27. TECHNIQUE FOR CALCULATING OPTIMUM NOISE-ABATEMENT TAKE-OFF PROFILES

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SUMMARY

An analytical technique has been developed for determining take-off and climbout profiles of jet aircraft that minimize the noise in a noise-sensitive area near an airport. Because the technique is analytical, it is especially suited to the study of the effect of such factors as engine noise characteristics, location of noise-sensitive area, and operational constraints on the optimum profile for noise abatement.

Two important elements of the technique are the division of the ground track of the profile into a section near the airport having low sensitivity to noise, followed by one that is noise sensitive, and the formulation of a criterion for comparing the noisiness of different profiles. The criterion used in this study was the average perceived noise along the noise-sensitive section of the ground track. Any other criterion could be used instead.

The technique was applied to the calculation of optimum profiles for a typical currently inservice jet transport. Although the complete specification of the profiles generally depends on the noise characteristics of the engines and on other factors, the optimum profiles calculated herein can be characterized by a period of acceleration as soon as possible after take-off, followed by a steep climb, which in turn is followed by thrust reduction when the noise-sensitive area or a specified altitude is reached. Before the transition from accelerating to climbing, the optimum profiles achieved an airspeed that permitted full retraction of flaps. This acceleration caused some altitude loss at the beginning of the noise-sensitive area, but the disadvantage of a slightly lower altitude can be outweighed by the advantage of greater thrust reduction that is possible in the clean airplane configuration. Thus, in the trade off between airspeed and altitude, gaining airspeed until it is permissible to retract flaps can be more important than gaining altitude, if the objective is to minimize the average perceived noise along the noise-sensitive ground track.

A piloted fixed-base simulation of take-off profiles demonstrated the reduction in average perceived noise that is possible with the optimum climbout profile. No unusual difficulties in flying this profile on the simulator were encountered by the pilot.

INTRODUCTION

In this paper a technique is developed for determining take-off and climbout profiles of jet aircraft that minimize the flyover noise in a noise-sensitive area located near an airport. The technique offers an analytical approach to the take-off noise minimization problem and differs therein from earlier work on this problem which consisted mainly of analyzing noise signatures obtained from flight testing preselected take-off profiles (refs. 1 and 2). The paper emphasizes the mathematical formulation of the problem and the interpretation of results in terms of basic aerodynamic theory and the properties of jet noise. This analysis yields an improved understanding of the interplay between the many factors that affect flyover noise and offers to reduce considerably the amount of flight testing needed to establish noise-abatement procedures.

The analytical technique developed in this paper for calculating take-off profiles that minimize flyover noise was closely guided by methods found in optimum-control theory. Thus, the development of the technique begins with a description of the take-off profile problem; then a numerical criterion for the noisiness of a take-off profile is formulated. The proposed criterion requires calculation of perceived noise level. For turbojet noise, the noise prediction method of the Society of Automotive Engineers (SAE) can be utilized, but for turbofan noise, measured noise data must be used until more accurate analytical methods for predicting fan noise become available.

By means of the dynamic programing algorithm programed on a digital computer, take-off profiles that minimize the chosen noise criterion are generated. These optimum profiles are then simplified in order to make them easier to fly along. Simplified optimum profiles are calculated for both turbofan- and turbojet-powered aircraft.

In order to keep the computational difficulties associated with the optimum-profile calculations within manageable bounds, it was necessary to use simplified equations of motion for the aircraft. The effect of these simplifications as well as the pilot's ability to fly along the profiles were evaluated in a piloted simulator study,

SYMBOLS

C_D	drag coefficient
C_L	lift coefficient
F	total thrust, pounds force

F_i	total thrust used to compute noise in ith segment of noise-sensitive section of ground track, pounds force
g	acceleration of gravity, 32.2 feet/second^2
h	altitude, feet
h_i	altitude used to compute noise in ith segment of noise-sensitive section of ground track, feet
i	integers representing segments of noise-sensitive section of ground track
J	performance function
L	total number of small segments of the noise-sensitive section of ground track
$N = 2^{(Z-40)/10}$	
S	wing reference area, feet2
$\Delta t_i = \frac{\Delta x}{V_i}$	
Δt_{ref}	reference time for duration of noise perceived in each short segment of noise-sensitive section of ground track, seconds
V	airspeed, knots
V_2	take-off safety speed, knots
V_i	airspeed used to compute noise in ith segment of noise-sensitive section of ground track, knots
W	gross take-off weight (GTOW), pounds
x	distance along ground track, feet
Δx	length of each short segment of noise-sensitive section of ground track, 750 feet

Z	perceived noise level, PNdB
α	angle of attack, radians
γ	flight-path angle, degrees
δ	flap-deflection angle, degrees
P	air density, slugs/foot^3

ANALYSIS

Description of Take-Off Profile Problem

For the purposes of this study, the ground track corresponding to the take-off and climbout path of an aircraft consists of two major sections. The first section, which is assumed to have low sensitivity to noise but which could have a limitation on maximum sideline noise, begins at brake release and ends at the beginning of the noise-sensitive area. Typical values for its length are 3 to 5 miles.

The section of the ground track that traverses the noise-sensitive area is designated as the second section, and it is typically 4 to 8 miles long. The length of both sections generally depends on conditions existing at a particular airport. A complete take-off and climbout path showing the location of the two sections along the ground track is illustrated in figure 1.

The entire ground track is taken to be a straight line parallel to the runway. This assumption is justified whenever the noise-sensitive area cannot be avoided by early turning maneuvers either because of unfavorable terrain or because the airport is closely surrounded by populated areas, all sensitive to noise.

In simplest terms, the objective of the study is to determine the flight path, subject to the operating limits of the aircraft, that is least annoying to people living in the noise-sensitive area. In determining this optimum flight path one must take into consideration the assumption that the ground track is composed of two adjacent sections, one having low sensitivity to noise and the other being noise sensitive. The implication is that the flight path cannot be optimized independently for each section since the flight path over the first section strongly affects the noise over the second section through the altitude, airspeed, and power setting at the beginning of the second section.

Mathematical Criterion for Noisiness of Profile

Application of optimum-control theory to the noise problem requires the defining of a performance function or noise criterion to be minimized by the take-off profiles. An obvious requirement of the performance function is that it be closely related to the perceived noise level in the noise-sensitive area, but this restriction still permits considerable latitude in its selection. For instance, one could simply establish as the noise criterion the perceived noise level in units of PNdB measured at the boundary of the noise-sensitive area (point B in fig. 1 as well as in subsequent figs.) after thrust has been reduced to meet some specified flight condition. This performance function has the advantage of simplicity and would be adequate if the noise-sensitive area were concentrated at a single point on the ground track. However, if the noise-sensitive section of the ground track is several miles long, as it is at most airports, minimizing the-noise at only a single point may be unrealistic since this procedure discriminates against other points along the noise-sensitive section.

The performance function defined below attempts to overcome the difficulty with measuring the noise at only a single point by averaging the noise levels perceived along all points of the noise-sensitive second section. An approximate but sufficiently accurate method for computing the average noise along the second section is to divide this noise-sensitive section into L short segments, to compute the maximum flyover noise for each segment, and then to average these values. Except for the factor $\sqrt{\frac{\Delta t_i}{\Delta t_{ref}}}$, which models the duration effects and which is discussed in greater detail subsequently, the essential content of the performance function is

$$J = 10 \log_2 \frac{1}{L} \sum_{i=1}^{L} N(F_i,h_i,V_i) \sqrt{\frac{\Delta t_i}{\Delta t_{ref}}} \tag{1}$$

The quantity N rather than Z is used in equation (1) because it puts heavier penalty on high noise levels than the logarithmically dependent Z does, and also because it is convenient computationally.

It is well known that the subjectively judged noisiness of a sound depends not only on acoustic power and spectral content but also on duration (ref. 3). The complete relationship between duration and perceived noise is too complex to be considered herein, but an approximation sufficient for the purpose of this study is to assume that doubling the exposure time of a noise increases the perceived noise level by 5 PNdB. Since the values of N entering into equation (1) are antilogarithmically related to the perceived noise level in PNdB units, the equivalent operation on them for a 5-PNdB increase per doubling of duration is multiplication by $\sqrt{\frac{\Delta t_i}{\Delta t_{ref}}}$. The reference duration Δt_{ref} is arbitrary in this study, since it affects J only by a constant and thus has no influence on the determination of the profiles.

Method for Computing Perceived Noise

The performance function discussed in the previous section requires calculations of the perceived noise level as a function of thrust, altitude, and airspeed. For turbojet noise, the SAE noise prediction method described in references 4, 5, and 6 is employed to perform these calculations with the use of parameters of a typical turbojet engine. This method is reasonably accurate for predicting the maximum flyover noise for turbojet engines with standard exhaust nozzles. Fortunately, high accuracy in predicting noise levels is not needed in this study, since in minimizing such functions as the performance measure for noisiness, the absolute value of the function to be minimized is irrelevant. The important items in the minimization are the trade offs among the factors of thrust, altitude, and airspeed that enter into the evaluation of the function, Hence, a model for jet noise that preserves these trade offs, as the SAE model does, is sufficient.

For profile calculations involving turbofan noise, however, the currently available SAE noise prediction method, even with refinements introduced specifically to model the fan-generated noise, is inadequate. Comparison with measured turbofan noise data has often shown considerable error both in the absolute value of the calculated noise and in the accuracy of the trade offs between altitude and thrust. As just pointed out, it is the latter type of error that prohibits the use of this method for optimum-profile calculations. Instead, measured noise data as functions of thrust and altitude for a currently inservice turbofan were used directly. These data are reproduced in figure 2. The effect of changes in airspeed on noise, which is not given in figure 2, was assumed to be the same as for turbojet noise calculated by the SAE prediction method.

Since the optimum profiles are influenced mainly by the trade offs in the noise measure among altitude, thrust reduction, airspeed, and duration, it is instructive to tabulate the change in perceived noise caused by a doubling of each of these quantities. The following table shows the results for turbojet- and turbofan-generated noise:

Factors affecting flyover noise	Change in perceived noise caused by doubling of quantity, PNdB, for –	
	Turbojet	Turbofan
Thrust	9 to 15	4 to 9
Altitude	-9	-9
Airspeed	-2 to -8	Unknown
Duration	2 to 6	2 to 6

Particularly noteworthy is the fact that sensitivity of the noise to thrust changes is considerably higher for turbojets than for turbofans. This difference in noise characteristics of the two engine types has some effect on the optimum-climbout procedures.

Equations of Motion and Method for Computing Optimum Profiles

In order to evaluate the noise criterion described previously, one must be able to generate histories of altitude and airspeed along the ground track corresponding to a specific take-off procedure. In this study, it is convenient to describe a take-off procedure by giving the thrust, the flight-path angle, and the flap-deflection angle as functions of the distance along the ground track. These three quantities are the only control variables that need to be considered, since the ground track of the climbout is assumed to be a straight line, with lateral control maneuvers therefore excluded. The equations of motion are then given as

$$\frac{dV}{dx} = \frac{g}{WV \cos \gamma}\left[F \cos \alpha - \frac{1}{2}\rho S V^2 C_D(\alpha, \delta) - W \sin \gamma\right] \tag{2}$$

and

$$\frac{dh}{dx} = \tan \gamma \tag{3}$$

These equations are in standard form except that x plays the role of the independent variable rather than time, as is usually the case. The angle of attack α, which is needed to solve equation (2), is calculated by solving the following equation for α:

$$\frac{1}{2}\rho S V^2 C_L(\alpha, \delta) - W \cos \gamma + F \sin \alpha = 0 \tag{4}$$

Equation (4) is based on the assumption that the centripetal acceleration during changes in flight-path angle is negligibly small. This equation can be solved for α by approximating $\sin \alpha$ with α and by approximating C_L with a linear function in α and δ. Equations (2) to (4) allow one to generate take-off profiles if the initial altitude and airspeed are specified and if the control variables F, γ, and δ are assigned specific functions of x. However, these equations were used only for generating the part of the profile that begins at a point where the aircraft has reached a 400-foot altitude and the take-off safety speed V_2 (point A in fig. 1). The location of this point can be calculated for a particular aircraft by using the procedure described in its flight manual.

Calculating take-off profiles that minimize the chosen criterion for noisiness can now be interpreted as a problem in optimum control in which the state variables are airspeed and altitude and the control variables are thrust, flight-path angle, and flap-deflection angle. Solutions of this optimum-control problem were obtained by implementing the dynamic programing algorithm on a digital computer (ref. **7).**

RESULTS OF THEORETICAL CALCULATIONS

General Properties of Optimum Profiles

Optimum profiles computed for the previously introduced noise criterion have been found to depend strongly on the engine type (whether turbojet or turbofan), on the length of the first and second sections of ground track, and on operational constraints. Although influenced by many variables, an optimum profile, represented herein as a history in altitude-airspeed coordinates, typically has the form shown in figure **3.** For the purposes of this study, the optimum profile is assumed to begin at a point, marked A in figure **3,** where the aircraft has achieved the take-off safety speed V_2 and a 400-foot altitude, since before reaching this airspeed and altitude no unusual maneuvers are permitted. Thus, starting at point **A,** the aircraft accelerates in level flight until a certain climb speed, which is often close to the minimum drag speed, has been attained. During this acceleration period, flaps are retracted as soon as the minimum speed for flap retraction is achieved. After the acceleration period, the aircraft enters a steep climb that is essentially constant until a point just ahead of the noise-sensitive area, At that point, the climb steepens to become a decelerating climb. Then a large power reduction occurs as the noise-sensitive area is penetrated at point B. The remainder of the profile, although depending somewhat on the length of the second section of the ground track, consists of a slightly decelerating climb at the minimum permissible power setting.

Profiles that minimize the perceived noise only at the beginning of the noise-sensitive area (point B), although not shown herein, have similar characteristics. In such profiles, the climb speed has been found to depend more strongly on the length of the first section than in the previously discussed profiles; here the climb speed decreases and eventually approaches V_2 as point B moves toward point A.

Simplification of Optimum Profiles

In assessing the practical value of noise-optimum profiles, one must consider the difficulty that a pilot would experience in flying along them and the number of parameters required to describe them. Examined in this light, the optimum profile shown in figure **3** is too complicated and therefore must be simplified before it can be put to practical use. Such simplification of optimum profiles is found to be necessary in most practical applications of optimal control.

The optimum profile between points A and B consists essentially of a period of maximum acceleration followed by a period of maximum climb. Therefore, a logical choice for a simplified optimum profile would be one that accelerates as fast as possible to a certain airspeed, then climbs at constant airspeed, and finally enters a reduced-power flight near the beginning of the noise-sensitive area. Such a simplified optimum profile is indicated in figure 3 by the dashed line. In effect, this simplified profile needs to be optimized only over two parameters, namely the climb speed and the amount of thrust reduction. The computation is thus simplified, and a profile that is easier for the pilot to fly along is produced.

However, the decisive test of acceptability of the simplified optimum profile is given by the penalty measured in terms of the noise generated by it in comparison with the minimum noise. In all profiles examined, the noise generated by the simplified profile exceeds that of the optimum profile by less than 0.5 dB. Hence, only simplified optimum profiles are presented in the next section, although they are also referred to as optimum profiles.

Minimizing Average Noise Over Noise-Sensitive Area With Altitude Constraints

Profiles that minimize the performance function defined by equation (1) are presented. The profile calculations were performed for a typical large jet transport powered by either turbofans or turbojets at a gross weight of 280 000 pounds. The length of the first section of ground track was chosen as 21 000 feet and that of the second section as 26 000 feet. In the second section, thrust was constrained to be not less than that needed to maintain level, unaccelerated flight at the chosen climb speed. Constraints on the thrust based on maintaining some nonzero rate of climb over the noise-sensitive area have also been investigated and were found to yield similar results. Since operational or safety reasons may dictate that the aircraft first achieve some minimum altitude above ground level before power is reduced in a noise-abatement climbout, the optimum profiles were calculated subject to the constraint that maximum power reduction not take place until an altitude of 1500 feet is attained. However, some power reduction is assumed to take place at point B, even if the aircraft is at a lower altitude, in order not to exceed an upper limit on the perceived noise that is assumed to exist at the beginning of the noise-sensitive area. Thrust after this initial power reduction was taken as 40 000 pounds, which is assumed to satisfy the maximum noise limitation.

In calculating the optimum profiles a search was conducted not only over all climb speeds but also over the amount of thrust reduction after a 1500-foot altitude is achieved in order to find the combination that minimizes equation (1). But it was found that the optimum thrust after final thrust reduction for either turbofans or turbojets was always very close to the smallest thrust allowed by the constraint. It should be pointed out, however, that for second-section lengths much longer than 5 miles, the optimum thrust after power reduction does increase above the smallest value allowed by the constraint.

Figure 4 shows optimum profiles for turbofan- and turbojet-powered aircraft and also a nonoptimum profile consisting of a steep climbout with maximum take-off flaps. The optimum profiles, which at initial penetration of the noise-sensitive area obviously produce more noise than the steep-climbout profile, nevertheless produce a lower average noise value. The chief reason is the fact that with fully retracted flaps, considerably less thrust is required to maintain level, unaccelerated flight than with 25° take-off flaps. As mentioned previously, noise produced by an overflight of a jet aircraft depends very strongly on the amount of thrust developed by its engines, along with the altitude and airspeed of the aircraft. Thus, acceleration to airspeeds at which flaps can be retracted, even at the expense of some altitude loss, permits flight at lower thrust levels than is possible with take-off flaps and thereby helps to reduce the noise. In addition to the effect of airspeed and flap setting on noise through the influence of these factors on thrust reduction, an increase in airspeed also helps to reduce the perceived noise by decreasing its duration, The computed profiles in figure 4 optimize the trade off between using the available thrust to gain altitude, on the one hand, and to gain airspeed, on the other, so that the average noise level produced along the noise-sensitive section of the ground track is minimized. The steep climbout profile attains the minimum altitude of 1500 feet just as the noise-sensitive area is penetrated at point B and, therefore, immediately permits maximum power reduction to take place. The optimum profiles, however, first accelerate to the indicated climb speeds, then climb at full power as far as the noise-sensitive area and continue to climb at reduced power to 1500 feet, where the final power reduction occurs. The values of thrust used on the three profiles after power reduction has occurred are given in figure 4.

Figure 4 also gives the differences between the values of the performance function (eq. (1)) obtained for the steep-climbout profile and those obtained for the optimum profiles. The noise reduction is higher for the turbojet than for the turbofan because of the greater effect of thrust reduction on turbojet noise.

The profiles as well as the reductions in average noise level given in figure 4 are based on the performance of a large, currently inservice jet transport and on representative noise characteristics of jet engines. These data are presented herein to illustrate the theory and could easily be recalculated for specific aircraft and airport conditions.

SIMULATION RESULTS

This section describes the results of flying noise-abatement profiles on a fixed-base simulator for a large jet transport. Flying the profiles on a simulator was thought to provide an independent means of checking the basic theory developed herein and to uncover difficulties that a pilot may encounter in flying along the optimum profiles. A gross weight of 300 000 pounds was simulated in this study.

The performance function was again assumed to be the average perceived noise along the second ground-track section. In agreement with current operational thinking, an altitude constraint of 1500 feet was again placed on the initial point for power reduction. However, in the theoretical study, thrust after final power reduction could be as low as necessary to maintain level flight at constant airspeed, whereas in the simulation, the pilot's goal after achieving a 1500-foot altitude was a throttle setting that would yield a 500-ft/min rate of climb and would thereby ensure a positive climb gradient over the noise-sensitive area. This positive climb gradient is desirable for operational and safety reasons.

The noise-level calculations performed in the simulation were based on the turbofan noise data given in figure 2. The perceived noise level calculated from this figure was reduced by 1.5 PNdB for the optimum profile in order to account for airspeed and duration differences between the steep-climbout and optimum profiles.

The profiles are shown in figure 5; one is a steep-climbout noise-abatement profile described previously, and the other is a typical pilot's approximation of an optimum profile. The beginning of the noise-sensitive area was assumed to be located at point B, 29 000 feet from brake release, where the steep-climbout noise-abatement profile achieves a 1500-foot altitude. Figure 5(a) shows that the optimum profile requires an additional distance of 2500 feet along the ground track to achieve the same altitude. This distance is the penalty for accelerating from V_2+18 knots, with 25° flaps used on the steep-climbout profile, to V_2+50 knots, with retracted flaps used on the optimum profile. As expected, along this section of the ground track the perceived noise is higher for the, optimum than for the steep-climbout profile (see fig. 5(c)). However, once the aircraft reaches 1500 feet on the optimum profile, the pilot reduces the thrust 28 percent more than on the steep-climbout profile, as shown in figure 5(b). This greater thrust reduction causes the noise level to drop and remain below that of the steep-climbout profile for the remaining 24 000 feet of the noise-sensitive area.

Generally, the pilot encountered no exceptional difficulty in flying along the simplified optimum profiles. However, practice did improve his timing in initiating flap retraction and his accuracy in setting the throttles for a 500-ft/min rate of climb.

CONCLUDING REMARKS

A rational technique for determining take-off and climbout profiles that minimize the annoyance from jet take-off operations in communities located along the climbout path has been established. What distinguishes this technique from others used in the past is the mathematical formulation of the problem and its solution by purely analytical methods. The technique permits computation of optimum take-off trajectories for a particular aircraft operating from a particular airport subject to prescribed operational constraints.

A mathematically defined criterion for noisiness of a take-off procedure was formulated and then used as a basis for arriving at noise-optimum profiles. The criterion was taken as the average noise level, including a penalty on the duration of the noise, produced by an overflight of a noise-sensitive area. Any other criterion can be easily used instead.

The technique was applied to the calculation of optimum take-off profiles for a typical large jet transport. Although the optimum profiles were found to depend upon many factors, such as the noise characteristics of the jet engines and the length of sections of ground track, some generally valid properties of the profiles can be discerned. The optimum profiles calculated have a period of acceleration as soon as possible after take-off, followed by a steep climb, which in turn is followed by thrust reduction when the noise-sensitive area or a specified altitude is reached. Before the transition from accelerating to climbing, the optimum profiles achieve an airspeed that permits full retraction of flaps. This acceleration causes some altitude loss at the beginning of the noise-sensitive area, but the disadvantage of a slightly lower altitude can be outweighed by the advantage of greater thrust reduction that is possible in the clean airplane configuration. Thus, in the trade off between airspeed and altitude, gaining airspeed until it is permissible to retract flaps can be more important than gaining altitude, if the objective is to minimize the average noise along the noise-sensitive ground track.

A piloted fixed-base simulation of take-off profiles demonstrated the reduction in average perceived noise that is possible with the optimum climbout profile. No unusual difficulties in flying along this profile were encountered by the pilot.

REFERENCES

1. Copeland, W. Latham; Hilton, David A.; Huckel, Vera; Dibble, Andrew C., Jr.; and Maglieri, Domenic J.: Noise Measurement Evaluations of Various Take-Off - Climbout Profiles of a Four-Engine Turbojet Transport Airplane. NASA TN D-3715, 1966.

2. Galloway, W. J.; Pietrasanta, A. C.; and Pearsons, K. S.: Study of the Effect of Departure Procedures on the Noise Produced by Jet Transport Aircraft. ADS-41, FAA, Oct. 1965.

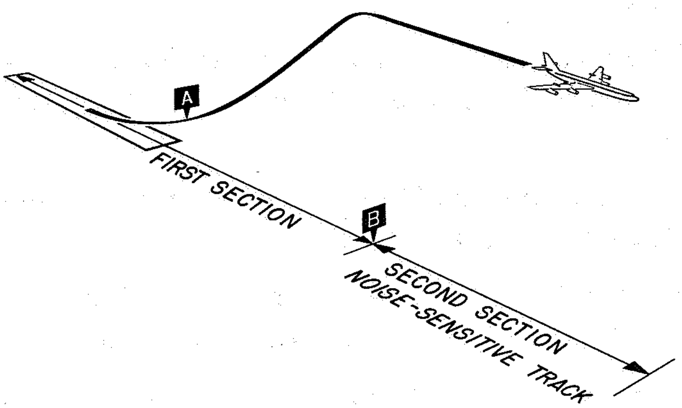
3. Pearsons, Karl S.: The Effects of Duration and Background Noise Level on Perceived Noisiness. ADS-78, FAA, Apr. 1966.

4. Anon.: Definitions and Procedures for Computing the Perceived Noise Level of Aircraft Noise. ARP 865, Soc. Automot. Eng., Oct. 15, 1964.

5. Anon.: Jet Noise Prediction. AIR 876, Soc. Automot. Eng., July 10, 1965.

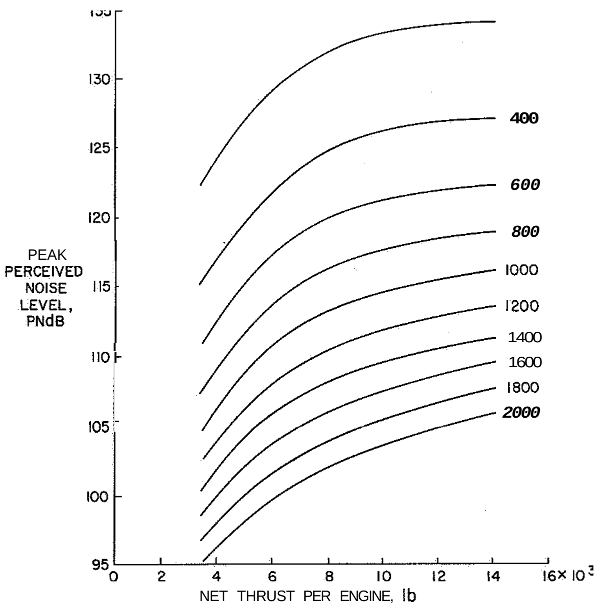
6. Lee, Robert; Farrell, James; Henry, George; and Lowe, Albert: Procedures for Estimating the Effects of Design and Operational Characteristics of Jet Aircraft on Ground Noise. NASA CR-1053, 1968.

7. Bellman, Richard: Dynamic Programming. Princeton Univ. Press, 1957.



TWO-SECTION CLIMBOUT

Figure 1



PERCEIVED NOISE AS A FUNCTION OF THRUST AND ALTITUDE FOR A CURRENTLY INSERVICE TURBOFAN

Figure 2

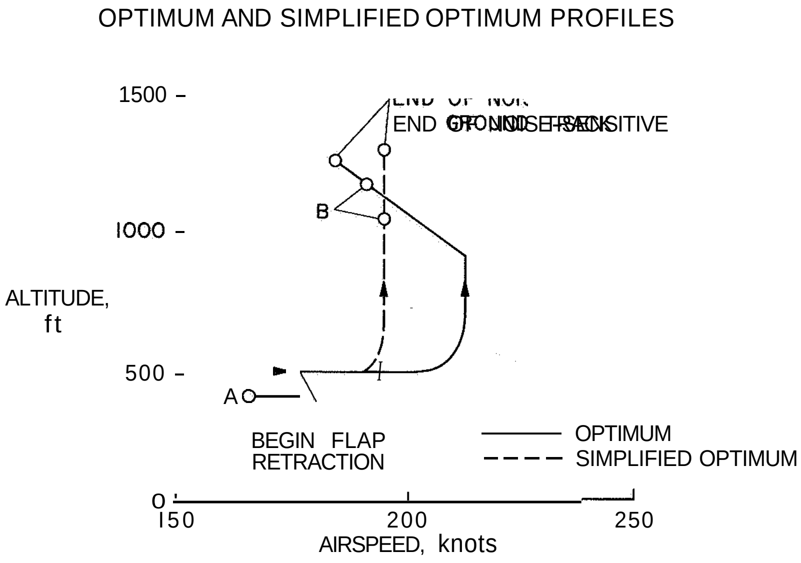


Figure 3

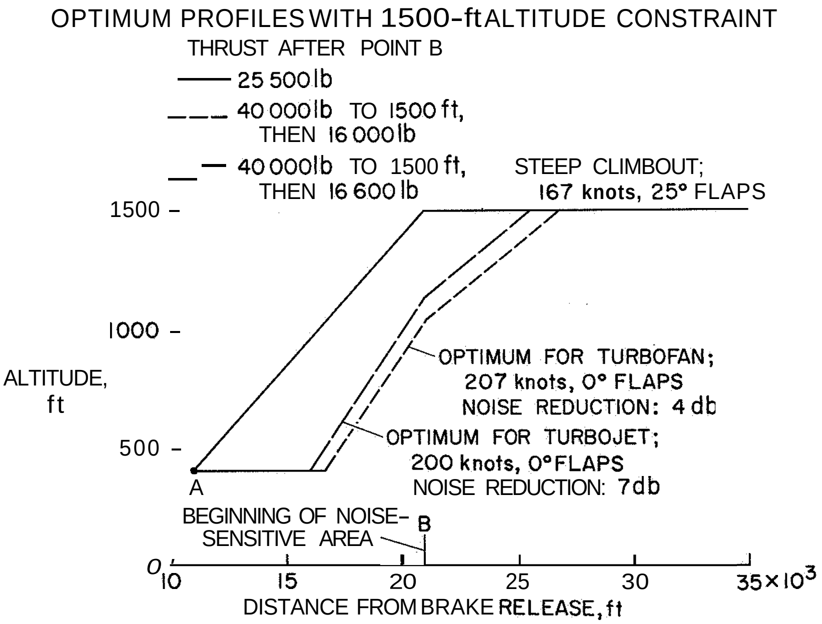
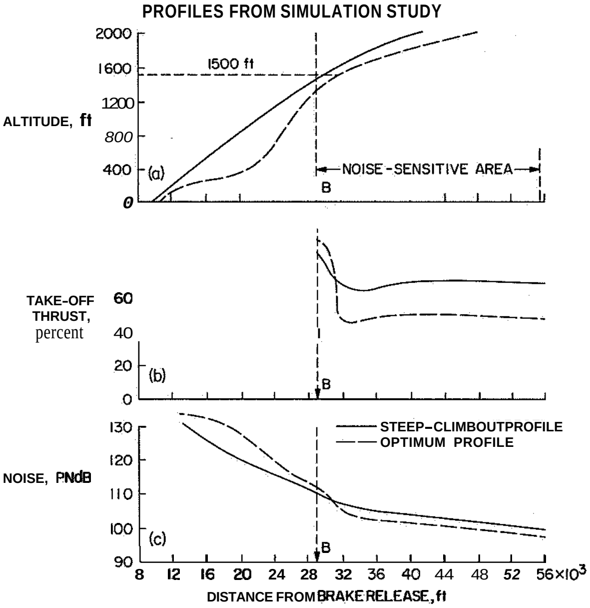


Figure 4


PROFILES FROM SIMULATION STUDY

NOTE: GTOW = 300 000 lb; TAKE-OFF THRUST = 15 600 lb/ENGINE
STEEP-CLIMBOUT PROFILE 25° FLAPS, V_2 + 18 knots
OPTIMUM PROFILE: BEGIN FLAP RETRACTION AT V_2 + 15 knots; FINAL CLIMB SPEED, V_2 + 50 knots

Figure 5