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ON THE STRUCTURE OF THE GEOMAGNETIC FIELD AT GREAT DISTANCES FROM EARTH

by

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by A. E. Antonova &
V. P. Shabanskiy

SUMMARY

The shape of the magnetosphere boundary is determined and the magnetic field lines are computed within the framework of the 2-dipole approximation model, taking into account the current sheet in the magnetosphere tail for various parameters characterizing both the solar wind and the current sheet.

The remote regions of the magnetosphere from the daytime side are determined, in which the field intensity along the lines of force has respectively two and three minima contrary to the single one for the lines of force of inner regions. Projections are presented of these regions on the equatorial plane and on the Earth's surface.

*
* *

THE TWO-DIPOLE APPROXIMATION. - We shall make use of the simplified model of the magnetosphere [1, 2], of which the shape of the daytime boundary and the magnetic field are determined with the help of two dipoles: the initial dipole with the Earth's magnetic moment $M = 8.06 \cdot 10^{25}$ gauss·cm^3 and the auxiliary dipole, imitating the perturbing action of the solar wind with the magnetic moment $M_{CF} = J_1{}^3 M$, located at a distance a from the initial dipole from the daytime side along the line Earth-Sun. The shape of the daytime boundary of the magnetosphere and the perturbation field are therefore determined by two parameters a and j_1, which may be appropriately selected from the comparison of the magnetosphere boundary, obtained in the model, with the experimentally found one. On the other hand, parameters a and j_1 may be linked with the solar wind parameters, i. e. the dynamic P_d and hydrostatic pressure P.

(*) O STRUKTURE GEOMAGNITNOGO POLYA NA BOL'SHIKH RASSTOYANIYAKH OT ZEMLI

A similar magnetosphere model was first considered in the work [3]; however, the value of the magnetic moment of the auxiliary dipole and its range were chosen to a certain degree in a random fashion and corresponded to values $j_1^3 = a = 28$, for which the daytime boundary of the magnetosphere was situated at a distance $L_1 \sim 7.5\ R_E$. In the absence of current in the neutral sheet of the magnetosphere tail, whose field we shall take into account below, the magnetosphere region in the two-dipole model occupies a finite volume and is determined by assortment of field lines, closed on the initial dipole M.

Let us consider the case when the Earth's dipole is oriented perpendicularly to the direction of the solar wind [1, 2]. Then the field B in the frontal point of the magnetosphere boundary will be

$$B = 2fB_M, \tag{1}$$

where $\underline{f}$ is a numerical factor; B_M is the field of the dipole M and is determined by the equality of the magnetic $(B^2/8\pi)$ and dynamic gas pressure

$$P_d = 2mv^2N\cos^2\chi,$$

$\underline{m}$ being the mass of the proton, $\underline{v}$ is the wind velocity, χ is the angle between $\vec{v}$ and the normal to magnetosphere surface, N is the number of particles in 1 cm^3. The equality of pressures gives the geocentric distance to the frontal point of the magnetosphere in Earth's radii

$$L_1 = \left(\frac{f^2B_0^2}{2\pi P_d}\right)^{1/6}, \tag{2}$$

where $B_0 = 0.31$ gauss is the field on the equator on the Earth's surface, numerically coinciding the the value of the magnetic moment M, expressed in gauss $\cdot R_E^3$.

The approximation (4) presumes that the relation (1) between the limiting value of the field B and the component along the boundary of the unperturbed field B_M is fulfilled for all the points of the boundary with constant coefficient $\underline{f}$. This allows us to determine analytically the equatorial and the meridional cross-section of the magnetosphere [5, 6].

If we choose for the perturbed field B the sum of the fields of the auxiliary and initial dipoles, we shall have at f = constant, from (1), for the equatorial cross-section of the magnetosphere boundary the circumference

$$j^3r^3 = R^3, \quad j_1 = j(2j-1)^{1/3}, \quad R = (r^2 + a^2 - 2ar\cos\Lambda)^{1/2} \tag{3}$$

where $\underline{r}$ is the radius-vector from the coordinate origin in which the dipole M is placed; R is the equatorial distance from the boundary to the dipole j_1^3M; Λ is the longitude. The radius of the circumference (3) is $\rho = aj/(j^2-1)$; its center is shifted relative to the dipole M toward the side opposite to dipole M_{CF}, by the distance $a/(j^2-1)$. Hence there are three characteristic distances L_1, L_2, L_3 from the center of the Earth to the corresponding day morning (in the direction perpendicular to the line Earth - Sun) and night-time magnetosphere boundary

$$L_1 = a/(j+1), \quad L_3 = a/(j-1), \quad L_2 = (L_1 L_3)^{1/2} = a/(j^2-1)^{1/2}. \tag{4}$$

Resolving relative to a and j, we have

$$j = \frac{L_3 + L_1}{L_3 - L_1} = \frac{(L_2/L_1)^2 + 1}{(L_2/L_1)^2 - 1}, \qquad a = L_1(j+1) = L_1 \frac{2L_2^2}{L_2^2 - L_1^2}. \tag{5}$$

In the approximation of [4], the value of L_3 is evidently determined by the hydrostatic pressure (since the dynamic pressure is absent on the night side), i. e.

$$L_3 = L_1 (P_d / P)^{1/6}. \tag{6}$$

Concomitantly with (2) and (4), Eq,(6) yields a single-valued determination of parameters a and j by solar wind parameters.

The shape of the equatorial cross-section of the magnetosphere in the two-dipole model (shifted circumference) coincides sufficiently well with the shape of magnetosphere cross-section of [5, 6] in the approximation (1) for the same parameters P_d, P, as is shown by comparison in [1, 2]. However, the approximation (1) is poorly fulfilled even for the equator. If on the daytime side f $\sim$ 1 (as for a plane boundary), on the night side f is closer to the value f $\sim$ $^3/_2$(just as for a spherical surface with dipole at the center of the sphere). For that reason, when determining a and j, it is better to make use of the characteristic dimensions of L_1 and L_2 rather than of the combination L_1 and L_3. The more so, since in the presence of a current sheet in the magnetosphere tail, the value of L_3 loses all sense as an independent characteristic.

Limiting ourselves to the first two harmonics of the expansion by L/a of the perturbation field (L being the equatorial distance), we shall obtain for the total field in the equatorial plane at f = 1

$$B = \frac{M}{L^3} + \frac{\alpha M}{L_1^3}\left(1 + \beta \frac{L}{L_1} \cos \Lambda\right), \tag{7}$$

where

$$\alpha = j^3/(j+1)^3, \qquad \beta = 3/(j+1). \tag{8}$$

For example, at L_1 = 10, L_2 = 16, we shall obtain a = 33.3, j = 2.33, $\alpha \approx 0.35$, $\beta = 0.90$; at L_1 = 10, L_2 = 14, we shall obtain a = 40.8, j = 3.08, $\alpha = 0.42$, $\beta = 0.74$. Comparison with the model by Mead [7], for which L_1 = 10, L_2 = 14.5, while the numerical coefficients correspond to $\alpha = 0.8$, $\beta = 0.85$, we see that in the Mead model β practically coincides with its value in the two-dipole model, and α is approximately two times greater. Basically, this is the consequence of smaller dimensions of Mead's model magnetosphere in the direction North-South. Note also that expansion (7) for the two-dipole model is valid for greater L than in the Mead model, for in the latter the expansion parameter is the quantity L/L_1, which is substantially greater than L/a.

CURRENT FIELD IN THE TAIL'S NEUTRAL SHEET. Let us introduce a solar-magnetospheric Cartesian system of coordinates in which the Earth is placed at its origin, the axis x is directed at the Sun and the axis z - along the magnetic moment of the Earth's projection on a plane perpendicular to x. Let z be positive in the northerly direction; the axis y is then directed toward the evening. At a sufficiently great distance from the Earth the neutral sheet constitutes a plane parallel to the plane xy. In correspondence with the observations of [8], the distance z_1 to the sheet may be described by the empirical relation [9]

$$z_1 = C \sin\psi \tag{9}$$

where ψ is the geomagnetic latitude of the axis x (or the angle between the dipole axis and axis z); $C \approx 8$ is a numerical factor.

The neutral sheet with constant surface current density J_T, directed along the axis y will be chosen in the form of a plate $z = z_1$ with coordinates x_T, y_T, varying within the limits $-y_1 < y_T < y_1$, $x_2 < x_T < x_1$ (x_1, x_2, x_T are negative quantities). Then the components of magnetic field intensity induced by the plate at the point x, y, z, are

$$B_{zT} = -\frac{J_T}{c}\ln\frac{A_1^- A_2^+}{A_1^+ A_2^-},$$

$$B_{xT} = -\frac{J_T}{c}\left\{\operatorname{arctg}\frac{Y^- X_1}{ZR_1^-} - \operatorname{arctg}\frac{Y^- X_2}{ZR_2^-} + \operatorname{arctg}\frac{Y^+ X_2}{ZR_2^+} - \operatorname{arctg}\frac{Y^+ X_1}{ZR_1^+}\right\}, \tag{10}$$

$$A_i^\pm = Y^\pm + R_i^\pm, \quad R_i^\pm = [X_i^2 + (Y^\pm)^2 + Z^2]^{1/2},$$
$$X_i = x - x_i, \quad Y^\pm = y \pm y_1, \quad Z = z - z_1 \quad (i = 1, 2).$$

In spherical coordinates r, θ, Λ

$$B_{rT} = B_{xT}\sin\theta\sin\Lambda + B_{zT}\cos\theta,$$

$$B_{\theta T} = B_{xT}\cos\theta\sin\Lambda - B_{zT}\sin\theta, \quad B_{\Lambda T} = B_{xT}\cos\Lambda. \tag{11}$$

Following are the cases that may be considered: 1) an infinitely extended plate in the direction y ($y_1 = \infty$) for finite x_2 and 2) an infinitely prolonged plate in the night time direction, with current ($x_2 = -\infty$) at finite y_1. The first case is considered on the basis of the Mead model for the explanation of the high-latitude boundary of trapped radiation [10] and the determination of the shape of the meridional (with respect to xz) cross-section of magnetic drift shells at great geocentric distances [11]. The cases (2) is apparently more realistic; its estimate is made in [12]. Alongside with the current in the plate, the current closing it on the morning side along the northern and southern cylindrical surface of the magnetosphere tail is also taken into account in [12]. However, inasmuch as in our case the field perturbation by the outer magnetosphere boundary is fully accounted for by the field B_{CF} of the auxiliary dipole, the inclusion of the closing current is not necessary.

SHAPE OF THE MAGNETOSPHERE AND OF THE FIELD LINES. The field intensity is determined at any point of the magnetosphere by the sum of the fields

$$\mathbf{B} = \mathbf{B}_M + \mathbf{B}_{CF} + \mathbf{B}_T \tag{12}$$

of the initial and auxiliary dipoles in a plate with current in the magnetosphere tail (10), (11). If we denote

$$V_M = -\frac{B_0}{r^2}(\cos\theta\cos\psi + \sin\theta\cos\Lambda\sin\psi), \tag{13}$$

$$V_{CF} = -\frac{j^3 B_0 r\cos\theta}{(r^2 + a^2 - 2ar\sin\theta\cos\Lambda)^{3/2}} \tag{14}$$

as the field potentials of respectively the initial and auxiliary dipoles, then we shall have in spherical coordinates (assuming also $j_1 = j$, i. e. $f = 1$)

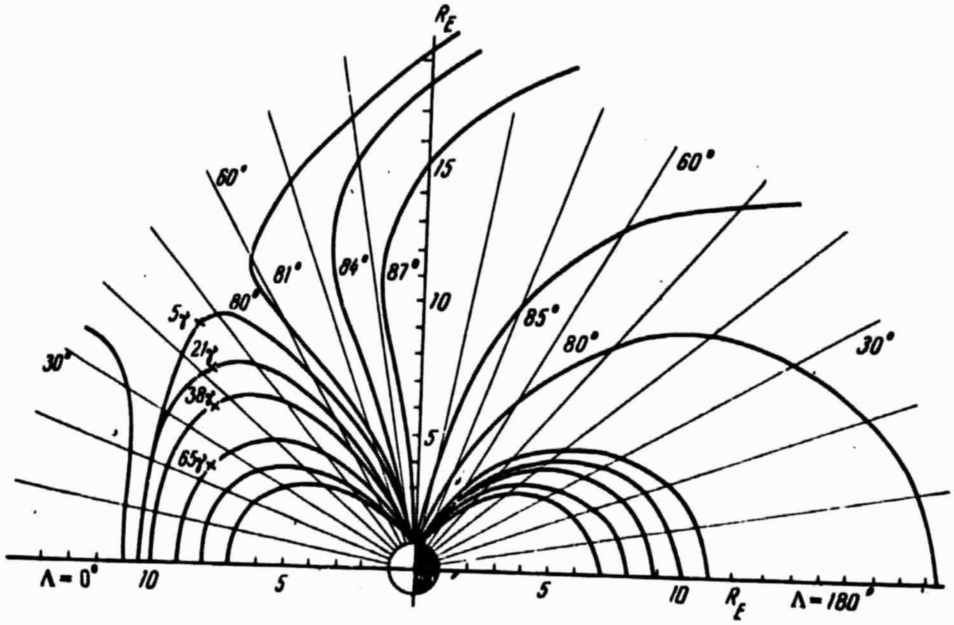
$$B_r = -\frac{\partial V}{\partial r} + B_{rT}, \quad B_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} + B_{\theta T},$$
$$B_\Lambda = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial\Lambda} + B_{\Lambda T}, \quad V = V_M + V_{CF}. \tag{15}$$

In this way the field intensity at any point is found analytically. For the determination of magnetic field lines we have the equations

$$\frac{dr}{ds} = \frac{B_r}{B}, \quad \frac{d\theta}{ds} = \frac{1}{r}\frac{B_\theta}{B}, \quad \frac{d\Lambda}{ds} = \frac{1}{r\sin\theta}\frac{B_\Lambda}{B}, \tag{16}$$

where ds is an element of line of force length. Integration of system (16) was performed on a BESM-4 and M-20 electronic computers by the Runge-Kutta method with constant step, automatic selection of the step, with absolute and relative precision on the step 10^{-5} at automatic control of calculation correctness. The preliminary results of these calculations are brought out in [13, 14]. Let us consider separately the two cases of dipole M, parallel to and inclined to the axis z.

Case $M \uparrow\uparrow z$. Plotted in Fig.1a is magnetosphere cross section in the central meridian plane xz in the absence of current in the magnetosphere tail for parameter values $a = 33.0$, $j^3 = 12.7$, $\psi = 0$. The southern half of the magnetosphere is symmetrical with respect to the northern half relative to the plane $z = 0$. The latitude of intersection with the Earth's surface and the magnitude of field intensity in the minimum are indicated on the field lines. The latitude of the point of the Earth's surface $\phi = 90° - \theta$, in which the field lines, departing into the tail and closing on the daytime side, are separating, is $\phi \sim 80°$. The midday boundary is at $L_1 \sim 10.4$. The field's "neutral" point, where $\mathbf{B}_{CF} + \mathbf{B}_M \approx 0$, is determined by the coordinates $r_N \approx 1.2\,L_1$, $\varphi_N \approx 50°$. The neutral point exists also in the Mead model [7]; however, as a consequence of not taking into account the isotropic pressure component in the solar wind, its coordinates are $r_N \approx 0.93\,L_1$, $\varphi_N \approx 70°$. Here also the neutral point is the results of summation of the fields of two dipoles, of which one — the field of the auxiliary dipole — has no real sense beyond the magnetosphere.


R_E
15
10
5
60°
81°
84°
87°
80°
85°
80°
60°
30°
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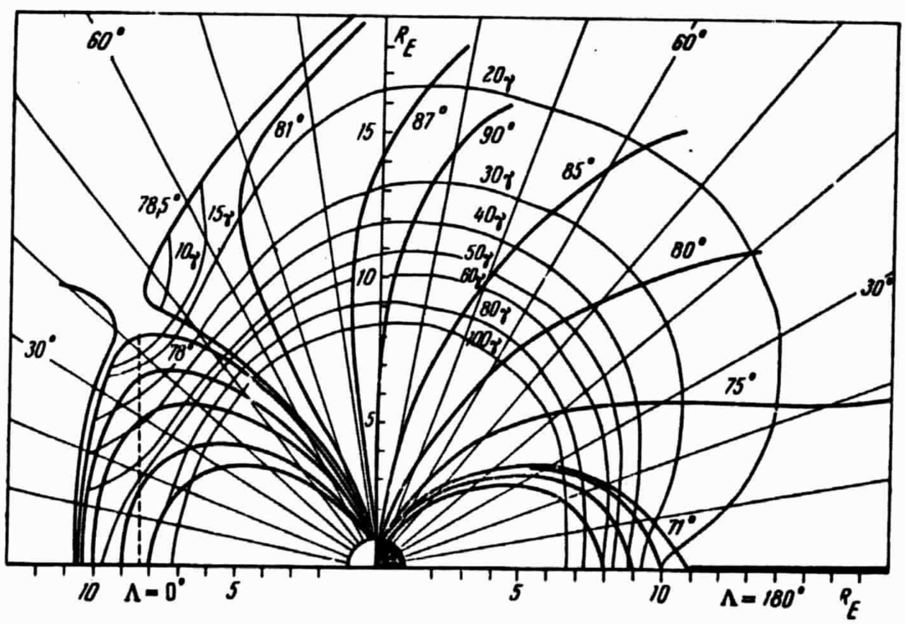
5γ
21γ
38γ
65γ
Λ = 0°
10
5
5
10
R_E
Λ = 180°

1a


R_E
60°
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Λ = 180°
R_E

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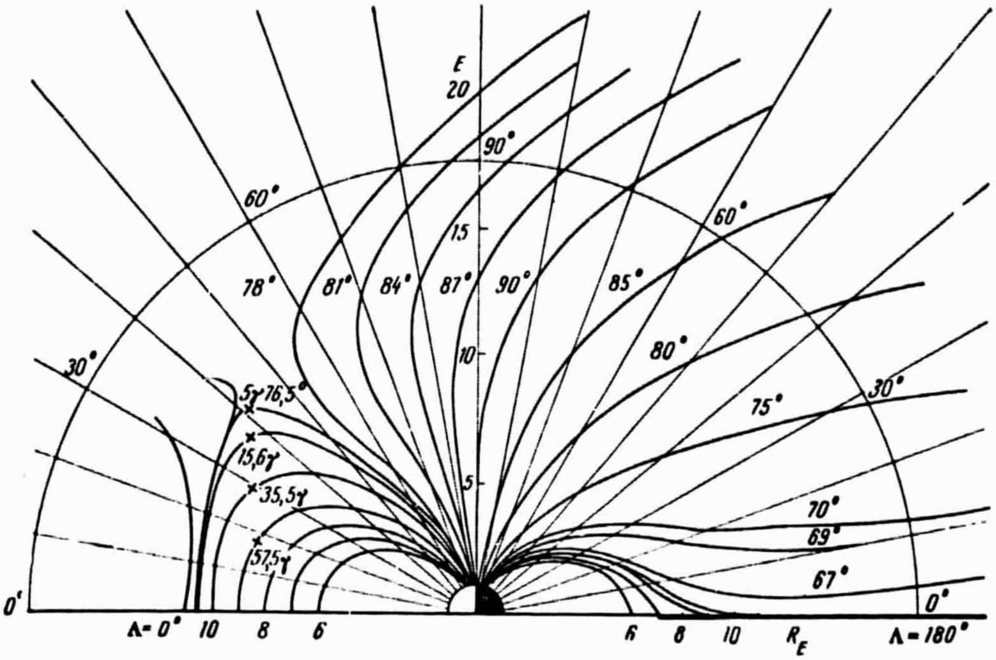

Fig. 1e

As in the Mead model, the magnetic field minimum for the lines of force passing near the daytime boundary of the magnetosphere is not on the equator, as is for less perturbed and deeper field lines, but is shifted toward higher latitudes.

It is evident that part of the field lines closed on dipole M and passing near the neutral point, do not really exist, for the magnetic pressure in the minimum may result to be smaller than the gas pressure P_d + P. Since for the extreme lines of force, closing on the daytime side, the minimum is attained at the point where the tangent to the field line is directed nearly parallelwise to solar wind, the limiting factor here will be the isotropic part of pressure P($P_d \sim \cos^2\chi$, where χ is the angle between the normal to magnetosphere surface and the solar wind direction). One should think that in the vicinity of the "funnel" formed by the lines of force departing to the daytime side and into the tail, the isotropic pressure P of solar wind is, generally speaking, greater than in other parts of the magnetosphere surface, for in the funnel, open toward the side of solar wind, there takes place an additional deceleration of the corpuscular flux. As to the field lines in the funnel that depart into the tail, the field minimum in them is attained at a point where χ is small. This is why the existence of the extreme field line of the tail is determine mainly by the pressure P. Since practically nothing is known there about the real solar wind pressure in the vicinity of the funnel, one may conditionally assume for the field lines bounding the magnetosphere, those having at the minimum an intensity B $\sim$ 10 - 20 γ. Such also is the magnitude of field intensity in remote regions of magnetosphere tail, being also determined by the isotropic component of pressure P.

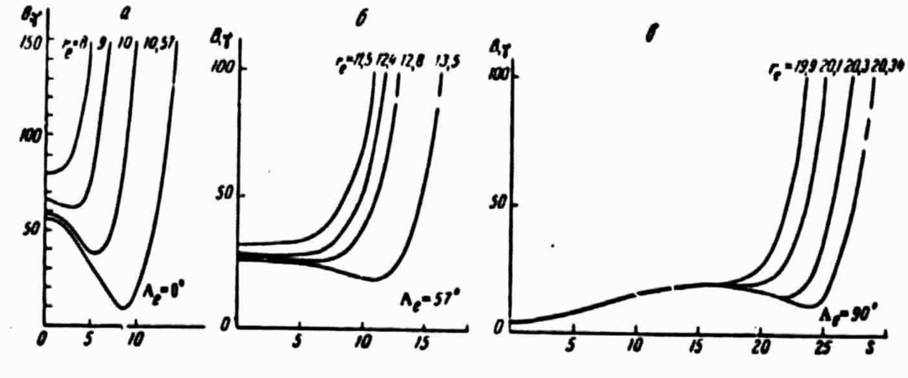

Fig.2

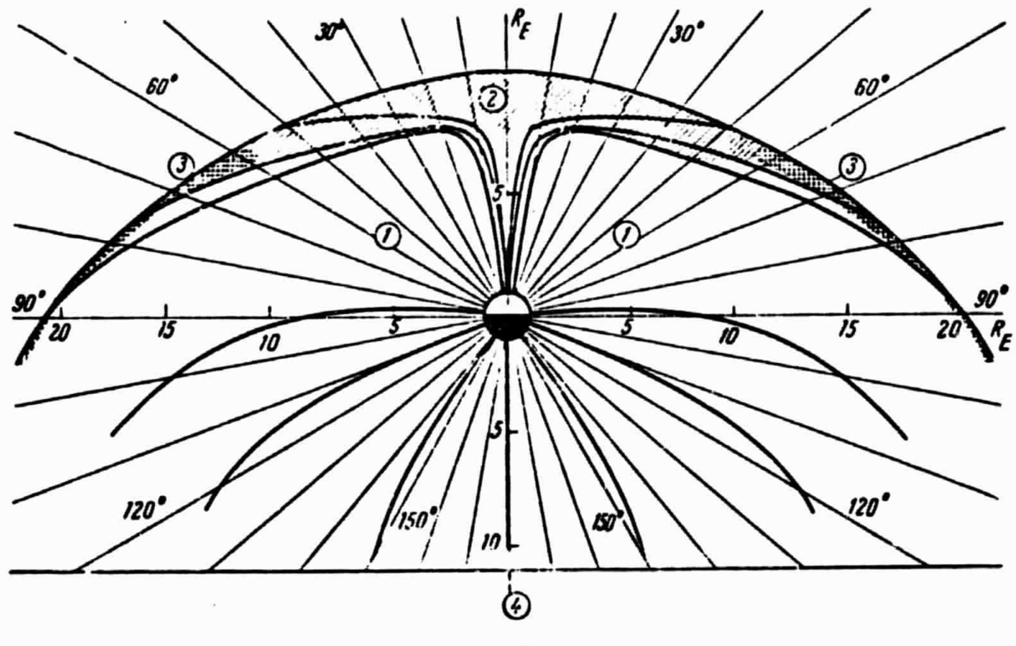

Fig.3a

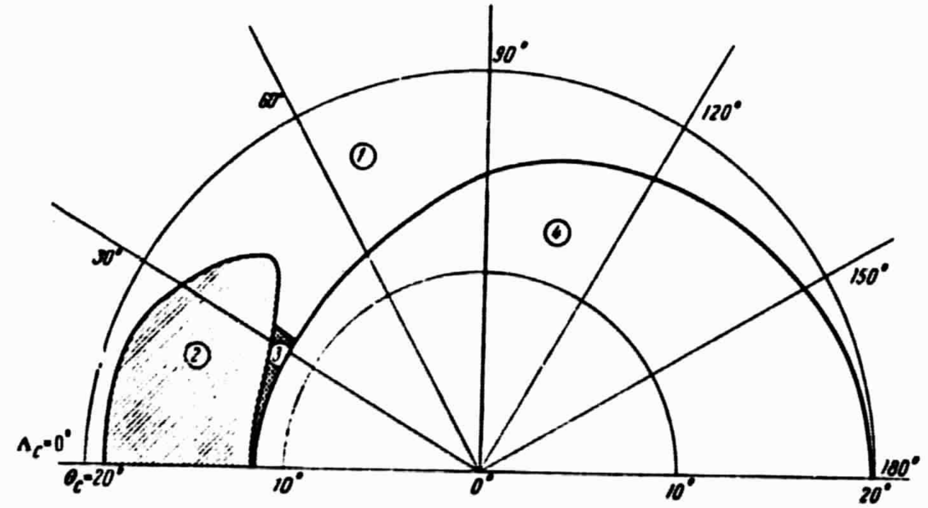

Fig.3

Shown in Fig.1б is the cross-section along the central meridian of the magnetosphere with current sheet in the tail. The plate has for parameters $z_1 = 0$; $y_1 = \infty$; $x_1 = -11$; $x_2 = -51$. The current J_T in the plate yields at the plate the tangential component $B_t = 2\pi J_T/c = 16\gamma$. The neutral point of the field, situated near the plate's edge x_1 , is projected along the field line on the Earth's surface at latitute $\phi_n = 70.5°$ from the nighttime side. The presence of current in the tail lowers the latitude of magnetosphere boundary from the daytime side to $\phi_d \approx 78°$. Presented here also are the lines of equal B (thin lines) and the line (strokes) passing through the points of field minima on the lines of force. The line minima's intersection with axis x determines the line of force with equatorial distance L, limiting the region of inner field lines with minimum on the equator.

An example of a magnetosphere with stronger current in the tail is shown in Fig.1 . Here $z_1 = 0$, $y_1 = 20$, $x_1 = -7.1$, $x_2 = -57.1$, $B_t = 45\gamma$, $\phi_n = 65°$ and $\phi_d = 76.5°$.

The neutral point in the central meridian cross-section near x_1 belongs to the netral line of the field lying in the plane xy. This line separates the region of closed field lines from that of field lines departing into the tail and proceeding either parallelwise to the current sheet, or undergoing in it a sharp break. Alongside with the daytime boundary of the magnetosphere the lines of force passing in the vicinity of the neutral point, bound the region of closed field lines, which could be call the heart of the magnetosphere. The remaining part is the tail of the magnetosphere. The projection of heart's boundary on the Earth's surface yields the polar oval.

Figure 2 shows examples of the course of field intensity along the lines of force departing from the Earth at various latitudes ϕ and longitudes Λ (s being the line of force's arc counted from the geomagnetic equator in Earth's radii, r_e and Λ_e are respectively the geocentric distance and the longitude of the point of intersection of the geomagnetic equator by the field lines). The closer to the midday meridian, the wider the region in which the field minimum lies outside the equator. The lines of force of the heart (core) emerging from Earth near the polar oval from the daytime side, have three field minima: one on the equator and two at high latitudes both in northern and southern hemispheres (Fig.2B). These lines of force deviate particularly strongly to the night side, undergoing, as they drift away from Earth, an inflection by longitude (Fig.3a). A strong inflection along longitude makes understandable the presence of three minima of field intensity along the lines of force. Near Earth these lines have two minima at high latitudes, also characteristic of the frontal field lines, while as they get closer to the equator, they depart to the night side and have still one more minimum on the equator characteristic of the lines of force from the night side (or for lines in the depth of the magnetosphere).

The equatorial cross-sections of regions 1, 2, 3 of the core, characterized by the fact that in them the magnetic field lines have respectively 1, 2 and 3 minima of field intensity are represented in Fig.3a, where shown also is the region of magnetosphere tail 4 with current sheet and the projection of the near-boundary field lines of the core of the magnetosphere for the case 1б. Shown in Fig.3б is the polar oval and the projection of these regions' field along the lines of force.on the Earth's surface. θ_c and Λ_c are the geomagnetic polar angle and the longitude of field line intersection with the ground.

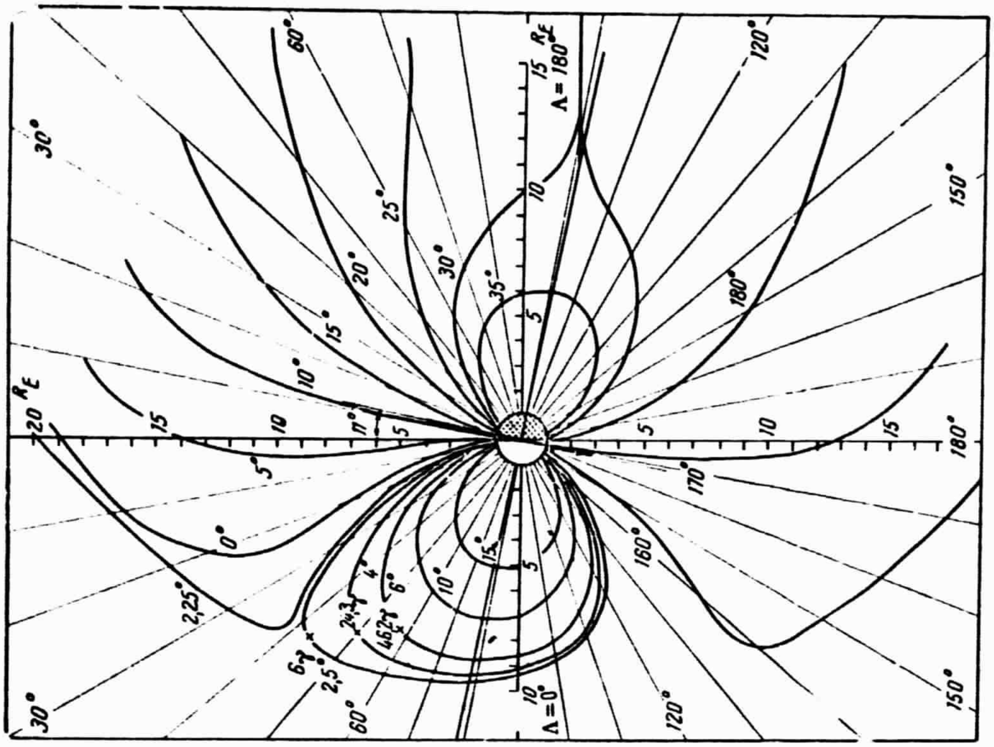

Fig. 4б

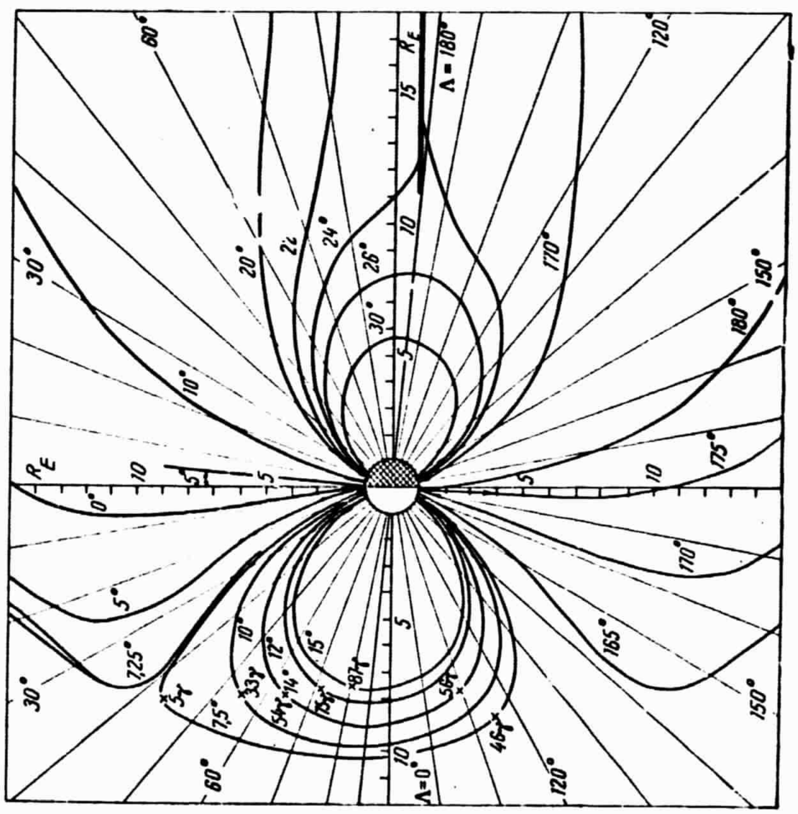

Fig. 4a

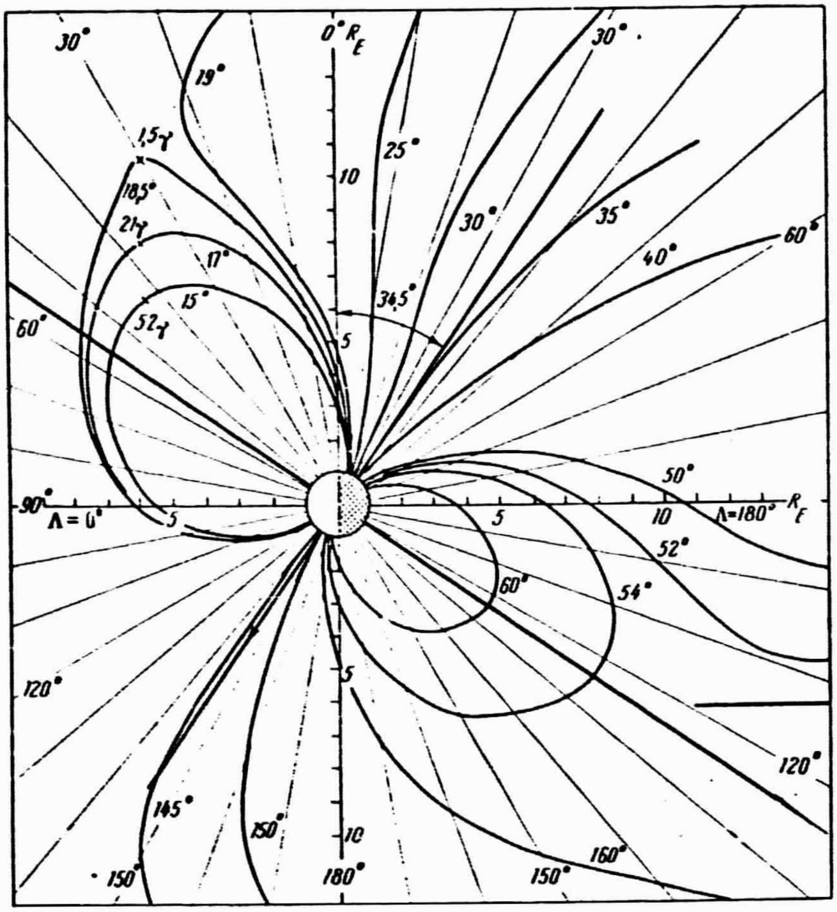

Fig. 4e

<u>THE CASE OF A DIPOLE OBLIQUE TO AXIS Z</u>. It is evident that the two-dipole approximation is in this case rougher than in the case considered above. First of all the question arises about the disposition and orientation of the auxiliary dipole, which would best correspond to the real case of initial dipole inclination to axis <u>z</u> when the solar wind is directed along the axis <u>x</u>. The case $M_{CF}\uparrow\uparrow M$ may be considered as generalization of the method of representation for an ideally conducting plane: it may be seen from formulas (3) and (4) that the field of the auxiliary dipole serves in the equatorial region as an imaginary picture of the initial dipole field in a spherical mirror [1]. In the image method for the plane x = L_1 the imaginary dipole should be inclined in the opposite side at the same angle ψ as for the initial dipole. But for points outside the equator the image method for the spherical mirror is not equivalent to the two-dipole app oximation, and the analogy could not be propagated further.

Taking account of the well known conditionality of the resulting pattern, we shall admit that for not too great angles ψ the auxiliary dipole remains as formerly (ψ = 0). The fact that this is not the best assumption, we may see, for example, from the limiting case ψ = 90°: indeed, in the real case there should obviously be axial symmetry relative to axis x, whereas here the symmetry is only relative to the plane xz, this symmetry being absent relative to the plane xy. We shall place the plate with current at the distance $z = z_1$, in accord with formula (9). We shall neglect the current sheet's inflection as it approaches the Earth, which obviously exists in reality.

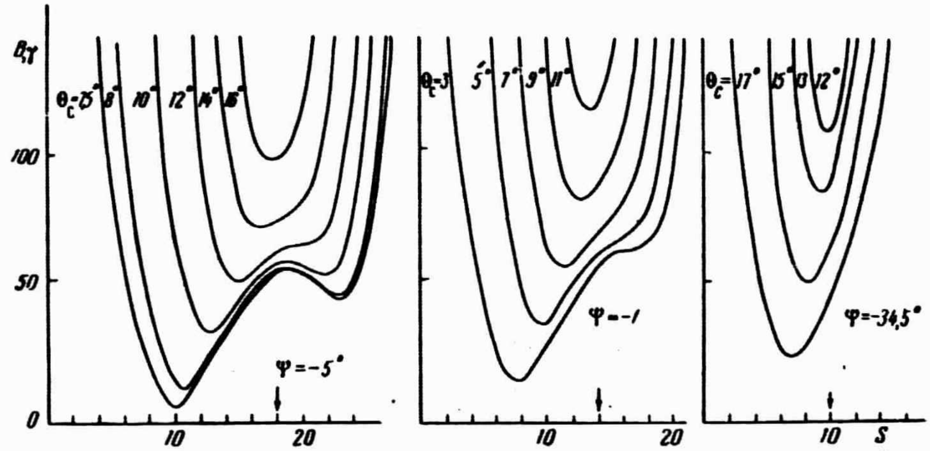

Fig.5

Shown in Figures 4a, б, в are the boundary of the magnetosphere and the field lines in the central meridian plane for various inclination angles: $\psi = -5, -11, -34{,}5°$ ($z_1 = 1.5, 2.2, 6.2$) respectively for parameters $x_1 = -11$, $x_2 = -51$, $y_1 = \infty$

As may be seen from Fig.5, analogous to Fig.2, but with increased inclination angle ($\psi = -5, -11, -34.5°$ respectively) the depth of one of the field intensity minima along the field lines from region 2 decreases and in the final count the minimum vanishes altogether. Here the arrow indicates the position of the geomagnetic equator and θ_c is the polar angle.

**** THE END ****

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References follow ../..

R E F E R E N C E S

1. V. P. SHABANSKIY. Space Research, 5, 125, 1965.

2. V. P. SHABANSKIY. Geomagnetizm i Aeronomiya, 7. 205, 1967.

3. E. W. HONES. J.Geophys. Res., 68, 1209, 1963.

4. D. B. BEARD. Ibid. 65, 3559, 1960.

5. J. R. SPREITER, R. R. BRIGGS. Ibid. 67, 2883, 1962.

6. J. R. SPREITER, B. J. HUYETT. Ibid. 68, 1631, 1963.

7. G. D. MEAD. Ibid. 69, 1181, 1964.

8. K. W. BEHANNON, N. F. NESS. Ibid. 71, 2337, 1966.

9. T. MURAYAMA. Ibid, 71, 5547, 1966.

10. D. J. WILLIAMS, G. D. MEAD. Ibid, 70, 3017, 1965.

11. J. G. ROEDERER. Ib., 72, 981, 1967

12. G. L. SISCOE. Plan. Space Sci., 14, 947, 1966.

13. A. E. ANTONOVA, V. P. SHABANSKIY. Sb."Kosmicheskiye Luchi", No.11. Izd-vo "NAUKA", (IN PRINT)

14. A. E. ANTONOVA, V. P. SHABANSKIY. Izv AN SSSR, ser.fiz. 32, No.11, 1968. (still not issued)

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