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CHARRING ABLATION PERFORMANCE IN TURBULENT FLOW

Volume I - Analytical and Experimental Studies

D2-114031-1

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Prepared by

R. S. Gaudette, E. P. del Casal, P. A. Crowder

THE BOEING COMPANY
Space Division
Seattle, Washington

September 1967

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Manned Spacecraft Center
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PREFACE

This report documents work completed for the National Aeronautics and Space Administration under contract NAS9-6288, "Charring Ablation Performance in Turbulent Flow," issued through the Manned Spacecraft Center, Houston, Texas 77058.

NASA technical monitor was Mr. D. M. Curry of the Thermal Technology Branch of the Structures and Mechanics Division. The Boeing Company program manager was Mr. V. Deriugin, Head of Structural Heating of Spacecraft Mechanics and Materials Technology of the Space Division.

The authors express their appreciation to Mr. Deriugin for his original conception of the program, helpful suggestions and imparted vision; to R. L. Stevens for his able manipulation of the arc plasma test facility; to R. L. Colony for his contributions in numerical analysis and computer programming; and to B. E. Nelson for contributions to the problem description.

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CHARRING ABLATION PERFORMANCE

IN TURBULENT FLOW

by

R. S. Gaudette, E. P. del Casal, and P. A. Crowder

SUMMARY

The ablation performance of Avcoat 5026-39HC/G in turbulent boundary layer flow is investigated analytically and experimentally. An integral method is used to determine the heat, mass, and momentum transfer in the equilibrium boundary layer with mass injection for the laminar through fully turbulent flow regimes. Phenomena occurring on the surface of or within the ablating material are correlated when possible on the basis of analytical models developed, as well as experimental data generated under this contract and available elsewhere. Solutions of these two analyses are coupled through the surface temperature and mass injection rate. Overall ablation performance can then be described for various environmental conditions. Specifically, the Apollo heat shield performance is predicted at selected body locations for two trajectories.

The results obtained in this program represent an advance in the state-of-the-art of predicting the ablation performance of complex epoxy novolac and silica-based ablation materials and a first in obtaining turbulent flow conditions and ablation data in a plasma jet on a flat plate at enthalpies largely in excess of 2000 Btu/lb.

The results show that ablation performance can be predicted with reasonable confidence using semi-empirical correlation techniques verified by experimental data; however, extrapolation to regimes significantly beyond those verified and to other materials will require additional experimental data.

Two computer programs were developed in the course of the present study. One predicts the heating rate to the ablating surface and the other the ablation performance (surface recession, char layer thickness, etc.) of the ablation material investigated (Avocat 5026 - 39HC/G) in laminar and turbulent flow. The description of the analytical and experimental work performed is given in Volume I, whereas a detailed description and write-up of the computer programs developed are given in Volume II of this report.

INTRODUCTION

In predicting ablation performance of the Apollo heat shield several problem areas have required further study. Among the several factors to be

accounted for have been: prediction of heat transfer rates in all flow regimes with boundary layer injection; introduction of a transition criterion in boundary layer analysis; prediction of surface recession in turbulent flow; and the development of experimental techniques to obtain ground test data to verify predictions for ablation performance in a turbulent boundary layer.

Studies performed under contract NAS 9-6288 related to these problems are presented herein.

SYMBOLS

A	stoichiometric factor defined in eq. (B11)
A_c	cross-section area
A_i	chemical representation of species i (reactant); overall diffusivity for species i
A_k	Arrhenius factor for gas combustion
a	parameter in eq. (3); constant on page 22
a_o	parameter in eq. (21) for smooth incompressible flow; speed of sound
B	activation energy
B_i	chemical representation of species i (product)
B'	blowing parameter based on Stanton number, $\dot{m}_w/H$
b	parameter in eq. (3)
b_o	parameter in eq. (22)
C_f	local skin friction coefficient, $2\tau_w/\rho_e u_e^2$
C_j	mass flux fraction of combustible species "j" undergoing instantaneous combustion
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
c	constant in eq. (21)
D_i	diffusivity of species "i" into mixture
D_{12}	binary diffusion coefficient
d	constant in eq. (21)

F	ratio of wall mass flux to free stream mass flow, $\rho_w v_w / \rho_e u_e$
f	fraction of pyrolysis gas combusting
g	gravitational constant
h	heat transfer coefficient based on enthalpy, $\rho_e u_e St$
H	total enthalpy, $h + (u^2/2gJ)$
h, Δh	specific enthalpy
I_A	arc current
J	Joules' constant
K	elemental mass fraction
K_s	roughness height
$K_{1,2,...}$	empirical constants in eqs. (26), (27), and (34)
k	thermal conductivity; universal constant, l/δ
Le	Lewis number
l	mixing length
M	molecular weight
M_e	Mach number at the boundary layer edge
M	mass flow rate
$\dot{m}$	mass flux, ρv
m	constant on page 24
N	function in eq. (15)
n	constant on page 24
P	total pressure
Pr	Prandtl number
Pr*	Prandtl number based on ratio of total eddy diffusivities of momentum and energy
p_d	duct static pressure (end of closed duct)
p_m	altitude chamber static pressure

p	static pressure
$\dot{Q}_A, \dot{Q}_W$	energy input to arc heater and heat loss to cooling water
q	constant in eq. (22)
$\dot{q}, \dot{q}_0$	heat flux at the surface
Re	Reynolds number
r	constant in eq. (22)
r_0	radius of revolution of body
S	shape factor for velocity
Sc	Schmidt number
S_H	thermal shape factor
S_m	displacement shape factor
St	Stanton number
$\dot{s}$	surface recession rate, $\dot{m}/\rho_c$
T	static temperature
u	local velocity component in x-direction
u	scaled local velocity, u/u_e
V_A	arc voltage
v	local velocity component in y-direction
W	water mass flow rate
w_i	mass fraction of species i
w_i	sink or source term in species conservation equation
x	coordinate along surface
y	coordinate normal to ablator surface within the boundary layer
y^+	boundary layer shear distance, $\sqrt{\tau_w/\rho_e} \delta/\mu_e$
y_m^+	boundary layer shear thickness
Z	compressibility factor
z	distance normal to ablator surface within solid, $z = -y$

α	dimensionless mass transfer parameter, $2F/C_F$
α_w	$\dot{m}_c/\dot{m}_p$
α_i	stoichiometric coefficient of species i (reactant)
β	velocity gradient
β_i	stoichiometric coefficient of species i (product)
β_j	stoichiometric mass ratio
δ	ratio of specific heats, C_P/C_V
δ	boundary layer thickness
ϵ	emissivity; exponent in eq. (5)
ϵ_M	eddy diffusivity of momentum
ξ	ratio of $ v' $ to $ u' $
ξ	ratio of Stanton number to skin friction
η	scaled distance from surface, y/δ
Λ	pyrolysis gas combustion parameter, defined in eq. (B12)
λ	reduced radius, r/R
μ	molecular viscosity
ν	kinematic viscosity
ρ	density
σ	Stefan-Boltzmann constant
τ	shear stress
$\phi_{1,2,\dots}$	"function of," eqs. (26), (27), (31), (32), and (33)
ϕ	dimensionless ratio, e.g., ϕ_μ
ψ	blocking function
ω	ratio of mixing length to distance from wall, l/y

Subscripts:

b bulk condition

c	char condition; solid combustible species (in Appendix B)
D	condition based on displacement thickness
e	condition at boundary layer edge
g	gas
H	conditions based on total enthalpy
i	species "i" in chemical reactions of the type $\sum_i \alpha_i A_i = \sum_i \beta_i B_i$
j	species "j"
M	conditions based on momentum thickness
m	conditions based on displacement thickness
max	maximum
min	minimum
o	oxidant
p	pyrolysis gases
pyr	pyrolysis reaction
r	pyrolysis gas species reacting with oxidant
R	condition based on hydraulic radius
rad	refers to radiation to ablator surface
s	stagnation condition
sub	refers to sublimation
surf	refers to surface
T	conditions based on static temperature
vp	virgin plastic
W	water flow
X	conditions at local value of x
δ	conditions referred to boundary layer thickness
μ	viscosity

ρ	density
σ	smooth wall
0	condition at no blowing
0,w	wall conditions
∞	free stream or undisturbed condition
-	approaching from negative direction
+	approaching from positive direction

Superscripts:

(*)	total quantity (molecular plus turbulent)
',"	dummy variable indicators in integrations
'	fluctuating variable (with time)
—	average quantity

Other:

[]	concentration of (in terms of partial pressure)
O()	order of magnitude of
(') _g () _s	gas, solid states, respectively (for chemical species)

GENERAL APPROACH TO ANALYSIS

The overall problem in predicting ablation performance has been divided into two analysis regimes: 1) description of the gas phase heating environment; and 2) description of the condensed phase (ablator) phenomena. Conditions at the ablator-gas phase interface serve to couple the two analyses in order to obtain a solution of the whole problem. When possible, the analyses have been compared with experimental data in order to deduce necessary empirical forms and constants. Appendix D presents a summary description of the approach taken in programming the solutions to the two problems. A detailed description of the computer program developed is given in Volume II of this report.

Boundary Layer Analysis

The most successful semi-empirical models for turbulent shear flow are the mixing length models typified by those of Prandtl, von Karman, and Taylor. Although each is based on a different hypothesis, their degree of success in correlating experimental data and predicting turbulent flow behaviour has not been significantly different. The merits and weaknesses of the different mixing length models are discussed in many standard reference texts dealing with turbulent flow; one can, for example, refer to Hinze (ref. 1), Pai (ref. 2) Schlichting (ref. 3), and others.

Using Prandtl's expression for the mixing length, calculations were made (refs. 4 and 5) to determine the effect of mass injection on heat transfer in turbulent boundary layer flows. In these analyses, it was assumed that the turbulent boundary layer can be divided into two regions, namely, a laminar sublayer and a turbulent core. In addition, it was assumed that the shear stress across the boundary layer is constant and that the flow is locally similar. The theory predicts a reduction in skin friction and wall heat transfer in the presence of mass transfer that compares favorably with experiment.

To account for shear stress variation across the boundary layer, van Driest (ref. 6) solved the heat transfer problem by assuming a linear shear distribution. Only the case of an impermeable wall was considered. Using available data from the literature, Spalding, et al. (ref. 7) were able to develop satisfactory correlations taking into account mass transfer and compressibility effects on turbulent boundary layer heat transfer.

Recently, Kendall, et al. (ref. 8) analyzed the incompressible boundary layer using a continuous expression for the mixing length. The boundary layer was divided into two regions, the law of the wall region and the wake region. In the law of the wall region, the flow was assumed to be locally similar, whereas in the wake region, two-dimensional effects were included. The two regions were matched at their interface so that the velocity and boundary layer growth were continuous. Their formulation may possibly be modified to take into account boundary layer transition and compressible flow effects.

Recently, del Casal and Koh (ref. 9) treated the incompressible turbulent boundary layer flow problem by using a continuous expression for the mixing length based on Prandtl's hypothesis and by assuming a linear stress distribution, thus by-passing the procedure of matching several regions within the boundary layer. The results obtained in this analysis agree very favorably with experimental data on incompressible turbulent boundary layer flow. In addition, they suggest a method of semi-empirically treating the whole boundary layer (i.e., the laminar, transition, and fully turbulent regimes) using a single expression for eddy viscosity. In view of the very favorable results obtained for incompressible flow and the relative simplicity of the method, it was modified for the present program to include the effects of mass transfer, compressibility, pressure gradients, and variable physical properties.

The expression for the mixing length used here is

$$l = y \left\{ k - [k - \omega(0)] e^{-\frac{\eta}{\eta_1}} \right\} \quad (1)$$

where

$$\omega(0) = \lim_{y \rightarrow 0} \left(\frac{l}{y} \right) \quad (2)$$

and

$$\frac{1}{\eta_1} = \phi = \frac{y_m^+ - a}{b} \quad (3)$$

where η_1 is the normalized laminar sub-layer thickness.

The parameters a and b determine respectively the initiation of transition and the extent of the transition region. The eddy diffusivity of momentum thus becomes

$$\epsilon_m = \nu k^2 y^{+2} \left\{ 1 - \left[1 - \frac{\omega(0)}{k} \right] e^{-\phi \eta} \right\}^2 \frac{du^+}{dy^+} \quad (4)$$

The above expression was originally used by Gill and Scher (ref. 10), without the term containing $\omega(0)$, who obtained excellent correlations for the velocity profiles in fully developed flow through smooth pipes and channels. Lee and Gill (ref. 11) applied the same expression to consider heat and mass transfer for both constant and variable property turbulent flow in smooth channels.

The novelty of the expression for the eddy viscosity, ϵ_M , used by Gill and Scher (ref. 10) rests in its ability to predict with sufficient accuracy the flow behavior in the turbulent as well as transition regimes for bounded flow. (In the present formulation, allowance can be made for rough walls, $\omega(0) \neq 0$; this, however, will not be considered here.) This suggests an immediate applicability to external flows. Since the viscous damping factor,

$\left\{ 1 - \left[1 - \frac{\omega(0)}{k} \right] e^{-\phi \eta} \right\}^2$ is a function of boundary layer thickness, wall mass

transfer, and external flow conditions, and since this factor determines the flow behaviour in the laminar sublayer as well as the transition or buffer sublayer, it would not be unreasonable to assume that the same expression applies to the determination of the flow behaviour of the laminar, transition, and turbulent regions of the boundary layer. An analogy between transition, layer-wise and region-wise, is thus hypothesized. This was demonstrated by del Casal and Koh (ref. 9) when they applied the same logic to the incompressible flat plate. It is again used here.

Mathematical formulation of the problem. - The problem concerns itself with the simultaneous transfer of heat, mass, and momentum from a surface immersed in a fluid stream under conditions such that the flow adjacent to the surface (the boundary layer) is turbulent. A schematic diagram of the physical configuration is given in figure 1. The surface is assumed to consist of a decomposable material (the ablator) such that under severe thermal environments, the gross heat flux to the surface will be balanced by the absorption and conduction of energy by the decomposing material, the reduction of heat flux by the blocking effect of mass injection, and the re-radiation of energy into the external stream. The equations governing the transport processes in such a system are given by the two-dimensional boundary layer equations which may be found in any standard reference text on the subject (see, for example, Dorrance, ref. 12). Numerical solutions to the boundary layer equations together with their associated conditions, may be obtained by suitable numerical schemes if expressions for the eddy diffusivities of momentum, energy, and mass are known. It is usual practice to assume that the turbulent Prandtl and Schmidt numbers are unity.

Instead of a rigorous numerical solution to the exact equations, an integral approach is used here. In terms of accuracy, the integral method is second only to the exact solution and in terms of labor, it is an order of magnitude less than the exact numerical scheme.

The integral boundary layer equations are:

Continuity

$$\frac{dRe_D}{dRe_x} + Re_D \left[\frac{d \ln \mu_e}{dRe_x} - (1 - S_m) \frac{d \ln r_o^e}{dRe_x} - S_m \frac{d \ln (\rho_e \mu_e)}{dRe_x} \right] = F_S - F_0 \quad (5)$$

Species Conservation

$$\frac{dRe_{Di}}{dRe_x} - Re_{Di} \left[\frac{d \ln \mu_e}{dRe_x} - (1 - S_{mi}) \frac{d \ln r_o^e}{dRe_x} - S_{mi} W_{ie} \frac{d \ln (\rho_e \mu_e W_{ie})}{dRe_x} \right] = F_{is} - F_{io} - \frac{Re_s}{Re_i} \quad (6)$$

Momentum

$$\frac{dRe_M}{dRe_x} + Re_M \left[\frac{d \ln \mu_e}{dRe_x} + \frac{d \ln r_o^e}{dRe_x} + (1 + S) \frac{d \ln \mu_e}{dRe_x} \right] = F_0 + \frac{C_f}{2} \quad (7)$$

Energy

$$\frac{dRe_T}{dRe_x} + Re_T \left[\frac{d \ln \mu_e}{dRe_x} + \frac{d \ln r_o^e}{dRe_x} + (1 + S_H) \frac{d \ln H_e}{dRe_x} \right] = \dot{q}_0 + F \left(1 - \frac{H_w}{H_e} \right) \quad (8)$$

Discontinuities in the enthalpy and species profiles such as would occur if combustion flame fronts occur in the boundary layer offer no particular difficulties. These will not be discussed here. A combustion model based on

the concept of reaction fronts is given in Appendix A.

Method of solution. - To obtain sufficiently accurate information on the transfer coefficients, i.e., the heat, mass, and momentum transfer coefficients (skin friction coefficient), approximate profiles for the velocity, enthalpy, and species concentrations must first be generated and then used in the integral equations. For turbulent boundary layer flows, the assumption of local similarity (which in this case means that the velocity, enthalpy, and species concentration profiles are essentially solely dependent on the scaled distance from the wall, y/δ) is very close to reality. Thus, the velocity, enthalpy, and species concentration distributions may be obtained from the asymptotic boundary layer equations where all derivatives with respect to the coordinate along the surface are omitted:

Continuity

$$\frac{d(\rho v)}{dy} = 0 \quad (9)$$

Species Conservation

$$\frac{d}{dy} \left\{ (\rho v) w_i - \rho D_i^* \frac{dw_i}{dy} \right\} = \rho w_i \quad (10)$$

Momentum

$$\tau - \tau_w = (\rho v)_w u + \int_0^y \left[(\rho v) \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \int_0^{y'} \frac{\partial}{\partial x} (\rho u) dy'' \right] dy' + \frac{\partial p}{\partial y} y \quad (11)$$

Energy

$$\frac{d}{dy} \left\{ \left(\sum \left[(\rho v) w_i - \rho D_i^* \frac{dw_i}{dy} \right] \left[h_i - \frac{u^2}{2gJ} \right] \right) - k \frac{\partial T}{\partial y} - u\tau \right\} = 0 \quad (12)$$

The boundary conditions are:

$$y=0 : u=0, T=T_w, (\rho v) \neq 0, \text{ and } w_i = w_{i,0} \quad (13)$$

$$y=\delta : u=u_e, T=T_e, w_i = w_{i,e} \quad (14)$$

Solutions to the above equations may be obtained in closed form which upon rearrangement yield the velocity profile,

$$\tilde{u} = \frac{2L_m^+}{u_m^+} \int_0^\eta \frac{\frac{1}{\phi_\mu} \left\{ 1 - N(\eta') - \left(\alpha - \frac{2\delta}{c_f} \frac{d \ln u_e}{dx} \right) [\tilde{u} - N(\eta')] \right\} d\eta'}{1 - \sqrt{1 - \frac{4\phi_\rho}{\phi_\mu^2} \left\{ \eta'^+ \frac{u_m^+}{k} \left[1 - \left(1 - \frac{u(0)}{k} \right) e^{-\phi_\eta'} \right] \right\}^2 \left\{ 1 - N(\eta') - \left(\alpha - \frac{2\delta}{c_f} \frac{d \ln u_e}{dx} \right) [\tilde{u} - N(\eta')] \right\}}} d\eta' \quad (15)$$

where:

$$1 - N(\eta) = \frac{\tau}{\tau_w} \quad (16)$$

for incompressible flow along a flat plate; the enthalpy profile,

$$\frac{H - H_w}{H_e - H_w} = e^{\int_0^{\tilde{u}} \frac{\alpha Pr^* d\tilde{u}'}{1 + \alpha \tilde{u}'}} \int_0^{\tilde{u}} \frac{\alpha Pr^*}{1 + \alpha \tilde{u}'} \left[\frac{5 Pr^*}{1 + \alpha \tilde{u}'} \sum \frac{(\alpha_e^* - 1) h_i}{H_e - H_w} \frac{dw_i}{d\tilde{u}'} - \frac{(Pr^* - 1)}{H_e - H_w} \frac{\tilde{u}_e^2}{gJ} \tilde{u}' \right] d\tilde{u}' \quad (17)$$

where:

$$\zeta = \frac{\alpha \left[1 + e^{\int_0^1 \frac{\alpha Pr^* d\tilde{u}'}{1 + \alpha \tilde{u}'}} \int_0^1 \left[\sum \frac{(\alpha_e^* - 1) h_i}{H_e - H_w} \frac{dw_i}{d\tilde{u}'} + \frac{(Pr^* - 1)}{H_e - H_w} \frac{u_e^2}{gJ} \tilde{u}' \right] e^{-\int_0^{\tilde{u}'} \frac{\alpha Pr^* d\tilde{u}''}{1 + \alpha \tilde{u}''}} d\tilde{u}' \right]}{e^{\int_0^1 \frac{\alpha Pr^* d\tilde{u}'}{1 + \alpha \tilde{u}'}} - 1} \quad (18)$$

and the species concentration profiles,

$$\frac{w_i - w_{i,w}}{w_{i,e} - w_{i,w}} = \frac{e^{\int_0^{\eta} A_i d\eta'} - 1}{e^{\int_0^1 A_i d\eta'} - 1} \left[1 + e^{\int_0^1 A_i d\eta'} \int_0^1 \frac{A_i \int_0^{\eta'} \rho w_i \delta d\eta''}{\rho_e u_e F_o(\Delta w_i)} e^{\int_0^{\eta'} A_i d\eta''} d\eta' \right] - e^{\int_0^1 A_i d\eta'} \int_0^1 \frac{A_i \int_0^{\eta'} \rho w_i \delta d\eta''}{\rho_e u_e F_o(\Delta w_i)} e^{\int_0^{\eta'} A_i d\eta''} d\eta' \quad (19)$$

where

$$A_i = \frac{F_o Re_s Sc_i}{\rho_u (1 + \frac{\epsilon}{D_i})} \quad (20)$$

equations (10), (11), and (12) are then used in the integral equations (15), (17), and (19) to determine the heat, mass, and momentum transfer coefficients as functions of distance along the surface.

Determination of the transition region.— As previously stated, the parameters a and b in the expression for the eddy diffusivity of momentum indicate, respectively, the point along the boundary layer where transition starts and the extent of the transition region. The parameter a may be considered as the lower limit for the momentum thickness below which turbulence cannot occur. Thus, for $y_m^+ \leq a$, the flow is always laminar. On the other hand, if $y_m^+ > a$, the flow may no longer be purely laminar, in all probability it will start becoming turbulent. The extent to which it has become turbulent is determined by the ratio $(y_m^+ - a)/b$ where $(y_m^+ - a)$ may be looked upon as a "turbulence inducing potential" and b as the "viscous impedance" of the boundary layer that opposes the turbulence. To illustrate the point, if b is very small (as in the case of large mass injection), once y_m^+ becomes larger than a , the transition from laminar to fully turbulent flow is almost abrupt whereas for very large b the transition is very gradual. It must be pointed out here that for the very first time a unifying expression is being used that is applicable to the whole boundary layer spectrum, the laminar, transition, and fully turbulent regimes.

The critical transition point for boundary layer flow is known to be dependent on several parameters: the momentum thickness, free-stream-to-wall temperature ratio, unit Reynolds number, free stream Mach number, mass transfer, free stream pressure gradient, surface roughness, and free stream

turbulence (refs. 13 and 14). The parameters a and b are thus dependent on all the variables mentioned. In the present program, only a cursory attempt has been made to obtain suitable expressions for a and b to encompass the effects of the different turbulence inducing mechanisms. For incompressible flow along an impermeable and smooth flat plate, a and b are constant and respectively equal to approximately 80 and 22. To account for heat and mass transfer effects and compressibility, the following expressions are used:

$$a = a_0 \left(\frac{T_e}{T_w} \right)^c \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^d \quad (21)$$

$$b \exp \left(\frac{V_o^2}{2} \right) = b_0 \left(\frac{T_w}{T_e} \right)^q \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^r \quad (22)$$

where $a_0 = 80$, $b_0 = 22$, and c , d , q , and r are empirically determined constants. For the present program, $c = 0$, $d = \frac{1}{2}$, $q = r = \frac{1}{2}$.

Charring Ablator Analysis

With the external environment described by the boundary layer program, it remains to describe the performance of the ablative material itself. The basic approach taken is to satisfy a heat balance, with mass flux terms based on correlations developed from test data. The overall direction of this analysis is to predict the surface recession rate.

An approximate heat balance at the surface is given by:

Heat In:

$$\begin{aligned} & \boxed{\text{NET CONVECTION}} + \boxed{\text{PYROLYSIS GAS COMBUSTION}} + \boxed{\text{CHAR COMBUSTION}} + \boxed{\text{RADIATION TO SURFACE}} \\ & \psi \dot{q}_o + f \dot{m}_p \Delta H_{c,p} + \dot{m}_c \Delta H_{c,c} + \dot{q}_{\text{rad}} \end{aligned}$$

(23)

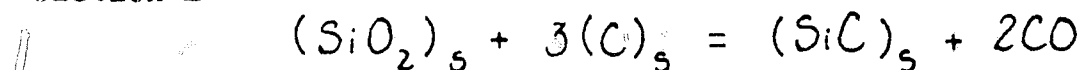
= Heat Out:

$$\begin{aligned} & \boxed{\text{RADIATION FROM SURFACE}} + \boxed{\text{HEAT STORED IN PYROLYSIS GASES}} + \boxed{\text{SENSIBLE HEAT STORED IN REMOVED SURFACE MATERIAL}} \\ & = \epsilon \sigma T_w^4 + \dot{m}_p [\Delta H_{\text{pyr}} + \bar{C}_{p,p} (T_w - T_o)] + \bar{C}_{p,c} (T_w - T_o) (\dot{m}_c + \dot{m}_{\text{sh}} + \dot{m}_{\text{sub}}) \\ & + \boxed{\text{SUBLIMATION}} \\ & + \dot{m}_{\text{sub}} \Delta H_{\text{sub}} \end{aligned}$$

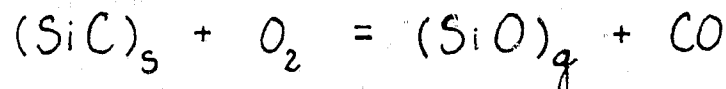
This balance neglects the contributions due to sensible heat storage and heat conduction through the ablator back wall, both of which are negligible in the test data available for analysis (using extent property values). The net hot wall convected heat $\psi \dot{q}_s$ is calculated from the boundary layer program by iteration with the heat balance, and should include roughness effects on heat transfer. For analysis of ablation test data, a preliminary estimate of ψ may be determined from figure 2 (reproduced from ref. 22), in order to deduce an estimate of roughness effects on heat transfer when these are a priori unknown. (The importance of roughness effects is demonstrated in Table X, where for the experiment conducted for this contract, it was deduced that convective heat transfer increased by as much as 291% due to surface roughness.)

The combustion terms are derived from a priori chemical models (see Appendix B). For a siliceous char former such as the Apollo material, the following system of reactions is assumed to occur simultaneously in air at temperatures above circa 3500°R:

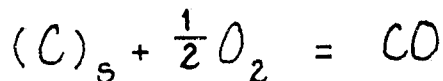
Reaction 1



Reaction 2

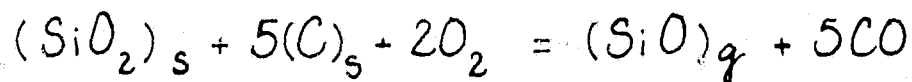


Reaction 3



On the basis of char composition, the surface combustion becomes:

Reaction 4



The Arrhenius kinetic constants for reactions 1 and 2 have not been firmly established in the literature (see refs. 16, 17, 18, and 19). At temperatures below ~3500°R, there is experimental evidence that the principal reaction is 3; a white siliceous residue remains in this case, suggesting that in-depth oxidation of carbon takes place within the char. While this reaction appears to occur under the test conditions achieved under this contract, the surface recession noted is considered to be due to shear removal of the siliceous residue, and only indirectly to reaction 3. On the basis of TGA data and char compositions, the pyrolysis gases have an assigned formula $C_2H_{4.5}O$ suggesting the presence of acetylene (C_2H_2) and water (H_2O), and/or methane (CH_4) and carbon monoxide (CO). Such components have been detected in gases produced by flash pyrolysis of an epoxy resin (ref. 20). The rate of combustion of the surface materials seems to be appreciably reduced by competing pyrolysis gas combustion. This reduction may be accounted for in a relationship of the type:

$$\dot{m}_c = \dot{m}_{c,max} - \Lambda \dot{m}_p \quad (24)$$

where $\dot{m}_{c,max}$ is a maximum uncorrected surface mass flux due to oxidation in the pertinent reaction model and regime (kinetic-, transition-, or diffusion-controlled), and Λ is a correction factor designed to take into account the pyrolysis gas combustion, but which also appears to account for shrinkage. A correlation for Λ is presented in "Correlation of Test Data". By removing oxygen, the combustion processes also affect the rate of sublimation of SiO_2 , which thermodynamically sublimes principally to $(\text{SiO})_g + \frac{1}{2} \text{O}_2$. The shear removal term has to be correlated (e.g., "Correlation of Test Data") since the effective shear strength is unknown for a melting, multicomponent charred residue undergoing oxidation. For the purpose of reducing the complexity of iterating for ψq_0 , the mass flux of pyrolysis gases may be obtained by correlation rather than by a transient charring ablator analysis. It is also hoped that, in this way, accounting in detail will be avoided for phenomena such as char densification, sintering, intumescence, unknown in situ char thermal conductivity, internal reactions between char components and/or pyrolysis gas species, and unknown pyrolysis gas specific heat.

EXPERIMENTAL STUDY

Wind Tunnel Facilities

All testing was conducted in the Boeing Miniarc E arc-heated plasma facility. Associated with the arc heater is a two-dimensional Mach 2.5 nozzle and a 1- by 4-inch rectangular duct which is attached to the arc jet plenum chamber. The plasma tunnel exhausts into a vacuum chamber capable of altitude simulation to 260,000 feet. Power sources for the arc are four 40-kilowatt selenium rectifier units, four 250-kilowatt motor generators, and three 200-kilowatt engine-driven generators. Photographs of the facility are seen in figures 3 and 4.

A diagram of the plasma tunnel facility is seen in figure 5. The heat source is a tungsten-copper constricted arc heater. Downstream of the arc heater are the plenum chamber and the two-dimensional supersonic nozzle. The tunnel configuration downstream of the nozzle is dependent upon whether laminar or turbulent flow is desired in the test section. For laminar flow, the duct section and test section are attached to the nozzle section. In the turbulent flow configuration, the duct extension section followed by the boundary layer trip section are placed between the nozzle section and the duct and test sections. This allows for fully developed turbulent flow to be established in the duct by the time the flow enters the test section.

The test section is designed to accept either 4- by 8- by 2-inch thick flat plate ablation specimens or the specially designed calibration plate. The test section is an open channel; that is, the top surface of the duct has been removed. During tunnel operation the altitude chamber pressure was adjusted to

equal the static pressure of the duct flow just upstream of the test section in order to minimize disturbance of the boundary layer profile of the flow entering the test section. The open channel allows for displacement of the centerline streamline in accordance with the mass injected into the boundary layer during the ablation process as well as providing unrestricted motion-picture and total radiation pyrometer coverage of the specimen.

The basic instrumentation utilized included water-cooled asymptotic calorimeters and pressure taps for facility calibration, platinum-platinum 13% rhodium thermocouples for instrumenting the ablation specimens, a total radiation pyrometer for determining char surface temperatures, and a motion picture camera for recording test events. In addition, a total pressure probe was used to determine pressure profiles of the plasma flow at the entrance to the test section.

Photographs of the plasma tunnel in the turbulent flow configuration and in the laminar flow configurations are seen in figures 6 and 7, respectively.

Wind Tunnel Calibration

The flow field of the duct section and the test section were calibrated at conditions bracketing the laminar and turbulent flow conditions of the ablation tests for both nitrogen and reconstituted air atmospheres.

Figure 5 is a diagram of the plasma tunnel showing calorimeter and pressure tap locations for both the laminar and turbulent flow configurations.

Calibration of the duct section was accomplished by using five water-cooled asymptotic calorimeters and five static pressure ports located as shown in figure 5 along the "model side" of the duct wall. It is noted that the instrumentation of the duct section in the laminar flow configuration can be utilized to characterize the flow in the duct extension section for the turbulent flow configuration for identical conditions of arc power and mass flow rate.

The calibration plate of figure 5 is shown in greater detail in figure 8. This plate was fabricated to characterize the plasma flow through the tunnel test section. It contains five water-cooled asymptotic calorimeters and four static pressure ports located as shown in figures 5 and 8.

The results of the calibration runs are shown in figure 9. Plotted as functions of distance from the nozzle exit plane (the downstream edge of the nozzle section) are the cold-wall heat transfer rates as measured by the water-cooled asymptotic calorimeters and the wall static pressure. Figures 9a - 9d are for turbulent flow in nitrogen and reconstituted air at a mass flow rate of $0.075 \text{ lb}_m/\text{sec}$; figures 9e - 9h are for turbulent flow in nitrogen and reconstituted air at a mass flow rate of $0.100 \text{ lb}_m/\text{sec}$; and figures 9i - 9l are for laminar flow in nitrogen and reconstituted air at a mass flow rate of $0.060 \text{ lb}_m/\text{sec}$. The bulk total enthalpy called out in figure 9 represents the average total enthalpy over the test section as determined from the heat balance.

Figure 9 is presented as part of the evidence of the existence of turbulent flow in the plasma tunnel. The turbulent heat transfer plots show a slow decrease in laminar heating at the entrance of the duct section followed by transition to turbulent flow. The turbulent heat transfer rate then decays slowly through the test section of the tunnel. The shift in the point of transition in the downstream direction with lower Reynolds number is also evident. A comparison of the nitrogen heating rate curves for 0.075 lb_m/sec and 0.10 lb_m/sec show the upstream shift in the transition point due to increased Reynolds number.

Another device used for calibrating the plasma tunnel was a total pressure probe. The information obtained is an additional indication of the existence of turbulent flow. The cylindrical pressure probe of 1/4-inch diameter was used to record the pressure history of the plasma flow at the entrance to the test section. Traverses of the flow field were made across the 1-inch dimension of the 1- by 4-inch duct at two planes; one at the duct centerline and the second at a plane 1-inch to the right of the duct centerline as viewed by an upstream observer. All pressure surveys were made with the head of the probe 1/4-inch downstream of the plane of the duct-test section interface with neither the calibration plate nor an ablation specimen in position in the test section. Figure 10 shows typical laminar and turbulent profiles.

Analytically, results of computations based on reference 21 give a critical Reynolds number of 6500 based on a duct hydraulic diameter D . The tunnel conditions achieved during calibration show Reynolds numbers up to 18,500, further substantiating the existence of turbulent flow.

A method for determining the edge conditions for a turbulent boundary layer inside a duct is given in Appendix C. Using these values and the measured heat fluxes, values for the Stanton number were obtained. A comparison between the experimentally obtained Stanton numbers with those predicted by the turbulent boundary layer computer program is shown in figure 11. Excellent agreement is obtained.

The selected ablation specimen test conditions provide for the greatest possible spread in heat transfer rate and shear environment while staying within the operating envelope of the plasma facility. Table I shows a matrix of the conditions achieved during the testing of instrumented models.

Model Design and Construction

Twenty test specimens were fabricated as 4- by 8-inch flat plates. The thickness of all models was 2 inches. Instrumentation of the specimens was confined to an area much smaller than 4- by 8-inches to reduce possible sources of error due to corner, edge, and perturbation effects. The first 3 inches of the model were left uninstrumented to minimize flow establishment effects in the transition between duct and open channel flow. A 1-inch margin was maintained in both sides of the model to minimize conduction effects, and a 2-inch margin was left at the rear to reduce edge effects. This left a strip of 3-inches along the stream direction and 2-inches wide in which to locate thermocouples.

The thermocouples used in this test were constructed of 3-mil platinum-platinum 13% rhodium wire. The thermocouple beads were formed using an inert gas electrical discharge welder. Figure 12 shows the location and arrangement of the thermocouples.

Recessing the forward thermocouples ensures that they will not protrude into the stream and disturb the downstream data. Arranging three thermocouples at the same level side-to-side provides triplication of data along a near-isotherm. As seen in figure 12, the depths of the thermocouples fell into three groups. The first group with thermocouples at nominal depths of 0.05, 0.10, and 0.15 inches was used for the laminar flow tests in both nitrogen and reconstituted air. The second group with thermocouples at nominal depths of 0.05, 0.15, and 0.30 inches was used for the turbulent flow tests in nitrogen atmospheres. The third group with thermocouples at nominal depths of 0.10, 0.30, and 0.60 inches was used for turbulent flow tests in reconstituted air atmospheres. This placement of thermocouples was based on computer analyses using the CHARM program (ref. 22) and tests of uninstrumented models. The ultimate objective of this placement of the thermocouples was to have the shallowest thermocouples completely eroded away during the test time, the mid-depth thermocouples remain in the char layer, and the deepest thermocouples to remain at or near the final degradation plane.

Figure 13 shows a typical ablation model in position on its mounting plate. The models were bonded by a high temperature epoxy cement to the aluminum model holder. This assembly was then bolted into position in the test section of the plasma tunnel during testing.

Figure 14 shows an enlargement of a typical thermocouple installation. As can be gathered from this figure, the method of thermocouple installation coupled with the 0.003-inch wire diameter required a high degree of skill in model instrumentation. The requirements for small wire diameter and location of the leads nearly along isotherms to minimize errors due to heat conduction in the thermocouple wires, in keeping with the findings of Dow (ref. 23), dictated the procedure followed here. Prior to testing, all models were X-rayed in order to precisely locate the thermocouple beads in relation to the model surface. Table II gives the actual depths of all thermocouples as obtained from the model X-rays. It should be noted that the underside of the model holder was enclosed in a water-cooled copper box which protected the instrumentation leads from the recirculating hot gases inside the altitude chamber of the plasma facility.

Model Testing

The procedure followed in testing the instrumented ablation models is discussed in this section. Table I is a matrix of the conditions attained during testing.

The first step in the test of a given model is installation of the model in the test section and hookup and verification of all instrumentation. The bolt holes in the aluminum model holder were drilled slightly oversized to permit centering of the model in the test section. In no case was the gap

between the ablation material and adjacent water-cooled copper side wall of the test section large enough to permit significant flow of the plasma gases into the gap. Extreme care was exercised in assuring no steps between the lower surface of the duct section and the surface of the model.

The tunnel was sealed and the vacuum system activated. The facility high pressure water system for cooling was turned on and the entire system was checked for the proper coolant flows as well as checking for any water or air leaks.

In all cases the arc was initiated in a nitrogen atmosphere at minimum mass flow rate and arc power. After successful initiation of the arc, the power and mass flow rates were increased to the preselected values for that run. For those runs utilizing reconstituted air atmospheres the plenum chamber nitrogen mass flow was decreased and the oxygen mass flow was increased until the proper nitrogen-oxygen balance in the total tunnel flow was achieved after the desired arc power level was reached. At this point the pressure in the altitude chamber P_m , was manually increased until it was 95 to 96 percent of the duct static pressure P_d , as measured by the static pressure port located at the downstream end of the duct. This time was considered to be time zero for recession. Data were taken and recorded at one-minute intervals to the end of the run. The average length of time between initiation of the arc and "time zero" was 30 seconds. It is felt that this time period did not have significant effect on the ablation performance of the run since a majority of the time was at low power level in a nitrogen atmosphere under highly expanded conditions in the test section. During calibration runs and uninstrumented model runs it was found that heat rates and surface temperatures of the model were strong functions of the balance between the duct pressure and the altitude chamber pressure. As the chamber pressure P_m , approaches the duct pressure P_d , a shock wave forms at the downstream end of the duct section (the upper surface of the duct) which impinges on the model or calibration plate surface. As P_m/P_d decreases below 0.95, the surface temperatures of an ablating model and the heat rates recorded by the calorimeters of the calibration plate drop very rapidly due to the expanding flow.

The average length of run was 500 seconds. The progress of each run was monitored visually as well as by means of 16 mm color motion pictures. The criterion for termination of a run was flexible but in general was based on stabilization of the temperature response of the deepest thermocouple.

Upon termination of the run, the tunnel was shut down except for the nitrogen gas which was permitted to flow for 60 seconds to help cool the model and to suppress post-test combustion.

Post-test examination included a continuity check of all thermocouples and still photographs of the models. At the conclusion of the test program all models were sectioned lengthwise in order to obtain accurate surface recession and char layer thickness.

Figures 15, 16, and 17 are post-test photographs of the ablation specimens. The models of figure 15 were tested under laminar flow conditions. Models 1 and 2 were tested in nitrogen atmospheres and models 3 and 4 were tested in

oxygen-rich (30% by weight) reconstituted air atmospheres. The models of figure 16 were tested under turbulent flow conditions in nitrogen atmospheres. The models of figure 17 were tested under turbulent flow conditions in reconstituted air atmospheres.

Data Reduction and Presentation

Calibration data for the plasma tunnel have been discussed in some detail previously, Figure 9 shows calibration curves for various tunnel configurations, mass flow rates, power levels, and atmospheres. Due to the large amount of heating rate data generated by checkout runs, calibration runs, and model testing, the millivolt-temperature calibration curves for each of the ten calorimeters (five in the duct and five in the calibration plate) were programmed on a computer. This provided rapid, accurate conversion from the millivolt output of the calorimeter to the appropriate cold-wall heat transfer rate.

Another computer program was written to calculate the test section total enthalpy from the heat balance for each run from inputs of the arc power, the cooling water mass flow rate, the temperature change of the coolant, and the tunnel mass flow rate.

The millivolt outputs of the eleven thermocouples per model were recorded on a Minneapolis-Honeywell Visicorder recording oscillograph. The photo-sensitive strip chart output of this machine records continuously the millivolt thermocouple output as a deflection from the reference line, the reference line being calibrated to the ice bath reference temperature. The thermocouple millivolt outputs were read from the strip charts at a maximum time increment of 10 seconds (smaller increments if required by rapid changes, pulses, or oscillations in output) on a strip chart reading device (Oscar J. Reader).

Generation of the time-temperature response curve for each thermocouple of each ablation model was greatly expedited through the use of a computer and automatic plotting machines. The time-millivolt history of each thermocouple along with the conversion table of millivolts to degrees Fahrenheit for platinum-platinum 13% rhodium thermocouples with a reference temperature of 32°F were input by punched card into the computer. Conversion from millivolts to degrees Rankine and plotting instructions for a Model 562 Orthomat drafting machine were accomplished by the computer. The Orthomat drafting machine faired a parabolic curve through all data points. The thermocouple time-temperature histories so generated are shown in figures 47 through 66 in Appendix C. Figure 67 shows a comparison of a typical Orthomat curve with data from the thermocouple strip chart.

The surface temperature of each ablation specimen was measured by a Leeds and Northrup Ray-O-Tube total radiation pyrometer and was recorded continuously on a strip chart. Figures 18 through 22 show the surface temperature histories of all of the ablation specimens based on an emissivity of 0.75.

Figures 23 through 27 show the original specimen shape, the final surface contour, and the final char virgin plastic interface as obtained from ablation models which have been sectioned along their centerplane.

Tables III and IV are summaries of tunnel data and ablation data, respectively. The methods of obtaining the boundary layer edge conditions are found in Appendix C.

The experimental surface temperature history and average surface recession rate along with the char and virgin material ablation properties were input to the CHARM program (ref. 22) for matrix runs 5, 9, 10, 11, 12, and 13. Table V lists the ablation properties which were used in the program. Table VI compares the actual and the calculated char layer thicknesses. Figures 20 through 33 show the actual and calculated time-temperature histories for selected thermocouples. It is suspected that development of crevices during turbulent flow ablation and the existence of endothermic reactions within the char lead to the discrepancies observed.

CORRELATION OF TEST DATA

The correlation of test data started with data obtained in laminar flow supplied by NASA-Houston. This formed the basis for turbulent flow correlations obtained in the course of this program.

Correlations

The basic ablation analysis used for the Apollo ablative material has already been discussed. For this particular material, three correlations describing different aspects of performance have been developed in lieu of theoretical relationships requiring exact physical details. When used with a heat balance, these correlations predict surface recession.

Equation (24) may be written as:

$$\dot{m}_c = AK_{0.2} \rho_e u_e St_o - \Lambda \dot{m}_p \quad (25)$$

in the diffusion-limited regime as shown in Appendix B. The term $\Lambda \dot{m}_p$ represents the reduction in surface recession attributed to pyrolysis gas combustion, but which must also be considered to include at least the effects of shrinkage or sintering. It is assumed that 1) combustion rates depend on some type of Arrhenius factor, viz., $A_k(P,T)e^{-B/T}$, and 2) combustion products depend on pressure, temperature, and the amount of pyrolysis gas present (as measured by $\dot{m}_p$); i.e., $\Lambda = \phi(\dot{m}_p^\alpha T_w^\beta P^\gamma K_1^{10^4/T_w})$.

A dimensional analysis yields

$$\Lambda = \phi_1 \left(\frac{\dot{m}_p T_w^{\frac{1}{2}}}{P} K_1 \frac{10^4}{T_w} \right) \quad (26)$$

The group $\frac{\dot{m}_p T_w^{1/2}}{P}$ is arrived at in the following manner. Letting Λ be dimensionless, the term $a \dot{m}_p^\alpha T_w^\beta P^\gamma$ is to have powers α , β , γ , and dimensional constant a such that it is dimensionless. In order to show that dimensions may be in terms of lb_m , ft , and sec , the factors T_w and P need be multiplied by the dimensional groups $\frac{R g_c}{M} \frac{\text{ft}^2}{\text{sec}^2 \cdot ^\circ\text{R}}$ and

$g_c \frac{\text{lb}_m}{\text{lb}_f} \frac{\text{ft}}{\text{sec}^2}$, respectively. (R is the gas constant with dimensions

$\text{ft} \cdot \text{lb}_f / \text{mole} \cdot ^\circ\text{R}$, M is a molecular weight, $\text{lb}_m / \text{mole}$, and g_c is the gravitational constant.) Hence the dimensional constant a becomes

$\left(\frac{R g_c}{M}\right)^\beta \cdot (g_c)^\gamma$. The powers α , β , and γ are solutions of the following equations derived from consideration that each unit (lb_m , ft , and sec) be raised to the zero power.

$$1 = \left(\dot{m}_p \frac{\text{lb}_m}{\text{ft}^2 \cdot \text{sec}}\right)^\alpha \left(\frac{R g_c}{M} T_w \frac{\text{ft}^2}{\text{sec}^2}\right)^\beta \left(P g_c \frac{\text{lb}_m}{\text{ft} \cdot \text{sec}^2}\right)^\gamma$$

$$\begin{aligned} \alpha + \gamma &= 0 & (\text{lb}_m) \\ -2\alpha + 2\beta - \gamma &= 0 & (\text{ft}) \\ -\alpha - 2\beta - 2\gamma &= 0 & (\text{sec}) \end{aligned}$$

Arbitrarily selecting α to be 1, then $\gamma = -1$, and $\beta = \frac{1}{2}$; hence the appropriate grouping is $\frac{\dot{m}_p T_w^{1/2}}{P}$.

For the purpose of ease in data analysis, St_o was used in eq. (25), rather than St , under the assumption that the correlation developed would automatically take care of blocking effects.

From stagnation region data available (ref. 24) a value of 0.12 for K_1 was determined to give the "smoothest" fit for the correlation for Λ (shown in figure 34) for use in eq. (25). Figure 35 shows $\dot{m}_c$ determined from this correlation as a function of T_w and P for $\dot{q}_o / H_s P^{1/2} = 0.1$.

(The value for $\dot{m}_p$ was determined from the correlation of eq. (34).) These results may be compared to those shown on figures 28 and 29 of reference 15. The mass fluxes used from ref. 24 are those observed at 150 seconds, and seem to represent an average value for the complete duration of each experimental run. Since surface recession can be predicted by this correlation at conditions where a kinetic and transition surface combustion regime may exist, it will be used throughout all combustion regimes, even though the term

$A \rho_c u_c St_o K_{o,c}$ represents only a diffusion-limited recession. If, in applications, the correlation predicts negative recession, $\dot{m}_c$ should be set at 0. For the experimental conditions attained for this contract, the chemical recession model predicts $\dot{m}_c = 0$; the recession noted may thus be directly

attributed to shear failure alone.

To arrive at the appropriate parameters to correlate shear removal, eq. (14) of ref. 25 for the case of negligible gradient of the pressure gradient was utilized:

$$\dot{s}_{sh} = \frac{2kc^2}{\mu_{cw} \rho_c^2 c_{pc}^2 n^2 \dot{s}^2} \frac{dT_w}{dx}$$

where n is an exponent in the approximate relationship for viscosity as a function of temperature, $\mu/\mu_o = (T_o/T)^n$, where the subscript o refers to some reference condition. For a flat plate, $T_w \propto Re_x^m$ where x is measured from some fictitious leading edge for the experimental conditions encountered in this work, and m is an exponent ($\approx \frac{1}{2}$ for laminar flow, $\approx 1/5$ for turbulent flow). Thus

$$\frac{dT_w}{dx} \propto m (Re_x)^{m-1} \frac{dRe_x}{dx} \propto T_w \approx \psi H_o u_e$$

for the experimental cases analyzed, assuming dRe_x/dx is approximately the same for all runs at the model distance at which data was taken (6 inches downstream). The melt layer surface viscosity may be correlated by $\mu_{cw} \propto \exp(-B\mu/T_w)$.

Hence the expression for shear recession may be written:

$$\dot{s}_{sh} \propto \frac{2kc^2 e^{B\mu/T_w}}{\rho_c^2 c_{pc}^2 n^2 \dot{s}^2} \psi H_o u_e$$

or rearranging:

$$\dot{m}_{sh} = \rho_c \dot{s}_{sh} \propto \sqrt[3]{\frac{2kc^2 e^{B\mu/T_w}}{c_{pc}^2 n^2} \rho_c \psi H_o u_e}$$

Since properties are not constant with temperature, shear stress, etc., this relationship may best be written as:

$$\dot{m}_{sh} = \phi_2 \left(\psi H_o u_e K_2^{10^4/T_w} \right) \quad (27)$$

Eq. (27) represents the form developed for correlation of the experimental data taken under this contract. For the data available $\psi \rightarrow 1.0$, and u_e was calculated as shown in Appendix C. Figure 36 shows the correlation obtained for $K_2 = 0.5$. It is applicable for an air environment only, since it appears that an oxidized char has much different strength properties from those observed in a nitrogen environment.

A correlation between T_w , $\dot{m}_p$, and total surface recession rate was developed from theoretical considerations. For flow through a porous matrix,

the temperature profile is approximately given by ref. 26:

$$T - T_0 = (T_w - T_0) e^{\frac{-\dot{m}_p C_p z}{k_c}} \quad (28)$$

Letting $T_0 \rightarrow 0$

$$T \approx T_w e^{\frac{-\dot{m}_p C_p z}{k_c}} \quad (29)$$

Assuming that $\dot{m}_p$ is related to the temperature gradient at the reaction zone, and that the distance z to the pyrolysis zone from the surface is related to

$$\dot{s}_p - \dot{s}_{surf} \equiv \dot{m}_p / \rho_p - \dot{m}_{surf} / \rho_c \quad (30)$$

a dimensional correlation will have the form:

$$\dot{m}_p = \phi_3 \left(-k_c \frac{\partial T}{\partial x} \right) = \phi_3 \left(T_w \dot{m}_p C_p e^{\frac{-\dot{m}_p C_p z}{k_c}} \right) \quad (31)$$

$$= \phi_4 \left[\dot{m}_p T_w K_3 \dot{m}_p (\dot{m}_p / \rho_p - \dot{m}_{surf} / \rho_c) \right] \quad (32)$$

$$= \phi_5 \left[T_w K_3 \dot{m}_p (\dot{m}_p / \rho_p - \dot{m}_{surf} / \rho_c) \right] \quad (33)$$

Since for the Apollo material $\rho_p \approx \rho_{surf} \approx 16.5$ (where ρ_p denotes the density of the gas-forming fraction of the virgin plastic) eq. (33) may be rewritten as:

$$\dot{m}_p = \phi_5 \left[T_w K_4 10^6 \dot{m}_p (\dot{m}_p - \dot{m}_{surf}) \right] \quad (34)$$

where the exponent 10^6 is utilized for convenience in hand calculations. For a value of $K_4 = 1.10$, both flat plate and stagnation region data (using average mass fluxes) are correlatable using this function, as shown in figure 37. That the mass flux difference $(\dot{m}_p - \dot{m}_{surf})$ is important in correlating data is also foretold by steady state ablation theory (see ref. 27).

The approximate heat balance, eq. (23), when combined with the above

correlations, eqs. (25), (26), (27), and (34), yields a set of five simultaneous equations with five unknowns ($\dot{m}_p$, $\dot{m}_c$, T_w , $\dot{m}_{surf}$, and $\mathcal{L}$) which may be solved for $\dot{m}_p$, $\dot{m}_{surf}$, and T_w . The parameters $\dot{m}_p$ and T_w are then used as boundary conditions in the boundary layer program. Final solution is obtained by iteration. Table V presents the necessary property values for the charring ablator analysis and the boundary layer program. In place of the heat balance eq. (23), and the $\dot{m}_p - T_w$ correlation eq. (34), a transient charring ablator program may be essentially used to determine $\dot{m}_p$ and T_w .

In general, heat fluxes to the rough charring ablator surfaces are somewhat higher than calibrated values obtained by means of calorimeters (which exhibit a much smoother cooler surface). Surface roughness appears, then, to influence heat transfer significantly. Appendix A shows that all of the data lie at or below the upper bound of heat flux for a completely rough surface with protuberances equal in height to surface recession. It appears that roughness effects can increase convective heat transfer by a factor of two or more. Unfortunately, no trends have been discovered to help predict what roughness factor (x/K_s) ensues from a given ablation environment and honeycomb orientation. As a plausible estimate, the height of protuberances may be taken as 1/3 of the average surface recession of the filler material within the honeycomb cells. Figure 11 compares predicted (see Appendix A for calculations) and experimentally measured smooth wall heat fluxes without mass injection of this contract. Figure 38 compares predicted with experimental recession data in both cases using experimental heat fluxes with roughness effects as deduced from eq. (23). Table VII compares predicted surface temperature and recession rate (for both calibrated smooth wall and experimental heat fluxes) to experimental values.

Discussion of Limits of Applicability

It should be noted that the overall analysis as applied does not predict effects of 1) nonequilibrium gas phase chemistry; 2) external atmospheres other than air (the ablator performance is somewhat different in N_2 and Ar atmospheres (ref. 24), since chemical processes at and in the char are dependent on the type of boundary layer species present); 3) radiation heat transfer from the boundary layer or shock layer (although it is allowed for, but cannot be predicted by this analysis); and 4) history of surface roughness on heat transfer. While the correlation of eq. (33) empirically accounts for heretofore uncharacterized complex internal physiochemical processes (e.g., sintering, kinetics of internal char reactions) of the ablation material, the effects on ablator performance of each of all of these processes have not been delineated.

PERFORMANCE PREDICTION FOR APOLLO FLIGHT TRAJECTORIES

Application of the boundary layer program and charring ablator routine has led to prediction of surface recession performance of the Apollo vehicle as shown in Figures 39 through 44. The two trajectory inputs were supplied by NASA-MSC, whose designations "202" and "501" are used (Tables VIII and IX).

Calculations were carried out at several points in time at various body locations. Figure 45 depicts the surface energy balance components (cf. eq. (23)) during the as 201 trajectory, station 2, for the transition criterion $a = 60$. The effect of transition criterion (value of a) on cold wall unblocked heat flux is shown in figure 46 for the cases: as 202 trajectory; stations 1, 2, 5; and $a = 20$ and 60 . Effects of pressure gradients and radiation heat transfer to the surface were accounted for, using supplied NASA-MSC data. Roughness effects are not included, since heights of protuberances cannot yet be predicted.

CONCLUSIONS AND RECOMMENDATIONS

From the analysis and experiments performed in this program, the following conclusions may be drawn:

- (1) The surface recession of a char-forming silica-filled ablator in an oxidizing atmosphere is accelerated by melt-layer shear removal. It is possible to delineate surface recession due to shearing and recession due to char oxidation. Correlatable data for recession attributable to shearing were obtained for the entire experimental range considered.
- (2) It is possible to correlate the ablation performance of complicated organic compounds such as epoxy novolacs (AVCOAT 5026 - 30H/CG) used on Apollo by applying comparatively simple semi-empirical techniques. Detailed analysis of the various chemical and physical processes involved may be bypassed and only macroscopic performance considered. Three successful correlations were developed, from which may be obtained the mass flux of pyrolysis gases, surface temperature, virgin plastic recession, surface recession and effects of boundary layer combustion. As in all semi-empirical techniques, caution must be observed in extrapolating the correlations obtained here to other materials and conditions significantly different from the range considered in the program.
- (3) Supersonic turbulent boundary layer flow over a flat plate with free stream enthalpies in excess of 2000 Btu/lb was obtained.
- (4) Surface roughness plays a major role on the heat transfer to the surface of a char forming ablator. Serious underprediction of the heat fluxes may occur if it is ignored.
- (5) A computer program based on an integral solution of the boundary layer equations was developed and successfully applied for the calculation of smooth wall heat fluxes to an ablating surface in laminar and turbulent flow.

In the course of the program further possible refinements in analysis of phenomena related to ablation have been deemed important enough to warrant further study. Among these have been: accounting for in-depth oxidation of char arising from diffusion of oxygen into the char; description of in-depth pressure profiles, spallation mechanisms, pressure-diffusion effects, and

diffusion of gases through porous virgin plastic; development of various char densification models, including sintering of silica, shrinkage of the carbon matrix, and deposition of carbon; allowance for various chemical reaction options within or on the char, including gas phase, condensed phase, and heterogeneous reactions; and improvement of models for surface removal via shear. Attempts were made in this study to develop experimentally corroborated models for shear removal and sintering. Preliminary results in both instances were qualitatively plausible, but due to the complexity of the problems, final solution was not completed. Rather than using the two separate computer programs to solve the ablation problem, an overall integral method should be developed for transient ablation, eliminating much of the work involved in matching conditions at the gas-solid interface.

Further improvements in experimental techniques appear desirable. Future tests might be run with lengthened models to ensure minimal upstream effects on surface recession caused by structural collapse at the downstream vertical face. Roughened calorimeters should be designed and used to determine more accurately the heat transfer to typical surfaces. A transpiring, roughened porous calibration plate should be utilized to verify predictions of blowing and roughness effects at least in the transition regime. By careful positioning of thermocouples, the effect of the honeycomb on internal temperature profiles may be studied.

TABLE I MATRIX OF TEST CONDITIONS ATTAINED

RUN NO. AND MODEL NO.	FLOW CONDITION	ATMOSPHERE	ARC CURRENT (AMPS)	MASS FLOW RATE (LBM/SEC)	H (BTU/LBM)	REYNOLDS NO. BASED ON HYDRAULIC DIAMETER
1	Laminar	Nitrogen	1000	0.060	2582	6426
2	Laminar	Nitrogen	1200	0.060	3130	5357
3	Laminar	Air	1000	0.043	3065	4040
4	Laminar	Air	1200	0.043	3396	3975
5	Turbulent	Air	800	0.075	1933	9996
6	Turbulent	Air	800	0.075	2045	9705
7	Turbulent	Air	1000	0.075	2264	9367
8	Turbulent	Air	1000	0.075	2376	9198
9	Turbulent	Air	1200	0.075	2779	8767
10	Turbulent	Air	1200	0.075	2560	8947
11	Turbulent	Air	1000	0.100	2078	12890
12	Turbulent	Air	1000	0.100	1942	13115
13	Turbulent	Air	1200	0.100	2304	12332
14	Turbulent	Air	1200	0.100	2333	12255
15	Turbulent	Air	1000	0.115	1943	15419
16	Turbulent	Air	800	0.125	1610	18261
17	Turbulent	Nitrogen	900	0.075	2670	8928
18	Turbulent	Nitrogen	1200	0.075	3227	8382
19	Turbulent	Nitrogen	1200	0.100	2957	11255
20	Turbulent	Nitrogen	900	0.125	1720	18525

TABLE II
ACTUAL THERMOCOUPLE DEPTHS IN INCHES
FROM THE ORIGINAL SURFACE PLANE

MODEL NUMBER	TC 1	TC 2	TC 3	TC 4	TC 5	TC 6	TC 7	TC 8	TC 9	TC 10	TC 11
1	.210	.135	.130	.080	.075	.130	.110	.050	.125	.085	.065
2	.150	.100	.110	.090	.065	.165	.110	.035	.140	.105	.070
3	.170	.155	.110	.055	.080	.160	.090	.035	.145	.085	.060
4	.140	.160	.090	.045	.065	.145	.120	.040	.180	.120	.050
5	.705	.695	.380	.110	.125	.580	.335	.175	.680	.335	.095
6	.695	.665	.345	.115	.155	.565	.340	.100	.680	.310	.110
7	.655	.685	.370	.170	.135	.570	.355	.120	.665	.335	.095
8	.700	.705	.340	.125	.105	.585	.375	.135	.640	.325	.095
9	.670	.690	.345	.095	.110	.575	.330	.115	.770	.435	.090
10	.750	.690	.375	.125	.185	.555	.370	.175	.700	.335	.115
11	.700	.655	.350	.150	.100	.555	.330	.125	.690	.340	.110
12	.695	.635	.390	.130	.135	.555	.345	.125	.635	.320	.095
13	.780	.715	.370	.130	.165	.650	.375	.185	.680	.310	.120
14	.705	.730	.355	.105	.120	.565	.315	.125	.660	.335	.085
15	.750	.700	.355	.120	.160	.620	.370	.150	.675	.320	.100
16	.695	.690	.410	.150	.125	.575	.345	.155	.675	.360	.150
17	.345	.390	.150	.090	.090	.395	.175	.060	.320	.165	.035
18	.325	.425	.215	.052	.075	.345	.150	.085	.305	.175	.045
19	.330	.330	.155	.045	.095	.350	.210	.050	.305	.165	.045
20	.290	.345	.155	.070	.040	.325	.160	.050	.300	.170	.070

TABLE III
PLASMA TUNNEL DATA

RUN NO. AND MODEL NO.	$\dot{m}/A$ (LB _m /ft ² sec)	$\dot{q}_{cw}$ (BTU/ft ² sec)	R_T (LB _f /ft ²)	P_m (LB _f /ft ²)	P_d (LB _f /ft ²)	$\frac{P_m}{P_d}$
1	2.160	9.3	1032	69	73	.94
2	2.160	11.6	1110	78	81	.97
3	1.548	14.5	804	66	69	.96
4	1.548	17.0	849	72	76	.94
5	1.548	17.0	1141	116	122	.95
6	2.700	18.3	1136	117	121	.96
7	2.700	22.8	1231	125	132	.95
8	2.700	22.4	1230	128	131	.98
9	2.700	29.6	1309	136	142	.96
10	2.700	29.7	1309	136	143	.95
11	3.600	21.9	1549	161	165	.98
12	3.600	22.2	1542	159	166	.96
13	3.600	25.1	1671	170	179	.95
14	3.600	25.2	1650	170	178	.96
15	4.140	22.6	1737	171	186	.92
16	4.500	18.0	1721	170	186	.91
17	2.700	21.0	1282	120	123	.97
18	2.700	28.8	1369	122	125	.98
19	3.600	30.0	1774	168	172	.98
20	4.500	23.4	1837	165	179	.92

TABLE IV
ABLATION DATA

RUN NO. AND MODEL NO.	$\dot{q}_{cw}$ (BTU/FT ² SEC)	H (BTU/LB _m)	T max, $\epsilon = 0.75$ (o_R)	$\dot{s}$ (IN) SEC	s (IN)	CHAR THICKNESS (IN)	TIME* (SEC)
1	4.3	2582	2020	2.22E-4	0.04	0.09	180
2	11.6	3130	2080	1.07E-4	0.03	0.18	280
3	14.5	3065	2570	1.75E-4	0.07	0.33	400
4	17.0	3396	2540	1.88E-4	0.06	0.21	320
5	18.0	1933	2810	4.19E-4	0.27	0.54	645
6	18.3	2045	2810	2.70E-4	0.15	0.48	555
7	22.8	2264	2900	4.39E-4	0.25	0.53	570
8	22.4	2376	2870	4.33E-4	0.21	0.49	485
9	29.6	2779	3220	6.08E-4	0.38	0.55	625
10	29.7	2560	3270	6.90E-4	0.40	0.45	580
11	21.9	2078	3460	7.58E-4	0.39	0.55	515
12	22.2	1942	3320	6.98E-4	0.45	0.55	645
13	25.1	2304	3430	9.22E-4	0.59	0.47	690
14	25.2	2335	3390	1.019E-4	0.54	0.42	530
15	22.6	1943	2780	6.95E-4	0.49	0.34	705
16	18.0	1610	2780	5.43E-4	0.28	0.37	515
17	21.0	2670	2820	1.23E-4	0.08	0.58	650
18	28.8	3227	2980	0.91E-4	0.03	0.48	330
19	30.0	2957	3100	1.28E-4	0.05	0.58	390
20	23.4	1720	2540	1.40E-4	0.07	0.44	500

* Zero time for recession is considered to be when the surface temperature has reached 90% of the temperature at the first data point of the run. The first data point of a run was taken when nominal test conditions were achieved.

TABLE V.- PROPERTY VALUES

SYMBOL	DEFINITION	VALUE	UNITS	SOURCE	REMARKS
A	Stoichiometric coefficient in eq. (B11)	1.866	-	Appendix B	
A_{sub}	Frequency factor	$o(10^8)$	$\text{lb}_m/\text{ft}^2\text{-sec}$	Appendix B	Based on equilibrium evaporation with evaporation coefficient of $o(10^{-2})$.
B_{sub}	Activation temperature for sublimation of silica	103,600	$^{\circ}\text{R}$	Appendix B	
$\bar{c}_{\text{p,c}}$	Specific heat of charred residue	1.25	$\text{Btu}/\text{lb}_m\text{-}^{\circ}\text{R}$		Chosen to give predicted temperature profiles in agreement with experimental data of this contract. Endothermic reaction implied.
$\bar{c}_{\text{p,p}}$	Specific heat of pyrolysis gases	0.62	$\text{Btu}/\text{lb}_m\text{-}^{\circ}\text{R}$		Assumed that of C_2H_2 at 1000 $^{\circ}\text{K}$.
$\bar{c}_{\text{p,v.p.}}$	Specific heat of virgin plastic	0.43	$\text{Btu}/\text{lb}_m\text{-}^{\circ}\text{R}$		
f	Fraction of pyrolysis gases burnt	1.0			Assumed
$\Delta H_{\text{c,c}}$	Heat of combustion of char:				
	a. via reaction	2130	Btu/lb_m	Appendix B	
	b. via reaction	-855	Btu/lb_m	Appendix B	
$\Delta H_{\text{c,p}}$	Heat of combustion of pyrolysis gases via reaction	6154	Btu/lb_m	Appendix B	
ΔH_{pyr}	Heat of pyrolysis of virgin plastic	250	Btu/lb_m	Reference 24	
ΔH_{sub}	Heat of sublimation of silica	126,800	Btu/lb_m	Reference 30	
K_1	Empirical constant in eq. (26)	0.12		This work	Chosen to give monotonic curves in figures 29, 30, and 31.
K_2	Empirical constant in eq. (27)	1.10			
K_4	Empirical constant in eq. (34)	0.50			

TABLE V.--PROPERTY VALUES (Concluded)

SYMBOL	DEFINITION	VALUE	UNITS	SOURCE	REMARKS
T_o	Initial ablator temperature	535	$^{\circ}\text{R}$		Assumed
$w_{O_2,e}$	Mass fraction of oxygen at boundary layer edge	0.23	--	Reference 33	
ϵ	Charred residue emissivity	0.75	--	Reference 24	
ρ_c	Density of charred residue	16.5	lb_m/ft^3	Reference 24	} Calculated from TGA data
ρ_p	Density of decomposable fraction of virgin plastic	17.5	lb_m/ft^3	Reference 24	
Air properties				{ Reference 32 Reference 33	
$k_{v.p.}$	Thermal conductivity of virgin plastic at				
	a. $T = 460^{\circ}\text{R}$	6.5×10^{-2}	$\text{Btu}/\text{ft}\cdot\text{hr}\cdot^{\circ}\text{R}$	Reference 24	
	b. $T = 1060^{\circ}\text{R}$	6.8			
	c. $T = 2460^{\circ}\text{R}$	7.6			
k_c	Thermal conductivity of char at				
	a. $T = 460^{\circ}\text{R}$	4.0×10^{-2}	$\text{Btu}/\text{ft}\cdot\text{hr}\cdot^{\circ}\text{R}$	Reference 24	Selected from available static data to give predicted in situ temperature profiles in agreement with experimental data of this contract.
	b. $T = 1060^{\circ}\text{R}$	7.6			
	c. $T = 2460^{\circ}\text{R}$	32.0			
	d. $T = 4210^{\circ}\text{R}$	61.0			
	e. $T = 5460^{\circ}\text{R}$	15.0			
	f. $T = 6460^{\circ}\text{R}$	0.68			
Pyrolysis rate				Reference 24	

$$\frac{dp}{dt} = \begin{cases} 1.97 \times 10^{-7} \rho_p \left(\frac{\rho_p - \rho_c}{\rho_p} \right)^{2.0} e^{-\frac{19900}{T}}; & T > 1000^{\circ}\text{R} \\ 0; & T \leq 1000^{\circ}\text{R} \end{cases}$$

TABLE VI

ACTUAL AND PREDICTED CHAR LAYER THICKNESS FOR
MATRIX RUNS 5, 9, 10, 11, 12, and 13

Matrix Run Number	Experimental Char Layer Thickness, inches	Predicted Char Layer Thickness, inches
5	0.54	0.53
9	0.55	0.53
10	0.45	0.50
11	0.55	0.48
12	0.47	0.47
13	0.47	0.46

TABLE VII
COMPARISON OF PREDICTED TO EXPERIMENTAL PARAMETERS
(FOR AIR ONLY)

Run No.	Experiment		Predicted Rough Wall		Predicted Smooth Wall	
	$\dot{m}_{sh}$ lb _m /ft ² sec	T_w °R	$\dot{m}_{sh}$ lb _m /ft ² sec	T_w °R	$\dot{m}_{sh}$ lb _m /ft ² sec	T_w °R
3	2.41×10^{-4}	2570	2.39×10^{-4}	2614	2.26×10^{-4}	2457
4	2.58	2540	2.58	2615	2.44	2574
5	5.76	2810	5.39	2812	5.22	2711
6	3.72	2760	5.60	2871	5.28	2743
7	6.03	2900	6.78	2903	6.57	2868
8	5.95	2870	7.06	3067	6.21	2878
9	8.36	3220	9.85	3217	8.54	3025
10	9.48	3270	12.3	3265	8.87	3001
11	10.4	3460	11.4	3448	6.07	2746
12	9.59	3320	10.9	3331	6.32	2739
13	12.7	3430	12.4	3445	8.05	2988
14	14.0	3390	11.2	3352	7.23	2875
15	9.56	2780	7.02	2760	7.48	2819
16	7.48	2780	6.19	2744	5.56	2637

TABLE VIII

AS202 POST FLIGHT TRAJECTORY (Ref. 24)

Time, sec	Velocity, ft/sec	Altitude, ft
0	27,197	400,471
52	27,287	318,645
102	27,238	257,895
152	25,991	218,542
162	25,514	215,163
172	23,009	213,385
202	23,596	216,171
252	27,158	234,742
302	21,622	251,377
352	21,339	255,962
402	21,045	245,322
452	20,348	221,345
472	19,757	209,760
492	18,882	198,440
512	17,704	188,524
532	16,265	181,083
552	14,788	175,904
572	13,384	171,780
592	12,019	167,550
612	10,707	162,611
632	9,470	156,212
652	8,230	147,978
672	6,922	138,078
692	5,544	127,500
712	4,205	116,749
732	3,039	105,665
752	2,080	94,126

TABLE IX

AS 501 NOMINAL FLIGHT TRAJECTORY (Ref. 24)
(Selected data)

Time, sec	Velocity, ft/sec	Altitude, ft
0	35,004	400,000
28	35,086	286,420
60	33,409	191,821
84	27,019	175,431
120	23,423	196,520
152	21,893	220,818
232	21,159	265,656
336	20,908	260,380
456	17,605	174,066
532	9,050	138,894
604	3,800	105,540

TABLE X

COMPARISON OF HEAT TRANSFER COEFFICIENTS OBTAINED FROM TESTS TO VALUES ATTRIBUTABLE TO SURFACE ROUGHNESS EFFECTS

Run no.	f	$N_G \cdot 10^3$	$N_H \cdot 10^3$	Re_R	$Re_x \cdot 10^5$	h/h_G	h/h_G	$h/h_{G,max}$
3	0	4.00	5.20	86	1.75	1.10	1.30	1.30
4	0	3.55	3.89	74	1.74	1.10	1.09	1.25
5	0	9.50	18.43	594	2.90	1.30	1.94	2.70
6	0	9.04	15.16	312	2.90	1.25	1.68	1.90
7	0	10.08	16.52	470	2.76	1.25	1.64	2.20
8	0	9.26	14.68	376	2.76	1.20	1.59	1.85
9	1.00	10.30	12.81	654	2.61	1.40	1.24	2.25
9	0.30	10.30	17.37	654	2.61	1.40	1.68	2.25
9	0	10.30	19.14	654	2.61	1.40	1.85	2.25
10	1.00	11.25	16.00	696	2.77	1.60	1.42	2.30
10	0.60	11.25	18.13	696	2.77	1.60	1.61	2.30
10	0.40	11.25	18.81	696	2.77	1.60	1.83	2.30
11	1.00	10.68	31.07	1088	3.51	1.75	2.91	3.70
12	1.00	11.62	29.22	1300	3.64	1.95	2.51	>> 3
13	1.00	10.85	25.00	1565	3.27	2.20	2.30	↓
14	1.00	10.77	21.92	1440	3.27	2.15	2.04	
14	0.80	10.77	24.10	1440	3.27	2.15	2.24	
15	0	12.10	17.60	1656	4.12	2.00	1.45	
16	0	11.91	24.52	1133	4.76	1.90	2.06	

^aBased on $k_s = 1/3$ (total recession of surface).^bBased on $k_s = 1.0$ (total recession of surface).^cActual value from test data.

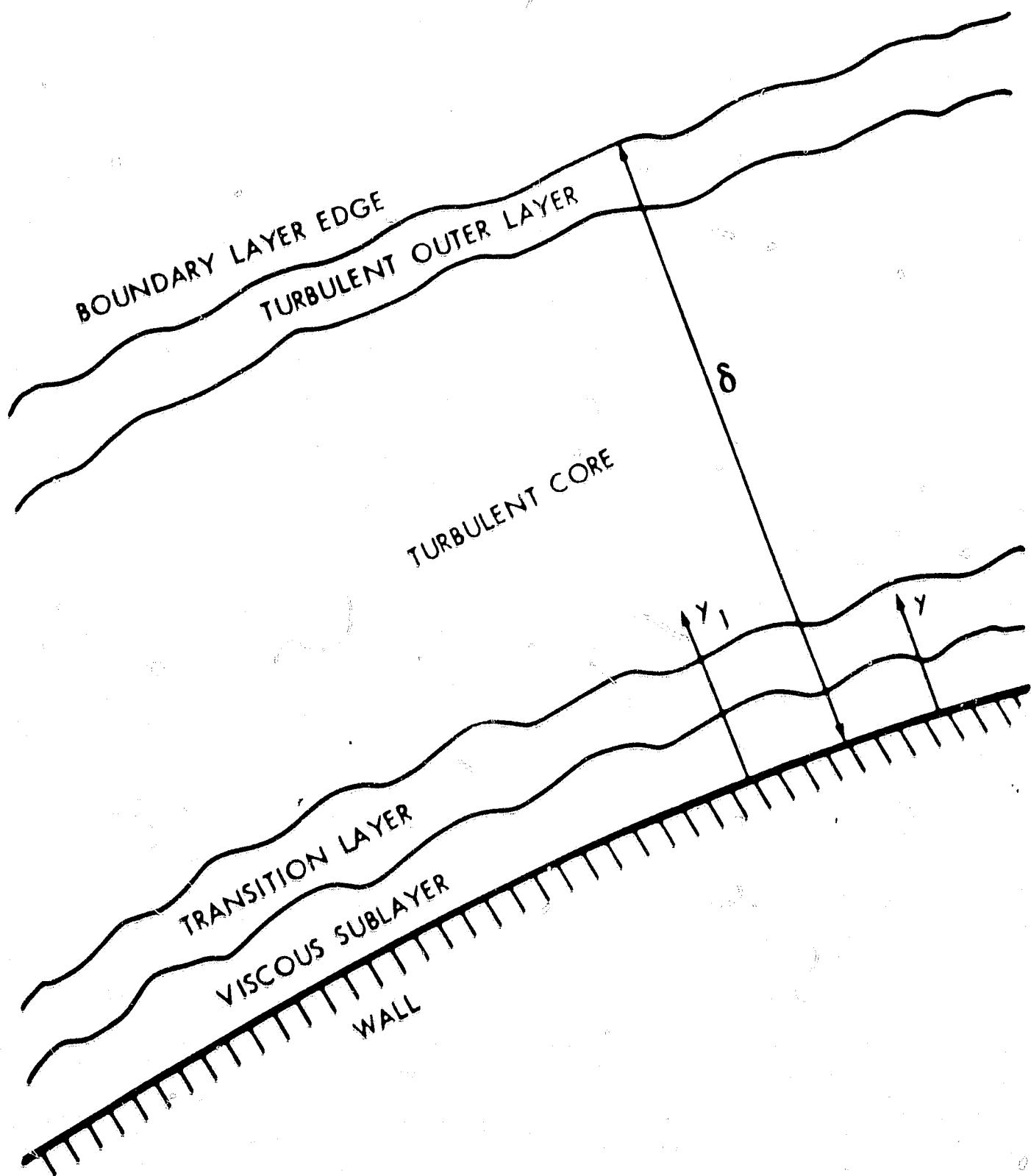


Figure 1.- Boundary-Layer Model

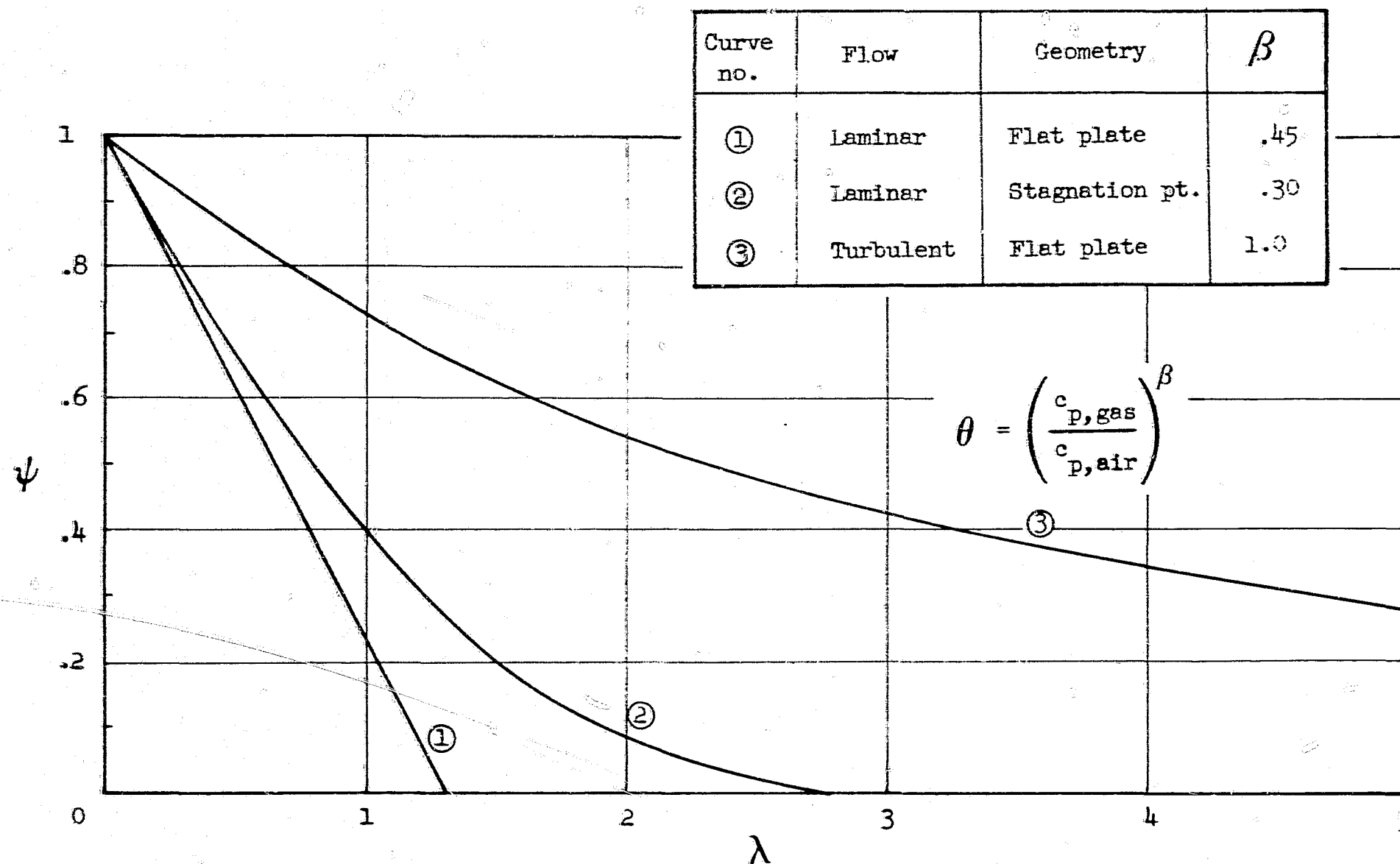


Figure 2.- Preliminary blocking curves used in analysis of experimental data.
(Taken from ref. 22)

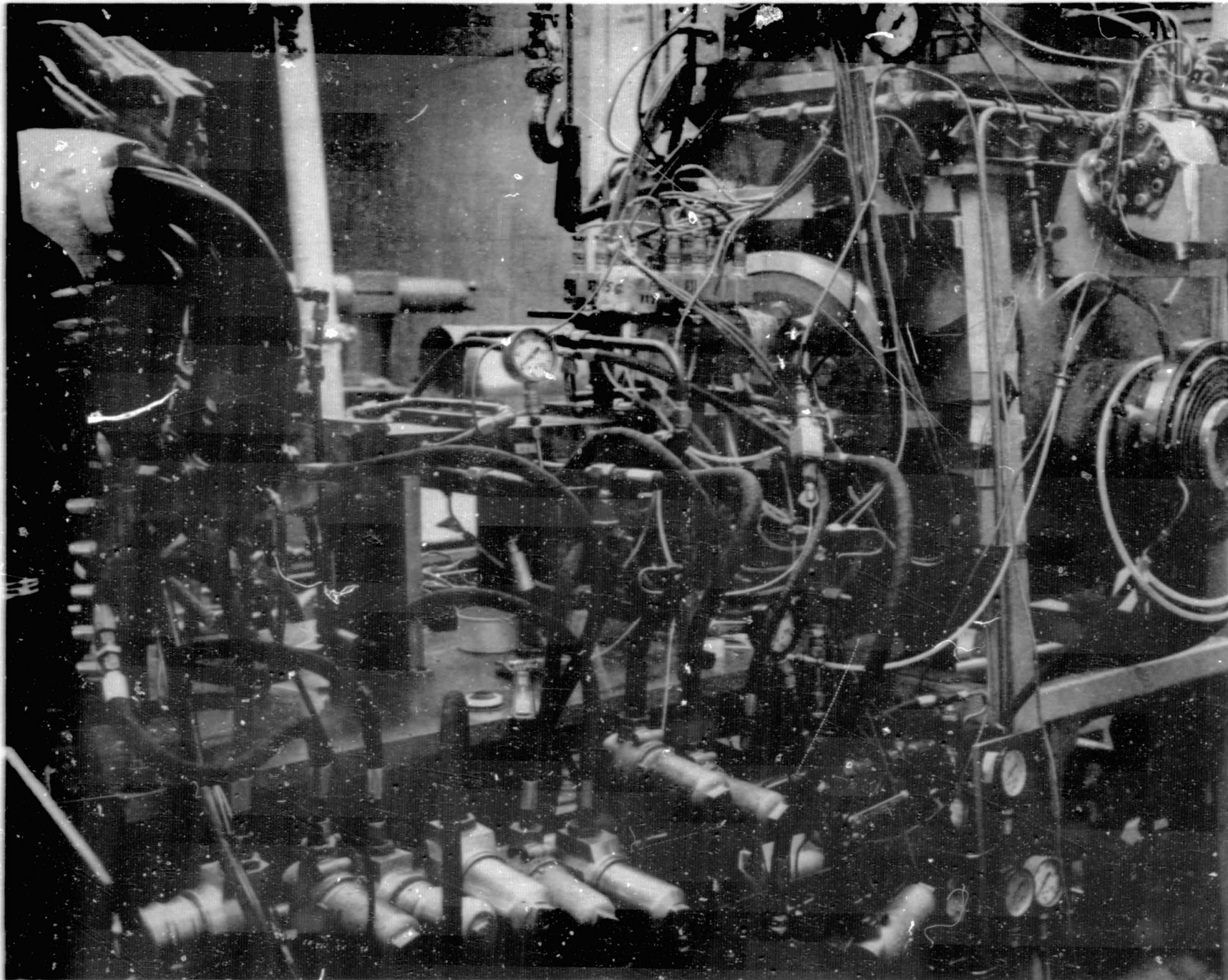


Figure 3.- Plasma tunnel facility

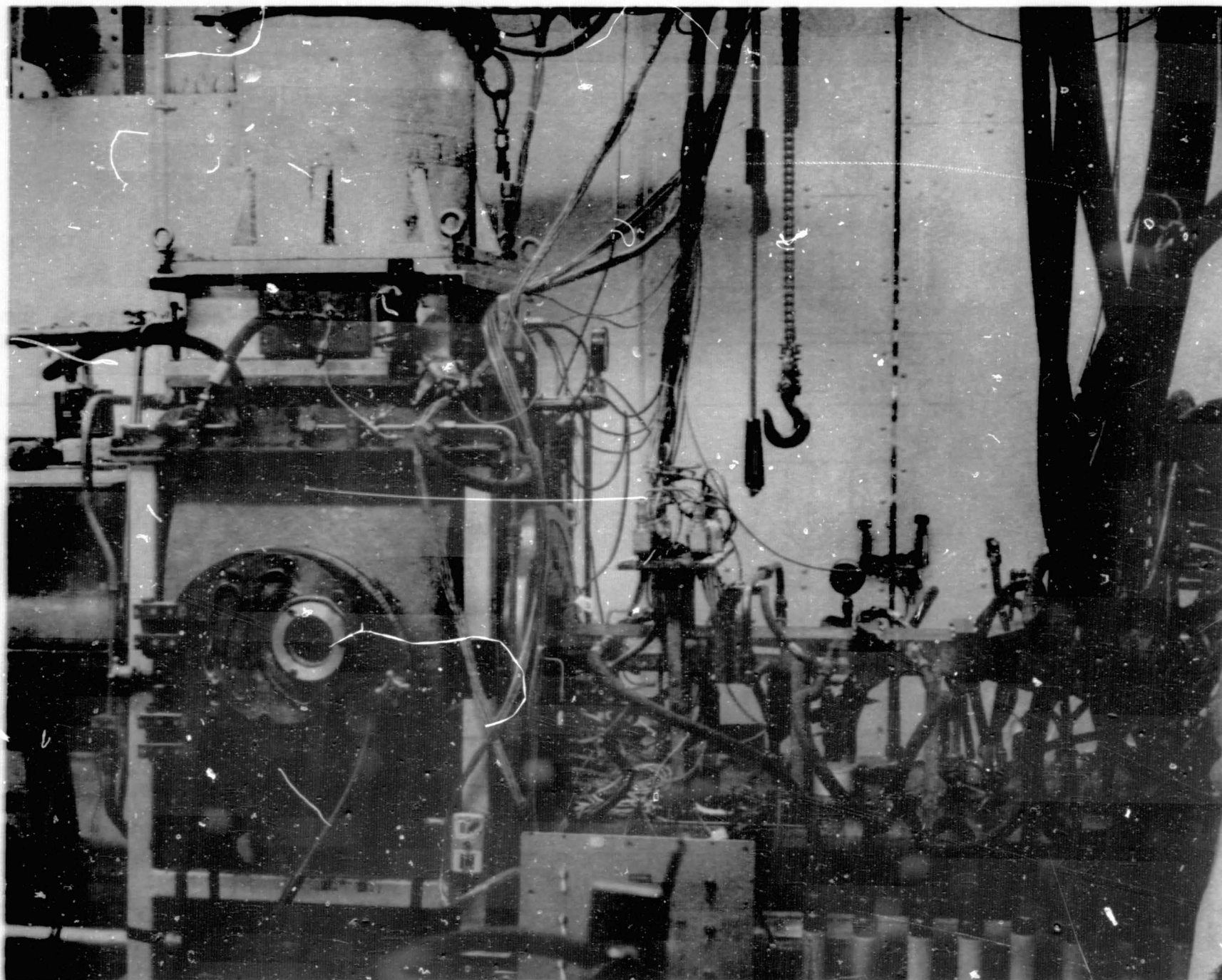


Figure 4.- Plasma tunnel facility - side view

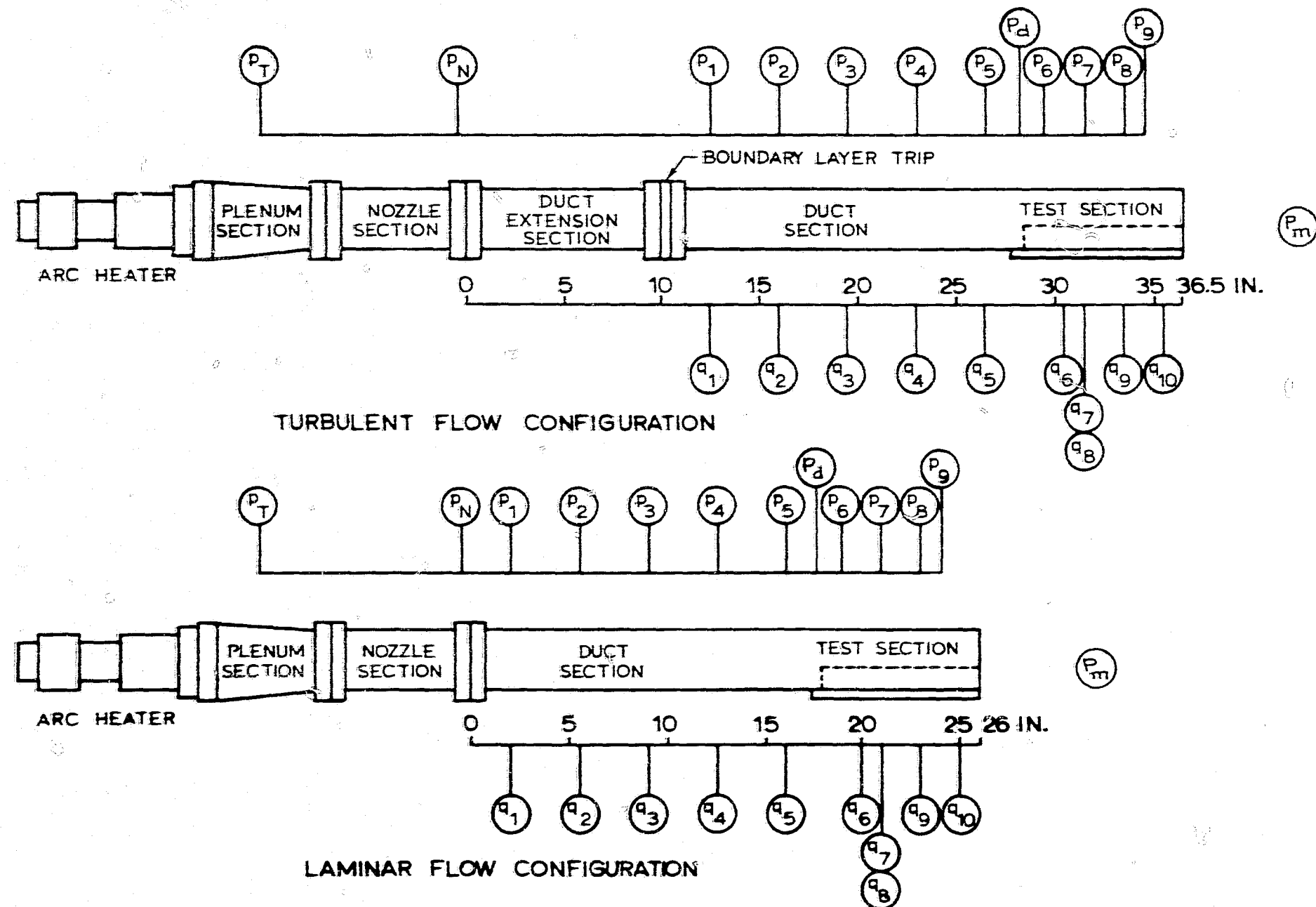


Figure 5

Diagram of the Plasma Tunnel Facility
in Laminar and Turbulent Flow Configurations

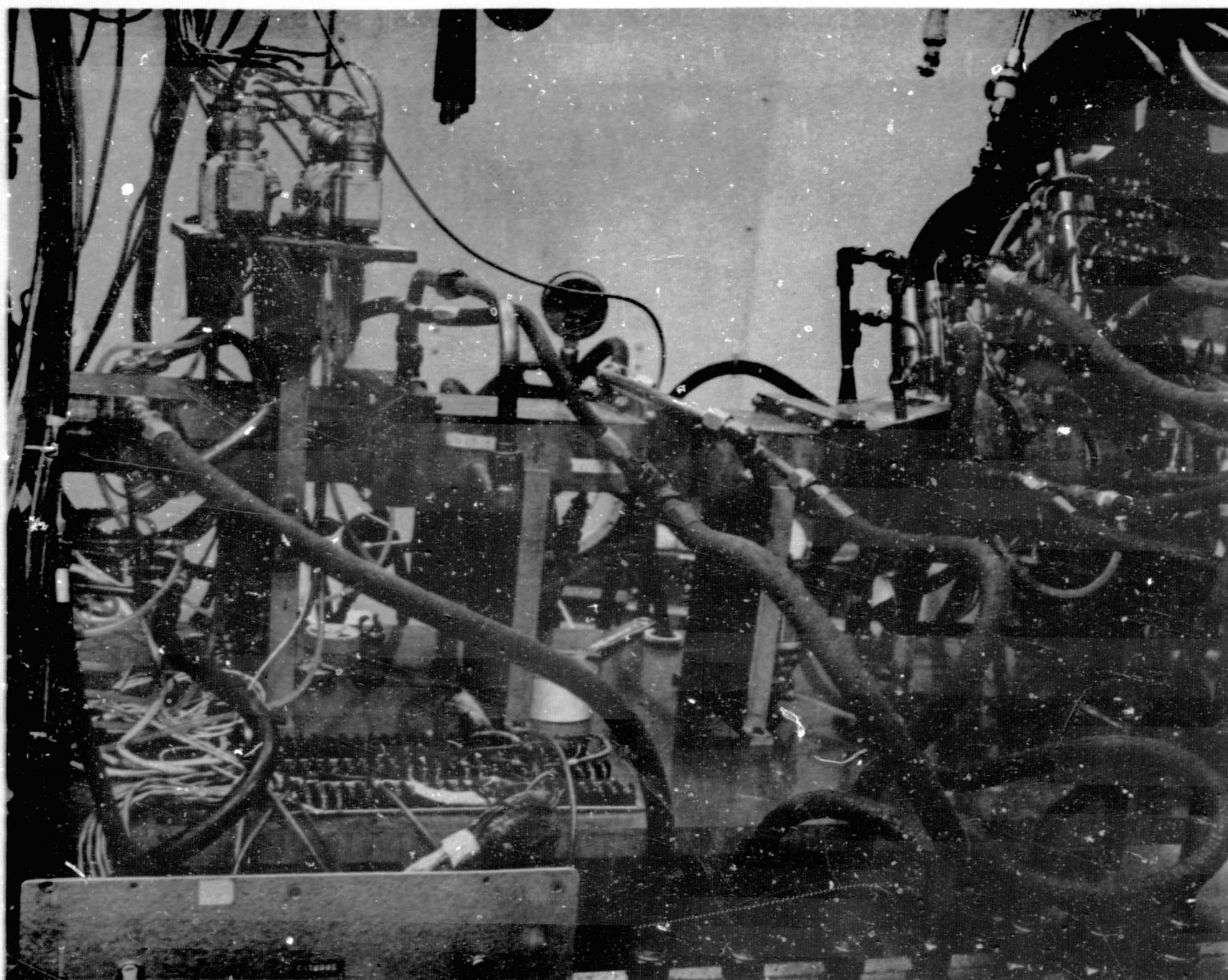


Figure 6.- Plasma tunnel in turbulent flow configuration

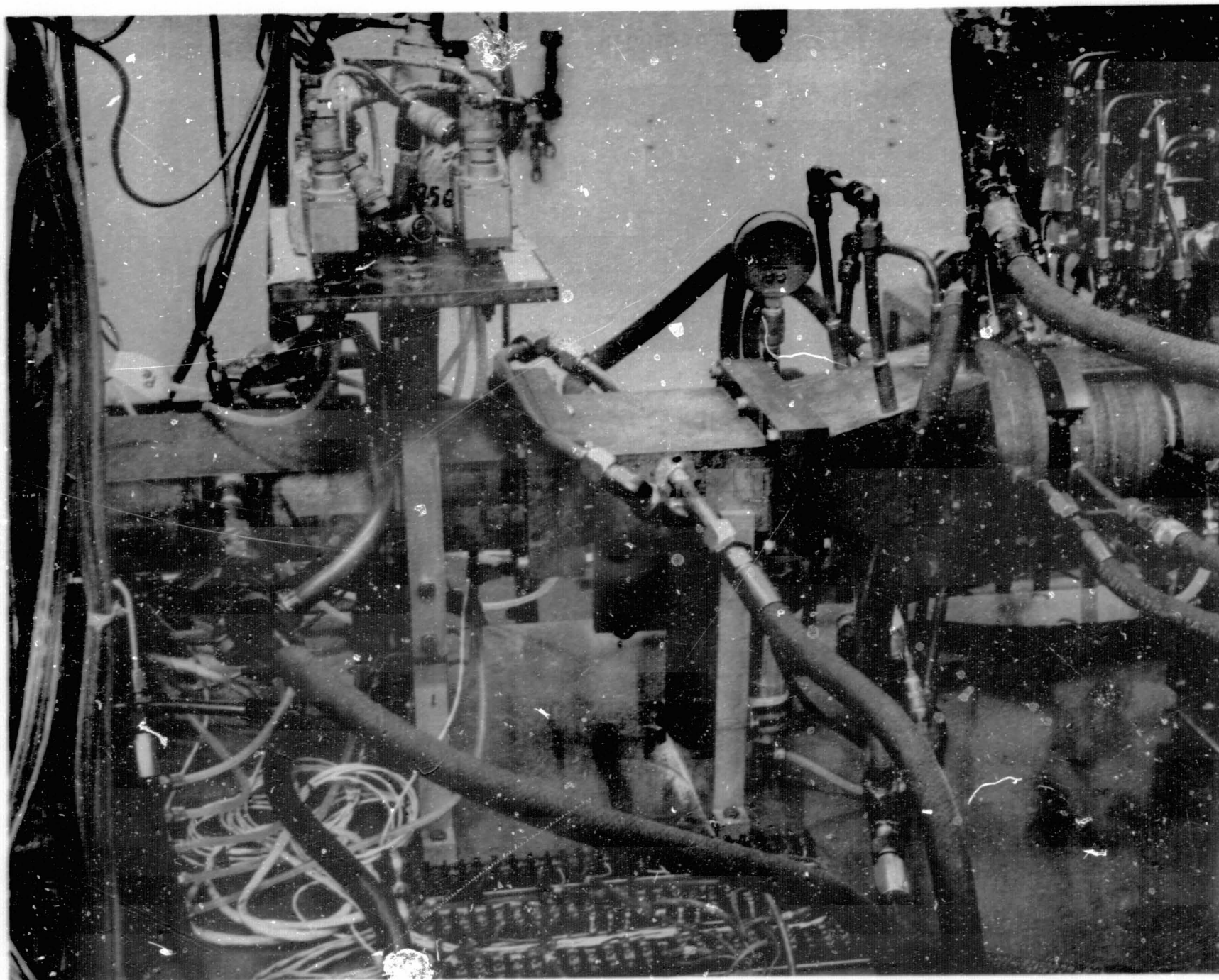


Figure 7.- Plasma tunnel in laminar flow configuration

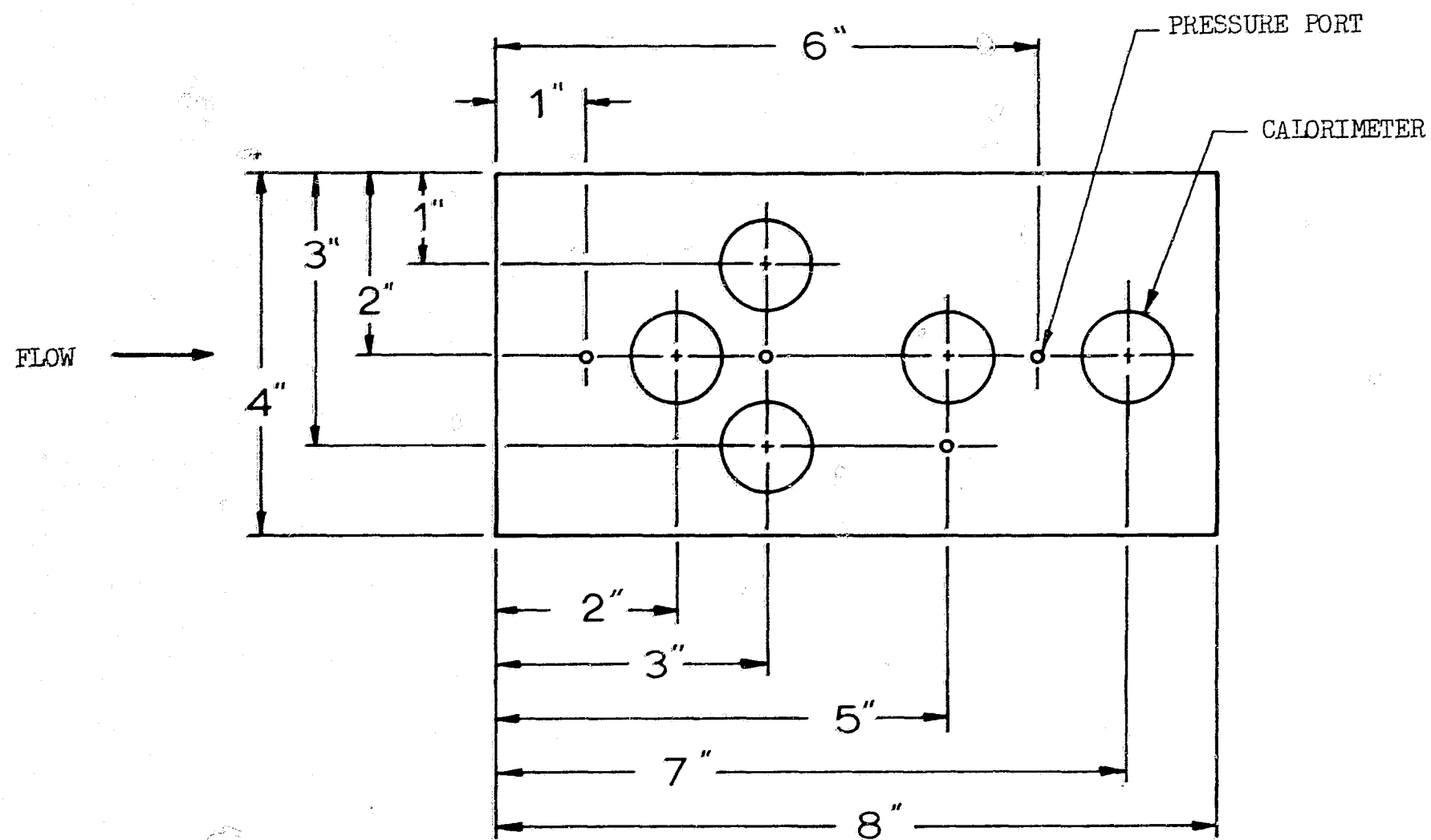


Figure 8
Test Section Calibration Plate

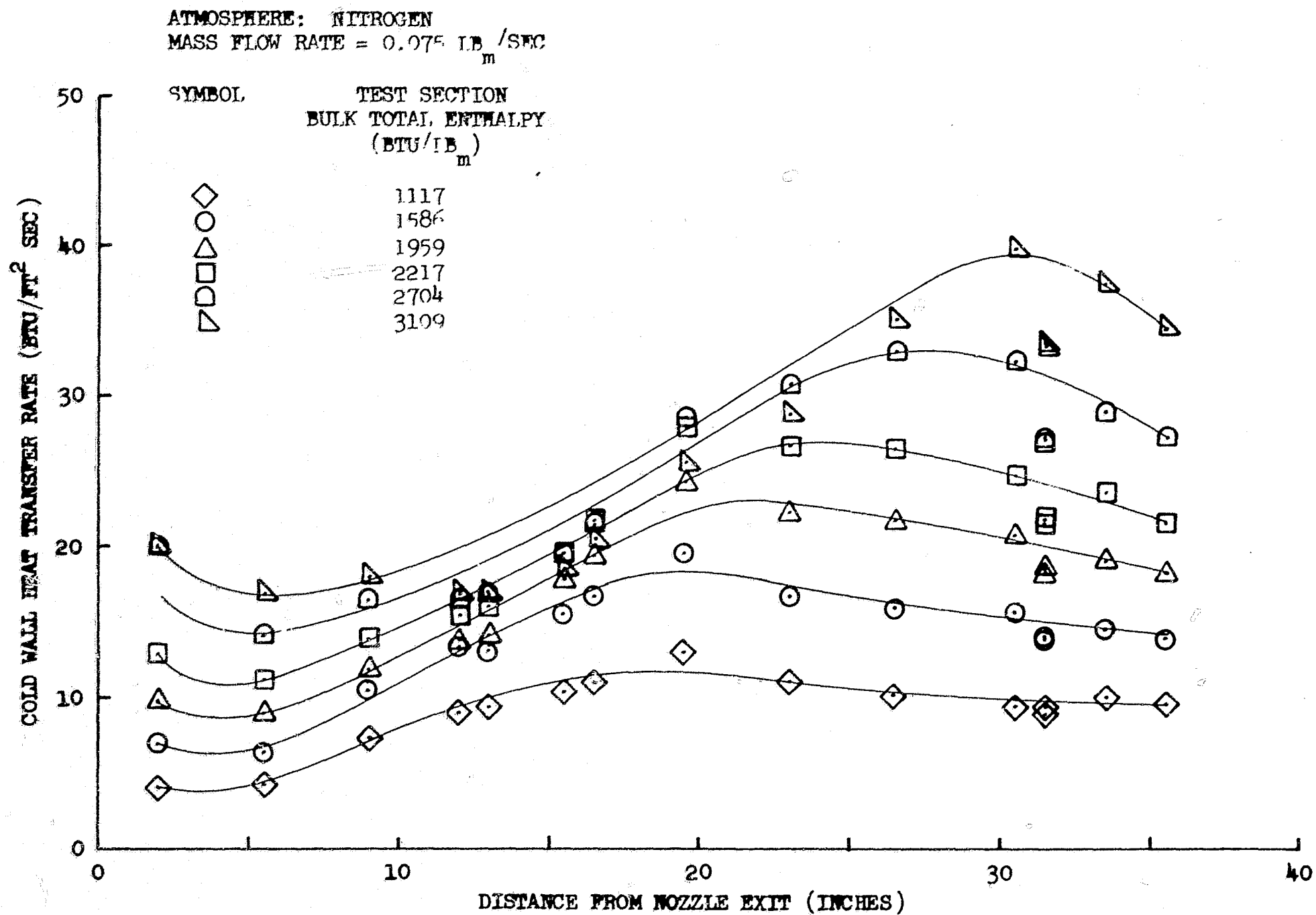


Figure 9a
 Plasma Tunnel Heat Transfer Calibration Data

ATMOSPHERE: NITROGEN
 MASS FLOW RATE = 0.075 LB_m/SEC

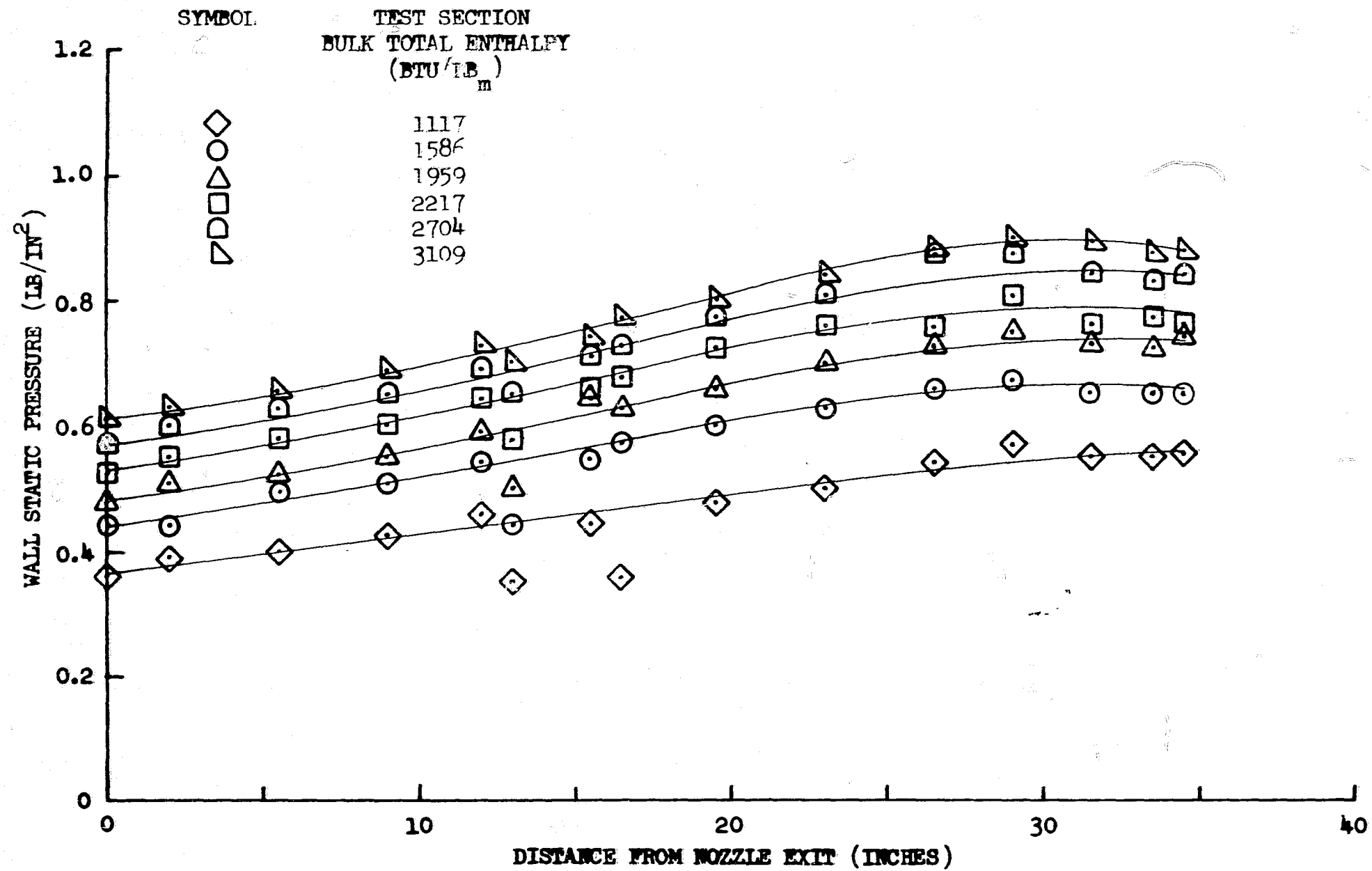


Figure 9b

Plasma Tunnel Pressure Calibration Data

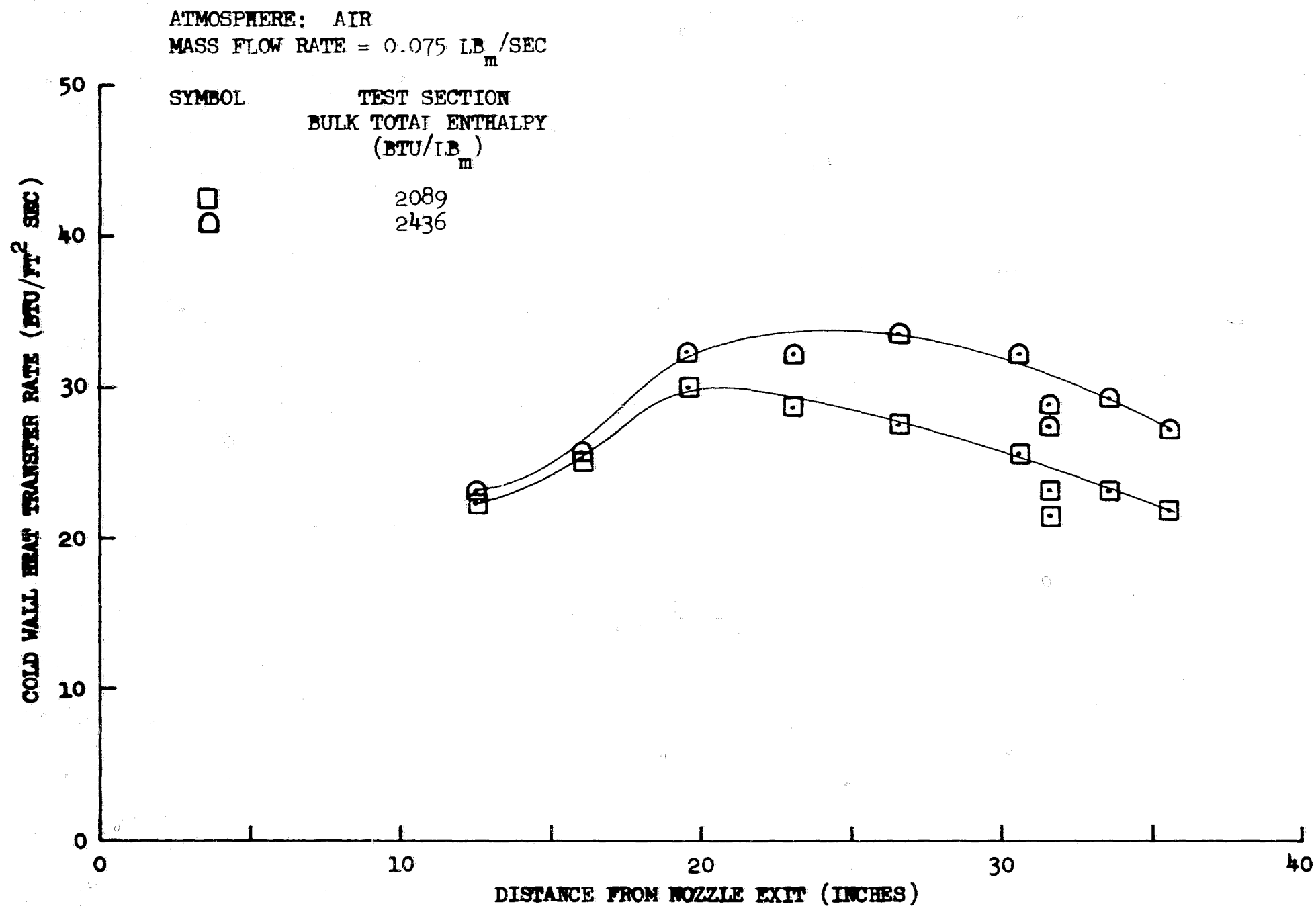


Figure 9c

Plasma Tunnel Heat Transfer Calibration Data

ATMOSPHERE: AIR
 MASS FLOW RATE = 0.075 LB / SEC
_m

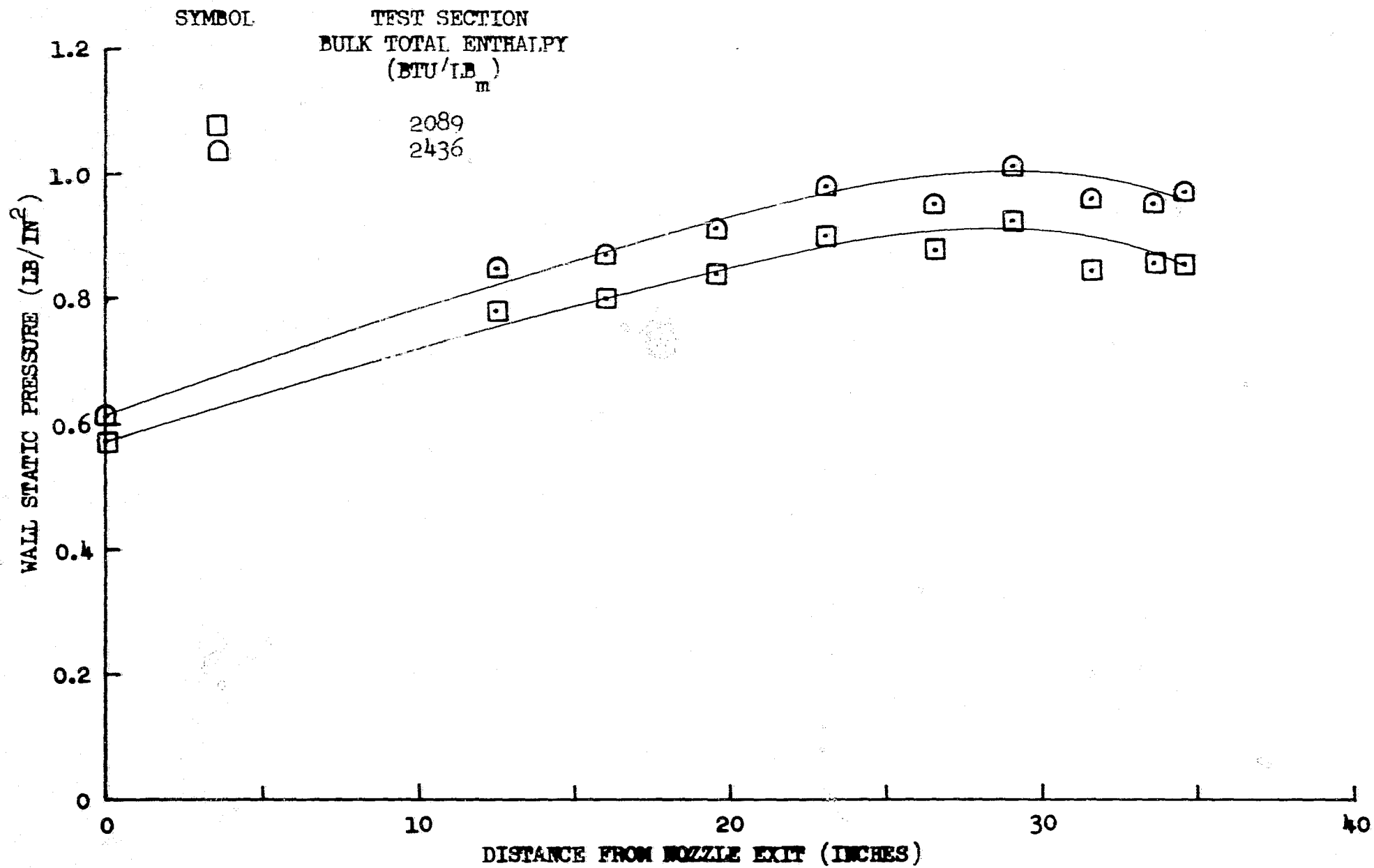


Figure 9d

Plasma Tunnel Pressure Calibration Data

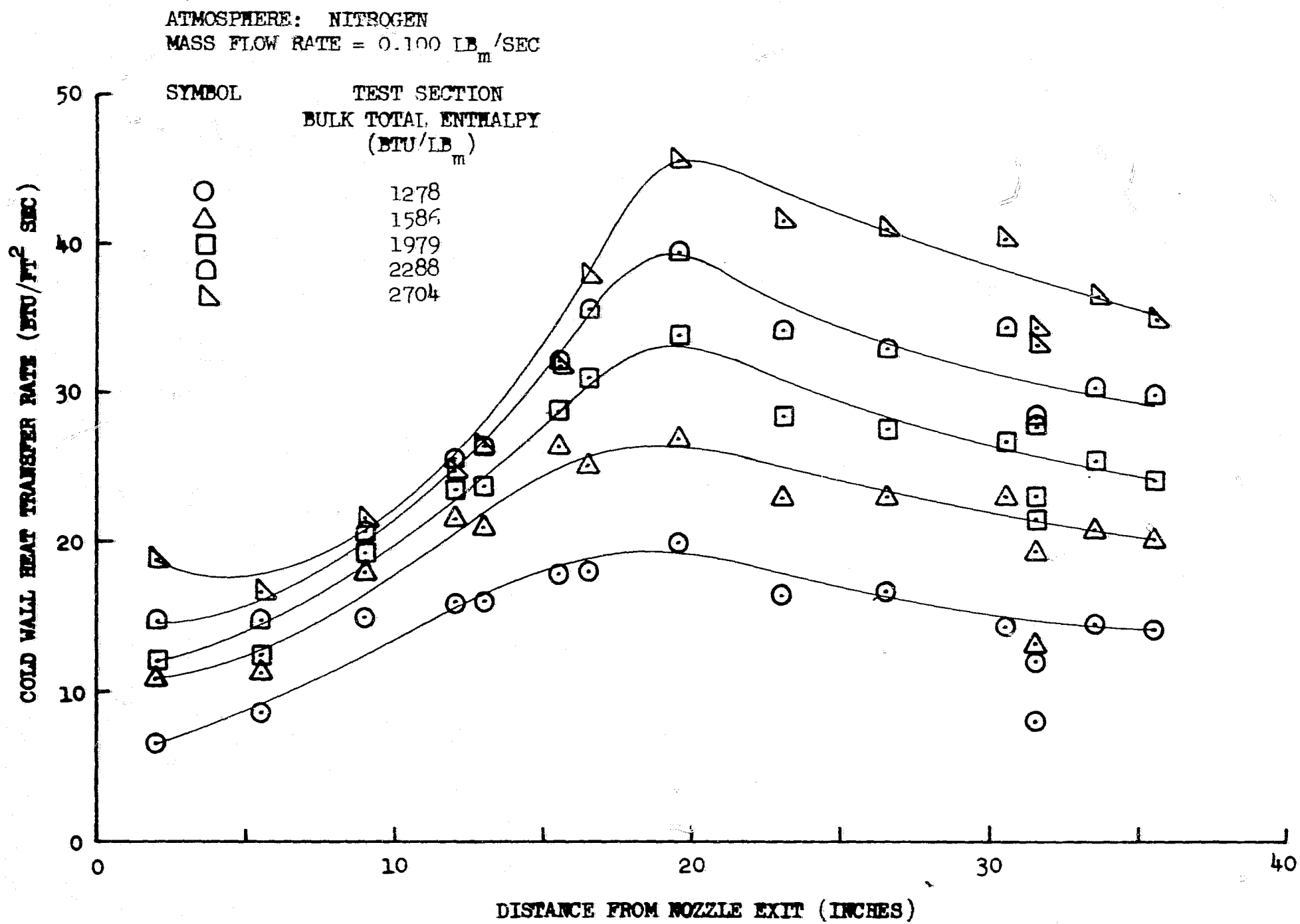


Figure 9e

Plasma Tunnel Heat Transfer Calibration Data

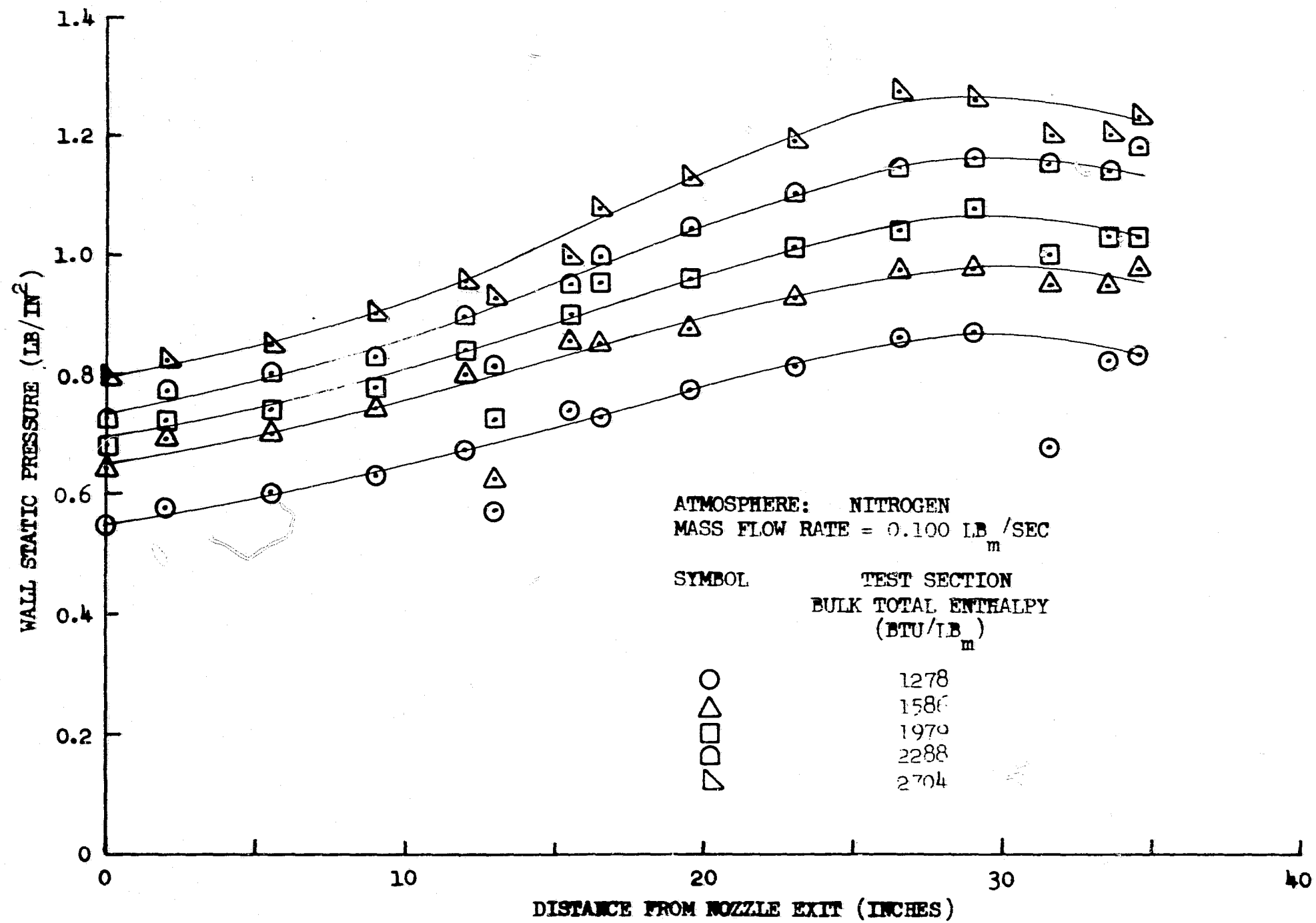


Figure 9f
Plasma Tunnel Pressure Calibration Data

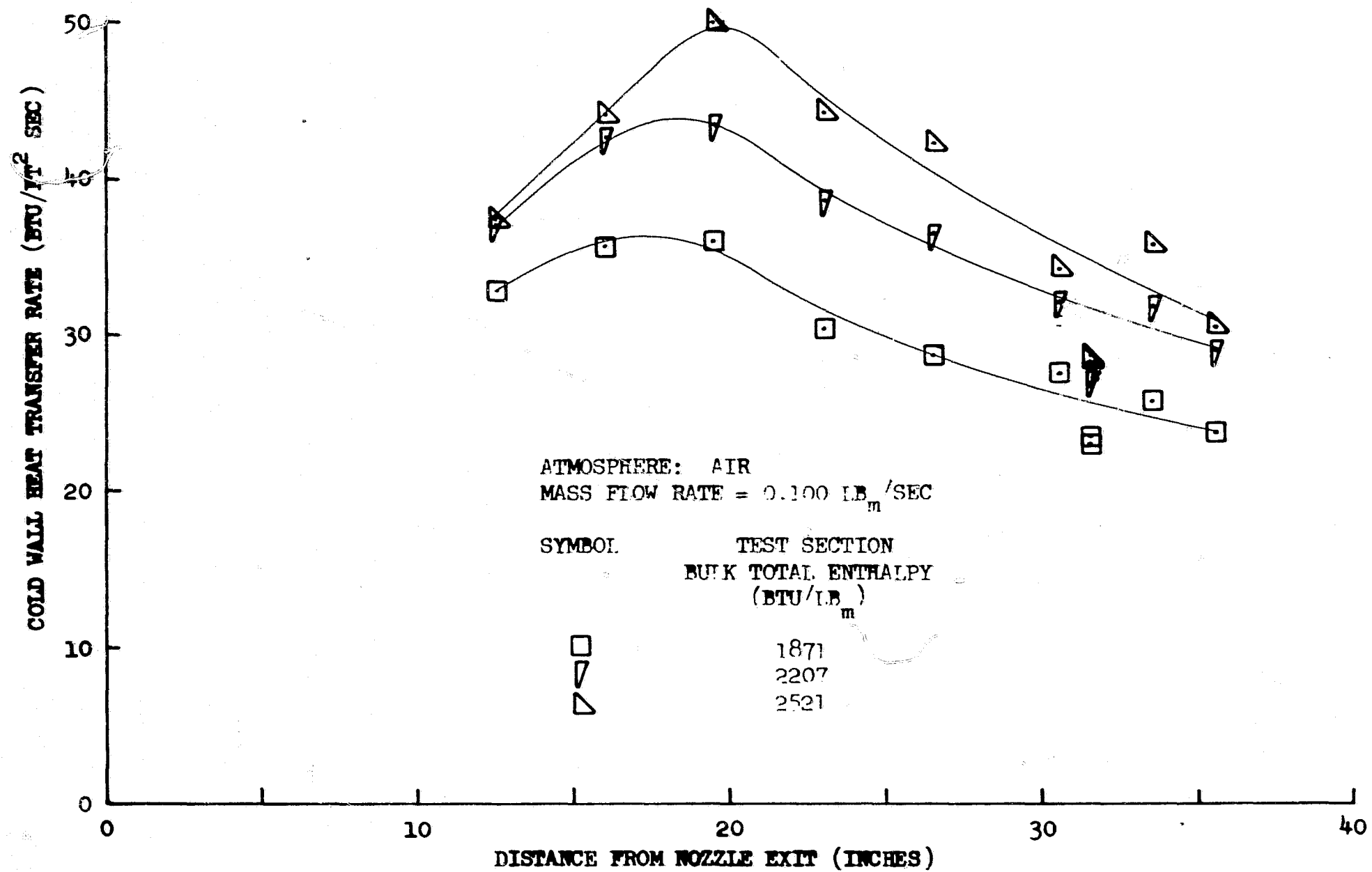


Figure 9g
 Plasma Tunnel Heat Transfer Calibration Data

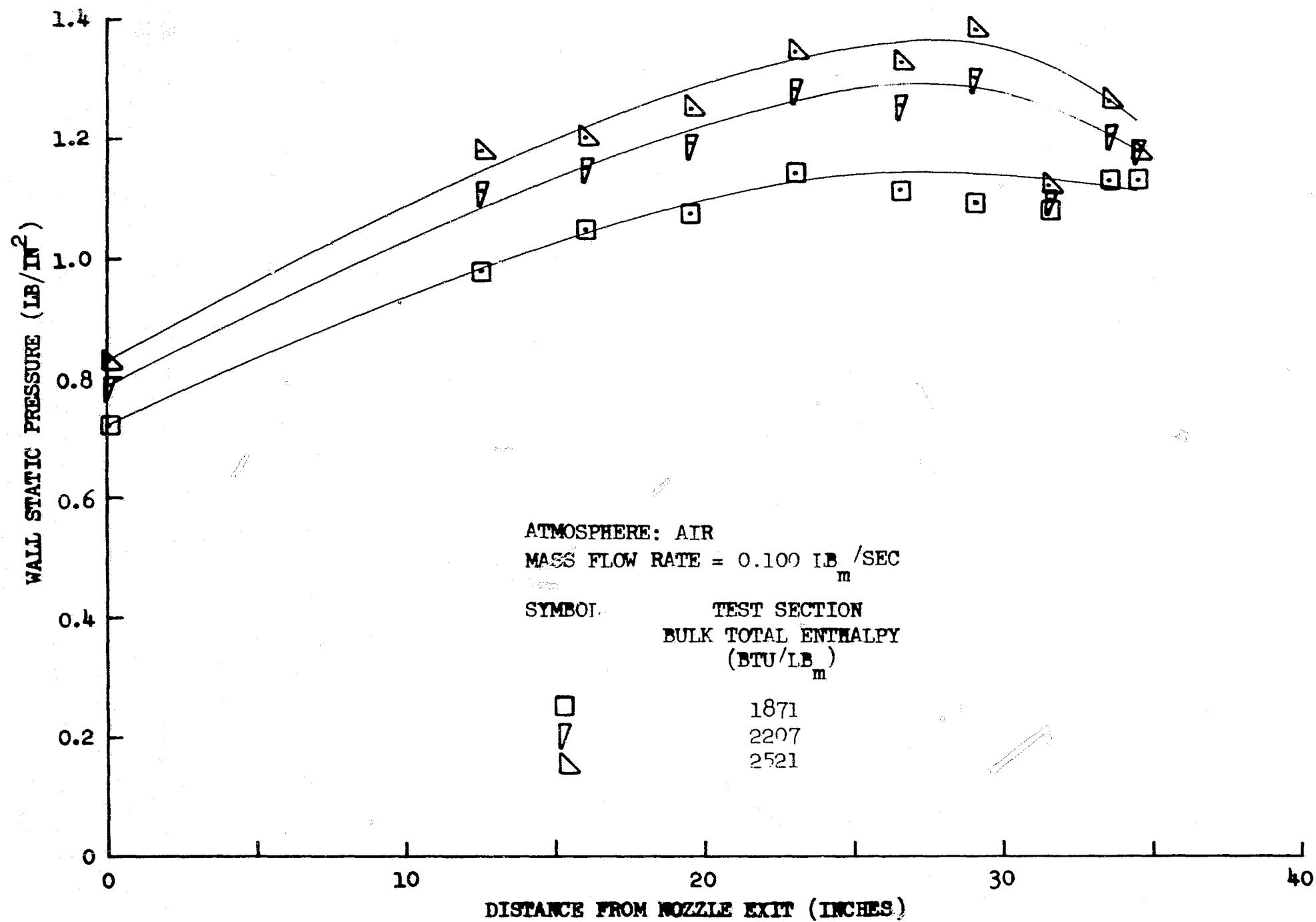


Figure 9h

Plasma Tunnel Pressure Calibration Data

ATMOSPHERE: NITROGEN
 MASS FLOW RATE = $0.060 \text{ lb}_m/\text{SEC}$

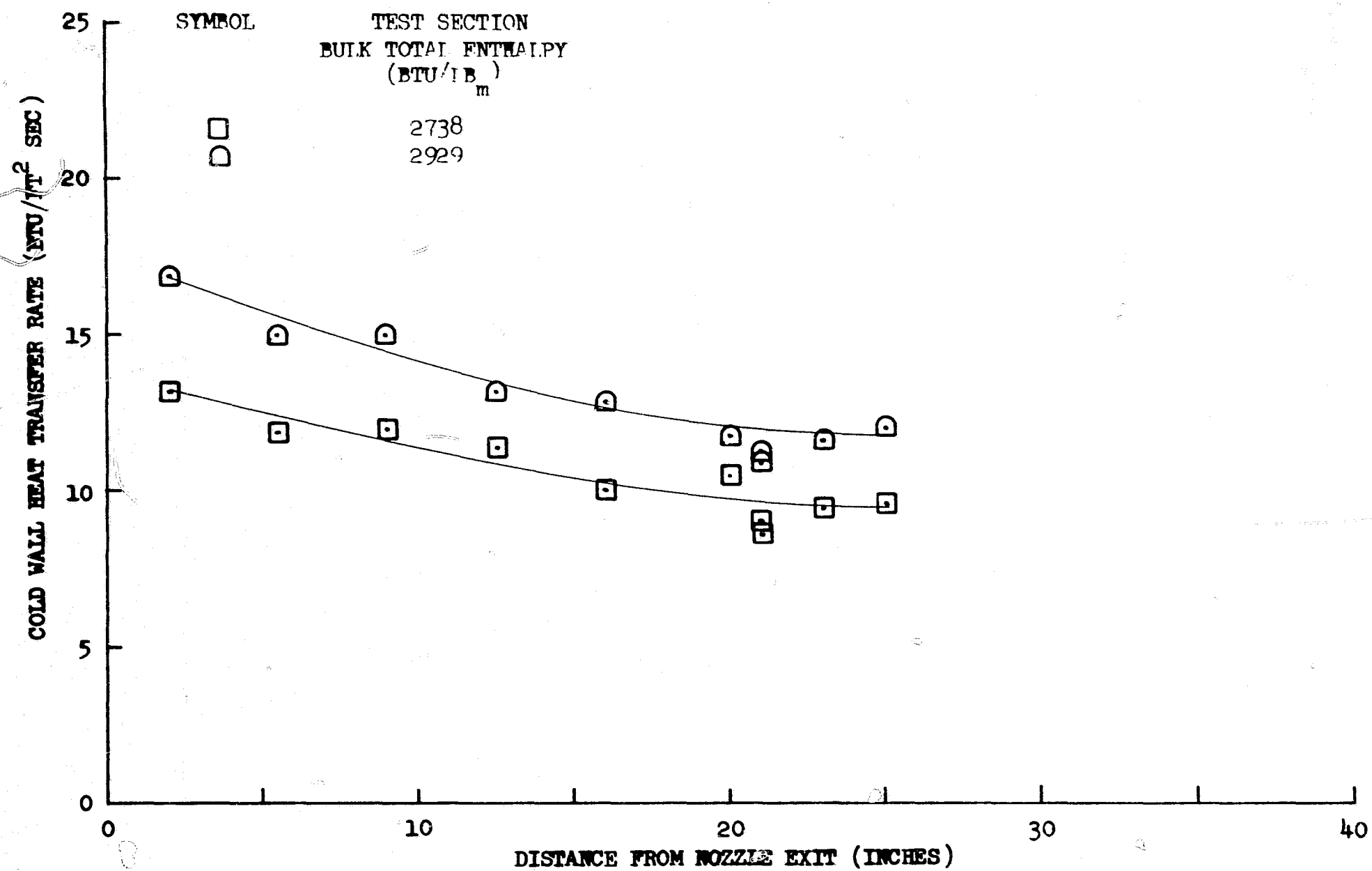


Figure 9i

Plasma Tunnel Heat Transfer Calibration Data

ATMOSPHERE: NITROGEN
MASS FLOW RATE = 0.000 lb_m/SEC

SYMBOL TEST SECTION
 BULK TOTAL ENTHALPY
 (BTU/lb_m)

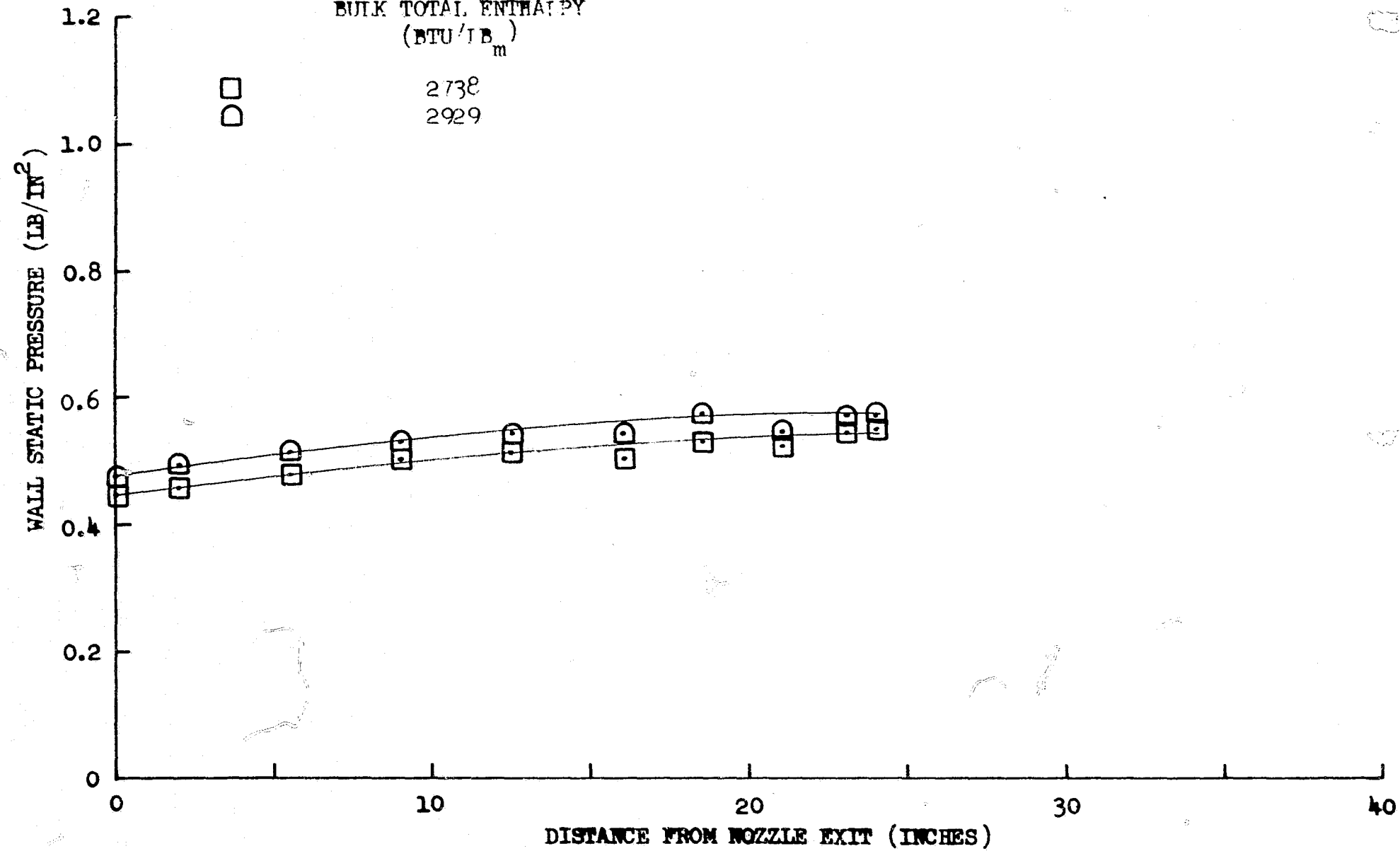


Figure 9j
Plasma Tunnel Pressure Calibration Data

ATMOSPHERE: AIR
MASS FLOW RATE = 0.000 LB_m/SEC

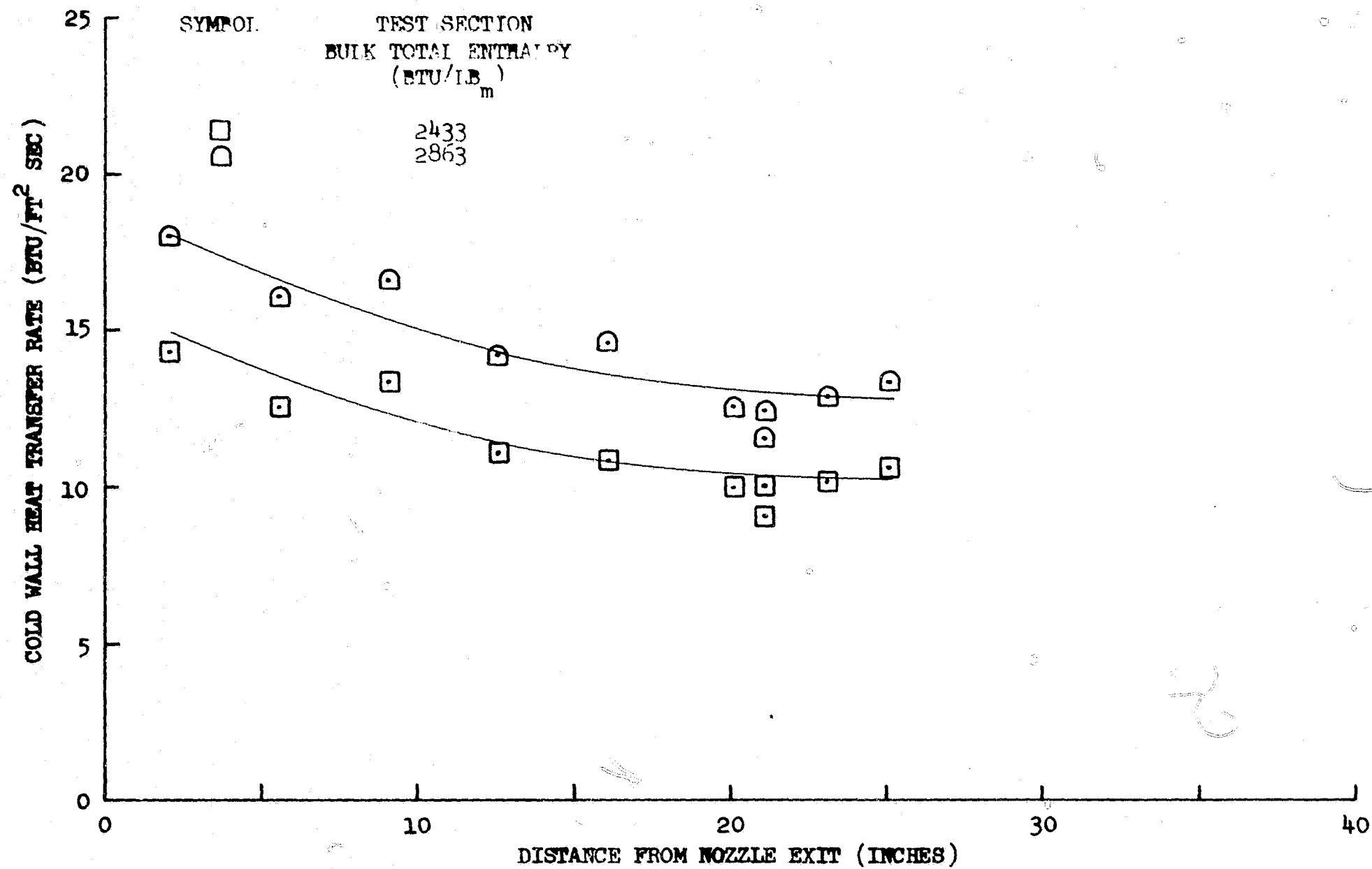


Figure 9k

Plasma Tunnel Heat Transfer Calibration Data

ATMOSPHERE: AIR
MASS FLOW RATE = 0.060 lb_m/SEC

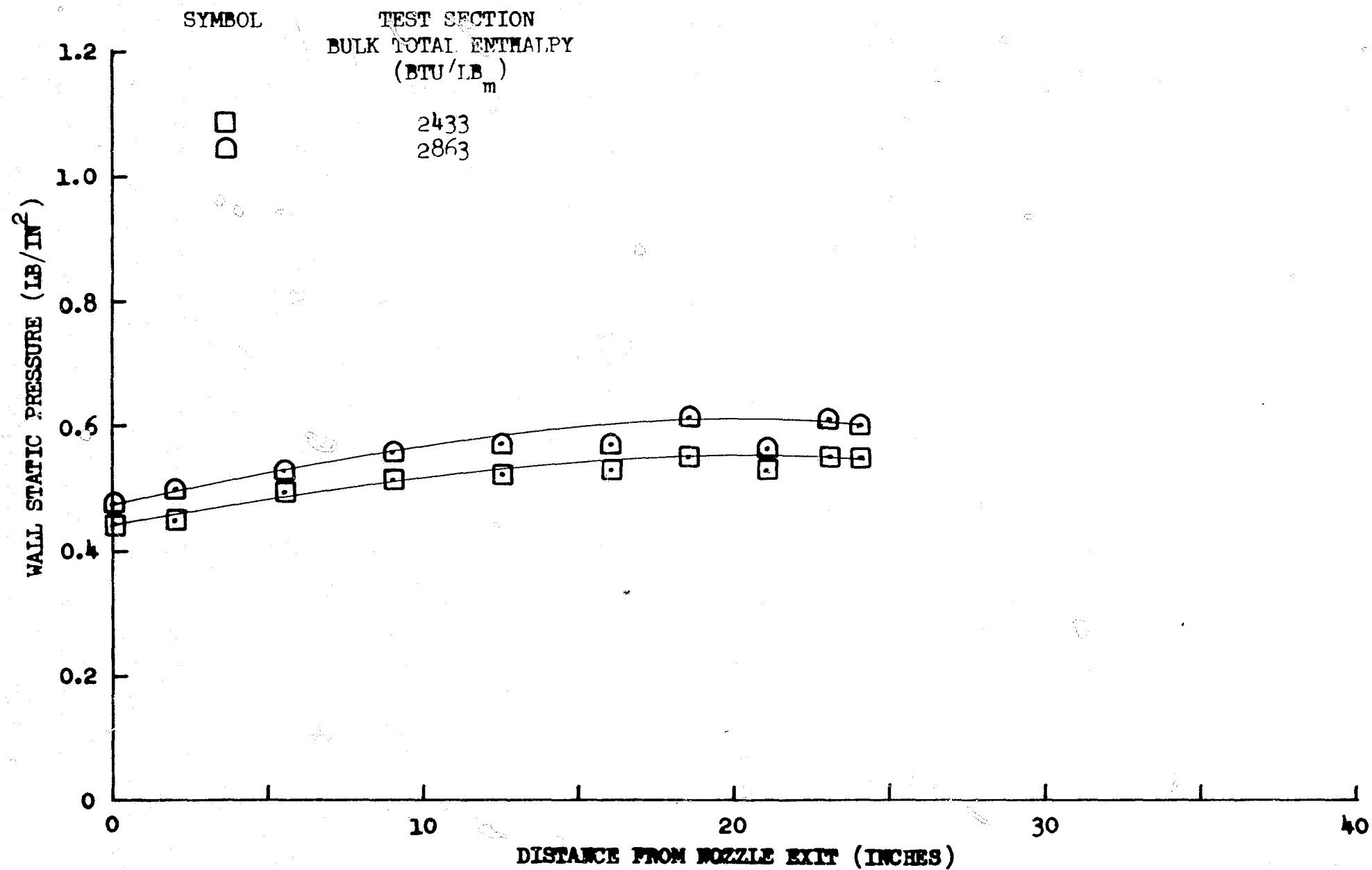


Figure 91

Plasma Tunnel Pressure Calibration Data

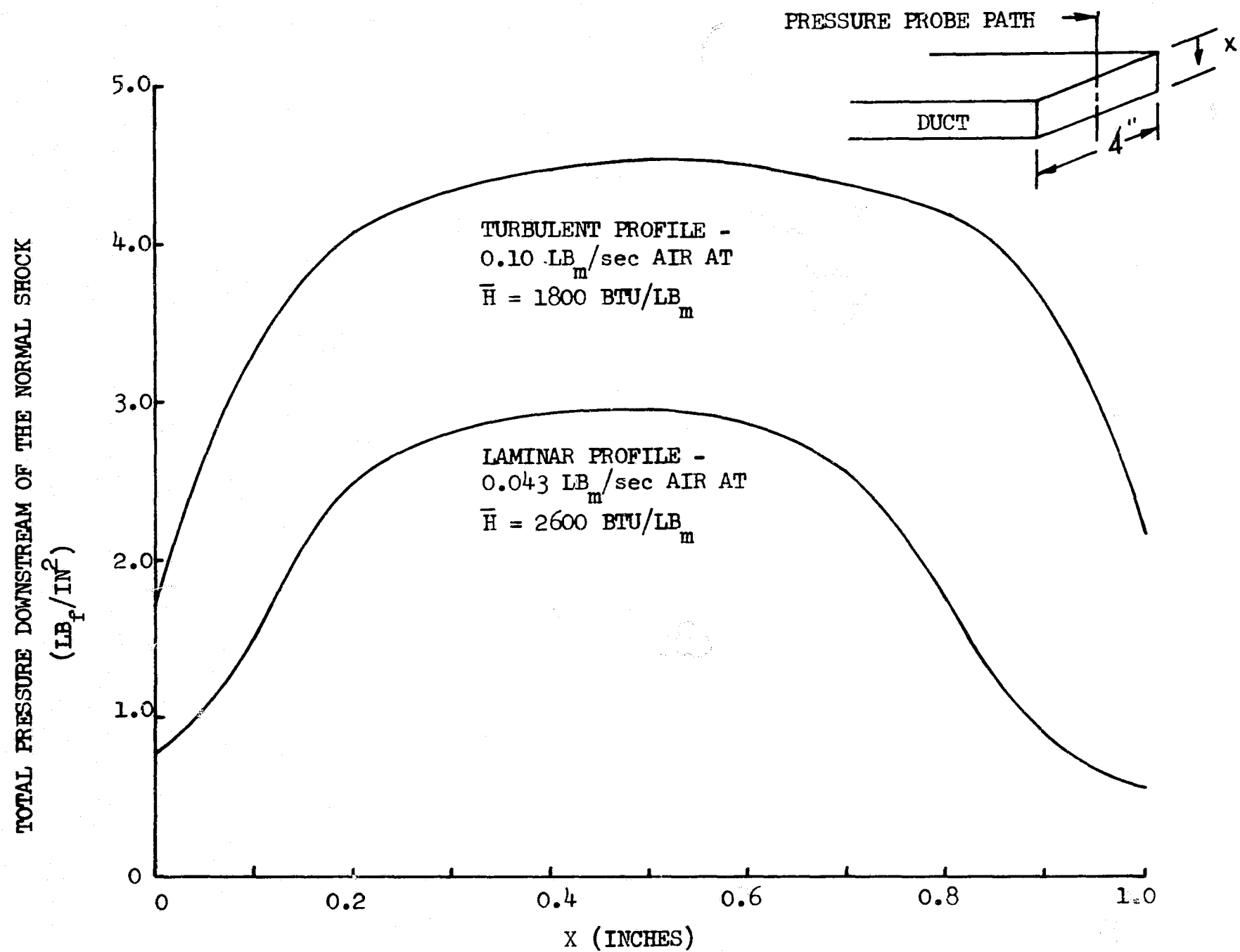


Figure 10

Centerline pressure profiles in
the tunnel model section.

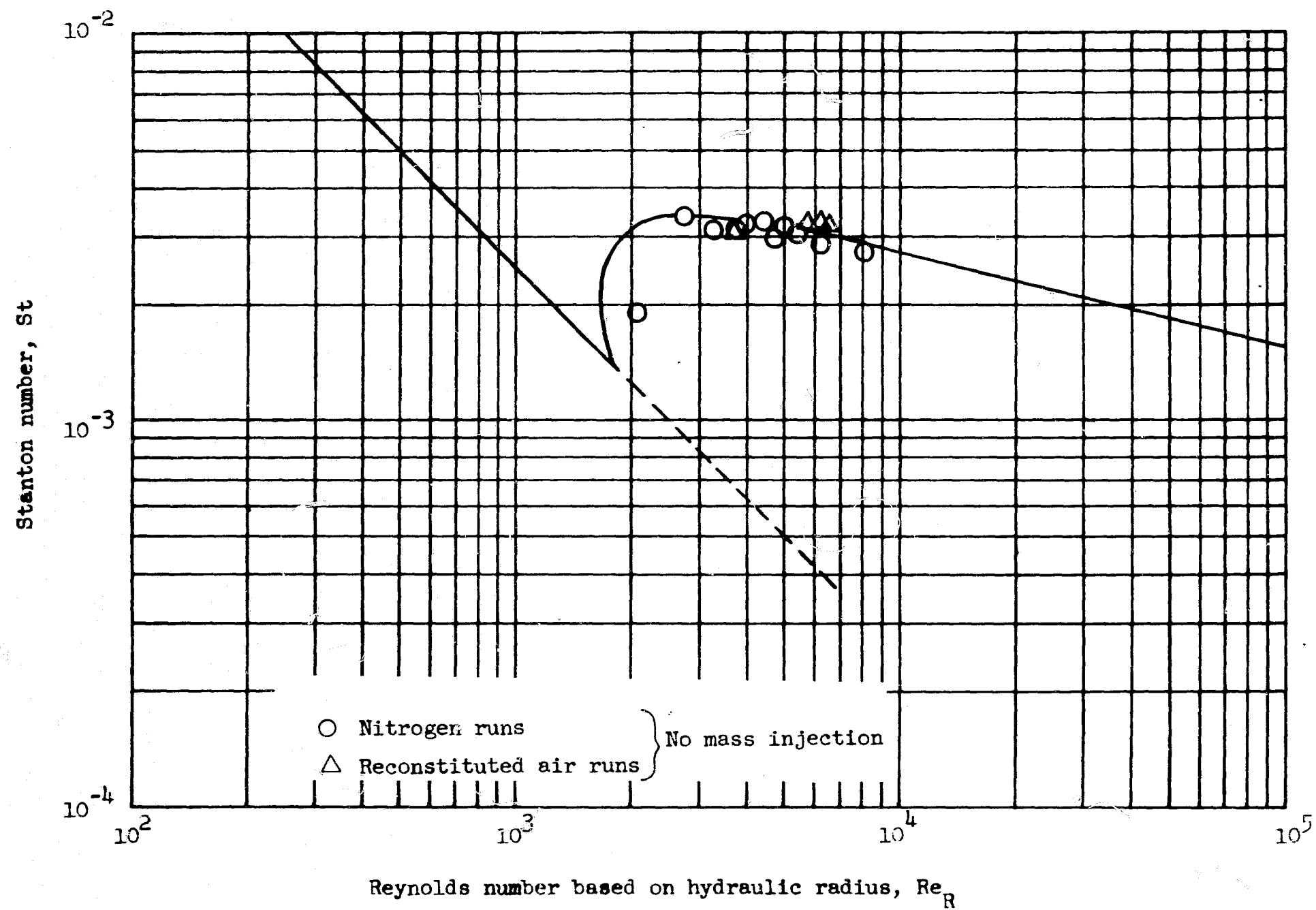
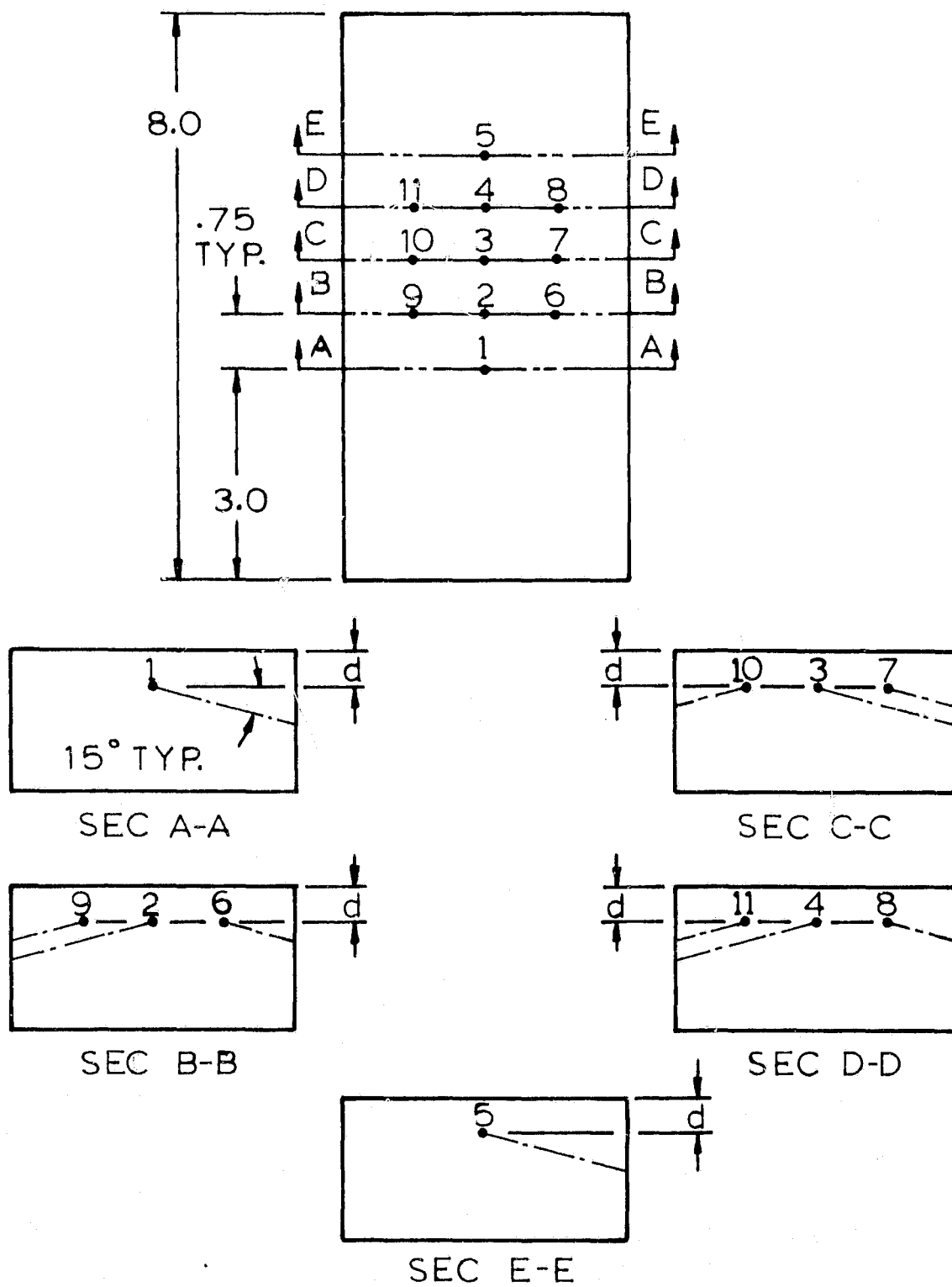


Figure 11.- Comparison of experimental heat transfer to computer prediction of heat transfer (non-blowing case)



GROUP	NOMINAL TC DEPTH d		
	TC 1,2,6,9	TC 3,7,10	TC 4,5,8,11
1	0.05	0.10	0.15
2	0.05	0.15	0.30
3	0.10	0.30	0.60

Figure 12

Ablation Model Thermocouple Arrangement

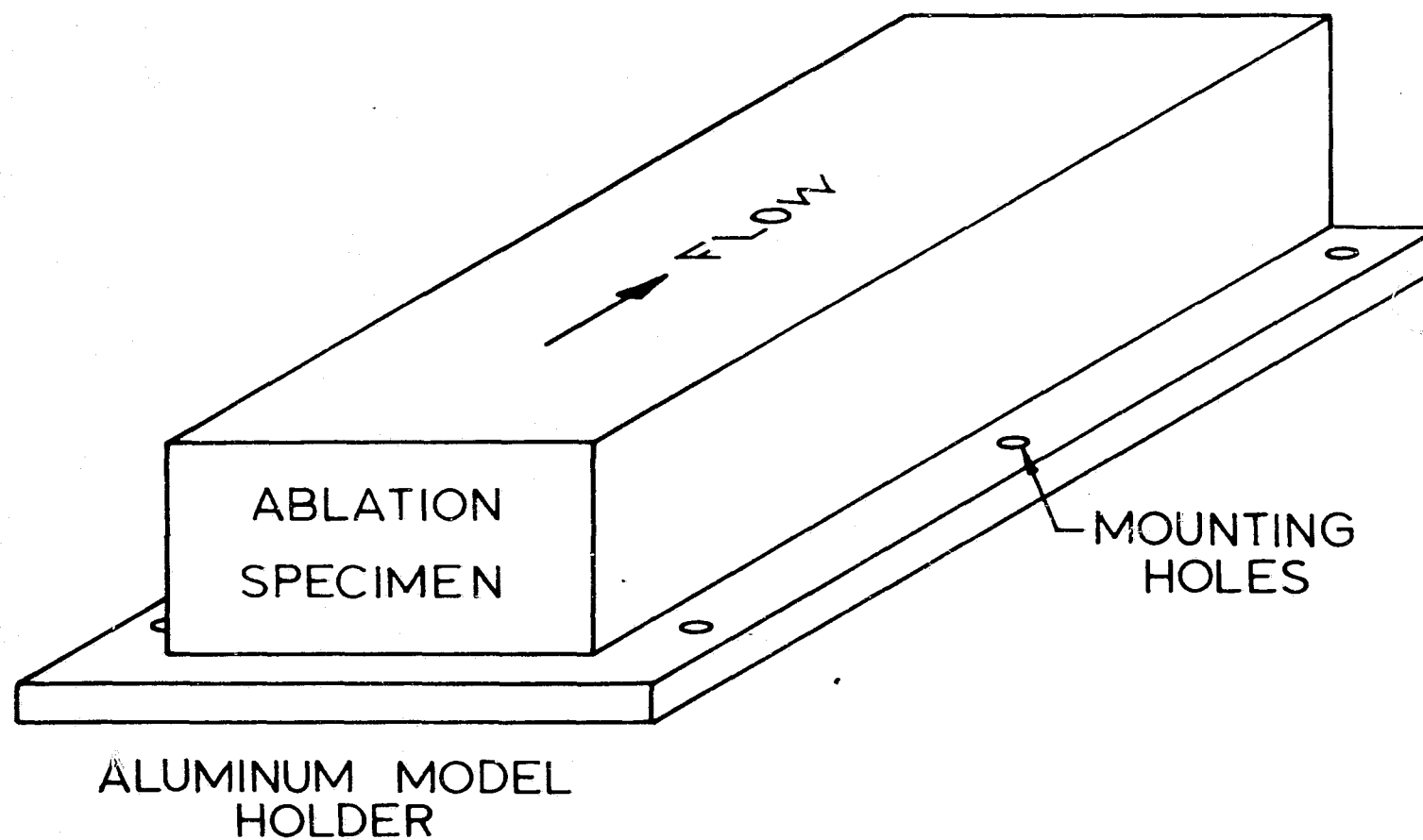


Figure 13

Typical Ablation Model on its Mounting Plate

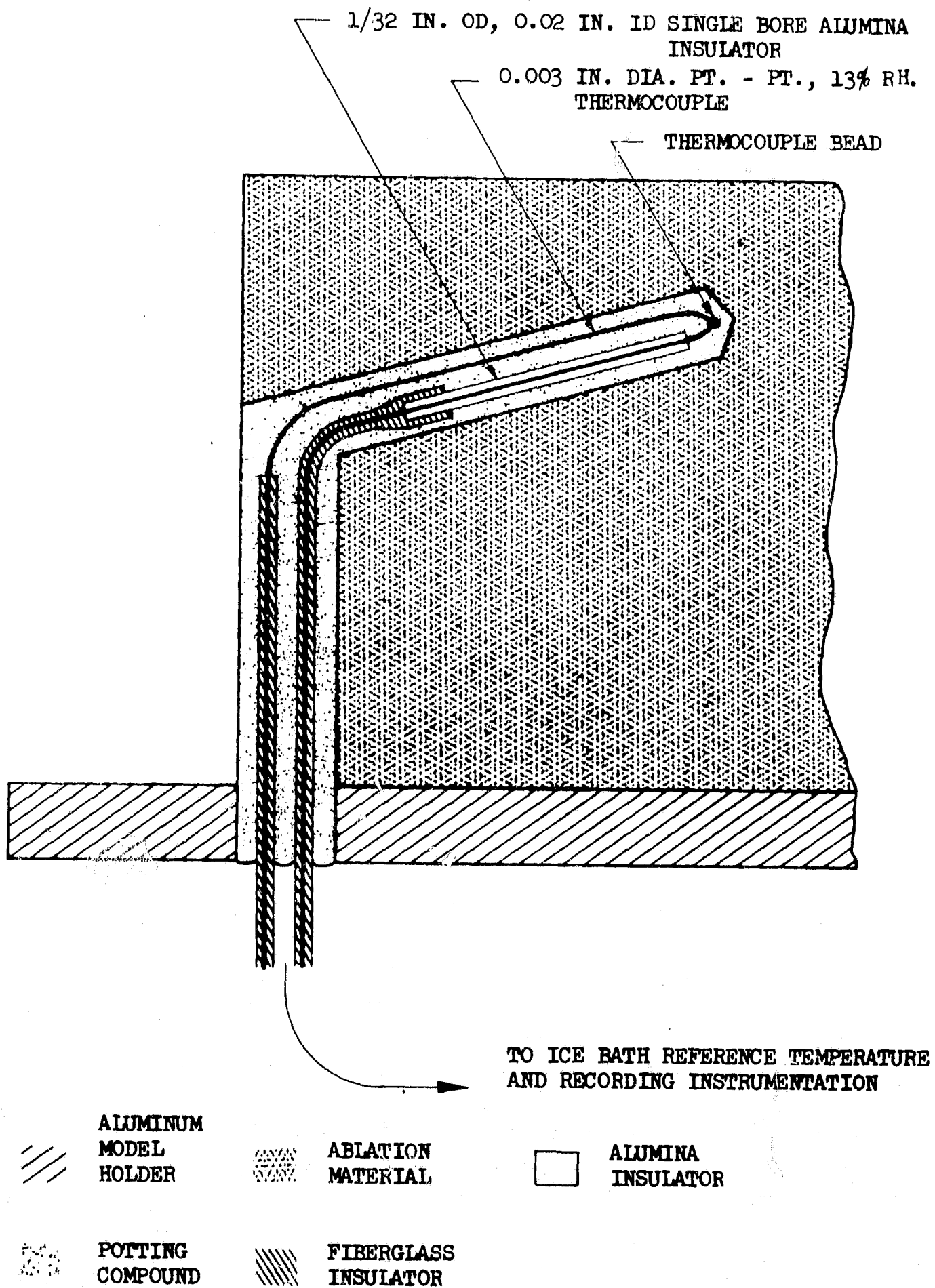
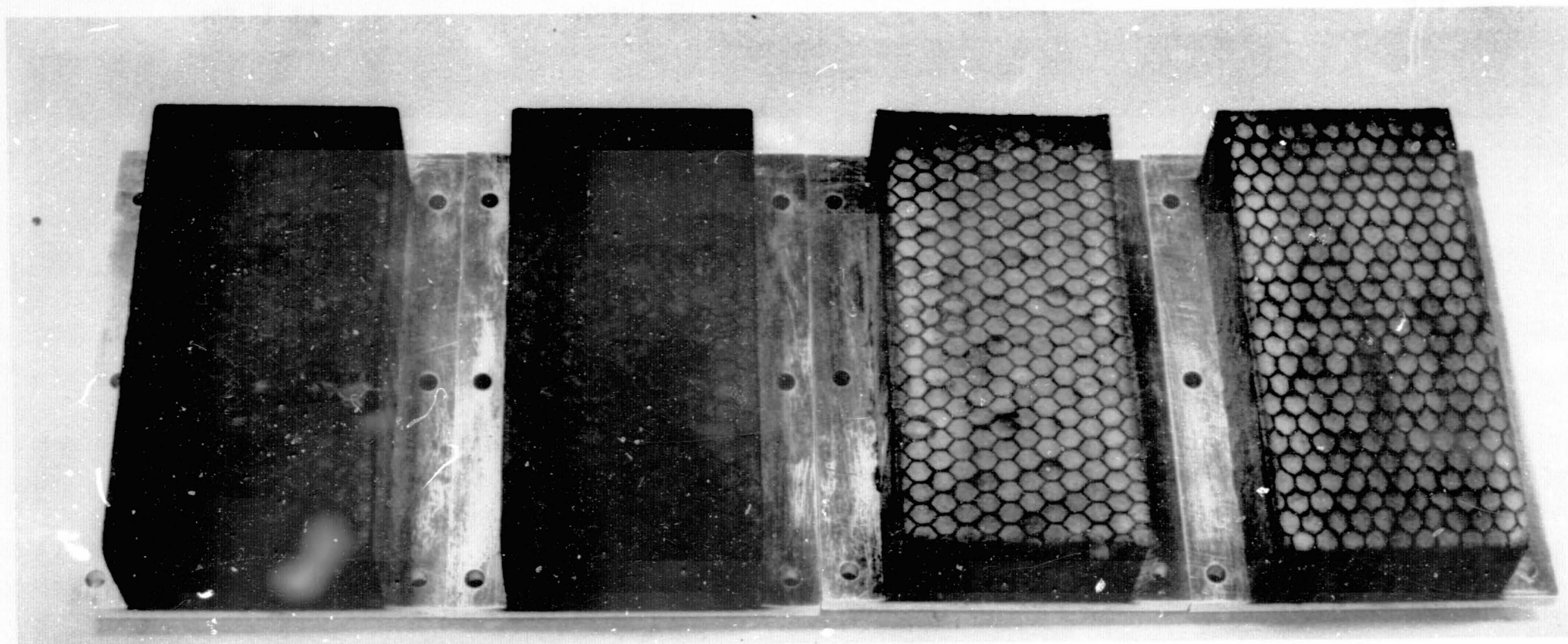


Figure 14
Typical Thermocouple Installation



Model 1

$$\dot{q}_0 = 4.3$$

$$H_{0,\sigma} = 0.0038$$

Model 2

$$\dot{q}_0 = 11.6$$

$$H_{0,\sigma} = 0.0039$$

Model 3

$$\dot{q}_0 = 14.5$$

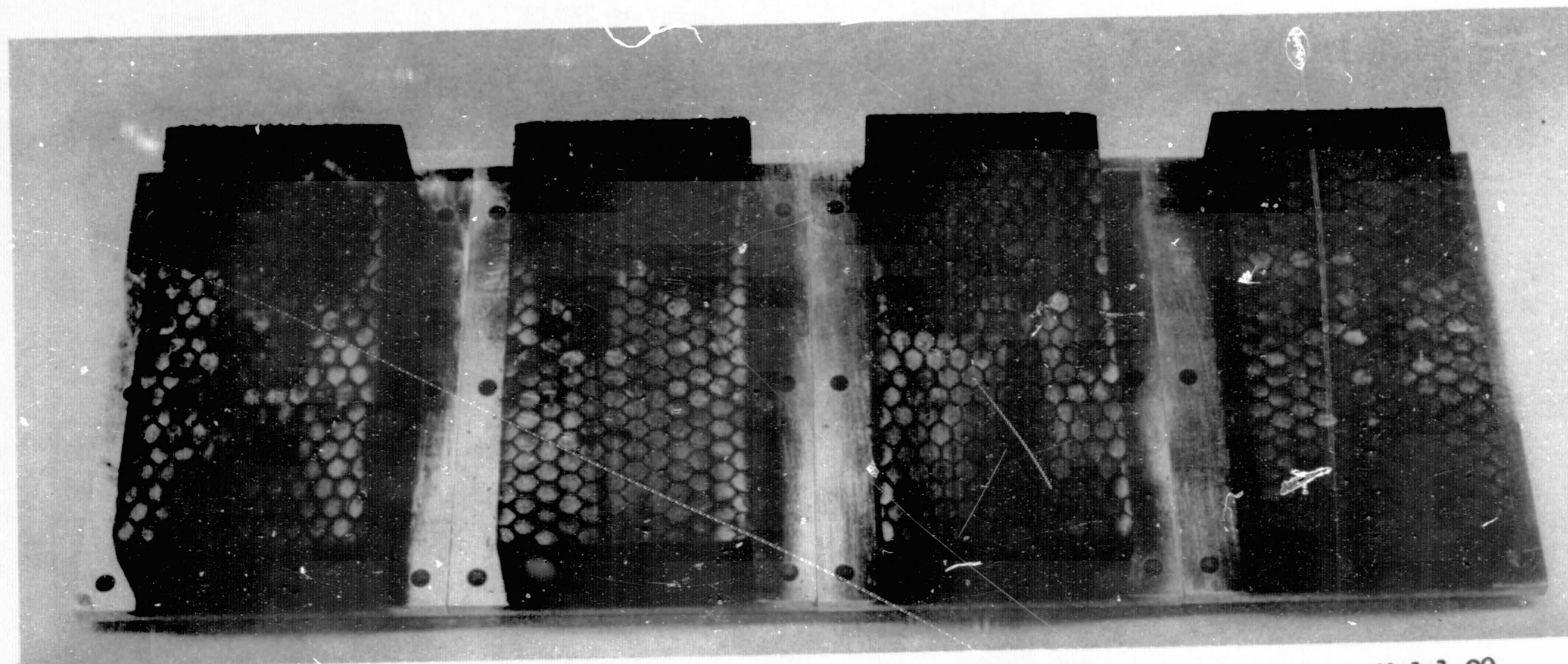
$$H_{0,\sigma} = 0.0049$$

Model 4

$$\dot{q}_0 = 17.0$$

$$H_{0,\sigma} = 0.0052$$

Figure 15.- Photograph of laminar models 1 - 4



Model 17

$$\dot{q}_0 = 21.0$$

$$H_{0,s} = 0.0097$$

Model 18

$$\dot{q}_0 = 28.0$$

$$H_{0,s} = 0.0109$$

Model 19

$$\dot{q}_0 = 30.0$$

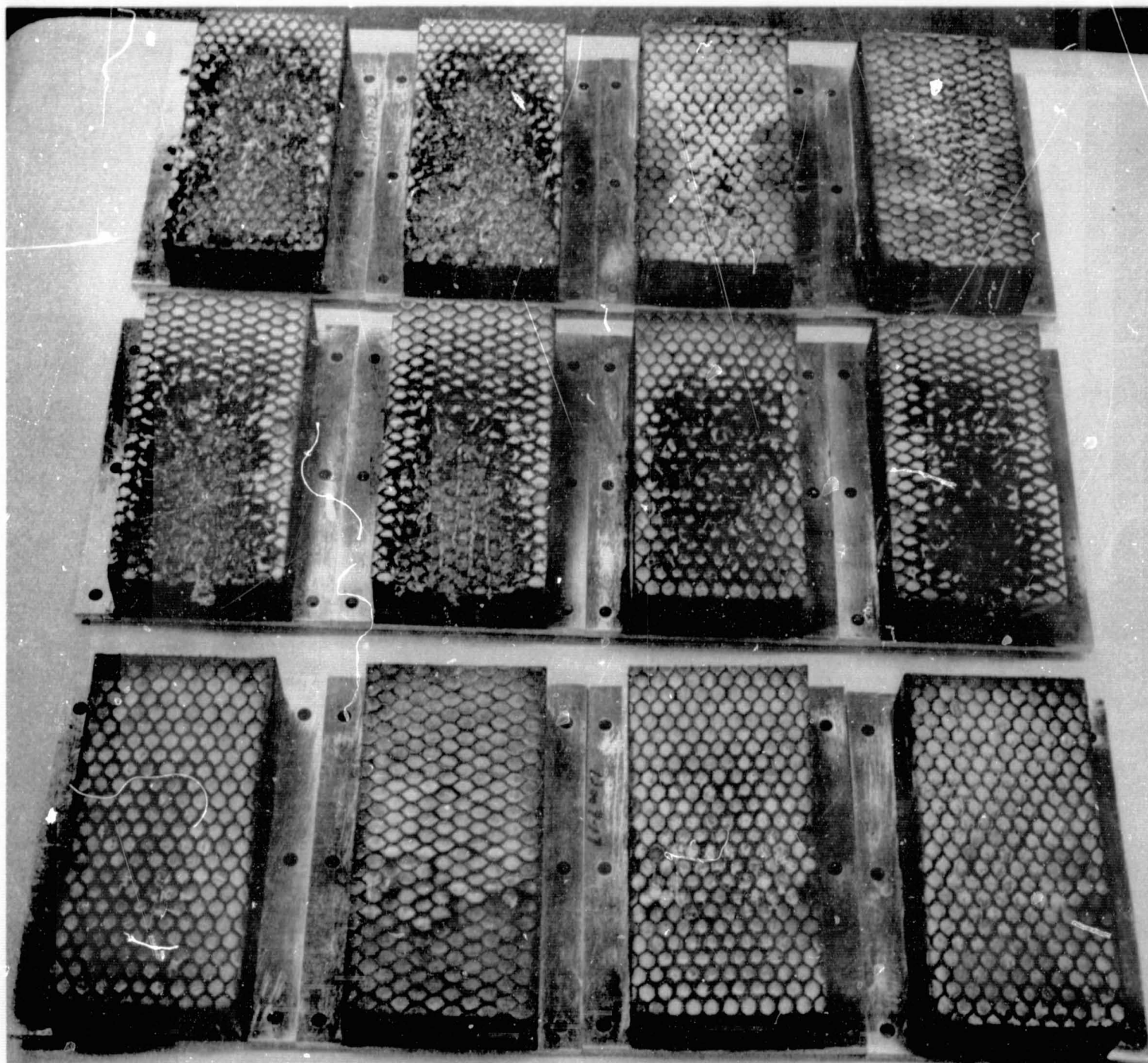
$$H_{0,s} = 0.0124$$

Model 20

$$\dot{q}_0 = 23.4$$

$$H_{0,s} = 0.0173$$

Figure 16.- Photograph of turbulent nitrogen models 17 - 20



Model	14	13	15	16
$\dot{q}_0$	25.2	25.1	22.6	18.0
$H_{0,\sigma}$	0.0131	0.0132	0.0147	0.0143
Model	9	10	11	12
$\dot{q}_0$	29.6	29.7	21.9	22.2
$H_{0,\sigma}$	0.0127	0.0138	0.0130	0.0141
Model	5	6	7	8
$\dot{q}_0$	18.0	18.3	22.8	22.4
$H_{0,\sigma}$	0.0116	0.0111	0.0123	0.0114

Figure 17.- Photograph of turbulent air models 5 - 16

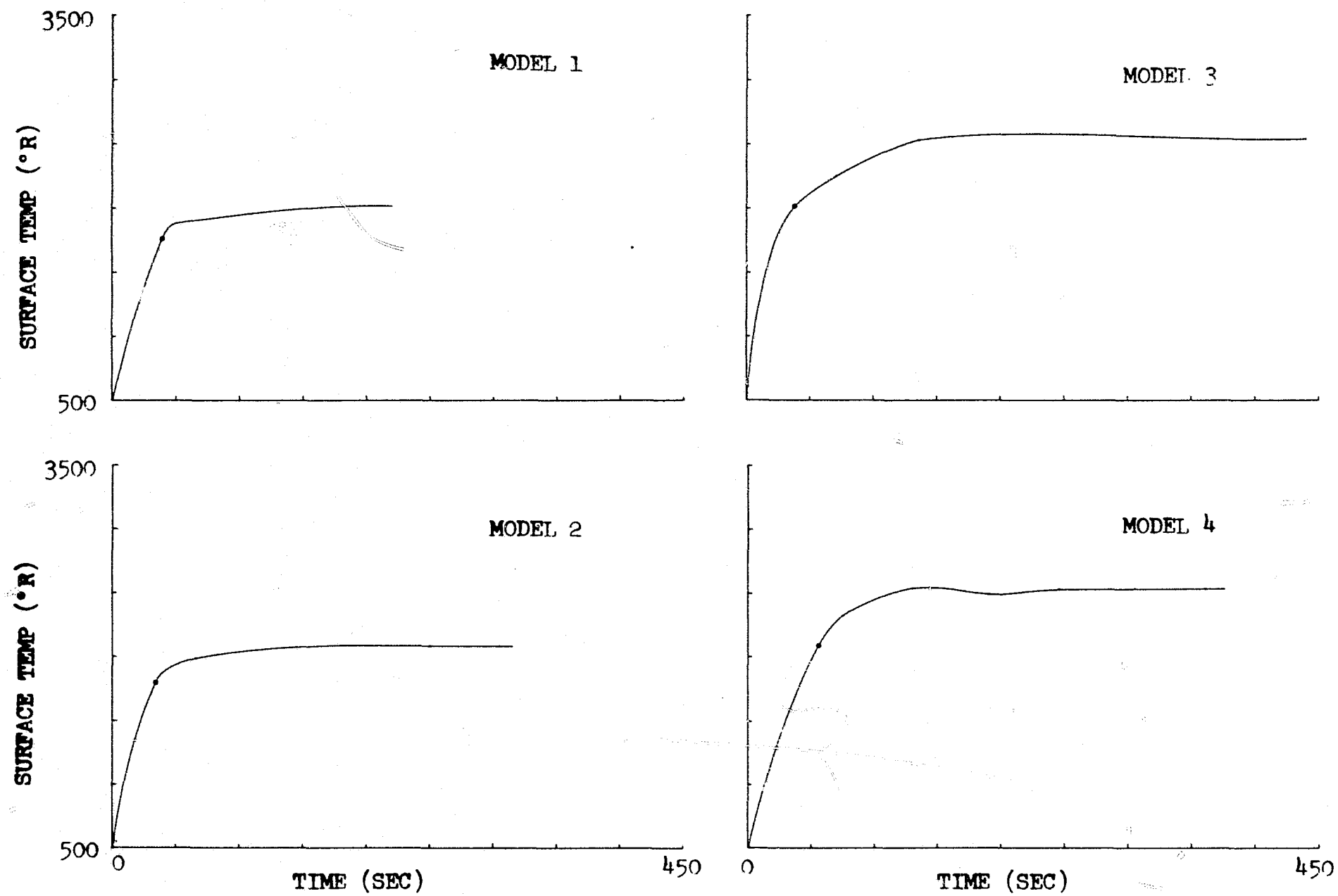


Figure 18

Surface Temperature Histories of Laminar Flow
Models 1 - 4 for a Surface Emissivity of 0.75

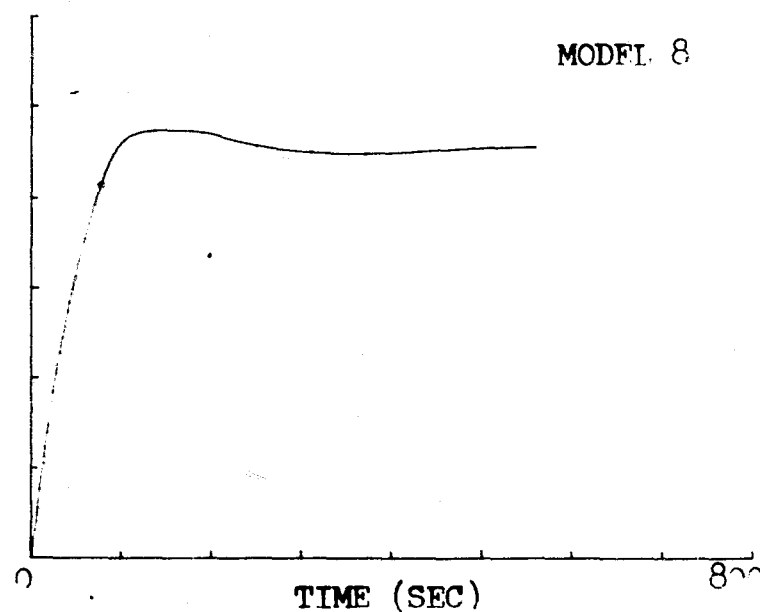
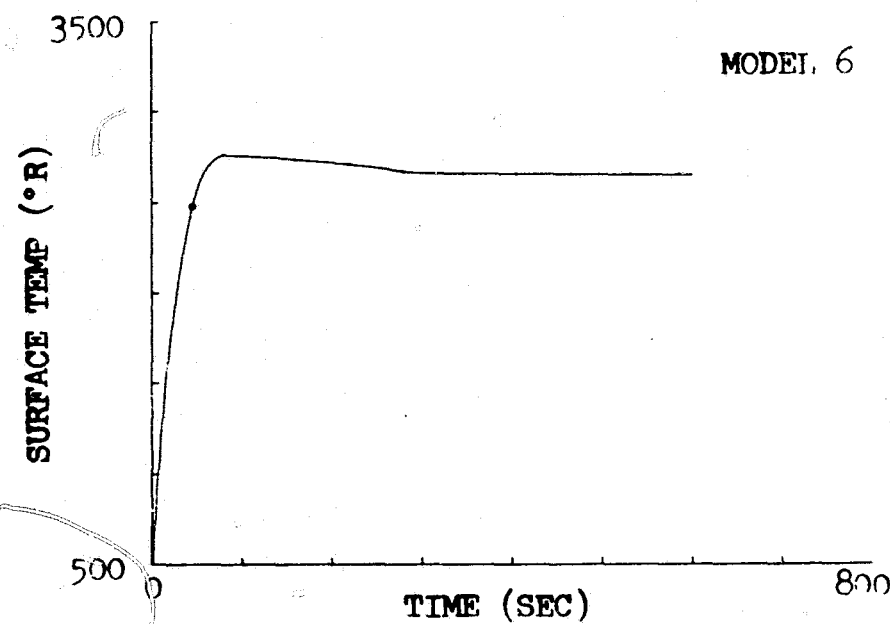
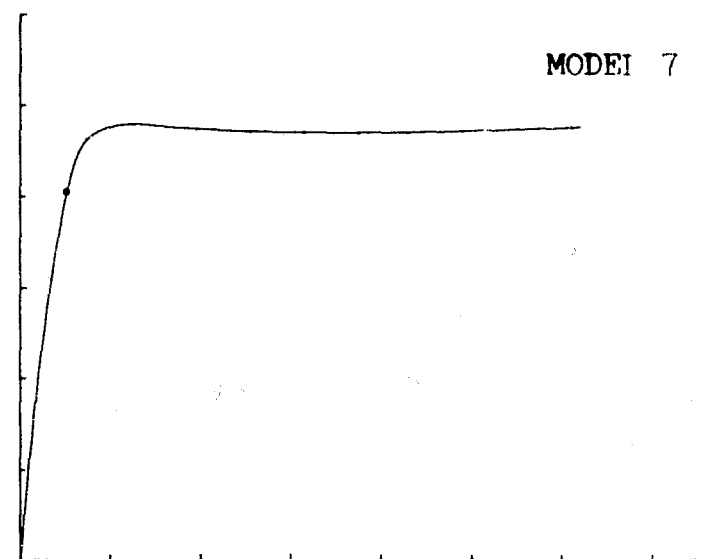
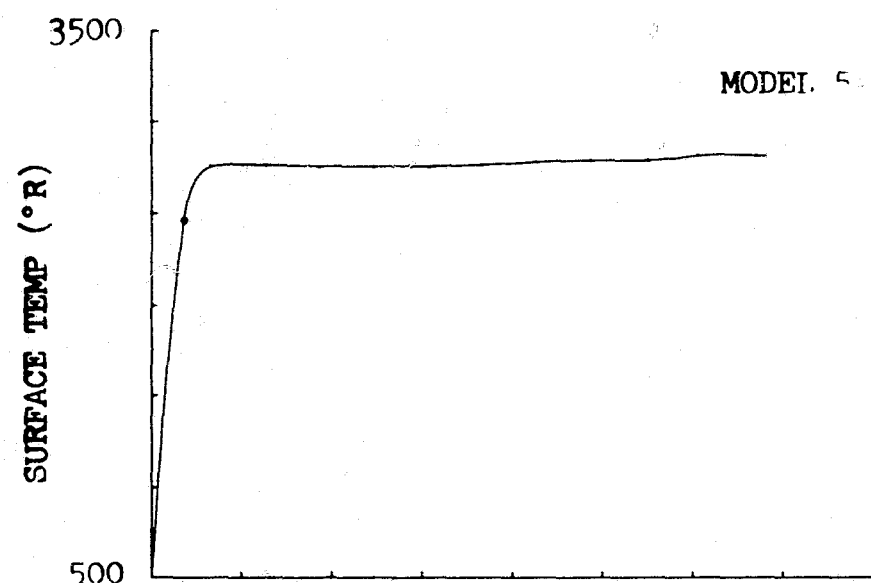


Figure 19
Surface Temperature Histories of Turbulent Air
Models 5 - 8 for a Surface Emissivity of 0.75

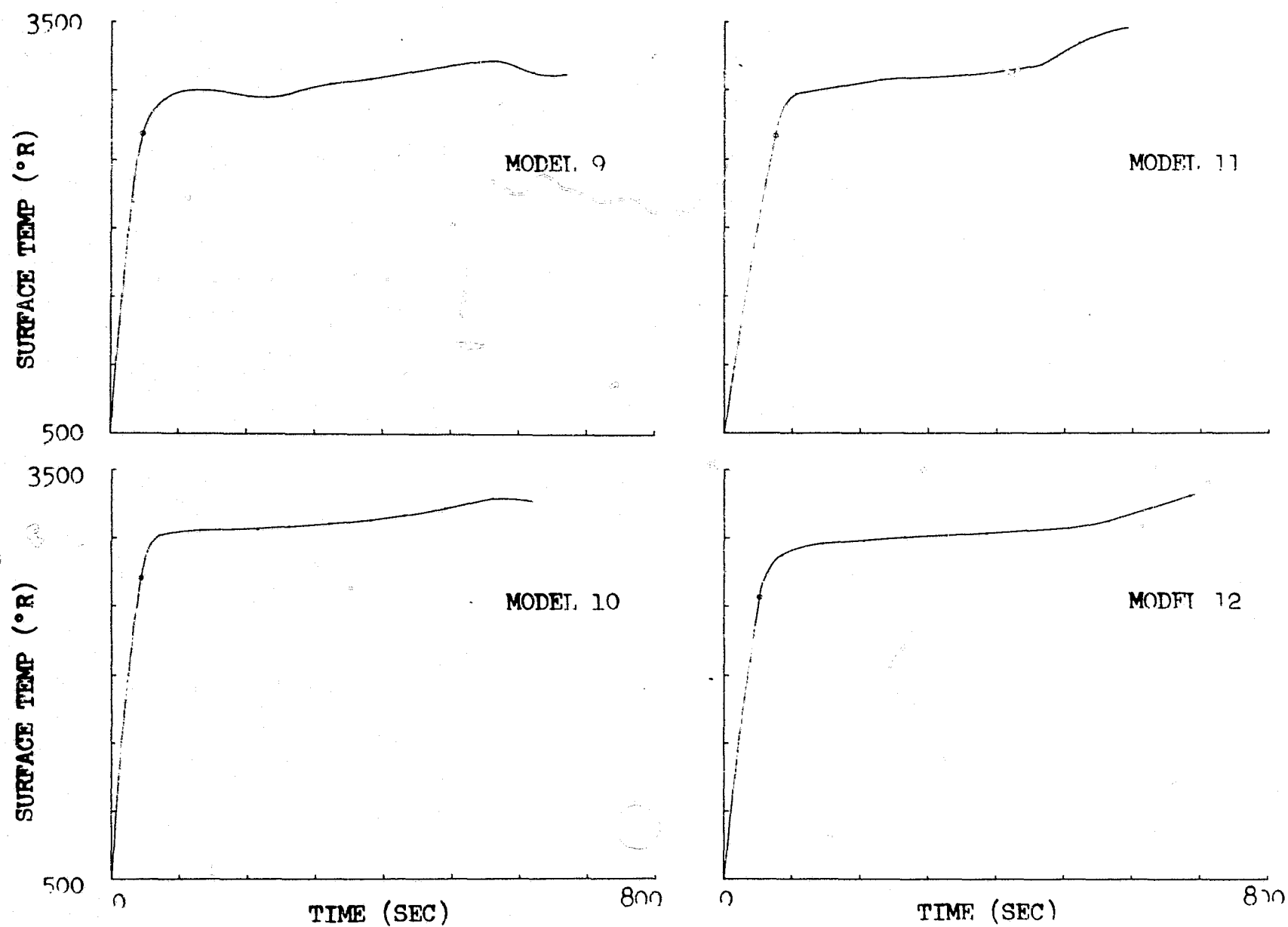


Figure 20
Surface Temperature Histories of Turbulent Air
Models 9 - 12 for a Surface Emissivity of 0.75

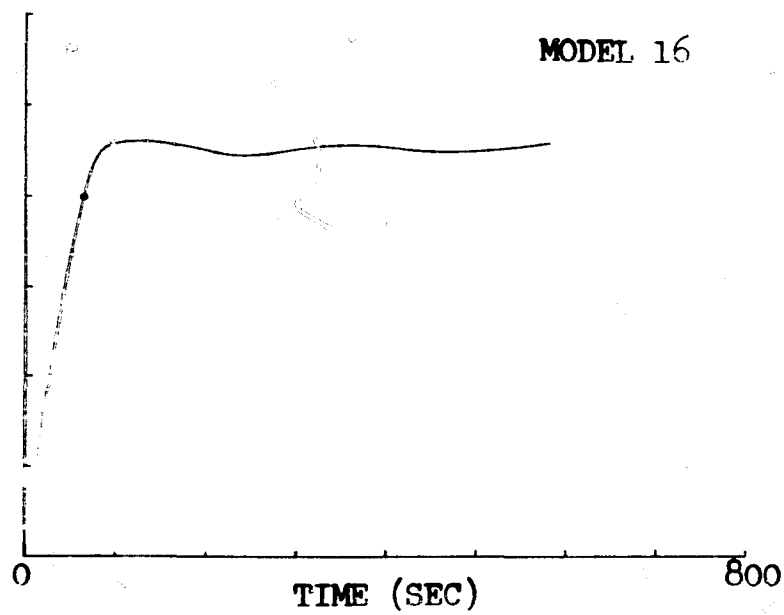
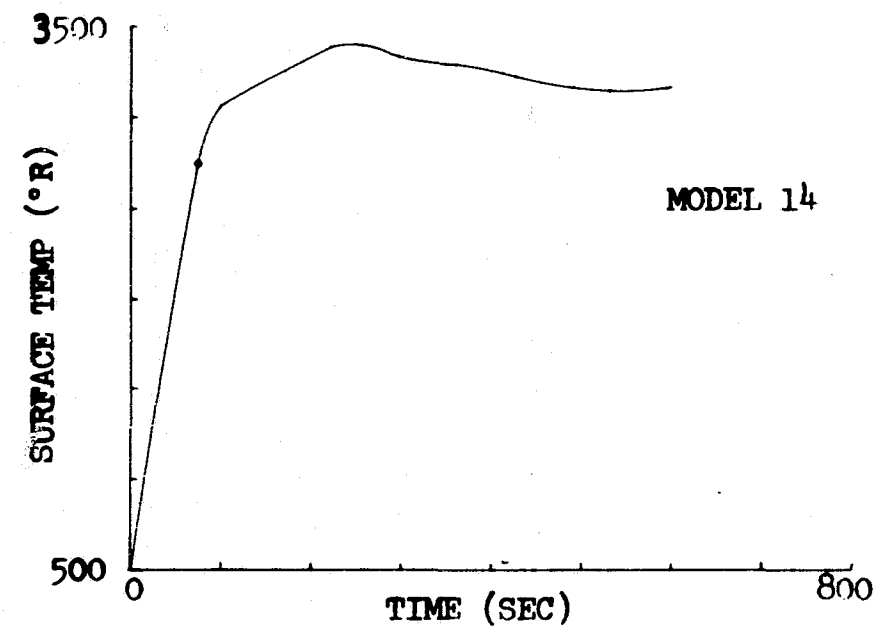
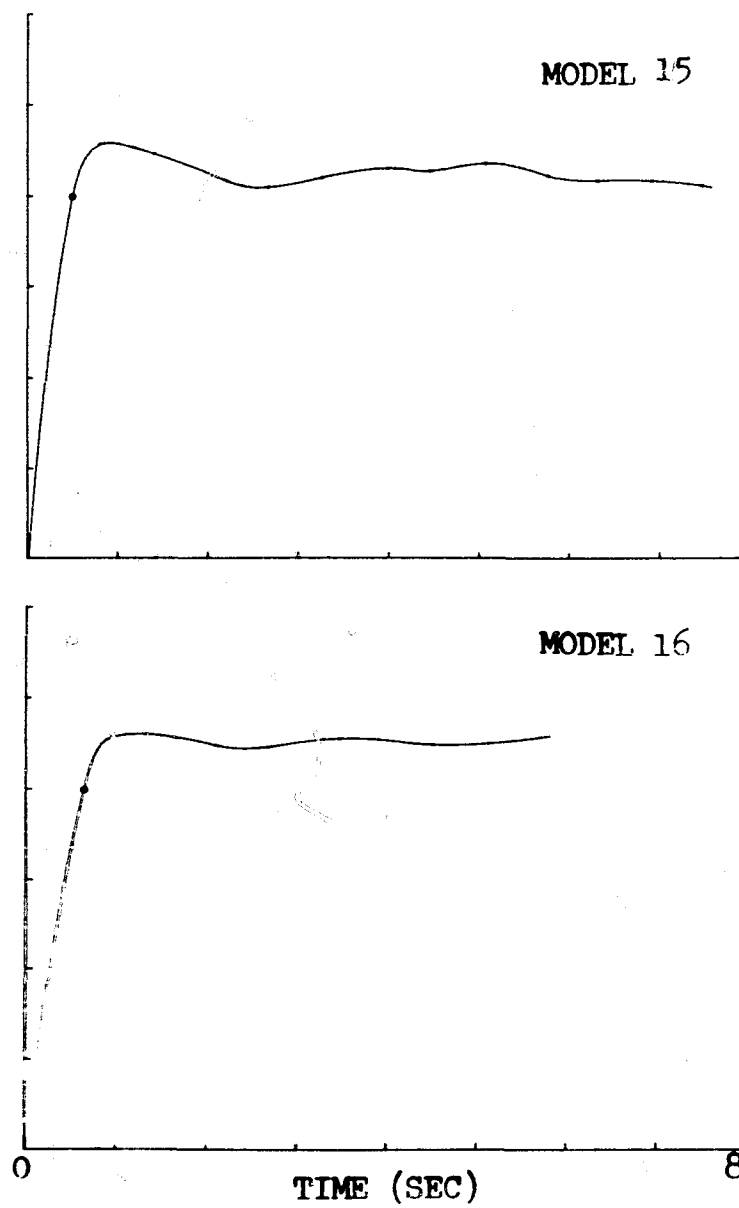
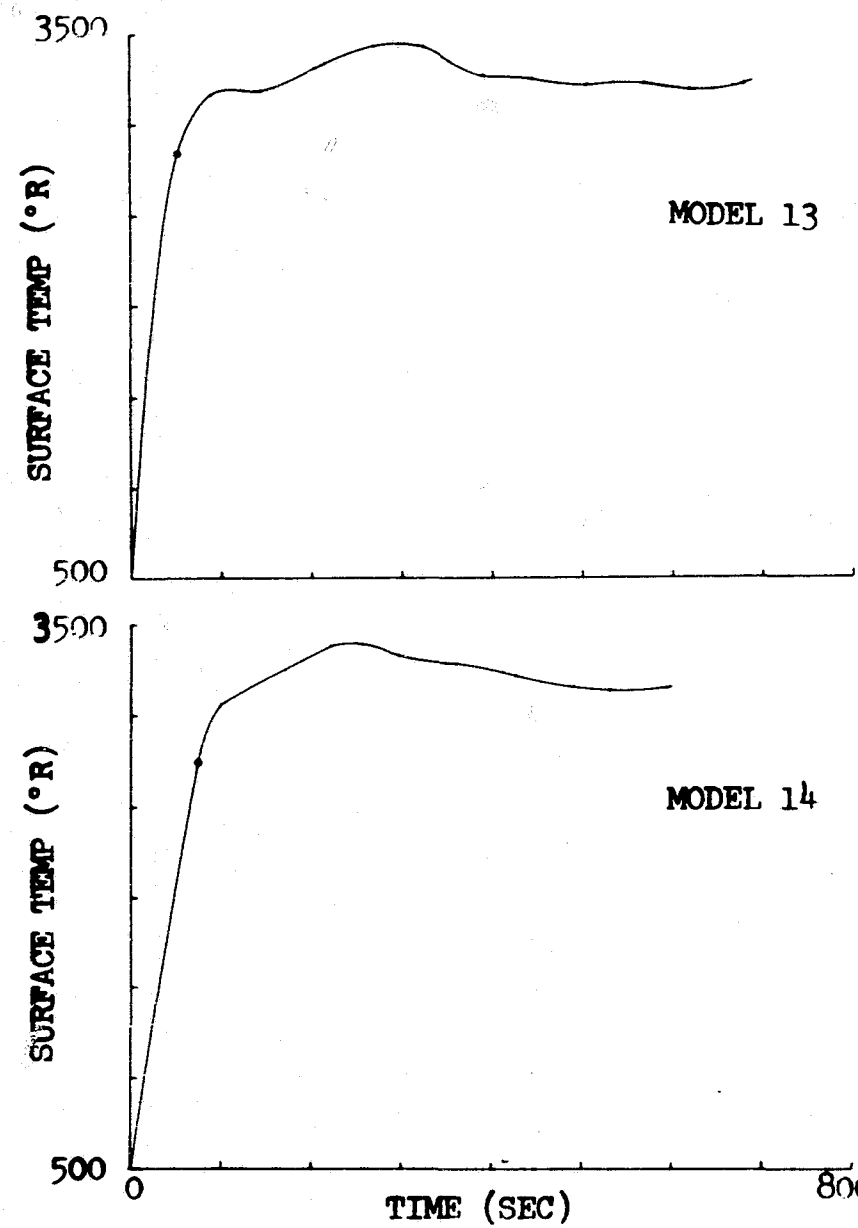


Figure 21
Surface Temperature Histories of Turbulent Air
Models 13 - 16 for a Surface Emissivity of 0.75

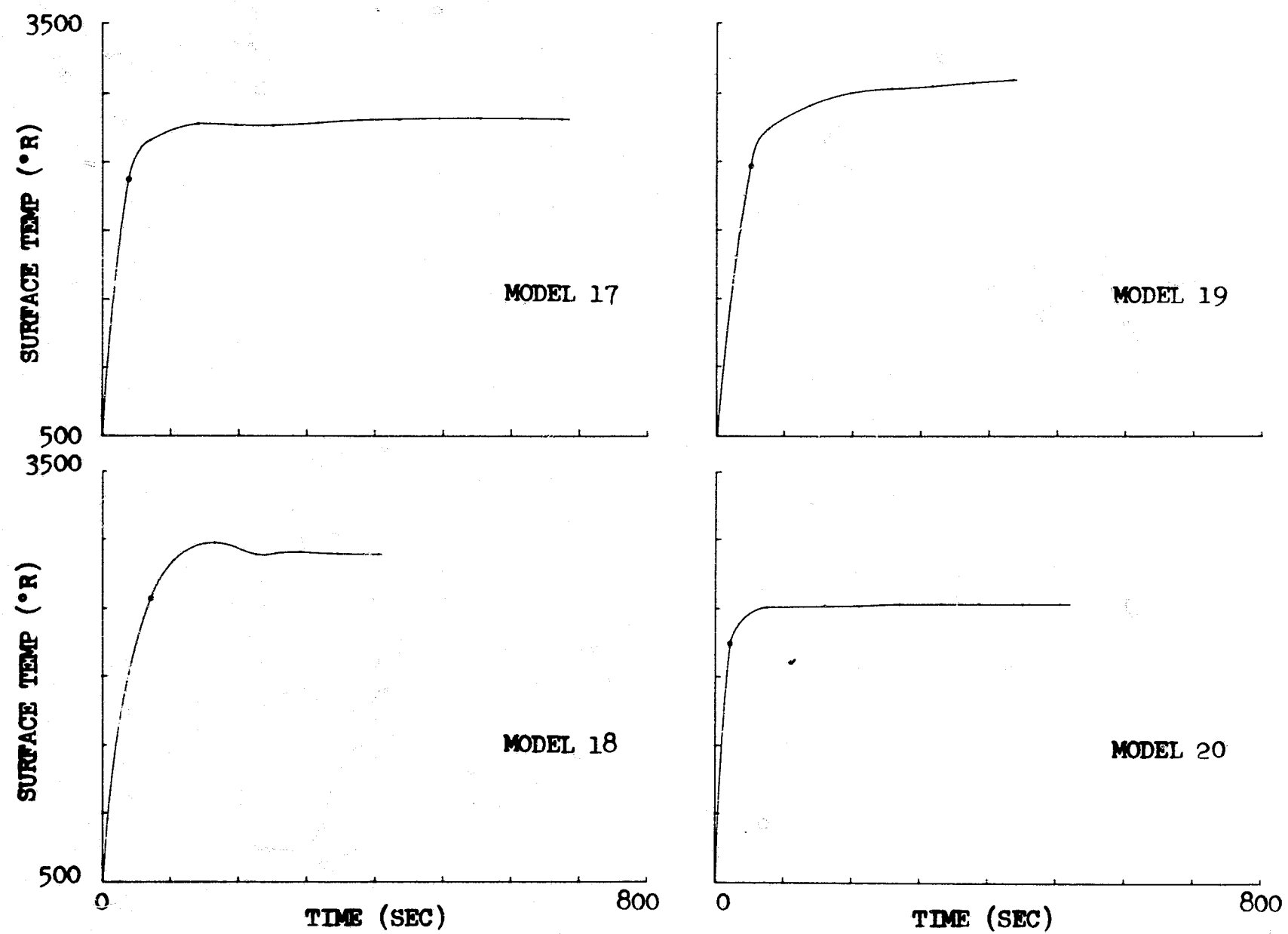


Figure 22

Surface Temperature Histories of Turbulent Nitrogen
Models 17 - 20 for a Surface Emissivity of 0.75

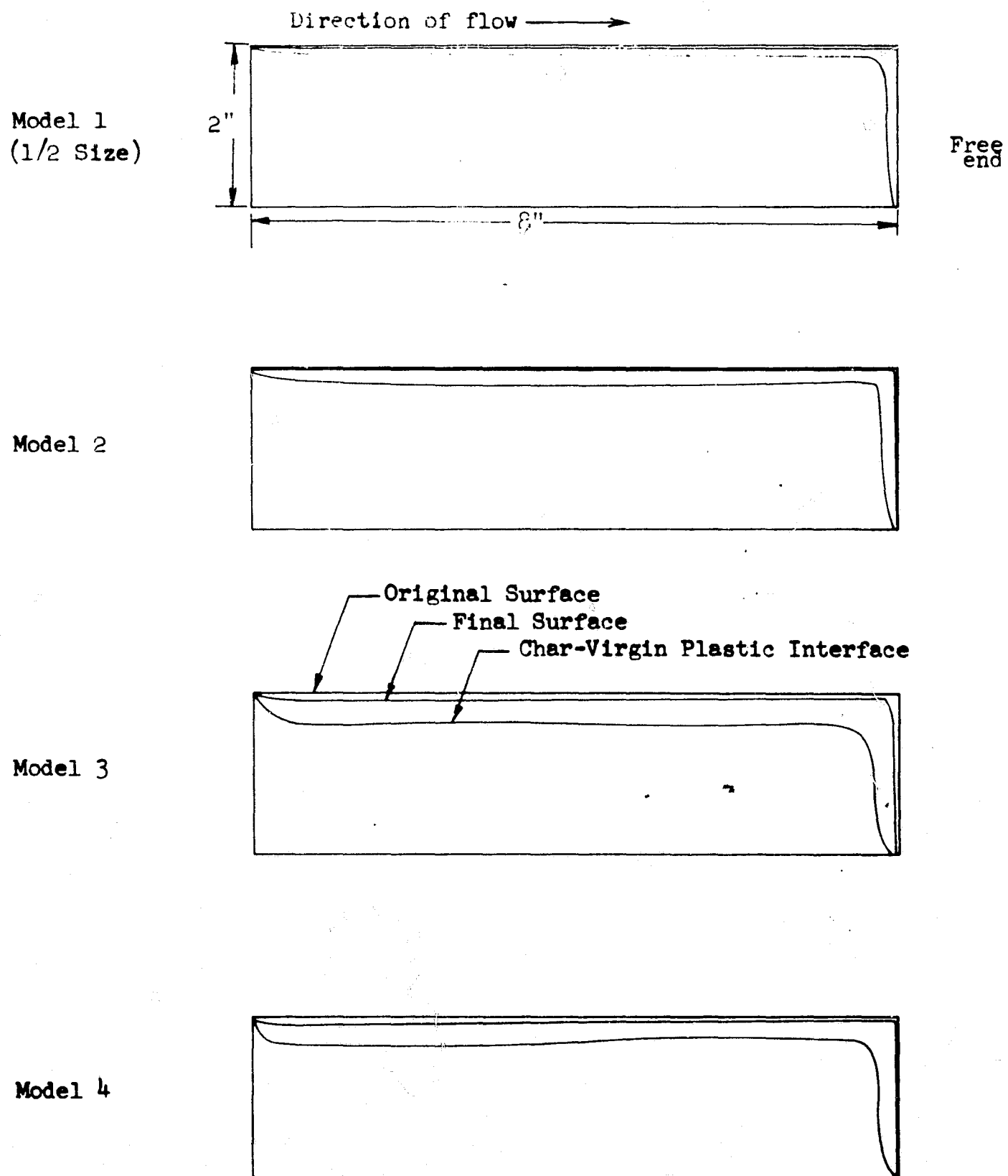


Figure 23
Original and Final Surface Profiles and
Char-Virgin Plastic Interfaces for
Laminar Models 1 - 4

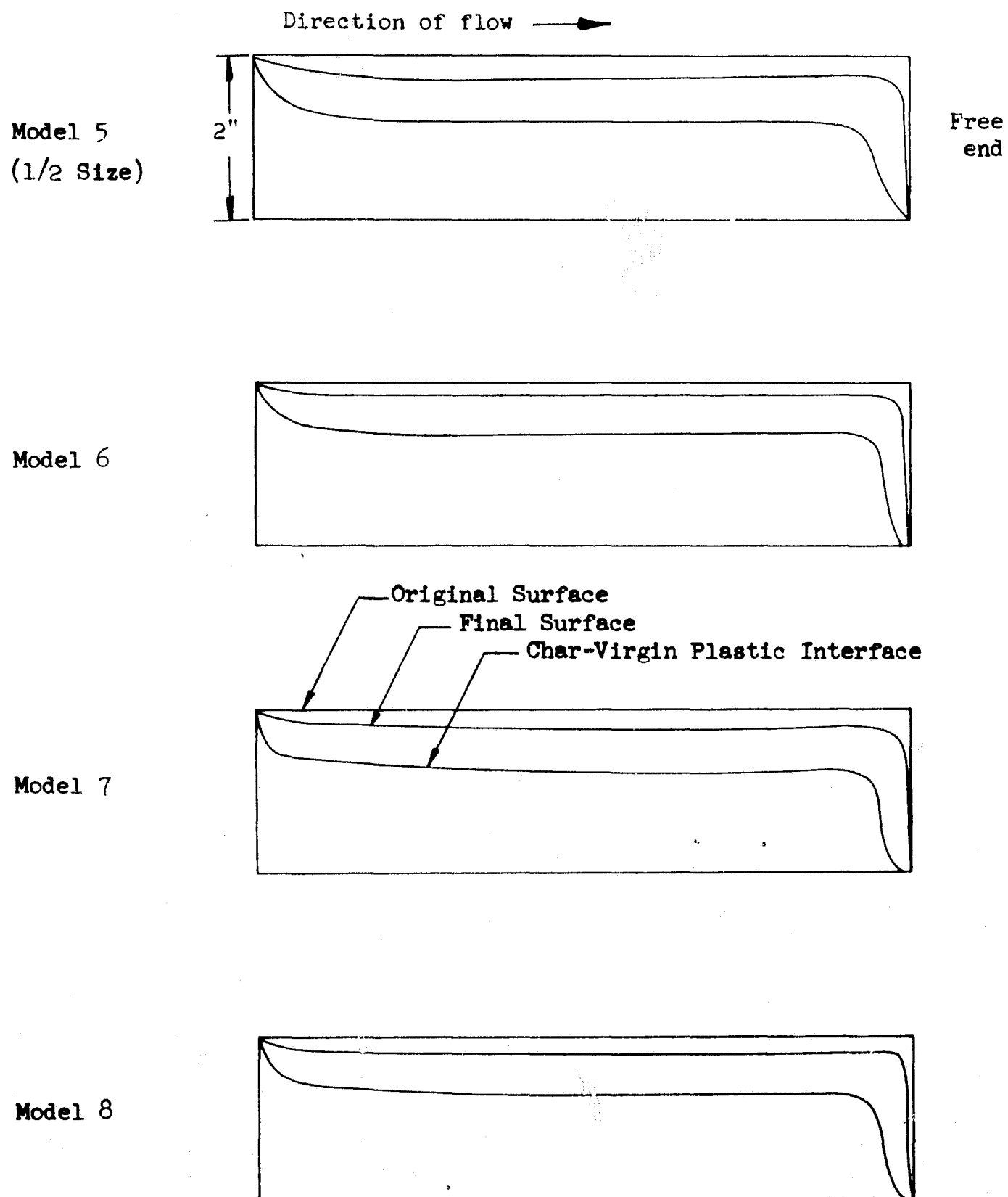


Figure 24

Original and Final Surface Profiles and
Char- Virgin Plastic Interfaces for
Turbulent Air Models 5 - 8

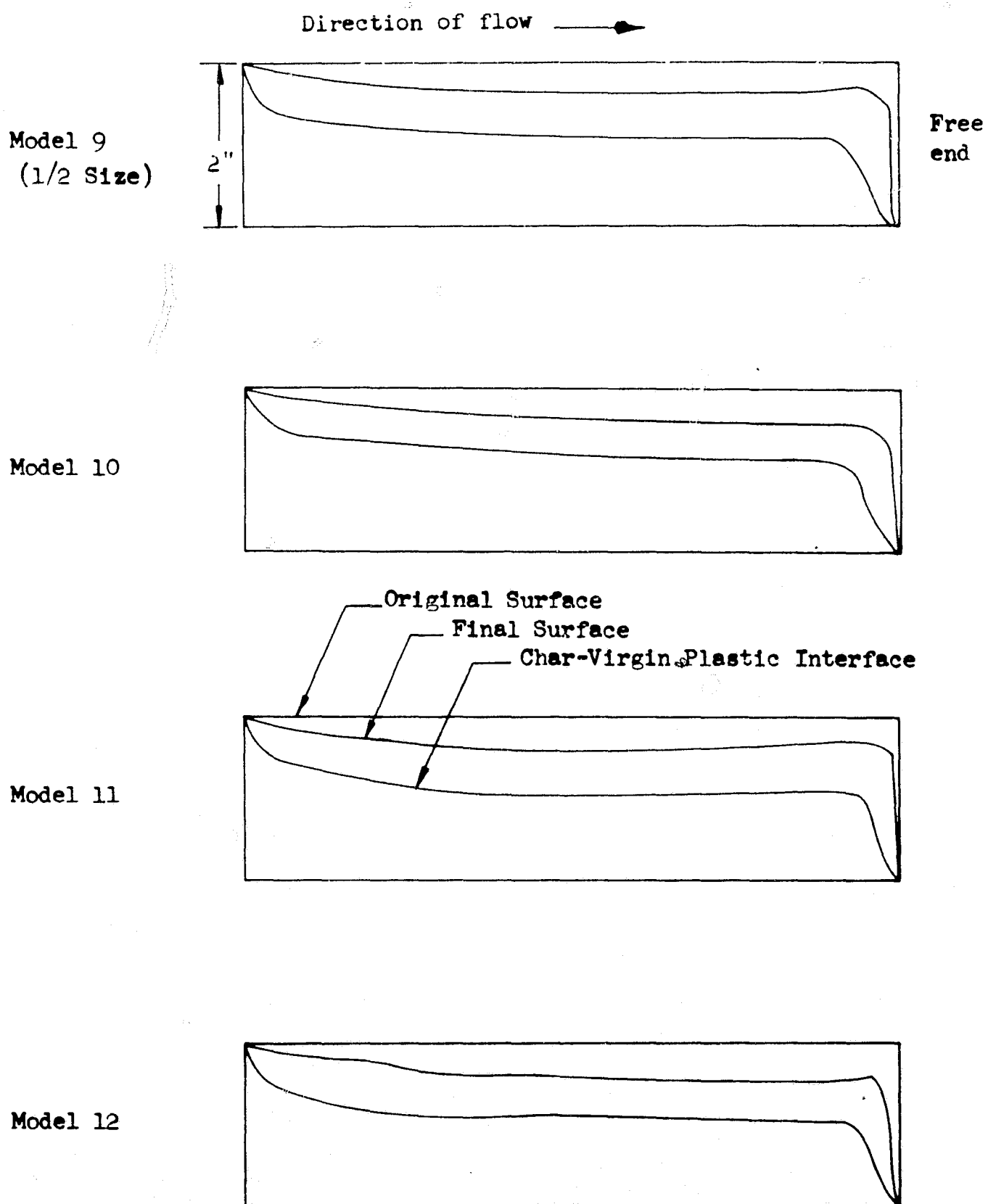


Figure 25

Original and Final Surface Profiles and
Char-Virgin Plastic Interfaces for
Turbulent Air Models 9 - 12

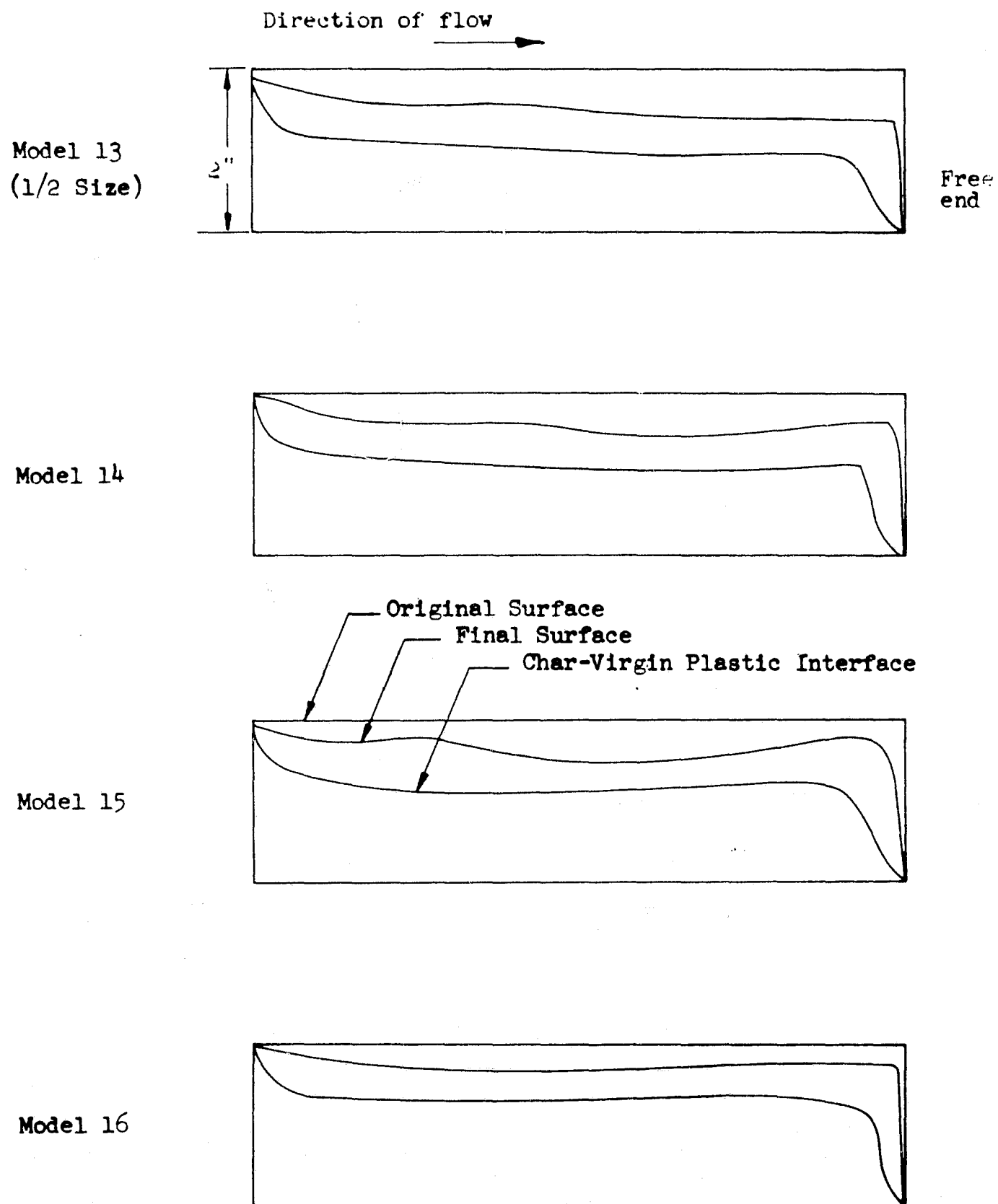
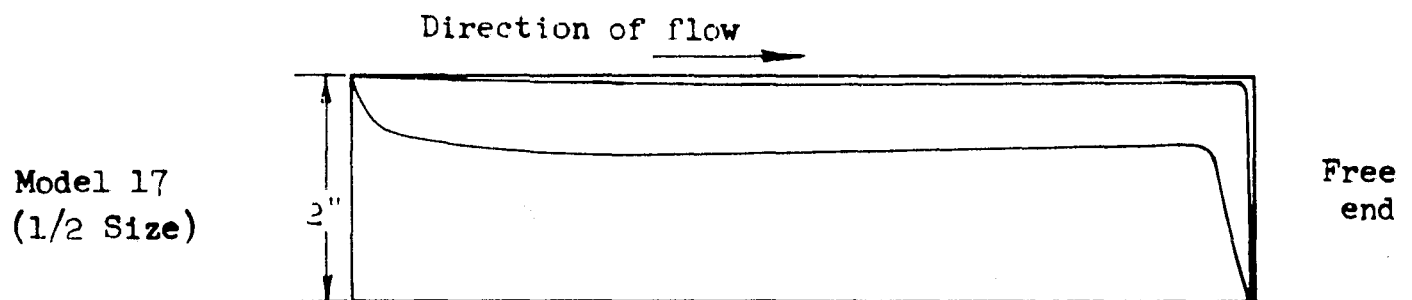
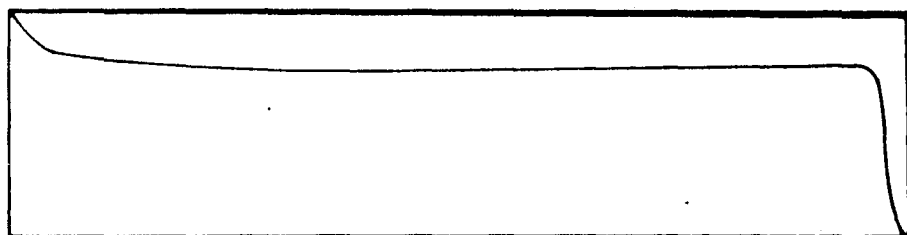


Figure 26

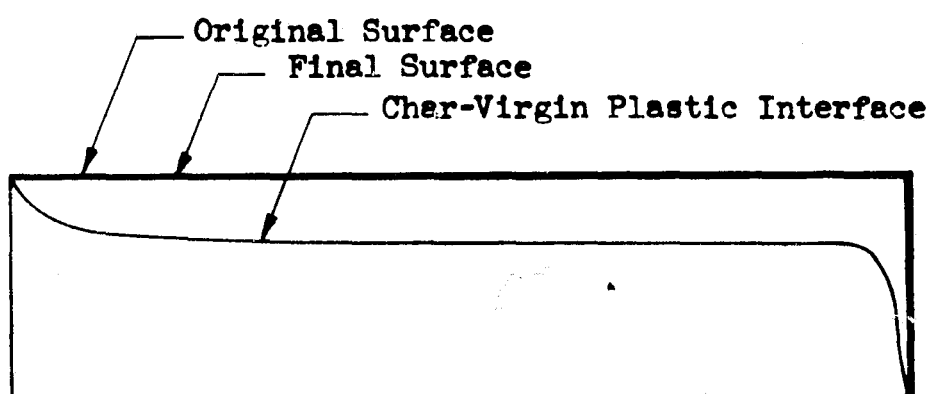
Original and Final Surface Profiles and
Char-Virgin Plastic Interfaces for
Turbulent Air Models 13 - 16



Model 18



Model 19



Model 20

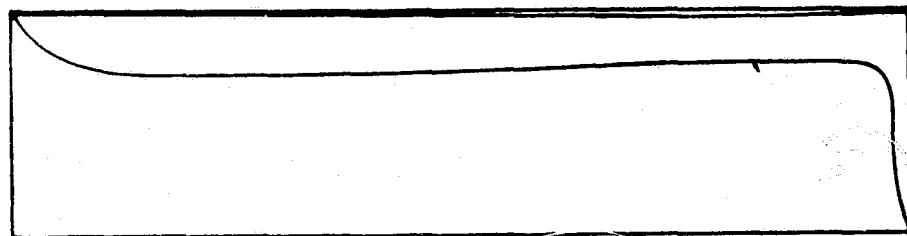


Figure 27

Original and Final Surface Profiles and
Char-Virgin Plastic Interfaces for
Turbulent Nitrogen Models 17 - 20

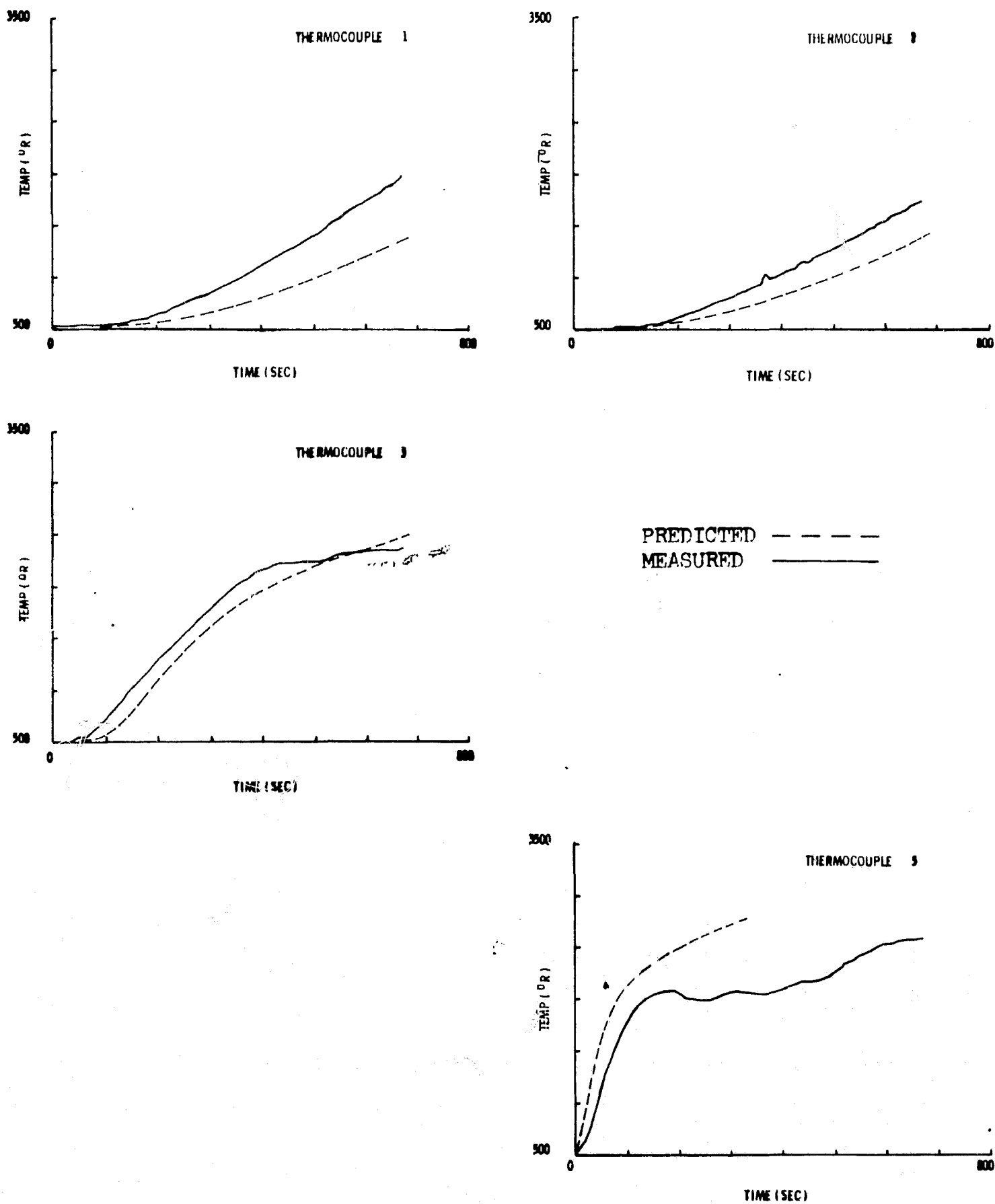


Figure 28a

Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 5

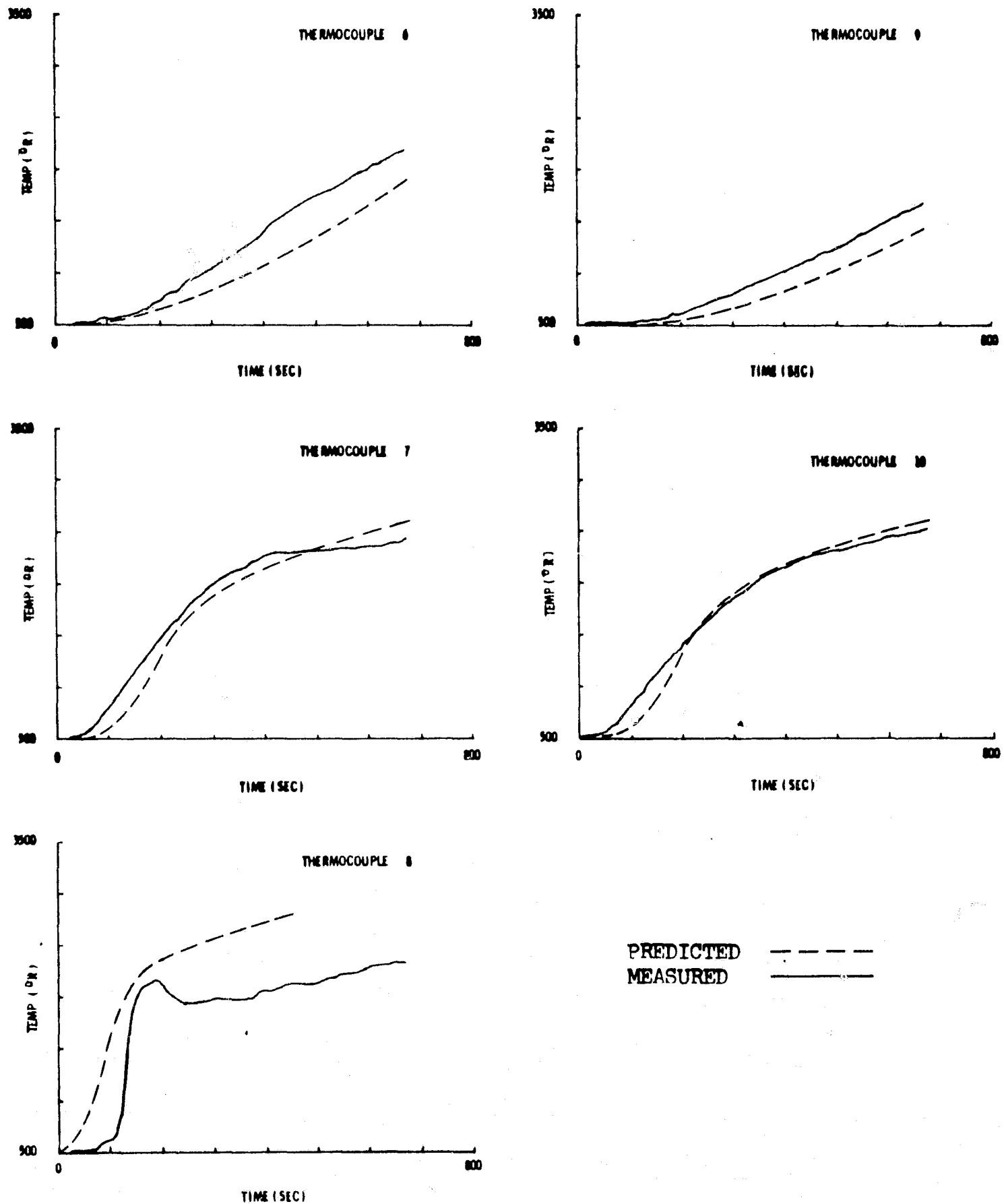
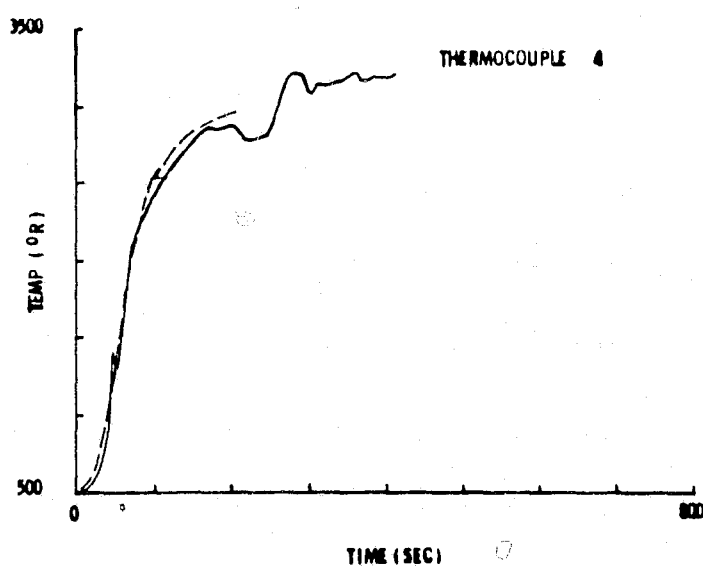
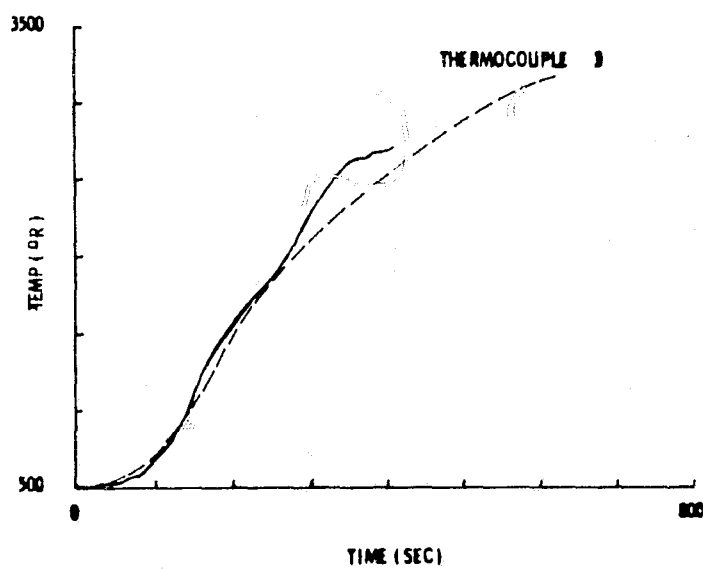
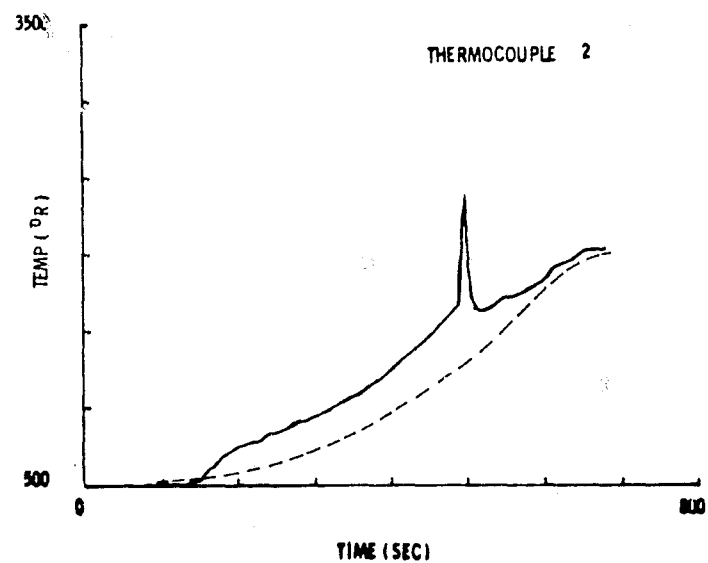


Figure 28b
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 5



PREDICTED ———
MEASURED ———

Figure 29a

Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 9

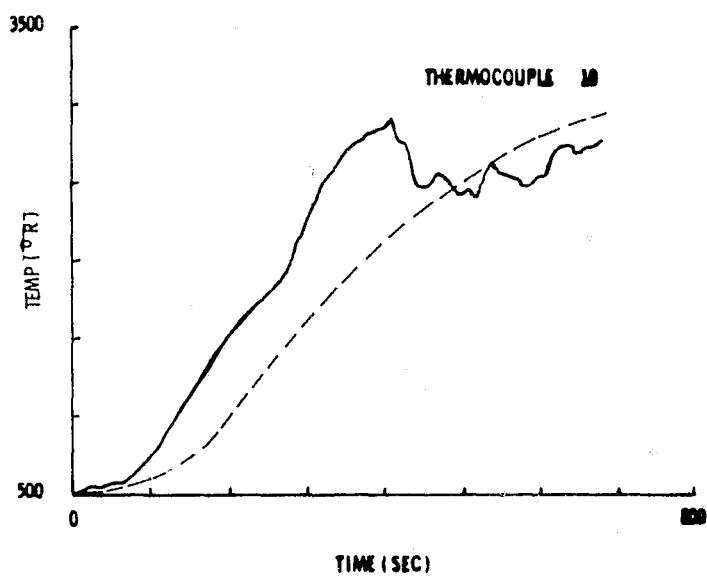
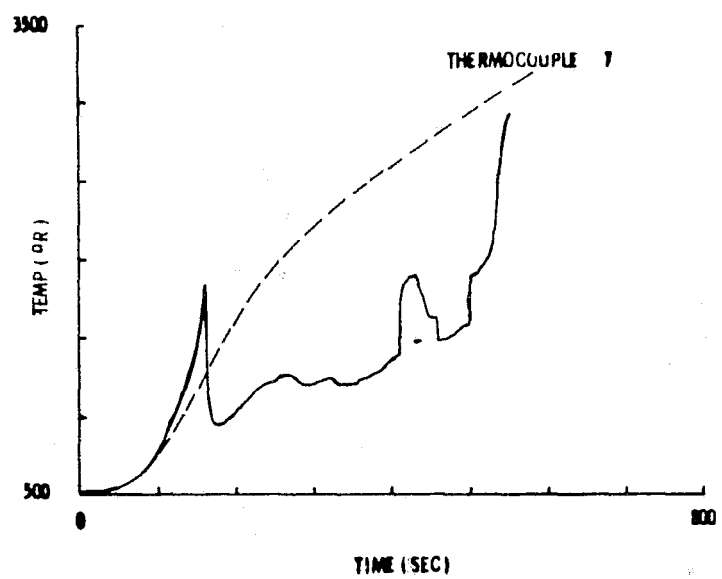
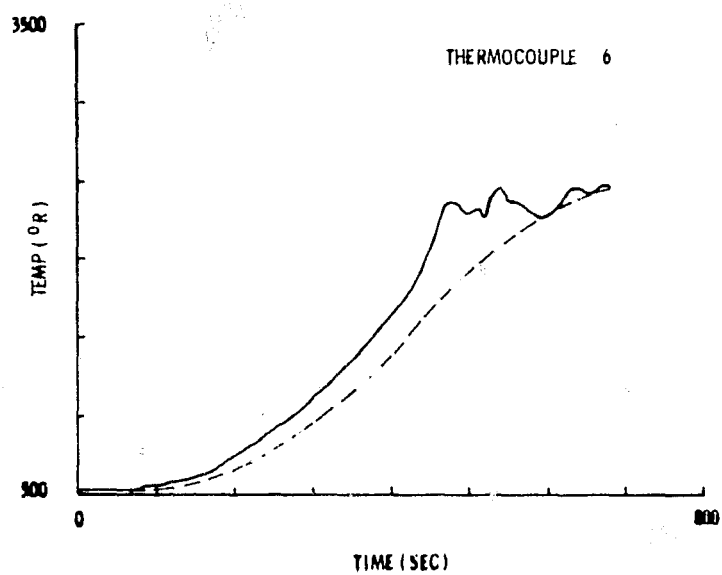


Figure 29b
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 9

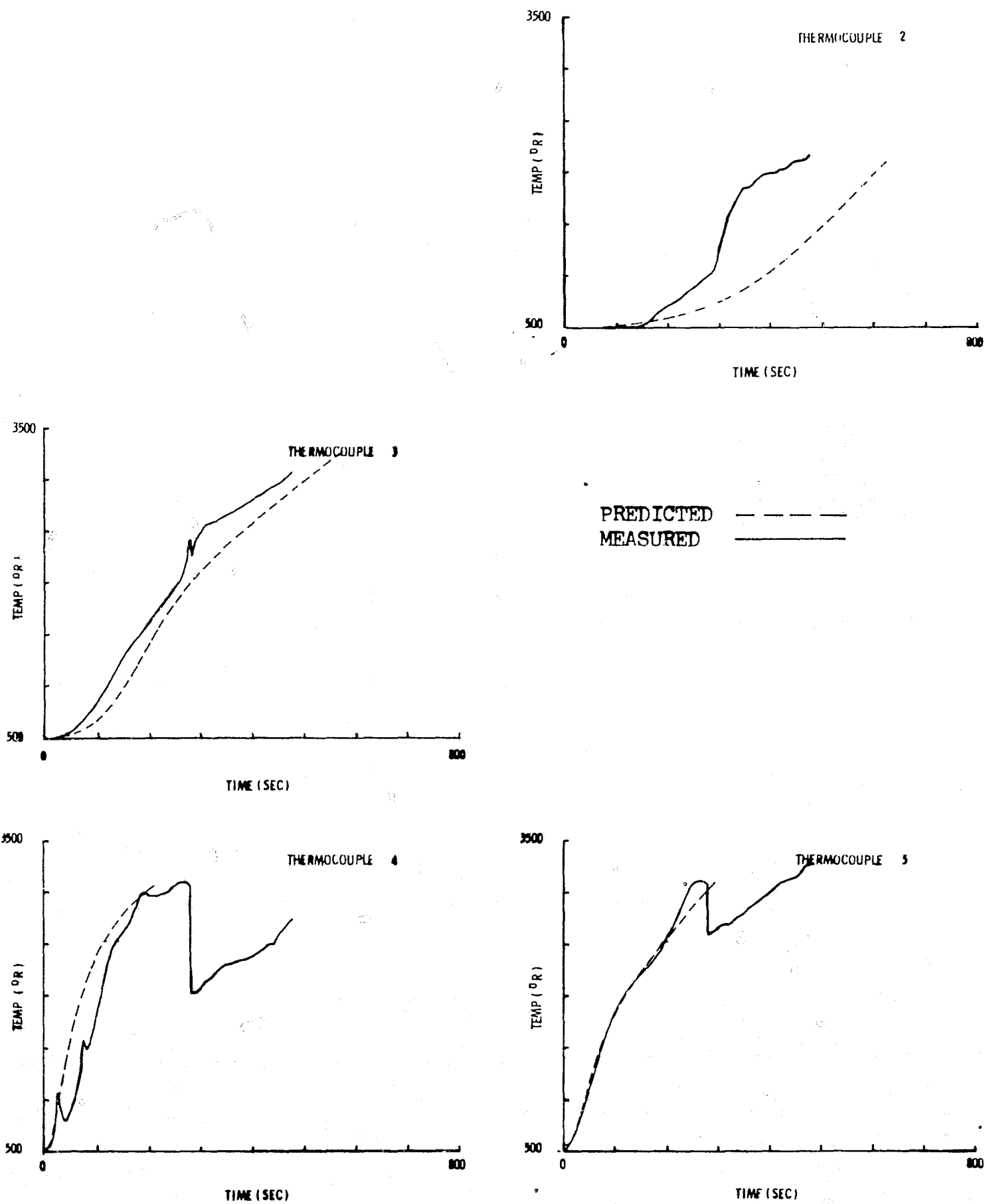


Figure 30a.

Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 10

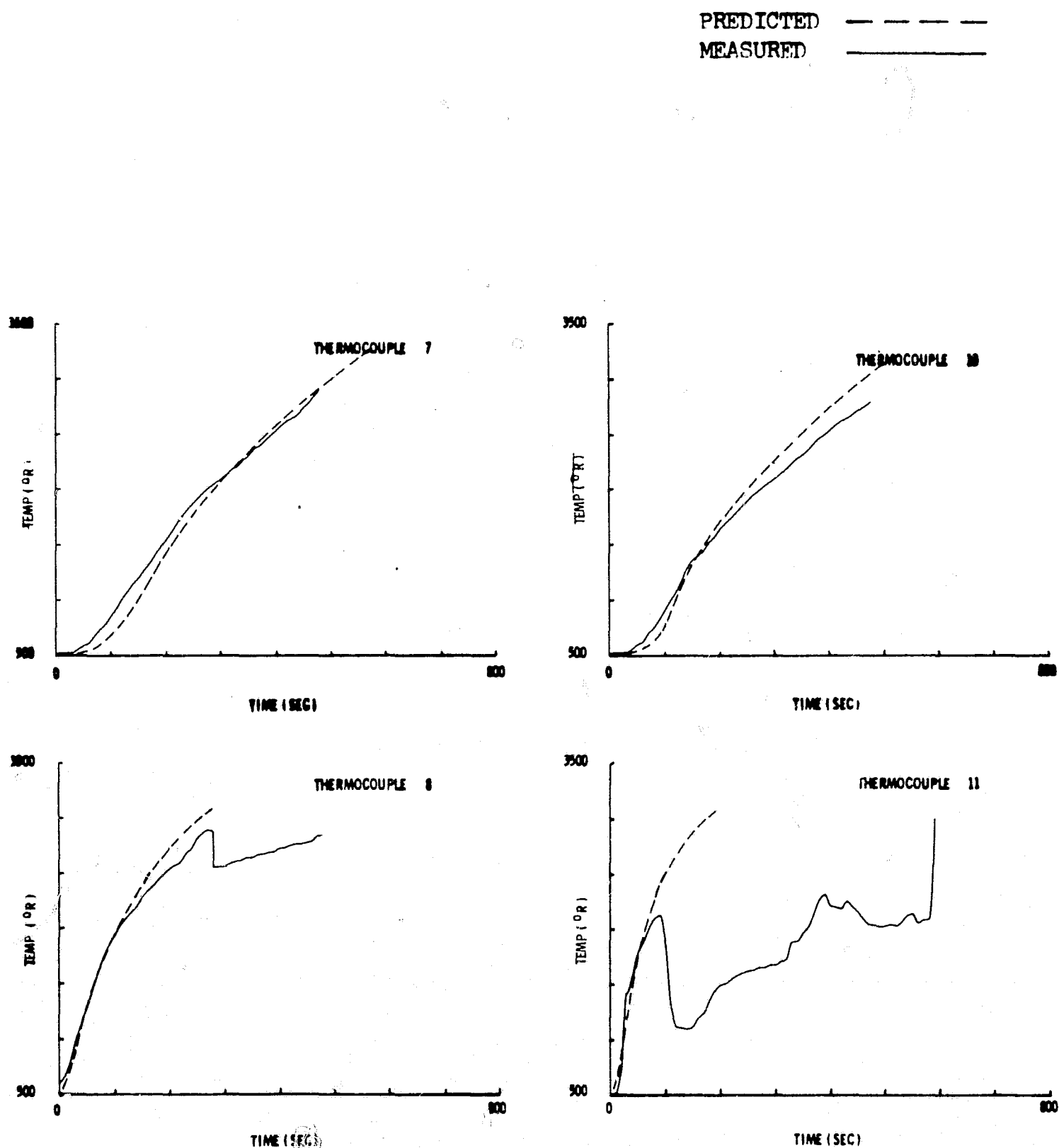
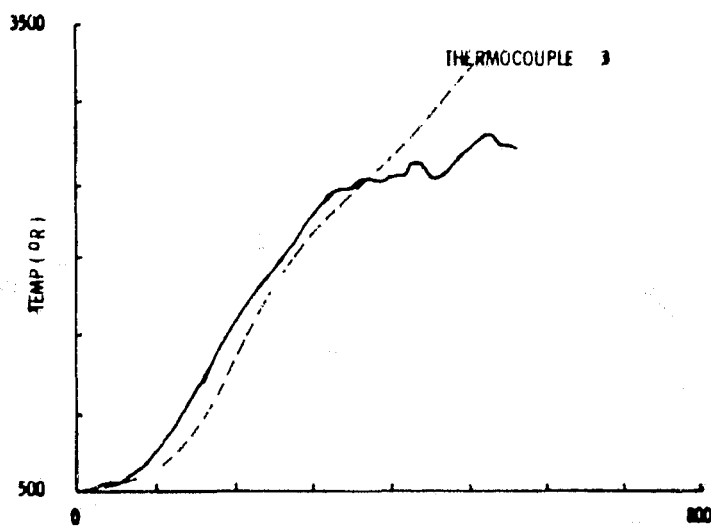
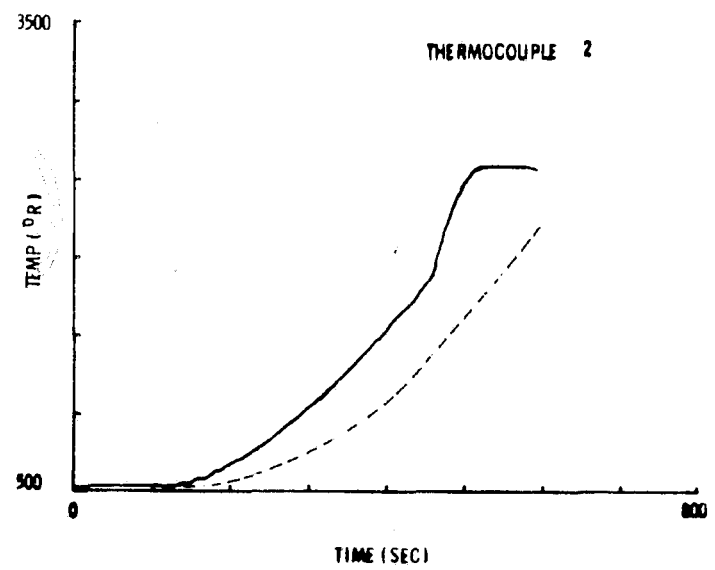


Figure 30b
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 10



PREDICTED — — — — —
MEASURED —————

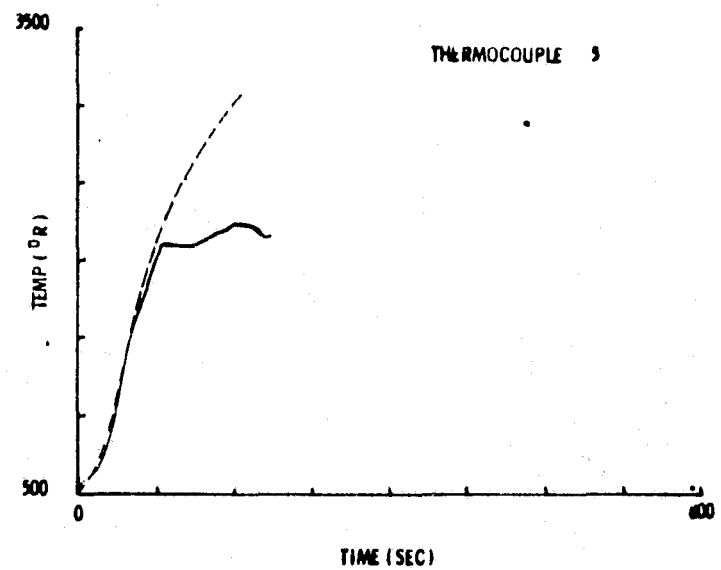


Figure 31a
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 11

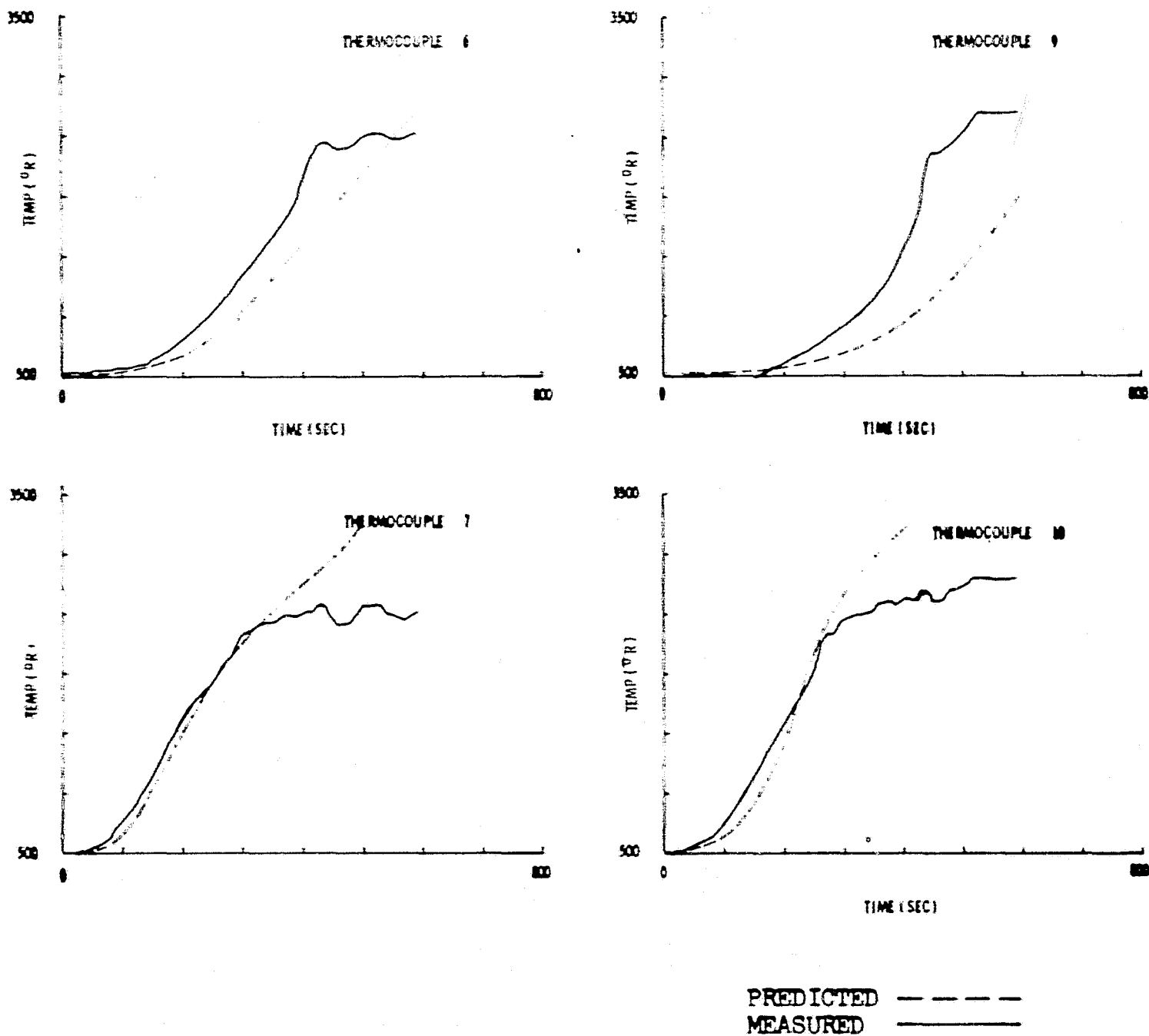


Figure 31b
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 11

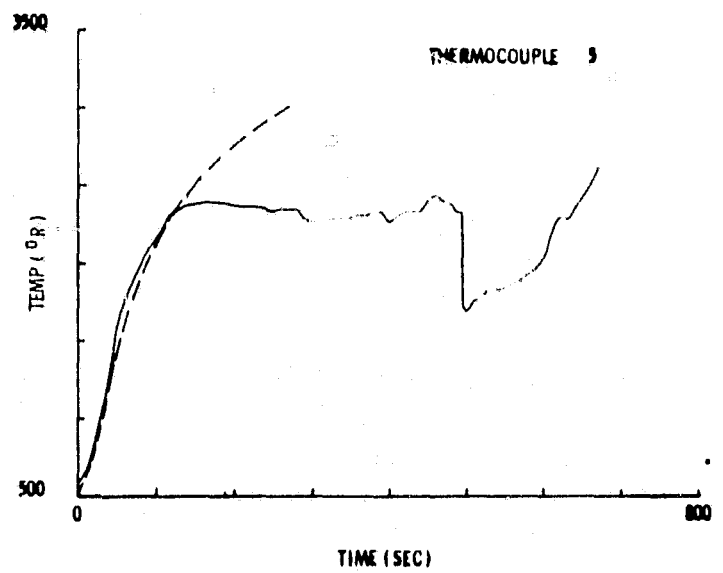
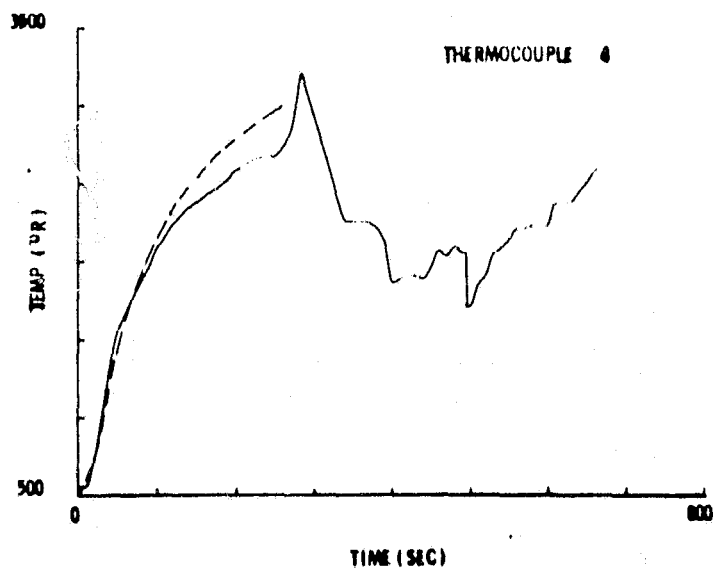
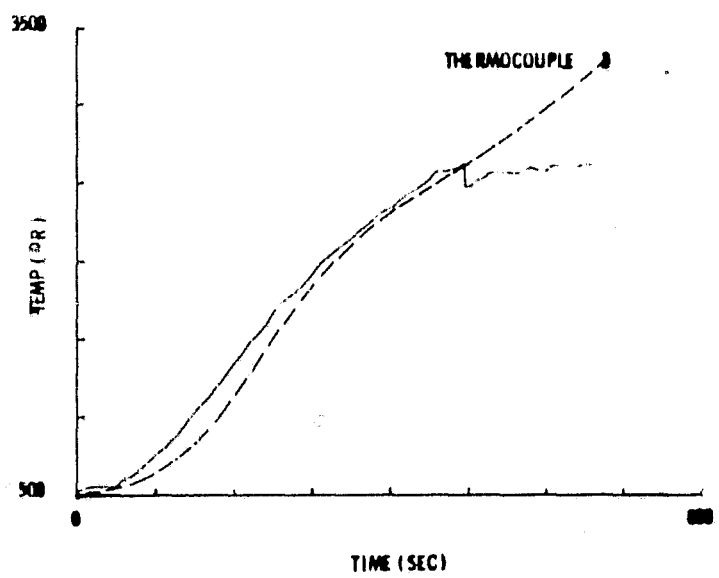
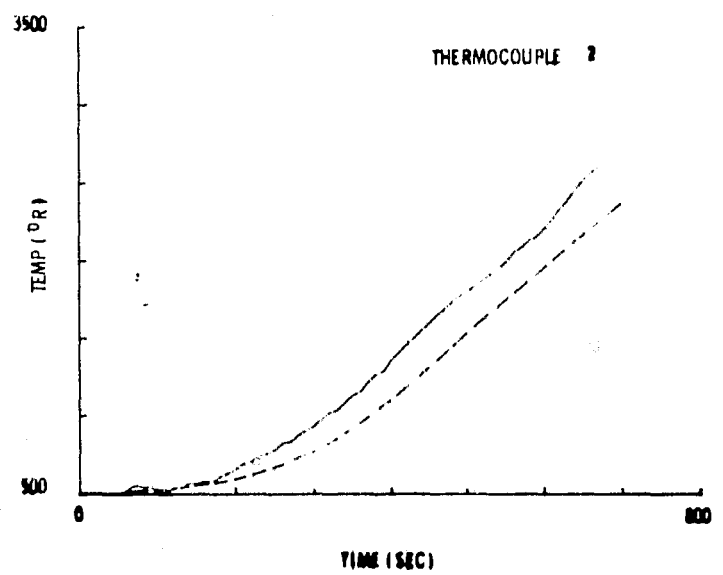
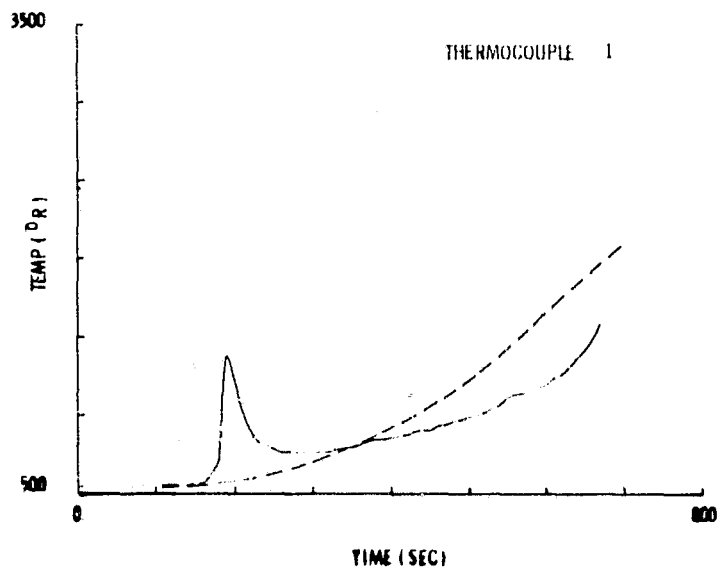


Figure 32a
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 12

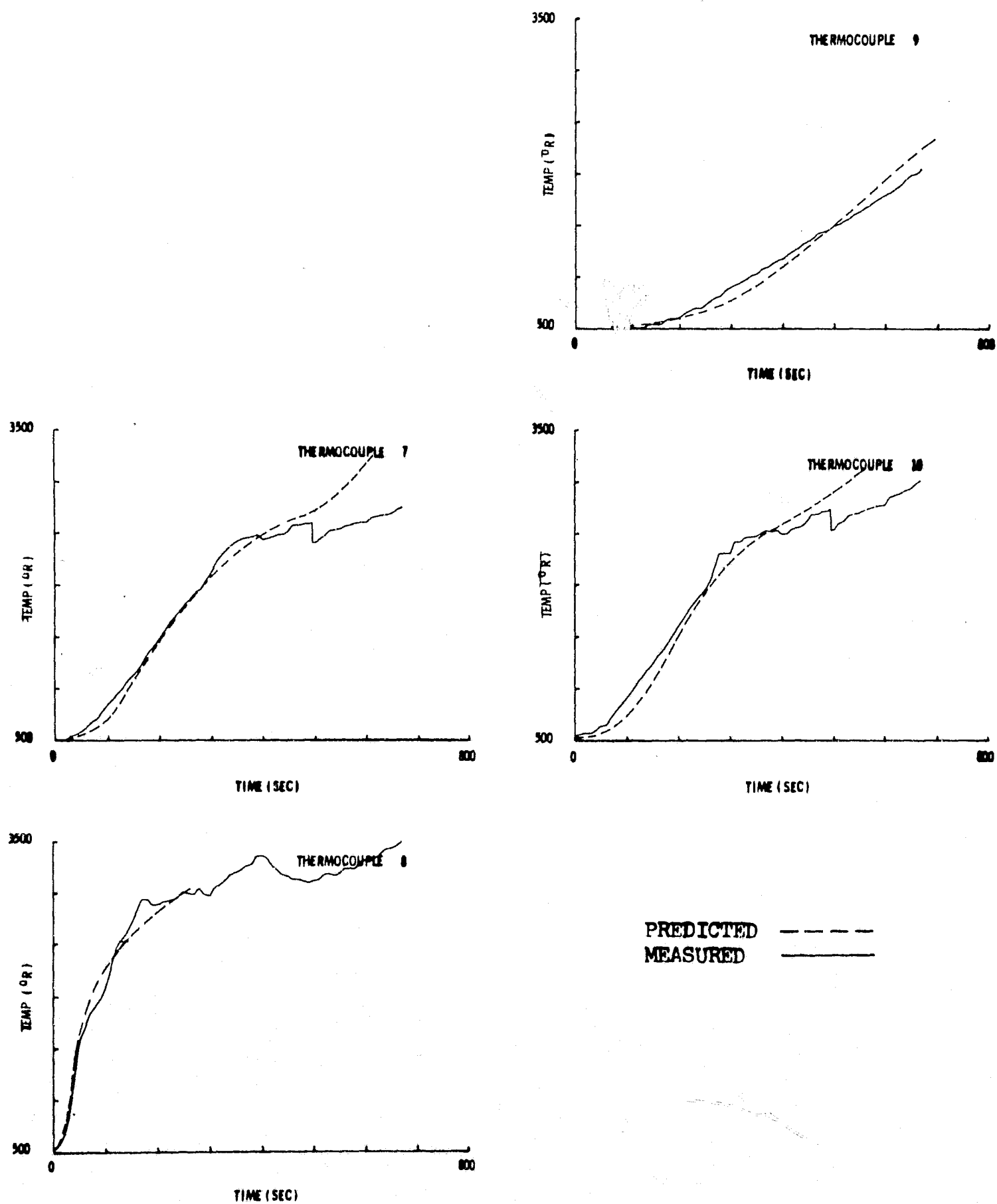
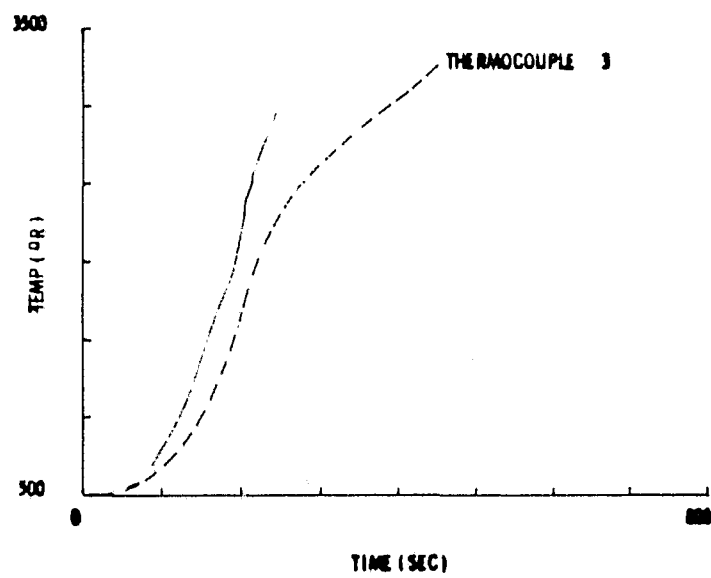


Figure 32b
 Comparison of Predicted Temperature Response
 with the Thermocouple Response of Model 12



PREDICTED ---
MEASURED —

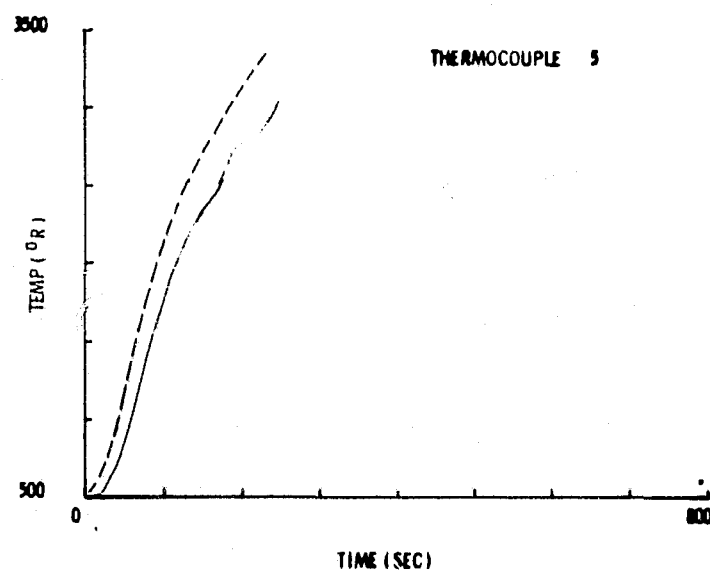
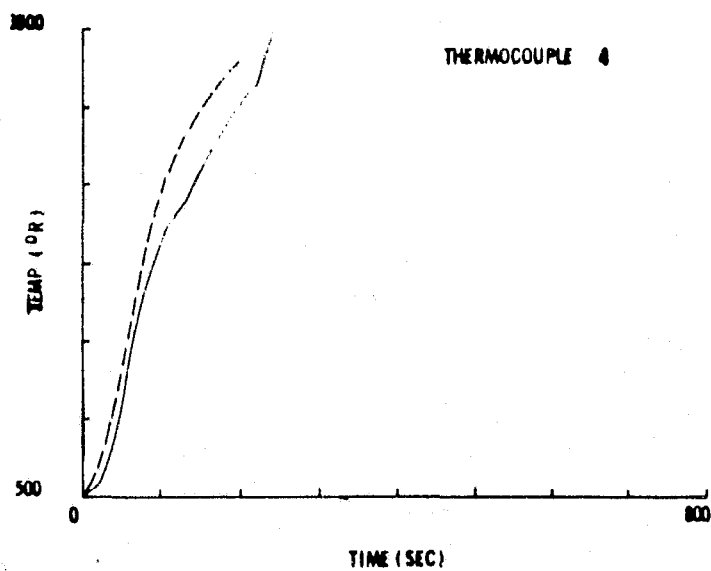
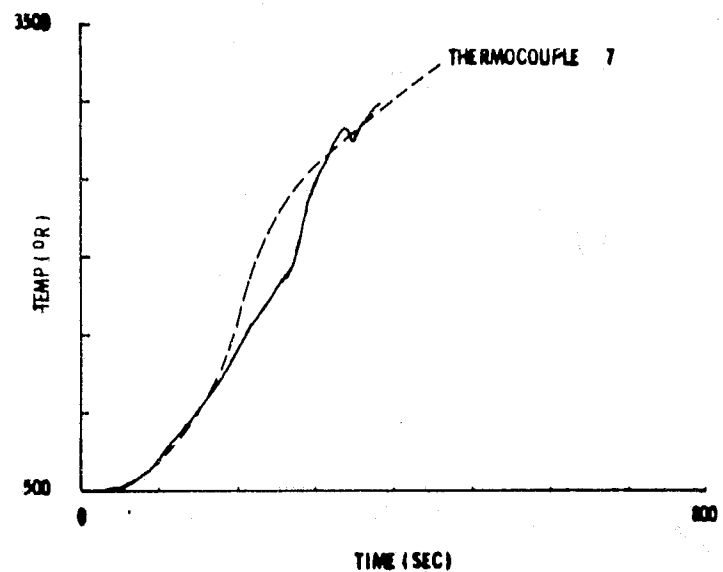
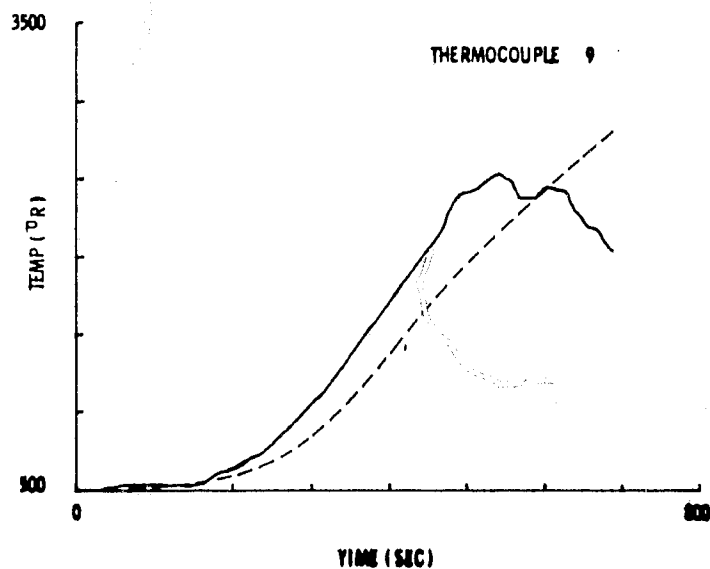
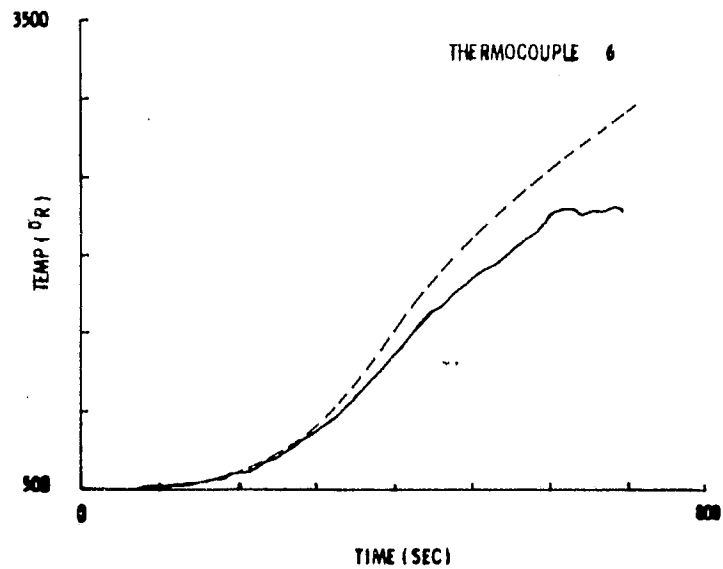


Figure 33a.

Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 13



PREDICTED ———
MEASURED ———

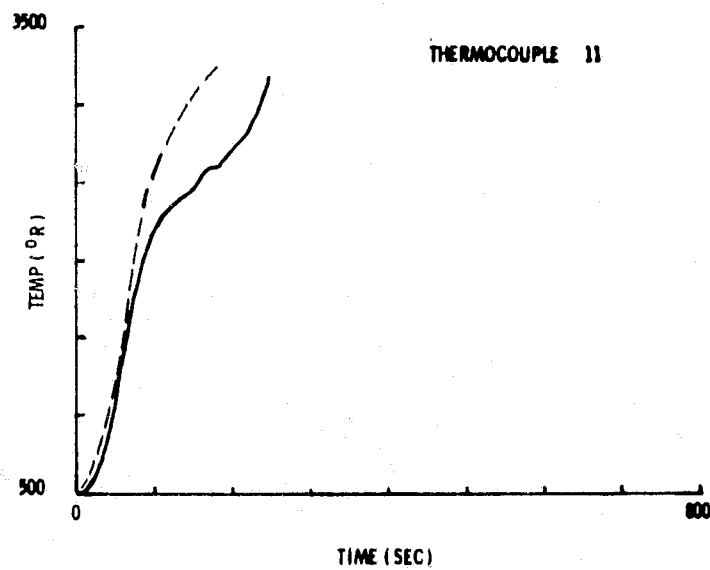
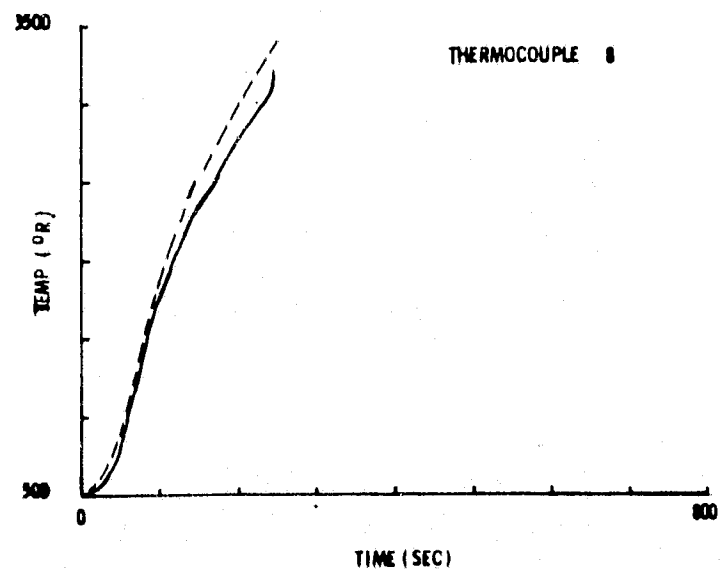


Figure 33b
Comparison of Predicted Temperature Response
with the Thermocouple Response of Model 13

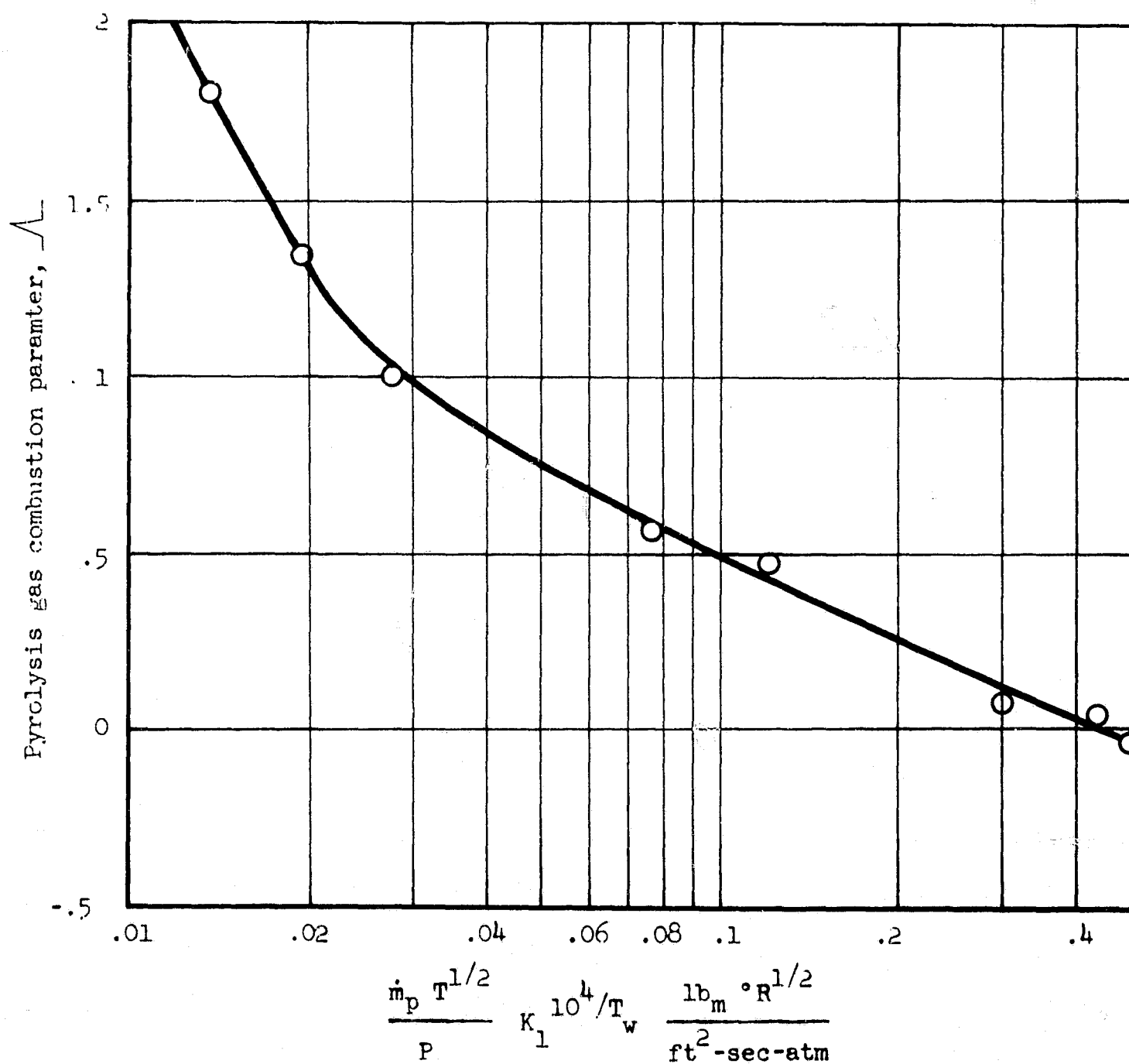


Figure 34.- Correlation of pyrolysis gas combustion parameter

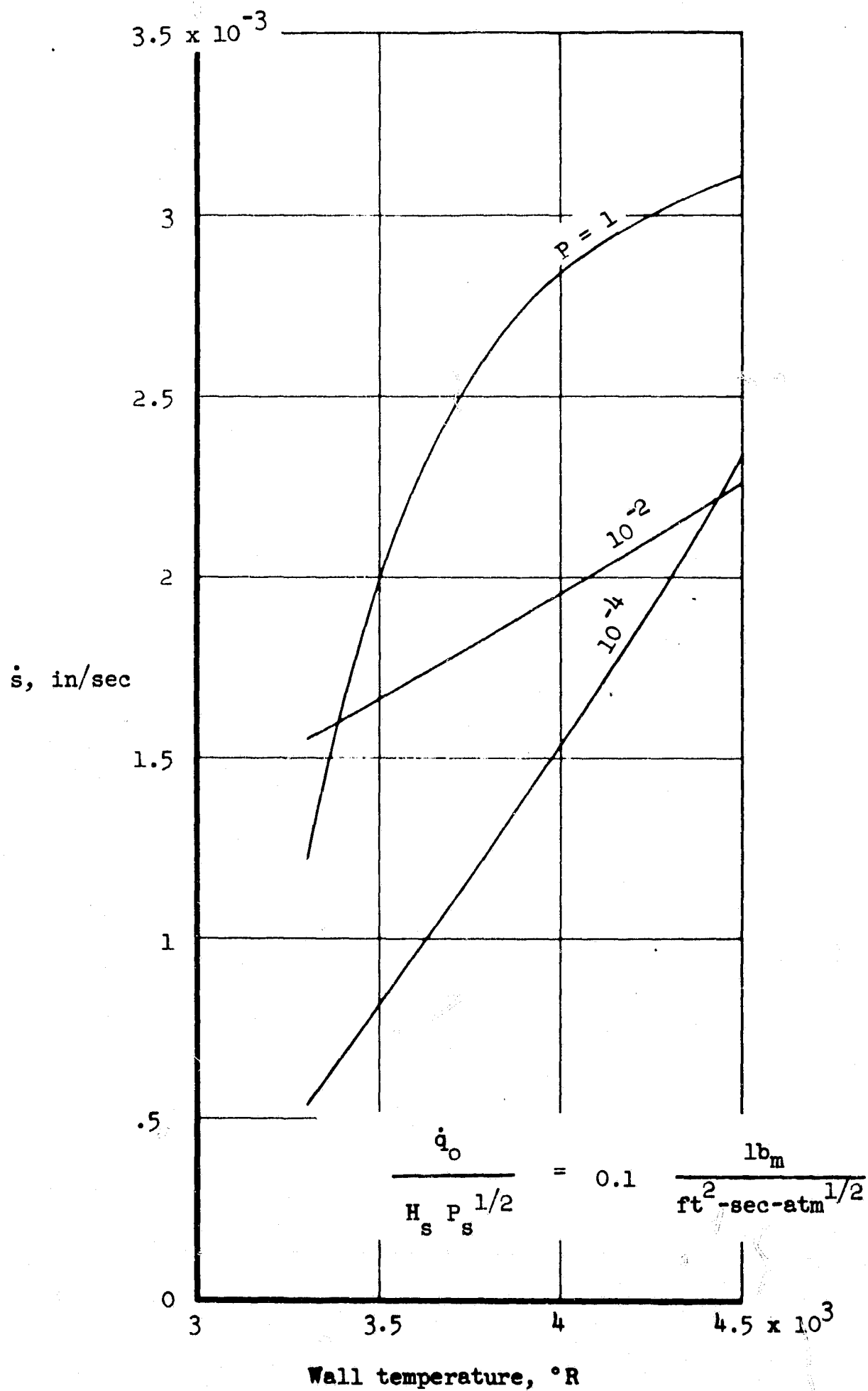


Figure 35.- Predicted steady state surface recession due to solid combustion only
Based on eqs. (25), (26), and (34)

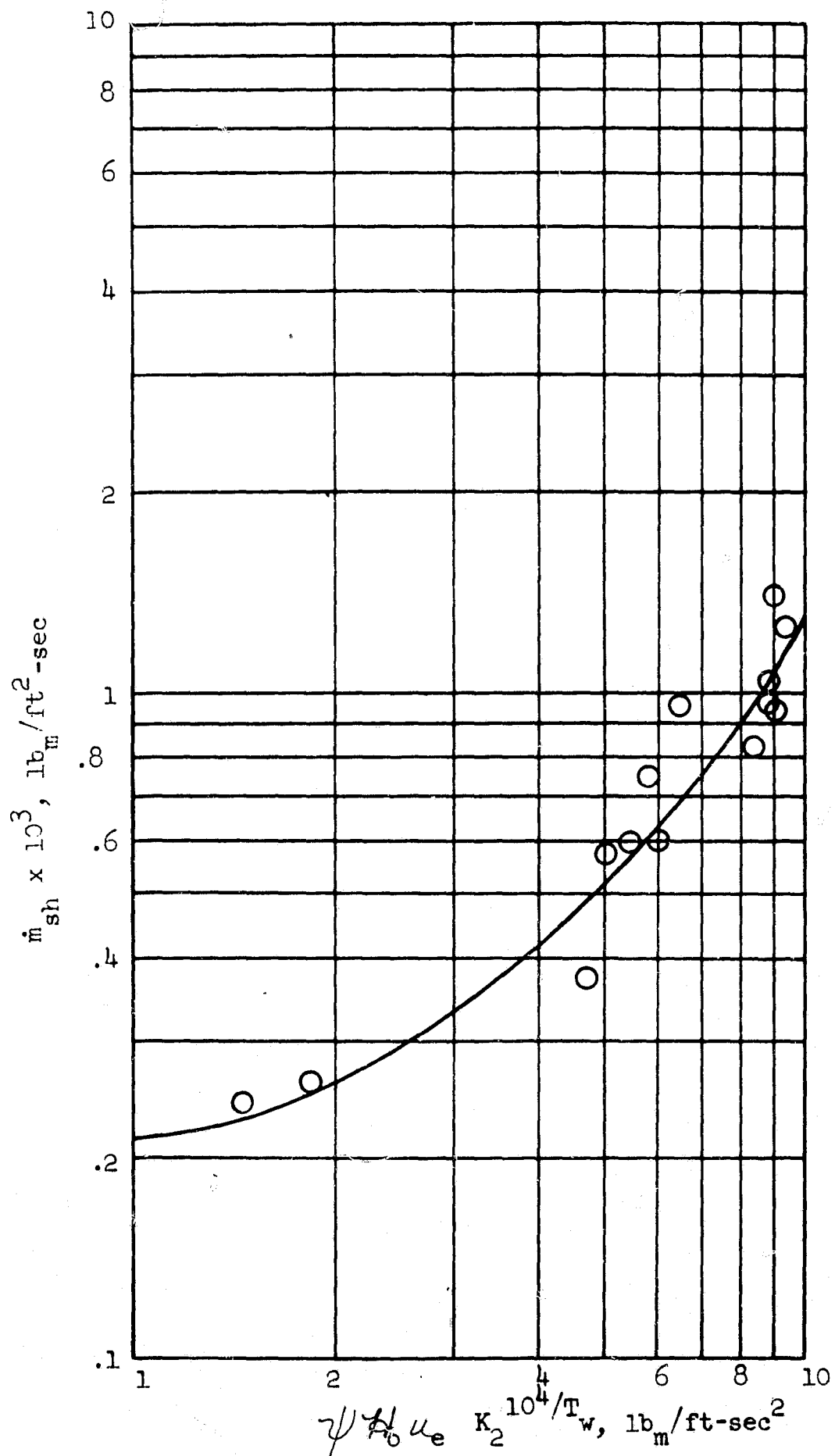


Figure 36.- Correlation of surface recession in an oxidizing shear environment (based on smooth, cold wall H_0).

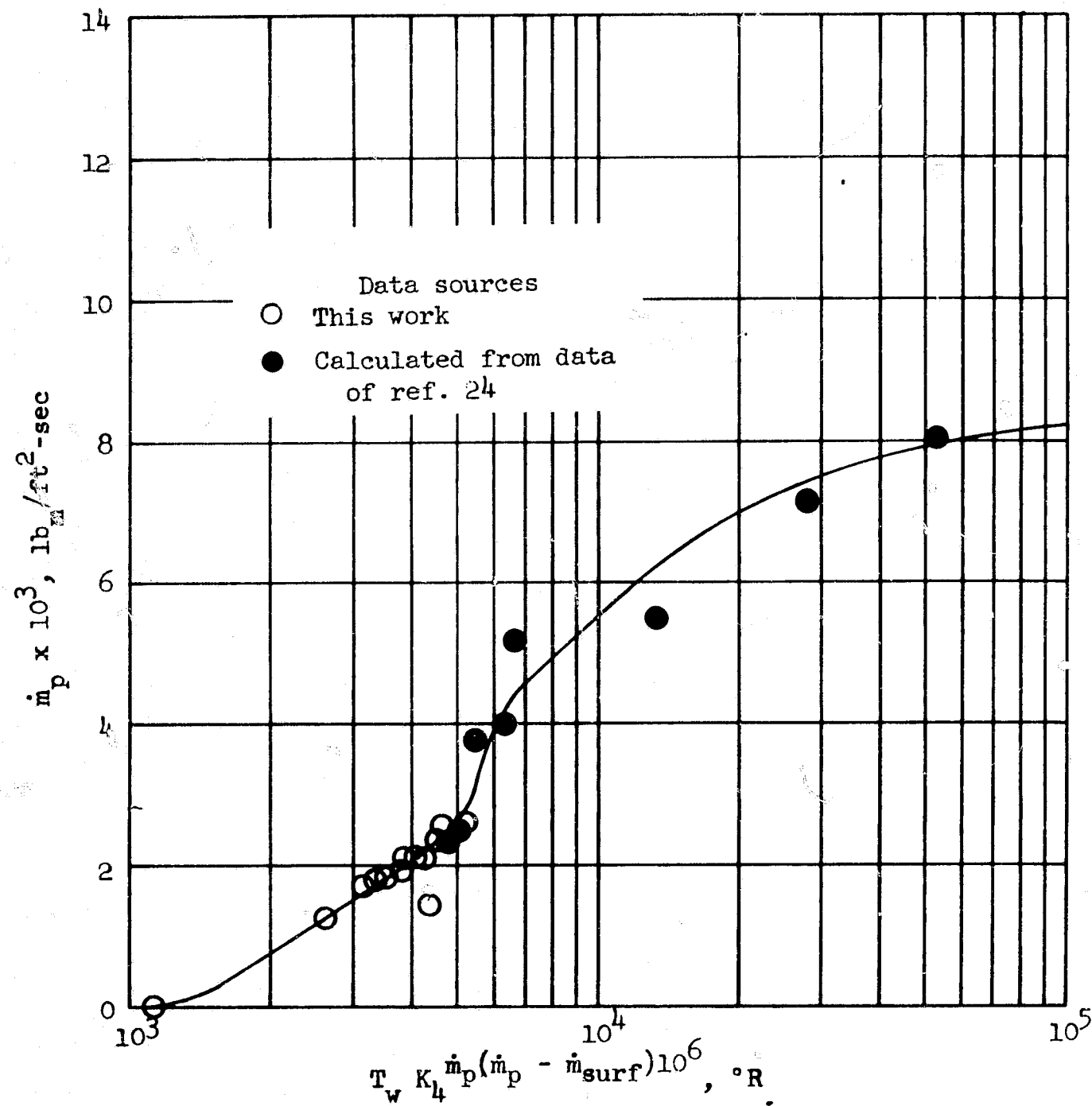


Figure 37.- Correlation of ablator condensed phase performance
 $\dot{m}_p, \dot{m}_{\text{surf}},$ and T_w

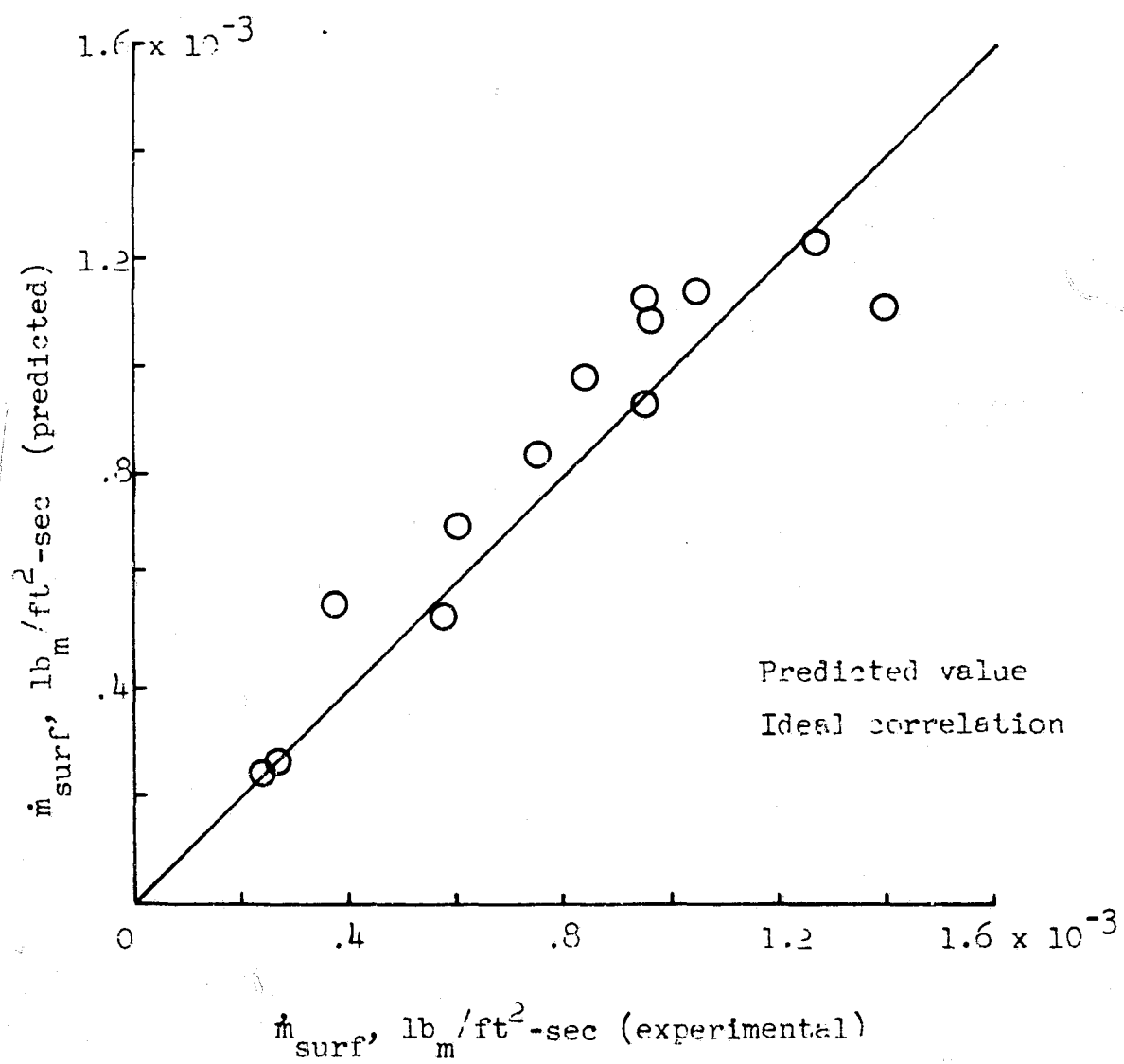


Figure 38.- Comparison of predicted and experimental ablator performance for turbulent flow

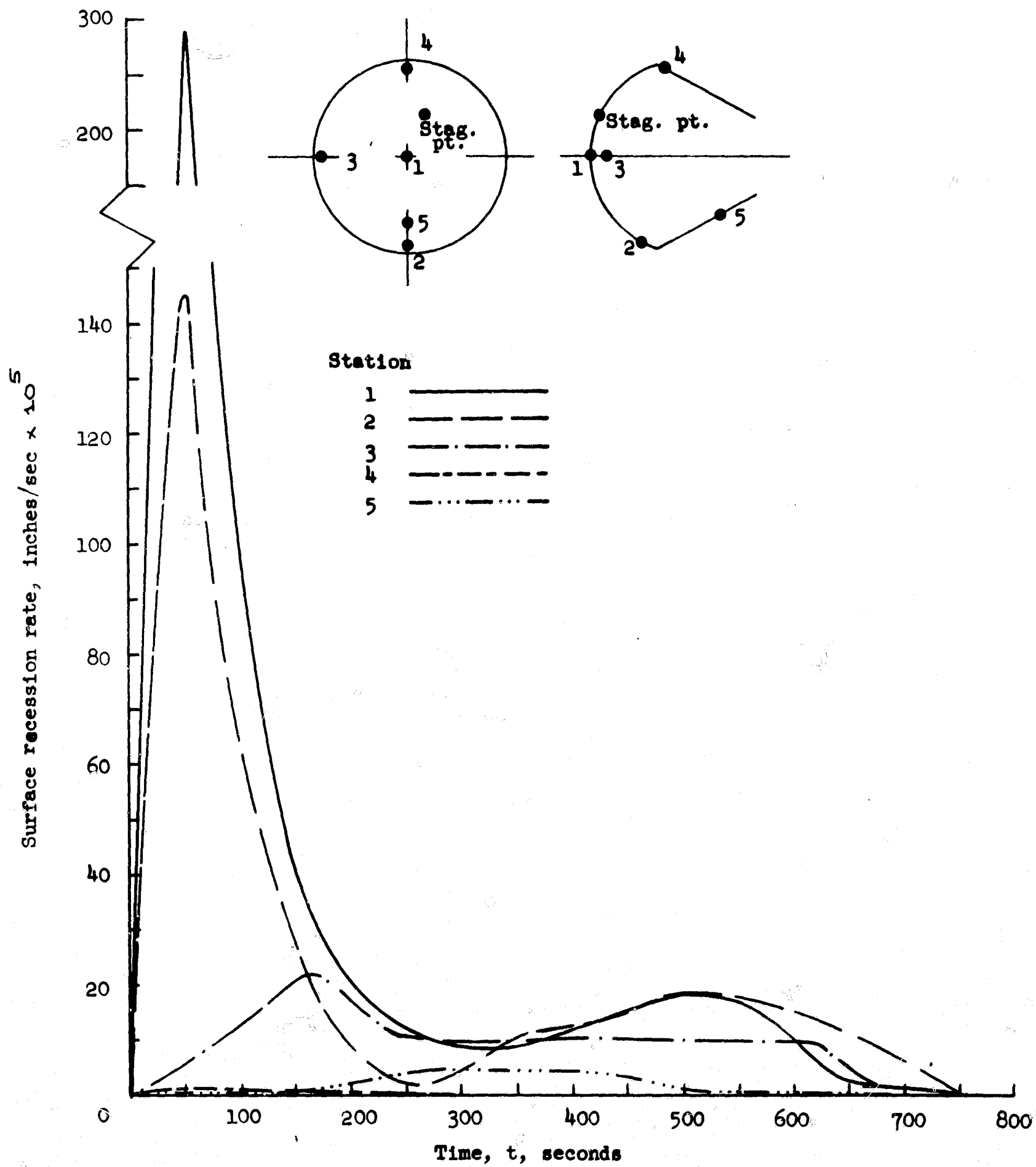


Figure 39
Apollo Surface Recession Rate
During 202 Trajectory

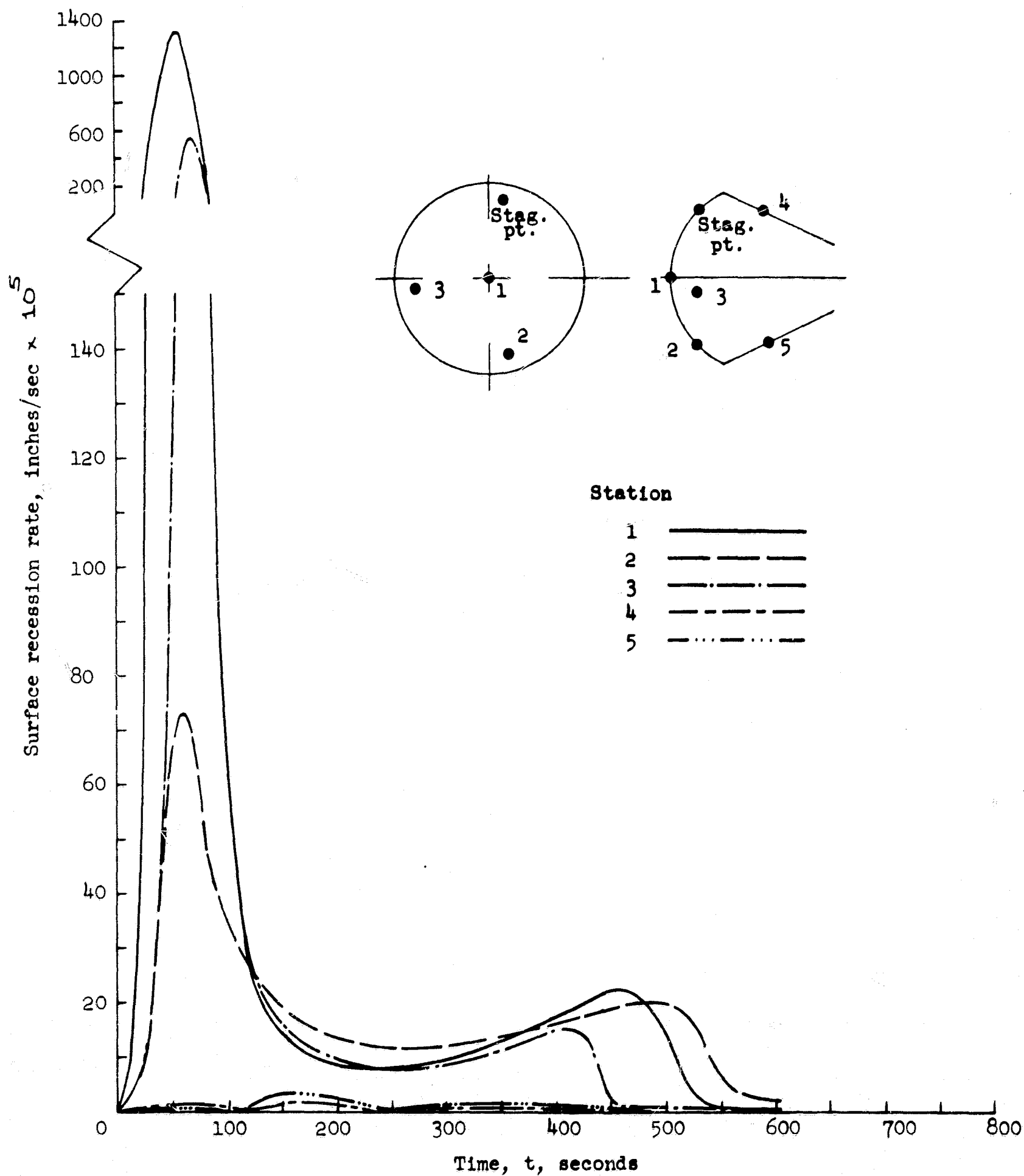


Figure 40
Apollo Surface Recession Rate
During 501 Nominal Trajectory

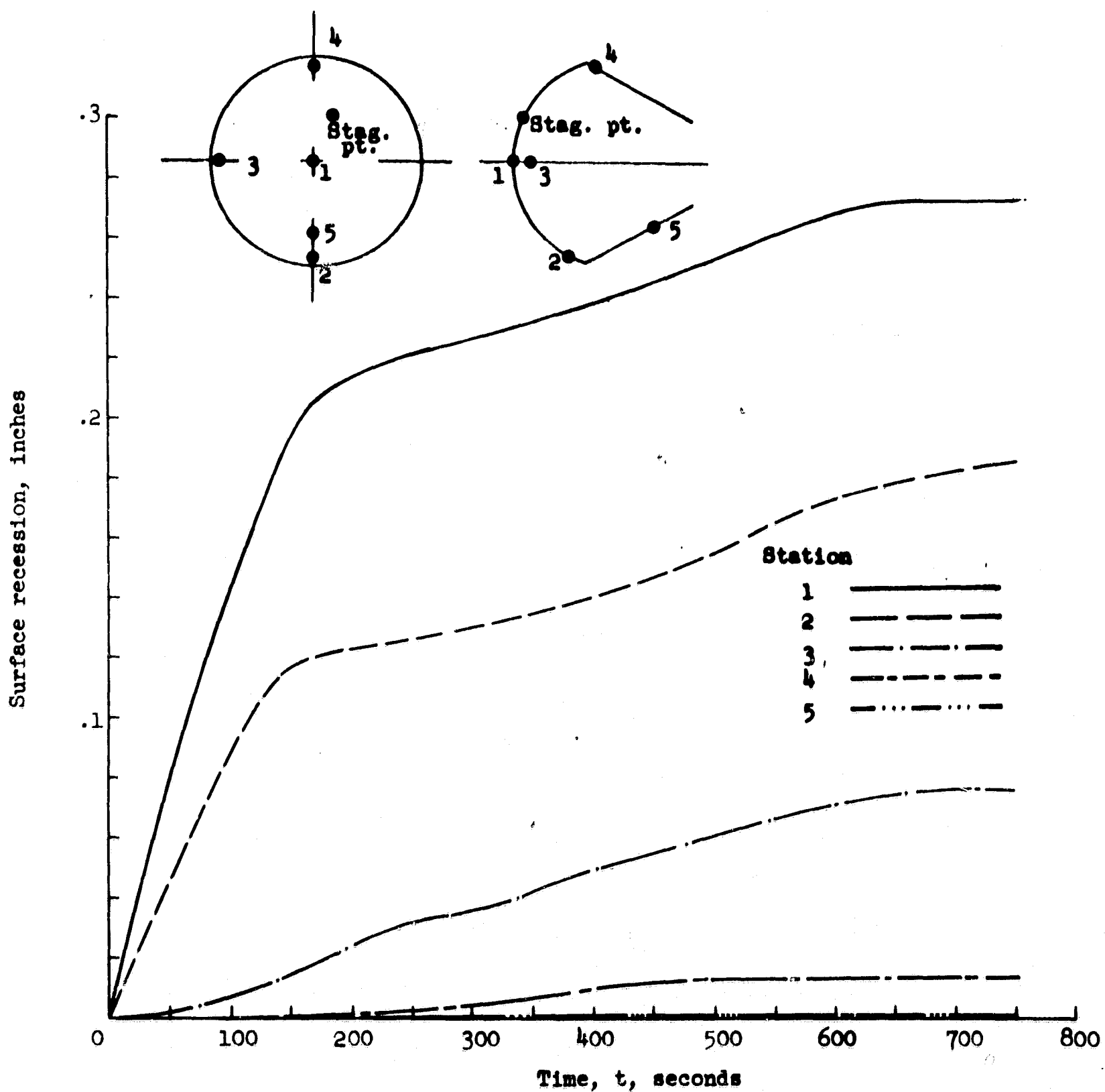


Figure 41
Apollo Surface Recession
During 202 Trajectory

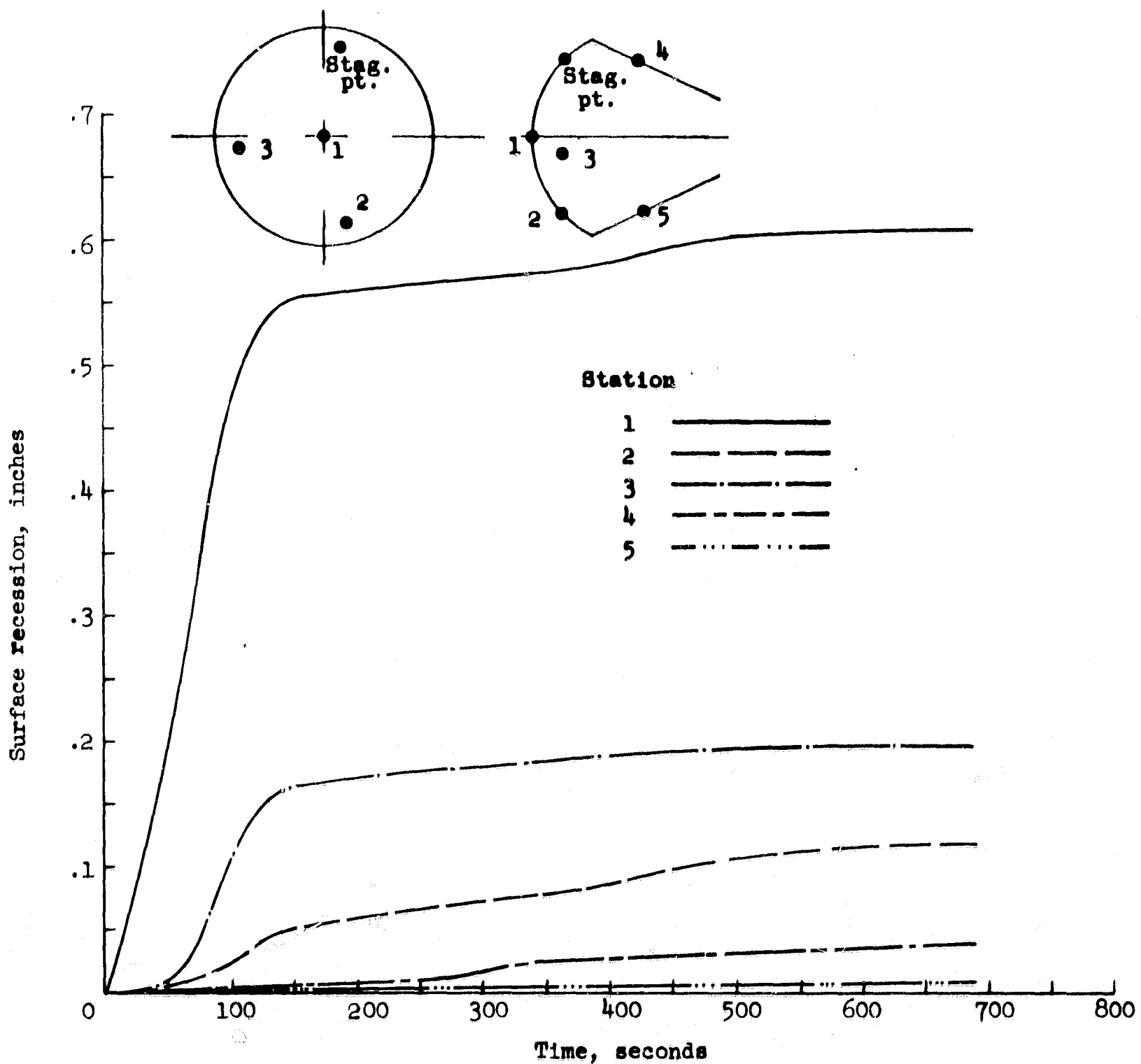


Figure 42
Apollo Surface Recession
During 501 Nominal Trajectory

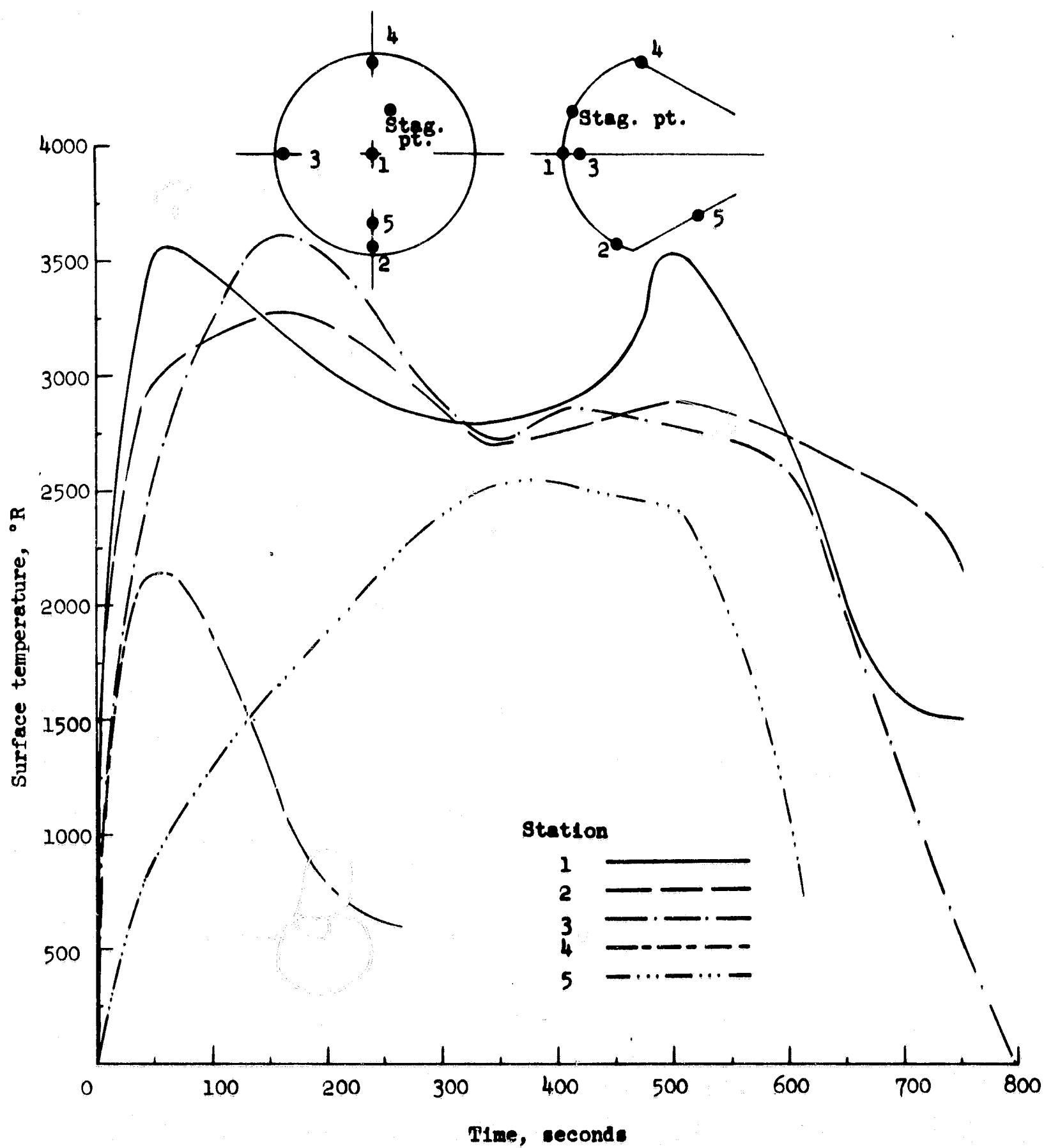


Figure 43
Apollo Surface Temperature
During 202 Trajectory

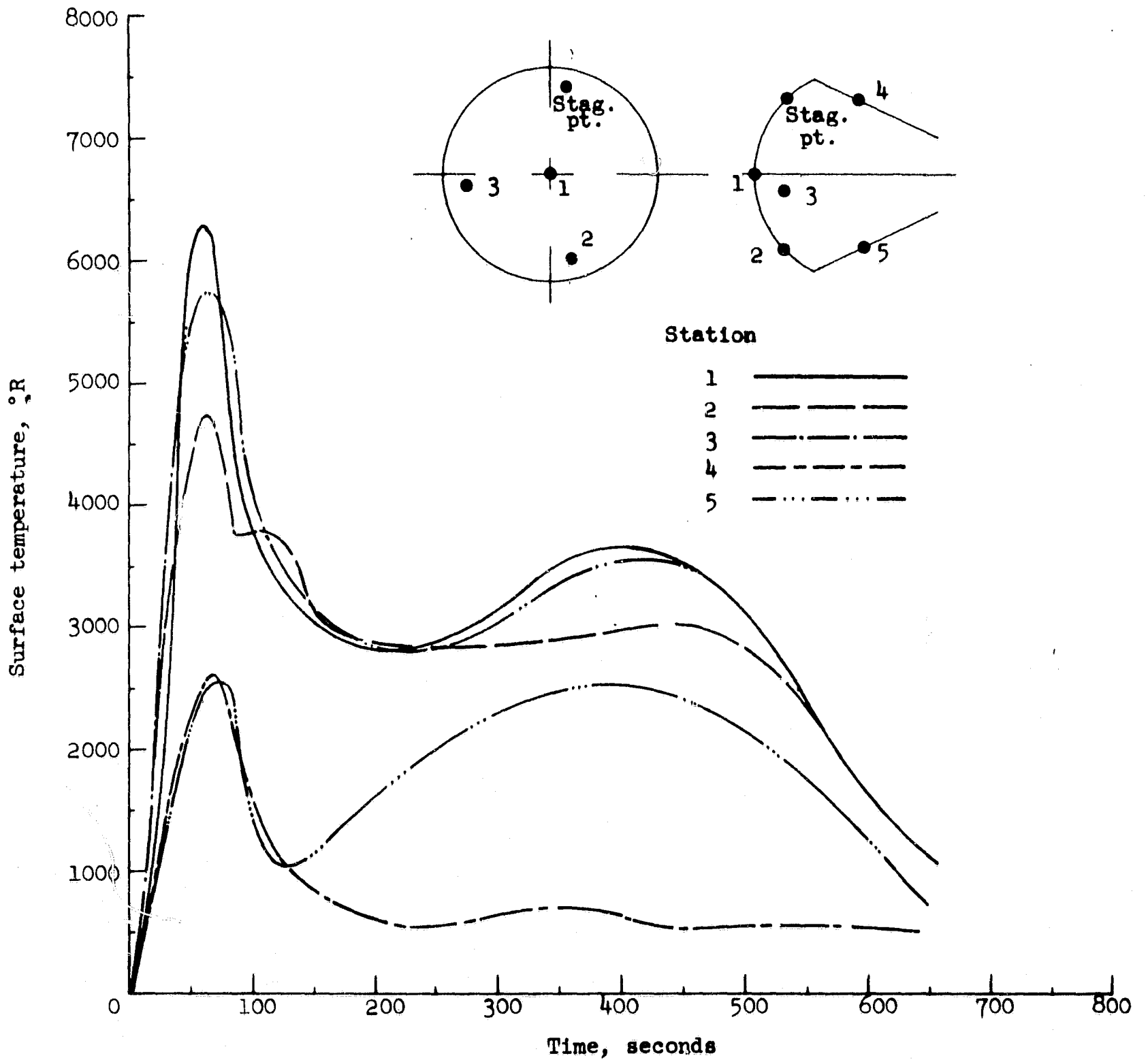


Figure 44

Apollo Surface Temperature
During 501 Nominal Trajectory

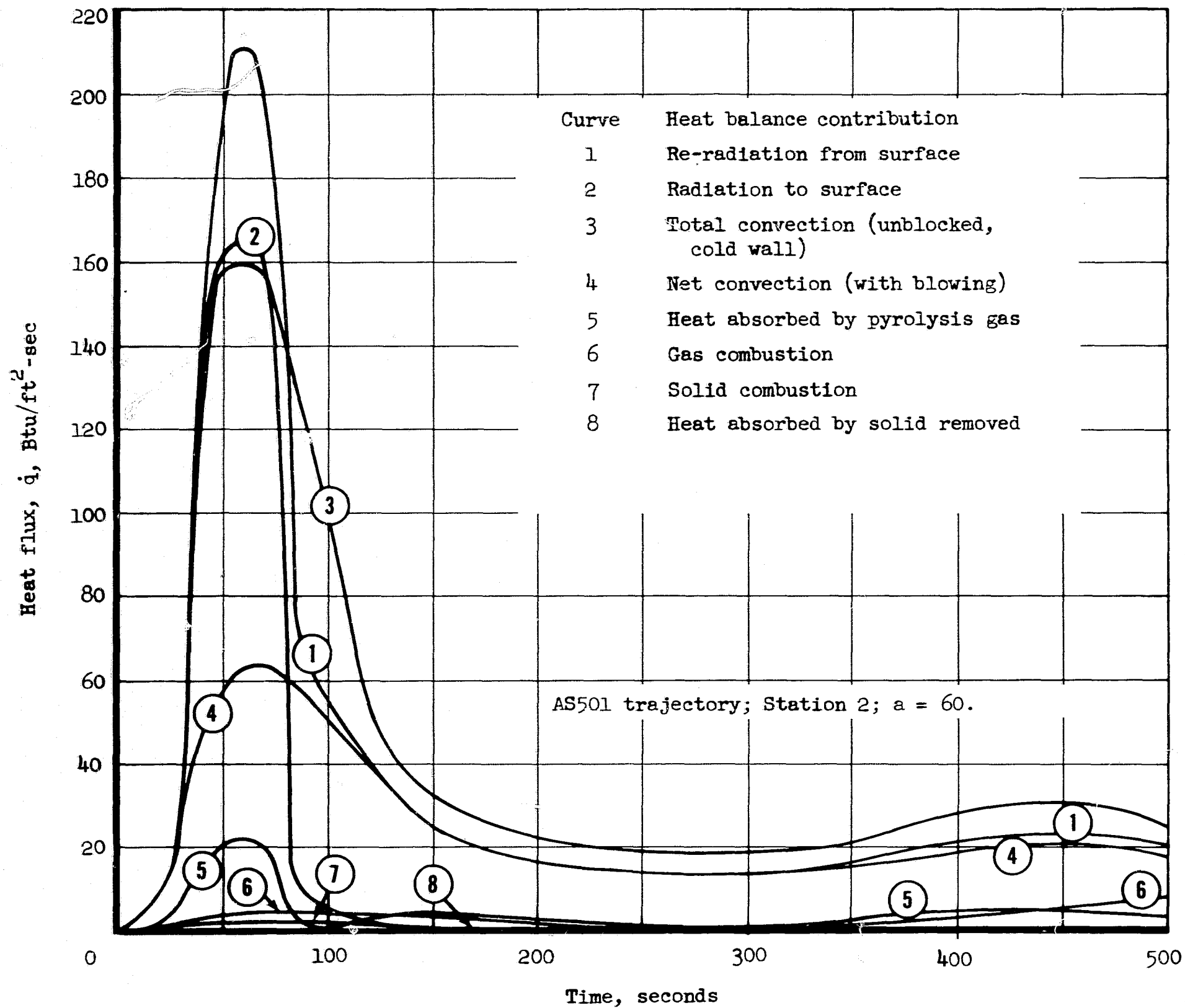


Figure 46.- Surface energy balance components.

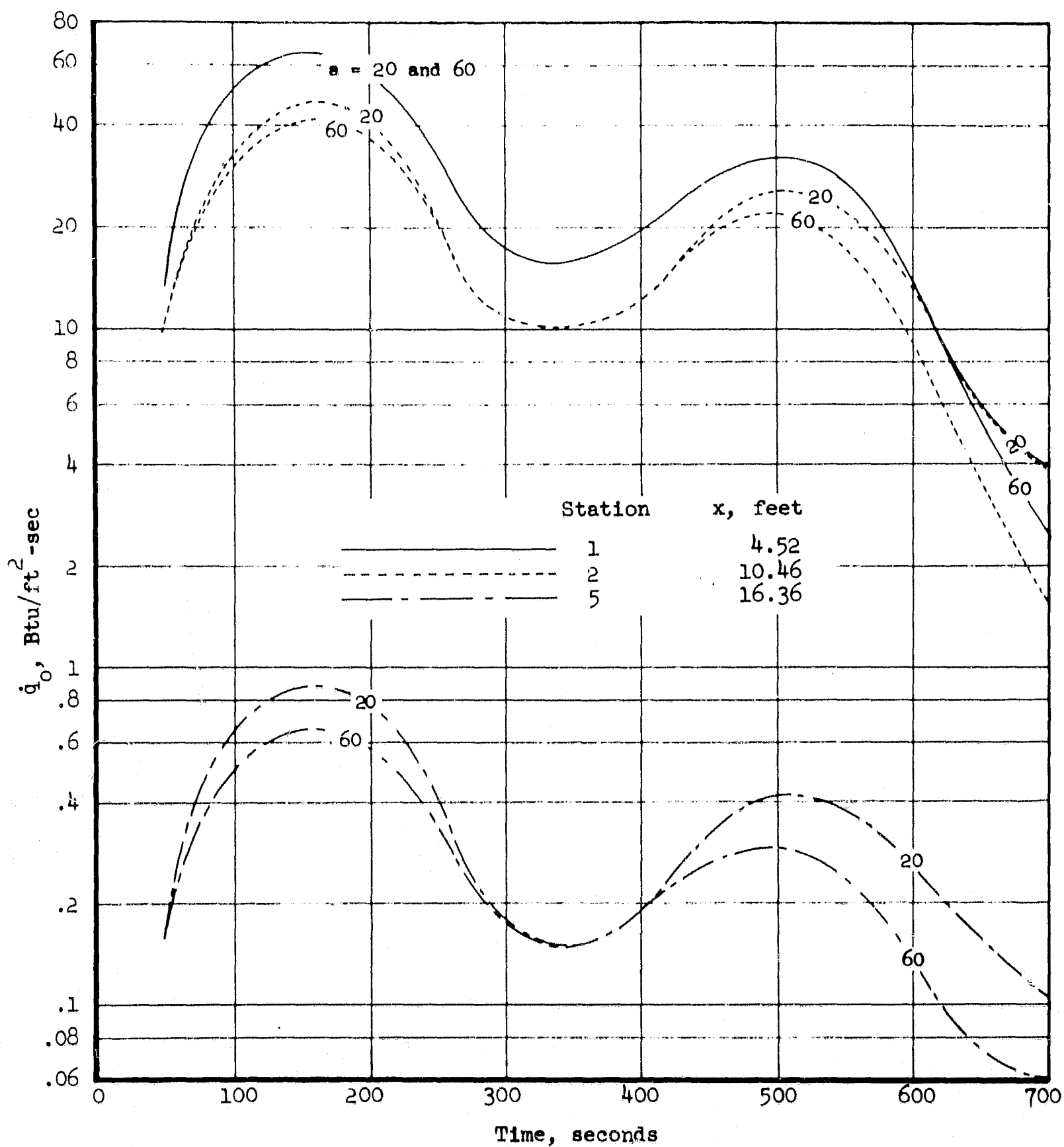


Figure 46.- Comparison of Apollo cold wall, unblocked convective heat transfer rates for two transition criteria; $a = 20$ and $a = 60$.

APPENDIX A

TURBULENT FLOW HEAT TRANSFER PREDICTION

Derivation of Mixing Length Expression

Assuming that a fluid in turbulent motion satisfies the Navier-Stokes equations instantaneously, it can be readily shown that the shear stress tensor may be defined as having two components: the viscous stresses and the turbulent or Reynolds stresses. For a steady or quasi-steady one-dimensional flow, the total stress, τ , in the direction of flow (neglecting triple correlations) is:

$$\tau = -\mu \frac{d\bar{u}}{dy} - \overline{(\rho v)' u'} \quad (A1)$$

Applying Prandtl's reasoning, the momentum is a transportable quantity. Thus, if a unit mass of fluid with velocity $u(y)$ is transported through a length ℓ , the "mixing length" (analogous to the mean free path in the kinetic theory of gases), then:

$$\bar{u}(y) + u' = \bar{u}(y + \ell) \quad (A2)$$

Approximating the value of $u(y + \ell)$ by a Taylor series expansion, and assuming ℓ to be small:

$$\bar{u}(y + \ell) = \bar{u}(y) + \ell \left. \frac{\partial \bar{u}}{\partial y} \right|_y \quad (A3)$$

so that:

$$u' = \bar{u}(y + \ell) - \bar{u}(y) \approx \ell \frac{\partial \bar{u}}{\partial y} \quad (A4)$$

Using van Driest's approximations (ref. 6)

$$-\overline{(\rho v)' u'} \approx -\bar{\rho} \overline{u' v'} \quad (A5)$$

and Prandtl's assumption that the velocity fluctuations u' and v' are of the same order of magnitude:

$$|v'| \sim |u'| \quad (A6)$$

so that:

$$v' = \xi |u'|, \quad 0 \leq \xi \leq 1 \quad (A7)$$

and the expression for the shear stress, τ , then becomes:

$$-\tau \approx \mu \frac{\partial \bar{u}}{\partial y} + \xi \rho \ell^2 \left| \frac{d\bar{u}}{dy} \right|^2 \quad (A8)$$

which is the very successful relationship used by Prandtl in considering wall turbulence. Rewriting the above expression in Boussinesq's form:

$$-\tau \approx (\mu + \rho \epsilon_M) \frac{d\bar{u}}{dy} \quad (A9)$$

where ϵ_M is the so called "eddy diffusivity of momentum".

Prandtl assumed that the mixing length ℓ is directly proportional to the distance from the wall, i.e.,

$$\ell = K y \quad (A10)$$

with K being a universal constant. He was able to obtain the logarithmic law of the wall which is applicable to the turbulent core where viscous stresses are negligible. The linear assumption fails in the transition and viscous sublayers. To calculate the wall transfer coefficients, it is necessary to modify the linear relationship in the vicinity of the wall to take into account viscous damping.

The usual practice is to use a two-layer model, wherein it is assumed that the viscous sublayer and turbulent core intersect abruptly at a certain distance δ_1 from the wall. The solution then becomes simply that of matching conditions at δ_1 . The introduction of δ_1 necessitates the evaluation of an arbitrary constant, which is very fortunate because it may be used to fit experimental data. Although the two-layer model is useful, its reliability is restricted.

From the success achieved by Prandtl in using a linear relationship for the mixing length, it follows that if δ is the hypothetical boundary layer thickness (see figure 1), then:

$$\omega(\delta) \equiv l/\delta \equiv k \quad (A11)$$

The ratio of the mixing length to the distance from the wall may be expressed as a function of the scaled distance provided local similarity is valid:

$$\omega(y) \equiv \omega_1(\eta) \equiv l/y \equiv l/\eta\delta \quad (A12)$$

Supposing that at a certain distance y from the wall the influence of viscosity becomes negligible, it would then be possible to obtain an approximate expression for $\omega(y)$ about y , by a Taylor series expansion. It would be reasonable to assume that $y \ll \delta$ so that $\eta_1 = y/\delta \ll 1$ and if $\eta < \eta_1$, then $\eta \ll 1$. We can thus pick any $\eta < \eta_1$, and expand $\omega_1(\eta)$ about η to obtain $\omega_1(\eta_1)$:

$$\omega_1(\eta_1) = \omega(\eta) + (\eta_1 - \eta) \frac{d\omega_1}{d\eta} + \dots \quad (A13)$$

Neglecting second and higher-order derivatives of ω_1 with respect to η , then:

$$\omega_1(\eta_1) \approx \omega(\eta) + (\eta_1 - \eta) \frac{\partial \omega_1}{\partial \eta} \quad (A14)$$

Assuming:

$$\eta \ll \eta_1$$

$$\omega_1(\eta_1) \approx \omega_1(\eta) + \eta_1 \frac{d\omega_1(\eta)}{d\eta} \quad (A15)$$

and since:

$$\omega_1(\eta_1) = k$$

$$\text{and } \eta_1 \frac{d\omega_1}{d\eta} \approx k - \omega_1(\eta) \quad (A16)$$

Then, integrating the above expression, we have:

$$\ln \left[\frac{k - \omega_1(\eta)}{k - \omega_1(0)} \right] = \frac{\eta}{\eta_1} \quad (A17)$$

or:

$$\omega = \frac{l}{y} = k - [k - \omega(0)] e \quad (A18)$$

Allowing $\lim_{y \rightarrow 0} \frac{l}{y} = 0$ then:

$$l = ky \left[1 - e^{-\frac{y}{\eta_1}} \right] \quad (A19)$$

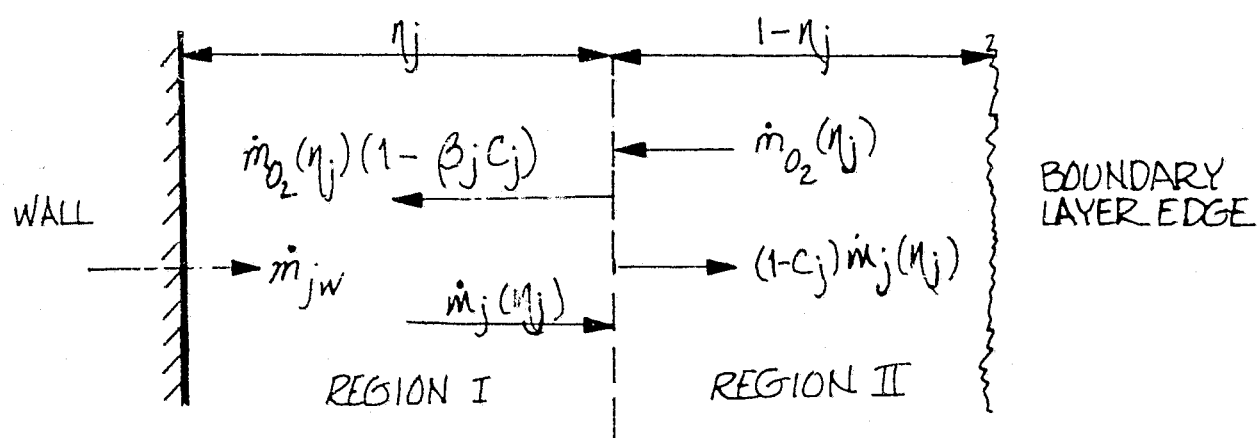
which is a form first empirically proposed by van Driest (ref. 6) and subsequently modified by Gill and Scher (ref. 10). The difference between the expressions of van Driest and Gill and Scher rests in the evaluation of the coefficient $1/\eta_1$. Gill's expression seems to be the more flexible since it contains two adjustable parameters and indeed, van Driest's expression is a special case of Gill's expression as seen from:

$$\phi = \frac{1}{\eta_1} = \frac{y_m^+ - a}{b} \quad (\text{Gill and Scher}) \quad (A20)$$

$$\phi = \frac{1}{\eta_1} = \frac{y_m^+}{b} \quad (\text{van Driest}) \quad (A21)$$

Simplified Boundary Layer Combustion Model

In the simplified combustion model, it is assumed that the combustion of each combustible species takes place in single planes. A schematic diagram is shown below.



If a fraction of the mass flux of the combustible species "j" undergoes instantaneous combustion at the plane "j", the equation for the conservation of species (j) may be applied to regions I and II separately and matched at the reaction front (the plane "j"). The same must, of course, be done with the oxygen species. The simplicity of the model rests on the fact that the sink or source terms in the species equation may be set to zero ($\dot{w}_j = 0$). Matching conditions at the interface are:

$$w_j(\eta_j)_- = w_j(\eta_j)_+ \quad (A22)$$

$$\dot{m}_j (\eta_j)_+ = \dot{m}_j (\eta_j)_- (1 - C_j) \quad (A23)$$

$$W_{O_2} (\eta_j)_- = W_{O_2} (\eta_j)_+ \quad (A24)$$

$$\dot{m}_{O_2} (\eta_j)_- = \dot{m}_{O_2} (\eta_j)_+ (1 - \beta_j C_j) \quad (A25)$$

Where β_j is the stoichiometric ratio of oxygen to species "j". The unknown η_j 's can be determined from the simultaneous solution of the concentration profiles for the combustible species and oxygen species. The model can handle any number of combustible species provided the C_j 's are known.

In the boundary layer computer program developed, the simplified model is included as a subroutine. Allowance is made for the presence of five species in the boundary layer: oxygen, nitrogen, pyrolysis gases, products of combustion, and an inert species coming from the matrix. The subroutine has not been used since the correlations that have been developed for the steady state performance of the charring ablator take into account the effects of boundary layer combustion.

Effect of Wall Roughness on Heat Flux

It is well known that wall roughness has a very pronounced effect on the transfer mechanisms in a turbulent boundary layer. Systematic studies have been made by Nikuradse (ref. 28) and others to take into account wall roughness effects on the skin friction coefficient. It is known that the skin friction is a very strong function of the roughness factor χ/K_s , where χ is the distance from the leading edge and K_s is the average height of the wall protuberances. For certain values of (χ/K_s) , the effects of roughness completely dominate the transfer mechanisms, the flow is then said to be in the "completely rough" regime. In the completely rough regime, the local skin friction for a turbulent boundary layer is given in reference 3 as:

$$\left(\frac{C_f}{2}\right)_R = \frac{1}{2} \left[2.87 + 1.58 \log_{10} \frac{\chi}{K_s} \right]^{-2.5} \quad (A26)$$

provided that:

$$10^2 < \frac{\chi}{K_s} < 10^6$$

We may rewrite the above equation to read:

$$\left(\frac{C_f}{2}\right)_R = \frac{1}{2} \left[2.87 + 1.58 \log_{10} \frac{Re_x}{Re_R} \right]^{-2.5} \quad (A27)$$

where Re_x is the local Reynolds number and Re_R is the "roughness Reynolds" number ($\rho_e u_e K_s / \mu_e$). Equations (A26) or (A27) are strictly valid when Re_R , say from zero to a large value, i.e., in the completely rough regime, equation (A27) may still be used provided a virtual origin x_0 is chosen appropriately, thus:

$$\left(\frac{C_f}{2}\right)_R \approx \frac{1}{2} \left[2.87 + 1.58 \log_{10} \frac{Re_x - Re_{x,0}}{Re_R} \right]^{-2.5} \quad (A28)$$

For the present purposes, we shall set $Re_{x,0} = 0$ since this would give us a lesser value of the skin friction.

We are interested in the ratio of the skin friction for the fully rough regime $(C_f/2)_R$ to that for a smooth wall $(C_f/2)_\sigma$. Making use of (A27), we have

$$\frac{C_f}{(C_f)_\sigma} = \left[\frac{1 + 0.238 \ln(Re_x / Re_R^0)}{1 + 0.238 \ln(Re_x / Re_R)} \right]^{2.5} \quad (A29)$$

where Re_R^0 is obtained from

$$\left(\frac{C_f}{2}\right)_\sigma = \frac{1}{2} \left[2.87 + 0.689 \ln \left(\frac{Re_x}{Re_R^0} \right) \right]^{-2.5} \quad (A30)$$

$$\left(\frac{C_f}{2}\right)_\sigma \approx a_1 Re_x^{-0.2} \quad (A31)$$

and

$$a_1 \approx 0.0288$$

In the tests carried out in this program the Reynolds number based on boundary layer thickness (Re_δ) varied from 2000 to 8000:

$$2000 \lesssim \frac{\rho_e u_e \delta}{\mu_e} \lesssim 8000 \quad (A32)$$

which corresponds to $200,000 \lesssim Re_x \lesssim 500,000$. For the runs using air as the free-stream gas, the average height of the protuberances was always greater than 0.03-inch (with the exception of two low wall temperature runs possibly), i.e.,

$$K_s \gg 0.03 \quad (A33)$$

The minimum value would be $Re_R \approx 80$.

Equation (A29) will not apply if

$$\left(\frac{C_f}{2}\right)_\sigma > \frac{1}{2} \left[2.87 + 0.69 \ln \frac{Re_x}{Re_R} \right]^{-2.5} \quad (A34)$$

$$Re_R^\circ > Re_R \quad (A35)$$

To show that the test runs carried in this program all fall in the completely rough regime, one need only show that

$$Re_{x,max} / Re_{R,min} \leq (\chi/K_s)_{max} \quad (A36)$$

where $(\chi/K_s)_{max}$ is the maximum roughness factor corresponding to $(Re_x)_{max}$ that will make the flow completely rough. From figure 21.12 of reference 1,

$$\left(\frac{\chi}{K_s}\right)_{max} = 7000$$

Thus,

$$\frac{(Re_x)_{max}}{(Re_R)_{min}} = \frac{500,000}{80} = 6250 \quad (A37)$$

and certainly less than 7000.

From the Reynolds analogy,

$$(\tau_w / \rho_e u_e^2) / (\dot{q}_w / \rho_e u_e \Delta H) \approx Pr^{2/3} \quad (A38)$$

or

$$(C_f/2) / St \approx Pr^{2/3} \quad (A39)$$

Now,

$$\frac{C_f}{(C_f)_\sigma} = \frac{St}{(St)_\sigma} = \frac{St \rho_e u_e}{(St)_\sigma \rho_e u_e} = \frac{H}{H_\sigma} \quad (A40)$$

where

$$H = St \rho_e u_e = \frac{\dot{q}_w}{H_e - H_w} \quad (A41)$$

For the test runs in this program:

$$\frac{C_f}{(C_f)_\sigma} = \frac{h}{h_\sigma} = \frac{\dot{q}_w}{(\Delta H)} = \frac{(\Delta H)_\sigma}{\dot{q}_{w,\sigma}} \quad (A42)$$

where the subscript " σ " refers to smooth cold wall conditions. The values of $\dot{q}_w$ for the air runs were obtained from an overall heat balance at the wall (see eq. (23) of this report).

Calculations were carried out to compare values of h/h_σ based on the actual experimental data to those obtained using eq. (A29). It was assumed that the probable heights of the protruberances were of the order of a third of the total recession. A method of calculating boundary layer edge conditions within the duct is given in Appendix C. Using these calculated values and assuming that the boundary layer thickness in the exit of the duct does not considerably differ from half the thickness of the duct, the Reynolds number based on boundary layer thickness is obtained. The corresponding value of Re_x for a flat plate may then be determined from the turbulent boundary layer solutions (e.g., ref. 9). Table X gives the results of such calculations. In all cases, the increase in Stanton number may be attributable to surface roughness. It should also be noted that surface roughness may greatly affect heat transfer in the stagnation region. A recent report by Dunavant and Stone (ref. 35) presents data showing that at some conditions an increase in heat transfer of up to at least 80% over a smooth wall level is possible. At the test conditions in ref. 24, it is estimated that heat transfer may be increased by as much as 60% by roughness development during stagnation region ablation. Clearly further work is necessary in developing prediction methods for roughness formation and resulting effects.

Solution of the Asymptotic Boundary Layer Equations

The approximate profiles for velocity, enthalpy and specie concentration used in the integral boundary layer equations were obtained from the asymptotic boundary layer equations. The solution of the asymptotic equations are presented in this section:

Continuity

$$\frac{d}{dy} \rho v r_0^E = 0$$

$$(\rho v) = (\rho v)_0 \quad (A43)$$

Species Conservation

$$(\rho v)_0 \frac{dw_i}{dy} = \frac{d}{dy} (L_i^*) + \rho \dot{w}_i \quad (A44)$$

where

$$L_i^* = \rho D_i^* \frac{dw_i}{dy}$$

$$= \rho D_i \left(1 + \frac{\epsilon}{D_i} \right) \frac{dw_i}{dy}$$

In dimensionless form (A44) becomes;

$$\frac{d\phi_i}{d\eta} = \frac{d}{d\eta} \left[A_i^{-1} \frac{d\phi_i}{d\eta} \right] + \frac{\rho \dot{w}_i \delta}{F_0 (\rho_e u_e) (w_{ie} - w_{i0})} \quad (A45)$$

where

$$\phi_i = \frac{w_i - w_{i0}}{w_{ie} - w_{i0}}$$

$$A_i^{-1} = \frac{\phi_\mu \left(1 + \frac{\epsilon}{D_i} \right)}{F_0 Re \delta Sc_i}$$

Integrating (A45) once,

$$\frac{d: i}{d\tau} - A_i : i, = A_i \left[\frac{1}{A_{i0}} \frac{d: i}{d\tau} \right]_0 - \int_0^{\tau} \frac{\dot{w}_i d\tau}{e^{\int_0^{\tau} A_i d\tau} F_0 (w_{ie} - w_{i0})} \quad (A46)$$

Another integration yields:

$$\begin{aligned} : i = & - \frac{1}{A_{i0}} \frac{d: i}{d\tau} \Big|_0 + \int_0^{\tau} \frac{\dot{w}_i d\tau}{e^{\int_0^{\tau} A_i d\tau} F_0 (w_{ie} - w_{i0})} \\ & + \frac{1}{A_{i0}} \frac{d: i}{d\tau} \Big|_0 \frac{\int_0^{\tau} A_i d\tau}{e^{\int_0^{\tau} A_i d\tau}} - \int_0^{\tau} \frac{\int_0^{\tau} A_i d\tau}{e^{\int_0^{\tau} A_i d\tau}} \frac{\dot{w}_i}{F_0 (w_{ie} - w_{i0})} d\tau \end{aligned}$$

Now,

$$1 = \frac{1}{A_{i0}} \frac{d: i}{d\tau} \Big|_0 \left(e^{\int_0^1 A_i d\tau} - 1 \right) - e^{\int_0^1 A_i d\tau} \int_0^1 \frac{A_i \int_0^{\tau} \dot{w}_i d\tau}{e^{\int_0^{\tau} A_i d\tau} F_0 (w_{ie} - w_{i0})} e^{-\int_0^{\tau} A_i d\tau} d\tau$$

so that,

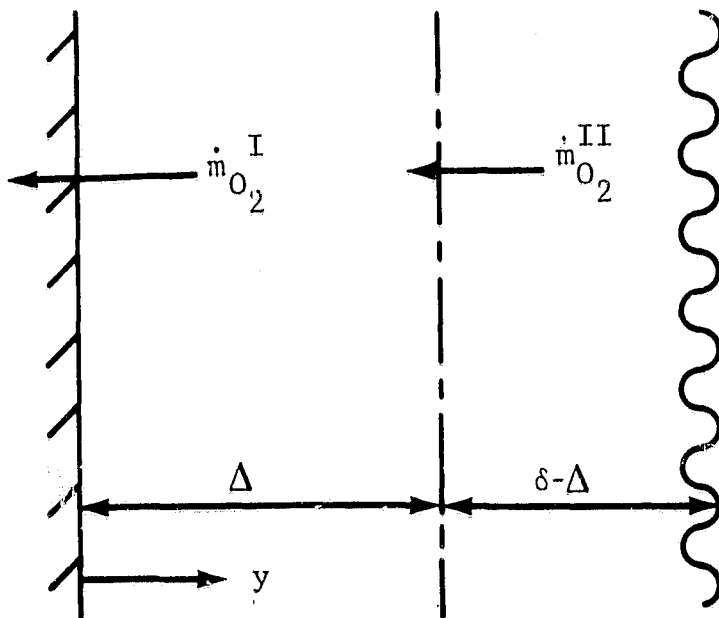
$$\left(\frac{1}{A_{i0}} \right) \frac{d: i}{d\tau} \Big|_0 = \frac{1 + e^{\int_0^1 A_i d\tau} \int_0^1 \frac{A_i \int_0^{\tau} \dot{w}_i d\tau}{e^{\int_0^{\tau} A_i d\tau} F_0 (w_{ie} - w_{i0})} e^{-\int_0^{\tau} A_i d\tau} d\tau}{e^{\int_0^1 A_i d\tau} - 1} \quad (A47)$$

Thus,

$$\begin{aligned} : i = & \frac{e^{\int_0^1 A_i d\tau} - 1}{e^{\int_0^1 A_i d\tau} - 1} \left[1 + e^{\int_0^1 A_i d\tau} \int_0^1 \frac{A_i \int_0^{\tau} \dot{w}_i d\tau}{e^{\int_0^{\tau} A_i d\tau} F_0 (w_{ie} - w_{i0})} e^{-\int_0^{\tau} A_i d\tau} d\tau \right] \quad (A48) \\ & - e^{\int_0^1 A_i d\tau} \int_0^1 \frac{A_i \int_0^{\tau} \dot{w}_i d\tau}{e^{\int_0^{\tau} A_i d\tau} F_0 (w_{ie} - w_{i0})} e^{-\int_0^{\tau} A_i d\tau} d\tau \end{aligned}$$

The preceding equation is too complicated and requires too much information. For the present purposes, a less rigorous model will be used for boundary layer combustion. Five species are assumed present: Oxygen, an inert boundary layer species, the products of combustion, the inert and combustible ablator gases.

Consider the O_2 species:



Region I:

$$(\rho v)_0 \frac{dw_{O_2}}{dy} = \frac{d}{dy} \left[\rho D_{O_2} \left(1 + \frac{\epsilon}{D_{O_2}} \right) \frac{dw_{O_2}}{dy} \right]$$

$$\frac{dw_{O_2}}{d\eta} = \frac{d}{d\eta} \left[\frac{\rho D_{O_2} \left(1 + \frac{\epsilon}{D_{O_2}} \right) dw_{O_2}}{(\rho v)_0 \delta} \frac{d\eta}{d\eta} \right]$$

$$\frac{w_{O_2} - (w_{O_2})_0}{(w_{O_2})_c - (w_{O_2})_0} = \frac{e^{\int_0^{\eta} A_{O_2} d\sigma} - 1}{e^{\int_0^{\eta_c} A_{O_2} d\sigma} - 1} \quad 0 \leq \eta \leq \eta_c$$

$$\eta = \frac{y}{\delta}$$

$$(w_{O_2})_0 = O_2 \text{ mass fraction at wall}$$

$$\left(w_{O_2}\right)_c = O_2 \text{ mass fraction at combustion plane}$$

$$A_{O_2} = \frac{F_0 Re Sc_{O_2}}{\left(1 + \frac{\epsilon}{D_{O_2}}\right)}$$

$$r_c = \frac{\dot{m}}{4\pi}$$

Also, for $O_{2,c}$:

$$\begin{aligned} -\dot{m}_{O_2} &= (\rho v)_0 w_{O_2} - D_{O_2} \left(1 + \frac{\epsilon}{D_{O_2}}\right) \frac{1}{\delta} \frac{dw_{O_2}}{dr} \\ &= (\rho v)_0 \left\{ \left[w_{O_2}\right] - \left[\left(w_{O_2}\right)_c - \left(w_{O_2}\right)_0\right] \frac{\int_0^r A_{O_2} d\sigma}{\left(\int_0^{r_c} A_{O_2} d\sigma\right)_{-1}} \right\} \end{aligned}$$

so that

$$-\dot{m}_{O_2}^I = -\frac{\dot{m}_{O_2}^I}{(\rho v)_0} = \left(w_{O_2}\right)_0 - \left[\left(w_{O_2}\right)_c - \left(w_{O_2}\right)_0\right] \frac{1}{\left[\int_0^{r_c} A_{O_2} d\sigma\right]_{-1}}$$

$$\left(w_{O_2}\right)_0 = \frac{\left(w_{O_2}\right)_c}{\int_0^{r_c} A_{O_2} d\sigma} - \dot{m}_{O_2}^I \left[\frac{\int_0^{r_c} A_{O_2} d\sigma}{\int_0^{r_c} A_{O_2} d\sigma} \right]_{-1}$$

$$= \left[\left(w_{O_2}\right)_c + \dot{m}_{O_2}^I\right]_c e^{-\int_0^{r_c} A_{O_2} d\sigma} - \dot{m}_{O_2}^I$$

L

$$\beta_{O_2}^I = - \left(w_{O_2} \right)_c \left[\frac{1}{\int_0^{\eta_c} \frac{1}{A_{O_2}} d\sigma - 1} \right] \quad (A49)$$

Region II: $\eta_c \leq \eta \leq 1$

$$- \dot{m}_{O_2}^{II} = (\rho v)_0 w_{O_2} - (\rho v)_0 \left[\frac{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma}{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma - 1} \right] \left[\left(w_{O_2} \right)_e - \left(w_{O_2} \right)_c \right]$$

$$\begin{aligned} - \beta_{O_2}^{II} &= w_{O_2} - \left[\left(w_{O_2} \right)_e - \left(w_{O_2} \right)_c \right] \frac{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma}{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma - 1} \\ &= \left(w_{O_2} \right)_c - \left[\left(w_{O_2} \right)_e - \left(w_{O_2} \right)_c \right] \frac{1}{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma - 1} \end{aligned}$$

$$= \left(w_{O_2} \right)_c \left[\frac{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma}{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma - 1} \right] - \frac{\left(w_{O_2} \right)_e}{\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma - 1}$$

$$\left(w_{O_2} \right)_c = \left[\left(w_{O_2} \right)_e + \beta_{O_2}^{II} \right] e^{-\int_{\eta_c}^1 \frac{1}{A_{O_2}} d\sigma} - \beta_{O_2}^{II} \quad (A50)$$

Different cases may now be treated. A particular case of interest here is complete combustion.

When the combustion is complete:

$$\left(w_{O_2}\right)_c = 0$$

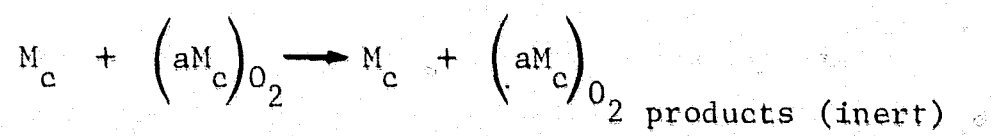
$$\beta_{O_2}^I = 0$$

Let

w_{IM} = inert (combustion-wise) species from ablator

w_c = combustible species

w_{O_2} = oxygen species



The reaction is:

$\left(M_c\right)$ pounds of combustible + $\left(aM_c\right)$ pounds of oxygen $\rightarrow \left[M_c(1+a)\right]$ pounds inert

Now at I:

$$\dot{m} = (\rho v)_0 = \dot{m}_c + \dot{m}_I$$

at II:

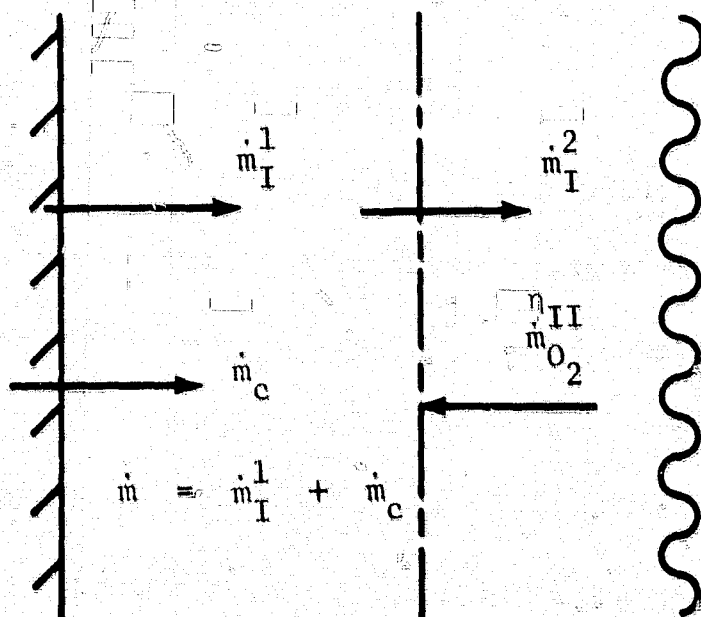
$$\dot{m} = \dot{m}_I^2 - \dot{m}_{O_2}^{II}$$

So that:

$$\dot{m}_{O_2} = \frac{1}{a} \dot{m}_c$$

$$\dot{m}_I^2 = \dot{m}_I^1 + a\dot{m}_{O_2}^{II}$$

In schematic form:



$$\dot{m} = \dot{m}_I^1 + \dot{m}_c = \dot{m}_I^2 - \dot{m}_{O_2}^{II}$$

$$\dot{m}_I^2 = \dot{m}_I^1 + \dot{m}_c + \dot{m}_{O_2}^{II}$$

$$= \dot{m}_I^1 + \dot{m}_c (1+a)$$

since

$$\dot{m}_{O_2} = \frac{1}{a} \dot{m}_c, \quad \beta_{O_2}^{II} = \frac{\dot{m}_c}{a\dot{m}} = \frac{1}{a} F_c$$

Thus:

$$\left[\left(w_{O_2} \right)_e + \frac{F_c}{a} \right] e^{-\int_{\eta_c}^1 A_{O_2} d\sigma} = \frac{F_c}{a}$$

$$e^{-\int_{\eta_c}^1 A_{O_2} d\sigma} = \frac{\frac{F_c}{a}}{\left(w_{O_2} \right)_e + \frac{F_c}{a}} = \frac{1}{1 + a \frac{\left(w_{O_2} \right)_e}{F_c}}$$

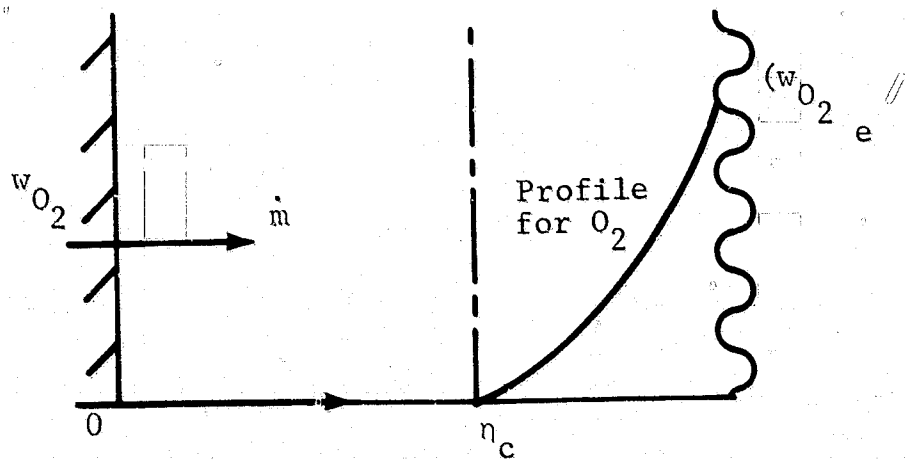
$$\int_{\eta_c}^1 A_{O_2} d\sigma = \ln \left[1 + a \frac{\left(w_{O_2} \right)_e}{F_c} \right]$$

$$F_0 \operatorname{Re} \delta \int_{\eta_c}^1 \frac{Sc_{O_2}}{\phi_\mu \left(1 + \frac{\epsilon}{D_{O_2}} \right)} d\sigma = \ln \left[1 + a \frac{\left(w_{O_2} \right)_e}{F_c} \right] \quad (A51)$$

This relation determines η_c

$$w_{O_2} = 0 \quad \eta \leq \eta_c$$

$$\frac{w_{O_2}}{\left(w_{O_2} \right)_c} = \frac{e^{\int_{\eta_c}^{\eta} A_{O_2} d\sigma} - 1}{\int_{\eta_c}^1 A_{O_2} d\sigma - 1} \quad \eta_c \leq \eta \leq 1$$



In a straightforward manner, we can derive the profiles for the inert and combustible species. Thus:

$$(\rho v)_0 \frac{dw_I}{dy} = \frac{d}{dy} \left[\rho D_I \left(1 + \frac{\epsilon}{D_I} \right) \frac{dw_I}{dy} \right]$$

$$\frac{dw_I}{d\eta} = \frac{d}{d\eta} \left[A_I^{-1} \frac{dw_I}{d\eta} \right]$$

$$\frac{w_I - (w_I)_0}{(w_I)_c - (w_I)_0} = \frac{e^{\int_0^\eta A_I d\sigma} - 1}{e^{\int_0^{\eta_c} A_I d\sigma} - 1}$$

$$\dot{m}_I^1 = (\rho v)_0 w_I - (\rho v)_0 \left[(w_I)_c - (w_I)_0 \right] \frac{\left[e^{\int_0^\eta A_I d\sigma} - 1 \right]}{\left[e^{\int_0^{\eta_c} A_I d\sigma} - 1 \right]}$$

$$\frac{\dot{m}_I^1}{(\rho v)_0} = w_I - \left[(w_I)_c - (w_I)_0 \right] \frac{e^{\int_0^\eta A_I d\sigma} - 1}{e^{\int_0^{\eta_c} A_I d\sigma} - 1}$$

$$K_I^1 = \frac{\dot{m}_I^1}{(\rho v)_0} = \left(w_I\right)_0 - \left[\left(w_I\right)_c - \left(w_I\right)_0\right] \left(\frac{1}{e^{\int_0^{\eta_c} A_I d\sigma} - 1}\right)$$

$$= \left(w_I\right)_0 \left[\frac{e^{\int_0^{\eta_c} A_I d\sigma}}{e^{\int_0^{\eta_c} A_I d\sigma} - 1}\right] - \left(w_I\right)_c \left(\frac{1}{e^{\int_0^{\eta_c} A_I d\sigma} - 1}\right)$$

$$\left(w_{Im}\right)_0 = \left[\left(w_I\right)_c - \beta_I^1\right] e^{-\int_0^{\eta_c} A_I d\sigma} + \beta_I^1$$

$$\left(w_{Im}\right)_c = \left[\left(w_I\right)_e - \beta_I^2\right] e^{-\int_{\eta_c}^1 A_I d\sigma} + \beta_I^2$$

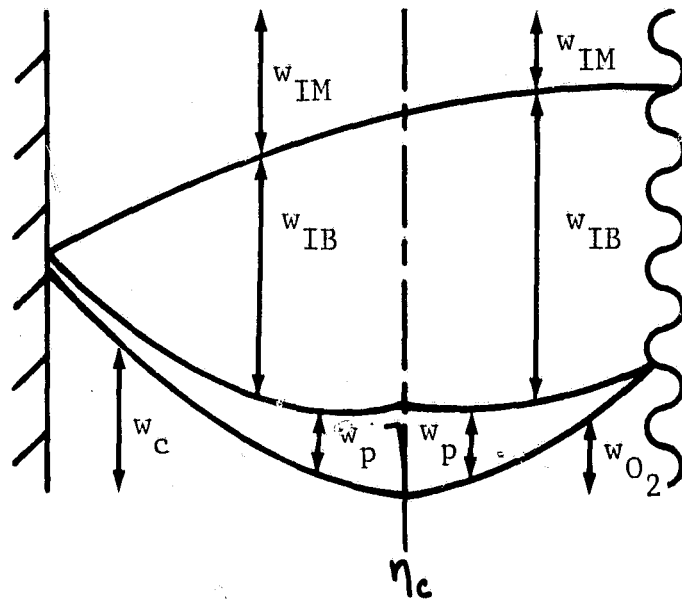
$$\beta_I^2 = \beta_I^1$$

$$\left(w_p\right)_0 = \left(w_p\right)_c e^{-\int_0^{\eta_c} A_p d\sigma}$$

$$\left(w_p\right)_c = \left[\left(w_p\right)_e - \beta_p\right] e^{-\int_{\eta_c}^1 A_p d\sigma} + \beta_p$$

$$\left(w_c\right)_0 = \left[\left(w_c\right)_c - \beta_c\right] e^{-\int_0^{\eta_c} A_c d\sigma} + \beta_c$$

$$= \beta_c \left[1 - e^{-\int_0^{\eta_c} A_c d\sigma}\right]$$



$$\frac{w_e - (w_c)_0}{(-w_c)_0} = \frac{e^{\int_0^{\eta_c} A_c d\sigma} - 1}{e^{\int_0^{\eta_c} A_c d\sigma} - 1}$$

$$w_c = (w_c)_0 \left[1 - \frac{e^{\int_0^{\eta_c} A_c d\sigma} - 1}{e^{\int_0^{\eta_c} A_c d\sigma} - 1} \right]$$

$$= \beta_c \left[e^{\int_0^{\eta_c} A_c d\sigma} - e^{\int_0^{\eta_c} A_c d\sigma} \right] e^{-\int_0^{\eta_c} A_c d\sigma}$$

$$= \beta_c \left[1 - \frac{e^{\int_0^{\eta_c} A_c d\sigma}}{e^{\int_0^{\eta_c} A_c d\sigma}} \right]$$

$$\frac{w_p - (w_p)_0}{(w_p)_c - (w_p)_0} = \frac{e^{\int_0^{\eta_c} A_p d\sigma} - 1}{e^{\int_0^{\eta_c} A_p d\sigma} - 1} \quad 0 \leq \eta \leq \eta_c$$

$$\begin{aligned}
\left(w_p\right)_c &= \left(w_p\right)_0 = - \left(w_p\right)_c e^{-\int_0^{\eta_c} A_p d\sigma} + \left(w_p\right)_c \\
&= \left(w_p\right)_c \left[1 - e^{-\int_0^{\eta_c} A_p d\sigma}\right] \\
&= \left\{ \left[\left(w_p\right)_e - \beta_p\right] e^{-\int_{\eta_c}^1 A_p d\sigma} + \beta_p \right\} \left\{1 - e^{-\int_0^{\eta_c} A_p d\sigma}\right\} \\
&= \beta_p \left[1 - e^{-\int_{\eta_c}^1 A_p d\sigma}\right] \left[1 - e^{-\int_0^{\eta_c} A_p d\sigma}\right] \\
\left(w_p\right)_0 &= \beta_p \left[1 - e^{-\int_{\eta_c}^1 A_p d\sigma}\right] e^{-\int_0^{\eta_c} A_p d\sigma}
\end{aligned}$$

$$\begin{aligned}
w_p &= \beta_p \left[1 - e^{-\int_{\eta_c}^1 A_p d\sigma}\right] \left\{ e^{-\int_0^{\eta_c} A_p d\sigma} + \left[1 - e^{-\int_0^{\eta_c} A_p d\sigma}\right] \left[\frac{e^{\int_0^{\eta_c} A_p d\sigma} - 1}{e^{\int_0^{\eta_c} A_p d\sigma} - 1} \right] \right\} \\
&= \beta_p \left[1 - e^{-\int_{\eta_c}^1 A_p d\sigma}\right] \left\{ e^{-\int_0^{\eta_c} A_p d\sigma} \right\} \left\{ e^{\int_0^{\eta_c} A_p d\sigma} \right\} \\
&= \beta_p \left[1 - e^{-\int_{\eta_c}^1 A_p d\sigma}\right] \frac{e^{\int_0^{\eta_c} A_p d\sigma}}{e^{\int_0^{\eta_c} A_p d\sigma}}
\end{aligned}$$

$$\frac{w_p - \left(w_p\right)_c}{-\left(w_p\right)_c} = \frac{e^{\int_{\eta_c}^{\eta_A} p d\sigma} - 1}{e^{\int_{\eta_c}^1 p d\sigma} - 1}$$

$$w_p = \left(w_p\right)_c \left[1 - \frac{e^{\int_{\eta_c}^{\eta_A} p d\sigma} - 1}{e^{\int_{\eta_c}^1 p d\sigma} - 1} \right] \quad \eta_c \leq \eta \leq 1$$

$$w_p = \beta_p \left[1 - \frac{e^{\int_{\eta_c}^{\eta_A} p d\sigma} - 1}{e^{\int_{\eta_c}^1 p d\sigma} - 1} \right] \quad (A52)$$

$$\left(w_{Ie}\right)_0 = \left(w_{Ie}\right)_e e^{\int_0^1 A_{Ie} d\sigma}$$

$$\frac{w_{Ie} - \left(w_{Ie}\right)_0}{\left(w_{Ie}\right)_e - \left(w_{Ie}\right)_0} = \frac{e^{\int_0^{\eta_A} A_{Ie} d\sigma} - 1}{e^{\int_0^1 A_{Ie} d\sigma} - 1}$$

$$\begin{aligned} w_{Ie} &= \left(w_{Ie}\right)_e e^{-\int_0^1 A_{Ie} d\sigma} + \left(w_{Ie}\right)_e \left(1 - e^{-\int_0^1 A_{Ie} d\sigma} \right) \left[\frac{e^{\int_0^{\eta_A} A_{Ie} d\sigma} - 1}{e^{\int_0^1 A_{Ie} d\sigma} - 1} \right] \\ &= \left(w_{Ie}\right)_e e^{-\int_0^1 A_{Ie} d\sigma} \left\{ e^{\int_0^{\eta_A} A_{Ie} d\sigma} \right\} \end{aligned}$$

$$w_{Ie} = \left(w_{Ie} \right)_e \frac{\int_0^{\eta} A_{Ie} d\eta}{\int_0^1 A_{Ie} d\eta}$$

$$\begin{aligned} \left(w_{IM} \right)_0 &= - \beta_{IM} \frac{\int_0^1 A_{IM} d\eta}{e} + \beta_{IM} \\ &= \beta_{IM} \left[1 - \frac{\int_0^1 A_{IM} d\eta}{e} \right] \end{aligned}$$

$$\frac{w_{IM} - \left(w_{IM} \right)_0}{- \left(w_{IM} \right)_0} = \frac{\int_0^{\eta} A_{IM} d\sigma - 1}{\int_0^1 A_{IM} d\sigma - 1}$$

$$\begin{aligned} w_{IM} &= \left(w_{IM} \right)_0 \left[1 - \frac{\int_0^{\eta} A_{IM} d\sigma - 1}{\int_0^1 A_{IM} d\sigma - 1} \right] \\ &= \beta_{IM} \frac{\int_0^1 A_{IM} d\sigma}{e} \left[\frac{\int_0^1 A_{IM} d\sigma}{e} - \frac{\int_0^{\eta} A_{IM} d\sigma}{e} \right] \\ &= \beta_{IM} \left[1 - \frac{\int_0^{\eta} A_{IM} d\sigma}{\int_0^1 A_{IM} d\sigma} \right] \end{aligned} \tag{A53}$$

$$\sum w_i = 1$$

Region I:

$$\beta_{IM} \left[1 - \frac{\int_0^{\eta} A_{IM} d\sigma}{\int_0^1 A_{IM} d\sigma} \right] + \left(w_{Ie} \right)_e \frac{\int_0^{\eta} A_{Ie} d\sigma}{\int_0^1 A_{Ie} d\sigma} + \beta_p \left[1 - \frac{\int_0^1 A_p d\sigma}{\int_0^{\eta} A_p d\sigma} \right] \left[\frac{\int_0^{\eta} A_p d\sigma}{\int_0^{\eta} A_p d\sigma} \right]$$

$$\left(w_{Ie}\right)_e = w_{Ie} \frac{e^{\int_0^{\eta_c} \text{Ad}\sigma}}{e^{\int_0^1 \text{Ad}\sigma}}$$

$$w_c = \beta_c \left[1 - \frac{e^{\int_0^{\eta_c} \text{Ad}\sigma}}{e^{\int_0^1 \text{Ad}\sigma}} \right] \quad 0 \leq \eta \leq \eta_c \quad (\text{A54})$$

$$w_c = 0 \quad \eta_c \leq \eta \leq 1$$

$$w_{02} :$$

$$0 \leq \eta \leq \eta_c \quad w_{02} = 0$$

$$\eta_c \leq \eta \leq 1 \quad w_{02} = \left(w_{02}\right)_e \frac{\left[\frac{e^{\int_{\eta_c}^{\eta} \text{Ad}\sigma} - 1}{e^{\int_{\eta_c}^1 \text{Ad}\sigma} - 1} \right]}{e^{\int_{\eta_c}^{\eta} \text{Ad}\sigma} - 1}$$

$$w_p$$

$$0 \leq \eta \leq \eta_c \quad w_p = \beta_p \left[1 - \frac{e^{\int_{\eta_c}^1 \text{Ad}\sigma}}{e^{\int_0^1 \text{Ad}\sigma}} \right] \frac{e^{\int_0^{\eta_c} \text{Ad}\sigma}}{e^{\int_0^{\eta_c} \text{Ad}\sigma}} \quad (\text{A55})$$

$$\eta_c \leq \eta \leq 1 \quad w_p = \beta_p \left[1 - \frac{e^{\int_{\eta_c}^{\eta} \text{Ad}\sigma}}{e^{\int_{\eta_c}^1 \text{Ad}\sigma}} \right]$$

$$\left(w_p\right)_c + \left(w_{IM}\right)_c + \left(w_{Ie}\right)_c = 1$$

$$\beta_p \left[1 - \frac{e^{\int_{\eta_c}^1 \text{Ad}\sigma}}{e^{\int_0^1 \text{Ad}\sigma}} \right] + \beta_{IM} \left[1 - \frac{e^{\int_0^{\eta_c} \text{Ad}\sigma}}{e^{\int_0^1 \text{Ad}\sigma}} \right] + \left(w_{Ie}\right)_e \frac{e^{\int_0^1 \text{Ad}\sigma}}{e^{\int_0^1 \text{Ad}\sigma}} = 1$$

$$+ \beta_c \left[1 - \frac{\int_0^{\eta_c} A_c d\sigma}{\int_0^{\eta_c} A_c d\sigma} \right] = 1 \quad 0 \leq \eta \leq \eta_c$$

$$\left(w_{Ie} \right)_e \frac{\int_0^{\eta} A_{Ie} d\sigma}{\int_0^1 A_{Ie} d\sigma} - \beta_{IM} \frac{\int_0^{\eta} A_{IM} d\sigma}{\int_0^1 A_{IM} d\sigma} + \beta_p \left[1 - \frac{\int_{\eta_c}^1 A_p d\sigma}{\int_0^{\eta_c} A_p d\sigma} \right] \frac{\int_0^{\eta} A_p d\sigma}{\int_0^{\eta_c} A_p d\sigma}$$

$$- \beta_c \frac{\int_0^{\eta} A_c d\sigma}{\int_0^{\eta_c} A_c d\sigma} = 0$$

$$\dot{m} = \dot{m}_{IM} + \dot{m}_p - \dot{m}_{O_e}$$

$$= \dot{m}_{IM} + \dot{m}_c$$

$$\dot{m}_p = \dot{m}_c + \dot{m}_{O_2}$$

$$= \dot{m}_c \left(1 + \frac{1}{a} \right)$$

$$= \dot{m}_c \left(\frac{1+a}{a} \right)$$

$$\beta_p = \beta_c \left(\frac{1+a}{a} \right)$$

To make things even simpler, assume:

$$A_p \approx A_{Ie} \approx A_{IM} \approx A_{O_2} \approx A_c = A$$

Thus:

$$w_{IM} = \beta_{IM} \left[1 - \frac{\int_0^{\eta} A d\sigma}{\int_0^1 A d\sigma} \right]$$

$$\beta_p + \beta_{IM} + \left[\left(w_{Ie} \right)_e - \left(\beta_{IM} + \beta_p \right) \right] e^{-\int_{\eta_c}^1 Ad\sigma} = 1$$

$$1 = \beta_p + \beta_{IM} - \beta_{O_2} = \beta_c + \beta_{IM}$$

$$\beta_c = \beta_p - \beta_{O_2}$$

$$\beta_p = \beta_c + \beta_{O_2}$$

$$\beta_c + \beta_{O_2} + \beta_{IM} + \left[\left(w_{Ie} \right)_e - \left(\beta_{IM} + \beta_{O_2} + \beta_{O_2} \right) \right] e^{-\int_{\eta_c}^1 Ad\sigma} = 1$$

$$\beta_{IM} + \beta_c = 1$$

$$\beta_{O_2} + \left[\left(w_{Ie} \right)_e - \left(1 + \beta_{O_2} \right) \right] e^{-\int_{\eta_c}^1 Ad\sigma} = 0$$

$$1 - \left(w_{O_2} \right)_e = \left(w_{Ie} \right)_e$$

$$\beta_{O_2} + \left[1 - \left(w_{O_2} \right)_e - \left(1 + \beta_{O_2} \right) \right] e^{-\int_{\eta_c}^1 Ad\sigma} = 0$$

$$\beta_{O_2} - \left[\left(w_{O_2} \right)_e + \beta_{O_2} \right] e^{-\int_{\eta_c}^1 Ad\sigma} = 0$$

$$\beta_{O_2} = \left[\left(w_{O_2} \right)_e + \beta_{O_2} \right] e^{-\int_{\eta_c}^1 Ad\sigma}$$

(A56)

Consider now the energy equation:

$$\frac{d}{dy} \left\{ \sum \dot{m}_i h_i \right\} + \frac{d}{dy} \sum \dot{m}_i \left(\frac{u^2}{2gJ} \right) = \frac{d}{dy} \left(k^* \frac{dT}{dy} \right) + \sum \rho \dot{w}_i h_i + \frac{d}{dy} (\tau u) \quad (A57)$$

or

$$\frac{d}{dy} \left[k^* \frac{dT}{dy} + \sum \left[\rho D_i^* \frac{dw_i}{dy} \right] h_i + \frac{u\tau}{gJ} - (\rho v)_0 H \right] = 0 \quad (A58)$$

where

$$H = \sum w_i h_i + \frac{u^2}{2gJ}$$

$$\begin{aligned} \frac{dH}{dy} &= \sum \left[w_i \frac{dh_i}{dy} + h_i \frac{dw_i}{dy} \right] + \left(\frac{u}{gJ} \right) \frac{du}{dy} \\ &= \left(\sum w_i c_{p_i} \right) \frac{dT}{dy} + \sum h_i \left(\frac{dw_i}{dy} \right) + \left(\frac{u}{gJ} \right) \frac{du}{dy} \\ &= c_p \frac{dT}{dy} + \sum h_i \left(\frac{dw_i}{dy} \right) + \left(\frac{u}{gJ} \right) \frac{du}{dy} \end{aligned}$$

$$k^* \frac{dT}{dy} = \frac{k^*}{c_p} \left[\frac{dH}{dy} - \sum h_i \left(\frac{dw_i}{dy} \right) - \frac{u}{gJ} \left(\frac{du}{dy} \right) \right]$$

$$\frac{d}{dy} \left\{ \frac{k^*}{c_p} \frac{dH}{dy} + \sum \left[\rho D_i^* - \frac{k^*}{c_p} \right] \left(h_i \frac{dw_i}{dy} \right) + \left(u^* - \frac{k^*}{c_p} \right) \frac{u}{gJ} \left(\frac{du}{dy} \right) \right\}$$

$$= (\rho v)_0 \frac{dH}{dy}$$

$$\frac{d}{dy} \left\{ \frac{k^*}{c_p} \left[\frac{dH}{dy} + \sum \left(\frac{\rho D_i^* c_p}{k^*} - 1 \right) h_i \left(\frac{dw_i}{dy} \right) + \left(\frac{u^* c_p}{k^*} - 1 \right) \frac{u}{gJ} \left(\frac{du}{dy} \right) \right] \right\}$$

$$= (\rho v)_0 \left(\frac{dH}{dy} \right)$$

$$\frac{k^*}{C_p} \left[\frac{dH}{dy} + \sum (Le_i^* - 1) h_i \frac{dw_i}{dy} + (Pr^* - 1) \frac{u}{gJ} \frac{du}{dy} \right] = q_w + (\rho v)_0 (H - H_0) \quad (A59)$$

where

$$q_w = \left[k \frac{dT}{dy} + \sum \rho D_i h_i \frac{dw_i}{dy} \right]_0$$

Now,

$$\tau_w + (\rho v)_0 u \quad (A60)$$

or

$$\mu^* \frac{du}{dy} = \tau_w + (\rho v)_0 u \quad (A61)$$

Dividing (A59) by (A61),

$$\frac{k^*}{\mu^* C_p} \left[\frac{dH}{du} + \sum (Le_i^* - 1) h_i \frac{dw_i}{du} + (Pr^* - 1) \left(\frac{u}{gJ} \right) \right] = \frac{q_w + (\rho v)_0 (H - H_0)}{\tau_w + (\rho v)_0 u}$$

$$\frac{dH}{d\tilde{u}} + \sum (Le_i^* - 1) h_i \left(\frac{dw_i}{d\tilde{u}} \right) + (Pr^* - 1) \frac{\tilde{u}}{gJ} u_e^2 = Pr^* \left(\frac{q_w u_e}{\tau_w} \right) \left[\frac{1 + \frac{(\rho v)_0 (H_e - H_0)}{\rho_e u_e q_w} \rho_e u_e \phi_H}{1 + \alpha \tilde{u}} \right]$$

$$\frac{d\phi_H}{d\tilde{u}} + \sum \frac{Le_i^* - 1}{(H_e - H_0)} h_i \frac{dw_i}{d\tilde{u}} + \frac{(Pr^* - 1)}{(H_e - H_0)} \frac{\tilde{u}}{(gJ)} u_e^2 = Pr^* \frac{2S\tau}{C_f} \left[\frac{1 + \frac{\alpha}{\zeta} \phi_H}{1 + \alpha \tilde{u}} \right]$$

$$\frac{d\phi_H}{d\tilde{u}} - \frac{\alpha \text{Pr}^* \phi_H}{1 + \alpha \tilde{u}} = \frac{\zeta \text{Pr}^*}{1 + \alpha \tilde{u}} - \sum \frac{(\text{Le}_i^* - 1) h_i}{H_e - H_o} \frac{dw_i}{d\tilde{u}} - \frac{(\text{Pr}^* - 1)}{H_e - H_o} \frac{u_e^2}{gJ} \tilde{u}$$

$$\phi_H = e^{\int_0^{\tilde{u}} \frac{\alpha \text{Pr}^*}{1 + \alpha \tilde{u}} d\tilde{u}} \int_0^{\tilde{u}} e^{-\int_0^{\tilde{u}} \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}} \left[\frac{\zeta \text{Pr}^*}{1 + \alpha \tilde{u}} - \sum \frac{(\text{Le}_i^* - 1) h_i}{H_e - H_o} \left(\frac{dw_i}{d\tilde{u}} \right) - \frac{(\text{Pr}^* - 1)}{H_e - H_o} \frac{u_e^2}{gJ} \tilde{u} \right] d\tilde{u}$$

$$1 = e^{\int_0^1 \frac{\alpha \text{Pr}^*}{1 + \alpha \tilde{u}} d\tilde{u}} \int_0^1 e^{-\int_0^{\tilde{u}} \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}} \left[\frac{\zeta \text{Pr}^*}{1 + \alpha \tilde{u}} - \sum \frac{(\text{Le}_i^* - 1) h_i}{[H_e - H_o]} \frac{dw_i}{d\tilde{u}} - \frac{(\text{Pr}^* - 1)}{[H_e - H_o]} \frac{u_e^2}{gJ} \tilde{u} \right] d\tilde{u}$$

$$= e^{\int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}} \int_0^1 e^{-\int_0^{\tilde{u}} \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}} \frac{\zeta \text{Pr}^*}{1 + \alpha \tilde{u}} d\tilde{u} - e^{\int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}} \int_0^1 \left[\sum \frac{(\text{Le}_i^* - 1) h_i}{(H_e - H_o)} \frac{dw_i}{d\tilde{u}} + \frac{(\text{Pr}^* - 1)}{(H_e - H_o)} \frac{u_e^2}{gJ} \tilde{u} \right] d\tilde{u}$$

$$= e^{\int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}} \int_0^1 \frac{\zeta \text{Pr}^*}{1 + \alpha \tilde{u}} d\tilde{u}$$

$$\frac{1}{\alpha} \left[\frac{\int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}}}{e^{-1}} \right] = 1 + e^{-1} \int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}} \int_0^1 \left[\sum \frac{(\text{Le}_i^* - 1) h_i}{H_e - H_o} \left(\frac{dw_i}{d\tilde{u}} \right) \right]$$

$$+ \frac{(\text{Pr}^* - 1)}{H_e - H_o} \left[\frac{u_e^2}{gJ} \tilde{u} \right] e^{-1} \int_0^{\tilde{u}'} \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}} d\tilde{u}$$

$$= \frac{\alpha \left[1 + e^{-1} \int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}}{1 + \alpha \tilde{u}} \int_0^1 \left[\sum \frac{(\text{Le}_i^* - 1) h_i}{(H_e - H_o)} \frac{dw_i}{d\tilde{u}} + \frac{(\text{Pr}^* - 1)}{(H_e - H_o)} \frac{u_e^2}{gJ} \tilde{u} \right] e^{-1} \int_0^{\tilde{u}'} \frac{\alpha \text{Pr}^* d\tilde{u}'}{1 + \alpha \tilde{u}'} d\tilde{u} \right]}{e^{-1} \int_0^1 \frac{\alpha \text{Pr}^* d\tilde{u}'}{1 + \alpha \tilde{u}'} d\tilde{u} - 1}$$

$$\alpha \rightarrow 0$$

$$\rightarrow \frac{\alpha \left[1 + \int_0^1 \left[\sum \frac{(\text{Le}_i^* - 1) h_i}{(H_e - H_o)} \frac{dw_i}{d\tilde{u}} + \frac{(\text{Pr}^* - 1)}{(H_e - H_o)} \left(\frac{u_e^2}{gJ} \right) \tilde{u}' \right] d\tilde{u}' \right]}{\int_0^1 \frac{\text{Pr}^* d\tilde{u}'}{1 + \alpha \tilde{u}'}}$$

$$= \frac{\left[1 + \int_0^1 \left[\sum \frac{(\text{Le}_i^* - 1) h_i}{(H_e - H_o)} \frac{dw_i}{d\tilde{u}} + \frac{(\text{Pr}^* - 1)}{(H_e - H_o)} \frac{u_e^2}{gJ} \tilde{u}' \right] d\tilde{u}' \right]}{\int_0^1 \text{Pr}^* d\tilde{u}}$$

For the case of an impermeable and inert surface:

$$1 + \frac{\int_0^1 \frac{(\text{Pr}^* - 1)}{H_e - H_o} \left(\frac{u_e}{gJ} \right)^2 (\tilde{u}) d\tilde{u}}{\int_0^1 \text{Pr}^* d\tilde{u}}$$

APPENDIX B

CHAR-FORMING ABLATOR PERFORMANCE PREDICTION

Effect of Boundary Layer Combustion on Surface Recession

This derivation is an extension of the basic approach of Lees (ref. 29). For an elemental species diffusing to the wall when all molecular species have the same binary diffusivity:

$$(\dot{m}_j)_w = - \left(\rho D_{12} \frac{\partial \tilde{K}_j}{\partial y} \right)_w + (\tilde{K}_j \dot{m})_w \quad (B1)$$

When similarity exists between enthalpy and concentration:

$$\frac{\tilde{K}_j - \tilde{K}_{jw}}{\tilde{K}_{je} - \tilde{K}_{jw}} = \frac{H_s - H_w}{H_{se} - H_w} \quad (B2)$$

Hence

$$\left(\frac{\partial K_j}{\partial y} \right)_w = \frac{\tilde{K}_{je} - \tilde{K}_{jw}}{h_{se} - h_w} \left(\frac{\partial H_s}{\partial y} \right)_w \quad (B3)$$

Substituting eq. (B3) into eq. (B1):

$$\dot{m}_{jw} = - \rho D_{12} \frac{\tilde{K}_{je} - \tilde{K}_{jw}}{h_{se} - h_w} \left(\frac{\partial H_s}{\partial y} \right)_w + (\tilde{K}_j \dot{m})_w \quad (B4)$$

Now for $Le = 1$, $\rho D_{12} = k/c_p$ and since:

$$St = \frac{k}{c_p} \left(\frac{\partial h_s}{\partial y} \right)_w / \left[\rho_e u_e (H_{se} - H_w) \right]$$

then

$$\dot{m}_{jw} = - \rho_e u_e St (\tilde{K}_{je} - \tilde{K}_{jw}) + (K_j \dot{m})_w \quad (B5)$$

Assuming no storage,

$$\dot{m}_{jw} = \dot{m}_p \tilde{K}_{jp} + \dot{m}_c \tilde{K}_{jc}$$

Substituting this expression into eq. (B5) and letting $\mathcal{H} = \rho_e u_e St$ and $\dot{m}_w = \dot{m}_p$:

$$\dot{m}_p \tilde{K}_{jp} + \dot{m}_c \tilde{K}_{jc} = -\mathcal{H} \tilde{K}_{je} + \mathcal{H} \tilde{K}_{jw} + \tilde{K}_{jw}(\dot{m}_c + \dot{m}_p)$$

Solving for $\tilde{K}_{jw}$,

$$\tilde{K}_{jw} = \frac{\dot{m}_p \tilde{K}_{jp} + \dot{m}_c \tilde{K}_{jc} + \mathcal{H} \tilde{K}_{je}}{\mathcal{H} + \dot{m}_c + \dot{m}_p}$$

Dividing numerator and denominator by $\dot{m}_c + \dot{m}_p$, letting $(\dot{m}_c + \dot{m}_p)/\mathcal{H} = B'$ and $\dot{m}_c/\dot{m}_p = \alpha_w$ and noting that $\dot{m}_c/\dot{m}_w = \alpha/(1+\alpha)$ and $\dot{m}_p/\dot{m}_w = \frac{1}{1+\alpha}$:

$$\tilde{K}_{jw} = \frac{[\tilde{K}_{jp}/(1+\alpha_w)] + [\alpha_w/(1+\alpha_w)] \tilde{K}_{jc} + \tilde{K}_{je}/B'}{\frac{1}{B'} + 1}$$

Rearranging:

$$\tilde{K}_{jw} = \frac{B'(\tilde{K}_{jp} + \alpha \tilde{K}_{jc})/(1+\alpha_w) + \tilde{K}_{je}}{B' + 1} \quad (B6)$$

where it is assumed that no diffusion takes place in the porous solid, i.e., beyond the wall ($y \leq 0$).

By eq. (B1), for the case of oxygen from air:

$$\dot{m}_{O_2,w} = -\dot{m}_w (\tilde{K}_{O_2,e} - \tilde{K}_{O_2,w})/B' + (K_{O_2,w}) \dot{m}_w \quad (B7)$$

since for a frozen boundary layer, molecular and elemental species mass fractions are similar.

Assuming all O_2 diffusing from the boundary layer edge is consumed at the wall, or in homogeneous reactions, $K_{O_2,w}$. Thus:

$$\dot{m}_{O_2,w} = -\frac{\dot{m}_w}{B'} K_{O_2,e} = \mathcal{H} K_{O_2,e} \quad (B8)$$

Another expression for $\dot{m}_{O_2,w}$ is obtained from the fluxes of gaseous products which have reacted with O_2 at the wall. For the case of C_s burning to CO or CO_2 .

$$\dot{m}_{O_2,w} = -\dot{m}_c \left(\frac{\alpha_{ow} M_{ow}}{\alpha_{cw} M_{cw}} \right) - \sum_i f_i \tilde{K}_{ip} \left(\frac{\alpha_{oi} M_{oi}}{\alpha_{ri} M_{ri}} \right) \dot{m}_p \quad (B9)$$

From eqs. (B8) and (B9), the wall mass flux due to combustion is:

$$\dot{m}_c = \mathcal{H} \tilde{K}_{O_2,e} \left(\frac{\alpha_{cw} M_{cw}}{\alpha_{ow} M_{ow}} \right) - \sum_i f_i K_{ip} \left(\frac{\alpha_{oi} M_{oi}}{\alpha_{ri} M_{ri}} \right) \dot{m}_p \left(\frac{\alpha_{cw} M_{cw}}{\alpha_{ow} M_{ow}} \right) \quad (B10)$$

Defining

$$A \equiv \frac{\alpha_{cw} M_{cw}}{\alpha_{ow} M_{ow}} \quad (B11)$$

and

$$\Lambda \equiv \sum_i f_i \tilde{K}_{ip} \left(\frac{\alpha_{oi} M_{oi} \alpha_{cw} M_{cw}}{\alpha_{ri} M_{ri} \alpha_{ow} M_{ow}} \right) \quad (B12)$$

eq. (B10) becomes

$$\dot{m}_c = \cancel{K} K_{o_2} A - \Lambda \dot{m}_p \quad (B13)$$

On the basis of the observed recession rate and virgin plastic-char interface recession rate ($\dot{m}_c = \rho_c \dot{S}_c$; $\dot{m}_p = \rho_p \dot{S}_p$) values of Λ were calculated from the data of reference 24, assuming that data at 150 seconds were representative of average performance and that chemical model A of the following section applied. This value of Λ was then correlated as shown in figure 34.

Chemical Models for Combustion

Silica Reduction - Carbon Oxidation, Model A. - The reactions assumed are:

		Δh at 2000 °K, cal. (reference 30)
Reaction 1	$SiO_2 = (Si)_s + O_2$	-225,900
Reaction 2	$(Si)_s + (1/2) O_2 = (SiO)_g$	+36,700
Reaction 3	$5 \left[(C)_s + (1/2) O_2 = CO \right]$	$5 \cdot 28,430$
Overall reaction	$(SiO_2)_s + 5(C)_s + 2O_2 = (SiO)_g + 5CO$	-57,000
Reaction 4		-855 Btu/lb _m residue

For reaction 4, the value of $\alpha_{cw} M_{cw}$ is due to contributions from silica and carbon, while the value of $\alpha_{ow} M_{ow}$ is due to oxygen.

Thus

$$A = \frac{\alpha_{cw} M_{cw}}{\alpha_{ow} M_{ow}} = \frac{(1 \times 60) + (5 \times 12)}{2 \times 32} = 1.875.$$

There is some question as to the applicability of this model at the conditions attained in this contract, especially in light of the presence of white siliceous residue sometimes observed on the ablation models. Therefore, a second model was derived for this case.

Carbon Combustion Model B, with Recession of Residual Silica. - The reaction assumed is:



with silica removed mechanically, or sintered. Since about 1/2 of the residue by weight is carbon, the heat of combustion becomes 2130 Btu/lb_m residue.

The use of this reaction leads to the same correlation for Λ , provided eq. (B13) is changed to read:

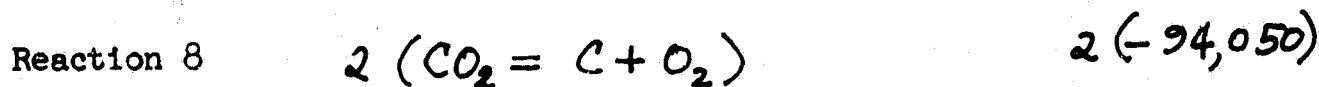
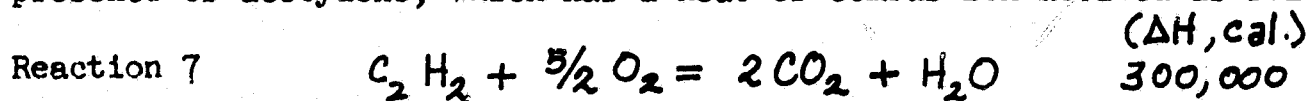
$$\frac{1}{2} \dot{m}_c = \frac{1}{2} K_{o_2,e} A Le - \dot{m}_p \Lambda \quad (B14)$$

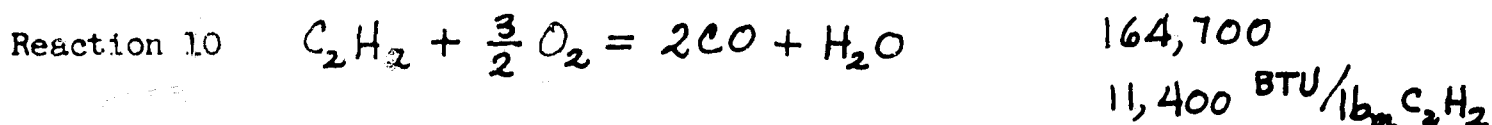
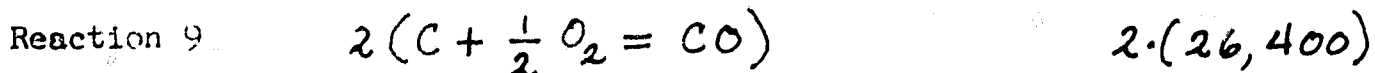
The inclusion of the Lewis number is suggested in ref. 34. The effective Lewis numbers used for dissociated air are available in ref. 33. For reaction 5,

$$A = \frac{\alpha_{cw} M_{cw}}{\alpha_{ow} M_{ow}} = \frac{1 \times 12}{\frac{1}{2} \times 32} = 0.75$$

Reaction 5 is assumed to occur in the turbulent ablation tests conducted for this contract.

Heat of Combustion of Pyrolysis Gases - The pyrolysis gases have an ascribed approximate formula $C_2H_{4.5}O$, based on elemental analysis of virgin plastic and residue. Assuming that oxygen is present as H_2O , substances having an overall formula $C_2H_{2.5}$ remain for combustion. This formula suggests the presence of acetylene, which has a heat of combustion derived as follows:

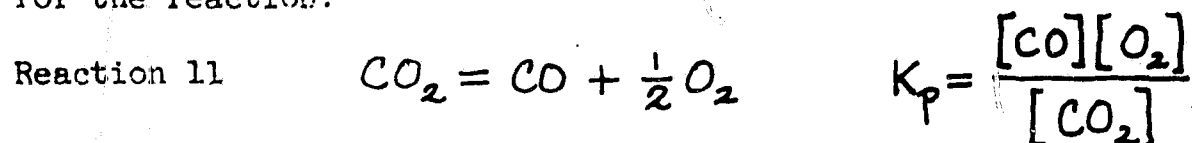




It is assumed that the $C_2H_{2.5}$ portion of the pyrolysis gases has this value; hence the heat of combustion for the pyrolysis gases becomes:

$$\Delta H_{c,p} = 11,400 \times \frac{26.5}{44.5} = 6800 \text{ BTU/lb}_m \text{ gases (B15)}$$

where 26.5 and 44.5 represent the formula weights of $C_2H_{2.5}$ and $C_2H_{4.5}O$, respectively. The heat of combustion will be considerably higher if free hydrogen is present, or if CO_2 is a combustion product. For the Apollo ablation data available, CO is the thermodynamically preferred product; hence, heats of combustion based on CO as a product are used in present heat balances. This practice is justified as follows. An estimate of the ratio of the concentration of CO to that of CO_2 is given by the equilibrium constant for the reaction:



values for which may be obtained from ref. 36. The ratio $[CO] / [CO_2]$ is a function of $[O_2]$. For diffusion-limited wall combustion, $[O_2]_w = 0(10^{-4})$ atm. at 1 atm total pressure in air, as predicted by a nonequilibrium laminar boundary layer program available at Boeing (ref. 37). Hence, for example at $3240^\circ R$, $[CO] / [CO_2] = 0(5)$. Since K_p increases with temperature, so will $[CO] / [CO_2]$. Also, since pyrolysis gas combustion causes a decrease in $[O_2]$, the ratio $[CO] / [CO_2]$ becomes greater.

Silica Sublimation

The usual gas kinetic theory rate of vaporization is given by:

$$\dot{m}_{sub,i} = 121.56 \alpha_v (P_{i,sat} - P_i) \sqrt{M/T} \quad (B16)$$

where P is in atm., T in $^\circ R$, and m in $lb_m/ft^2\text{-sec}$. From data in ref. 38 the vapor pressure of silica may be expressed as:

$$P_{sat} = 4.2 \times 10^4 \times 10^{-\frac{45000}{T}} \quad (B17)$$

A rough estimate of the rate of silica sublimation, assuming $P_i = 0$,

that silica is the only species vaporizing, and that $\alpha_w = 10^{-2}$ (see ref. 39)

$$\dot{m}_{\text{SUB}} = \frac{4 \times 10^{10} e^{-103,600/T}}{T^{1/2}} \quad (\text{B18})$$

At $\sim 6000^\circ\text{R}$ equation (B18) may be cast as:

$$\dot{m}_{\text{SUB}} = 0 \left(5 \times 10^8 e^{-103,600/T} \right) \quad (\text{B19})$$

APPENDIX C

TURBULENT FLOW ABLATION TESTS

Determination of Test Section Conditions

The methods for determining the test section conditions for those runs utilizing reconstituted air atmospheres are discussed in this appendix. The assumption of equilibrium air in the test section is made and the results are based on equilibrium air properties as found in ref. 32.

The average total enthalpy and the static pressure of the flow at the entrance to the test section are known. The average total enthalpy may be expressed as:

$$H_b = h_b + \frac{u_b^2}{2gJ} K \quad (C1)$$

$$\text{where } K = 2 \int_0^1 \frac{\rho u}{\rho_b u_b} \left(\frac{u}{u_b} \right)^2 \lambda d\lambda$$

In addition, the continuity equation gives:

$$\frac{\dot{m}}{A_c} = \rho_b u_b \quad (C2)$$

A value for the static enthalpy h_b is assumed. Using the charts of ref. 32 with P_∞ and h_b , the static temperature, entropy, and compressibility may be determined for this state. Using the equation of state, the free-stream density may be determined as:

$$\rho_b = \frac{P_b}{ZRT_b} \quad (C3)$$

Equation (C1) is solved for u_b :

$$u_b = \left[(2gJ/K) (H_b - h_b) \right]^{1/2} \quad (C4)$$

Equation (C2) is solved for :

$$u_b = \frac{\dot{m}/A_c}{\rho_b} \quad (C5)$$

and the two average free-stream velocities are compared. A new value for the

static enthalpy is assumed and the process is continued until a balance of the velocity is achieved. Based on these average conditions, the free-stream ratio of specific heats, the sonic velocity, and the Mach number can be obtained.

Consider fully developed turbulent flow through a circular section of cross-sectional area equal to that of the channel area. Let it be assumed that the velocity distribution in the circular section follows the one-seventh power law, i.e.:

$$\frac{u_b}{u_e} \approx 2 \int_0^1 (1-\lambda)^{\frac{1}{7}} \lambda d\lambda \quad (c6)$$

where:

$\lambda = r/R$
 $u_b = \text{bulk average velocity}$
 $u_e = \text{core velocity}$
 $R = \text{section radius}$
 $r = \text{local radius}$

Performing the indicated integration:

$$\frac{u_b}{u_e} = \frac{98}{120} = 0.8167 \quad (c7)$$

Hence, the core velocity and total enthalpy may be obtained from:

$$u_e = u_b / 0.8167 \quad (c8)$$

and

$$H_e = h_\infty + \frac{u_e^2}{2gJ} \quad (c9)$$

In addition, the Mach number based on the core velocity is:

$$M_e = \frac{u_e}{a_o} \quad (c10)$$

Plasma Tunnel Heat Balance

The average total enthalpy of the plasma flow at the entrance to the test section is obtained from a heat balance between the energy input to the arc heater and the heat absorbed by the tunnel cooling water.

The energy input to arc heater $\dot{Q}_A$ is the product of the arc current I_A , the arc voltage V_A , and a factor of 0.0009481 to convert from watts to Btu/sec:

$$\dot{Q}_A = (0.0009481)(I_A)(V_A) \quad (C11)$$

The heat transferred from the plasma flow to all components of the plasma tunnel up to the entrance to the test section is manifested by a rise in the temperature of the water coolant circulating through these components. The water flow W , in pounds per second, was measured by a Potter Turbine Flowmeter. If the temperature rise of the coolant is ΔT_c and the water specific heat is assumed to be 1.0, then the heat loss to the water $\dot{Q}_w$, is:

$$\dot{Q}_w = (W)(\Delta T_c) \quad (C12)$$

If the plasma mass flow rate $\dot{M}$ (lb_m/sec) and enthalpy of the constituent gases is H_g (Btu/lb_m), then the average total enthalpy of the plasma flow at the entrance to the test section becomes

$$H_b = \frac{\dot{Q}_A - \dot{Q}_w}{\dot{M}} + H_g \quad (C13)$$

It should be emphasized that this is an average total enthalpy. The "core" or boundary layer edge value will be higher due to the velocity and temperature profiles in the flow. This was discussed in the previous section. As mentioned previously, eq. (C13) was programmed to facilitate data reduction for all checkout, calibration, and test runs.

Experimental and Analytical Ablator Temperature Response

Figures 47 through 66 show the thermocouple time-temperature responses as generated by the Orthomat drafting machine from the time-millivolt output of each of the platinum-platinum 13% rhodium thermocouples.

The Boeing CHARM Computer program (reference 22) was run for selected runs in order to verify the thermocouple responses. The surface temperature history and the average surface recession rate along with the char and virgin material ablation properties were input. Table V lists those properties. Table II gives the actual thermocouple depths for all models as determined from model X-rays. Figures 28 through 33 show the actual and calculated time-temperature histories for selected thermocouples of matrix runs 5, 9, 10, 11, 12, and 13. It is seen that, in general, excellent agreement between experimental and predicted temperature response is obtained for those thermocouples which are located at the shallow and intermediate depths. The predicted

temperatures for those thermocouples at the greatest depths, however, are somewhat lower than the test values. The primary property affecting the temperature response of the deep thermocouples is the virgin plastic thermal diffusivity. The virgin plastic thermal conductivity and specific heat of Table V used in generating the temperature responses of figures 23 through 28 were supplied by NASA/MSC. Table VI lists the actual and the predicted char layer thicknesses for matrix runs 5, 9, 10, 11, 12, and 13.

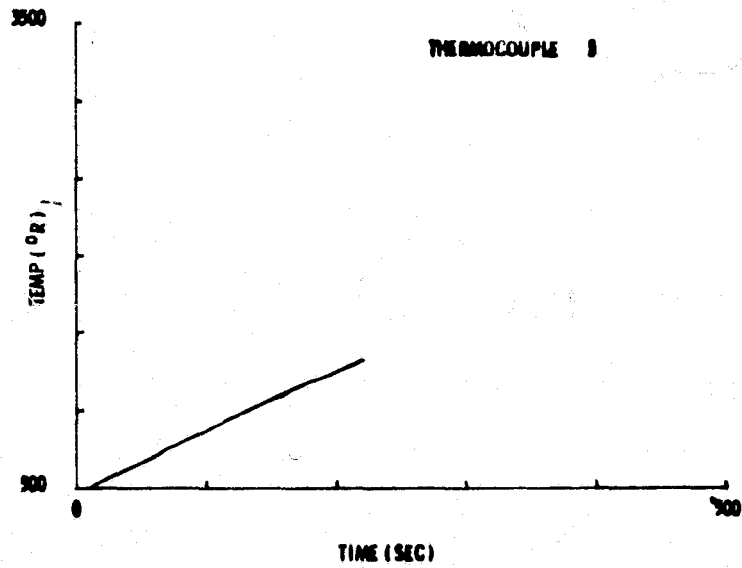
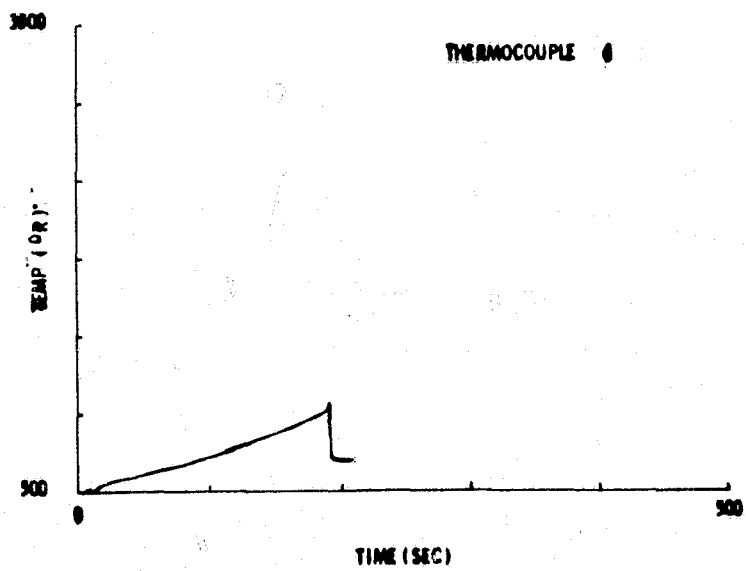
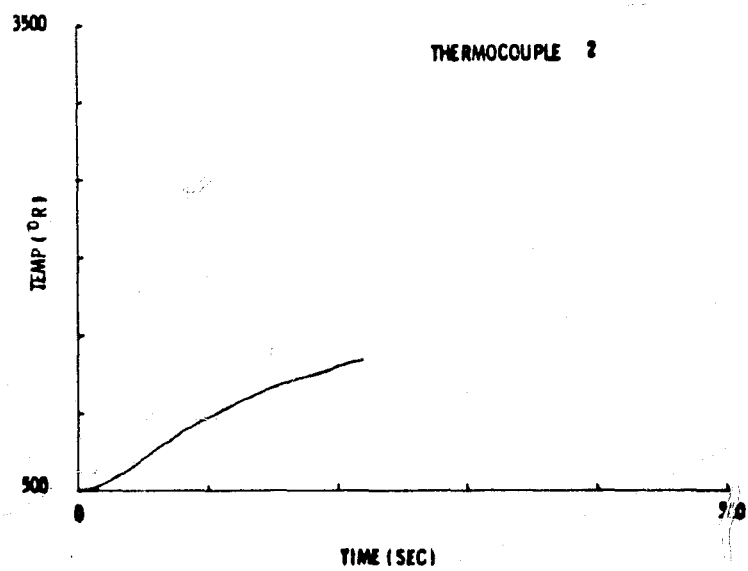
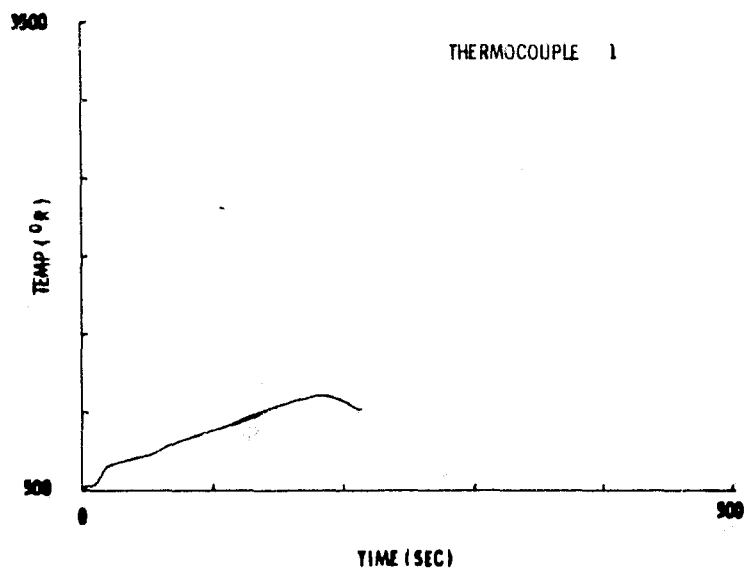


Figure 47a

Temperature Response of Model 1

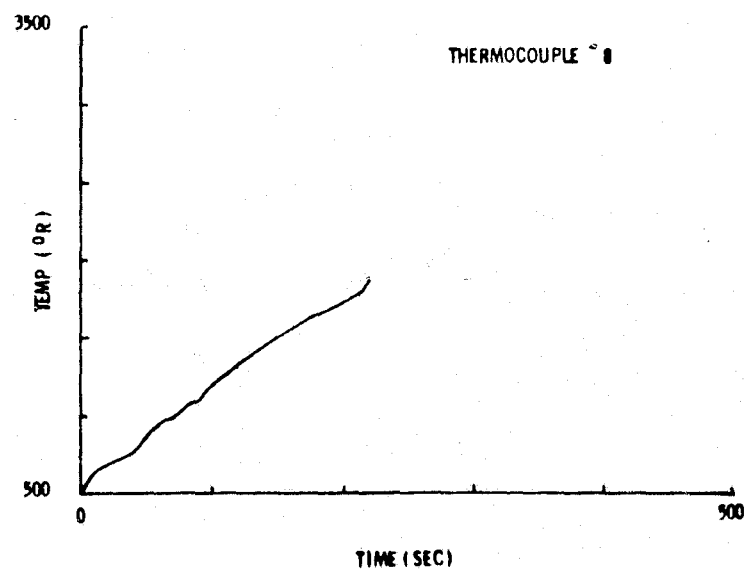
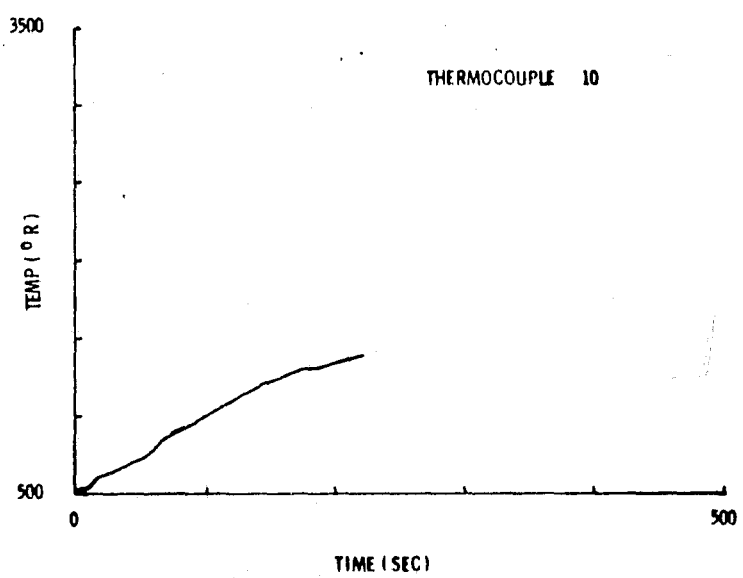
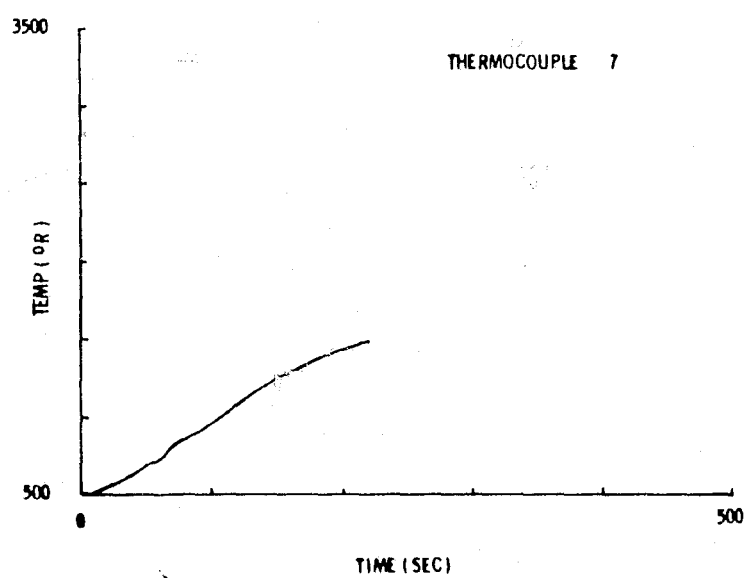
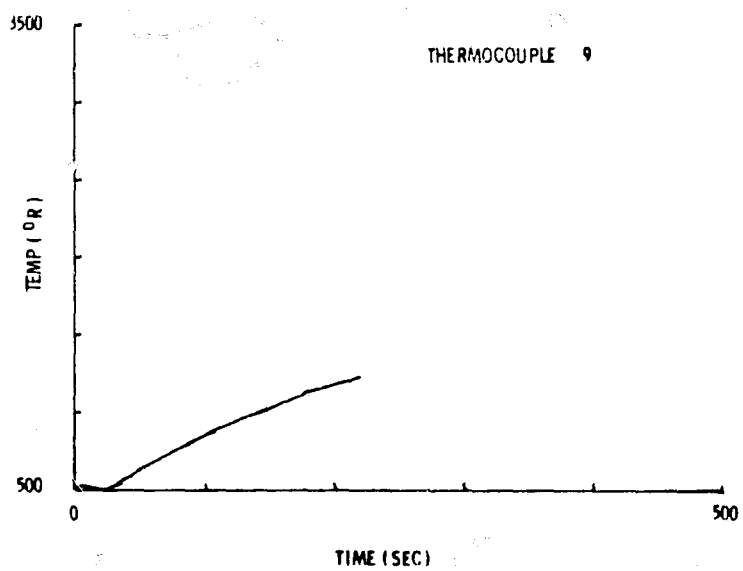
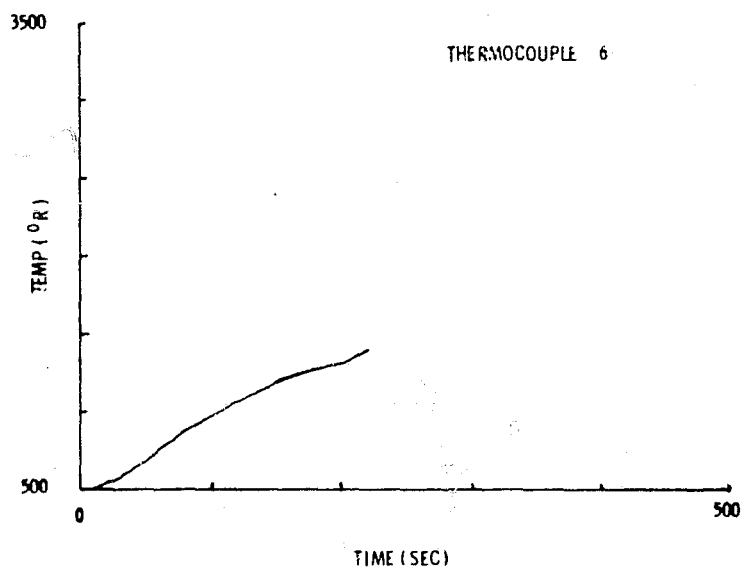


Figure 47b
Temperature Response of Model 1

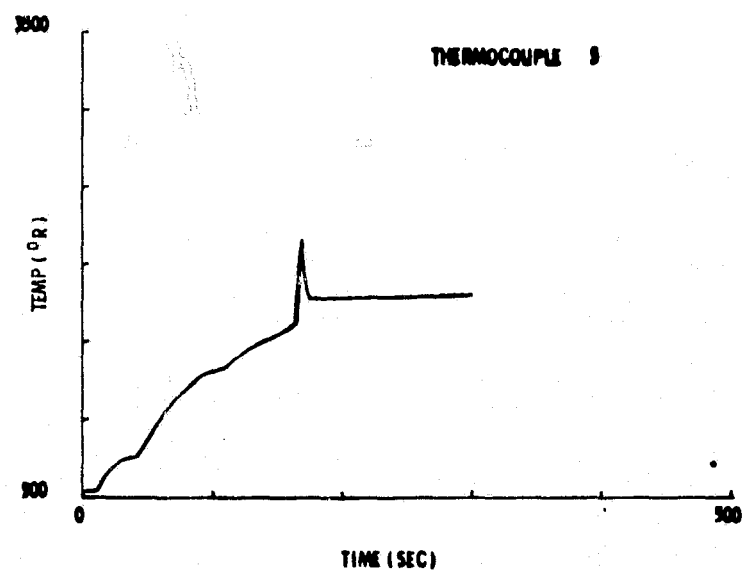
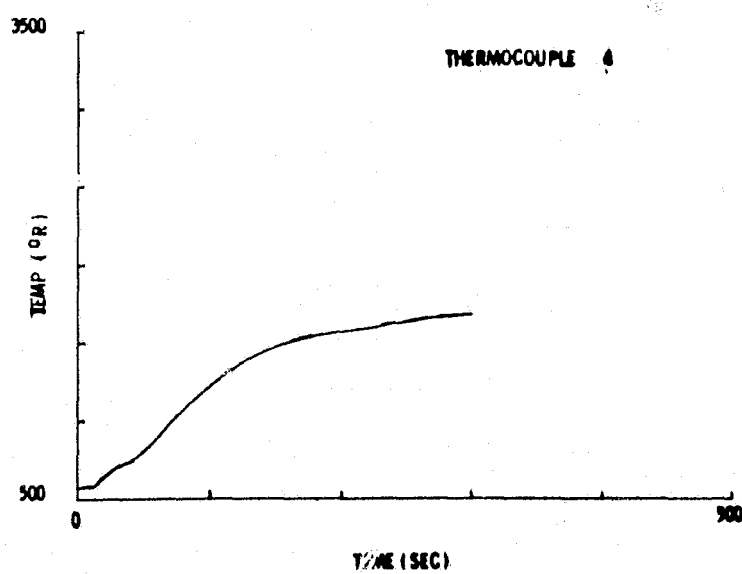
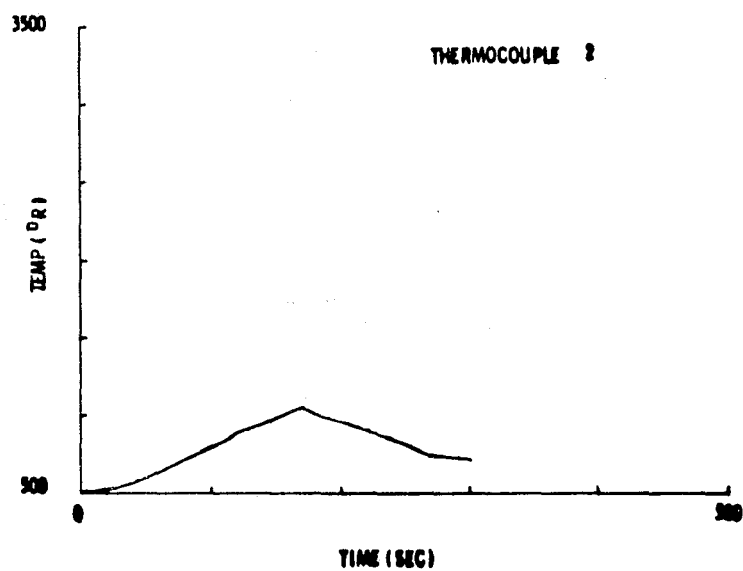
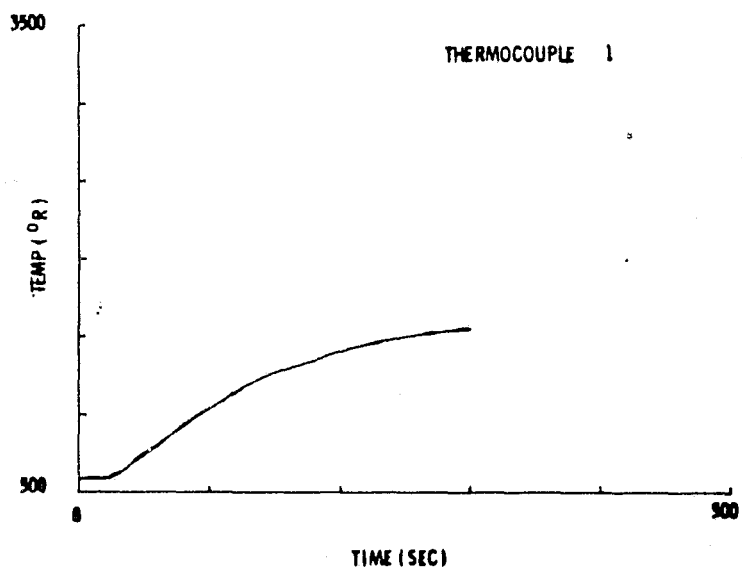


Figure 48a
Temperature Response of Model 2

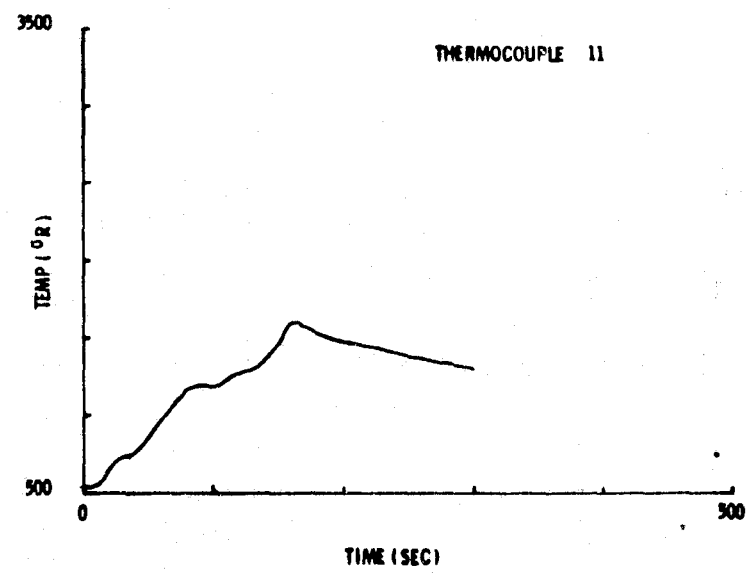
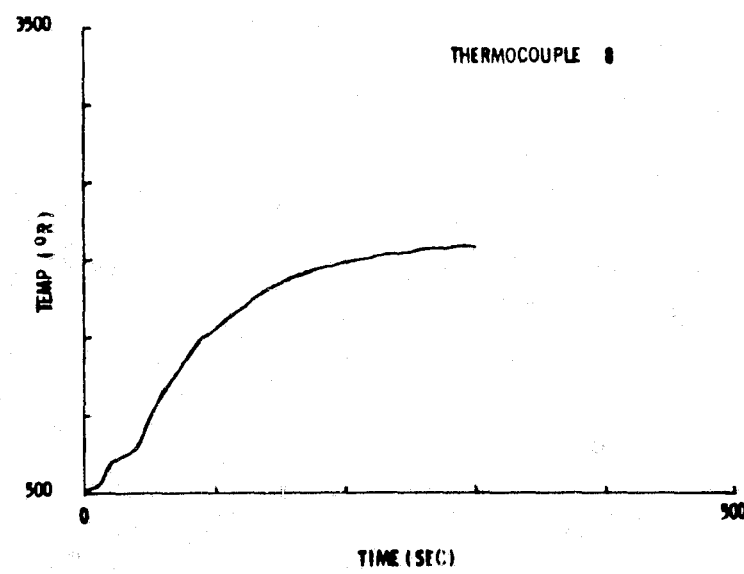
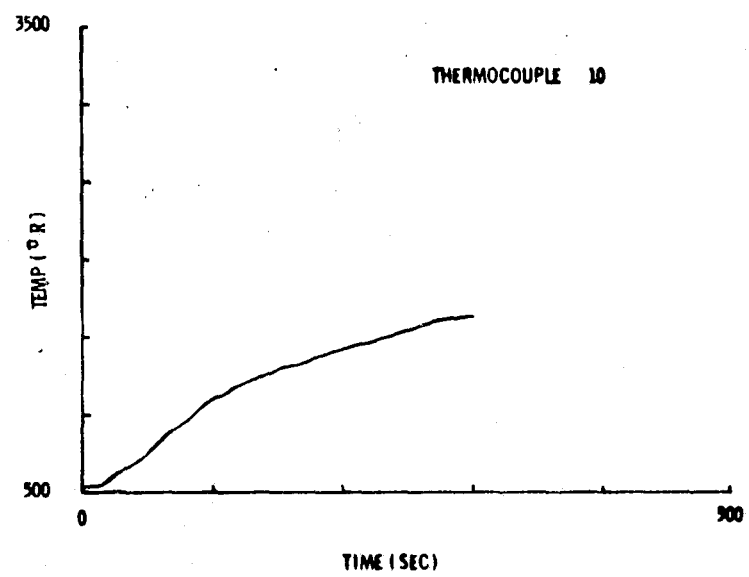
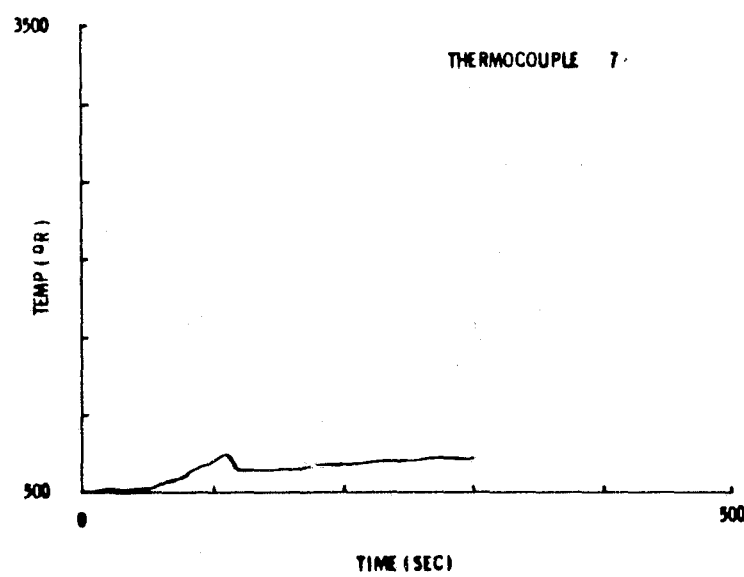
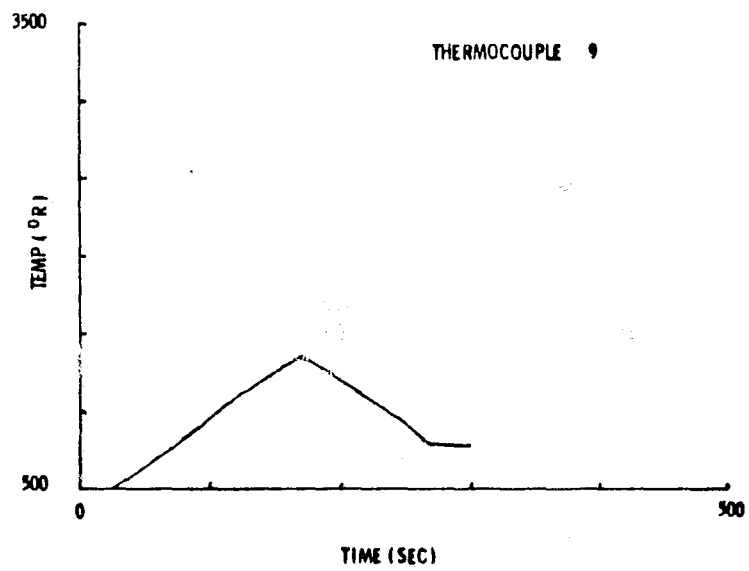
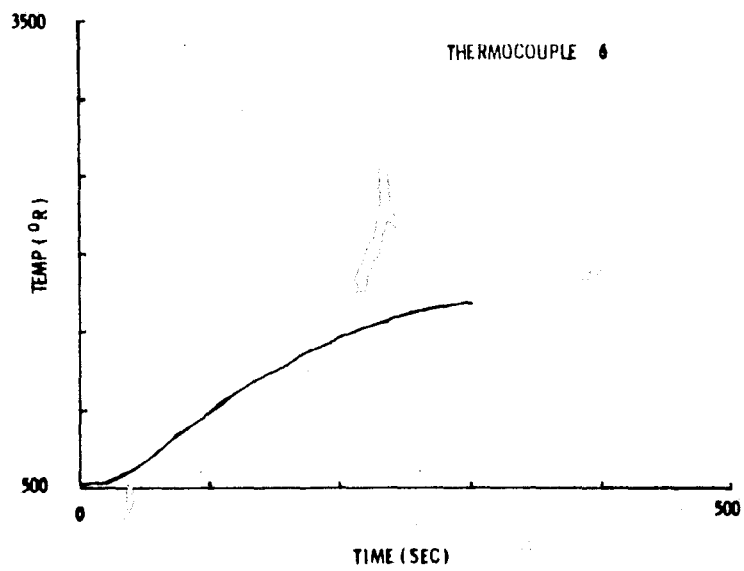


Figure 48b

Temperature Response of Model 2

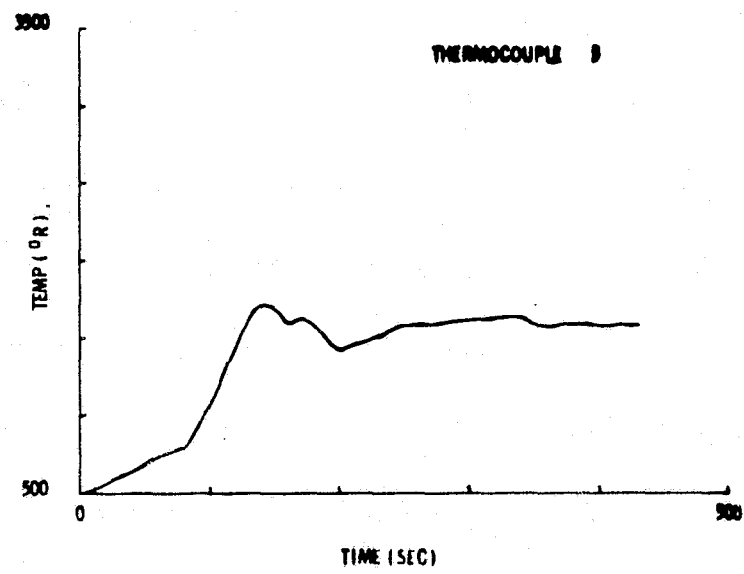
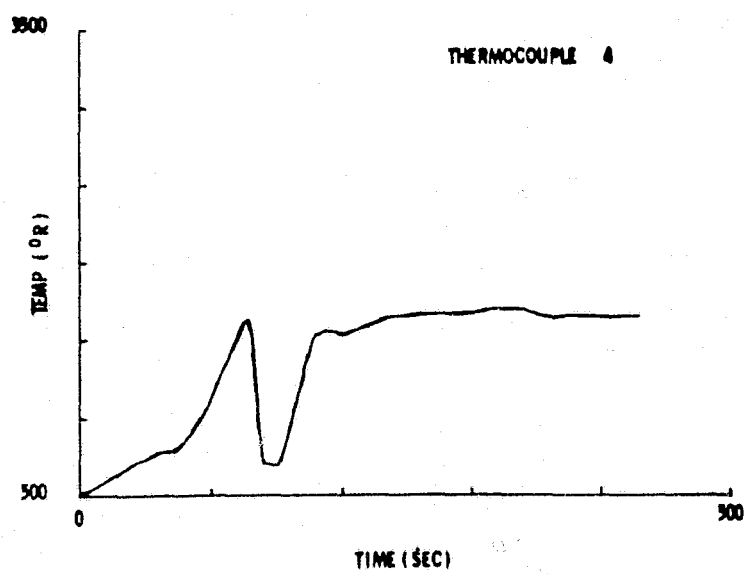
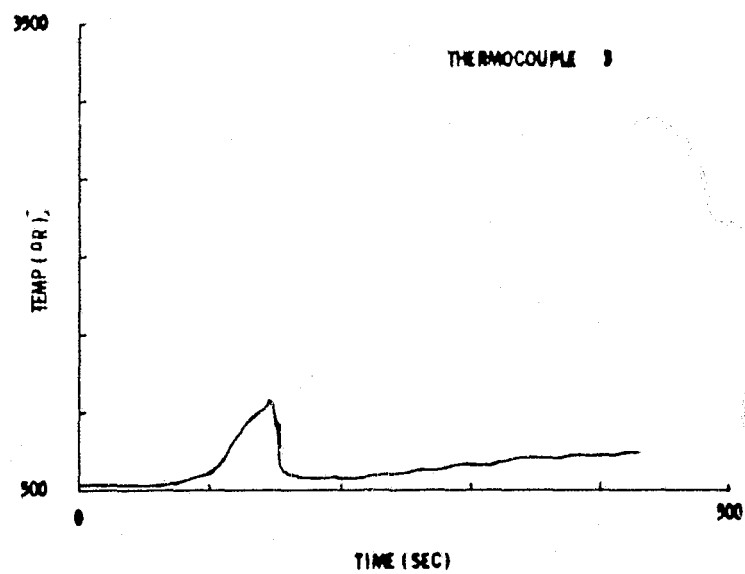
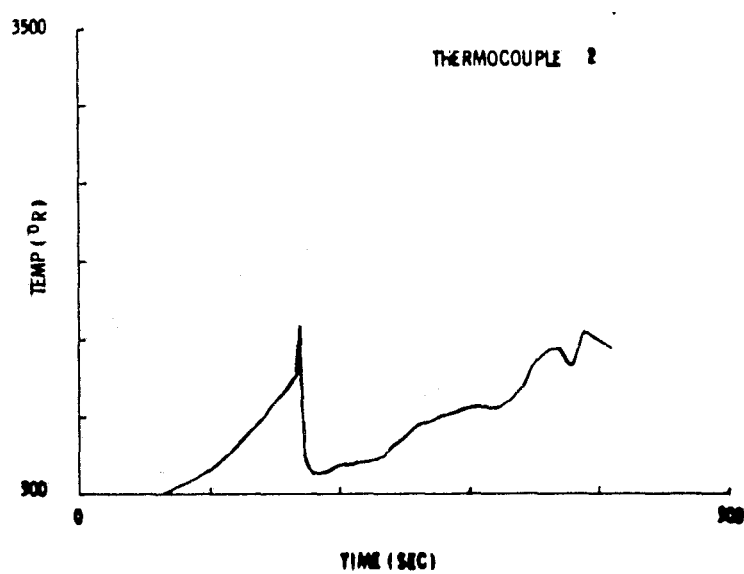
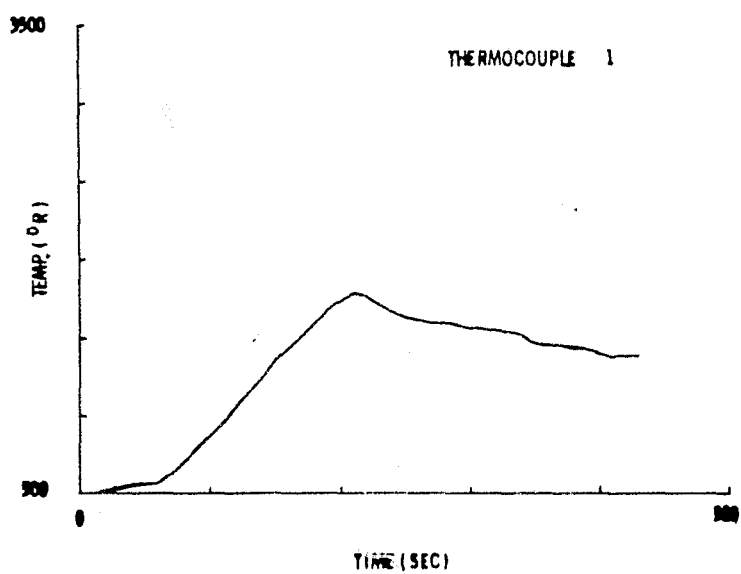


Figure 49a

Temperature Response of Model 3

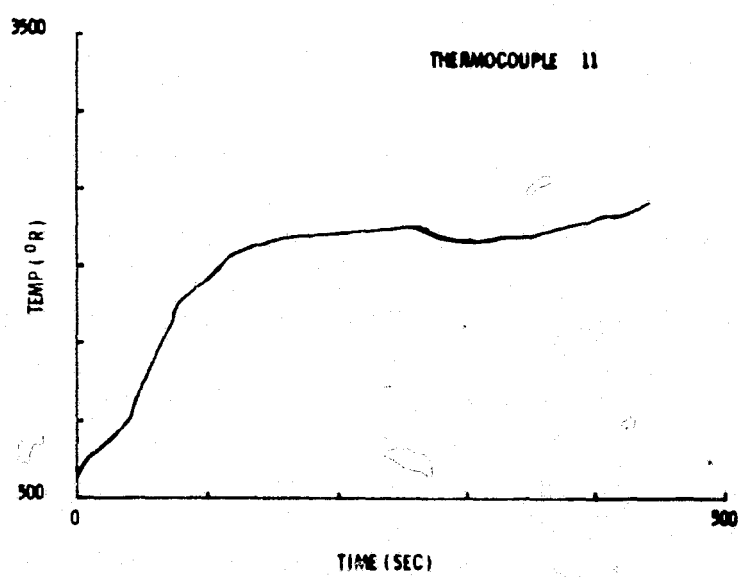
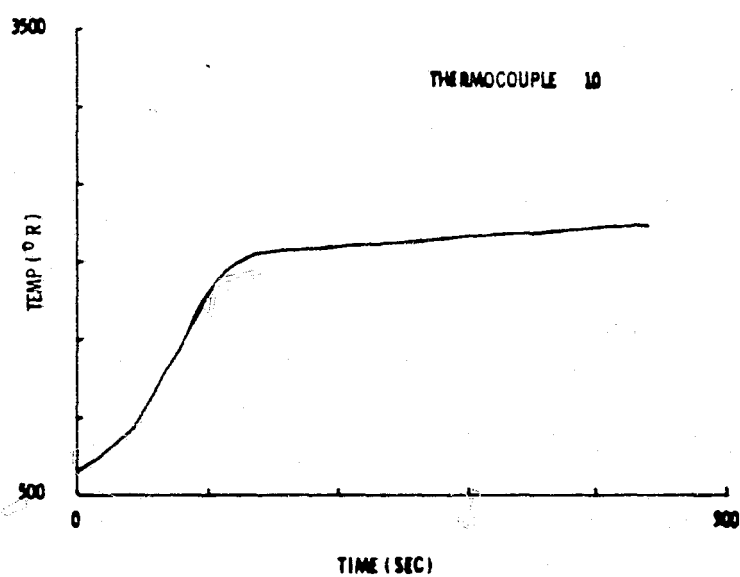
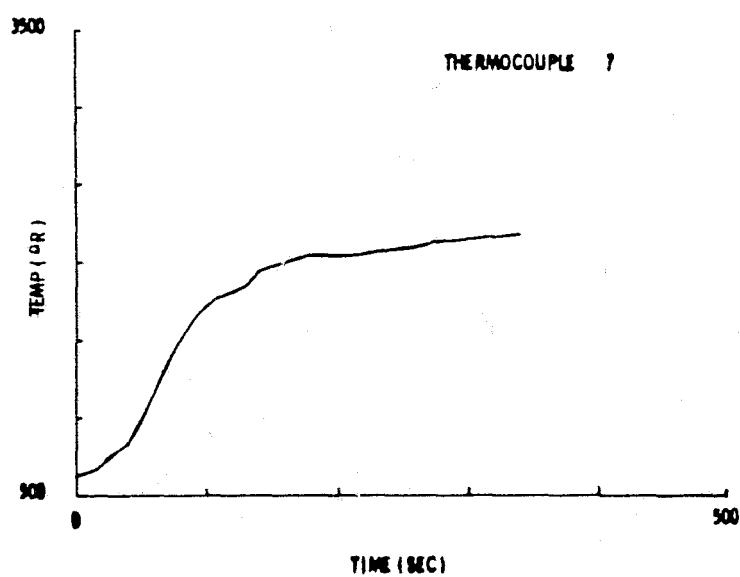
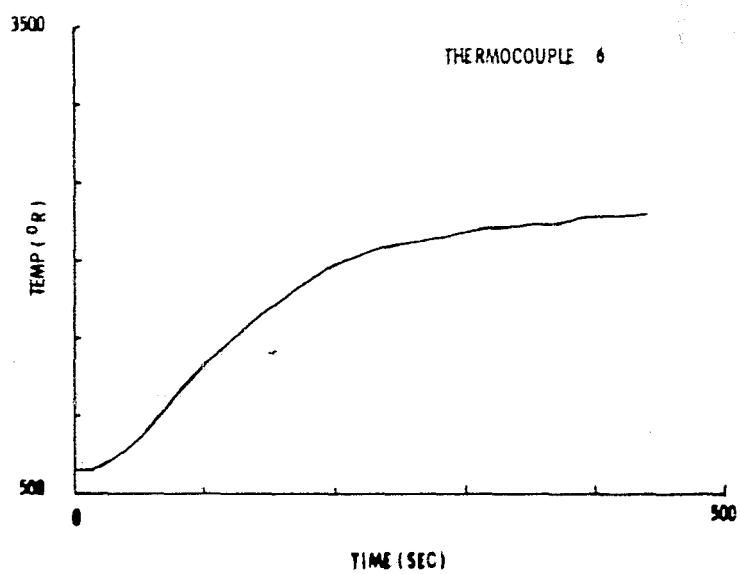


Figure 49b
Temperature Response of Model 3

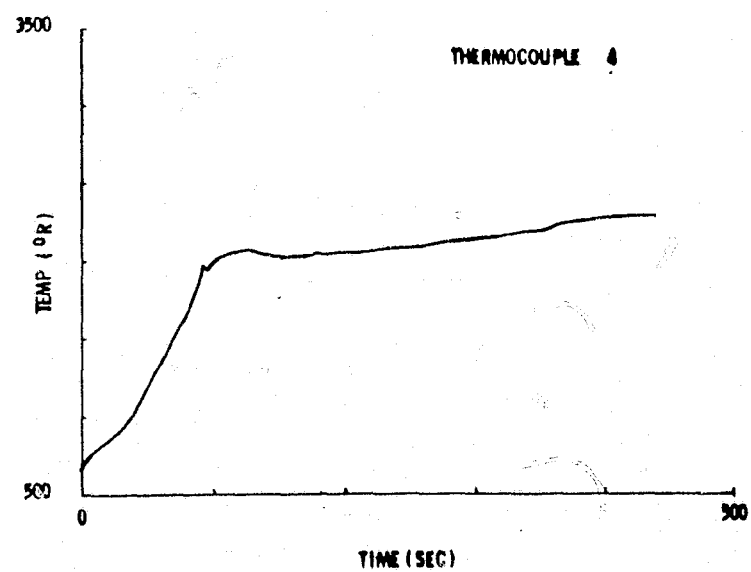
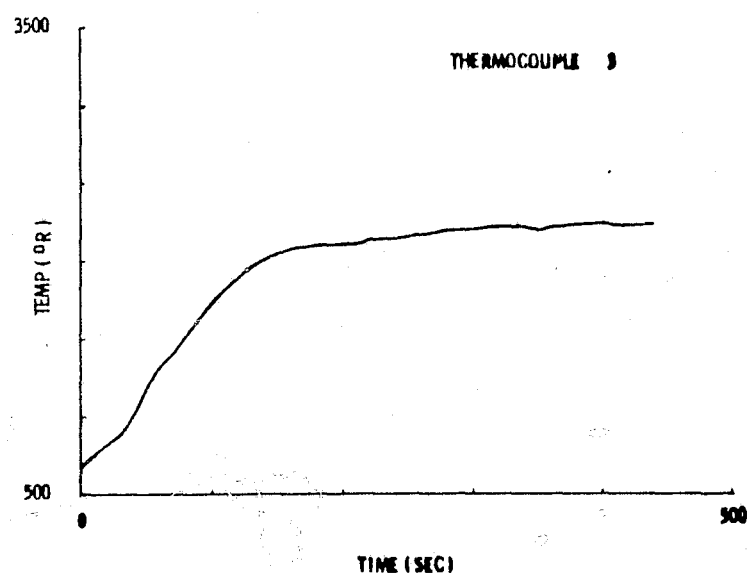
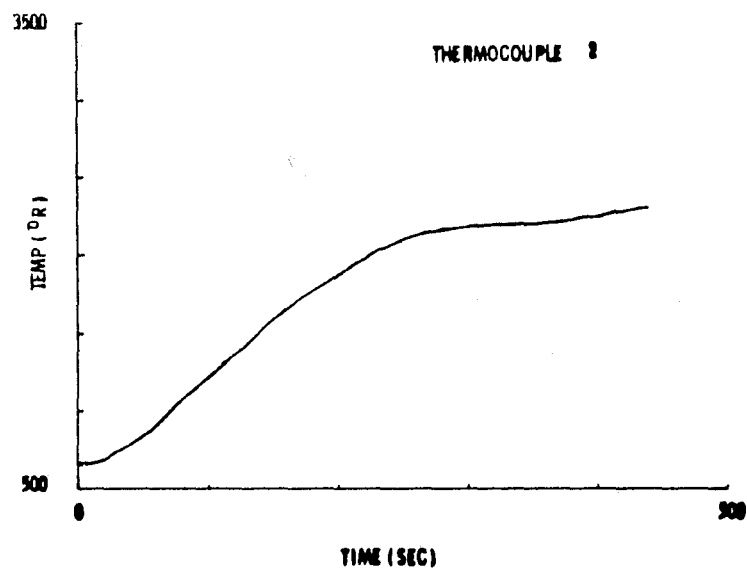
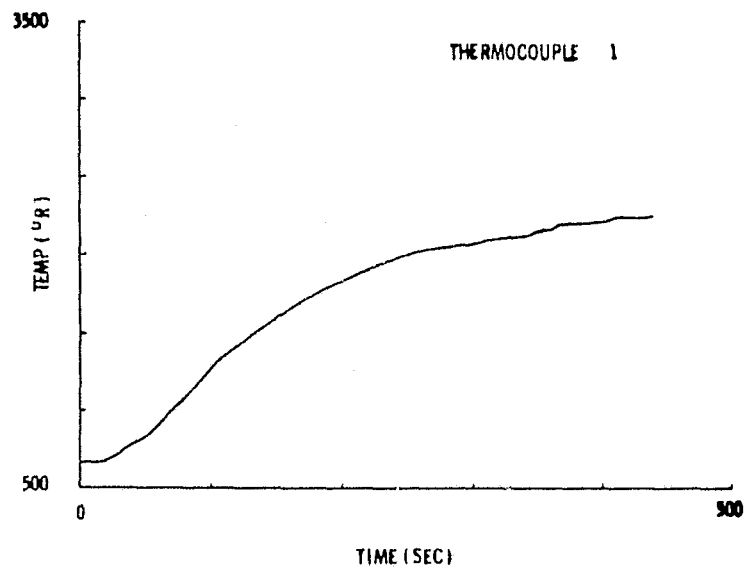


Figure 50a

Temperature Response of Model 4

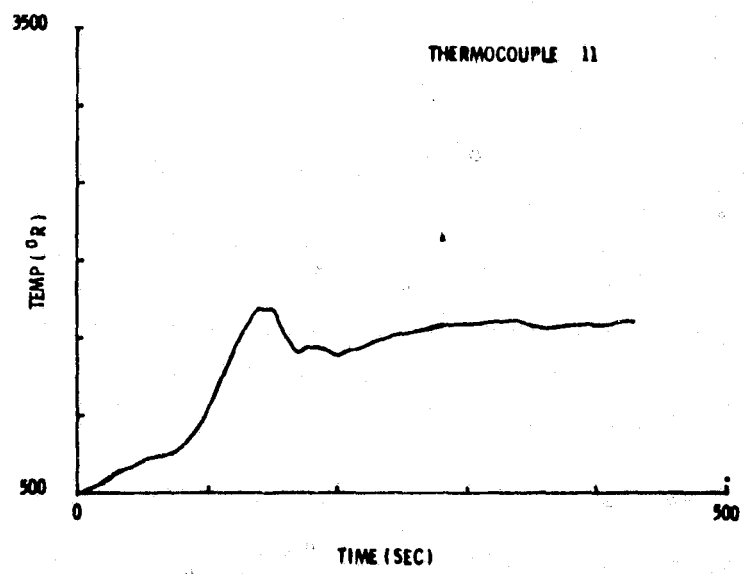
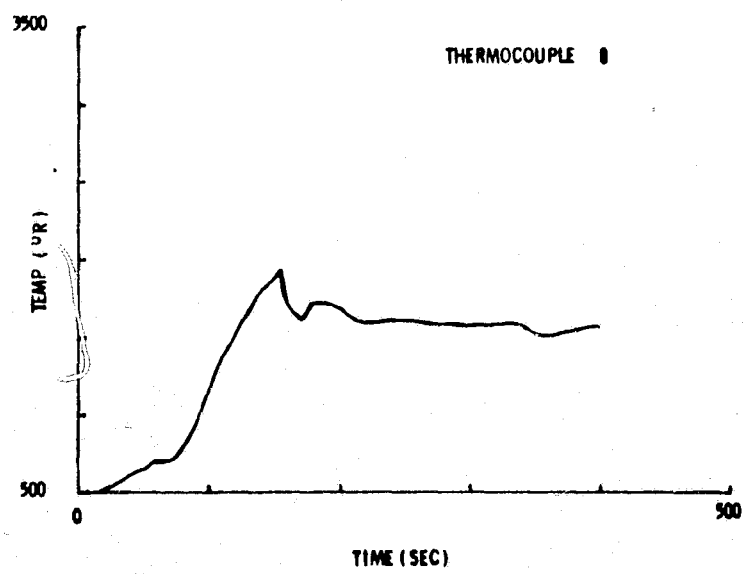
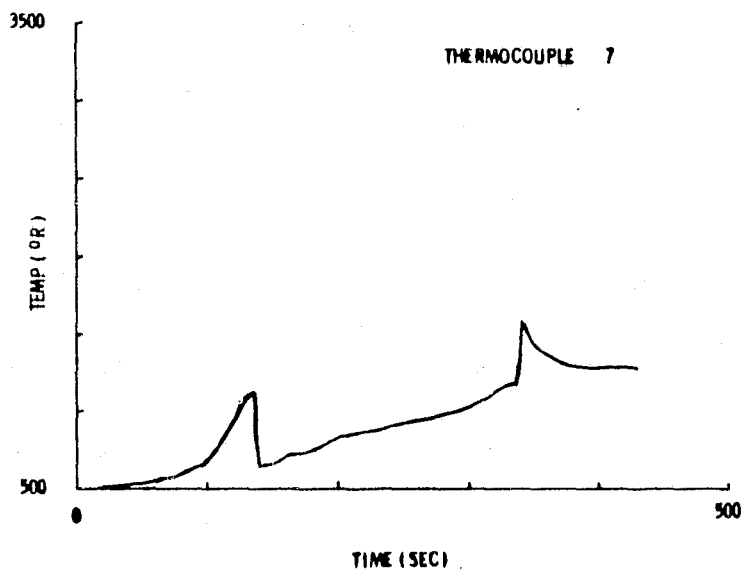


Figure 50b
Temperature Response of Model 4

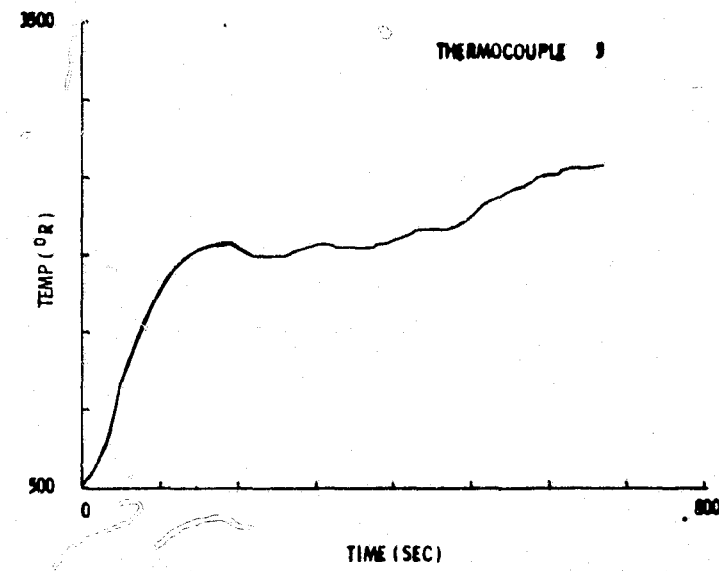
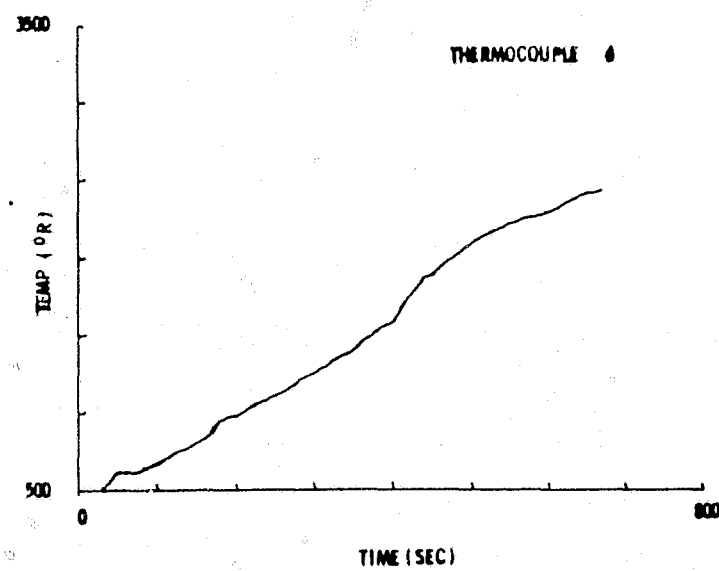
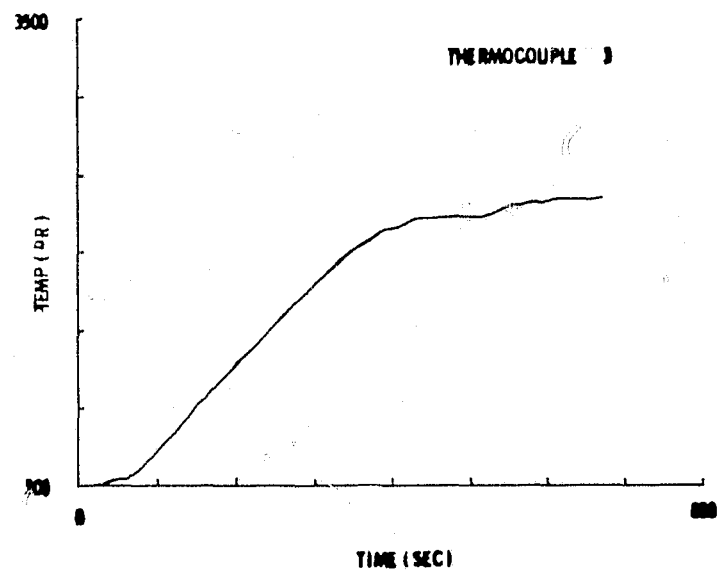
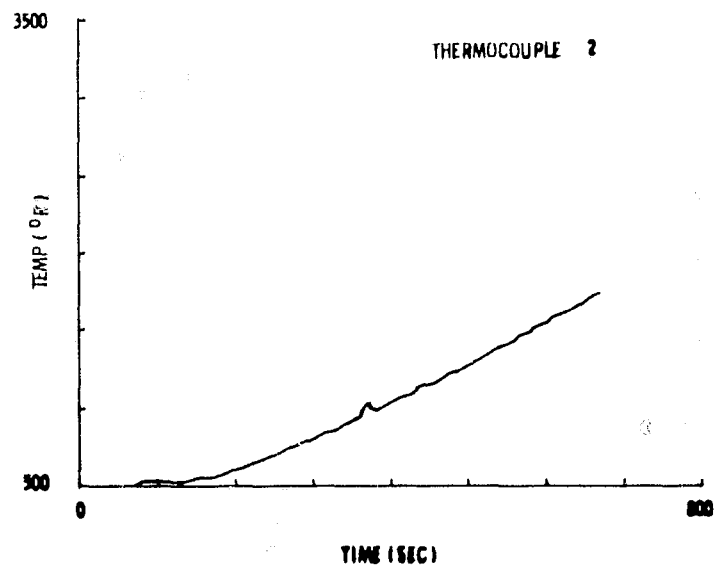
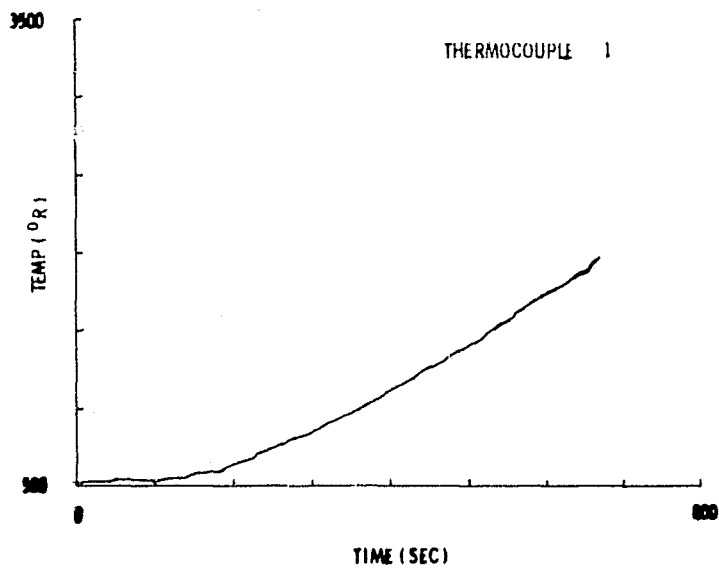


Figure 51a.

Temperature Response of Model 5

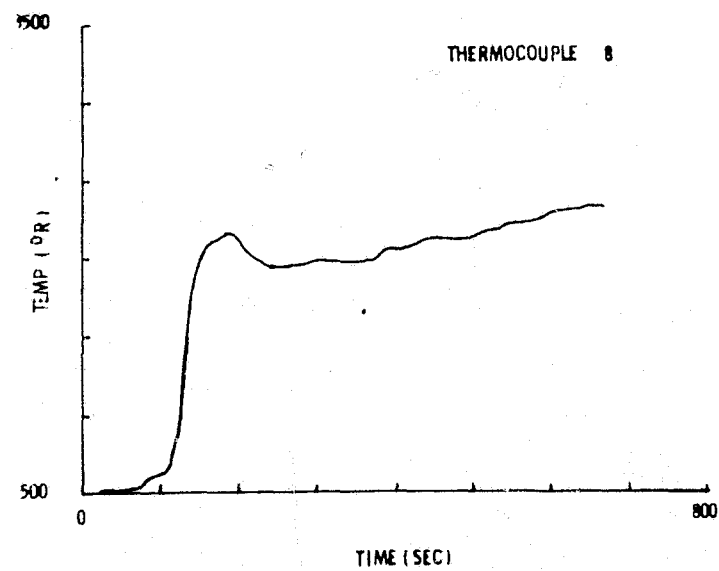
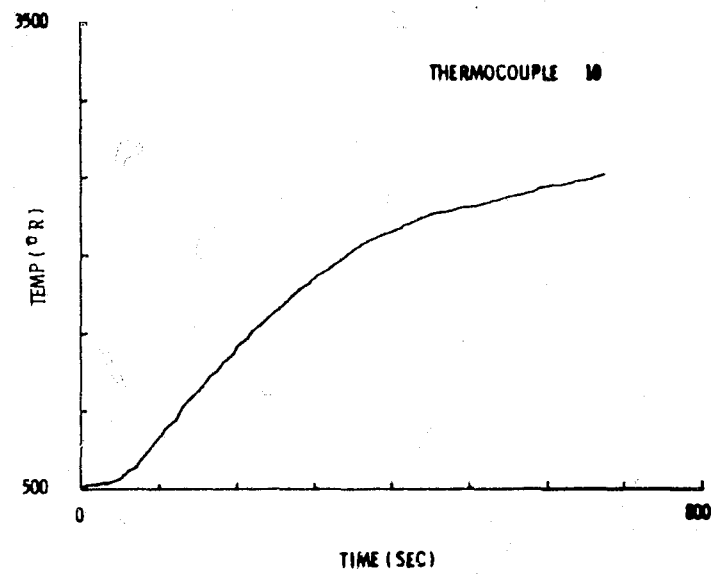
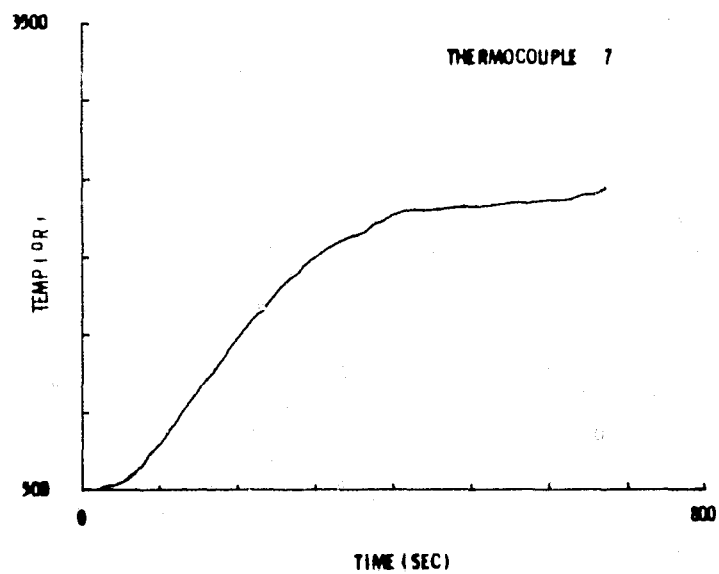
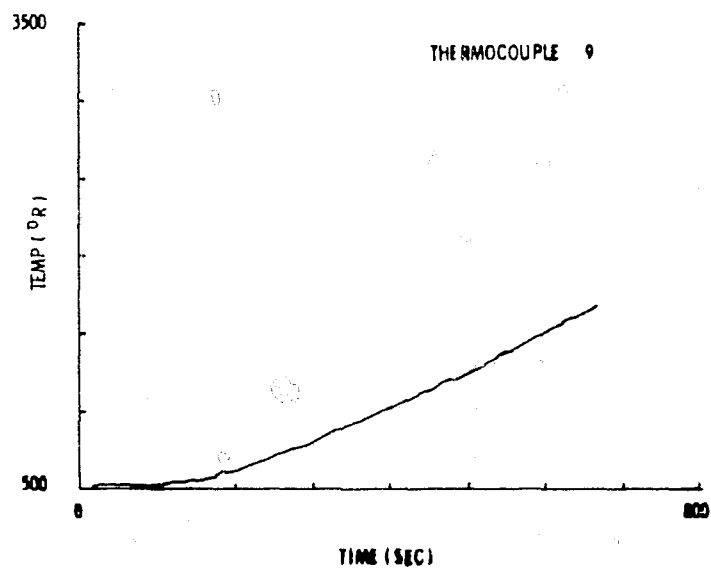
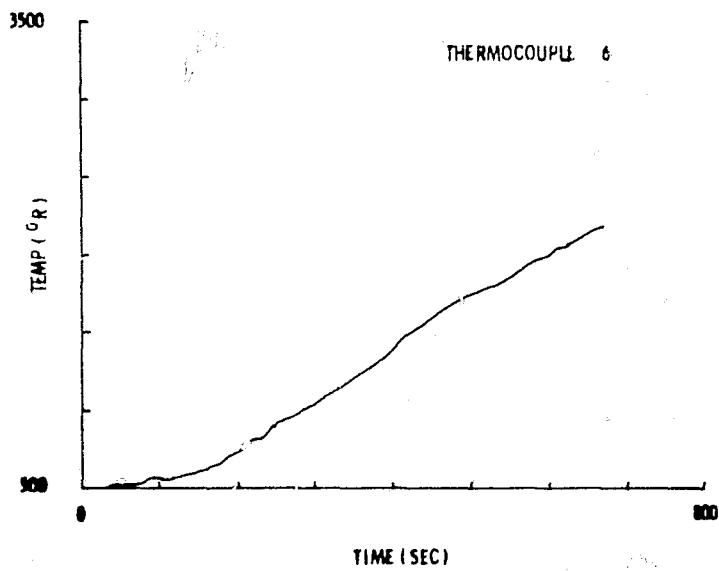


Figure 51b

Temperature Response of Model 5

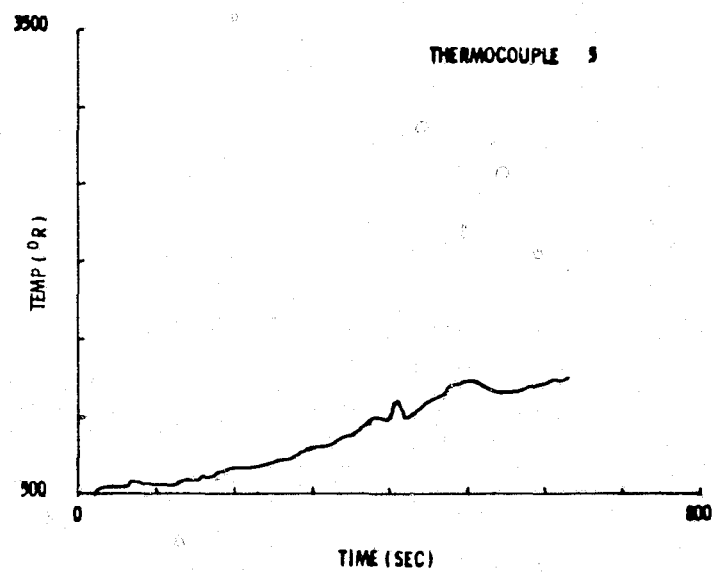
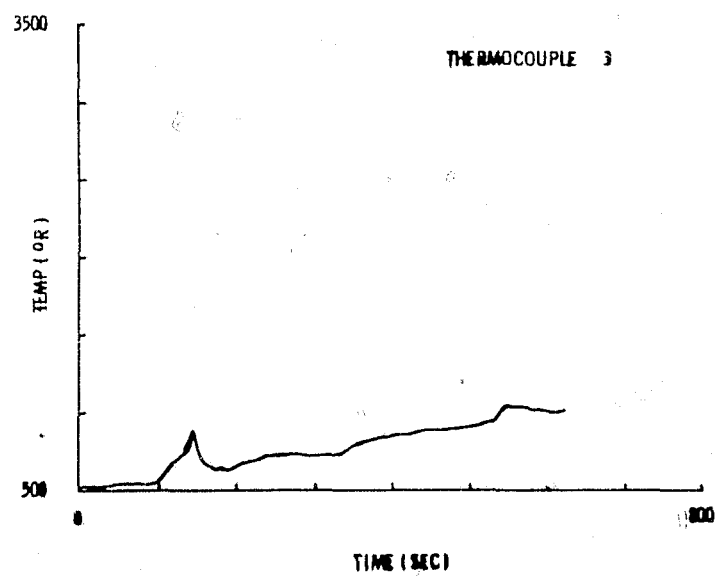
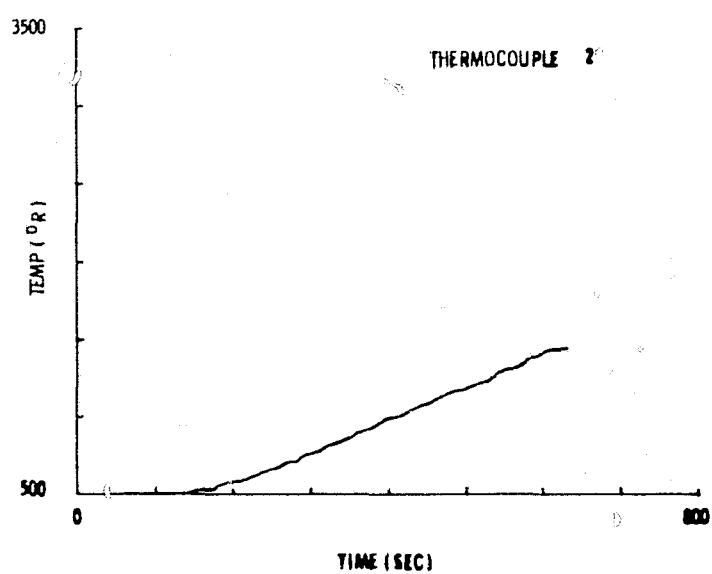
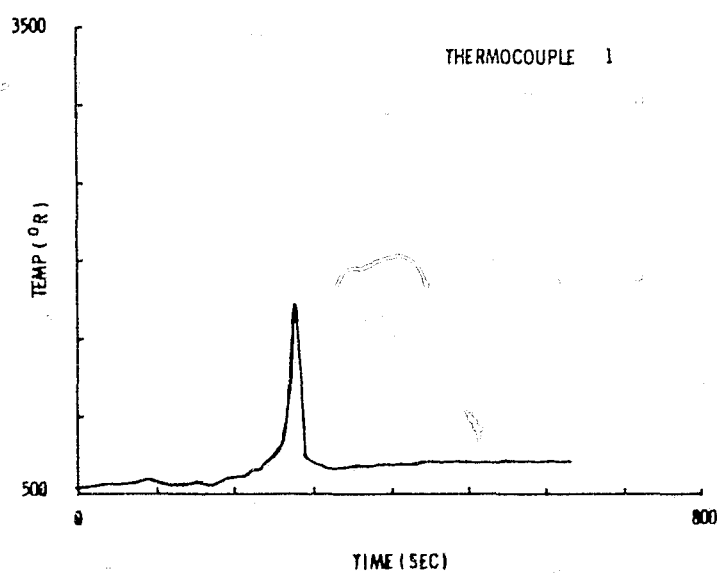


Figure 52a

Temperature Response of Model 6

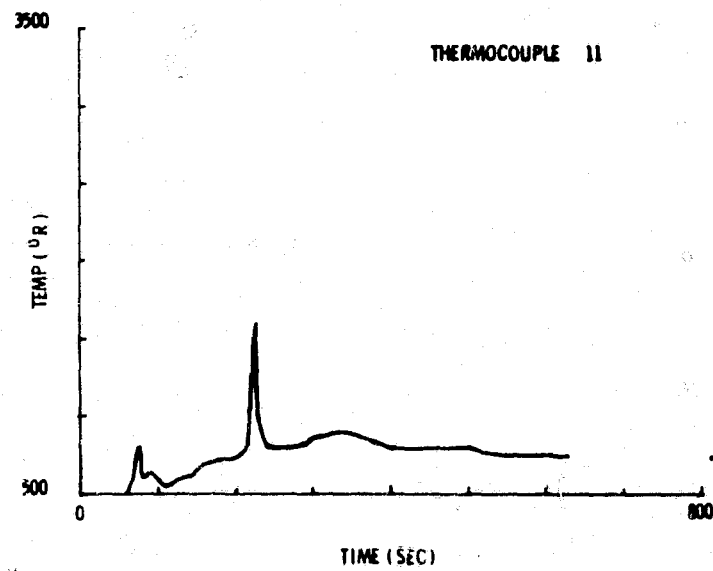
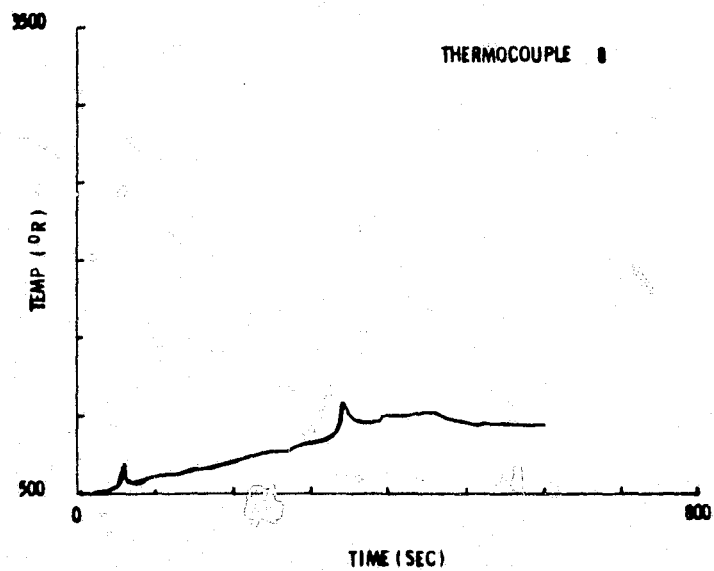
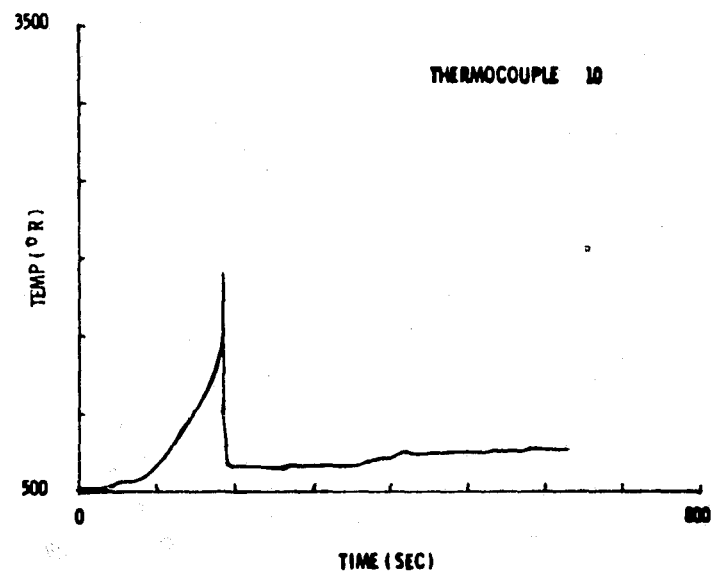
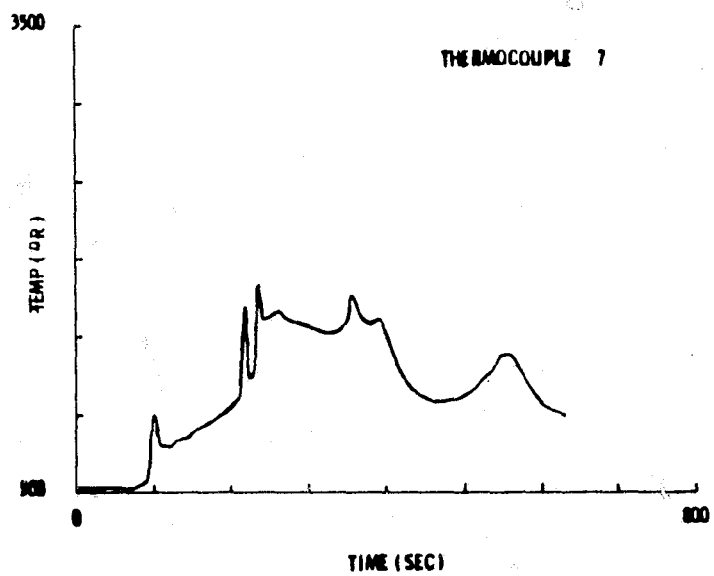
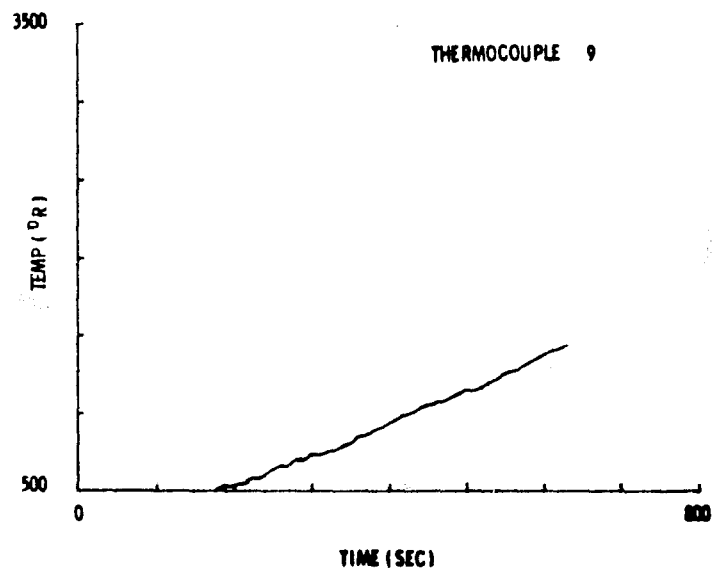
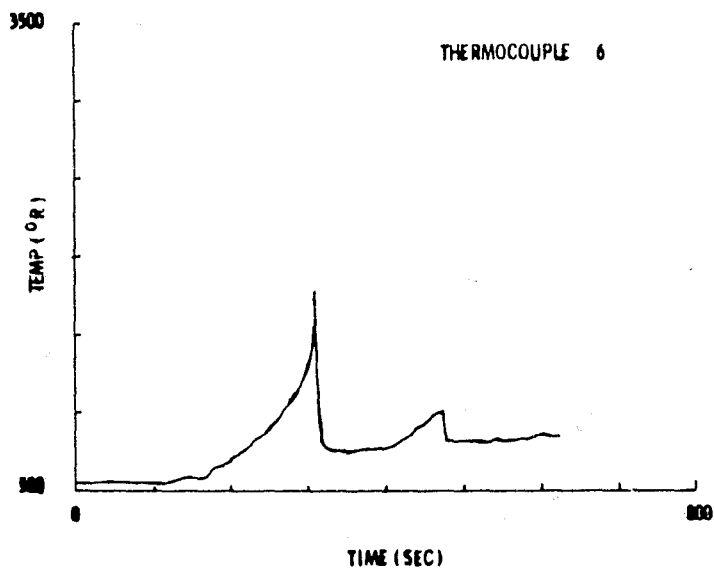


Figure 52b
Temperature Response of Model 6

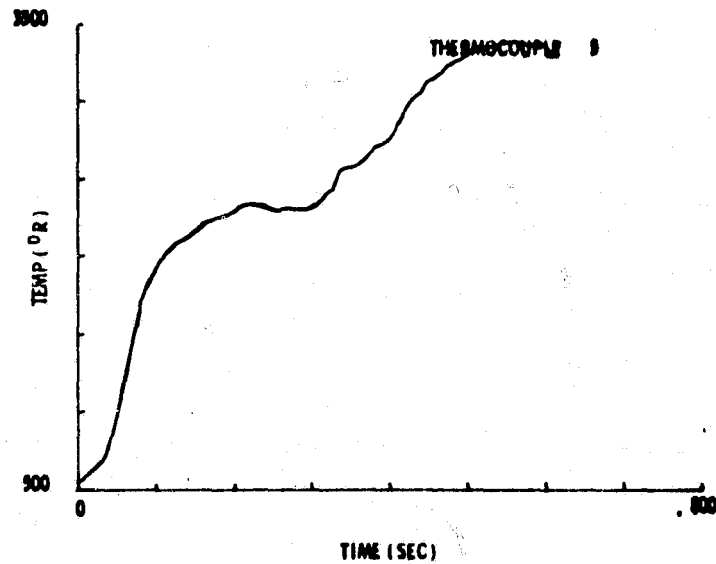
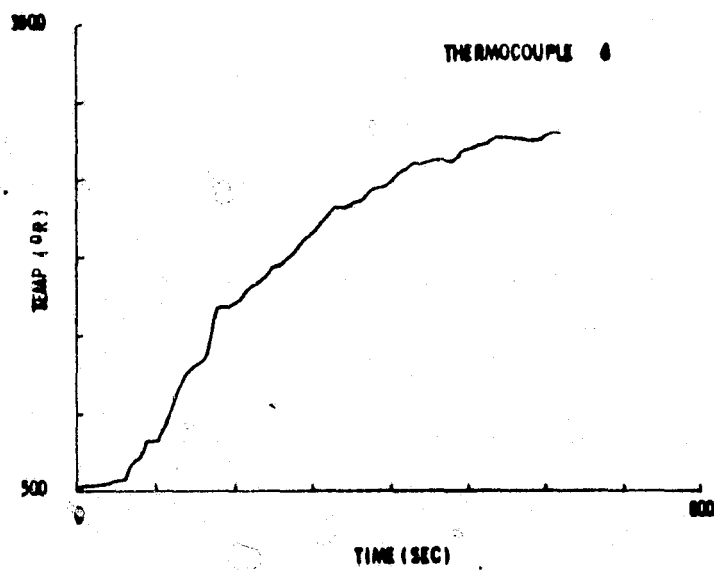
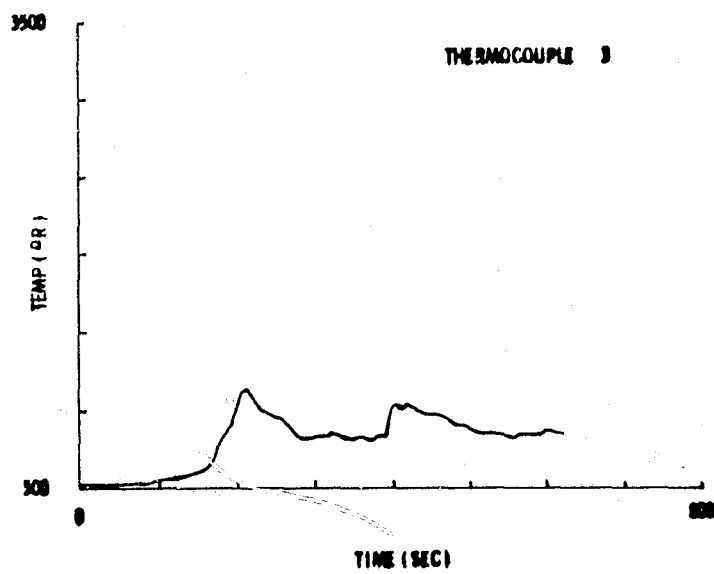
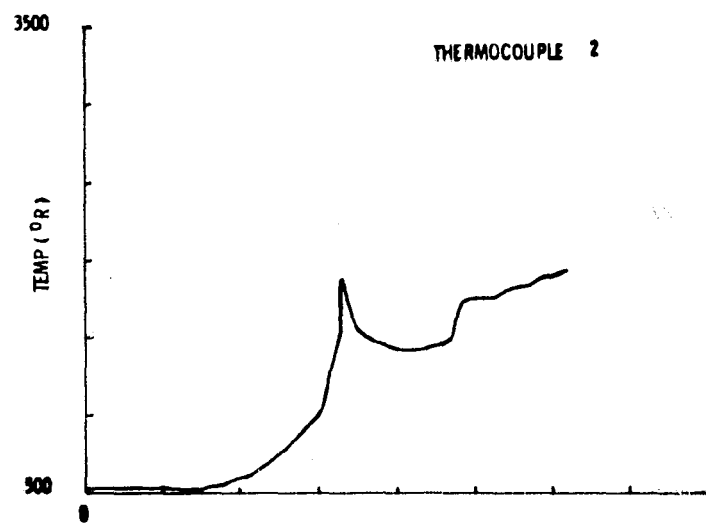
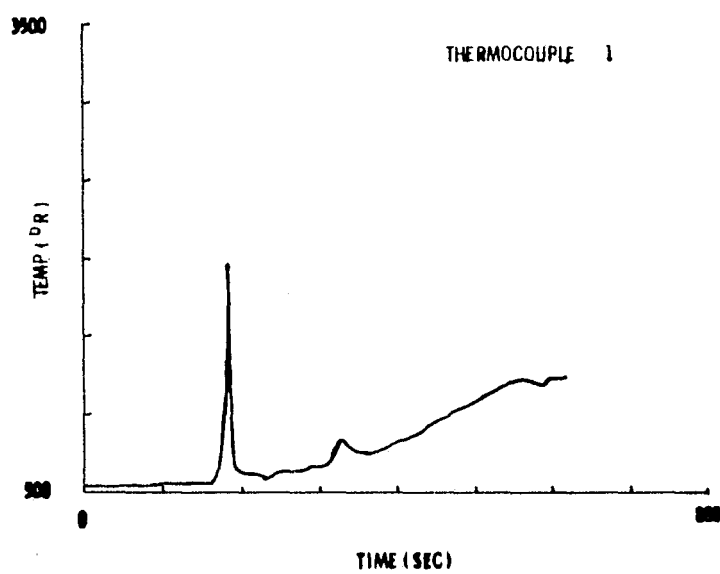


Figure 53a

Temperature Response of Model 7

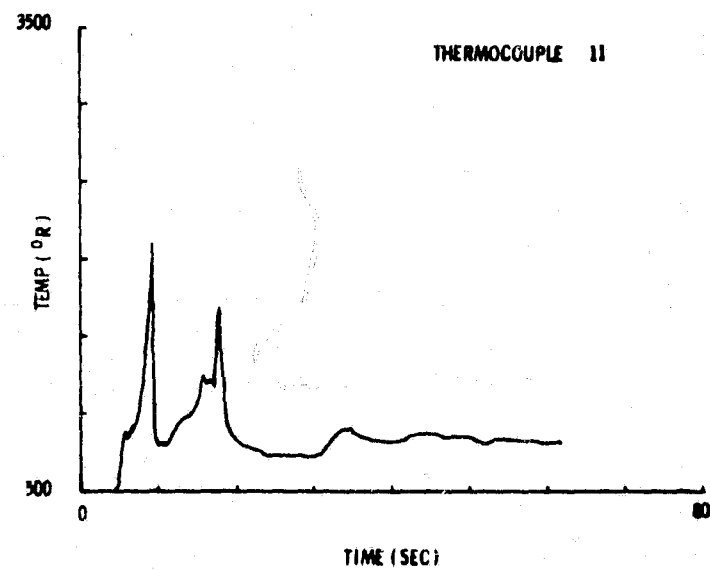
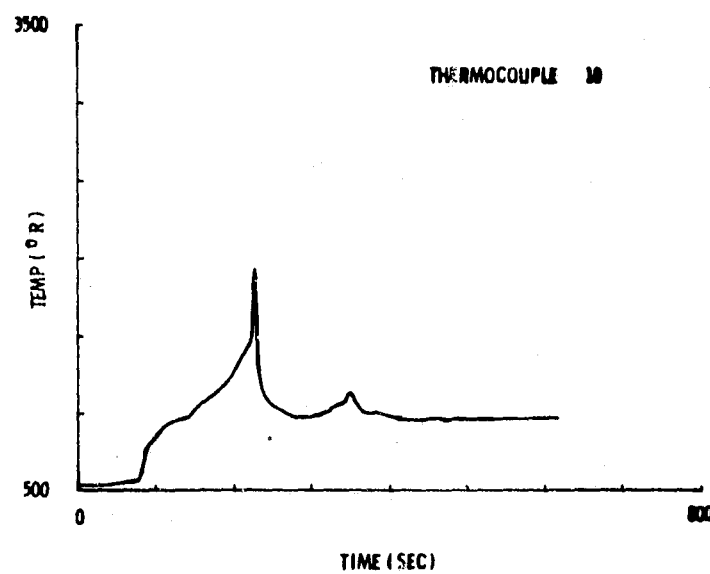
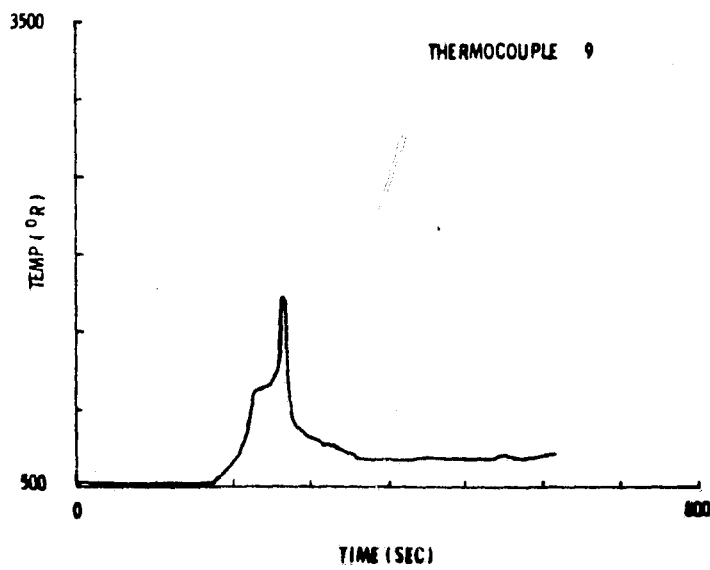
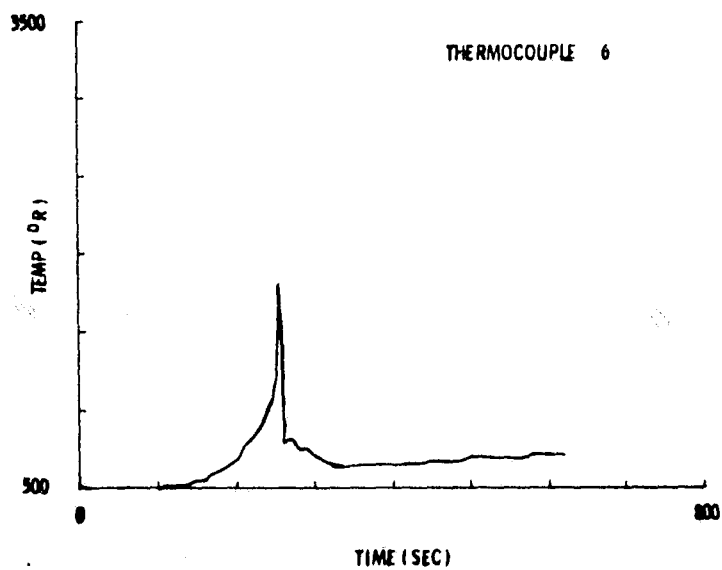


Figure 53b
Temperature Response of Model 7

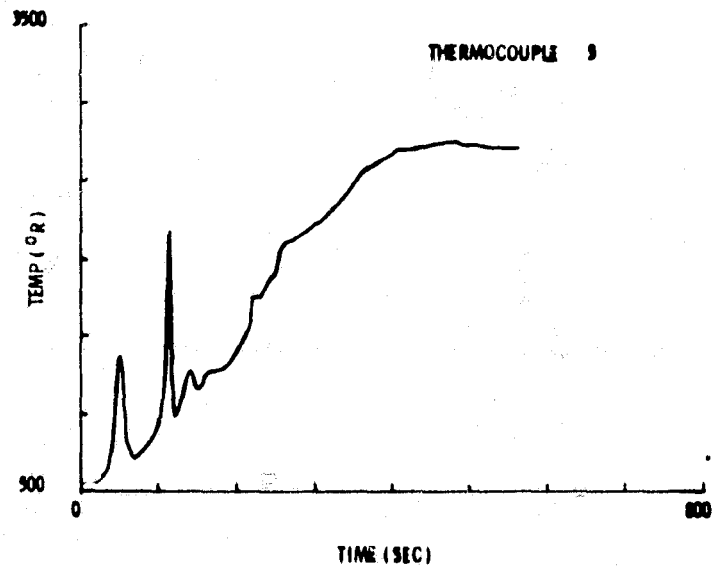
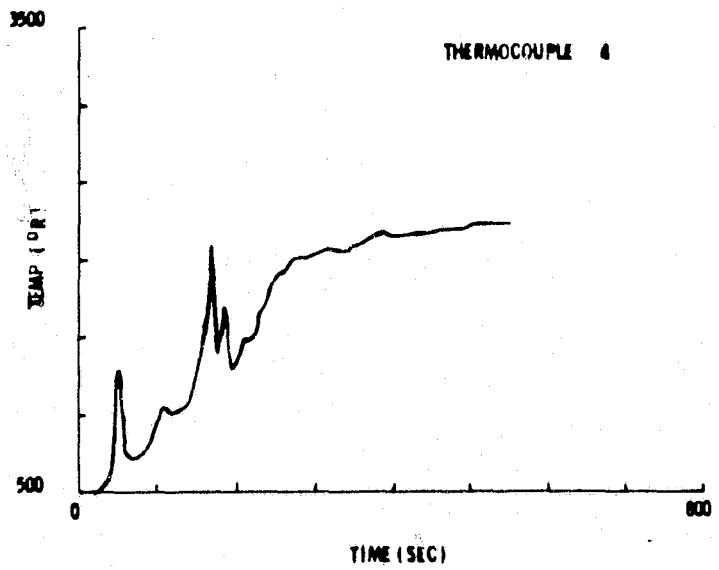
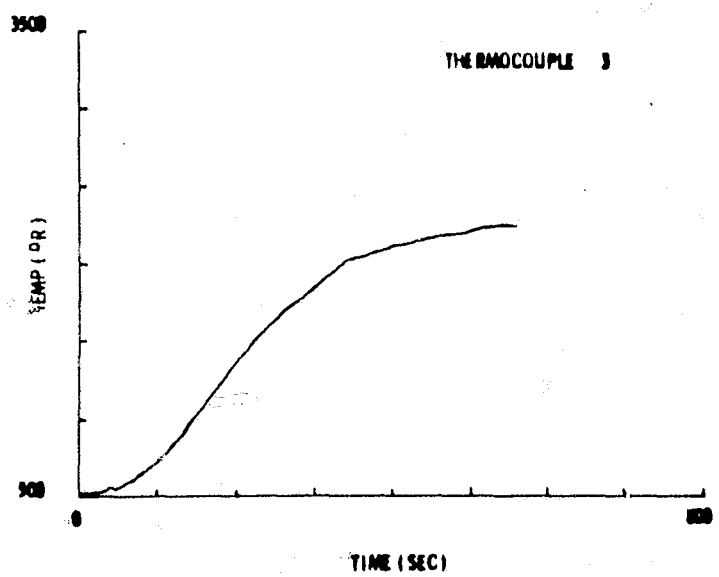
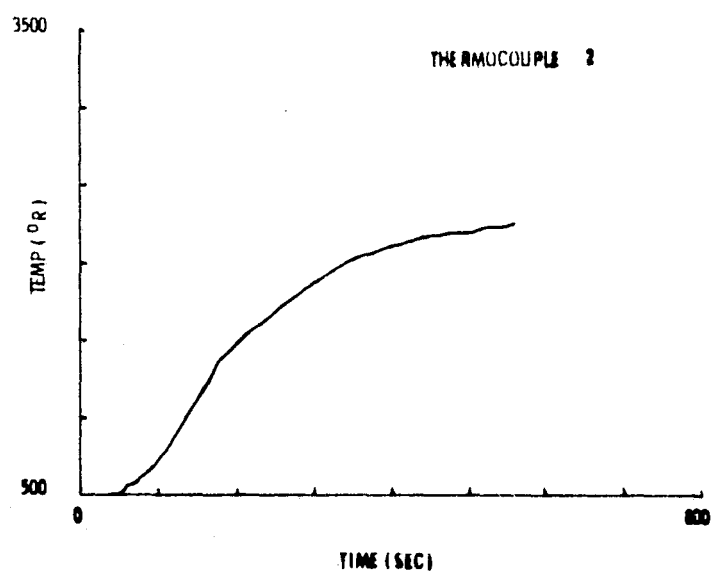
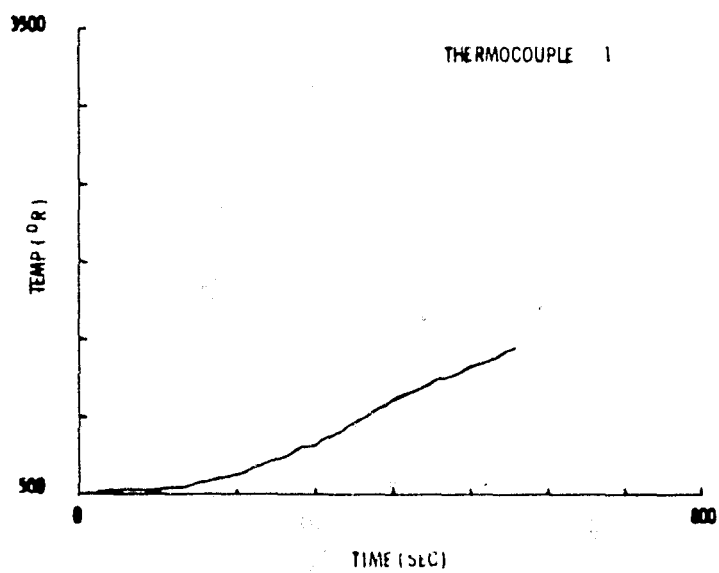


Figure 54a

Temperature Response of Model 8

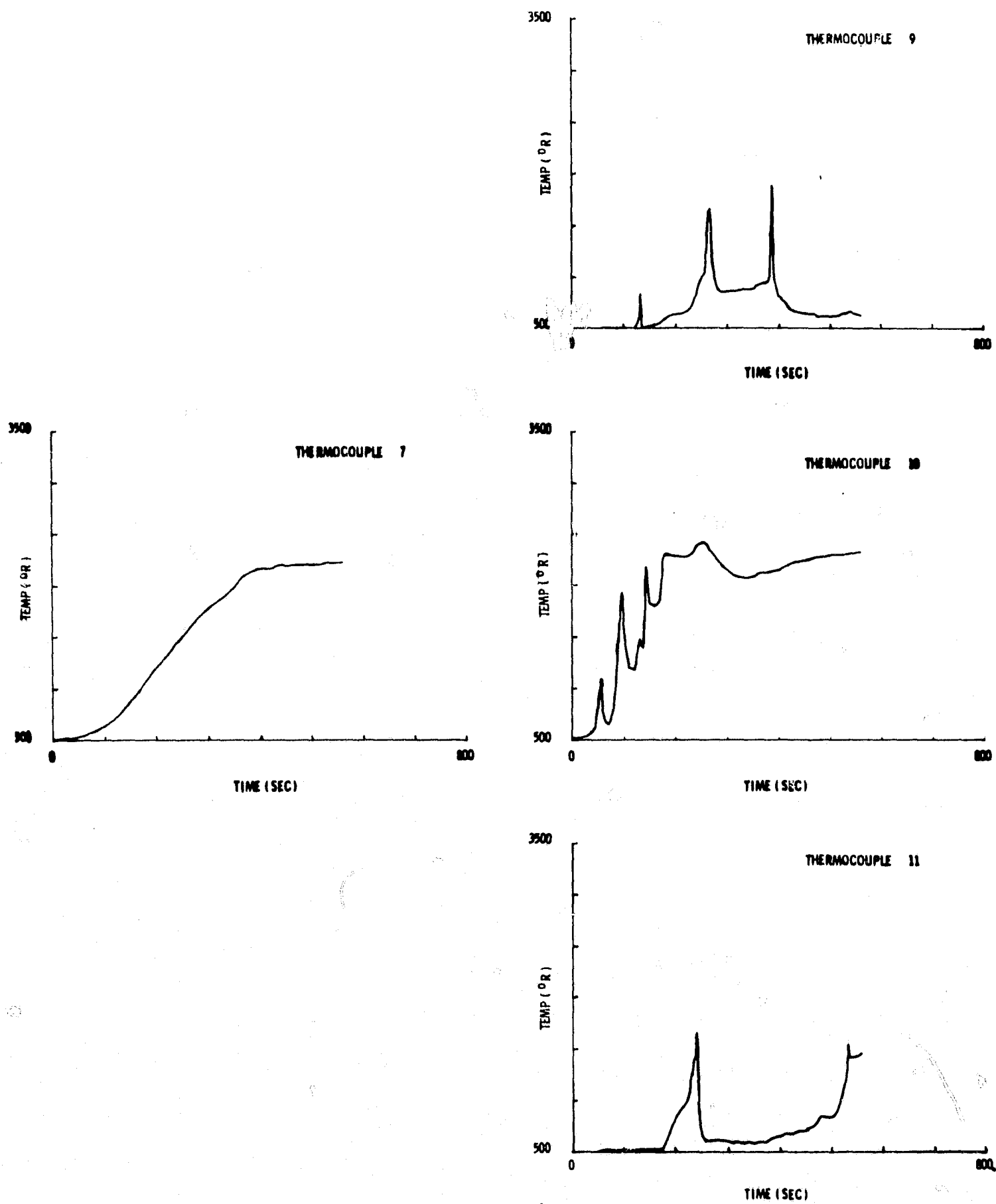


Figure 54b

Temperature Response of Model 8

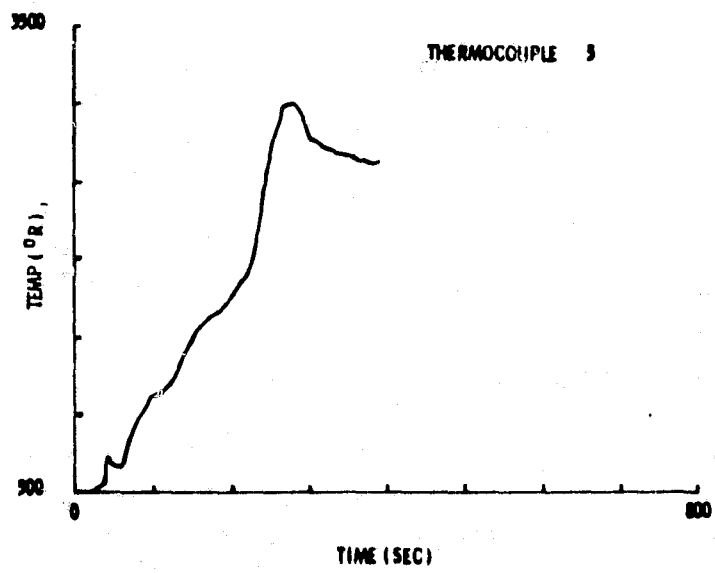
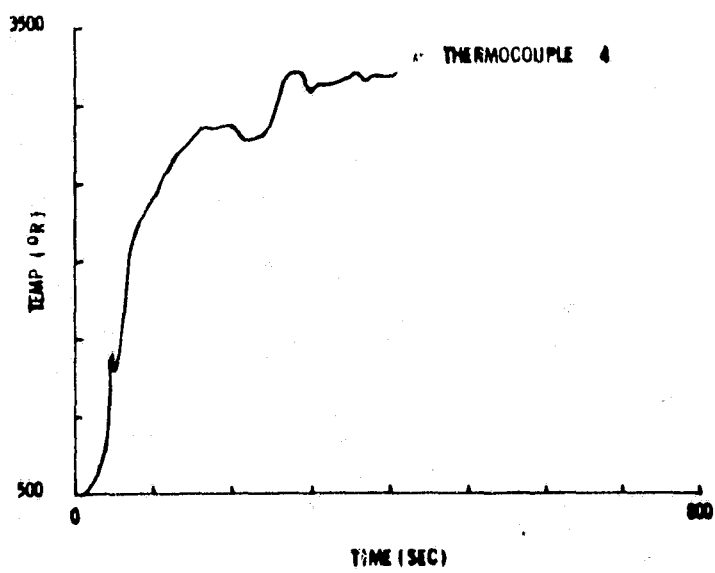
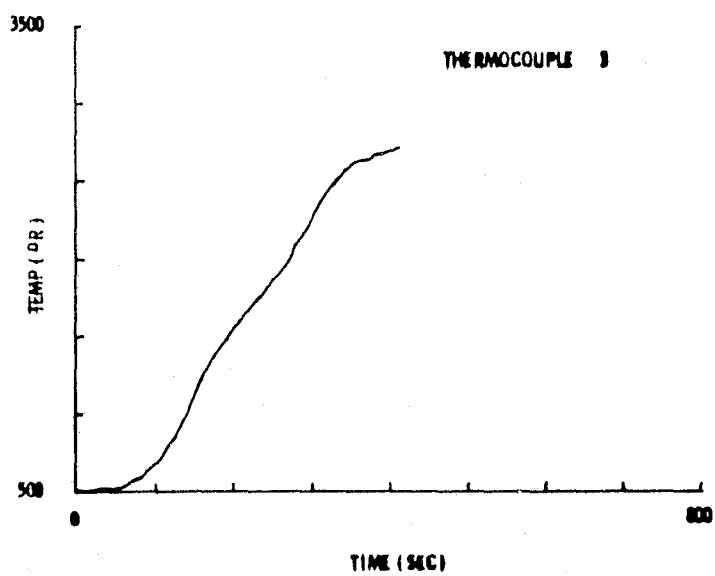
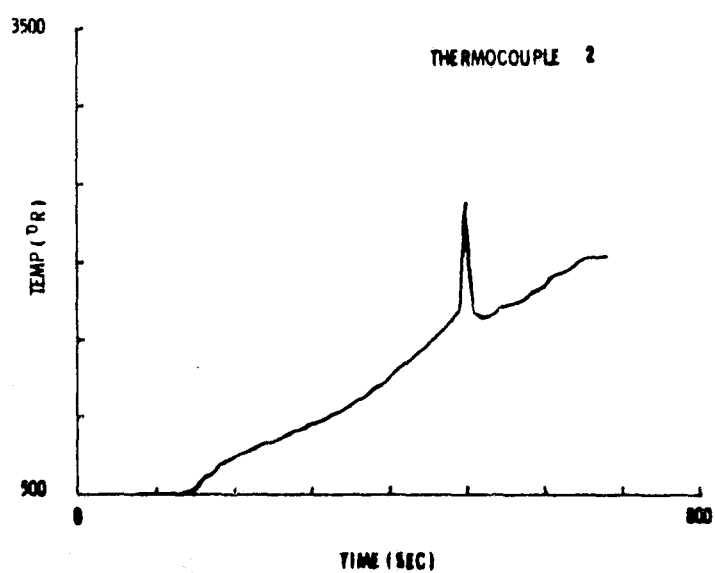
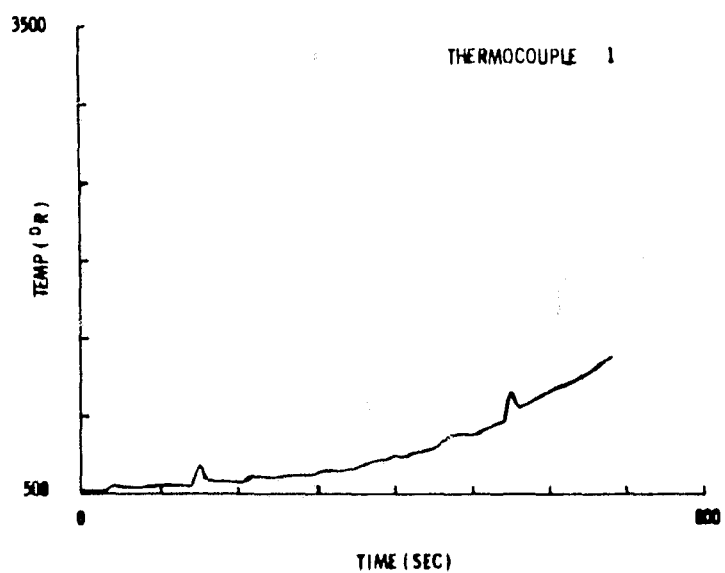


Figure 55a

Temperature Response of Model 9

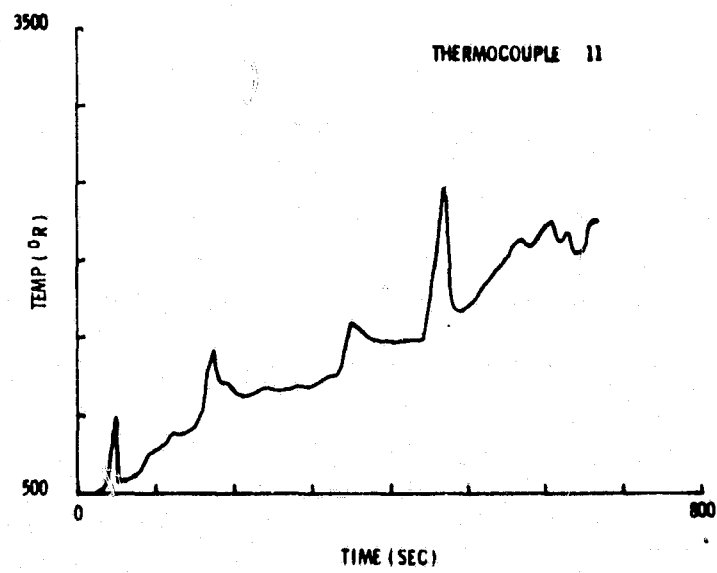
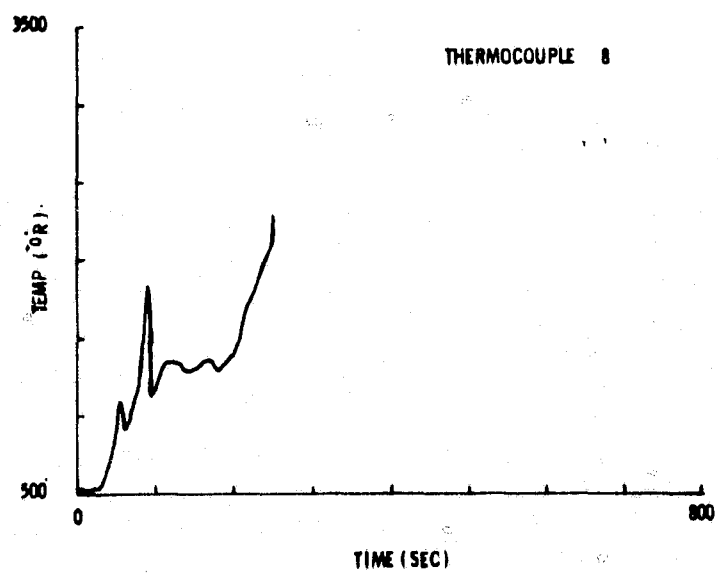
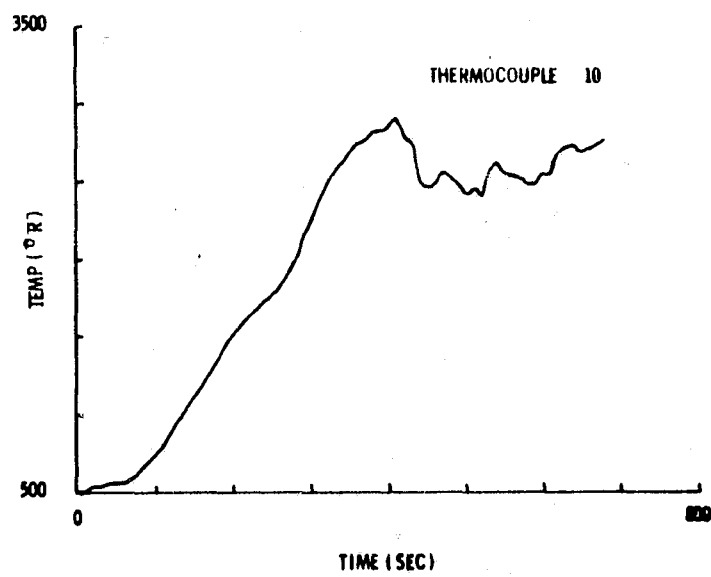
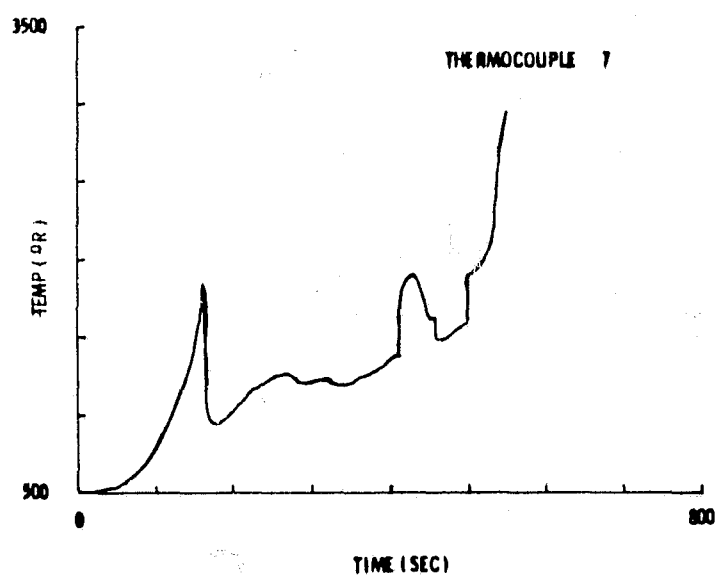
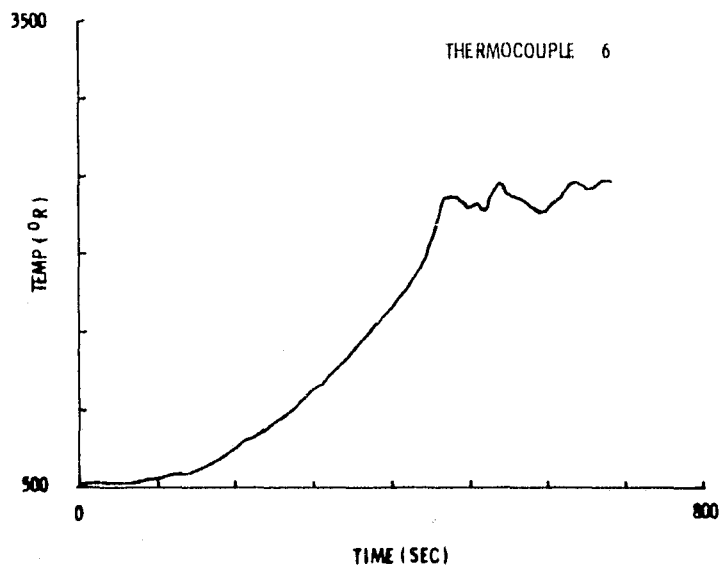


Figure 55b

Temperature Response of Model 9

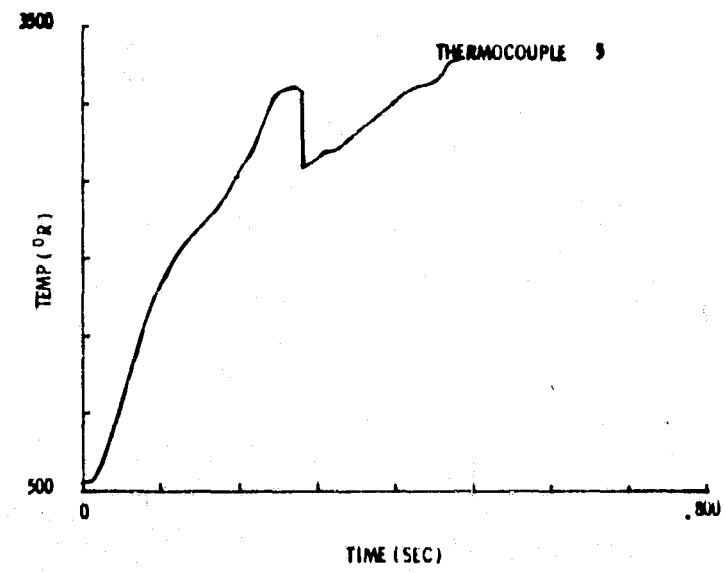
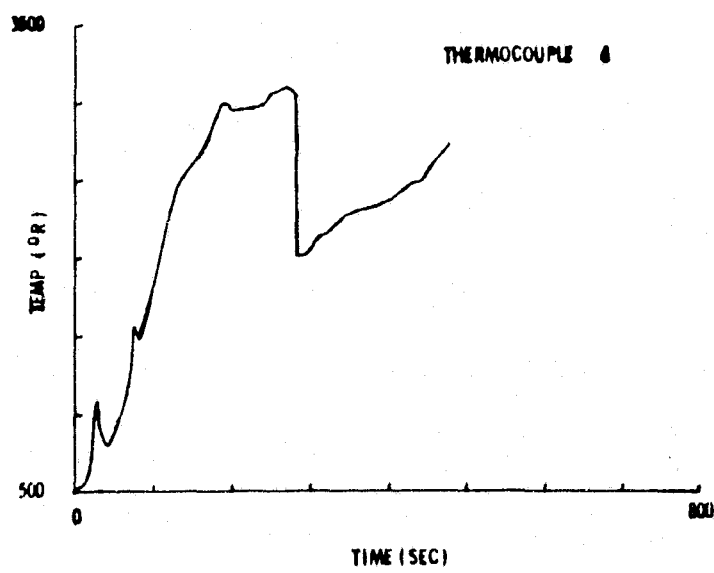
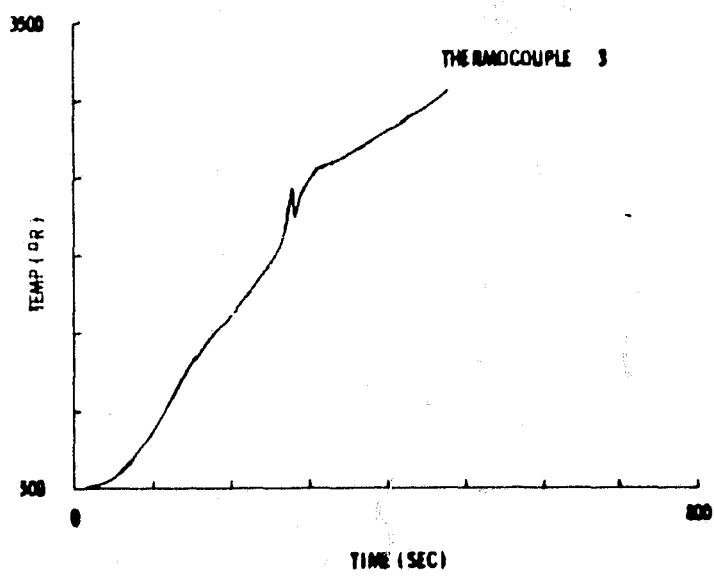
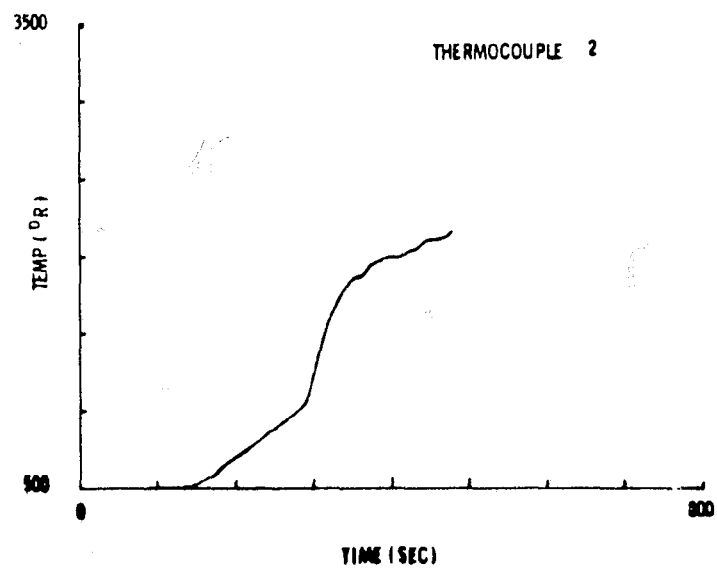
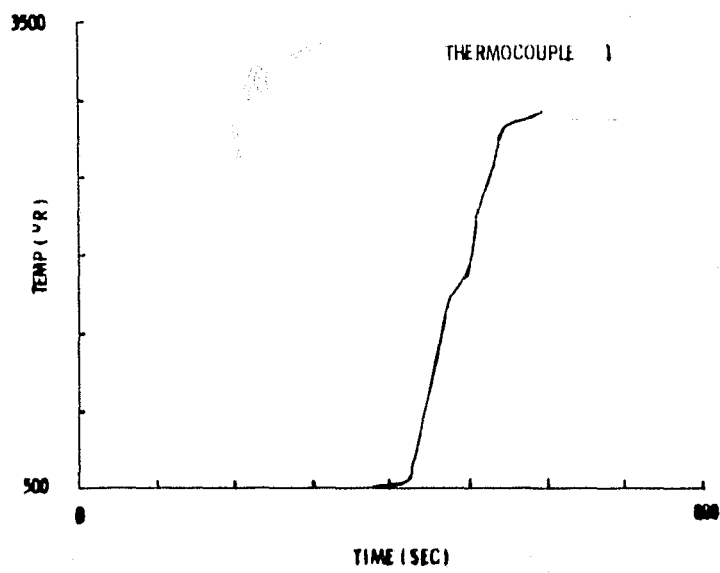


Figure 56a
Temperature Response of Model 10

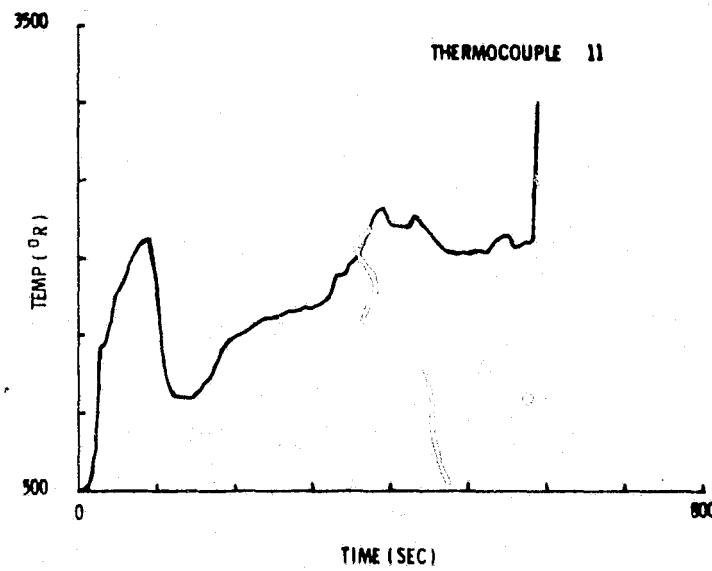
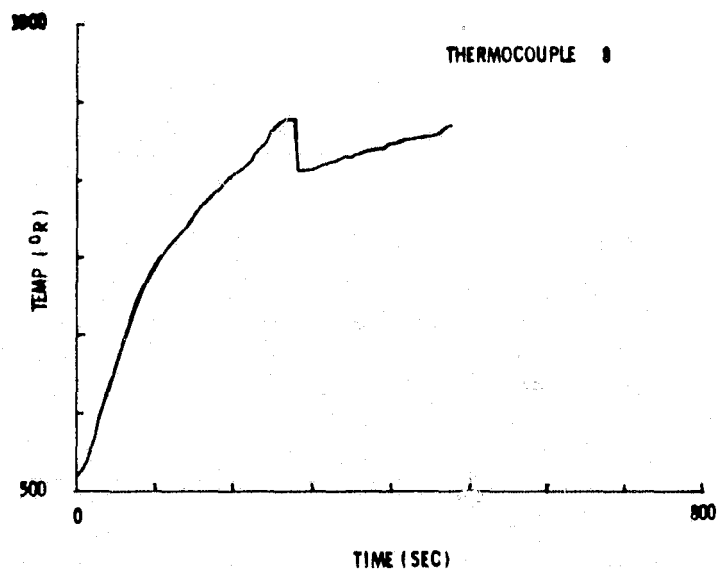
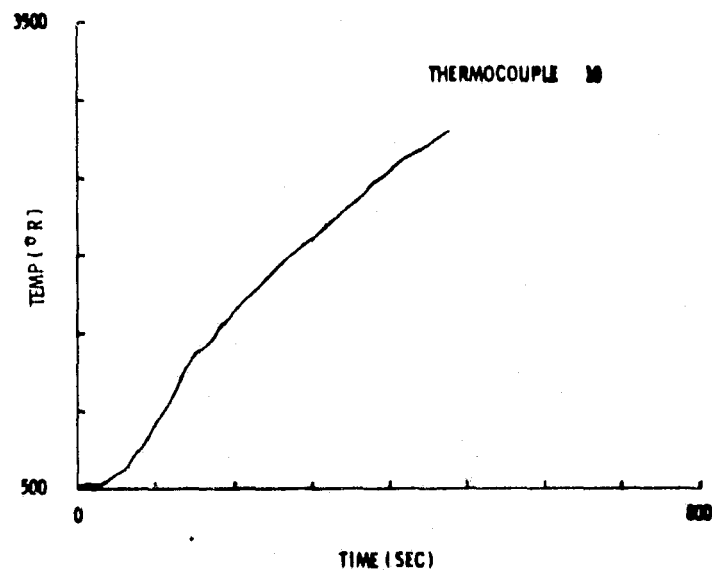
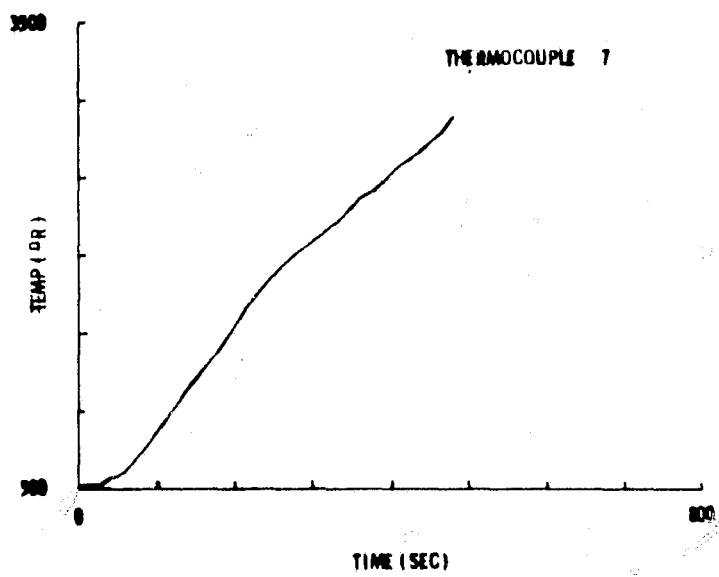


Figure 56b

Temperature Response of Model 10

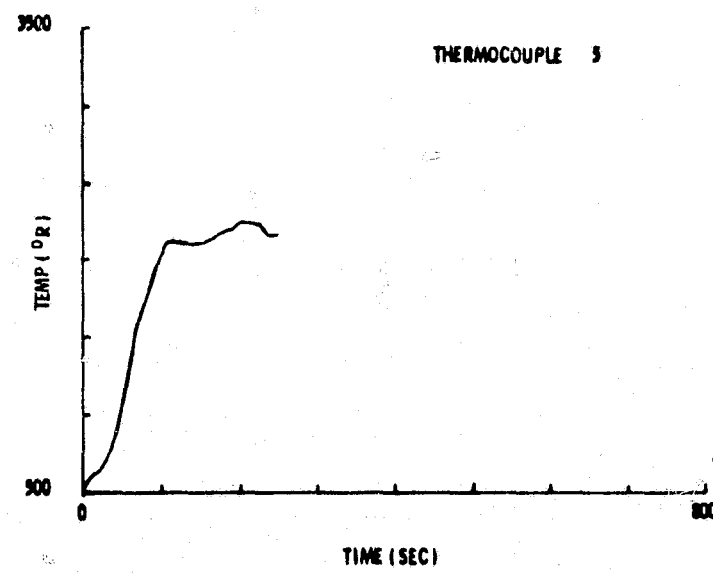
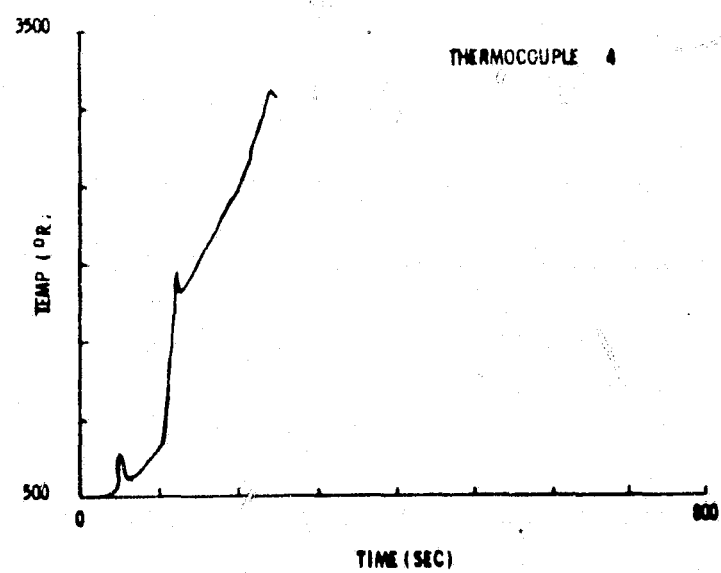
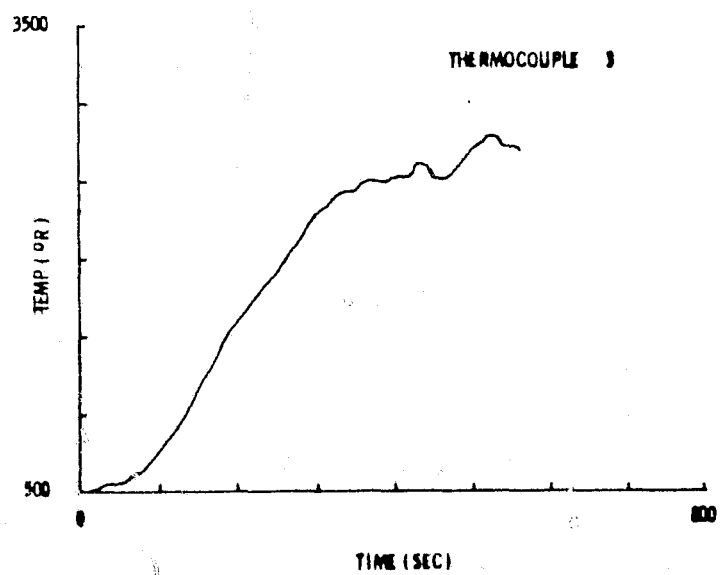
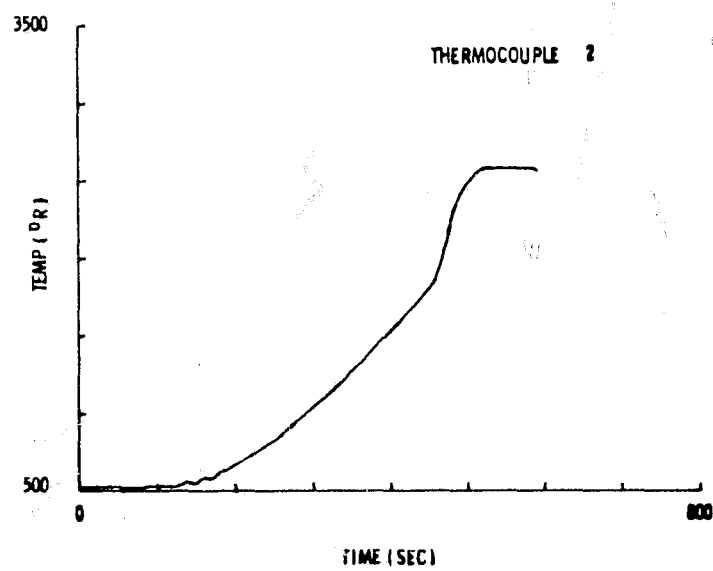
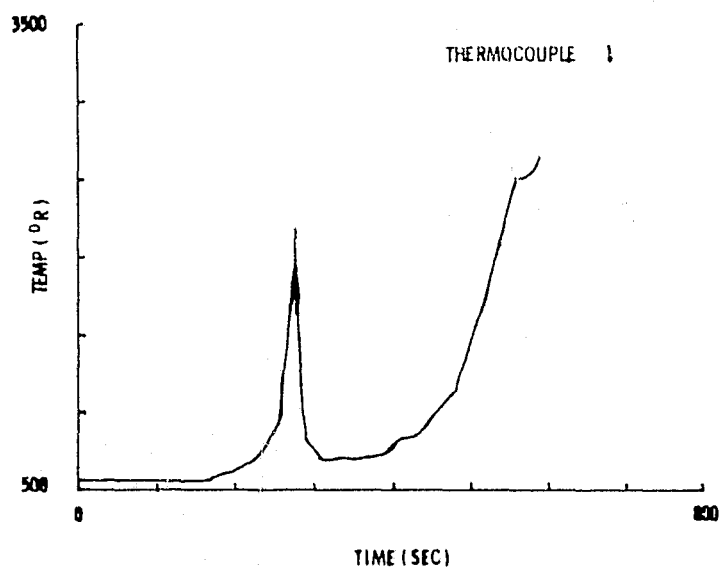


Figure 57a

Temperature Response of Model 11

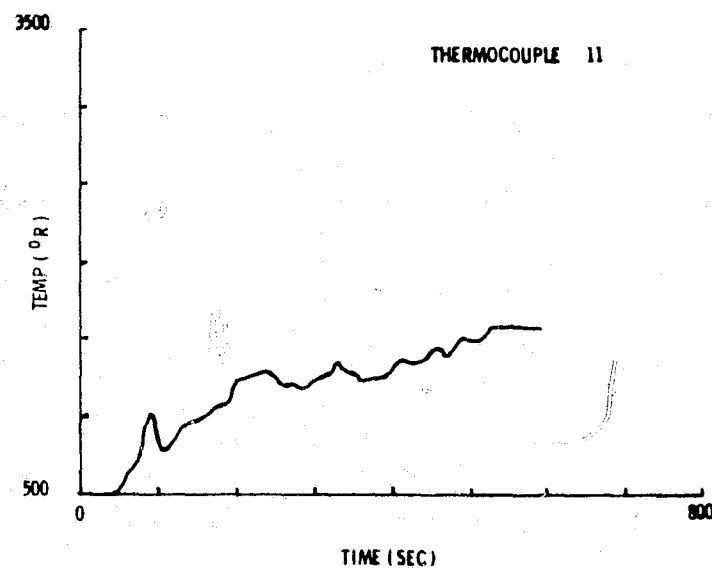
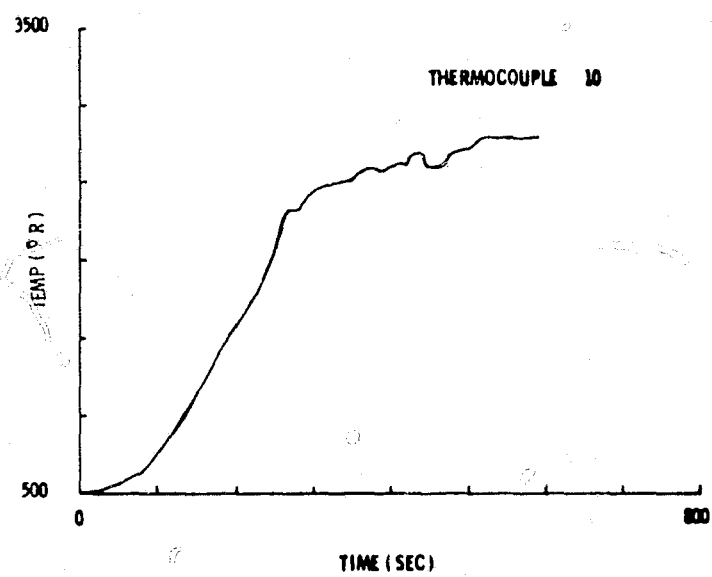
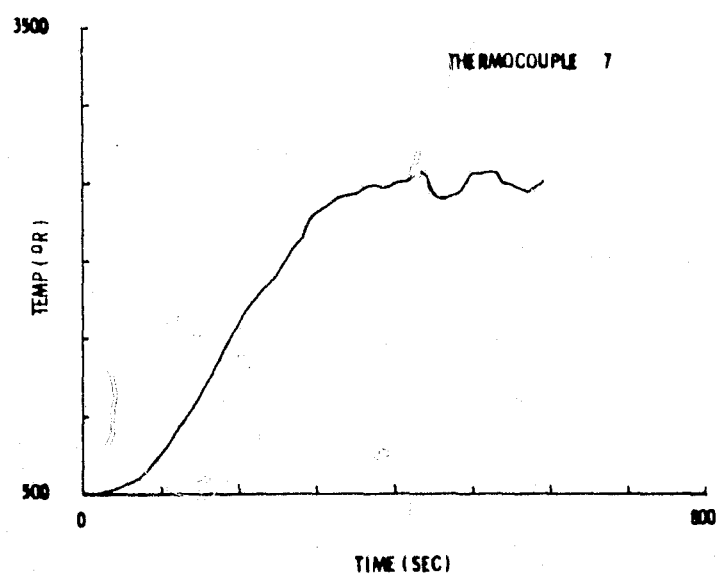
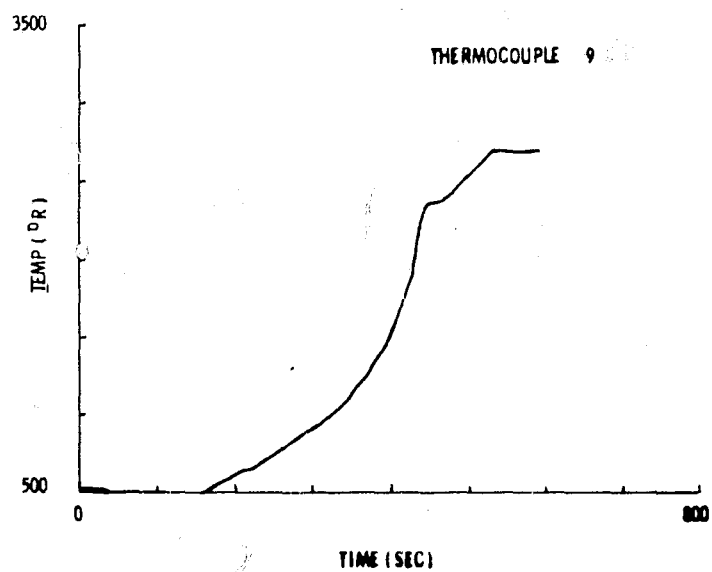
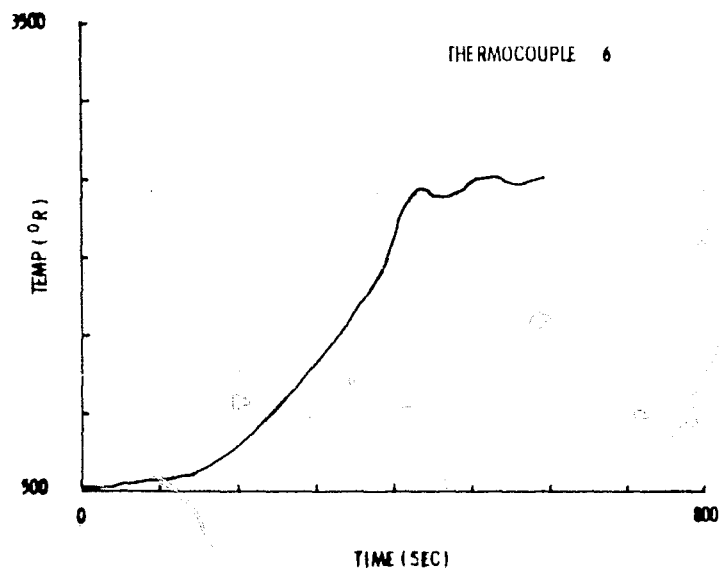


Figure 57b

Temperature Response of Model 11

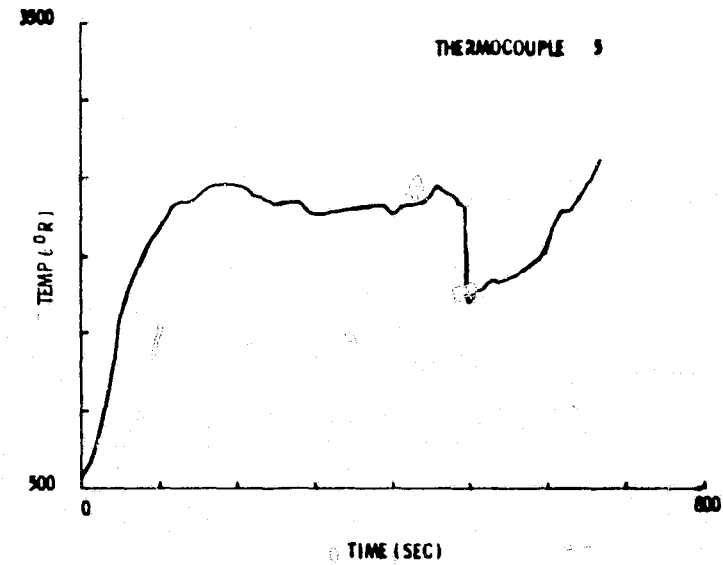
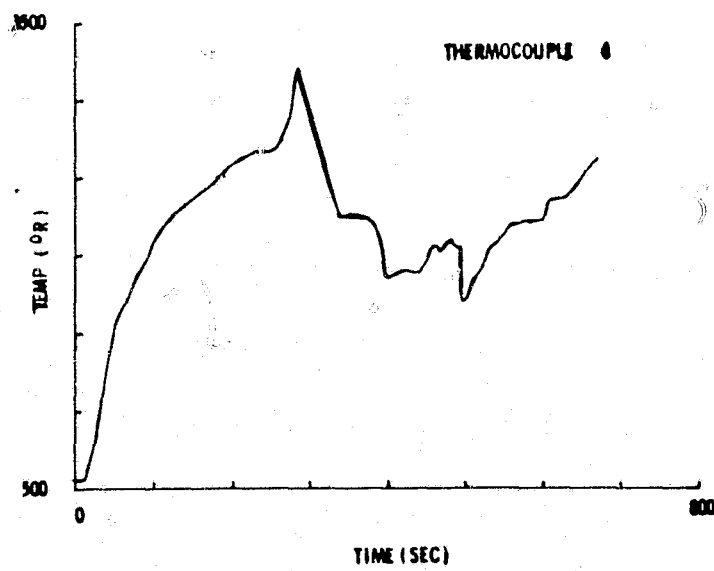
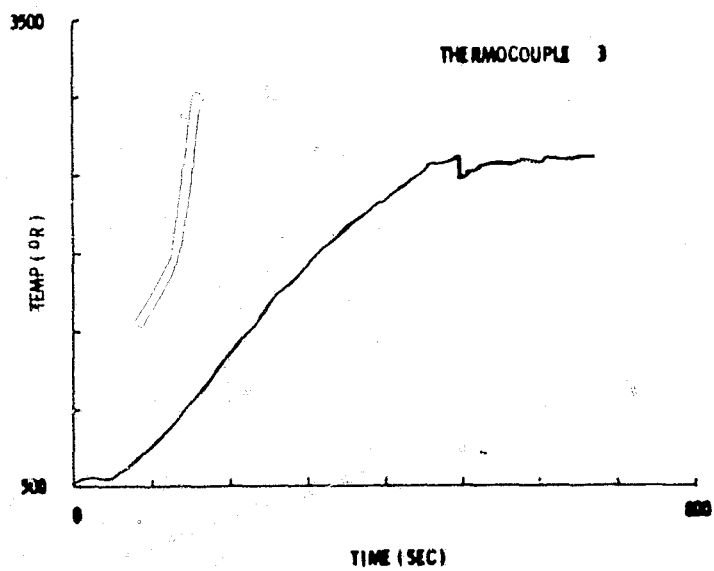
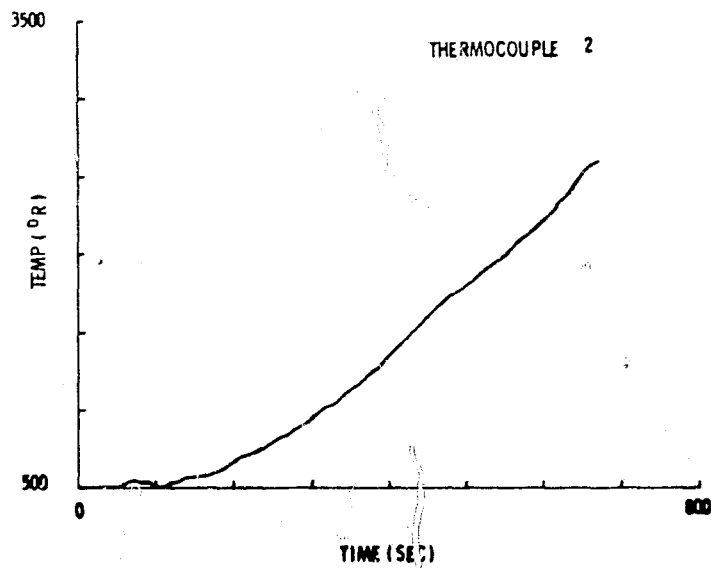
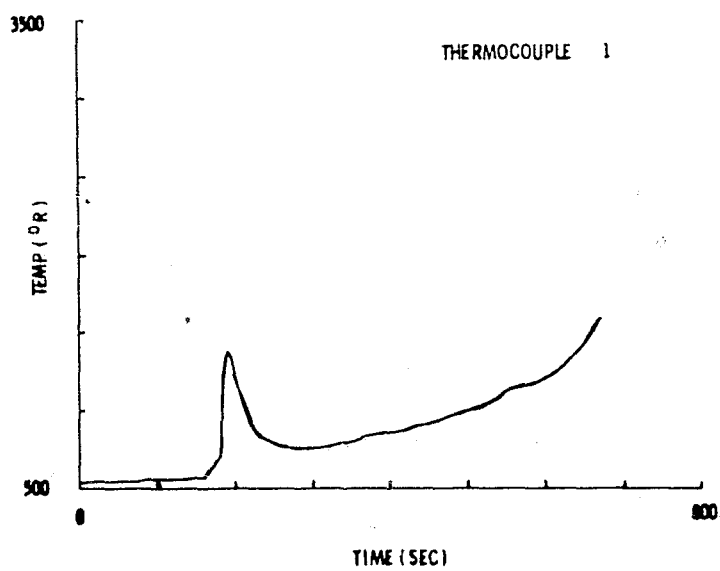


Figure 58a

Temperature Response of Model 12

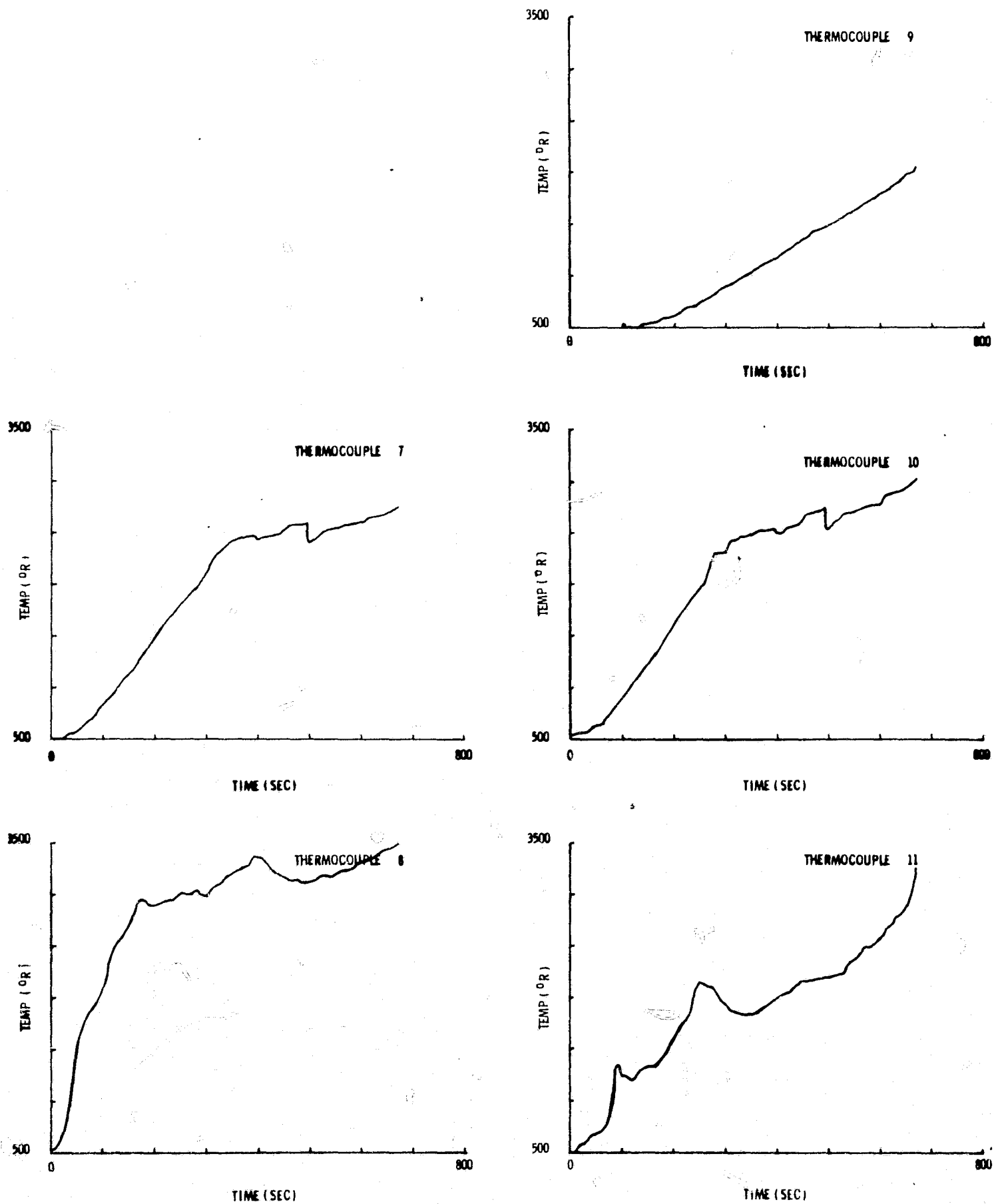


Figure 58b

Temperature Response of Model 12

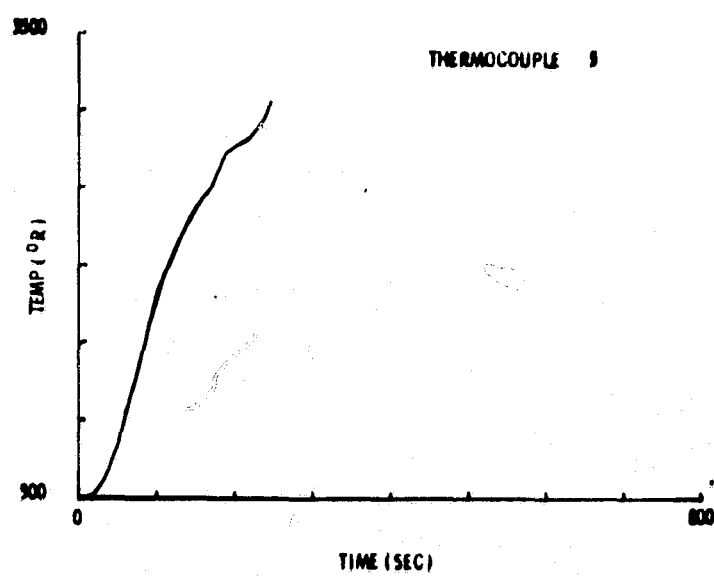
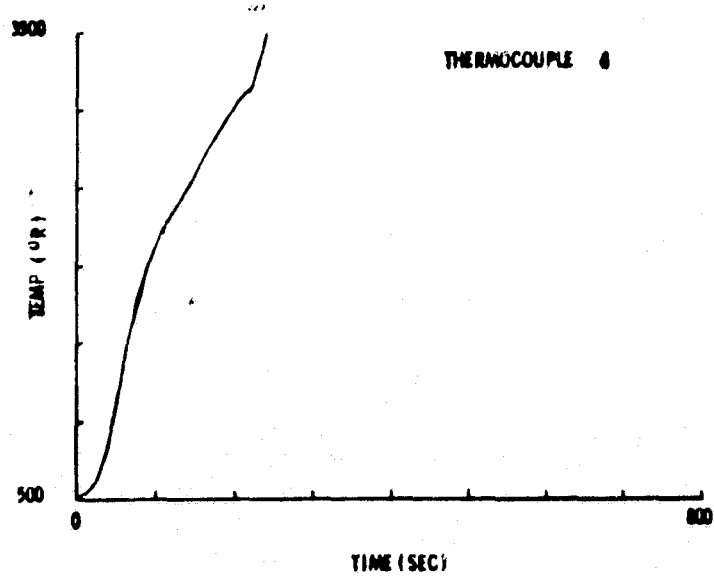
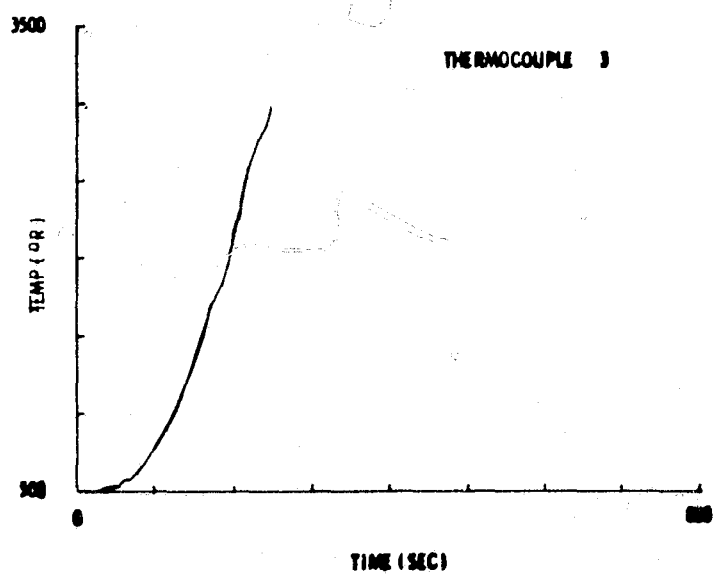


Figure 59a

Temperature Response of Model 13

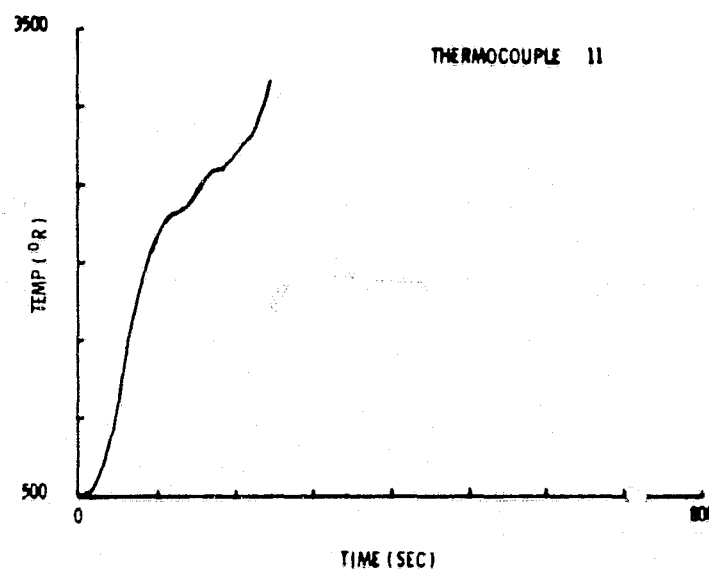
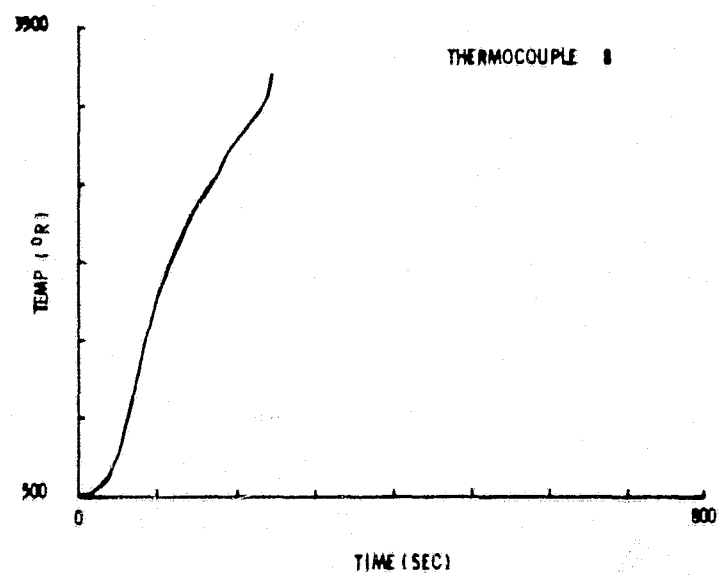
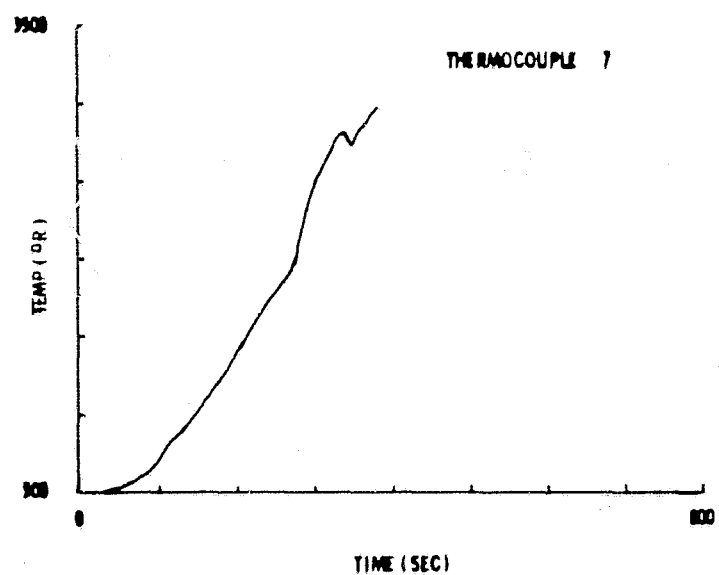
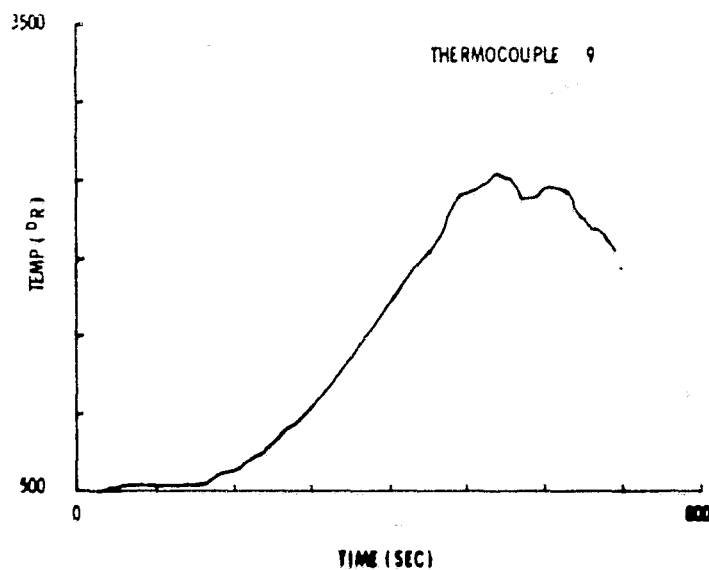
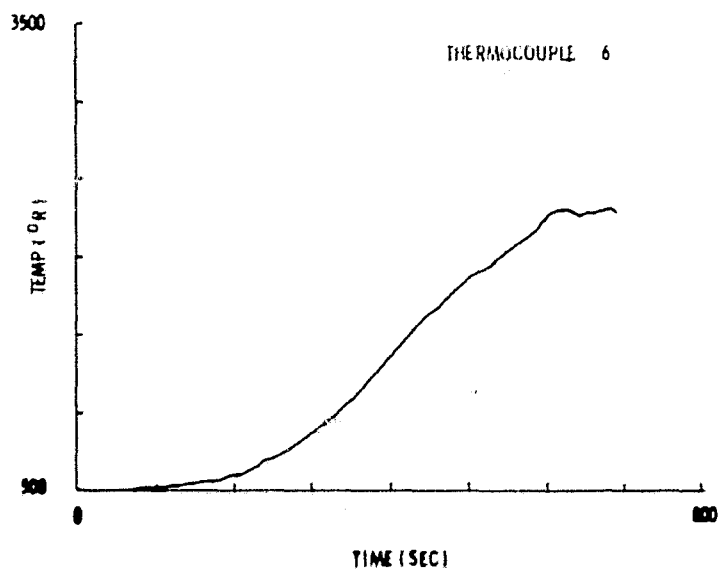


Figure 59b

Temperature Response of Model 13

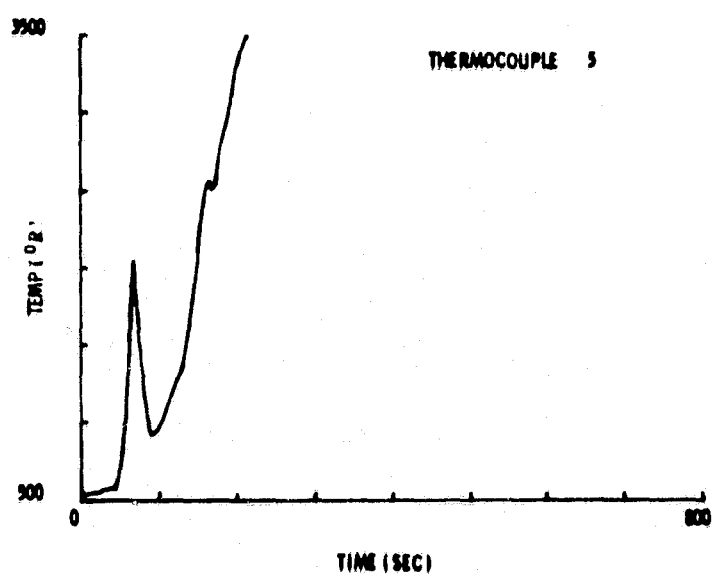
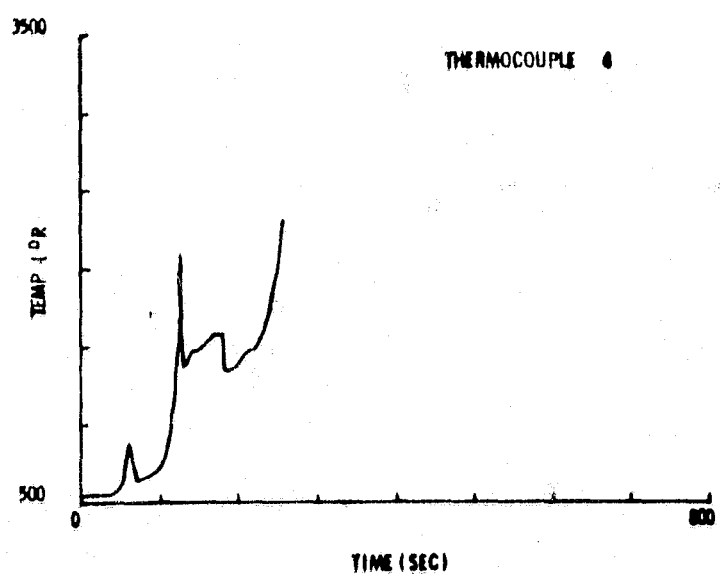
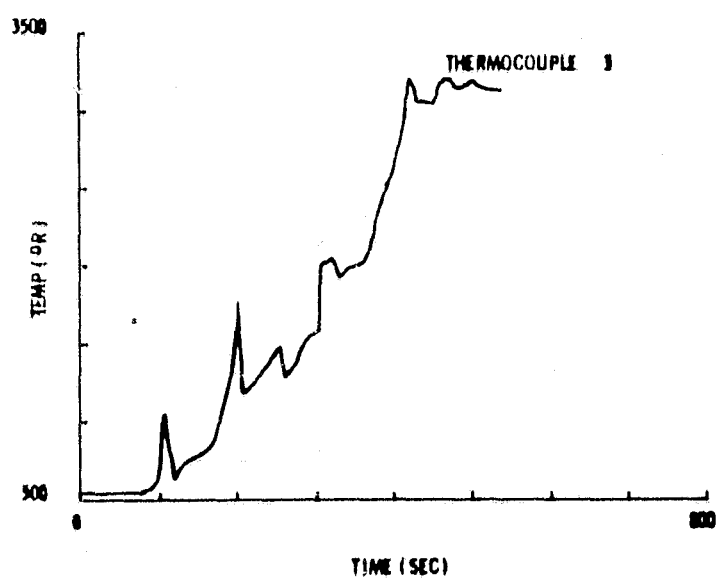
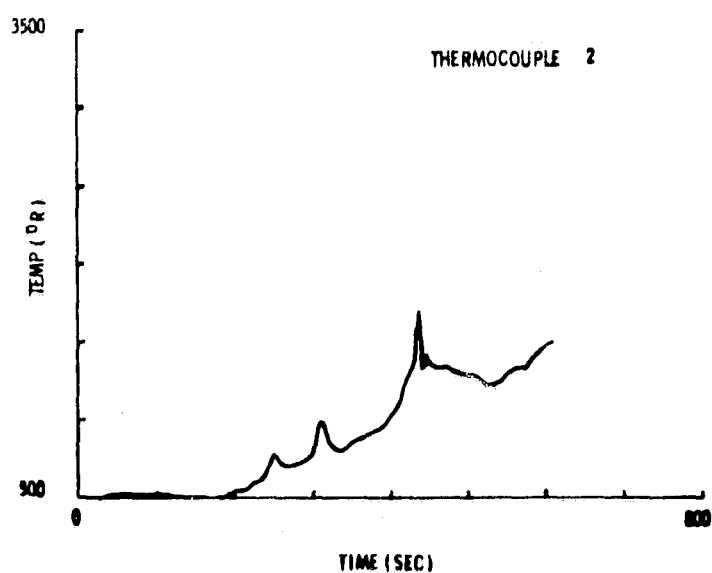
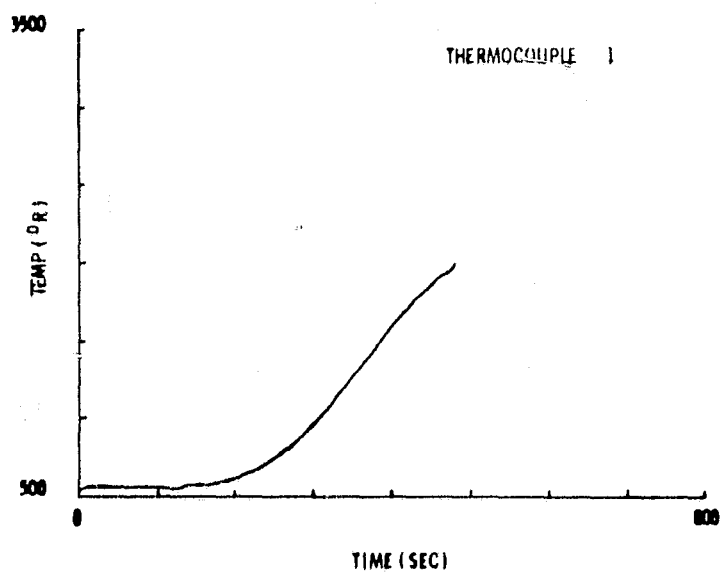


Figure 60a

Temperature Response of Model 14

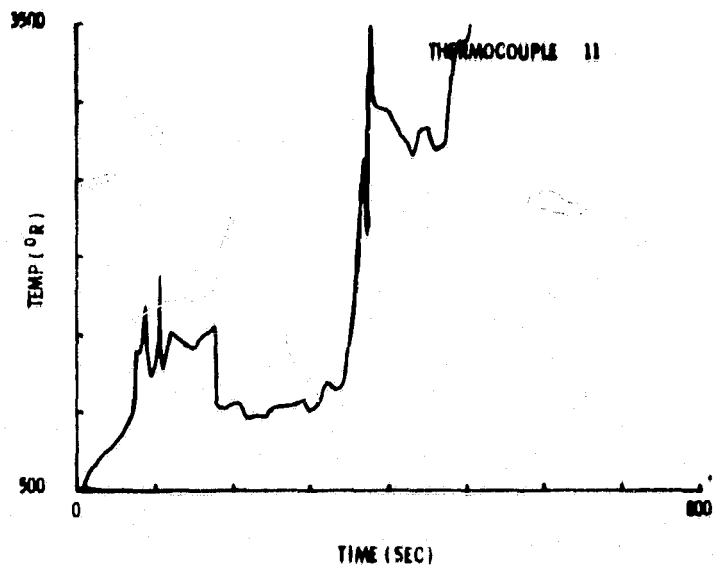
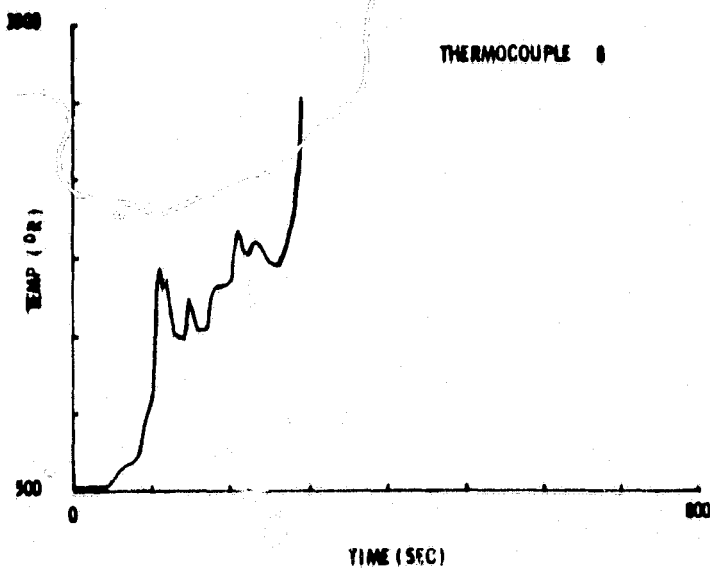
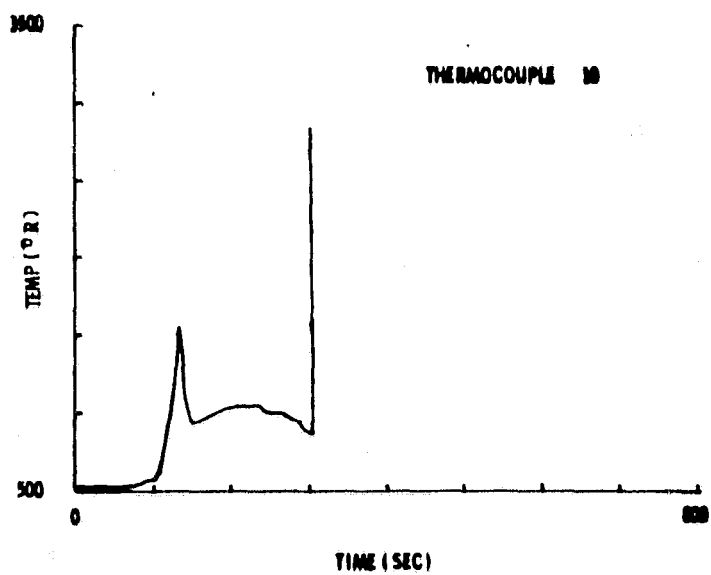
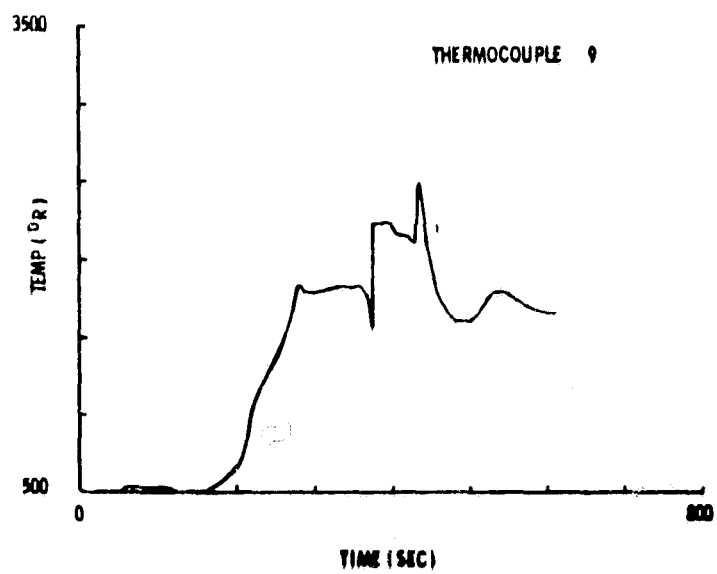
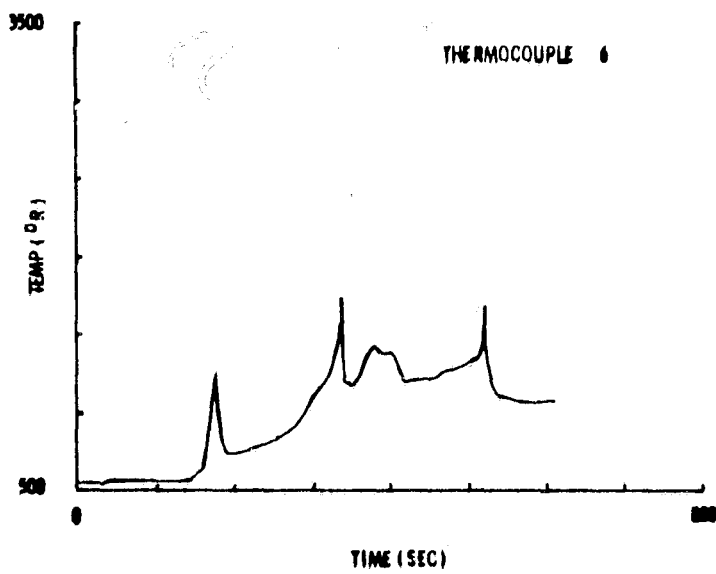


Figure 60b

Temperature Response of Model 14

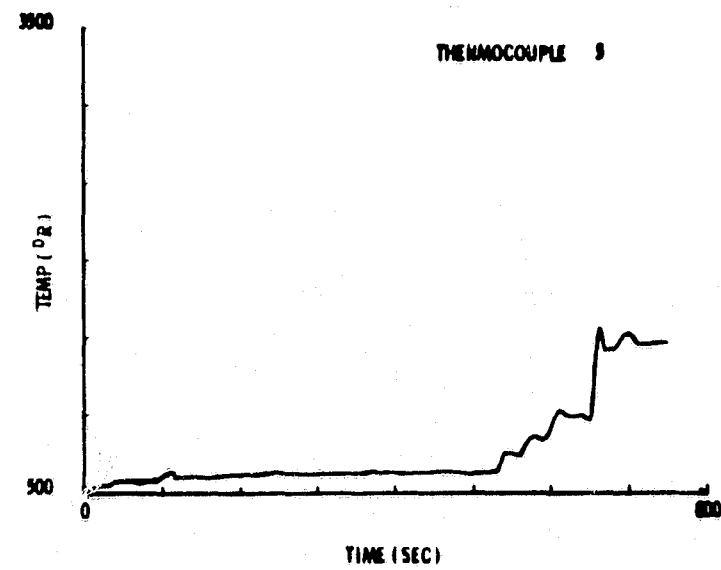
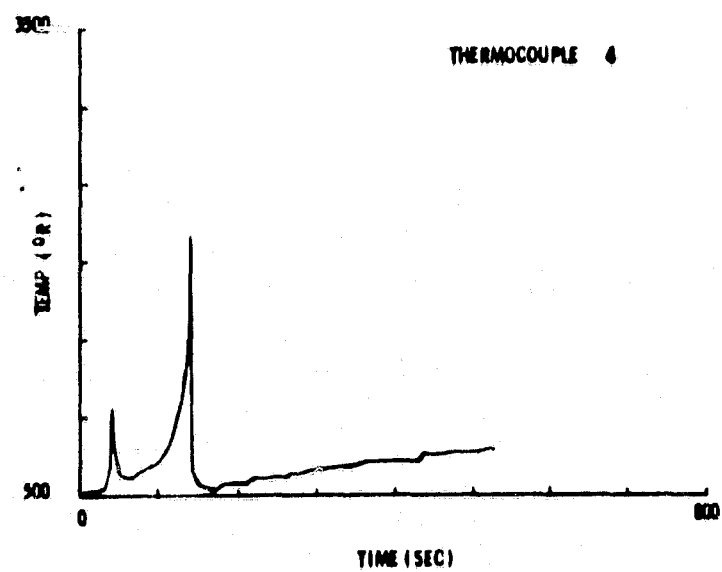
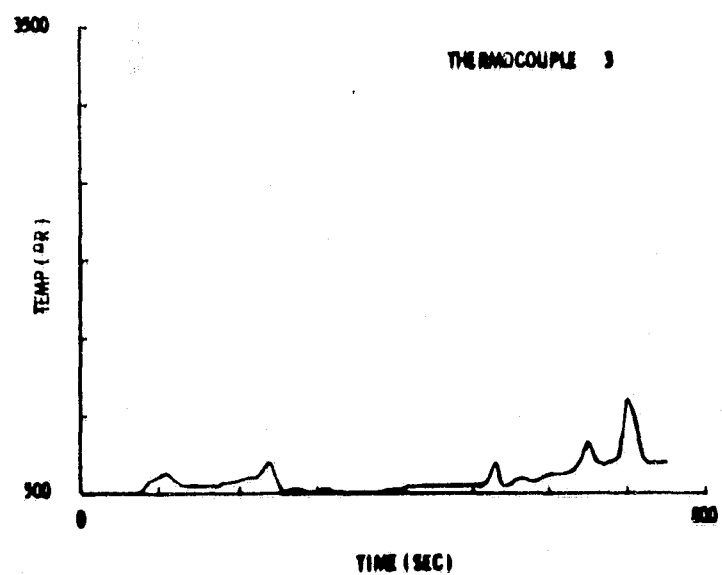
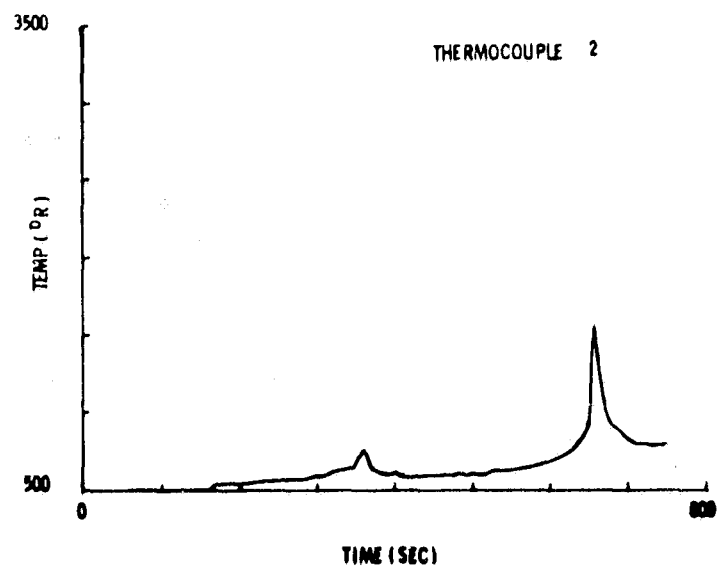
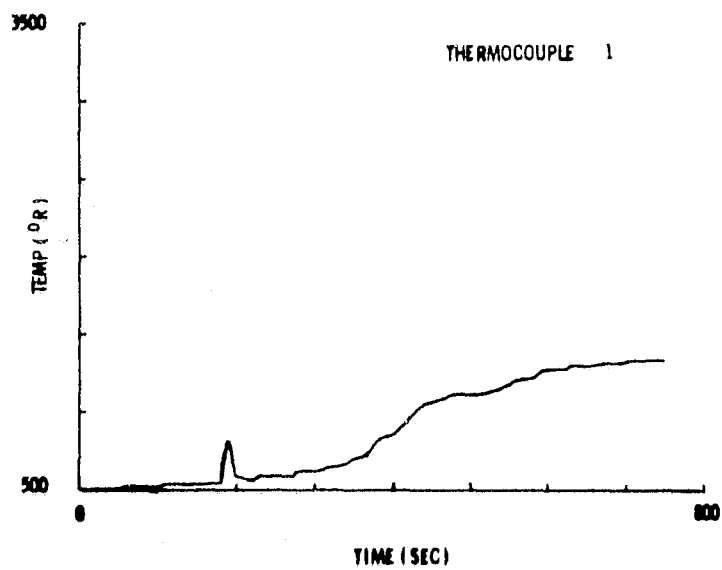


Figure 61a

Temperature Response of Model 15

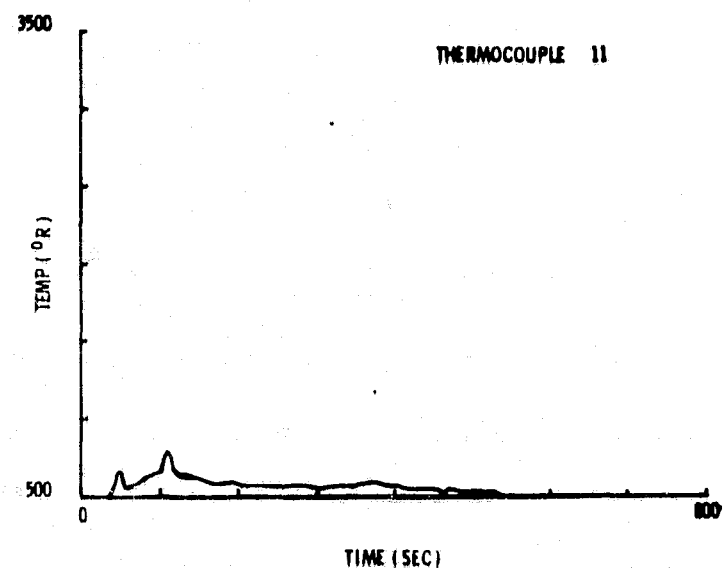
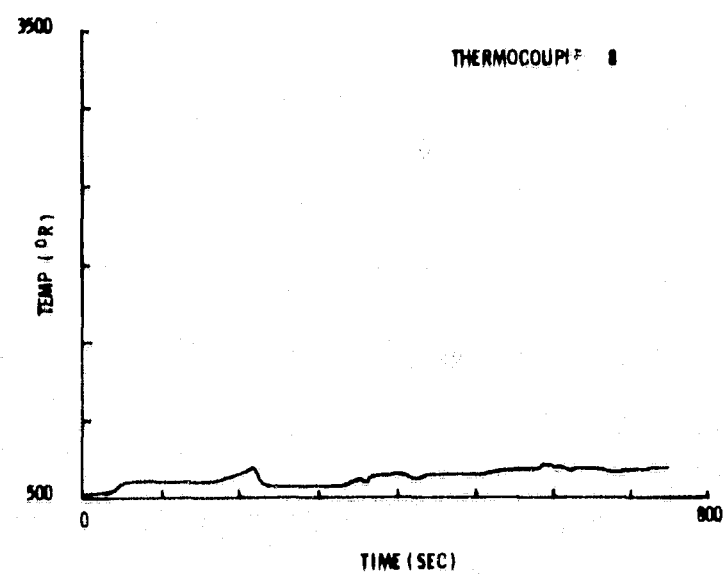
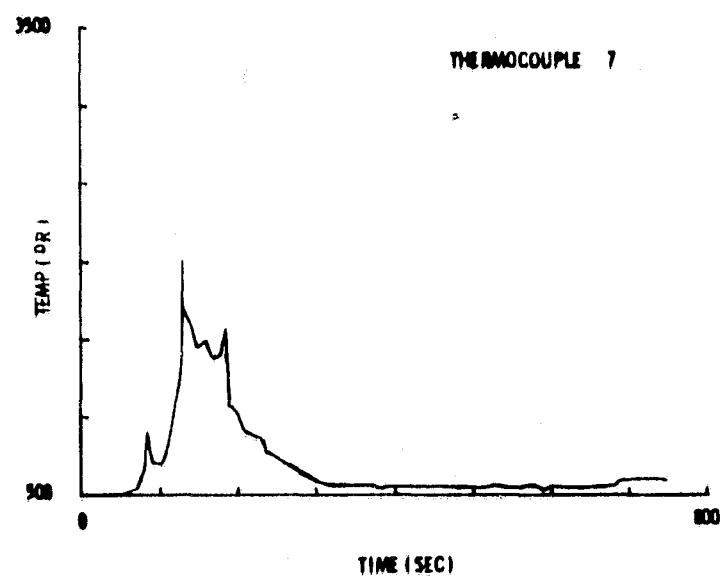
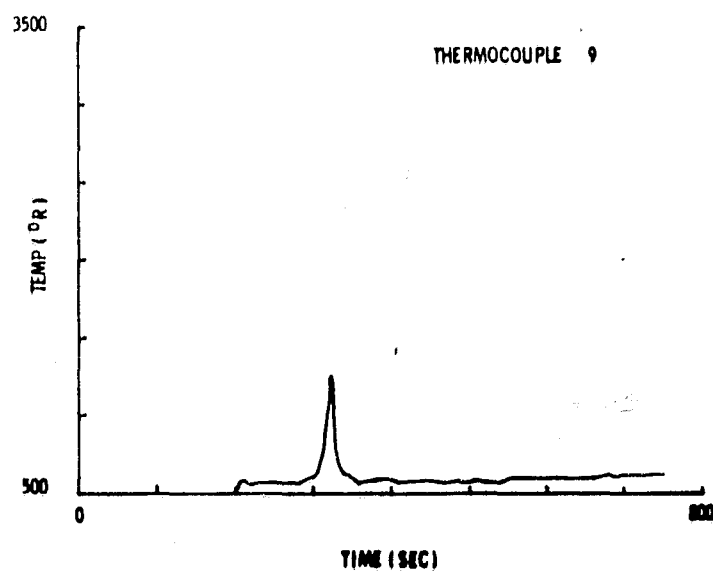
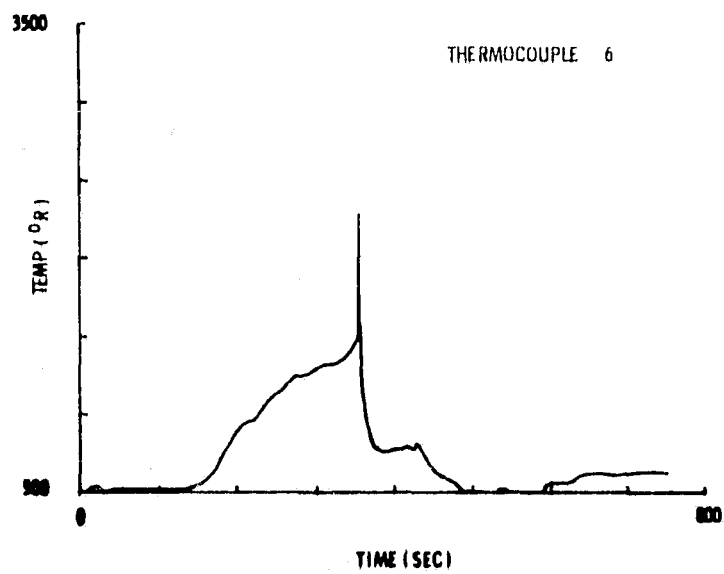


Figure 61b

Temperature Response of Model 15

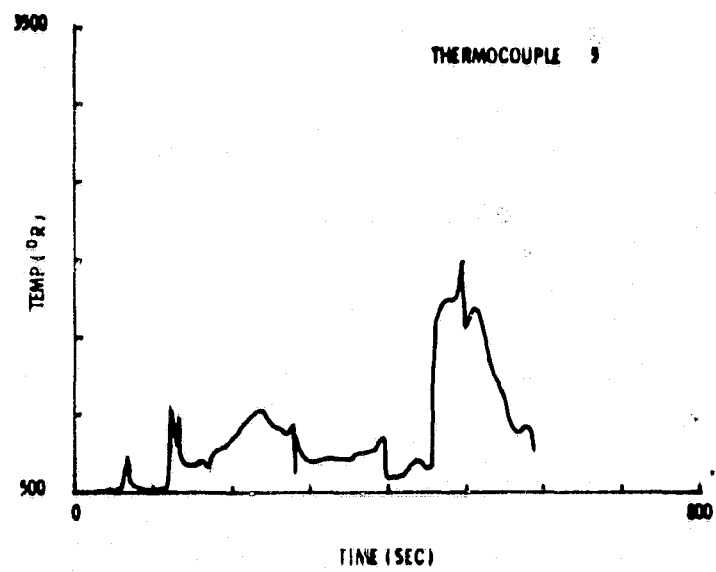
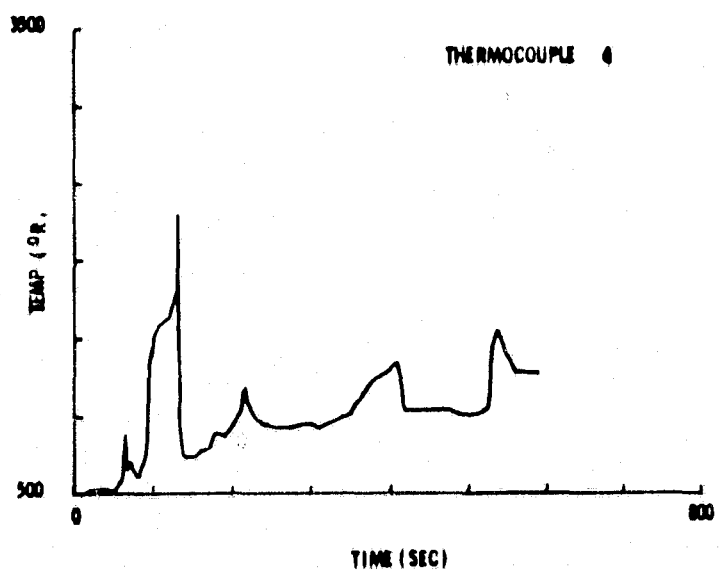
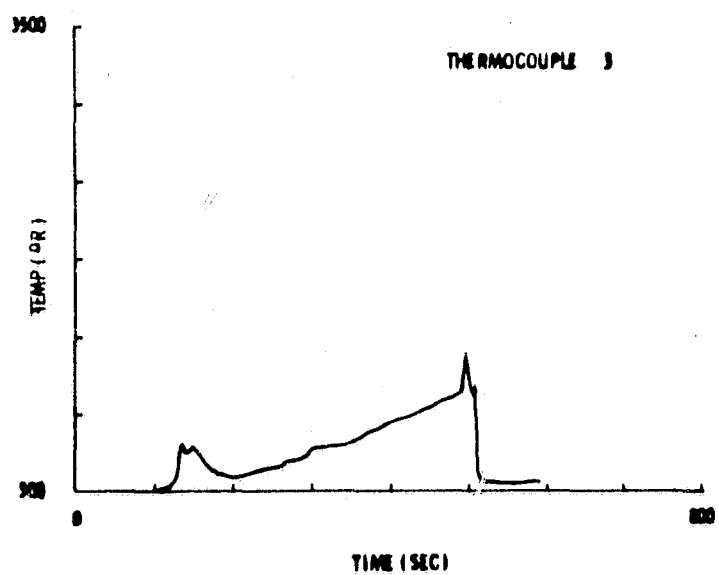
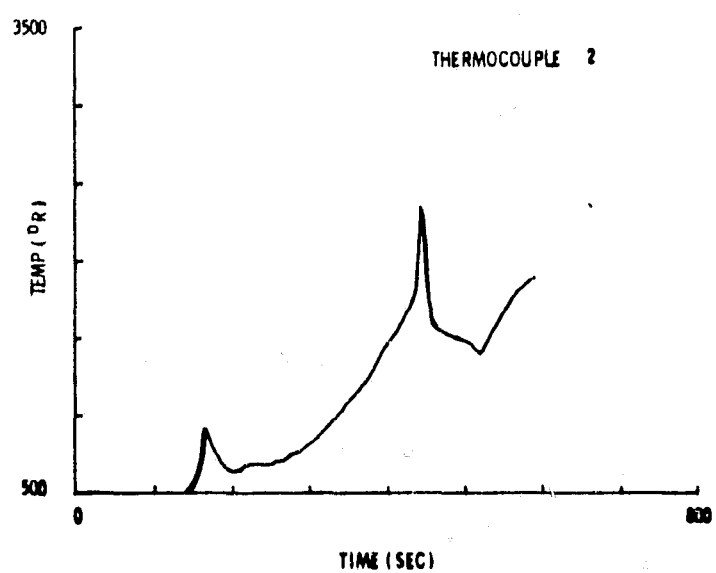
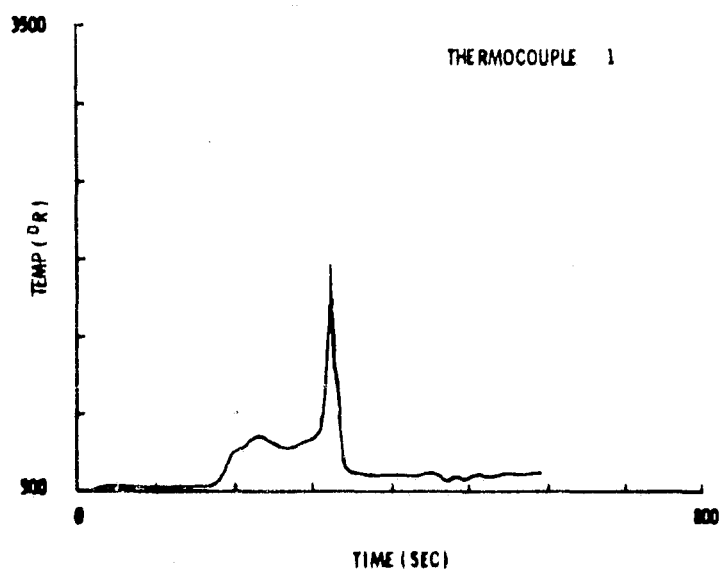


Figure 62a

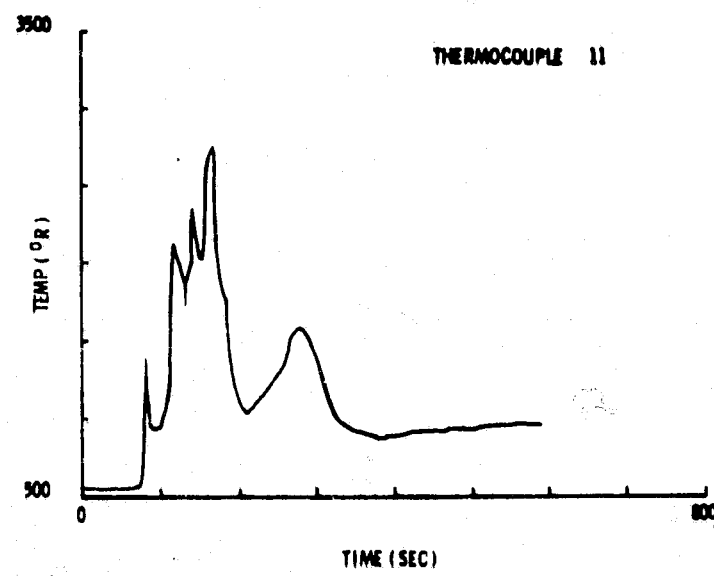
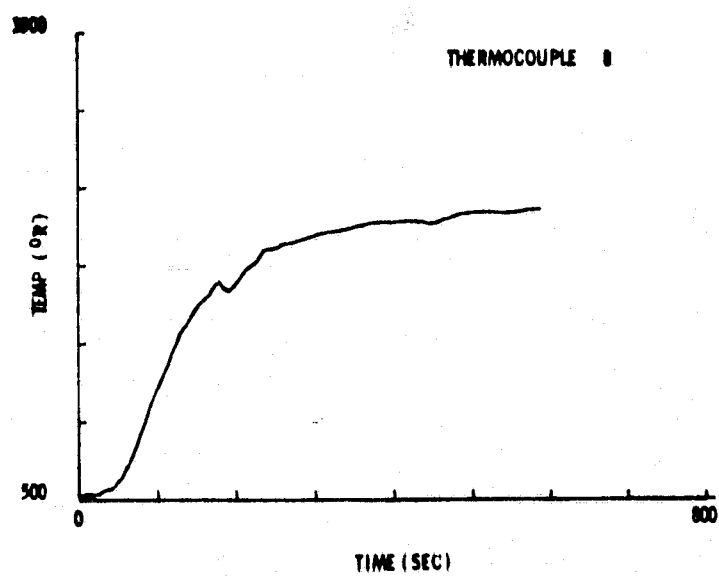
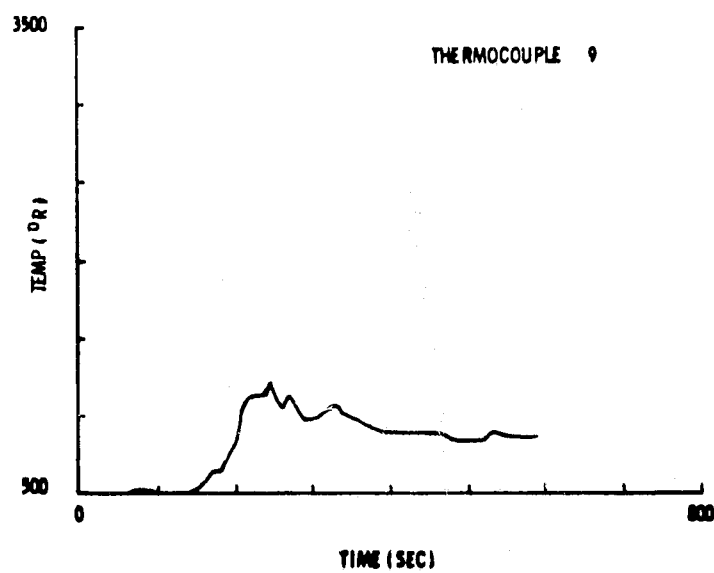
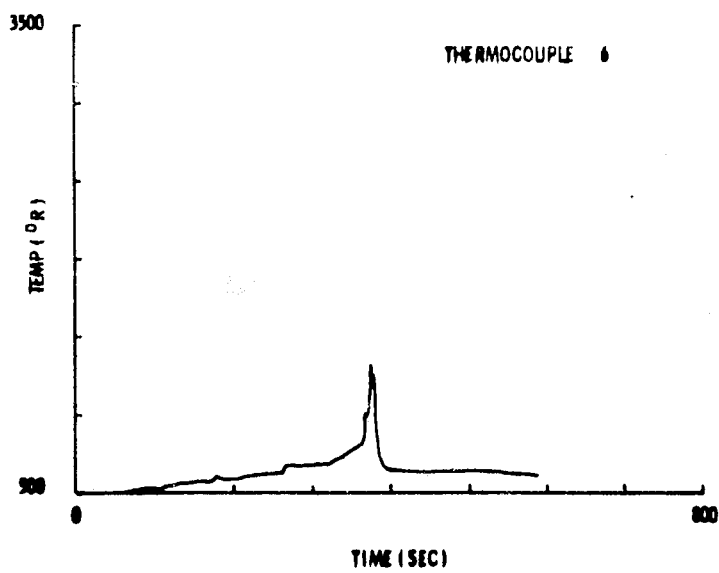


Figure 62b

Temperature Response of Model 16

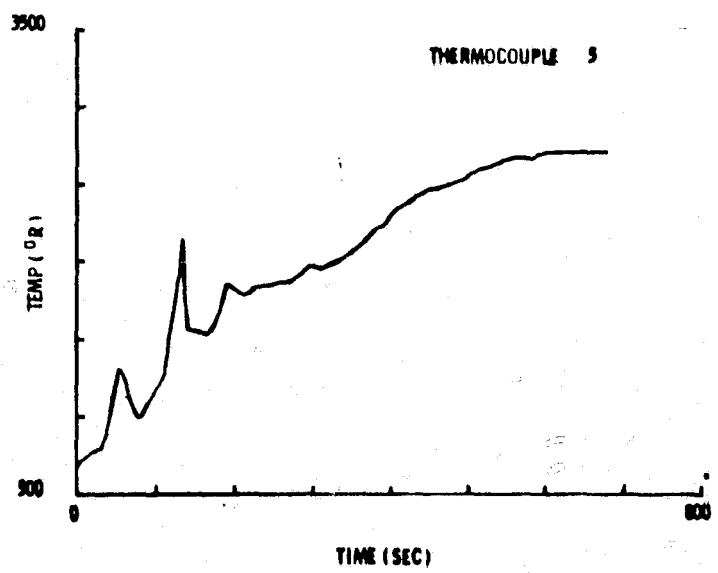
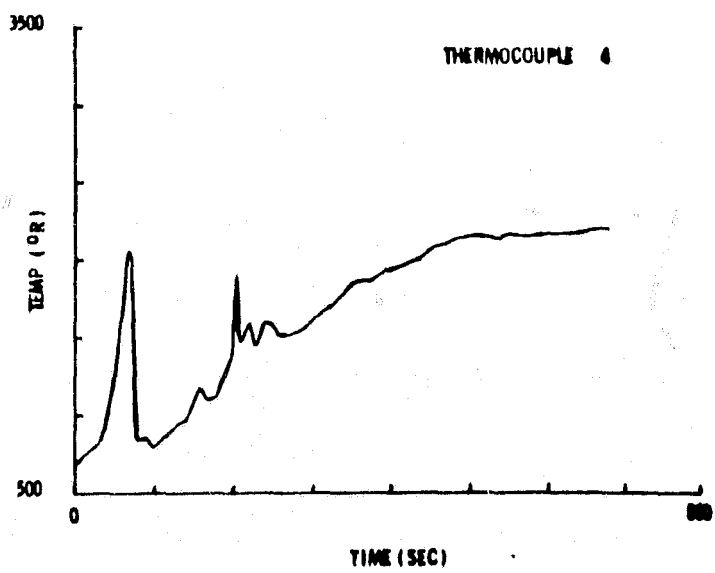
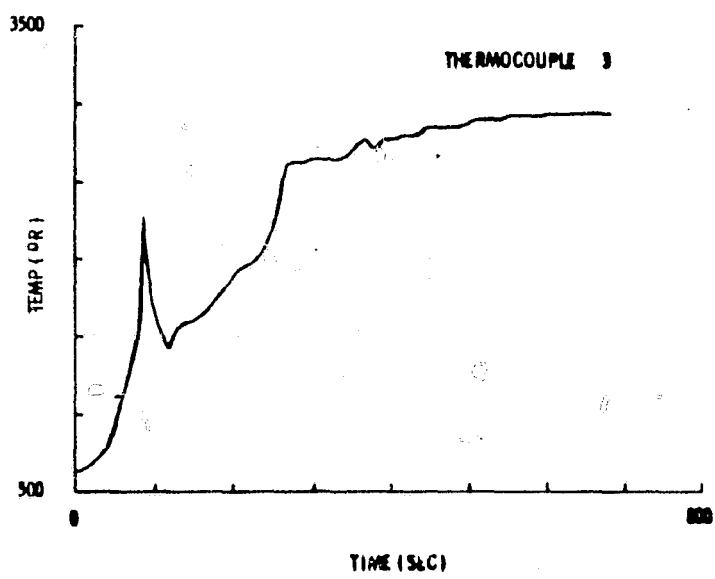
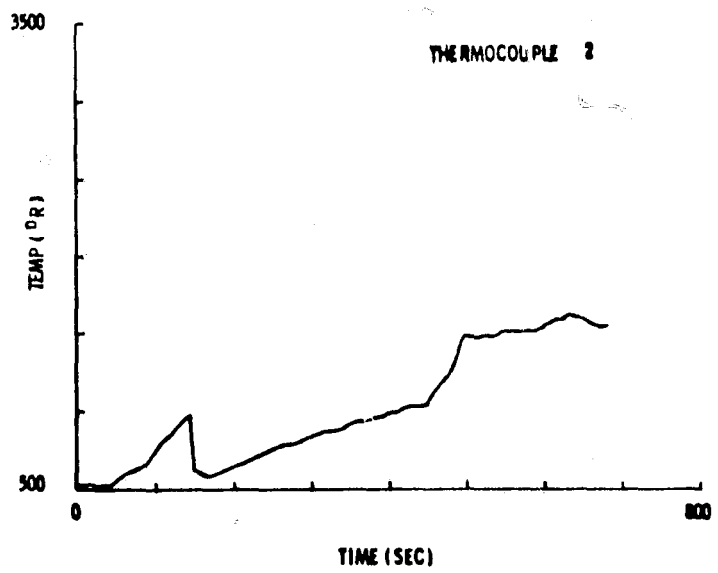


Figure 63a

Temperature Response of Model 17

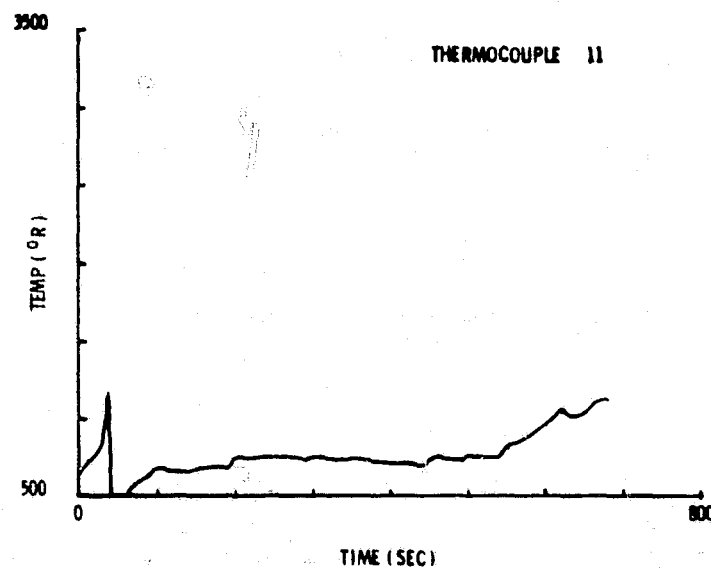
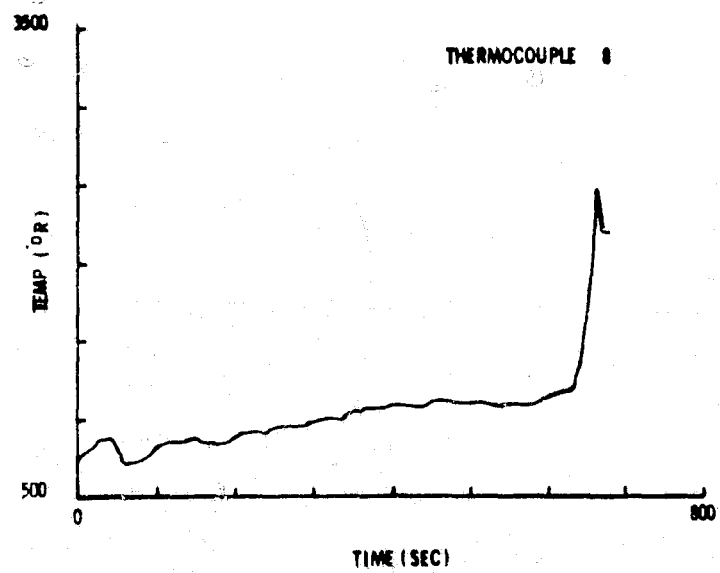
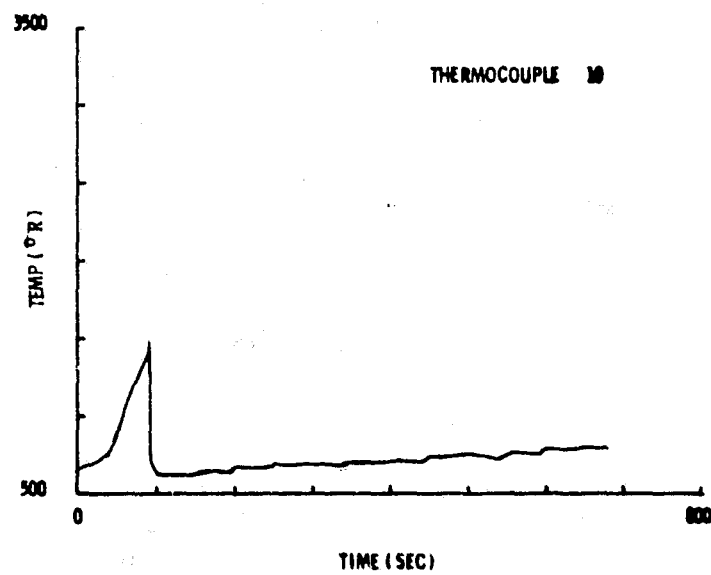
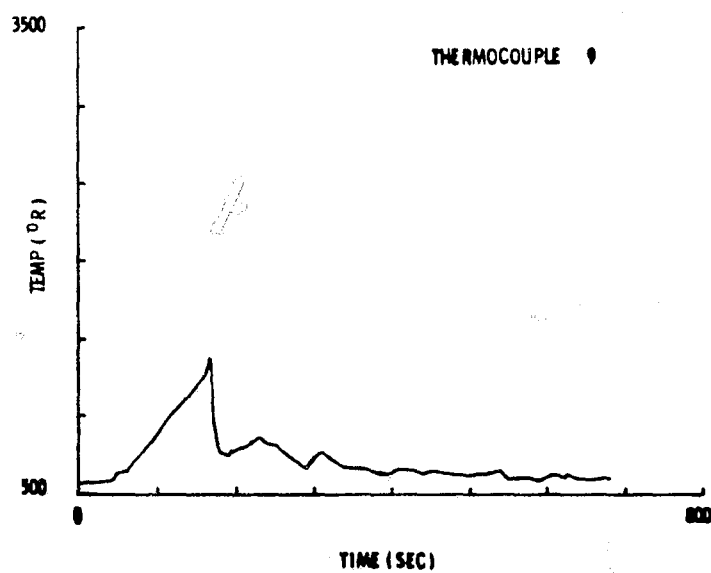
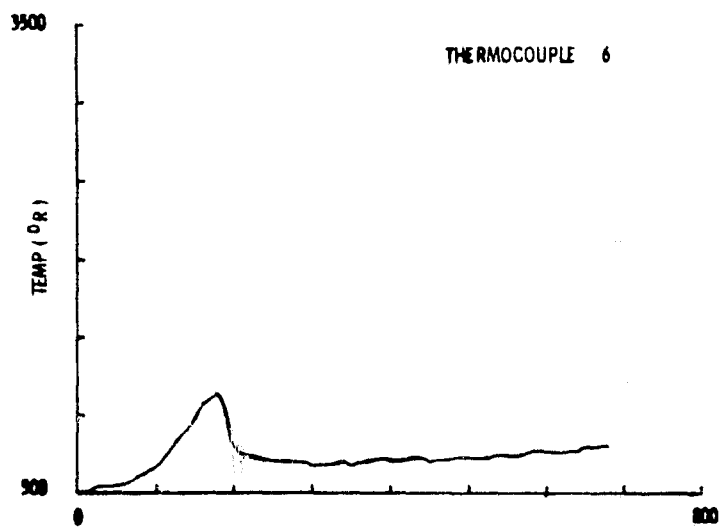


Figure 63b

Temperature Response of Model 17

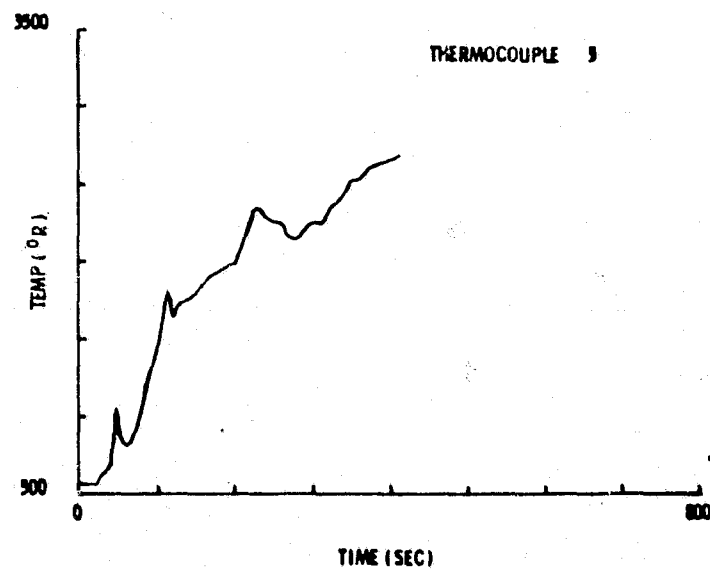
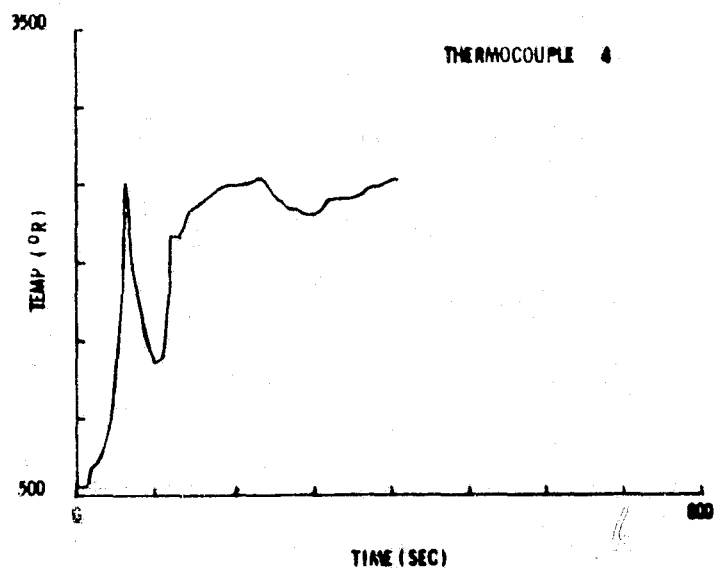
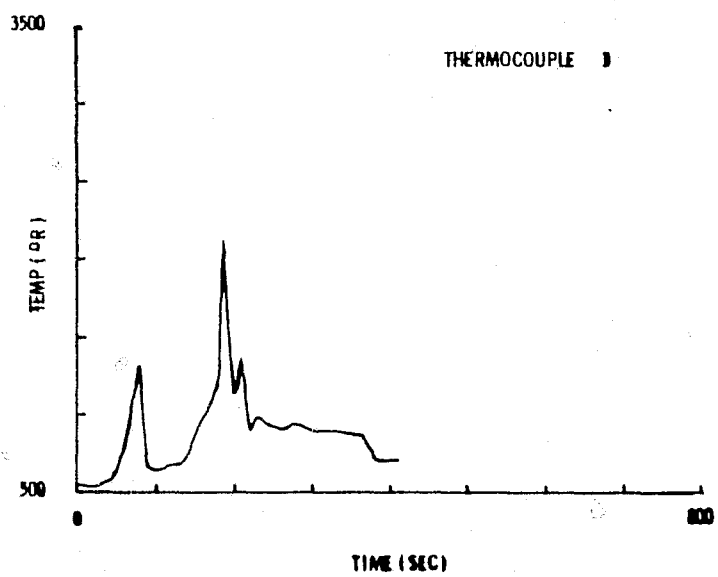
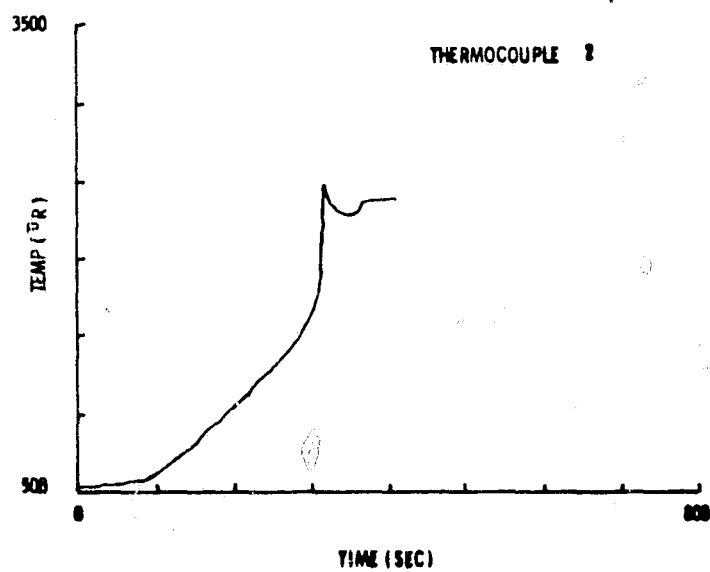
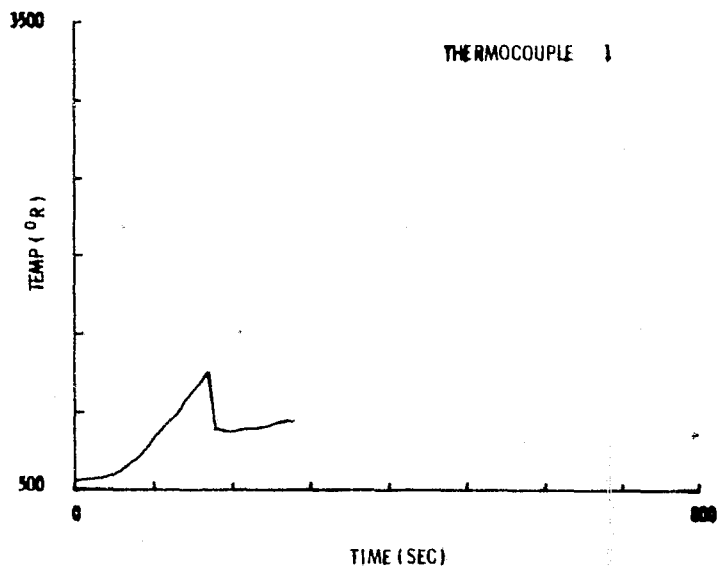


Figure 64a
Temperature Response of Model 18

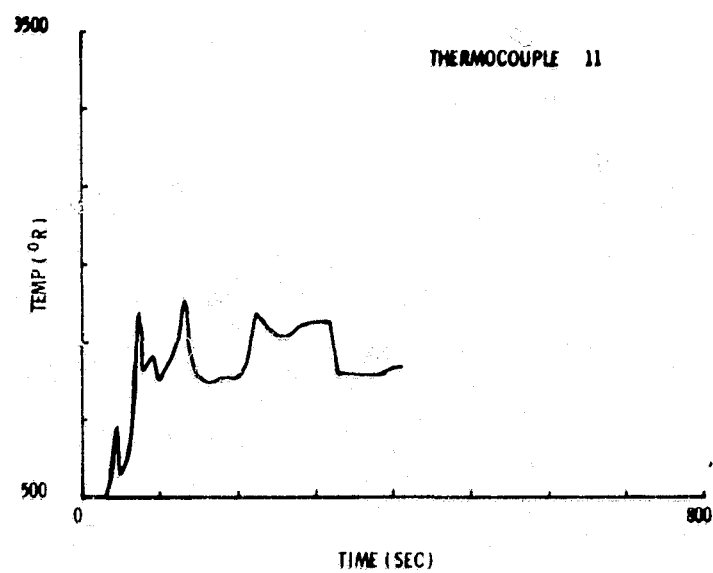
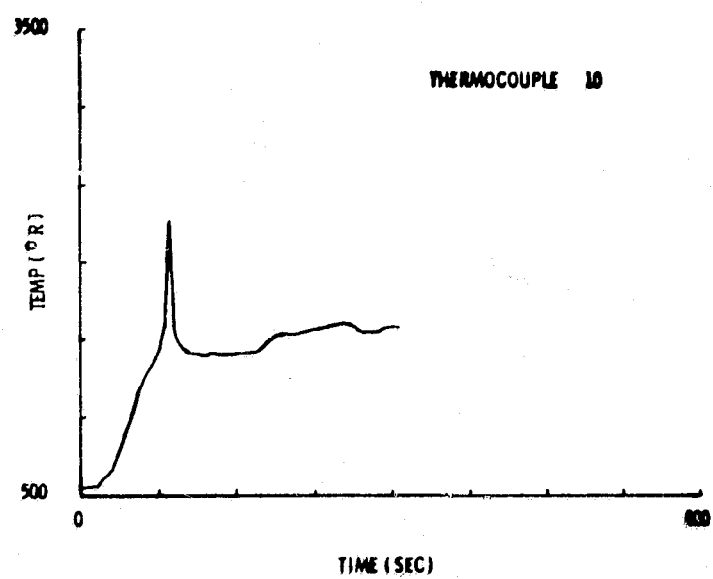
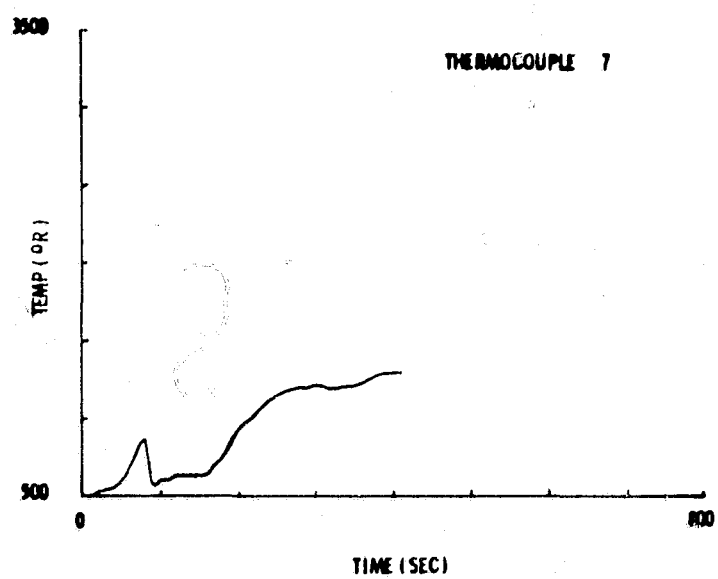
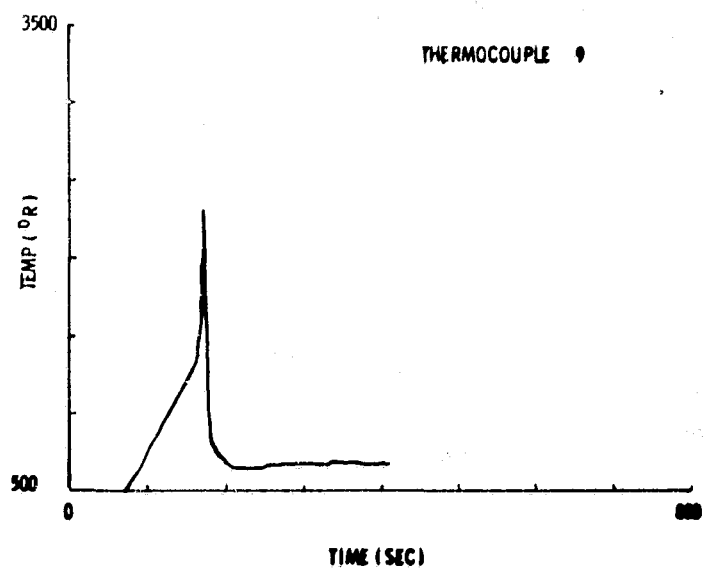


Figure 64b

Temperature Response of Model 18

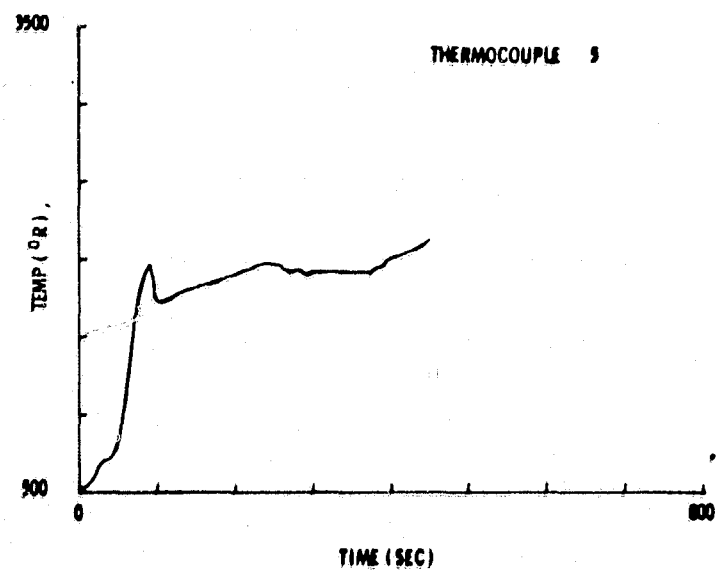
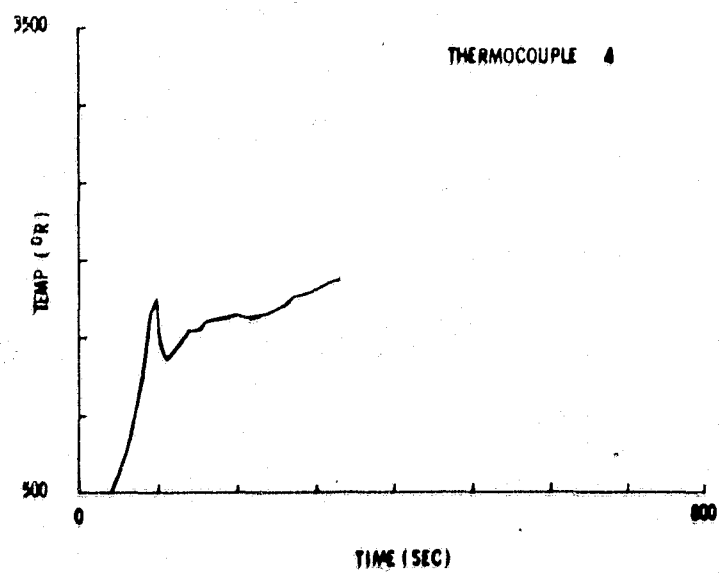
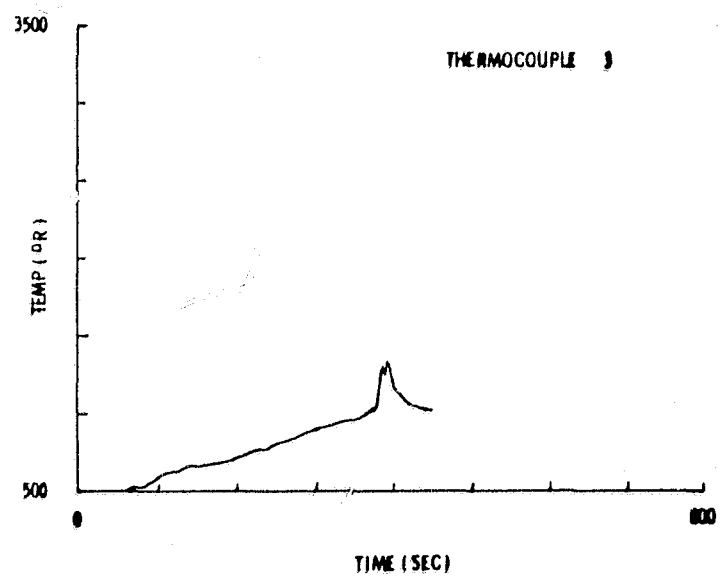
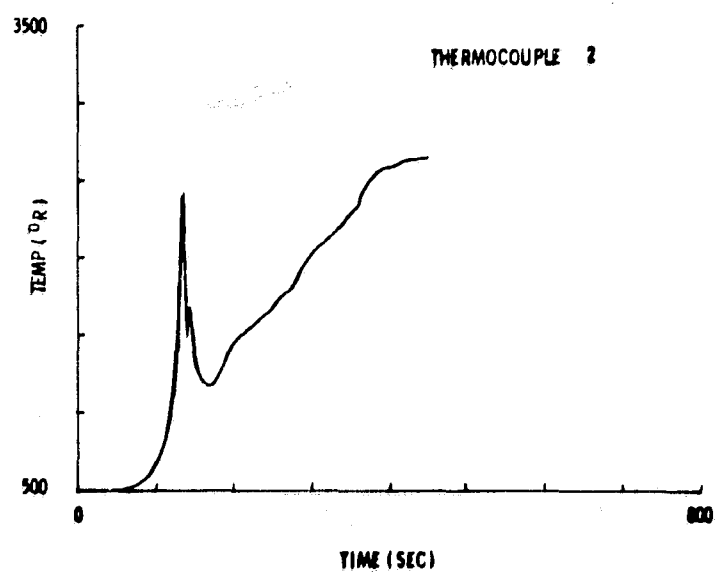
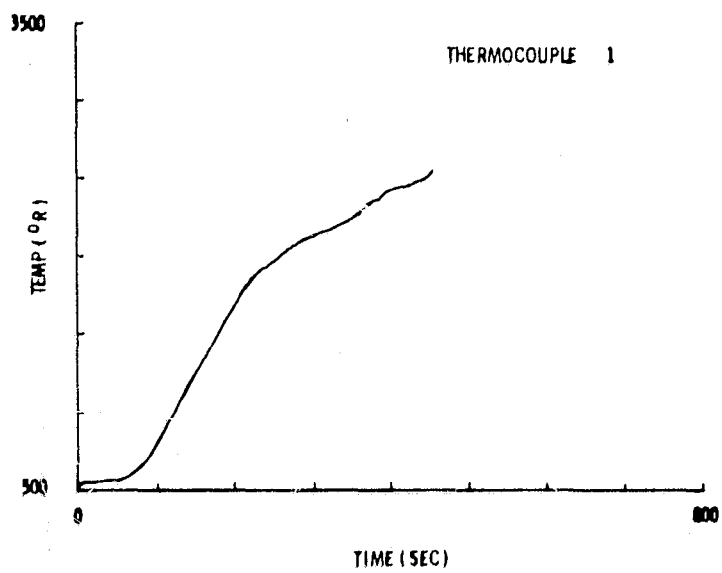


Figure 65a

Temperature Response of Model 19

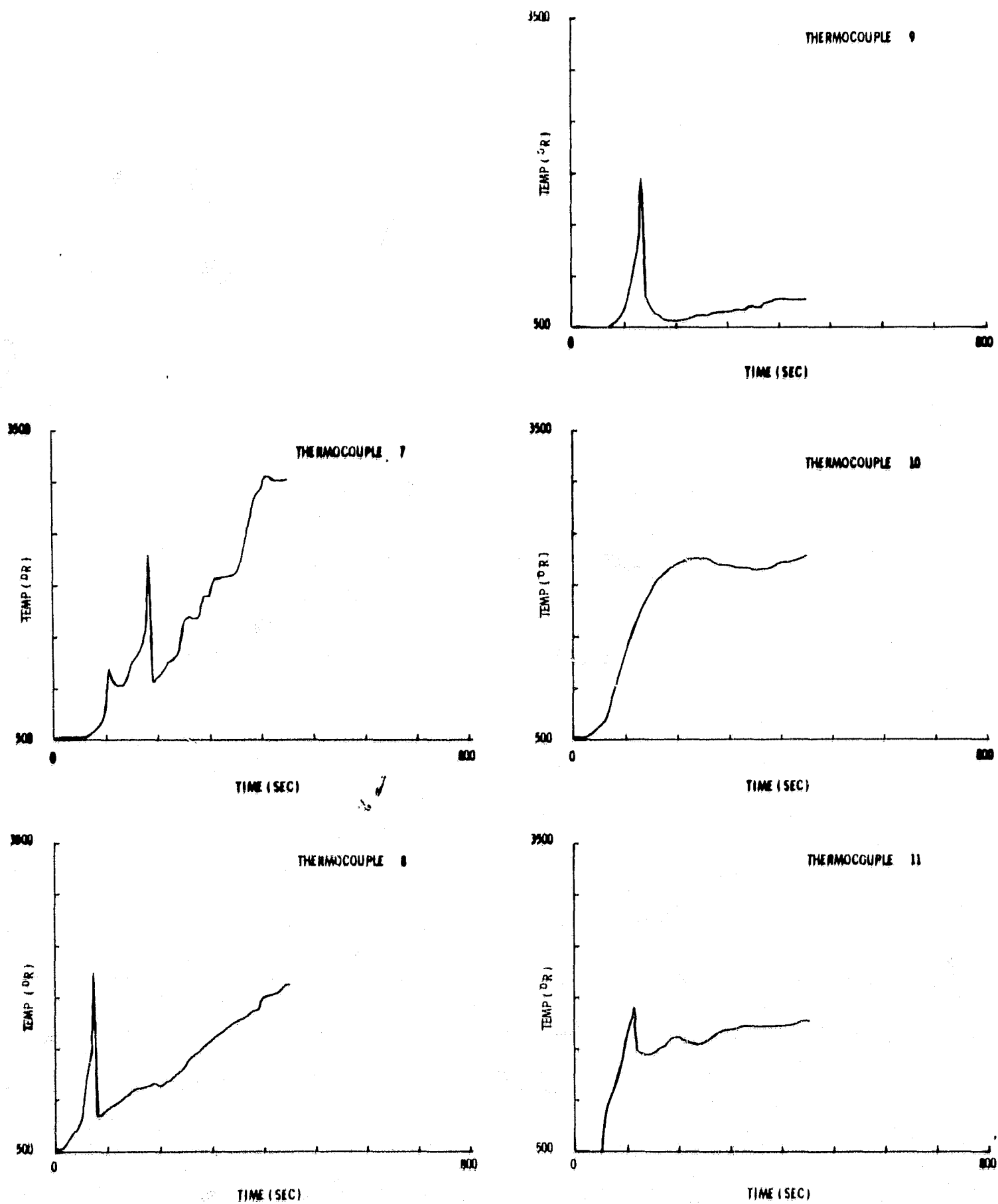


Figure 65b

Temperature Response of Model 19

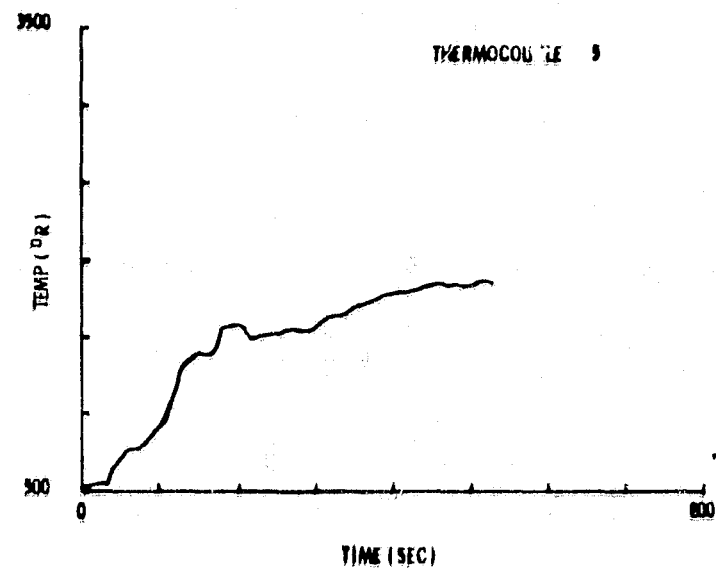
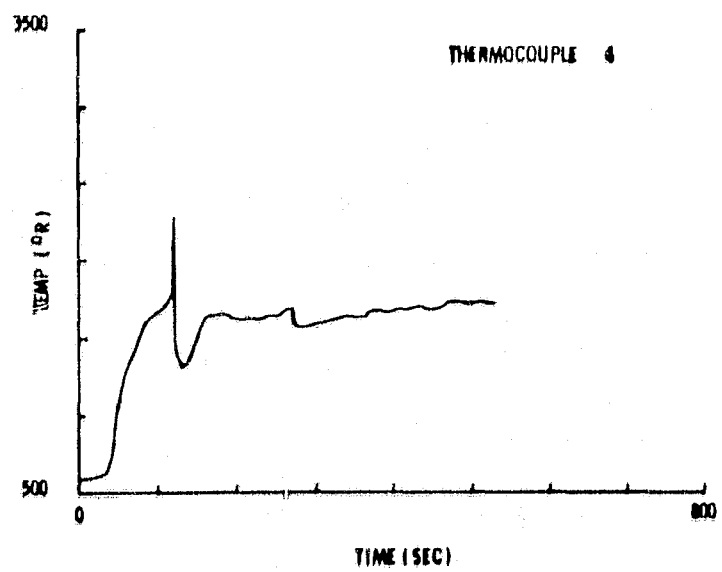
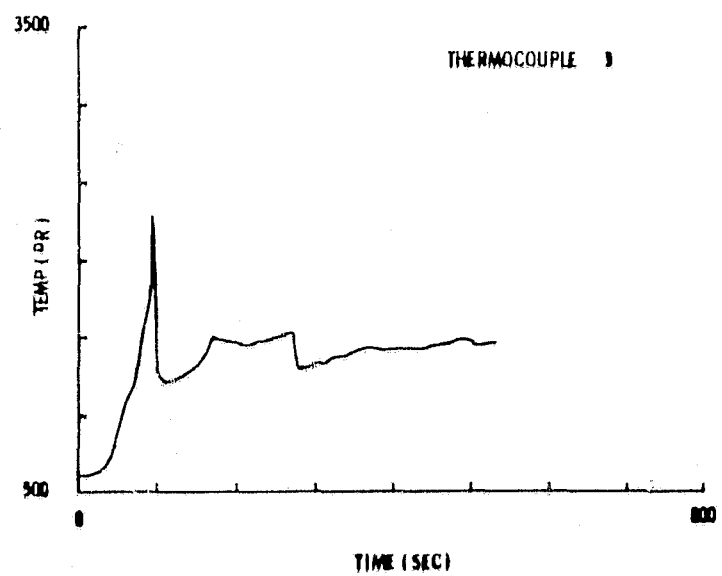
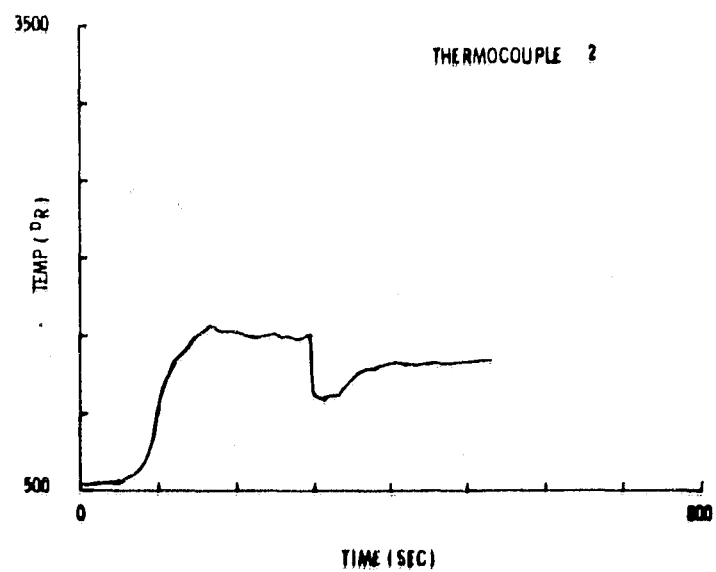
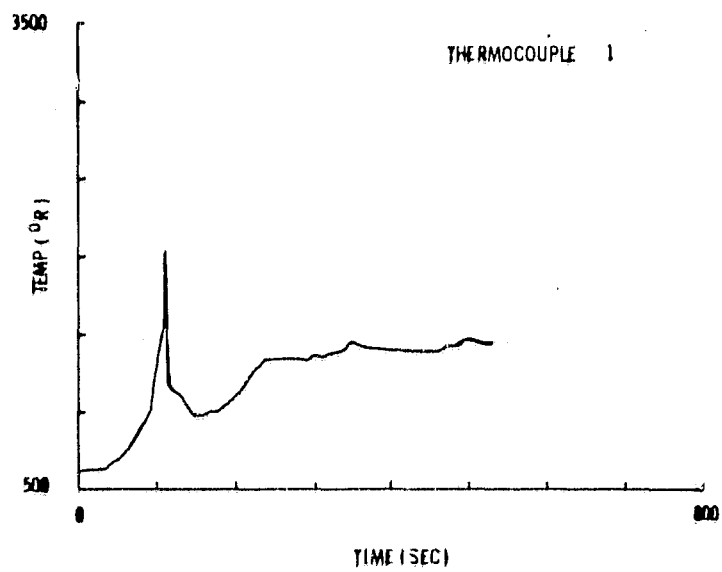


Figure 66a

Temperature Response of Model 20

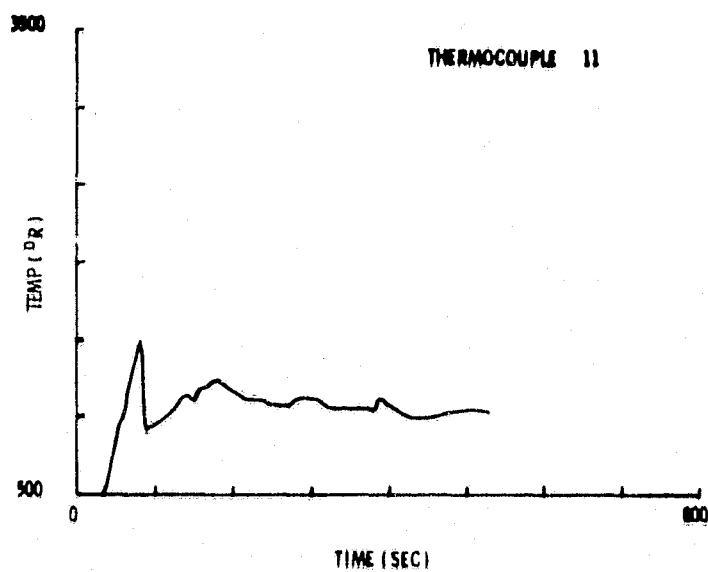
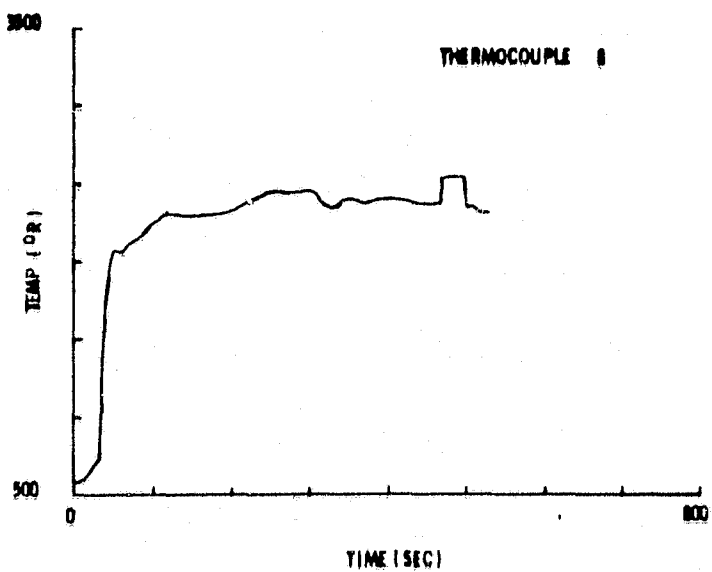
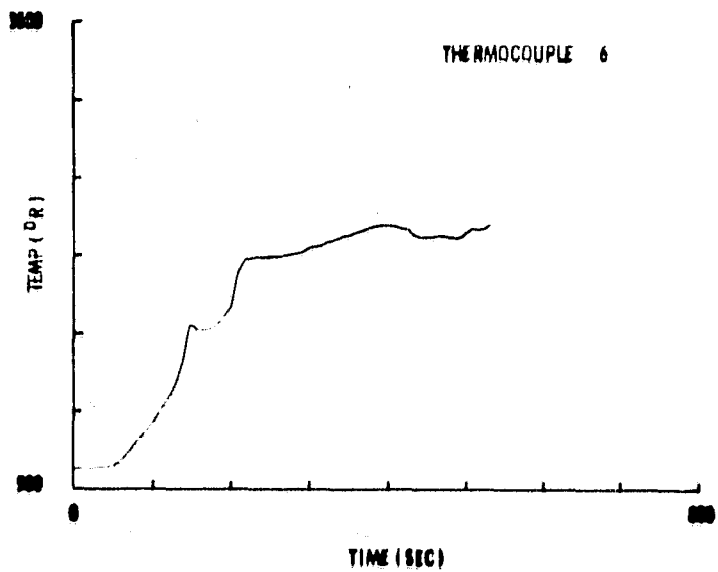


Figure 66b

Temperature Response of Model 20

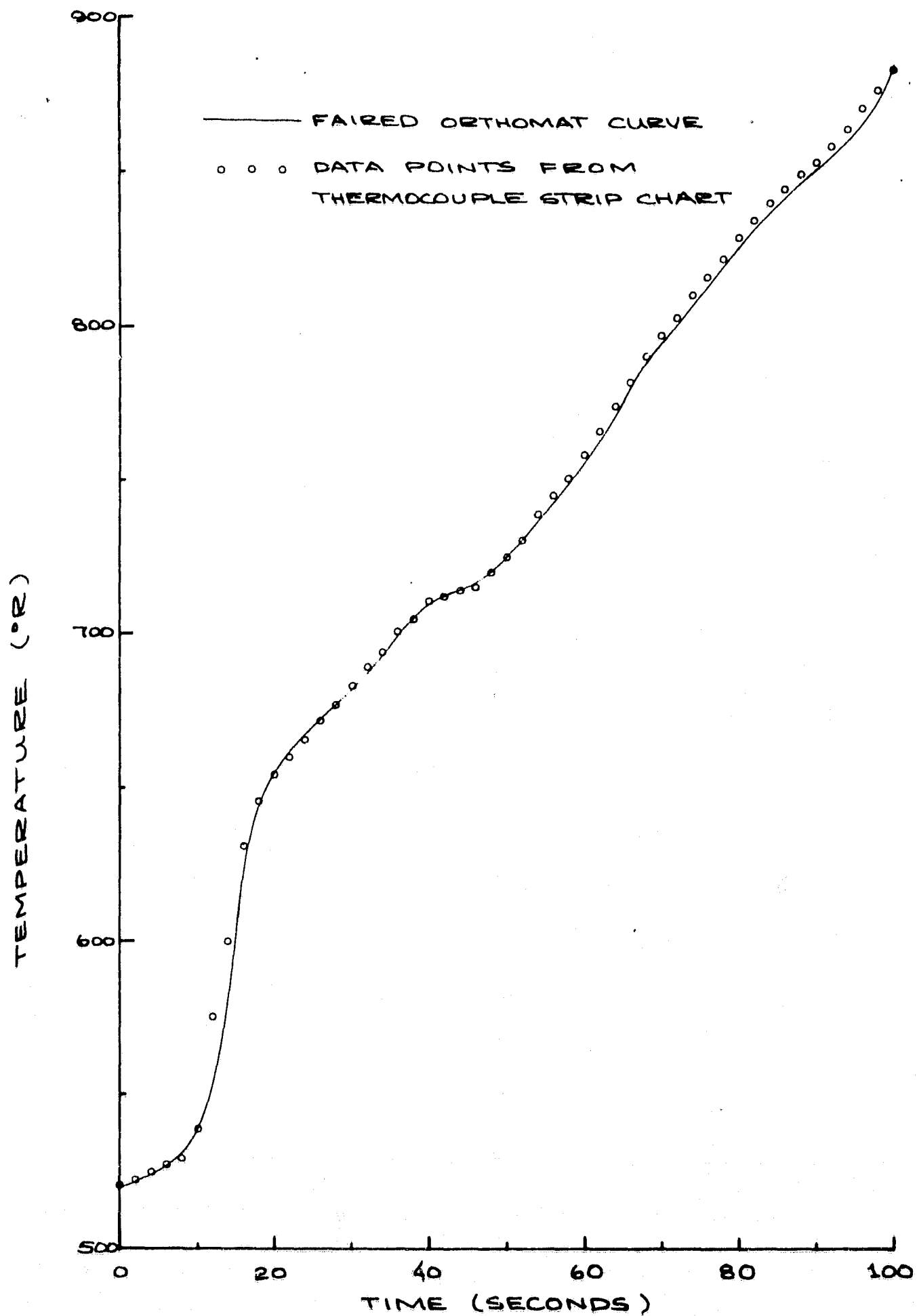


Figure 67

Comparison of faired Orthomat curve with data from thermocouple strip chart of Model 1, thermocouple 1.

APPENDIX D

SUMMARY OF NUMERICAL ANALYSIS AND COMPUTER PROGRAM

In the present program, equations of the following type are considered:

$$\xi = \int_{f(\xi_0)}^{f(\xi)} [f(\xi), \xi] df(\xi) \quad (D1)$$

where

$$f(\xi) = \int_0^1 [g(\xi, \eta), \xi, \eta]_1 d\eta \quad (D2)$$

$$g(\xi, \eta) = \int_0^1 [g(\xi, \eta), \xi, \eta]_2 d\eta \quad (D3)$$

In equations (5) to (8), $f(\xi)$ may represent $Re_m(\xi)$, $Re_b(\xi)$, etc., $g(\xi, \eta)$ represents $\tilde{u}$, τ , etc. Equation (D1) is the integrated form of equations (5) to (8) as can be seen by simply letting $\xi = Re_x$ (or x) and $f = Re_m(\xi)$, etc.

The solution of the integral equation (D1) requires that the lower limit of integration $f(\xi_0)$ be known or for that matter any one value of f at a specified ξ . In the present case, $f(\xi)$ is taken at the stagnation point. If it is assumed that the boundary layer thickness at the stagnation point is zero (as is done here), then

$$y_m^+ = 0 \text{ at } Re_x = 0$$

Equations (D1) and (D3) are solved by successive approximations. An algorithm may be formulated as follows:

- (i) Associate x , Re_m , $C_f/2$, . . . with $y_m^+ = 0$;
- (ii) Determine all functions depending on x such as u_e , h_e , . . . ;
- (iii) Select a new y_m^+ from an a priori determined array;
- (iv) Approximate eq. (D3) as

$$g_n(\xi, \eta) = \int_0^1 [g_{n-1}(\xi, \eta), \xi(y_m^{+, old}), \eta]_2 d\eta \quad (D4)$$

until suitable convergence is attained.

- (v) Approximate eq. (D1) as

$$\xi(y_m^{+, new}) = \xi(y_m^{+, old}) + \sum_{k=1}^K \Delta \xi_k \quad (D5)$$

where

$$\Delta \xi_k = \int_{f(\xi_{k-1})}^{f(\xi_k)} (\xi_{k-1}, f) df$$

- (vi) Either: (a) Exhaust y_m^+ array (end of case)
 (b) Exceed a prescribed maximum value of x (end of case)
 (c) Select another y_m^+ (go to iii)

In step (v) it is assumed that u_m^+ , Re_D and y_m^+ vary linearly with Re_m . Functions of x such as ρ_e , r , T_w , T_e , α , u_e , p_e and μ_e are defined in tables. Where derivatives are required (e.g., $d \ln u_e / d Re_x$) the differentiation is done numerically.

The consistency of the algorithm is demonstrated by solving the flow field with different arrays of y_m^+ . The heat transfer and skin-friction coefficients are not significantly affected by these choices.

The numerical solution of the five algebraic equations for the charring ablator is done by iteration on first order approximations. In general, the convergence properties are excellent. A few cases have diverged but these usually represent data for which the equations are not applicable. Perhaps the most perplexing problem is the existence of multiple roots in the neighborhood of the desired solution. Superficial efforts to eliminate the undesirable roots have only demonstrated the existence of roots not in the neighborhood. An effective method of selecting the proper root is to use appreciably underestimated first guesses for $\dot{m}_p$ and T_w , a value of 6.0 for $\mathcal{L}$, 10^{-5} for $\dot{m}_c$ and 10^{-6} for $\dot{m}_{sh}$.

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