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SUMMARY

Buckling equations are solved for a simply supported equatorial segment of a spherical shell loaded in the axial direction by its own weight. The Galerkin method is used to obtain a critical-thickness parameter for buckling as a function of the shell material and geometric properties and of the relative proportions of the shell weight supported at the upper and lower edges. Results of the present method are compared with those of a simplified approach in which the spherical shell segment is assumed to buckle if the maximum compressive stress is greater than the critical compressive stress for a complete spherical shell loaded by uniform external pressure. This comparison indicates that the simplified approach yields larger thicknesses than the method presented in the present report. The particular proportions of the shell weight which should be supported at the upper and lower edges to minimize the tendency to buckle are also determined. Results have been used in the design of a large rectangular reinforced-plastic shell segment which simulates the lunar surface in the lunar orbit and landing approach simulator at the Langley Research Center.

INTRODUCTION

The lunar orbit and landing approach simulator at the Langley Research Center incorporates a large rectangular segment of a thin-walled spherical shell which simulates the lunar surface. (See fig. 1.) This shell segment is loaded by its own weight and can be supported completely at either its upper or lower edge or can be supported in various proportions at these edges. (See fig. 2.) To design this shell segment properly, its stability must be considered. The stability of the rectangular segment can be approximated by the solution to the buckling equations for a complete circumferential segment of a spherical shell centered at the equator (equatorial segment) and loaded by its own weight; this problem is considered herein.

A simplified approach to the problem is to assume that the spherical shell segment buckles if the maximum compressive stress, as predicted by linear membrane theory, is greater than the critical compressive stress for a complete spherical shell loaded by uniform external pressure. The purpose of this paper is to present a more realistic solution, to compare this solution with the simplified approach, and to determine how best to distribute the supporting line reactions at the upper and lower edges of the shell so that buckling tendencies are minimized.

In the present paper a modified form of the buckling equations for shallow shells as given in reference 1 is utilized. These basic equations can be derived by a method given in reference 2 in which the shallow shell is represented as a flat plate with an initial deflection. To solve the present problem, the shell geometry is specialized to an equatorial segment of a spherical shell, and the Galerkin method is used to solve the resulting equations for a critical-thickness parameter. Plots are given showing the critical-thickness parameter for buckling as a function of shell material and geometric properties and of support conditions and comparing the present results with results of the simplified approach mentioned previously.

SYMBOLS

A_m, A_q, A_M	constants $\left(A_1, A_2, \ldots, A_M\right)$
$\{A\}, \{B\}$	$(M \times 1)$ column matrices
$\left[a_{qm}\right], \left[b_{qm}\right]$	$(M \times M)$ matrices
C	constant of integration
D	flexural stiffness of shell wall, $\dfrac{Et^3}{12\left(1 - \mu^2\right)}$
E	Young's modulus
I	unity matrix
k	proportion of weight of equatorial segment supported at upper edge
L	length of equatorial segment
M	number of terms in assumed truncated series solution
m,q	integers

N integer

N_x,N_y,N_{xy} membrane stress resultants (see fig. 3)

n number of complete circumferential waves

p_x,p_y,p_n components of body force acting in the meridional (also in the axial), circumferential, and normal directions, respectively

R radius of equatorial segment

t shell thickness

U,V,W meridional, circumferential, and normal displacements of the spherical shell, respectively

x,y,z axial, circumferential, and radial coordinates, respectively

α specific-weight parameter, $\gamma L/E$

$\beta = R/L$

γ weight per unit volume of shell-wall material (on earth, gravitational acceleration is 32.17 ft/sec^2 or 9.807 m/sec^2)

∇^2,∇^4 Laplacian and bi-Laplacian operators, respectively

λ thickness parameter, $\dfrac{t^2}{12\left(1 - \mu^2\right)R^2}$

λ_{cr} critical-thickness parameter obtained by the present method

λ^* critical-thickness parameter obtained by the simplified approach

μ Poisson's ratio

Φ stress function

Subscripts:

A quantities associated with prebuckling state

B quantities associated with buckling state

BUCKLING OF THE EQUATORIAL SEGMENT

Buckling Equations and Boundary Conditions

A narrow equatorial segment of a thin-walled spherical shell is shown in figure 2. This shell segment is simply supported by vertical line reactions at its upper and lower edges. It is of interest to determine how thick this spherical segment must be to support its own weight without buckling.

For the equatorial segment the nonlinear equilibrium equations which include body-force terms can be written as follows:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + p_x = 0 \tag{1a}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + p_y = 0 \tag{1b}$$

$$D\nabla^4 W + \frac{N_x + N_y}{R} - \frac{\partial}{\partial x}\left(N_x \frac{\partial W}{\partial x}\right) - \frac{\partial}{\partial y}\left(N_y \frac{\partial W}{\partial y}\right) - \frac{\partial}{\partial x}\left(N_{xy} \frac{\partial W}{\partial y}\right) - \frac{\partial}{\partial y}\left(N_{xy} \frac{\partial W}{\partial x}\right) + p_n = 0 \tag{1c}$$

The coordinate system of the shell segment is shown in figure 3; the y coordinate is the equator of the spherical shell. Membrane stress resultants, displacements, and body forces are also indicated in figure 3. Equations (1) are equivalent to equations (6) of reference 1 except that body forces are not included in reference 1. To obtain buckling equations, the stress resultants and normal displacement in equations (1) are regarded as being composed of a prebuckling component and a buckling component, a small change which occurs during buckling. Thus, the assumption is made that

$$N_x = N_{x,A} + N_{x,B} \tag{2a}$$

$$N_y = N_{y,A} + N_{y,B} \tag{2b}$$

$$N_{xy} = N_{xy,A} + N_{xy,B} \tag{2c}$$

$$W = W_A + W_B \tag{2d}$$

When equations (2) are substituted into equations (1) and products of buckling quantities are disregarded, the following equations are obtained:

$$\frac{\partial N_{x,A}}{\partial x} + \frac{\partial N_{xy,A}}{\partial y} + \frac{\partial N_{x,B}}{\partial x} + \frac{\partial N_{xy,B}}{\partial y} + p_x = 0 \tag{3a}$$

$$\frac{\partial N_{y,A}}{\partial y} + \frac{\partial N_{xy,A}}{\partial x} + \frac{\partial N_{y,B}}{\partial y} + \frac{\partial N_{xy,B}}{\partial x} + p_y = 0 \tag{3b}$$

$$\begin{aligned}
&D\nabla^4\left(W_A + W_B\right) + \frac{N_{x,A} + N_{y,A} + N_{x,B} + N_{y,B}}{R} - \frac{\partial}{\partial x}\left(N_{x,A}\frac{\partial W_A}{\partial x}\right) - \frac{\partial}{\partial y}\left(N_{y,A}\frac{\partial W_A}{\partial y}\right) \\
&- \frac{\partial}{\partial x}\left(N_{xy,A}\frac{\partial W_A}{\partial y}\right) - \frac{\partial}{\partial y}\left(N_{xy,A}\frac{\partial W_A}{\partial x}\right) - \frac{\partial}{\partial x}\left(N_{x,B}\frac{\partial W_A}{\partial x}\right) - \frac{\partial}{\partial y}\left(N_{y,B}\frac{\partial W_A}{\partial y}\right) \\
&- \frac{\partial}{\partial x}\left(N_{xy,B}\frac{\partial W_A}{\partial y}\right) - \frac{\partial}{\partial y}\left(N_{xy,B}\frac{\partial W_A}{\partial x}\right) - \frac{\partial}{\partial x}\left(N_{x,A}\frac{\partial W_B}{\partial x}\right) - \frac{\partial}{\partial y}\left(N_{y,A}\frac{\partial W_B}{\partial y}\right) \\
&- \frac{\partial}{\partial x}\left(N_{xy,A}\frac{\partial W_B}{\partial y}\right) - \frac{\partial}{\partial y}\left(N_{xy,A}\frac{\partial W_B}{\partial x}\right) + p_n = 0
\end{aligned} \tag{3c}$$

Buckling equations can be obtained from equations (3) on the basis of the following considerations: (1) The prebuckling terms alone must satisfy the nonlinear equilibrium equations (eqs. (1)). (2) As is conventional in shell buckling theory, the prebuckling state is assumed to consist of stresses given by the linear membrane theory and normal displacements which are independent of x and y. The resulting buckling equations are

$$\frac{\partial N_{x,B}}{\partial x} + \frac{\partial N_{xy,B}}{\partial y} = 0 \tag{4a}$$

$$\frac{\partial N_{y,B}}{\partial y} + \frac{\partial N_{xy,B}}{\partial x} = 0 \tag{4b}$$

$$D\nabla^4 W_B + \frac{N_{x,B} + N_{y,B}}{R} + p_x\frac{\partial W_B}{\partial x} + p_y\frac{\partial W_B}{\partial y} - N_{x,A}\frac{\partial^2 W_B}{\partial x^2} - N_{y,A}\frac{\partial^2 W_B}{\partial y^2} - 2N_{xy,A}\frac{\partial^2 W_B}{\partial x\,\partial y} = 0 \tag{4c}$$

Equations (4a) and (4b) can be satisfied identically by defining a stress function Φ_B such that

$$N_{x,B} = \frac{\partial^2 \Phi_B}{\partial y^2}$$

$$N_{y,B} = \frac{\partial^2 \Phi_B}{\partial x^2}$$

$$N_{xy,B} = -\frac{\partial^2 \Phi_B}{\partial x \, \partial y}$$

The stress function Φ_B is determined by a compatibility equation obtained from reference 3

$$\nabla^4 \Phi_B = \frac{Et}{R} \nabla^2 w_B \tag{5}$$

The boundary conditions for buckling at the upper and lower edges of this equatorial segment are taken to be

$$N_{x,B} = 0 \tag{6a}$$

$$v_B = 0 \tag{6b}$$

$$w_B = 0 \tag{6c}$$

$$\frac{\partial^2 w_B}{\partial x^2} = 0 \tag{6d}$$

at $x = \pm\frac{L}{2}$. With these boundary conditions assumed, the term $\frac{N_{x,B} + N_{y,B}}{R}$ in equation (4c) can be replaced by the quantity $\frac{Et}{R^2} w_B$, as shown in appendix A. Since the equatorial segment is loaded axisymmetrically and no lateral load is applied, it can be shown, as in appendix B, that

$$p_x = -\gamma t \tag{7a}$$

$$p_y = 0 \tag{7b}$$

$$N_{x,A} = \gamma t\left[x + \left(k - \frac{1}{2}\right)L\right] \tag{7c}$$

$$N_{y,A} = -\gamma t\left[2x + \left(k - \frac{1}{2}\right)L\right] \tag{7d}$$

$$N_{xy,A} = 0 \tag{7e}$$

where γ is the weight per unit volume of the shell-wall material and k is the proportion of the total shell weight supported at the upper edge. Equation (4c) now becomes

$$D\nabla^4 W_B + \frac{Et}{R^2} W_B - \gamma t \frac{\partial W_B}{\partial x} - \gamma t\left[x + \left(k - \frac{1}{2}\right)L\right]\frac{\partial^2 W_B}{\partial x^2} + \gamma t\left[2x + \left(k - \frac{1}{2}\right)L\right]\frac{\partial^2 W_B}{\partial y^2} = 0 \tag{8}$$

The governing buckling equations are equations (5) and (8) with unknowns the stress function Φ_B and the normal displacement function W_B and with boundary conditions given by equations (6). The problem is to determine the characteristic values of the shell thickness t for which solutions to equations (5) and (8) exist that satisfy the boundary conditions, equations (6).

Solution of Equations by Galerkin Method

Truncated series expressions for W_B and Φ_B are assumed as follows:

$$W_B = \cos\frac{ny}{R}\sum_{m=1}^{M} A_m \sin\frac{m\pi\left(x + \frac{L}{2}\right)}{L} \tag{9a}$$

$$\Phi_B = -\cos\frac{ny}{R}\sum_{m=1}^{M}\frac{\frac{Et}{R}A_m}{\left(\frac{n}{R}\right)^2 + \left(\frac{m\pi}{L}\right)^2}\sin\frac{m\pi\left(x + \frac{L}{2}\right)}{L} \tag{9b}$$

Equations (9) satisfy equation (5) and the boundary conditions given by equations (6) identically. They also satisfy equation (8) identically in the y-direction.

Substituting equations (9) into equation (8) and dividing through by $\cos\frac{ny}{R}$ yields

$$\sum_{m=1}^{M}\left(\left\{D\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n}{R}\right)^2\right]^2+\gamma t\left[x+\left(k-\frac{1}{2}\right)L\right]\left(\frac{m\pi}{L}\right)^2\right.\right.$$

$$\left.-\gamma t\left[2x+\left(k-\frac{1}{2}\right)L\right]\left(\frac{n}{R}\right)^2+\frac{Et}{R^2}\right\}A_m\sin\frac{m\pi\left(x+\frac{L}{2}\right)}{L}$$

$$\left.-\gamma t\left(\frac{m\pi}{L}\right)A_m\cos\frac{m\pi\left(x+\frac{L}{2}\right)}{L}\right)=0 \qquad (10)$$

Now, by the Galerkin method equation (10) is multiplied by

$$\sin\frac{q\pi\left(x+\frac{L}{2}\right)}{L} \qquad (q=1,2,3,\ldots,M)$$

and the resulting equation is integrated over the interval $-\frac{L}{2}\leqq x\leqq\frac{L}{2}$ to yield

$$\left\{D\left[\left(\frac{q\pi}{L}\right)^2+\left(\frac{n}{R}\right)^2\right]^2+\gamma tL\left(k-\frac{1}{2}\right)\left[\left(\frac{q\pi}{L}\right)^2-\left(\frac{n}{R}\right)^2\right]+\frac{Et}{R^2}\right\}\frac{L}{2}A_q$$

$$-\sum_{\substack{m=1\\m\neq q}}^{M}\gamma t\left\{\left[\left(\frac{m\pi}{L}\right)^2-2\left(\frac{n}{R}\right)^2\right]\left(\frac{L}{\pi}\right)^2\frac{2}{q^2-m^2}+1\right\}\frac{qm}{q^2-m^2}$$

$$\times\left[1-(-1)^{m+q}\right]A_m=0 \qquad (q=1,2,3,\ldots,M) \qquad (11)$$

For determining the critical-thickness parameter of a shell, it is convenient to express equation (11) in terms of nondimensional parameters as follows:

$$\left\{\frac{\alpha\left(k - \frac{1}{2}\right)\left[n^2 - (q\pi\beta)^2\right] - 1}{\left[n^2 + (q\pi\beta)^2\right]^2} - \lambda\right\}A_q$$

$$- \sum_{\substack{m=1 \\ m+q\ \mathrm{odd}}}^{M} 4\alpha \frac{\left[2n^2 - (m\pi\beta)^2\right]\frac{2}{\pi^2\left(q^2 - m^2\right)} - \beta^2}{\left[n^2 + (q\pi\beta)^2\right]^2} \frac{qm}{q^2 - m^2} A_m = 0 \qquad (q = 1,2,3,\ldots,M) \qquad (12)$$

where

$$\alpha = \frac{\gamma L}{E}$$

$$\beta = \frac{R}{L}$$

$$\lambda = \frac{t^2}{12\left(1 - \mu^2\right)R^2}$$

Equation (12) represents M homogeneous algebraic equations. In matrix notation, these equations are

$$\left[\left[a_{qm}\right] - \lambda I\right]\left\{A\right\} = 0 \qquad (13)$$

where the elements of $\left[a_{qm}\right]$ are

$$a_{qm} = \frac{\alpha\left(k - \frac{1}{2}\right)\left[n^2 - (q\pi\beta)^2\right] - 1}{\left[n^2 + (q\pi\beta)^2\right]^2} \qquad (m = q)$$

$$a_{qm} = -4\alpha \frac{\left[2n^2 - (m\pi\beta)^2\right]\frac{2}{\pi^2\left(q^2 - m^2\right)} - \beta^2}{\left[n^2 + (q\pi\beta)^2\right]^2} \frac{qm}{q^2 - m^2} \qquad \begin{pmatrix} m \neq q \\ m + q\ \mathrm{odd} \end{pmatrix}$$

$$a_{qm} = 0 \qquad \begin{pmatrix} m \neq q \\ m + q\ \mathrm{even} \end{pmatrix}$$

and

$$\{A\} = \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_M \end{Bmatrix}$$

Now, by noting that

$$\left[2n^2 - (m\pi\beta)^2\right]\frac{2}{\pi^2\left(q^2 - m^2\right)} - \beta^2 = \left[4n^2 - (m\pi\beta)^2 - (q\pi\beta)^2\right]\frac{1}{\pi^2\left(q^2 - m^2\right)}$$

equation (13) can be put in the more convenient symmetric form

$$\left[\left[b_{qm}\right] - \lambda I\right]\{B\} = 0 \tag{14}$$

where the elements of $\left[b_{qm}\right]$ are

$$b_{qm} = b_{mq} = \frac{\alpha\left(k - \frac{1}{2}\right)\left[n^2 - (q\pi\beta)^2\right] - 1}{\left[n^2 + (q\pi\beta)^2\right]^2} \qquad (m = q)$$

$$b_{qm} = b_{mq} = -4\alpha\frac{\left[4n^2 - (m\pi\beta)^2 - (q\pi\beta)^2\right]}{\left[n^2 + (m\pi\beta)^2\right]\left[n^2 + (q\pi\beta)^2\right]}\frac{qm}{\pi^2\left(q^2 - m^2\right)^2} \qquad \begin{pmatrix} m \neq q \\ m + q \text{ odd} \end{pmatrix}$$

$$b_{qm} = b_{mq} = 0 \qquad \begin{pmatrix} m \neq q \\ m + q \text{ even} \end{pmatrix}$$

and

$$
\{B\} = \begin{Bmatrix} \left[n^2 + (\pi\beta)^2\right] A_1 \\ \left[n^2 + (2\pi\beta)^2\right] A_2 \\ \cdot \\ \cdot \\ \cdot \\ \left[n^2 + (m\pi\beta)^2\right] A_m \\ \cdot \\ \cdot \\ \cdot \\ \left[n^2 + (M\pi\beta)^2\right] A_M \end{Bmatrix}
$$

Equation (14) is a standard eigenvalue problem, the nontrivial solutions of which yield M characteristic values of the thickness parameter λ as a function of α, β, k, and n. For a given α, β, and k, the limiting value of λ obtained by maximizing with respect to n as $M \rightarrow \infty$ yields the critical value λ_{cr}; that is, for λ greater than λ_{cr} the shell does not buckle. In practice this critical value of λ can be determined by increasing M until the maximum value of λ with respect to n converges to its limiting value.

NUMERICAL RESULTS AND DISCUSSION

Critical values of the thickness parameter λ have been determined over the range of the specific-weight parameter $10^{-7} \leqq \alpha \leqq 10^{-4}$ for radius-length ratios β of 3, 5, and 10 and for upper-edge support conditions of $k = 0, 1/4, 1/2$, or 1. Since most metallic materials have a γ/E of the order 10^{-8} per inch (4×10^{-7} per meter), and most plastics have a γ/E of the order 10^{-7} per inch (4×10^{-6} per meter), a value of α equal to 10^{-7} might indicate a metallic shell 10 inches (25 cm) long, and a value of α equal to 10^{-4} might indicate a plastic shell 1000 inches (25 meters) long.

The number of terms in the truncated series solution M required for convergence varied from 30 for $\alpha = 10^{-4}$ to 70 for $\alpha = 10^{-7}$. For support conditions where $k = 1$ or 1/2, convergence was found to be reasonably rapid. For $k = 1$ and $\alpha = 10^{-4}$, typical values of n at which λ became maximum were 169 for $\beta = 10$ and 137 for $\beta = 3$;

for $k = 1/2$ and $\alpha = 10^{-4}$, λ became maximum at $n = 213$ for $\beta = 10$ and at $n = 170$ for $\beta = 3$.

For the conditions where $k = 1/4$ or 0, however, convergence was found to be extremely slow when the truncated series solution was taken in the form of equations (9). This difficulty was removed by using a modified truncated series solution of the form

$$W_B = \cos\frac{ny}{R}\sum_{m=N}^{N+M} A_m \sin\frac{m\pi\left(x + \frac{L}{2}\right)}{L}$$

The eigenvalue λ was then maximized with respect to both n and N. For $k = 1/4$ or 0, this eigenvalue maximized at $n = 0$ while the value of N varied from 1 for $\alpha = 10^{-4}$ to 518 for $\alpha = 10^{-7}$, and the value of M again varied from 30 to 70.

When more than one-quarter of the weight of the equatorial segment is supported at the upper edge of the segment, the buckling mode tends to be composed of few waves in the axial direction (x-direction) and many waves circumferentially (y-direction). When more than three-quarters of the segment weight is supported at the lower edge, on the other hand, the buckling mode tends to be axisymmetric (no waves circumferentially) and composed of a few or many waves in the axial direction.

The critical-thickness parameter obtained by the simplified approach λ^* is determined in appendix B as

$$\lambda^* = \left(\frac{1}{2} - \frac{k}{2}\right)^2 \alpha^2 \qquad (0 \leqq k \leqq 1/4)$$

$$\lambda^* = \left(\frac{1}{4} + \frac{k}{2}\right)^2 \alpha^2 \qquad (1/4 \leqq k \leqq 1)$$

The ratio λ_{cr}/λ^* as a function of the specific-weight parameter α for radius-length ratios β of 3, 5, and 10 is presented in figures 4, 5, 6, and 7 for $k = 0$, 1/4, 1/2, and 1, respectively. The ratios λ^*/α^2 and λ_{cr}/α^2 are presented as functions of k for $\alpha = 10^{-7}$, 10^{-6}, 10^{-5}, and 10^{-4} in figures 8, 9, and 10 for $\beta = 3$, 5, and 10, respectively.

The ratio λ_{cr}/λ^* is less than unity for all conditions considered in this report as shown in figures 4, 5, 6, and 7. These figures also show that λ_{cr}/λ^* increases as the radius-length ratio and the specific-weight parameter decrease. In addition, these figures show that λ_{cr}/λ^* is significantly lower for $k = 1$ or 1/2 than for $k = 1/4$ or 0. The proportion of the shell weight which should be supported at the upper edge to minimize the shell thickness (or the tendency to buckle) is indicated by the lowest points on the curves for λ^*/α^2 and λ_{cr}/α^2 in figures 8, 9, and 10. The lowest value of λ^*/α^2

is always at $k = 1/4$ regardless of the value of α whereas the lowest values of λ_{cr}/α^2 always occur at values of k greater than 1/4. The optimum value of k for λ_{cr}/α^2 increases as the radius-length ratio and specific-weight parameter increase. For $\beta = 10$ and $\alpha = 10^{-4}$ the optimum value of k is a little less than 1/2.

SUMMARY OF RESULTS

Linear buckling equations have been used to study the problem of buckling of a simply supported equatorial segment of a thin-walled spherical shell loaded in the axial direction by its own weight. By using these equations the following results, which can be used in the design of an equatorial or rectangular segment of a spherical shell, were obtained:

1. The simplified approach, in which the shell segment is assumed to buckle if the maximum compressive stress is greater than the critical compressive stress for a complete sphere loaded by uniform external pressure, leads to larger critical thicknesses than those obtained by the method of this report. Differences between the results of the simplified approach and those of the present method are greater for the higher values of radius-length ratio and specific-weight parameter; that is, the simplified approach is more conservative for these cases.

2. The optimum support conditions of the shell segment depend on the geometry and material properties of the shell. For the range of parameters considered, the optimum proportion of the shell weight supported at the upper edge increased from approximately one-fourth to one-half as the radius-length ratio increased from 3 to 10 and the specific-weight parameter increased from 10^{-7} to 10^{-4}.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 29, 1968,
124-08-06-13-23.

APPENDIX A

MODIFICATION OF THE EQUATION FOR THE EQUILIBRIUM OF FORCES NORMAL TO THE SHELL MIDDLE SURFACE

Equation (4c) represents the equilibrium of forces in the direction normal to the middle surface of the shell segment. In this appendix it is shown that with the boundary conditions assumed in the present report, the term $\frac{N_{x,B} + N_{y,B}}{R}$ in equation (4c) can be replaced by the quantity $\frac{Et}{R^2} W_B$.

The relationship between $\frac{N_{x,B} + N_{y,B}}{R}$ and the stress function Φ_B can be written as follows:

$$\frac{N_{x,B} + N_{y,B}}{R} = \frac{1}{R}\left(\frac{\partial^2 \Phi_B}{\partial y^2} + \frac{\partial^2 \Phi_B}{\partial x^2}\right) \equiv \frac{\nabla^2 \Phi_B}{R} \tag{A1}$$

The stress function Φ_B is related to the normal displacement W_B through the compatibility equation, equation (5),

$$\nabla^4 \Phi_B = \frac{Et}{R} \nabla^2 W_B$$

or

$$\nabla^2 \left(\nabla^2 \Phi_B - \frac{Et}{R} W_B\right) = 0 \tag{A2}$$

Now, for any function ψ in a two-dimensional region S, if $\nabla^2 \psi = 0$ over S and $\psi = 0$ on all boundaries of S, then $\psi \equiv 0$ in S. If $\nabla^2 \Phi_B - \frac{Et}{R} W_B$ can be shown to be zero on the boundaries of the shell, then this quantity is also zero throughout the shell and $\frac{N_{x,B} + N_{y,B}}{R}$ can be replaced by $\frac{Et}{R^2} W_B$ in the equilibrium equation.

The value of $\nabla^2 \Phi_B - \frac{Et}{R} W_B$ at $x = \pm\frac{L}{2}$ can be examined with the aid of the following stress-resultant-displacement equations obtained from reference 1:

APPENDIX A

$$N_{x,B} = \frac{Et}{1 - \mu^2}\left[\frac{\partial U_B}{\partial x} + \mu \frac{\partial V_B}{\partial y} + (1 + \mu)\frac{W_B}{R}\right] \tag{A3a}$$

$$N_{y,B} = \frac{Et}{1 - \mu^2}\left[\frac{\partial V_B}{\partial y} + \mu \frac{\partial U_B}{\partial x} + (1 + \mu)\frac{W_B}{R}\right] \tag{A3b}$$

$$N_{xy,B} = \frac{Et}{2(1 + \mu)}\left(\frac{\partial U_B}{\partial y} + \frac{\partial V_B}{\partial x}\right) \tag{A3c}$$

With the boundary conditions assumed herein (eqs. (6)), $N_{x,B} = V_B = W_B = 0$ at $x = \pm\frac{L}{2}$. If $V_B = 0$ at $x = \pm\frac{L}{2}$, then $\frac{\partial V_B}{\partial y} = 0$ at $x = \pm\frac{L}{2}$ and from equation (A3a), $\frac{\partial U_B}{\partial x} = 0$ at $x = \pm\frac{L}{2}$. Furthermore, from equation (A3b), $N_{y,B} = 0$ at $x = \pm\frac{L}{2}$; therefore,

$$\nabla^2\Phi_B - \frac{Et}{R} W_B = \frac{\partial^2\Phi_B}{\partial x^2} + \frac{\partial^2\Phi_B}{\partial y^2} - \frac{Et}{R} W_B = N_{y,B} + N_{x,B} - \frac{Et}{R} W_B = 0$$

at $x = \pm\frac{L}{2}$, and the conditions for replacement are fulfilled.

APPENDIX B

STRESS DISTRIBUTION DETERMINED BY LINEAR MEMBRANE THEORY

The linear membrane equilibrium equations for the symmetrically loaded equatorial segment shown in figure 2 are solved to determine the prebuckled stress state. When $N_{xy} = 0$ and derivatives with respect to y are zero, these equations are

$$\frac{\partial N_{x,A}}{\partial x} + p_x = 0 \tag{B1}$$

$$\frac{N_{x,A} + N_{y,A}}{R} - p_n = 0 \tag{B2}$$

The body-force functions p_x and p_n are obtained by projecting the external loads, herein the weight of the shell, in directions tangent to and normal to the shell. By assuming $\frac{1}{2}\left(\frac{x}{R}\right)^2$ is negligible, the following expressions for p_x and p_n are obtained:

$$p_x = -\gamma t \tag{B3}$$

$$p_n = -\frac{\gamma t}{R} x \tag{B4}$$

Substituting equation (B3) into equation (B1) and integrating yields

$$N_{x,A} = \gamma t x + C \tag{B5}$$

and substituting equations (B4) and (B5) into equation (B2) yields

$$N_{y,A} = -2\gamma t x - C \tag{B6}$$

where C is a constant determined by the value of $N_{x,A}$ at the upper or lower edge of the shell. By assuming $\frac{1}{4}\left(\frac{L}{R}\right)^2$ is negligible, the values of $N_{x,A}$ at the upper and lower edges become

$$N_{x,A} = \gamma t L k \qquad \left(x = \frac{L}{2}\right) \tag{B7}$$

$$N_{x,A} = \gamma t L(k - 1) \qquad \left(x = -\frac{L}{2}\right) \tag{B8}$$

Substituting either of these two values for $N_{x,A}$ into equation (B5) and solving for C gives

$$C = \gamma tL\left(k - \frac{1}{2}\right) \tag{B9}$$

Substituting equation (B9) into equations (B5) and (B6) yields the required membrane-stress resultants

$$N_{x,A} = \gamma t\left[x + \left(k - \frac{1}{2}\right)L\right] \tag{B10}$$

$$N_{y,A} = -\gamma t\left[2x + \left(k - \frac{1}{2}\right)L\right] \tag{B11}$$

The maximum compressive-stress resultants are:

For $x = -\frac{L}{2}$,

$$N_{x,A} = \gamma tL(k - 1) \qquad (0 \leqq k \leqq 1/4)$$

For $x = \frac{L}{2}$,

$$N_{y,A} = -\gamma tL\left(k + \frac{1}{2}\right) \qquad (1/4 \leqq k \leqq 1)$$

By assuming that the spherical shell segment buckles if the maximum compressive stress is greater than the critical compressive stress for a complete sphere loaded by uniform external pressure, that is,

$$N_{x,A} \text{ or } N_{y,A} = -\frac{Et^2}{R\sqrt{3(1 - \mu^2)}}$$

the following critical values for λ are obtained:

$$\lambda^* = \left(\frac{1}{2} - \frac{k}{2}\right)^2 \alpha^2 \qquad (0 \leqq k \leqq 1/4) \tag{B12}$$

$$\lambda^* = \left(\frac{1}{4} + \frac{k}{2}\right)^2 \alpha^2 \qquad (1/4 \leqq k \leqq 1) \tag{B13}$$

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1. Stein, Manuel; and McElman, John A.: Buckling of Segments of Toroidal Shells. AIAA J., vol. 3, no. 9, Sept. 1965, pp. 1704-1709.

2. Mushtari, Kh. M.; and Galimov, K. Z.: Non-Linear Theory of Thin Elastic Shells. NASA TT F-62, The Israel Program for Scientific Translations, 1961, pp. 152-156. (Available from CFSTI.)

3. Reissner, Eric: Stresses and Small Displacements of Shallow Spherical Shells. I. J. Math. Phys., vol. XXV, no. 1, Feb. 1946, pp. 80-85.

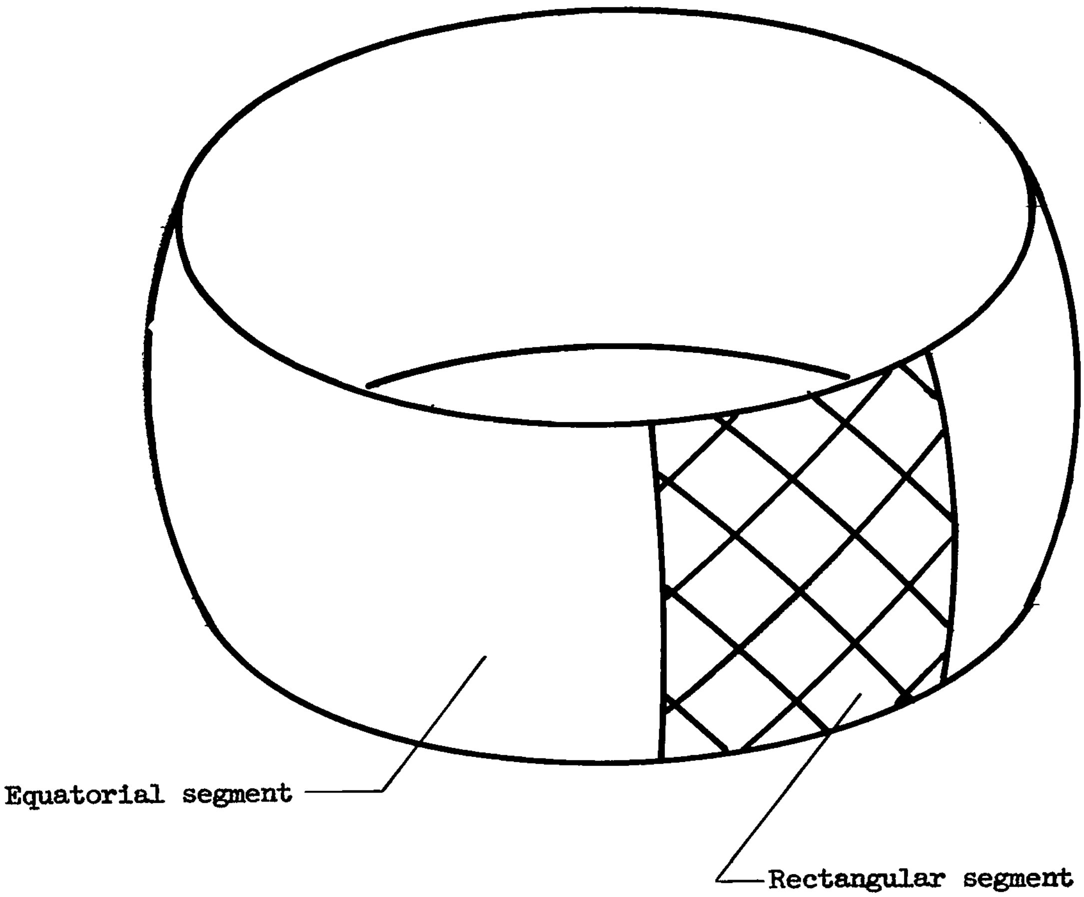

Figure 1.- Equatorial segment of spherical shell showing rectangular segment which simulates lunar surface in the lunar orbit and landing approach simulator at the Langley Research Center.

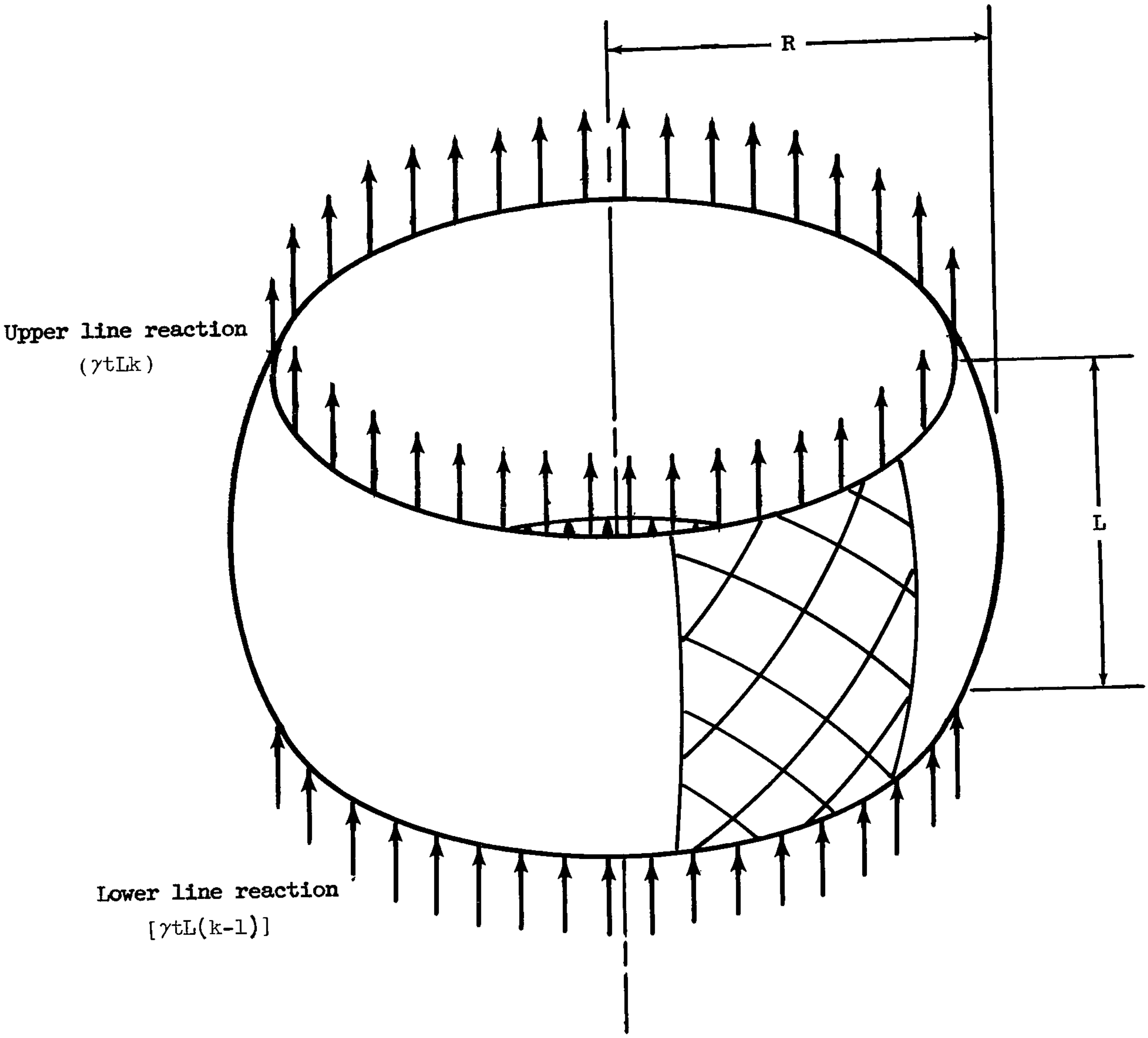

Figure 2.- Geometry and support conditions of an equatorial segment of a spherical shell.

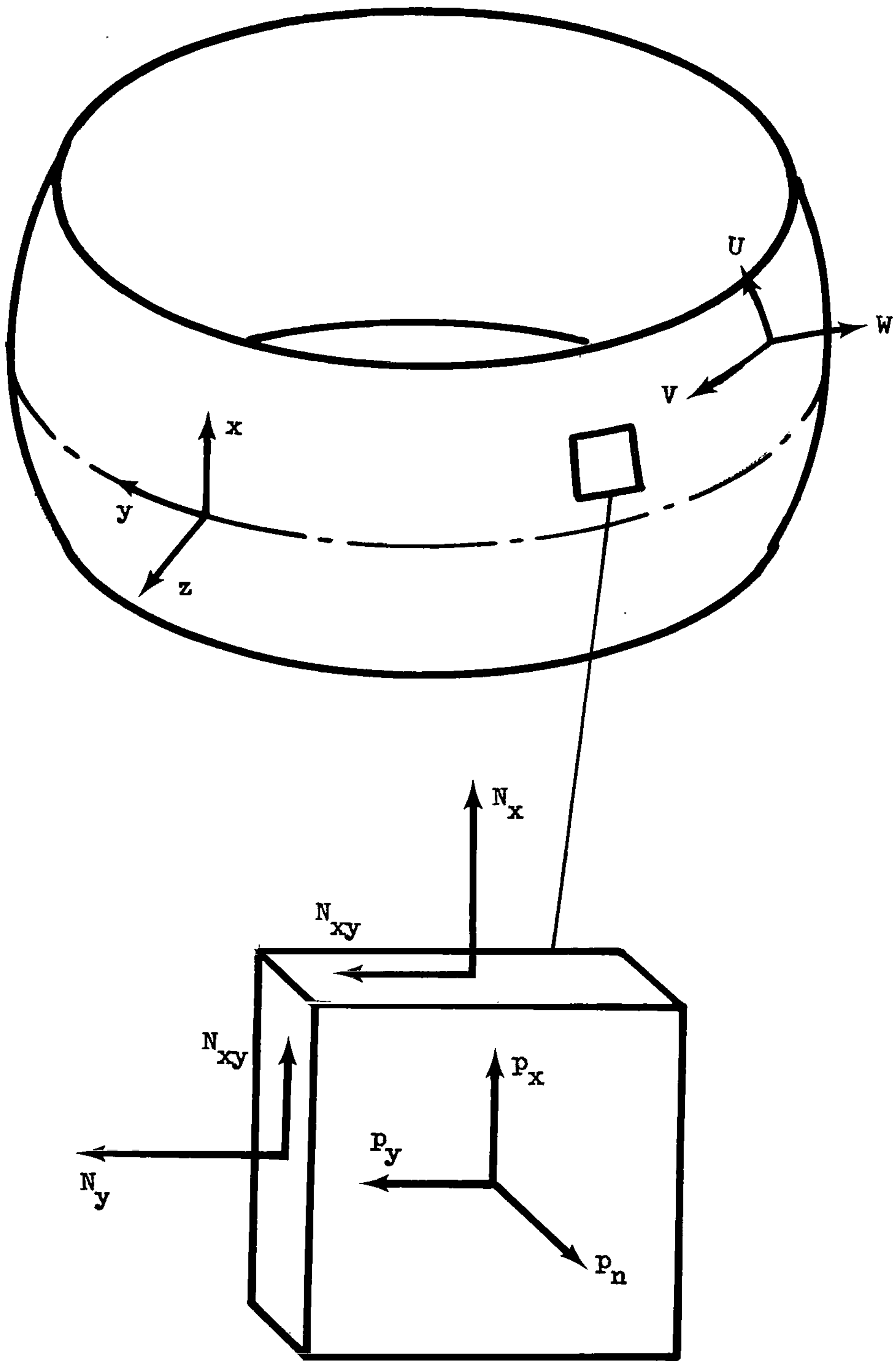

Figure 3.- Coordinate system, membrane stress resultants, displacements, and body forces of spherical segment.

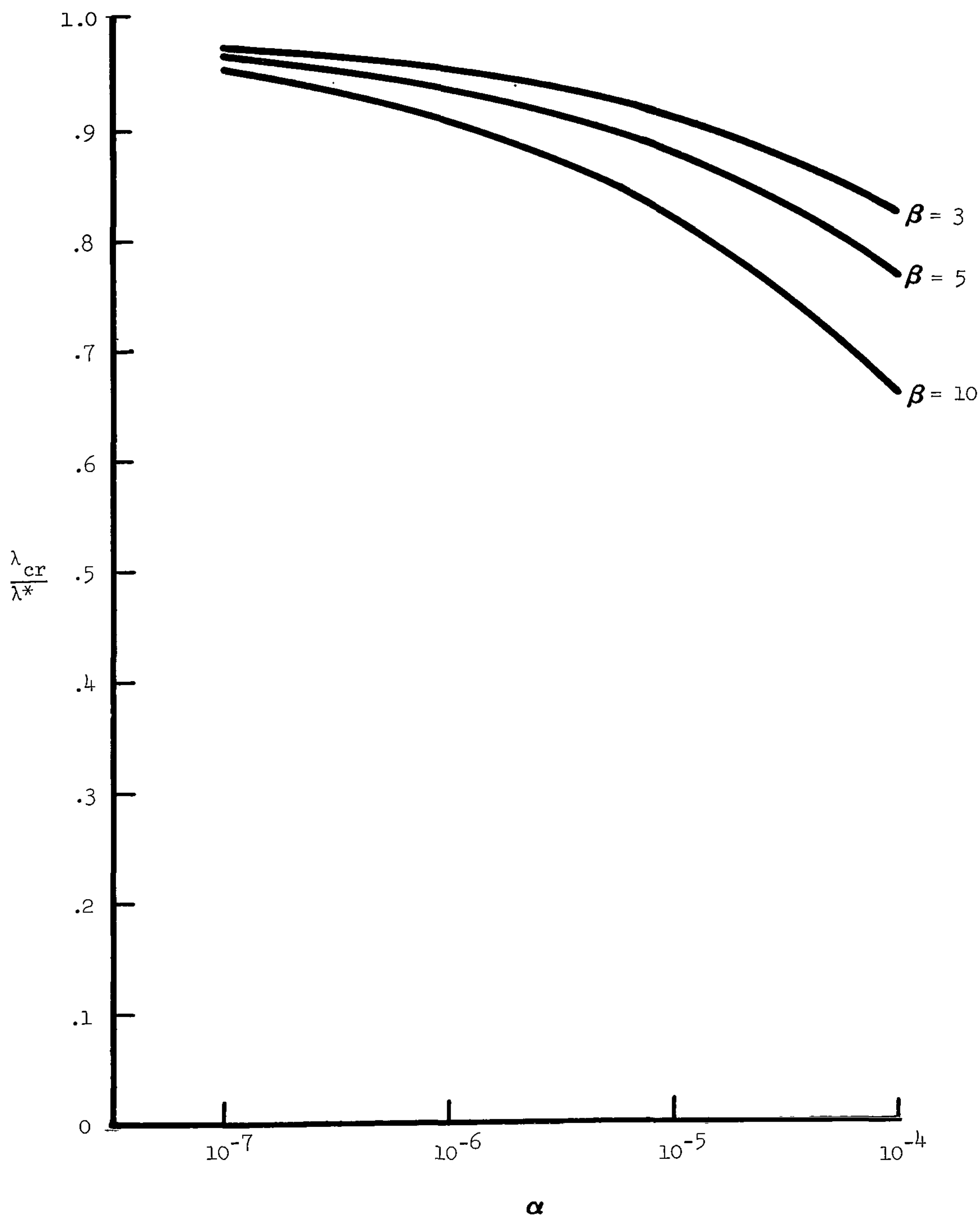

Figure 4.- Ratio of the critical values of the thickness parameter obtained by the present method λ_{cr} to those obtained by the simplified approach λ^*. $k = 0$; $\lambda^* = \frac{1}{4}\alpha^2$.

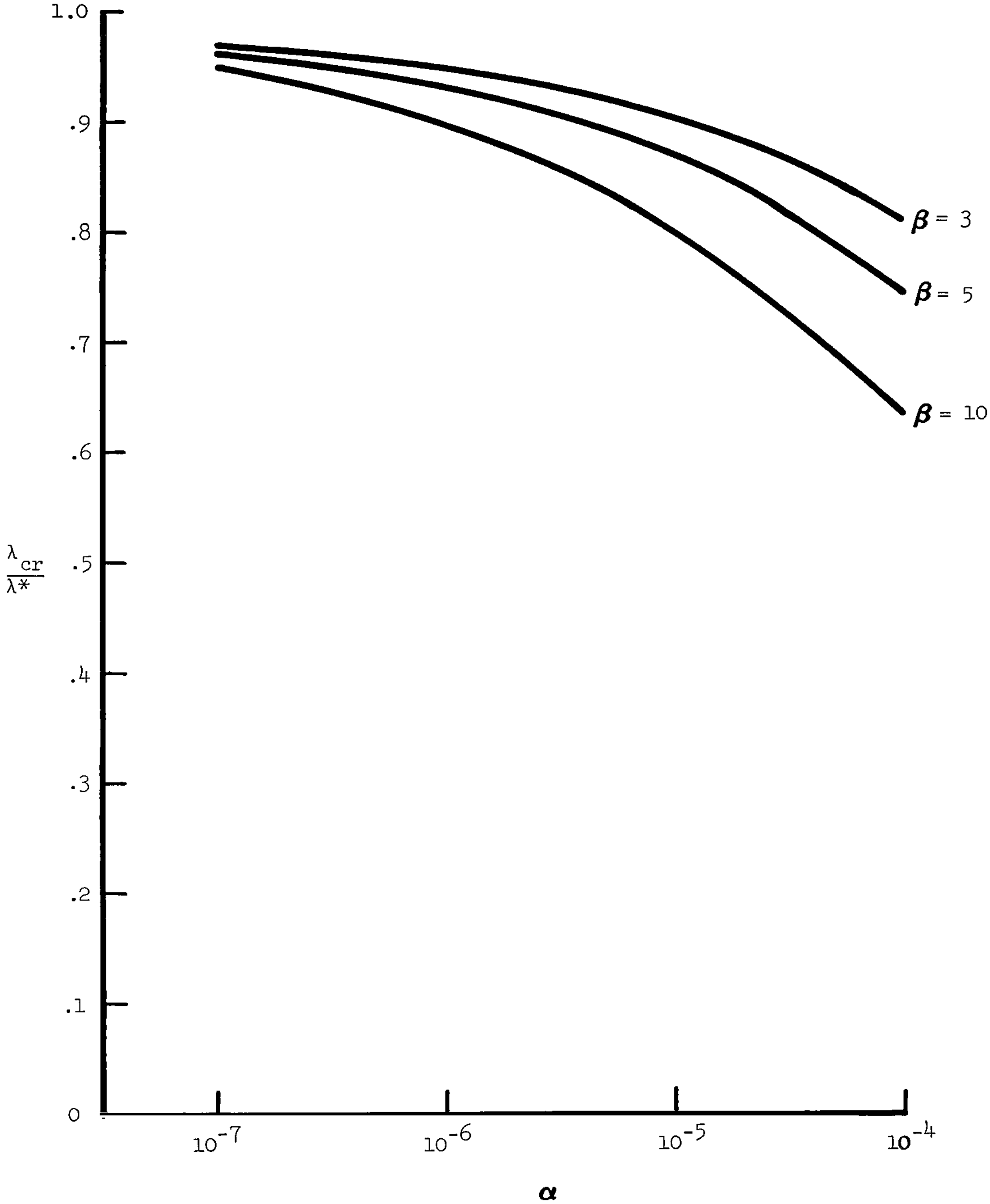

Figure 5.- Ratio of the critical values of the thickness parameter obtained by the present method λ_{cr} to those obtained by the simplified approach λ^*. $k = \frac{1}{4}$; $\lambda^* = \frac{9}{64}\alpha^2$.

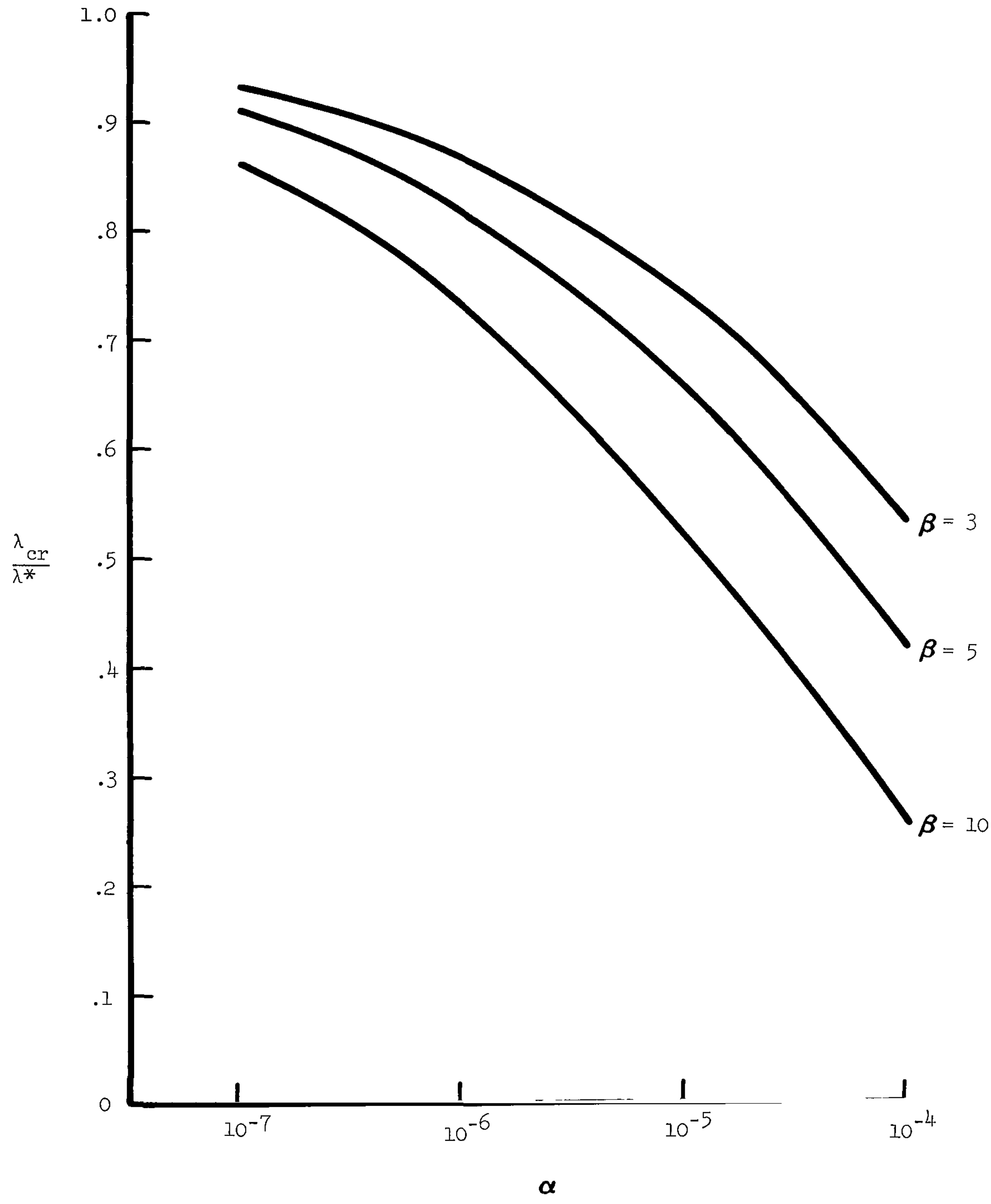

Figure 6.- Ratio of the critical values of the thickness parameter obtained by the present method λ_{cr} to those obtained by the simplified approach λ^*. $k = \frac{1}{2}$, $\lambda^* = \frac{1}{4}\alpha^2$.

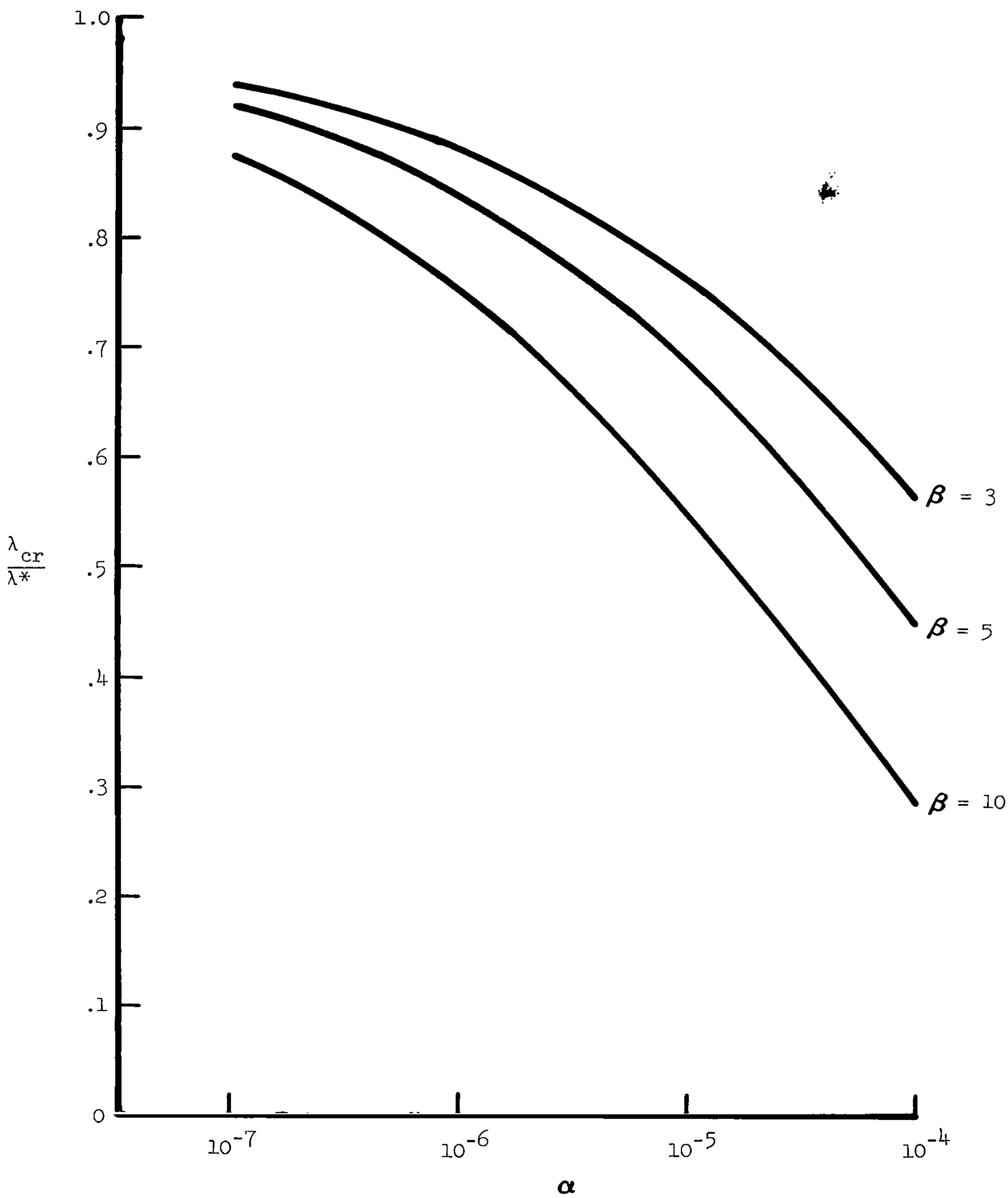

Figure 7.- Ratio of the critical values of the thickness parameter obtained by the present method λ_{cr} to those obtained by the simplified approach λ^*. $k = 1$; $\lambda^* = \frac{9}{16}\alpha^2$.

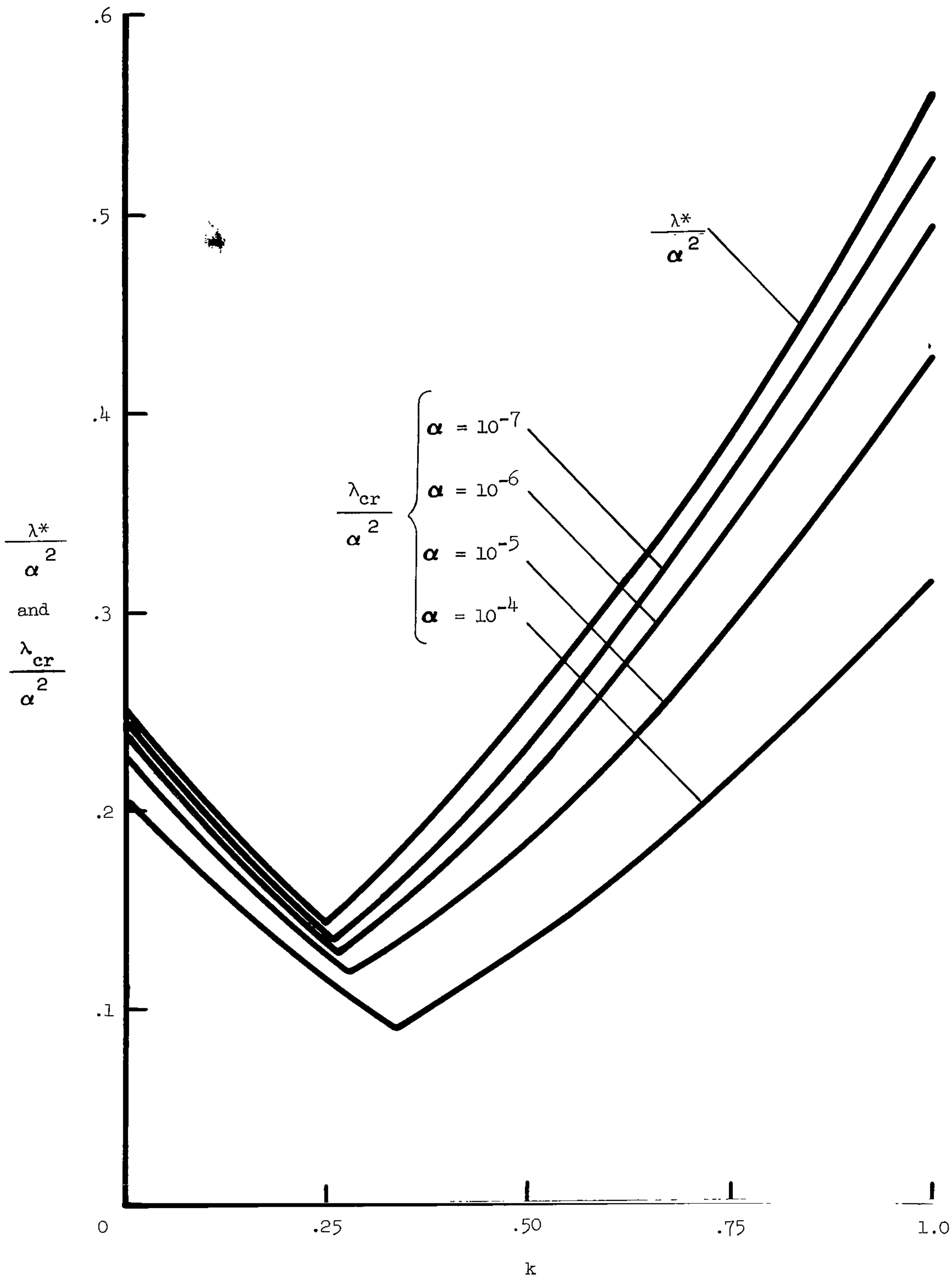

Figure 8.- Critical values of the thickness parameters λ_{cr} and λ^* as a function of the support conditions k with $\beta = 3$.

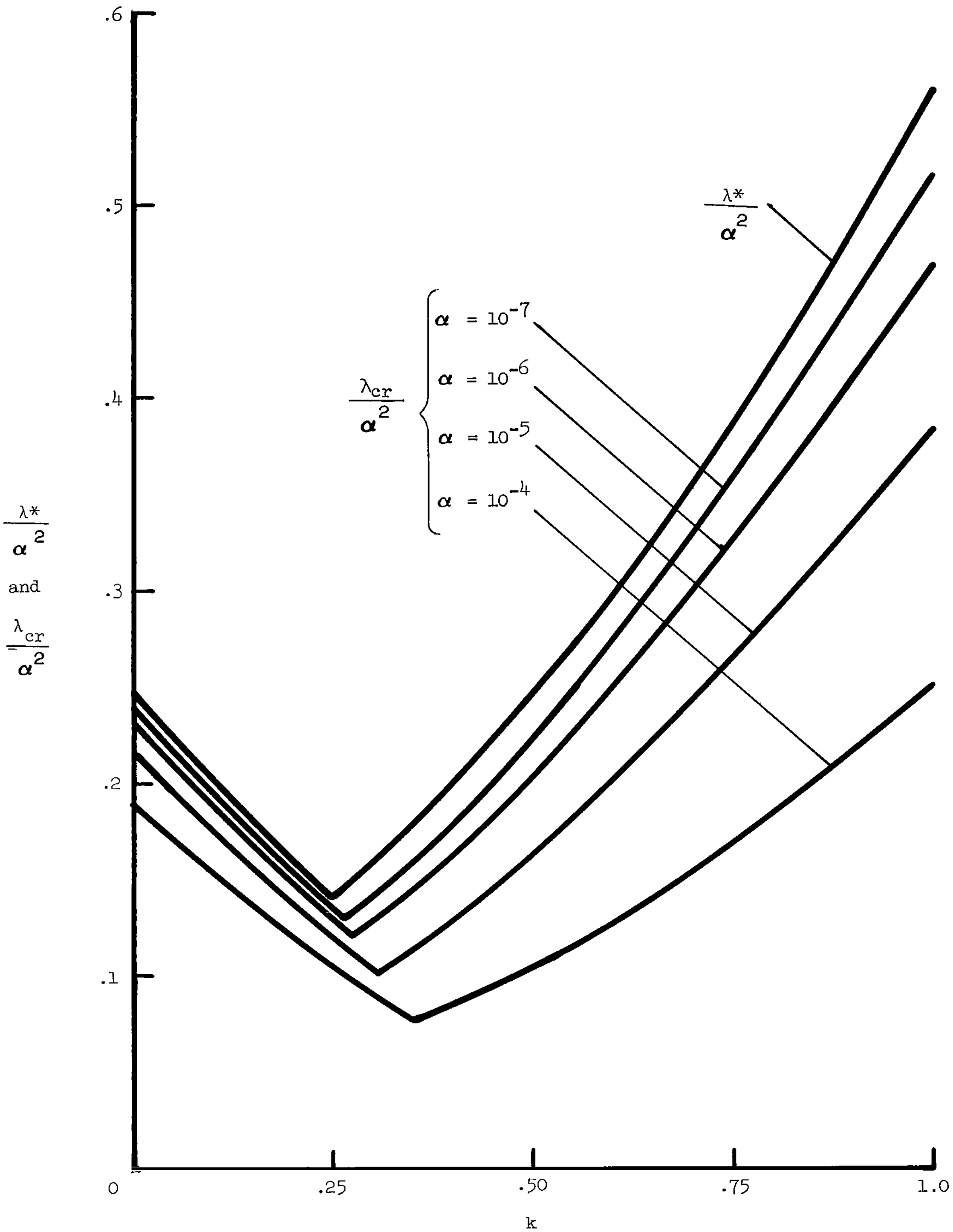

Figure 9.- Critical values of the thickness parameters λ_{cr} and λ^* as a function of the support conditions k with $\beta = 5$.

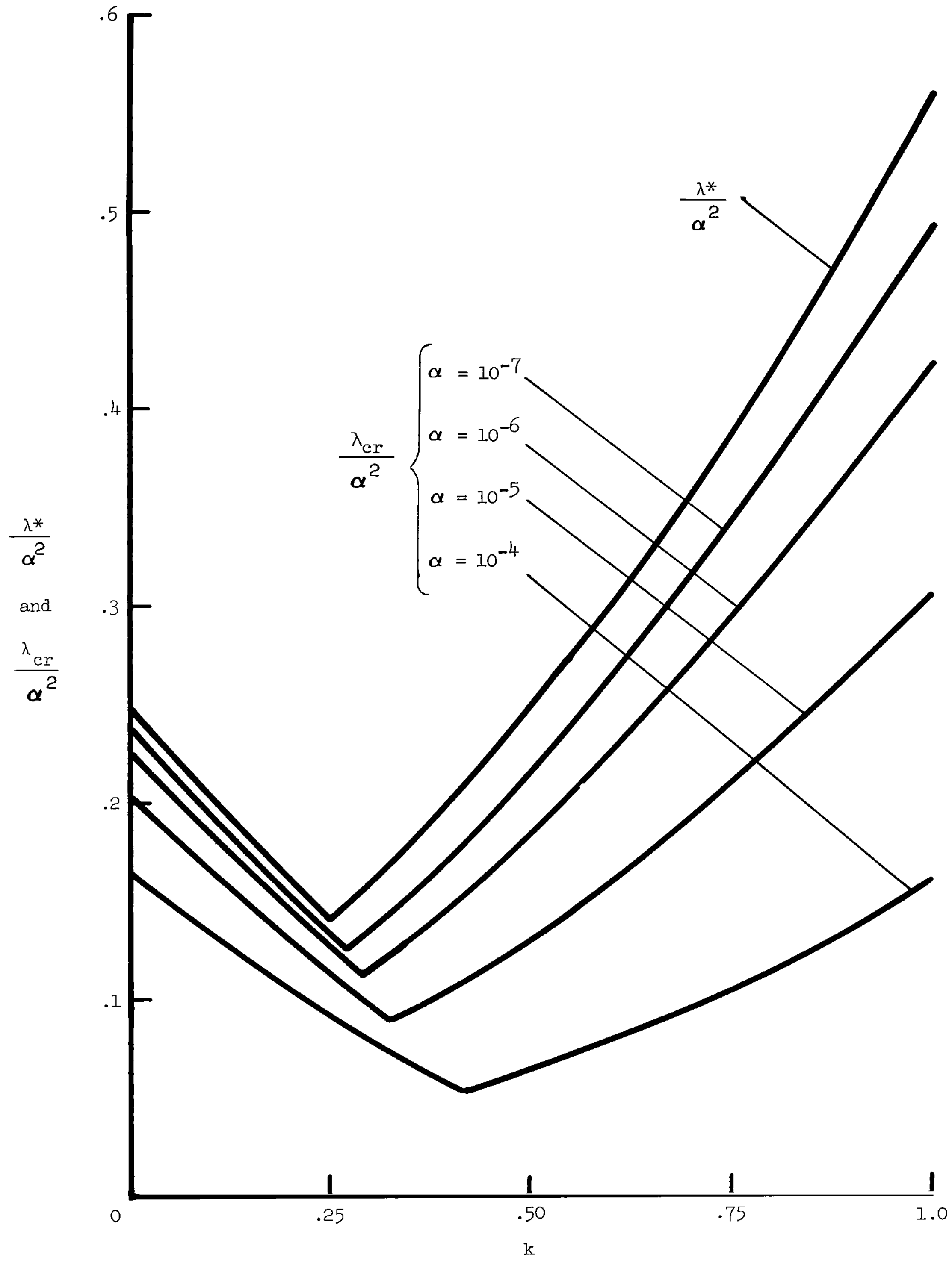

Figure 10.- Critical values of the thickness parameters λ_{cr} and λ^* as a function of the support conditions k with $\beta = 10$.

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