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**SYNTHESIS METHODS FOR
MANUAL AEROSPACE CONTROL SYSTEMS
WITH APPLICATIONS TO SST DESIGN**

by Walter W. Wierwille and James R. Knight

Prepared by
CORNELL AERONAUTICAL LABORATORY, INC.
Buffalo, N. Y.
for Langley Research Center



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ABSTRACT

This report deals with the development of methods for preliminary design (or synthesis) of manual aerospace control systems. Previous to this research effort, almost all manual control work had been in the areas of modeling and analysis. Therefore, it was necessary to develop new approaches to achieve synthesis. Two approaches were studied in detail: a programming approach and a man-machine performance approach.

The programming approach to synthesis places emphasis on moving the poles or eigenvalues of a closed-loop flight control system toward positions considered most desirable from a handling qualities point of view. These poles or eigenvalues are functions of each adjustable parameter in the compensators and various sensor feedback paths. To adjust the various parameters optimally, a programming procedure was developed that was implemented by means of digital computer programs. Three programs have been developed. The first performs synthesis in a single flight regime, determining optimum fixed values for the adjustable parameters. The second performs synthesis in an arbitrary number of flight regimes simultaneously, again determining a single set of fixed-values for the adjustable parameters. The third program also performs synthesis in an arbitrary number of flight regimes simultaneously, but allows specified parameters to vary from one regime to another. Thus, the second program produces a preliminary manual flight control system design that is fixed, while the third produces a design in which some of the parameters are scheduled or adapted with flight regime.

The programming approach was applied to the longitudinal dynamics of an SST for six flight regimes. It was found that all three programs operated effectively in moving the closed-loop system poles. The first and third programs allowed large improvements in pole positions (when compared with open-loop positions), whereas the second allowed only moderate improvement. Pilot ratings, obtained in a man-machine system simulation for a flight control system synthesized by the second program, showed moderate improvement over ratings obtained for the open-loop system.

The programming approach was also applied to the synthesis of a lateral-directional flight control system of an SST. The feasibility of designing the longitudinal and the lateral-directional manual flight control systems by the programming approach is clearly demonstrated.

The second approach to manual control system synthesis, called the man-machine performance approach, places emphasis on optimizing the performance of the man-machine system combination. Man-machine system performance is quantified by means of a performance measure which assesses various errors and signal excursions as a function of time.

The man-machine performance approach appears most promising for development of new flight control systems or systems where the number of adjustable parameters is small. Preliminary experimental work showed that a pilot was capable of sufficiently stable performance to allow man-machine performance to be evaluated as a function of adjustable parameters. Experimental work also showed that pilot rating and man-machine system performance were closely related.

MATHEMATICAL SYMBOLS

- A feedback loop gain of a feedback control system
- $A_{i/jk}$ coefficient of an unfactored characteristic equation (see equation 41)
- $D_o(s) = D_G(s) D_F(s)$
- $D_o'(s)$ defined by equation 8
- $D_o''(s)$ defined by equation 12
- $D_i(s)$ polynomial of open-loop poles with one adjustable pole factor removed
- F square matrix of coefficients
- G control input matrix
- $G(s) = \frac{N_G(s)}{D_G(s)}$, polynomial transfer function of the forward loop dynamics of a feedback control system
- I number of given and desired pole pairs to be included in the performance measure
- $I(\alpha)$ variational function of α
- K forward loop gain of a feedback control system
- $K_o = KA$; an adjustable parameter
- $K_{n\gamma}, K_r, K_\alpha, K_\beta, K_\theta, K_{\dot{\theta}}, K_\phi, K_{\dot{\phi}}$ parameters to be optimally adjusted by the programming procedure
- K_o' defined by equation 25
- $N_o(s) = N_G(s) N_F(s)$
- $N_i(s)$ polynomial of open-loop zeros with one adjustable zero factor removed
- $\frac{N_F(s)}{D_F(s)}$ polynomial transfer function of the feedback dynamics of a feedback control system
- $N_o'(s)$ defined by equation 9
- $N_o''(s)$ defined by equation 13

- P a parabolic expression; performance measure the human operator uses
- $P_N(s)$ power spectral density of a disturbance input
- S Laplace transform independent variable
- S_{D_i} a desired pole position with real part R_{D_i} and imaginary part I_{D_i}
- S_{G_i} a given pole position with real part R_{G_i} and imaginary part I_{G_i}
- S_p an adjustable open-loop pole
- $W(e)$ (see $f[e]$)
- a, b, c, a_i, b_i, c_i coefficients of an equation for a parabola
- a_m, b_m, c_m, d_m coefficients of a linear combination of environmental parameters
- a_n an adjustable parameter in an automaton
- $e(t)$ an error signal
- $f[e] = W^2(e)$, a nonnegative zero-memory function of error
- g an initial setting of an adjustable parameter a_m ; subscript indicating gust input quantity; acceleration of gravity, 32.2 ft/sec^2
- g_i coefficient in $D_o(s)$
- g_{ij} parameter in a factored polynomial
- h_i coefficient in $N_o(s)$
- i subscript for ordering the given and desired pole pairs; a counting integer
- j the imaginary number $\sqrt{-1}$; a counting integer
- k number of flight conditions; a counting integer
- k_o multiplying coefficient in a factored polynomial
- k_i performance measure weighting coefficient
- m order of the numerator, $N_o(s)$; a counting integer

n order of the denominator, $D_o(s)$; total number of adjustable parameters in any given flight regime; a counting integer
 $p_E(e)$ probability density function of error over the time interval $T_1 \leq t \leq T_2$
 r yaw rate (r/s); number of adjustable parameters to be scheduled with flight regime; a counting integer
 u control vector
 x state vector
 $x_{o1}(t), x_{o2}(t), x_1(t), x_2(t)$ defined in block diagrams of Figure 32
 y_r output of an automaton
 z feedback gain matrix
 Δ an increment in one of the adjustable parameters
 $\Delta N(s)$ transform of a disturbance input
 Δn_z incremental normal acceleration (g)
 ΔV incremental airspeed (ft /sec)
 $\Delta \alpha$ incremental angle of attack (rad)
 $\Delta \delta_e$ incremental elevator excursion (rad)
 $\Delta \delta_s$ incremental stick motion (rad)
 $\Delta \theta$ incremental pitch angle (rad)
 τ_o, τ_a, τ_r parameters to be optimally adjusted by the programming procedure
 τ_z an adjustable open-loop zero
 $\alpha \eta(e)$ variation in $W(e)$
 α_m an adjustable parameter
 β sideslip angle (rad)
 δ_a aileron deflection (rad)
 δ_r rudder deflection (rad)

ζ damping of an abstract short-period pole pair
 ζ_a, ζ_r damping of an actuator pole pair
 ζ_p damping of the phugoid pole pair
 ζ_{sp} damping of the short-period pole pair
 ζ_{spD} desired short-period damping
 $\eta_i(\alpha_m)$ a perturbing function that makes the performance measure nonparabolic as a function of α_m
 θ Lagrange multiplier measure
 $\theta_i(s)$ transform of the input of an abstract system
 θ_n man-machine performance measure value
 $\theta_o(s)$ transform of the output of an abstract system
 θ_p performance measure representing the total error between given and desired pole pairs
 $\theta_{p0}, \theta_{pu}, \theta_{pl}$ values of the performance measure θ_p at equally spaced increments Δ in an adjustable parameter
 $\theta_{PF}, \theta'_{PF}$ value of the performance measure after a major parameter adjustment
 λ Lagrange multiplier
 ϕ Euler angle in roll (rad)
 ϕ_n transformation output of an automaton
 ω_o corner frequency of gust input (r/s)
 ω_a, ω_r natural frequency of an actuator pole pair (r/s)
 ω_n natural frequency of an abstract short-period pole pair (r/s)
 ω_{np} natural frequency of the phugoid pole pair (r/s)
 ω_{nsp} natural frequency of the short-period pole pair (r/s)
 ω_{nspD} desired short-period natural frequency (r/s)

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PART I A PROGRAMMING APPROACH TO SYNTHESIS OF MANUAL AEROSPACE CONTROL SYSTEMS

1. DEVELOPMENT OF A PROGRAMMING APPROACH

A. BACKGROUND

The design of a manual flight control system for an aircraft is a task involving a great number of considerations. Cost, reliability, integration, safety, proper handling qualities, and structural damping are among the important factors that have to be considered. Because of the complexity of flight control systems, it has usually been the practice to design them using the analog computer as the central tool. Such factors as cost and reliability are considered by choosing only control configurations which are known a priori to be satisfactory from that point of view. Thus, design of manual flight control systems is based on a sophisticated form of trial and error.

The synthesis procedure described in Part I of this document was developed with the idea of improving the process of manual flight control system design. This new procedure takes into account many of the realities and practicalities of the situation. To some extent the development of precise analytical theory has been sacrificed for capability of incorporating practical aspects of the problem. Nevertheless, a theoretical framework is developed and experimentally verified.

The mathematical techniques used in the development of the programming approach to synthesis are conventional. The theory and its application should be understandable to anyone having a background in feedback control systems.

It should be mentioned that the research reported in this document represents the first known attempt at developing a mechanized manual control

synthesis procedure per se. Most previous work in manual control has in reality been "analysis". The area of manual control synthesis was virtually untrodden prior to this research study. Because this research was initial and because of limited scope, not all problems could be investigated. The section on conclusions and recommendations points up several topics suitable for future investigation.

B. NATURE OF MANUAL FLIGHT CONTROL SYSTEM DESIGN

Generally, the flight control system for a new aircraft is designed after the airframe and actuators have been developed. The aerodynamicists and aeronautical engineers try to develop the airframe in a way that will require the least control system compensation for dynamic characteristics. Here the idea is that the aircraft will be made as controllable as is physically possible. Should any stability enhancing component fail, the aircraft hopefully would remain controllable. Further, by developing the airframe for least compensation, various feedback or feedforward gains used for modifying the dynamics can be minimized in magnitude. Structural modes are then least excited and sensor noise has a minimal effect. Of course, the final airframe configuration represents a balanced compromise for the many flight regimes in which it will be used. Although a strong effort is made to maintain good dynamic characteristics in all flight regimes, it is often the case that the good characteristics in one regime must be compromised somewhat to improve the characteristics of another regime. Because the final airframe represents a multiple-trade-off, balanced, and well-planned dynamic system, it must be considered as frozen; that is, changes in the airframe may not be made by the control system designer unless it can be clearly shown that there is no other alternative.

Actuators used for driving the control surfaces of the airframe usually have only a secondary effect on the dynamics of the entire aircraft. Their bandwidths are usually greater by a factor of five or more than those associated with the airframe itself. Although the effect of actuator dynamics is small, it is not negligible. Modification of the servo characteristics of

the actuators is generally not considered acceptable unless it can be shown that the characteristics are inadequate from a power or speed of response standpoint. Thus, the dynamics of the various control surface actuators can be considered as fixed, just as the airframe dynamics are considered as fixed. In other words, compensation within the control system is generally not accomplished by modifying the dynamics of the actuators.

The airframe and actuators, which are fixed, can be specified by means of equations of motion for various flight conditions. These equations can be written in matrix or transfer function form or in state-space form. Thus, a block diagram of this portion of the manual flight control system can be drawn with all parameters specified.

The remainder of the manual control system consists of feedback sensors, feedback gains, controller feel servos and forward compensation. The designer may choose these subsystem components subject to a great number of practical constraints. Among the most important constraints are the following: 1) feedback gains must be kept below certain values to avoid exciting structural bending modes, 2) the aircraft must conform to accepted standards of aeronautical control system design and handling qualities, 3) control surfaces must not be driven into saturation because of overcompensation of the manual control system, 4) only certain limited types of stability augmentation may be used, and 5) the manual control system must be effective over all flight regimes and should require an absolute minimum of adaptive components, preferably none.

C. STEPS OF THE PROGRAMMING SYNTHESIS PROCEDURE

The major problem which must be solved is the development of a rational method for designing manual flight control systems that take into account the above stated practical constraints. This method should make manual control system synthesis a more automated and more scientific procedure, while remaining practical.

It is believed that most of the practical constraints on manual control system design can be incorporated into a synthesis procedure by completing the form of the system block diagram and then specifying the

ranges of values which various parameters may assume. The block diagram approach allows the designer to lay out the form of the manual control system. Unfeasible or undesirable configurations are thereby eliminated at the outset. For example, only those sensor feedbacks which are actually obtainable should be included in the diagram, thereby automatically eliminating undesirable feedback paths. Figure 1 shows a block diagram of a possible manual flight control system for the longitudinal dynamics of an SST. The diagram has been laid out in a way which accounts for many of the practical constraints. The airframe and actuator dynamics are considered fixed. The remainder of the diagram is of fixed-form. Particular attention has been given to sensor signals that are available. Pitch angle and pitch angle rate can generally be sensed without difficulty, and the signals are largely noise-free. The pitch rate signal can be filtered by a lead-lag network thereby allowing an approximation of pitch angle acceleration as well as pitch rate to be fed back. Normal acceleration and angle-of-attack signals can be sensed, but are usually somewhat noisy. Therefore, they should not be differentiated, nor can lead be applied to them. In fact, it is usually necessary to smooth these signals somewhat, as indicated by the lags associated with τ_c . Forward compensation is obtainable by performing operations on the signal summing point by means of simple electrical circuits. The feel servo for pitch input can be considered as a separate system by working with stick position as the input to the pitch dynamics.

Bounds may be placed on each of the free parameters in the fixed-form portion of the block diagram. These bounds again aid in ensuring that only practical configurations will be considered. Feedback gains and compensator lead can be held within realistic limits, thereby avoiding control surface limiting and excitation of structural modes.

The dynamics of the manual flight control system as outlined in the block diagram will be dependent on the settings of the adjustable parameters in the fixed-form portion of the system. The problem is one of specifying these parameters in a way that yields maximum improvement in the man-machine system for all flight regimes.

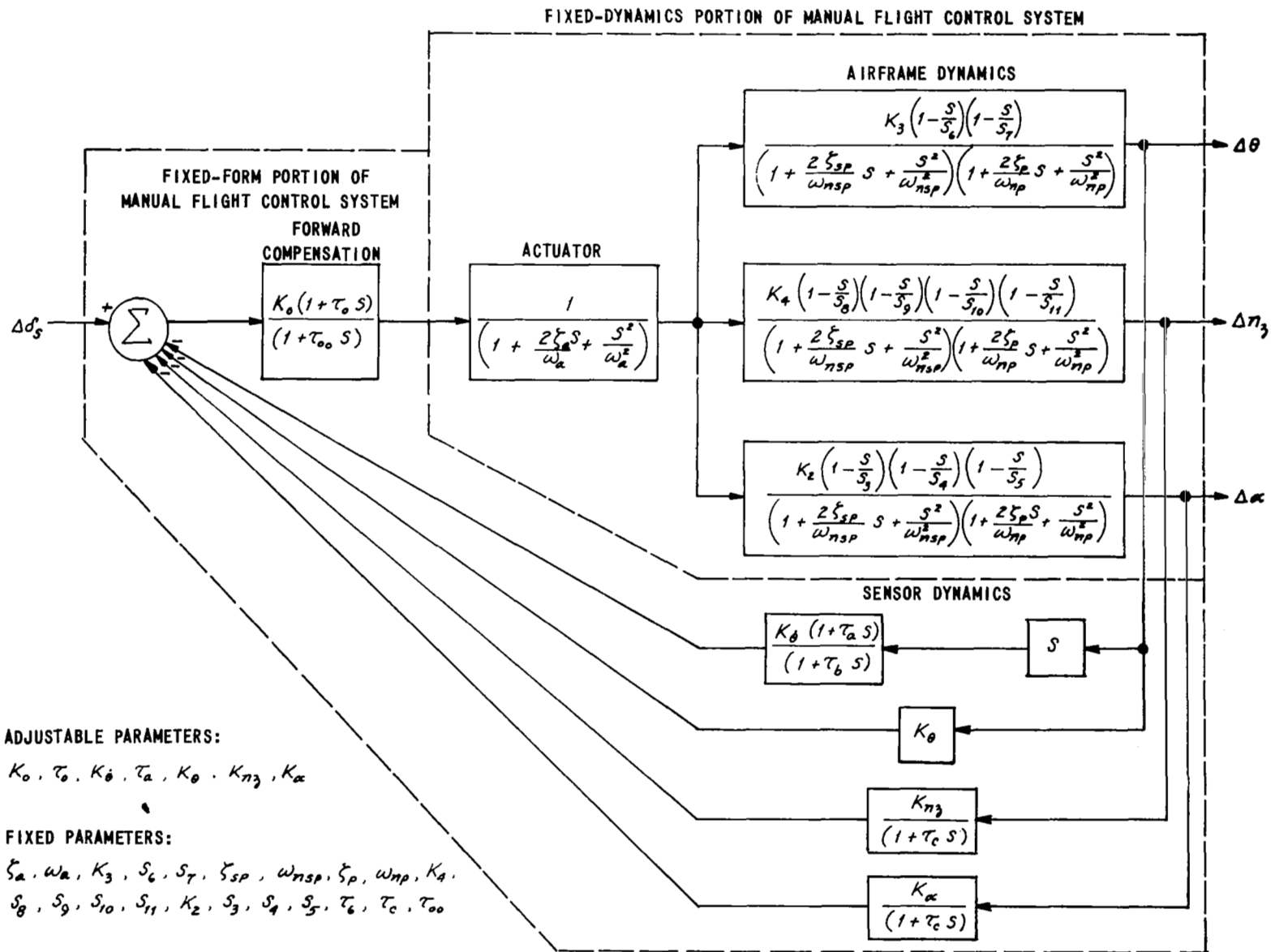


Figure 1 CONSTRAINING BLOCK DIAGRAM FOR LONGITUDINAL DYNAMICS OF AN SST

As is well known, the overall man-machine system dynamics of any vehicle are dependent on loop closure by the human. In the case of longitudinal dynamics of an aircraft, the pilot performs loop closure by maintaining a desired pitch angle through proper stick commands. He also closes an altitude loop that maintains, or attains, a desired altitude. It is possible to develop a dynamic model of the pilot in this endeavor; but the model parameters will be dependent on flight control system parameters. Rather than develop such an adaptive model, it is probably better for well-defined problems such as aircraft control to synthesize the vehicle dynamics in a way that accounts for subjective handling qualities and does not model the human operator per se. Subjective handling qualities information can be used as a means of introducing human operator dynamics into a flight control system synthesis procedure.

Generally, handling qualities research has resulted in relationships between pilot rating and the poles or eigenvalues of the manual control system*. Acceptable ranges for the damping and natural frequencies of the complex pole pairs and time-constants associated with the real poles are therefore known. Since the various feedback and compensation parameters will make pole movement possible, the synthesis procedure can be developed around the idea of moving the poles into the most desirable areas from a handling qualities point of view.

The first step in the actual synthesis procedure should involve the choice of desired pole locations for each flight regime. These may be the same for all regimes, or they may vary with regime. In the longitudinal dynamics case, there are two important pole pairs, the short-period pair and the phugoid pair. The closed-loop longitudinal dynamics will contain other poles of lesser importance. The actuator poles will fall at higher

* For the reader unfamiliar with the definition of eigenvalues, they can be considered as analogous in multi-input-multi-output systems to poles in single-input-single-output systems. The programming approach to manual control synthesis is applicable to both types of systems.

frequencies, as will some of the poles of the sensor dynamics filters. Thus, it should be possible, from a handling qualities point of view, to choose the most desirable location for the short-period pole pair and the phugoid pole pair for each flight regime. The remaining poles can be constrained to fall at much higher frequencies and with sufficient damping by choosing their desired positions at these higher frequencies.

It is worth noting here that handling qualities research in longitudinal dynamics still deals with the two dominant pole pairs, short-period and phugoid, as a means of specifying proper longitudinal handling qualities. If the order of the closed-loop longitudinal flight control system is high, the objective becomes one of pushing all poles except the short-period and phugoid pairs up to high enough frequencies to make their effect on dynamics of secondary importance. Thus, moving the poles to desired positions fits both handling qualities concepts and control systems concepts.

Often, however, additional handling qualities information of secondary importance is given in other forms, such as rise times, ratios of certain steady-state parameters, and controller sensitivities. In this initial study these additional specifications are taken into account by constraining the configuration and parameter values on the system block diagram. More direct methods of incorporation are left as a problem for future study.

Once the desired pole positions have been specified for each flight regime, the next step is to adjust the various free parameters and gains in the fixed-form portion of the manual control system to bring about closest correspondence between the desired and actual closed-loop pole positions for each individual flight regime. Here the idea is to determine whether or not the chosen configuration is capable of exhibiting the desired closed-loop positions.

As is well known from control theory, in general every state variable must be fed back with its gain freely chosen if all possible closed-loop pole positions are to be possible. Thus, if only certain feedback signals are available and if feedback gain ranges are limited, then certain pole locations become inaccessible. It must therefore be considered generally

impossible to place the actual closed-loop poles directly over the desired closed-loop poles when the feedback configuration is fixed and parameters are limited. Accordingly, it becomes necessary to choose a criterion and perform a systematic optimization process which adjusts the various parameters. The objective of the optimization process is to adjust the gains in a way which minimizes the error criterion, thereby allowing the closest match between desired and actual pole locations.

The optimization process can be performed by an iteration or programming procedure, which successively approaches a minimum. As with any programming procedure, extreme care must be taken. It is possible that more than one local minimum exists, and determination of the global minimum requires investigation of all local minima. The programming procedure developed for manual control synthesis is described in detail in Chapters 2 and 3. Briefly, the characteristic equation of the closed-loop system is computed at each adjustment of a parameter. The iteration process is used to adjust the parameters so as to bring the closed-loop characteristic equation into closest correspondence with the desired characteristic equation.

After the optimization process has been carried out individually for each flight regime, the designer must make the decision as to whether or not the optimal pole configurations are sufficiently close to the desired pole configurations. Is the designer willing to settle for the pole proximity that the optimization process yields? If he is, then for each flight regime a satisfactory solution to the manual flight control problem has been found. If he is not, then additional feedback, compensation, or gain ranges, must be allowed. Of course, considerable judgment is required in deciding the best course of action. Some insight as to which way to proceed can be obtained by noting which parameters have reached limiting values and by noting which state variables are not available for feedback. If the manual control configuration is carefully studied, it should be possible to introduce the necessary modifications so that a second use of the optimization process brings the closed-loop poles into satisfactory correspondence with the desired poles for each individual flight condition. It is worth pointing out that there can be no other avenue of approach than changing the configuration,

if after the first optimization process the closed-loop pole locations are unsatisfactory.

Since the above procedure would result in satisfactory closed-loop dynamics for each flight regime, the next step is to develop a multiple regime configuration. At the end of the above procedure, a set of parameters for each flight regime will have been obtained. These parameters produce the desired closed-loop dynamics in each flight regime; but, of course, they will no doubt vary from one regime to another. As pointed out earlier, the manual control system must be effective over all flight regimes and should require an absolute minimum of gain-scheduled components, preferably none. Thus, an attempt should be made to perform a multiple regime synthesis resulting in one optimum set of fixed parameters. Since it would be known at this point in the synthesis that the configuration is adequate for each flight regime individually, the problem becomes one of determining whether or not gain-scheduled components must be used.

The multiple regime synthesis procedure may be developed as an extension of individual regime synthesis. First, a criterion is chosen which represents the sum of the pole position errors over all individual flight conditions. For those conditions where the open-loop airframe dynamics are worst (from a handling qualities point of view) the error may be weighted more heavily. However, after an iteration in a given parameter, the characteristic equation for each flight condition is computed. From these characteristic equations, the sum error criterion would be evaluated. Again, a programming procedure is developed; but in this case only one set of optimum fixed parameters would be obtained.

After the multiple regime optimization procedure has been carried out, the designer must again make a decision as to whether or not the optimal pole configurations are sufficiently close to the desired pole configurations in each flight regime for the single fixed set of optimum parameters. If he considers the poles sufficiently close, the synthesis procedure is completed, and it will not be necessary to use gain-scheduled components in the design. If he decides that the poles are too far from the desired positions, he must

accept the fact that at least one of the parameters must be scheduled.* Under the latter condition the design is not completed and further steps will be required.

Examination of the optimal set of parameters for each individual flight condition should make clear which gains are varying widely from one regime to another. These parameters are the most probable cause of the unsatisfactory pole locations when multiple regime fixed parameter synthesis is attempted. By allowing some of these parameters to vary with flight condition in the synthesis procedure, an acceptable manual control system can probably be developed.

Suppose that one or more parameters are to be scheduled or made variable with flight condition, and the remainder are to be held constant. The variable parameters could be selected from among those which vary most widely in the individual flight regime syntheses. By using both the sum error criterion and the individual error criteria that have already been selected for the multiple and individual regime syntheses, it would be possible to develop a scheduled parameter synthesis procedure. If there are n total parameters in any given flight regime and r of these are to be variable, then the total number of adjustable parameters will be $n - (k-1)r$, where k is the number of flight conditions tested. Suppose that in an iteration process similar to those used for the multiple and individual regime syntheses, the fixed parameters are adjusted according to the sum error criterion, and the scheduled parameters are adjusted according to each individual regime criterion separately. When evaluating the sum criterion, the value of the variable parameters for each individual flight condition is used. Thus, the sum error criterion will have a smaller final value than will the multiple regime synthesis (with fixed parameters). It should be noted that adjusting one of the scheduled parameters using the individual criterion will also result in a reduction of the sum criterion. Consequently, if a convergent iteration process can be developed for the individual and multiple regime synthesis procedures described above, one can also be developed for the adaptive multiple regime synthesis procedure.

* In this report, a scheduled parameter is one which is permitted to vary with flight regime.

Of course, considerable judgment will be required in using the gain-scheduled multiple regime synthesis procedure. Which parameters should be made adaptive? How can one implement the adaptive parameters for reliable, fail-safe operation? How sensitive are the pole locations to small changes in scheduled parameters that may be required by the function generating mechanism? The computed information that is a by-product of synthesis procedures would be helpful in answering questions of this type.

The only remaining task is that of adjusting the controller (stick, pedals, etc.) gains so that good man-machine servo response is obtained. Generally there will be some latitude in the forward loop and feedback loop gains, if they are changed in a compensatory fashion. If the forward gain is reduced while the feedback gains are increased, it is possible to maintain the optimum closed-loop pole locations while changing controller gain. The correct gain can be chosen from previous aircraft design experience and from elementary man-servo principles.

D. SUMMARY OF STEPS OF THE OVERALL SYNTHESIS PROCEDURE

1. Obtain airframe equations for all representative flight regimes. Specify the actuator dynamics.
2. Complete the block diagram of the manual control system configuration including allowable feedback and compensation paths. Choose parameter ranges from practical considerations.
3. From handling qualities studies determine desired locations of poles or eigenvalues for each flight regime.
4. Perform individual flight regime syntheses. Compute closest location of actual poles to desired poles for each flight condition. Determine whether or not there are unsatisfactory conditions (actual poles cannot be placed sufficiently close to desired poles).
5. If an unsatisfactory condition exists, introduce other feedback or compensation paths to make possible the desired dynamics.

6. After satisfactory pole locations are obtained for each flight condition, attempt a multiple regime synthesis with one set of fixed parameters. Compute closest location of actual poles to desired poles for all flight conditions. Determine whether or not there are unsatisfactory conditions (actual poles cannot be placed sufficiently close to desired poles).
7. If an unsatisfactory condition exists, perform a scheduled parameter multiple regime synthesis. Determine which parameters are to be considered fixed and which variable. Compute closest pole locations.
8. Once a satisfactory system is obtained, adjust control input gain by compensatory changes between forward loop gain and all feedback loop gains.

2. A PARAMETER ADJUSTMENT PROCEDURE

A. BACKGROUND

A great deal of research effort has been devoted to the development of optimization procedures which either maximize or minimize a functional whose parameters may be adjusted. Linear programming¹, nonlinear programming², steepest descent or ascent procedures³, gradient methods⁴, and several other methods⁵ have been applied to problems of this type. The programming approach to manual control system synthesis as outlined in Chapter 1 requires that some type of procedure be developed for adjusting various system parameters in order that optimum system performance might be obtained. This chapter describes a certain programming procedure that has been developed specifically for the manual control synthesis problem. The procedure uses many of the principles described in the above cited references. However, it is also original in several aspects, particularly those involved in the logic for dealing with unusual situations and those involved in taking advantage of the common properties of all manual control synthesis problems. The procedure is relatively straightforward and can be understood by anyone with a background in control theory.

B. PERFORMANCE MEASURE

Implicit in any optimization procedure is the selection of a measure of goodness or performance. The functional which is maximized or minimized by adjustment of parameters must represent this measure. If the functional is not chosen in accordance with the correct assessment of goodness or performance then the optimum parameter values of the functional will not necessarily represent an optimum in performance. Therefore, the functional must be selected so as to represent system performance.

A second consideration in the choice of a functional for representing system performance involves the complexity of the optimization procedure and assurance that at least one solution does exist. If a complicated functional is chosen, the computational process will be more difficult. Therefore, it is advantageous to use a relatively simple functional if feasible from a performance measure standpoint. Also, since it is possible to

specify a functional that does not exist for certain values of the adjustable parameters, it is possible that for certain constraints on adjustable parameters a solution does not exist. In other words, there may be no feasible solution for a given problem. To avoid this situation, it is only necessary to restrict the class of functionals to those which have some finite value for any finite setting of adjustable parameters. Then, even though the adjustable parameters are constrained, there must be at least one solution to the problem.

As pointed out earlier, handling qualities research has resulted in specification of the desired pole or eigenvalue positions of the manual control system. Thus, the manual control synthesis procedure can be developed around the idea of moving the system poles into the most desirable areas from a handling qualities point of view. The functional for optimization enters the manual control synthesis problem by quantifying the measure of difference between closed-loop pole positions for any given setting of adjustable parameters and the desired pole positions as determined from handling qualities research. It appears that a weighted sum of squared differences is adequate as a functional for this problem:

$$\theta_p = \sum_{i=1}^I k_i (S_{D_i} - S_{G_i})(\overline{S_{D_i} - S_{G_i}}) \quad (1)$$

where $S_{G_i} = R_{G_i}(\alpha_1, \alpha_2, \dots, \alpha_m, \dots) + j I_{G_i}(\alpha_1, \alpha_2, \dots, \alpha_m, \dots)$ (2)

and $S_{D_i} = R_{D_i} + j I_{D_i}$ (3)

In this equation, θ_p represents the functional to be minimized, each k_i is a position weighting constant, each S_{D_i} is a desired pole position, and each S_{G_i} is a given pole position. The parameters $\alpha_1, \alpha_2, \dots, \alpha_m, \dots$ are to be adjusted to minimize θ_p .

It should be noted that the manual control synthesis problem requires careful association of each given pole with a certain desired pole.

To illustrate the importance of correct association, consider the effect of one particular improper association. Suppose that for the longitudinal aircraft dynamics problem the desired short-period pole pair were associated with the phugoid given pole pair. Then, minimization of the functional would force the given phugoid pole pair into the desired short-period pole pair region. Such a procedure is contrary to fundamental principles of flight control system design and would result in an impractical flight control system. Therefore, each given pole must be associated with one and only one desired pole by carefully ordering the poles in the subscript, i . The problem of ordering requires thorough consideration in the development of a digital program for optimization and will be discussed in greater detail later. Ordering, which appears not to have been studied previously by other researchers, is as important in manual control synthesis as the programming procedure itself.

C. ROLE OF THE ROOT LOCUS

The root locus has been used for many years as a standard graphical method for determining the closed-loop poles of a feedback control system as a function of loop gain. The technique is widely used in the aeronautical industry because of certain incidental properties of the root locus. Perhaps the most important incidental property is that the root locus handles unstable and nonminimum phase dynamics without any alterations to the method whatsoever. This property is very important, for it often happens that one pole pair, for example the phugoid pair, is slightly unstable in the open loop for one or more flight regimes. Another incidental property of great importance is that the root locus can be drawn for either positive or negative loop gain. Whereas the root-locus method inherently possesses these incidental properties, the Nyquist, Bode, and Nichols methods must be extended or made more complicated to incorporate them.

The conventional root-locus method is used to produce a complex-frequency plane locus of the poles or eigenvalues as a function of system loop gain. In aircraft manual control system synthesis problems, gain in a forward or feedback path is but one of three types of adjustable parameters

encountered. The other two types of adjustable parameters involved are:
 1) adjustment of an open-loop pole in either a forward or feedback path, and
 2) adjustment of an open-loop zero in either the forward or feedback path.
 These latter two types of adjustment do not fall within the realm of the conventional root locus method; however, the method can be extended to cover them. Reference 6 carries a very good account of this extension. For completeness, it will be summarized in the following paragraphs.

As is well known, a feedback control system has a closed-loop transfer function given by

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{K \frac{N_G(s)}{D_G(s)}}{1 + K \frac{N_G(s)}{D_G(s)} \cdot A \frac{N_F(s)}{D_F(s)}} \quad (4)$$

where K is the forward-loop gain, $\frac{N_G(s)}{D_G(s)}$ is a ratio of polynomials representing forward-loop dynamics, A is the feedback-loop gain, and $\frac{N_F(s)}{D_F(s)}$ is a ratio of polynomials representing the feedback-loop dynamics. The locus of closed-loop poles is obtained by solving for the roots of the equation

$$D_G(s) D_F(s) + K A N_G(s) N_F(s) = 0 \quad (5)$$

This equation may be written as

$$D_o(s) + K_o N_o(s) = 0 \quad (6)$$

where $D_o(s)$ is the polynomial made up of all open-loop poles, K_o is the total loop gain, and $N_o(s)$ is the polynomial made up of all open-loop zeros.

Suppose that a factor with an adjustable open-loop zero is introduced in equation (5); that is, $N_o(s)$ is replaced by $N_r(s)(1 + \tau_3 s)$ where τ_3 is adjustable. (This procedure is equivalent to introducing an adjustable zero into either $N_G(s)$ or $N_F(s)$.) The equation for the closed-loop poles then becomes

$$\left[D_o(s) + K_o N_i(s) \right] + \tau_3 \left[K_o s N_i(s) \right] = 0 \quad (7)$$

This new equation can be treated by the root-locus method by letting

$$D_o'(s) \equiv D_o(s) + K_o N_i(s) \quad (8)$$

and

$$N_o'(s) \equiv K_o s N_i(s) \quad (9)$$

and then solving for the roots of

$$D_o'(s) + \tau_3 N_o'(s) = 0 \quad (10)$$

as a function of the "gain" τ_3 . Because (6) and (10) are similar in form, it may be concluded that the root-locus method can be applied to the case where an open-loop zero is adjustable.

Similarly, suppose that a factor with an adjustable open-loop pole is introduced in equation (5); that is, $D_o(s)$ is replaced by $D_i(s) \cdot \left(1 + \frac{s}{s_p}\right)$ where s_p is adjustable. The equation for the closed-loop poles becomes

$$s D_i(s) + s_p \left[D_i(s) + K_o N_o(s) \right] = 0 \quad (11)$$

This new equation can be treated by the root-locus method by letting

$$D_o''(s) = s D_i(s) \quad (12)$$

and

$$N_o''(s) = D_i(s) + K_o N_o(s) \quad (13)$$

and then solving for the roots of

$$D_o''(s) + s_p N_o''(s) = 0 \quad (14)$$

as a function of s_p . Because (14) and (6) are similar, the root locus

method can be applied to the case where an open-loop pole is adjustable*.

In summary, the root-locus method has advantages which make it suitable as a basis for manual control system synthesis. The most important of these are:

1. graphical presentation of pole or eigenvalue positions which can then be compared with desired positions,
 2. ability to handle nonminimum phase and unstable open and closed-loop configurations,
 3. ability to handle positive and negative forward and feedback path gains,
- and
4. ability to handle (by straightforward extension) adjustable open-loop poles and zeros.

D. COMMON PROPERTIES OF THE MINIMIZATION PROCEDURE

It has been shown above that all parameter adjustments, whether they involve open-loop gain, zero, or pole movement, result in closed-loop pole motion that can be described by the root locus method. In this section it will be shown, that for the performance measure of equation (1), the error as a function of any adjustable parameter possesses certain general properties. These properties are useful in the development of a minimization procedure. They are largely a result of the common elements of root-locus plots.

To gain insight into the character of the error measure as a function of an adjustable parameter, a simple example will be studied. Consider a system whose forward path dynamics are

$$KG(s) = \frac{K}{s(s^2 + 2s + 2)} \quad (15)$$

* It is important to note that for simplicity, a pole should be adjusted in accordance with its frequency ζ_p , and not in accordance with its associated time constant.

and whose feedback is unity. Assume that the positions of the desired closed-loop poles are

$$s_{D1}, s_{D2} = -0.26 \pm j 1.50 \quad (16)$$

$$s_{D3} = -1.30 \quad (17)$$

If K is considered as the adjustable parameter, then the usual root-locus can be drawn for positive and negative values of K , as shown in Figure 2. Correspondingly, the performance measure can be computed as a function of the adjustable parameter K . Figure 3 shows a plot of θ_p versus K for $k_1, k_2 = 0.5$ and $k_3 = 1.0$.

Useful concepts can be obtained by examining Figure 3. First, contrary to intuition, there is but a single minimum. The minimum distance between the given real pole and the desired real pole is seen in Figure 2 to occur at $K = 1.8$ approximately. Similarly, the minimum distance between the upper given complex pole and the upper desired complex pole is seen to occur at $K = 3.9$ approximately. Because these two values of K are unequal, one might expect to observe two relative minima, one in the vicinity of $K = 1.8$ and another in the vicinity of $K = 3.9$. However, as the plot shows, there is but one minimum, and it occurs at $K = 2.0$. Secondly, it is observed that in the vicinity of the minimum, θ_p could be considered as approximately parabolic, and could probably be described adequately by the expression

$$\theta_p = a_1 + b_1 K + c_1 K^2 \quad (18)$$

where a_1, b_1 and c_1 are constants. The third important concept is, that for values of K far from the value where θ_p is minimized, the relation between θ_p and K is approximately linear. Thus, there appears to be an asymptotic relationship that comes into play for K far from the minimum. In the following paragraphs of this section, these concepts will be elaborated in a more general framework. The performance measure that has been chosen for this manual control synthesis procedure is actually

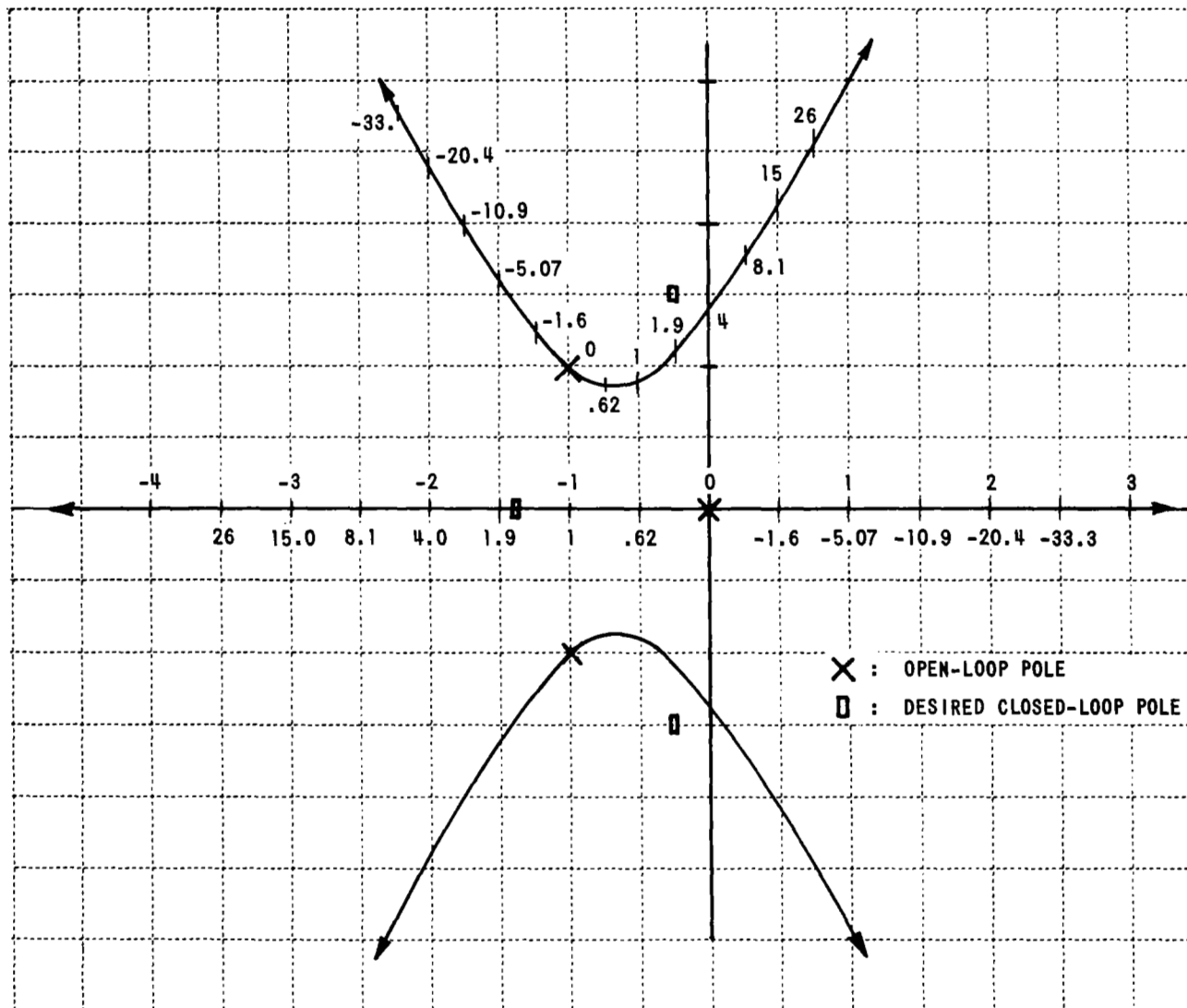


Figure 2 POSITIVE- AND NEGATIVE-GAIN ROOT LOCUS OF $KG(s) = \frac{K}{s(s^2 + 2s + 2)}$

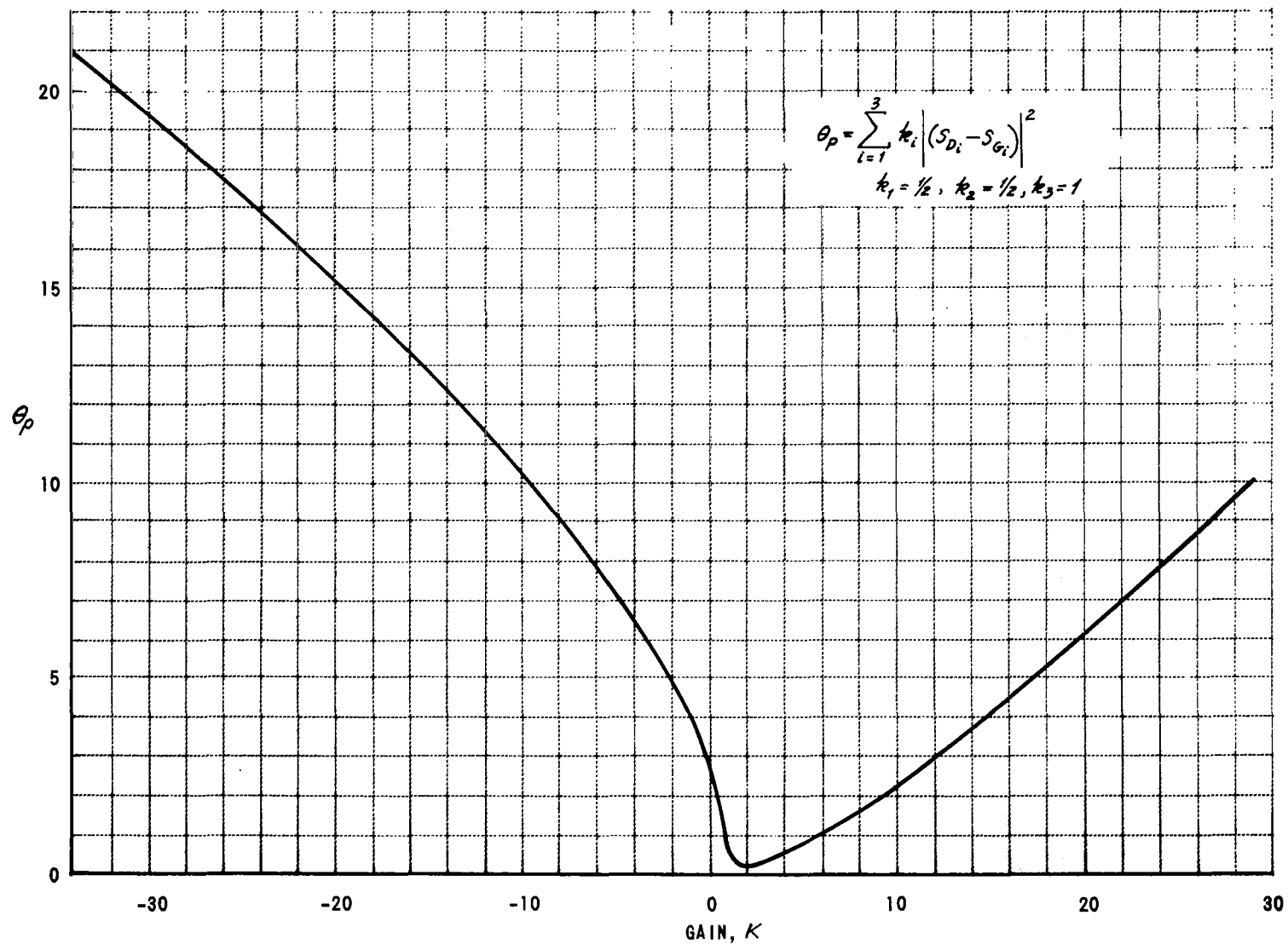


Figure 3 PERFORMANCE MEASURE, θ_p , AS A FUNCTION OF K FOR LOCUS OF FIGURE 2

a weighted sum of squared distances between corresponding desired and given closed-loop poles. While this measure is therefore a quadratic function of distances, it is not a parabolic function of the adjustable parameters. An analytical expression of the closed-loop pole positions as a function of an adjustable parameter is theoretically obtainable⁷; however, the relationship is nonlinear and too complicated to use in practice. Therefore, it is necessary to resort to simpler relations to describe the cause of the performance measure curve shape.

It was pointed out in the example above that there is but one minimum, and that in the vicinity of the minimum the performance measure is approximately parabolic. Actually, in most cases the same phenomenon will be observed. Consider the contribution to the performance measure of the difference between one desired pole and one given pole. As the given pole locus nears the desired pole, the squared distance between them as a function of the adjustable parameter is approximately parabolic. This relationship is generally valid even though the root-locus is based on nonlinear relationships. The reason for this situation is that the root locus exhibits an approximate "linearity" for small adjustments of parameters.

If the root-locus plot of Figure 2 is examined, it is seen that each given pole passes a desired pole with an approximately linear relation between given pole movement and change in gain. Linear movement of this type will result in a parabolic contribution to θ_p .

It is true of course that the parabolic relationship is only approximate for each associated desired pole and given pole. To analyze this situation, suppose that the performance measure θ_p is written as a summation of approximately parabolic functionals. Each functional is assumed the result of squared distance between one desired pole and its associated given pole. Then,

$$\theta_p = \sum_{i=1}^I \left\{ (a_i + b_i \alpha_m + c_i \alpha_m^2) [1 + \eta_i(\alpha_m)] \right\} \quad (19)$$

where the $\eta_i(\alpha_m)$ are perturbing functions which make each squared

distance somewhat nonparabolic*.

Suppose for the moment that each perturbing function is zero. Then a very important fact comes to light. It is seen that even though the individual parabolas (for each desired pole to given pole squared distance) may be of various shapes, the overall performance measure remains an exact parabola. In other words, the sum of parabolas of various shapes with minima at various positions is a parabola. This is easily observed by rearranging equation (19 with $\eta_i(\alpha_m)$ set equal to zero:

$$\theta_p = \left(\sum_{i=1}^I a_i \right) + \left(\sum_{i=1}^I b_i \right) \alpha_m + \left(\sum_{i=1}^I c_i \right) \alpha_m^2 \quad (20)$$

Therefore, if the squared distances between each desired pole and its corresponding given pole were exactly parabolic, one would expect to obtain an exact parabolic expression for θ_p with a corresponding single minimum.

Of course, in the realistic situation the $\eta_i(\alpha_m)$'s are generally nonzero; that is the squared distance between each desired pole and its associated given pole is somewhat nonparabolic. It is possible, however, to extend the results given above, so that certain statements can be made about the realistic situation. Suppose each $\eta_i(\alpha_m)$ is nonzero, is an independent sample of the same random process, and has zero mean. Then by using the theorems of sample averaging, it is possible to show that whereas the parabolic content of θ_p increases approximately linearly with I (the number of poles), the standard deviation of the nonparabolic content of θ_p increases with the square root of I. It may therefore be concluded that, if the perturbing functions $\eta_i(\alpha_m)$ are independent of one another, the performance measure θ_p approaches a parabola (as a function of α_m) as I becomes large. The perturbing functions will

* This performance measure appears slightly less general than the original, because the original contained a group of weighting constants, k_i . However, these constants may be considered as incorporated by modifying the values of a_i , b_i and c_i .

generally be approximately independent of one another in the area of the minimum of each individual squared distance, and therefore, θ_p does indeed approach a parabola. For these reasons, in the majority of instances, one minimum will exist in any adjustable parameter.

In the above discussion of parabolic minima, the validity of justification weakens as the adjustable parameter, say α_m , moves farther away from the minimum. This weakening is the result of "nonlinearity" of the root-locus. If Figure 2 is again observed, it can be seen that ever increasing amounts of gain are required to move a given pole a fixed distance. For example to move the given real pole from -2.0 to -2.5 requires an additional gain of 4.1, whereas to move the same pole from -2.5 to -3.0 requires an additional gain of 6.9. Therefore, one should expect distinctly nonparabolic behavior of the measure θ_p when a given adjustable parameter is far from its minimum. Figure 3, clearly exhibits this nonparabolic behavior when α_m is not in the vicinity of the minimum. In the development of a minimization procedure, it is as important to understand the behavior far from the minimum as it is to understand the behavior near the minimum.

As is well known, the root-locus exhibits asymptotic behavior for large values of loop gain. This behavior is helpful in describing the shape of the error functional θ_p far from the minimum. Suppose that in equation (6) $N_o(s)$ is of order m and $D_o(s)$ is of order n in s . Then

$$(g_0 + g_1 s + g_2 s^2 + \dots + g_n s^n) + K_o (h_0 + h_1 s + h_2 s^2 + \dots + h_m s^m) = 0 \quad n > m \quad (21)$$

For large s and large K_o , dividing by s^m yields the following approximate equation:

$$(g_n s^{n-m} + g_{n-1} s^{n-m-1} + \dots + g_1 s) \approx -K_o h_m \quad (22)$$

The polynomial can be factored as follows:

$$K_o (s + g_{10}) (s + g_{11}) \dots (s + g_{1(n-m)}) \approx -K_o h_m \quad (23)$$

Consequently, as S becomes large, the S terms dominate the root values $g_{10}, g_{11}, \dots, g_{1(n-m)}$ allowing the expression for the root locus to be written approximately as

$$S^{n-m} \approx K_o' \quad (24)$$

where

$$K_o' = \frac{-K_o h_m}{k_o}$$

Thus,

$$S \approx (K_o')^{1/(n-m)} \quad (25)$$

There are $n-m$ poles which approach infinity as the $(n-m)^{th}$ root of normalized gain. If it is assumed that all desired poles are located near the origin (when compared with the large values of the given pole positions), the performance measure may be written as

$$\theta_p \approx \sum_{i=1}^I k_i \left| (K_o')^{2/(n-m)} \right| = \left| (K_o')^{2/(n-m)} \right| \sum_{i=1}^I k_i \quad (26)$$

Therefore, θ_p approaches a $(2/(n-m))^{th}$ power curve as a function of the normalized gain K_o' . Since K_o' is proportional to the adjustable gain K_o , θ_p approaches the $(2/(n-m))^{th}$ power of the adjustable parameter K_o .

In the example described earlier (Figures 2 and 3), $m=0$ and $n=3$. Therefore, one should expect the values of θ_p to approach at $2/3$ power curve for large values of K . In Figure 3, θ_p appears to approach an approximately linear function of gain. However, the values of gain for which the plot is constructed are only moderately large. For larger values, one could expect a tapering to a $2/3$ power curve.

Generally, for a gain adjustment root-locus, the number of poles exceeds the number of zeros. Therefore $(2/(n-m))$ may take on the values 2, 1, $2/3$, $1/2$, $2/5$, etc. For a zero adjustment root-locus, all of the same powers are possible, plus one additional. This additional value occurs because $N_o'(S)$ in equations (9) and (10) introduces an extra zero into the root-locus; therefore, in some cases $n=m$. Under this condition, there are no poles in the locus that move toward infinity. θ_p then approaches

a finite constant as τ_3 becomes large. If the position of an open-loop pole is adjusted, equation (11) shows that the number of poles in the locus always exceeds the number of zeros by one. Therefore, for pole adjustment θ_p always approaches a second power curve function of $\mathcal{S}_p$; that is $\frac{2}{n-m} = 2$. It may be concluded that in no case will the curvature of θ_p exceed that of a squared function of an adjustable parameter (when the parameter is far from the value where the minimum occurs).

E. ELEMENTS OF THE MINIMIZATION PROCEDURE

In the last section it was shown that a single, approximately parabolic minimum can be expected in θ_p when any given parameter is adjusted. Further, the curvature of θ_p is between the zero and the second power of the adjustable parameter when the parameter is far from the minimum. Because of these properties, a particularly simple iteration process can be developed for performing the minimization. While most minimization procedures developed previously have high efficiency as a primary goal, the procedure developed for manual control system synthesis has reliability as its primary goal. The major objective in manual control synthesis is to obtain solutions under the most general circumstances possible. A minimization procedure that is highly efficient, but fails on occasion, is of little value in the synthesis problem. The objective is to perform control system synthesis with the digital computer, and therefore, all contingencies must be programmed as logic sequences. A "no compute" condition is unacceptable because it means that the original program must be modified to obtain a solution.

To overcome the above stated difficulty, a serial adjustment procedure was developed. This procedure involves the cyclical adjustment of only one parameter at a time. While the procedure is probably less efficient than others developed previously, it lends itself to coverage of the greatest number of contingencies and computational difficulties. As pointed out earlier, the pole association problem must be given careful consideration in the synthesis procedure. Each time one or more parameters are adjusted, the closed-loop poles of the system will move. After the new pole positions

are found by the computer, they must be ordered properly to obtain correct association with the desired poles. By adjusting only one parameter at a time, the difficulty of performing this association is minimized. While certain operations that the human designer performs may appear relatively straightforward, (such as interpreting the progression of closed-loop roots on a root-locus diagram) these operations are actually very complex and must be carefully programmed for solution by a digital computer. Thus, from an overall viewpoint, efficiency is but one facet of the manual control synthesis procedure.

The minimization procedure is most easily explained in terms of usual and unusual conditions. In the usual condition, θ_p will be assumed approximately parabolic. Suppose that θ_p is evaluated for a given initial setting, g , of the adjustable parameter α_m , yielding a value θ_{p0} . Afterward, α_m is increased and decreased by an amount Δ to yield two other points on θ_p , designated θ_{pU} and θ_{pL} . (See Figure 4.) A unique parabola may be passed through these three points. It will be given by

$$P = a + b\alpha_m + c\alpha_m^2 \quad (27)$$

where
$$c = \frac{\theta_{pU} - 2\theta_{p0} + \theta_{pL}}{2\Delta^2} \quad (28)$$

$$b = \frac{\theta_{pU} - \theta_{p0}}{\Delta} - 2gc - c\Delta \quad (29)$$

and

$$a = \theta_{p0} - bg - cg^2 \quad (30)$$

The minimum of this parabola occurs at

$$\frac{\partial P}{\partial \alpha_m} = 0 \quad (31)$$

yielding

$$\alpha_m \Big|_{min} = -\frac{b}{2c} \quad (32)$$

Therefore, given the coordinates of the three points on the curve θ_p , the value of α_m at which the minimum (of the fitted parabola) occurs is

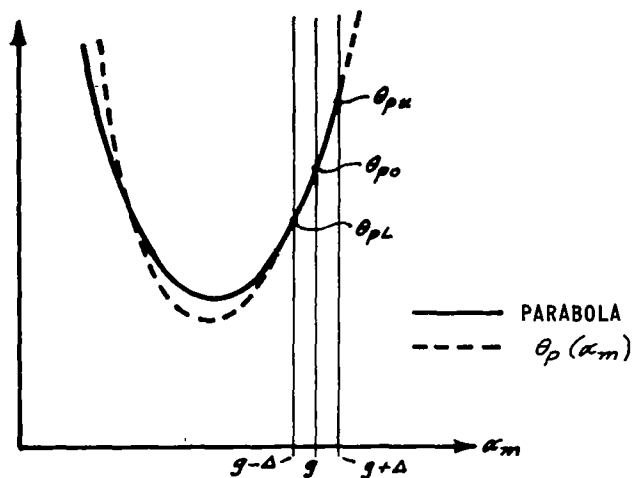


Figure 4 PARABOLIC FITTING IN THE USUAL SITUATION

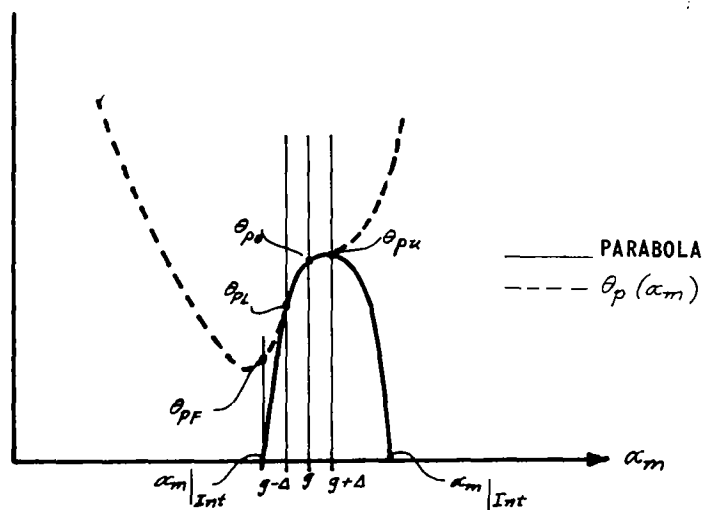


Figure 5 UNUSUAL CONDITION IN WHICH THE FITTED PARABOLA HAS NO FINITE MINIMUM

easily computed using equations (28), (29), and (32). It is clear that, if there is approximate correspondence between the parabola and the actual curve of θ_p , a significant reduction in the value of θ_p can be obtained by setting α_m to the value given by equation (32). In other words, an approximate minimum will be obtained. The cyclical procedure is developed by performing the above operation sequentially on each adjustable parameter. After the computation of the minimum for a given parameter, the optimum value is substituted and the computation for the next parameter is undertaken. At the end of adjustment of each parameter, the process is repeated. It has been shown previously that, if the performance measure is quadratic and satisfies certain other elementary conditions, this type of iteration process is convergent and yields the true multidimensional minimum.^{8, 9} Therefore, if θ_p is approximately parabolic in each adjustable parameter, this iteration procedure will almost certainly converge to the minimum*.

The above procedure represents the normal or usual optimization procedure used for manual control system synthesis. However, this normal procedure may fail in several different ways at some point in the iteration process. To avoid having the program fail to attain a solution, these unusual cases have been analyzed and the proper logic developed to allow the program to continue.

One unusual case is illustrated in Figure 5. This is the case in which the fitted parabola has a maximum instead of a minimum. Detection of this unusual case is straightforward.

$$\frac{\partial^2 \rho}{\partial \alpha_m^2} = 2c \quad (33)$$

If c is positive, the fitted parabola has a minimum;

If c is negative, the fitted parabola has a maximum, and

If c is zero, the fitted parabola degenerates to a straight line.

When c is negative, the optimization procedure can be continued by

* The assumption is being made that the second derivative of θ_p with respect to each parameter is positive.

examining the intersection of the parabola with the abscissa (α_m axis). These intersections occur at

$$P = a + b\alpha_m + c\alpha_m^2 \quad (34)$$

or

$$\alpha_m \Big|_{Int.} = \frac{-b \pm \sqrt{b^2 - 4ca}}{2c} \quad (35)$$

from which two real solutions will always be obtained. By choosing the solution that is nearer to g , and considering this as the new adjusted value of α_m , the optimization procedure can be continued. Figure 5 shows that θ_{PF} , the value of θ_p evaluated at $\alpha_m \Big|_{Int.}$ is smaller than θ_{p0} and is in an approximately parabolic region. In other words, the optimization procedure has been thrust back into the normal condition while at the same time reducing the value of θ_p .

Figure 6 shows another unusual condition that is related to the one shown in Figure 5. As was pointed out earlier, when α_m is far from the minimum, the curve of θ_p versus α_m may assume the powers 2, 1, and certain fractions less than one. In those cases where fractional powers are encountered, the fitted parabola will have a maximum instead of a minimum. However, as Figure 6 shows, the same procedure may be used as is used for the condition illustrated in Figure 5. No change in programming is required; however, several iterations may be necessary before the usual condition is reached (in which the fitted parabola has a minimum).

In those cases where the curve of θ_p approaches a straight line (a first power curve), it will be found that c is approximately zero. Under these conditions a single value of $\alpha_m \Big|_{Int}$ can be computed from the equation

$$P = a + b\alpha_m = 0 \quad (36)$$

$$\alpha_m \Big|_{Int.} = -\frac{a}{b} \quad (37)$$

If this value of $\alpha_m \Big|_{Int}$ is substituted as the new value of the adjusted

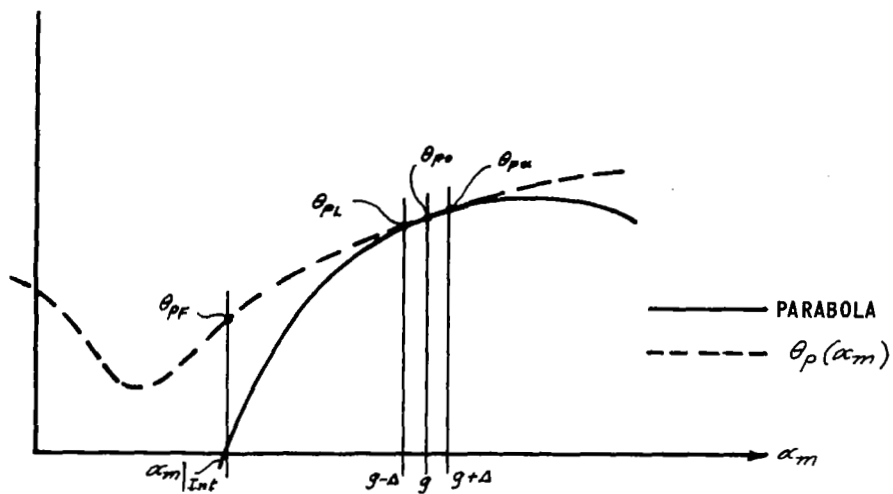


Figure 6 UNUSUAL CONDITION CAUSED BY FRACTIONAL POWER CURVE IN θ_p FAR FROM MINIMUM

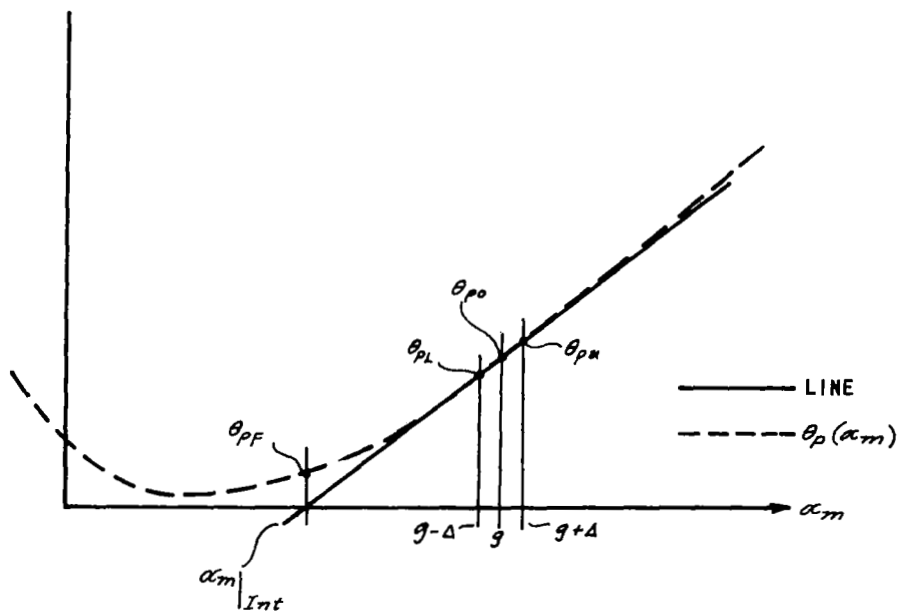


Figure 7 UNUSUAL CONDITION IN WHICH $c \approx 0$.

parameter, the iteration process can be continued in the usual manner (see Figure 7). Detection of this condition is easily checked by examining the value of c . If it is smaller in magnitude than a prescribed value, equation (37) can be used instead of equation (35).

When the curve of θ_p approaches a second power curve (a parabola) no special problem arises, because c must be positive. Therefore, the usual condition prevails.

There are isolated conditions where the fitted parabola has a minimum, but the minimum of the parabola is less than zero (see Figure 8). Although it is possible to deal with this situation without making a change in the original procedure, to do so would slow the process of convergence considerably. This condition is easily detected by evaluating P at $\alpha_m|_{min}$. If P is less than zero, then equation (35) may be used to compute the two intersections of P with the abscissa. By again choosing the value of $\alpha_m|_{Int}$ that is the closer to g , a reduction in the performance measure θ_p may generally be effected.

Finally, there are several isolated conditions in which the above logic would lead to an increase in the value of θ_p . Figure 9 shows an example in which θ_p would be increased by the iteration process. This example involves a situation similar to that shown in Figure 8, except that θ_{pF} is larger than θ_{p0} in Figure 9. Since this condition may arise in a number of different ways, it is convenient to deal with it by actually comparing the value of θ_p before and after the adjustment of a parameter. Then, if θ_p is increased by the parameter adjustment, appropriate remedial action may be taken. A particularly powerful and general method for dealing with this condition is to choose a value of α_m halfway between the values prior to and after the adjustment. If the evaluation of θ_p at this midpoint is less than the value of θ_p prior to adjustment, this value of α_m is used as the new adjusted value. If the midpoint value of α_m is still larger, a new midpoint is computed halfway between the value of α_m prior to adjustment and the just computed midpoint. This process is continued until a value of θ_p is obtained that is less than the value prior to adjustment. The power of this technique is a result of 1) its ability to

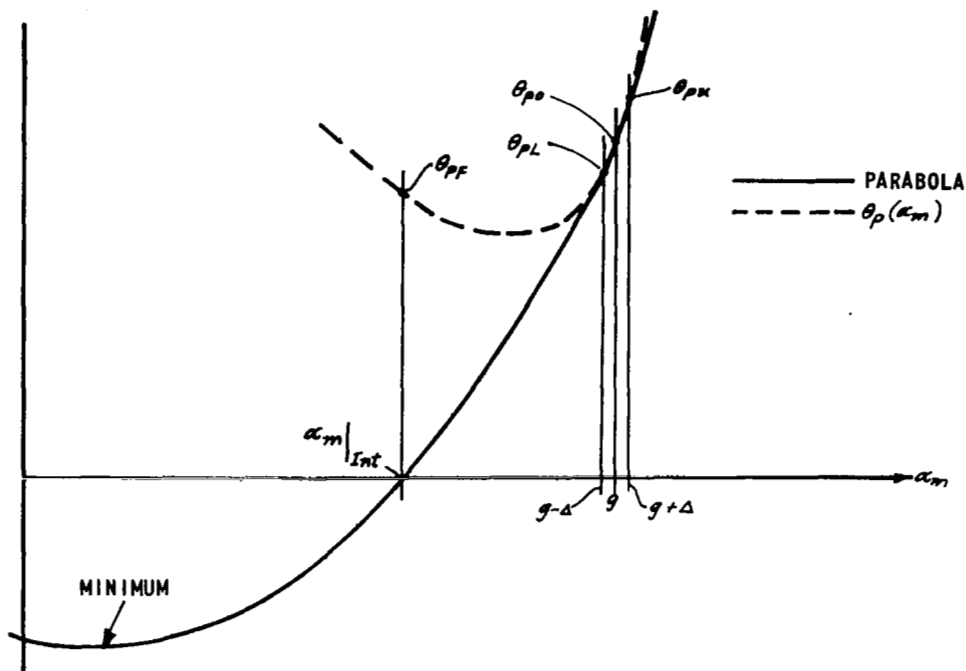


Figure 8 UNUSUAL CONDITION CAUSED BY FITTING PARABOLA
WHOSE MINIMUM IS LESS THAN ZERO

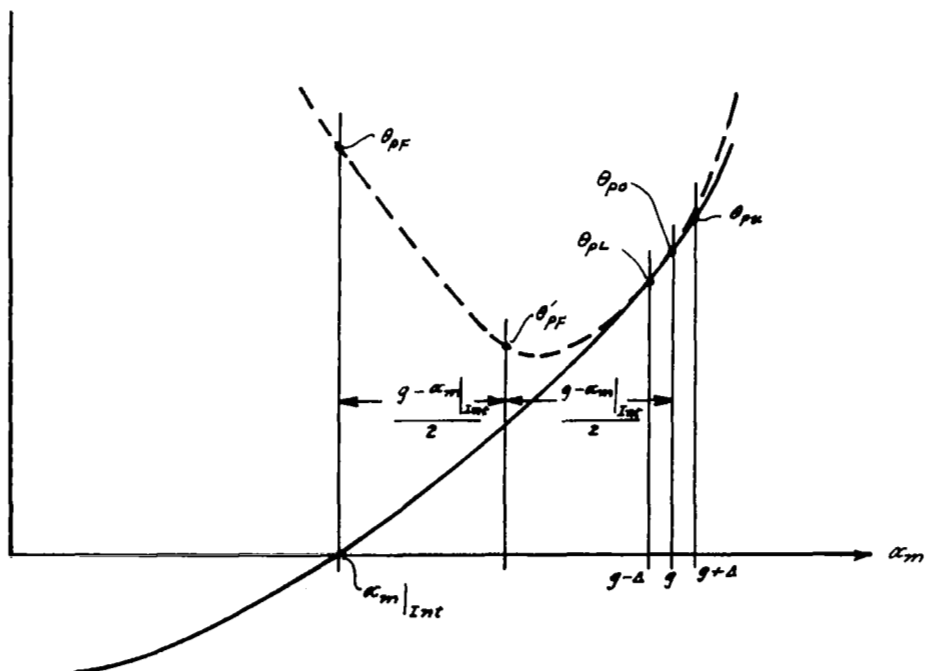


Figure 9 UNUSUAL CONDITION CAUSING INCREASE IN
PERFORMANCE MEASURE

seek out a minimum that lies between the values of α_m prior to and after adjustment, and 2) its general applicability.

It is seen that care has been taken to insure that the iteration process converges even though unusual conditions are encountered. By adjusting each parameter individually, the greatest possible number of contingencies can be taken into account. If all parameters are simultaneously adjusted, it is unlikely that the reliability of the minimization process could be made as great.

It should be noted that the iteration procedure described requires selection of the parameter Δ . This parameter determines the spacing between the points used for the parabolic fit. Because it is difficult to determine a suitable value of Δ (for each α_m), an adaptive procedure has been developed. In this adaptive procedure, $c\Delta^2$ is compared with θ_p just after a gain adjustment. If $c\Delta^2 > 0.1\theta_p$, Δ is decreased by a factor of 2 for use in the subsequent adjustment of the same parameter. If $c\Delta^2 < 0.0001\theta_p$, Δ is increased by a factor of 2 for the subsequent adjustment. In other words, Δ is increased as curvature of the fitted parabola becomes small, and Δ is decreased as the value of curvature becomes large. This procedure allows rapid strides when far from the minimum and fine adjustment when near the minimum.*

F. BEHAVIOR OF THE ADJUSTMENT PROCEDURE

In Figures 2 and 3 a relatively simple example was illustrated in order that the framework of the minimization procedure might be easily explained. It is necessary however to verify that the character of the performance measure curve is as expected for high-order realistic synthesis problems. To obtain this information, two approaches were pursued. First, a complex root locus (for both positive and negative gain) was constructed for the longitudinal dynamics of an SST in one flight regime. Then desired pole positions were superimposed and the pitch angle feedback gain adjusted. Subsequently θ_p vs α_m (in this case $\Delta\theta$ feedback) was computed and plotted. Secondly, the digital computer program for performing individual regime synthesis was modified for purposes of testing. After

* At the start of the iteration process, the value of each Δ is set equal to one-tenth the initial value of α_m . (The initial value of each α_m is read from the data cards.)

a certain number of iteration cycles, the program was stopped. Then a curve of θ_p versus α_m was computed and plotted by advancing α_m through equal increments across a wide range of values. The results of both of these studies are reported in this section.

In the first approach a positive and negative gain root locus was constructed for open-loop longitudinal dynamics for the Mach 0.23 heavy, low altitude flight condition:

$$KG(s) = \left[\frac{K}{\left(\frac{s^2}{\omega_a^2} + \frac{2\zeta_a s}{\omega_a} + 1 \right)} \right] \left[\frac{1}{\left(\frac{s}{s_r} + 1 \right) \left(\frac{s}{s_{r1}} + 1 \right)} \right] \left[\frac{\left(\frac{s}{s_6} + 1 \right) \left(\frac{s}{s_7} + 1 \right)}{\left(\frac{s^2}{\omega_{nsp}^2} + \frac{2\zeta_{nsp} s}{\omega_{nsp}} + 1 \right) \left(\frac{s^2}{\omega_{np}^2} + \frac{2\zeta_{np} s}{\omega_{np}} + 1 \right)} \right] \quad (38)$$

where

$$\begin{array}{ll} \zeta_a = 0.707 & \zeta_{sp} = 0.778 \\ \omega_a = 10 \text{ rad/sec} & \omega_{nsp} = 1.04 \text{ rad/sec} \\ s_r = 10 \text{ rad/sec} & \zeta_p = 0.062 \\ s_{r1} = 12 \text{ rad/sec} & \omega_{np} = 0.144 \text{ rad/sec} \\ s_6 = 0.033 \text{ rad/sec} & \\ s_7 = 0.457 \text{ rad/sec} & \end{array} \quad (39)$$

K is the loop gain with $\Delta\theta$ feedback. ($K = \alpha_m$)

In these dynamics the first bracketed quantity represents actuator dynamics, the second bracketed quantity represents feedback instrumentation dynamics, and the last bracketed quantity represents the airframe dynamics.

The desired pole positions were set at

$$\begin{array}{ll} \zeta_a \rightarrow 0.46 & \zeta_{sp} \rightarrow 0.62 \\ \omega_a \rightarrow 11.9 \text{ rad/sec} & \omega_{nsp} \rightarrow 2.5 \text{ rad/sec} \\ s_r \rightarrow 11. \text{ rad/sec} & \zeta_p \rightarrow 0.30 \\ s_{r1} \rightarrow 13. \text{ rad/sec} & \omega_{np} \rightarrow 0.15 \text{ rad/sec} \end{array} \quad (40)$$

A plot of θ_p versus α_m for uniform weighting of the pole position errors ($k_i = 1, i = 1, 2, \dots, 8$) was constructed. It is shown in Figure 10. The plot exhibits a single minimum and appears to approach a fractional power curve as indicated in theory. It is interesting to note that the

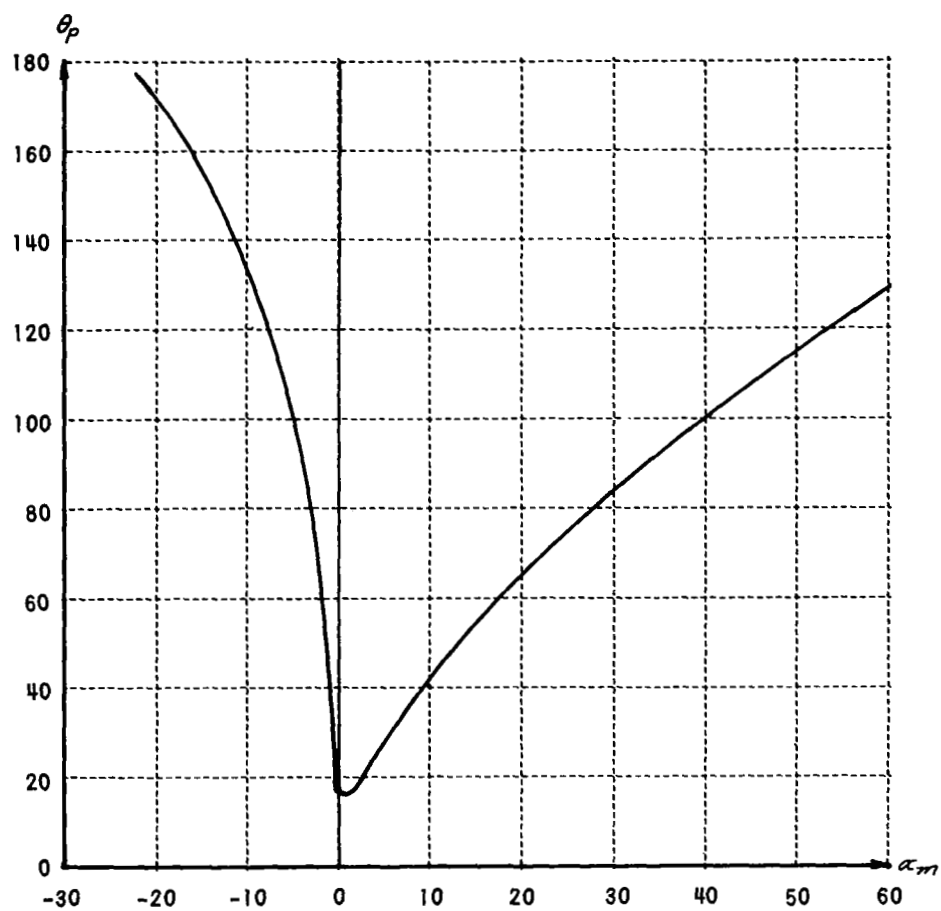


Figure 10 PLOT OF θ_p VS. α_m OBTAINED BY GRAPHICAL PROCEDURE (UNIFORM WEIGHTING).

minimum occurs at $\alpha_m = 0$ approximately. Thus, feeding back pitch angle alone would not allow improvement in the system dynamics.

The same root-locus plot was used to obtain a plot of θ_p versus α_m for the condition in which the pole position errors are not uniformly weighted. In this latter case the errors in the phugoid pair were weighted with a factor of 10^3 , errors in the short-period pair with a factor of 10^3 , and all other poles with a factor of 10. The plot of θ_p versus α_m then took the form shown in Figure 11. It is seen that two relative minima occur, one at $\alpha_m = -3.0$ and the other at $\alpha_m = 2.0$. Therefore, it becomes clear that exceptions to the general rule of a single minimum will occur.

The cause of the relative minimum at $\alpha_m = -3.0$ was determined. It can be explained as follows. As α_m becomes negative and increases in magnitude, the short period pole pair approaches the real axis and the pair becomes real. At $\alpha_m = -2.0$ approximately, one of these poles breaks abruptly away from the real axis once again. This behavior causes a temporary decrease in the value of θ_p in the region $-3 \leq \alpha_m \leq -2$. Then for $\alpha_m < -3$, the value of θ_p again increases. It is seen that abrupt breakaway of a heavily weighted pole may cause a second relative minimum.

The effect of this second relative minimum on the optimization would probably be unnoticeable. First, the second minimum is very narrow and might be missed altogether. Second, even if the adjustment procedure settled at the second minimum during one iteration, chances are good that adjustment of other parameters would change the position of the second minimum. As a result, the lower minimum would probably be found. Since the lower minimum is broader, it is less susceptible to variations caused by other parameters.

The results of the digital computer study became available shortly after the graphically obtained results were completed. The computer results were developed for the block diagram of SST longitudinal dynamics in Figure 1. The flight condition used for the graphical results was also

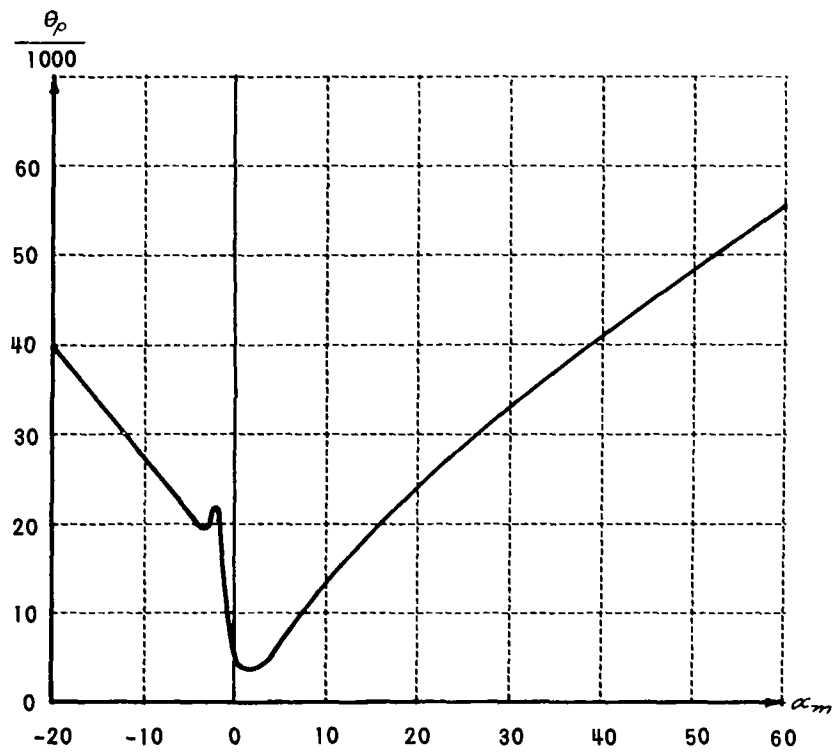


Figure 11 PLOT OF θ_ρ VS. α_m OBTAINED BY GRAPHICAL PROCEDURE
(NONUNIFORM WEIGHTING)

used for the computer results. In addition, a similar (although not identical) set of desired closed-loop pole positions was chosen. Weightings of the pole position errors were as follows: phugoid pair, 10^6 ; short-period pair, 10^3 ; filter poles, 10; actuator pair, 1.

The computer was programmed to perform one adjustment of each adjustable parameter. Then, the computer was programmed to step off equal increments in one adjustable parameter and compute the value of θ_p at each value. Figures 12, 13, and 14 show the computer-plotted results. Referring to the diagram of Figure 1, Figure 12 corresponds to step-adjustment of K_{n_2} , 13 corresponds to step-adjustment K_{α} , and 14 corresponds to step-adjustment K_{δ} . Each of these plots exhibits a single minimum and appears to correspond well with theory.

Finally, an attempt was made to uncover a condition in which more than one minimum exists. By performing a group of search operations, one such condition was found. Again referring to the diagram of Figure 1, K_{δ} was adjusted, K_{θ} was then adjusted, and then θ_p was plotted as a function of K_{θ} . Figure 15 shows the computer plot obtained. It is seen that three relative minima exist. It may be concluded that, while a single minimum will ordinarily exist, there are conditions under which two or more relative minima might occur. The digital program which performs the minimization will seek out the lowest minimum in most cases.

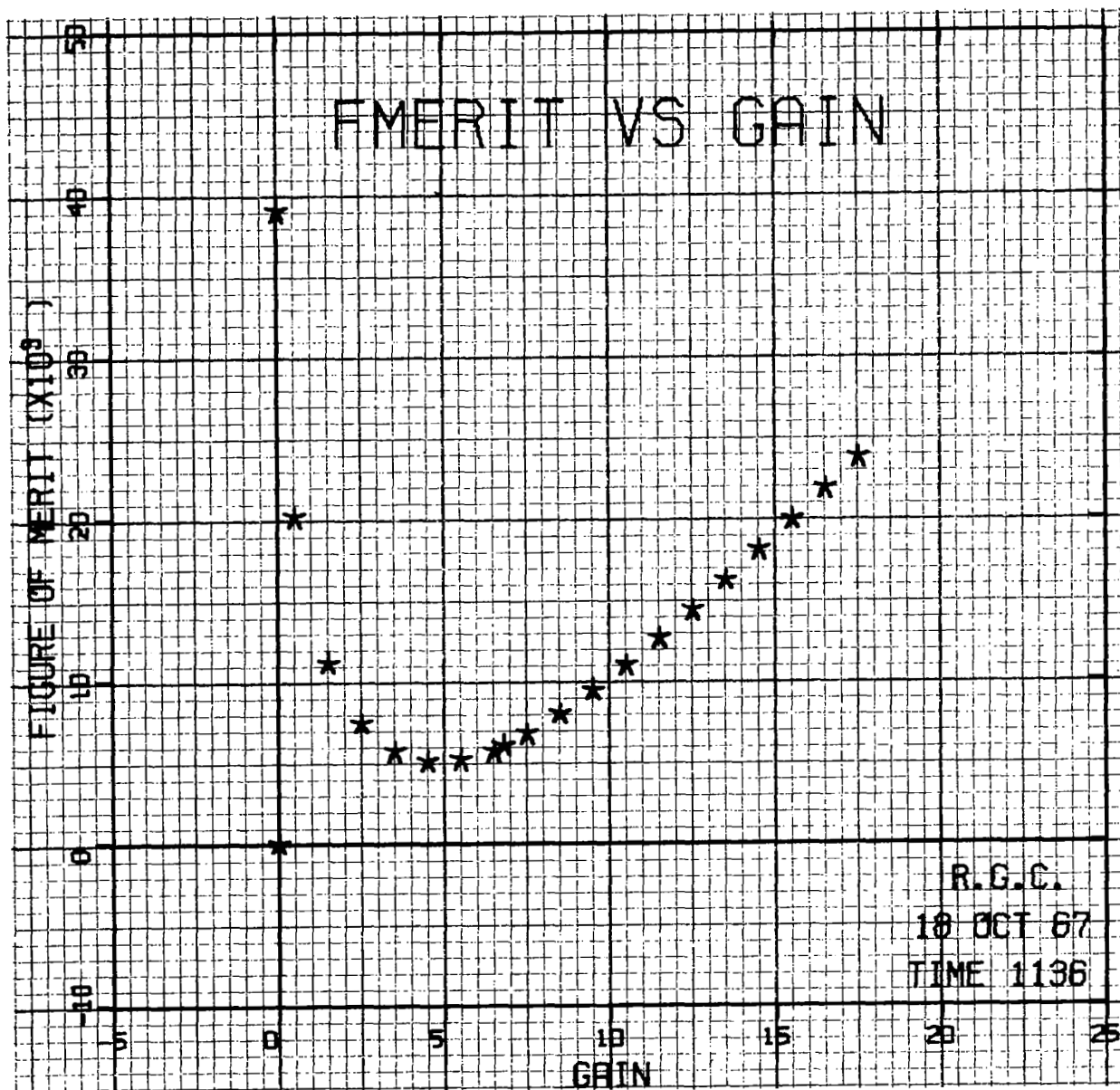


Figure 12 COMPUTER PLOT OF θ_p vs α_m AFTER ONE ADJUSTMENT OF EACH PARAMETER, ($\alpha_m = K_{n2}$)

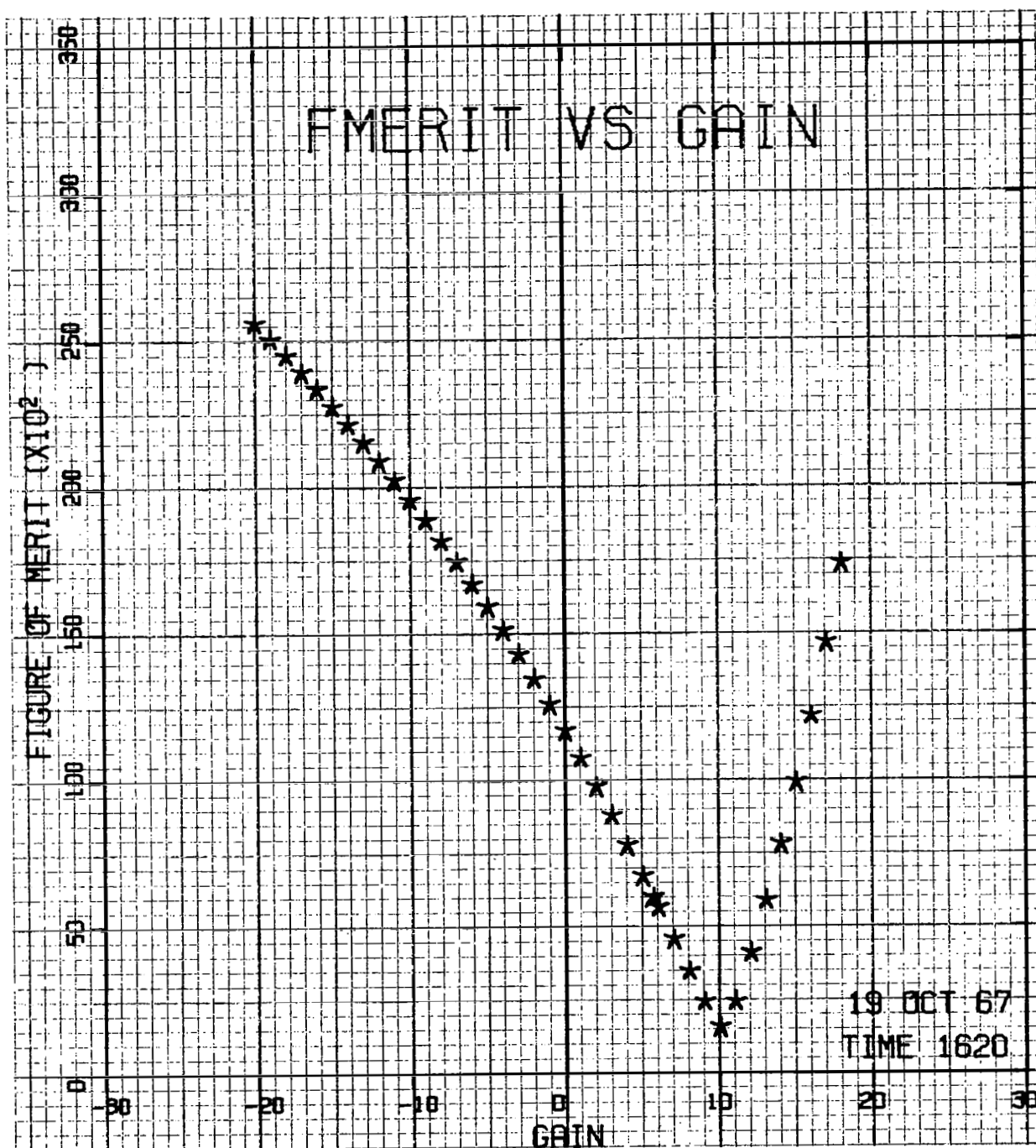


Figure 13 COMPUTER PLOT OF θ_p vs a_m AFTER ONE ADJUSTMENT OF EACH PARAMETER, ($a_m = K_R$)

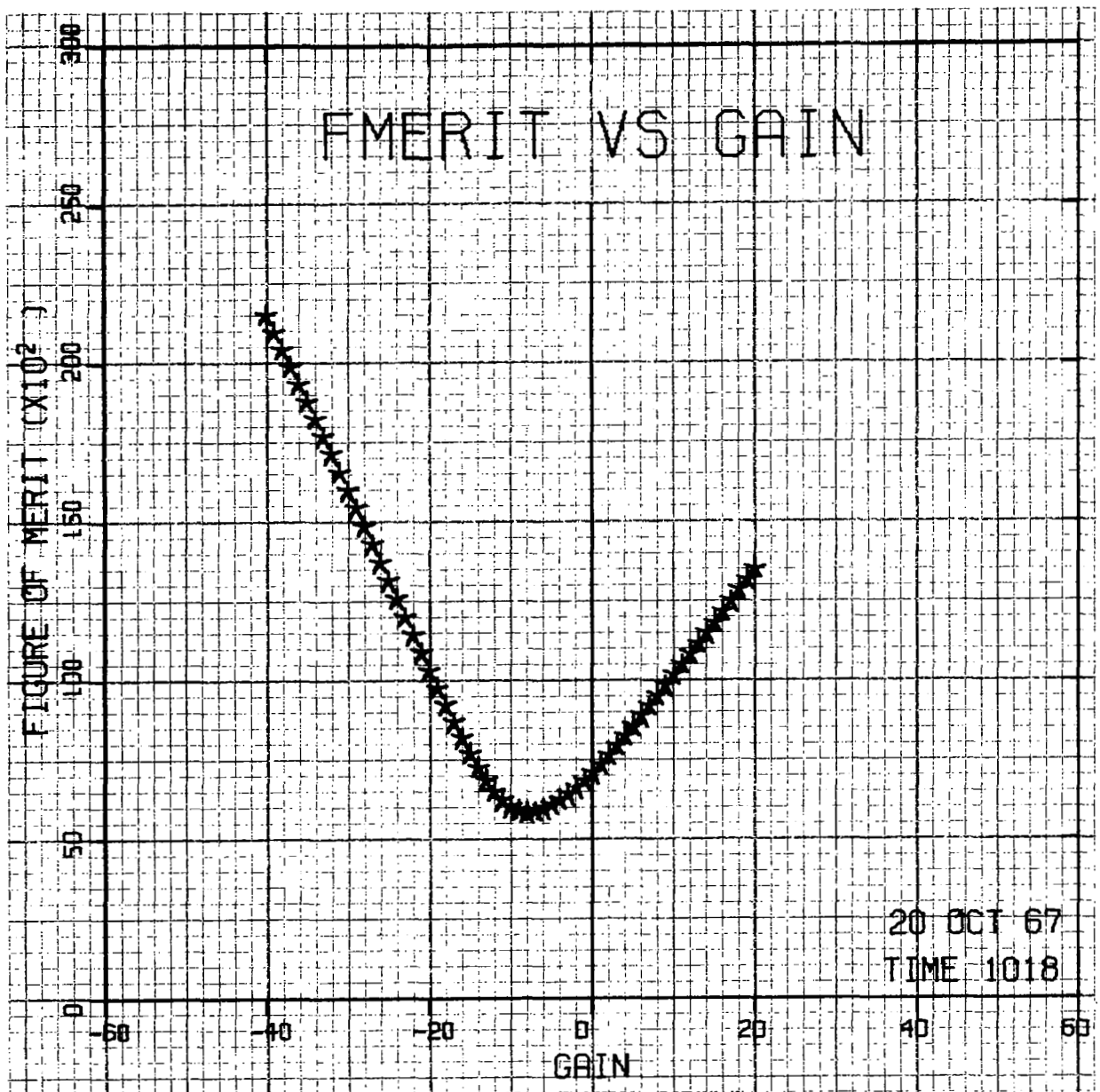


Figure 14 COMPUTER PLOT OF θ_p vs α_m AFTER ONE ADJUSTMENT OF EACH PARAMETER, ($\alpha_m = K_{\theta}$)

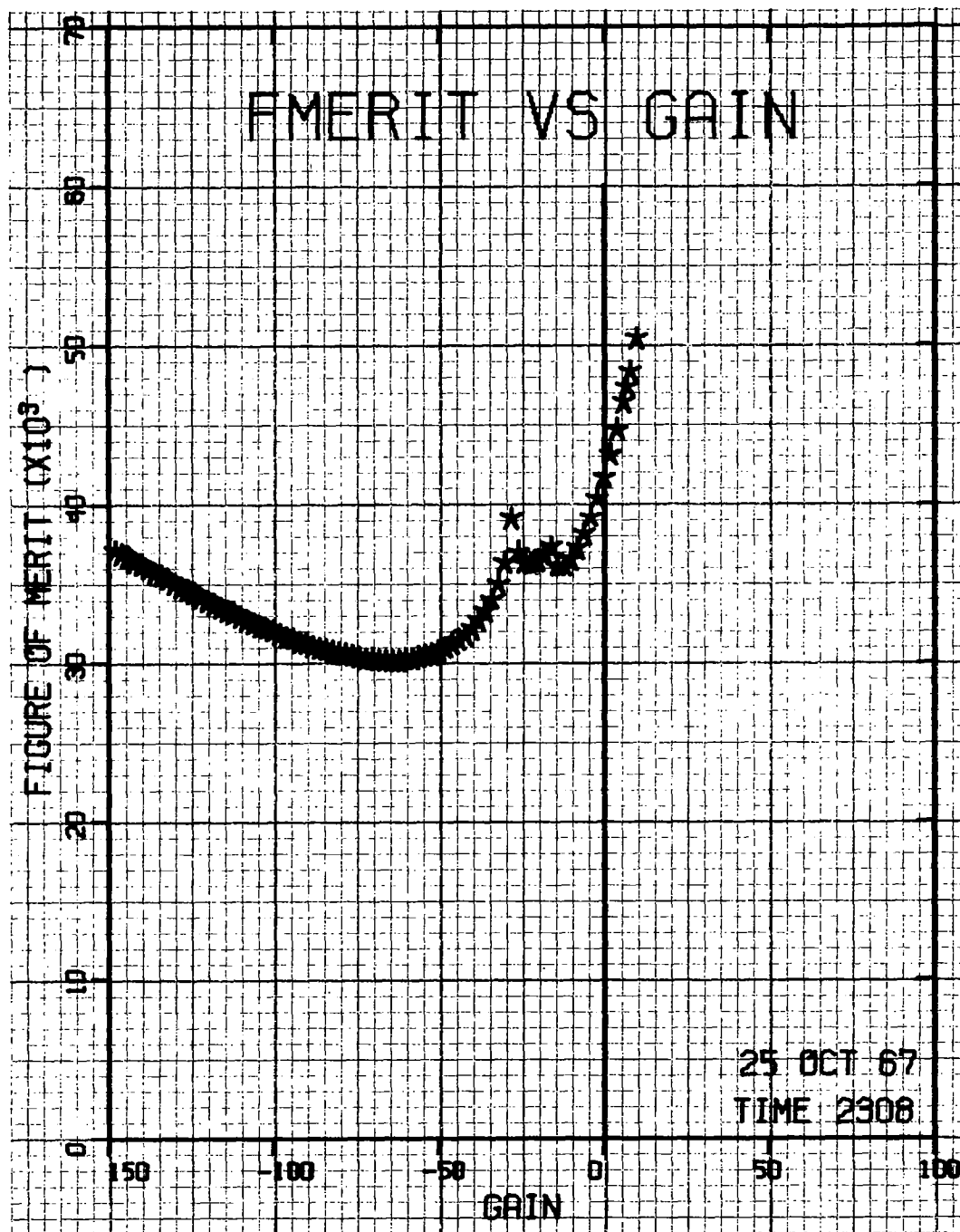


Figure 15 COMPUTER PLOT OF θ_p vs α_m FOR A CONDITION IN WHICH THREE RELATIVE MINIMA WERE FOUND

3. A DESCRIPTION OF THE DIGITAL COMPUTER PROGRAMS FOR SYNTHESIS

In the course of investigation of a programming approach to manual control system synthesis, three digital programs for performing synthesis were developed. The first of these was used for single-regime synthesis and also for debugging and study of concepts. It formed a stepping stone to the multi-regime synthesis program. This second program is used for multi-regime synthesis in which each adjustable parameter is to take on a single value that does not vary with flight regime. In other words, the resulting manual flight control system does not have scheduled parameters. This second program can be used for individual regime synthesis if desired, and therefore it supersedes the first program. The third program performs multi-regime synthesis also, but allows the incorporation of scheduling of any one or combination of adjustable parameters. Each parameter that is not designated as scheduled takes on a single optimum fixed value for all flight regimes. Here the idea is to allow the designer the choice of which parameters are to be scheduled while maintaining all others fixed. The third program computes all parameters optimally whether fixed or scheduled. The latter two programs are described in the remainder of this chapter.

A. GENERAL TASKS

To evaluate the performance measure, three tasks are necessary: (1) the characteristic equation for each flight regime must be put in the form of a single polynomial; (2) the roots of these polynomials must be found; and (3) the roots must be ordered or put into correspondence with the desired pole positions. Specifically, the program must match each of the desired pole positions with the correct root of the characteristic equation when the performance measure is calculated. When the performance measure for three equally spaced values of one parameter have been computed, they can be used to obtain a new value of the parameter that will reduce the value of the performance measure. The new value of the parameter is stored in the factor array and the process is repeated for the next parameter.

B. BRIEF DESCRIPTION OF THE DIGITAL PROGRAM

The initial step in the program is the calling of subroutine READIN. If there are no data, the program stops; otherwise control is passed to the subroutine MULT, which produces the characteristic equations from the factors of the data card input.

FACTOR, the next subroutine, finds the roots of the characteristic equations and stores them in a temporary array. The roots are then ordered by distance from the origin; for complex pairs the root having a positive imaginary part is ordered first. Plots can be generated of the original pole positions if they are desired. Plotting is under the user's control by means of a data card input. Next, the first performance measure value is computed. At this point in the program, the counters NCOUNT, NGAIN, and ITEST are updated. NCOUNT is the number of iterations (adjustments of each parameter) that have been made; ITEST is 1 or 2 if a test increase or decrease of the parameter is being used, respectively; and ITEST is 3 if the parameter is being adjusted to the minimum of the parabola. NGAIN is the index of the parameter being adjusted.

The function of subroutine NUGAIN is to compute the new value of a parameter and to store it in the factor array. For a scheduled parameter a separate value is computed for each flight regime. This is the first operation in the normal looping procedure of the program.

The second operation in the normal looping procedure is to form the characteristic equations in subroutine MULT. Subroutine FACTOR then produces the roots of the characteristic equations. At this point, there could be a deviation from the normal procedure. With certain root configurations, the factoring process will not produce all of the roots and there will be a branch to FAILGN, which is an entry point in subroutine NUGAIN.

Normally the next subroutine entered is ORDER, which compares each of the roots generated by FACTOR with each of those produced for that flight regime by the last major parameter change and stores them in a new array in the position corresponding to the old root to which they were closest. In the event that there are two roots closest to one of the old

roots, and there is not a branch in the root locus, FAILGN is called. Otherwise normal processing continues with subroutine MERIT computing the new figure of merit. If ITEST is 3, a check is made on the performance measure value to insure that it has not increased by more than one percent because of the nonparabolic nature of the actual performance measure versus parameter curve. If it has, FAILGN is called; otherwise the ordered roots are stored in the old root array, the counters are updated, and the loop starts again.

The action that FAILGN takes depends on ITEST and whether or not the given parameter is scheduled. If it is scheduled, it is changed only for those flight regimes that were not successful. If ITEST is 3, a major parameter change is being made and the change is reduced by one half. The new parameter value is the average of the old setting and of the setting that caused the failure. In this way, the state of the program can return arbitrarily close to the state resulting from the last successful parameter change. An arbitrary maximum of 25 successive calls to FAILGN is set to prevent excessive use of machine time in case of machine or program error. If FAILGN is called with ITEST equal 1 or 2, the value of the test change in parameter is decreased to one tenth of its previous value and the iteration process is begun again for this parameter.

When a specified number of successful iterations have been completed, plots may be made of the final root positions and control passed to the READIN subroutine for the next set of data.

C. DETAILED DESCRIPTION OF THE SUBROUTINES

1. READIN

The first data card contains the following integers: MAXGN, the number of adjustable parameters; MAXCT, the number of iterations; MAXFLT, the number of flight regimes; NROOTS, the number of roots of the characteristic equation (or the order of the overall system); NSKIP, the number of iterations between major printouts; and IIFPLT, which determines whether or not plots of pole locations are to be made. The second card contains a title for the run and the third contains an identifier for the

first flight regime. The fourth card and the several that follow contain the factors that make up the characteristic equation for the first flight regime, and integers related to the factors. It was found convenient to reduce each characteristic equation to the form

$$\left\{ \begin{aligned} & (A_{111} + A_{112} S + A_{113} S^2) (A_{121} + A_{122} S + A_{123} S^2) \dots (\quad) \\ & + (A_{211} + A_{212} S + A_{213} S^2) (\quad) \dots (\quad) + \dots \\ & + (A_{NSUM11} + A_{NSUM12} S + A_{NSUM13} S^2) (\quad) \dots (\quad) \end{aligned} \right\} = 0 \quad (41)$$

First order (real) factors are handled as special cases with the second order coefficient zero. In this form the denominator is the sum of products of second order factors. Thus, the fourth card contains the number of products that are summed to get the denominator of the transfer function. The fifth card contains the number of factors in the first product. The following cards contain the factors in the first product. The number of factors and a list of the factors alternate in this way until the end of the denominator is reached.

The next input is the list of desired pole positions arranged in order, the first being closest to the origin. For complex pairs, the root with a positive imaginary part is again ordered first. These roots will then be arranged in the same order as the roots from the original ordering subroutine. The next card contains the weighting factors to be used in calculating the value of the performance measure. They must be in the same order as the desired pole positions.

The input subroutine then reads the identifier for the next flight regime (if there is one), the factors, the desired pole positions, and the weights associated with that regime. The last inputs are the upper and lower limits on the adjustable parameters. The third program (which allows parameter scheduling) reads a set of limits for each flight regime and a logic variable for each adjustable parameter. The logic variable determines whether a given parameter is to be scheduled or not.

The input subroutine also prints out the input data with appropriate titles and contains the entry point, THE END, to which control is passed when certain conditions arise such as 25 successive calls to FAILGN or an ordering failure in the first iteration.

2. MULT

The subroutine MULT takes each factor of a product in turn and multiplies it by the product of the previous factors. The resulting product is developed in the same array. When each product is completed, it is added to the characteristic equation array. After the characteristic equation for each flight regime is formed, control returns to the main program.

3. FACTOR

Subroutine FACTOR uses the coefficients of the characteristic equations to produce the roots for each flight regime. Two methods are available to find the roots of the characteristic equations. If the first fails, the second is used. In this way, the likelihood of not being able to find the roots for any reasonable airplane-like configuration is made very small.

If it does happen that the roots cannot be found in the constant parameter multiple regime program, FAILGN is called without attempting the remaining flight regimes. In the scheduled parameter program, the variables FAIL and FALFLT (NFLIT) are set to "true", and the program attempts to find the roots for the next flight regime. (NFLIT is the index of the flight regime for which computations are being made.) When all regimes have been attempted, FAILGN is called if the variable FAIL is true.

4. ORDER

The normal operation of the ordering subroutine is reasonably straightforward. The distance from each of the unordered roots to one of the old roots is found, and the unordered root with the smallest distance is stored in the position of the ordered array corresponding to the old root. When this is completed, MERIT is called. The number returned from MERIT (FDUMYT) is added to FDUMY, and the entire process is repeated for the next flight regime. FDUMY then becomes the cumulative performance

measure value. It is stored in TESTUP if ITEST is 1, TESTDN if ITEST is 2, or FMERIT if ITEST is 3. When ITEST is 3, the ordered roots are also stored in the permanent array.

Complications in this program may occur because there are occasions when one of the unordered roots is closest to two or more of the old roots. In some cases excessive root movement has taken place and FAILGN is called to reduce the parameter change. In other cases a branch occurs in the root locus: these cases must be recognized and handled properly. To recognize a branch in the root locus, the positions of four roots must be noted: the unordered root that was closest to two of the old roots, both of these old roots, and the unordered root that was not closest to any of the old roots at the end of the ordering process. The position of the unordered root closest to two of the old roots is labeled JSAVE. The position of the root not closest to any of the old roots is labeled JXSAVE. The position of the old root matched with the unordered root in the JSAVE position is IXSAVE, and the position of the second old root that is closest to this unordered root is NOTERM. To have this information available, the USE array is filled with the number of the old root to which each new root is matched, and the ISAVE array is used to store the positions of new roots that are matched twice. The variable KSCREW is used to indicate whether there have been two old roots closest to one of the new roots, and KBAD is used to show that the old root and the unordered root (which is closest to it and has been previously matched) are both real or are both complex. In either of these cases there has not been a branch in the root locus, and FAILGN will be called. At the end of the initial loops that match the roots, KSCREW is checked. If KSCREW is zero, all roots have been matched successfully and MERIT is called. If KSCREW is one, there is at least one possible branch in the root locus. In this case, KBAD is checked to see if there was a branch, and the second position in the ISAVE array is checked to see if there was no more than one possible branch. If there is one branch in the root locus, the USE array is scanned to find the input root that was not matched. The empty spot in the ordered array is then filled. Subsequently, MERIT is called. After the return from MERIT, the first position in the

ISAVE array is checked. If it is zero, there were no branches, the roots are properly ordered, and FDUMYT is added to FDUMY. If the first position in ISAVE is not zero, the positions of the two unordered roots are reversed; FDUMYT is saved in location FTEMP; and a new FDUMYT is calculated. If the new FDUMYT is smaller than FTEMP, the roots are now in the correct location and processing continues normally. If FDUMYT is larger than FTEMP, the roots are returned to their original order. If the two values of the performance measure are equal, the roots are ordered as in the original ordering; that is, for real roots the smallest magnitude first, and for complex pairs the root with the positive imaginary part first. After the roots have been ordered, FDUMYT is added to FDUMY and processing continues with the next flight regime.

In the scheduled parameter program, both a multi-regime performance measure value and those for the individual flight regimes must be preserved. These are needed to compute the new values for scheduled parameters. In this program FDUMY, TESTUP, TESTDN and FMERIT are dimensioned variables with the first entry associated with the first flight regime, the second with the second regime, etc., and the last containing the total values. When a check is made to insure the performance measure value has not increased for a major parameter change, the value associated with the individual flight regime must be checked in a scheduled parameter case, and the multi-regime value must be checked in a fixed parameter case.

Another difference between the fixed parameter program and the scheduled parameter program is that in those places where the fixed parameter program calls FAILGN, the scheduled parameter program sets FAIL and FALFLT (NFLIT) equal to true and continues with the next flight regime. In this way only those adaptive parameters that cause problems need be changed when FAILGN is called at the end of the subroutine.

5. NUGAIN

The function of the NUGAIN subroutine is to compute the new value for a parameter and store it in the factor array. When NUGAIN is

entered for the first time, the GAIN array is initialized by inserting the value of the appropriate factor coefficient, and the DELGAN array is set to .1. A branch is then made to make either a test parameter change if ITEST is 1 or 2, or a major gain change if ITEST is 3. If ITEST is 1, and the last major parameter change was successful, the present value of the parameter is stored in the OLDGAN array. This value is used as the reference point from which other parameters are calculated. It is also the value to which the parameter converges if FAILGN is called. If ITEST is 1, the new parameter is normally computed by $GAIN = OLDGAN (1 + DELGAN)$, that is, DELGAN is a percentage change in the parameter. However, if OLDGAN is zero, DELGAN is treated as the actual change in the parameter value. If ITEST is 2, the parameter change is similar, but in the negative direction.

If ITEST is 3, a major parameter change is being made. First the counter NFAIL is rezeroed, and IFAIL (a logical variable) is set to false. Then the coefficients of the parabola that passes through the three points TESTDN, FMERIT, and TESTUP are computed. The three values of the independent variable are assumed to be -1, 0, and +1 respectively. The magnitude of the coefficient c is then checked. If it is less than one part in 10^6 of FMERIT, the computation of the coefficient c is relatively inaccurate and a straight line is fitted to the end points. The new parameter value is chosen as the root of the straight line equation. The next step is a check for a value of c less than zero. If such a value is found, the two roots of the equation are computed and the parameter value is chosen to be that at the closer root. If c is found to be greater than zero, the minimum of the upright parabola is computed. If the minimum is less than zero, then the two roots are used as above. If not, then the normal parameter value computation takes place. The new value is that value at the minimum of the parabola passed through the three points. After the parameter computation, the parameter is checked to insure that the limits are not exceeded. If they are, the value is set at the limit that is exceeded.

After the new parameter value has been computed, the magnitude of c is checked to determine if the magnitude of DELGAN should be changed. DELGAN is doubled if the magnitude of c is less than one part in 10^4 of the performance measure value, and halved if it is greater than one part in 10. This will prevent the curvature of the parabola from being too small and will also not let the three points be so far apart that the minimum is not accurately identified. In every case, after the new parameter value has been computed, it is stored in the appropriate places in the factor array; then control is returned to the main program.

In the scheduled parameter program, the parameter value changing process is the same. However, the variables GAIN, DELGAN, OLDGAN, UPLIM, and DNLIM are two-dimensional variables, where one of the independent variables is flight regime. TESTUP, TESTDN, and FMERIT are also made functions of flight regime so that scheduled parameter values can be computed for each regime. For fixed parameter computations in the scheduled parameter program, a single calculation is made using the total values of TESTUP, TESTDN, and FMERIT, and that value is stored in all of the flight regime factor arrays.

When FAILGN is entered with NCOUNT equal to 1, the program is unable to find the initial set of roots or is unable to order them. In this case, the program is halted, and it must be restarted with a new set of initial conditions. If NCOUNT is not 1, IFAIL is set to true, and NFAIL is increased by 1. If NFAIL is greater than 25, the program is halted. Then, if ITEST is 1 or 2, ISAVE is set to false so that the value of OLDGAN will not be changed. The value of DELGAN is reduced by 10. Control is then returned to the main program where the iteration process is started again for this variable. If ITEST is 3, the gain increment is reduced by one half, the new value is stored in the factor array, and control returns to the main program just before the MULT subroutine. In the case of a scheduled parameter only those values of DELGAN or GAIN are changed that correspond to true values of FALFLT.

4 . THE MULTIPLE MINIMA SEARCH PROGRAMS

In Chapter 2 it was shown that when using the programming approach to synthesis, one could ordinarily expect to encounter a single minimum in the adjustment of any parameter. However, it was also pointed out that occasionally, more than one local minimum will be encountered. This chapter describes two additional programs that were developed to search for multiple minima. These programs are intended to be used for diagnosis when more than one local minimum is suspected or when trouble is encountered in reaching a solution with the main digital programs described in Chapter 3.

It must be understood that, without analytical methods for determining the positions of local minima, every point in the multi-dimensional parameter space must be tested to insure finding all minima. Even with high-speed digital computers, such an approach is unfeasible from the standpoint of computation time or resources. To illustrate, suppose that 30 values in each of 6 parameters are to be tested. The performance measure would have to be computed and compared at 30^6 or 7.29×10^8 points. In view of the fact that the main synthesis program generally performs less than 1000 evaluations of the performance measure in achieving an optimum set of parameters, it is clear that direct search is impossible.

If one does less than a global search over the entire parameter space, one runs the risk of missing some of the minima. Nevertheless, there are certain simpler search procedures that would be helpful in uncovering more than one minimum and in determining an unusually shaped performance measure surface.

A. DIGITAL SEARCH PROGRAMS

The first program for multiple minima search involved the adjusting of only one parameter while holding all others constant. It will be recalled that, during the development of the adjustment procedure for the programming approach described in Chapter 2, a digital computer

program was developed for evaluating the performance measure versus an adjustable parameter. That program formed the basis for development of a more general single parameter search program for multi-regime fixed parameter optimization.

This search program is best described by the way in which it is used. First, one begins with the main multi-regime program described in Chapter 3. On data cards for the main program, one specifies the number of iterations to be performed between the beginning of the program and the point at which a single parameter search is to be performed. After that number of iterations is completed, the main program enters all necessary data on tape and then continues its normal procedure of finding the minimum. Actually, the necessary data can be entered onto tape at a number of points, thereby allowing search any number of times and after any number of iterations. Later the search program itself uses the data on tape as inputs. While all other adjustable parameters are held at fixed values, one of the parameters is adjusted in equal increments between the limits specified for that parameter. The performance measure is evaluated at each increment and then a plot of performance measure versus the adjustable parameter is plotted. The parameter for which the search was conducted is then reset to its value just prior to search and the next parameter is adjusted in equal increments. This process continues until all parameters have been adjusted in increments. After the search is completed for one block of data, a second search is performed on the next block. The process continues until parameter searches have been performed on all data blocks.

The second program for multiple minima search was designed to search along the positive and negative gradient direction of the performance measure. The idea involved in developing this program was to search in the direction where the greatest change in performance measure is taking place as a function of the adjustable parameters. Accordingly, one might also expect the character of the performance measure to change most rapidly in this direction. The gradient search program was designed to operate on the same data blocks read out on tape from the main multi-regime synthesis program. For any given block, the first operation

performed by the gradient program is the computation of the gradient itself. This is done by evaluating separately the performance measure at two close points in each adjustable parameter. The adjustable parameter having the largest magnitude of partial derivative is designated as the "lead" parameter, and it is this parameter that governs the stepping increments. The lead parameter is adjusted through a specified number of steps between its given limits. Every other adjustable parameter is simultaneously stepped with an increment that is proportional to the ratio of its partial derivative to the partial derivative of the lead parameter. During the stepping operation, limits on adjustable parameters other than the lead parameter are neglected. Accordingly, the parameters are simultaneously adjusted in a fixed gradient direction for increments between the lower and upper limits of the lead parameter. The gradient search program plots the value of the performance measure along the gradient direction versus the increments of the lead parameter. As with the first search program, the second (gradient) program will perform a search for each data block until they are depleted.

B. EXPERIMENTAL STUDY

Both programs were tested experimentally to ascertain their proper operation and to gain further insight into the root-moving program's operation. The first program was tested in conjunction with the multi-regime fixed-parameter synthesis of longitudinal SST dynamics. After five iterations of the main program, the data were read out on tape and used as an input for the single parameter search procedure.

Figures 16 through 21 are the plots of performance measure versus adjustable parameters obtained by applying the first program. These plots show no evidence of a second minimum. Apparently, the longitudinal synthesis problem is well behaved. It is worth noting that exceptions to equally incremented spacing are exhibited on the plots. These exceptions are caused by use of the "failgain" routine which reduces the parameter changes so that proper ordering of the system poles may be accomplished.

The gradient search program was tested on both single and

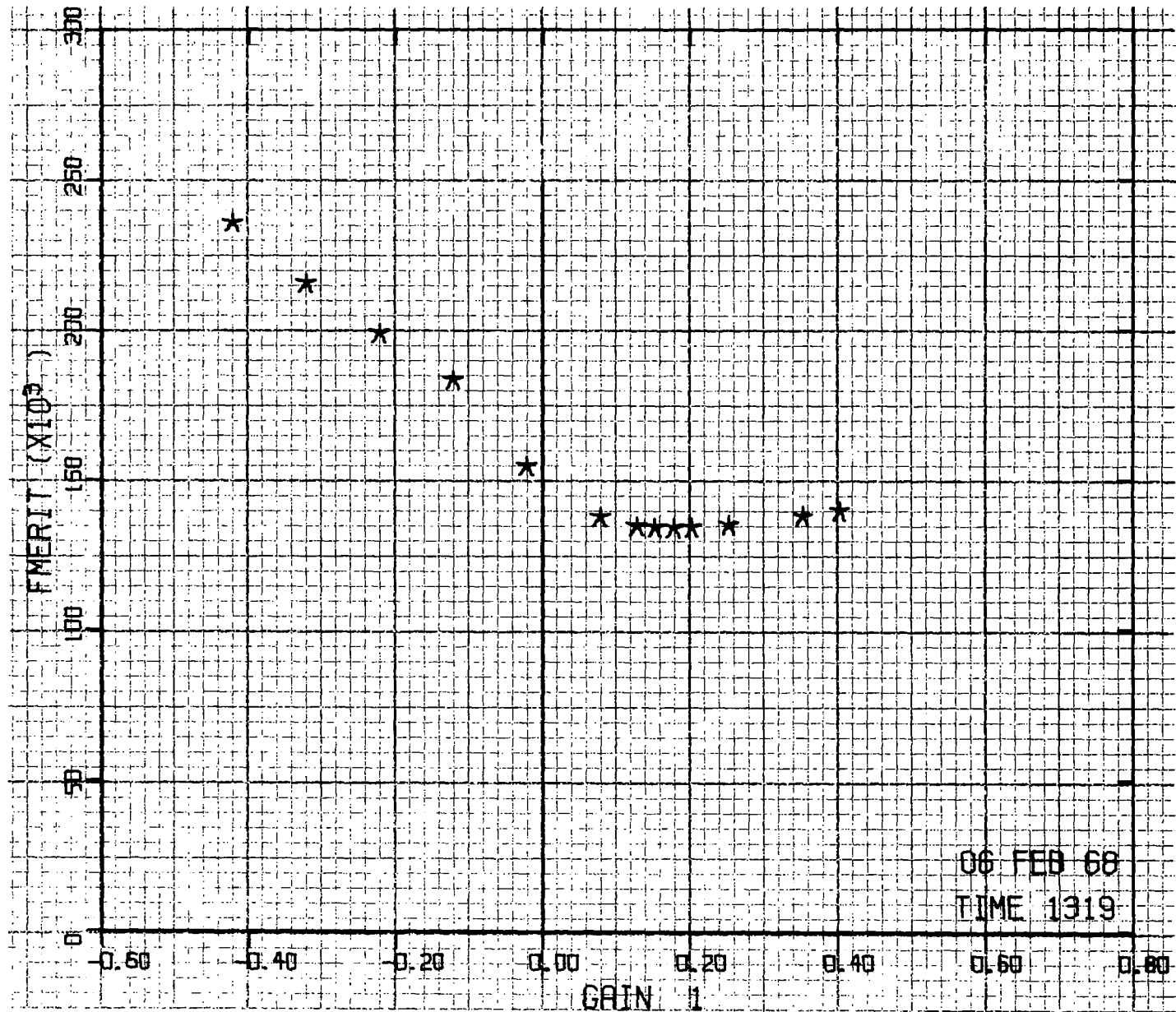


Figure 16 PERFORMANCE MEASURE VALUE vs PARAMETER τ_2

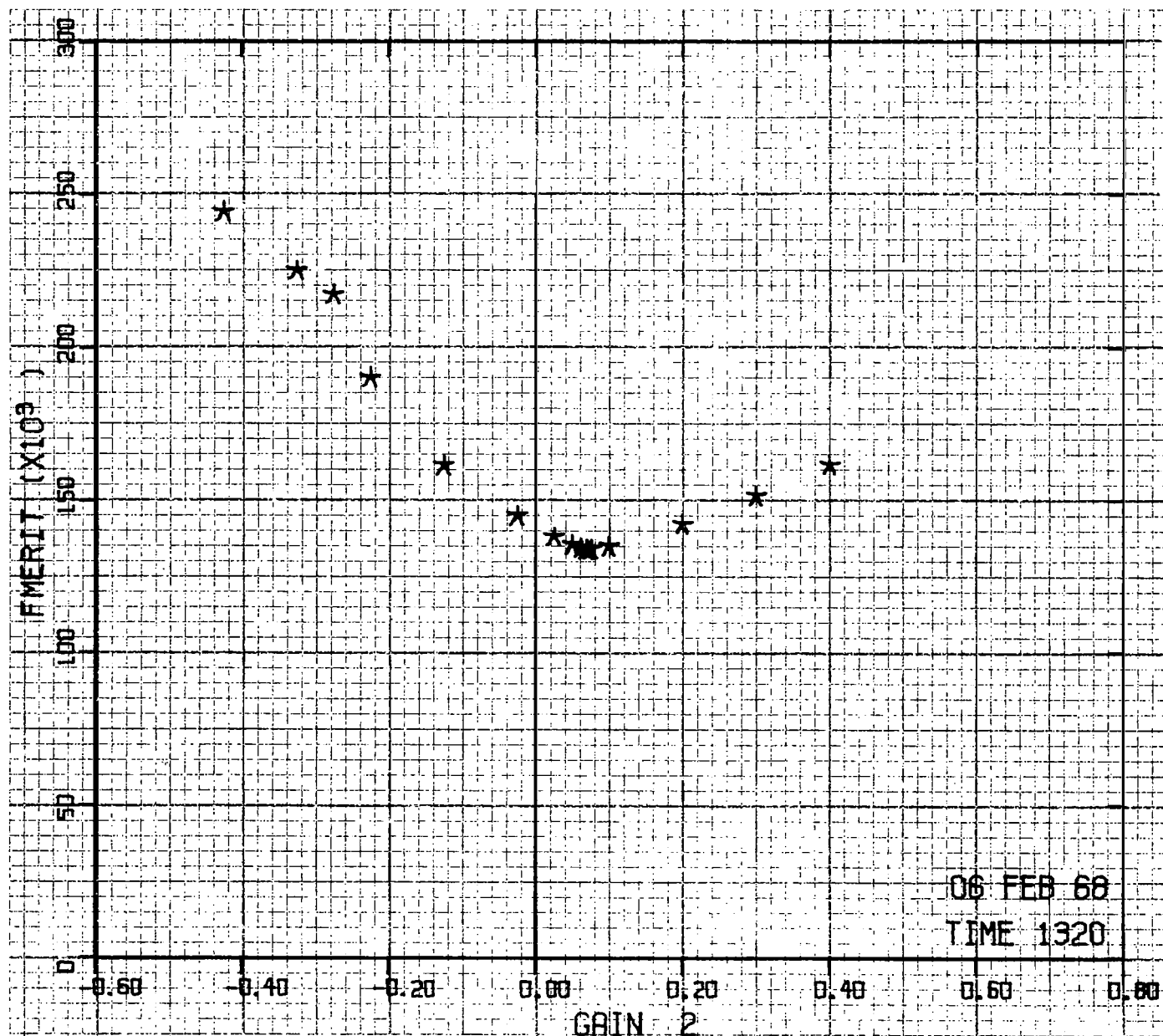


Figure 17 PERFORMANCE MEASURE VALUE vs PARAMETER τ_c

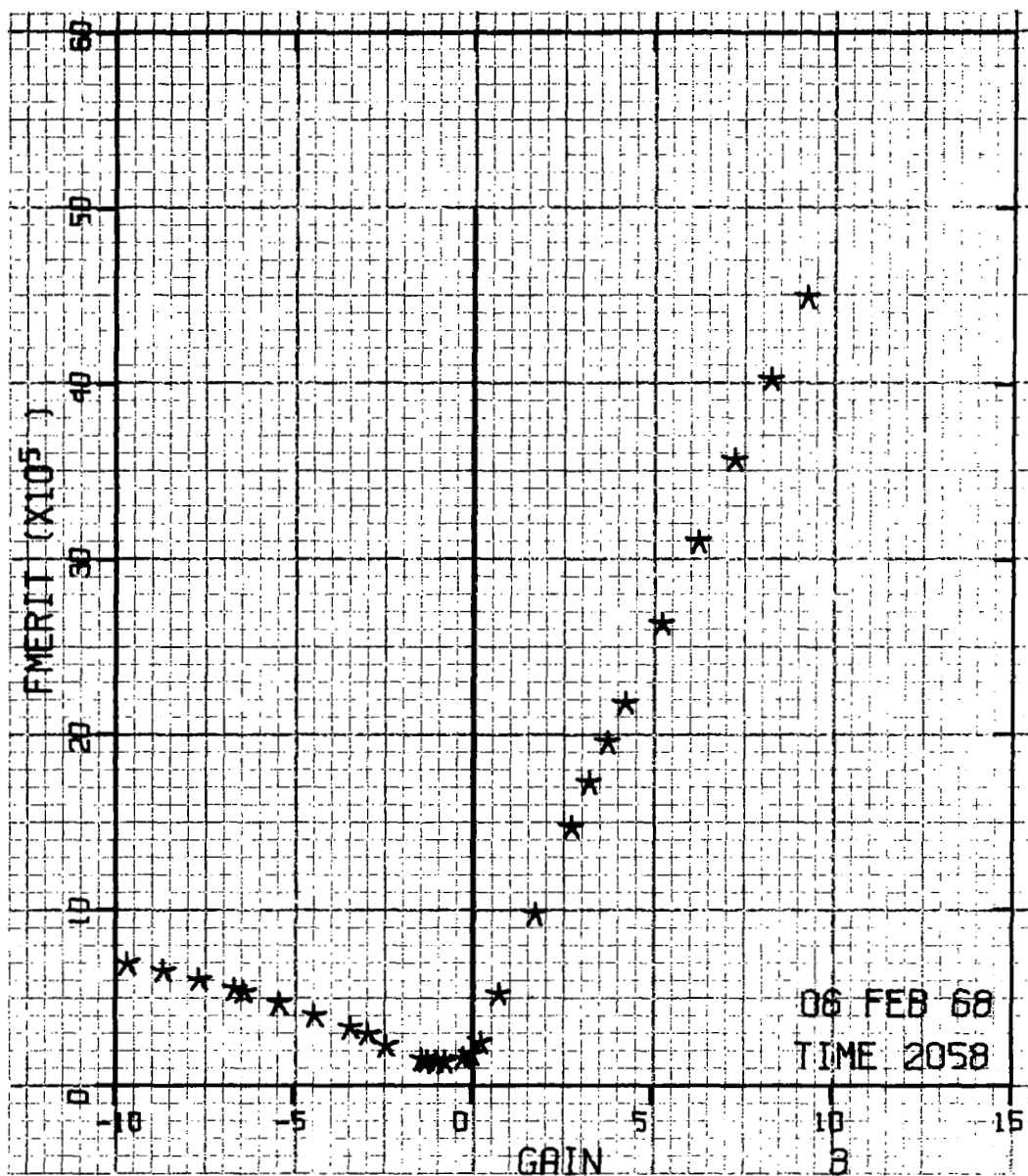


Figure 18 PERFORMANCE MEASURE VALUE vs PARAMETER K_0

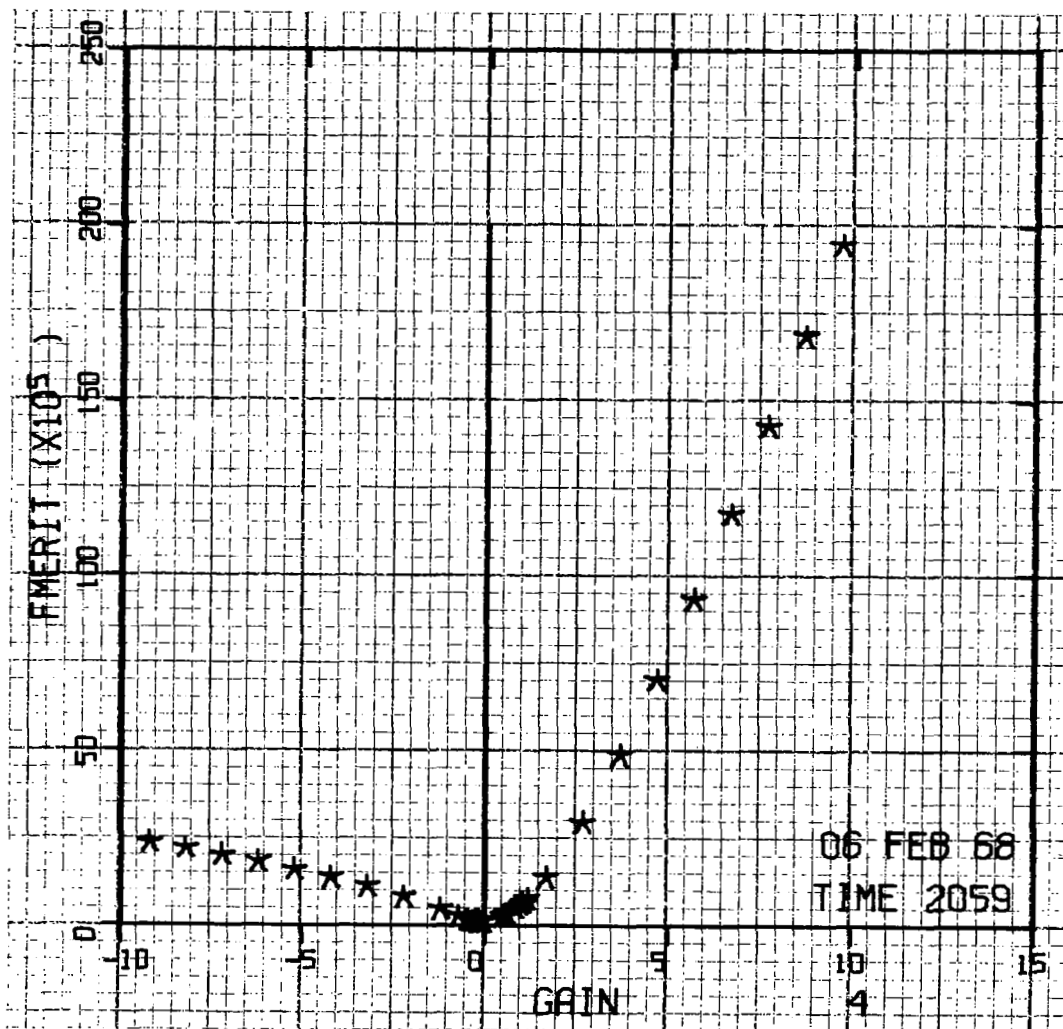


Figure 19 PERFORMANCE MEASURE VALUE vs PARAMETER K_0

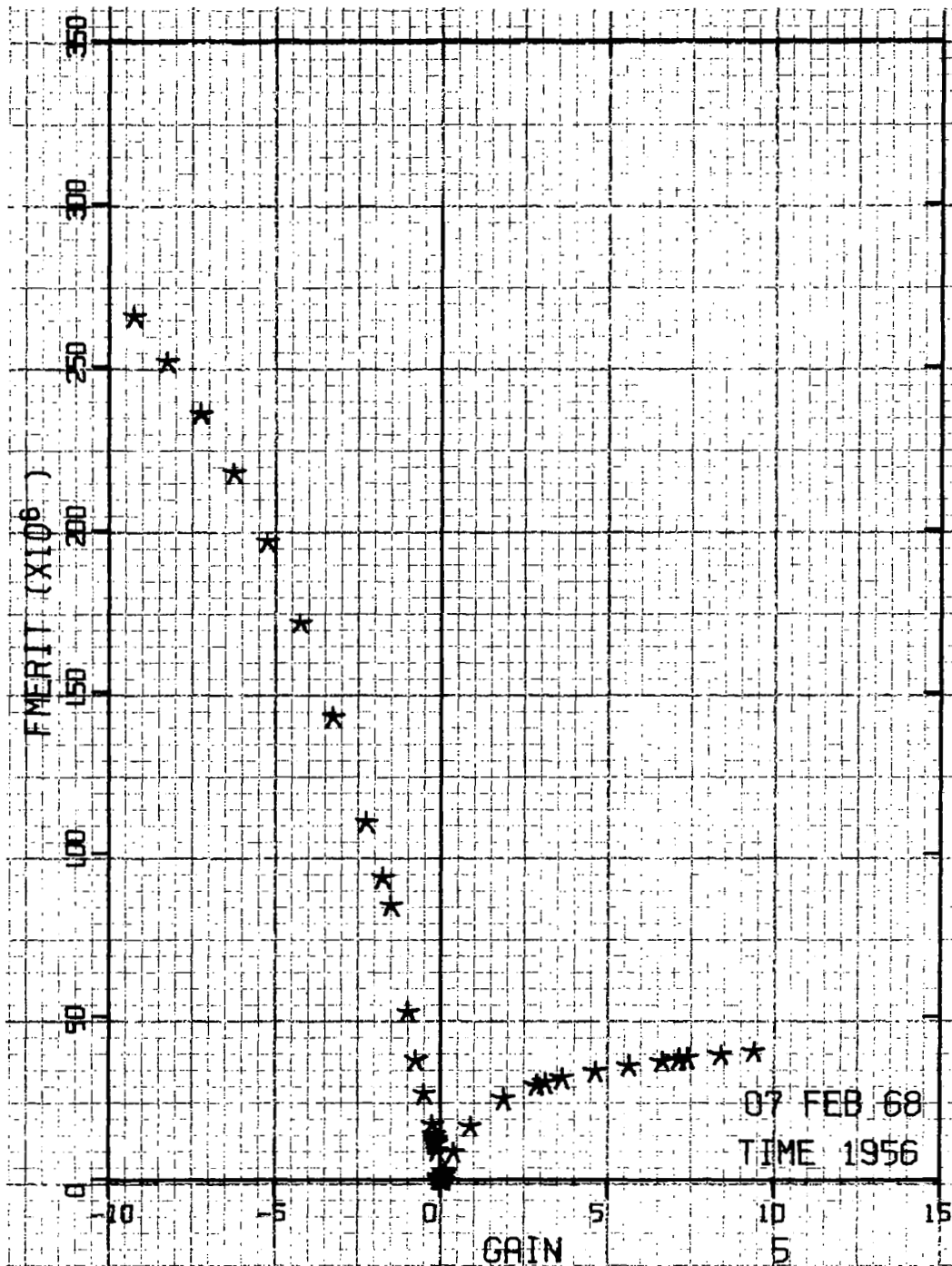


Figure 20 PERFORMANCE MEASURE VALUE vs PARAMETER K_{n3}

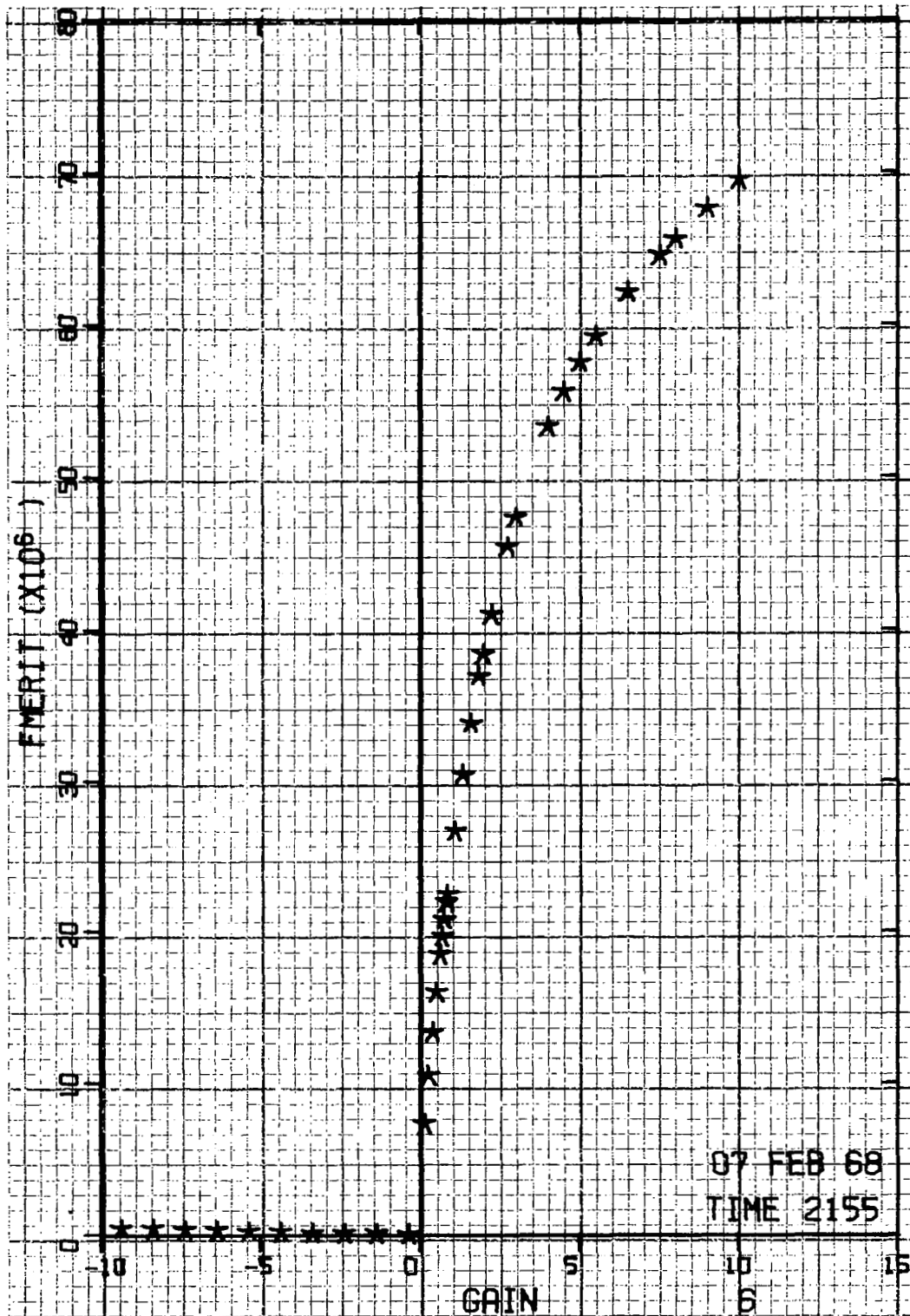


Figure 21 PERFORMANCE MEASURE VALUE vs PARAMETER K_x

multi-regime data. Figure 22 is a plot of a gradient search for the 3H70* single regime longitudinal synthesis case. The parameter τ_a is the lead parameter. Here again the search program was entered after five iterations of the main program. It is seen that no evidence of unusual behavior or a second minimum exists. The gradient search program was then applied to the same problem as the first search program, that is, multi-regime synthesis of longitudinal SST dynamics. Figure 23 is a plot of performance measure versus the lead parameter, K_{n3} . Here, certain interesting results are obtained: an apparent global minimum near zero, and approximate inflection point at $K_{n3} = 2.0$, and evidence of a relative minimum for $K_{n3} < -10$. During this computer run, the locations of the system poles were printed out after every step. It becomes clear from examination of these pole positions that the response of closed-loop system would be "non-airplane-like" for any value of performance measure over 6×10^6 . Thus, the inflection point and the relative minimum are occurring in regions that are uninteresting from an aircraft handling qualities point of view. While no further action was taken because of the nearness to zero of the global minimum, it would be possible to write a very minor program to begin the main program iteration with adjustable parameters set at the inflection point exhibited in Figure 23. Accordingly, iteration toward a second minimum could be attempted.

It is quite clear that the two programs operate properly and would be very helpful in diagnosis of difficulties. Experience with the nonlinear programming approach thus far indicates that such programs will not generally be required. However, for diagnosis or simply to insure that nothing has been missed, these programs are helpful.

* Mach 3.0, 70,000 ft., heavily loaded case.

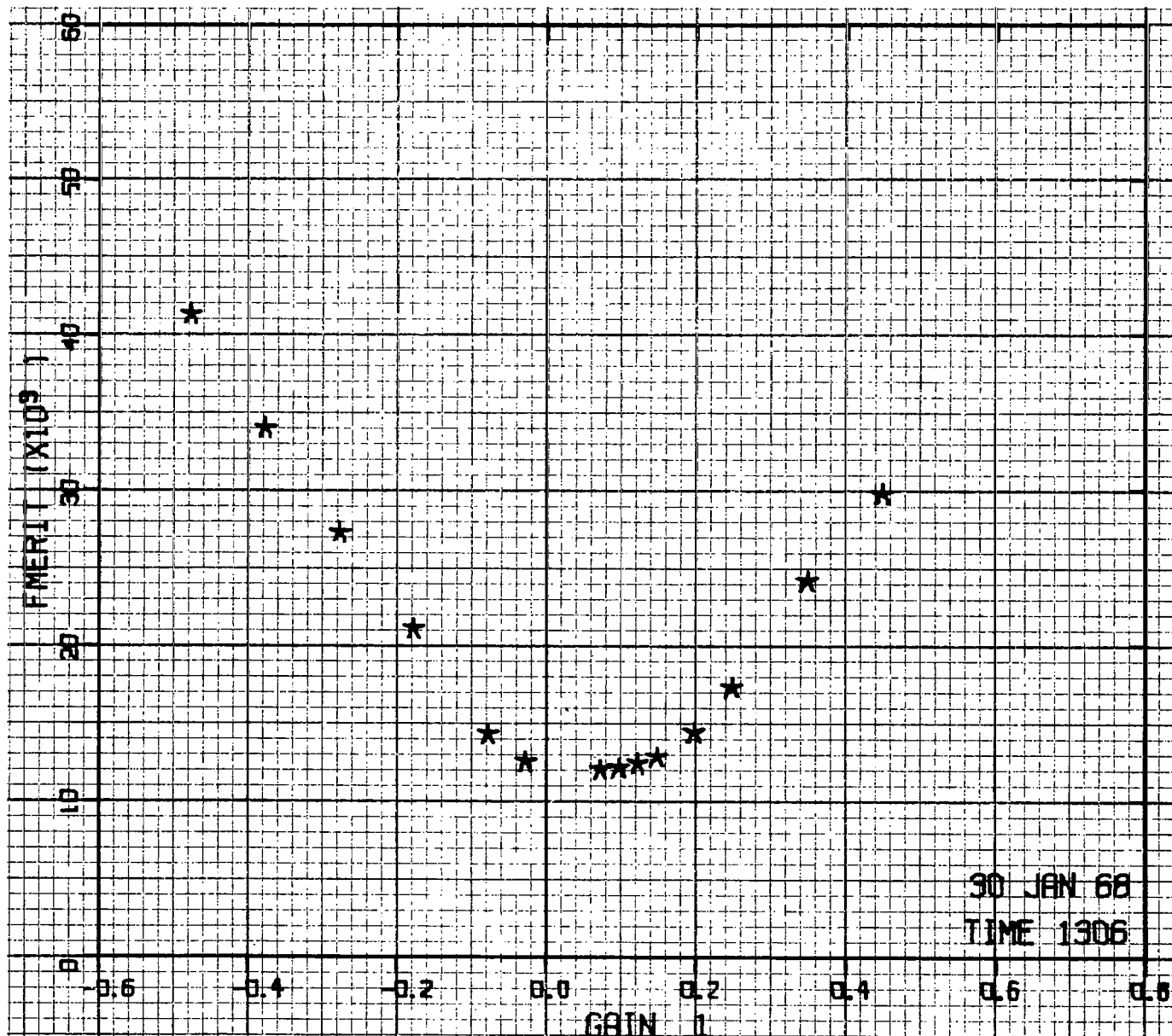


Figure 22 PERFORMANCE MEASURE VALUE vs GRADIENT WITH τ_a AS LEAD PARAMETER. (SINGLE REGIME, 3H70 CASE)

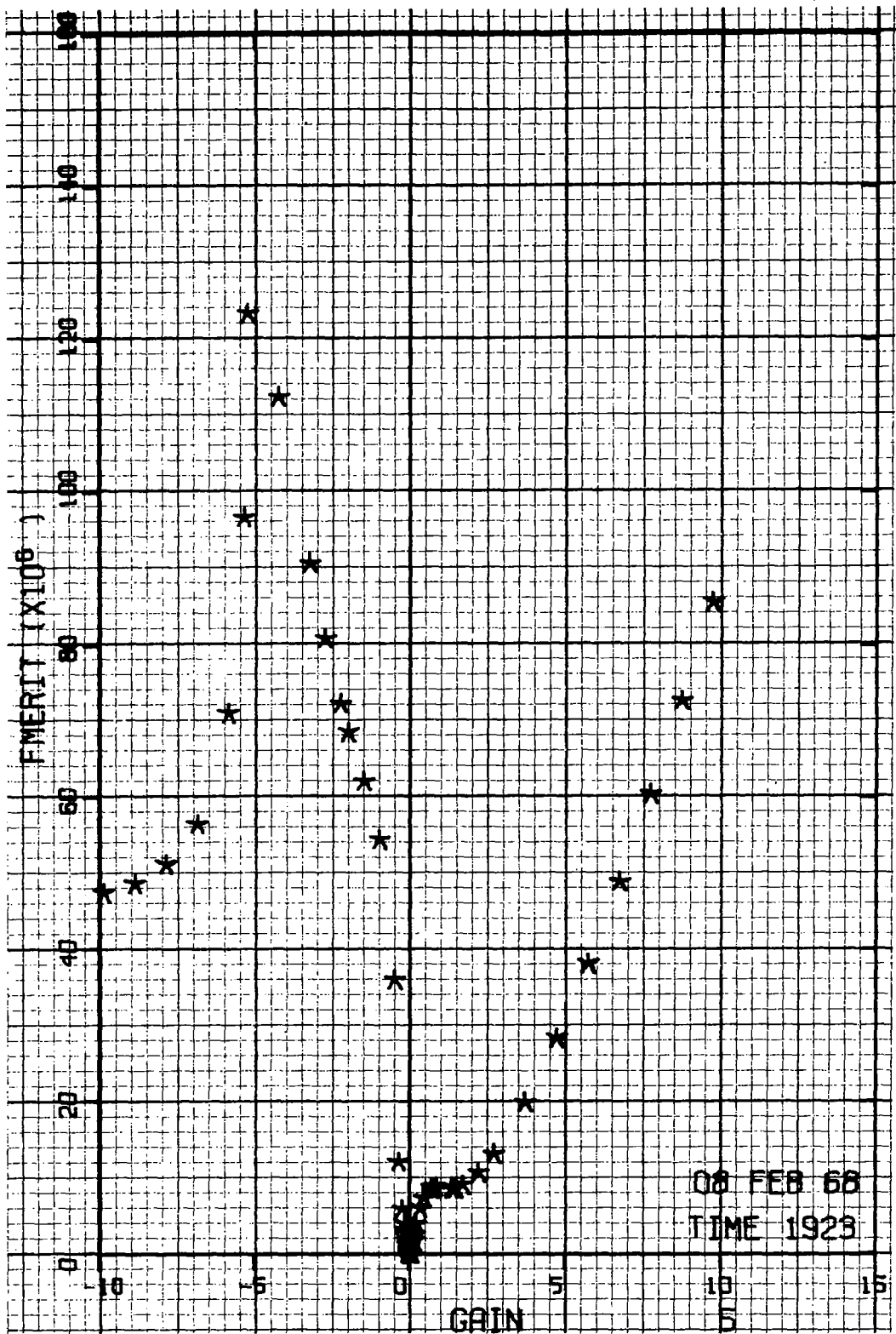


Figure 23 PERFORMANCE MEASURE VALUE vs GRADIENT WITH K_{n3} AS LEAD PARAMETER. (FIXED PARAMETER, MULTI-REGIME CASE)

5. APPLICATION OF THE PROGRAMMING APPROACH TO THE MANUAL LONGITUDINAL FLIGHT CONTROL SYSTEM OF AN SST

From the inception of this project on synthesis of manual control systems, it was believed that the synthesis procedures should be developed with a specific application in mind. In this way it was hoped that a general procedure might result that would take into account the realities and practicalities of manual flight control system design. The particular application receiving major emphasis involved the longitudinal dynamics of an SST.

A. EQUATIONS OF MOTION

Previously, the Flight Research Department of Cornell Aeronautical Laboratory had developed SST equations of motion in six typical flight regimes¹⁰. These equations had also been presented in transfer function form. As a result, they were particularly convenient for application to the manual control system synthesis problem*.

Figure 1 shows the block diagram of the longitudinal SST dynamics in transfer function form. The transfer function form is particularly convenient when there is only one control input point. When more than one input is involved and there is crosscoupled feedback, it becomes more difficult to develop an automated procedure for obtaining the characteristic equation in factored form. To avoid this, it is desirable to use either the equations of motion directly or a state-space approach. Application to the lateral-directional dynamics as discussed in Chapter 7 makes use of the latter approach.

For completeness, the equations of motion and associated parameters are included herein. (Table 1.) The equations include gust inputs which will be neglected in the present chapter, but will be used in the simulation study of Chapter 6. The equations assume constant thrust and no head-on gust component. Data for a given flight regime are designated by a three-part code: the first part indicating mach number, the second indicating aircraft gross weight, and the third indicating altitude.

* Reference 10 contains the original equations for a typical SST. These equations have been modified and updated recently. The updated equations are the ones used in the present study and report.

For example, .8H40 represents a speed of mach 0.8, heavy loading, and an altitude of 40,000 feet.

TABLE 1
EQUATIONS OF MOTION AND ASSOCIATED PARAMETERS
FOR THE LONGITUDINAL DYNAMICS OF AN SST.

Drag Equation -

$$\Delta \dot{V} = -D_V \Delta V - V_T D_\alpha \Delta \alpha - V_T \left(D_\alpha + \frac{g}{V_T} \right) \Delta \alpha_g - g \Delta \theta$$

Z Force Equation -

$$\Delta \dot{\alpha} = -\frac{\alpha_T}{V_T} \Delta \dot{V} + \frac{1}{V_T} \beta_V \Delta V + \beta_\alpha (\Delta \alpha + \Delta \alpha_g) + \Delta \dot{\theta} + \beta_\theta \Delta \theta + \beta_{\delta_e} \Delta \delta_e$$

Pitching Moment Equation -

$$\Delta \ddot{\theta} = M_q (\Delta \dot{\theta} + \Delta \dot{\alpha}_g) + M_\alpha (\Delta \alpha + \Delta \alpha_g) + M_{\dot{\alpha}} (\Delta \dot{\alpha} + \Delta \dot{\alpha}_g) + M_{\delta_e} \Delta \delta_e$$

Vertical Acceleration Equation -

$$\Delta \eta_z = \frac{V_T}{g} (\Delta \dot{\alpha} - \Delta \dot{\theta} + \frac{\alpha_T}{V_T} \Delta \dot{V}) + \theta_T \Delta \theta$$

<u>Flight Regime</u>	V_T (ft/sec)	W (lb)	D_V	D_α	α_T (rad)	β_V
.23H0	257	4.3×10^5	.0386	.00884	.253	-.257
.8H40	777	4.3×10^5	.00490	-.01435	.105	-.0831
3H70	2920	4.3×10^5	-.00050	.00085	.0555	-.0221
.365M10	393	3.4×10^5	.0150	-.0035	.116	-.164
.23L0	257	2.4×10^5	.0263	.00622	.138	-.257
1.4L40	1360	2.4×10^5	.0041	-.000361	.0256	-.0473

<u>Regime</u>	β_α	β_θ	β_{δ_e}	M_q	M_α	$M_{\dot{\alpha}}$
.23H0	-.501	-.0317	-.0720	-.658	-.741	-.439
.8H70	-.366	-.00435	-.0561	-.643	-1.49	-.427
3H70	-.154	-.000610	-.00477	-.158	-.782	-.105
.365M10	-.669	-.00951	-.103	-.801	-1.12	-.534
.23L0	-.842	-.0173	-.129	-.771	-.697	-.514
1.4L40	-.922	-.000607	-.0595	-.850	-6.08	-.566

TABLE 1, continued

Regime	M_{δ_e}	θ_T	K_3^*	$S_6 (r/s)$	$S_7 (r/s)$	
.23H0	-.823	.253	-.530	-.0328	-.457	
.8H40	-1.98	.105	-1.08	-.00863	-.326	
3H70	-.753	.0555	.369	.000622	-.149	
.365M10	-1.56	.116	-.996	-.01598	-.6162	
.23L0	-.965	.138	-.799	-.0238	-.805	
-.4L40	-2.68	.0256	-1.28	-.00412	-.797	

Regime	K_4	$S_8 (r/s)$	$S_9 (r/s)$	$S_{10} (r/s)$	$S_{11} (r/s)$	K_2
.23H0	-.136	1.70	-2.87	.0583	-.0186	-1.11
.8H40	-.147	2.88	-3.96	.00806	-.0060	-1.33
3H70	-.0576	4.72	-4.99	.000685	.00157	-.965
.365M10	-.1155	2.395	-3.752	.01608	-.009456	-1.392
.23L0	-.0109	1.78	-3.11	.0202	-.00205	-1.38
1.4L40	-.0191	5.29	-6.71	.000461	-.00316	-.441

Regime	$S_3, S_4 (r/s)$	$S_5 (r/s)$	ζ_{sp}	ζ_p	$\omega_{nsp} (r/s)$	$\omega_{np} (r/s)$
.23H0	-.0180+j0.168	-12.1	0.778	.0619	1.04	.144
.8H40	-.00241+j0.0579	-36.0	0.548	.0344	1.31	.0542
3H70	+.000251+j0.0156	-158.	0.232	-.0194	.897	.0154
.365M10	-.00710+j0.1121	-15.95	0.780	.0456	1.289	.09466
.23L0	-.0113+j0.167	-8.25	0.918	.0457	1.17	.126
1.4L40	-.00204+j0.0331	-46.0	0.446	.0613	.4278	.0315

The following fixed parameters were chosen:

$$\begin{aligned} \zeta_a &= 0.500 & \tau_{o_o} &= 0.10 \text{ sec.} & \tau_c &= 0.08 \text{ sec.} \\ \omega_a &= 20 (r/s) & \tau_b &= 0.07 \text{ sec.} \end{aligned}$$

B. INPUT INFORMATION FOR THE DIGITAL PROGRAMS

Once the system block diagram is drawn and the various fixed parameters specified, the problem of manual control synthesis becomes one of choosing the optimum values of the adjustable parameters, K_o , τ_o , K_{θ} , τ_a , K_{θ} , K_{n_2} and K_x . The parameter K_o can be set equal to unity during the optimization process. Later, its value may be changed along with inverse changes in K_{θ} , K_{θ} , K_{n_2} and K_x thereby allowing stick gain to be adjusted

* K_3 and all of the following parameters in the table are associated with Fig. 1.

without change in the closed-loop pole positions.* The adjustable parameters are to be chosen in a way which brings about closest correspondence between the closed-loop pole positions and a desired set of pole positions determined from handling qualities information.

Three sets of computer runs were made for the synthesis of the SST longitudinal dynamics. First, a single regime run was made for each of the six flight regimes. Second, a multiple regime run with fixed parameters was made. Finally, a multiple regime run with scheduled parameters was made. For these three sets of runs, the desired pole positions and performance measure weighting factors given in Tables 2 and 3 were used.

TABLE 2
DESIRED POLE POSITIONS FOR ALL LONGITUDINAL DYNAMICS RUNS

Phugoid:	$-.05 \pm j .05$ (rad/sec)	
Short period:**	.23H0,	$-1.09 \pm j 1.45$
	.8H40,	$-1.66 \pm j 2.22$
	3H70,	$-2.21 \pm j 2.94$
	.365M10,	$-1.58 \pm j 2.10$
	.23L0,	$-1.40 \pm j 1.86$
	1.4L40,	$-3.43 \pm j 4.57$
Filters:	-10, -12, -14	
Actuator:	$-10 \pm j 15$	

* Experience has shown that K_o usually has to be increased. Thus, the feedback gains are decreased.

** The desired short period pole pairs were determined by the following handling qualities rule. $\omega_{\eta_{SPD}}^2$ is set equal to the open loop steady-state ratio of $\Delta\eta_z/\Delta\alpha$, with speed changes neglected. ζ_{SPD} is set for slightly less than critical damping, at 0.6.

TABLE 3
PERFORMANCE MEASURE POLE ERROR WEIGHTS USED
FOR LONGITUDINAL DYNAMICS RUNS

.23H0 Flight Regime:

	<u>Phugoid</u>	<u>Short Period</u>	<u>Feedback Filters</u>	<u>Actuator</u>
Single Regime	4×10^5	10^3	10	10^2
Multi-Regime (Fixed)	10^6	10^3	10	10^2
Multi-Regime (Scheduled)	10^6	10^3	10	10^2

.8H40 Flight Regime:

Single Regime	4×10^5	10^3	10	10^2
Multi-Regime (Fixed)	4×10^5	10^3	10	10^2
Multi-Regime (Scheduled)	4×10^5	10^3	10	10^2

3H70 Flight Regime:

Single Regime	4×10^6	4×10^3	20	40
Multi-Regime (Fixed)	10^7	3×10^4	20	40
Multi-Regime (Scheduled)	10^7	3×10^4	20	40

.365M10 Flight Regime:

Single Regime	4×10^5	3×10^3	10	10^2
Multi-Regime (Fixed)	4×10^5	3×10^3	10	10^2
Multi-Regime (Scheduled)	4×10^5	3×10^3	10	10^2

.23L0 Flight Regime:

Single Regime	4×10^5	10^3	10	10^2
Multi-Regime (Fixed)	4×10^5	3×10^3	10	10^2
Multi-Regime (Scheduled)	4×10^5	3×10^3	10	10^2

1.4L40 Flight Regime:

Single Regime	4×10^5	10^3	10	10^2
Multi-Regime (Fixed)	3×10^6	10^3	10	10^2
Multi-Regime (Scheduled)	3×10^6	10^3	10	10^2

The desired pole positions and the performance measure weights were obtained through handling qualities information and from experience with earlier versions of the digital programs used for synthesis. If a designer were working with a totally new problem, a few preliminary computer runs would probably be required to determine desired pole positions and performance measure weights.

All computer runs were programmed to perform 20 iterations in each adjustable parameter. There is no guarantee that significant error reduction will not take place beyond 20 iterations; however, the rate of convergence can be checked following any given run.

C. COMPUTED RESULTS

As might be expected, an enormous amount of computer output information was generated. It is estimated that for the three sets of runs made, 40,000 lines were printed. It becomes necessary to reduce these data considerably for presentation. This reduction task has been attempted in the following paragraphs of this section.

Perhaps the most instructive computer output involves the initial and final values of the performance measures obtained for each run. Table 4 contains this information. For the single regime runs, it is seen that the performance measure values are greatly reduced by the iteration process. In other words, one could expect to find the final closed-loop poles positions in close proximity to their desired positions. Of course, for the single regime case, no compromise must be made; that is, all six adjustable parameters may be used to move the nine closed-loop poles toward their desired positions.

The multi-regime synthesis with adjustable parameters set at single values for all flight regimes (termed multi-regime, fixed) does not yield great reductions in either the performance measures of the individual regimes or the total measure for all regimes. The total measure is reduced by about one third. Consequently, while some improvement does take place with optimum fixed parameters, one would not expect to see

TABLE 4
PERFORMANCE MEASURE VALUES FOR THE VARIOUS COMPUTER RUNS

<u>SINGLE REGIME</u>			
<u>REGIME</u>	<u>INITIAL</u>	<u>FINAL</u>	<u>RATIO</u>
.23H0	9744.3	494.7	.0507
.8H40	6835.3	145.1	.0212
3H70	98225.8	29148.7	.297
.365M10	18025.5	241.1	.0134
.23L0	11520.5	357.4	.0310
1.4L40	22389.4	3687.9	.165
Total	166740.8	34074.9	

<u>MULTI-REGIME (FIXED)</u>			
<u>REGIME</u>	<u>INITIAL</u>	<u>FINAL</u>	<u>RATIO</u>
.23H0	22014.5	20989.1	.964
.8H40	6385.3	4531.9	.710
3H70	586506.2	319027.5	.545
.365M10	18025.5	9978.4	.554
.23L0	20650.1	80385.3	3.89
1.4L40	37308.0	14813.9	.397
Total	690889.6	449726.1	

<u>MULTI-REGIME (SCHEDULED)</u>			
<u>REGIME</u>	<u>INITIAL</u>	<u>FINAL</u>	<u>RATIO</u>
.23H0	22014.5	9693.4	.440
.8H40	6385.3	3187.3	.499
3H70	586506.2	84298.6	.144
.365M10	18025.5	8715.3	.483
.23L0	20650.1	5774.6	.280
1.4L40	37308.0	23800.8	.637
Total	690889.6	135470.	

close correspondence between desired and final pole positions. It should be noted that these results are as expected, since only six parameters are adjusted to bring about correspondence between 54 pairs of given and desired poles. It is worth noting that for the .23L0 regime, the performance measure value was increased by the optimization process. This increase occurred because the total (overall) performance measure value could be most greatly decreased by so doing.

For the scheduled-parameter multi-regime synthesis run, the values of K_{n3} and K_{κ} were permitted to vary from one flight regime to another. All other parameters were to be optimally chosen, but were to be held constant from regime to regime. Table 4 shows that this run produces performance measure reduction values between those of the fixed multi-regime synthesis and those of the single regime syntheses. It can be safely stated that, as more parameters are scheduled, the single-regime performance measure reduction values will be approached. For the particular scheduled multi-regime run made, one could expect moderate discrepancies between final computed pole positions and corresponding desired positions.

In addition to the initial and final values of the performance measures, the value of each measure was printed after every iteration. To obtain information on the convergence of the reduction process, some of the performance measures were plotted as a function of number of iterations. Typical of the results obtained are those shown in Figures 24 and 25. Two of the runs exhibited the step-like behavior depicted in Figure 24 by the .23H0 case. This case exhibits monotone decreasing behavior (as do all of the computer runs), but the performance measure is reduced largely in bursts or at intervals. This behavior is probably the result of nonlinear root-locus behavior as a function of adjustable parameters. In contrast the .8H40 case in Figure 24 and the fixed-parameter, multi-regime run of Figure 25 both exhibit the more typical behavior of less and less performance measure reduction with each iteration. These latter cases are similar to those observed earlier for reduction of a purely quadratic performance measure^{8,9}.

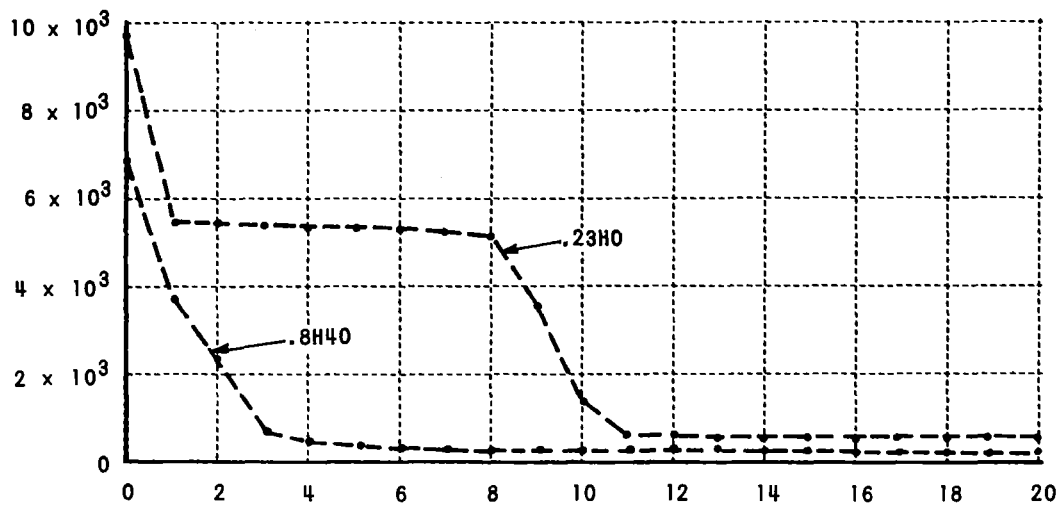


Figure 24 PERFORMANCE MEASURE VS. NUMBER OF ITERATIONS IN EACH PARAMETER FOR SINGLE REGIME RUNS

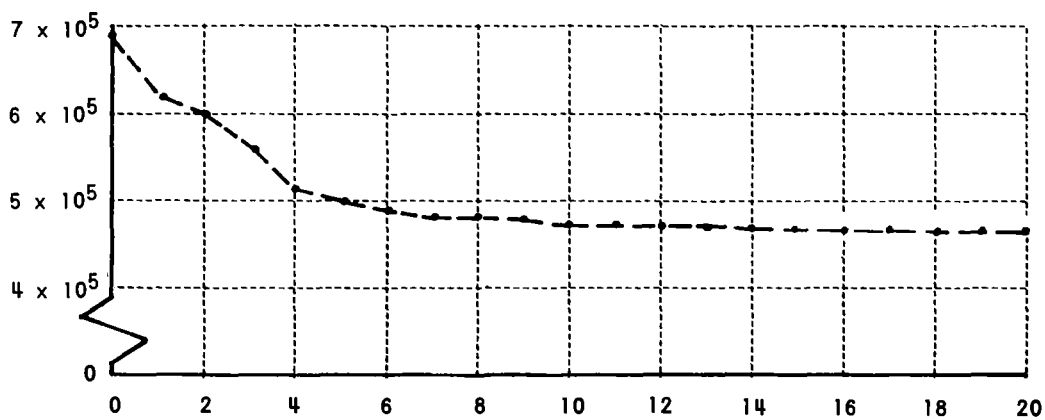


Figure 25 PERFORMANCE MEASURE VS. NUMBER OF ITERATIONS IN EACH PARAMETER FOR FIXED-PARAMETER MULTI-REGIME RUNS

Final pole positions for the various synthesized dynamics are given in Table 5. The various runs for each regime are grouped together for ease in interpretation. The columns labeled "Initial" in Table 5 give the closed-loop pole positions for small, but nonzero adjustable parameter values. It must be remembered that the order of the overall system changes as soon as the adjustable parameters move away from zero. Consequently, the complement of initial pole positions must be computed for adjustable parameter values near zero, but not at zero. These initial pole positions were used to compute the initial performance measure values given in Table 4. A comparison of the remaining columns in Table 5 with information in Table 2 allows one to examine the closeness of the synthesized pole positions to corresponding desired positions.

For the single regime cases, the synthesized and desired pole positions are very close in most cases. The synthesized short-period poles, which have heavy bearing on handling qualities evaluations, are in close proximity to the desired short-period pole positions. The phugoid pairs are also close to desired positions with the exception of the 3H70 case, and the remaining poles are in an area where they would cause no difficulty. It is quite clear that the single regime synthesis procedure is operating properly and that the six adjustable parameters allow adequate pole movement.

The multi-regime runs show greater disparities between desired and synthesized pole pairs. As might be expected, the scheduled parameter run produces somewhat better results than does the fixed parameter run, particularly for the phugoid pole pairs. However, some of the short-period pairs in each run are at considerable distances from their corresponding desired positions: the .23L0 case for the fixed parameter run, and the .23H0 case for the scheduled parameter run. It is interesting to note that a pair of filter poles become complex in the .23H0 and 3H70 cases for the scheduled parameter run. It is clear that the ORDER subroutine is able to deal with branching operations of this type.

Insight may also be gained by examining the final values of the

TABLE 5
SYNTHESIZED SYSTEM POLE POSITIONS

Flight Regime .23H0

	<u>Initial</u>	<u>Single Regime</u>	<u>Multi-Regime (Fixed)</u>	<u>Multi-Regime (Scheduled)</u>
Phugoid	$-.00696 \pm j 0.142$	$-.0525 \pm j .0524$	$-.0302 \pm j .149$	$-.0836 \pm j .0876$
Short Period	$-8.04 \pm j 0.674$	$-1.02 \pm j 1.32$	$-1.35 \pm j 1.35$	$-.946 \pm j .227$
Filters	-9.70 -12.8 -14.3	-8.72 -11.4 -15.2	-8.59 -12.3 -14.3	$-12.9 \pm j 1.52$ -14.5
Actuator	$-10.0 \pm j 17.3$	$-10.5 \pm j 17.2$	$-10.2 \pm j 17.0$	$-8.08 \pm j 17.4$

Flight Regime .8H40

Phugoid	$+ .486 \times 10^6 \pm j .0518$	$-.0428 \pm j .0429$	$-.0187 \pm j .0542$	$-.0173 \pm j .0512$
Short Period	$-.690 \pm j 1.167$	$-1.66 \pm j 2.21$	$-2.55 \pm j 2.13$	$-2.22 \pm j 1.79$
Filters	-9.44 -13.2 -14.2	-9.98 -11.8 -13.8	-5.54 -12.1 -14.3	-11.9 -13.7 -15.7
Actuator	$-10.0 \pm j 17.4$	$-9.63 \pm j 16.6$	$-10.6 \pm j 16.6$	$-6.21 \pm j 17.3$

Flight Regime 3H70

Phugoid	$+ .00101 \pm j .0166$	$-.00781 \pm j .00890$	$-.00501 \pm j .0119$	$-.00272 \pm j .0111$
Short Period	$-.217 \pm j .798$	$-2.12 \pm j 2.96$	$-.504 \pm j 1.82$	$-2.03 \pm j 3.09$
Filters	-10.3 -12.2 -14.3	-7.09 -9.07 -17.6	-8.32 -12.6 -14.3	$-5.35 \pm j 5.93$ -14.0
Actuator	$-10.0 \pm j 17.3$	$-9.60 \pm j 16.7$	$-10.5 \pm j 17.1$	$-14.1 \pm j 16.1$

TABLE 5, continued

Flight Regime .365M10

	<u>Initial</u>	<u>Single Regime</u>	<u>Multi-Regime (Fixed)</u>	<u>Multi-Regime (Scheduled)</u>
Phugoid	$-.00156 \pm j .0911$	$-.0539 \pm j .0539$	$-.0313 \pm j .0981$	$-.0353 \pm j .0942$
Short Period	$-.987 \pm j .886$	$-1.57 \pm j 2.09$	$-2.40 \pm j 1.63$	$-1.98 \pm j 1.33$
Filters	-9.43 -13.2 -14.2	-8.85 -11.2 -14.9	-6.68 -12.2 -14.3	-12.1 -13.8 -15.8
Actuator	$-10.0 \pm j 17.4$	$-10.3 \pm j 16.9$	$-10.4 \pm j 16.7$	$-6.54 \pm j 17.5$

Flight Regime .23L0

Phugoid	$-.00228 \pm j .128$	$-.0585 \pm j .0653$	$-.00641 \pm j .0558$	$-.035 \pm j .0729$
Short Period	$-1.08 \pm j .386$	$-1.48 \pm j 1.96$	$-.948 \pm j 5.45$	$-1.86 \pm j 1.17$
Filters	-9.48 -13.0 -14.3	-10.0 -12.0 -14.2	-8.35 -12.7 -14.3	-11.6 -13.9 -15.1
Actuator	$-10.0 \pm j 17.4$	$-9.83 \pm j 16.9$	$-10.8 \pm j 16.6$	$-7.32 \pm j 17.4$

Flight Regime 1.4L40

Phugoid	$-.000518 \pm j .0294$	$-.0243 \pm j .0222$	$-.0328 \pm j .0198$	$-.0261 \pm j .0175$
Short Period	$-1.02 \pm j 2.55$	$-3.01 \pm j 4.42$	$-4.43 \pm j 4.77$	$-3.59 \pm j 2.11$
Filters	-9.38 -13.6 -14.1	-7.00 -9.53 -11.8	-2.61 -12.0 -14.4	-10.7 -14.7 -17.0
Actuator	$-10.0 \pm j 17.4$	$-12.4 \pm j 16.0$	$-10.6 \pm j 16.2$	$-5.12 \pm j 17.5$

adjustable parameters, as given in Table 6 for the various computer runs. Perhaps the most important finding is that the adjustable parameters generally assume small values. While it is true that some of the parameters did occasionally touch the limits during the early stages of the iteration processes, none of the parameters take on limited final values. The small values of these parameters indicate: (1) that the corresponding flight control systems could probably be implemented and (2) that the iteration process is generally searching inside the boundaries of the parameter space to obtain an optimum. Another interesting finding is that those parameters that are permitted to vary from regime to regime will do so. Several of these parameters vary widely in both sign and magnitude. The two scheduled parameters K_{η_2} and K_{α} were selected on the basis of their wide variations during the single regime runs. For the scheduled-parameter multi-regime run, these parameters also vary widely.

D. ADDITIONAL INFORMATION ON LONGITUDINAL DYNAMICS SYNTHESIS

At an earlier stage in the study of longitudinal dynamics, computer runs were made to determine the effect of deleting feedback from $\Delta \alpha$. There was some indication that deleting this feedback would not appreciably increase the performance measure value if $\Delta \eta_2$ and $\Delta \theta$ (and their derivatives) were used as feedbacks. To determine the effect of deleting feedback from $\Delta \alpha$, three pairs of single regime runs were made with K_{α} set equal to zero for one member of each pair. In each of the three cases, deleting feedback from $\Delta \alpha$ caused an increase in the performance measure value. The ratios of the performance measure pairs were 1.63, 1.20, and 1.39. It may be concluded that there is some merit in including feedback from $\Delta \alpha$. Also, it might be conjectured that in the multi-regime case, deletion of this feedback would produce larger increases in the performance measure value.

As mentioned earlier, the scheduled-parameter multi-regime computer run produced poor matches between certain final pole positions and their corresponding desired positions. To remedy this situation, a

TABLE 6
FINAL ADJUSTABLE PARAMETER VALUES*

<u>Single Regime</u>						
Flight Regime	τ_a	τ_o	$K_{\dot{\theta}}$	K_{θ}	K_{n_2}	K_{α}
.23H0	.261	.0919	-.825	-.682	.413	.645
.8H40	-.00667	.0760	-.958	-.801	-.0899	-2.07
3H70	.136	.476	-1.00	-.0116	-.195	-5.34
.365M10	.124	.0947	-.890	-.599	.153	-.421
.23L0	.0625	.100	-.902	-.795	.0126	-.229
1.4L40	-.0546	.440	-.0968	-.271	.00879	-.0157

<u>Multi-Regime (Fixed)</u>						
	τ_a	τ_o	$K_{\dot{\theta}}$	K_{θ}	K_{n_2}	K_{α}
	-.03261	.0693	-1.13	-.291	-.0506	-1.79

<u>Multi-Regime (Scheduled)</u>						
Flight Regime	τ_a	τ_o	$K_{\dot{\theta}}$	K_{θ}	K_{n_2}	K_{α}
.23H0	.0706	.392	-1.02	-.427	.00755	.426
.8H40	.0706	.392	-1.02	-.427	.0221	-3.19
3H70	.0706	.392	-1.02	-.427	-.3052	-4.73
.365M10	.0706	.392	-1.02	-.427	.00427	-2.55
.23L0	.0706	.392	-1.02	-.427	-.0155	-.2679
1.4L40	.0706	.392	-1.02	-.427	.0203	-3.16

*

For all runs the allowable ranges on the adjustable parameters were as follows:

$$\begin{array}{lll}
 -0.5 \leq \tau_a \leq 0.5 & -10 \leq K_{\dot{\theta}} \leq 10 & -10 \leq K_{n_2} \leq 10 \\
 -0.5 \leq \tau_o \leq 0.5 & -10 \leq K_{\theta} \leq 10 & -10 \leq K_{\alpha} \leq 10
 \end{array}$$

second scheduled-parameter run was made with increased weighting on poorly matched poles. The result was that the positions of the poorly matched poles were improved, at the cost of slight degradation of other pole positions. It is therefore feasible to improve the overall dynamics (from a subjective point of view) by adjusting the performance measure weighting after observing the results of a preliminary computer run.

One of the subroutines that could be called from the main programs was capable of plotting pole positions in the complex S-plane. This subroutine was used in the earlier stages of the longitudinal dynamics study, but was not used for the final runs presented in detail in this chapter. Figure 26 is a sample plot for a final single regime run (.8H40).

Once the programs had been completed and debugged, no further difficulties were encountered in their operation. In every case that was attempted, a solution was obtained. It is important to note that the printout of every run indicated some type of unusual situation was encountered; that is, root-locus branching, inverted parabola, inaccurate quadratic root solution, increase in performance measure, or failure of the first factor routine to find all roots of the characteristic equation. In other words, the logic built into the digital computer programs was indeed required to obtain a solution. It may be concluded that program reliability considerations are at least as important as efficiency considerations.

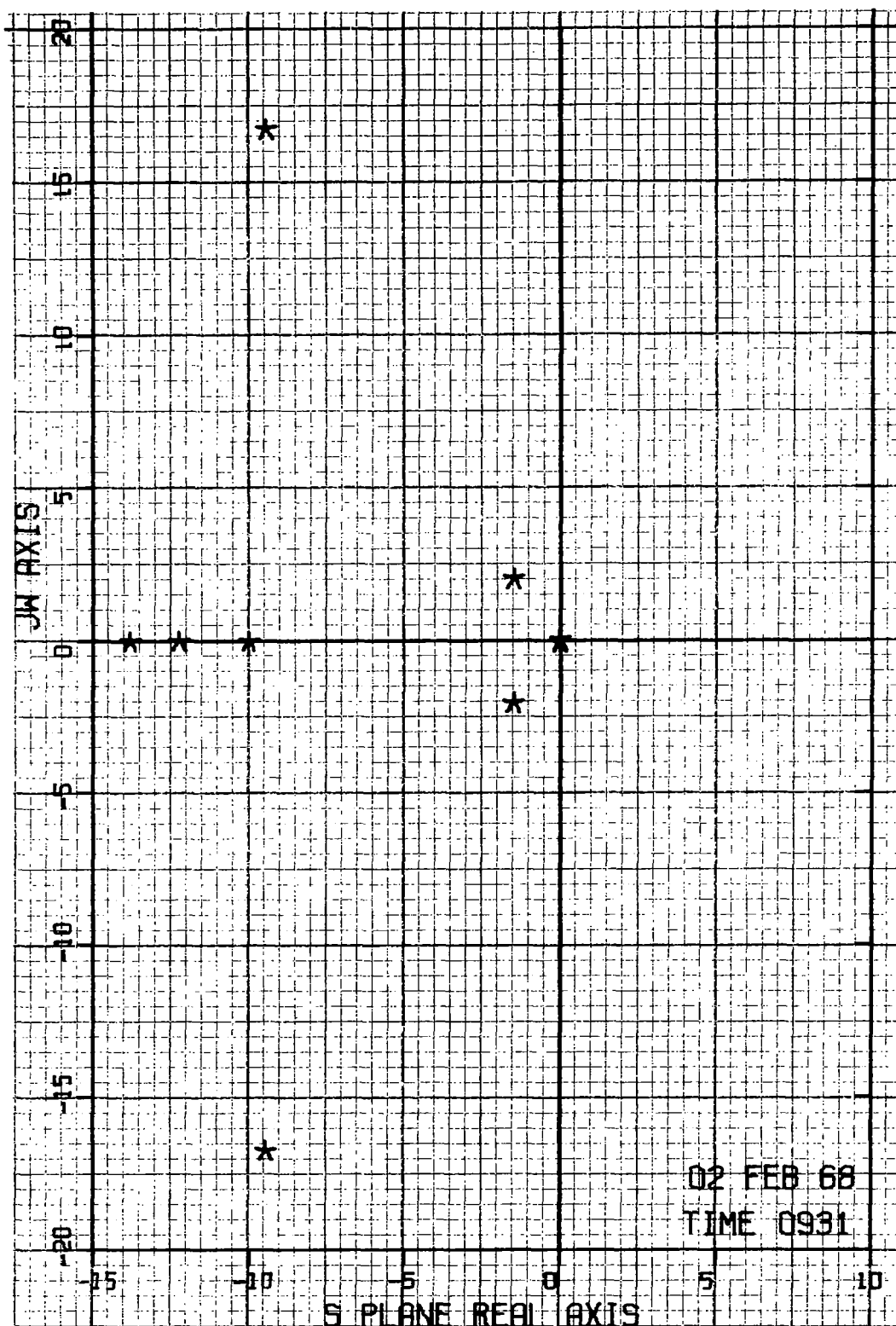


Figure 26 A TYPICAL S-PLANE POLE PLOT

6. EXPERIMENTAL STUDY OF HANDLING QUALITIES AND DISTURBANCE RECOVERY CHARACTERISTICS FOR THE SYNTHESIZED SST LONGITUDINAL DYNAMICS

The longitudinal dynamics synthesis study described in Chapter 5 and the programming approach to manual control synthesis in general place emphasis on moving the closed-loop poles or eigenvalues to those positions most appropriate from a handling qualities viewpoint. As a result of this emphasis on pole motion, there are several questions that require answering. What is the effect of parameter adjustment on the zeros (or the input signal derivative introduction) of the overall system dynamics? Does the introduction of additional signal paths, as required in the parameter adjustment procedure, affect the sensitivity of the overall system to gust disturbances? And, does an improvement in pilot rating actually occur for the synthesized dynamics? To answer these questions, a detailed analog simulation was conducted with a pilot controlling the simulated longitudinal SST dynamics.

A. DESCRIPTION OF THE EXPERIMENTS

The longitudinal dynamics were simulated in full, including the drag equation which results in the low-frequency phugoid dynamics. The actuator, forward compensator, feedback signals and feedback filters were also simulated, thereby allowing a precise analog simulation of the overall synthesized system dynamics. While a transfer function form was used to develop the characteristic equation in the digital programs, the analog simulation solved the SST dynamics directly through the equations of motion given in Chapter 5.

Gust disturbances were introduced into the simulation by choosing appropriately shaped random noise signals for the inputs $\Delta \alpha_g$, $\Delta \dot{\alpha}_g$, and $\Delta \dot{\theta}_g$. In accordance with conventional turbulence theory, a turbulence scale number of 1000 feet was used for all flight regimes. The corner frequency in radians per second for the power spectral density of $\Delta \alpha_g$ is then given by $\omega_o = \frac{V_T}{1000}$, where V_T is the true airspeed in feet per second. Thus, the output of an uncorrelated random noise generator could be passed through a single-pole filter, whose corner frequency is ω_o , to

produce $\Delta \alpha_g$. The disturbance signal $\Delta \dot{\theta}_g$ can be set equal to $-\Delta \dot{\alpha}_g$ without loss of generality. However, before introducing $\Delta \dot{\alpha}_g$ and $\Delta \dot{\theta}_g$ into the simulation, these quantities must be smoothed slightly to account for fuselage averaging of very choppy gust components. For an SST approximately 300 feet long, the corner frequency of the single-pole smoothing filter applied to either $\Delta \dot{\alpha}_g$ or $\Delta \dot{\theta}_g$ should be set at $5.0 \omega_0$. Amplitude of the gust disturbance signal was set for each flight regime so that the turbulence simulated would be rated as severe.

Once the overall system dynamics had been programmed, stick gain was adjusted. It was found that including the compensator and feedback paths reduced stick sensitivity in each flight regime (for the fixed-parameter, multi-regime case). Since the stick sensitivity was not to be scheduled, K_o had to be set at a single value for all flight regimes. The value was determined by summing the overall system impulse responses for each regime at a point in time just following the short-period transient. K_o was set so that the open-loop sum and the closed-loop sum were equal. In other words, the average pitch rate (following initial transient) was made equal for open and closed loop systems. It was found that $K_o = 3.0$. All feedback path gains were reduced by a factor of 3.0 so that the closed-loop poles would not be moved from the positions specified by the digital computer.

Simulation tests were limited to the fixed-parameter, multi-regime synthesis case. This synthesis would be expected to yield only moderate improvement in flight control system characteristics because the poles of the synthesized flight control system are only moderately close to the desired pole positions. However, results obtained for the fixed-parameter, multi-regime synthesis would be conservative, in that either the single regime syntheses or the scheduled-parameter, multi-regime synthesis would produce pole positions that are closer to the desired pole positions.

The pilot who performed the evaluations had been a career air force officer. He was a college graduate in business administration, but had 1-1/2 years of college level engineering work. Of course, he had completed all necessary flight school training. His flight experience

included 1,500 hours in high-performance fighter aircraft and 1,500 hours in heavy transport aircraft. He had not performed as a test pilot previously.

The pilot received pitch information from an oscilloscope simulation of an artificial horizon display. The display included the "horizon", a pair of fixed angled bars, and an edge scale with 0, 5, 10, and 15 degree markings in each pitch direction. The pilot placed inputs into the dynamics by means of a side-arm controller which possessed spring centering and viscous damping. While most heavy aircraft use column position as the pitch input signal, it was felt that no important experimental differences would occur through substitution of the side-arm controller.

The specific purpose of the experiment was to compare in each regime the controllability of the actuator-airframe system with the overall synthesized flight control system. It was believed that a comparison of this type would exhibit the improvement, if any, produced by introduction of the synthesized flight control system. Controllability was to be evaluated in terms of pilot rating, according to the following commonly used scale:

<u>Category</u>	<u>Adjective</u>	<u>Number</u>
Acceptable and Satisfactory	Excellent	1
	Good	2
	Fair	3
Acceptable but Unsatisfactory	Fair	4
	Poor	5
	Bad	6
Unacceptable	Bad	7
	Very Bad	8
	Dangerous	9
Unflyable		10

The pilot was given the open-loop (airframe-actuator) system and the closed-loop (complete flight control) system in each regime separately. However, to avoid possible biases, the pilot was told simply to evaluate the two different configurations.* He was not told that one was open-loop and

* The pilot was informed of the flight regime for each simulation run.

the other closed-loop. Furthermore, for some of the flight regimes, the first configuration was the open-loop system, while for other flight regimes, the first configuration was the closed-loop system. This counterbalancing aided in obtaining a fair comparison over all flight regimes even though some learning or fatigue may have occurred.

Two tests were performed in each flight regime. The first test involved pilot rating of each configuration in still air, and the second involved pilot rating in turbulence. The r.m.s. value of the disturbance signal, $\Delta \alpha_g$, was kept the same for the pair of configurations in any given regime. A comparison of open-loop and closed-loop gust sensitivity could then be made for each flight regime.

The still air experiment and the turbulence experiment were conducted in the same manner. The pilot controlled configuration 1 for one minute and then controlled configuration 2 for one minute. This procedure was then repeated in the same order, that is, configuration 1 and then configuration 2. At the beginning of each run the pilot was explicitly told the configuration number. After the four runs, the pilot was asked to choose a rating from the scale for configuration 1 and a rating from the scale for configuration 2. The four runs for the turbulence experiment immediately followed the four runs for the still air experiment. Data were taken for only one flight regime at a time, with approximate half-hour breaks between regimes.

B. EXPERIMENTAL RESULTS

The results of the experiments are given in Tables 7 and 8. Perhaps the most surprising results contained in these tables are the relatively good ratings given to the open-loop configurations. It had been anticipated that pilot ratings for the open-loop configurations would be poorer because the system poles do not match those specified by handling qualities information. There are several possible reasons for these relatively good ratings. First, it may be that the zeros of the airframe transfer functions compensate in such a way as to make the poles appear in different places. Second, the analog simulation was fixed-base; the

TABLE 7

PILOT RATINGS FOR SIMULATED STILL AIR
FLIGHT SIMULATION

Flight Condition	W/O Flight Control System (Open-Loop)	With Flight Control System (Closed-Loop)
.23H0	2	2
.8H40	2	2
3H70	2	1
.365M10	1	2
.23L0	1	2
1.4L40	<u>4</u>	<u>2</u>
Total:	12	Total: 11

TABLE 8

PILOT RATINGS FOR SEVERE TURBULENCE
FLIGHT SIMULATION

Flight Condition	W/O Flight Control System (Open-Loop)	With Flight Control System (Closed-Loop)
.23H0	5	3
.8H40	7	4
3H70	6	2
.365M10	2	3
.23L0	1	5
1.4L40	<u>8</u>	<u>6</u>
Total:	29	Total: 23

addition of motion cues might have changed the ratings. Finally, it may be that the ratings of a career pilot are somewhat different from those of a test-pilot or pilot engineer. The good ratings obtained for the open-loop configuration indicate that further work ought to be done in determining the desired response characteristics for flight control systems.

Tables 7 and 8 show that moderate improvements in pilot ratings are obtained through the use of the fixed-parameter, multi-regime synthesis procedure. In the still air condition, only a single point of improvement occurs. However, both the open-loop and the closed-loop configurations have very good response according to pilot rating. In severe turbulence, there is moderate improvement with the flight control system in use. It can therefore be stated that the synthesized system does not increase the sensitivity of the aircraft to turbulence*. Moreover, it is likely that the improved response afforded by the synthesized flight control system makes the aircraft easier to control in turbulence. Hence, better pilot ratings are obtained.

It should be reiterated that the fixed-parameter, multi-regime synthesis procedure produced only moderate correspondence between synthesized system poles and desired poles for the longitudinal dynamics. Therefore only moderate improvements in pilot rating are to be expected.

C. TRANSIENT RESPONSE STUDY

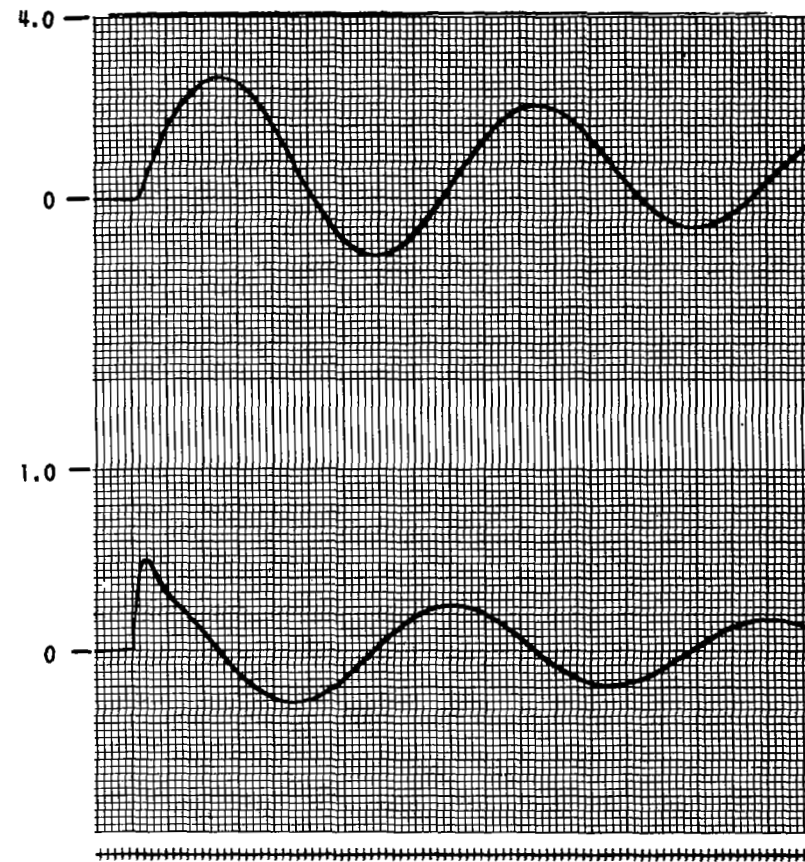
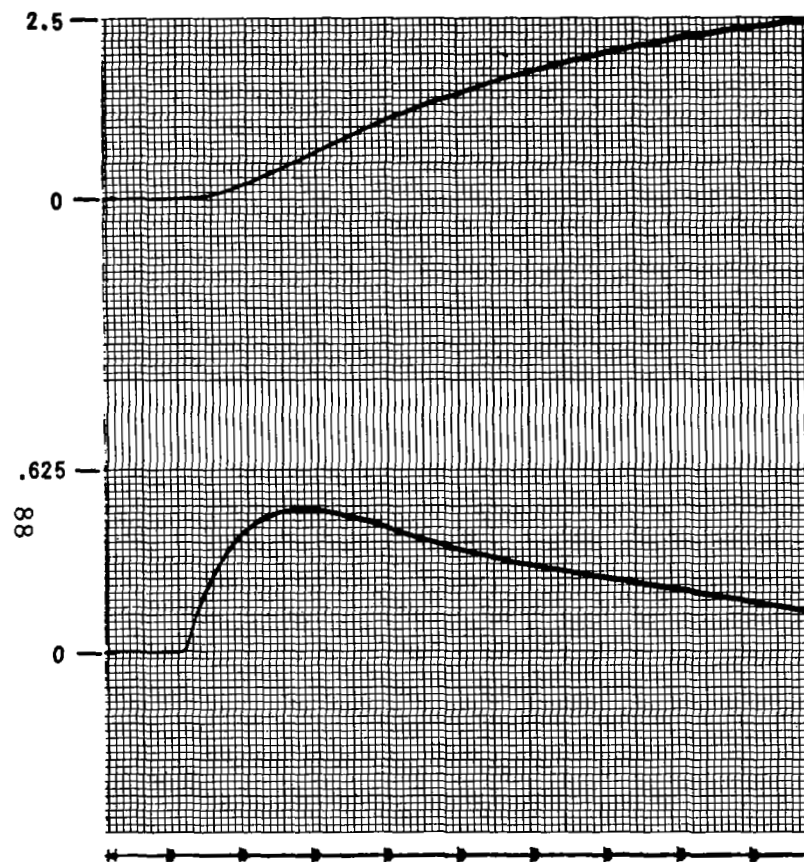
After the piloted simulation studies had been completed, the simulation was used to obtain the transient response of each open and closed-loop system in each regime. Both impulse response and step response recordings were made. The objective in taking these data was to allow a final check on the correctness of the simulation, the pole locations obtained from the synthesis program, and the pole locations computed earlier for the open-loop airframe. These responses were

* It is worth noting in Table 8 that the .23L0 case is the one for which the performance measure was increased by the optimization process. This explains why the pilot gave the closed-loop system a considerably poorer rating in this regime.

carefully checked for short-period damping and frequency as well as phugoid damping and frequency. No discrepancies were found.

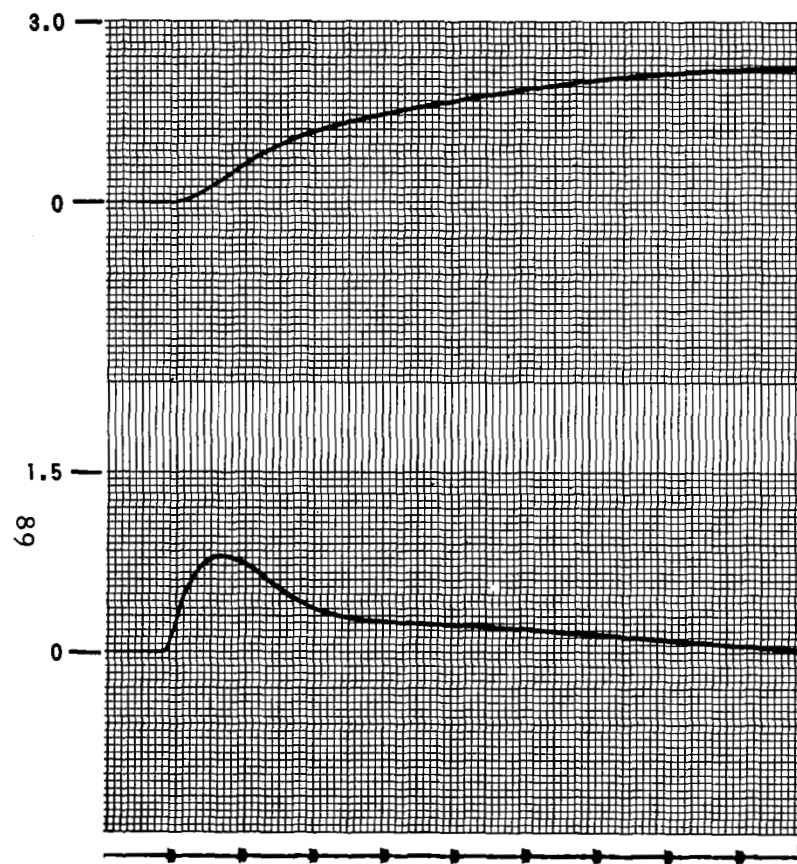
The recordings also made possible an examination of the overall system response per se. Since the zeros of the transfer functions were not directly controlled in the synthesis procedure, there was some question as to whether they might not move in a way that would produce detrimental transient response. Examination of the recordings showed no unusual or detrimental characteristics. In fact, the changes that occurred in going from open-loop to closed-loop response were almost entirely in system gain, and in the damping and natural frequencies of the short-period and phugoid pole pairs.

Figures 27 and 28 show exemplary transient response data. Each figure contains two separate plots of the same response. One second timer markings appear at the bottom of each recording. The left-hand figures aid in evaluating the short-period dynamics and the right-hand figures aid in evaluating the phugoid dynamics. Comparison of the left-hand figures shows that the closed-loop system possesses a faster rise time, indicating a higher frequency for the short-period pole pair. Comparison of the right-hand figures exhibits the higher damping of the phugoid pole pair for the closed-loop system.



UPPER: STEP RESPONSE IN RAD.PER RAD.
 LOWER: IMPULSE RESPONSE IN RAD./SEC. PER RAD.

Figure 27 OPEN-LOOP STEP AND IMPULSE RESPONSES OF SST DYNAMICS FOR .23HO FLIGHT REGIME



UPPER: STEP RESPONSE IN RAD. PER RAD.

LOWER: IMPULSE RESPONSE IN RAD./SEC. PER RAD.

Figure 28 CLOSED LOOP STEP AND IMPULSE RESPONSES OF SST FLIGHT CONTROL SYSTEM
FOR .23HO FLIGHT REGIME

7. PRELIMINARY APPLICATION OF THE PROGRAMMING APPROACH TO THE MANUAL LATERAL-DIRECTIONAL FLIGHT CONTROL SYSTEM OF AN SST

While major emphasis in the application of this research study was placed on the longitudinal dynamics of an SST, an equally important application is synthesis of the lateral-directional dynamics. In order that insight might be gained in the synthesis of lateral-directional dynamics, a preliminary application of the programming approach was made. It is to be emphasized that this application to lateral-directional dynamics is only preliminary.

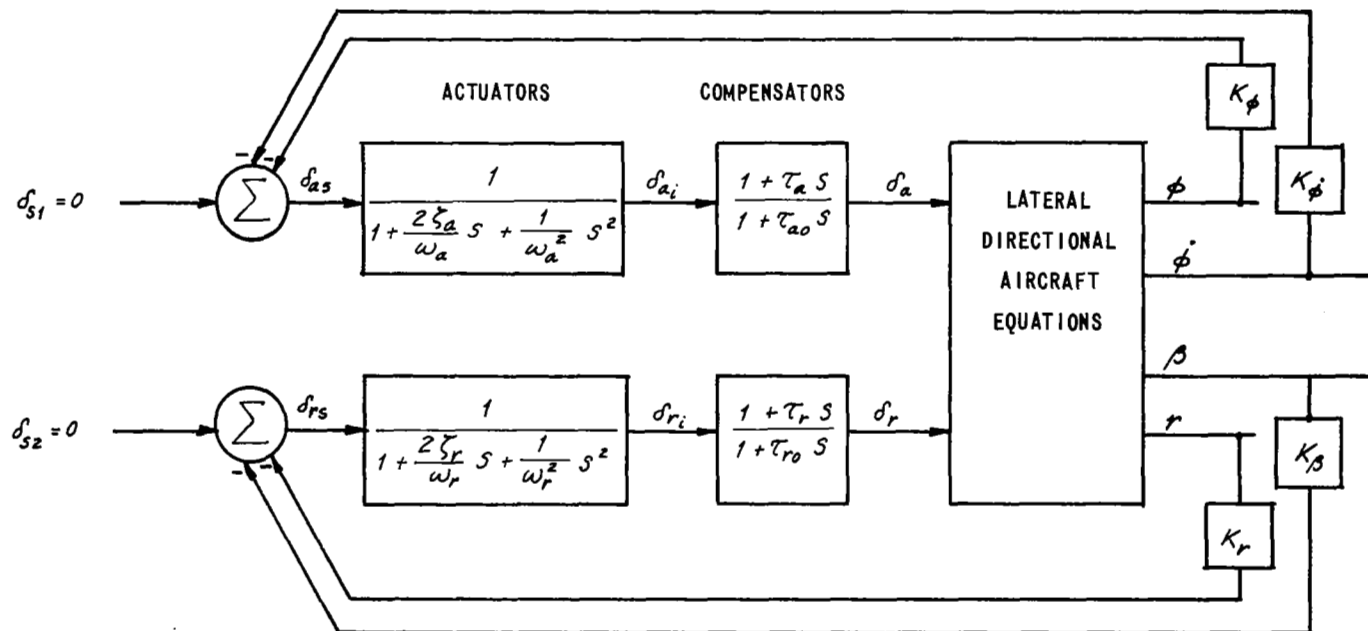
A. SPECIFICATION OF THE LATERAL-DIRECTIONAL SYNTHESIS PROBLEM

Whereas the longitudinal dynamics of an aircraft, including flight control system, can generally be considered as a single input, multi-output system, the lateral directional dynamics must be considered as a dual input, multi-output system. Consequently, one must deal with the closed loop characteristics in terms of an eigenvalue equation instead of a pole equation. Actually, the difference lies in the approach one must use to obtain a closed-loop characteristic equation which yields the eigenvalues.

Following the approach described in Chapter 1, one begins by drawing a block diagram of the overall system including all feedback parameters, aircraft, actuator, and compensator dynamics. In Figure 29, the block diagram of such a system has been drawn, however the relative positions of compensator and actuator have been reversed to make a state-space equation representation possible. (Reversing these two components during the optimization process presents no problem in implementation.)

The equations of motion of the aircraft itself are included on the diagram instead of the transfer function form used earlier. For the system shown in Figure 29, it is possible to write the equations in state-space form. A particularly convenient state vector is given by

$$x' = \begin{bmatrix} \phi & \dot{\phi} & \beta & r & \delta_{ai} & \delta_{ai} & \delta_{ri} & \delta_{ri} & \delta_a & \delta_r \end{bmatrix} \quad (42)$$



ADJUSTABLE PARAMETERS

 $\tau_a, \tau_r, K_\phi, K_{\dot{\phi}}, K_\beta, K_r$

FIXED PARAMETERS

 $\zeta_a, \omega_a, \tau_{a0}, \zeta_r, \omega_r, \tau_{r0}$

NOTE: Positions of the actuators and compensators are purposely reversed.

AIRCRAFT EQUATIONS

SIDE FORCE EQUATION:

$$-\alpha_T \dot{\phi} - \frac{g}{V_T} \phi + (1 + \alpha_T^2) r + \beta - Y_\beta \beta = Y_{\delta_r} \delta_r$$

ROLLING MOMENT EQUATION:

$$\ddot{\phi} - L_p \dot{\phi} - \left(\frac{I_{xz}}{I_{xx}} + \alpha_T \right) \dot{r} + (\alpha_T L_p - L_r) r - L_\beta \beta = L_{\delta_r} \delta_r + L_{\delta_a} \delta_a$$

YAWING MOMENT EQUATION:

$$-\frac{I_{xz}}{I_{zz}} \ddot{\phi} - N_p \dot{\phi} + \left(1 + \alpha_T \frac{I_{xz}}{I_{zz}} \right) \dot{r} + (\alpha_T N_p - N_r) r - N_\beta \beta = N_{\delta_r} \delta_r + N_{\delta_a} \delta_a$$

Figure 29 BLOCK DIAGRAM FOR SST LATERAL-DIRECTIONAL FLIGHT CONTROL SYSTEM SYNTHESIS

where the equations of the system are

$$\dot{x} = Fx + Gu, \quad u = z'x \quad \text{and} \quad u = \begin{bmatrix} \delta_{as} \\ \delta_{rs} \end{bmatrix} \quad (43)$$

Since there are 10 state-variables, the overall closed-loop system will possess 10 eigenvalues. These are obtained by evaluating the following determinant

$$\left| Is - F - Gz' \right| = 0 \quad (44)$$

This determinant yields a characteristic equation in S, that is, a polynomial which when solved for S yields the eigenvalues.

To use the digital programs for synthesis based on the programming approach, the desired eigenvalue positions and all fixed parameters in the block diagram must be specified. For this preliminary lateral-directional study, synthesis in only one regime was attempted. The 0.23H0 case was used, for which the following parameter values were specified:

System Parameters:

$$\zeta_a = 0.5 \quad \tau_{a0} = 1/10 \text{ sec}$$

$$\zeta_r = 0.5 \quad \tau_{r0} = 1/11 \text{ sec}$$

$$\omega_a = 20 \text{ rad/sec}$$

$$\omega_r = 15 \text{ rad/sec}$$

$$\alpha_T = 0.253 \text{ rad}$$

$$V_T = 257 \text{ ft/sec}$$

$$y_\beta = -.0351$$

$$y_{\delta_r} = .0174$$

$$L_p = -1.55$$

$$I_{xx} = 1.88 \times 10^6 \text{ s. ft.}^2$$

$$I_{zz} = 12.9 \times 10^6 \text{ s. ft.}^2$$

$$I_{xz} = 48.6 \times 10^3 \text{ s. ft.}^2$$

$$L_r = .492$$

$$L_\beta = -7.42$$

$$L_{\delta_r} = .495$$

$$L_{\delta_a} = -1.93$$

$$N_p = .00273$$

$$N_r = -.139$$

$$N_\beta = .468$$

$$N_{\delta_r} = -.287$$

$$N_{\delta_a} = -.0835$$

Desired Eigenvalue Positions and Error Weighting:

	<u>Position</u>	<u>Error Weight</u>
Aileron Actuator	$-10 \pm j 15$	10
Rudder Actuator	$-8 \pm j 12$	10
Aileron Compensator	-10	10
Rudder Compensator	-12	10
Dutch Roll Pair*	$-0.3 \pm j 2.5$	100
Spiral	-.01	10000
Roll	-2.5	100

The multi-regime fixed parameter program was used for the synthesis. Data cards were punched so that synthesis was performed for a single regime only.

The initial value of the performance measure (the value obtained for very small, but nonzero adjustable parameters) was 675.14, indicating that the initial eigenvalues were not far from their desired positions. The program performed 20 iterations in each parameter. At the end of the program, the feedback gains had been adjusted so that the following data were obtained:

Performance measure value: 197.8

Closed-loop eigenvalue positions:

Aileron Actuator	$-9.598 \pm j 16.99$
Rudder Actuator	$-7.794 \pm j 12.86$
Aileron Compensator	-9.449
Rudder Compensator	-10.45
Dutch Roll Pair	$-0.3769 \pm j 1.903$
Spiral	-0.008725
Roll	-2.401

Adjustable parameter values:

$\tau_a = .06922$	$K_\phi = .04705$	$K_\delta = 5.000$ (limited)
$\tau_r = -.05827$	$K_\phi = -.5930$	$K_r = -1.341$

* The desired damping of the Dutch roll pair was purposely set at a lower value than is used in practice. This was done to see if the pair could be moved with the chosen set of adjustable parameters.

These data show that considerable improvement in the closed-loop eigenvalue positions is attained. Moreover, all of the feedback gains except K_β are small; K_β was limited at 5.00.* It was noted that the value of the performance measure had reached a nearly steady-state value after 8 iterations of each adjustable parameter; little improvement occurred in the remaining 12 iterations of each parameter.

By means of this example it is clearly demonstrated that: (1) the programming approach is applicable to lateral-directional aircraft dynamics, (2) state-space representations, which must be used for the multi-input case, are handled with equal facility, and (3) substantial improvement is possible in moving the closed-loop eigenvalues. A large number of subjects remain to be investigated in the lateral-directional case. These are enumerated in Chapter 11 under conclusions and recommendations.

* Limits on $\frac{\tau_a}{\tau_{a0}}$, $\frac{\tau_r}{\tau_{r0}}$, K_ϕ , K_δ , K_β , and K_r were all set at ± 5.0.

PART II A MAN-MACHINE PERFORMANCE-CRITERION APPROACH TO MANUAL AEROSPACE CONTROL SYSTEM SYNTHESIS

8. DEVELOPMENT OF A MAN-MACHINE PERFORMANCE APPROACH

A. BACKGROUND

In Part I of this report the fundamental concept of synthesis involves the modification of aircraft-flight control system dynamics to bring them into correspondence with handling qualities specifications. Thus, the "man" in the man-machine system is taken into account through these specifications, and therefore, the man enters the design of the system indirectly.

There is a second approach to manual control system synthesis which accounts more directly for interaction of the man with the system. Suppose that in some way a measure of the man-machine system performance were attained as a function of the flight control system parameters. It would then be possible to select the best parameter values by minimizing the performance measure. The statement could then be made that, for the conditions tested and for the performance measure chosen, the man-machine system is optimum.

The measure of man-machine system performance might include a group of terms, each representing the error or excursion of a certain variable. For example, a designer might want to minimize pitch angle, pitch angle rate, control surface motion, and pilot controller (stick) motion while the aircraft is experiencing turbulence. By using a weighted sum of mean-square values, for example, a performance measure could be computed for each setting of parameters.

B. THREE LEVELS OF SYNTHESIS BY MAN-MACHINE SYSTEM PERFORMANCE

Synthesis based on man-machine system performance can be carried out in several different ways and with varying degrees of sophistication. In this section synthesis at three different levels will be described.

1. Model-Automaton Approach

Suppose that an aircraft (or other vehicular) system is given wherein certain feedback and equalization network parameters are to be determined optimally. The first step is to obtain dynamic models of the human operator or pilot for various settings of the adjustable parameters. These models may be obtained by means of a man-machine system simulation.

The second step involves association of the system dynamics for each setting of parameters with its corresponding human operator model. Since the human is adaptive, the model can be expected to change with each different setting of parameters. A device such as a continuous pattern recognizer or group of automata can be used to perform the association function. Basically, the function of the automata is to accept a sequence of numbers representing the values of the adjustable parameters and to yield a second sequence of numbers which represent the settings of parameters in the human operator model.

An automaton is a device for performing a continuous nonlinear zero-memory transformation. (See Figure 30.) A group of parameters $a_1, a_2, \dots, a_N$ internal to the automaton are adjusted in a way which brings y_r into closest correspondence with the desired output value. The process of bringing about correspondence by adjusting the internal parameters is often called "training". It is not difficult to show that if y_r is to approximate most closely in a mean square sense a desired set of values for certain sequences of inputs, the optimum settings for the a_i 's are obtained by the solution of simultaneous linear algebraic equations.

One automaton would be required for each parameter to be adjusted in the human operator model. Each automaton would be trained over the entire group of simulation runs. The input sequences would be composed of the flight control system adjustable parameters for each run, and the output sequences would be composed of the human operator model parameters for each corresponding run. After each automaton had been trained, it could be inserted in an overall man-machine system model.

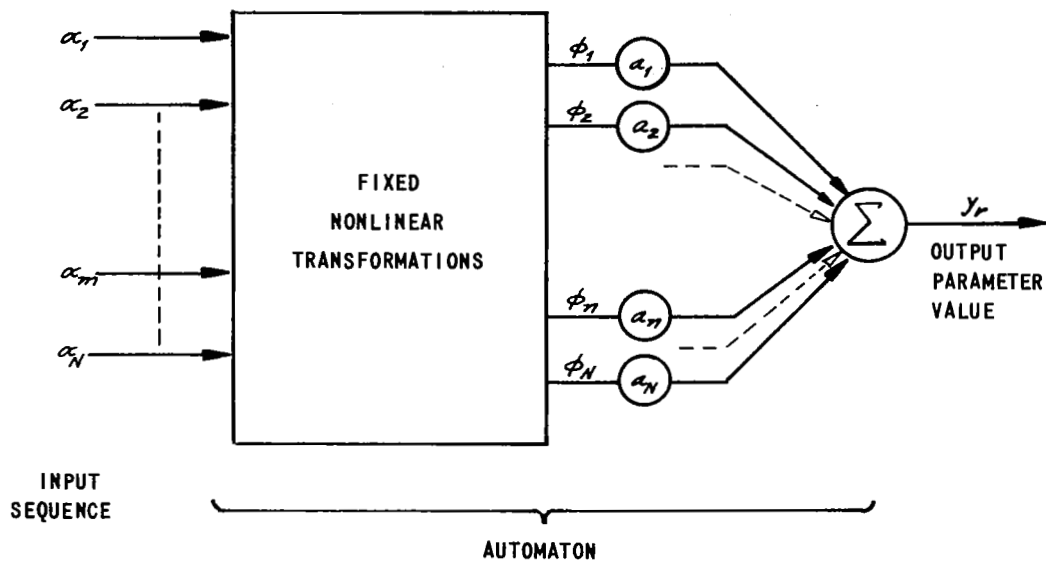


Figure 30 A CONTINUOUS AUTOMATON

Correspondingly, for each setting of the flight control system adjustable parameters, the human operator dynamic model would be automatically adjusted. (See Figure 31.)

Two important and useful properties of continuous automata are their ability to perform smoothing and interpolation. It is impossible to obtain a human operator model for every possible setting of flight control system adjustable parameters. First, a severe dimensionality problem is encountered. Suppose, for example, that six parameters are to be adjusted. If each parameter is permitted to take on 10 different values, then 10^6 man-machine simulation runs are required. Secondly, continuous adjustment of any single parameter requires an infinite number of simulation runs. Fortunately, an automaton is capable of performing interpolation because it will produce a continuous output for continuous adjustment of the input. Therefore, it is not necessary to train an automaton for all possible input sequences; representative input sequences may be used. Smoothing, the other useful property of an automaton can be used to reduce the effects of learning and fatigue that will occur in gathering data from man-machine simulation. Suppose that the sequences of flight-control system adjustable parameters are selected at random (within the constraint bounds) instead of in an orderly fashion. The effects of learning and fatigue would then be scattered randomly throughout the adjustable parameter space. Since automata are capable of smoothing, they will attenuate any point to point variation that does not involve a large trend. Accordingly, point to point variation caused by learning and fatigue will be attenuated.

Once the human operator models have been obtained and the automata trained, the actual synthesis procedure may begin. The objective is to adjust the parameters of the forward and feedback compensators (Figure 31) in a way that minimizes a criterion of performance. To reach this objective, inputs and disturbances of a stochastic nature might be applied to the overall system model and the performance measure evaluated. Then by using some type of minimum seeking strategy, the parameters could be adjusted until a minimum is attained. In practice this procedure proves

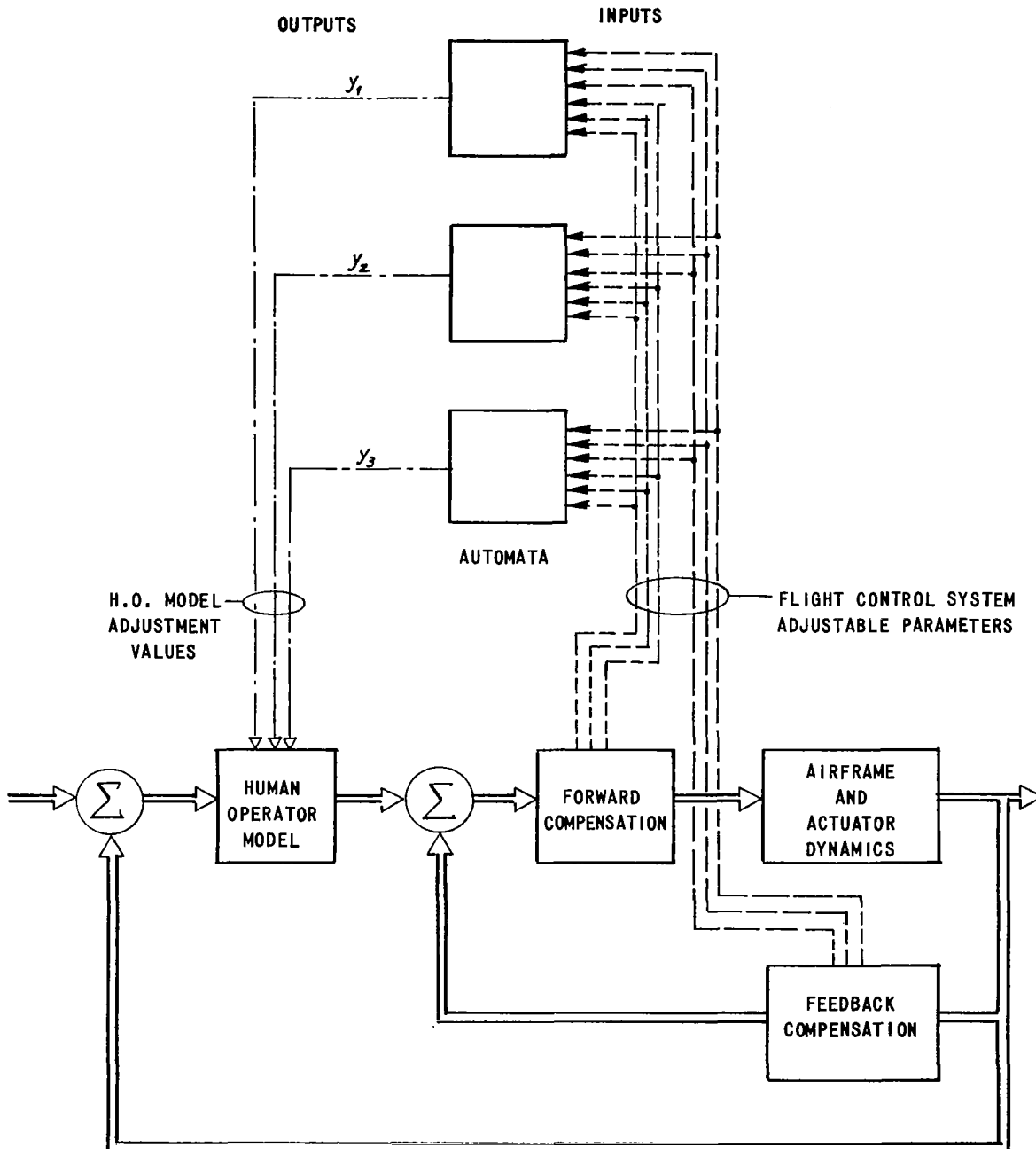


Figure 31 USE OF AUTOMATA TO ADJUST HUMAN OPERATOR MODEL FOR VARIOUS SETTINGS OF FLIGHT CONTROL SYSTEM ADJUSTABLE PARAMETERS

difficult because long lengths of stochastic inputs and disturbances are required to obtain a stable estimate of the performance measure.

The problem of evaluating the performance measure can be handled by taking advantage of an analogy that exists between stochastic and deterministic systems. There are some restrictions on its use however. First, it applies only to linear constant-coefficient systems. Second, the stochastic inputs and disturbances must be stationary. Aircraft flight control systems will generally meet these restrictions, so that no problem need arise. It should be noted that in Figure 31, if all the components other than the automata are linear, then the entire system is linear for any setting of the adjustable parameters. In other words, the nonlinear automata do not destroy the linearity properties of the man-machine system model. Generally, equations of motion used in describing an aircraft are linear. Actuators and compensators are also usually describable as linear systems. Therefore, if the human operator model is a linear model, the stochastic-deterministic analogy may be used. Restricting the inputs and disturbances to stationary random processes is generally acceptable, since they are usually described in terms of stationary spectral densities anyway.

There is one further restriction on the use of the analogy; namely, that the performance measure selected must be restricted to a sum of weighted mean square values. Other measures such as mean absolute values may not be used. It is believed that this restriction would not generally prove troublesome because experience in statistics has shown that minimization of one properly chosen criterion produces approximately the same results as minimization of another properly chosen criterion for the same problem.

The analogy itself is illustrated in Figure 32. It is not difficult to show the mean square value of $x_1(t)$ is equal in amplitude to the integral square value of $x_2(t)$, even though the units on the two quantities are different. Therefore, the mean square value of $x_1(t)$ may be obtained by evaluating the integral square value of $x_2(t)$. The usefulness of the analogy arises from the fact that mean square values require long time-averages

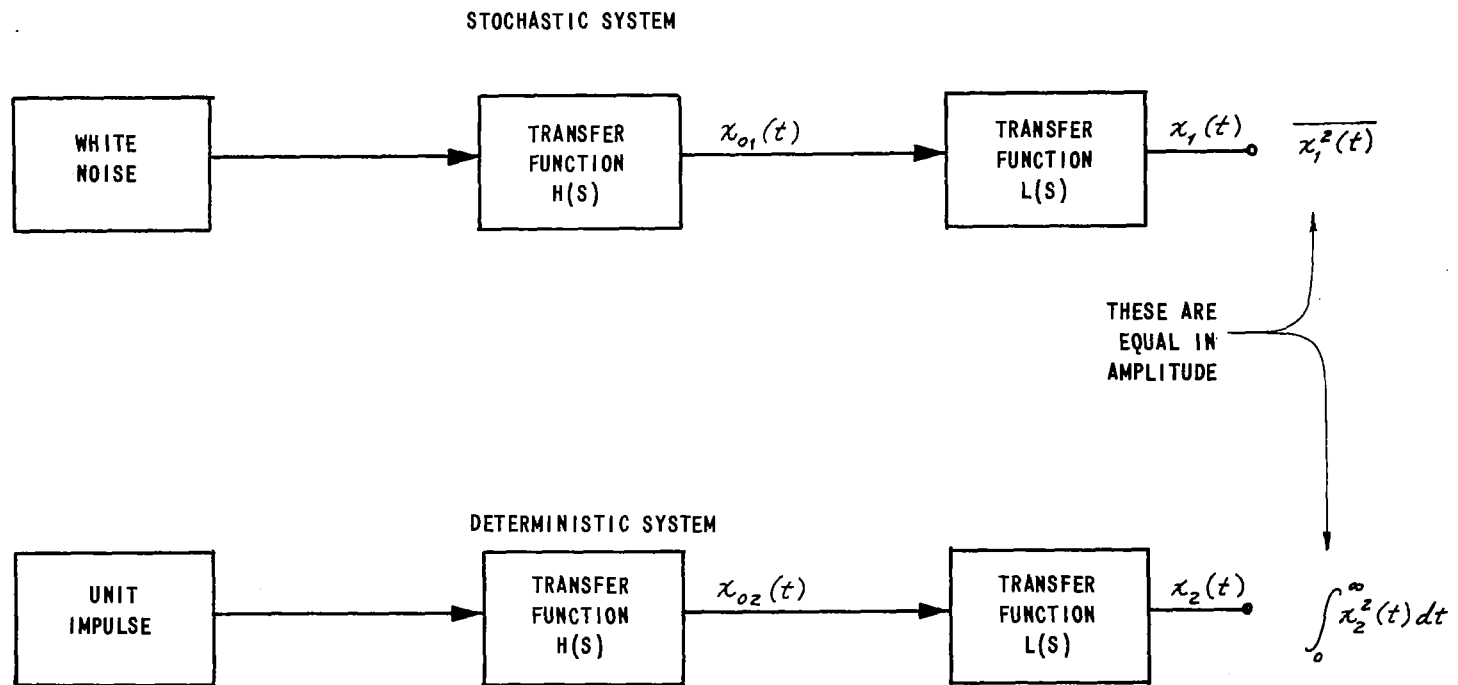


Figure 32 ILLUSTRATION OF THE STOCHASTIC-DETERMINISTIC ANALOGY

to compute, whereas integral square values are based on transient signals that are usually very short. Accordingly, great computational savings can be obtained.

Generally input signals and disturbances are not composed of white noise; that is, their spectra are shaped. The analogy still applies if one chooses the transfer function $H(s)$ so that the spectrum of $x_{oi}(t)$ matches the spectrum of the input or disturbance. In that case one uses the analogy by applying an impulse to the transfer function $H(s)$. The output of $H(s)$ is then applied as the signal or disturbance to the control system, and the desired integral square values are computed. These values are exactly equal to the true mean square values for the stochastic inputs and disturbances. In other words, $L(s)$ represents the total dynamics between the point in the man-machine system model where the input or disturbance is applied and the point where the mean square value is to be evaluated.

Because the performance measure may be quickly evaluated by means of the analogy for any set of adjustable parameters, the search for a minimum may be brought within reasonable computation times for hybrid or digital equipment. Many of the programming methods for seeking out minima could therefore be applied. Once the minimum is found, the corresponding set of parameters are considered optimum and are used for the manual flight control system. The automata and the human operator model are discarded. Synthesis of this type would represent an optimum in a true man-machine performance sense.

2. Model-Discrete Adjustment Approach

The above described synthesis procedure represents a rather sophisticated approach to manual control system synthesis. The question arises as to whether a somewhat simpler approach might be developed that incorporates a good portion of the same philosophy. Simplifications can be made with certain sacrifices in the generality and quality of results.

Suppose that the automata were removed. The consequences of this action would be twofold. First, the performance measure could be evaluated only at the discrete points for which data were taken. Secondly,

the human operator model would have to be adjusted by a "table look-up" approach for each setting of adjustable parameters. A designer could no longer rely on the interpolative feature of the automata. He would have to take data for all settings of the adjustable parameters that he wishes to include in the optimization space. Since the performance measure could be evaluated only at the points for which data were taken, the optimum system would have to be selected from among those discrete points.

While the restrictions caused by eliminating the automata may seem rather stringent, there are certain smaller problems where the approach would be quite applicable. For example, suppose that for a given problem there are three adjustable parameters. If each of these is assigned four discrete values, then 64 data runs are required. A priori testing may show that regions of parameters produce dynamics that are totally unsatisfactory to the human operator, and therefore the four allowable values of each parameter may be chosen over a relatively narrow range. Accordingly, a reasonably good optimization process may be developed even though each adjustable parameter is restricted to a small number of values.

In taking data at the discrete points, one would have to expect learning and fatigue to become important. These effects may be minimized even though automata are not used. If the data runs are selected at random from the parameter space, the effects of learning and fatigue will again appear as a point to point random variable in the performance measure when reordered as a function of parameter values. Smoothing may then be incorporated into the plot of performance measure versus adjustable parameters.

The optimization process in the model-discrete adjustment approach becomes one of determining which of the discrete points produces the smallest value of the performance measure. This process is a relatively simple point to point comparison. As pointed out earlier, since interpolation is not easily introduced in this procedure, the optimization process is limited to selection of one discrete sequence of adjustable parameter values.

3. Direct Performance Measurement Approach

The model-discrete adjustment approach described above makes use of the stochastic-deterministic analogy and human operator modeling to determine the optimum system within the discrete parameter space. Another alternative is to directly evaluate the performance measure in the man-machine system simulation. Rather than develop a model of the human operator from input and output records, it is possible to perform timed data runs for each discrete setting of adjustable parameters and compute the performance measure during the run. The optimization process then becomes one of selecting that setting of adjustable parameters that produces the smallest value of the performance measure.

Learning and fatigue may again be controlled by randomizing the order in which the adjustable parameter settings are used in the man-machine simulation. Smoothing may thereby be incorporated into the performance measure plot.

This direct performance measurement approach is actually rather straightforward. It illustrates the fact that manual control synthesis, if sufficiently simplified, becomes a direct multi-dimensional search.

C. RELATIONSHIP BETWEEN HANDLING QUALITIES INFORMATION AND MAN-MACHINE SYSTEM PERFORMANCE

The synthesis procedures outlined in this chapter are based on optimization of man-machine system performance, whereas the synthesis procedure of Part I is based on meeting handling qualities specifications in a feasible manner. One may inquire as to whether a relationship exists between these two approaches.

There is little information available on the relationship between an optimum in man-machine performance and an optimum from a handling qualities point of view. It is probably true, since an aircraft with proper handling qualities is quite controllable and exhibits good performance, that there is some relationship between it and the man-machine system optimum. Thus, in the experimental study of the performance approach to synthesis, special provisions were made to determine the relationship between handling qualities and man-machine system performance.

9. EXPERIMENTAL STUDY OF THE MAN-MACHINE PERFORMANCE APPROACH

A. BACKGROUND

An experimental study of the performance approach to synthesis was conducted to determine the feasibility of using the approach for design of flight control systems. Because of limited scope of effort and larger emphasis placed on the programming approach to synthesis, the experimental study for the performance approach had to be kept rather uncomplicated. Accordingly, experiments were devised for testing the third level of synthesis, the direct performance measurement approach. In addition, data were taken to examine the relationship between handling qualities and man-machine system performance. The major aspect of the man-machine performance approach to synthesis that was not studied experimentally had to do with the training and use of automata.

B. EXPERIMENT 1

In the first experiment the longitudinal dynamics of an SST at mach 3.0, 70,000 feet, and heavily loaded were simulated on an analog computer. The equations of motion used were the same as those used for the 3H70 case described in Chapter 5, except that airspeed was assumed constant and the actuator dynamics were changed somewhat as follows:

$$\Delta \ddot{\delta}_e + \frac{2\zeta}{\omega_a} \Delta \dot{\delta}_e + \frac{1}{\omega_a^2} \Delta \ddot{\delta}_e = \Delta \delta_s \quad (45)$$

where $\omega_a = 10$ rad/sec and $\zeta_a = 0.707$

Disturbances were introduced into the simulation through aerodynamic terms in the equations of motion, thus again simulating severe turbulence.

It was decided to allow two feedback paths with adjustable gains, K_θ and K_{η_2} . The first of these took its input from $\Delta \dot{\theta}$ and the second from $\Delta \eta_2$. Both were fed back and summed with the stick signal δ_s at the input to the actuator. For each setting of the two adjustable parameters the gain of the stick signal δ_s was set so that the steady state output rate of $\Delta \theta$ for a given fixed stick excursion would always be the same. In other words, stick sensitivity was normalized for each setting of adjustable

parameters.

The two adjustable parameters were set at equal increments across the range for which stable man-machine system operation could be achieved. Data were taken for 27 combinations of the two feedback parameters.

The pilot used for the experiment was the same pilot who performed in the experimental study for the programming approach to synthesis. His description is given in Chapter 6. The pilot was instructed to fly the aircraft under the given turbulent condition, maintaining attitude in the same way he would in an actual aircraft. He was told that a large number of configurations would be tested and that he would be asked for his pilot rating of each configuration immediately following a data run for that configuration. At the end of each run, the pilot was given the pilot rating scale (which is given in Chapter 6) and asked to select a number from the chart representing the response of the aircraft under the turbulent condition.

After the configurations to be tested had been selected, the experimental runs were presented in random order. In this way, the effects of learning and fatigue could be spread randomly over the parameter space.

For each configuration, a three minute practice run was made. Then, after a one minute rest, a three minute data run was made. During the data run, the following mean square values were computed: $\overline{(\Delta \theta)^2}$, $\overline{(\Delta \dot{\theta})^2}$, $\overline{(\Delta \sigma_e)^2}$, and $\overline{(\Delta \sigma_s)^2}$. These mean square values and the pilot rating were obtained for each configuration (or setting of adjustable parameters).

There were two objectives in the experiment. The first was simply to determine which configurations were optimum for certain performance measures. The second was to study the relationship between pilot opinion and man-machine system performance.

Four different performance measures were evaluated for each configuration. They were defined as

$$\begin{aligned}
\theta_1 &= \overline{(\Delta\theta)^2} \\
\theta_2 &= \overline{(\Delta\delta_s)^2} \\
\theta_3 &= \overline{(\Delta\theta)^2} + \frac{1}{4} \overline{(\Delta\delta_s)^2} \\
\theta_4 &= \overline{(\Delta\theta)^2} + \frac{1}{4} \overline{(\Delta\delta_s)^2} + \frac{1}{4} \overline{(\Delta\dot{\theta})^2} + \frac{1}{8} \overline{(\Delta\delta_e)^2}
\end{aligned} \tag{46}$$

It should be noted that all of these measures are computable from a single experiment in which $\overline{(\Delta\theta)^2}$, $\overline{(\Delta\dot{\theta})^2}$, $\overline{(\Delta\delta_e)^2}$, and $\overline{(\Delta\delta_s)^2}$ are recorded. Thus, the run need not be repeated to study a new performance measure.

The results of the experiment are shown graphically in Figures 33 and 34. In Figure 33, the impulse response of each configuration is plotted with corresponding values of feedback, pilot opinion, and performance measures given directly below the plot. Accordingly, one may obtain an overall view of the results.

The figure shows that both pilot opinion ratings and performance measure values are smallest for the plots in the lower right hand portion. The plot in the lower right hand corner is associated with the lowest values of θ_1 , θ_2 , and θ_3 . In addition, θ_4 is third from smallest and the pilot opinion rating is second from smallest among all the configurations. The next three configurations in the bottom row all have low values for the performance measures and low pilot opinion ratings. Thus, if the two adjustable parameters take on the values $-10 \leq K_{\theta} \leq -4$ and $K_{n3} \approx 0$, the system will produce optimum man-machine performance.

Figure 33 makes it clear that there is a relationship between pilot opinion and man-machine system performance. To determine this relationship a series of nonparametric statistical tests were performed. A Spearman rank correlation coefficient was obtained between pilot rating and each one of the performance measures θ_1 , θ_2 , θ_3 , and θ_4 .¹¹ The results are as follows:

	Pilot Op. vs. θ_1	Pilot Op. vs. θ_2	Pilot Op. vs. θ_3	Pilot Op. vs. θ_4
Spearman rank Correlation	.9558	.6278	.9653	.7886

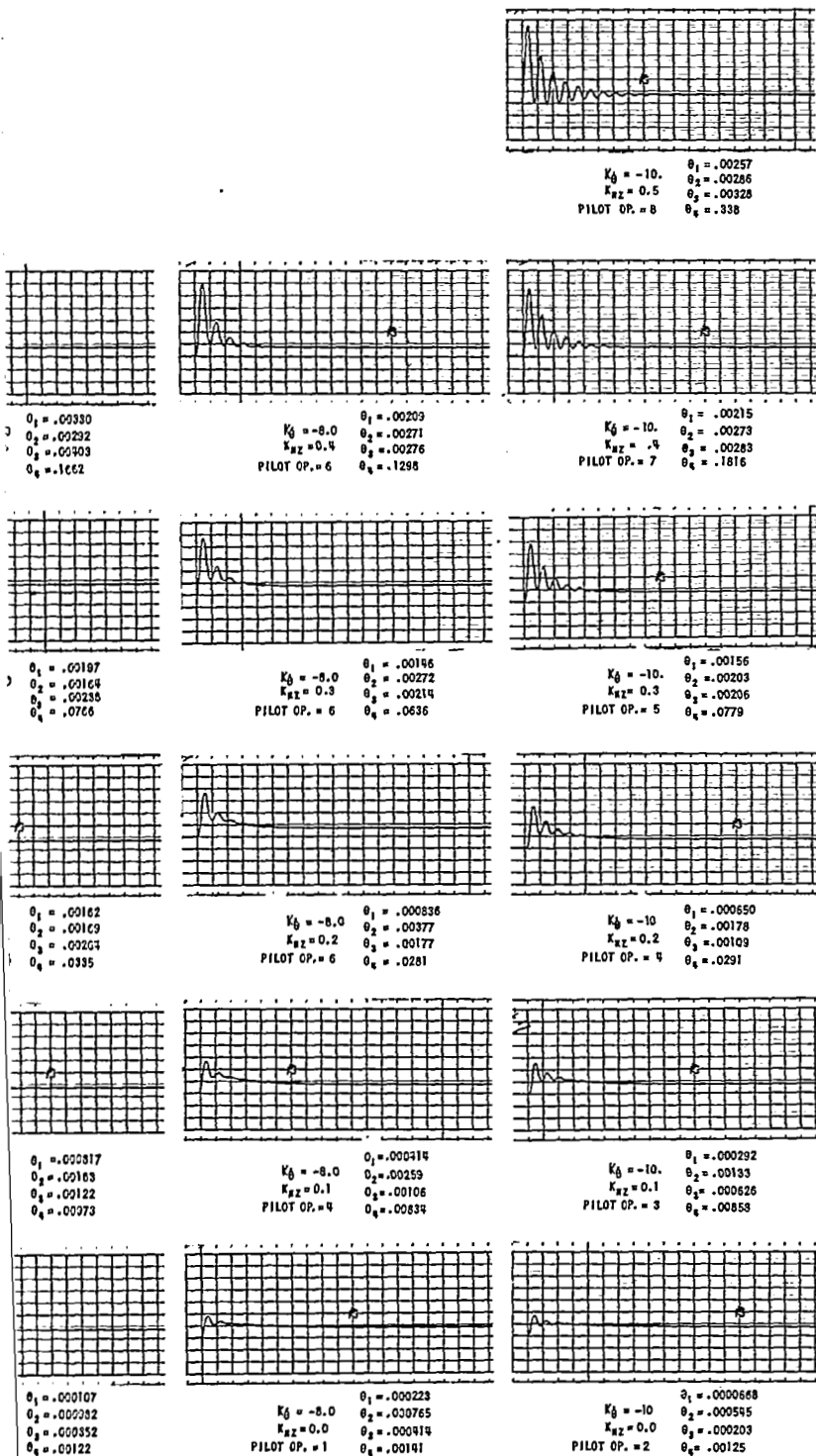


Figure 33

IMPULSE RESPONSE PLOTS, FEEDBACK VALUES, PILOT RATINGS, AND PERFORMANCE MEASURE VALUES FOR EACH RUN OF EXPERIMENT 1

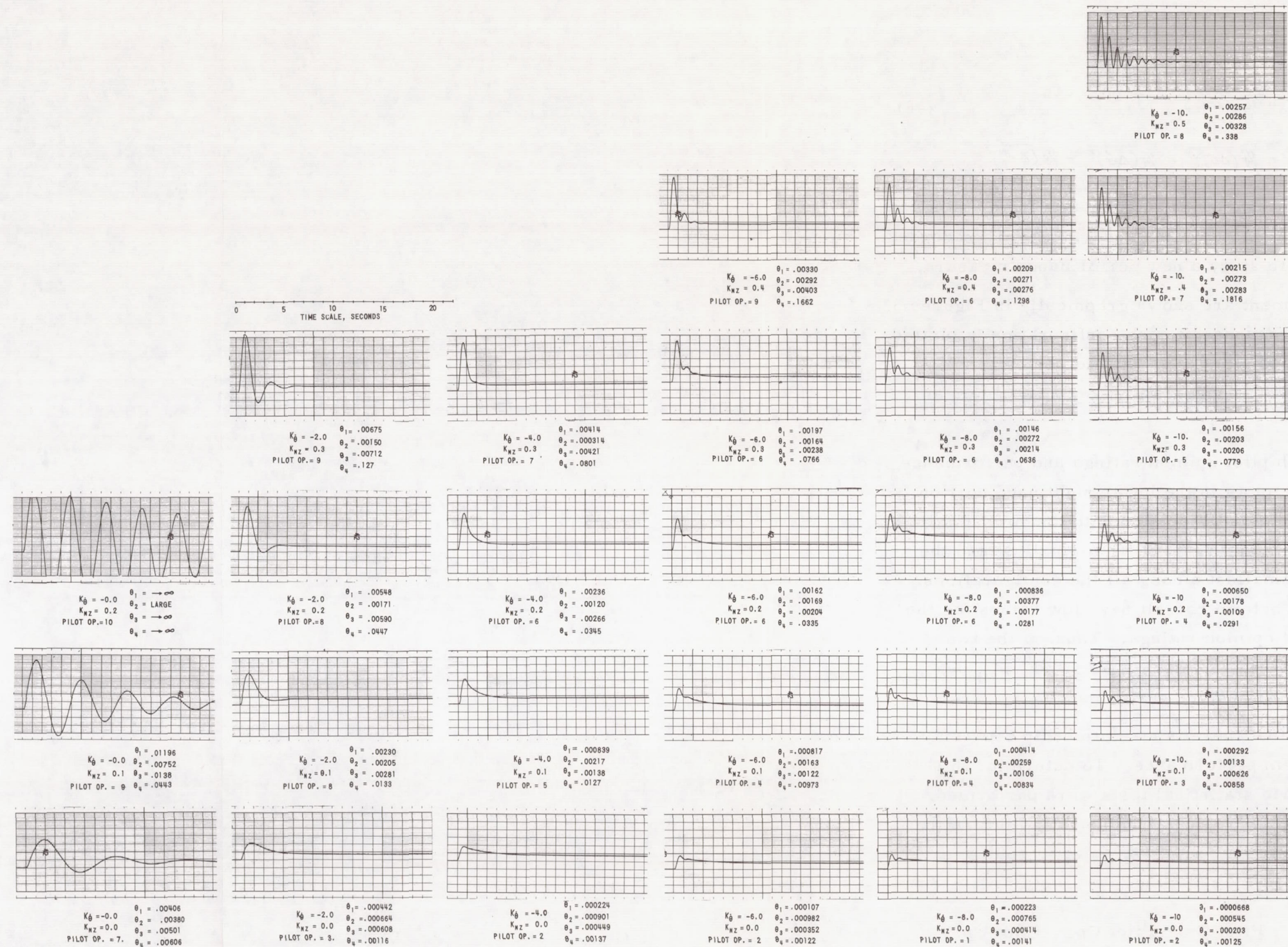


Figure 33
IMPULSE RESPONSE PLOTS, FEEDBACK VALUES, PILOT RATINGS, AND PERFORMANCE MEASURE VALUES FOR EACH RUN OF EXPERIMENT 1

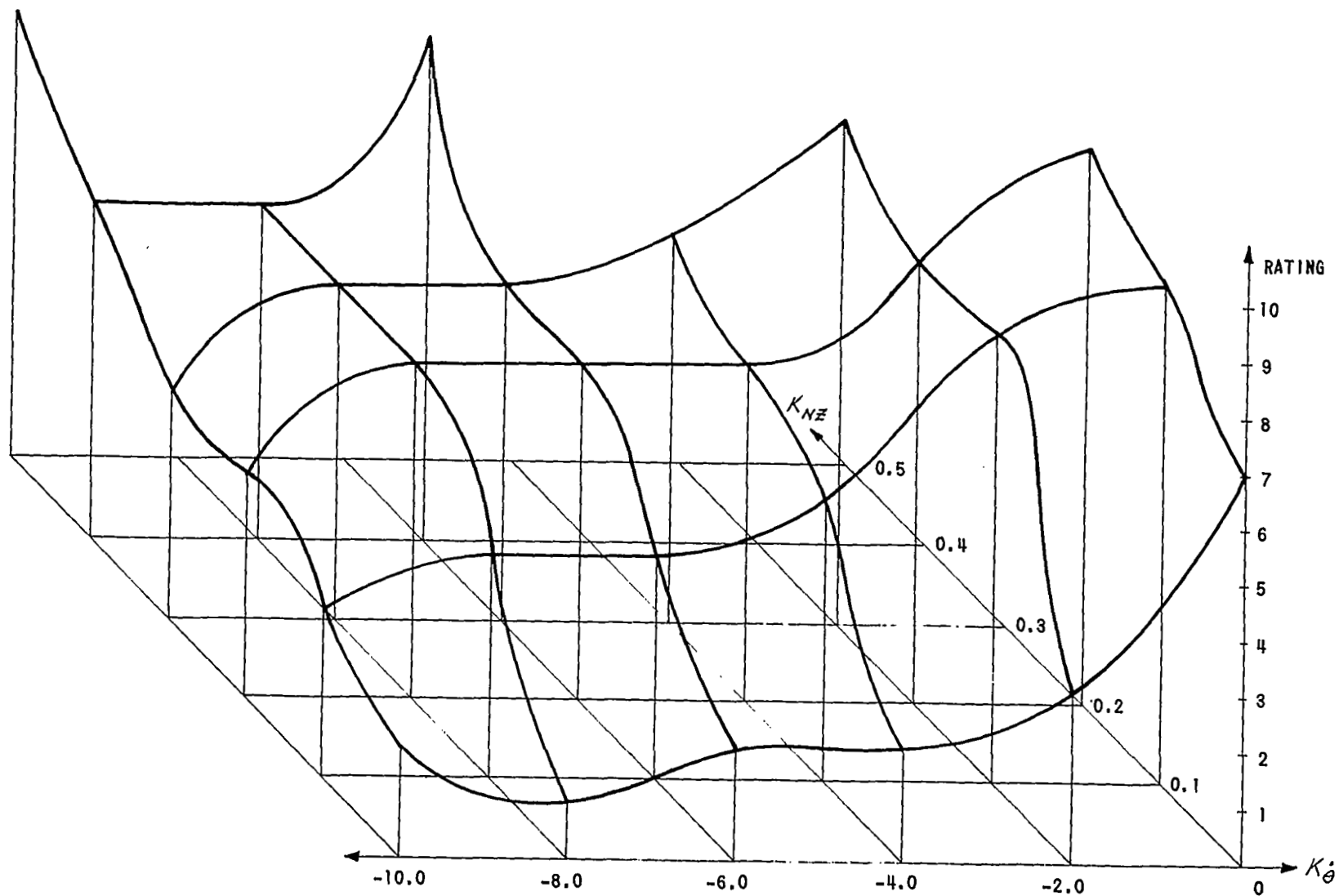


Figure 34 PILOT RATING AS A FUNCTION OF FEEDBACK PARAMETER VALUES; EXPERIMENT 1.

For 27 data points, the Spearman rank correlation is significant at the $p = .05$ level for a value of .323, and it is significant at the $p = .01$ level for a value of .456. Thus, all four man-machine performance measures are highly correlated with pilot opinion. The correlation of θ_1 and θ_3 with pilot opinion represents an extremely strong relationship, indicating that a flight control system rated as optimum by a pilot will also be nearly optimum in a man-machine performance sense. The importance of this result is that the pilot may be relied upon to determine the approximate area in which optimum man-machine system performance may be found.

It is also worth noting that those responses considered as optimum by the pilot (pilot rating of 1 or 2) are systems that respond rapidly and without appreciable overshoot. The natural frequency of the optimum responses is approximately twice as high as that specified by handling qualities specifications.

C. EXPERIMENT 2

In the second experiment, a somewhat more abstract longitudinal aircraft configuration was simulated on the analog computer. Its dynamics contained a short-period pole pair whose damping and natural frequency could be adjusted. The dynamics were given in transform by the equation

$$\Delta\theta(s) = \frac{\Delta N(s)}{s} + \Delta d_s(s) \frac{(1 + 0.3s)}{s \left(1 + \frac{2\zeta}{\omega_n} s + \frac{s^2}{\omega_n^2}\right)} \quad (47)$$

where ζ and ω_n represent the damping and natural frequency of the short-period pole pair, and where ΔN represents the disturbance input. The spectrum of the disturbances was given by

$$P_N(s) = \left| \frac{s}{\left(1 + \frac{s}{0.5}\right)(1 + s)} \right|^2 \quad (48)$$

Here again, the disturbance input was chosen to simulate an aircraft in severe turbulence.

In this experiment d' and ω_n were chosen as the adjustable parameters. Five values of each were chosen, resulting in a total of 25 configurations to be simulated. The values of d' and ω_n were centered

but spread widely about the values of δ' and ω_n considered optimum in handling qualities specifications.

The same pilot was used for this second experiment. Instructions given to the pilot and the procedures followed were almost identical to those of Experiment 1. The exceptions were the following. The pilot was told that the aircraft dynamics were for a hypothetical aircraft in the small transport class. And, during the data run, the following mean square values were computed: $\overline{(\Delta\theta)^2}$, $\overline{(\Delta\dot{\theta})^2}$, and $\overline{(\Delta\delta_s')^2}$.

There were several objectives involved in this second experiment. The first of these involved determining which configurations were optimum for certain performance measures. The second was to further study the relationship between pilot rating and man-machine system performance. The third was to determine whether pilot rating could be predicted on the basis of the configuration's nearness to meeting handling qualities specifications. The methods for reaching the first two objectives were similar to those used in the first experiment. To reach the third objective, one of the investigators selected a pilot opinion rating number for each configuration. The rating was based on the nearness of the damping and natural frequency to those specified by handling qualities research. These predicted opinions were permanently recorded so that they could be compared with the pilot ratings after the experimental data were taken.

Again, four different performance measures were evaluated for each configuration. They were

$$\begin{aligned}\theta_1 &= \overline{(\Delta\theta)^2} \\ \theta_2 &= \overline{(\Delta\dot{\theta})^2} \\ \theta_3 &= \overline{(\Delta\theta)^2} + \frac{1}{4} \overline{(\Delta\dot{\theta})^2} \\ \theta_4 &= \overline{(\Delta\theta)^2} + \frac{1}{4} \overline{(\Delta\dot{\theta})^2} + \frac{1}{4} \overline{(\Delta\delta_s')^2}\end{aligned}\tag{49}$$

All of these measures could be obtained from the mean square values computed during the experiment.

The results of the experiment are shown graphically in Figures 35 and 36. In Figure 35, the impulse response of each configuration is plotted with corresponding values of ζ , ω_n , pilot opinion, predicted pilot opinion, and performance measure values given directly below the plot. In this experiment, optimum values of the four performance measures are spread somewhat. The optimum value of θ_1 occurs for $\zeta = 0.1$ and $\omega_n = 9.0$, and the optimum value for θ_2 occurs for $\zeta = 0.60$ and $\omega_n = 9.0$. Both were given the lowest value of pilot opinion rating, that is, 3. The measures θ_3 and θ_4 both assume optimum values for $\zeta = 0.35$ and $\omega_n = 3.0$, for which the pilot opinion rating is 4. The pattern of optimum values is not as clear cut in this experiment as in Experiment 1. However, it is seen that the pilot preferred rapidly responding dynamics that are somewhat underdamped. The performance measure values are also low for rapid somewhat underdamped responses.

Predicted pilot opinion is accurate for approximately half the configurations, but is rather inaccurate for the remainder of them. Particularly large errors exist in the area where handling qualities work would have predicted good pilot ratings, namely, in the region of $\zeta = 0.6$ and $\omega_n = 3.0$. Also, underdamped, fast responding systems such as $\zeta = 0.1$, $\omega_n = 9.0$ were predicted to have poor pilot ratings, whereas they produced good pilot ratings.

To more carefully analyze the relationship between pilot opinion and man-machine performance, Spearman rank correlations were again computed. The results are as follows:

	Pilot Op. vs. θ_1	Pilot Op. vs. θ_2	Pilot Op. vs. θ_3	Pilot Op. vs. θ_4
Spearman rank Correlation	.8632	.7292	.8686	.8752

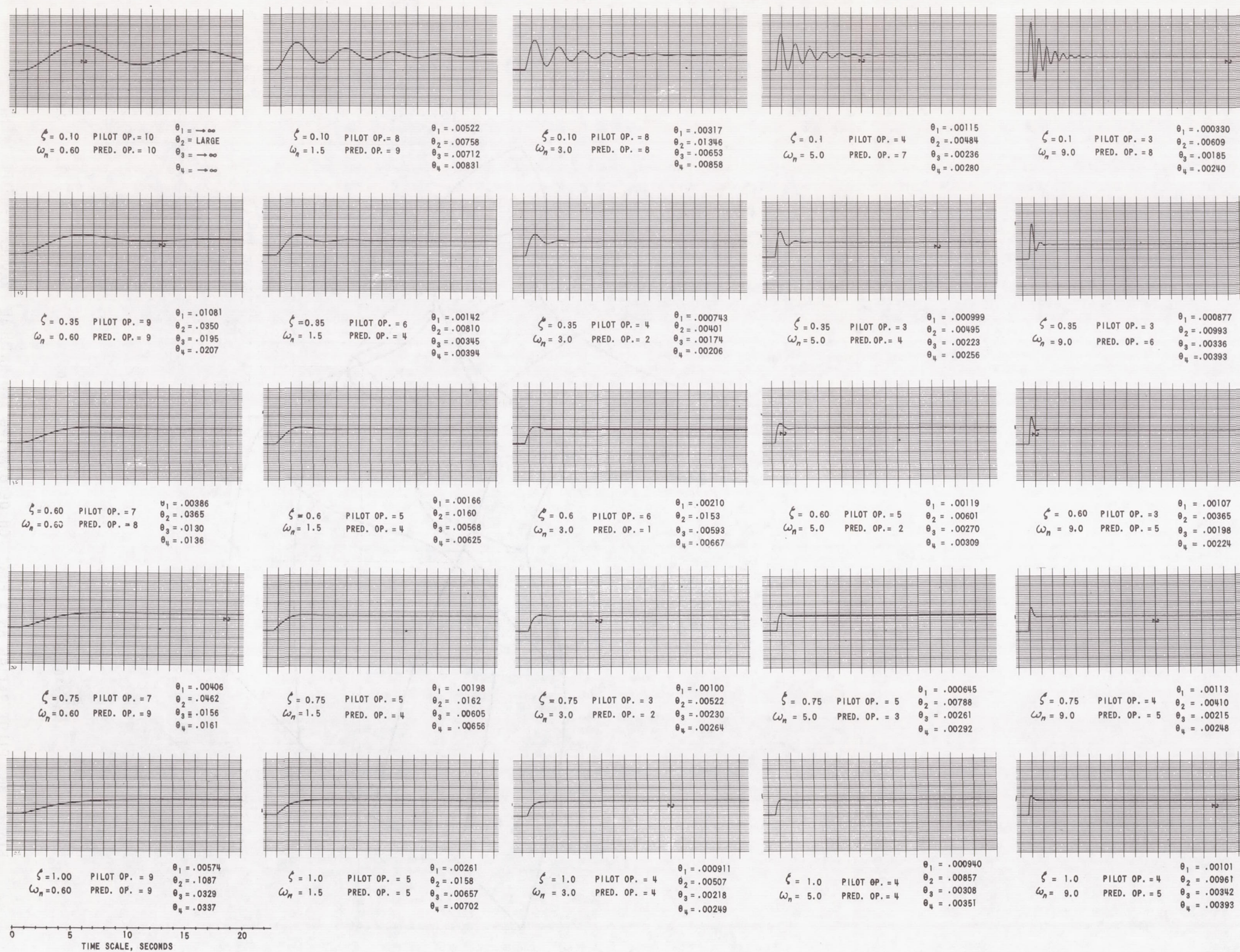
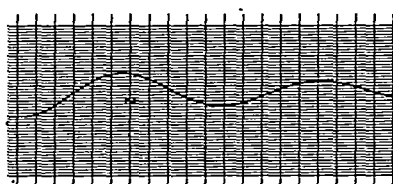
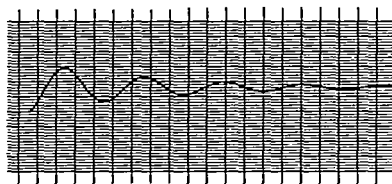


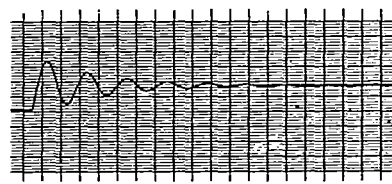
Figure 35
IMPULSE RESPONSE PLOTS, DAMPING AND NATURAL
FREQUENCIES, PREDICTED AND ACTUAL PILOT RATINGS,
AND PERFORMANCE MEASURE VALUES FOR EACH RUN
OF EXPERIMENT 2



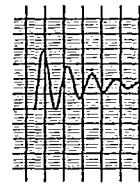
$\zeta = 0.10$ PILOT OP. = 10
 $\omega_n = 0.60$ PRED. OP. = 10
 $\theta_1 = \infty$
 $\theta_2 = \text{LARGE}$
 $\theta_3 = \infty$
 $\theta_4 = \infty$



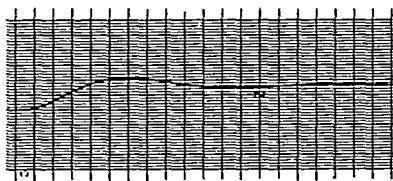
$\zeta = 0.10$ PILOT OP. = 8
 $\omega_n = 1.5$ PRED. OP. = 9
 $\theta_1 = .00522$
 $\theta_2 = .00758$
 $\theta_3 = .00712$
 $\theta_4 = .00831$



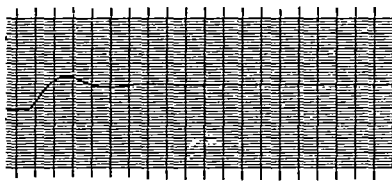
$\zeta = 0.10$ PILOT OP. = 8
 $\omega_n = 3.0$ PRED. OP. = 8
 $\theta_1 = .00317$
 $\theta_2 = .01346$
 $\theta_3 = .00653$
 $\theta_4 = .00858$



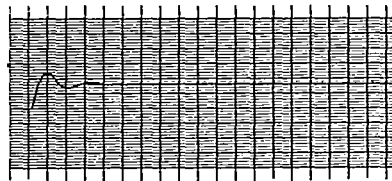
$\zeta = 0.1$
 $\omega_n = 5.0$



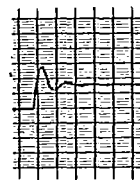
$\zeta = 0.35$ PILOT OP. = 9
 $\omega_n = 0.60$ PRED. OP. = 9
 $\theta_1 = .01081$
 $\theta_2 = .0350$
 $\theta_3 = .0195$
 $\theta_4 = .0207$



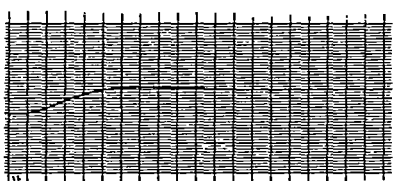
$\zeta = 0.35$ PILOT OP. = 6
 $\omega_n = 1.5$ PRED. OP. = 4
 $\theta_1 = .00142$
 $\theta_2 = .00810$
 $\theta_3 = .00345$
 $\theta_4 = .00394$



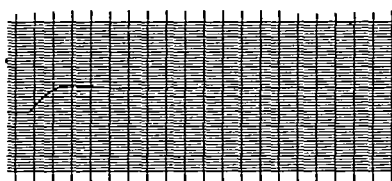
$\zeta = 0.35$ PILOT OP. = 4
 $\omega_n = 3.0$ PRED. OP. = 2
 $\theta_1 = .000743$
 $\theta_2 = .00401$
 $\theta_3 = .00174$
 $\theta_4 = .00206$



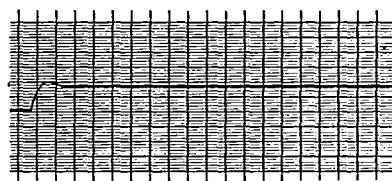
$\zeta = 0.35$
 $\omega_n = 5.0$



$\zeta = 0.60$ PILOT OP. = 7
 $\omega_n = 0.60$ PRED. OP. = 8
 $\theta_1 = .00386$
 $\theta_2 = .0365$
 $\theta_3 = .0130$
 $\theta_4 = .0136$



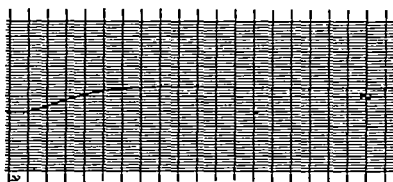
$\zeta = 0.6$ PILOT OP. = 5
 $\omega_n = 1.5$ PRED. OP. = 4
 $\theta_1 = .00166$
 $\theta_2 = .0160$
 $\theta_3 = .00568$
 $\theta_4 = .00625$



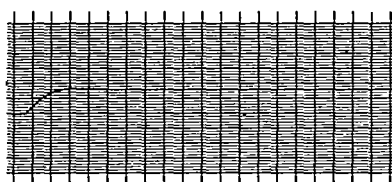
$\zeta = 0.6$ PILOT OP. = 6
 $\omega_n = 3.0$ PRED. OP. = 1
 $\theta_1 = .00210$
 $\theta_2 = .0153$
 $\theta_3 = .00593$
 $\theta_4 = .00667$



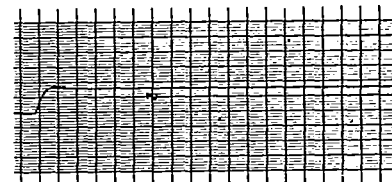
$\zeta = 0.60$
 $\omega_n = 5.0$



$\zeta = 0.75$ PILOT OP. = 7
 $\omega_n = 0.60$ PRED. OP. = 9
 $\theta_1 = .00406$
 $\theta_2 = .0462$
 $\theta_3 = .0156$
 $\theta_4 = .0161$



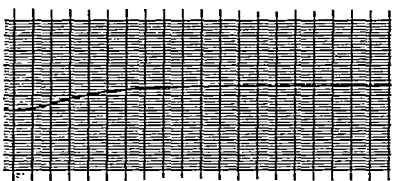
$\zeta = 0.75$ PILOT OP. = 5
 $\omega_n = 1.5$ PRED. OP. = 4
 $\theta_1 = .00198$
 $\theta_2 = .0162$
 $\theta_3 = .00605$
 $\theta_4 = .00656$



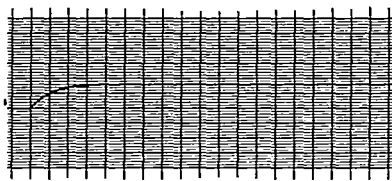
$\zeta = 0.75$ PILOT OP. = 3
 $\omega_n = 3.0$ PRED. OP. = 2
 $\theta_1 = .00100$
 $\theta_2 = .00522$
 $\theta_3 = .00230$
 $\theta_4 = .00264$



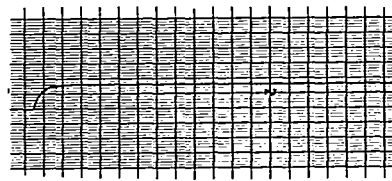
$\zeta = 0.75$
 $\omega_n = 5.0$



$\zeta = 1.00$ PILOT OP. = 9
 $\omega_n = 0.60$ PRED. OP. = 9
 $\theta_1 = .00574$
 $\theta_2 = .1087$
 $\theta_3 = .0329$
 $\theta_4 = .0397$



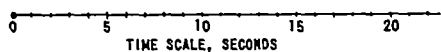
$\zeta = 1.0$ PILOT OP. = 5
 $\omega_n = 1.5$ PRED. OP. = 5
 $\theta_1 = .00261$
 $\theta_2 = .0158$
 $\theta_3 = .00657$
 $\theta_4 = .00702$



$\zeta = 1.0$ PILOT OP. = 4
 $\omega_n = 3.0$ PRED. OP. = 4
 $\theta_1 = .000911$
 $\theta_2 = .00507$
 $\theta_3 = .00218$
 $\theta_4 = .00248$



$\zeta = 1.0$
 $\omega_n = 5.0$



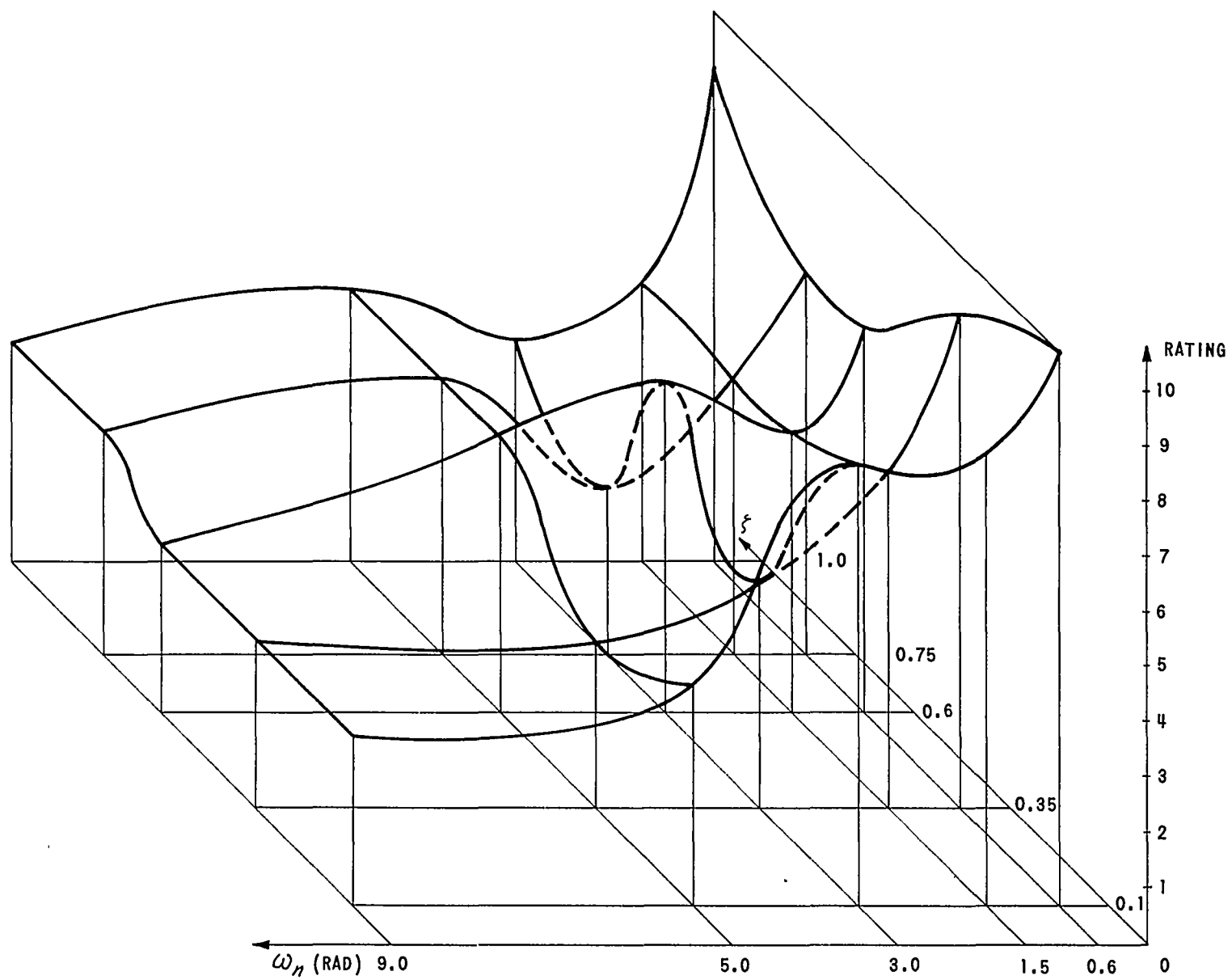


Figure 36 PILOT RATING AS A FUNCTION OF DAMPING AND NATURAL FREQUENCY; EXPERIMENT 2.

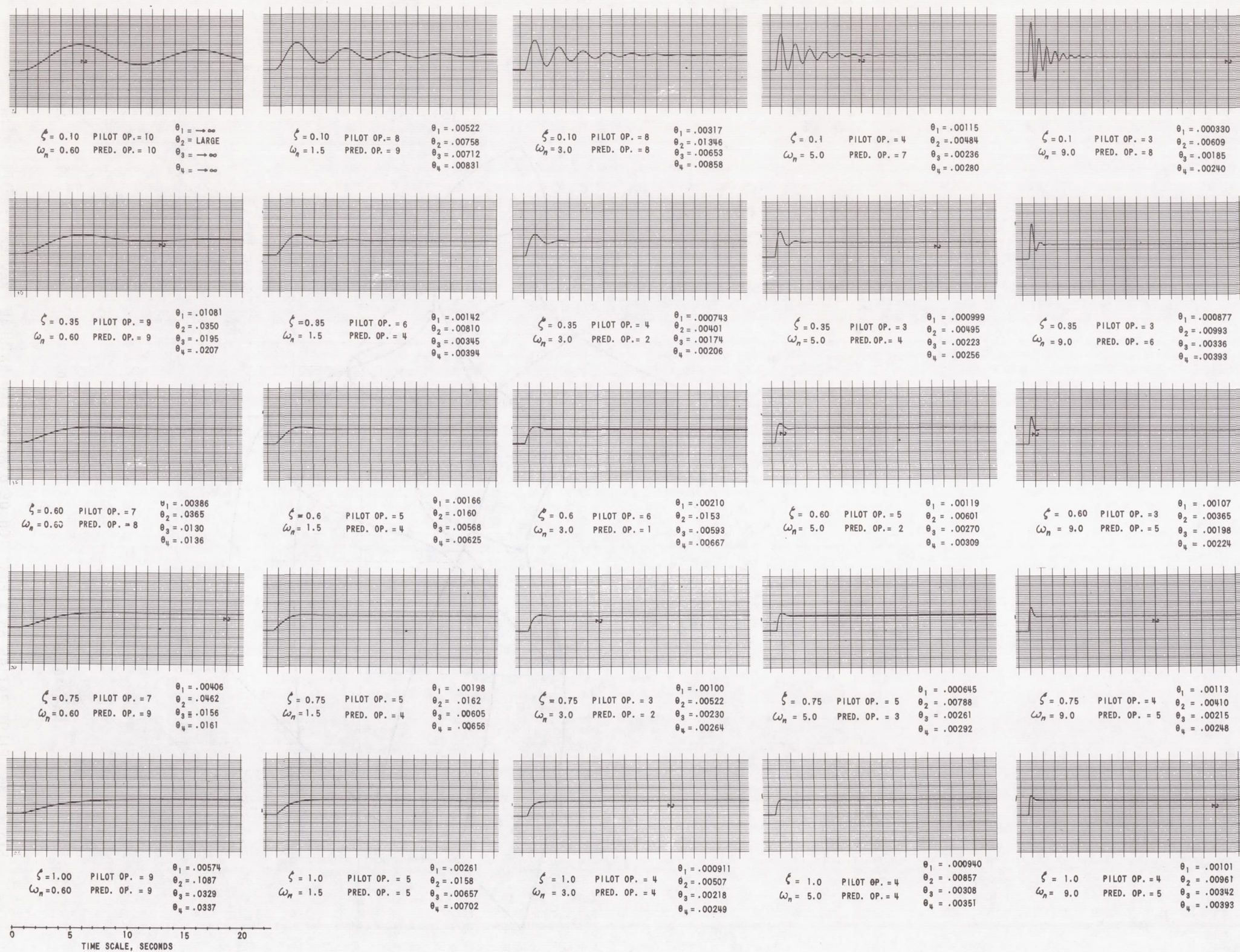


Figure 35
IMPULSE RESPONSE PLOTS, DAMPING AND NATURAL
FREQUENCIES, PREDICTED AND ACTUAL PILOT RATINGS,
AND PERFORMANCE MEASURE VALUES FOR EACH RUN
OF EXPERIMENT 2

For 25 data points, the Spearman rank correlation is significant at the $p = .05$ level for a value of .337, and it is significant at the $p = .01$ level for a value of .475. Therefore, the results of the analysis indicate that pilot opinion and man-machine system performance are very strongly related. θ_1 and θ_2 exhibit very high values of correlation in both Experiments 1 and 2. Once again there is a strong indication that the pilot could be used to find the area in which optimum man-machine performance would be obtained.

To determine more precisely the relationship between predicted pilot opinion and actual pilot opinion, a Spearman rank correlation was computed for the 25 opinion pairs. The result was

	Predicted Pilot Op. vs. Pilot Op.
Spearman rank Correlation	.4652

which is significant at the $p = .05$ level. Consequently, it may be concluded that while a relationship exists between the two functions, important errors do occur in predicting pilot opinion.

D. DISCUSSION

The two experiments performed to test the man-machine performance approach to synthesis have made a number of concepts clear. First, a pilot is capable of sufficiently steady behavior during long experimental simulations to make man-machine system optimization possible. Second, experimentally determined optima do not necessarily correspond closely to the optima predicted from handling qualities information. However, pilot opinion and man-machine performance are closely related.

Following these experiments, the pilot was asked how he arrived at his pilot opinion ratings. He said that his major concern was pitch error. The larger the pitch error in the experiment, the poorer would be his rating. He also considered other factors to a lesser extent: the amount of stick motion required (roughly equivalent to his own workload) and any

peculiarity such as sluggishness or low frequency oscillation. Thus, a criterion such as pitch angle error, θ_1 , or pitch angle error plus stick excursion, θ_3 , can be considered as the performance measure the pilot was using in making his evaluation.

PART III RELATED MATERIAL

10. MEASUREMENT OF THE HUMAN OPERATOR'S SELECTION OF A PERFORMANCE MEASURE

One of the themes that seems to run throughout this report and perhaps synthesis of manual aerospace control systems in general is that of determining the relationship between pilot opinion and man-machine system performance. In Part II it was found that these two quantities were closely related. To better understand this relationship, it will be necessary to determine by some direct method the performance measure by which the pilot controls a system.

In this chapter a new method of measuring the human operator's performance criterion will be presented. The method is based on a mathematical derivation that fits an error weighting criterion to the man-machine system signals. While this method must be considered as preliminary and will require further work before it is applied, it appears to hold considerable promise.

Let the class of performance measures the human operator uses be defined as

$$P \equiv \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f[e(t)] dt \quad (50)$$

where T_1 is the initial point,

T_2 is the final point in time over which the performance measure is to be determined

and $f[e]$ is the nonlinear zero memory gain function which represents the weighting of the error, $e(t)$. It is this nonlinear zero memory function that is considered to be the human operator's error criterion.

It is necessary that $f[e]$ be non-negative for all values of e so that certain values of error do not reduce the error measure. This non-negativity can be assured in the derivation by defining an auxiliary function as follows:

$$f[e] = W^*[e] \quad (51)$$

where $W[e]$ is real for all e . Note that this definition in no way limits the function $f[e]$ except for non-negativity.

The performance measure P may now be written as

$$\begin{aligned} P &= \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} W^2[e(t)] dt \\ &= \int_{e_L}^{e_u} W^2[e] p_E(e) de \end{aligned} \quad (52)$$

where $p_E(e)$ is the first-order probability density function and e_u is the maximum upper bound, and e_L is the minimum lower bound of the tracking error signal, $e(t)$, over the finite interval from T_1 to T_2 .*

If one attempts to minimize P directly, the result will be $P = 0$, since by allowing $W(e)$ to vanish identically over the interval, the integrand is zero. It is necessary to subject the function $W(e)$ to a constraint in order to obtain a meaningful answer. The choice of the constraint must be such that it does not bias the true performance criterion of the human operator. One choice might be the normalization of the area under the criterion function $f[e]$. However, generally speaking, the value of f becomes large as e becomes large in magnitude. Therefore, the integral of $f(e)$ will diverge.

The fact that $f[e]$ becomes large for large e can be used to determine a meaningful constraint. If the function $f[e]$ is inverted, then the extremes tend to zero. Accordingly, the area under the inverse function should be finite. Therefore, choose the constraint

$$\int_{-\infty}^{\infty} \frac{de}{f[e]} = \int_{-\infty}^{\infty} \frac{de}{W^2[e]} = 1.0 \quad (53)$$

which will normalize the solution obtained without biasing it. The problem is now in a form which is amenable to a Lagrange multiplier minimization.

* If $e(t)$ is a continuous function (which it is for all practical cases), then for a finite interval $T_1 \leq t \leq T_2$, the probability density function exists and is nonzero over the interval $e_L < e < e_u$. Therefore, the inverse of the probability density function exists over the interval $e_L < e < e_u$. This fact will be required in the subsequent derivation.

A simple form of the calculus of variations may be used to obtain the function, $f[e]$. The following derivation outlines the method of solution:

The Lagrange-multiplier measure is formed:

$$\theta = \int_{-\infty}^{\infty} \left[W^2(e) p_E(e) + \frac{\lambda}{W^2(e)} \right] de \quad (54)$$

The function $W(e)$ is subjected to a variation:

$$I(\alpha) = \int_{-\infty}^{\infty} \left\{ \left[W(e) + \alpha \eta(e) \right]^2 p_E(e) + \lambda \left[W(e) + \alpha \eta(e) \right]^{-2} \right\} de \quad (55)$$

The variation is then minimized:

$$\frac{\partial I(\alpha)}{\partial \alpha} \bigg|_{\alpha=0} = 0 \quad (56)$$

$$\int_{-\infty}^{\infty} \eta(e) \left[W(e) p_E(e) - \lambda W^{-3}(e) \right] de = 0 \quad (57)$$

The fundamental theorem of the calculus of variations is applied:

$$W(e) p_E(e) = \frac{\lambda}{W^3(e)} \quad (58)$$

$$f[e] = \frac{\sqrt{\lambda}}{\sqrt{p_E(e)}}$$

The Lagrange multiplier, λ , is determined:

$$\int_{-\infty}^{\infty} \frac{de}{f[e]} = \int_{-\infty}^{\infty} \frac{\sqrt{p_E(e)}}{\sqrt{\lambda}} de = 1.0 \quad (59)$$

$$\sqrt{\lambda} = \int_{-\infty}^{\infty} \sqrt{p_E(e)} de$$

The multiplier is substituted to obtain the final solution:

$$f[e] = \frac{\int_{-\infty}^{\infty} \sqrt{p_E(e)} de}{\sqrt{p_E(e)}} \quad (60)$$

It is found that, based upon this derivation, the human operator's criterion of error is inversely proportional to the square root of the probability density function of error. This is a very meaningful result, for it shows that the human operator tends to maintain the error signal at those amplitude values which he considers to have least weight.

To illustrate the effectiveness of this equation for determination of an error criterion, a hypothetical example is chosen. Suppose that one human operator is instructed simply to make the error as small as possible, and that a second human operator is instructed to make the error small, but never allow it to become negative. The probability densities of the errors for the two subjects might be similar to those shown in Figures 37a and 37b. Then, accordingly, the error criteria of the human operators would be those shown in Figures 38a and 38b.

It is seen that the error criteria appear to properly reflect the conditions of the experiment. The above approach to performance measure determination will apply equally well to the derivatives of the error signal. Therefore, performance measures for rate of change and acceleration of error may also be determined.

This approach to error criterion determination appears promising and can probably be generalized. Several topics should be investigated, in particular:

1. Constraints should be incorporated which are based upon the behavioral characteristics of the human operator instead of being based upon normalization.
2. Generalizations should be made for the simultaneous determination of more than one error criterion, or simultaneous determination of error criteria and constraint criteria.
3. Experimental study which verifies and applies the theory should be performed.

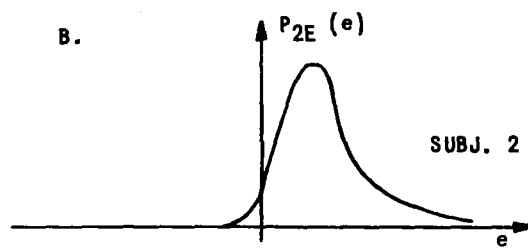
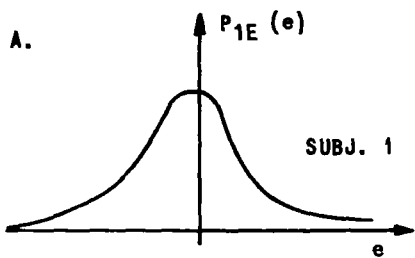


Figure 37 HYPOTHETICAL ERROR PROBABILITY DENSITIES FOR
SUBJECTS INSTRUCTED DIFFERENTLY

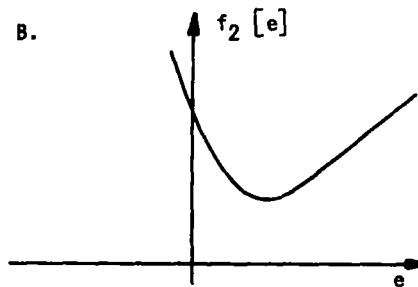
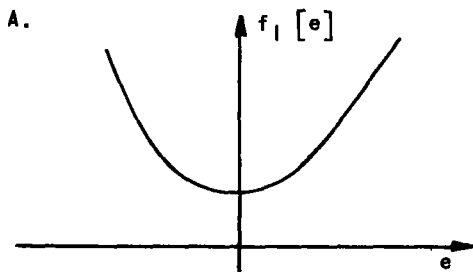


Figure 38 ERROR CRITERIA FOR SUBJECTS INSTRUCTED DIFFERENTLY

11. CONCLUSIONS AND RECOMMENDATIONS

A great deal has been learned in this initial study of synthesis methods for manual aerospace control systems. Both the programming approach and the man-machine performance approach to synthesis have been found to be feasible. In addition, a number of important new concepts have been discovered. The major conclusions drawn from the investigation will be briefly reviewed, and recommendations for future work will be made.

A. CONCLUSIONS

The programming approach to manual control system synthesis has clearly demonstrated the ability of programming techniques to move closed-loop poles or eigenvalues of a flight control system toward desired positions. Single regime, fixed-parameter multi-regime, and scheduled-parameter multi-regime synthesis procedures have been developed. These procedures and their corresponding digital computer programs have been evolved in a way which will allow wide application to flight-control problems. Furthermore, they form a solid foundation for more complex programs that could be developed later.

The digital computer programs for performing synthesis operate in a reasonably efficient manner. Because pole motion is generally a non-linear function of adjustable parameters, the optimization process must include a variety of contingencies to handle unusually shaped performance measure curves.

In practical flight control problems several of the state-variables are usually unmeasurable, dictating that only certain filtered feedback and compensation paths be used. Analytical techniques generally require feedback from all state-variables. The programming approach used in this research does not require that all state-variables be measurable. Moreover, limits may be specified for the adjustable parameters.

In the preliminary applications of the scheduled-parameter multi-regime synthesis procedure, it has been found that the adjustable parameters take on widely different values if they are allowed to vary from one regime to

another. This explains why the fixed-parameter multi-regime program was unable to reduce the performance measure value drastically. Since the optimum conditions were in conflict from one regime to another, the best that could be done was to choose some compromise value for each adjustable parameter. The result is a rather shallow optimum that does not lower the performance measure value by more than perhaps 50 percent.

For the examples studied, the adjustable parameters have been found to assume relatively small optimum values. This result is encouraging because it indicates that the various closed-loop flight control configurations can be implemented. Problems associated with saturation of electronic components and excitation of bending modes would probably not be severe.

Pilot ratings, taken from a man-machine simulation of the longitudinal fixed-parameter multi-regime flight control system for an SST, were moderately improved over the open-loop ratings. In still air, no important difference was found between open-loop and closed-loop configurations. However, in severe turbulence the closed-loop configurations were preferred by the pilot. Moderate improvement was all that was expected because the overall performance measure for the fixed-parameter, multi-regime synthesis was reduced by only 35 percent. A major finding of the simulation study was that the closed-loop flight control system decreased the sensitivity of the aircraft to turbulence.

The simulation study also yielded the unexpected result that the longitudinal SST dynamics possess reasonably good handling qualities without a flight control system. In the still-air experiment, the open-loop dynamics are rated nearly as good as the closed-loop dynamics. This result is surprising because the open-loop poles are not close to the positions specified by handling qualities information. The cause of these good ratings can only be conjectured, and further experimental work is required.

The lateral-directional manual flight control system of an SST can also be synthesized by means of the programming method. However, because the lateral-directional dynamics possess two control inputs, it is desirable to place the equations of motion in state-space form. The characteristic

equation may then be obtained by straightforward evaluation of a determinant. In the preliminary lateral-directional study, the closed-loop dynamics were obtained for a single-regime synthesis. It was found that the performance measure decreased from 675 to 198, indicating considerable improvement in the eigenvalue positions. However, single regime synthesis does not require a great deal of compromise; it might therefore be conjectured that substantial multi-regime improvement in eigenvalue positions will occur only if some of the parameters are scheduled.

Finally, in regard to the programming approach, it is feasible to develop search programs to find multiple minima. However, because of the dimensionality problem, the entire parameter space cannot be searched. Two programs were developed for multiple minima search: the first adjusts only one parameter at a time and produces plots of performance measure versus the parameter; the second adjusts all parameters simultaneously along a fixed gradient and produces a single plot of performance measure versus a designated lead parameter. When these programs were used to study the performance measure surface for longitudinal SST dynamics, it was found that the surfaces were usually well behaved for parameter values in the range where airplane-like responses are obtained. However, occasionally, a surface with more than one minimum was encountered.

The man-machine performance approach to manual control system synthesis appears promising from the initial tests that were performed. The approach may be particularly valuable when the number of adjustable parameters is not large, perhaps two or three, or when a new flight control problem is to be studied. Two different experiments were performed and the results obtained were similar. First, it was found that a pilot was capable of sufficiently uniform performance over a long sequence of runs that a man-machine performance surface as a function of adjustable parameters may be obtained. Randomizing the order in which configurations are presented will cause the effects of learning and fatigue to be randomly distributed over the

performance surface. Smoothing may then be introduced to reduce the manifestations of these effects. It was found that in each experiment a reasonably clear-cut optimum existed for man-machine performance. Thus, optimization of a man-machine system per se appears possible.

Perhaps the most important discovery made in the entire project involved the relationship between pilot rating and man-machine system performance. For completeness of data, it was decided to obtain a pilot rating along with the man-machine performance data for each experimental run. As an afterthought, a nonparametric Spearman rank correlation coefficient was computed between pilot rating and man-machine performance measure values. It was found that for a measure of pitch error, or pitch error and stick motion, extremely high correlation values were obtained. It may therefore be stated that there was almost a one to one correspondence between pilot rating and man-machine system performance. In other words, the pilot rated each aircraft configuration to a great extent on the basis of man-machine system performance.

An additional study was performed along with the second experiment to determine if a relationship exists between pilot opinion and predicted pilot opinion as determined from handling qualities information. It was found that even though there was a significant correlation ($p \leq .05$), predicted pilot opinion was often considerably in error. It may be concluded that in the experiments performed during this project, pilot rating and man-machine performance were closely related, whereas pilot rating and handling qualities information were only moderately related.

B. RECOMMENDATIONS FOR FUTURE WORK

While it is believed that this initial study has accomplished a great deal of work in manual control system synthesis, important new topics and extensions of those already investigated require further investigation. In this section, recommendations will be made for what is believed to be the most fruitful course of future investigation.

The programming approach to manual control system synthesis has been developed to the point where it could be applied to preliminary flight control system design. In the research study just completed, equations of motion for a typical SST were used. It is suggested that future work on manual control system synthesis be performed directly on the equations of motion of the United States SST, presently under development by the Boeing Company. Use of these equations would not be detrimental in any way to the manual control research effort, and perhaps, may result in information that would be of value to the designers of this aircraft.

Since it appears that fixed-parameter multi-regime synthesis as applied in the present study does not afford large reduction in performance measure values, further work should be done on the fixed-parameter approach. In the longitudinal dynamics case it is suggested that thrust be incorporated as a control input. Controlling thrust should allow the phugoid dynamics to be improved without large sacrifices in positioning of the short-period pole pair. Another suggestion for improving the capability of the fixed-parameter approach is to develop a post-optimization program for pulling pole positions within hard constraints. The idea here is to make the procedure directly compatible with handling qualities contour curves while at the same time not penalizing a given pole pair as long as it is within the constraint bounds. Thus, maximum effort can be applied to those pole pairs outside the constraint bounds.

Emphasis should also be placed on refining the scheduled-parameter, multi-regime synthesis program. The area where the most work is needed presently is in the scheduling process itself. Thus far, parameters that have been allowed to change with flight regime have not been constrained in any way. Consequently, their values are not simple functions of such regime-dependent parameters as airspeed, altitude, dynamic pressure, and gross weight. In any final configuration, it would be necessary to approximate the scheduled parameter values as functions of regime-dependent parameters. Rather than introduce this source of approximation, it would be better to constrain the scheduled parameters in the optimization process.

The computer output would then be usable directly in the flight control system design. To illustrate, suppose that a feedback parameter α_m is to be scheduled, but is to be composed of some linear combination of airspeed, altitude, and gross weight:

$$\alpha_m = a_m + b_m V_T + c_m h + d_m W \quad (61)$$

It would then be possible to adjust a_m , b_m , c_m , and d_m in a way that minimizes the value of the performance measure over all flight regimes. In other words, the parameters a_m , b_m , c_m , and d_m become the new adjustable parameters. The advantage of this procedure is that the corresponding synthesis is directly implementable because V_T and h can be measured and W can be entered by the crew. If it is found that the linear combination given above is inadequate, different regime-dependent parameters may be used or higher order terms may be introduced. The major complication in the digital program would be the increased number of adjustable parameters.

While pilot ratings are predominantly dependent on pole or eigenvalue positions, the zeros of the dynamics enter the ratings also. There are two ways in which the zeros might be adjusted. First, desired positions for zeros as well as poles might be chosen, and then a performance measure minimized which includes both zero position errors and pole position errors. Second, the zeros might be positioned by placing error criteria on the transient response. In other words, the performance measure might include errors between given and desired pole positions as well as errors between given and desired transient response curves. It must be remembered, however, that feedback does not usually change the positions of zeros. Thus, the forward loop compensators would play the major role in zero movement. It should also be noted that the zeros of a system change with the input-output signal pair, whereas the eigenvalues do not.

Turbulence sensitivity is a major concern in the development of any flight control system. This research study has shown that better pilot ratings were obtained in turbulence when the closed-loop longitudinal system was operating. It would be better, however, to account for turbulence in the synthesis procedure by minimizing sensitivity to gust inputs. Perhaps the inclusion of transient response testing and turbulence immunity can be combined.

Two other problems of importance are digital program efficiency and excitation of bending modes. It is believed that neither of these problems was severe in the present study. In future studies, it will be important to keep them in mind. The former will eventually limit the sophistication of the digital programs that can be developed and the latter will limit the degree of compensation that can be used in a flight control system.

In the lateral-directional synthesis problem a great deal of work remains. What sensors and compensators afford the greatest performance measure value reduction? How does one incorporate lateral-directional handling qualities information that is in a form other than desired eigenvalue positions? What performance measure component weighting should be used? These are some of the questions that could be answered by a thorough study. Actually, the lateral-directional synthesis problem would be ideally suited for testing the more advanced synthesis methods suggested in this section.

It is strongly suggested that the relation between pilot rating and man-machine performance be investigated further. The initial studies performed in this research effort indicate that pilot rating is best when measured man-machine system performance is best. A more thorough experimental study should be conducted with several human subjects. The objective of the experimental study should be to determine more precisely the relationship between pilot rating and man-machine system performance. The importance of such a relationship lies in the human's ability to determine the parameter ranges where an optimum in man-machine performance will occur.

A somewhat disquieting note in the present study has been that a configuration that is optimum from the viewpoint of handling qualities specifications may not result in the highest possible pilot rating. Evidence of this can be found in the inaccuracy of predicting pilot rating from system characteristics (Experiment 2, Chapter 9). The causes for this might be very complex; nevertheless, attention should be given to this matter in future studies. Hopefully, optima in handling qualities specifications, pilot ratings, and man-machine system performance could be brought into correspondence. If this can be achieved, then the two different synthesis procedures developed in the present research study would actually become a single procedure as seen from two different viewpoints.

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