X-615-68-334
PREPRINT

NASA TM X- 63381

SYNCHROTRON RADIATION FROM ELECTRONS IN HELICAL ORBITS

KUNITOMO SAKURAI

AUGUST 1968

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

SYNCHROTRON RADIATION FROM ELECTRONS IN HELICAL ORBITS

Kunitomo Sakurai*
NASA Goddard Space Flight Center
Greenbelt, Maryland

ABSTRACT

This paper considers correction factors of some of the commonly used results of synchrotron radiation theory both in vacuo and in ambient plasmas. These revisions are important when the spectral distribution of synchrotron radiation and the wave frequency range in which the influence of ambient plasmas becomes serious are considered.

*NAS-NRC Postdoctoral Resident Research Associateship

Synchrotron radiation from energetic electrons is very important in explaining various characteristics of solar radio type IV bursts and galactic radio emission (for example, Kundu 1965; Ginzburg and Syrovatskii, 1964, 1965). In recent time, both the effect of helical motion of radiating electrons (Epstein and Feldman, 1967) and the influence of ambient plasmas (Ramaty and Lingenfelter, 1967; Ramaty, 1968) on the characteristics of synchrotron radiation have attracted attention.

The influence on the characteristics of synchrotron radiation of the helical motion of radiating electrons will be here estimated for both cases, (1) in vacuo and (2) in the plasma medium.

Spectral Distribution of Synchrotron Radiation

The angular distribution (angle θ) of the radiation intensity from electrons of arbitrary energy in helical orbits with the pitch angle Ψ, for the S-th harmonic component is given by

$$\frac{dI_s}{d\Omega} = \frac{s^2 e^2 \omega_H^2}{2\pi c} \frac{\beta^2(1-\beta^2)}{(1-\beta\cos\theta\cos\Psi)^4} \left\{ \left[\sin\Psi J_s'\left(s \frac{\beta\sin\theta\sin\Psi}{1-\beta\cos\theta\cos\Psi}\right)\right]^2 + \left[\frac{\cos\theta-\beta\cos\Psi}{\sin\theta} J_s\left(s \frac{\beta\sin\theta\sin\Psi}{1-\beta\cos\theta\cos\Psi}\right)\right]^2 \right\} \qquad (1)$$

where ω_H is the gyrofrequency which is defined by eH_o/m_oc and $c\beta$ is the velocity of electrons, (e.g., Takakura, 1960; Kundu, 1965; Bekefi, 1966), and H_o is the strength of external magnetic field.

The total intensity of radiation of the s-th harmonic is obtained, by integrating with respect to the solid angle, as follows:

$$I_s = \int\left(\frac{dI_s}{d\Omega}\right)d\Omega$$

$$= \frac{2se^2\omega_H^2}{c}\ \frac{1-\beta^{*2}}{\beta^*}\left[\beta^{*2}J'_{2s}(2s\beta^*)\right.$$

$$\left.-s(1-\beta^{*2})\int_0^{\beta^*}J_{2s}(2s\xi)d\xi\right], \qquad (2)$$

where $d\Omega=2\pi\sin\theta d\theta$ and $\beta^*=\beta\sin\Psi/\sqrt{1-\beta^2\cos^2\Psi}$.

In the above equation, we define an apparent Lorentz factor γ^* in order to proceed with the discussion of the case for $\Psi=\pi/2$, which has been already dealt with by many authors (for example, Schott, 1912; Schwinger, 1949; Francia, 1959; Landau and Lifshitz, 1961). This apparent Lorentz factor is defined as

$$\gamma^*=\frac{1}{\sqrt{1-\beta^{*2}}} = \gamma\ \sqrt{1-\beta^2\cos^2\Psi} \qquad (3)$$

where $\gamma=1/\sqrt{1-\beta^2}$, is the true Lorentz factor. If we assume $\gamma>>1$, i.e. $\beta\simeq 1$, γ^* is reduced to $\gamma\sin\Psi$ according to equation (3). It thus follows that $\gamma^*>>1$ because the pitch angle Ψ is taken as constant in the above discussion as far as the calculation of the spectral distribution is concerned.

For $\gamma^*>>1(\therefore \beta^*\simeq 1)$ and $s>>1$, the first and the second terms in the parenthesis on the right-hand side of equation (2) are expressed as

$$J'_{2s}(2s\beta^*) = \frac{1}{\sqrt{3}\pi\gamma^{*2}} K_{2/3}(2s/3\gamma^{*3}) \tag{4}$$

and

$$s(1-\beta^{*2})\int_0^{\beta^*} J_{2s}(2s\xi)d\xi$$
$$= \frac{1}{2\sqrt{3}\pi\gamma^{*2}} \int_\alpha^\infty K_{1/3}(\eta)d\eta, \tag{5}$$

by making use of the approximate formulae in Watson's book (1945), where $\alpha=2s/3\gamma^{*3}$. By using the equality

$$2K_{2/3}(\alpha) - \int_\alpha^\infty K_{1/3}(\eta)d\eta = \int_\alpha^\infty K_{5/3}(\eta)d\eta \tag{6}$$

the right-hand side of equation (2) can be rewritten for the case $\gamma>>1$ as follows:

$$I_s = \frac{e^2}{\sqrt{3}} \frac{\omega_H^2}{\pi c} \frac{s}{\gamma^{*4}} \int_\alpha^\infty K_{5/3}(\eta)d\eta$$
$$= \frac{e^2\omega_H^2}{\sqrt{3}\pi c} \frac{s}{\gamma^4 \sin^4\psi} \int_\alpha^\infty K_{5/3}(\eta)d\eta \tag{7}$$

Here we define the characteristic harmonic number s_c by $3/2.\gamma^{*3}(=3/2.\gamma^3\sin^3\psi)$, analogous to the discussion for the case $\psi=\pi/2$ as considered by Schwinger (1949). When this number s_c

is substituted into equation (7), the following equation is obtained:

$$I_s = \frac{\sqrt{3}e^2\omega_H^2}{2\pi\gamma c} \frac{1}{\sin\psi} \frac{s}{s_c} \int_{s/s_c}^{\infty} K_{5/3}(\eta)d\eta \tag{8}$$

If the relation $I_s = \omega_H/2\pi\gamma\ I_f$ is taken into account, the spectral distribution with respect to the wave frequency is obtained as follows:

$$I_f = \frac{2\sqrt{3}\pi e^2}{c} f_H \frac{1}{\sin\psi} \frac{f}{f_c} \int_{f/f_c}^{\infty} K_{5/3}(\eta)d\eta \tag{9}$$

where $f=\omega/2\pi$, $f_H=\omega_H/2\pi$ and $f_c=\omega_c/2\pi$. This result is the same as has earlier been obtained by many authors except for a factor $(\sin\psi)^{-1}$. In fact, it is known that, as the pitch angle of radiating electrons becomes smaller, the peak intensity radiated into the direction of the instantaneous velocity vector of these electrons becomes higher (Sakurai and Ogawa, 1968).

In the above equation, the characteristic frequency f_c is given by

$$f_c = \frac{3}{2} f_H\ \gamma^2\sin^2\psi = \frac{3}{4\pi} \frac{eH_o}{m_o c}\ \gamma^2\sin^2\psi, \tag{10}$$

which is different from the result by Epstein and Feldman (1967) by a factor sin ψ. The reason is due mainly to the vagueness in their definition of gyrofrequency. Whenever we observe the Doppler-shifted gyrofrequency, we must always define this frequency by

$$\omega_H^* = \frac{eH_o}{m_o c} \frac{\sin\psi \sqrt{1-\beta^2}}{1-\beta\cos\psi\cos\theta} \qquad (11)$$

since only this component $H_o\perp(=H_o\sin\psi)$ is effective in the gyration of an electron. If the form $H_o\perp$ is used, equation (10) reduces to

$$f_c = \frac{3}{4\pi} \frac{eH_o}{m_o c} \gamma^2 \sin\psi \qquad (11a)$$

In conclusion, equation (9) must always be used in considering the spectral characteristics of synchrotron radiation from relativistic electrons, in which case the frequency f_m, where the emission intensity I_f attains the maximum, is given by

$$f_m \simeq 0.29\ f_o \qquad (12)$$

where f_c also contains the factor $\sin^2\psi$. Therefore, this frequency always becomes smaller, than the case for $\psi=\pi/2$. This discrepancy becomes serious when the pitch angle ψ approaches zero.

The factor $\sin\psi$ does not appear in the polarization equation defined by Ginzburg and Syrovasitii (1965). It thus follows that this factor only affects the spectral distribution of the emission power.

Synchrotron Radiation in Plasmas

The characteristics of synchrotron radiation are somewhat modified in the presence of ambient plasmas as has been considered by Ginzburg and Syrovatskii (1965) and Ramaty and Lingenfelter (1967). In dealing with the synchrotron radiation from energetic electrons in helical orbits in ambient plasmas, it must, however, be remarked that the power radiated per unit frequency interval is altered as follows:

$$I_f = \frac{2\sqrt{3}\pi e^2 f_H}{c} \frac{1}{\sin\psi} \cdot \frac{1}{[1+(1-\mu^2)\gamma^2]^{\frac{1}{2}}} \frac{f}{f_{c1}} \int_{f/f_{c1}}^{\infty} K_{5/3}(\eta)d\eta \quad (13)$$

where the refractive index $\mu^2 = 1-f_p{}^2/f^2$, the plasma frequency $f_p{}^2 (= \frac{e^2 n_e}{\pi m_o}$; n_e, electron number density) and the characteristic frequency in this case is given by

$$f_{c1} = \frac{3eH_o}{4\pi m_o c} \gamma^2 \sin^2\psi [1+(1-\mu^2)\gamma^2]^{\frac{1}{2}} \quad (14)$$

In equation (13), the influence of ambient plasmas is neglected when

$$(1-\mu^2)\gamma^2 << 1. \quad (15)$$

Then, from this equation, it follows that

$$f^2 >> \frac{4}{3} \frac{n_e ec}{H_o \sin^2\psi} f_c. \quad (16)$$

Since the essential part of synchrotron radiation is confined to the frequency range $f \sim f_c$, the criterion

$$f >> f_o \equiv \frac{4n_e ec}{3H_o \sin^2\psi} \quad (17)$$

must be satisfied for the influence of ambient plasmas to be neglected.

Consequently, since the factor $(\sin^2\psi)^{-1}$ is larger than unity unless $\psi = \pi/2$, the critical frequency f_0 becomes larger by this factor in comparison with the case where the pitch angle ψ of radiating electrons is $\psi = \pi/2$. It is, therefore, impossible to neglect it in estimating the influence of ambient plasmas on the characteristics of synchrotron radiation from energetic electrons in helical orbits. The criterion given by equation (17) differs by a factor $(\sin^2\psi)^{-1}$ from that given by Ramaty and Lingenfelter (1967).

REFERENCES

Bekefi, G., Radiative Processes in Plasmas, John Wiley, New York (1966).

Epstein, R. I. and Feldman, P. A., Synchrotron radiation from electrons in helical orbits, Ap. J., 150, L109-110(1967).

Francia, C. T. di, Introduction to the theory of synchrotron radiation, Radioastronomia Solare, 414-423, ed. by Righini, G., Societa Italia di Fisica (1959).

Ginzburg, V. L. and Syrovatskii, S. I., The Origin of Cosmic Rays, Macmillan, New York (1964).

Ginzburg, V. L. and Syrovatskii, S. I., Cosmic magneto-bremsstrahlung, Ann. Rev. Astron. Astrophys., 3, 297-350, (1965).

Kundu, M. R., Solar Radio Astronomy, John Wiley, New York (1965).

Landau, L. D. and Lifshitz, E. M., The Classical Theory of Fields, Pergamon, Oxford (1961).

Ramaty, R., Influence of the ionized medium on synchrotron emission of intermediate electrons, J. Geophys. Res., 73, 3573-3582 (1968).

Ramaty, R. and Lingenfelter, R. E., The influence of the ionized medium on synchrotron emission spectra in the solar corona, J. Geophys. Res., 72, 879-883 (1967).

Sakurai, K. and Ogawa, T., Some characteristics of gyro-synchrotron radiation from energetic electrons and the evolution of solar radio type IV bursts, Uchusen-Ken-kyu, 12, 503-531 (1968).

Schott, G. A., Electromagnetic Radiation, Cambridge Univ., Cambridge (1912).

Schwinger, J., On the classical radiation of accelerated electrons, Phys. Rev., 75, 1912-1925 (1949).

Takakura, T., Synchrotron radiation from intermediate energy electrons in helical orbits and solar radio bursts at microwave frequencies, Pub. Astron. Soc. Japan, 12, 352-375 (1960).

Watson, G. N., A Treatise on the Theory of Bessel Functions, Cambridge University, Cambridge (1945).