THE INSTABILITY OF STELLAR STRUCTURES INTERMEDIATE

BETWEEN WHITE DWARFS AND NEUTRON STARS.

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ABSTRACT

Drake has suggested that apparent multimillisecond periodicities in pulsar signals may correspond to vibrations of neutron stars. We have reexamined stellar models in the appropriate density range with an improved equation of state, and we conlude that suitable stable structures are unlikely.

At a recent seminar, Drake (Drake 1968) has reported a superposition of 10 and 15 millisecond periods on the pulses received from the pulsars AP 2015 and CP 1919. He is reported to have suggested that these periods represent the intrinsic vibrational period of a neutron star. However, neutron star periods much in excess of one millisecond would be very unlikely in a neutron star (Tsuruta, Wright, and Cameron 1965). A period of order 10 milliseconds would be roughly characteristic of a body with a mean density near 10^{13} gm/cm^3, two orders of magnitude smaller than the mean density of a typical neutron star. The composite white dwarf-neutron star model curve of Tsuruta and Cameron (1966) does show a subsidiary mass peak between 10^{13} and 10^{14} gm/cm^3. However, because the treatment of the physics in this density range was extremely crude, no significance was claimed for this peak by the above authors. However, the suggestion by Drake raises an important question as to the possible existence of stable structures in this density range. In this note we report on an investigation of this question with an improved but still crude treatment of the relevant physics.

The following physical phenomena play a key role in the equation of state. In the density range up to about 3×10^{11} gm/cm^3 only ions and electrons are present under cold degenerate conditions. In this range the electron Fermi level approaches 30 MeV, and electron capture reduces the charge to mass ratio of the nuclei.

The neutron binding energy of the nuclei approaches zero; the point where it becomes zero has been denoted the "neutron drip line" by J.A. Wheeler. At still higher densities and electron Fermi energies, electron capture continues, and neutrons leave the nuclei. Gradually a degenerate neutron gas is established in the volume between the nuclei, and a significant neutron Fermi energy is established. At this stage the nuclei will contain unbound neutrons, since neutrons can only be removed from the nuclei with energies in excess of the neutron Fermi level. Of course, neutrons will be readily exchanged between the free neutron sea and the nuclear interior, but the density of unbound neutrons in the nuclear interior will considerably exceed that outside, since the Fermi level of the neutrons inside the nucleus will be much greater than that outside, and thus the number of momentum states per unit energy interval near the Fermi surface will be much greater inside than outside.

We have treated the physics in the following very approximate way. We write $\mathcal{M} = M - A$ for the mass excess of a nucleus. We use the simplest possible form of the mass equation:

$$\mathcal{M}(A,Z) = \mathcal{M}_z Z + \mathcal{M}_n (A-Z) + \alpha A + \beta (A-2Z)^2/A + \gamma A^{2/3} + \delta Z^2/A^{1/3} \qquad (1)$$

Here $\mathcal{M}_z$ and $\mathcal{M}_n$ are the mass excesses of the proton and neutron, and the last four terms on the right hand side represent respectively the volume binding energy, the volume symmetry energy, the surface energy, and the coulomb energy. This equation is applicable to nuclei with sharp edges. In what follows we shall make some approximations valid for large mass numbers A.

The beta decay energy is

$$Q = \mathcal{M}(Z,A) - \mathcal{M}(Z+1,A) \tag{2}$$

This leads to the expression

$$\frac{Z}{A} \approx \frac{-Q-(\mathcal{M}_z-\mathcal{M}_n) + 4\beta - 4\beta/A - \delta/A^{1/3}}{8\beta + 2\delta A^{2/3}} \tag{3}$$

We use values of the coefficients given by Green (1954): $\alpha = -15.756$ MeV, $\beta = 23.694$ MeV, $\gamma = 17.794$ MeV, and $\delta = 0.7103$ MeV. It may be seen that the last two terms of the numerator of (3) are small compared to the preceeding ones, and that the second term in the denominator is fairly small compared to the first. Hence for a given value of the beta decay energy the ratio Z/A is not strongly dependent on mass number. Taking as typical $A \sim 100$, we have

$$\frac{Z}{A} \approx \frac{94-Q}{220} \tag{4}$$

The neutron binding energy is

$$B = \mathcal{M}_n + \mathcal{M}(Z,A-1) - \mathcal{M}(Z,A) \tag{5}$$

This leads to the expression

$$\frac{Z^2}{A^2} \approx \frac{3(B+\alpha+\beta) + 2\delta/A^{4/3}}{12\beta + \delta A^{2/3}} \tag{6}$$

It may be seen that the last terms in the denominator and numerator of (6) are small compared to the preceeding terms, and hence for a given neutron binding energy the ratio Z/A is not strongly dependent on mass number. Again taking as typical $A \sim 100$, we have

$$\frac{Z^2}{A^2} \approx \frac{B+7.9524}{100} \tag{7}$$

The neutron drip line (B=0) occurs at Z/A = 0.282, according to (7). From (4), the corresponding beta decay energy (or, equivalently, electron capture threshold) is Q = 31.96 MeV.

An equation of state can now be constructed, with the pressure and density parametrically depending on the electron Fermi energy E_e^F. The latter quantity specifies n_e and P_e, the electron number density and pressure. The total number of protons (inside nuclei) per cm^3 is equal to n_e, for charge equality. We rewrite (4) as

$$\frac{Z}{A} = \frac{94 - E_e^F}{220} \tag{8}$$

This specifies the number of neutrons in nuclei per cm^3, and hence the mass density of the nuclei. The ionic pressure can be neglected. If $E_e^F \leq 31.96$ MeV, the equation of state is thus completely specified. If $E_e^F > 31.96$ MeV, then free neutrons are also present. We calculate the neutron Fermi energy E_n^F from a modification of (7):

$$\frac{Z^2}{A^2} = \frac{-E_n^F + 7.9524}{100} \tag{9}$$

From E_n^F we also calculate the number density of free neutrons in the internuclear region and the neutron pressure P_n, which will also apply to the region occupied by nuclei. We assign the nuclei a density 4×10^{14} gm/cm^3, calculate the fractional volume occupied by the nuclei, and hence find the mass of the neutrons in the internuclear region. The total pressure $P = P_e + P_n$. The equation of state is thus completely specified.

It may be noted from equation (8) that $Z/A \rightarrow 0$ as $E_e^F \rightarrow 94$ MeV. This indicates, since the total number of protons per unit volume is fixed by the value of E_e^F, that ever increasing numbers of neutrons are absorbed into the nuclei. If individual nuclear charges become too small, such nuclei will fuse by pycnonuclear reactions. Thus the fractional volume occupied by the nuclei approaches unity as the total density approaches 4×10^{14} gm/cm^3 and as $E_n^F \rightarrow 7.9524$ MeV.

This equation of state is obviously not a good approximation near a density of 4×10^{14} gm/cm^3 and certainly cannot be used beyond this density. However, this limit is quite sufficient for the present investigation, which is intended only to bring out the essential physical behavior of the equation of state. A more refined investigation should take into account higher terms in the nuclear symmetry energy, the rounded edge of the nucleus, corrections to the surface energy when a neutron sea is present, corrections to the coulomb energy due to the electrons, formation of a crystalline lattice, and nucelar potential energies of the free neutrons.

With this equation of state the equations of hydrostatic equilibrium were integrated to obtain stellar models. Figure 1 shows the relation between stellar mass and central density, showing the well known dwarf mass peak and the region up to a central density of 4 X 10^{14} gm/cm^3. It may be seen that there is no second mass peak in the higher density region, although the mass starts rising slowly above log ρ=13.5.

In the Tsuruta-Cameron procedure the neutronization of nuclei was assumed to create new nonrelativistic particles as the matter was compressed, thus assuring a large value of the adiabatic index Γ_1. Their secondary peak resulted from these assumptions. In this improved procedure, the neutronization process initially creates many nonrelativistic neutrons, but these are progressively absorbed on nuclei as compression occurs at the higher densities. Hence Γ_1 remains low The values of Γ_1 obtained for the equation of state described here are shown in Figure 2. It may be seen that Γ_1 attains a maximum value of 1.24 in the neutronization region at log ρ = 12.8. To create a secondary meass peak, major portions of a star would require Γ_1 > 4/3, which is not possible here.

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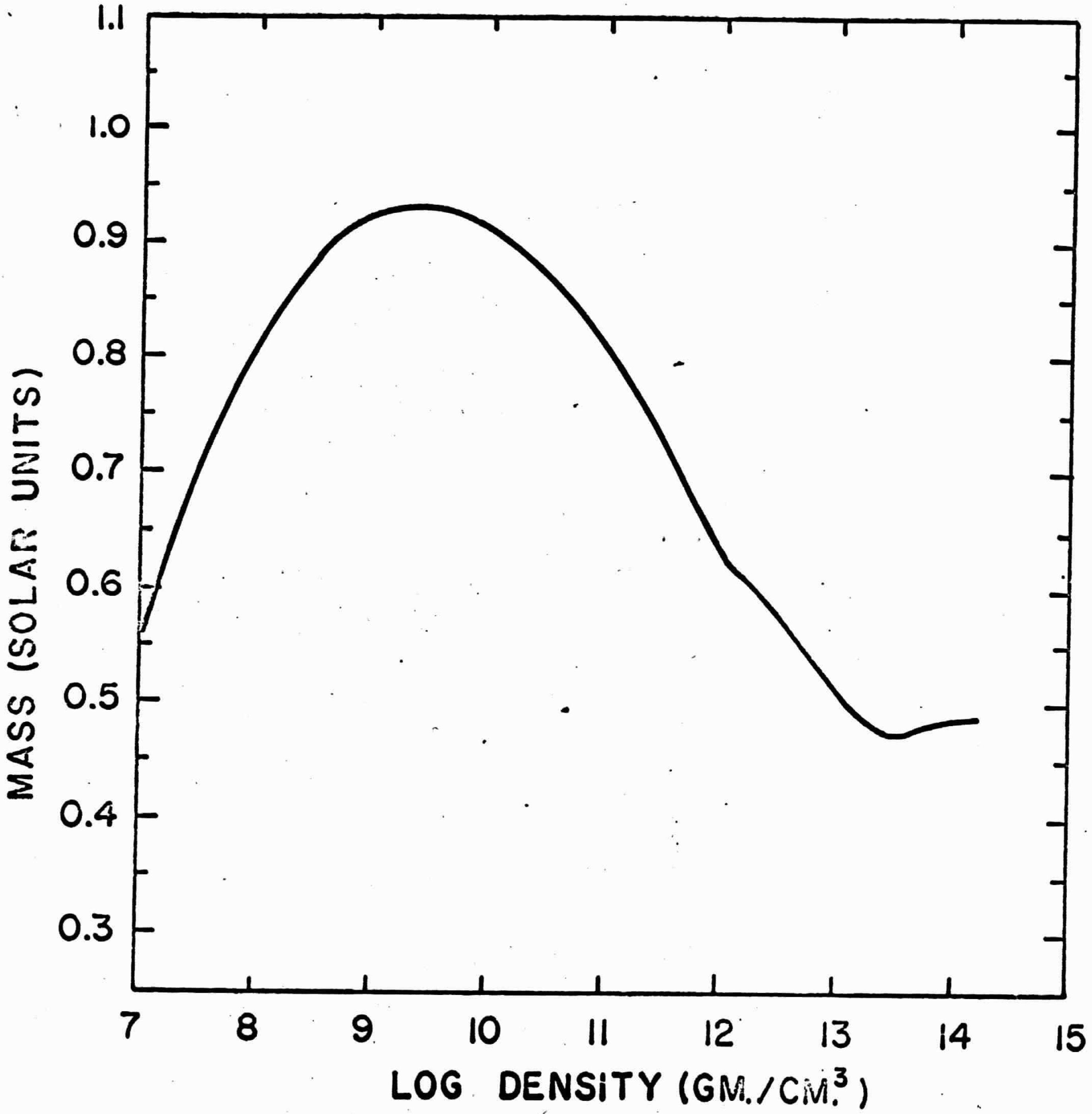
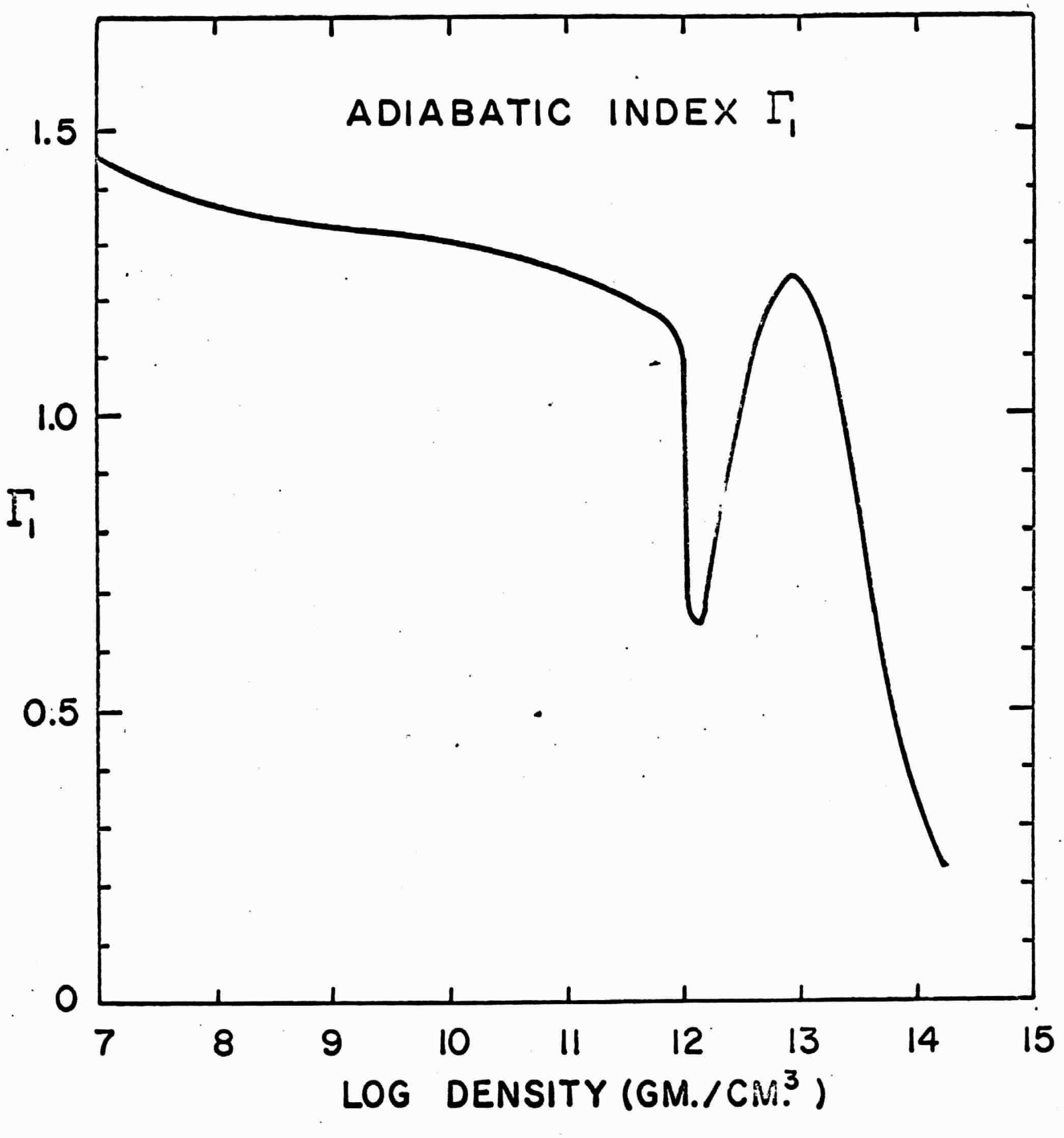
FIGURE CAPTIONS

Figure 1. Mass of a degenerate star plotted as a function of central density, in the range below nuclear density.

Figure 2. Adiabatic Index Γ_1 in the density range up to nuclear density corresponding to the equation of state developed in this paper.



MASS (SOLAR UNITS)
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
7
8
9
10
11
12
13
14
15
LOG DENSITY (GM./CM.³)



ADIABATIC INDEX Γ_1
1.5
1.0
Γ_1
0.5
0
7
8
9
10
11
12
13
14
15
LOG DENSITY (GM./CM.^{3})