4. NONLINEAR ACOUSTIC THEORY FOR THIN POROUS SHEETS

By William E. Zorumski and Tony L. Parrott

NASA Langley Research Center

SUMMARY

A nonlinear theory is presented which predicts the acoustic properties of thin porous sheets of material in a high-intensity noise environment. The nonlinear theory utilizes two experimental measurements, the low-intensity impedance measurement and the flow-resistance measurement. The basis of the theory is described and results of the theory are compared with experimental data. It is concluded that use of this theory may eliminate the need for impedance tests in the high-intensity range and that nonlinearity can be a useful property of an acoustic absorber.

4

INTRODUCTION

The purpose of this paper is to discuss some of the basic concepts underlying the acoustical properties of duct-lining materials for airplane engines and nacelles and to construct a mathematical model for these materials which will predict their absorption of high-intensity sound. Sound intensities of the order of 140 to 160 dB are often found in the interior of modern design airplane engines. These acoustic intensities are in the nonlinear range where the classical linear theory of acoustic absorption needs to be modified. This paper presents an acoustic theory for the behavior of thin porous sheets of material in a high-intensity noise environment. The nonlinear theory utilizes two fundamental acoustic measurements, the impedance measurement and the flow-resistance measurement.

For the past 30 years, acoustic materials have been characterized by their impedance, a complex number which uniquely relates the magnitude and phase of a reflected wave to an incident wave. In figure 1 this quantity Z, measured in rayls, is defined as the complex ratio of the pressure to the velocity p/V of the sound waves at the sample. The real or acoustic resistance part of the impedance R is due to viscous action in the sample, and the imaginary or reactive part X is due to the momentum of the air within the sample. The device used to measure the impedance is a simple standing-wave tube or impedance tube. The acoustic intensity or sound pressure level SPL where the impedance begins to depend on SPL determines the nonlinear range for the material. This intensity is often about 120 dB for the thin sheet materials being discussed.

Figure 2 shows another measure of the properties of an acoustic material. The steady flow resistance R, also measured in rayls, is the ratio of the pressure drop across a sample to the velocity through the sample $\Delta p/V$. In the linear range this ratio is constant, whereas it depends on velocity in the nonlinear range. In the linear region the acoustic and steady-flow velocities are approximately equal; in the nonlinear region they are related by the nonlinear theory.

In the linear range of material behavior, the impedance tube is easy to use, acceptably accurate, and does not need to be supplemented by the flow-resistance test. In the high-intensity or nonlinear range, the impedance tube may still be used; however, many tests are required since the properties of the absorber now depend on SPL and frequency. Also, because of design limitations, the intensity which may be achieved with impedance tubes is often less than that for which data are desired. It is clear that alternate theoretical and experimental methods are needed for extending acoustic properties into the nonlinear range. The following theory shows how the steady-flow test and a low-intensity impedance-tube test may be used to obtain information about material behavior in the high-intensity or nonlinear region.

SYMBOLS

c	speed of sound
d	depth
e	base of natural log system
f	frequency
$i = \sqrt{-1}$	
M	Mach number
N_{Re}	Reynolds number
m,k	integers used in summation limits and corresponding subscripts and superscripts
p	pressure
Δp	change in pressure
R	acoustic resistance; steady-flow resistance

SPL	sound pressure level
t	time
V	acoustic velocity
v	velocity
w	width
X	acoustic reactance
X_L	low-intensity acoustic reactance
x	distance
Z	acoustic impedance
α_N	normal absorption coefficient
ρ	mass density
Φ_a, Φ_b	velocity potential Φ for compartments a and b, respectively

THEORY

The nonlinear acoustic theory is developed from the laws of mechanics and from dimensional analysis as shown in figure 3. Consider a very thin porous sheet which is enclosed by a hypothetical control volume. The thickness of the sheet is assumed to be small compared with the wavelength of sound. On either side of the sheet, there are small acoustic velocities and pressures which must be related. The law of conservation of mass requires that the acoustic velocity, normal to sheet, is the same on both sides. This requirement exists essentially because the sample is too thin to store mass; thus, an inflow on one side must be immediately matched by an outflow on the other side. This simple statement is the first part of the nonlinear theory.

The second statement of the theory is obtained from the momentum law. When this law is applied to the control volume enclosing the screen, it is found that the pressure change, or drop across the screen, depends on two terms which must be determined experimentally. The first term is the steady-flow pressure drop. Dimensional analysis indicates that this term will depend on the acoustic Reynolds and Mach numbers. Experimental investigation has shown that, for a given sample, the steady-flow pressure drop is a nonlinear function of the acoustic velocity because Reynolds and Mach numbers are proportional to this quantity. The second term is the time rate of change of momentum inside the sample. This term has been found to depend on acoustic velocity and frequency and is related to the reactance measured in the low-intensity impedance-tube test.

The two instantaneous laws which relate acoustic pressures and velocities across the material form the nonlinear acoustic theory. These laws hold continuously in time, in contrast to impedance laws which hold as averages over a cycle. This theory may be compared with the standard impedance theory by using it to solve the impedance-tube problem. In figure 4, a theoretician's view of the impedance tube is shown. The tube is terminated by a speaker at one end and a rigid backing at the other. At these points impedance boundary conditions are used. The tube is separated into two compartments by the material sample. In each part, a velocity potential Φ must satisfy a time-dependent wave equation. The instantaneous laws for velocity and pressure are used across the sample. It is not possible to find a simple harmonic solution to this problem because of the nonlinear effect of the material. It is possible, however, to find a periodic solution by expanding the acoustic velocity at the sample $V(t)$ in terms of a Fourier series. The problem is then to find the coefficients V_k of this series, which are the complex velocities of all the harmonic waves at the screen. The tedious details of solving this problem are omitted here. This solution leads to a nonlinear definition of impedance, which is described by figures 5 and 6.

Experimentally measured pressure differentials $\Delta p/\rho c^2$ for a typical sample are shown in figure 5 as a function of velocity V/c. The extended dashed line indicates how the pressure differential could vary if the material were linear. Actual measurements fall along a curve which bends upward and thus indicates nonlinearity. Shown in figure 6 is the general nonlinear impedance definition which uses these experimental data. The nonlinear impedance Z_k is defined as the ratio of the Fourier pressure coefficient p_k to the velocity coefficient V_k in a periodic wave. The subscript k denotes the kth harmonic of the fundamental period; thus, this equation actually defines a set of impedances Z_1, Z_2, ... The integral term gives the Fourier coefficient p_k. It depends on the experimental function Δp which, in turn, is a function of the complex velocity coefficients V_k. This relationship shows the dependence of the impedance on the amplitude and phase of all harmonics in a wave. In a boundary-value problem, such as the impedance-tube problem, the impedance must be found by an iteration procedure; this procedure also solves the problem. This property is characteristic of a nonlinear problem. The additional term X_L is the low-intensity reactance which is due to internal momentum. If the linear experimental curve is used in the general definition, it is found that the real part is simply the flow resistance R for small velocities, and the imaginary part is the low-intensity reactance X_L. When the nonlinear experimental curve is used, the integral term has both real and imaginary parts R_k and X_k so that the total reactance, as well as resistance, varies with SPL and frequency.

RESULTS

The theory has been compared with experimental data by making high-intensity impedance-tube tests on the sample of material whose pressure-differential curve is shown in figure 5. This material was selected because flow-resistance tests indicated that it is more nonlinear than several other popular materials and could be expected to show nonlinear features clearly. In figure 7 is shown a typical pressure signal at the face of the sample. This particular signal had a fundamental frequency of 1250 Hz and a fundamental SPL of 153 dB. The distortion of the wave was due to a third harmonic with a SPL of 135 dB. This curve qualitatively demonstrates the nonlinear effect. Figure 8 shows the comparison of experimental and theoretical impedance data for the fundamental wave. The theoretical curves were obtained by assuming a pure incident wave of 1250 Hz and various intensities. The data are presented in nondimensional form in terms of the characteristic impedance ρc of air which is 41.6 rayls (cgs). Theoretical data above 145 dB may be subject to correction because of the uncertainty in the numerical accuracy. This uncertainty is indicated by the short-dashed lines. The measured and computed resistance disagree slightly for low intensities, but agreement improves in the high-intensity range. This low-intensity discrepancy could be easily corrected by the artifice of replacing the low-velocity flow resistance by the low-intensity acoustic resistance. The reactance data automatically correspond with theory for low intensities and fall in the vicinity of the theoretical curves for high intensities, but high-intensity reactance points are scattered between 0 and 0.4 and it was not possible to reach intensities which would clearly show the theoretical dip in the reactance curve.

Many people have assumed that nonlinearity is an undesirable feature in an acoustic absorber. An attempt has been made to determine whether this assumption is true by computing the absorption of a pure wave of varying intensity which is normally incident on a nonlinear absorber. In figure 9, the absorption coefficient, which is the ratio of absorbed wave energy to incident wave energy, is plotted against a frequency parameter. For the sample used here, the absorption is low for low intensities and increases with intensity up until 150 dB. After 150 dB, the maximum absorption decreases slightly, whereas the low- and high-frequency absorption still increases. The resulting flat-topped curve has a widened absorption spectrum which is useful when there are significant variations of frequency. Consequently, nonlinearity can be used to advantage in absorber design.

CONCLUSIONS

The foregoing discussion can be summarized in the following statements: First, the new nonlinear theory uses instantaneous laws instead of impedances. The use of these laws leads to a generalized definition of impedance which has been used to compare this theory with experimental data. Limited data indicate that the theory is fairly accurate. The theory, therefore, may eliminate the need for high-intensity impedance testing. The material would be completely characterized by a low-intensity impedance-tube test and a steady-flow test. Finally, the theory indicates that nonlinearity can be useful.

ACOUSTIC IMPEDANCE

DEFINITION

$$Z = \frac{p}{V} = R + iX$$

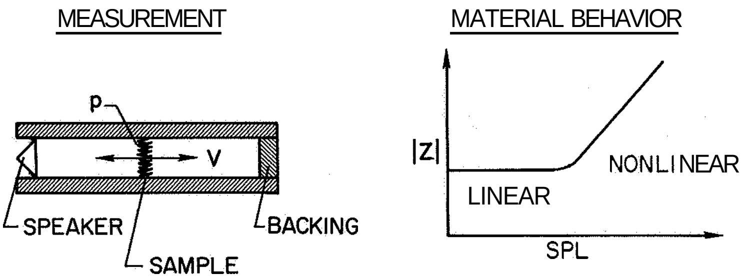


Figure 1

FLOW RESISTANCE

DEFINITION

$$R = \frac{\Delta p}{\dot{V}}$$

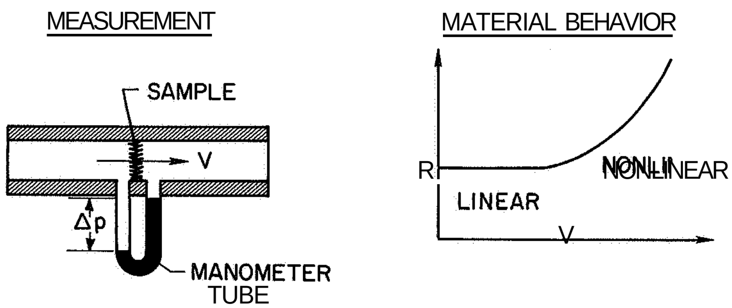


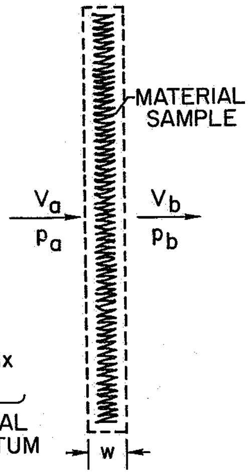
Figure 2

BASIS OF THE THEORY

INSTANTANEOUS ACOUSTIC LAWS

CONSERVATION OF MASS:

$$V_a = V_b$$

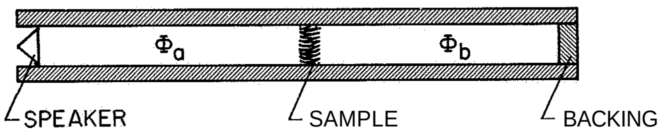


CONSERVATION OF MOMENTUM:

$$p_a - p_b = \underbrace{f(N_{Re}, M)}_{\text{PRESSURE DROP}} + \underbrace{\frac{\partial}{\partial t} \int_0^w \rho v dx}_{\text{INTERNAL MOMENTUM}}$$

Figure 3

IMPEDANCE-TUBE THEORY



WAVE EQUATIONS:

$$\frac{\partial^2 \Phi_a}{\partial x^2} = \frac{\partial^2 \Phi_a}{\partial t^2} \; ; \; \frac{\partial^2 \Phi_b}{\partial x^2} = \frac{\partial^2 \Phi_b}{\partial t^2}$$

NONLINEAR LAWS:

$$V_a(t) = V_b(t); \quad p_a(t) - p_b(t) = \Delta p\left[V(t)\right] + \frac{\partial}{\partial t} \int_0^w \rho v dx$$

PERIODIC SOLUTION:

$$V(t) = \sum_{k=-\infty}^{\infty} V_k e^{ikt}$$

Figure 4

MEASURED FLOW RESISTANCE
5-RAYL MATERIAL SAMPLE

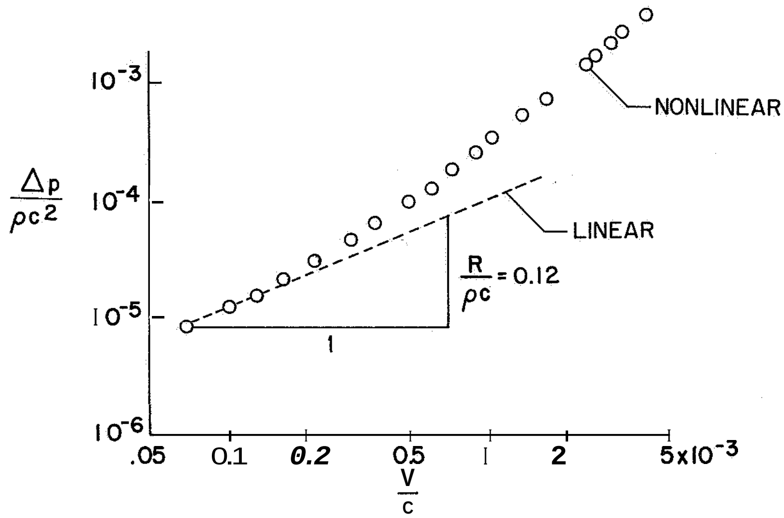


Figure 5

IMPEDANCE DEFINITIONS

GENERAL:

$$Z_k = \frac{p_k}{V_k} = \frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-ikt}\,\Delta p\left[\sum_{m=-\infty}^{\infty} V_m e^{imt}\right]dt}{V_k} + ikX_L$$

LINEAR:

$$Z_k = \underbrace{R}_{\text{FLOW RESISTANCE}} + \underbrace{ikX_L}_{\text{INTERNAL MOMENTUM}}$$

NONLINEAR:

$$Z_k = \underbrace{R_k + iX_k}_{\text{FLOW RESISTANCE}} + \underbrace{ikX_L}_{\text{INTERNAL MOMENTUM}}$$

Figure 6

EXAMPLE OF PRESSURE SIGNAL IN IMPEDANCE TUBE

153 dB

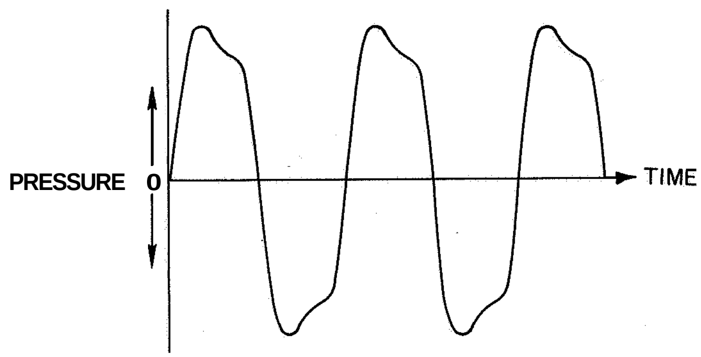


Figure 7

COMPARISON OF THEORY AND EXPERIMENT

5-RAYL MATERIAL; 1250 Hz

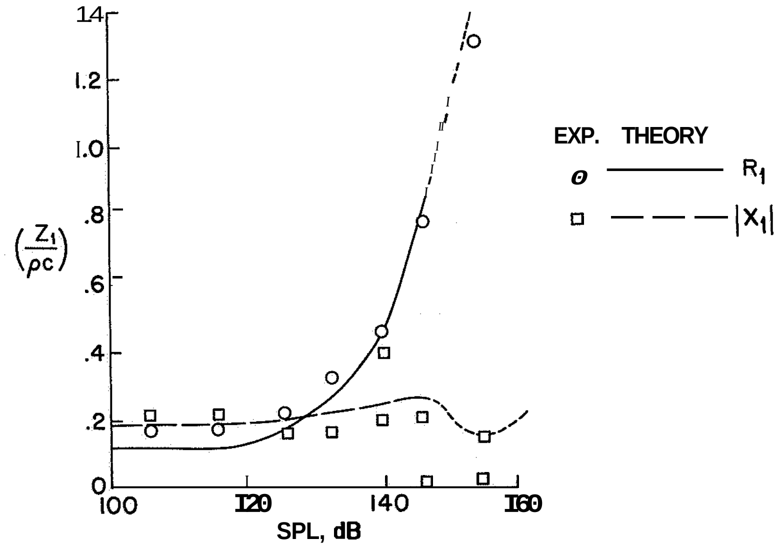


Figure 8

EXAMPLE OF NORMAL-INCIDENCE ABSORPTION

$$\alpha_N = \frac{\text{ABSORBED ENERGY}}{\text{INCIDENT ENERGY}}$$

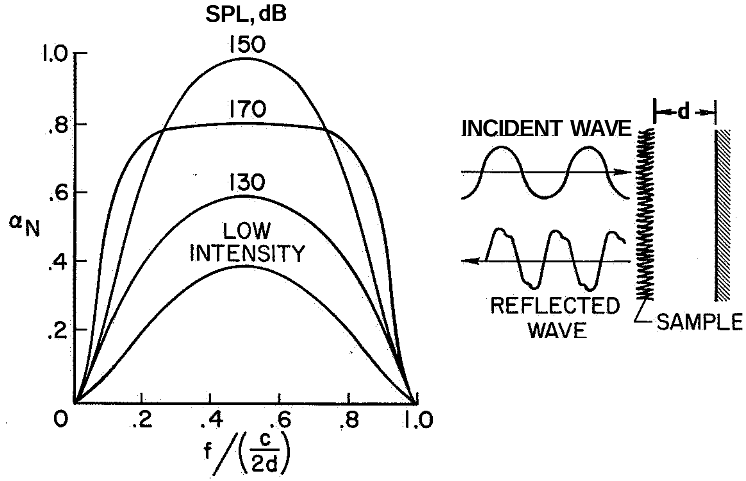


Figure 9