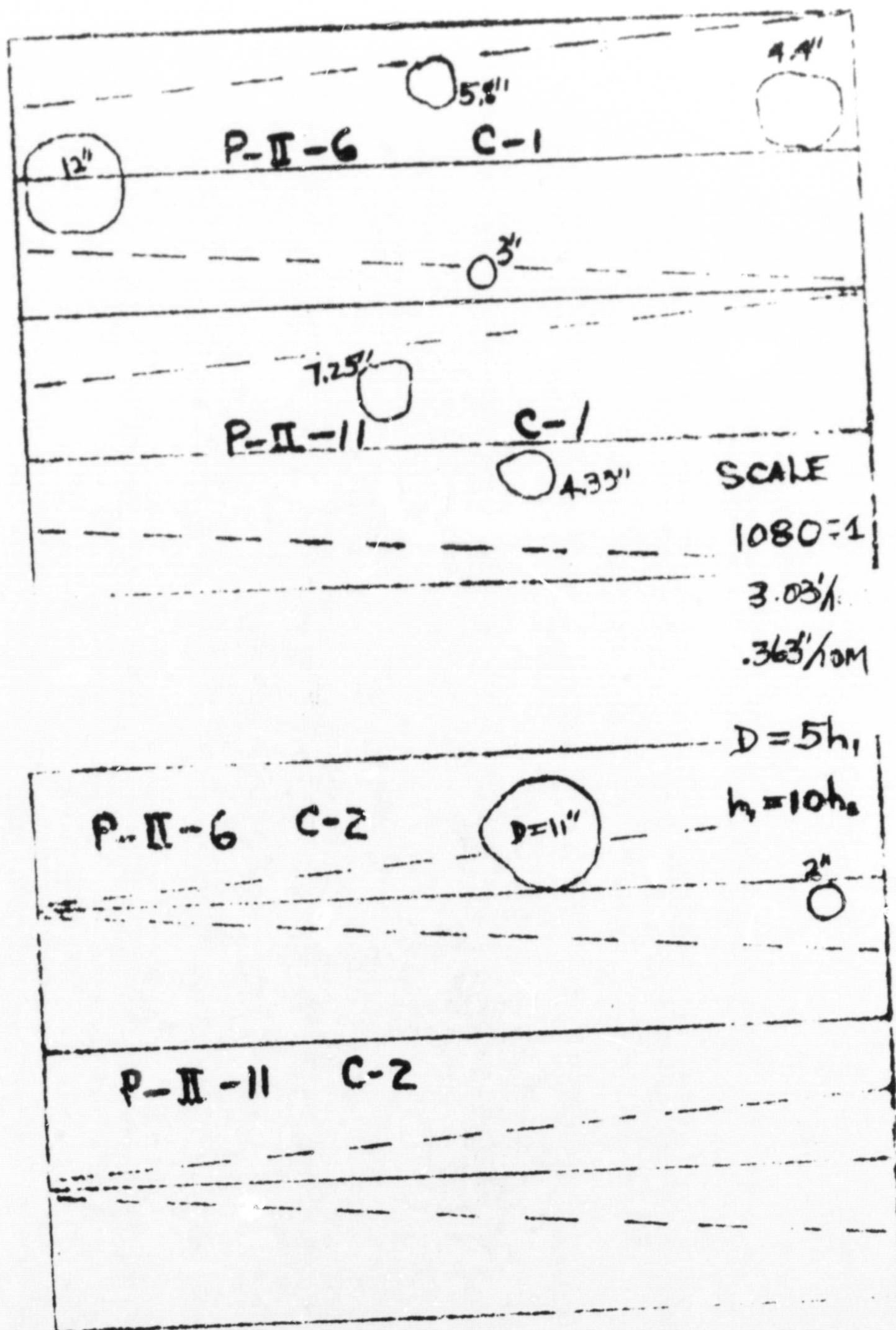


Figure 3-13

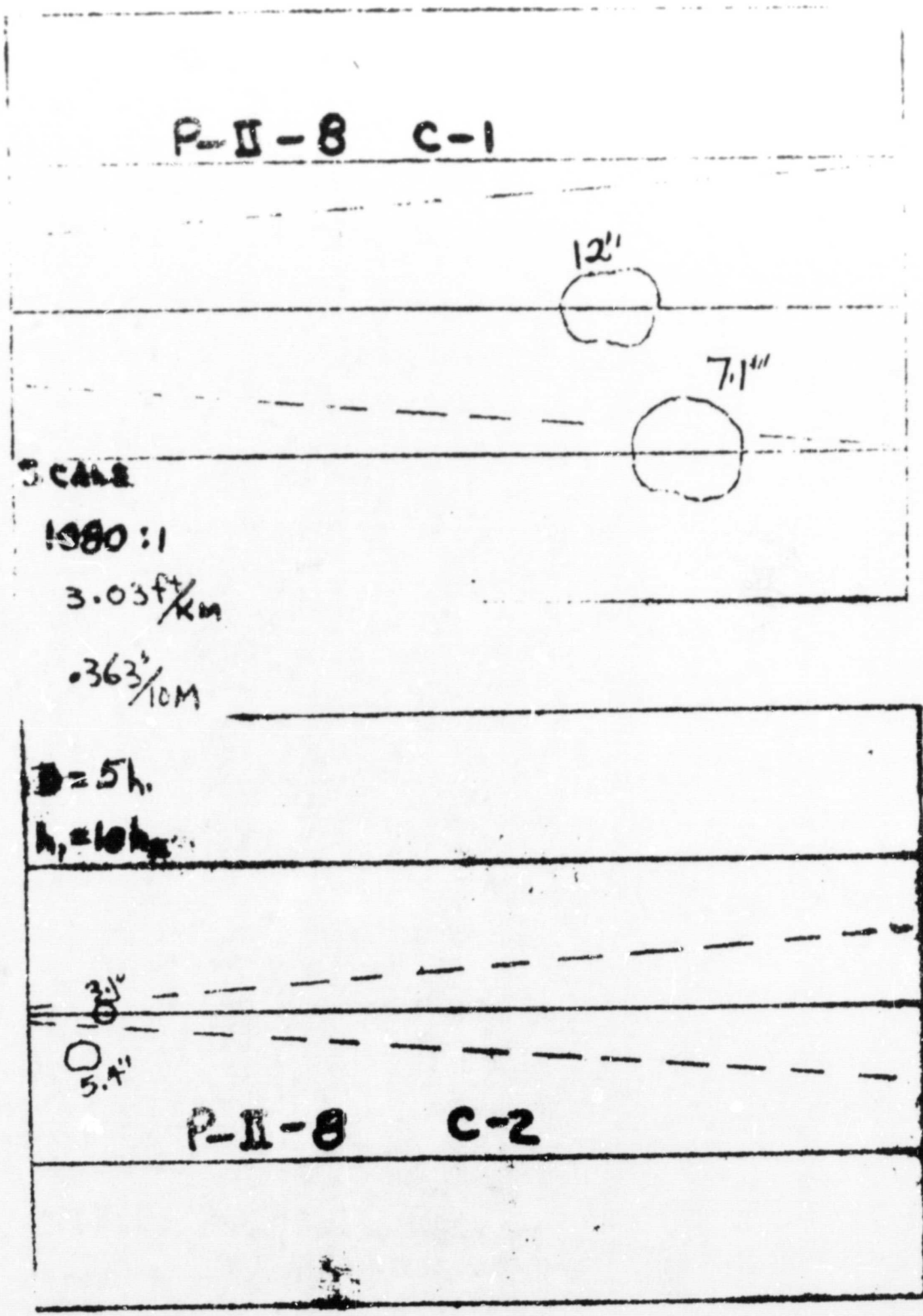


Figure 3-14

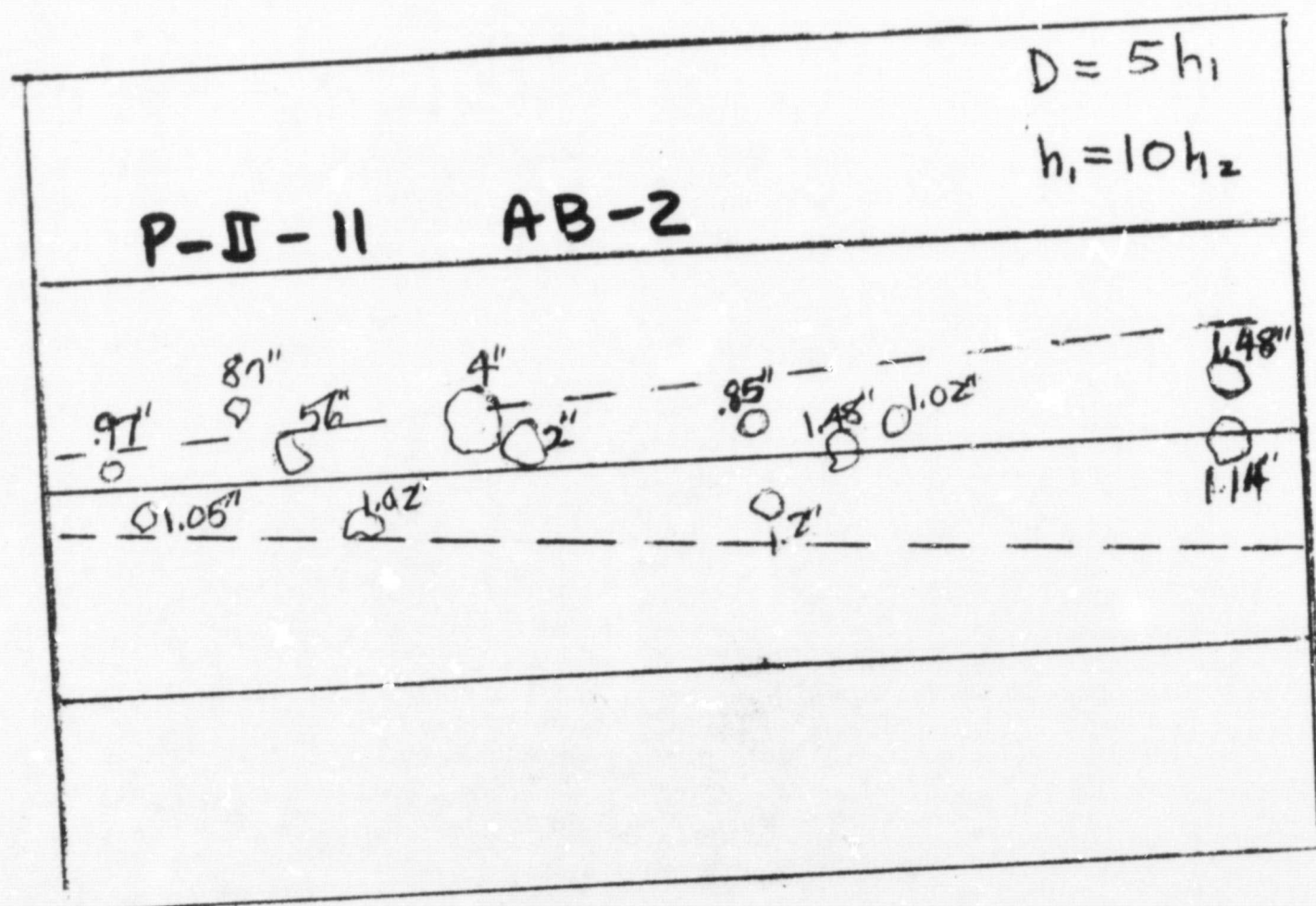
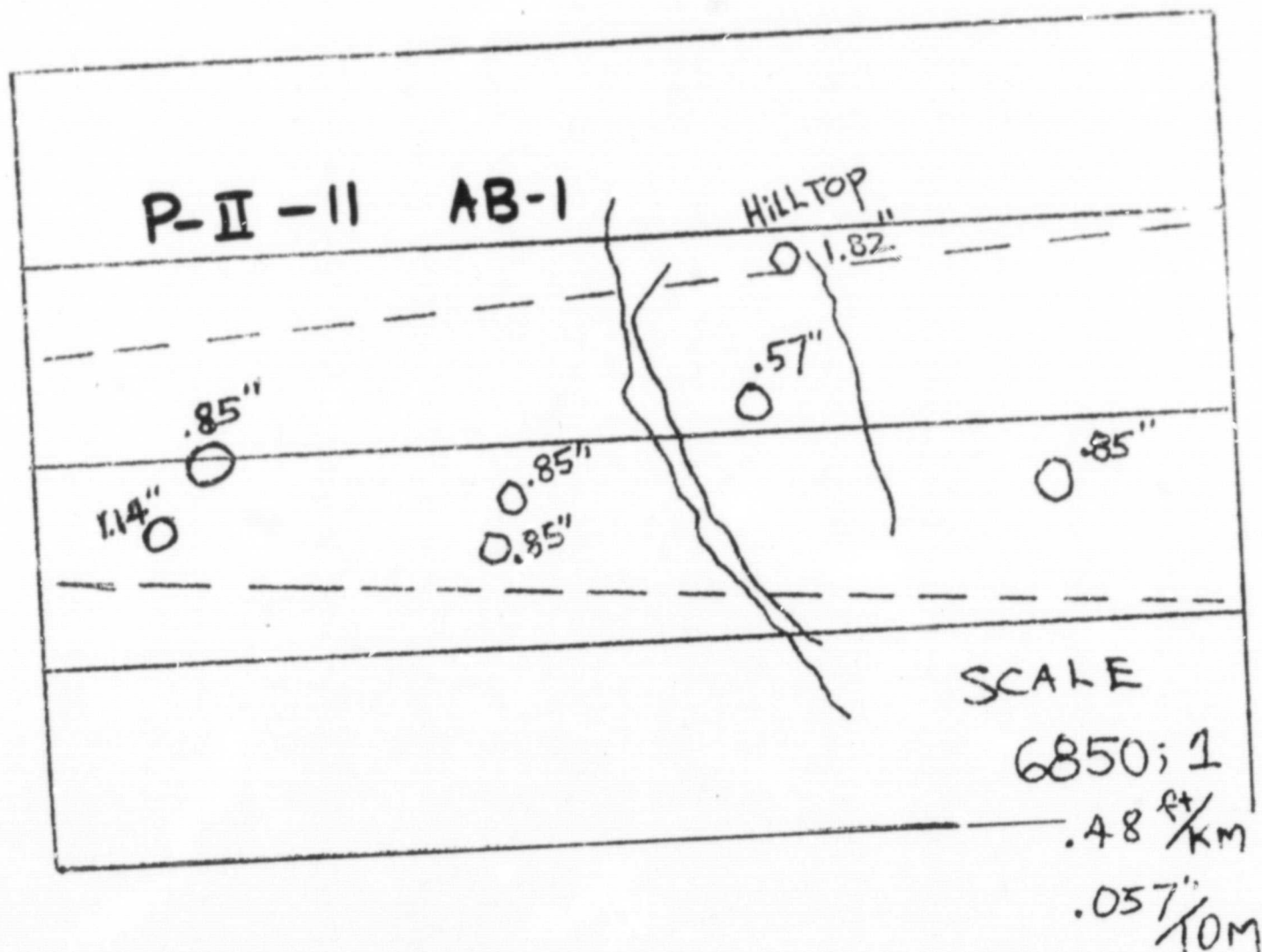


Figure 3-15

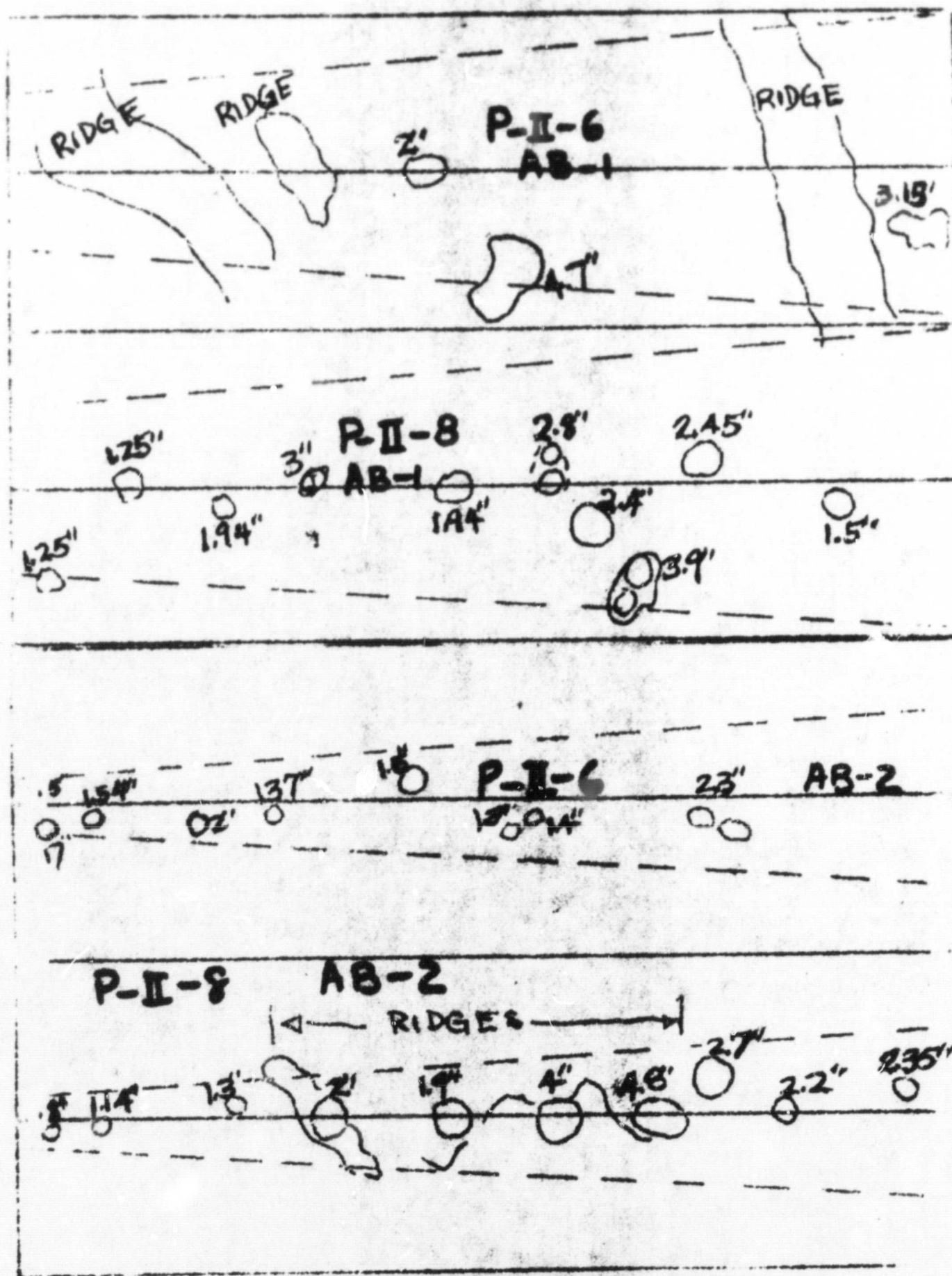


Figure 3-16

$$D = 5 h_1$$

$$h_1 = 10 h_2$$

3.3 The Worst Case Lunar Surface Model

The lunar surface features are known to be of many different geometrical shapes, such as:

- a) Rills (valley)
- b) Ridges
- c) Hills
- d) Craters
- e) Boulders
- f) Ridge intersecting craters
- g) Multiple craters, and
- h) Boulders or rocks inside craters

It is therefore desirable to design a surface model consisting of all the above mentioned elements in such an arrangement as to bias the altimeter radar signal in the worst possible way. Such a model shall be constructed for ultrasonic simulation of the landing module radar signal return.

The reflection data obtained from this mode of very rough surface will be compared with that obtained from the landing-site models in order to check the validity of the existing LM landing radar simulation system now under progress.

Furthermore, a set of scattering patterns for a range of incident angles shall be obtained for correlation analysis in order to establish a criteria to indicate the presence of certain rough features.

A sketch of a possible worst case lunar surface model is shown in Fig 3-17 where the most significant features are the rill and ridge which run side by side. This arrangement shall be used to examine how the altimeter radar signal would be backscattered from such widely different variety of reflectors and their inter-structures as the corner reflector formed by the intersection of the rill and the ridge.

The biasing of the altimeter radar signal depends on the surface roughness, the radar altimeter antenna orientation and the range from the lunar surface. Fig 3-18 shows the radar beam coverage of the lunar surface as the vehicle is descending along the trajectory in the form of the areas covered by the radar beam in one averaging period which covers 30 frequency modulating cycles. This information is necessary in order to determine the importance of the hazardous features existing along the landing approach of the landing module trajectory.

0 200 m 400 m

1000000

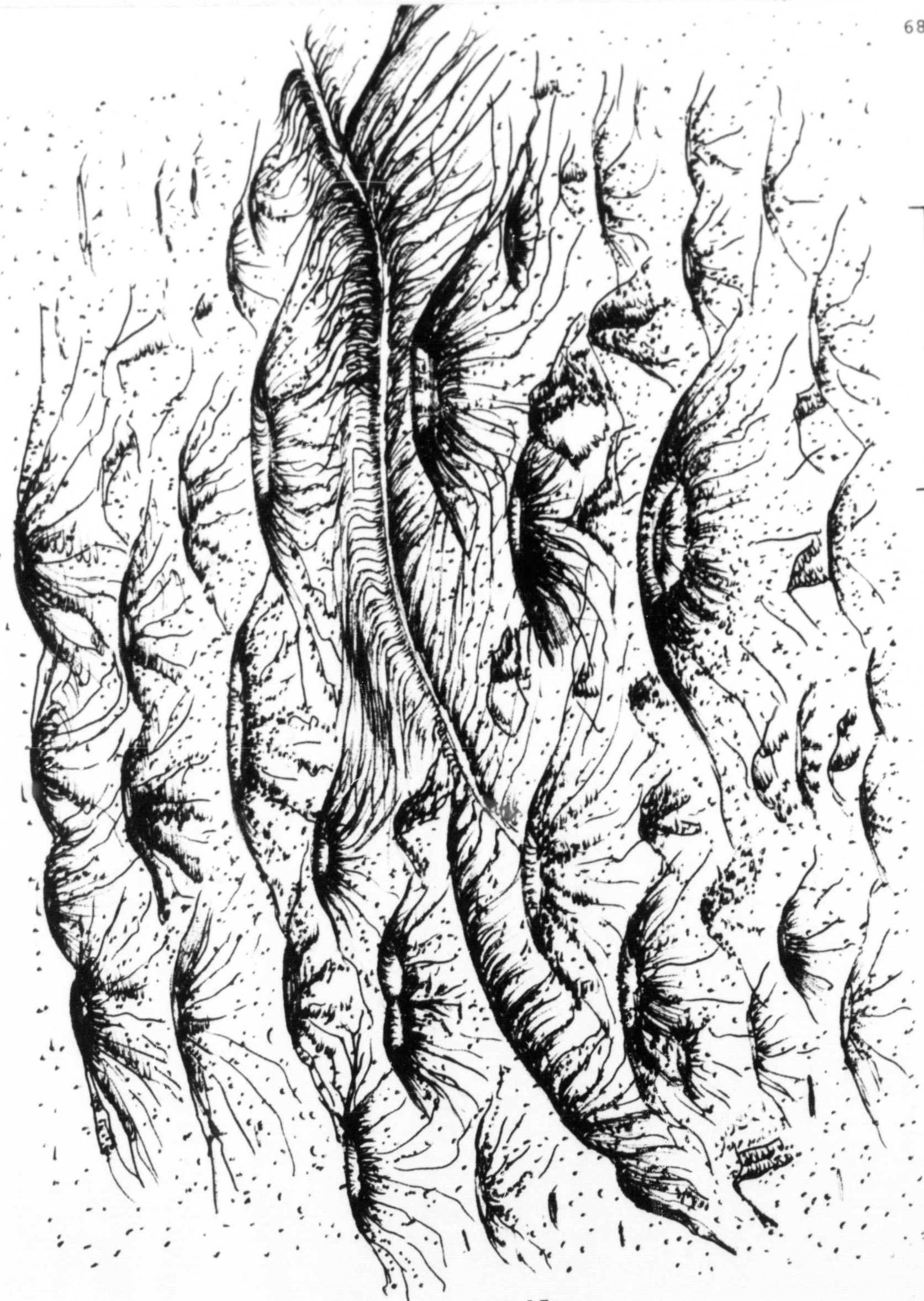


Fig. 3-17

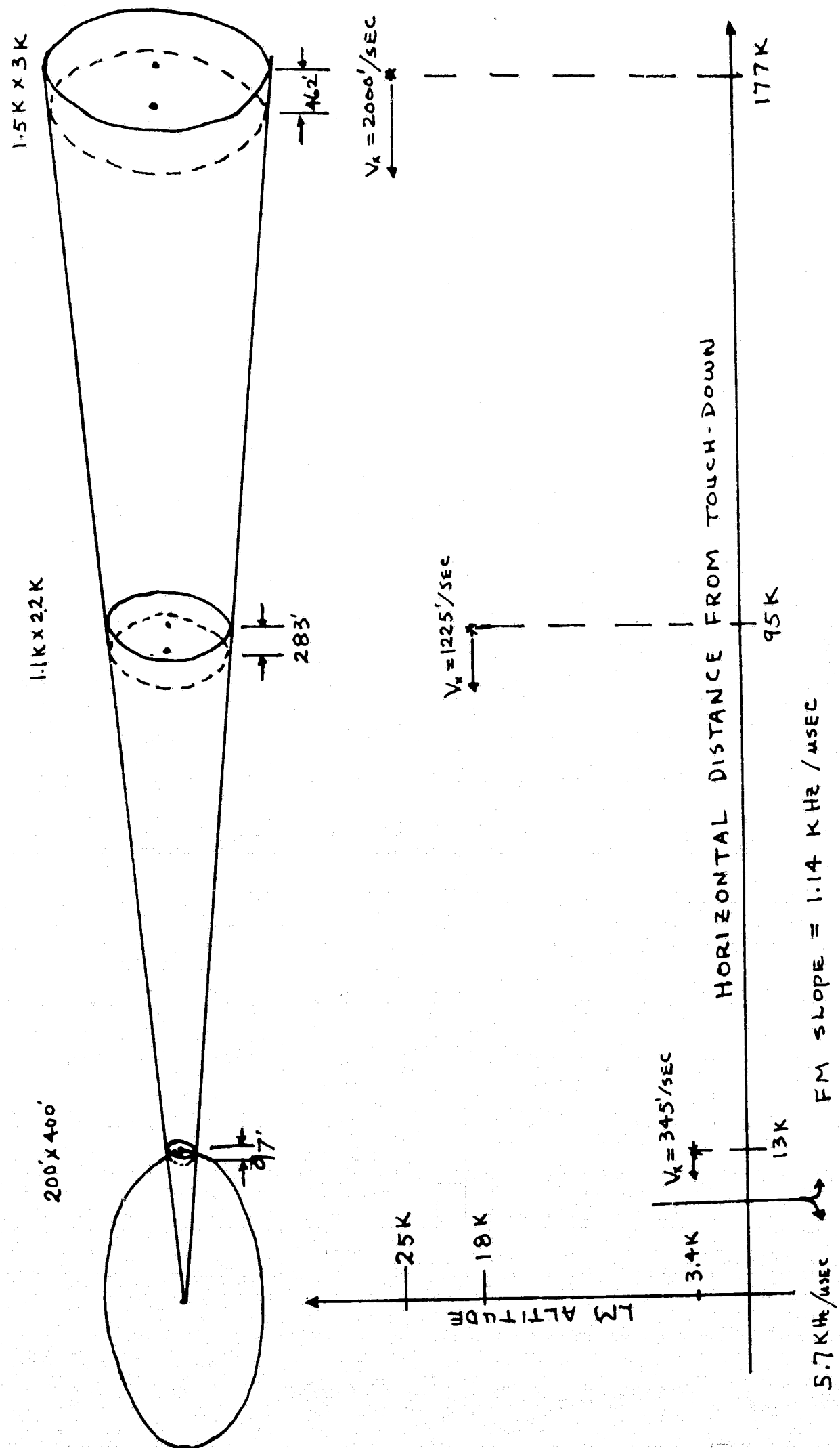


Fig. 3-18

CHAPTER IV

RADAR RETURN FROM LUNAR SURFACE

4.1 Radar Reflectivity Model for LM

Previously, lunar surface models have consisted of randomly varying surface heights and isotropic surface height distance correlation functions or some slope distribution with isotropic property of sectional correlation functions, and finally a superposition of various types of surfaces such as, for example, small and large scale. These models do not seem to represent the craters which are superimposed over the random height variations. It is this aspect of rimmed craters superimposed on the two-dimensional distribution of surface height with each component surface having its own statistical parameters and yet being statistically independent of each other which is used to calculate the monostatic radar return for LM system at various angles of incidence. The unique features of this surface model are that a random variation of the period between the rimmed craters allows their realistic spatial distribution as well as size distribution which has been found to be exponential from Orbitor I and II photographic data of medium and fine resolutions. For near vertical incidence the return power is calculated using scalar Helmholtz integral and then reduced to a few special cases such as when the randomly varying main crater diameter tends to zero and the random period or distance between crater

centers to infinity. The results seem to indicate that this model may be used to generate almost all types of randomly rough surfaces.

4.2 Lunar Landing Module Reflectivity Model

Previous lunar surface models consisted of randomly varying surface heights with isotropic surface height spatial correlation functions or sectional correlation functions or some other surface slope distribution. Furthermore, a superposition of various types of surfaces such as small and large scale roughness were also employed. These models do not seem to represent explicitly the lunar craters which are superimposed on the random height variations. It is this aspect of rimmed craters superimposed on a two-dimensional distribution of surface heights with each component surface being a random variable but statistically independent of each other that is used to calculate the monostatic radar return for the Lunar Landing Module guidance and control radars at various angles of incidence. The unique feature of this surface model is that a random variation of the spacing between the rimmed craters allows a realistic spatial distribution as well as size distribution which has been found to be exponential from Orbitor I and II photographic data of medium and fine resolutions. The near-vertical incidence radar return power is calculated using a scalar Helmholtz integral and the results are then reduced to a few special cases such as

one generated when the randomly varying main crater diameter tends to zero and the random period or distance between crater tends to infinity. The results seem to indicate that this model may be used to generate a wide variety of randomly rough surfaces for a calculation of their radar return.

4.2.1 Summary

An electromagnetic wavefront incident on an obstacle induces a charge thereon which causes each point on its surface to radiate secondary waves in all directions. Each point acts as a Huygens source and this phenomena is termed scattering, and physical optic techniques may be used in order to analyze the scattering. The re-radiation from a surface illuminated by an electromagnetic wavefront depends on such surface properties as its permittivity, permeability and conductivity. In theory, a complete knowledge of the surface parameters and its roughness would yield its scattering pattern for a given source of the incident energy, although the calculation procedure is very complicated in the case of a randomly rough surface. Therefore, certain assumptions given below must be made to simplify the mathematical manipulation and to ensure the validity of the resulting solution.

- a) The incident wave is monochromatic
- b) The polarization of the incident wave is linear

- c) The distance between the source and the reflecting surface is large compared to the surface dimension, so that the wavefront may be treated as a plane wave, and
- d) The edge effect, shadowing, and multiple scattering are assumed negligible.

In spite of these restrictions and assumptions, a complete solution of the problem of scattering has not been found as yet although considerable work has been done in the area by physicists and engineers for the last twenty-five years.

Theoretically, a terrain may be thought of as a randomly spaced array of scatterers which yield a certain radar scattering pattern, and conversely, knowing the scattering pattern, an unknown terrain may be specified. This technique of surface roughness detection by employing radar backscattering cross-section has attracted considerable interest in recent years.

The terrain illuminated by a radar is quite often randomly rough, therefore it is necessary to use probability techniques in its description; consequently, the radar return obtained from it is also described statistically. The present investigation of the lunar surface radar return assumes that it is a perfectly conducting surface at high

Frequencies. The Fresnel reflection coefficient for horizontally polarized plane wave is

$$R^- = \frac{\cos \vartheta - \sqrt{\gamma^2 - \sin^2 \vartheta}}{\cos \vartheta + \sqrt{\gamma^2 - \sin^2 \vartheta}} \quad (4-1)$$

where ϑ is the local angle of incidence and

$$\gamma = \sqrt{\frac{\frac{\epsilon}{\epsilon_0} + i 60 \lambda c}{\frac{\mu}{\mu_0} + i \frac{L_m}{\mu_0}}} \quad (4-2)$$

as shown in Fig. 4-1, where various terms are defined. For a perfectly conducting surface, c and γ are infinite and

$$R^- = -1. \quad (4-3)$$

The early lunar models were based on the earth-based lunar radar echoes, and some investigators such as Senior and Siegel (1959) regarded the moon as a quasi-smooth scatterer. A hypothetical surface model consisting of corner reflectors, cone-like projections of small included angle, and flat areas, was also used by them to calculate the radar scattering cross-section, employing an approximate theoretical form of the power reflection coefficient.

Katz and Spetner (1960) presented two statistical models of rough surfaces, namely the Specular-Point and Random-Scatterer model. In the Specular-Point model, the rough surface was assumed to be a perfect conductor and the surface height function was assumed to be not only continuous but also to have continuous derivatives with gaussian distribution of surface slopes. The radar return from the

surface was supposed to be predominantly contributed by large facets oriented perpendicular to the incident radiation, and remaining contributions coming from small isotropic scatterers.

In the Random-Scatterer model, the radar return was assumed to be generated mainly by a large number of statistically independent and incoherent scatterers, such as a random array of corner reflectors, distributed upon a non-reflecting plane surface.

Brown (1960) assumed the lunar radar echo to be made up of specular and Lambert scatter components. And the lunar surface was interpreted to be made up of two parts, one consisting of a statistical array of mirror-like reflectors having exponential probability density function and causing the specular return component and the random roughness contributing to the Lambert scatter component of the return power.

Hayre (1962) used a lunar model which consists of randomly varying surface heights (gaussian density function), surface slope distribution and the height-distance autocovariance function which were calculated for various terrains. These statistical functions are considered to describe a rough surface sufficiently well. Later in 1965, Hayre, based on the above statistical parameters, suggested a composite roughness to describe the lunar surface. The

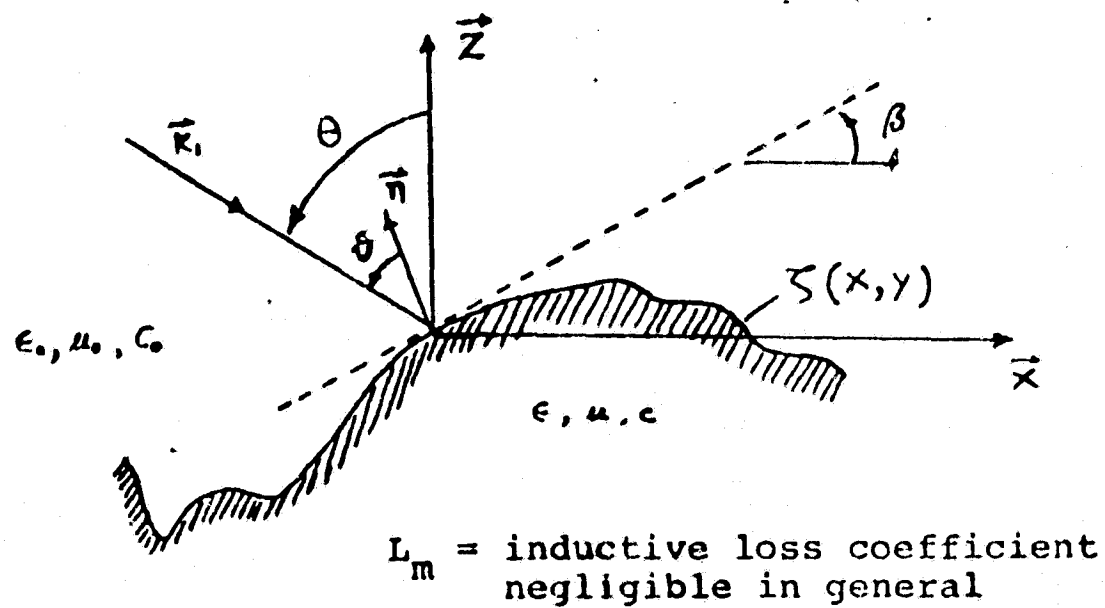


Fig. 4-1. The Local Scattering Geometry.
 ($\vec{n}$ is the local normal. ϑ is the local angle of incidence and θ_1 is the overall angle of incidence defined with respect to $\vec{z}$.)

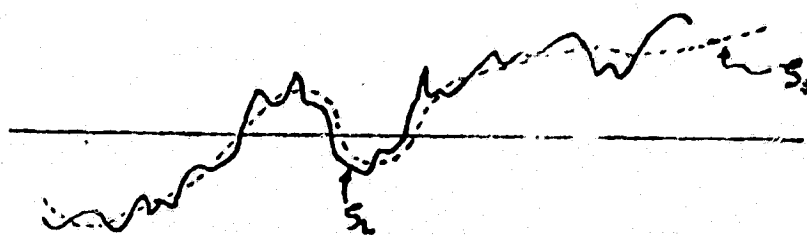


Fig. 4-2. A Rough Surface Composed of Small-Scale ξ_s and Large-Scale Roughness ξ_L .

superposition model considers a slowly varying large-scale roughness and a rapidly varying small-scale roughness within the limits of the validity of the Kirchhoff integral approach, and it is assumed that they are correlated (Fig. 4-2). The interdependency of the two rough profiles seems to represent a surface model in a more realistic manner.

A review of the lunar models of the past indicates that they were good estimates of the lunar surface roughness, as interpreted from the earth-based radar lunar echoes. However, these cannot represent such lunar surface features as craters and their rims for the LM radar reflectivity calculations.

The requirement of a high degree of accuracy in calculation of radar signal degradation by the lunar surface initiated the formulation of a new lunar model, which is discussed later. The flexibility and general nature of the model components and their statistical parameters suggested the use of the same approach for a description of many other rough features. Finally, a mathematical solution for the average radar return power including the lunar surface effects is presented.

4.2.2 Lunar Surface Modeling

The Apollo guidance and control signals during landing are primarily conditioned by the lunar surface roughness. An accurate analysis of lunar surface roughness in the radar-echo study provides the means to predict the effects of the former on the guidance and control commands which are generated by a computer on board the module after processing the radar return signals from the velocity and altimeter radars. The lunar surface roughness is expected to bias the lunar landing module (LM) radar signals used for guidance and control during its landing. It is this aspect of lunar radar echo-degradation of LM velocity and altimeter beams which is discussed in this paper.

Although the early scientific efforts had provided valuable information on surface roughness of the moon, the application of their data was limited for various reasons including the lack of full scale verification. Even the recently acquired earth-based radar information about the lunar surface was not sufficiently detailed, partly due to the libration of the moon and partly due to the fading caused by the inhomogeneous ionospheric medium.

The availability of a high-energy source for a high-frequency radar was in itself a problem. Consequently, radar

observations could not supply reliable evidence on the nature of the lunar surface structures of the order of a meter or smaller in dimensions. For example, according to Hayre's criterion (1961), a random rough surface is considered smooth for vertical incidence if

$$\sigma \leq \frac{\lambda}{12} \quad (4-4)$$

where

σ is the standard deviation of the surface undulation in meters, and

λ is the incident wavelength in meters.

If λ is 4 meters, a surface with a standard deviation of 0.3 meters is considered smooth. As a result, only the large scale roughness features could be obtained from the lunar radar echoes.

In 1961, Hayre and Moore conducted a laboratory experiment and acoustically simulated the lunar echoes. Nonlinear acoustic modeling techniques were used to model the lunar surface on a set of scaled down aluminum spheres. The acoustic return data was used to formulate a statistical model composed of a large-scale roughness and small-scale roughness with their reasonably complete statistical description including cross-correlation between these two components. The general Kirchhoff's solution with its usual assumptions was used in their theoretical model to obtain a mathematically

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manageable expression for radar cross-section. However, their model was not designed to describe such irregular and nonuniform surface features as lunar ridges in general, and craters and rims in particular.

Some other attempts have been made to model the lunar backscatter cross-section (Hayre, 1962, Muehlman, 1966). Muehlman based his results on two point radar data from Surveyor I. The simplified radar scattering cross-section equation includes weighted statistical parameters of the lunar surface such as standard deviation, the decorrelation constant and incident angle. Recent U. S. Ranger, Surveyors, and Orbiter I and II photographs of the surface of the moon show it to contain nonuniform roughness with its statistics varying from region to region. Thus, these previous models do not suffice for this study of degradation of radar signals for guidance systems of Apollo landing.

With the advent of the successive Orbiter missions, more detailed lunar surface photographs have now been obtained. Information extracted from the Orbiter photographs and their mosaic overlays is believed to be sufficiently complete to establish a set of roughness parameters corresponding to each particular lunar site. These parameters are the large-scale and small-scale random roughness, crater-size probability distribution with inter-relationship among the crater dimensions,

such as the diameter, depth and rim height and rim base width. Mosaic surface data from a number of preselected LM landing sites have been carefully analyzed and their roughness parameters seem to be statistically independent from one site to the next. Therefore, a surface model composed of uncorrelated random and periodic structure with random or fixed parameters is formulated.

The present model is a superposition of a gaussian random rough surface, and a periodic crater outline surface which can be made random as well. The random surface $\xi_R(x,y)$ is assumed to have a mean value of zero and variance of σ as shown in Fig. 4-3. Its autocorrelation function is assumed to be $\exp \frac{-r}{B}$ (Hayre, 1961) where r is the distance between two points measured on the X-Y plane, and B is the decorrelation distance. The periodic surface $\xi_p(x,y)$ is defined differently over three zones within a typical interval, as shown in Fig. 4-4 and 4-5.

In Zone I

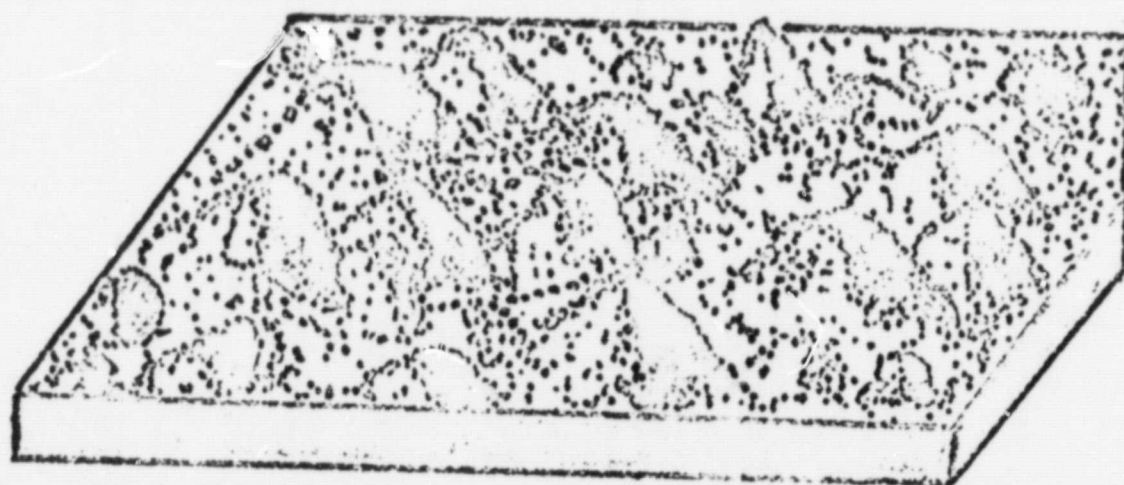
$$\xi_p(x,y) = 0 \quad \text{for } x^2 + y^2 > d_2^2 \quad (4-5)$$

In Zone II

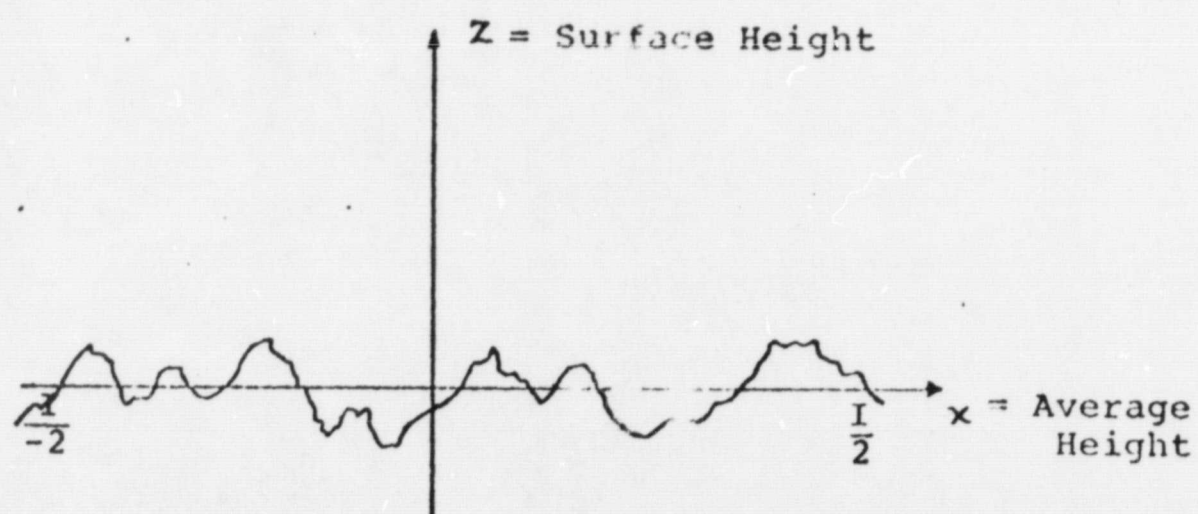
$$\xi_p(x,y) = h_2 \sin \frac{\pi(\sqrt{x^2 + y^2} - d_1)}{d_2 - d_1} \quad \text{for } d_1^2 < x^2 + y^2 < d_2^2 \quad (4-6)$$

In Zone III

$$\xi_p(x,y) = -h_1 \cos \frac{\pi \sqrt{x^2 + y^2}}{2d_1} \quad \text{for } x^2 + y^2 < d_1^2 \quad (4-7)$$



Three-Dimensional Representation



Profile Cross-Section Along an Arbitrary Line

Fig. 4-3. A Random Surface Component.

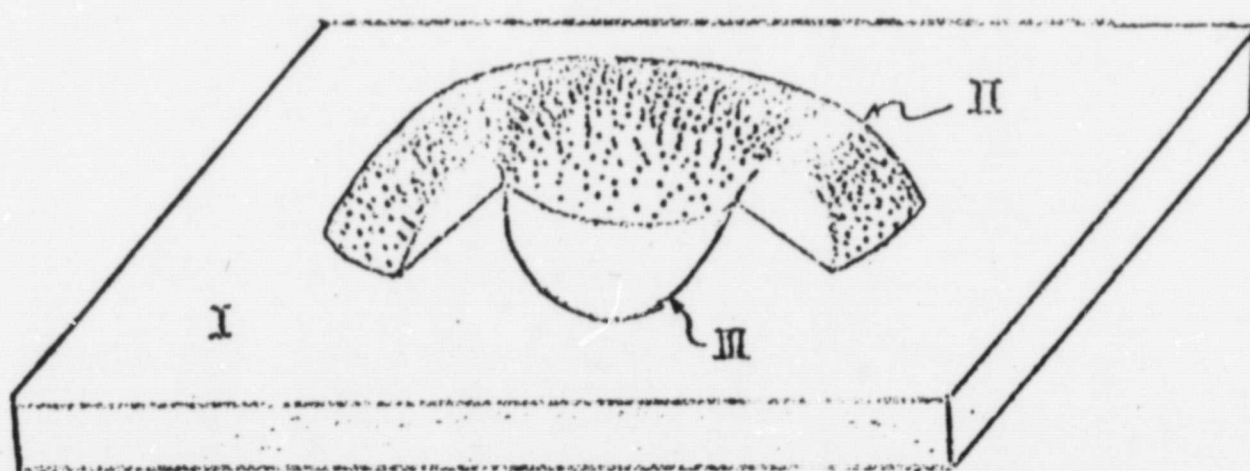


Fig. 4-4. A Periodic Surface Component

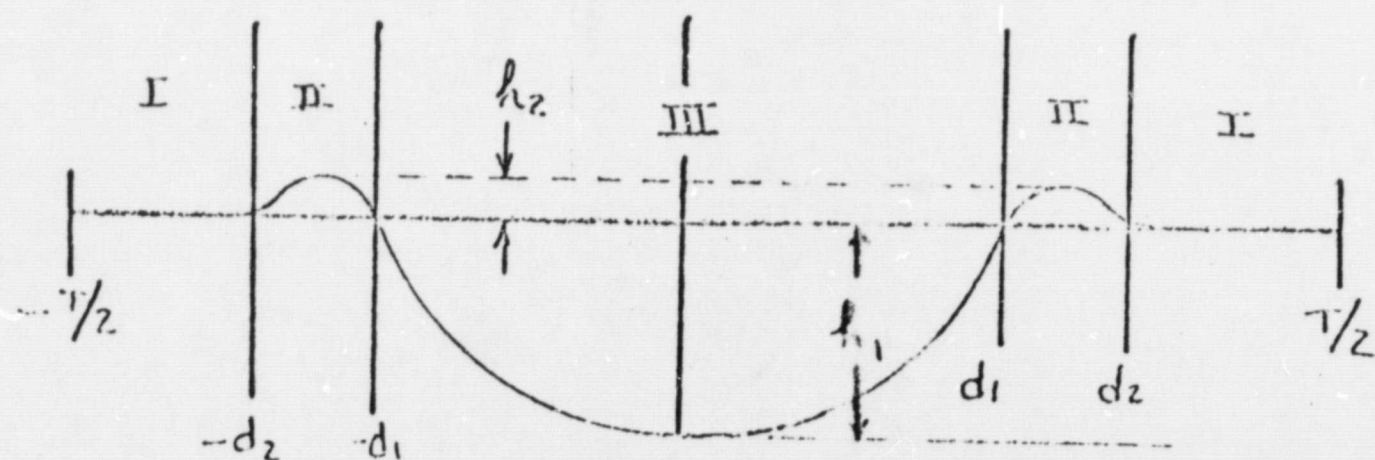


Fig. 4-5. A Cross-Section View of the Periodic Surface

The component ξ_R represents the general small-scale or large-scale roughness, while the component ξ_P may be designed to represent craters, boulders, and hills. An appropriate selection of the variables d_1 , d_2 , h_1 , and h_2 , and the combination of ξ_R and ξ_P may be devised to fit the description of most types of lunar surface features. For instance, by varying σ/λ and θ/λ , ξ_R may be varied from smooth to a very rough surface.

If h_1 and h_2 approach zero, the surface ξ_P becomes mirror-like. Similarly, a suitable choice of h_1 , h_2 , d_1 , and d_2 may be made to represent most surface rills and protrusions. The flexibility of this particular model in terms of functions ξ_R and ξ_P renders a manipulable means for mathematical calculation of radar return signals.

The quasi-periodic component ξ_P may assume any geometrical function which is most appropriate for the terrain description in keeping with the assumption of negligible shadowing and multiple reflections, etc. A careful analysis of the Orbiters' high resolution photographic mosaics indicates that the sinusoidal surface function very nearly matches the description of the lunar surface. The sinusoidal function is superior geometrically and flexible mathematically. For example, the function

$$f(x) = h \sin \frac{\pi x}{T} \quad (4-8)$$

where

h = the surface height, and

T = the surface period

may be assigned any value for h and T or a mode random to generate a realistic surface. Furthermore, h and T may either be made dependent on each other or statistically independent. In the case of the lunar crater, it is found that the rim height, rim base width, and crater depth are all almost linearly related to the crater diameter on the average, such as:

$$\begin{aligned} D/10 < h_1 < D/3, & \quad h_2 \approx .1h_1 \\ d_1 \approx .5D, & \quad d_2 \approx .55D. \end{aligned} \quad (4-9)$$

Thus, the crater diameter is the only random variable for the quasi-periodic surface component, such that given a crater diameter the quasi-periodic surface is completely specified.

The scalar Helmholtz integral is used to calculate the LM radar return power since ξ_R and ξ_P describe a complete set of statistics for this lunar module landing zone. Radar signal degradation is computed for each section along each of the landing approaches. This surface modeling is primarily limited by the resolution in interpreting the Orbiter and Surveyor lunar photographs.

The surface model generated by superposition of ξ_R and ξ_P is a posteriori stipulation based on the Surveyor and

Orbiter pictures; nevertheless, its statistical generality permits a very realistic surface representation and the prediction of radar return from it. Further research on the model is expected to yield more information on the scattering phenomena, such as secondary scattering and point-target diffraction.

4.2.3 Radar Return from the One-Dimensional New Lunar Surface Model

A knowledge of radar doppler and velocity signal degradation is necessary for appropriate guidance and control of LM landing used in the Apollo program. A study of several possible landing sites shows that the distribution and size of craters and boulders are random, and the probability distribution of their sizes appears to be exponential in shape as shown in Fig. 4-6. A set of close-up pictures of the lunar surface taken by Russian Lunar 9 and American Surveyors and Orbiters I and II indicate that its local surface undulations are very rough in terms of the wavelength ($\lambda = 3$ cm) of the incident signal in the case of LM radars.

A surface roughness function denoting heights above and below the mean level can be described as the superposition of a number of rough surfaces such as

$$\xi = \xi_1 + \xi_2 + \xi_3 + \dots \quad (4-10)$$

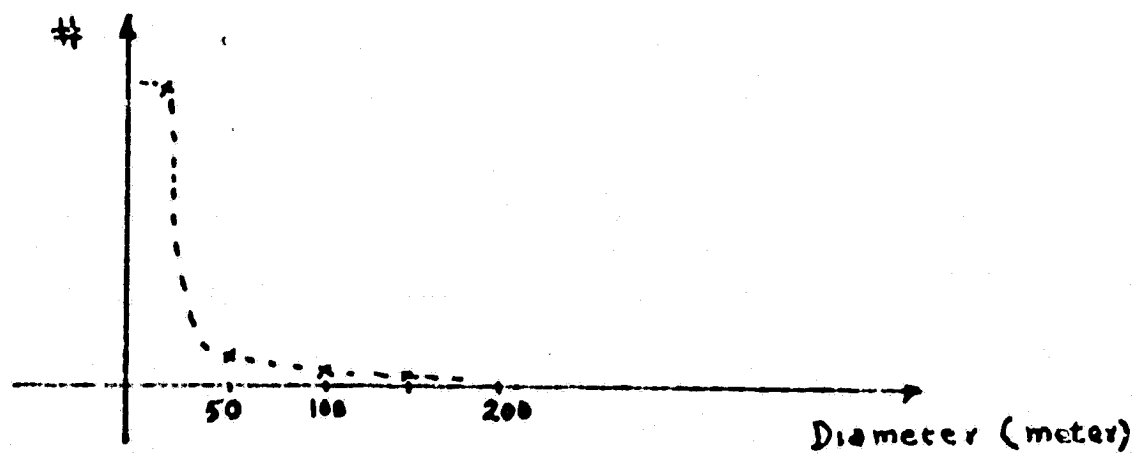


Fig. 4-6. Crater Number Vs. Crater Diameter Curve

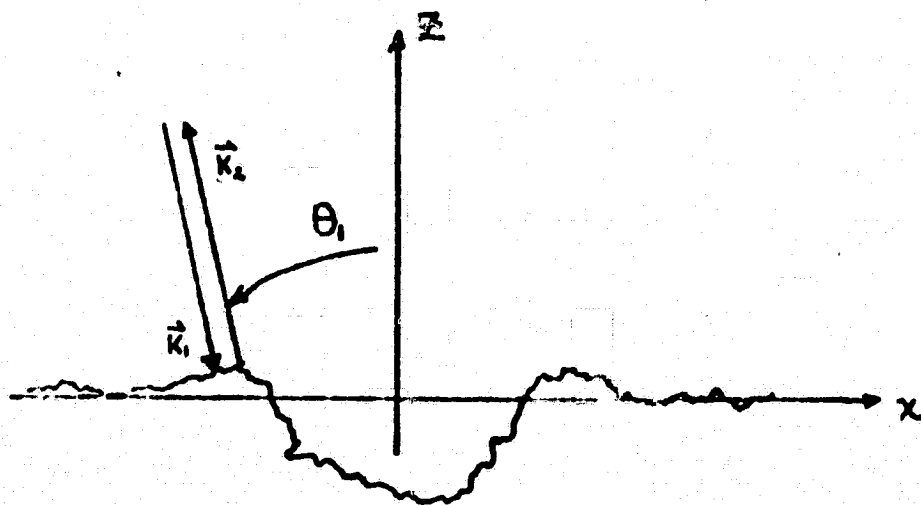


Fig. 4-7. Monostatic Radar Signal Return Geometry

The landing sites on the lunar surface may be modeled by such a superposition of a random roughness and a quasi-periodic surface in an interval T as a zero-th order model as shown in Fig. 4-3 and Fig. 4-5 where ξ_r is random gaussian component with zero mean and ξ_p is periodic with an exponential probability density D of the crater diameter at the average elevation of the crater. The dimensions d_1 , d_2 , h_1 , and h_2 are all functions of D , and T is fixed in order to maintain the identity of ξ_r and ξ_p ; because if T was randomly varying ξ_p would tend to become ξ_r .

As a first order approximation, the surface is assumed invariant along the y -axis, and the radar return power can be calculated from this one-dimensional mode of the lunar surface. The general Kirchhoff integral is applied under the following assumptions:

- a) The radar beam cross-section has dimensions large as compared to T .
- b) The radius of curvature of the random rough surface is much greater than the wavelength of the incident radiation.
- c) The incident radiation is a horizontally polarized plane wave.
- d) Shadowing, multiple scattering and depolarization effect is neglected.
- e) ξ_r and ξ_p are statistically independent.

$$E_2(\theta_1; \theta_2) = \frac{E_{20} F}{2L} \int_{-L}^L \exp(-\bar{v} \cdot \bar{r}) dx \quad (4-11)$$

where

$E_2(\theta_1; \theta_2)$ is the reflected field from the surface.

θ_1 and θ_2 are angles of incident and reflection as measured from the outward surface normal.

E_{20} is the field reflected in the direction of specular reflection ($\theta_2 = \theta_1$) by a smooth, perfectly conducting plane of the same dimensions under the same angle of incidence at the same distance.

$$F = \sec \theta_1 \frac{1 + \cos(\theta_1 + \theta_2)}{\cos \theta_1 + \cos \theta_2}$$

$4L^2$ is taken as a sampling area

$$\bar{v} = \bar{k}_1 - \bar{k}_2$$

$$v_x = k (\sin \theta_1 - \sin \theta_2)$$

$$v_z = -k (\cos \theta_1 + \cos \theta_2)$$

$$\bar{r} = x\bar{a}_x + (\xi_R + \xi_P) \bar{a}_z$$

ξ_R and ξ_P are functions of x denoting the height of the rough surface.

The scattering geometry is shown in Fig. 4-7 where

$$\theta_2 = -\theta_1 \text{ and } F = 1/\cos^2 \theta_1.$$

The calculation of the monostatic scattered field with

$L = nT$ from Eq. (4-11) is

$$E_z(\theta_1; \theta_2) = \frac{E_{z0} F}{2n\tau} \sum_{\mu=-n}^{n-1} \int_{\mu\tau}^{(\mu+1)\tau} \exp[iV_z x + iV_z(\xi_R + \xi_p)] dx \quad (4-12)$$

$$E_z(\theta_1; \theta_2) = \frac{E_{z0} F}{T} \frac{1}{2n} \sum_{\mu=-n}^{n-1} \exp(i2\pi\mu p) \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_z x + iV_z(\xi_R + \xi_p)] dx \quad (4-13)$$

The generalized grating equation is

$$\begin{aligned} p &= \frac{T}{\lambda} (\sin\theta_1 - \sin\theta_2) \\ &= \frac{\tau V_z}{2\pi} \end{aligned} \quad (4-14)$$

where P is any real number satisfying the above grating equation for a given θ_1 and θ_2 . The substitution of P in Eq. (4-13) yields:

$$E_z(\theta_1; \theta_2) = \frac{W E_{z0} F}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_z x + iV_z(\xi_R + \xi_p)] dx \quad (4-15)$$

$$W = \frac{1}{2n} \sum_{\mu=-n}^{n-1} \exp(i2\pi\mu p) \quad (4-16)$$

The average return power is

$$\langle E_z E_z^* \rangle = \left\langle \frac{|W|^2 E_{z0}^2 F^2}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_z(x_1 - x_2) + iV_z(\xi_{R1} - \xi_{R2}) + iV_z(\xi_{p1} - \xi_{p2})] dx_1 dx_2 \right\rangle \quad (4-17)$$

where

$$\begin{aligned} \xi_{R1} &= \xi_R(x_1) & \xi_{R2} &= \xi_R(x_2) \\ \xi_{p1} &= \xi_p(x_1) & \xi_{p2} &= \xi_p(x_2) \end{aligned} \quad (4-18)$$

or

$$\begin{aligned} \langle E_z E_z^* \rangle &= \left\langle \frac{|W|^2 E_{z0}^2 F^2}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_z(x_1 - x_2) + iV_z(\xi_{p1} - \xi_{p2})] \right. \\ &\quad \left. \langle \exp[iV_z(\xi_{R1} - \xi_{R2})] \rangle_{\xi_R} dx_1 dx_2 \right\rangle_{\xi_p} \end{aligned} \quad (4-19)$$

The average is taken over the statistically independent random variables ξ_R and ξ_p .

Now

$$\langle \exp[iV_2(\xi_{R_1} - \xi_{R_2})] \rangle_{\xi} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x_1; x_2) \exp[iV_2(\xi_{R_1} - \xi_{R_2})] dx_1 dx_2 \quad (4-20)$$

where

$$p(x_1; x_2) = \frac{1}{2\pi\sigma^2\sqrt{1-c^2}} \exp\left[-\frac{x_1^2 - 2cx_1x_2 + x_2^2}{2\sigma^2(1-c^2)}\right] \quad (4-21)$$

The characteristic function evaluated from Eq. (4-20) using Eq. (4-21) is (Middleton, 1960):

$$\begin{aligned} \langle \exp[iV_2(\xi_{R_1} - \xi_{R_2})] \rangle &= \exp\left[-\sigma^2 V_2^2 \left(1 - \exp\frac{-|x_1 - x_2|}{B}\right)\right] \\ &= \exp-(\sigma V_2)^2 \sum_{m=0}^{\infty} \frac{(\sigma V_2)^{2m}}{m!} \exp\left(\frac{-m|x_1 - x_2|}{B}\right) \end{aligned} \quad (4-22)$$

A transformation of x_1 and x_2 into x_2 and τ yields

$$x_1, x_2 \rightarrow x_2, \tau \quad (4-23)$$

$$dx_1, dx_2 \rightarrow |\mathcal{J}| dx_2 d\tau \quad (4-24)$$

$$|\mathcal{J}| = 1 \quad (4-25)$$

A substitute of the Jacobian in the average power Eq. (4-19)

yields

$$\begin{aligned} \langle E_1 E_2^* \rangle &= \left\langle \frac{|W|^2 E_{20}^2 F^2}{T^2} \exp-(\sigma V_2)^2 \sum_{m=0}^{\infty} \frac{(\sigma V_2)^{2m}}{m!} \right. \\ &\quad \left. \int_{-\frac{T}{2}-x_2}^{\frac{T}{2}-x_2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp\left(iV_2\tau - \frac{m|\tau|}{B}\right) \cdot \exp[iV_2(\xi_{R_1} - \xi_{R_2})] dx_2 d\tau \right\rangle_{\xi} \end{aligned} \quad (4-26)$$

The limits make it necessary to integrate over τ before integrating over X_2 . However, by assuming that $B \ll \frac{T}{2}$ and that the integral of τ has value only at very small value of τ , it would not be erroneous to extend the limits to infinity instead of from $-\frac{T}{2} - X_2$ to $\frac{T}{2} - X_2$, since

$$\int_{-\frac{T}{2} - X_2}^{\frac{T}{2} - X_2} f(\tau) d\tau \simeq \int_{-\infty}^{\infty} f(\tau) d\tau \quad (4-27)$$

Since the limits of integration on τ are approximated to be minus infinity to plus infinity, taking into account the divergent term representing the average return power as shown below:

$$\langle E_1 E_1^* \rangle = \langle E_1 \rangle \langle E_1^* \rangle + D \{E_1\} \quad (4-28)$$

After simplifying, Eq. (4-1) becomes

$$E_2 = \frac{WE_{20}F}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [iV_x X + iV_z (\xi_k + \xi_p)] dx \quad (4-29)$$

the average of which is

$$\langle E_2 \rangle = \left\langle \frac{WE_{20}F}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp (iV_x X + iV_z \xi_k) \langle \exp (iV_z \xi_p) \rangle dx \right\rangle \quad (4-30)$$

The characteristic function is calculated to be (Middleton, 1960)

$$\langle \exp (iV_z \xi_k) \rangle = \exp -\frac{1}{2} (\sigma_{V_z})^2 \quad (4-31)$$

and

$$\langle E_z \rangle = \left\langle \frac{WE_{z0}F}{T} \exp -\frac{1}{2}(\sigma V_z)^2 \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp(iV_x x + iV_z \xi_p) dx \right\rangle_{\xi_p} \quad (4-32)$$

Let

$$\begin{aligned} I &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp(iV_x x + iV_z \xi_p) dx \\ &= I_1 + I_2 + I_3 \end{aligned} \quad (4-33)$$

with

$$\begin{aligned} I_1 &= \int_{-\frac{T}{2}}^{-d_2} \exp(iV_x x) dx + \int_{d_2}^{\frac{T}{2}} \exp(iV_x x) dx \\ &= \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \end{aligned} \quad (4-34)$$

$$\begin{aligned} I_2 &= \int_{-d_1}^{-d_2} \exp[iV_x x + iV_z h_2 \sin \frac{\pi(x-d_1)}{d_2-d_1}] dx + \int_{d_1}^{d_2} \exp[iV_x x + iV_z h_2 \sin \frac{\pi(x-d_1)}{d_2-d_1}] dx \\ &= \sum_{p=-\infty}^{\infty} J_p(V_z h_2) \left[\frac{e^{-iV_x d_1} \cdot e^{ip\pi} - e^{-iV_x d_2} + e^{iV_x d_2} \cdot e^{ip\pi} - e^{iV_x d_1}}{i(V_x + \frac{p\pi}{d_2-d_1})} \right] \end{aligned} \quad (4-35)$$

$$\begin{aligned} I_3 &= \int_{-d_1}^{d_1} \exp[iV_x x - iV_z h_1 \cos \frac{\pi x}{2d_1}] dx \\ &= \sum_{f=-\infty}^{\infty} i^f J_f(-V_z h_1) \frac{\sin(V_x + \frac{f\pi}{2d_1})d_1}{(V_x + \frac{f\pi}{2d_1})/2} \end{aligned} \quad (4-36)$$

Substituting Eq. (4-34), (4-35), and (4-36) into Eq. (4-32), one obtains

$$\begin{aligned} \langle E_z \rangle &= \left\langle \frac{WE_{z0}F}{T} \exp -\frac{1}{2}(\sigma V_z)^2 \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ &\quad \left. \left. + \sum_{p=-\infty}^{\infty} J_p(V_z h_2) \left[\frac{e^{-iV_x d_1} \cdot e^{ip\pi} - e^{-iV_x d_2} + e^{iV_x d_2} \cdot e^{ip\pi} - e^{iV_x d_1}}{i(V_x + \frac{p\pi}{d_2-d_1})} \right] \right\} \right\rangle \end{aligned}$$

$$+ \sum_{f=-\infty}^{\infty} i^f J_f(-V_z h_1) \frac{\sin(V_x + \frac{f\pi}{2d_1})d_1}{(V_x + \frac{f\pi}{2d_1})/2} \Bigg\} >_{\mathfrak{F}_p} \quad (4-37)$$

Taking the conjugate of Eq. (4-37), one obtains

$$\begin{aligned} \langle E_2^* \rangle = & \left\langle \frac{WE_2 F}{T} \exp -\frac{1}{2}(\sigma V_z)^2 \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_z h_2) \left[\frac{-e^{iV_x d_1} \cdot e^{ip\pi} + e^{iV_x d_2} - e^{-iV_x d_2} \cdot e^{ip\pi} - e^{-iV_x d_1}}{i(V_x + \frac{p\pi}{d_1 - d_2})} \right] \\ & \left. \left. + \sum_{f=-\infty}^{\infty} i^f J_f(V_z h_1) \frac{\sin(V_x - \frac{f\pi}{2d_1})d_1}{(V_x - \frac{f\pi}{2d_1})/2} \right\} \right\rangle_{\mathfrak{F}_p} \quad (4-38) \end{aligned}$$

The second term in Eq. (4-28) is now calculated to be as

$$\begin{aligned} D\{E_2\} = & \left\langle K_1 \int_{-\infty}^{\infty} \exp[iV_x \tau - \frac{m\tau}{B}] \right. \\ & \left. \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_z(\mathfrak{F}_p - \mathfrak{F}_2)] dx_2 d\tau \right\rangle_{\mathfrak{F}_p} \quad (4-39) \end{aligned}$$

with

$$K_1 = \frac{|W|^2 E_2^2 F^2}{T^2} \exp -(\sigma V_z)^2 \sum_{m=1}^{\infty} \frac{(\sigma V_z)^{2m}}{m!} \quad (4-40)$$

Since $\mathfrak{F}_p$ and $\mathfrak{F}_2$ are the only functions in the integrand containing x_2 , it may be evaluated separately.

Let

$$\begin{aligned} I &= \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_z(\mathfrak{F}_p - \mathfrak{F}_2)] dx_2 \\ &= I_1 + I_2 + I_3 \quad (4-41) \end{aligned}$$

$$I_1 = \int_{-\frac{T}{2}}^{-d_1} dx_2 + \int_{d_1}^{\frac{T}{2}} dx_2 \quad (4-42)$$

$$I_2 = \int_{-d_2}^{-d_1} \exp \left[i V_2 h_2 \left\{ \sin \frac{\pi (-\tau - x_2 - d_1)}{d_2 - d_1} - \sin \frac{\pi (-x_2 - d_1)}{d_2 - d_1} \right\} \right] dx_2 \\ + \int_{d_1}^{d_2} \exp \left[i V_2 h_2 \left\{ \sin \frac{\pi (\tau + x_2 - d_1)}{d_2 - d_1} - \sin \frac{\pi (x_2 - d_1)}{d_2 - d_1} \right\} \right] dx_2 \quad (4-43)$$

$$I_3 = \int_{-d_1}^{d_1} \exp \left[-i V_2 h_1 \left\{ \cos \frac{\pi (\tau + x_2)}{2d_1} - \cos \frac{\pi x_2}{2d_1} \right\} \right] dx_2 \quad (4-44)$$

Further integration yields:

$$I_1 = T - 2d_2, \quad (4-45)$$

$$I_2 = 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} (i)^{p+q} J_p(V_2 h_2) J_q(-V_2 h_2)$$

$$\left[\frac{\sin \frac{\pi}{2} (p+q)}{\frac{\pi}{2} (p+q)} \right] \cos \frac{p \pi \tau}{d_2 - d_1}, \text{ AND} \quad (4-46)$$

$$I_3 = 2d_1 \sum_{f=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{f+g} J_f(-V_2 h_1) J_g(V_2 h_1)$$

$$\left[\frac{\sin \frac{\pi}{2} (f+g)}{\frac{\pi}{2} (f+g)} \right] \exp \left(\frac{f \pi \tau}{2d_1} \right). \quad (4-47)$$

$$D\{E_2\} = \left\langle K_1 \int_{-\infty}^{\infty} \exp \left[i V_2 \tau - \frac{m|\tau|}{B} \right] (I_1 + I_2 + I_3) d\tau \right\rangle_{\mathcal{P}} \\ = \left\langle K_1 (I'_1 + I'_2 + I'_3) \right\rangle_{\mathcal{P}} \quad (4-48)$$

with
$$\begin{aligned}
 I_1' &= \int_{-\infty}^{\infty} \exp(iV_x \tau) \cdot \exp\left(-\frac{m|\tau|}{B}\right) (T - 2d_2) d\tau \\
 &= 2(T - 2d_2) \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) \cos V_x \tau d\tau \\
 &= 2(T - 2d_2) \frac{\frac{m}{B}}{\left(\frac{m}{B}\right)^2 + V_x^2}, \quad (4-49)
 \end{aligned}$$

$$\begin{aligned}
 I_2' &= 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{p+g} J_p(V_2 h_2) J_g(-V_2 h_2) \\
 &\quad \left[\frac{\sin \frac{\pi}{2}(p+g)}{\frac{\pi}{2}(p+g)} \right] \int_{-\infty}^{\infty} \exp(iV_x \tau) \cdot \exp\left(-\frac{m|\tau|}{B}\right) \cos \frac{p\pi\tau}{d_2 - d_1} d\tau \\
 &= 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{p+g} J_p(V_2 h_2) J_g(-V_2 h_2) \\
 &\quad \left[\frac{\sin \frac{\pi}{2}(p+g)}{\frac{\pi}{2}(p+g)} \right] \left[\frac{\frac{m}{B}}{\left(\frac{m}{B}\right)^2 + \left(V_x + \frac{p\pi}{d_2 - d_1}\right)^2} + \right. \\
 &\quad \left. + \frac{\frac{m}{B}}{\left(\frac{m}{B}\right)^2 + \left(V_x - \frac{p\pi}{d_2 - d_1}\right)^2} \right], \quad \text{AND} \quad (4-50)
 \end{aligned}$$

$$\begin{aligned}
 I_3' &= 4d_1 \sum_{f=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{f+g} J_f(-V_2 h_1) J_g(V_2 h_1) \\
 &\quad \left[\frac{\sin \frac{\pi}{2}(f+g)}{\frac{\pi}{2}(f+g)} \right] \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) \cos\left(V_x + \frac{f\pi}{2d_1}\right) \tau d\tau
 \end{aligned}$$

$$= 4d_1 \sum_{f=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{f+g} J_f(-V_2 h_1) J_g(V_2 h_1) \left[\frac{\sin \frac{\pi}{2}(f+g)}{\frac{\pi}{2}(f+g)} \right] \left[\frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x + \frac{f\pi}{2d_1})^2} \right] \quad (4-51)$$

The variance of the return signal is,

$$\begin{aligned} D\{E_2\} = & \langle K_1 \left\{ 2(T - 2d_2) \frac{\frac{m}{B}}{(\frac{m}{B})^2 + V_x^2} \right. \\ & + 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} (i)^{p+q} J_p(V_2 h_2) J_q(-V_2 h_2) \\ & \left. \left[\frac{\sin \frac{\pi}{2}(p+q)}{\frac{\pi}{2}(p+q)} \right] \left[\frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x + \frac{p\pi}{d_2 - d_1})^2} + \frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x - \frac{p\pi}{d_2 - d_1})^2} \right] \right. \\ & + 4d_1 \sum_{f=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{f+g} J_f(V_2 h_1) J_g(V_2 h_1) \\ & \left. \left. \left[\frac{\sin \frac{\pi}{2}(f+g)}{\frac{\pi}{2}(f+g)} \right] \left[\frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x + \frac{f\pi}{2d_1})^2} \right] \right\} \right\rangle_p \quad (4-52) \end{aligned}$$

The complete solution for the averaged radar return is the DC term, which is the product of Eq. (4-37) and (4-38), plus the variance of Eq. (4-52).

In the absence of craters or hills, i. e., $\xi_p = 0$

$$\langle E_2 \rangle \langle E_2^* \rangle = |W|^2 E_{10}^2 F^2 \exp -(\sigma V_0)^2 \left[\frac{\sin V_x \frac{T}{2}}{V_x \frac{T}{2}} \right]^2 \quad (4-53)$$

and

$$D\{E_2\} = |W|^2 E_{10}^2 F^2 \exp -(\sigma V_0)^2 \sum_{m=1}^{\infty} \frac{(\sigma V_0)^{2m}}{m!} \frac{2 \frac{m}{B}}{[(\frac{m}{B})^2 + V_x^2]} \quad (4-54)$$

Therefore,

$$\langle E_z E_z^* \rangle \Big|_{\xi=0} = |W|^2 E_{z0}^2 F^2 \exp -(\sigma V_x)^2 \left[\frac{\sin V_x \frac{T}{2}}{V_x \frac{T}{2}} + \sum_{m=1}^{\infty} \frac{(\sigma V_x)^{2m}}{m!} \frac{2 \frac{m}{B}}{\Gamma[(\frac{m}{B})^2 + V_x^2]} \right] \quad (4-55)$$

The factor $|W|^2$ for the pure random surface equals unity and the result checks well with that calculated by Hayre and Kaufman (1965).

In the absence of the general roughness, $\xi_R = 0$

$$\sigma = 0 \quad (4-56)$$

$$\frac{1}{B} = 0 \quad (4-57)$$

and Eq. (4-28) becomes

$$\begin{aligned} \langle E_z E_z^* \rangle \Big|_{\xi_R=0} = & \frac{|W|^2 E_{z0}^2 F^2}{T^2} \left\langle \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_x h_2) \left[\frac{e^{-iV_x d_1} e^{ip\pi} - e^{-iV_x d_2} + e^{iV_x d_2} e^{ip\pi} - e^{iV_x d_1}}{i(V_x + \frac{p\pi}{d_2-d_1})} \right] \\ & + \sum_{f=-\infty}^{\infty} i^f J_f(-V_x h_1) \frac{\sin(V_x + \frac{f\pi}{2d_1})d_1}{(V_x + \frac{f\pi}{2d_1})/2} \Big\} \Big\rangle \times \\ & \left\langle \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_x h_2) \left[\frac{e^{iV_x d_1} e^{ip\pi} + e^{iV_x d_2} - e^{iV_x d_2} e^{ip\pi} - e^{iV_x d_1}}{i(V_x + \frac{p\pi}{d_2-d_1})} \right] \\ & + \sum_{f=-\infty}^{\infty} i^f J_f(V_x h_1) \frac{\sin(V_x - \frac{f\pi}{2d_1})d_1}{(V_x - \frac{f\pi}{2d_1})/2} \Big\} \Big\rangle \quad (4-58) \end{aligned}$$

In the absence of both ξ_x and ξ_y , the surface becomes a mirror reflector, and its radar reflectivity is

$$\langle E_1 E_1^* \rangle \Big|_{\substack{\xi_x=0 \\ \xi_y=0}} = E_{z0}^2 F^2 \left[\frac{\sin \frac{V_x T}{2}}{\frac{V_x T}{2}} \right]^2 \quad (4-59)$$

For the monostatic case, one has

$$F^2 = \frac{1}{\cos^4 \theta_1} \quad (4-60)$$

$$V_x = \frac{4\pi}{\lambda} \sin \theta_1 \quad (4-61)$$

The curves of F^2 vs. θ_1 and $\left[\frac{\sin \frac{V_x T}{2}}{\frac{V_x T}{2}} \right]^2$ vs. θ_1 are presented in Fig. 4-4. If T is $\gg \lambda$, $\left[\frac{\sin \frac{V_x T}{2}}{\frac{V_x T}{2}} \right]^2$ decays very rapidly, and the product of the two curves may be approximated by an impulse, or

$$\langle E_1 E_1^* \rangle \Big|_{\substack{\xi_x=0 \\ \xi_y=0}} = E_{z0}^2 \delta(\theta_1) \quad (4-62)$$

The significant terms in the Eq. (4-28) for the conditions $\sigma V_2 \ll 1$, $\sigma V_2 \approx 1$ and $\sigma V_2 \gg 1$ which correspond to slight, moderate and very rough surfaces, respectively, have been discussed in great detail by Beckman (1963). In the case of the lunar surface where the general random surface appears to be very rough in terms of 3 cm wavelength (as shown in Surveyor pictures) σV_2 is indeed large compared to 1, Eq. (4-28) becomes

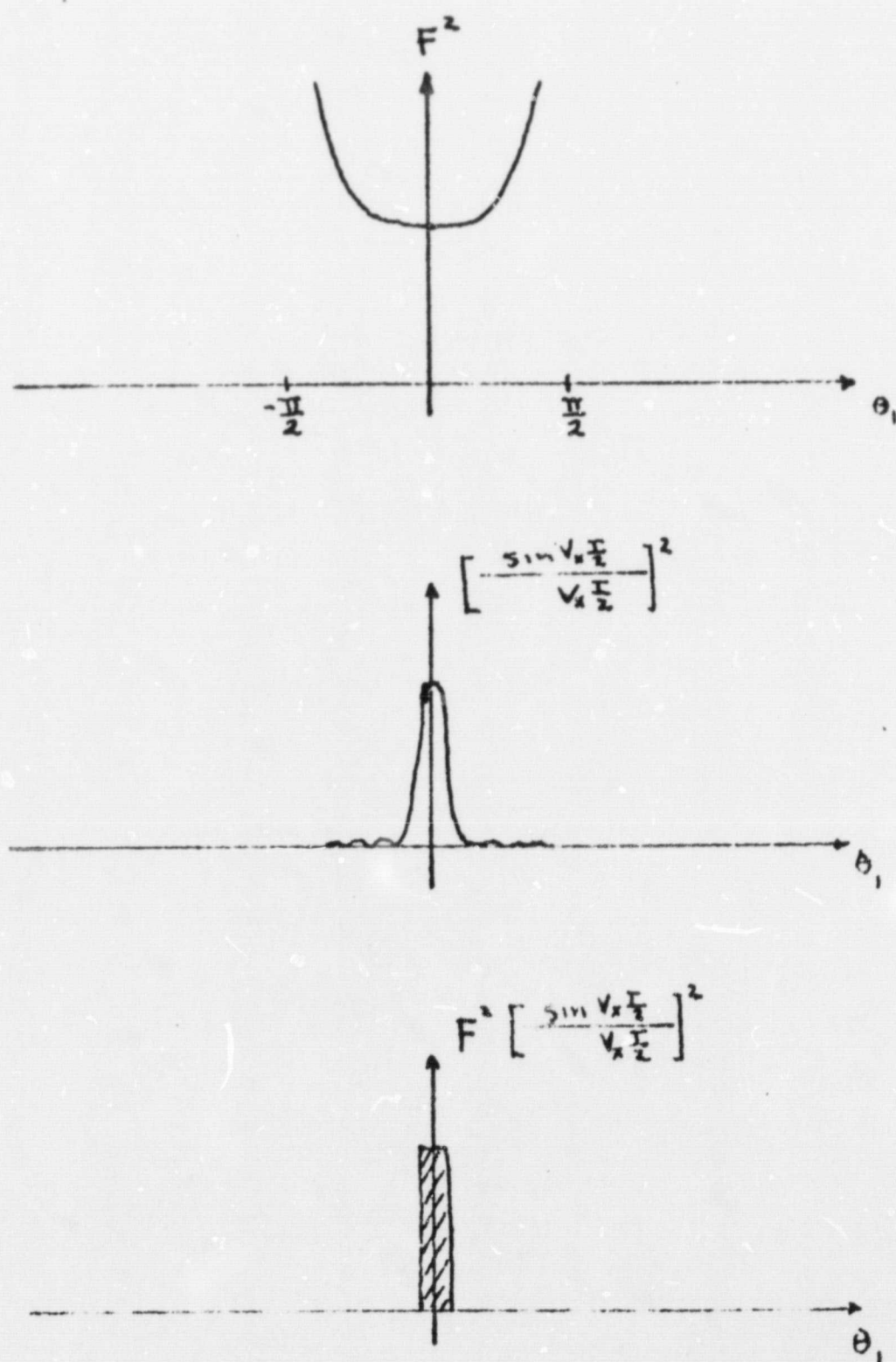


Fig. 4-8. The Impulse Approximation for the Monostatic Mirror Reflector

$$\langle E_1 E_1^* \rangle \approx D \{ E_2 \} \quad (4-63)$$

and

$$\exp -(\sigma v_z)^2 (1 - e^{-\frac{171}{B}}) \approx \exp -(\sigma v_z)^2 \frac{171}{B} \quad (4-64)$$

Substituting Eq. (4-64) in (4-39) for the calculation of $\langle E_2 E_2^* \rangle$, the resulting expression is

$$\begin{aligned} \langle E_2 E_2^* \rangle = & \left\langle \frac{|W|^2 E_{20}^2 F^2}{T^2} \left\{ 2(T - 2d_2) \frac{(\sigma v_z)^2 B}{(\sigma v_z)^4 + v_x^2 B^2} \right. \right. \\ & + 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} J_p^2(v_z h_2) \left[\frac{(\sigma v_z)^2 B}{(\sigma v_z)^4 + (v_x + \frac{p\pi}{d_2 - d_1})^2 B^2} \right. \\ & + \left. \left. \frac{(\sigma v_z)^2 B}{(\sigma v_z)^4 + (v_x - \frac{p\pi}{d_2 - d_1})^2 B^2} \right] + 4d_1 \sum_{f=-\infty}^{\infty} J_f^2(v_z h_1) \right. \\ & \left. \left. \left[\frac{(\sigma v_z)^2 B}{(\sigma v_z)^4 + (v_x + \frac{f\pi}{d_1 - d_2})^2 B^2} \right] \right\} \right\rangle_p \quad (4-65) \end{aligned}$$

In Fig. 4-9, a curve is plotted using Eq. (4-65) with the following parameters:

$$\lambda = 3 \text{ cm}$$

$$\sigma = .3 \text{ cm}$$

$$B = 3 \text{ cm}$$

$$|W|^2 = 1$$

$$T = 30 \text{ meters}$$

$$P_p(0) = .05 \exp(-.050)$$

$$d_1 = .5D$$

$$d_2 = .55D$$

$$h_1 = .2D$$

$$h_2 = .02D$$

The results of other models (Hayre, 1962; Muhleman, 1964) and experimental results (Evans and Pettingel, 1963) dealing with lunar radar return are also plotted in Fig. 4-9, and a comparison of these with results obtained by the model used in this work shows very good comparison.

4.2.4 Radar Return from the Two-Dimensional New Lunar Surface

Model

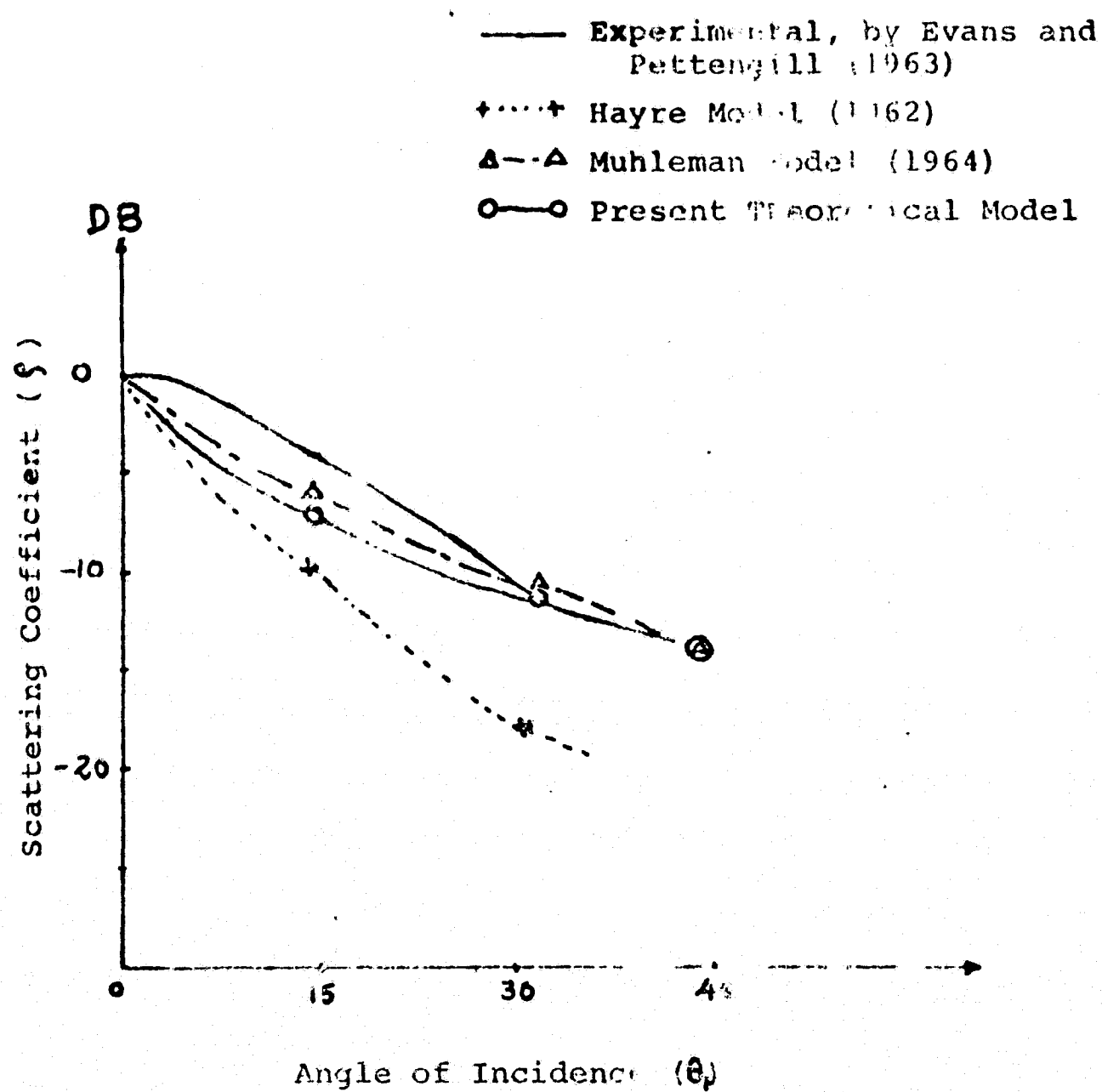
The calculations of average radar return power in the last chapter were made for one-dimensional lunar surface model in order to simplify the mathematical procedure. The two-dimensional case is presented in this chapter.

The data on the LM trajectory shows that the altimeter beam has an incident angle (θ_1) variation from zero to forty degrees with respect to the outward average lunar surface normal. Since the radar system is a monostatic case, the reflection angle (θ_2) is the negative of the incident angle (θ_1) and the lateral deviation angle (θ_3) is zero. (See Fig. 4-7.) The close-up pictures taken by Surveyor II indicate that the general surface is very rough in terms of the incident wavelength (3 cm). The surface function is taken as a composite of a random and a periodic roughness. (See Figs. 4-3, 4-4 and 4-5.)

The assumptions similar to those made for one-dimensional case are made here as well.

The return field E_2 for a two-dimensional surface is given by (Beckman, et al, 1963):

$$E_2 = \frac{E_{i0} F}{A} \int_{-L_x}^{L_x} \int_{-L_y}^{L_y} \exp(i \vec{V} \cdot \vec{r}) \, dx \, dy$$



$$\delta = \frac{\langle E_s E_s^* \rangle}{|E_{i0}|^2}$$

Fig. 4-9. Scattering Coefficient Vs. the Angle of Incidence

$$= \frac{W_x W_y E_{20} F}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [i V_x x + i V_y y + i V_z (\xi_p + \xi)] dx dy \quad (4-66)$$

where

$$W_x = \frac{\sin (2n p \pi)}{2n \sin (p \pi)} \exp (-i p \pi) \quad (4-67)$$

$$W_y = \frac{\sin (2m q \pi)}{2m \sin (q \pi)} \exp (-i q \pi) \quad (4-68)$$

$$n = \frac{L_x}{T}, \quad m = \frac{L_y}{T} \quad (4-69)$$

$$p = \frac{T V_x}{2\pi}, \quad q = \frac{T V_y}{2\pi} \quad (4-70)$$

E_{20} is the normal specular return from a smooth surface

$$F = \frac{1 + \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \theta_3}{\cos \theta_1 (\cos \theta_1 + \cos \theta_2)} \quad (4-71)$$

$$V_x = \frac{2\pi}{\lambda} (\sin \theta_1 - \sin \theta_2), \quad V_y = \frac{-2\pi}{\lambda} \sin \theta_2 \sin \theta_3 \quad (4-72)$$

$$V_z = \frac{-2\pi}{\lambda} (\cos \theta_1 + \cos \theta_2) \quad (4-73)$$

The average power is therefore given by

$$\begin{aligned} \langle E_z E_z^* \rangle &= \left\langle \frac{|W_x|^2 |W_y|^2 E_{20}^2 F^2}{T^4} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [i V_x (x_1 - x_2) \right. \\ &\quad \left. + i V_y (y_1 - y_2) + i V_z (\xi_{p1} - \xi_{p2})] \right. \\ &\quad \left. \langle \exp [i V_z (\xi_{p1} - \xi_{p2})] \rangle_{\xi} dx_1 dx_2 dy_1 dy_2 \right\rangle_{\xi_p} \quad (4-74) \end{aligned}$$

$$\begin{aligned} \xi_{x_1} &= \xi(x_1, y_1), & \xi_{x_2} &= \xi(x_2, y_2) \\ \xi_{y_1} &= \xi(x_1, y_1), & \xi_{y_2} &= \xi(x_2, y_2) \end{aligned} \quad (4-75)$$

Since the roughness is isotropic, its correlation between two points on the $z = 0$ plane is a function of only the magnitude of the distance between the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Let the distance between the two points P_1 and P_2 be τ , where

$$\tau = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (4-76)$$

Then $\langle \exp[iV_z(\xi_1 - \xi_2)] \rangle$ can be treated as a one-dimensional second order random process, and the characteristic function in terms of τ is found as

$$\Phi(\tau) = \exp -(\sigma V_z)^2 \sum_{m=0}^{\infty} \frac{(\sigma V_z)^{2m}}{m!} \exp \left(-\frac{m\tau}{\delta} \right) \quad (4-77)$$

A set of new variables may be defined as follows:

$$x_2 = \tau \cos \phi \quad (4-78)$$

$$y_2 = \tau \sin \phi \quad (4-79)$$

$$\phi = \tan^{-1} \frac{y_1 - y_2}{x_1 - x_2} \quad (4-80)$$

Now, transforming x_1, x_2, y_1, y_2 into x_1, y_1, τ, ϕ

where $x_1 = x_1$

$$x_2 = x_1 - \tau \cos \phi$$

$$y_1 = y_1$$

$$y_2 = y_1 - \tau \sin \phi \quad (4-81)$$

one obtains

$$dx_1 dx_2 dy_1 dy_2 = |J| dx_1 dy_1 d\tau d\phi \quad (4-82)$$

where

$$\begin{aligned} |J| &= \begin{vmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial \tau} & \frac{\partial x_1}{\partial \phi} \\ \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial \tau} & \frac{\partial x_2}{\partial \phi} \\ \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial y_1} & \frac{\partial y_1}{\partial \tau} & \frac{\partial y_1}{\partial \phi} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial y_1} & \frac{\partial y_2}{\partial \tau} & \frac{\partial y_2}{\partial \phi} \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -\cos\phi & \tau\sin\phi \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -\sin\phi & -\tau\cos\phi \end{vmatrix} \\ &= \tau \cos^2\phi + \tau \sin^2\phi = \tau \end{aligned} \quad (4-83)$$

This transformation process introduces the corresponding change of variables and limits as

$$\xi_1 = \xi_p(x_1, y_1) \quad (4-84)$$

$$\xi_2 = \xi_p(x_1 - \tau \cos\phi, y_1 - \tau \sin\phi) \quad (4-85)$$

$$-\frac{\pi}{2} < x_1 < \frac{\pi}{2} \quad (4-86)$$

$$-\frac{\pi}{2} < y_1 < \frac{\pi}{2} \quad (4-87)$$

$$\sqrt{(x_1 - \frac{\pi}{2})^2 + (y_1 - \frac{\pi}{2})^2} < \tau < \sqrt{(x_1 + \frac{\pi}{2})^2 + (y_1 + \frac{\pi}{2})^2} \quad (4-88)$$

$$\tan^{-1}\left(\frac{y_1 - \frac{\pi}{2}}{x_1 - \frac{\pi}{2}}\right) < \phi < \tan^{-1}\left(\frac{y_1 + \frac{\pi}{2}}{x_1 + \frac{\pi}{2}}\right) \quad (4-89)$$

The limits of τ and ϕ are functions of x_1 and y_1 which vary from $-\frac{T}{2}$ to $\frac{T}{2}$ in each interval.

It is assumed that the autocorrelation C of $\mathcal{E}_R$ is insignificantly small for τ greater than $T/2$; and that the term $\exp(\frac{iV_1\tau}{B})$ converges very rapidly. Therefore, the integral has major contribution for small τ , and it is possible to extend the limits on τ from zero to infinity.

The limits on the angle ϕ are integrated from $\phi_1 = \tan^{-1}(\frac{y_1 + \frac{T}{2}}{x_1 + \frac{T}{2}})$ to $\phi_2 = \tan^{-1}(\frac{y_1 - \frac{T}{2}}{x_1 - \frac{T}{2}})$ where x_1 and y_1 take on values between $-T/2$ and $T/2$ respectively. However, for all values of x_1 and y_1 , ϕ is always bounded between zero and 2π .

And the transformed integral becomes

$$\langle E_1 E_2^* \rangle = \left\langle \frac{|w_1|^2 |w_2|^2 E_0 F^2}{T^4} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_0^\infty \exp [iV_x \tau \cos \phi + iV_y \tau \sin \phi + iV_z (\frac{T}{2} - \frac{T}{2})] \mathcal{E}(\tau) \tau d\tau d\phi dx_1 dy_1 \right\rangle_{\mathcal{E}} \quad (4-90)$$

In order to avoid a divergent integral, it is necessary to rearrange Eq. (4-90) by noting that

$$\langle E_1 E_2^* \rangle = \langle E_1 \rangle \langle E_2^* \rangle + D \{E_1\} \quad (4-91)$$

where

$$\begin{aligned} \langle E_1 \rangle &= \left\langle -\frac{w_1 w_2 E_0 F}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [iV_x x + iV_y y + iV_z \frac{T}{2}] \langle \exp (iV_z \frac{T}{2}) \rangle dx dy \right\rangle_{\mathcal{E}} \\ &= \left\langle -\frac{w_1 w_2 E_0 F}{T^2} \exp -\frac{1}{2} (0V_z)^2 \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [iV_x x + iV_y y + iV_z \frac{T}{2}] dx dy \right\rangle_{\mathcal{E}} \quad (4-92) \end{aligned}$$

After performing the polar transformation

$$\langle E_z \rangle = \left\langle \frac{W_x W_y E_{z0} F}{T^2} \exp -\frac{1}{2} (\sigma V_z)^2 \cdot [I_1 + I_2 + I_3] \right\rangle \quad (4-93)$$

(see Fig. 4-4) with

$$I_1 = T^2 - \pi d_1^2 \quad (4-94)$$

$$\begin{aligned} I_2 &= \int_0^{2\pi} \int_{d_1}^{d_2} \exp [iV_x r \cos \theta + iV_y r \sin \theta + iV_z h_2 \sin \frac{\pi(r-d_1)}{d_2-d_1}] r dr d\theta \\ &= 4\pi \sum_{p=1}^{\infty} \sum_{s=0}^{\infty} J_p(V_z h_2) \frac{(-1)^s}{s! s!} \frac{(V_x^2 + V_y^2)^s}{4^s} \\ &\quad \cdot \exp \left(\frac{-i2\pi p d_1}{d_2 - d_1} \right) \left[\frac{d_2^{2s+2}}{4s+4} - \frac{d_1^{2s+2}}{4s+4} - \sum_{a=0}^{2s+1} \frac{(2s+1)!}{a!} \right. \\ &\quad \left. \left(\frac{d_2 - d_1}{-i2\pi p} \right)^{2s+2-a} \left(e^{\frac{i2\pi p d_2}{d_2 - d_1}} d_2^a - e^{\frac{i2\pi p d_1}{d_2 - d_1}} d_1^a \right) \right] \quad (4-95) \end{aligned}$$

$$\begin{aligned} I_3 &= \int_0^{2\pi} \int_{d_1}^{d_2} \exp [iV_x r \cos \theta + iV_y r \sin \theta - iV_z h_1 \cos \frac{\pi(r-d_1)}{2d_1}] r dr d\theta \\ &= 4\pi \sum_{f=1}^{\infty} \sum_{s=0}^{\infty} i^f J_f(-V_z h_1) \frac{(-1)^s}{s! s!} \frac{(V_x^2 + V_y^2)^s}{4^s} \\ &\quad \left[\frac{d_1^{2s+2}}{4s+4} + \frac{(2s+1)!}{\left(-\frac{ifV_z}{2d_1}\right)^{2s+2}} \right. \\ &\quad \left. - \exp \left(\frac{ifV_z}{2} \right) \sum_{a=0}^{2s+1} \frac{(2s+1)!}{a!} \frac{d_1^a}{\left(-\frac{ifV_z}{2d_1}\right)^{2s+2-a}} \right] \quad (4-96) \end{aligned}$$

Thus, substituting Eq. (4-94), (4-95) and (4-96) in Eq. (4-93) is the solution for $\langle E_2 \rangle$. Similarly, $\langle E_2^* \rangle$ is obtained by taking the conjugate of the expressions for I_1 , I_2 and I_3 , the detailed calculations are not repeated here.

$$\langle E_2^* \rangle = \left\langle \frac{w_x^* w_y^* E_{20} F}{T^2} e^{-\frac{1}{2}(\sigma V_2)^2} [I_1^* + I_2^* + I_3^*] \right\rangle_{\tau} \quad (4-97)$$

$D\{E_2\} = \langle E_2 E_2^* \rangle - \langle E_2 \rangle \langle E_2^* \rangle$ is the variance of the complex scattering field, it may be written (Beckman, 1963) in a form similar to Eq. (4-90) as

$$D\{E_2\} = \left\langle \frac{|w_x|^2 |w_y|^2 E_{20}^2 F^2}{T^4} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \exp[iV_x \tau \cos \phi + iV_y \tau \sin \phi + iV_z(\tau_1 - \tau_2)] \cdot [\Phi(V_2, -V_2) - \Phi(V_2) \Phi^*(V_2)] \tau d\tau d\phi dx_1 dy_1 \right\rangle_{\tau} \quad (4-98)$$

where

$$[\Phi(V_2, -V_2) - \Phi(V_2) \Phi^*(V_2)] = e^{-\frac{1}{2}(\sigma V_2)^2} \sum_{m=1}^{\infty} \frac{(\sigma V_2)^{2m}}{m!} e^{-\frac{m\tau}{\theta}} \quad (4-99)$$

Let

$$K_1 = \frac{|w_x|^2 |w_y|^2 E_{20}^2 F^2}{T^4} \exp(-\frac{1}{2}(\sigma V_2)^2) \sum_{m=1}^{\infty} \frac{(\sigma V_2)^{2m}}{m!} \quad (4-100)$$

then

$$D\{E_2\} = \left\langle K_1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \exp(-\frac{m\tau}{\theta}) \cdot \exp[iV_x \tau \cos \phi + iV_y \tau \sin \phi + iV_z(\tau_1 - \tau_2)] dx_1 dy_1 \tau d\tau d\phi \right\rangle_{\tau} \quad (4-101)$$

Also, let

$$I(\tau, \phi) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp[iV_z(\tau_1 - \tau_2)] dx_1 dy_1 \quad (4-102)$$

From the geometry of the periodic surface, it is observed that $I(\tau, \phi)$ could be separated into three parts as (see Fig. 4-4 :

$$I_1(\tau, \phi) = T - \pi d_1^2 \quad (4-103)$$

$$I_2(\tau, \phi) = \iint_{A_1} \exp[iV_2(\xi_1 - \xi_2)] dx_1 dy_1 \quad (4-104)$$

$$I_3(\tau, \phi) = \iint_{\text{crater}} \exp[iV_2(\xi_1 - \xi_2)] dx_1 dy_1 \quad (4-105)$$

$$\left. \begin{aligned} \xi_1 &= \xi(x_1, y_1) \\ \xi_2 &= \xi(x_1 - \tau \cos \phi, y_1 - \tau \sin \phi) \end{aligned} \right\} \quad (4-106)$$

Now, performing the polar transformation for x_1, y_1 to r_1, θ , one obtains

$$x_1 = r_1 \cos \theta \quad (4-107)$$

$$y_1 = r_1 \sin \theta \quad (4-108)$$

$$dx_1 dy_1 = |J| dr_1 d\theta \quad (4-109)$$

$$|J| = \begin{vmatrix} \frac{\partial x_1}{\partial r_1} & \frac{\partial x_1}{\partial \theta} \\ \frac{\partial y_1}{\partial r_1} & \frac{\partial y_1}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r_1 \sin \theta \\ \sin \theta & r_1 \cos \theta \end{vmatrix} = r_1 \quad (4-110)$$

where r_1 and r_2 are

$$r_1 = \sqrt{x_1^2 + y_1^2} \quad (4-111)$$

$$\begin{aligned} r_2 &= \sqrt{x_2^2 + y_2^2} \\ &= \sqrt{r_1^2 + \tau^2 - 2r_1 \tau \cos(\theta - \phi)} \end{aligned} \quad (4-112)$$

the integral over Region II becomes

$$\begin{aligned} I_2(\tau, \phi) &= \int_0^{2\pi} \int_{d_1}^{d_2} \exp \left[iV_2 h_2 \sin \frac{\pi(r_2 - d_1)}{d_2 - d_1} \right. \\ &\quad \left. - iV_2 h_2 \sin \frac{\pi(r_1 - d_1)}{d_2 - d_1} \right] r_1 dr_1 d\theta \end{aligned}$$

$$= \int_0^{2\pi} \int_{d_1}^{d_2} \exp \left[i V_2 h_2 \sin \frac{\pi (r_2 - d_1)}{d_2 - d_1} - i V_2 h_2 \sin \frac{\pi (\sqrt{r_1^2 + r^2 - 2 r_1 r \cos(\theta - \phi)} - d_1)}{d_2 - d_1} \right] r_1 dr_1 d\theta \quad (4-113)$$

After making an expansion of the exponential in Bessel functions, I_2 becomes

$$I_2(\tau, \phi) = \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} J_{\mu}(V_2 h_2) J_{\nu}(-V_2 h_2) \exp \left[\frac{-i(\mu+\nu)\pi d_1}{d_2 - d_1} \right] \cdot \int_0^{2\pi} \int_{d_1}^{d_2} \exp \left[\frac{i\mu\pi r_1}{d_2 - d_1} \right] \cdot \exp \left[\frac{i\nu\pi}{d_2 - d_1} \sqrt{r_1^2 + r^2 - 2 r_1 r \cos(\theta - \phi)} \right] r_1 dr_1 d\phi \quad (4-114)$$

The origin of the reference coordinates is chosen at the center of the crater such that the integral around the rim and the crater is symmetrical to ϕ ; therefore, I_2 and I_3 are functions only of τ as given below:

$$I_2(\tau) = \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} J_{\mu}(V_2 h_2) J_{\nu}(-V_2 h_2) \exp \left[\frac{-i(\mu+\nu)\pi d_1}{d_2 - d_1} \right] \cdot \int_0^{2\pi} \int_{d_1}^{d_2} \exp \left[\frac{-i\mu\pi r_1}{d_2 - d_1} \right] \cdot \exp \left[\frac{i\nu\pi}{d_2 - d_1} \sqrt{r_1^2 + r^2 - 2 r_1 r \cos \theta} \right] r_1 dr_1 d\theta \\ = K_2 \left(d_2^{a+l+2n+2} - d_1^{a+l+2n+2} \right) \tau^{b-2n} \quad (4-115)$$

where

$$K_2 = \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} J_{\mu}(V_2 h_2) J_{\nu}(-V_2 h_2) \exp \left[\frac{-i(\mu+\nu)\pi d_1}{d_2 - d_1} \right] \cdot \sum_{l=0}^{\infty} \frac{1}{2^l} [1 - (-1)^{l+1}] \left[\frac{-i\nu\pi}{d_2 - d_1} \right]^l \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \frac{1}{a! b!} \left[\frac{i\pi}{d_2 - d_1} \right]^{a+b} \mu^a \nu^b B(a+l+2, 1) \cdot \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{i-b}{2} \right)_n \left(\frac{a+l+2}{2} \right)_n}{n! \left(\frac{a+l+4}{2} \right)_n} \quad (4-116)$$

with $B(x, y)$, the beta function, given by

$$B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt \quad (4-117)$$

and $(x)_n$, the hypergeometric series, given by

$$(x)_n = x(x+1)(x+2) \cdots (x+n-1) \quad (4-118)$$

The integral over Region III (the crater) is

$$I_3(\tau) = \iint_{\text{crater}} \exp[iV_2(\xi_1 - \xi_2)] dx_1 dy_1 \quad (4-119)$$

A transformation similar to that for I_2 yields the following value for I_3

$$I_3(\tau) = \int_0^{d_1} \int_0^{2\pi} \exp\left[-iV_2 h_1 \cos \frac{\pi r_1}{2d_1} + iV_2 h_1 \cos \frac{\pi r_1}{2d_1}\right] r_1 dr_1 d\theta \quad (4-120)$$

A substitution of

$$r_1 = \sqrt{r^2 + \tau^2} - \frac{r_1 \tau}{\sqrt{r^2 + \tau^2}} \cos \theta \quad (4-121)$$

into I_3 above yields

$$I_3(\tau) = \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} (i)^{\mu+\nu} J_{\mu}(-V_2 h_1) J_{\nu}(V_2 h_1) \cdot \int_0^{d_1} \exp\left[\frac{i\mu\pi r_1}{2d_1} + \frac{i\nu\pi}{2d_1} \sqrt{r^2 + \tau^2}\right] \cdot \int_0^{2\pi} \exp\left[-\frac{i\nu\pi}{2d_1} \frac{r_1 \tau}{\sqrt{r^2 + \tau^2}} \cos \theta\right] d\theta r_1 dr_1 \quad (4-122)$$

which integrated, yields

$$I_3(\tau) = K_3 d_1^{\alpha+2n+2} \tau^{b-2n} \quad (4-123)$$

where

$$K_3 = \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} (i)^{\mu+\nu} J_{\mu}(-V_2 h_1) J_{\nu}(V_2 h_1) \cdot$$

$$\sum_{l=0}^{\infty} \frac{1}{l!} \left[\frac{-i v \pi}{2 d_1} \right]^l [1 - (-1)^{l+1}] \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \frac{1}{a! b!}$$

$$\left[\frac{i \pi}{2 d_1} \right]^{a+b} \mu^a v^b B(a + l + 2, 1) \cdot$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{l-b}{2}\right)_n \left(\frac{a+l+2}{2}\right)_n}{n! \left(\frac{a+l+4}{2}\right)_n} \quad (4-124)$$

$$D\{E_2\} = \langle K_1 \int_0^{2\pi} \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) \cdot \exp(iV_x \tau \cos \phi + iV_y \tau \sin \phi) \cdot$$

$$[I_1 + I_2 + I_3] \tau d\tau d\phi \rangle_{\mathcal{F}_p} \quad (4-125)$$

But

$$\int_0^{2\pi} \exp(iV_x \tau \cos \phi + iV_y \tau \sin \phi) d\phi \simeq 2\pi J_0(\tau \sqrt{V_x^2 + V_y^2}) \quad (4-126)$$

$$\therefore D\{E_2\} = \langle K_1 2\pi \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) J_0(\tau \sqrt{V_x^2 + V_y^2})$$

$$[I_1 + I_2 + I_3] \tau d\tau \rangle_{\mathcal{F}_p}$$

$$= \langle 2\pi K_1 [I'_1 + I'_2 + I'_3] \rangle_{\mathcal{F}_p} \quad (4-127)$$

where

$$I'_1 = (T^2 - \pi d_1^2) \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) J_0(\tau \sqrt{V_x^2 + V_y^2}) \tau d\tau$$

$$= (T^2 - \pi d_1^2) \frac{\frac{m}{B}}{\left[\left(\frac{m}{B}\right)^2 + V_x^2 + V_y^2\right]^{3/2}}, \quad (4-128)$$

$$I'_2 = K_2 \left[d_2^{\alpha+l+2n+2} - d_1^{\alpha+l+2n+2} \right] \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) \cdot$$

$$J_0(\tau \sqrt{V_x^2 + V_y^2}) \tau^{b-2n+1} d\tau$$

$$= K_2 \left[d_2^{a+l+2n+2} - d_1^{a+l+2n+2} \right] (-1)^{b-2n+1} \frac{d^{(b-2n+1)}}{d \left(\frac{m}{B} \right)^{(b-2n+1)}} \left[\frac{1}{\left(\frac{m^2}{B^2} + V_x^2 + V_y^2 \right)^{1/2}} \right], \text{ AND } (4-129)$$

$$\begin{aligned} I'_3 &= K_3 d_1^{a+l+2n+2} \int_0^\infty \exp\left(-\frac{m\tau}{B}\right) J_0(\tau \sqrt{V_x^2 + V_y^2}) \tau^{b-2n+1} d\tau \\ &= K_3 d_1^{a+l+2n+2} (-1)^{b-2n+1} \frac{d^{(b-2n+1)}}{d \left(\frac{m}{B} \right)^{(b-2n+1)}} \left[\frac{1}{\left(\frac{m^2}{B^2} + V_x^2 + V_y^2 \right)^{1/2}} \right] \end{aligned} \quad (4-130)$$

The solution is

$$D\{E\} = \langle K, 2\pi [I'_1 + I'_2 + I'_3] \rangle_{\mathbb{R}^p} \quad (4-131)$$

The complete solution described in Eq. (4-91) is the product of Eq. (4-93), (4-97) plus (4-131).

The solution obtained above for the two dimensional scattering from the lunar surface is valid for all points in space. The monostatic radar where the parameters in $\langle E_2 E_2^* \rangle$ are

$$\theta_2 = -\theta_1 \quad (4-132)$$

$$\theta_3 = 0 \quad (4-133)$$

$$F = \frac{1}{\cos^2 \theta_1} \quad (4-134)$$

$$|W_y|^2 = 1 \quad (4-135)$$

$$V_y = 0 \quad (4-136)$$

$$|W_x|^2 = \left(\frac{\sin 2np\pi}{2n \sin p\pi} \right)^2 \quad (4-137)$$

$$V_x = \frac{4\pi}{\lambda} \sin \theta_1 \quad (4-138)$$

$$V_z = -\frac{4\pi}{\lambda} \cos \theta_1 \quad (4-139)$$

$$n = \frac{L_x}{T} \quad (4-140)$$

$$p = \frac{2T}{\lambda} \sin \theta_1 \quad (4-141)$$

The final solution of the radar return power $\langle E_2 E_2^* \rangle$ is expressed in terms of a set of conditions and terrain parameters. The radar antenna orientation with respect to the surface normal is substituted for θ_1 ; V_x and V_z and W_x is thus determined accordingly. The variance σ and the correlation distance B are the random surface characteristics, and may assume different values which can be fitted for the description of a certain section along the LM approach. The variables h_1 , h_2 , d_1 , and d_2 are linear functions of D , the crater diameter, which is a random variable with an independent probability density function $p(d)$ which is nonuniform over the whole landing track. However, $p(d)$ may be approximated to be piecewise uniform. Thus, with the aid of the computer, the required radar return degradation curve can also be calculated.

4.2.5 Conclusion

A very realistic lunar surface model description using a superposition of a sinusoidal quasi-periodic component and a randomly rough component surface is presented. This model

is believed to represent lunar surface conditions and the regularly existing rough features such as craters, hills and boulders as shown by the Surveyors and Orbiters photographs.

The solution of the radar signal return based on this new model is calculated using the scalar Helmholtz Integral. The result is verified for various conditions such as when the random component or the quasi-periodic component or both become zero, respectively. In all cases it is found that the result checks well with the work reported previously in the literature by other authors.

The one-dimensional and the two-dimensional radar return calculations are included with their implications. The scattering from a short range radar, where the beam cross-section area is small compared to the dimensions of the surface, the one-dimensional solution is appropriate and moreover its evaluation is simple, whereas for long range radar return it is necessary to use the two-dimensional solution. One must employ a spherical wave instead of a plane wave for very low altitude return.

The superposition of one or more quasi-periodic and random components as used for the lunar surface modeling may be extended to model several other types of terrain.

4.3 Spatially Anisotropic Random Surface Return

It is very common to employ an isotropic spatial surface height correlation function of one form or another in scalar calculation of radar return from a randomly rough surface. A study of certain types of surface roughness which may exist on the lunar surface such as hills and ridges, and others on the earth, it appears that in certain directions this correlation function is particularly biased, and therefore the azimuthal dependence of the surface must be accounted for in theoretical calculation. Such instances of azimuthal dependence also are found in the case of sea surface. It is this aspect of the surface feature that is studied by using topographical maps to show the azimuthal dependence of its spatial autocovariance. Theoretical study employing scalar Helmholtz integral calculations of the return power shows the specific difference between this and the isotropic case. Finally, a few surface models are used for laboratory ultrasonic simulation of this problem. The results of theoretical study are then compared with those obtained in laboratory simulation as well as with some obtained by aircraft flights over different terrains which exhibit such azimuthal characteristics.

4.4 LM Radar Return from an Anisotropic Lunar Surface

An anisotropic lunar surface roughness model is developed and its radar return is calculated to show the effect of this

anisotropy by comparing the results obtained for isotropic lunar surface. The anisotropic case is also shown to reduce to isotropic model and the results are exactly verified.

4.4.1 Summary

When a train of electromagnetic wave energy is incident upon a plane surface, the total energy is reflected, absorbed and transmitted at the surface boundary depending on its parameters. The LM radar receives only the reflected energy and thus the backscattering pattern of the surface may be plotted as a function of the incident angle (θ_1). A certain surface profile has a definite radar scattering pattern associated with it at a given frequency. The received field intensity is given as the sum of n scattered field components (Beckmann, 1963)

$$r e^{i\psi} = \sum_{j=1}^n A_j e^{i\phi_j} \quad (4-142)$$

where r = the resultant amplitude

ψ = the resultant phase

A_j = the amplitude contributed by the j^{th} scatterer, and

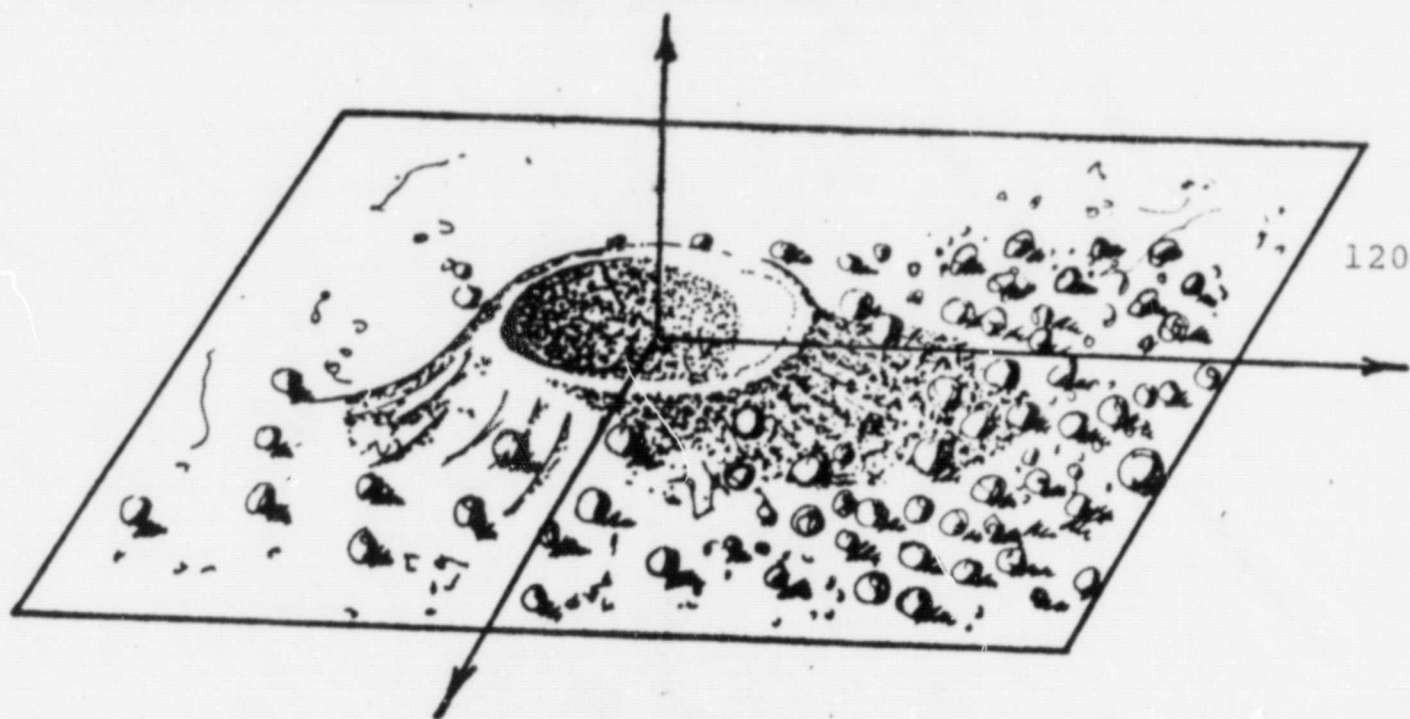
ϕ_j = the phase of the field from the j^{th} scatterer.

There are many surfaces with different roughness features which may yield the same scattering pattern so long as the amplitudes and the phases add up to be the same at all points in space, and therefore, the radar detection of terrain roughness is a sort of an estimation. And the mathematical solution

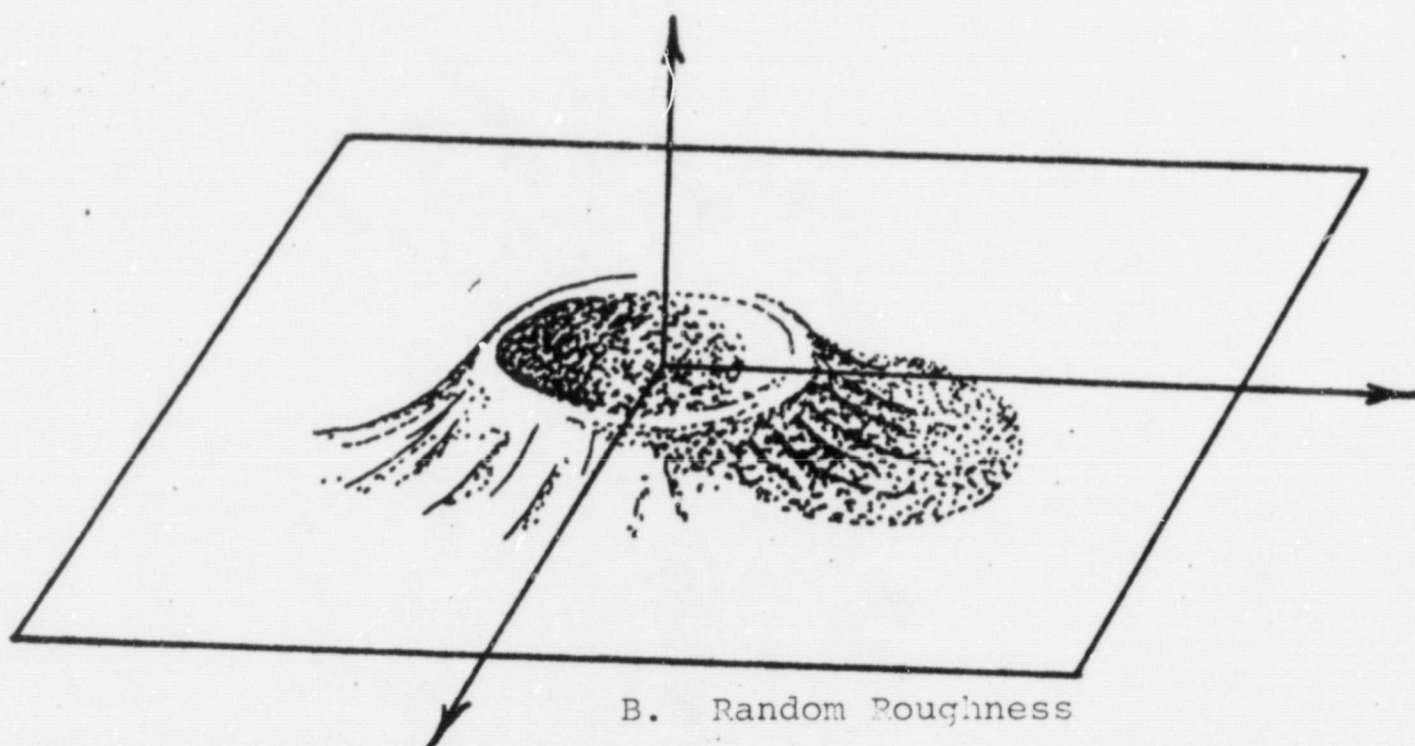
of radar return treats an unknown surface height variation to be a stochastic process.

The radar return from the lunar surface has been previously calculated in detail by Hayre and Tong (1967). Their statistical model is a periodic sinusoidally shaped crater superimposed on a general random surface with a gaussian distributed height function. In order to simplify the calculation, the craters and the general rough is assumed to be statistically independent, while the random surface itself is isotropically rough.

Many terrains occurring in nature are quite often anisotropic so far as their spatial roughness is concerned. A spatially anisotropic rough surface may be statistically described by its correlation function and its height probability density function. The following is a revised model of Hayre and Tong (1967) where an anisotropic rough component is superimposed on the periodically occurring craters of random diameters. The purpose of this revised model is to examine the effect on the radar return from a certain crater formation. For example, the craters on the maria appear to be mostly of impact origin and this is presumably true for the post-mare craters on the terrain. The primary craters are by definition, those due to impacts of cosmic masses. A hyper-velocity impact of a meteor at a certain incidence angle would generate a crater shown in Fig.4-10. The use of superposition method accounts for the two components as shown in Fig.4-11 where the random component ϵ_R appears to have a nonuniform roughness caused by the ejection of the surface material by such a meteor impact.



A. Typical Crater



B. Random Roughness

FIGURE 4-10.

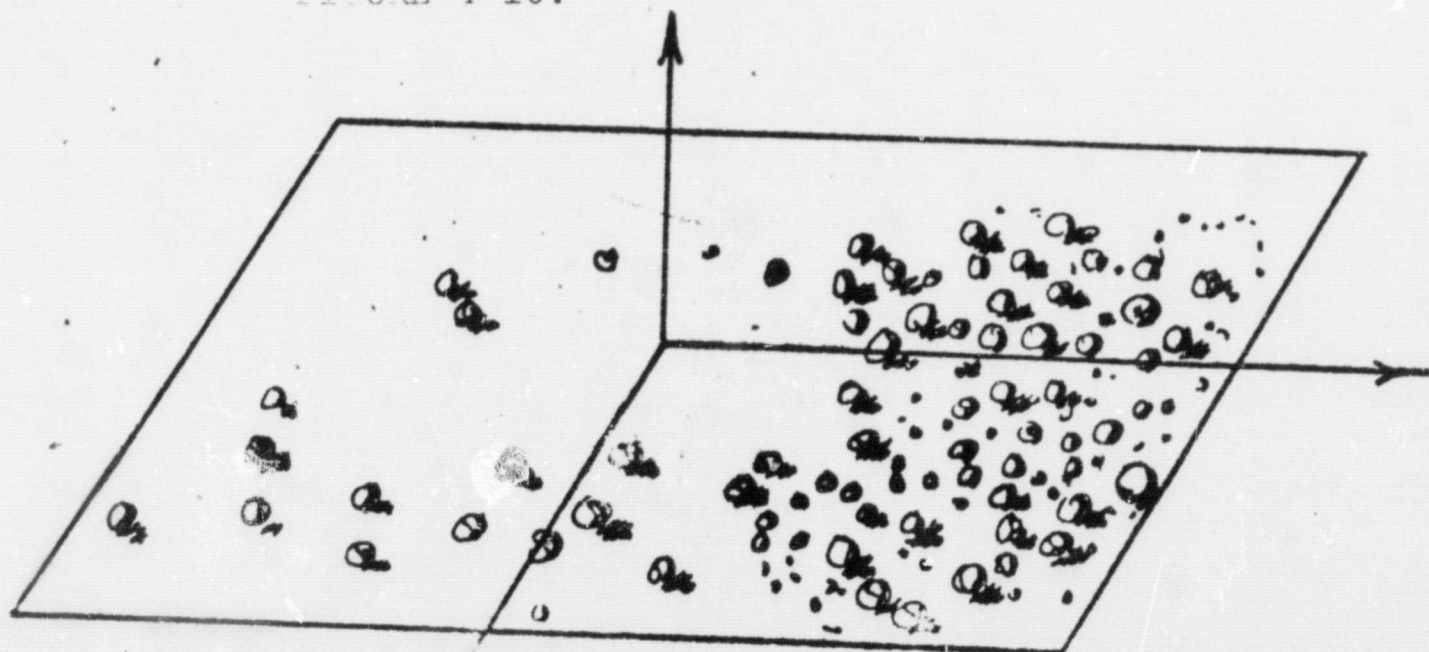


FIGURE 4-11. Cratered Random Rough Surface

When the crater within the area T by T is considered as a whole, the surface parameters are:

$$a. \quad \xi_p = \begin{cases} h_2 \sin \frac{\pi(\lambda - d_1)}{d_2 - d_1} & d_1 < r \leq d_2 \\ -h_1 \cos \frac{\pi R}{2d_1} & 0 < R \leq d_1 \\ 0 & d_2 \leq R \end{cases} \quad (4-143)$$

where $r = \sqrt{x^2 + y^2}$

assume $d_1 = C_1 D$

$d_2 = C_2 D$

$h_1 = C_3 D$

$h_2 = C_4 D$

with D being the diameter of the crater, which follows a certain density distribution $P(D)$.

b. ξ_R is gaussian random with

$$\langle \xi_R \rangle = 0 \quad (4-144)$$

$$P(\xi_R(z)) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{z^2}{2\sigma^2}} \quad (4-145)$$

$$C(\gamma, \phi) = e^{-\frac{\gamma^\beta}{B}} \cos \beta \phi \quad (4-146)$$

γ = projected distance on x-y plane between two sampling points (4-147)

ϕ = azimuth angle measured from the x-axis (4-148)

B = decorrelation constant

β = non-integer constant

The Stratton-Chu formula reduces to the Kirchhoff-Huygens integral for horizontal polarization of the incident field which is used here with the following assumptions:

a. The radar beam cross-section has dimensions large as compared to the surface period T .

- b. The radius of curvature of the random rough surface is much greater than the wavelength of the incident radiation.
- c. The incident radiation is a horizontally polarized plane wave.
- d. Depolarization effect is negligible.
- e. Shadowing, multiple scattering and edge effect are likewise neglected.
- f. ξ_R and ξ_p are statistically independent.

The scalar Helmholtz integral for the calculation of reflection based on the above stipulation is

$$E_2 = \frac{E_{20} F_3}{4L_x L_y} \int_{-L_x}^{L_x} \int_{-L_y}^{L_y} \exp(i\vec{v} \cdot \vec{r}) dx dy \quad (4-149)$$

$$E_2 = \frac{E_{20} F_3 W_x W_y}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp[iV_x x + iV_y y + iV_z(\xi_R + \xi_p)] dx dy \quad (4-150)$$

where E_2 = the reflected field from the surface

E_{20} = the field reflected in the direction of specular reflection ($\theta_1 = \theta_2$) by a smooth, perfectly conducting plane of the same dimensions under same angle of incidence at the same distance.

$$F_3 = \frac{1 + (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \sin \theta_3)}{\cos \theta_1 (\cos \theta_1 + \cos \theta_2)} \quad (4-151)$$

θ_1 = incident angle

θ_2 = reflection angle

θ_3 = deviation angle

$T \times T$ = a periodic sampling area illuminated by the radar beam

$$W_x = \frac{\sin(2\pi np)}{2n \sin p\pi} \exp(-ip\pi)$$

$$W_y = \frac{\sin(2\pi m q)}{2m \sin q\pi} \exp(-iq\pi)$$

$$n = \frac{L_x}{T}, \quad m = \frac{L_y}{T}$$

$$p = \frac{TV_x}{2\pi}, \quad q = \frac{TV_y}{2\pi}$$

$$V_x = \frac{2\pi}{\lambda} (\sin \theta_1 - \sin \theta_2) \quad (4-152)$$

$$V_y = \frac{-2\pi}{\lambda} (\sin \theta_2 \sin \theta_3) \quad (4-153)$$

$$V_z = \frac{-2\pi}{\lambda} (\cos \theta_1 + \cos \theta_2) \quad (4-154)$$

Then the average power return is given as

$$\langle E_2 E_2^* \rangle = \left\langle \frac{E_{10}^2 F_2^2 |w_x|^2 |w_y|^2}{T^2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [iV_x(x_1 - x_2) + iV_y(y_1 - y_2) \right. \quad (4-155)$$

$$\left. + iV_z(z_1 - z_2) \right] \langle \exp [iV_z(z_1 - z_2)] \rangle_{\mathbf{r}} dx_1 dx_2 dy_1 dy_2 \rangle_{\mathbf{r}}$$

$$\langle \exp [iV_z(z_1 - z_2)] \rangle_{\mathbf{r}} = e^{-\sigma^2 V_z^2} \sum_{m=0}^{\infty} \frac{\sigma^{2m} V_z^{2m}}{m!} (\cos \beta \phi)^m e^{-\frac{m\pi}{\theta}} \quad (4-156)$$

Through transformation and with the substitution of the previous equation, the average return power is

$$\langle E_1 E_2^* \rangle = \langle K_1 \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp(-\frac{m\tau}{\theta}) \cos^m \beta \phi \exp[iV_1 \tau \cos \phi + iV_2 \tau \sin \phi + iV_2 (\frac{\pi}{2} - \frac{\pi}{2})] dx_1 dy_1 \tau d\tau d\phi \rangle_{\frac{\pi}{2}} \quad (4-157)$$

$$K_1 = \frac{|W_1|^2 |W_2|^2 E_1^2 F_2^2}{T^4} \exp(-\sigma^2 V_2^2) \sum_{m=0}^{\infty} \frac{\sigma^{2m} V_2^{2m}}{m!} \quad (4-158)$$

$$\text{Let } I = I_1 + I_2 + I_3 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp[iV_2 (\frac{\pi}{2} - \frac{\pi}{2})] dx_1 dy_1 \quad (4-159)$$

$$\text{with } I_1 = T^2 - \pi d_1^2 \quad (4-160)$$

$$I_2 = K_2 (d_2^{a+l+2n+2} - d_1^{a+l+2n+2}) \tau^{b-2n} \quad (4-161)$$

$$K_2 = \sum_{u=0}^{\infty} \sum_{v=0}^{\infty} J_u(V_2 h_2) J_v(-V_2 h_2) \exp\left[\frac{-i(u+v)\pi d_1}{d_2 - d_1}\right] \\ \sum_{l=0}^{\infty} \frac{1}{l!} [1 - (-1)^{l+1}] \left[\frac{-iV\pi}{d_2 - d_1}\right]^l \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \frac{1}{a! b!} \\ \left[\frac{i\pi}{d_2 - d_1}\right]^{a+b} u^a v^b B(a+l+2, 1) \sum_{n=0}^{\infty} (-1)^n \frac{(\frac{l-b}{2})_n (\frac{a+l+2}{2})_n}{n! (\frac{a+l+4}{2})_n} \quad (4-162)$$

$$I_3 = K_3 d_1^{a+l+2n+2} \tau^{b-2n} \quad (4-163)$$

$$K_3 = \sum_{\mu=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} (1)^{\mu+\nu} J_{\mu}(-v_z h_1) J_{\nu}(v_z h_1) \sum_{l=0}^{\infty} \frac{1}{l!} \left[\frac{-iv_z \pi}{2d_1} \right]^l$$

$$\left[1 - (-1)^{l+1} \right] \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \frac{1}{a! b!} \left[\frac{iv_z \pi}{2d_1} \right]^{a+b} \mu^a \nu^b$$

$$B(a+l+2, 1) \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{l-b}{2}\right)_n \left(\frac{a+l+2}{2}\right)_n}{n! \left(\frac{a+l+4}{2}\right)_n} \quad (4-164)$$

Hence,

$$\langle E_z E_z^* \rangle = \langle K_1 \int_0^{2\pi} \int_0^{\infty} \exp\left(-\frac{m\tau}{B}\right) \cos^m \beta \phi \exp[i v_z \tau \cos \phi + i v_y \tau \sin \phi] (I_1 + I_2 + I_3) \tau d\tau d\phi \rangle_{\xi} \quad (4-165)$$

Integral with respect to ϕ ,

$$I_{\phi} = \int_0^{2\pi} \cos^m \beta \phi e^{i v_z \tau \cos \phi + i v_y \tau \sin \phi} d\phi \quad (4-166)$$

Through expansion,

$$\cos^m \beta \phi = \frac{1}{2^m} \sum_{t=0}^m \binom{m}{t} e^{i\beta\phi(m-2t)} \quad (4-167)$$

$$e^{i v_z \tau \cos \phi} = \sum_{p=-\infty}^{\infty} i^p J_p(v_z \tau) e^{ip\phi} \quad (4-168)$$

$$e^{i v_y \tau \sin \phi} = \sum_{g=-\infty}^{\infty} J_g(v_y \tau) e^{ig\phi} \quad (4-169)$$

$$I_{\phi} = \frac{1}{2^m} \sum_{t=0}^m \binom{m}{t} \sum_{p=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} i^p J_p(v_z \tau) J_g(v_y \tau) \int_0^{2\pi} e^{i\phi[(p+g) + \beta(m-2t)]} d\phi \quad (4-170)$$

$$= \frac{2\pi}{2^m} \sum_{t=0}^m \binom{m}{t} \sum_{p=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} i^p J_p(v_z \tau) J_g(v_y \tau) \frac{\sin \pi[(p+g) + \beta(m-2t)]}{\pi[(p+g) + \beta(m-2t)]}$$

Substituting I_p into equation (26), becomes

$$\langle E_2 E_1^* \rangle = \langle G (I'_1 + I'_2 + I'_3) \rangle_{\bar{z}_p} \quad (4-171)$$

where

$$G = K_1 \frac{2\pi}{2^m} \sum_{t=0}^m \binom{m}{t} i^p \frac{\sin \pi[(p+q) + \beta(m-2t)]}{\pi[(p+q) + \beta(m-2t)]} \quad (4-172)$$

$$I'_1 = \int_0^\infty (\tau^2 - \pi d_1^2) e^{-\frac{m\tau}{\theta}} J_p(V_x \tau) J_q(V_y \tau) \tau d\tau \quad (4-173)$$

$$I'_2 = K_2 (d_2^{a+l+2n+2} - d_1^{a+l+2n+2}) \int_0^\infty e^{-\frac{m\tau}{\theta}} J_p(V_x \tau) J_q(V_y \tau) \tau^{b-2n+1} d\tau \quad (4-174)$$

$$I'_3 = K_3 d_1^{a+l+2n+2} \int_0^\infty e^{-\frac{m\tau}{\theta}} J_p(V_x \tau) J_q(V_y \tau) \tau^{b-2n+1} d\tau \quad (4-175)$$

The integrals 4-173, 4-174, and 4-175 diverge for $m = 0$. It is necessary to calculate the average power as:

$$\langle E_2 E_1^* \rangle = D\{E_2\} + \langle E_2 \rangle \langle E_1^* \rangle \quad (4-176)$$

and

$$D\{E_2\} = \langle E_2 E_1^* \rangle - \langle E_2 \rangle \langle E_1^* \rangle \quad (4-177)$$

which has nearly the same expression as 4-157 except the m -series of k_1 now begins from 1 to infinity.

The completion of the integration of 4-173, 4-174, and 4-175 for $m > 0$, yields

$$I'_1 = (\tau^2 - \pi d_1^2) \frac{(V_x)^p (V_y)^b}{\Gamma(g+1) 2^{p+g} \left(\frac{m\tau}{B}\right)^{p+g+2}} \quad (4-178)$$

$$\sum_{\alpha=0}^{\infty} \frac{\Gamma(2+p+g+2\alpha)}{\alpha! \Gamma(p+\alpha+1)} F(-\alpha, -p-\alpha; g+1; \frac{V_y^2}{V_x^2}) \left(-\frac{V_x^2}{4\frac{m^2\tau^2}{B^2}}\right)^{\alpha}$$

$$I'_2 = K_2 (d_2^{a+l+2n+2} - d_1^{a+l+2n+2}) \frac{(V_x)^p (V_y)^b}{\Gamma(g+1) 2^{p+g} \left(\frac{m\tau}{B}\right)^{p+g+b-2n+2}}$$

$$\sum_{\alpha=0}^{\infty} \frac{\Gamma(b-2n+2+p+g+2\alpha)}{\alpha! \Gamma(p+\alpha+1)} F(-\alpha, -p-\alpha; g+1; \frac{V_y^2}{V_x^2}) \left(-\frac{V_x^2}{4\frac{m^2\tau^2}{B^2}}\right)^{\alpha} \quad (4-179)$$

$$I'_3 = K_3 d_1^{a+l+2n+2} \frac{(V_x)^p (V_y)^b}{\Gamma(g+1) 2^{p+g} \left(\frac{m\tau}{B}\right)^{p+g+b-2n+2}}$$

$$\sum_{\alpha=0}^{\infty} \frac{\Gamma(b-2n+2+p+g+2\alpha)}{\alpha! \Gamma(p+\alpha+1)} F(-\alpha, -p-\alpha; g+1; \frac{V_y^2}{V_x^2}) \left(-\frac{V_x^2}{4\frac{m^2\tau^2}{B^2}}\right)^{\alpha} \quad (4-180)$$

$$D\{E_2\} = \langle G(I'_1 + I'_2 + I'_3) \rangle_{\mathcal{F}_p} \quad m > 0 \quad (4-181)$$

$$\langle E_2 \rangle = \left\langle \frac{\omega_1 \omega_2 E_{10} F_3}{\tau^2} e^{-\alpha \frac{V_y^2}{V_x^2}} (I_a + I_b + I_c) \right\rangle_{\mathcal{F}_p} \quad (4-182)$$

$$\text{with } I_a = \tau^2 - \pi d_1^2 \quad (4-183)$$

$$I_b = 4\pi \sum_{p=1}^{\infty} \sum_{s=0}^{\infty} J_{2p}(V_b h_s) \frac{(-1)^s}{s! s!} \frac{(V_x^2 + V_y^2)^s}{2^{2s}} e^{-i \frac{2\pi p d_1}{d_2 - d_1}} \left[\frac{d_2^{2s+2} - d_1^{2s+2}}{s+1} - \sum_{a=0}^{2s+1} \frac{(2s+1)!}{a!} \left(\frac{-i 2\pi p}{d_2 - d_1} \right)^{2s+2-a} \left(e^{\frac{i 2\pi p d_2}{d_2 - d_1}} d_2^a - e^{\frac{i 2\pi p d_1}{d_2 - d_1}} d_1^a \right) \right] \quad (4-184)$$

$$I_c = 4\pi \sum_{f=1}^{\infty} \sum_{s=0}^{\infty} i^f J_f(-V_c h_s) \frac{(-1)^s}{s! s!} \frac{(V_x^2 + V_y^2)^s}{2^{2s}} \left[\frac{d_1^{2s+2}}{s+1} + \frac{(2s+1)!}{\left(-\frac{i f \pi}{2 d_1} \right)^{2s+2}} - i^f \sum_{a=0}^{2s+1} \frac{(2s+1)! d_1^a}{a! \left(-\frac{i f \pi}{2 d_1} \right)^{2s+2-a}} \right] \quad (4-185)$$

$$\langle E_z^* \rangle = \left\langle \frac{W_x^* W_y^* E_{z0} F_3}{T^2} e^{-(\sigma V_c)^2/2} (I_a^* + I_b^* + I_c^*) \right\rangle_p \quad (4-186)$$

I_a^* , I_b^* , and I_c^* are the conjugate of 4-183, 4-184, and 4-185

The average return power then is:

$$\langle E_z E_z^* \rangle = \left\langle \left[G (I_1' + I_2' + I_3') \right] + \left[\frac{|W_x|^2 |W_y|^2 E_{z0}^2 F_3^2}{T^4} e^{-(\sigma V_c)^2} (I_a + I_b + I_c) (I_a^* + I_b^* + I_c^*) \right] \right\rangle_p \quad (4-187)$$

4.4.2 Conclusion

The effect of the radar return from an anisotropic random scattering surface appears in the coefficient G , which is a function of the anisotropic index B . Furthermore, if B were

an integer, the resultant scattered energy would lose the effect of the anisotropy of the surface and its scattering cross-section would be similar to the case of an isotropic random surface. Similarly if B is equated to zero, the results check exactly with those obtained for the isotropic case.

4.5 Radar Backscattering From a Cratered Lunar Model

Previous lunar surface models consisted of randomly varying surface heights with isotropic spatial surface height correlation functions or sectional correlation functions or some other surface slope distribution. Furthermore, a super-position of various types of surfaces such as small and large scale roughness were also employed. These models do not seem to represent explicitly the lunar craters which are superimposed on the random height variations. It is this aspect of rimmed craters superimposed on a two dimensional distribution of surface heights with each component surface being a random variable but statistically independent of each other that is used to calculate the monostatic radar return for the Lunar Landing Module guidance and control radars at various angles of incidence. The unique feature of this surface model is that a random variation of the spacing between the rimmed craters allows a realistic spatial distribution as well as size distribution which has been found to be exponential from Orbiter I and II photographic data of medium and fine resolutions. The near-vertical

incidence radar return power is calculated using scalar Helmholtz integral and the results are then reduced to a few special cases such as one generated when the randomly varying main crater diameter tends to zero and the random period or distance between crater centers tends to infinity. The results seem to indicate that this model may be used to generate a wide variety of randomly rough surfaces for a calculation of their radar return.

4.5.1 Introduction

An electromagnetic wavefront incident on a surface causes each illuminated point on its surface to re-radiate secondary waves in all directions. Each point acts as a Huygens source, the phenomena is termed scattering, and physical optic techniques may be used in order to analyze the scattering. Such re-radiation depends on such surface properties as its permittivity, permeability and conductivity, as well as its surface features. In theory, a complete knowledge of the surface parameters and its roughness would yield its scattering pattern for a given source of the incident energy, although the calculation procedure is very complicated in the case of a randomly rough surface.

Theoretically, a terrain may be thought of as a randomly spaced array of scatterers which yield a certain radar scattering pattern, and conversely, knowing the scattering pattern, an unknown terrain may be specified. This technique of surface

roughness detection by employing radar backscattering cross-section has attracted considerable interest in recent years.

The terrain illuminated by a radar is quite often randomly rough; therefore it is necessary to use probability techniques in its description. Consequently, the radar return obtained from it is also described statistically. The present investigation of the lunar surface radar return assumes that it is a perfectly conducting surface at high frequencies. The Fresnel reflection coefficient for horizontally polarized plane wave is

$$R_1 = \frac{\cos \vartheta - \sqrt{\gamma^2 - \sin^2 \vartheta}}{\cos \vartheta + \sqrt{\gamma^2 - \sin^2 \vartheta}} \quad (4-188)$$

where ϑ is the local angle of incidence and

$$\gamma = \sqrt{\frac{\frac{\epsilon}{\mu_0} + i60\lambda C}{\frac{\mu}{\mu_0} + i\frac{4\pi}{\lambda}}} \quad (4-189)$$

as shown in Fig 4-12 where various terms are defined. For a perfectly conducting surface, C and γ are infinite and

$$R_1 = -1. \quad (4-190)$$

4.5.2 Lunar Surface Modeling

The Apollo guidance and control signals during its landing are primarily conditioned by the lunar surface roughness. An accurate analysis of lunar surface roughness in the radar-echo

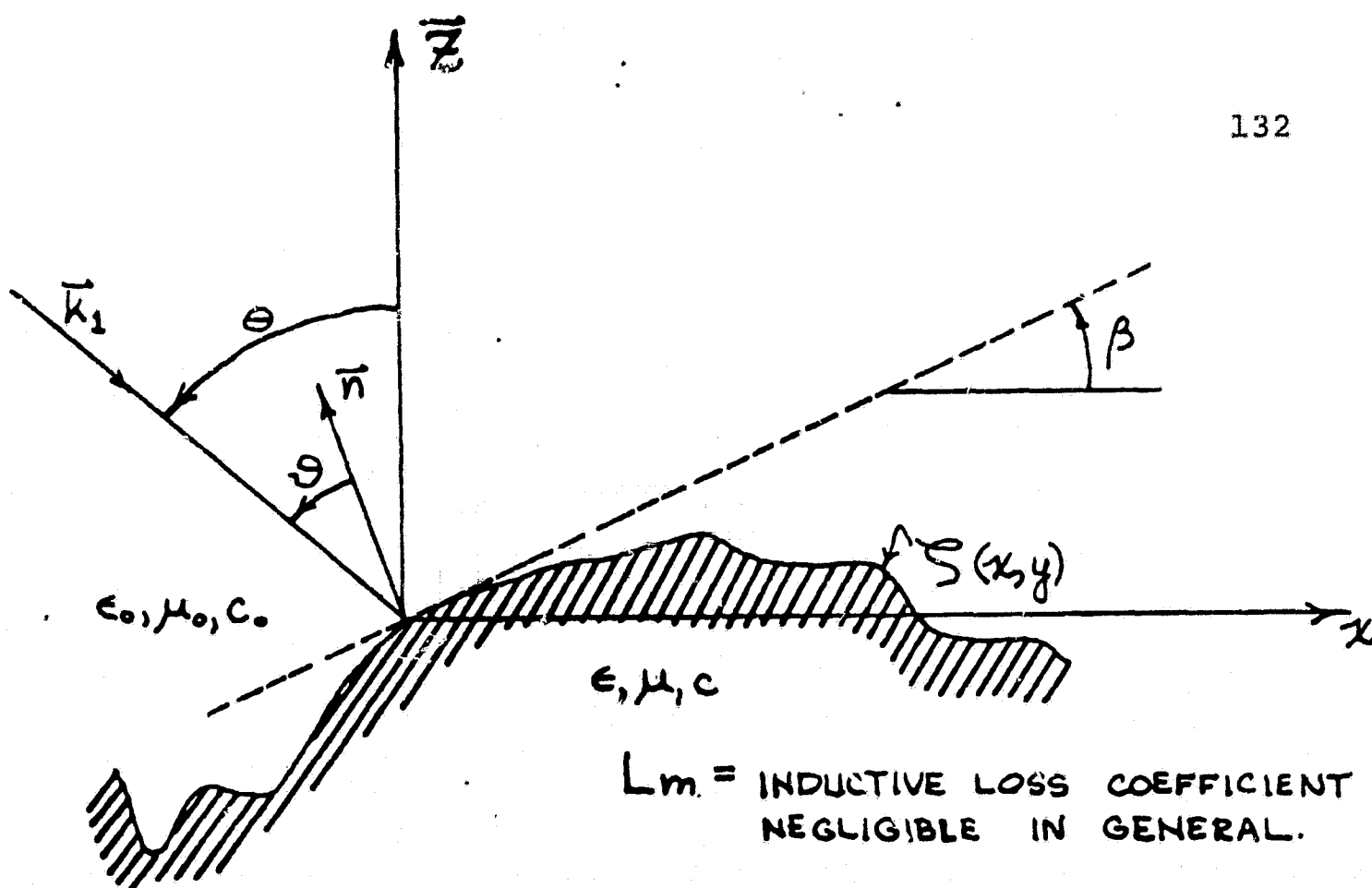


Fig. 4-12

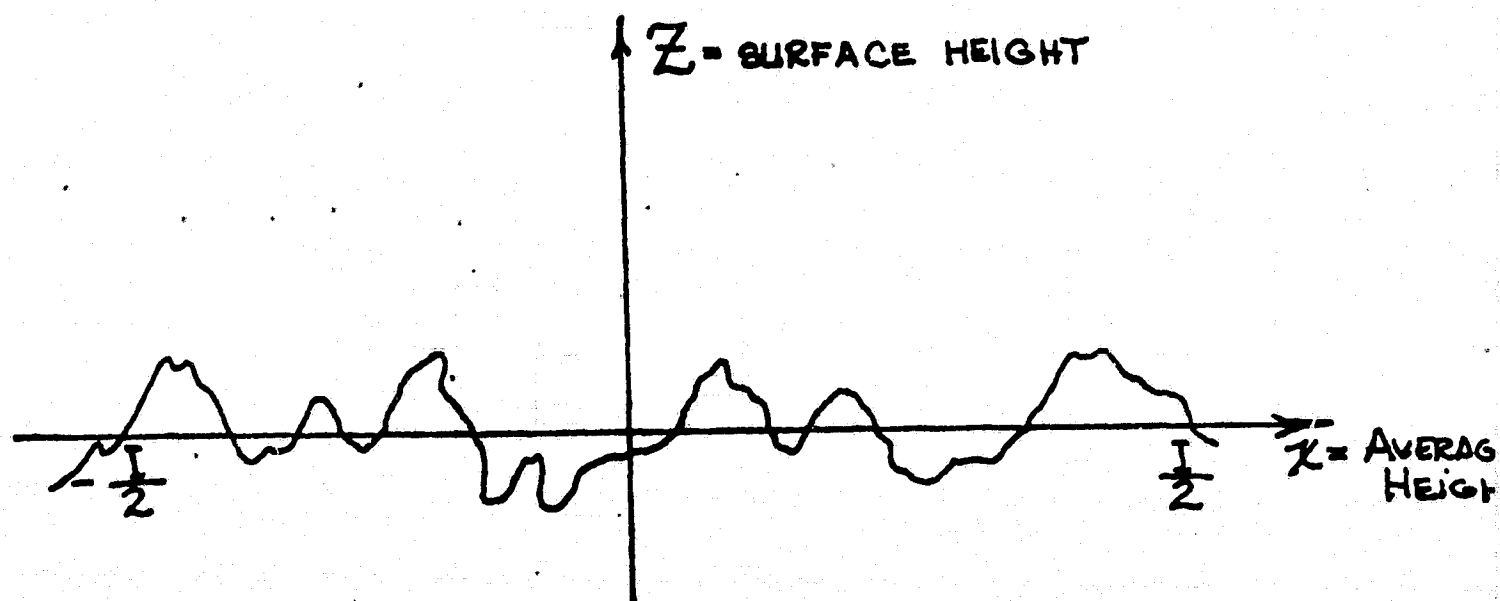


Fig. -13

study provides the means to predict the effects of the former on the guidance and control commands which are generated by a computer on board the module after processing the radar return signals from the velocity and altimeter radars. The lunar surface roughness is expected to bias the lunar landing module (LM) radar signals used for guidance and control during its landing. It is this aspect of lunar radar echo-degradation of LM velocity and altimeter beams which is discussed in this paper.

Although the early scientific efforts had provided valuable information on surface roughness of the moon, the application of their data was limited for various reasons including the lack of full scale verification. Even the recently acquired earth-based radar information about the lunar surface was not sufficiently detailed, partly due to the libration of the moon and partly due to the fading caused by the inhomogeneous ionospheric medium, and because of large areas contributing to radar return of earth based radars.

The availability of a high-energy source for a high-frequency radar was in itself a problem. Consequently, radar observations could not supply reliable evidence on the nature of the lunar surface structures of the order of a meter or smaller in dimensions. For example, according to Hayre's criterion (1961), a random rough surface is considered smooth for vertical incidence if

$$\sigma \leq \frac{\lambda}{12}$$

where

σ is the standard deviation of the surface undulation in meters, and

λ is the incident wavelength in meters.

If λ is four meters, a surface with a standard deviation of 0.3 meters is considered smooth. As a result, only the large scale roughness features could be obtained from the lunar radar echoes.

In 1961, Hayre and Moore conducted a laboratory experiment and acoustically simulated the lunar echoes. Nonlinear acoustic modeling techniques were used to model the lunar surface on a set of scaled down aluminum spheres. The acoustic return data was used to formulate a statistical model composed of a large-scale roughness and small-scale roughness with their reasonably complete statistical description including cross-correlation between these two components. The general Kirchhoff's solution with its usual assumptions was used in their theoretical model to obtain a mathematically manageable expression for radar cross-section. However, their model was not designed to describe such irregular and nonuniform surface features as lunar ridges in general, and craters and rims, in particular.

Some other attempts have been made to model the lunar backscatter cross-section (Hayre, 1962; Muehlman, 1966). Muehlman based his results on two point radar data from

Surveyor I. The simplified radar scattering cross-section equation includes weighted statistical parameters of the lunar surface such as standard deviation, the decorrelation constant and incident angle. Recent U. S. Ranger, Surveyors, and Orbiter I and II photographs of the surface of the moon show it to contain nonuniform roughness with its statistics varying from region to region. Thus, these previous models do not suffice for this study of degradation of LM radar signals.

With the advent of the successive Orbiter and Surveyor missions, more detailed lunar surface photographs have now been obtained. Information extracted from the Orbiter photographs and their mosaic overlays is believed to be sufficiently complete to establish a set of roughness parameters corresponding to each particular lunar site. These parameters are the large-scale and small-scale random roughness, crater-size probability distribution with inter-relationship among the crater dimensions, such as the diameter, depth, and rim height and rim base width. Mosaic surface data from a number of pre-selected LM landing sites have been carefully analyzed and their roughness parameters seem to be statistically independent from one site to the next. Therefore, a surface model composed of uncorrelated random and periodic structure with random or fixed parameters is formulated.

The present model is a superposition of a gaussian random rough surface, and a periodic crater outline surface

which can be made random as well. The random surface $\zeta_R(x,y)$ is assumed to have a mean value of zero and variance of σ as shown in Fig. 4-13. Its autocorrelation function is assumed to be $\exp(-\frac{\gamma}{B})$ (Hayre, 1961) where γ is the distance between two points measured on the x-y plane, and B is the decorrelation distance. The periodic surface $\zeta_p(x,y)$ is defined differently over three zones within a typical interval, as shown in Fig. 4-14 and Fig. 4-15.

In Zone I

$$\zeta_p(x,y) = 0 \quad \text{for } x^2 + y^2 > d_2^2 \quad (4-192)$$

In Zone II

$$\zeta_p(x,y) = h_2 \sin \frac{\pi(\sqrt{x^2+y^2} - d_1)}{d_2 - d_1} \quad \text{for } d_1^2 < x^2 + y^2 < d_2^2 \quad (4-193)$$

In Zone III

$$\zeta_p(x,y) = -h_1 \cos \frac{\pi\sqrt{x^2+y^2}}{2d_1} \quad \text{for } x^2 + y^2 < d_1^2 \quad (4-194)$$

As a first order approximation, the surface is assumed invariant along the y-axis, and the radar return power can be calculated from this one-dimensional mode of the lunar surface. The general Kirchhoff integral is applied under the following assumptions:

- a) The radar beam cross-section has dimensions large as compared to T.
- b) The radius of curvature of the random rough surface is much greater than the wavelength of the incident radiation.

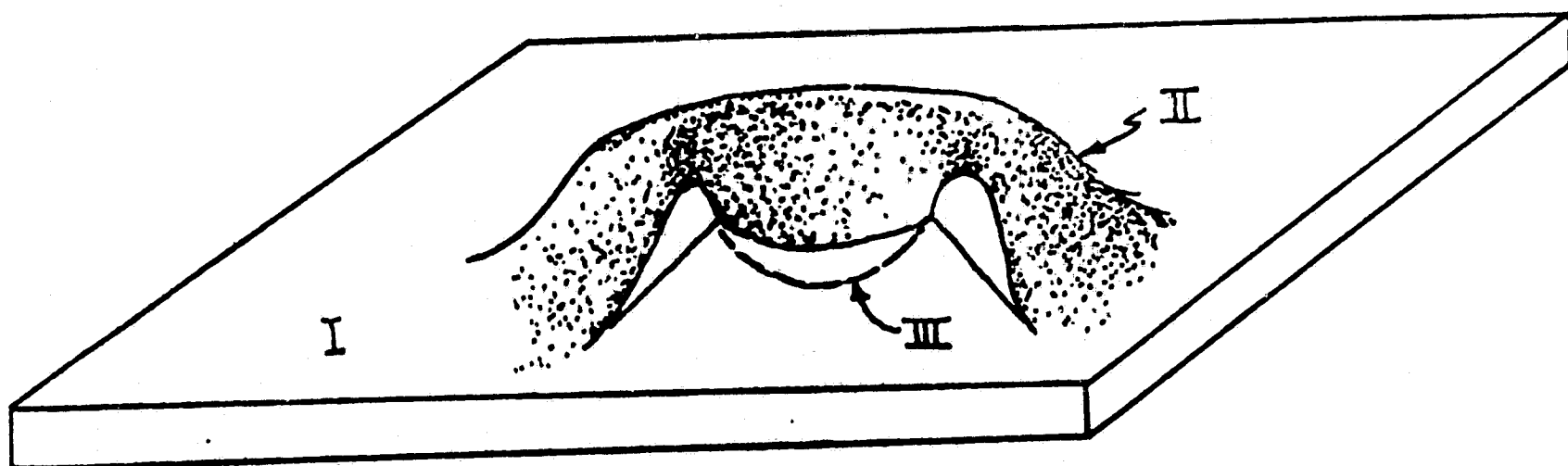


Fig. 14

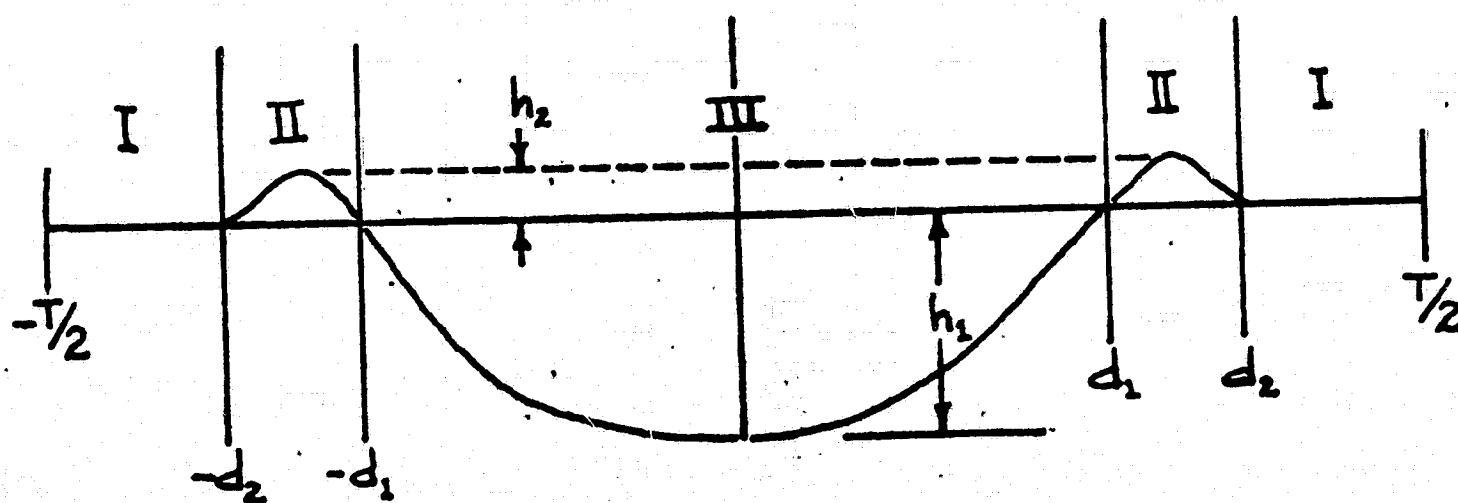


Fig. 15

- c) The incident radiation is a horizontally polarized plane wave.
- d) Shadowing, multiple scattering and depolarization effect is neglected.
- e) ξ_R and ξ_P are statistically independent.

The calculation of the monostatic backscatters energy with $L = n^T$ is

$$\langle E_1 E_1^* \rangle = \langle E_1 \rangle \langle E_1^* \rangle + D \{ E_1 \} \quad (4-195)$$

where $D \{ E_2 \} = \langle |E_2 - \langle E_2 \rangle|^2 \rangle$ is the variance of the complex variable E_2 .

$$E_1(\theta_1; \theta_2) = \frac{W E_{20} F}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp [L V_x x + i V_z (\xi_R + \xi_P)] dx \quad (4-196)$$

$$E_1(\theta_1; \theta_2) = \frac{E_{20} F}{2L} \int_{-L}^L \exp (i \vec{V} \cdot \vec{r}) dx \quad (4-197)$$

where

$E_2(\theta_1; \theta_2)$ is the reflected field from the surface.

θ_1 and θ_2 are angles of incident and reflection as measured from the outward surface normal.

E_{20} is the field reflected in the direction of specular reflection ($\theta_2 = \theta_1$) by a smooth, perfectly conducting plane at the same distance.

$$F = \sec \theta_1 \frac{1 + \cos (\theta_1 + \theta_2)}{\cos \theta_1 + \cos \theta_2}$$

$4L^2$ is taken as a sampling area

$$\bar{v} = \bar{k}_1 - \bar{k}_2$$

$$v_x = k (\sin \theta_1 - \sin \theta_2)$$

$$v_z = -k (\cos \theta_1 + \cos \theta_2)$$

$$\bar{r} = x \bar{a}_x + (\xi_R + \xi_P) \bar{a}_z$$

ξ_R and ξ_P are functions of x denoting the height of the rough surface.

The scattering geometry is shown in Fig 4-16 where

$$\theta_2 = -\theta_1 \text{ and } F = 1/\cos^2 \theta_1.$$

$$W = \frac{1}{2n} \sum_{u=-n}^{n-1} \exp(i2\pi up), \quad p = \frac{TV_z}{2\pi} \quad (4-198)$$

$$\begin{aligned} \langle E_s \rangle = & \left\langle \frac{WE_{so}F}{T} \exp\left(-\frac{\sigma^2 V_x^2}{2}\right) \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_x h_2) \left[\frac{e^{iV_x d_1} \cdot e^{ip\pi} - e^{-iV_x d_2} + e^{iV_x d_1} \cdot e^{ip\pi} - e^{-iV_x d_1}}{i(V_x + \frac{p\pi}{d_2 - d_1})} \right] \\ & \left. + \sum_{f=-\infty}^{\infty} i^f J_f(-V_x h_1) \frac{\sin(V_x + \frac{f\pi}{2d_1}) d_1}{(V_x + \frac{f\pi}{2d_1})/2} \right\} \right\rangle_{\xi_P} \quad (4-199) \end{aligned}$$

Taking the conjugate of Eq 4-199, one obtains

$$\begin{aligned} \langle E_s^* \rangle = & \left\langle \frac{W^* E_{so} F}{T} \exp\left(-\frac{\sigma^2 V_x^2}{2}\right) \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_x h_2) \left[\frac{-e^{-iV_x d_1} \cdot e^{ip\pi} + e^{iV_x d_2} - e^{-iV_x d_1} \cdot e^{ip\pi} - e^{-iV_x d_1}}{i(V_x + \frac{p\pi}{d_2 - d_1})} \right] \\ & \left. + \sum_{f=-\infty}^{\infty} i^f J_f(V_x h_1) \frac{\sin(V_x - \frac{f\pi}{2d_1}) d_1}{(V_x - \frac{f\pi}{2d_1})/2} \right\} \right\rangle_{\xi_P} \quad (4-200) \end{aligned}$$

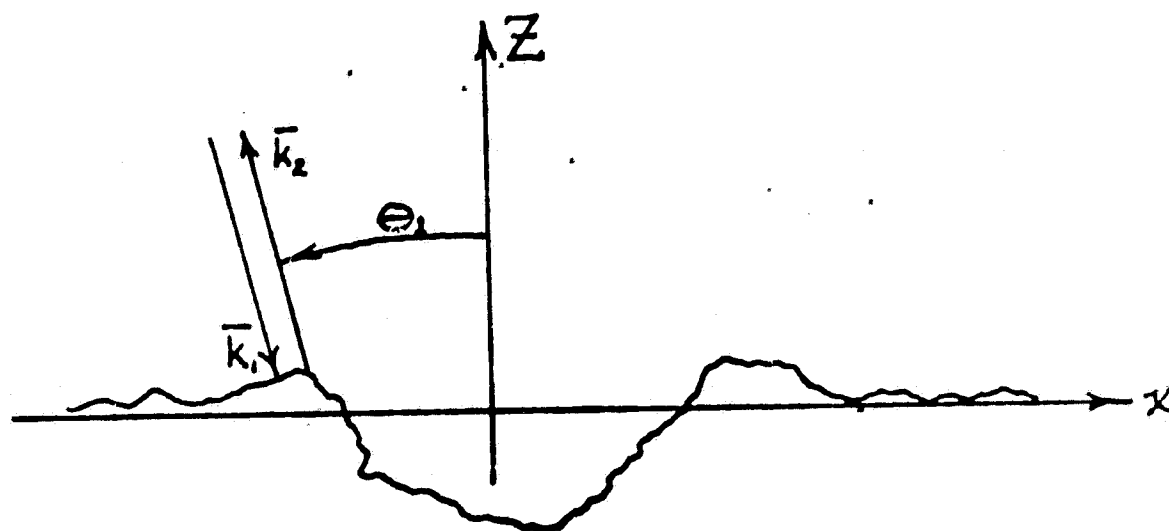


Fig. 4-16

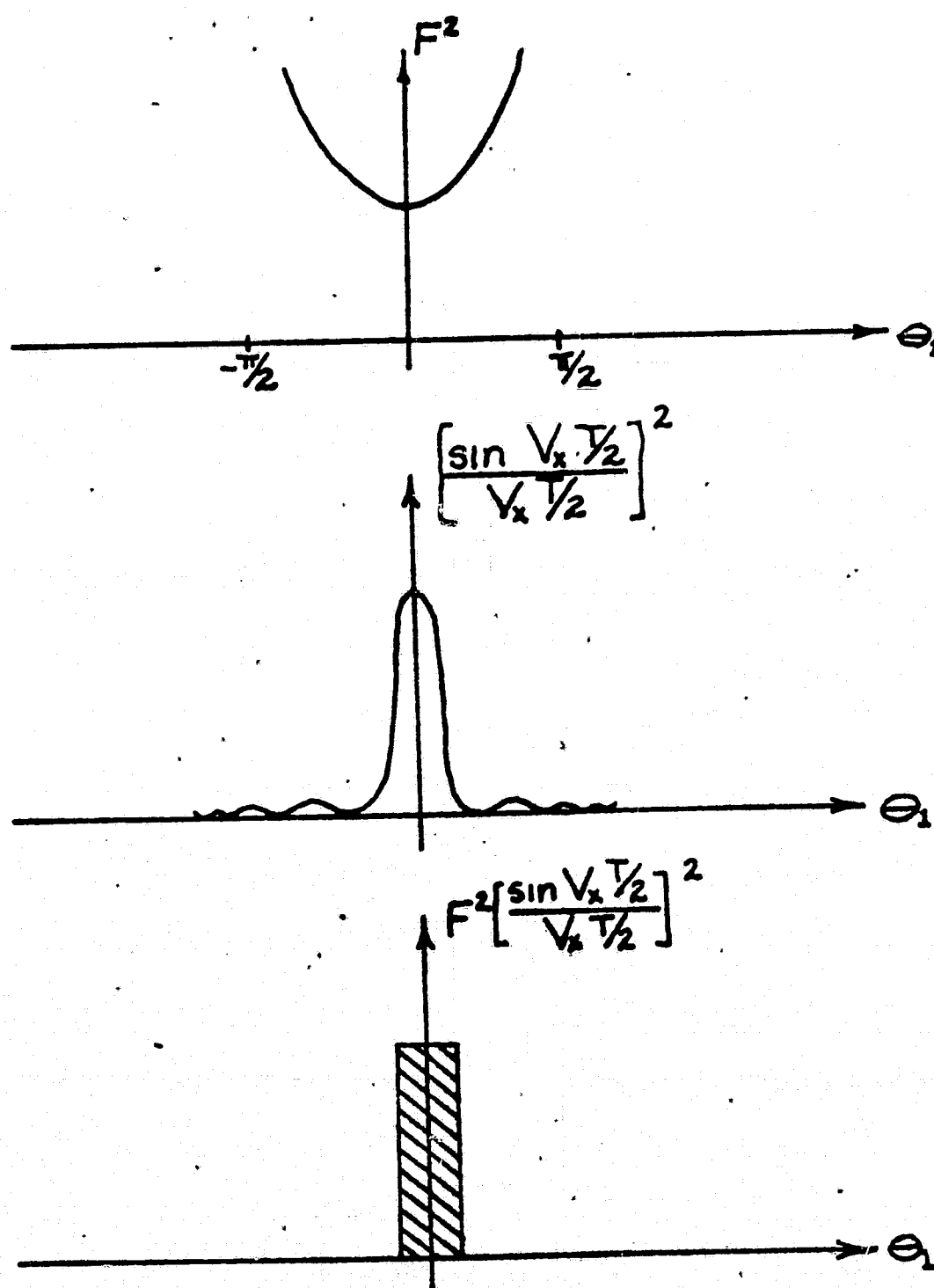


Fig. 4-17

The variance of the return signal is

$$\begin{aligned}
 D\{E_s\} = & \langle K_1 \left\{ 2(\tau - 2d_s) \frac{\frac{m}{B}}{(\frac{m}{B})^2 + V_x^2} \right. \\
 & + 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} (i)^{p+q} J_p(V_2 h_s) J_q(-V_2 h_s) \\
 & \cdot \left[\frac{\sin \frac{\pi}{2}(p+q)}{\frac{\pi}{2}(p+q)} \right] \left[\frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x + \frac{p\pi}{d_2 - d_1})^2} + \frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x - \frac{p\pi}{d_2 - d_1})^2} \right] \\
 & + 4d_1 \sum_{f=-\infty}^{\infty} \sum_{g=-\infty}^{\infty} (i)^{f+g} J_f(-V_2 h_1) J_g(V_2 h_1) \\
 & \cdot \left[\frac{\sin \frac{\pi}{2}(f+g)}{\frac{\pi}{2}(f+g)} \right] \left[\frac{\frac{m}{B}}{(\frac{m}{B})^2 + (V_x + \frac{f\pi}{2d_1})^2} \right] \left. \right\} \rangle_p \quad (4-201)
 \end{aligned}$$

$$K_1 = \frac{|W|^2 E_{s0}^2 F^2}{T^2} \exp(-\sigma^2 V_e^2) \sum_{m=1}^{\infty} \frac{(\sigma V_e)^{2m}}{m!} \quad (4-202)$$

The complete solution for the averaged radar return is the DC term, which is the product of Eq. 4-199 and Eq. 4-200 plus the variance of Eq. 4-201

In the absence of craters or hills, i.e., $\xi_p = 0$

$$\langle E_s \rangle \langle E_s^* \rangle = |W|^2 E_{s0}^2 F^2 \exp(-\sigma^2 V_e^2) \left[\frac{\sin V_e \frac{T}{2}}{V_e \frac{T}{2}} \right]^2 \quad (4-203)$$

and

$$D\{E_s\} = |W|^2 E_{s0}^2 F^2 \exp(-\sigma^2 V_e^2) \sum_{m=1}^{\infty} \frac{(\sigma V_e)^{2m}}{m!} \frac{2 \frac{m}{B}}{T [(\frac{m}{B})^2 + V_x^2]} \quad (4-204)$$

Therefore,

$$\begin{aligned} \langle E_z E_z^* \rangle \Big|_{z=0} = |w|^2 E_0^2 F^2 \exp(-\sigma^2 V_x^2) & \left[\left(\frac{\sin V_x \frac{T}{2}}{V_x \frac{T}{2}} \right)^2 \right. \\ & \left. + \sum_{m=1}^{\infty} \frac{(\sigma V_x)^{2m}}{m!} \frac{2 \frac{m}{B}}{\Gamma\left(\frac{m}{B} + V_x^2\right)} \right] \quad (4-205) \end{aligned}$$

The factor $|w|^2$ for the pure random surface equals unity and the result checks well with that calculated by Hayre and Kaufman (1965).

In the absence of the general roughness, $\beta_R = 0$

$$\sigma = 0 \quad (4-206)$$

$$\frac{1}{B} = 0 \quad (4-207)$$

and the expression for average power in the absence of random component is given as

$$\begin{aligned} \langle E_z E_z^* \rangle \Big|_{z=0} = \frac{|w|^2 E_0^2 F^2}{T^2} & \left\langle \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_x h_2) \left[\frac{e^{iV_x d_1} e^{ip\pi} - e^{-iV_x d_2} + e^{iV_x d_2} e^{ip\pi} - e^{iV_x d_1}}{i(V_x + \frac{p\pi}{d_2 - d_1})} \right] \\ & + \sum_{p=-\infty}^{\infty} i^p J_p(-V_x h_1) \frac{\sin(V_x + \frac{p\pi}{2d_1})d_1}{(V_x + \frac{p\pi}{2d_1})/2} \Bigg\} \Bigg\rangle_{\beta} \times \\ & \left\langle \left\{ \frac{\sin V_x \frac{T}{2} - \sin V_x d_2}{V_x/2} \right. \right. \\ & + \sum_{p=-\infty}^{\infty} J_p(V_x h_2) \left[\frac{-e^{-iV_x d_1} e^{ip\pi} + e^{iV_x d_2} - e^{-iV_x d_2} e^{ip\pi} + e^{-iV_x d_1}}{i(V_x + \frac{p\pi}{d_2 - d_1})} \right] \\ & + \sum_{p=-\infty}^{\infty} i^p J_p(V_x h_1) \frac{\sin(V_x - \frac{p\pi}{2d_1})d_1}{(V_x - \frac{p\pi}{2d_1})/2} \Bigg\} \Bigg\rangle_{\beta} \quad (4-208) \end{aligned}$$

In the absence of both β_R and β_P , the surface becomes a mirror reflector, and its radar reflectivity is

$$\langle E_1 E_2^* \rangle \Big|_{\substack{\beta_R=0 \\ \beta_P=0}} = E_{10}^2 F^2 \left[\frac{\sin V_k \frac{T}{2}}{V_k \frac{T}{2}} \right]^2 \quad (4-209)$$

For the monostatic case, one has

$$F^2 = \frac{1}{\cos^2 \theta_1} \quad (4-210)$$

$$V_k = \frac{4\pi}{\lambda} \sin \theta_1 \quad (4-211)$$

The curves of F^2 vs. θ_1 and $\left[\frac{\sin V_k \frac{T}{2}}{V_k \frac{T}{2}} \right]^2$ vs. θ_1 are presented in Fig. 4-17. If T is $\gg \lambda$, $\left[\frac{\sin V_k \frac{T}{2}}{V_k \frac{T}{2}} \right]^2$ decays very rapidly, and the product of the two curves may be approximated by an impulse, or

$$\langle E_1 E_2^* \rangle \Big|_{\substack{\beta_R=0 \\ \beta_P=0}} = E_{10}^2 \delta(\theta_1) \quad (4-212)$$

The significant terms in Eq. 4-205 for the conditions $\sigma_{V_2} \ll 1$, $\sigma_{V_2} = 1$ and $\sigma_{V_2} \gg 1$ which correspond to slight, moderate and very rough surfaces, respectively, have been discussed in great detail by Beckman (1963). In the case of the lunar surface where the general random surface appears to be very rough in terms of 3 cm wavelength (as shown in Surveyor pictures) σ_{V_2} is indeed large compared to 1.

Equation 4-205 becomes

$$\langle E_2 E_2^* \rangle = D \{ E_2 \} \quad (4-213)$$

AND

$$\exp\left[(-\sigma^2 V_2^2) \left(1 - e^{-\frac{\lambda}{B}}\right)\right] \approx \exp\left(-\sigma^2 V_2^2 \frac{\lambda}{B}\right) \quad (4-214)$$

Substituting Eq. 4-214 in the general expression for the average power, one obtains

$$\begin{aligned} \langle E_2 E_2^* \rangle = & \left\langle \frac{|W|^2 E_2^2 F^2}{T^2} \left\{ 2(T - 2d_2) \frac{(\sigma V_2)^2 B}{(\sigma V_2)^4 + V_2^2 B^2} \right. \right. \\ & + 2(d_2 - d_1) \sum_{p=-\infty}^{\infty} J_p^2(V_2 h_2) \left[\frac{(\sigma V_2)^2 B}{(\sigma V_2)^4 + (V_2 + \frac{p\pi}{d_2 - d_1})^2 B^2} \right. \\ & + \left. \left. \frac{(\sigma V_2)^2 B}{(\sigma V_2)^4 + (V_2 - \frac{p\pi}{d_2 - d_1})^2 B^2} \right] + 4d_1 \sum_{f=-\infty}^{\infty} J_f^2(V_2 h_1) \right. \\ & \left. \left. \left[\frac{(\sigma V_2)^2 B}{(\sigma V_2)^4 + (V_2 + \frac{f\pi}{2d_1})^2 B^2} \right] \right\} \right\rangle_{\mathcal{F}_p} \quad (4-215) \end{aligned}$$

In Fig. 4-18a curve is plotted using Eq. 4-215 with the following parameters:

$$\lambda = 3 \text{ cm}$$

$$\sigma = .3 \text{ cm}$$

$$B = 3 \text{ cm}$$

$$|W|^2 = 1$$

$$T = 30 \text{ meters}$$

$$P_f(\omega) = .05 \exp(-.05D)$$

$$d_1 = .5D$$

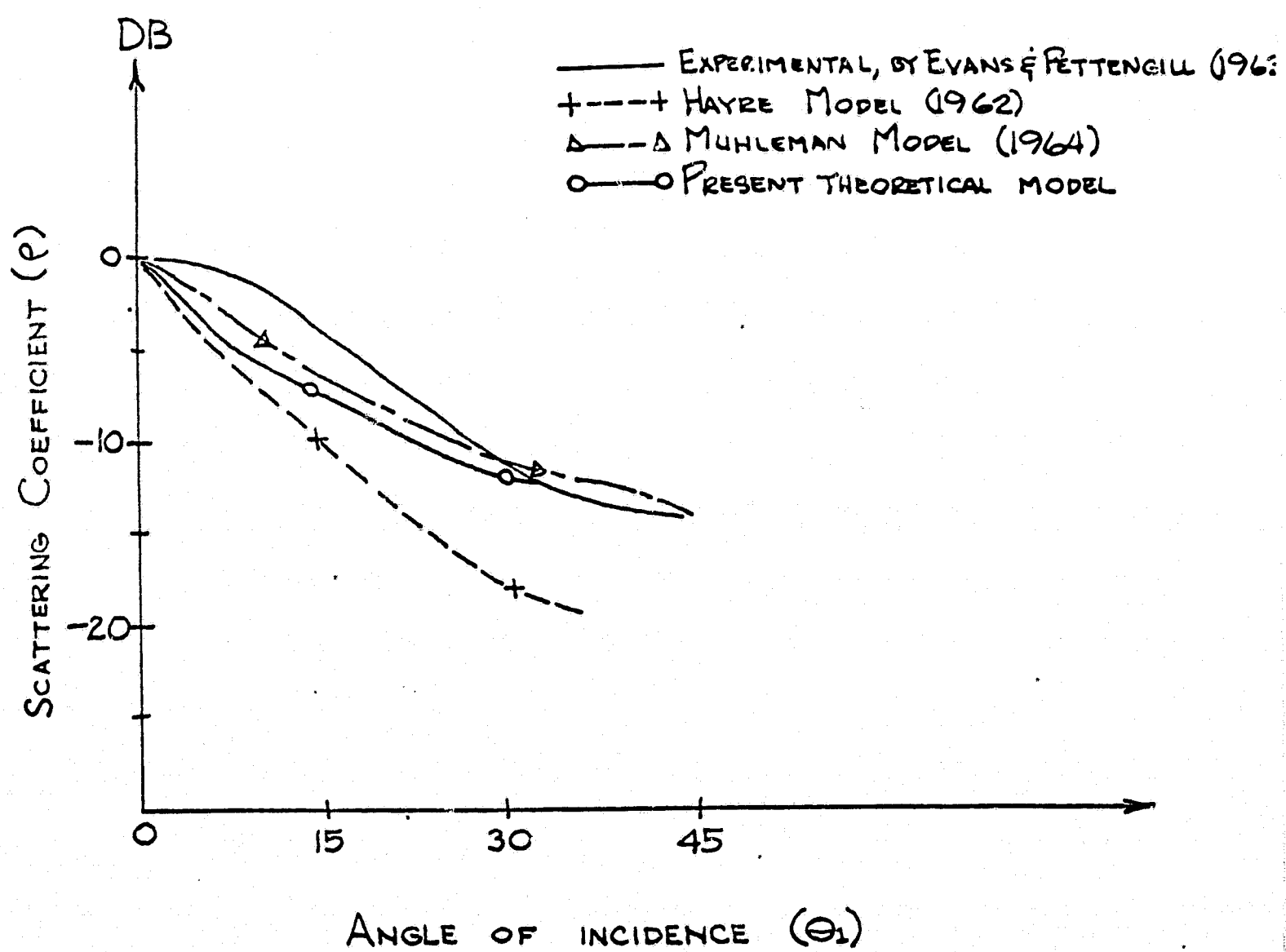


Fig. 4-18


$$d_2 = .55D$$

146

$$h_1 = .2D$$

$$h_2 = .02D$$

The results of other models (Hayre, 1962; Muhleman, 1964) and experimental results (Evans and Pettingel, 1963) dealing with lunar radar return are also plotted in Fig 4-18 and a comparison of these with results obtained by the model used in this work shows very good comparison.

4.5.3 CONCLUSION

This cratered model of randomly rough lunar surface is believed to be simple and yet inclusive of this important and common feature of the surface of the moon. Theoretical work seems to yield a realistic result in comparison and other theories.

CHAPTER V

LUNAR SURFACE BIAS ON LANDING MODULE

ALTIMETER SIGNAL

5.1 Modulation System of the FM Radar Altimeter

The radar frequency modulated altimeter in the Apollo landing module employs a continuous sawtooth modulated wave. During the process of landing, the altitude of the vehicle above the lunar surface or the distance between the transmitter and the illuminated surface varies linearly with the beat frequency resulting from mixing the backscattered signal with the transmitted signal whereas all other high frequencies are filtered out.

In order to make the beat frequencies significant along the entire landing trajectory, two different slopes of the sawtooth modulation are applied as shown in Fig. 5-1. The 700 microsecond portion upsweep is blanked out by the receiver. The frequency mixing process averages the results over thirty such 7000 microsecond long downsweeps, and the averaging period is 231 milliseconds. The turn-around error thus becomes insignificant in such an averaging process.

A monostatic near-vertical incidence radar operation is assumed and the distance traversed by the module during the signal travel to and from the lunar surface is negligible, as seen in Fig. 5-1 where at t_1 the transmitting signal is $A \sin 2\pi f t_1$

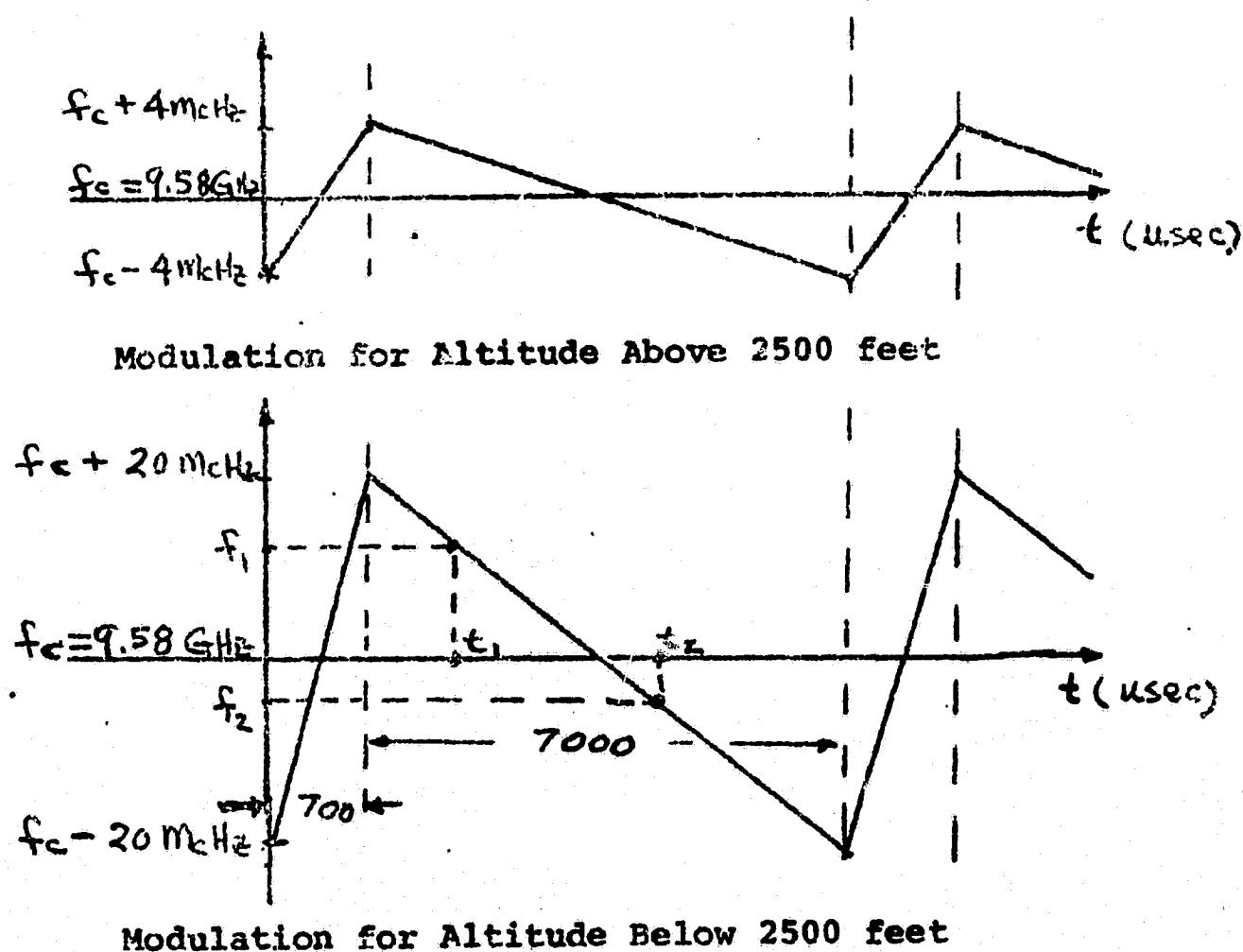


Fig. 5-1
Frequency Modulation

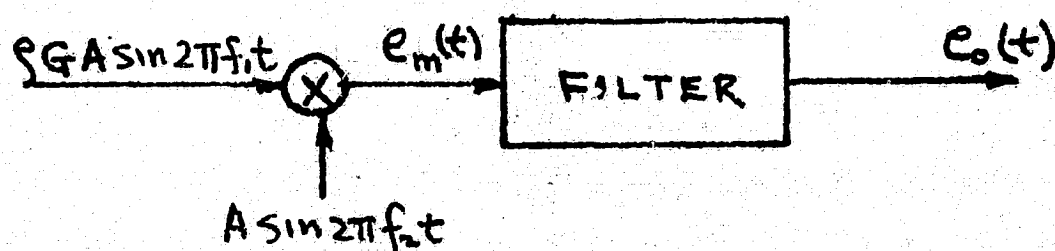


Fig. 5-2
Mixing and Filtering Operation

while at t_2 it is $A \sin 2\pi f_2 t$. If the height of the transmitter with respect to the surface is h , and

$$h = \frac{(t_2 - t_1)c}{2} \quad (5-1)$$

then the signal received at t_2 is $\mathcal{S} GA \sin 2\pi f_1 t$ where $\mathcal{S}$ is the scattering coefficient of the surface, G is the total gain of the antenna.

The mixing and filtering process as shown in Fig.5-2 is a simple multiplier, with its output

$$\begin{aligned} e_m(t) &= [A \sin 2\pi f_2 t] [\mathcal{S} GA \sin 2\pi f_1 t] \\ &= \mathcal{S} \frac{GA^2}{2} [\cos 2\pi(f_1 - f_2)t + \cos 2\pi(f_1 + f_2)t] \end{aligned} \quad (5-2)$$

which, when filtered in order to remove the high frequency content, becomes:

$$e_o(t) = \mathcal{S} \frac{GA^2}{2} \cos 2\pi f_B t \quad (5-3)$$

where

$$f_B = f_1 - f_2 \quad (5-4)$$

The output in Eq. 5-3 yields such important information about the target because its amplitude at the beat frequency output varies linearly with the scattering coefficient $\mathcal{S}$. Thus a knowledge of the backscattering power leads to an estimation

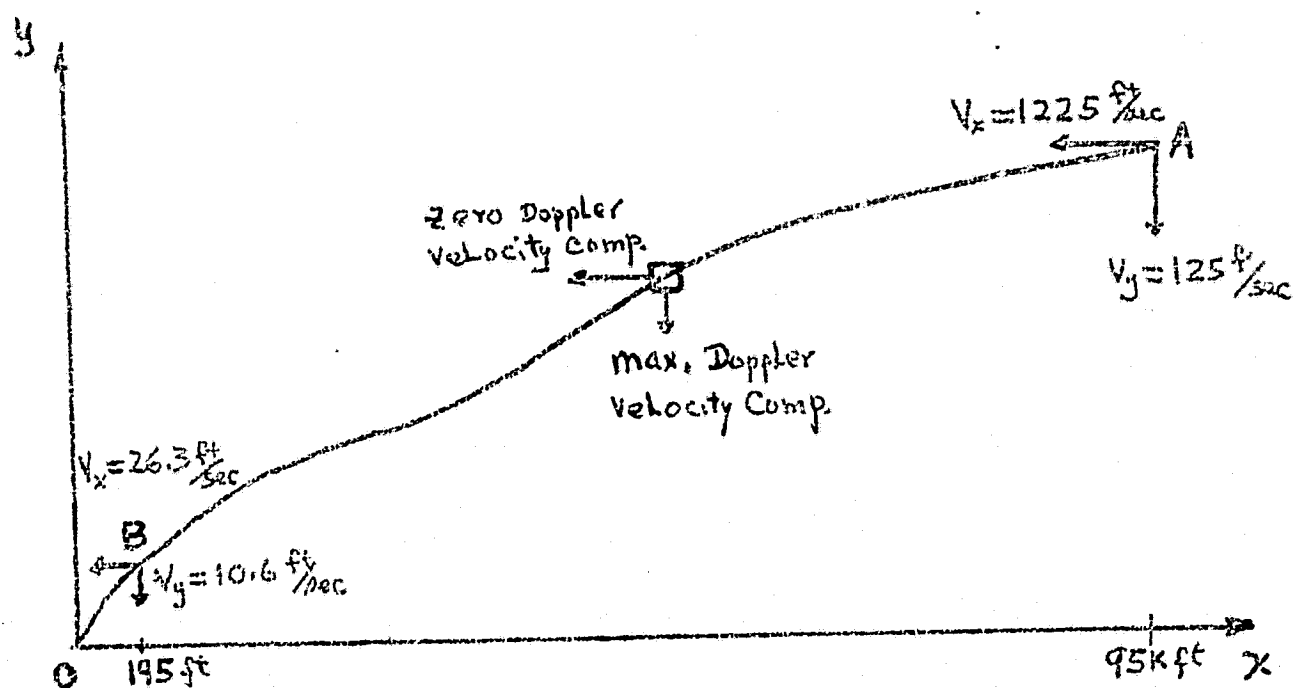


Fig. 5-3

Lunar Module Landing Trajectory

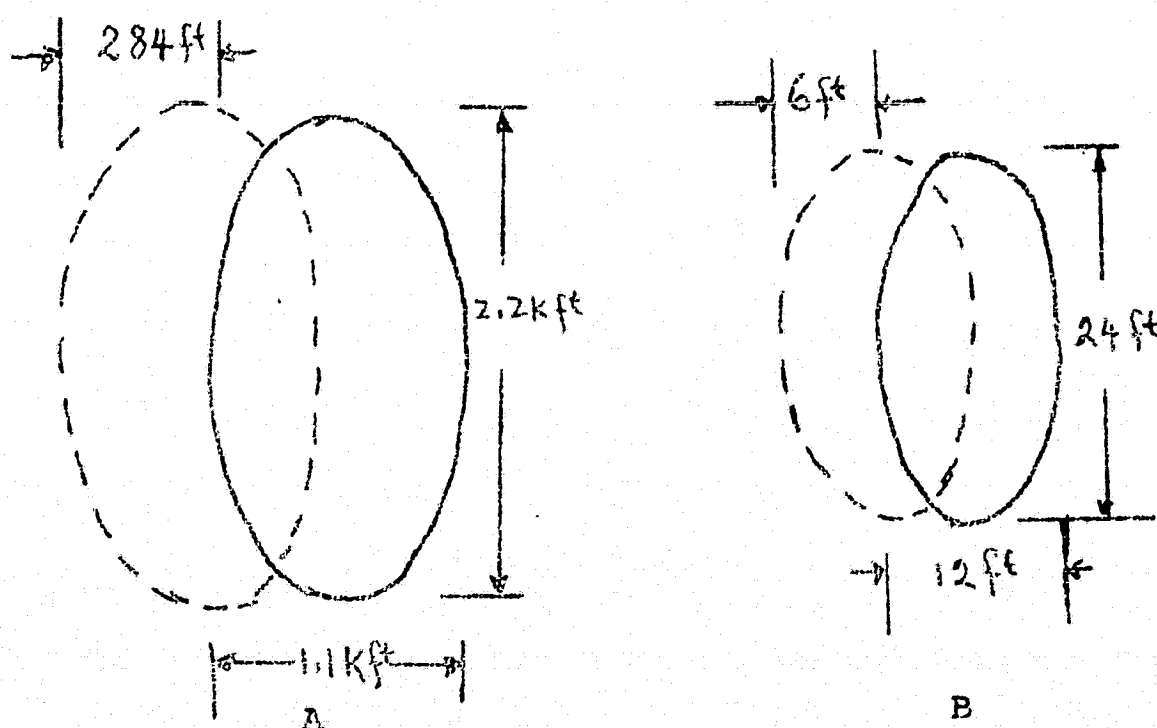


Fig. 5-4

Radar Beam on the Lunar Surface at Parts A and B
During One Averaging Period

of the lunar surface conditions in a probabilistic sense whereas the altitude h may be obtained from f_B as,

$$h = \frac{f_B C}{2\dot{f}} \quad (5-5)$$

where

$$\dot{f} = \frac{f_1 - f_2}{t_1 - t_2} \quad (5-6)$$

At high gate $\dot{f}$ is 1.14×10^3 cps/sec while at low gate, $\dot{f}$ is 5.76×10^3 cps/sec.

5.1.1 LM Mean Return Spectrum

The velocity of the Apollo module descending along the trajectory may be described by its two components as shown in Fig.5-3. The normal velocity V_y introduces doppler effect f_d as given below.

$$f_D = \frac{2V_y}{C} f_c \quad (5-7)$$

The beat frequency f_B when the transmitter and target are both stationary would be

$$f_B = \frac{2h}{C} \dot{f} \quad (5-8)$$

and in the presence of the doppler effect, the beat frequency would become

$$f_B = \frac{2h}{C} \dot{f} - f_D \quad (5-9)$$

The mean return power spectrum is expressed as (Rice, 1944, 1945)

$$\overline{P(f_B)} = \frac{\lambda^2 \omega_r^2 G^2(\theta) \zeta(\theta) \dot{f}^2}{8\pi^2 C^2 r_B^3} \quad (5-10)$$

for $\frac{2h}{c} \dot{f} - f_D < f_B < \frac{3h}{c} \dot{f} - f_D$

where

ω_r = the transmitting output power

$G(\theta)$ = the gain of the antenna

$\zeta(\theta)$ = the surface scattering coefficient

$\dot{f}$ = the time rate of frequency modulation

f_B = the beat frequency

f_D = the doppler frequency

and the expected frequency, $\bar{f}_B$ is

$$\bar{f}_B = \left[\frac{\int_{\frac{2h}{c} \dot{f} - f_D}^{\frac{3h}{c} \dot{f} - f_D} f^2 \overline{P(f_B)} df}{\int_{\frac{2h}{c} \dot{f} - f_D}^{\frac{3h}{c} \dot{f} - f_D} \overline{P(f_B)} df} \right]^{1/2} \quad (5-11)$$

It is therefore necessary to analyze the beat frequencies for each section of the trajectory because both the altitude and both velocity components of the vehicle are varying. These changes significantly effect the output of the altimeter.

Taking two points A and B on the trajectory as shown in Fig.5-3 one may write down all the necessary parameters as

$$V_x = 1225 \text{ feet/sec}$$

$$V_y = 125 \text{ feet/sec}$$

$$\Delta t = \frac{2h}{c} = 36 \mu\text{sec}$$

$$\dot{f} = 1.14 \times 10^3 \text{ cps}/\mu\text{sec}$$

$$\dot{f}\Delta t = 40 \times 10^3 \text{ cps}$$

$$f_D = \frac{2V_y}{c} f_c = 2.4 \times 10^3 \text{ cps}$$

During the averaging period of 231 milliseconds, the linear surface distance covered is

$$\begin{aligned} L &= 1225 \text{ ft/sec} \times 231 \times 10^{-3} \text{ sec} \\ &= 283 \text{ feet} \end{aligned} \quad (5-12)$$

The radar beam cross-section at A is shown in Fig.5-4. The average scattering coefficient $\langle \rho(\theta) \rangle$ may be calculated using a two-dimensional scattering geometry employing the superposition method (Hayre and Tong, 1967), and having calculated $\rho(\theta)$, $\overline{\rho(f_B)}$ and $\overline{f_B}$ can then be obtained from Eq.5-10 and Eq.5-11.

At point B, one has

$$V_x = 26.3 \text{ feet/sec}$$

$$V_y = 10.6 \text{ feet/sec}$$

$$\Delta t = \frac{2h}{c} = .41 \mu\text{sec}$$

$$\dot{f} = 5.76 \times 10^3 \text{ cps/sec}$$

$$\dot{f}\Delta t = 2.35 \times 10^3 \text{ cps}$$

$$f = 207 \text{ cps}$$

The radar beam cross-section at B is shown in Fig. 5-4.

$$\begin{aligned} L &= 26.3 \text{ feet/sec} \times 231 \times 10^{-3} \text{ sec} \\ &= 6 \text{ feet} \end{aligned}$$

Since the radar beam cross-section is small compared to the dimensions of the significant hazardous features, the scattering coefficient may be approximated by the one-dimensional solution. Again it is necessary to apply Eq. 5-10 and Eq. 5-11 for the calculation of the mean return power spectrum $\overline{P(f_B)}$ and the expected beat frequency $\overline{f_B}$.

5.1.2 Conclusion

This discussion indicates that the FM altimeter response may be analyzed to obtain the necessary information during the Apollo landing, namely the return energy level and the beat frequency. The return energy level generally represented by the scattering coefficient, indicates the surface roughness whereas the beat frequency is linearly proportional to the radar range. As seen from Eq. 5-10 and Eq. 5-11, this information is directly related to the scattering coefficient. And a reasonably exact solution has been derived by superposition of a random crater-array and a Gaussian rough surface (Hayre and Tong, 1967).

5.2 Probable Deviations in Altitude Reading Given by the LM Altimeter for the Most Rough Surface Along a Certain Given Trajectory

The spectrum of the backscattered energy for the Pre-frequency Modulated Altimeter may be analyzed to obtain the target range variations versus the target surface conditions. And the expected frequency, and consequently the associate altitude may be calculated (Rice, 1944) as

$$\langle f_B \rangle = \left[\frac{\int_{\frac{2hf}{c} - f_D}^{\frac{3hf}{c} - f_D} \frac{G^2(\theta) \sigma(\theta)}{f_B} df_B}{\int_{\frac{2hf}{c} - f_D}^{\frac{3hf}{c} - f_D} \frac{G^2(\theta) \sigma(\theta)}{f_B^3} df_B} \right]^{\frac{1}{2}} \quad (5-13)$$

where

- f_B = the static beat frequency
- h = the actual range
- f_D = the doppler frequency
- $\dot{f}$ = the FM slope
- c = velocity of light
- $G^2(\theta)$ = the two-way antenna gain
- $\sigma(\theta)$ = the target scattering coefficient

For the Landing Module (LM), the radar has a beam width of 3.5° by 7° and the orientation of the beam on the landing track is as shown in Fig 5-5 for vertical incidence case.

The return signal is averaged over a period of 231 milliseconds along the trajectory. The horizontal velocity of the LM during landing is small, and the surface covered by the radar in one period is comparable to 150% of the beam cross-section (Tong and Hayre, 1967). Considering that the pitch angle of the vehicle, and the antenna angle do not vary within an averaging period, it suffices to treat the system as stationary with respect to the horizontal axis. If the width of the landing track scanned by the radar is normalized, the average beat frequency may be approximated using a one-dimensional profile.

Fig. 5-6 Geometry of the Apollo radar-detection.

As shown in Fig. 5-6 the transformation of f_B to θ would be;

$$h = \frac{f_2 c}{2 \dot{f}} \quad \text{or} \quad f_B = \frac{2h \dot{f}}{c} \quad (5-14)$$

$$R_1 = \frac{f_1 c}{2 \dot{f}} \quad \text{or} \quad f_1 = \frac{2h \dot{f}}{c \cos \psi_1} \quad (5-15)$$

$$R_2 = \frac{f_2 c}{2 \dot{f}} \quad \text{or} \quad f_2 = \frac{2h \dot{f}}{c \cos \psi_2} \quad (5-16)$$

In general;
$$f_B = \frac{2h \dot{f}}{c \cos \theta} \quad (5-17)$$

and
$$df_B = \frac{2h \dot{f}}{c} \frac{\sin \theta}{\cos^2 \theta} d\theta \quad (5-18)$$

Substituting 5-15, 5-16, 5-17 and 5-18 in 5-13, one obtains,

$$\langle f_B \rangle = \frac{2h \dot{f}}{c} \left[\frac{\int_{\psi_1}^{\psi_2} G^2(\theta) \sigma(\theta) \tan \theta d\theta}{\int_{\psi_1}^{\psi_2} G^2(\theta) \sigma(\theta) \cos \theta \sin \theta d\theta} \right]^{1/2} \quad (5-19)$$

The expression $\frac{2h \dot{f}}{c}$ in Eq. 5-19 is the beat frequency corresponding to the actual altitude of the transmitter, whereas the square root term is the ratio of the detected altitude to the actual altitude. Since the percent deviation in altitude is the essential information, one needs only to carry the calculation of the square root part of Eq. 5-13.

Based on the information of the radar beamwidth, the antenna gain is approximated to be

$$G^2(\theta) = \cos^4 13\theta \quad (5-20)$$

and the scattering cross-section is chosen of point No. 8 Site P-II-6 of static runs of the acoustic simulation test and is shown in Fig. 5-7.

The percent deviation is plotted as shown in Fig. 5-8.

5.2.1 Conclusion

This calculation shows that the percent deviation of the altitude reading from the actual would vary considerably from 79% to 203% corresponding to angles of incidence of 0° (vertical incidence) to 60° degrees. A close examination of these results indicate the variation to be within $\pm 10\%$ for look angles of 0° , 5° , 10° , 20° , 25° , and 30° , whereas for 15° angle of incidence it is -22% and for angles greater than 30° , it increases almost monotonically to $+203\%$ at $\theta = 60^\circ$

Fig.5-7 $\sigma(\theta)$ of Point No. 8

Site - 6

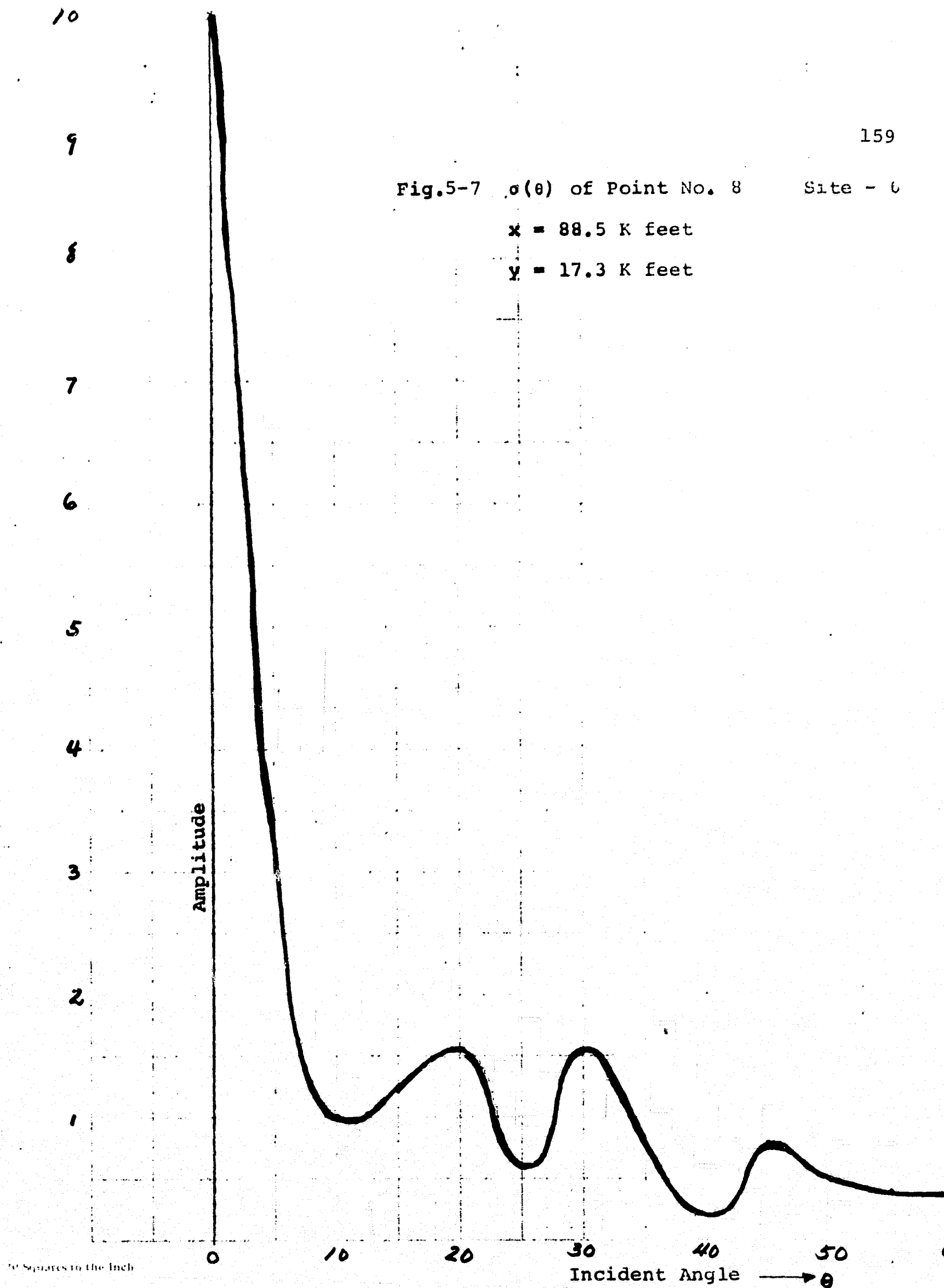
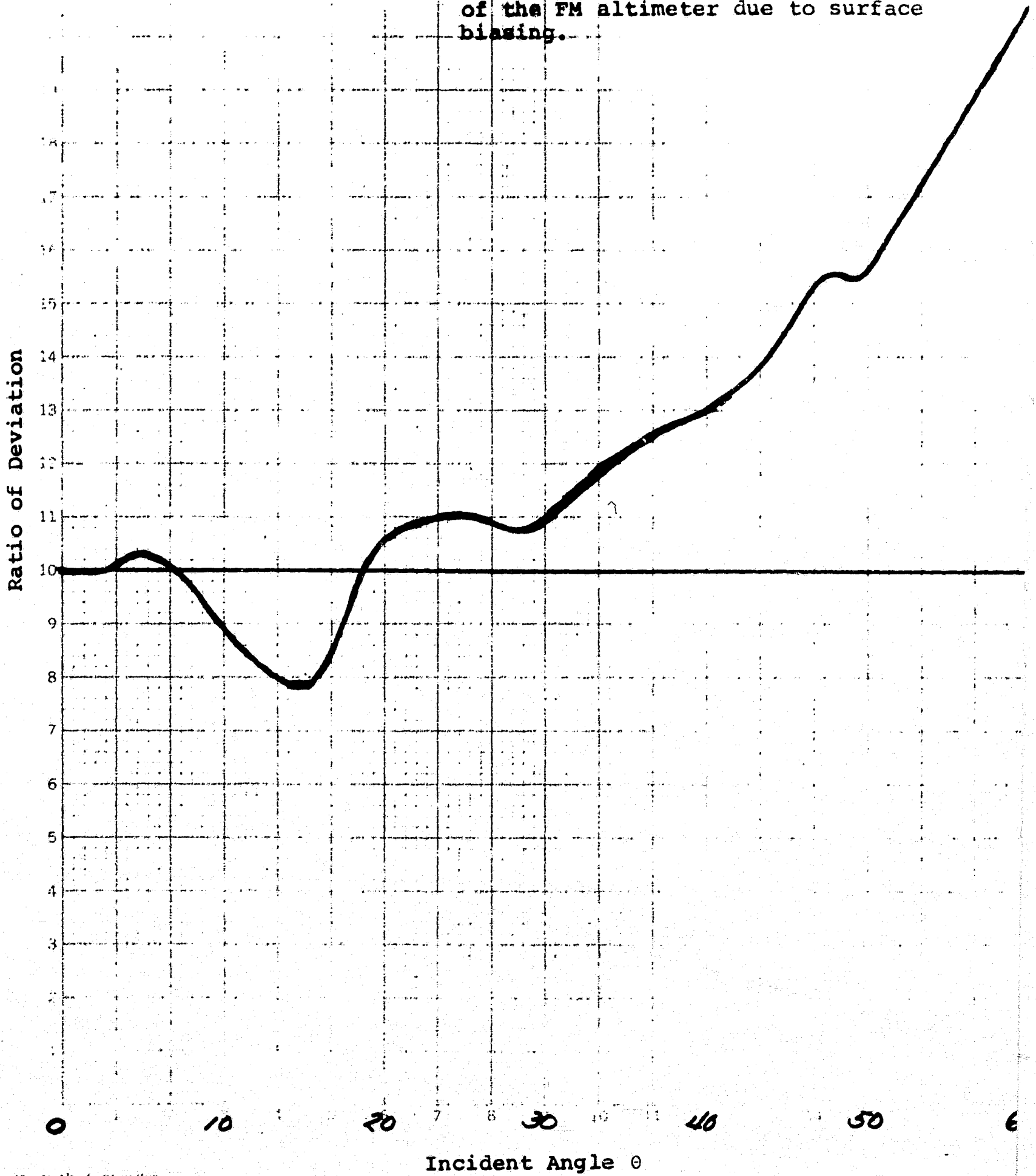
 $x = 88.5$ K feet $y = 17.3$ K feet

Fig. 5-8. The percent altitude deviation of the FM altimeter due to surface biasing.



CHAPTER VI

RADAR CROSS-SECTION AT POINTS ALONG TRAJECTORY

6.1 Simulated LM Static Reflectivity Data for Site P-II-6

A brief discussion of the LM acoustic simulation system and procedures used in taking static reflectivity data is given. A total of thirty points along the landing trajectory were selected for this data. The reflected field was plotted in dbs versus incident angles for quick reference and each set is referred to a difference 0°-reference return in db. This is later modified and all points have a common reference at maximum altitude.

6.1.1 Surface Model

The landing track with a length of 177 K feet is modeled on 36 feet long, 4 feet wide aluminum-base model composed of six sections 4' x 6' in size, as shown in Figs. 6-1 and 6-2. The scaling, design and construction have been reported in detail in a previous report (Hayre, Boyd and Tong, TR-67-8, 1967), and also in Chapter III.

6.1.2 Selection of Points

A total of 30 points were selected for the static data with appropriate consideration for low altitude effects. The distribution of the points along the landing trajectory is listed in Table 6-1 and shown in Fig. 6-3. Only a few data points were chosen at higher altitudes, whereas the point distribution along the trajectory becomes denser near the landing zone. The points were so chosen as to allow sufficient surface overlapping

from one point to the next so that the whole track would be illuminated in subsequent look angles at various points. Furthermore, the reflectivity information is more critical at low altitude than it is at high altitude because of the smaller area illuminated at latter altitudes.

6.1.3 Model Surface Mounting

The model surface is attached to a fixed metal frame inside the water tank. The metal frame is mechanically isolated from other structures such that the experiment is noise free (vibration).

6.1.4 Transmitter and Receiver Mounting

The transmitter and receiver are mounted on a rigid arm attached to a motor carrier. The carrier and the surface frame have the same mechanical reference so that any vibration will be self-cancelled.

6.1.5 Positioning of the Transmitter and Receiver

The X (lateral distance measured from the center of the landing zone) and Y (altitude from the lunar surface) position of the carrier are controlled by DC motors which have built-in mechanical feedback. These position reading signals are fed into digital encoders with a readout accuracy up to a hundredth of an inch. The angular control of the transmitter and receiver

is designed to change one degree per pulse input. The angular swing is from 0° to 70° toward the landing, as shown in Fig. 6-4. Figure 5-5 shows the overall electrical and mechanical layout of the entire system.

6.1.6 Data

A sample of raw data is presented in Fig 6-6 where the return signal envelope is plotted. The incident angle of the transmitter corresponds to the number of pulses and is also recorded in the same graphs. Fig 6-7 through Fig 6-36 are the return signal in db's plotted versus the incident angle of the transmitter and has a resolution of 1 degree.

Each plot of reflected field versus angle of incidence is plotted on a separate sheet with such details as altitude, distance from landing zone, and x,y positions on the model and model number. Each set of data was taken after the system was calibrated using flat plate model and repeatability of this data was assured before making these runs.

6.1.7 Conclusion

The LM acoustic simulation system has been carefully designed and built to avoid any extraneous vibration. The positioning and switching of all mechanical components have been operated and checked electronically to assure the continuity in the simulation process. Each set of data has been run and recorded repeatedly to the point until satisfactory

comparison is made between runs. Therefore it is believed that optimum accuracy has been achieved within our scope upon the present existing system.

The return signal has also been re-examined point by point throughout the physical model. The curves agree favorably with predictions by the scattering theory. Any unexpected fluctuation caused by the frame iron angle near the edge of the model structure is pin-pointed. Such error, however, may be neglected and its real level may be interpreted from its neighboring data points.

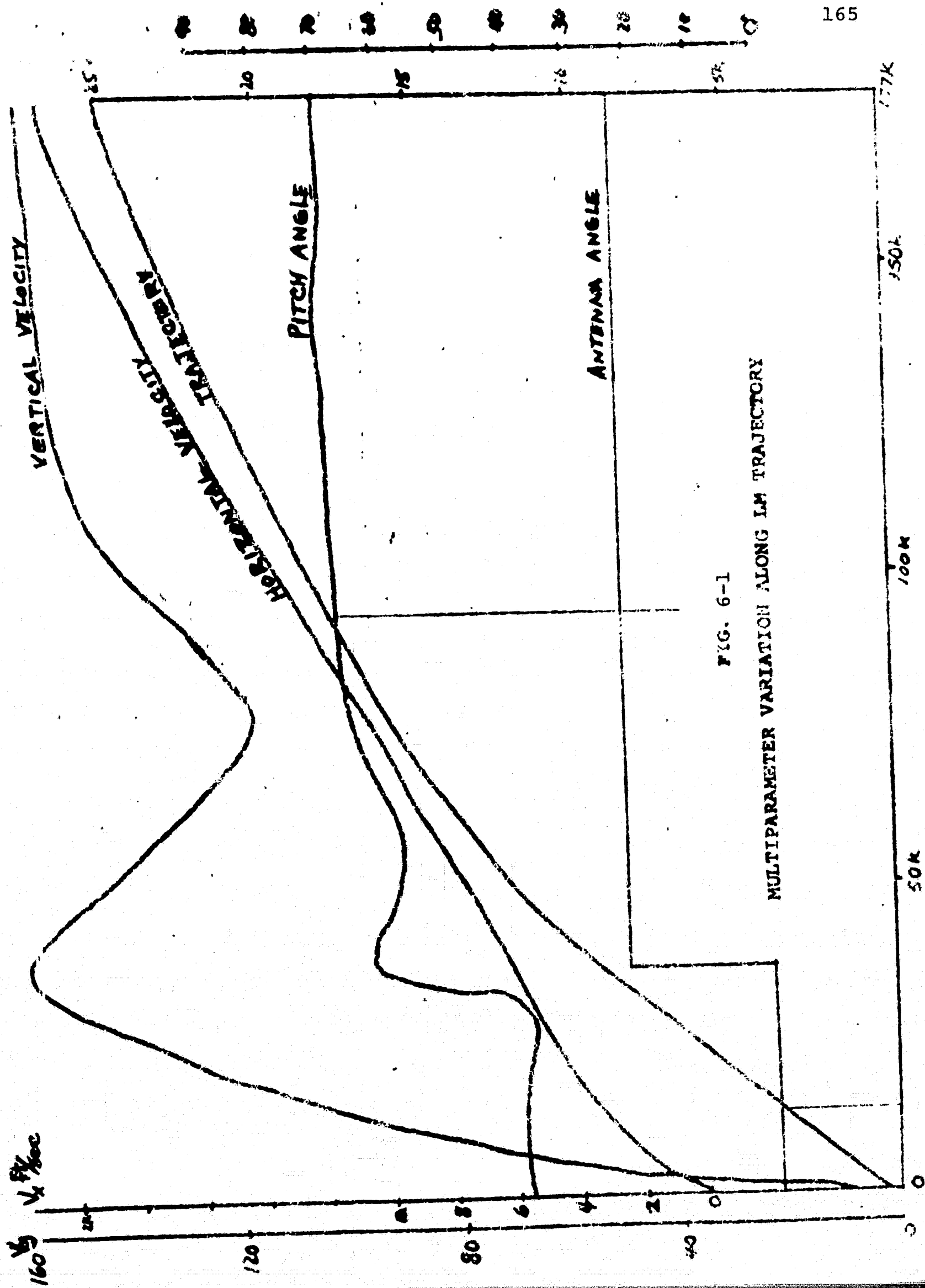
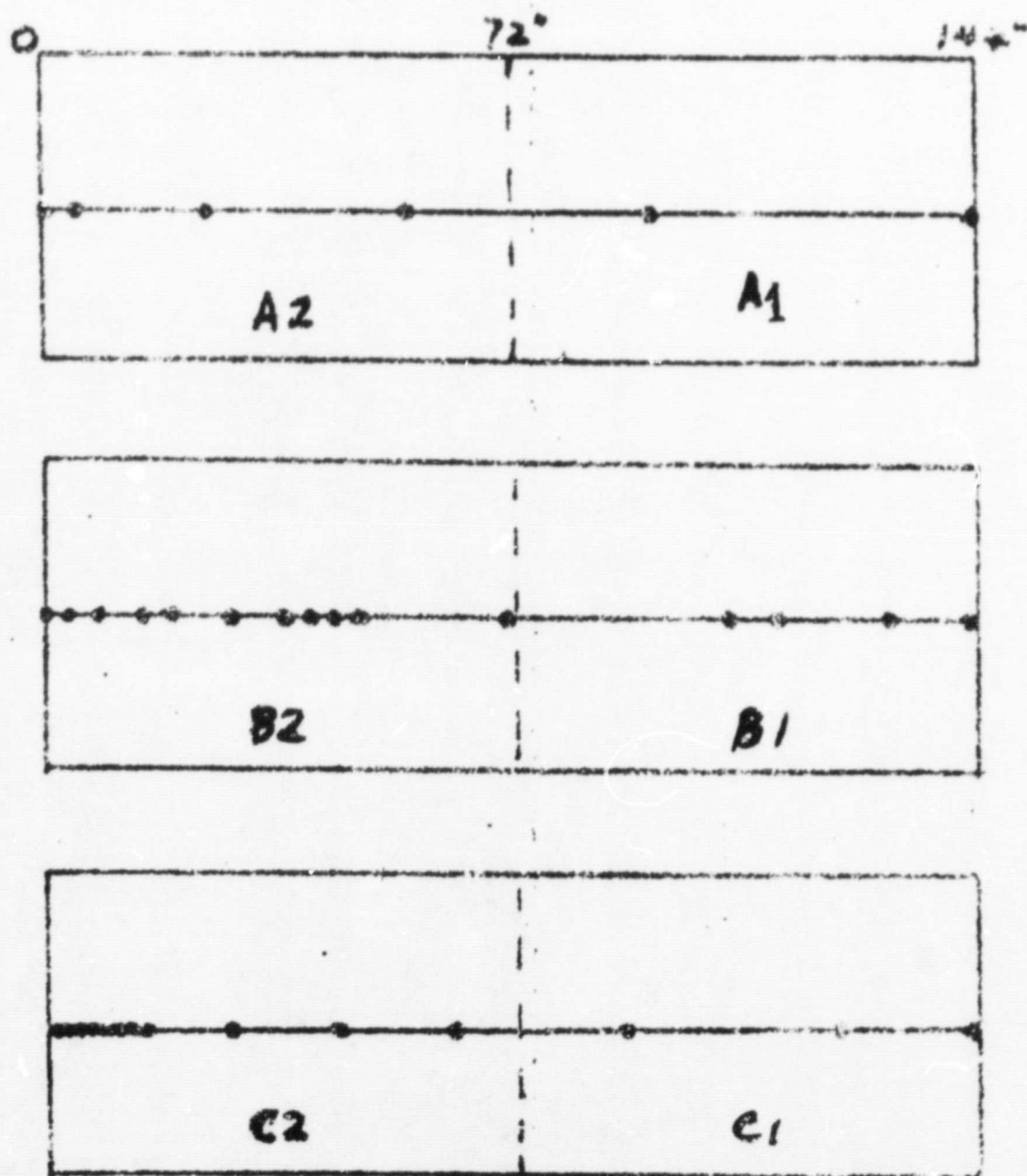
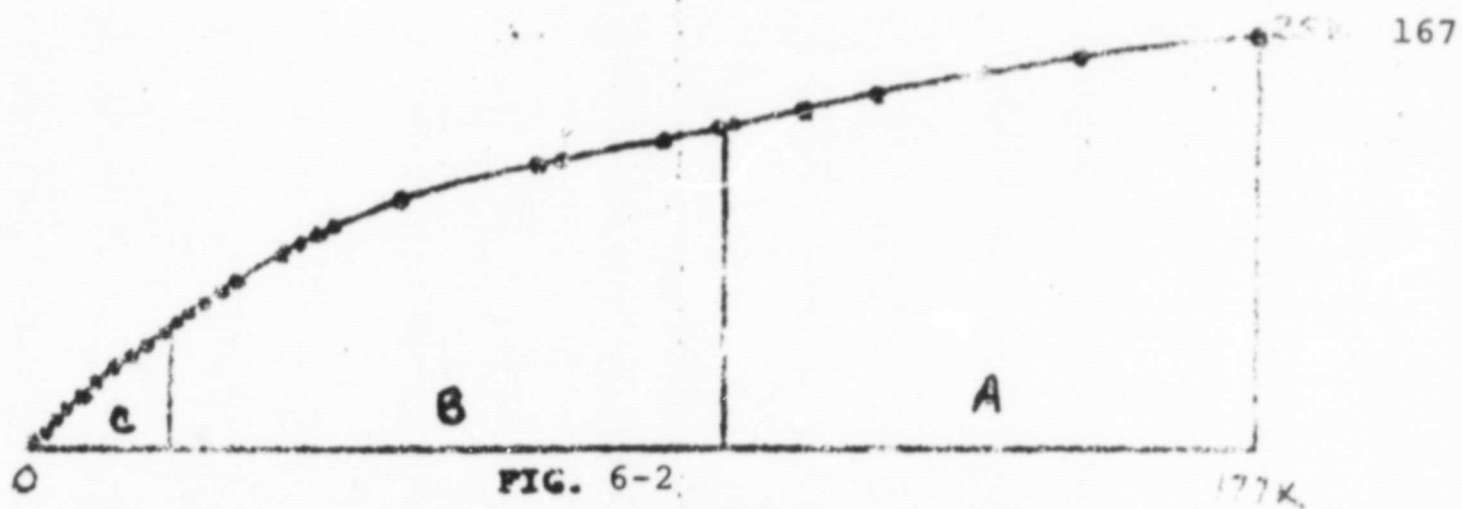


FIG. 6-1
MULTIPARAMETER VARIATION ALONG LM TRAJECTORY

DISTANCE (x', x) FROM CENTER OF LANDING ELLIPSE
x ft. x' in. y ft. y' in.

166

1	177 K	143.7372	25000	43.7956
2	150 K	96.3504	23000	40.2920
3	125 K	52.5547	20800	36.4380
4	107.5 K	21.8978	19200	33.6350
5	100 K	8.7591	18400	32.2336
6	95 K	0.0	17950	31.4453
7	95 K	143.7372	17950	31.4453
8	88.5 K	132.3504	17300	30.3066
9	78.3 K	113.9562	16200	28.3796
10	75 K	108.7007	15800	27.5780
11	53.9 K	71.9124	12800	22.4234
12	36.2 K	40.7299	9250	16.2044
13	34.9 K	38.4526	8938	15.6578
14	33.7 K	36.3504	8622	15.1042
15	32.5 K	34.2482	8310	14.5577
16	27.9 K	26.1898	7150	12.5255
17	25 K	21.1095	6500	11.3869
18	22.8 K	17.2555	5870	10.2832
19	18.3 K	9.3723	4800	8.4088
20	14.4 K	2.5401	3800	6.6569
21	12.95K	0.0	3400	5.9562
22	12950	143.8896	3400	37.7778
23	11000	122.2222	3000	33.3333
24	8100	90.0000	2300	25.5556
25	5740	63.7778	1760	19.5556
26	3870	43.0000	1290	14.3333
27	2430	27.0000	900	10.0000
28	1710	19.0000	700	7.7778
29	1380	15.3333	600	6.6667
30	740	8.2222	400	4.4444
31	670	7.4444	375	4.1444
32	246	2.7333	220	2.4444
33	195	2.1667	200	2.2222



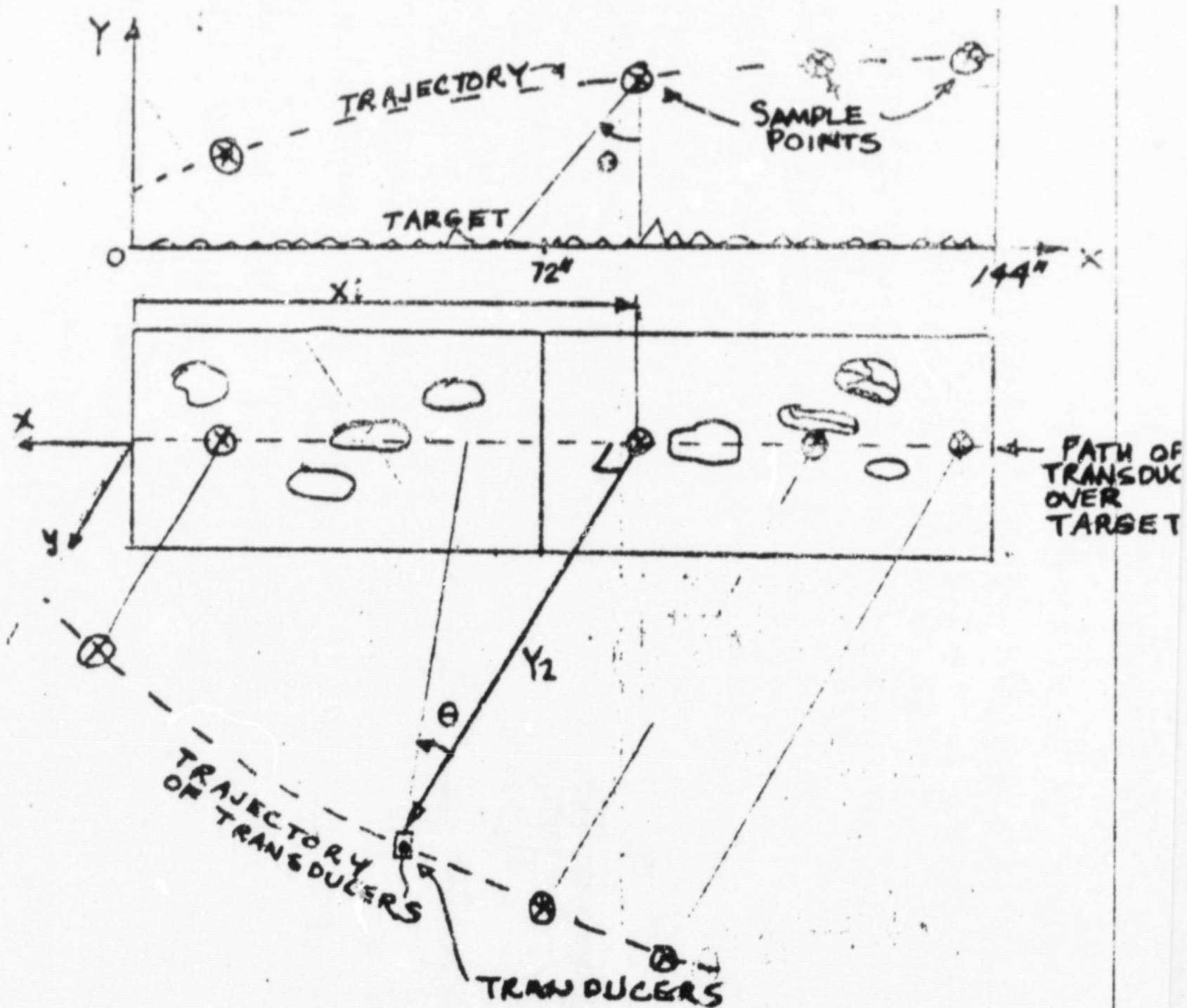


FIG. 6-4

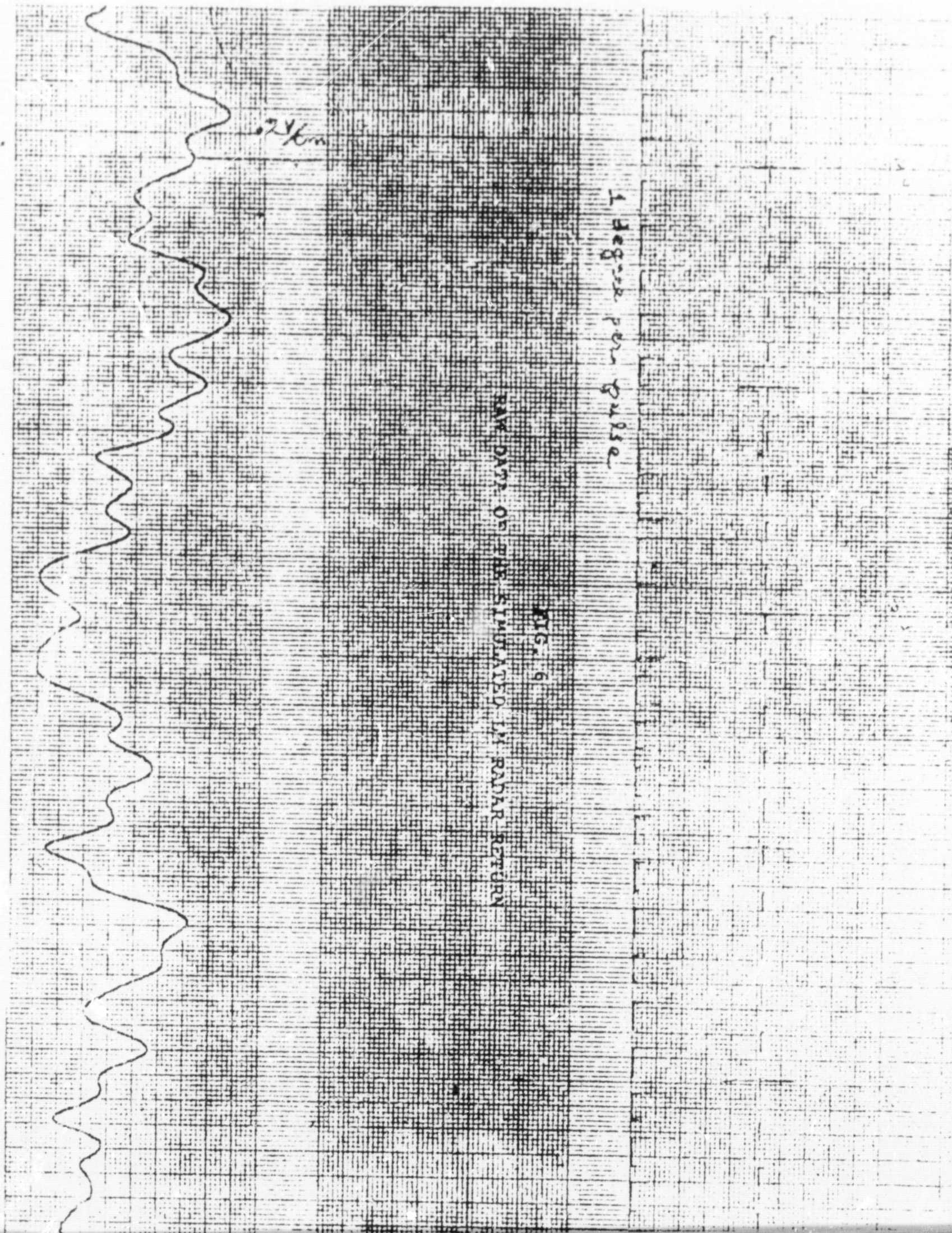
OVER VIEW OF THE RELATIVE LOCATIONS OF LM SIMULATED
RADAR PACKAGE AND TARGET SURFACE



BLOCK DIAGRAM OF INSTRUMENTATION FOR STATIC LM LANDING SIMULATION

Fig. 6-6

170



#1 P-II-4 A-1
 $X = 143.74''$
 $Y = 43.79''$
 $Sen = .1v/cm.$

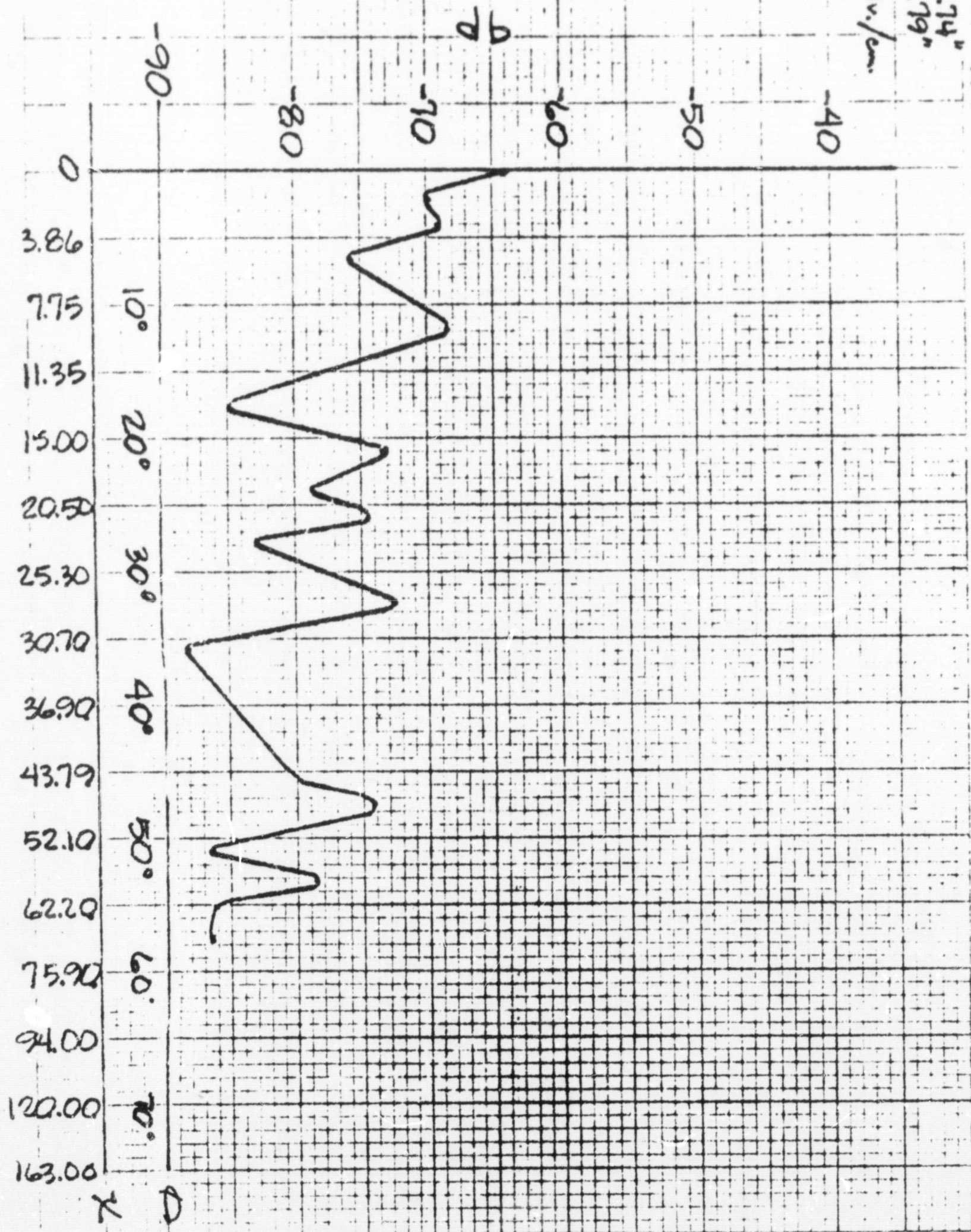
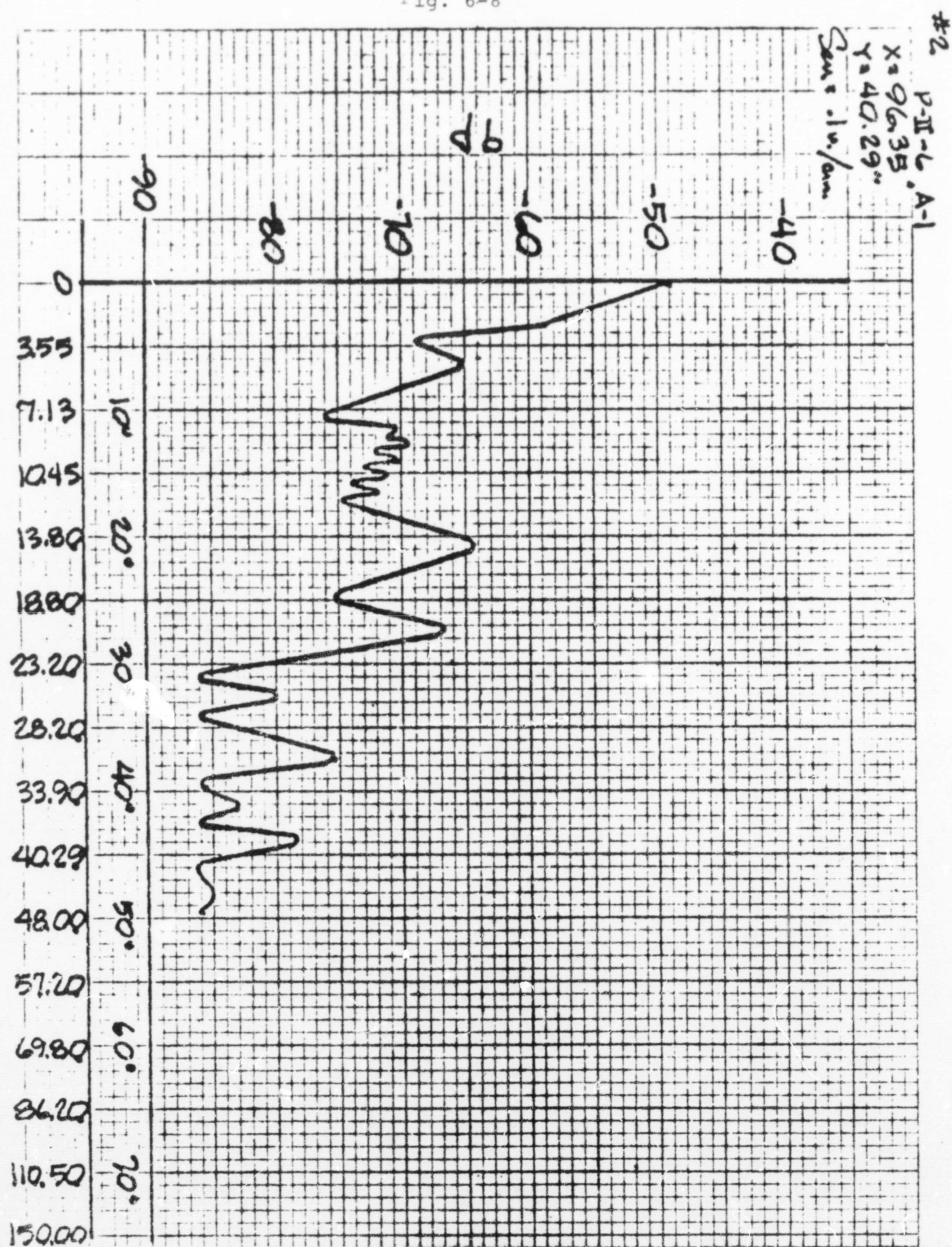


Fig. 6-7

Fig. 6-8

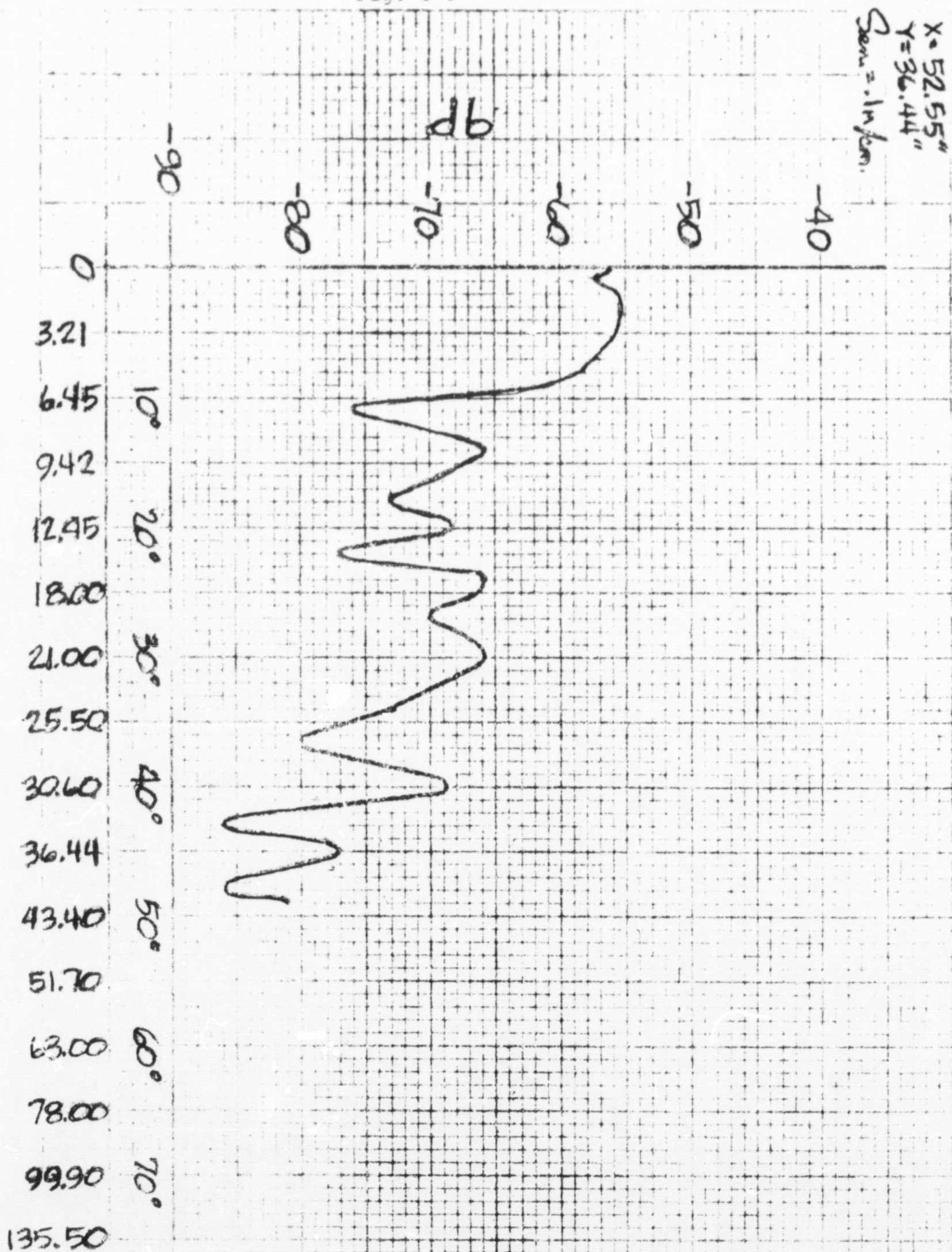


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Fig. 6-9

173

#3 P-II-6 A-2
X = 52.55"
Y = 36.44"
Scale = 1 in./cm.



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174

Fig. 6-10

#A
 $X = 21.89''$
 $Y = 33.64''$
 $Sen = 146mm$
 P-II-6 A-2

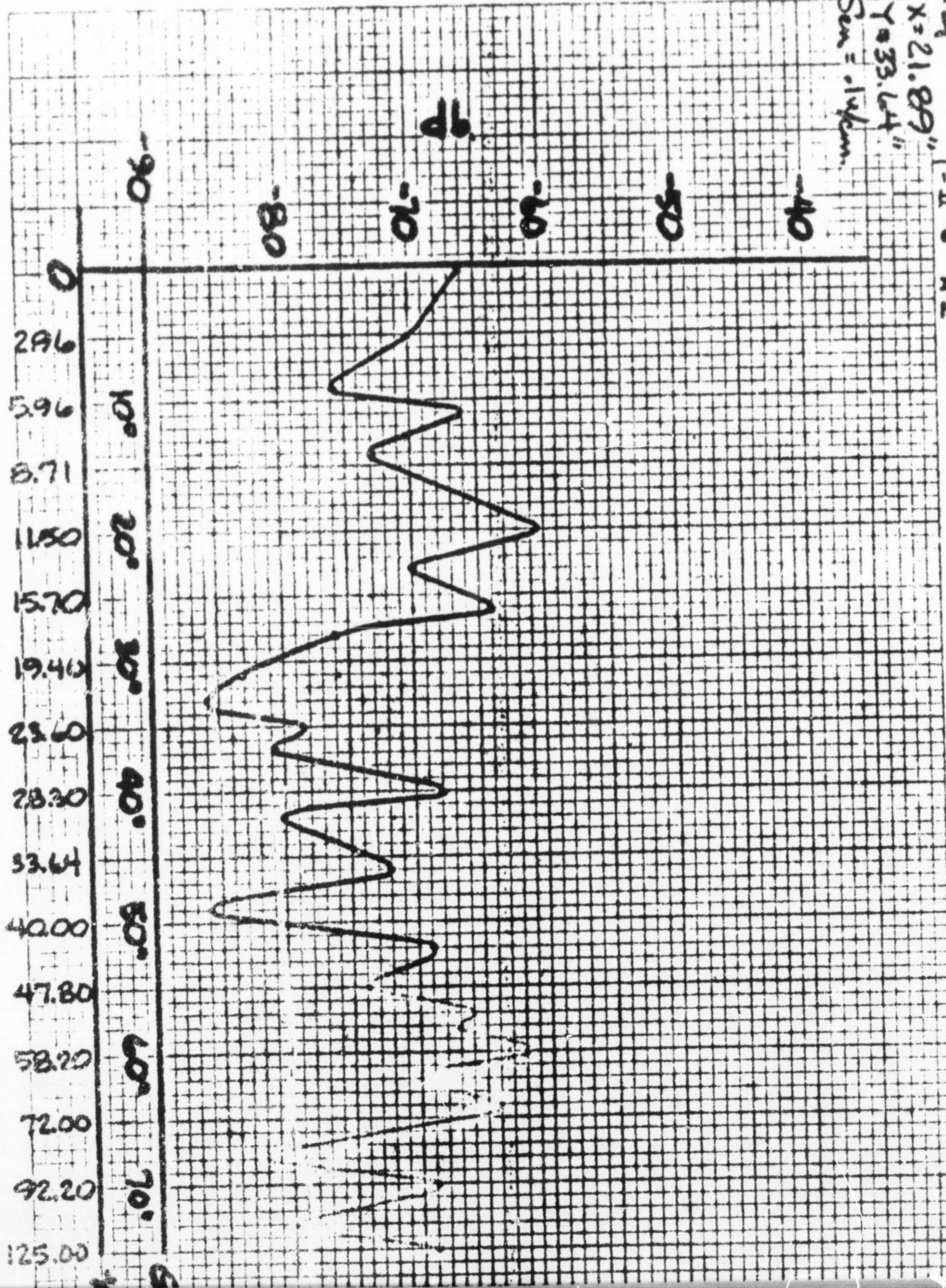
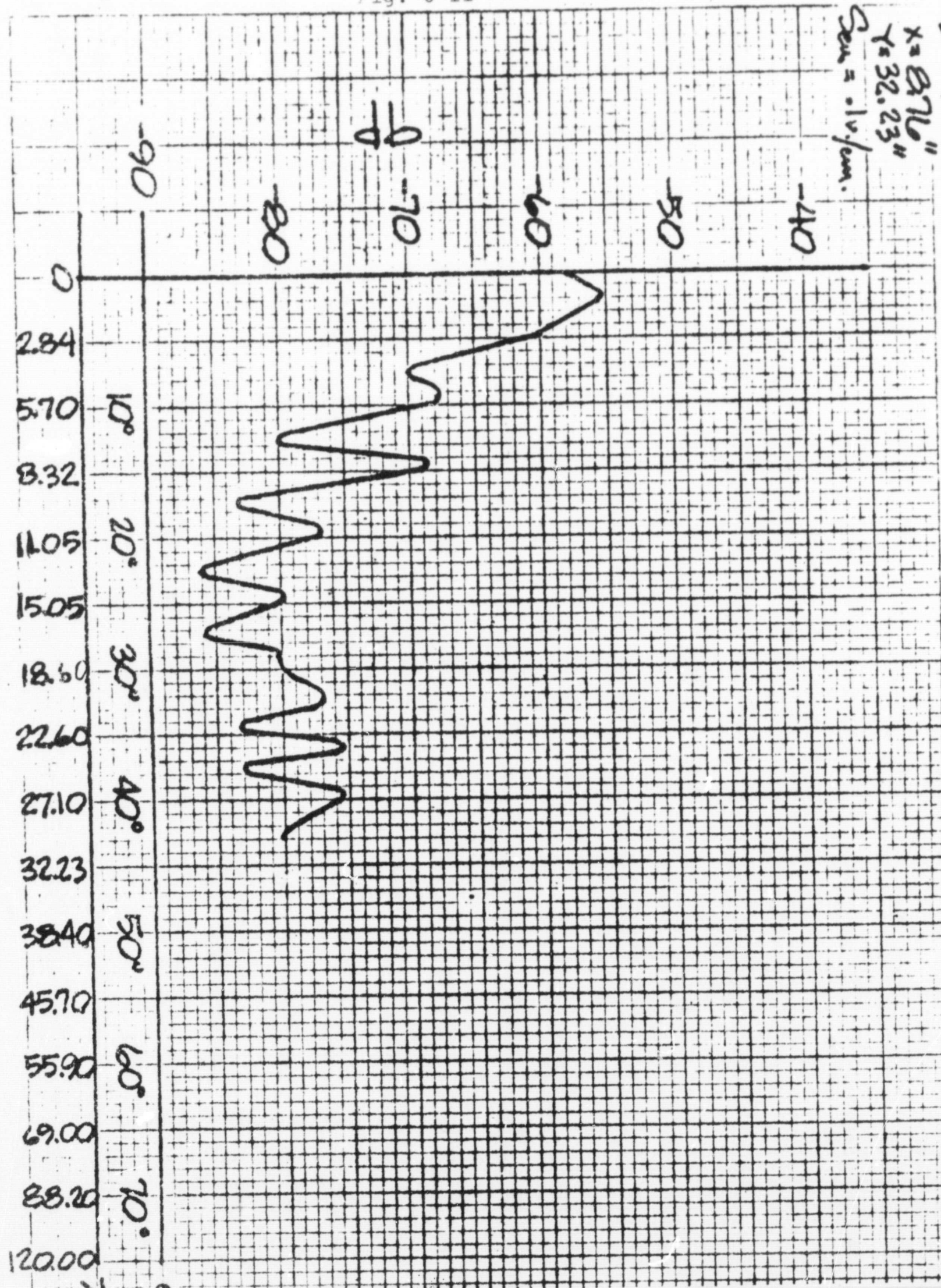


Fig. 6-11

#5 P-II-6 A-2

 $x = 8.716''$
 $y = 32.23''$
 $S_{\text{max}} = .1 \text{ v/cm.}$


12-286

Fig. 6-12

TRAJECTORY 1, MODEL 2
 TARGET R-II-6, B-1
 X=143.743" Row #2
 Y=31.193" Row #2
 Sensitivity 20/cm
 8.3127

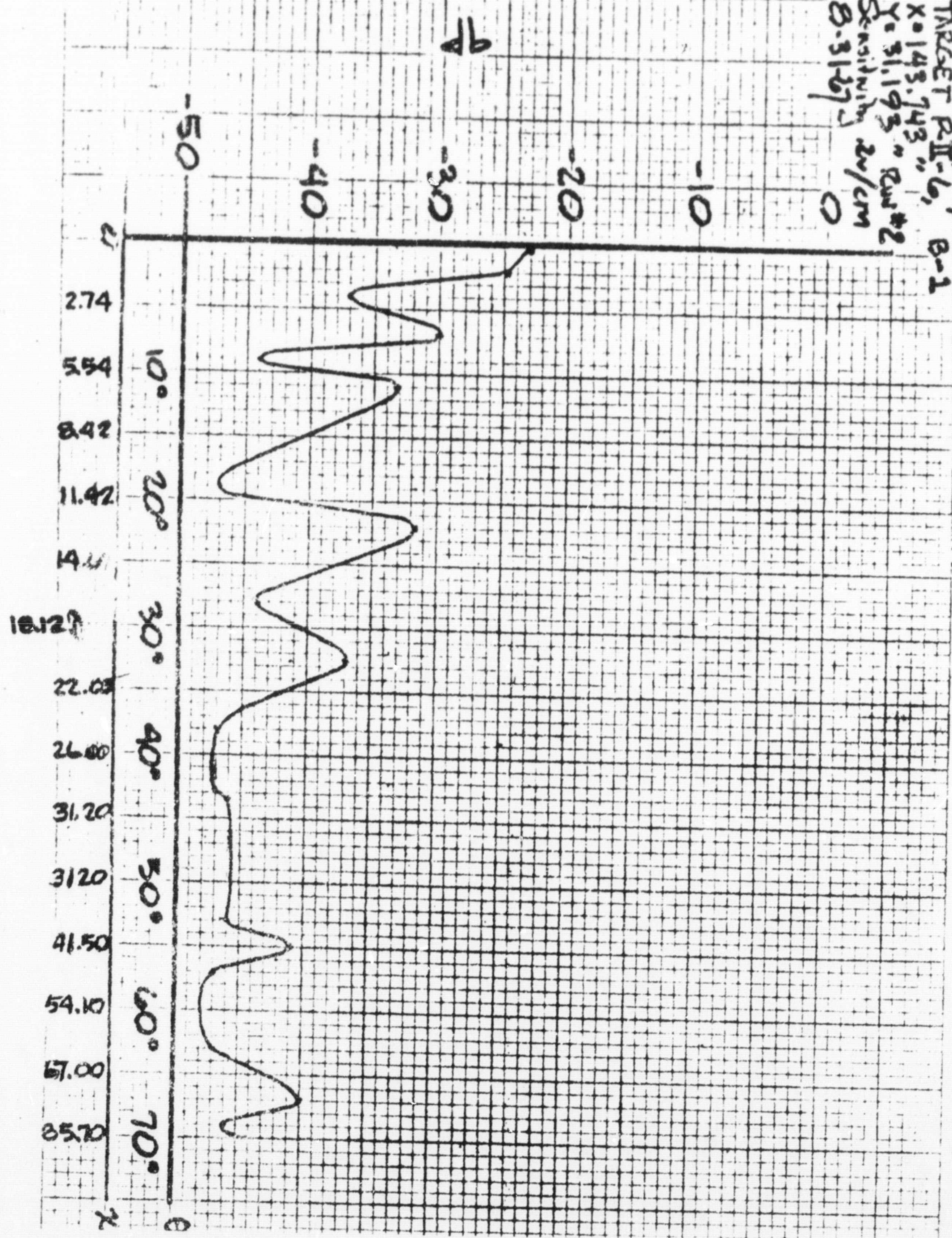


Fig. 6-13

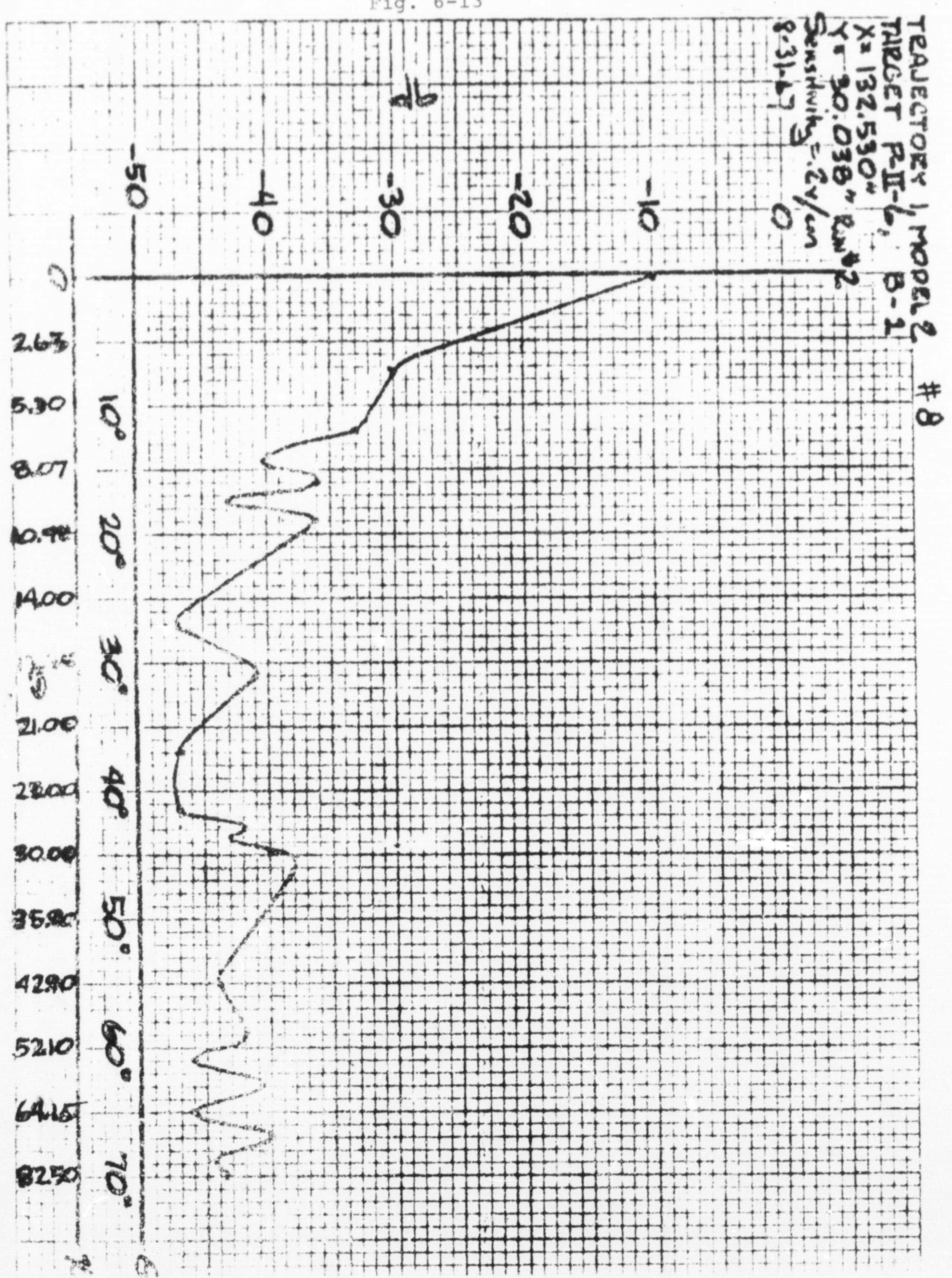
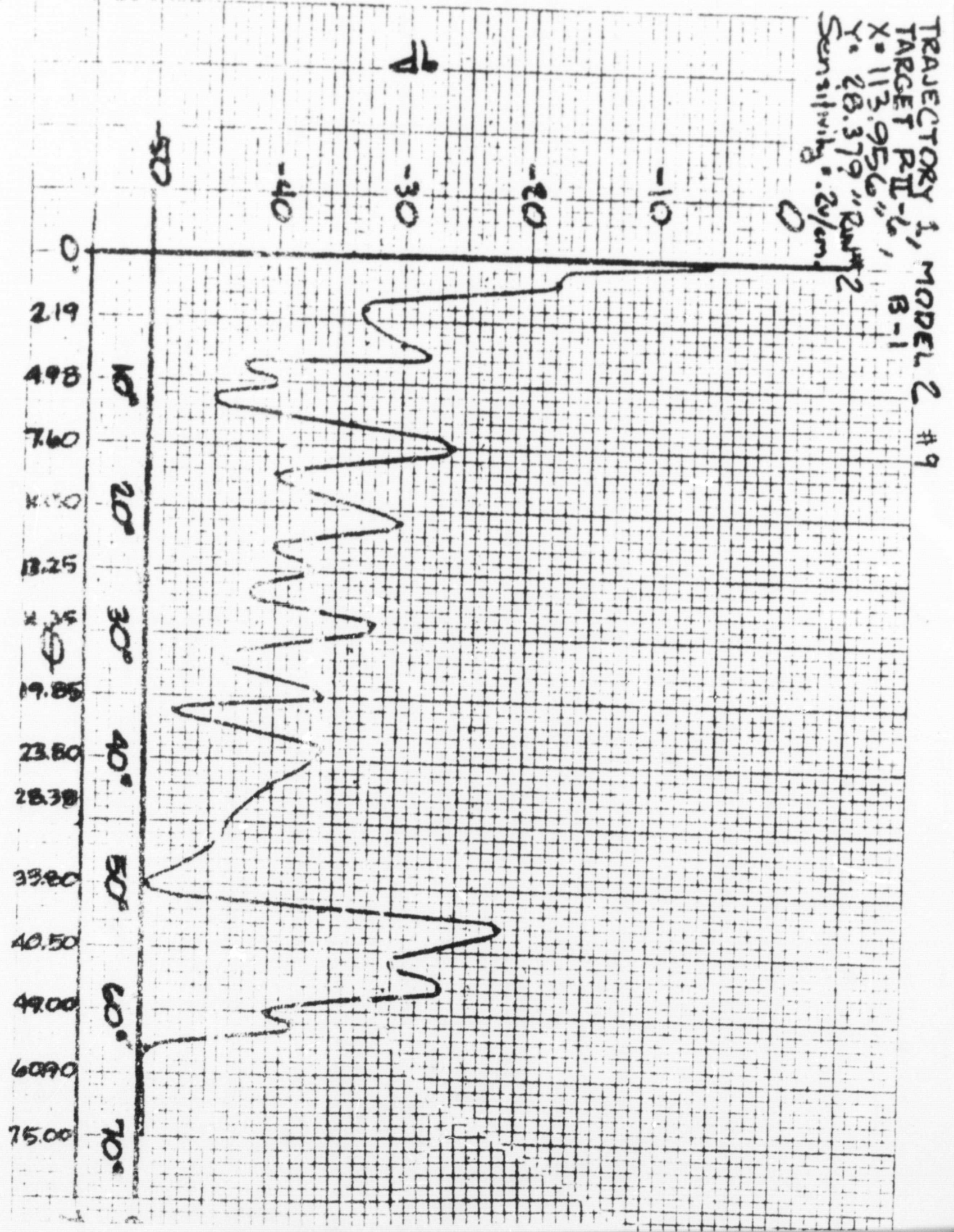


Fig. 6-14



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Fig. 6-15

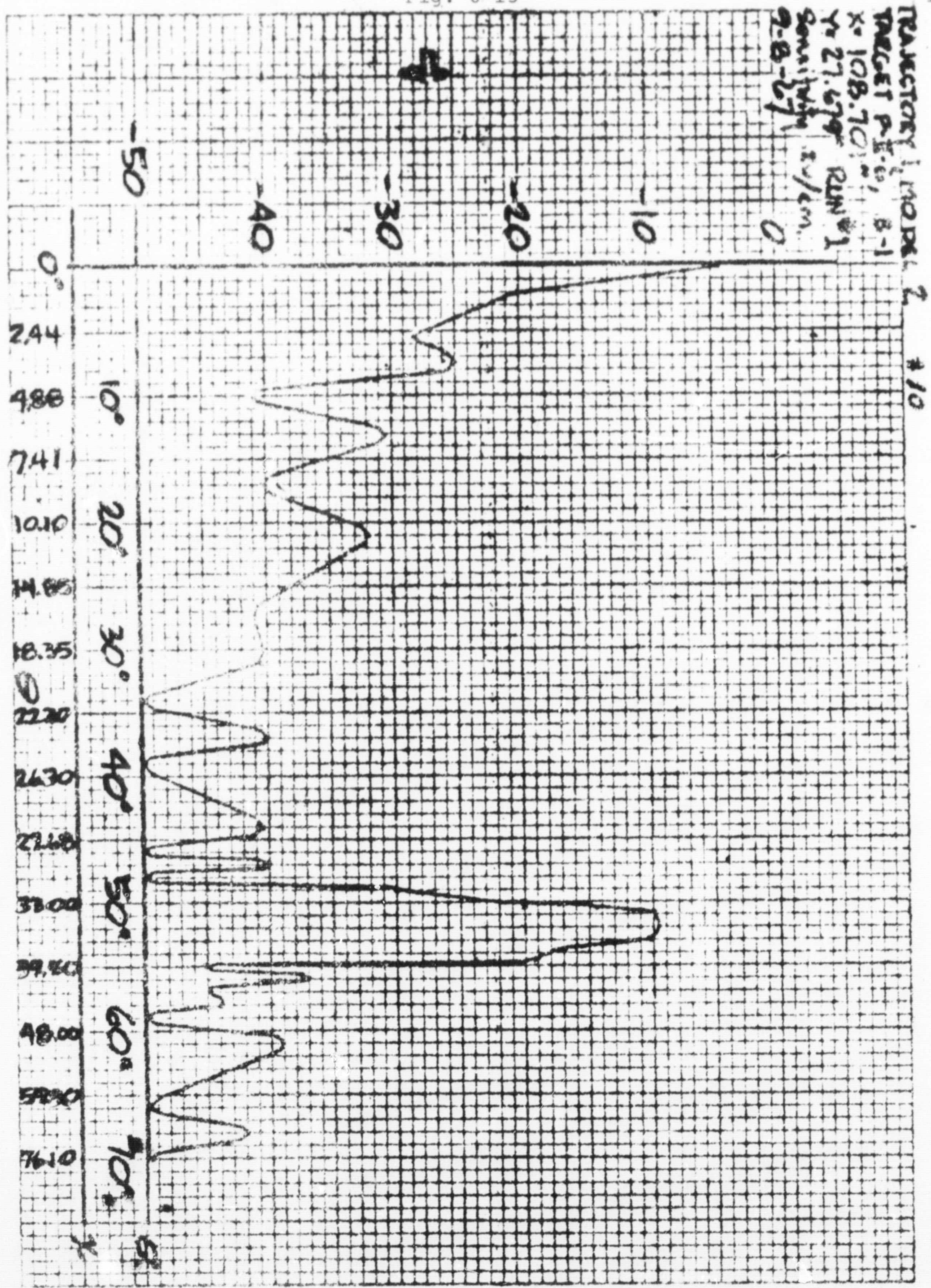


Fig. 6-16

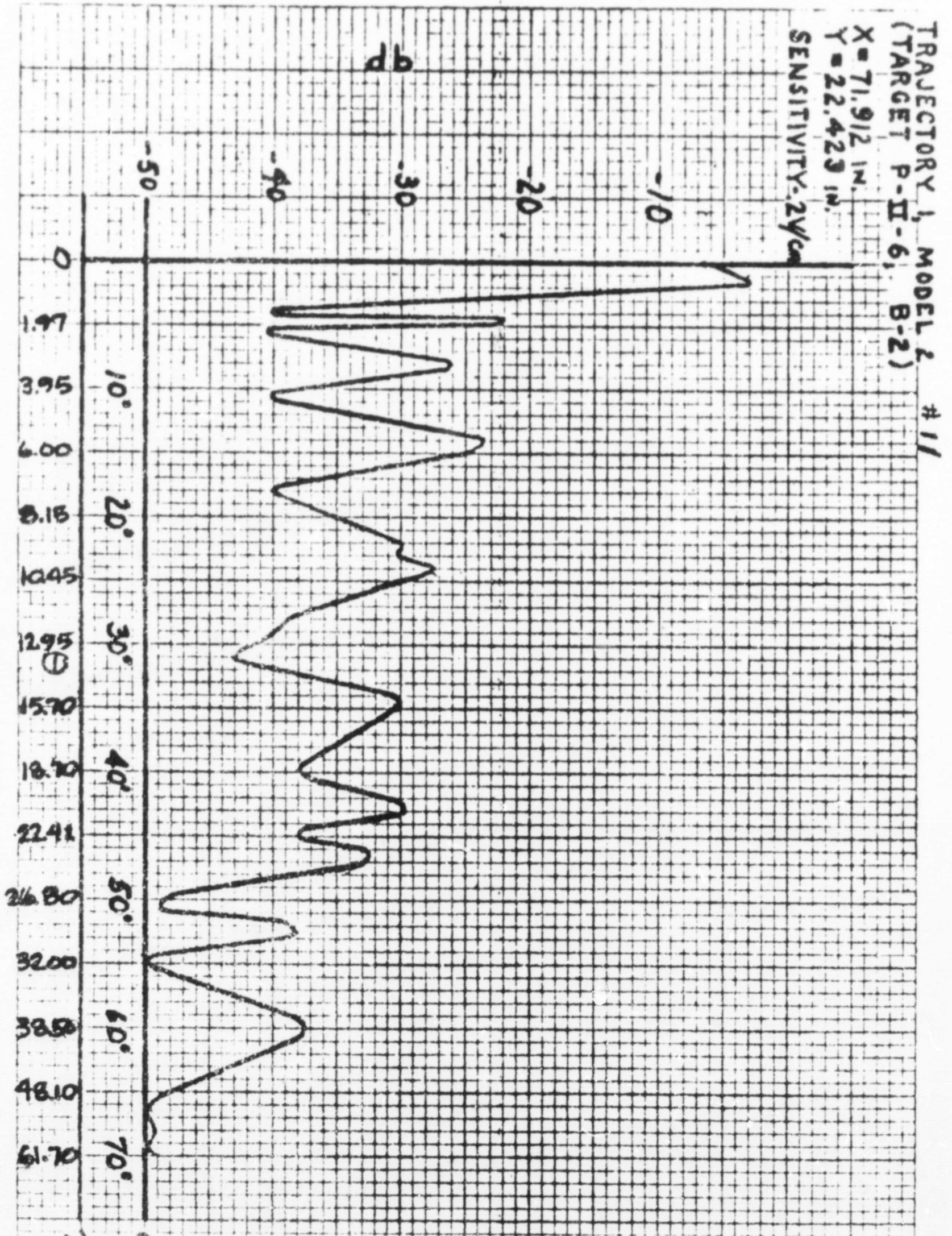


Fig. 6-17

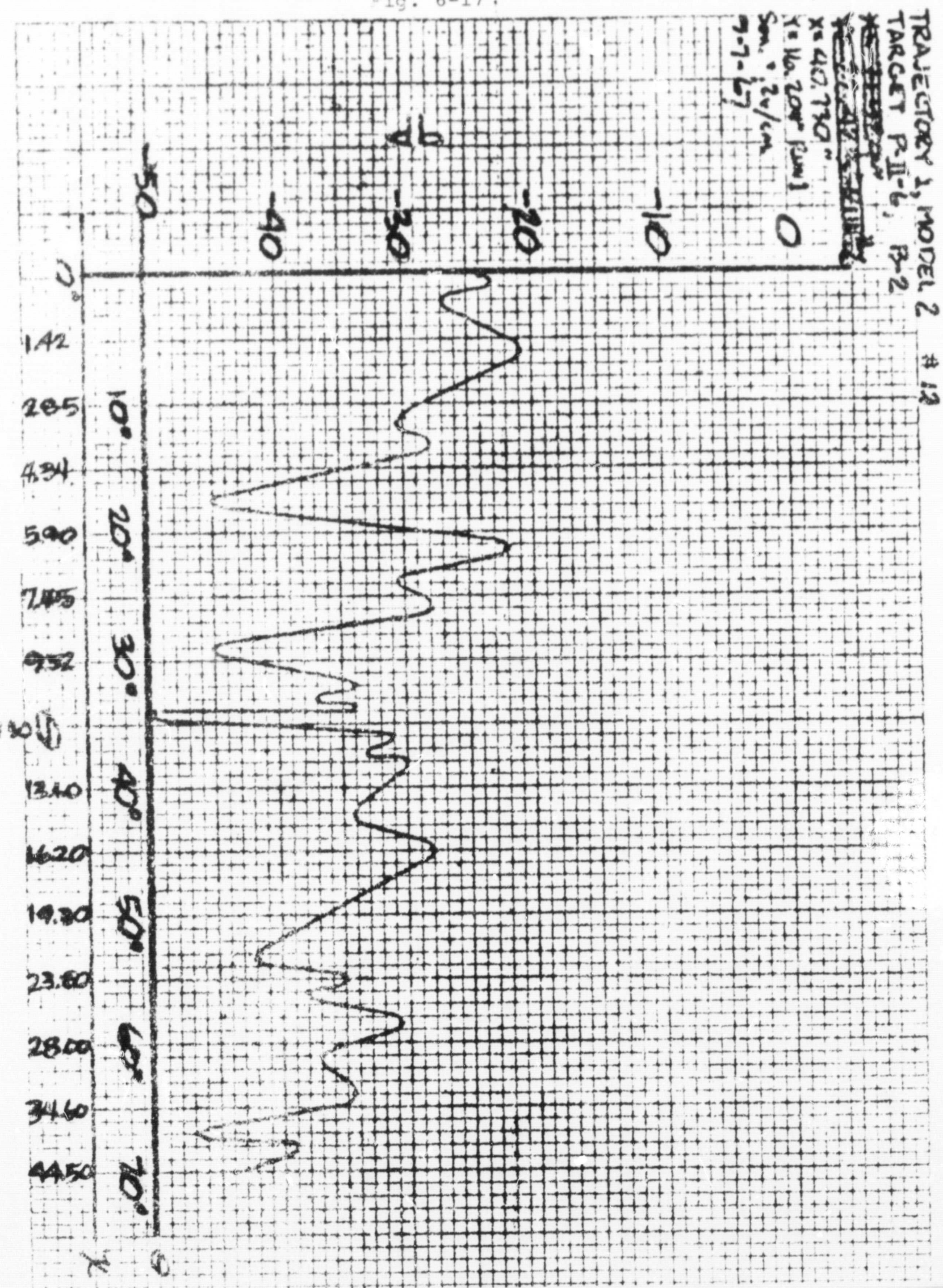


Fig. 6-18

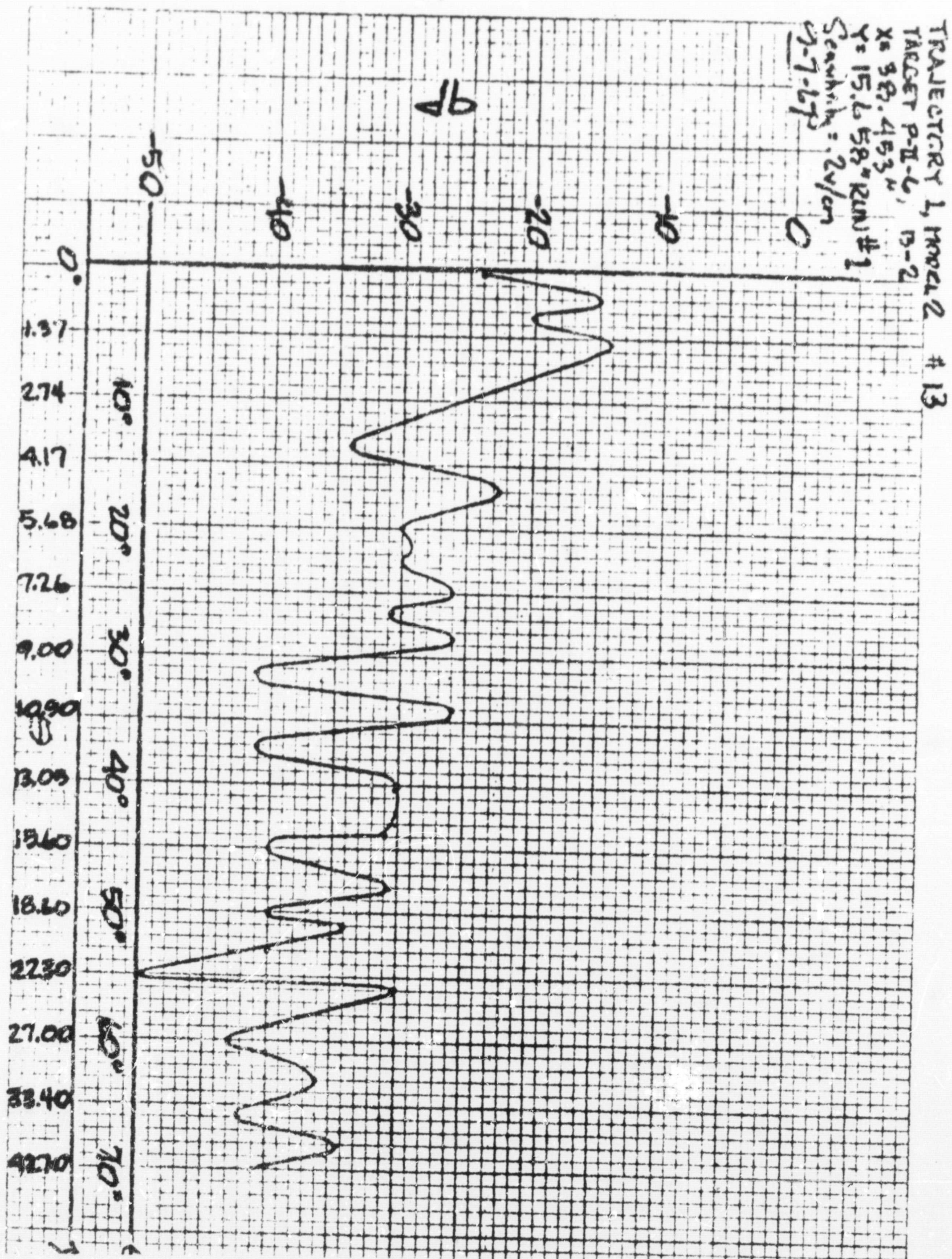


Fig: 6-19

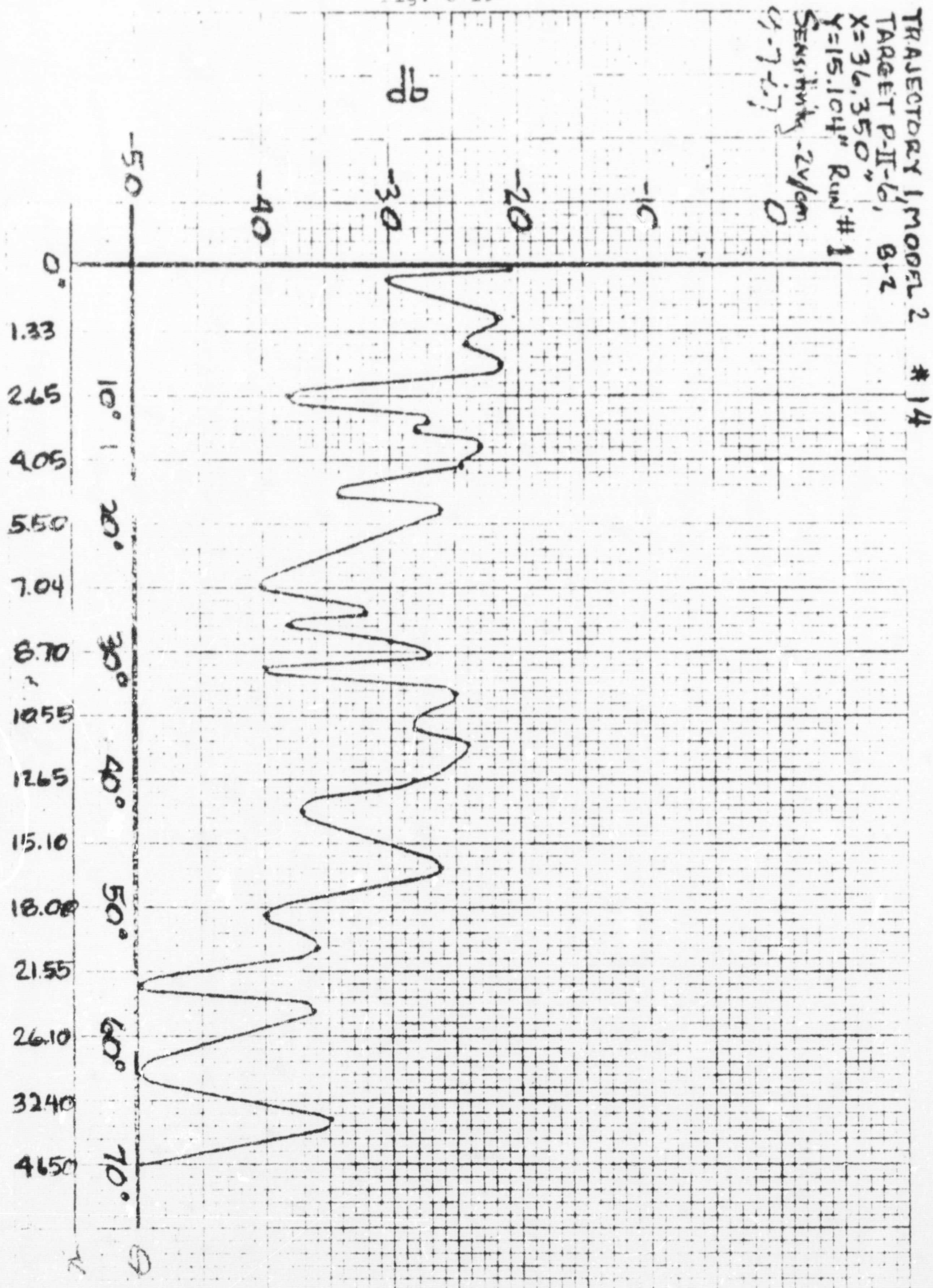


Fig. 6-20

TRAJECTORY 1, MODEL 2
 TARGET P-II-6, B-2
 X=54.248"
 Y=14.556"
 SENSITIVITY 24/100

#15

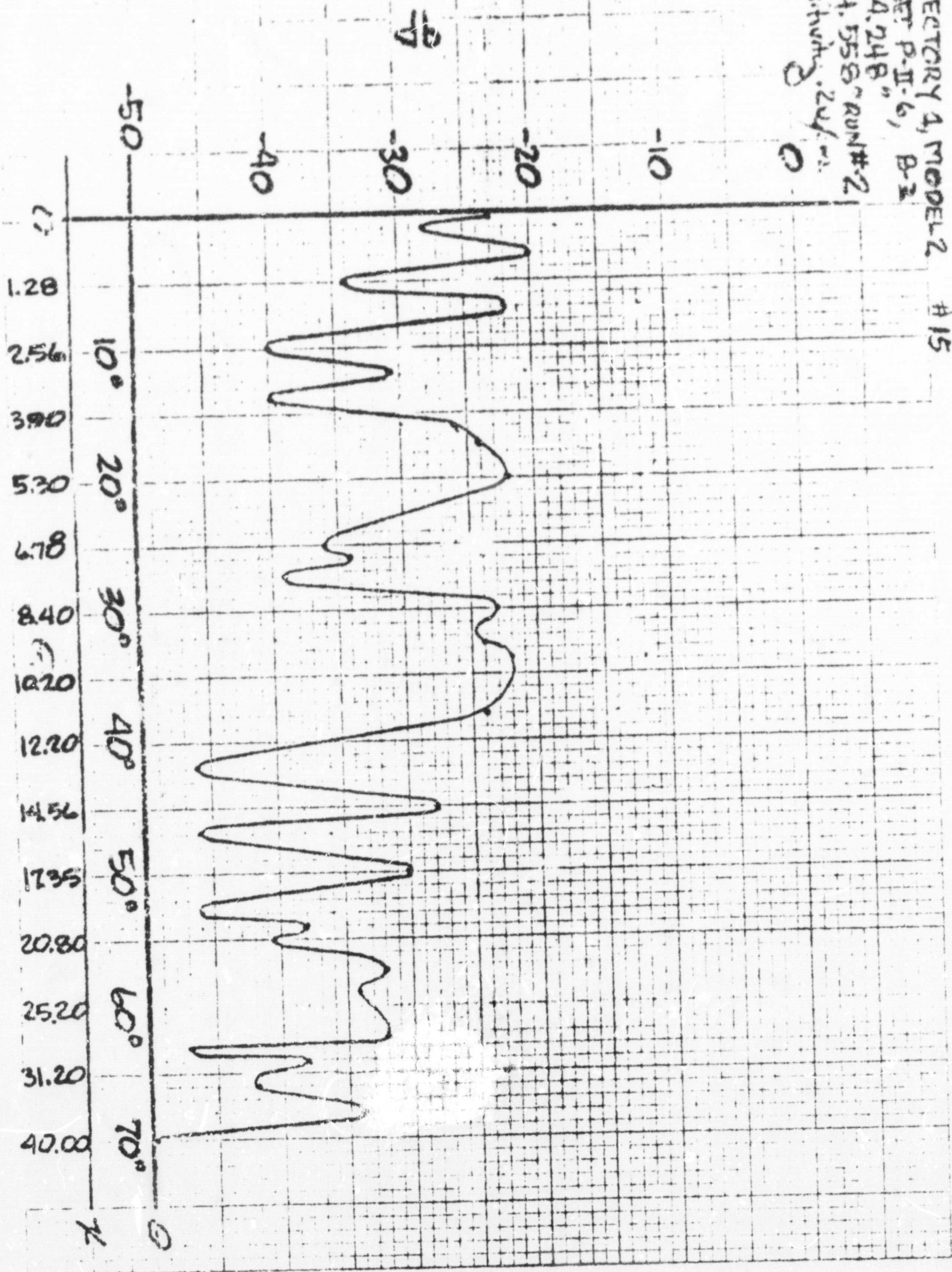


Fig. 6-21

TRAJECTORY 1, MODEL 2 #16
 TARGET BII-b, B-2
 X=26.190
 Y=12.526 " RUM 1
 Sensitivity 24/cm
 2.67

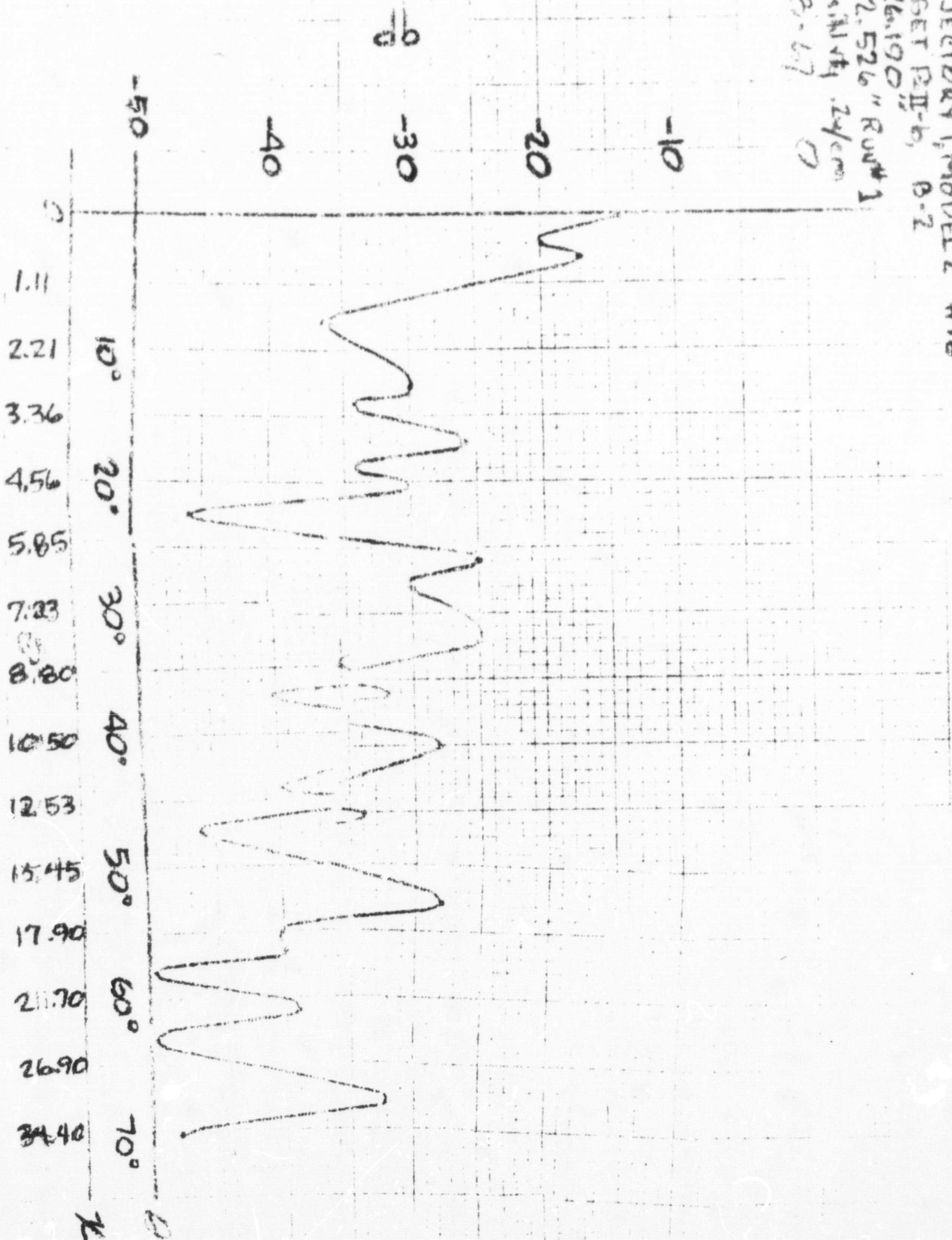
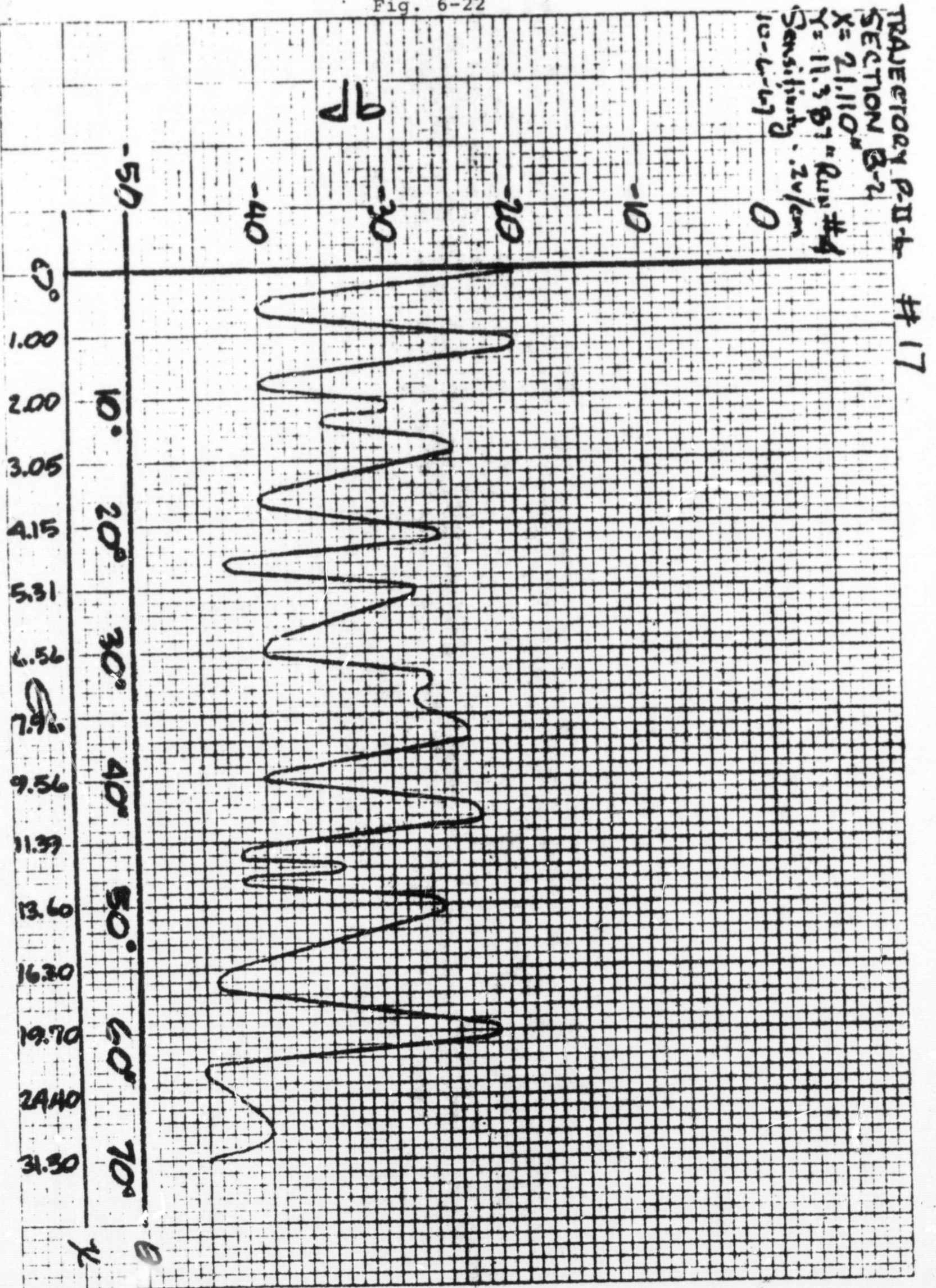


Fig. 6-22



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Fig. 6-23

46

TRAJECTORY P-II-6 #18
 ECT. B-2
 17.256"
 19.283" Run #3
 Sensitivity = 2v/cm
 7-6-67

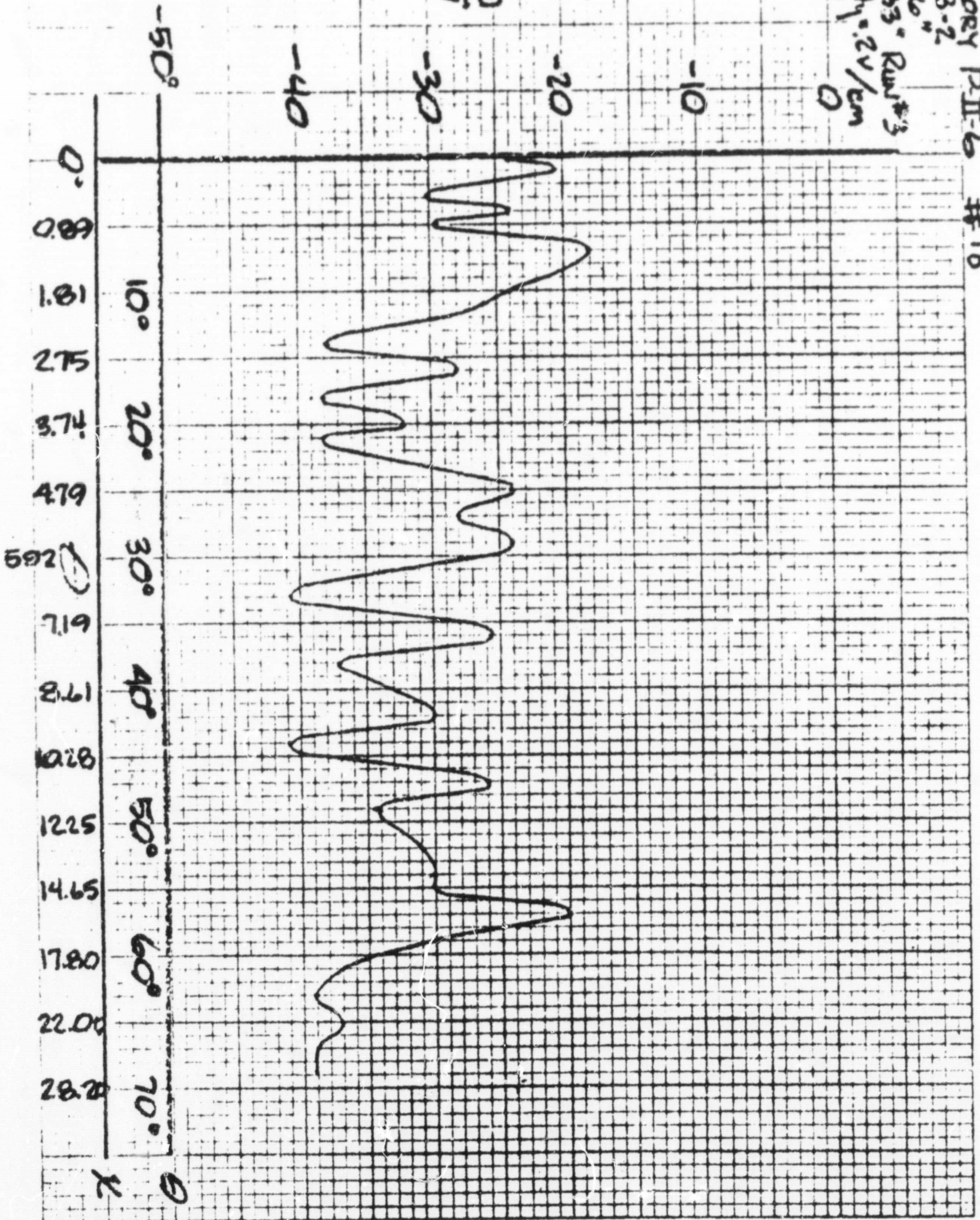


Fig. 6-24

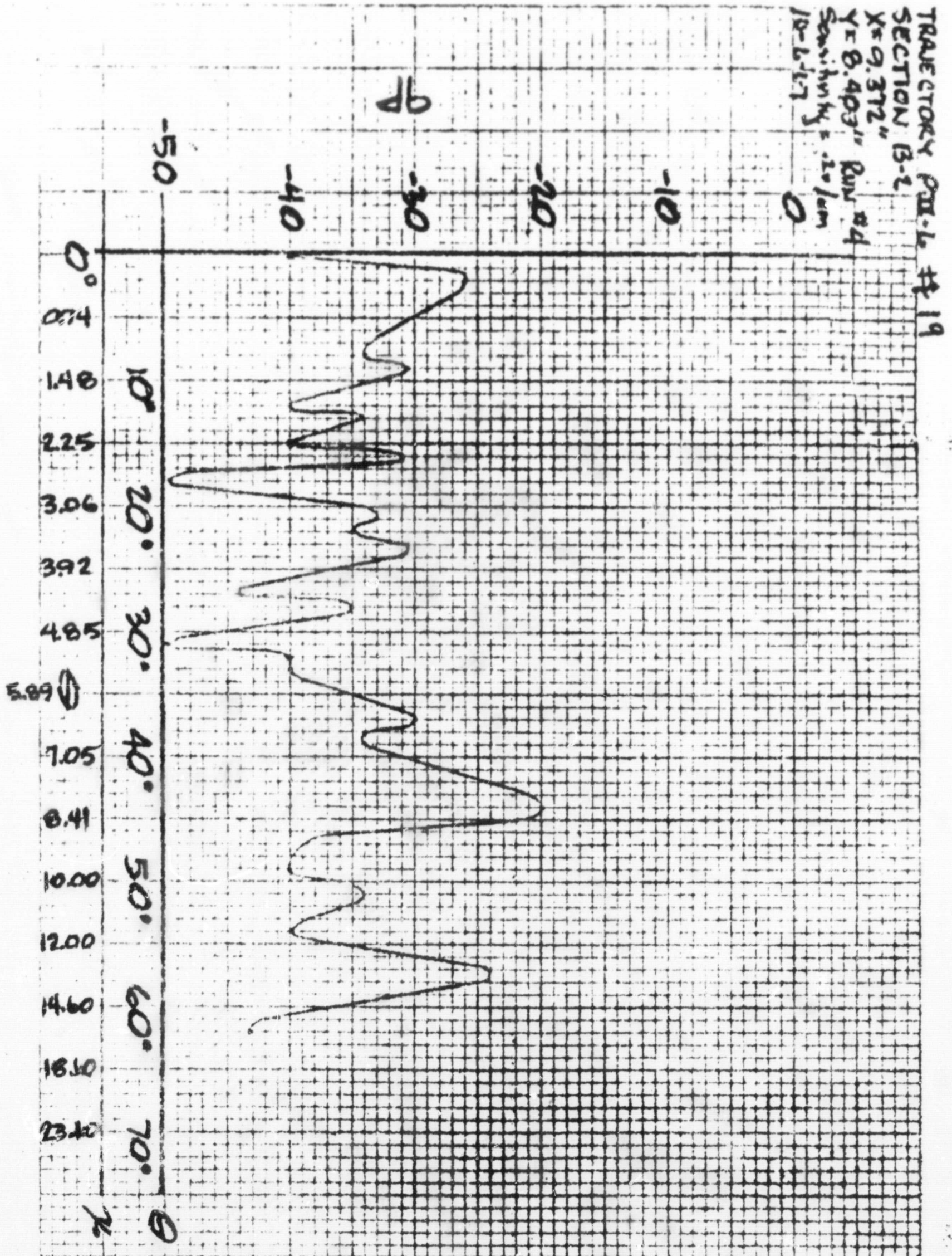
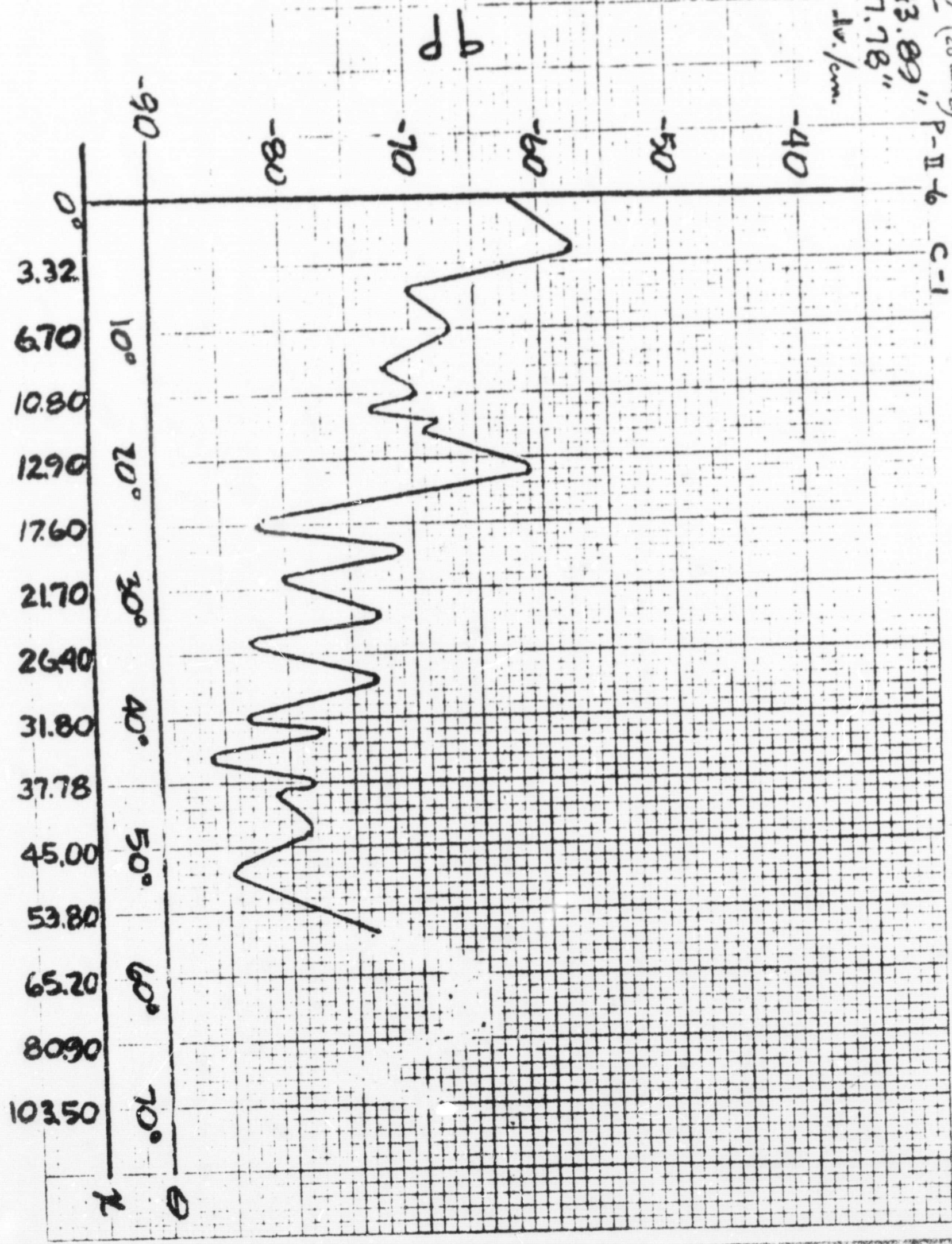


Fig. 6-25

$\lambda = 22$ (20 and 23)
 $X = 143.89$ "
 $Y = 37.78$ "
 $S_{\text{un}} = 14.1 \text{ cm.}$
P-II-6 C-1



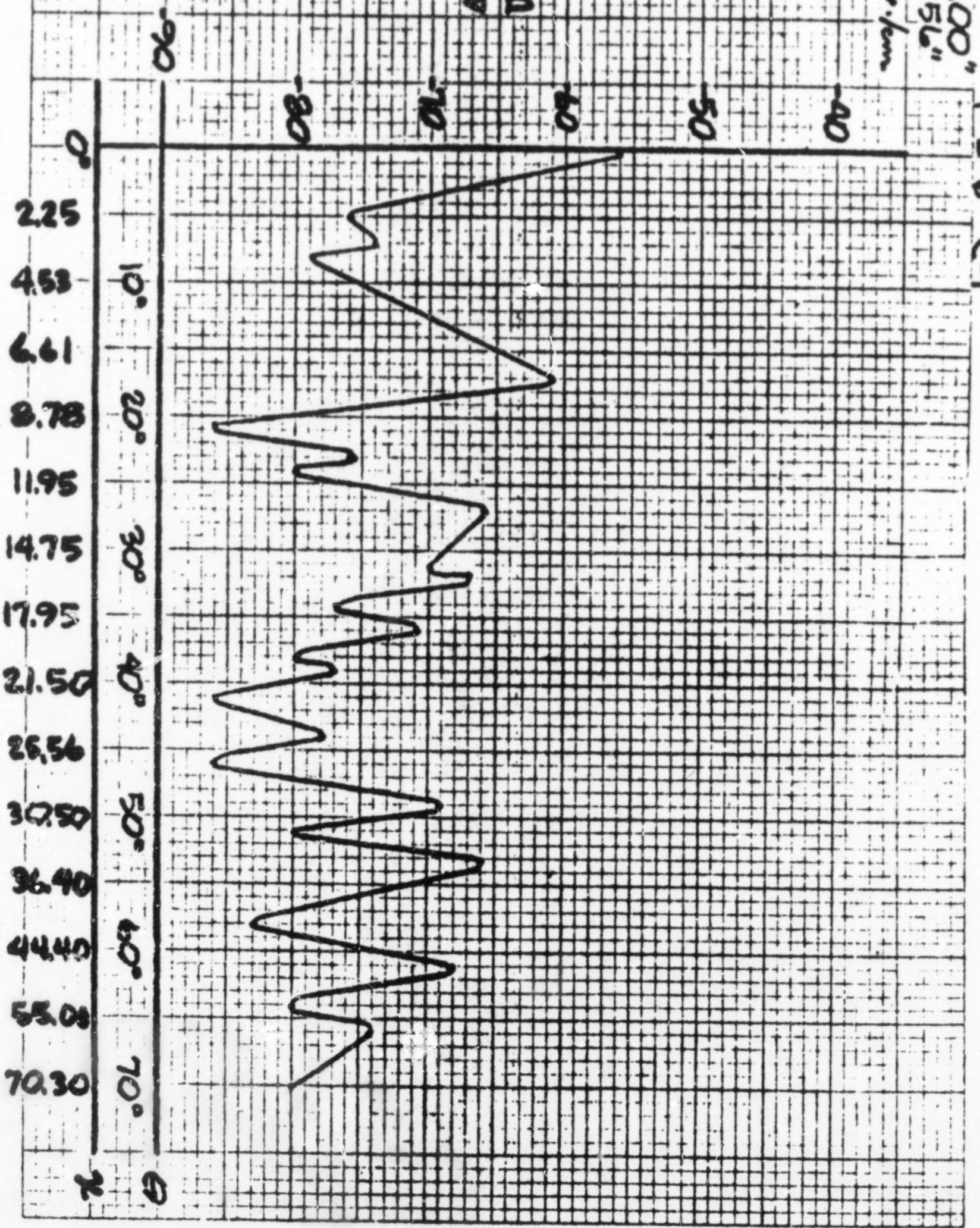
#24

P-II-6 C-1

X: 90.00"
Y: 25.56"
S: 14.4 km

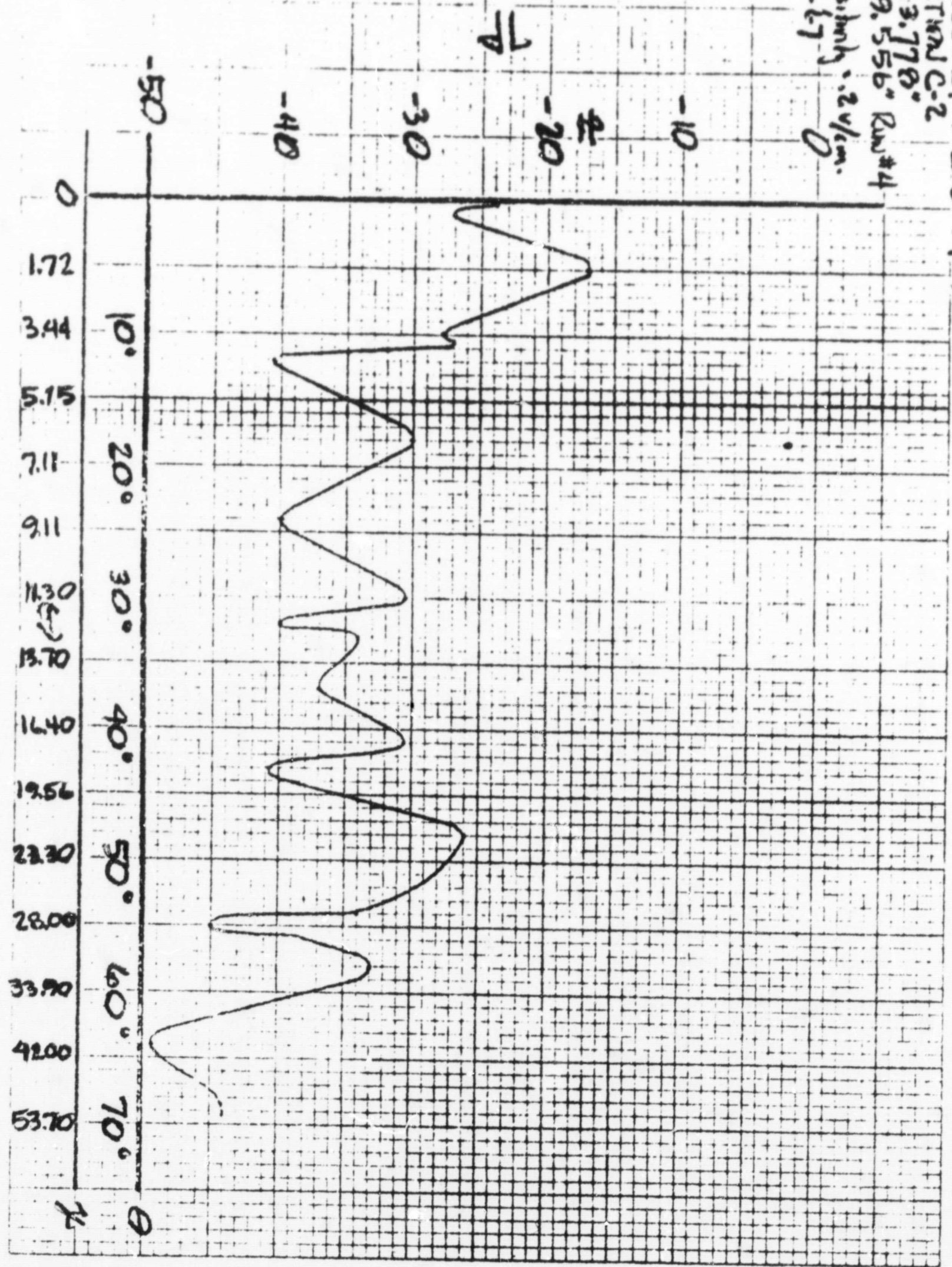
Fig. 6-26

db



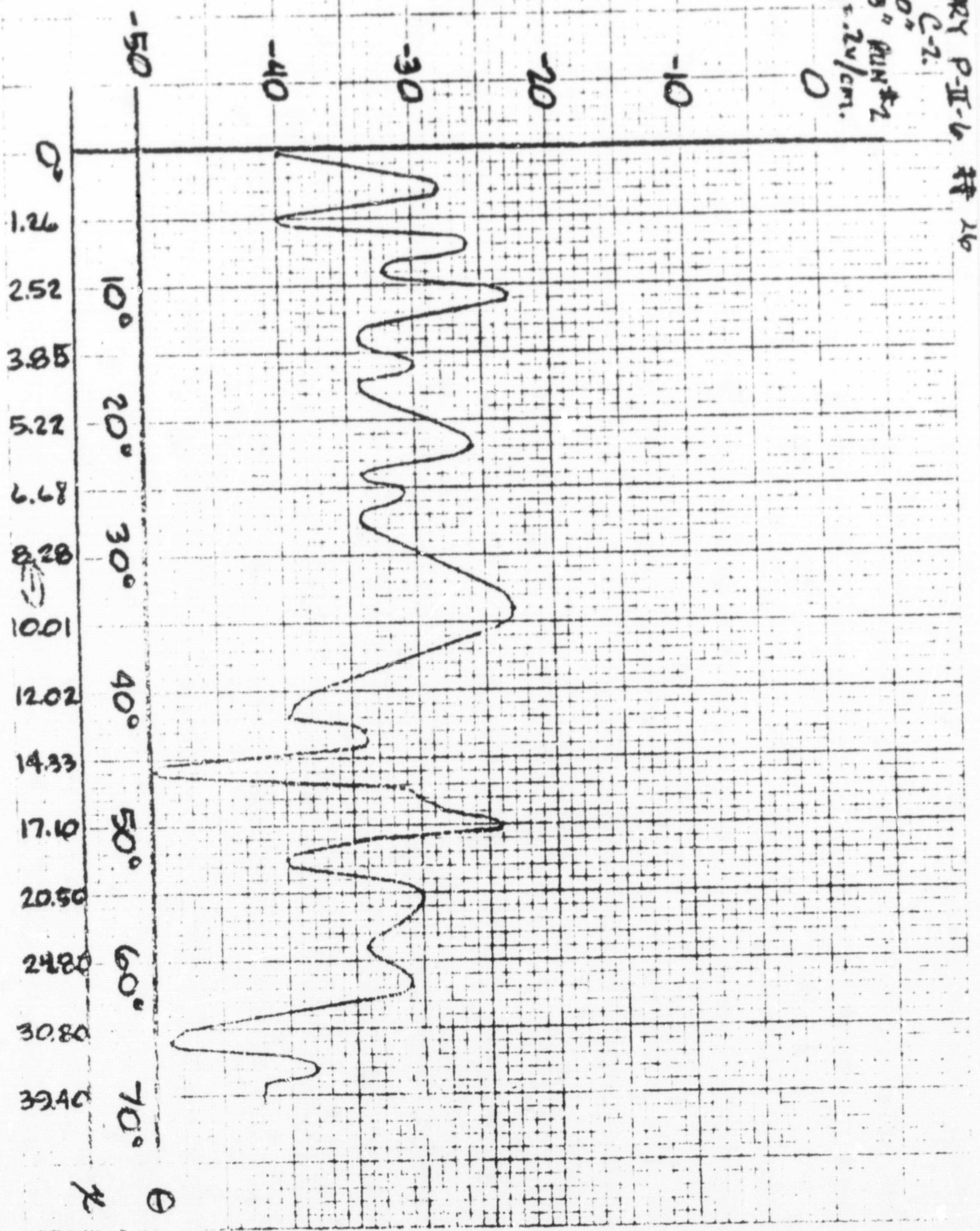
191
 TRAJECTORY P-II-6 # 25
 SECTION C-2
 X = 63.778°
 Y = 19.556° Run #4
 Sensitivity .2 v/cm.
 K-6-67

Fig. 6-27



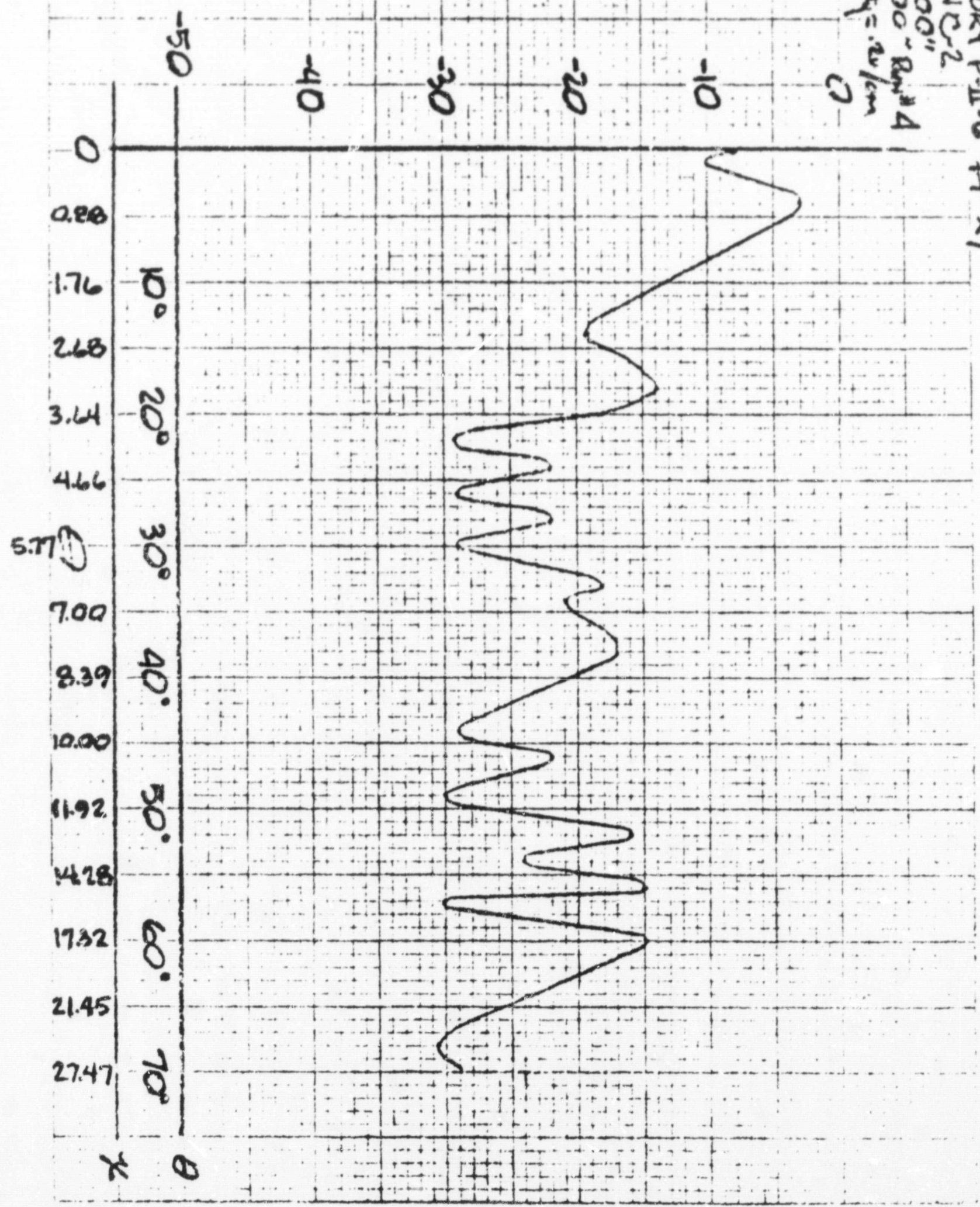
TRAJECTORY P-II-6 # 26
SECTION C-2.
X=43.000"
Y=14.333" RUN #2
Sensitivity = .2u/cm.
10-L-67

Fig. 6-28



TRAJECTORY P-II-6 # 47
 SECTION C-2
 27.000" RADIUS
 10.000" RADIUS
 SENSITIVITY = 20/CM
 193

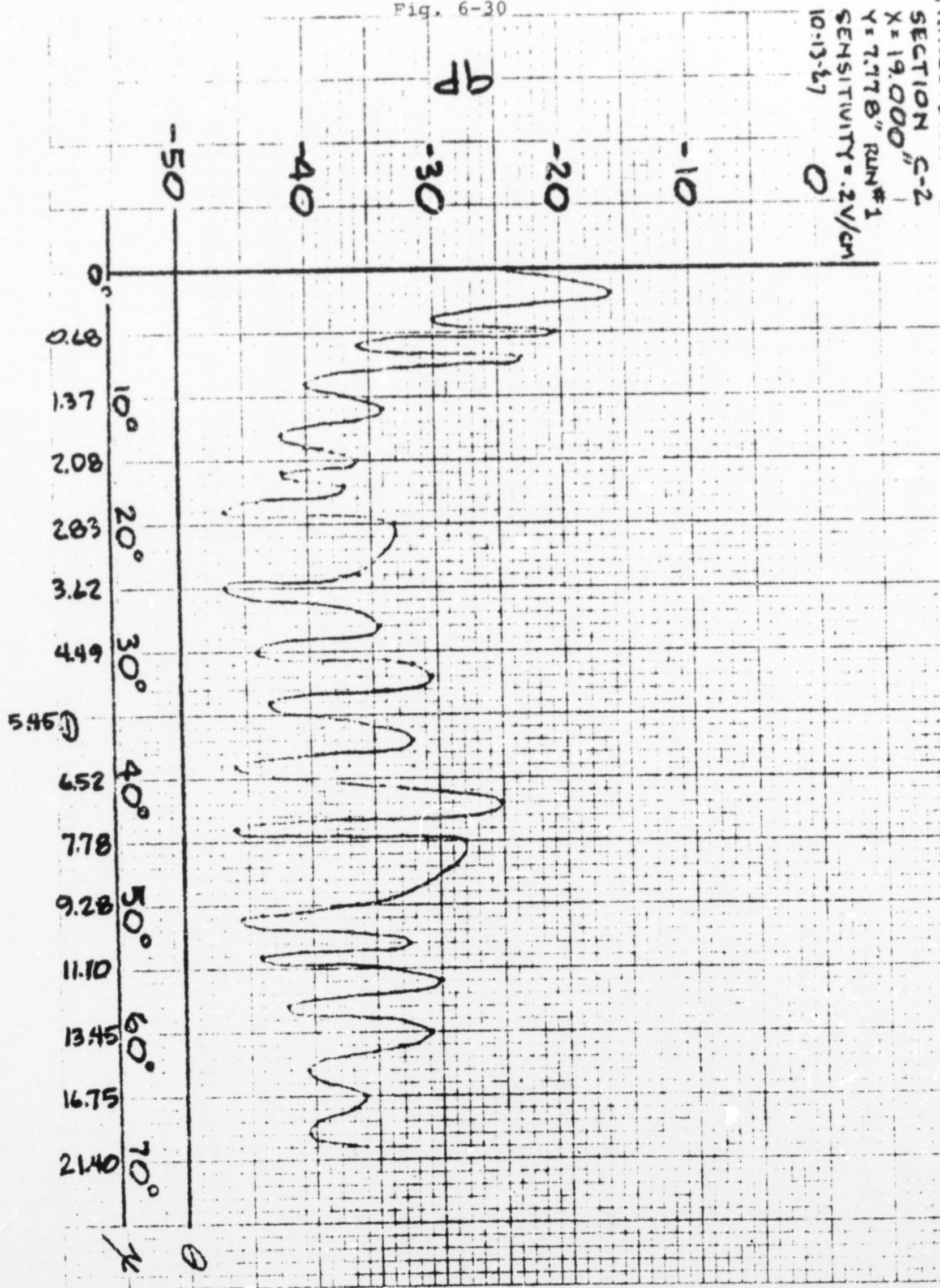
Fig. 6-29
91



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Fig. 6-30



TRAJECTORY P-II-6 # 29
SECTION C-2
X: 15.333"
Y: 6.667" AUL # 1
SENSITIVITY: .2V/cm
10-13-67

Fig. 6-31

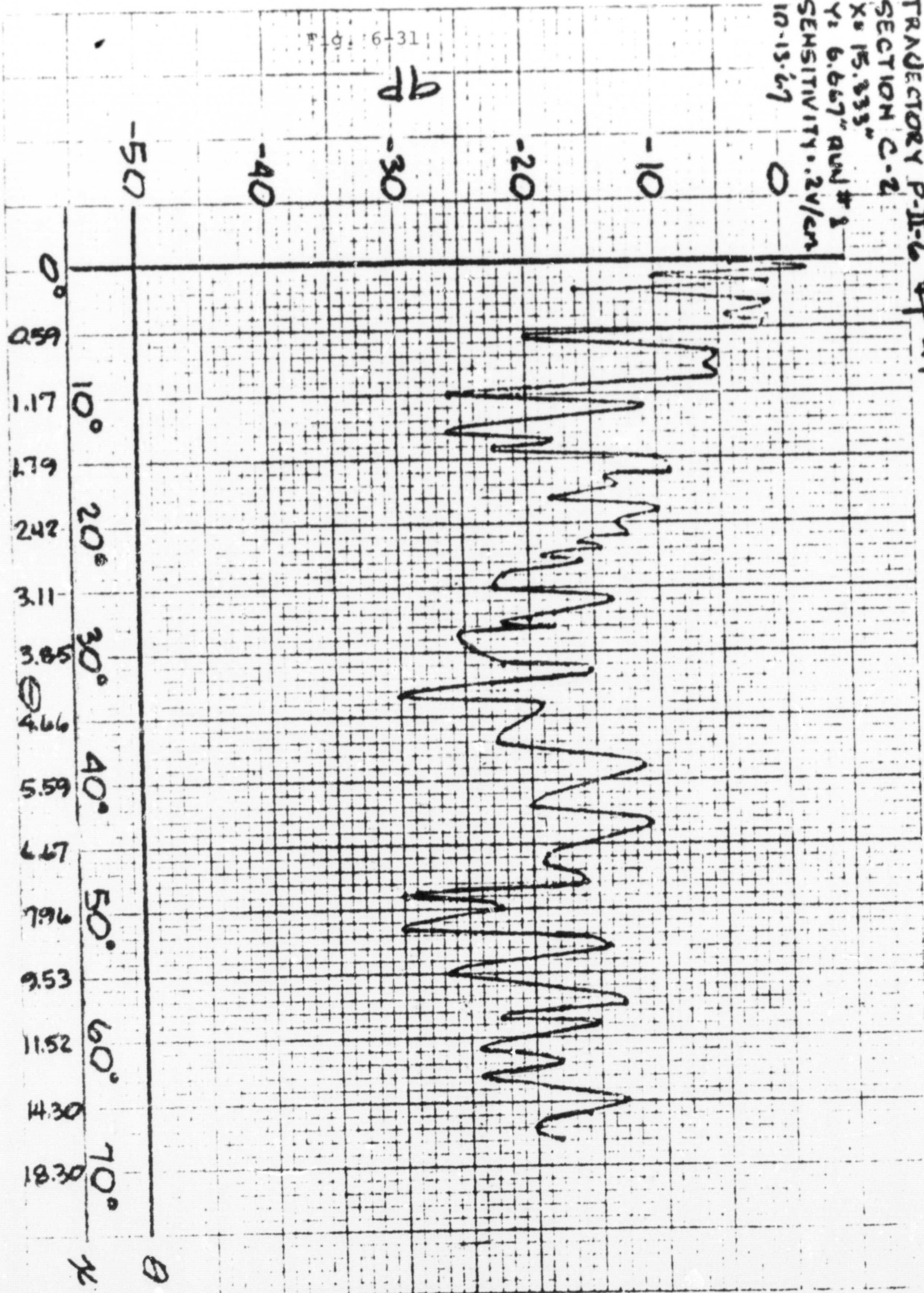
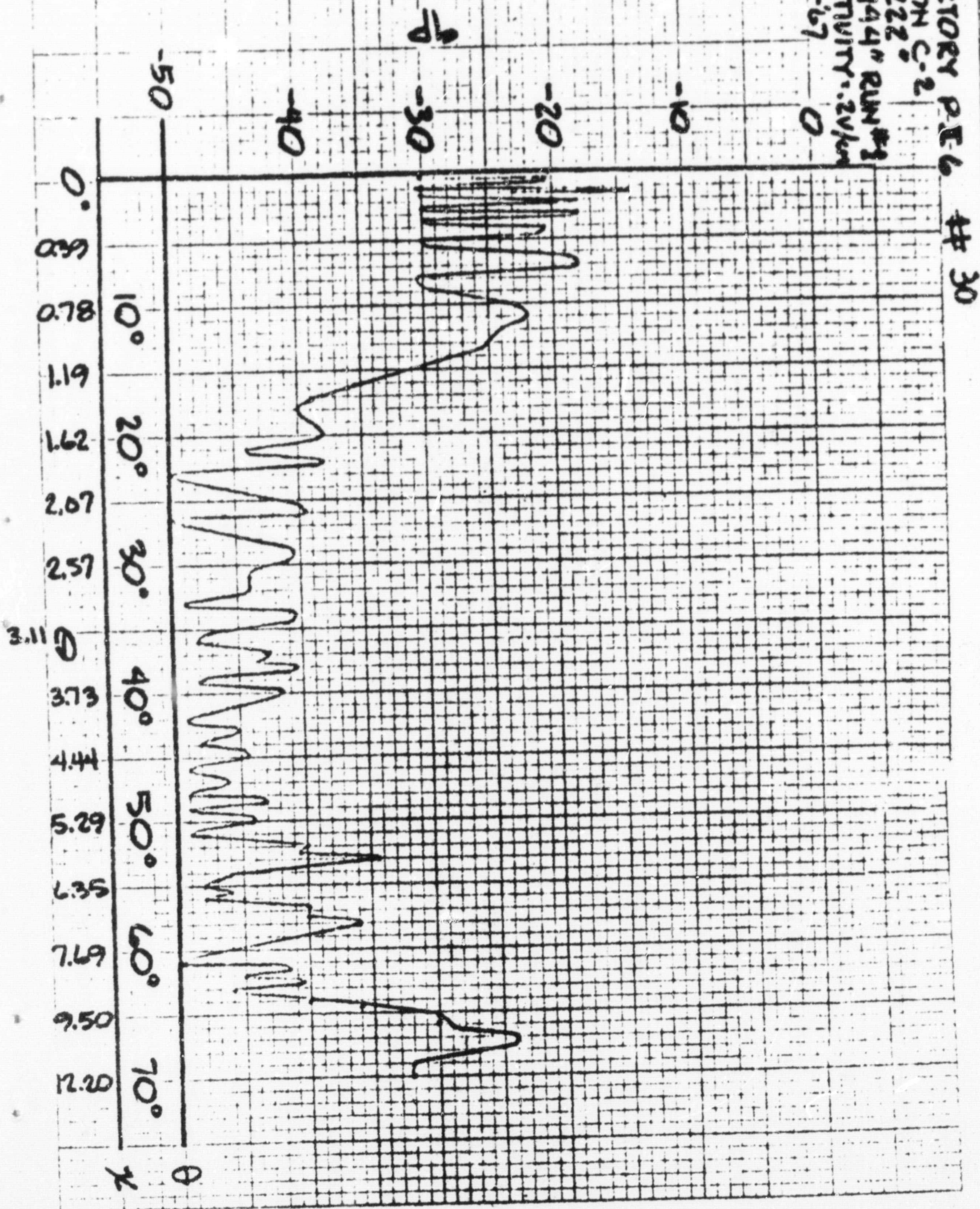


Fig. 6-32

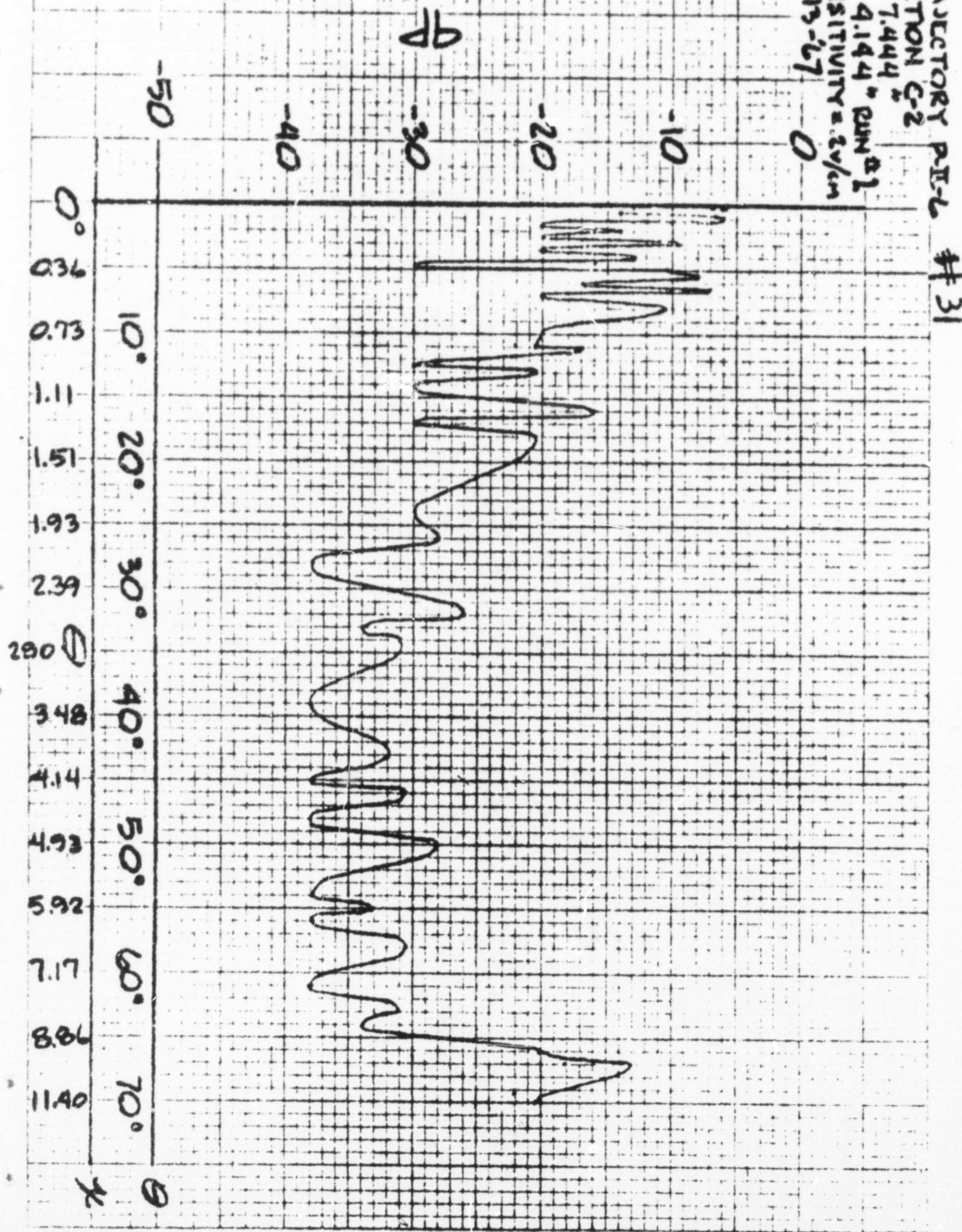
196

TRAJECTORY P.E.C. # 30
SECTION C-2
X = 8.222°
Y = 4.444° RUN #1
SENSITIVITY = 2V/Km
10-13-67



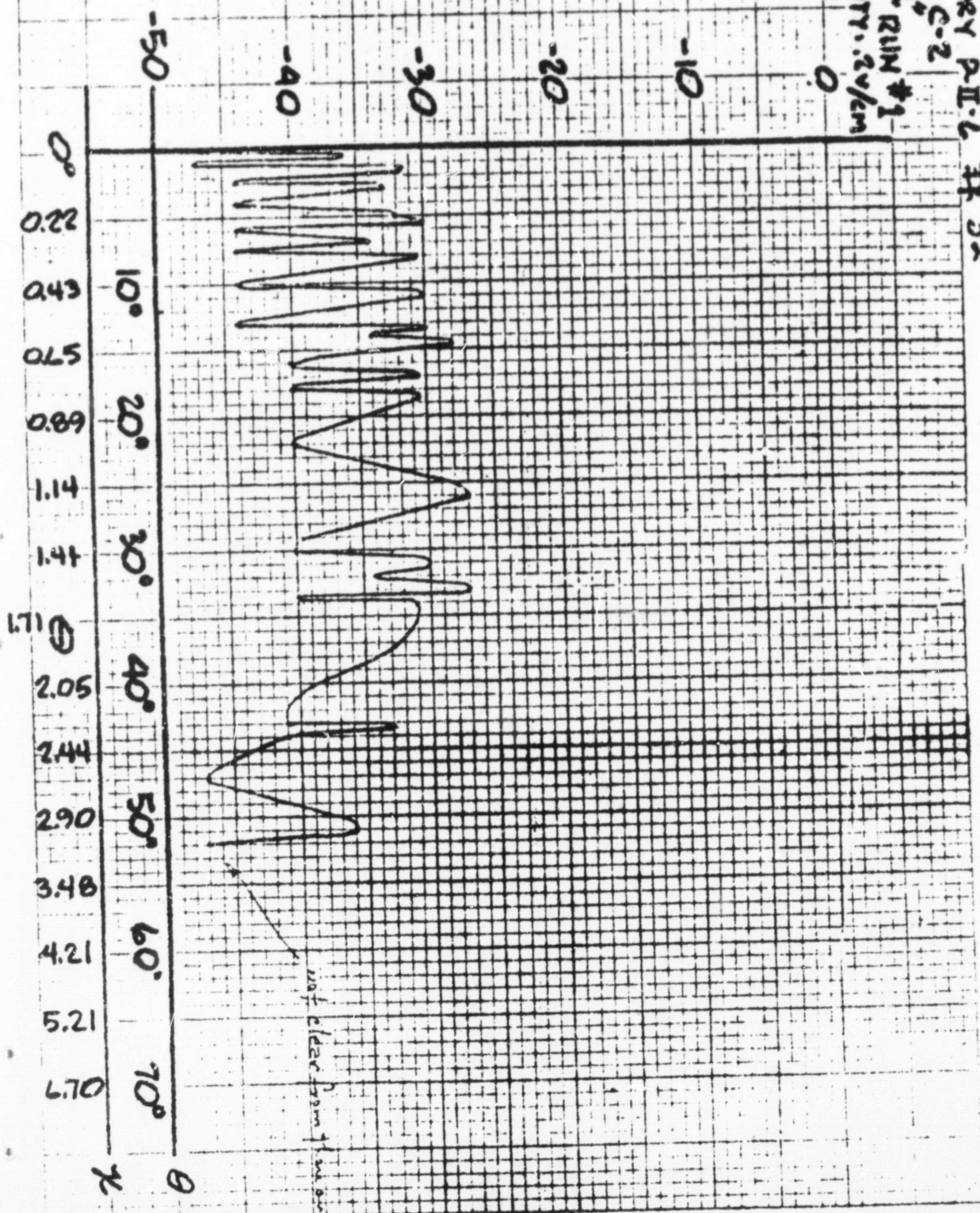
197
 TRAJECTORY P.E.-L
 SECTION C-2
 X = 7.444"
 Y = 4.144" RUN #2
 SENSITIVITY = 2v/cm
 10-13-67

Fig. 6-33



TRAJECTORY P-II-C # 32
 SECTION C-2
¹⁹⁸
 X=2.733"
 Y=2.444" RUN #1
 SENSITIVITY .2v/cm
 R-13-27

Fig. 6-34



TRAJECTORY P-T-6 #33
 SECTION 199
 X = 2.167
 Y = 2.000
 SENSITIVITY = 2.5
 10-13-67

Fig. 6-35

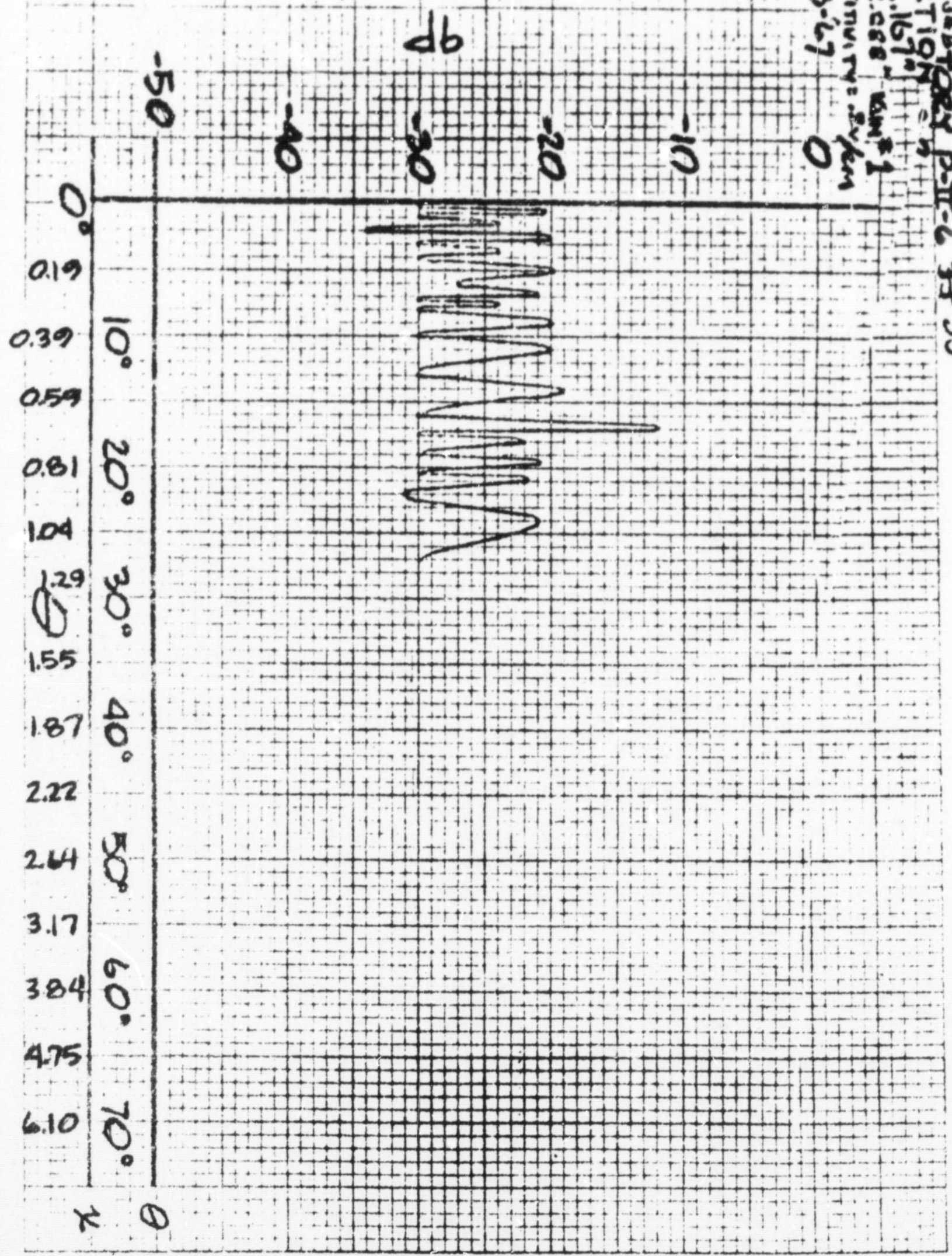


TABLE
MODEL AND FULL SCALE ALTITUDE (y' , y) AND HORIZONTAL
DISTANCE (x' , x) FROM CENTER OF LANDING ELLIPSE

	x ft.		x' in.	y ft.	y' in.
1	177	k	143.7372	25000	43.7956
2	150	k	96.3504	23000	40.2920
3	125	k	52.5547	20800	36.4380
4	107.5	k	21.8978	19200	33.6350
5	100	k	8.7591	18400	32.2336
6	95	k	0.0	17950	31.4453
7	95	k	143.7372	17950	31.4453
8	88.5	k	132.3504	17300	30.3066
9	78.3	k	113.9562	16200	28.3796
10	75	kk	108.7007	15800	27.6788
11	53.9	k	71.9124	12800	22.4234
12	36.2	k	40.7299	9250	16.2044
13	34.9	k	38.4526	8938	15.6578
14	33.7	k	36.3504	8622	15.042
15	32.5	k	34.2482	8310	14.5577
16	27.9	k	26.1898	7150	12.5255
17	25	k	21.1095	6500	11.3869
18	22.8	k	17.2555	5870	10.2832
19	18.3	k	9.3723	4800	8.4088
20	14.4	k	2.5401	3800	6.6569
21	12.95	k	0.0	3400	5.9562
22	12950		143.8896	3400	37.7
23	11000		122.2222	3000	33.2
24	8100		90.0000	2300	25.5
25	5740		63.7778	1760	19.5
26	3870		43.0000	1290	14.3
27	2430		27.0000	900	10.0
28	1710		19.0000	700	7.75
29	1380		15.3333	500	6.65
30	740		8.2222	400	4.45
31	670		7.4444	373	4.14
32	246		2.7333	220	2.44
33	195		2.1667	200	2.22

SUMMARY OF NORMALIZED ULTRASONIC SIMULATION
REFLECTIVITY DATA (RCS) FOR TRAJECTORY POINTS (P-II-6)

DEGREES CORRESPOND TO -DB														
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
0	- 8	- 7	-14	-12	-12	-14	-21	-21	-17	-19	-22	--	--	--
0	-19	-26	-19	-16	-26	-32	-34	-36	-34	-37	--	--	--	--
0	0	-15	-11	-12	-10	-10	-20	-13	-21	-25	--	--	--	--
0	- 4	- 6	- 6	+ 7	0	-15	-13	- 2	- 8	-14	- 8	--	--	--
0	- 3	-11	-12	-19	-22	-22	-23	-18	--	--	--	--	--	--
0	-12	-16	-18	-21	-15	-18	-19	-25	-23	-23	-16	-25	-20	-21
0	-15	-21	-29	-27	-35	-31	-36	-37	-28	-31	-34	-34	-36	-33
0	-27	-34	-20	-26	-31	-28	-32	-31	-37	-44	-22	-34	-44	-44
0	-23	-37	-32	-29	-34	-36	-43	-46	-41	-19	-41	-46	-46	-46
0	-32	-32	-20	-29	-24	-36	-25	-32	-32	-43	-44	-31	-42	-43
0	+ 2	- 5	-10	- 6	- 5	-19	-24	- 8	- 4	-13	-11	-11	-13	-20
0	+ 9	- 2	- 6	- 6	- 2	-11	- 5	- 6	-16	-16	-26	-19	-17	-17
0	- 2	-17	- 3	- 8	-19	- 6	- 7	- 7	-10	-18	-24	-22	-21	-29
0	-11	-17	-10	+ 1	-13	0	- 1	-15	- 5	- 7	-18	-10	-19	-27
0	-11	-19	-19	-16	-17	-12	-21	-14	-22	-23	-26	-36	-30	-34
0	- 5	-14	- 8	-10	- 8	-20	- 6	-20	-19	- 6	-23	- 2	-23	-25
0	- 5	0	- 9	- 8	0	- 2	- 4	- 9	-15	-10	- 6	-11	-13	-15
0	+10	+13	0	- 9	+ 3	- 5	+ 5	+ 9	+17	+ 3	+ 3	+ 2	--	--
0	+ 1	- 5	- 8	- 1	-20	-16	-19	-21	-16	-19	-16	- 4	- 4	- 8
0	-20	-19	- 9	-24	-21	-12	-19	-26	-27	-18	-16	-18	-22	-24
0	+ 7	- 4	-11	- 9	-15	- 7	-12	- 7	-11	- 3	-21	-14	-25	-20
0	0	+12	+ 8	+11	+ 9	+11	+16	+ 2	- 5	+16	+10	+ 7	- 8	- 2
0	+ 4	- 5	- 9	-10	-18	-21	-13	-13	-19	-20	-12	- 7	-18	-21
0	+ 3	-12	-12	- 9	-22	-20	-15	-17	- 4	-14	-11	- 7	-12	--
0	- 4	-22	- 6	- 9	-19	-20	-16	-11	-13	-19	-23	-17	- 9	--
0	0	+ 6	- 1	- 8	-12	-13	-15	-12	-14	-14	-18	-20	- 1	- 2
0	-23	-13	-19	-15	-22	-25	-24	-30	-31	-21	-26	-30	-22	-14
0	+13	+ 1	+11	+10	+17	+ 5	+14	+ 6	+ 1	+ 8	--	--	--	--
0	+10	0	+ 5	+ 2	+ 7	--	--	--	--	--	--	--	--	--

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DATA REFERRED TO REFLECTIVITY AT $\theta = 0$ AT POINT 1

ULTRASONIC (RCS) REFLECTIVITY DATA STATISTICS (P-II-6)

TABLE I-1

Point Number	1	2	3	4	5	6-7	8	9	10	11	12	13	14	15	16
0° (db) Exp. -64 (Ref.)	15	8	-2	4.6	5.3	5.8	6.4	7.5	8.0	11.6	17.3	17.9	18.5	19.2	21.7
0° Th.	Ref.	1.5	3.2	4.6	5.3	5.8	6.4	7.5	8.0	11.6	17.3	17.9	18.5	19.2	21.7
σ (db)	7.8	11.2	8.8	10.3	12.3	6.2	7.8	9.9	11.2	12.8	11.3	7.3	9.1	9.0	8.4
Corr Funct	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1	0.94	0.95	0.94	0.94	0.95	0.94	0.95	0.95	0.90	0.91	0.90	0.91	0.92	0.89	0.87
2	0.89	0.89	0.89	0.88	0.90	0.88	0.88	0.88	0.80	0.82	0.82	0.84	0.86	0.82	0.78
3	0.83	0.83	0.83	0.82	0.84	0.81	0.81	0.81	0.75	0.74	0.77	0.79	0.78	0.74	0.73
4	0.75	0.76	0.75	0.75	0.77	0.75	0.74	0.73	0.72	0.68	0.69	0.73	0.69	0.66	0.62
5	0.68	0.68	0.67	0.68	0.69	0.67	0.67	0.66	0.63	0.66	0.61	0.65	0.61	0.59	0.58
6	0.60	0.60	0.59	0.60	0.61	0.60	0.60	0.59	0.55	0.58	0.54	0.60	0.54	0.53	0.54
7	0.53	0.52	0.53	0.52	0.51	0.52	0.51	0.50	0.48	0.48	0.48	0.54	0.48	0.48	0.46
8	0.45	0.44	0.45	0.44	0.42	0.45	0.43	0.42	0.43	0.39	0.43	0.43	0.43	0.44	0.45
9	0.37	0.36	0.38	0.37	0.34	0.37	0.36	0.34	0.35	0.32	0.36	0.34	0.36	0.41	0.41
10	0.30	0.28	0.30	0.30	0.26	0.31	0.29	0.26	0.26	0.28	0.29	0.29	0.29	0.32	0.34
11	0.23	0.21	0.22	0.23	0.20	0.23	0.22	0.19	0.22	0.22	0.21	0.23	0.23	0.26	0.31
12	0.15	0.14	0.15	0.15	0.14	0.17	0.15	0.12	0.18	0.15	0.17	0.16	0.15	0.20	0.24
13	0.10	0.09	0.09	0.09	0.08	0.09	0.08	0.06	0.09	0.06	0.09	0.10	0.09	0.11	0.15
14	0.05	0.04	0.04	0.05	0.04	0.04	0.03	0.02	0.01	0.01	0.01	0.06	0.05	0.06	0.07

DATA REFERRED TO REFLECTIVITY AT $Q = 0$ AT POINT 1
ULTRASONIC (RCS) REFLECTIVITY DATA STATISTICS (P-II-6)

TABLE I-2

Point Number	17	18	19	20	21-22	24	25	26	27	28	29	30	31	32	33
0° (-db) Exp.	27	23	27	30	30	36	40	44	46	38	42	49	56	63	70
0° Theo.*	23.4	25.5	28.9	33	34.7	41.5	46.2	51.5	57.8	62	64.8	71.8	73	82	84
Std. Dev. (+db)	9.3	8.4	5.2	7.4	8.3	7.0	8.4	7.5	7.7	6.9	7.7	7.9	8.1	7.2	10
Corr. Funct.	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1	0.92	0.87	0.92	0.89	0.94	0.94	0.91	0.86	0.90	0.94	0.81	0.95	0.94	0.91	0.90
2	0.84	0.82	0.84	0.81	0.89	0.86	0.84	0.76	0.81	0.89	0.77	0.89	0.88	0.87	0.87
3	0.74	0.78	0.78	0.74	0.83	0.80	0.76	0.71	0.79	0.83	0.74	0.83	0.81	0.78	0.81
4	0.66	0.68	0.71	0.68	0.76	0.74	0.68	0.67	0.74	0.75	0.88	0.75	0.74	0.73	0.75
5	0.60	0.66	0.65	0.59	0.69	0.67	0.63	0.64	0.64	0.68	0.59	0.68	0.67	0.64	0.63
6	0.53	0.57	0.57	0.57	0.61	0.60	0.56	0.54	0.53	0.60	0.50	0.59	0.58	0.59	0.58
7	0.49	0.49	0.50	0.52	0.53	0.53	0.51	0.48	0.46	0.51	0.45	0.51	0.49	0.51	0.47
8	0.43	0.44	0.45	0.46	0.45	0.46	0.44	0.47	0.40	0.44	0.41	0.42	0.41	0.46	0.40
9	0.39	0.36	0.40	0.40	0.37	0.39	0.39	0.40	0.30	0.35	0.35	0.34	0.31	0.38	0.31
10	0.33	0.29	0.33	0.36	0.29	0.32	0.31	0.32	0.21	0.27	0.24	0.27	0.25	0.30	0.24
11	0.26	0.24	0.26	0.27	0.23	0.25	0.25	0.28	0.15	0.21	0.16	0.20	0.19	0.23	0.16
12	0.18	0.18	0.19	0.20	0.16	0.19	0.16	0.24	0.09	0.14	0.17	0.14	0.12	0.17	0.12
13	0.10	0.12	0.14	0.15	0.11	0.13	0.10	0.20	0.05	0.08	0.05	0.08	0.07	0.10	0.07
14	0.04	0.05	0.06	0.09	0.05	0.05	0.06	0.09	0.03	0.04	0.02	0.04	0.01	0.05	0.01

*R⁻⁴ Law

DATA REFERRED TO REFLECTIVITY AT $Q = 0$ AT POINT 1
 STATIC REFLECTIVITY DATA STATISTICS

θ	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
Mean (-db)	29.3	35.2	40.8	40.1	40.3	43.1	45.3	44.5	46.1	46.0	45.6	46.4	42.9	43.5	45.2
Std. Devn (+db)	19.7	19.3	19.4	18.2	18.2	19.1	18.3	21.3	19.9	19.1	19.9	18.3	14.4	12.9	14.6
Corr fctn	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1	0.81	0.84	0.90	0.90	0.89	0.89	0.89	0.89	0.88	0.88	0.89	0.91	0.91	0.94	0.94
2	0.73	0.77	0.79	0.83	0.83	0.80	0.80	0.80	0.80	0.80	0.80	0.82	0.85	0.90	0.89
3	0.59	0.67	0.72	0.75	0.74	0.74	0.75	0.72	0.74	0.74	0.73	0.76	0.78	0.84	0.84
4	0.50	0.61	0.68	0.68	0.69	0.70	0.68	0.67	0.69	0.67	0.67	0.69	0.74	0.81	0.81
5	0.38	0.55	0.63	0.62	0.61	0.65	0.62	0.61	0.64	0.62	0.64	0.66	0.73	0.79	0.79
6	0.39	0.55	0.61	0.58	0.59	0.62	0.59	0.58	0.61	0.61	0.60	0.66	0.71	0.77	0.76
7	0.39	0.53	0.58	0.57	0.57	0.59	0.59	0.57	0.59	0.61	0.60	0.65	0.70	0.74	0.73
8	0.41	0.51	0.57	0.55	0.56	0.54	0.58	0.55	0.56	0.58	0.57	0.61	0.66	0.71	0.70
9	0.41	0.50	0.55	0.53	0.54	0.52	0.57	0.53	0.55	0.56	0.56	0.61	0.63	0.67	0.66
10	0.38	0.47	0.51	0.49	0.50	0.49	0.52	0.47	0.51	0.51	0.53	0.57	0.57	0.62	0.62
11	0.35	0.41	0.48	0.47	0.46	0.46	0.46	0.45	0.45	0.47	0.47	0.51	0.55	0.58	0.58
12	0.32	0.41	0.46	0.44	0.44	0.45	0.43	0.41	0.44	0.46	0.46	0.50	0.52	0.55	0.55
13	0.31	0.40	0.45	0.44	0.43	0.45	0.42	0.40	0.44	0.44	0.43	0.47	0.50	0.52	0.52
14	0.34	0.40	0.43	0.43	0.43	0.42	0.41	0.39	0.42	0.43	0.42	0.45	0.47	0.49	0.48
15	0.39	0.39	0.42	0.44	0.44	0.43	0.43	0.42	0.43	0.42	0.41	0.43	0.45	0.46	0.45
16	0.42	0.41	0.42	0.43	0.45	0.43	0.45	0.42	0.43	0.43	0.42	0.42	0.41	0.43	0.42
17	0.42	0.40	0.40	0.43	0.42	0.41	0.42	0.42	0.41	0.42	0.41	0.41	0.41	0.40	0.39

DATA REFERRED TO REFLECTIVITY AT $\theta = 0$ AT POINT 1
 STATIC REFLECTIVITY DATA STATISTICS

	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
Mean. (-db)	29.3	35.2	40.8	40.1	40.3	43.1	45.3	44.5	46.1	46.0	45.6	46.4	42.9	43.5	45.2
Std. Devn (+db)	19.7	19.3	18.4	18.2	18.2	19.1	18.8	21.3	19.9	19.1	19.9	18.3	14.4	12.9	14.6
CORR fctn	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
18	0.39	0.39	0.39	0.40	0.41	0.41	0.41	0.40	0.40	0.40	0.40	0.39	0.39	0.39	0.38
19	0.34	0.37	0.37	0.37	0.37	0.40	0.39	0.39	0.40	0.41	0.38	0.37	0.37	0.35	0.34
20	0.27	0.29	0.31	0.31	0.32	0.33	0.32	0.31	0.33	0.34	0.30	0.30	0.32	0.30	0.29
21	0.19	0.20	0.23	0.24	0.23	0.25	0.26	0.24	0.24	0.26	0.24	0.23	0.24	0.23	0.22
22	0.20	0.19	0.21	0.21	0.22	0.24	0.25	0.22	0.23	0.24	0.22	0.21	0.22	0.19	0.18
23	0.16	0.16	0.19	0.19	0.19	0.21	0.24	0.21	0.21	0.21	0.22	0.19	0.19	0.15	0.15
24	0.19	0.18	0.20	0.19	0.19	0.20	0.22	0.20	0.20	0.20	0.20	0.19	0.19	0.13	0.13
25	0.19	0.16	0.17	0.15	0.16	0.16	0.21	0.17	0.17	0.19	0.18	0.17	0.16	0.10	0.11
26	0.18	0.15	0.14	0.14	0.14	0.13	0.18	0.15	0.16	0.19	0.17	0.15	0.15	0.10	0.08
27	0.12	0.11	0.11	0.10	0.10	0.09	0.13	0.10	0.12	0.14	0.12	0.10	0.10	0.07	0.06
28	0.12	0.09	0.09	0.08	0.08	0.06	0.10	0.07	0.09	0.10	0.09	0.06	0.06	0.05	0.04
29	0.05	0.03	0.04	0.03	0.04	0.03	0.05	0.04	0.05	0.05	0.05	0.03	0.03	0.02	0.02

CHAPTER VII

DYNAMIC DATA

The dynamic trajectory was simulated and the data for it was equivalent to scanning the target with fixed velocity, pitch angle, and elevation. This is substantiated by the theory that a sampling of the field from many points on a target which the transmitter is in motion is equivalent to sampling it at fixed altitude, with pitch angle and velocity fixed. In particular, when it is desired to obtain upper and lower limits for a reflectivity curve for certain altitudes is concerned, it is advisable to sample sufficiently often to obtain a target (statistically significant) population of data in order to establish these bounds. The data taken for this requirement was supplied on magnetic tapes to NASA and its summary is enclosed on the following pages.

Certain such dynamic data was reduced at the University of Houston and its statistical analysis will be performed under the 1968-69 contract.

TAPE NO. 1

Calibration--NASA Fixed Height--Speed

DATA

TARGET

P-II-A-B

TAPE NO. 2-1, 2F, 2

Run 1 = 31.5 - 3 + 26.28 25" Farside Carriage Setting
 Voltage = 25.3 (1.77 IPS)
 Y = 15K

Ch. 1 - M₁ Level set at .48 volt
 7 - E₁ Distribution Level set at Reference set at
 10 - 10 kc clock
 12 - Voice

Ch. 2 h = 17.52 V = 24.2
 V = 14.2

Ch. 2 - Rerun at V = 24.2 Volts

TAPE NO. 3

Calibration--Channel 1, 7 3V RMS or 100 kc Sine 1.69 IPS

Run 3 h = 13.14 V = 18.73 (4 F)
 Run 4 h = 8.76 V = 7.55
 Run 5 h = 8.76 V = 15.1
 Run 6 h = 8.76 V = 30.2

25" ----- 58 1/2"

Run 7 h = 4.38 V = 8.5

TAPE NO. 4

8F , 8F , 8F

8/30/68

Run 10 h = 2.63" V = 34.0 (70°)
 Calibration ±5 V dc

TAPE NO. 5

Run 9 h = 2.63" V = 17.0 (70°)
 Run 8 h = 2.63" V = 8.5 (70°)
 Run 11 F (Mixer h = 2.63" V = 7.32 (60°)
 in trouble)
 Run 12 h V = 5.32 (50°)

TAPE NO. 6

Calibration F ±5 V dc

Run 11F h = 2.63" V = 7.32 (60°)
 Calibration
 Run 11 h = 2.63" V = 4.56 (40°)

TAPE NO. 7

Run 13 half h = 2.63" V = 4.56 (40°)
 Run 13 half h = 2.63" V = 4.56 (40°)

Calibration

Run 14 h = 2.63" V = 4.0 0.17 IPS

Fixed Height and Angle Doppler Simulation

Actual Alt. Ft.	Model Alt. In.	Run #	Run Dur. Sec.	Sect./Angle	Along Beam Velocity - A	Model Vel. - IPF A/D x 12 (Cos 0)	Voltage
15 K	26.28	1	54	B-70	340	1.77	25.3
10 K	17.52	2	57	B-70	330	1.69	24.2
7.5K	13.14	3	74	B-70	250	1.28	18.73
5 K	8.76	4	205	B-70	90	.461	7.55
		5	102		180	.922	15.1
		6	51		360	1.84	30.2
2.5K	4.38	7	186	B-70	100	.512	8.5
1.0K	2.63	8	186	B-70	100	.512	8.5
		9	93		200	1.02	1.7
		10	46		300	2.04	3.4
		11	275	B-60	100	.349	7.32
		12	350	B-50	100	.274	5.32
		13	440	B-40	100	.217	4.56
		14	470	B-30	100	.202	4.36

Model B: X = Variation 25" - 120"
Y = Variation 2.25" - 48"

Doppler Scale Factor = 300

D_B = 6850 = Distance Scale Factor

Doppler TR Beam 3.37° x 8.04°
Rec Beam 7.39° x 12.15°

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APPENDIX A

ANALOGIES

The following is a well known NASA report from which this chapter is reproduced.

ANALOGIES

The analogies between various traveling wave phenomena have been recognized and used for many years.^{1,2} It is a common practice to utilize well developed techniques in a familiar subject to investigate similar phenomena in another area. In some applications the analogy may be completely valid and in other cases it may be incomplete or inexact, so the results must be interpreted with care. In this investigation, it is assumed that the medium through which the waves propagate is linear and isotropic when developing the analogies. It is, however, possible to use these modeling techniques to study propagation through a non-homogeneous medium.

It is shown in this investigation that certain problems concerning the scattering of electromagnetic waves from rough surfaces can be modeled in a system using acoustic waves. Either the dynamic pressure or the particle velocity (in the model system) can be made equivalent to the electric or magnetic field intensity in the real system.³ The analogy is investigated for the case of electromagnetic waves in air and boundary conditions are considered. It is assumed that the acoustic carrier frequency is low enough that losses in the water remain negligible.⁴

A.1 Wave Equations

Maxwell's equations in free space can be written as

$$\nabla \times \underline{E} = -\mu_0 \frac{\partial \underline{H}}{\partial t} \quad (\text{A-1})$$

and

$$\nabla \times \underline{H} = \epsilon_0 \frac{\partial \underline{E}}{\partial t} \quad (\text{A-2})$$

¹Jordan, op. cit.

²Meyer, op. cit.

³Moore, Traveling-Wave Engineering, op. cit., p. 49.

⁴The plane wave attenuation in fresh water is approximately 0.2 db per meter at a frequency of 1 mc and it increases approximately with the square of the frequency.

where $\underline{E}$ is the electric field intensity (the bar indicates a vector quantity),

$\underline{H}$ is the magnetic field intensity,

μ_0 is the permeability of free space, and

ϵ_0 is the dielectric constant of free space.

A corresponding pair of partial differential equations can be written to describe the pressure and velocity fields of an acoustic wave in a fluid.¹ Adiabatic compression and expansion are assumed² and the compressibility is considered to be much less than unity. These equations are

$$\nabla P = -\rho \frac{\partial \underline{U}}{\partial t} \quad (A-3)$$

and

$$\nabla \cdot \underline{U} = -K \frac{\partial P}{\partial t} \quad (A-4)$$

where P is the dynamic acoustic pressure,

$\underline{U}$ is the particle velocity,

ρ is the density of the fluid, and

K is the compressibility of the fluid.

The wave equations for the two systems are readily obtained from Equations A-1 and A-2, and from Equations A-3 and A-4. These wave equations are

$$\nabla^2 \underline{E} = \mu_0 \epsilon_0 \frac{\partial^2 \underline{E}}{\partial t^2} \quad (A-5)$$

and

$$\nabla^2 \underline{H} = \mu_0 \epsilon_0 \frac{\partial^2 \underline{H}}{\partial t^2} \quad (A-6)$$

for the electromagnetic wave, and

$$\nabla^2 P = \rho K \frac{\partial^2 P}{\partial t^2} \quad (A-7)$$

and

$$\nabla^2 \underline{U} + \nabla \times \nabla \times \underline{U} = \rho K \frac{\partial^2 \underline{U}}{\partial t^2} \quad (A-8)$$

¹Morse, Philip M., Vibration and Sound, McGraw-Hill, 1948, Chapter 7.

²The compression and expansion are said to be adiabatic when the temperature changes and the heat energy does not change. The acoustic frequencies used are in the range of one megacycle per second which is high enough that heat flow is negligible during one cycle.

for the acoustic wave. Equation A-8 can be simplified and written as

$$\nabla^2 \underline{U} = \rho_0 K \frac{\partial^2 U}{\partial t^2} \quad (\text{A-9})$$

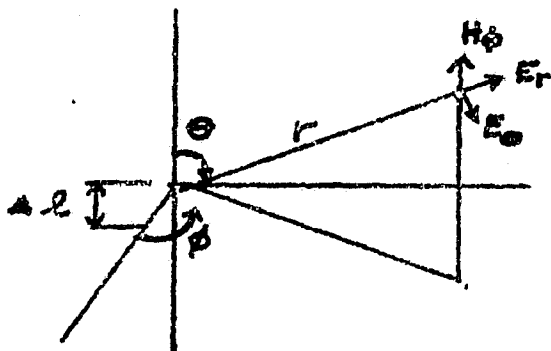
for cases where $\text{curl } (\text{curl } \underline{U})$ is small enough to neglect.¹

These wave equations form the basis for the useful analogy between acoustic and electromagnetic waves. It should be observed that electromagnetic waves are transverse while acoustic waves are longitudinal in nature. This basic difference between the two wave phenomena will prevent the analogy from being complete for all situations that might arise. Polarization effects are not present with acoustic waves. There are important phenomena, however, that are caused by the effects of interference. These effects are reproduced in the acoustic field and can be useful in predicting similar effects in the electromagnetic field.

A.2 Spherical Waves

The analogy between the electromagnetic wave and the acoustic wave is investigated first for the case of spherical geometry and for elementary sources. A short current element is used to establish an electromagnetic field. Several different acoustic sources are then considered and useful analogies are determined for each case.

A.2.1 Short Current Element - A short current element is located along the polar axis at the origin of a spherical coordinate system as shown in Figure A-1.



Short Current Element

Figure A-1

¹Weinstein, M. S., On the Failure of Plane Wave Theory to Predict the Reflection of a Narrow Ultra Sonic Beam, "Jour. Acoust. Soc. of Am., Vol. 24, No. 3, May 1952, pp. 284-87.

A solution for the field distributions about the short wire carrying a sinusoidally oscillating current,^{1,2} can be expressed in spherical coordinates as

$$E_R = \frac{I_m \Delta l \eta}{2\pi} e^{-jkr} \left(\frac{1}{r^2} + \frac{\lambda}{2\pi j r^3} \right) \cos \theta \quad (A-10)$$

$$E_\theta = j \frac{I_m \Delta l \eta}{2\lambda} e^{-jkr} \left(\frac{1}{r} + \frac{\lambda}{2\pi j r^2} - \frac{\lambda^2}{4\pi^2 r^3} \right) \sin \theta \quad (A-11)$$

and

$$H_\phi = j \frac{I_m \Delta l}{2\lambda} e^{-jkr} \left(\frac{1}{r} + \frac{\lambda}{2\pi j r^2} \right) \sin \theta \quad (A-12)$$

where I_m is the peak value of the current,

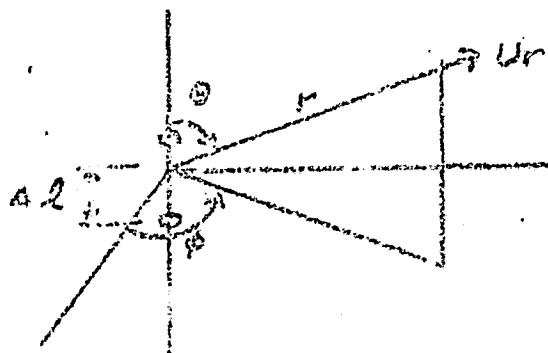
Δl is the length of the elementary conductor and is much shorter than a wavelength,

λ is the wave length,

k is the wave number, $2\pi/\lambda$, and

η is the intrinsic impedance of free space.

A.2.2 Double Acoustic Source - The analogies are developed first between the fields about a double acoustic source and the fields about the short current element. The double acoustic source consists of two elementary spherical sources operating 180° out of phase with each other and separated by a short distance Δl , where Δl is very small compared to the distance to the observation point.³ The geometry is given in Figure A-2.



Acoustic Double Source

Figure A-2

¹Sinusoidally varying signals are represented by $e^{j\omega t}$ and the time variation is suppressed in the equations.

²Schelkunoff, S. A. and Friis, H. T., Antennas, Theory and Practice, John Wiley and Sons, New York, 1952, p. 120.

³The pulsing spheres are much smaller than a wavelength; however, Δl is not restricted to dimensions less than a wavelength in size.

A solution for the pressure and velocity fields about the source¹, using wave Equations A-7 and A-8, can be expressed in spherical coordinates for the case of sinusoidal variations as

$$P_1 = \frac{\pi A U_0 \Delta l}{\lambda^2} z e^{-jkr} \frac{1}{r} \cos \theta \quad (A-13)$$

and

$$U_r = \frac{\pi A U_0 \Delta l}{\lambda^2} e^{-jkr} \left(\frac{1}{r} + \frac{\lambda}{2\pi j r^2} \right) \cos \theta \quad (A-14)$$

where P_1 is the dynamic pressure phasor,

U_r is the particle velocity phasor,

A is the area of one elementary sphere,

u_0 is the maximum value of the velocity of the surface of the source,

z is the specific acoustic impedance (product of the density and velocity of propagation), and

λ is the wave length.

There is an obvious partial analogy between the magnetic field intensity expressed by Equation A-12 and the particle velocity expressed by Equation A-14 with the exception that a $\sin \theta$ appears in one case and a $\cos \theta$ appears in the other case. The analogy between the electric field component, E_θ given by Equation A-11 and the pressure, P_1 , given by Equation A-13 is evident only when the distance r is larger so that terms of the order $1/r^2$ are negligible. There is no component of the acoustic field that compares to the radial component of the electromagnetic field given by Equation A-10.

There are several analogies between the electromagnetic wave and the acoustic wave that can be useful. The first analogy is

$$\frac{H_\theta}{j \sin \theta} \sim \frac{U_r}{\cos \theta} \quad (A-15)$$

where terms of order $1/r$ and $1/r^2$ are considered. The peak value of the current is proportional to the peak particle velocity in the acoustic system. The analogy breaks down at the points where θ goes to zero or $\pi/2$, but is valid in the region somewhat closer to the source than the other analogies because terms of order $1/r^2$ are retained. The value of r , however, must still remain much greater than Δl as required for the solutions A-13 and A-14 to be valid.

¹Randall, Robert H., An Introduction to Acoustics, Addison-Wesley, Cambridge, Mass., 1951, p. 58.

Three additional analogies can be obtained for the case where r is large, so that terms of the order $1/r^2$ and $1/r^3$ can be neglected. These analogies are tabulated in Table A-1. It should be observed from Figure A-2 and Equations A-13 and A-14 that the radiation field from the acoustic dipole is a maximum along the axis of the dipole. This presents practical difficulties in realizing a suitable dipole source, since the transducer will interfere with the wave propagation because of its physical size. Because of these difficulties in obtaining an acoustic dipole source of this type, it is desirable to investigate the possible analogies using other acoustic sources.

TABLE A-1

ANALOGIES - ACOUSTIC DOUBLE SOURCE AND SHORT CURRENT ELEMENT

Analogy Pair	EM System	Acoustic System	Analogous Terms
1	$\frac{H_\theta}{j \sin \theta}$	$\frac{U_r}{\cos \theta}$	$1/r \quad 1/r^2$
2	$\frac{E_\theta}{j \sin \theta}$	$\frac{P_1}{\cos \theta}$	$1/r$
3	$\frac{H_\phi}{j \sin \theta}$	$\frac{P_1}{\cos \theta}$	$1/r$
4	$\frac{E_\theta}{j \sin \theta}$	$\frac{U_r}{\cos \theta}$	$1/r$

A.2.3 Elementary Spherical Source - Consider now the acoustic fields about an elementary pulsing sphere. Solutions to the wave equations for this simple source can be written as

$$P_1 = j \frac{AU_0}{2\lambda} e^{-jkr} \frac{1}{r} \quad (A-16)$$

and

$$U_r = j \frac{AU_0}{2\lambda} e^{-jkr} \left(\frac{1}{r} + \frac{\lambda}{2\pi j r^2} \right) \quad (A-17)$$

The analogy between the particle velocity in Equation A-17 and the magnetic field intensity in Equation A-12 extends through terms of order $1/r^2$ as in the case of the double acoustic source, and the $\sin \theta$ term is missing. The analogy between pressure and electric or magnetic field intensity is valid only for the case of large range where terms of order $1/r^2$ and $1/r^3$ can be neglected. The $\sin \theta$ term is again missing in the expression for acoustic pressure.

The analogies that can be drawn between the acoustic wave equation solutions for an elementary spherical source and the electromagnetic fields about a short current carrying conductor are given in Table A-2. These analogies appear to be fully as useful as those involving the more complicated acoustic double source discussed earlier.

TABLE A-2

ANALOGIES - ELEMENTARY SPHERICAL SOURCE AND SHORT CURRENT ELEMENT

Analogy Pair	EM System	Acoustic System	Analogous Terms
1	$\frac{H_\theta}{\sin \theta}$	U_r	$1/r \quad 1/r^2$
2	$\frac{E_\theta}{\sin \theta}$	P_l	$1/r$
3	$\frac{H_\phi}{\sin \theta}$	P_l	$1/r$
4	$\frac{E_\phi}{\sin \theta}$	U_r	$1/r$

A.2.4 Acoustic Dipole - As the final example of an acoustic source consider an acoustic dipole which is defined as a sphere which vibrates along the polar axis in a prescribed manner. In this case the sphere is assumed to be small compared to a wavelength and the vibration is assumed to be sinusoidal as in the previous examples (refer to Figure A-2 for the geometry).

The dynamic pressure can be expressed as

$$P_1 = \frac{-\pi A U_0 a}{2\lambda^2} e^{-jkr} \left(\frac{1}{r} + \frac{j}{2\pi r^2} \right) \cos \theta \quad (A-18)$$

and the particle velocity can be expressed as

$$U_r = \frac{-\pi A U_0 a}{2\lambda^2} e^{-jkr} \left(\frac{1}{r} + \frac{2A}{2\pi j r^2} - \frac{2\lambda^2}{4\pi^2 r^3} \right) \cos \theta \quad (A-19)$$

where "a" is the radius of the spherical source.¹

In this example the dynamic pressure equation is analogous to the magnetic field intensity equation for terms varying as $1/r$ and $1/r^2$. In the previous examples, it was the vector particle velocity that was analogous to this extent. The analogy between the particle velocity and the θ component of the electric field intensity about the short current element is valid for large values of range so that higher order terms are negligible. The terms in $1/r^2$ and $1/r^3$ are different from the corresponding terms in Equation A-11 by only a factor of two. The analogies are tabulated in Table A-3.

A.3 Plane Waves

The problem of analogies is somewhat simplified when plane waves with sinusoidal time variations are being considered. The wave equations can be written in rectangular

TABLE A-3

ANALOGIES - ACOUSTIC DIPOLE AND SHORT CURRENT ELEMENT

Analogy Pair	EM System	Acoustic System	Analogous Terms
1	$\frac{H_\phi}{j \sin \theta}$	$\frac{P_1}{-\cos \theta}$	$1/r \quad 1/r^2$
2	$\frac{E_\theta}{j \sin \theta}$	$\frac{U_r}{-\cos \theta}$	$1/r$
3	$\frac{H_\phi}{j \sin \theta}$	$\frac{U_r}{-\cos \theta}$	$1/r$
4	$\frac{E_\theta}{j \sin \theta}$	$\frac{P_1}{-\cos \theta}$	$1/r$

¹Morse, op. cit., p. 318.

coordinates as

$$\frac{d^2 E_y}{dx^2} + \omega^2 \mu_0 \epsilon_0 E_y = 0 \quad (A-20)$$

and

$$\frac{d^2 H_z}{dx^2} + \omega^2 \mu_0 \epsilon_0 H_z = 0 \quad (A-21)$$

for a linearly polarized electromagnetic wave, and

$$\frac{d^2 P_1}{dx^2} + \omega^2 \rho_0 K P_1 = 0 \quad (A-22)$$

and

$$\frac{d^2 U_x}{dx^2} + \omega^2 \rho_0 K U_x = 0 \quad (A-23)$$

for the acoustic wave.¹ Solutions for the electric and magnetic field intensities are given by

$$E_y = E_m e^{-j\omega \sqrt{\mu_0 \epsilon_0} x} \quad (A-24)$$

$$H_z = H_m e^{-j\omega \sqrt{\mu_0 \epsilon_0} x} \quad (A-25)$$

and solutions for the dynamic acoustic pressure and particle velocity are given by

$$P_1 = P_m e^{-j\omega \sqrt{\rho_0 K} x} \quad (A-26)$$

$$U_x = U_m e^{-j\omega \sqrt{\rho_0 K} x} \quad (A-27)$$

The analogies for the plane wave case are tabulated in Table A-4.

TABLE A-4

PLANE WAVE ANALOGIES

Pair	EM System	Acoustic System
1	H_z	P_1
2	E_y	U_x
3	H_z	U_x
4	E_y	P_1

¹Moore, op. cit., EE-22.

The analogies for the case of linearly polarized plane electromagnetic waves and plane acoustic waves are complete without the addition of trigonometric terms. This desirable situation will not always prevail, however, as will be evident when boundary conditions are considered.

In the discussion to this point the scalar nature of pressure has presented no particular difficulty. The analogy is exact when considering the radiation fields in space, providing the appropriate trigonometric relations are taken into account and both components of the electromagnetic radiation field can be determined from measurements of either the dynamic pressure or the particle velocity in the acoustic field.

2.4 Plane-Wave Reflection

The modeling problem at hand is essentially one of duplicating in an appropriate manner the boundary conditions experienced in the real system, since the propagating waves are adequately modeled. The general conditions that must be satisfied at the interface are given first and then several examples are considered.

2.4.1 Boundary Conditions - Boundary conditions that must be satisfied have similar characteristics in the two systems. Neglecting shear stresses in the liquid,¹ the boundary conditions can be expressed as

$$\underline{n} \times (\underline{E}_1 - \underline{E}_2) = 0 \quad (A-28)$$

and

$$\underline{n} \times (\underline{H}_1 - \underline{H}_2) = 0 \quad (A-29)$$

for the electromagnetic field, and as

$$P_1 = P_2 \quad (A-30)$$

and

$$\underline{n} \cdot (\underline{U}_1 - \underline{U}_2) = 0 \quad (A-31)$$

¹The skin depth for the viscous wave in water at 20° C is about 0.5 micron.

for the acoustic field, where $\hat{n}$ is a unit vector normal to the surface. These equations state that the tangential components of $\underline{E}$ and $\underline{H}$ are continuous across the boundary as are the pressure, p , and the normal component of the particle velocity, $\underline{u}$.

The situation is considered first at theoretically perfect surfaces, and then realizable surfaces are investigated. Boundary conditions at a perfect conductor require the tangential electric field to be zero and the tangential magnetic field to be two times the incident field. On the other hand, at a perfectly elastic wall, (pressure release surface) the dynamic pressure is zero and the normal component of the total particle velocity is twice the normal component of the incident particle velocity. In each of these situations there is no wave propagation beyond the interface.

When a plane electromagnetic wave is normally incident on a perfectly conducting plane surface, the field components are both parallel to the surface and the electric field component can be made equivalent to either the pressure or the particle velocity in the acoustic wave. When the electric field is made equivalent to pressure, a perfectly elastic boundary is required in the acoustic system in order to obtain the proper phase relationships. On the other hand, a perfectly rigid boundary is required when the electric field is made equivalent to the particle velocity in the acoustic wave.

A.4.2 Snell's Law - The boundary conditions require that the phase velocities be identical along the interface in both media. This requirement results in Snell's law which applies equally well to the electromagnetic or acoustic wave and is expressed as

$$k_1 \sin \theta_1 = k_1 \sin \theta_r = k_2 \sin \theta_t \quad (A-32)$$

where θ_1 is the angle of incidence,

θ_r is the angle of reflection,

θ_t is the angle of transmission, and

k_1 and k_2 are the wave numbers ($2\pi/\lambda$) in the two media.

Snell's law for the acoustic wave is frequently written as

$$\sin \theta_t = \frac{v_2}{v_1} \sin \theta_1 \quad (A-33)$$

since the phase velocities v_1 and v_2 are commonly available for different materials.

Oblique Incidence Reflection - Linearly polarized plane electromagnetic waves will be considered in this section. In addition the electric field vector will be assumed to lie in the plane of incidence (TM wave). For this situation the tangential component of the electric field intensity is reduced by a factor, $\cos \theta$, where θ is the angle of incidence as measured from the normal to the surface. It is therefore necessary to let the vector particle velocity in the acoustic wave be equivalent to the electric field intensity in the electromagnetic wave, since the normal component of the particle velocity is also reduced by a factor, $\cos \theta$. The magnetic field intensity is parallel to the interface and the dynamic acoustic pressure is a scalar so neither of these quantities is affected by the angle of incidence.

The reflection coefficient is by definition the ratio of the tangential component of the reflected electric field to the tangential component of the incident electric field and can be written as

$$\frac{r_a}{r} = \frac{E_{r \tan}}{E_i \tan} \quad (A-34)$$

where the subscripts i and r refer to the incident and reflected fields respectively.¹ Superscripts a and w are used to refer to the electromagnetic field in air or to the acoustic field in water. In terms of the magnetic field the above reflection coefficient becomes

$$\frac{r_a}{r} = \frac{H_r \tan}{H_i \tan} \quad (A-35)$$

The reflection coefficient for the acoustic wave is defined here in a similar manner in terms of particle velocity as

$$\frac{r_w}{r} = \frac{U_r \text{ norm}}{U_i \text{ norm}} \quad (A-36)$$

The above reflection coefficient can also be expressed in terms of the dynamic pressure as

$$\frac{r_w}{r} = - \frac{P_{ir}}{P_{ii}} \quad (A-37)$$

¹The reflection coefficient is ordinarily complex.

At a perfect conductor the reflection coefficient for the E component of an electromagnetic wave is -1 as given by Equation A-34. Since the tangential component of the E field must be zero on the surface, the corresponding situation for the acoustic wave occurs on a perfectly rigid wall where the normal component of particle velocity goes to zero. Therefore, Equation A-36 corresponds to Equation A-34, and the positive sign is used. It is now necessary to use a minus sign on Equation A-35 and A-37, since the tangential component of H and the pressure both double at the corresponding perfect surfaces.

The concept of directional impedance is useful when considering oblique incidence reflection of plane waves. The impedance in the x direction can be expressed as

$$Z_{x1}^a = \frac{E_i \tan \theta_i}{H_i} = n_1 \cos \theta_i \quad (\text{A-38})$$

for the electromagnetic field and

$$Z_{x1}^w = \frac{V_i \text{ norm}}{P_{1i}} = Z_1 \cos \theta_i \quad (\text{A-39})$$

for the acoustic field. It should be observed that the acoustic impedance has been defined as the ratio of the particle velocity to the dynamic pressure in this equation. This definition of acoustic impedance is just the reciprocal of that which is ordinarily used.¹ Since impedance is a defined quantity either definition is satisfactory, however, one definition must be consistently used in a particular problem.

The reflection coefficient for electromagnetic waves can also be expressed in terms of the directional impedances as

$$r_a = \frac{Z_{x2}^a - Z_{x1}^a}{Z_{x2}^a + Z_{x1}^a} \quad (\text{A-40})$$

where the subscripts 1 and 2 refer to the media on opposite sides of the interface. For acoustic waves in water the reflection coefficient can be written in a similar manner² as

¹Some authors, however, have preferred to define specific acoustic impedance as the ratio of particle velocity to dynamic pressure, for example; Moore, Traveling-Wave Engineering, op. cit.

²The expression for the reflection coefficient comes out in the usual manner because it is defined here in terms of velocity and the impedance is defined as the ratio of particle velocity to dynamic pressure.

$$r_r^w = \frac{Z_{w2} - Z_{w1}}{Z_{w2} + Z_{w1}} \quad (A-41)$$

The reflection coefficient for either system can be written as

$$r_r = \frac{\cos \theta_t + \frac{Z_1}{Z_2} \cos \theta_i}{\cos \theta_t + \frac{Z_1}{Z_2} \cos \theta_i} \quad (A-42)$$

using Equation A-38 (replace the Z 's with n 's) or A-39. The reflection coefficient is properly modeled when the ratio of Z_1 to Z_2 in the acoustic system is the same as the ratio of n_1 to n_2 in the electromagnetic system. As an example, assume the dielectric constant of the earth to be $13 \epsilon_0$. The impedance is given by

$$n = \sqrt{\frac{\mu}{\epsilon (1 + \frac{\sigma}{j\omega\epsilon})}} \quad (A-43)$$

where μ is the permeability,

σ is the conductivity,

ϵ is the dielectric constant, and

ω is the angular frequency.

If it is assumed that the conductivity is small enough or that the frequency is high enough that $\sigma/j\omega \ll 1$ then the impedance is given by $n = \sqrt{\frac{\mu}{\epsilon}} = 105$ ohms. Since the impedance of air to the electromagnetic plane wave is about 377 ohms, the air-earth impedance ratio of R or $1/R$ results in the same magnitude for the reflection coefficient; only the sign is changed.

To summarize this section it should be noted that the TM wave can be replaced by a TE wave. In this case it is convenient to redefine the reflection coefficient in terms of dynamic pressure and to redefine the acoustic impedance as the ratio of the dynamic pressure to the particle velocity. This is done merely to maintain the symmetry of the equations in the real and model system.

The analogies for all cases where a plane linearly polarized electromagnetic wave is reflected from a perfectly conducting plane surface are shown in Figure A-3.

TABLE A -5

ACOUSTIC IMPEDANCE^a RATIOS FOR SEVERAL MATERIALS

<u>Material</u>	<u>Impedance Ratio</u>
Water/Steel	31/1
Water/Glass	12/1
Water/Magnesium	6.5/1
Water/Elm Wood	1/2.7
Water/Bakelite	2.5/1
Water/Polystyrene	1.7/1

^aAcoustic impedance is defined as the ratio U/p in this table.

A.5 Applications of the Analogies

To demonstrate an application of the analogies, it is desirable to consider a simple situation where the exact solution is known in both the real and the model systems

BOUNDARY CONDITIONS ON PERFECT PLANE SURFACES

	<u>Electromagnetic</u>	<u>Acoustic</u>
	<u>Perfect Conductor</u>	<u>Rigid Surface</u>
(TEM wave)		
Normal Incidence	$E_t = 0$ $H_t = 2H_i$	$U_n = 0$ $P_o = 2P_i$
		<u>Elastic Surface</u>
		$P_o = 0$ $U_n = 2U_i$
		<u>Elastic Surface</u>
(TE wave)	$E_t = 0$	$P_o = 0$
Oblique Incidence	$H_t = 2H_{it}$	$U_n = 2U_{in}$
		<u>Rigid Surface</u>
(TM wave)	$E_t = 0$	$U_n = 0$
Oblique Incidence	$H_t = 2H_i$	$P_o = 2P_i$

Figure A-3

If an investigation of the problem indicates that correct experimental results will be obtained in the simple case then an extension to a more complicated problem is reasonable. Since only the correctness of the analogy is being investigated, no scaling is considered in this example. Also, the magnitude of the magnetic field intensity at unit distance in Equation A-12 is set numerically equal to the magnitude of the particle velocity at unit distance in Equation A-14. The applications considered in Section A.5.1 are theoretical only and were not investigated experimentally.

A.5.1 The Analogy with the Elementary Acoustic Source -
Consider a spherical coordinate system located in free space with an elementary current dipole located at the origin as shown in Figure A-4.

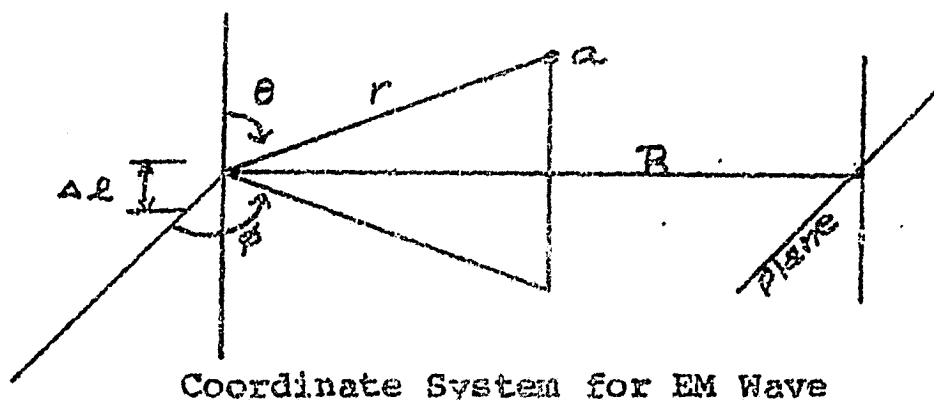
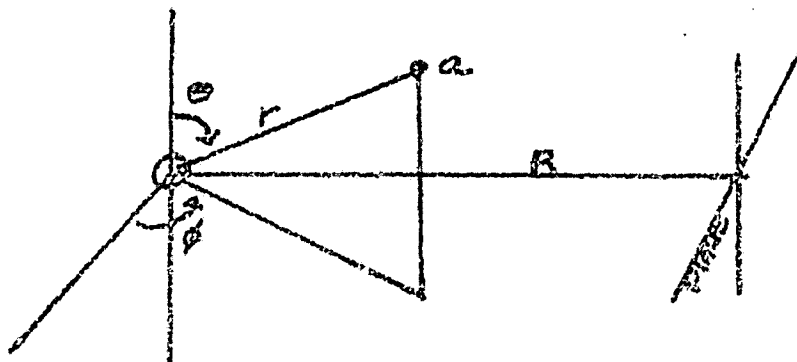


Figure A-4

The problem is to determine the field at any point "a" which is located far enough from the origin so that $r \gg \Delta l$. Assume that the acoustic model is set up as shown in Figure A-5.

Now the fields about the current element in the real system are specified by Equations A-10, A-11, and A-12; and in the model system they are given by Equations A-13, and A-14. If the analogy is considered to be between H_ϕ and U_r a measurement of the particle velocity in the radial direction in the model system will allow the magnitude of the magnetic field component in the ϕ direction in the real system to be determined using pair 1 from Table A-2.

The particle velocity can be easily measured using a properly calibrated narrow beam pressure transducer to probe the field at the point "a". Both the magnitude and the direction of the particle velocity can be determined by this method. After H_ϕ has been determined from the measurement in the model system, Maxwell's equations can be used to determine the remaining field components.



Coordinate System for Acoustic Wave

Figure A-5

Suppose now that in the real system a perfectly conducting infinite plane is located at a distance R from the origin as shown in Figure A-4. Let the line R be normal to the plane and pass through the origin of the coordinate system. Assume that the reflected fields are required at the location of the origin.

Using image theory and Equation A-12, the reflected magnetic field intensity at the origin is given by

$$H_\phi = j \frac{I_m \Delta l}{2\lambda} e^{-jk2R} \left(\frac{r}{2R} + \frac{\lambda r^2}{2\pi j (2R)^2} \right) \sin \theta \quad (\text{A-44})$$

where θ is equal $\pi/2$ (from the geometry),

r is the plane wave reflection coefficient, and

Γ is a reflection coefficient determined for the $1/r^2$ term.¹

For the special case of a perfectly conducting interface both r and Γ reduce to unity.

¹For a dipole oriented horizontally above a plane surface the reflected magnetic field for the radiation term is equivalent to that radiated from an image dipole modified by the plane wave reflection coefficient. The term which varies as $1/r^2$ is modified by a factor

$$\frac{n-1}{n+1} \left(\frac{n^2 - 6n + 9 - \frac{4}{n}}{(n-1)^2} \right)$$

where

$$n = \sqrt{\frac{\epsilon - j\frac{\sigma}{\omega}}{\epsilon_0}}$$

The plane wave reflection coefficient is given by

$$\Gamma = \frac{n-1}{n+1}$$

Moore, R. K., Normal Incidence Reflection of Dipole Radiation from a Plane Interface, Sandia Corp. Tech. Memo, 199-56-14. Sept., 1956.

Now in the model system the particle velocity is equivalent to the field intensity in the real system. Equivalent boundary conditions for particle velocity require a perfectly elastic infinite plane boundary located at a distance R from the origin as shown in Figure A-5. Again using image theory and Equation A-17 the reflected component of particle velocity at the origin is given by

$$U_r = j \frac{AU_0}{2\lambda} e^{-jk2R} \left(\frac{1}{2R} + \frac{\lambda}{2\pi j(2R)^2} \right) \quad (A-45)$$

It is required that the level and phase of the electromagnetic field be determined by physical measurements of the acoustic field. A pressure transducer with a very narrow antenna pattern can be used to measure the particle velocity and direction at the origin of the acoustic system. The results of the measurement will obviously give the magnitude of Equation A-45 and the direction will lie along the line R . The phase can be determined by comparing the received acoustic signal with the reference oscillator which controls the transmitter. In this model a component of particle velocity in the r direction corresponds to the component of magnetic field intensity in the ϕ direction. The magnitude of the particle velocity will be equal to the magnitude of the field intensity, in consistent units since the trigonometric factor ($\sin \phi$) is unity.

An alternative method of measurement is to use a transducer calibrated to measure pressure. The resulting measurement will be the magnitude of

$$P_1 = j \frac{AU_0}{2\lambda Z} e^{-jk2R} \frac{1}{2R} \quad (A-46)$$

which is Equation A-16 with the appropriate range inserted. It can be shown that the particle velocity for the case of sinusoidal variations is given by

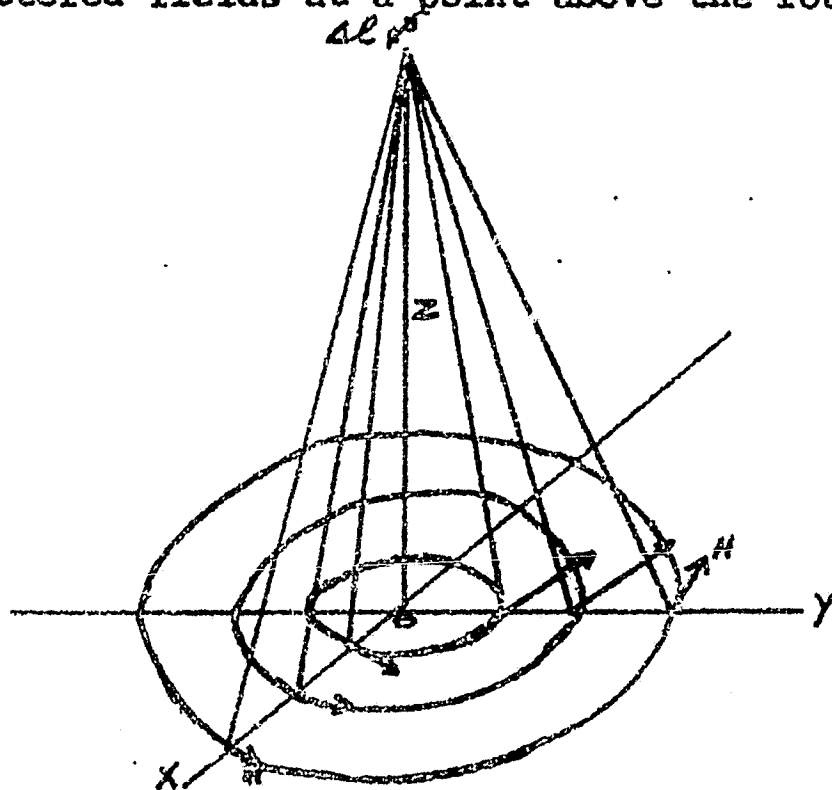
$$U_r = -\frac{1}{j\rho_0\omega} \frac{\partial P}{\partial r} \quad (A-47)$$

where P_1 is given by Equation A-16. Evaluating Equation A-47 results in Equation A-45 when the range, $2R$, is substituted for r .

A.5.2. Boundary Considerations - In the previous models the analogy is exact since the describing equations are identical in the two systems, except for factors which are taken into account in the transformation, and the boundary conditions are simple and equivalent. The complete electromagnetic

field can be determined from measurements made in the acoustic system because of the simplifications. This desirable situation will not continue when more interesting and useful problems are investigated.

Let a short dipole antenna carrying a sinusoidally varying current be located at the origin of a coordinate system as shown in Figure A-6. Now assume a perfectly conducting surface is placed at a distance, h , from the origin, where $h \gg \lambda$. Assume also that irregularities on the surface cause it to deviate from a plane surface by an amount comparable to a few wavelengths. A typical problem of the backscatter of electromagnetic waves from a rough surface is encountered when one attempts to calculate the scattered fields at a point above the rough surface.

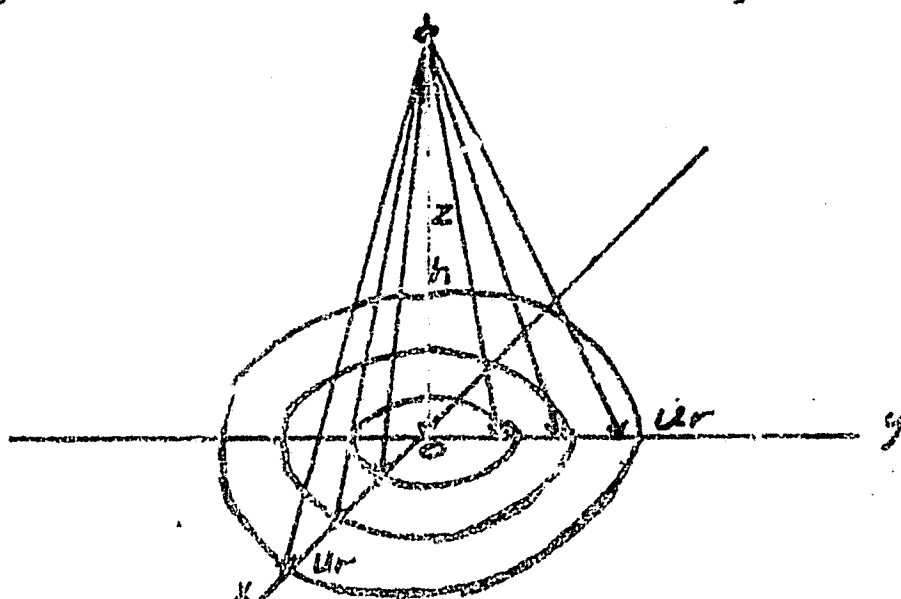


H-Field Incident on Plane

Figure A-6

In Figure A-6 the directions and relative magnitudes of the incident H vectors are shown as they reach the x,y plane where the rough boundary is located. The magnitude of the incident H vector varies inversely with the range to the point on the target. In addition the field is reduced by the size of the angle measured from the axis of the current element to the point on the target. This situation is to be compared to the incident particle velocity on the boundary in the case of an acoustic wave.

Figure A-7 shows an elementary spherical source located on the z axis at a distance h from the origin. The relative magnitude and direction of the incident particle velocities are shown in the x,y plane where the rough boundary is located. The magnitude of the velocities vary inversely with range and not with the angle of incidence. At each point on the target the particle velocity is normal to the direction of the corresponding H field vector in the electromagnetic field.



Particle Velocity Incident on Plane

Figure A-7

It is apparent that the intensity of the incident signal is not faithfully reproduced in the acoustic model in this example. The scattering elements of the y axis have an incident field that should be reduced by a factor

$$\cos (\tan^{-1} \frac{\lambda}{h})$$

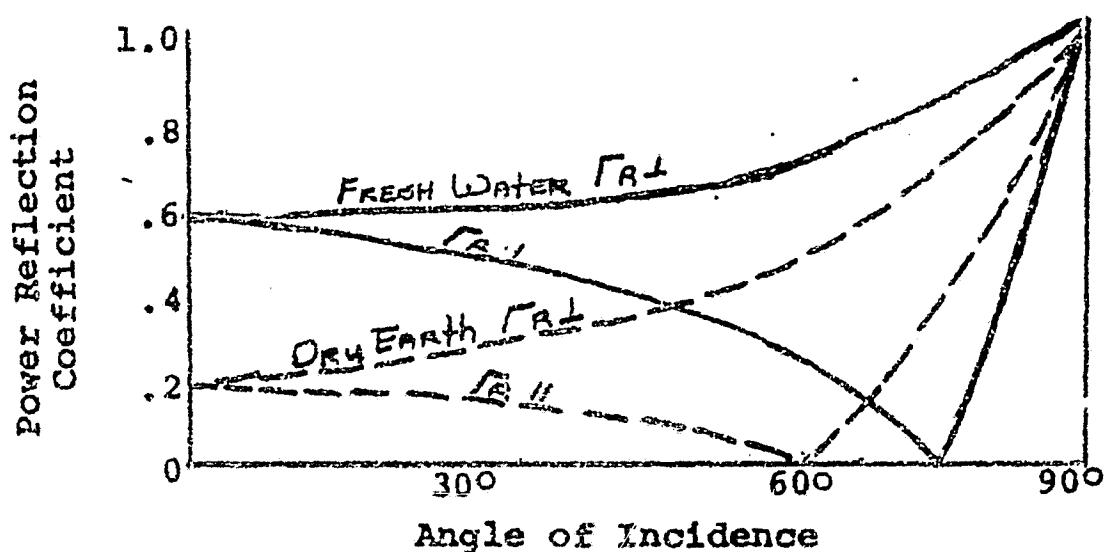
in each case. This is equivalent to saying that the antenna pattern of the elementary spherical acoustic source is different from the antenna pattern for the elementary current dipole. This situation can be remedied at least in part by choosing an acoustic source with a more appropriate antenna pattern.¹

There is, however, another discrepancy that cannot be removed by such a simple procedure. Consider a particular scattering element to consist of a small inclined plane surface. When this scattering model is used, the assumption

¹Hueter, T. F., and Bolt, R. H., Sonics, John Wiley and Sons, New York, 1955, p. 63.

is made that the surface consists of relatively plane areas that are several wavelengths in size; so that ray theory and the plane wave reflection coefficient can be used to approximate the direction, magnitude, and phase of the reflected wave. The incident magnetic field intensity can be separated into two components, one of which is in the plane of incidence on this particular scattering facet and one of which is normal to the plane of incidence. It is well known that the reflection coefficients for the two polarizations are different except for the case of normal incidence and at the grazing angle.

For the cases of fresh water and dry earth the approximate reflection coefficients for power for the two types of polarization¹ are shown in Figure A-8. Typical reflection coefficients² for sonic waves from several different boundary materials are given in Figure A-9.

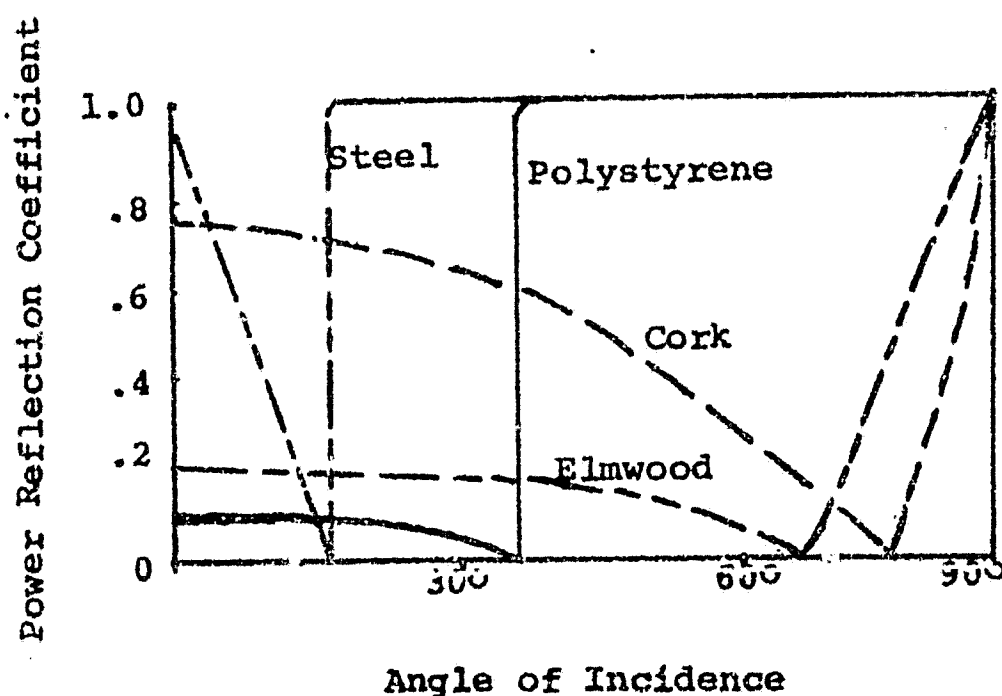


Reflection Coefficients for TM and TE Waves

Figure A-8

¹Stratton, J. A., Electromagnetic Theory, McGraw-Hill Book Company, New York, 1941, p. 510.

²These reflection coefficients are based upon an assumed lossless material.



Reflection Coefficient for Acoustic Waves

Figure A -9

The difference in reflection coefficients for the two polarizations is not too significant for many materials, until the angle of incidence exceeds 30 degrees. From the standpoint of plane-wave reflections from an infinite plane surface the error introduced by ignoring polarization is not appreciable for the case of near-vertical incidence. It is important however, to note that a scattering model based on the reflection coefficient of small plane facets is definitely limited in application. Use of this model requires that the slope of the surface change only slightly in a distance of several wavelengths. It may be desirable to use a different model to generate the scattered signal in which case the information on reflection coefficients will not be used.

A.6 A Scattering Model

In this section the acoustic-electromagnetic wave analogy is investigated for the case of rough targets where only the radiation field is considered. Radar signals are generally random in nature for this type of target; therefore, signal statistics such as the mean value and range of fading are of particular interest.

In the usual application the antenna pattern restricts radiation to a much smaller region than for the case of an elementary dipole and in many cases it is reasonable to assume that the pattern is circularly symmetric about a center line. The electric and magnetic field vectors are both at right

angles to the range vector between the antenna and the particular scattering element under consideration, and have a magnitude determined by the range and antenna gain in that direction. In the acoustic model of the situation, the particle velocity vector is in the direction of the range vector to the scattering element under consideration and has a magnitude determined by range and antenna gain in that direction. Therefore, the incident acoustic particle velocity on a scattering element always bears a definite relationship to the incident electromagnetic field vectors on the corresponding scattering element in the real system.

Consider a scattering model for the rough surface where the individual scattering elements are located in a radius manner about a mean elevation as shown in Figure A-10.

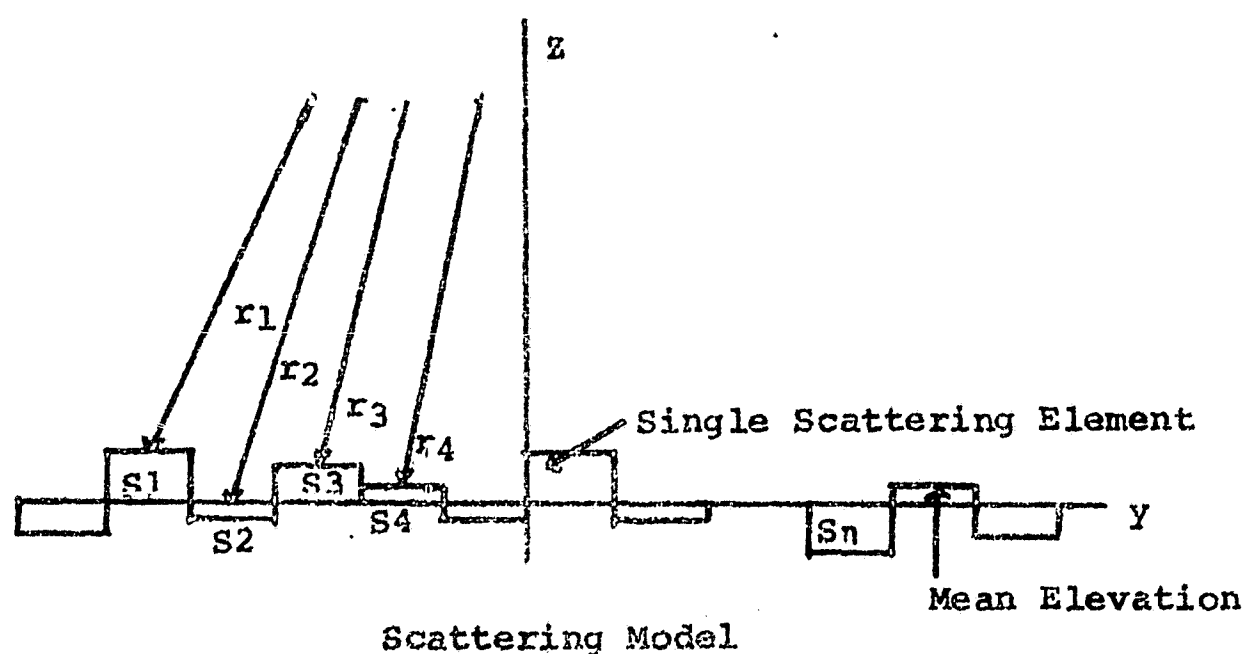


Figure A-10

In this rough surface model the range to each scattering element differs from the corresponding range to the mean elevation by some small amount, Δr_i . Each scatterer is assumed to be the same as though the scatterer were part of an infinite plane at that particular elevation. The electromagnetic wave is assumed to be polarized so that the H vector always lies parallel to the y,z plane as shown in Figure A-11. The tangential component of the H vector which is effective in exciting Huygen's current

sources in the boundary) varies with

$$\cos \theta_1 = \frac{1}{1 + \tan^2 \theta \sin^2 \phi} \quad (\text{A-48})$$

where θ_1 is measured between the incident H vector and the x,y plane.

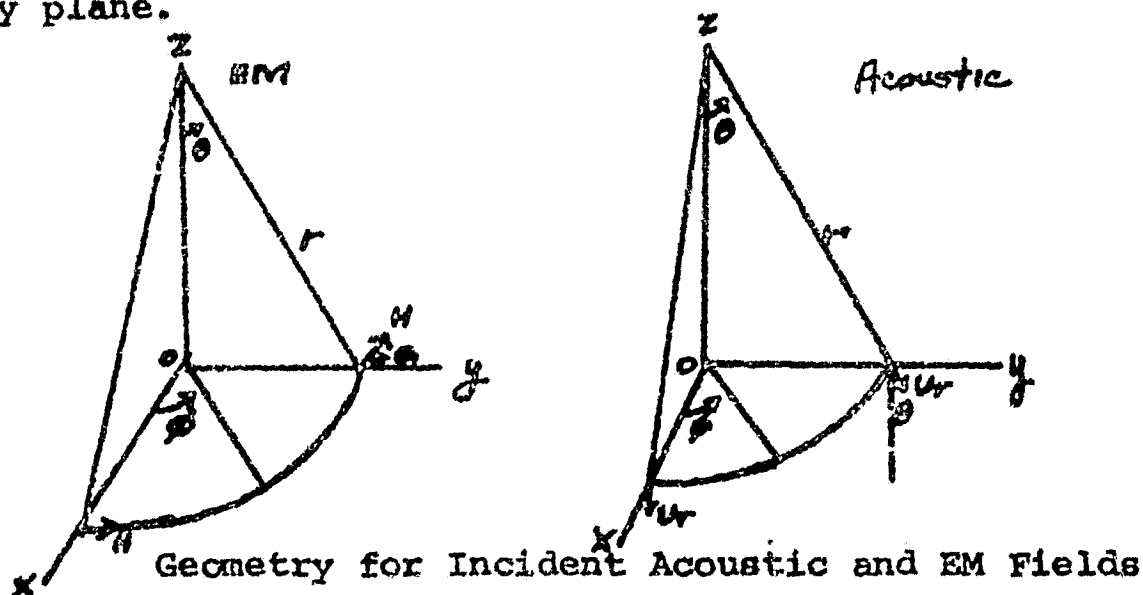


Figure A-11

The radiation from the small element for the electromagnetic wave can be considered as caused by the incremental conduction or displacement current excited in the element by the incident field. These current elements establish fields of the type described earlier in Equations A-10, A-11, and A-12. The radiation from the Huygen's current source varies with the sine of the angle, θ_2 , measured between its axis and the direction of the radiation. For signals received at the antenna this variation is given by

$$\sin \theta_2 = \sqrt{1 - \sin^2 \theta \cos^2 \phi} \quad (\text{A-49})$$

Now the magnetic field intensity received at the antenna from a single source on the plane can be expressed as

$$H = \frac{H_0}{2r} e^{-j2kr} \cos \theta_1 \sin \theta_2 \quad (\text{A-50})$$

where H_0 is the magnetic field intensity (radiation field) of the source at unit distance and α is a factor which accounts for absorption.

The geometry of Figure A-11 indicates that the normal component of particle velocity (which is effective in exciting Huygen's sources in the boundary) varies with $\cos \theta$ over the surface. The radiation from the Huygen's velocity source is isotropic (above the boundary) and

therefore does not vary with angle. Now, in a similar manner, the particle velocity at the antenna can be expressed as

$$U = \frac{2}{r} \frac{e^{-j2kr}}{2r} \propto \cos \theta \quad (\text{A-51})$$

where "2" is the particle velocity (radiation field) of the source at unit distance.

Equation A-50 can be written as

$$H = \frac{2}{r} \frac{e^{-j2kr}}{2r} \propto \cos \theta \left[\frac{(1 - \sin^2 \theta \cos^2 \phi)}{\cos^2 \theta + \sin^2 \theta \sin^2 \phi} \right]^{1/2} \quad (\text{A-52})$$

where the function outside the brackets corresponds to Equation A-51 for acoustic particle velocity and the quantity inside the brackets is unity for all values of θ and ϕ . The electromagnetic field intensity at the antenna due to a Huygen's current source is, therefore, the same as the particle velocity at the antenna due to a Huygen's velocity source for the scattering model considered. This particular rough surface model must be considered to be a "scalar model" since polarization effects are not evident.

A.7 Summary of Model Validity

It was shown that the pressure and velocity fields, about several acoustic sources, are analogous to the electric and magnetic field intensities about a short current element, when certain geometric factors are considered. In some cases the analogy applies to terms of order $1/r$ and $1/r^2$; in other cases it applies only to terms of order $1/r$.

The application of the acoustic-electromagnetic wave analogy, using the acoustic simulator, generally requires a directive antenna pattern and only the radiation fields are considered. In addition, it is necessary that the acoustic fields, scattered from appropriate model targets, have most of the characteristics of electromagnetic waves scattered from terrain or from some other object. The acoustic and electromagnetic fields are compared using a simple scattering model which assumes a distribution of Huygen's sources about a plane that is passed through the irregular surface. The acoustic and electromagnetic waves give the same results for this model, which shows that the model is scalar in nature and does not account for polarization effects. No restrictions are placed on the antenna

pattern or on the antenna orientation, except that the acoustic system must use an antenna pattern and orientation that are similar to those in the electromagnetic system.

The acoustic simulator, therefore, gives correct results to the extent that polarization effects can be neglected or can be considered by other means.

APPENDIX B

RADAR SYSTEM SCALE FACTORS

One application of the analogies between electromagnetic and acoustic waves allows a study of the scattering characteristics of a particular type of target using a model system. This is the first problem investigated with the acoustic simulator in this study. A further application involves a model of the complete radar system where circuitry such as duplexers and range gates are involved and the system performance is reproduced in detail. This type of simulation makes it possible to evaluate certain circuit design changes rapidly since full scale tests are not required.

A major characteristic of radar return from terrain is the fading due to interference of many signals of nearly the same frequency and amplitude.¹ At all but the lowest carrier frequencies the terrain acts principally as a scatterer of energy and the returned signal is therefore composed of a multitude of small signal components from the various scattering elements. The resulting fading in the received signal can be readily modeled by acoustic waves since the interference effects are common to all types of linear wave motion.

B.1 Pulsed Radar

The quantities which must be scaled when modeling a pulsed radar system are listed in Table B-1.² These quantities are based on the fundamental time and distance scales which are related by the basic equations of mechanics. Such factors as the radar cross-section per unit area for the

¹There has been some discussion of the importance of the interference phenomenon in explaining the angle dependence of the backscattering radar cross section of a target, especially over sea surfaces. Katz, I. and Spetner, L. M., "Polarization and Depression Angle Dependence of Radar Terrain Return," Jour. of Res. of N. B. S., Vol. 64D, No. 5, Sept. - Oct. 1960, pp. 483-86. While the angular dependence of the scattering pattern may not be classed as an interference phenomenon, the fading which occurs certainly is due to interference and the fading characteristics comprise an important part of the model study.

²Moore, R. K., Radar Design Using Acoustical Simulation as a Tool, Univ. of New Mexico, Engr. Exp. Sta. Rpt. EE-22, April, 1959. The scale factors for pulse and frequency modulated radar were first presented in this report.

target and antenna pattern are not scaled since their directional characteristics must be the same in the real and model systems. The scale factors represent the ratio of the indicated quantity in the real system in air to the same quantity in the model system in water.

The scale factor, V , is given by

$$V = \frac{V_a}{V_w} \approx 2 \times 10^5 \quad (B-1)$$

and is fixed by the ratio of the velocity of propagation of electromagnetic waves in air to that of acoustic waves in water. For most purposes, it is sufficiently accurate to use a value of 2×10^5 for this ratio of velocities. The range scale factor

$$R = \frac{R_a}{R_w} \quad (B-2)$$

and the time scale factor

$$J = \frac{T_a}{T_w} \quad (B-3)$$

are related to the velocity scale by the basic equation

$$R = VJ \quad (B-4)$$

TABLE B-1
SCALED PARAMETERS FOR PULSE RADAR

<u>Parameters</u>	<u>Scale Factor (Ratios)</u>
Velocity of propagation	V
Range	R
Time	J
Carrier Frequency	f_c
Wavelength	λ
Pulse Duration	d
Number of Cycles per Pulse	n
Repetition Rate	f_r
Antenna Velocity	u
Doppler Frequency	f_o

If the range scale is to be adjustable, then the model system cannot operate in real time. There is a definite advantage to scaling both time and range since a realistic range scale generally results in a time scale such that pulse widths are wider and repetition rates are lower in the model than in the real system. This serves to simplify the model circuitry problem.

The range scale, R , is selected first, based on the physical size limitation of the acoustic tank. This in turn determines the time scale, T , according to Equation B-4. The wavelength of the carrier frequency is scaled using the range scale, if linear modeling is employed. The carrier frequency scale factor, given by

$$f_c = \frac{f_a}{R} \quad (B-5)$$

is related to the basic time scale by

$$f_c = \frac{1}{T} \quad (B-6)$$

for linear modeling. In some cases, especially when the range is so large, this form of modeling becomes impractical since an unreasonably high carrier frequency is required in the model. The alternative in this situation is to employ non-linear modeling which scales the wavelength separately from the range. This is equivalent to making f_c a separate independent scale factor by definition, thus Equation B-6 will be considered as determining the frequency scale factor f_c only for the special case of linear modeling. In general the factor f_c will be independently chosen to best satisfy the modeling problem.

The wavelength scale is given by

$$\lambda = \frac{\lambda_a}{R} \quad (B-7)$$

which can be written as

$$\lambda = \frac{v_a^2 f_a^2}{v^2 f_c^2} = \frac{v}{f_c} \quad (B-8)$$

The pulse duration must be scaled so the pulse length in space is the same percentage of the total range in both the real and the model system. This criterion which is a

geometric similitude requirement is satisfied when the pulse width is scaled the same as time so the scale factor is written as

$$d = \frac{d^a}{d^w} = J \quad (B-9)$$

The number of cycles of carrier frequency per pulse is of importance since in some cases the model may be poor because of too few carrier cycles per pulse to properly define the pulse shape. The scale factor for the number of cycles per pulse is

$$n = \frac{n^a}{n^w} \quad (B-10)$$

which can be written as

$$n = \frac{d a f_c^a}{d^w f_c^w} = f_c \quad (B-11)$$

The repetition rate must be scaled so the distance in space between consecutive pulses is the same percentage of the total range in both the real and model systems. This insures that the signal standing wave in space is sampled at the proper interval. The condition is satisfied when the repetition frequency is scaled as

$$f_R = \frac{f_R^a}{f_R^w} = \frac{1}{J} \quad (B-12)$$

The scale factor for antenna velocity must be such that the antenna moves the same distance measured in wavelengths in both the real and model system in one unit of time. This requirement establishes the proper fading rate in the model. The antenna velocity scale factor can therefore be written as

$$\underline{v} = \frac{v^a}{v^w} = \frac{\lambda^a / t^a}{\lambda^w / t^w} \quad (B-13)$$

which becomes

$$\underline{v} = \frac{\lambda}{J} = \frac{v}{J} f_c \quad (B-14)$$

The scale factor for the Doppler frequency is given by

$$f_D = \frac{f_D^a}{f_D^w} \quad (B-15)$$

and since the Doppler frequency is proportional to the product of the carrier frequency and the antenna velocity divided by the velocity of propagation, Equation B-15 can be written as

$$f_D = \frac{\frac{V_a}{V} f_c}{\frac{V}{V} f_c} = \frac{1}{J} \quad (B-16)$$

The complete set of scaling relations for a pulse radar system is compiled in Table B-2.

TABLE B-2
SCALING RELATIONS FOR A PULSE RADAR SYSTEM

Parameter	Scale Factor	Relation to Independent Scale Factors
Velocity of Propagation	V	V
Range	R	VJ
Time	J	R/V
Carrier Frequency	f_c	f_c
Wavelength	λ	V/f_c
Pulse Duration	d	J
Number of Cycles per Pulse	n	Jf _v
Repetition rate	f _R	1/J
Antenna Velocity	$\frac{V}{J}$	V/Jf _c
Doppler Frequency	f _D	1/J

As an example of the application of these scaling factors, consider a pulse radar system with the parameters given in Table B-3. A model carrier frequency of 1 mc/sec and a model range of 1 meter are now selected to use in the acoustic system. The other scaled parameters in the model result from an application of the scaling relations shown in Table B-2.

In this example, the model carrier frequency is selected in a range convenient for the acoustic simulator. From Equation B-12 and Table B-2 it can be seen that an increase in the model carrier frequency will reduce the carrier frequency scale factor f_c , which will in turn increase the number of cycles per pulse and will reduce the antenna velocity in the model. Parameters such as the pulse duration, repetition rate and Doppler frequency will remain unaffected by this change. An increase in the range in the model will reduce the range scale factor, R , which will in turn reduce the time scale factor. This change will increase the pulse width, increase the number of cycles per pulse, reduce the repetition rate, reduce the antenna velocity and reduce the Doppler frequency. These changes are generally all desirable so it is advantageous to use the maximum permissible range in the model for many of the typical modeling problems.

TABLE B-3
EXAMPLE OF PULSE RADAR SCALING

Parameter	Radar (Parameters Given)	Model
Carrier Frequency	10^4 mc/s	1.0 mc/s
Wavelength	3 cm	0.15 cm
Range	2000 m	1 m
Pulse width	0.1 μ s	10 μ s
Cycles per Pulse	10^3	10
Repetition rate	3000 pps	30 pps
Antenna velocity	300 m/s	15 cm/s
Doppler frequency	20 kc/s	20 cps

If linear modeling is used in the previous example, the following situation exists. The range must still be selected to suit the size of the acoustic tank. Therefore the range will remain 1 meter in water. This results in a range scale of

$$R = \frac{R_a}{R_w} = \frac{2000}{1}$$

(B-17)

The time scale is determined from

$$J = \frac{R}{V} = 0.02 \quad (B-18)$$

and the frequency scale factor is

$$f_c = \frac{1}{J} = 100 \quad (B-19)$$

In this case linear modeling will require a model frequency of

$$f^w = \frac{f^a}{f_c} = \frac{10^{10}}{10^2} = 10^8 \text{ cps} \quad (B-20)$$

which is unreasonably high for several reasons. In order to improve the situation and retain linear modeling it is required that f_c be larger, perhaps as large as 10^4 . This results in a model carrier frequency of 1 mc/s which is very reasonable. The time scale now becomes 10^{-4} which gives a range scale of

$$R = VJ = 20 \quad (B-21)$$

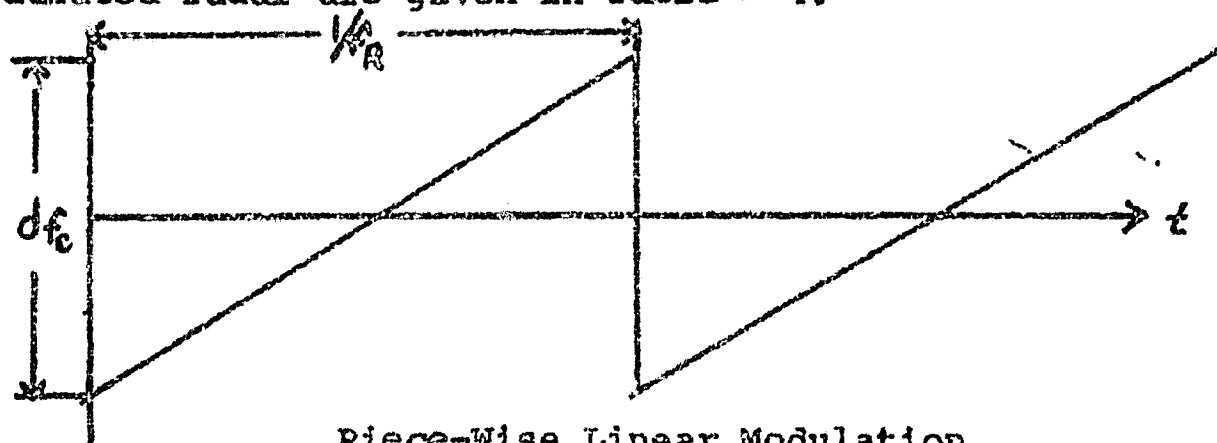
This would require a scaled range in the tank of 100 meters. The need for non-linear modeling for certain problems should now be evident.

B.2 FM Radar System

In frequency modulated radar systems the carrier frequency is swept in some manner over a portion of the frequency spectrum. The echo signals received from the target will occur at a frequency that is different from the instantaneous carrier frequency by an amount proportional to the time delay involved in making the round trip to the target (linear FM). The resulting difference frequency becomes the signal and is a measure of the range to the target. When the target is terrain, as is the case for a typical altimeter application, the signal consists of a multitude of separate signals received from the many scatterers in the illuminated region. Since the slant ranges to the different scatterers will vary, the return signal will consist of a spectrum of difference frequencies. Also if the altimeter is in motion there is a Doppler frequency shift added to each of the components of the return signal. Salient features of the signal can be scaled and modeled in order to study effects on the system operation.

A simple piece-wise linear frequency modulation, such as that shown in Figure B-1, is considered, although other types are frequently used.

Parameters which must be scaled in modeling a frequency modulated radar are given in Table B-4.



Piece-Wise Linear Modulation

Figure B-1

TABLE B-4

SCALED PARAMETERS FOR FM RADAR

Parameter	Scale Factor
Velocity of Propagation	V
Range	R
Time	T
Carrier Frequency	f_c
Wavelength	λ
Total Carrier Frequency Deviation	d
Repetition Rate	f_R
Difference Frequency	f_s
Number of Cycles of Difference Frequency per sweep	n
Antenna Velocity	V_a
Doppler Frequency	f_D

Here, as in the case of pulse radar, the scale factor V is fixed by the ratio of velocities of propagation of electromagnetic waves in air and acoustic waves in water. Also, the range scale is selected to accommodate the experiment in the available acoustic tank. The time scale, J , is therefore determined by the basic relation

$$J = R/V \quad (B-22)$$

In certain cases it may be feasible to use linear modeling, in which case the carrier frequency in the model is determined by the time scale J so that

$$f_c = \frac{1}{J} \quad (B-23)$$

The general problem, however, will require nonlinear modeling as in the pulse radar case so f_c will be determined by the ratio

$$f_c = \frac{f_c^a}{f_c^w} \quad (B-24)$$

where f_c^w is a convenient carrier frequency to use in the model. The wavelength scale is also given by the same expression as in the pulse radar case and is

$$\lambda = \frac{\lambda^a}{\lambda^w} = \frac{V}{f_c} \quad (B-25)$$

The total carrier frequency deviation, d , has dimensions of $[T]^{-1}$ and is therefore scaled as

$$d = \frac{d^a}{d^w} = \frac{1}{J} \quad (B-26)$$

The repetition rate should scale such that one cycle of the modulating frequency occupies a distance in space that is the same percentage of the range as in the real system. This requires that

$$f_R = \frac{f_R^a}{f_R^w} = \frac{1}{J} \quad (B-27)$$

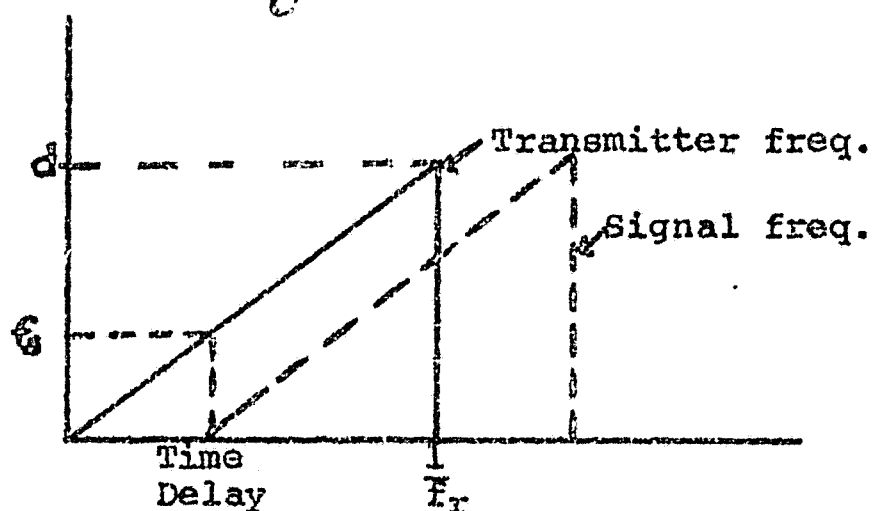
which is in agreement with the dimensions of the corresponding physical equation.

For the case of piece-wise linear modulation the difference frequency can be written as

$$f_s = \frac{f_s^a}{f_s^w} = tdf_R \quad (B-28)$$

as illustrated by the similar triangles in Figure B-2. The scale factor for the difference frequency can therefore be written as

$$f_s = tdf_R = \frac{1}{c} \quad (B-29)$$



FM Difference Frequency

Figure B-2

It is important in simulating a frequency modulated radar to maintain the number of cycles of the difference frequency per cycle of the modulation frequency, N , the same in the real and model systems. This requirement is satisfied when

$$N = \frac{f_s}{f_R} \quad (B-30)$$

in both the real and model systems since this relationship results in a scale factor

$$n = \frac{N_a}{N^w} = \frac{f_s^a / f_R^a}{f_s^w / f_R^w} = 1 \quad (B-31)$$

The antenna velocity must be scaled such that the antenna moves through a distance measured in wavelengths that is equivalent in one unit of time in both the real and model systems. The antenna velocity scale factor can be written as

$$\underline{v} = \frac{v^a}{v^w} = \frac{\lambda^a/t^a}{\lambda^w/t^w} \quad (\text{B-32})$$

which reduces to

$$\underline{v} = \frac{v}{vf_c} \quad (\text{B-33})$$

The Doppler frequency shift will change the difference frequency and will thus affect the range indication of the radar. It is important that the ratio of difference frequency to Doppler frequency be the same in the real and model systems in order to obtain correct results. The Doppler frequency is proportional to the product of the antenna velocity and the carrier frequency divided by the phase velocity and can be expressed as

$$f_D \approx \frac{vf_c}{\underline{v}} \quad (\text{B-34})$$

The corresponding scale factor can therefore be written as

$$f_D = \frac{\underline{v}f_c}{v} \quad (\text{B-35})$$

which is in terms of the antenna velocity scale factor. Now substituting the scale factor for antenna velocity into this relation the result is

$$f_D = \frac{1}{j} \quad (\text{B-36})$$

This result satisfies the requirement that the ratio of difference frequency to Doppler frequency remain constant in the real and model systems since $f_s/f_D = 1$. These results are now tabulated in Table B-5.

B.3 Doppler Frequency Radar

A Doppler frequency radar can be used to detect relative motion along a line joining the radar and the target. There are a number of variations of this type of radar; however,

only one particular kind is considered here. The transmitter operates continuously and transmits a signal toward the target. An echo signal is at the transmitter frequency if there is no relative velocity along the line joining the transmitter and receiver. When relative velocity exists, the echo is shifted in frequency by some amount due to the Doppler effect. When the echo is mixed with the transmitter frequency the beat difference frequency is directly proportional to the relative velocity between the radar and the target along the line which joins them.

TABLE B-5
SCALING RELATIONS FOR AN FM RADAR SYSTEM

Parameter	Scale Factor	Relation to Independent Scale Factors
Velocity of Propagation	V	V
Range	R	V
Time	J	R/V
Carrier Frequency	f_c	f_c
Wavelength	λ	V/f_c
Total Carrier Frequency Deviation	d	$1/J$
Repetition Rate	f_R	$1/J$
Difference Frequency	f_s	$1/J$
Number of Cycles of Difference Frequency Per Sweep	n	1
Antenna Velocity	u	V/Jf_c
Doppler Frequency	f_D	$1/J$

When the target is some irregular object such as an aircraft, the signal will fade due to echoes received from different scattering elements which are at slightly different ranges or where velocity components are slightly different. The spectrum of the returned signal gives rise to some ambiguity in the measured velocity. Model studies can show the statistics of the fluctuations of indicated velocity and provide a realistic method of evaluating indicators and read-out devices under controlled conditions.

The parameters to be scaled in modeling this radar are the same as for pulsed radar with a few omissions. It is assumed in this case that the radar is fixed and the target is in motion.

TABLE B -6
SCALED PARAMETERS FOR A DOPPLER RADAR

Parameter	Scale Factor (Ratios)
Velocity of Propagation	V
Range	R
Time	
Carrier Frequency	f_c
Wavelength	λ
Target Velocity	U
Doppler Frequency	f_D

The velocity scale factor is again fixed by the properties of the media in which the two wave phenomena propagate. Also, the range scale is selected to suit the physical dimensions of the model facility. The basic relations which are repeated for convenience are

$$V = 2 \times 10^5 \quad (B-37)$$

and

$$J = R/V \quad (B-38)$$

As in prior cases the carrier frequency can be scaled as $1/J$ if linear modeling is to be employed, or it can be scaled independently if the linear model results in an unrealistic value for the model carrier frequency. In either case the wavelength scale factor is

$$\lambda = V/f_c \quad (B-39)$$

which reduces to $\lambda = R$ for linear modeling.

The target velocity must scale in such a manner that the target moves through the distance equivalent to one wavelength in a corresponding amount of time in both the real and the model system. The scale factor for velocity can therefore be written as

$$\frac{L}{L'} = \frac{V}{V'} \quad (B-40)$$

which is the same as for the case of pulse radar. The Doppler frequency also scales the same as in the previous model so the scale factor is

$$f_D = \frac{1}{J} \quad (B-41)$$

The complete set of scaling relations for this Doppler radar model are compiled in Table B-7.

B.4 Forward Scatter Model

Communication links using forward scattering from inhomogeneities in the atmosphere are currently in use. Scattering takes place from the random distribution of "blobs" where the index of refraction is slightly different from that in the surrounding medium. The scattering tends to be more directive in the forward direction as the blob

TABLE B-7
SCALING RELATIONS FOR A DOPPLER RADAR SYSTEM

Parameter	Scale Factor	Scale Factor in Terms of Independent Scale Factors
Velocity of Propagation	V	V
Range	R	VJ
Time	J	R/V
Carrier Frequency	f_c	f_c
Wavelength	λ	V/Jf_c
Target Velocity	V'	V/Jf_c
Doppler Frequency	f_D	$1/J$

size increases with respect to the wavelength. A great deal of theoretical and experimental work has been directed in this area.^{1,2,3} The major interest in this area concerns the propagation of electromagnetic waves in an inhomogeneous distribution of heat sources on the bottom of the tank. The rising columns of heated water will provide an inhomogeneous medium with many of the characteristics found in the atmosphere. Stratified layers can easily be obtained by using thin mylar film in the tank to separate the different temperature or turbulent conditions. The thin film is essentially transparent to the sonic waves.

¹Wheeler, A. D., "Radio-Wave Scattering by Tropospheric Irregularities," *Jour. of Res. of N.B.S.*, Vol. 63D, No. 2, Sept. - Oct. 1959, pp. 205-33. This is an excellent review article on the subject of tropospheric scattering and contains an additional 81 references.

²Chernov, L. A., "Wave Propagation in a Turbulent Medium," McGraw-Hill Book Co., New York, 1960. (Russian Translation).

³Tatarski, V. I., "Wave Propagation in a Turbulent Medium," McGraw-Hill Book Co., New York, 1961. (Russian Translation).

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