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**DIELECTROPHORETIC CONTROL OF
PROPELLANT SLOSH IN LOW GRAVITY**

Final Report

**Covering the Period
January 5, 1966 to March 31, 1968**

Dynatech Report No. 769

by:

**Mathew Hurwitz
James R. Melcher
Advanced Systems Department**

Contract No. NAS 8-20553

Control No. 1-6-52-01028

Prepared for:

**National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama 35812**

March 31, 1968



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FOREWORD

This report covers the first phase of a program conducted by Dynatech Corporation, Cambridge, Massachusetts, under the NASA Contract NAS 8-20553 entitled "Research and Design of a Practical and Economical Dielectrophoretic System for the Control of Liquid Fuels under Low Gravity Environmental Conditions," initiated under the NASA PR No. 1-6-52-01028. Phase II of this program was a parametric design and optimization study of dielectrophoretic propellant control systems for orbital tankers and for lunar-mission vehicles. Phase III was an experimental study and demonstration of the safety of the dielectrophoretic system operating in liquid oxygen and liquid hydrogen propellants. Phases II and III have been presented under separate reports. Phase I, the subject of this report, was an analytical and experimental program to develop a dielectrophoretic baffle for the control of the fundamental slosh mode in propellant tanks under low gravity conditions.

Mr. Mathew Hurwitz was the principal investigator for Dynatech Corporation on Phase I of this program. The work was monitored by the Propulsion and Vehicle Engineering Laboratory, Marshall Space Flight Center, Huntsville, Alabama. Mr. Herman Beduerftig was Project Engineer.

We acknowledge the important contributions of Dr. J. R. Melcher and Mr. D. S. Guttman for several important experimental inputs to this program which they generated under NASA Grant NsG-368.

This report covers work performed from January 5, 1966 through March 31, 1968.

ABSTRACT

Analyses, experiments, and design evaluations were conducted to study the feasibility of using electrohydrodynamic (EHD) forces to stabilize a sloshing cryogenic propellant.

An electrode configuration was conceived which provides a significant stiffening influence on the liquid/vapor interface when located just below the sloshing surface. As the sloshing interface passes from the field-free region, outside the electrode array, to the region of strong electric field within the array, the EHD surface force "bangs" on abruptly. Experiments are described which demonstrate the influence of this EHD bang-bang phenomenon on modes of fluid oscillation. It is further demonstrated that liquids such as Freon 113 and liquid nitrogen can be suspended in a stable equilibrium over a gas in an adverse acceleration as great as 1-g. Experimental measurements of slosh frequency and interfacial stability are correlated with a theoretical model which includes the salient non-linear features of the dynamics. Experiments with ac and dc fields are reported. The use of dc fields is demonstrated to be potentially feasible for EHD propellant orientation systems.

A conceptual hardware design and weight analysis is presented for an EHD slosh baffle for the SIVB hydrogen tank.

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NOMENCLATURE

C	=	capacitance of analogue experiment
d	=	plate thickness
$\bar{E}$	=	electric field intensity
E_o	=	rms or dc electric field intensity between plane, parallel electrodes
F	=	$(\epsilon - \epsilon_o) E_o^2 / 2 \rho l$
$\bar{F}$	=	force density
f	=	frequency of oscillation (Hz/sec)
f_o	=	frequency of oscillation with no electric field
g	=	gravitational acceleration, also vehicle acceleration
g_o	=	vehicle acceleration prior to engine shut-off
g^*	=	acceleration of vehicle following engine cut-off
l	=	length of equivalent fluid pendulum
p	=	hydrostatic pressure
s	=	electrode spacing
T	=	period of oscillation
T_e	=	surface force density
T_1	=	1/4 of time slosh spends in bang-bang region
T_2	=	1/4 of time slosh spends in linear region
t	=	time
U	=	total potential energy
U_e	=	potential energy due to electric field intensity
U_g	=	potential energy due to gravity
$\bar{v}$	=	fluid velocity
γ	=	$2 \xi_m / s$ - slosh peak amplitude normalized to plate spacing
ϵ	=	permittivity of liquid
ϵ_o	=	permittivity of free space
$\epsilon(x, y, z, t)$	=	permittivity, a function of position x, y, z and time
λ	=	characteristic wavelength on interface
η	=	$E_o (\epsilon - \epsilon_o) / 2 \rho g s$

Nomenclature (cont.)

ξ	=	displacement of equivalent fluid pendulum
ξ_m	=	maximum average amplitude of slosh, and maximum deflection of equivalent pendulum
ξ_p	=	deflection at which potential has maximum value with g "adverse"
ρ	=	mass density of liquid
τ_e	=	total electric force per unit area acting on fluid pendulum
τ_g	=	gravity force per unit area acting on fluid pendulum
$\tau_{\max}$	=	maximum force per unit area on pendulum
ω_o	=	angular slosh frequency with no electric field

Section 1

INTRODUCTION

For some time it has been recognized that the storage of cryogenics for long-duration space missions may require some means for controlling the position of the liquid and vapor.¹ There are many classes of propellant control problems which must be dealt with: one, for example, is the suppression of slosh growth upon a sudden change or cut-off of engine thrust.² This program was conducted to show that electrohydrodynamic systems, which have been under serious development since 1961 as a means for propellant orientation,^{3,4,5} can also provide a significant influence on the dynamics of a sloshing surface. The non-linear, bang-bang stabilization phenomenon discussed here can be considered as an extreme case of the field gradient stabilization developed previously.⁶ It not only provides a practical engineering solution to an important class of propellant orientation problems, but makes possible a meaningful demonstration of EHD orientation in liquid-vapor systems (such as those involving cryogenics) in an earthbound laboratory environment.

In the design of dielectrophoretic liquid storage systems, it is natural to think in terms of replacing the effect of gravity with that of the electric field. Instead of the force density ρg , the electric field intensity $\bar{E}$ induces the force density

$$\bar{F} = \epsilon \nabla \left(\frac{1}{2} E^2 \right) \quad (1)$$

where ϵ is the liquid permittivity.

Although the force density written as Eq. (1) has the most obvious analogy to that due to gravity, it obscures the fundamental fact that the effect of the electric field on the incompressible fluid motions is completely determined by the fields at the free interface. This is true because forces which are purely the gradient of a pressure do not affect the incompressible dynamics of the liquid, and if the force density $-\nabla \frac{1}{2} \epsilon E^2$ is added to Eq. (1), the resulting dynamics will be unaltered, but the force density becomes^{7,8}

$$\bar{F} = - \frac{1}{2} E^2 \nabla \epsilon \quad (2)$$

In this form, it is clear that the electric polarization force density influences the liquid at the interface where ϵ undergoes an abrupt change, and therefore $\nabla\epsilon$ is not zero, as it is in the bulk. In some cases it is fruitful to think in terms of an analogy between dielectrophoretic forces and surface tension forces because, in a homogeneous liquid, the electric field produces only a surface force.

The "gravity-like" representation of Eq. (1) suggests that, to orient or control a liquid, the volume of interest should be filled with an electric field having as nearly as possible a constant E^2 gradient. By contrast, Eq. (2) suggests that the $\text{grad } E^2$ field could be confined to the region of the interface, since that is the only region where it will have any influence. Of course, if the interface can be anywhere in the tank initially, the two viewpoints do not differ. Thus, where total dielectrophoretic orientation is required, the field gradient stabilization approach⁶ would be adopted.*

There are classes of space missions, however, where slosh control or partial propellant orientation are required under conditions such that, at critical times, the interface position is essentially known. For example, a propellant tank may be accelerated into orbit with a programmed sequence of events. Then, as in the SIVB when the propulsion engines are shut down and low gravity conditions are established (by the ullage thrusters), the interface position is known. In such a situation it would be appropriate to design an electrohydrodynamic slosh baffle with fields confined to the region of the interface. This not only has the advantage of allowing the use of electrodes over a restricted volume of the total tank, but as will be shown here, makes possible the use of a greatly enhanced EHD "bang-bang" interaction with the fluid.

A significant spin-off from this research and development program is the discovery that dielectrophoretic propellant orientation with dc fields is potentially feasible. The experiments reported here show that liquids can be oriented against gases in the laboratory using the bang-bang mechanism, with the attendant advantage of being able to conduct earth-bound experiments under conditions that are conservative compared to those expected in space. In the past, it has been necessary to simulate liquid cryogenics oriented against their vapors (for example, by using liquid-liquid systems). Because of the many differences in the electromechanical properties of these two

*A review of related work concerned with dielectrophoretic orientation is given in Ref. 6.

two-phase systems, it has not been possible until now to obtain information on such important factors as the reliability of dc fields with cryogens. In liquid-liquid systems, dc fields invariably produce instability;^{9,10} however, the experiments conducted under this program have shown that a liquid/vapor interface can be controlled with dc fields without the incipience of catastrophic free-charge instabilities.

Section 2

TECHNICAL DISCUSSION

2.1 Bang-Bang Electrohydrodynamics

It follows from Eq. (2) that an interface between a liquid and its vapor is subject to the surface force density

$$T_e = \frac{1}{2} (\epsilon - \epsilon_0) E^2 \quad (3)$$

in a direction normal to the surface acting from the liquid to the vapor.

Figure 1 shows a typical electrode arrangement that can be used to suppress the amplitude of slosh. In this example, g is taken as acting in a direction that is favorable to the orientation (positive g), as would be the case, for example, at engine cut-off when the SIVB vehicle is injected into orbit. Sloshing occurs with the equilibrium position of the interface essentially coincident with the upper edges of the electrodes. These electrodes alternately have opposite potentials, so that between adjacent electrodes having the spacing, s , there is an essentially uniform electric field intensity, E_0 , independent of whether or not the liquid occupies that region.

As the liquid sloshes in the mode shown in Fig. 1, the interface on one side leaves the field region between the plates, while on the other side it enters the field region. At the instant shown in Fig. 1, the interface at (a) is essentially out of the electric field and free of any electric surface force, while that at (b) experiences the surface force given by Eq. (3). Note that this surface force tends to return the interface to the equilibrium position, and is constant so long as the depressed interface at (b) is between the electrodes. For surface displacements large compared to the plate spacing, s , the interface experiences an electric surface force $(\epsilon - \epsilon_0)E_0^2/2$ that either "bangs" full on, and tends to return it to equilibrium, or "bangs" full off.

The bang-bang interaction is at the opposite extreme from the case considered in Reference 6, in that gradients in E^2 are distributed over an interval of surface deflections which is small, rather than large, compared to possible excursions of interest.

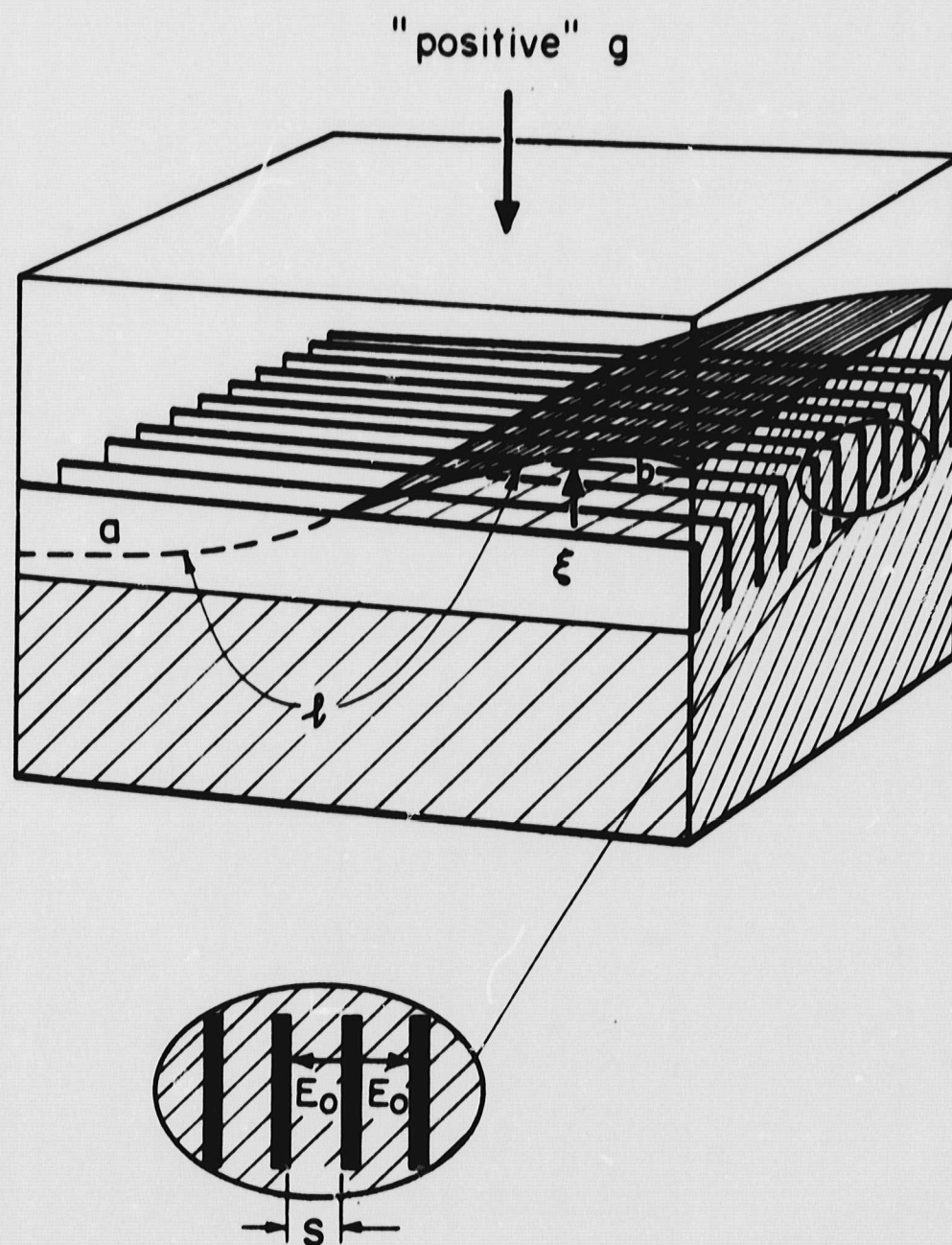


Fig. 1 Example of a dielectrophoretic slosh baffle that takes advantage of the bang-bang effect.

The slosh suppression of Fig. 1 illustrates the first of two aspects of this problem to be considered. The second is illustrated by Fig. 2, where it is incidental that the electrodes take on a coaxial rather than parallel plate configuration. The important point is that g acts adversely, or so as to disorient the liquid, and in addition to the question of slosh dynamics, there is the question of stability.

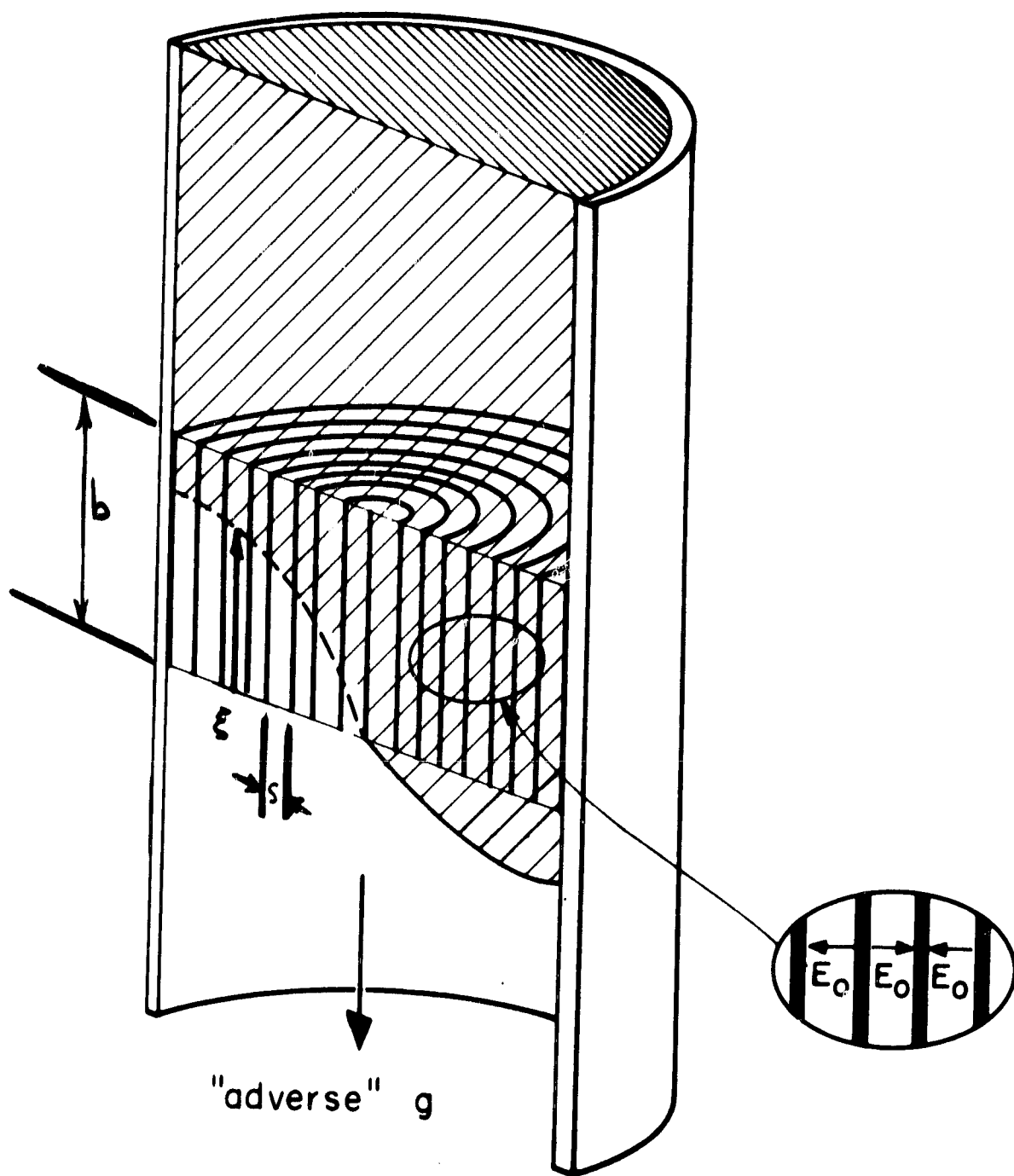


Fig. 2. Example of bang-bang orientation in the face of an adverse acceleration g

2.2 The Theoretical Model

The bang-bang interaction is inherently non-linear, and an exact analysis of the fluid motions would not only be extremely complex, but of little use in the engineering design of an orientation system. In Reference 6 it was shown that a simple pendulum model for the liquid system gave a surprisingly complete picture of the essential dynamics. It will be found that this is the case here as well.

The pendulum model for the bang-bang interaction is shown in Fig. 3, where ξ is an equivalent average of the slosh amplitude and l is the length of an equivalent pendulum which would have the same natural frequency as the system of Fig. 1, say, in the absence of the electric field. The length l would of course be different for different tank geometries and for different modes of slosh.¹¹

The equilibrium plane of the interface (the upper edge of each plane, parallel electrode) is at $x = 0$. Although the pendulum model of Fig. 3 shows only interfaces between two pairs of parallel electrodes, it is important to keep in view the fact that interfacial displacements of interest have characteristic wavelengths λ in the plane of the interface that are long compared to the interelectrode spacing, s . (The liquids wet the electrodes.) The effects of the electric field on the dynamics are determined by the degree to which the electric surface traction is altered as the liquid is displaced. There are two contributions to this change, one from the change in imposed electric field at the interface due to its change in position, and the second from the change in the electric field brought about by the surface deformation itself. It can be shown that these latter "self-field" effects are ignorable if:

$$\left| \frac{\epsilon - \epsilon_0}{\epsilon + \epsilon_0} \right| \left(\frac{s}{\lambda} \right) \ll 1 \quad (4)$$

In the dynamics of interest here, either slosh suppression or stability, it is the longest wavelengths that are the most dangerous, hence this approximation is justified. Furthermore, because $\lambda \gg s$, it is appropriate to ignore the detailed nature of the fringing field at the upper edges of the electrodes.



According to Eq. (2), there is no electric force in the bulk of the liquid, and so long as attention is confined to the bulk, the usual dynamic form of Bernoulli's equation is applicable.

$$\int_b^d \frac{\partial}{\partial t} \rho \bar{v} \cdot d\bar{l} + [p + \rho g x + \frac{1}{2} \rho v^2]_b^d = 0 \quad (5)$$

The limits of integration through the liquids are just below the interfaces, as shown in Fig. 3. The first term is approximated by the product of a pendulum length l and the velocity $\partial \xi / \partial t$, and Eq. (5) becomes:

$$\rho l \frac{d^2 \xi}{dt^2} = \tau_g + p_b - p_d \quad (6)$$

where $\tau_g = - \partial U_g / \partial \xi$ with $U_g = \rho g \xi^2$ in the case where g is positive. The same procedure shows that the pressures at a and c just above the interfaces are equal, because the gas in the upper region is of negligible density. (Note that if the force density of Eq. (1) is used, this is not true.) For convenience, p_a and p_c are taken as zero, so that p_d and p_b are respectively, the negatives of the electrical surface forces acting upward on the right and left interfaces, as given by Eq. (3). These subtract to give the total surface force $\tau_e = - p_d + p_b$ shown in Fig. 4 as a function of ξ .

The dependence on ξ of the total electrical surface force acting on the pendulum has been divided into two parts. There is a linear range $-\frac{s}{2} < \xi < \frac{s}{2}$ where the interface is still in the fringing fields, and

$$\tau_e = - (\epsilon - \epsilon_0) E_0^2 \xi / s \quad (7)$$

$$U_e = - \frac{1}{2} (\epsilon - \epsilon_0) E_0^2 s \left(\frac{\xi}{s} \right)^2 \quad (8)$$

where U_e is a potential function defined such that $\tau_e = - \partial U_e / \partial \xi$. Beyond the linear region, there is the bang-bang region $|\xi| > \frac{s}{2}$ where

$$\tau_e = - (\epsilon - \epsilon_o) E_o^2 / 2 \quad (9)$$

$$U_e = (\epsilon - \epsilon_o) E_o^2 s \left(\frac{\xi}{2s} - \frac{1}{8} \right) \quad (10)$$

The extent of the linear region assumed here is determined by capacitance measurements as described in Appendix A.

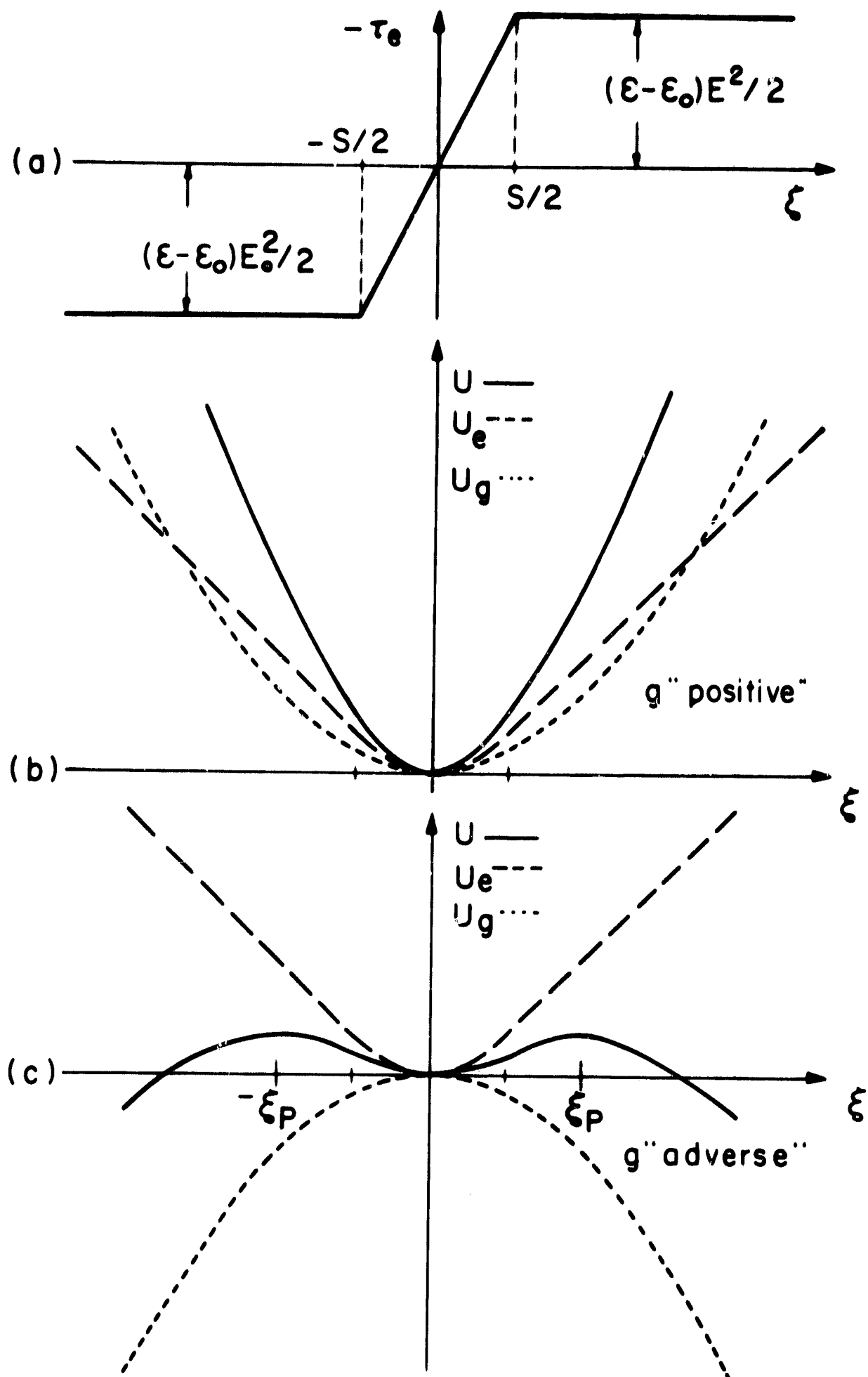


Fig. 4

- a) Total electrical surface force acting on pendulum of Fig. 3.
- b) Electric, gravitational, and total potential for positive acceleration
- c) as in (b), but with g adverse

2.3 Slosh Dynamics

The piecewise continuous potential U_e , as well as U_g with g positive and the total potential $U = U_e + U_g$ are sketched in Fig. 4b. This simple model makes it possible to relate the peak slosh amplitude ξ_m to the parameters of the electro-hydrodynamic baffle and hydrodynamic system. In the linear range, the pendulum equation becomes

$$\frac{d^2\xi}{dt^2} + \omega_o^2 \left(1 + \frac{2F}{3\omega_o^2} \right) \xi = 0 \quad (11)$$

where

$$F = (\epsilon - \epsilon_o) E_o^2 / 2\rho l \text{ and } \omega_o^2 = 2g/l$$

while in the bang-bang range

$$\frac{d^2\xi}{dt^2} + \omega_o^2 \xi = -F \quad (12)$$

From Eq. (11) it follows that, so long as the peak amplitude ξ_m is less than $s/2$, the slosh frequency is

$$\frac{f}{f_o} = \sqrt{1 + \eta^2} \quad (13)$$

where

$$f_o = \omega_o / 2\pi \text{ and } \eta = E_o \sqrt{(\epsilon - \epsilon_o) / 2\rho g s}$$

To determine the slosh frequency when the peak deflection is in the bang-bang range, T_1 and T_2 are respectively defined as 1/4 of the times required for the fluid pendulum to traverse the bang-bang and linear regions as it executes one cycle. Thus the period of oscillation is

$$T = 4 (T_1 + T_2) \quad (14)$$

To compute T_1 , assume that $\xi = \xi_m$ when $t = 0$. By definition of ξ_m , $d\xi/dt(0) = 0$ and from Eq. (12),

$$\xi = \left(\xi_m + \frac{F}{\omega_o^2} \right) \cos \omega_o t - \frac{F}{\omega_o^2} \quad (15)$$

Then, since $\xi = s/2$ when $t = T_1$, it follows that

$$\omega_o T_1 = \cos^{-1} [(\eta^2 + 1)/(\gamma + \eta^2)] \quad (16)$$

where $\gamma = 2\xi_m/s$

To compute T_2 , Eq. (11) is solved, given that when $t = 0$, $\xi = s/2$ and $d\xi/dt$ has the value given by Eq. (15) when $t = T_1$ (the time origin is shifted by T_1)

$$\xi = - \left(\xi_m + \frac{F}{\omega_o^2} \right) \frac{\sin \omega_o T_1}{\sqrt{1 + \eta^2}} \sin \omega_o \sqrt{1 + \eta^2} t + \frac{s}{2} \cos \omega_o \sqrt{1 + \eta^2} t \quad (17)$$

Then, when $t = T_2$, $\xi = 0$ and T_2 must satisfy the equation:

$$\frac{-(\gamma + \eta^2)}{\sqrt{1 + \eta^2}} \sin \omega_o T_1 \sin \omega_o \sqrt{1 + \eta^2} T_2 + \cos \omega_o \sqrt{1 + \eta^2} T_2 = 0 \quad (18)$$

Finally, Eq. (16) is used to eliminate T_1 from Eq. (18) and an expression is obtained for T_2

$$\omega_o T_2 = \frac{1}{\sqrt{1 + \eta^2}} \tan^{-1} \left\{ (1 + \eta^2) / [(\gamma + \eta^2)^2 - (\eta^2 + 1)^2] \right\}^{\frac{1}{2}} \quad (19)$$

Given the normalized electric field intensity η and normalized slosh amplitude γ , Eqs. (16) and (19) provide $\omega_o T_2$ and $\omega_o T_1$, which in turn determine the slosh

frequency f normalized to the frequency in the absence of the electric field.

$$\frac{f}{f_0} = \pi/2 (\omega_0 T_1 + \omega_0 T_2) \quad (20)$$

The slosh frequency is shown in Fig. 5. Note that, according to the piecewise continuous potential model, Eq. (20) is valid for $\gamma > 1$. For $\gamma < 1$, f/f_0 is found from Eq. (13) and is independent of γ , as shown in Fig. 5.

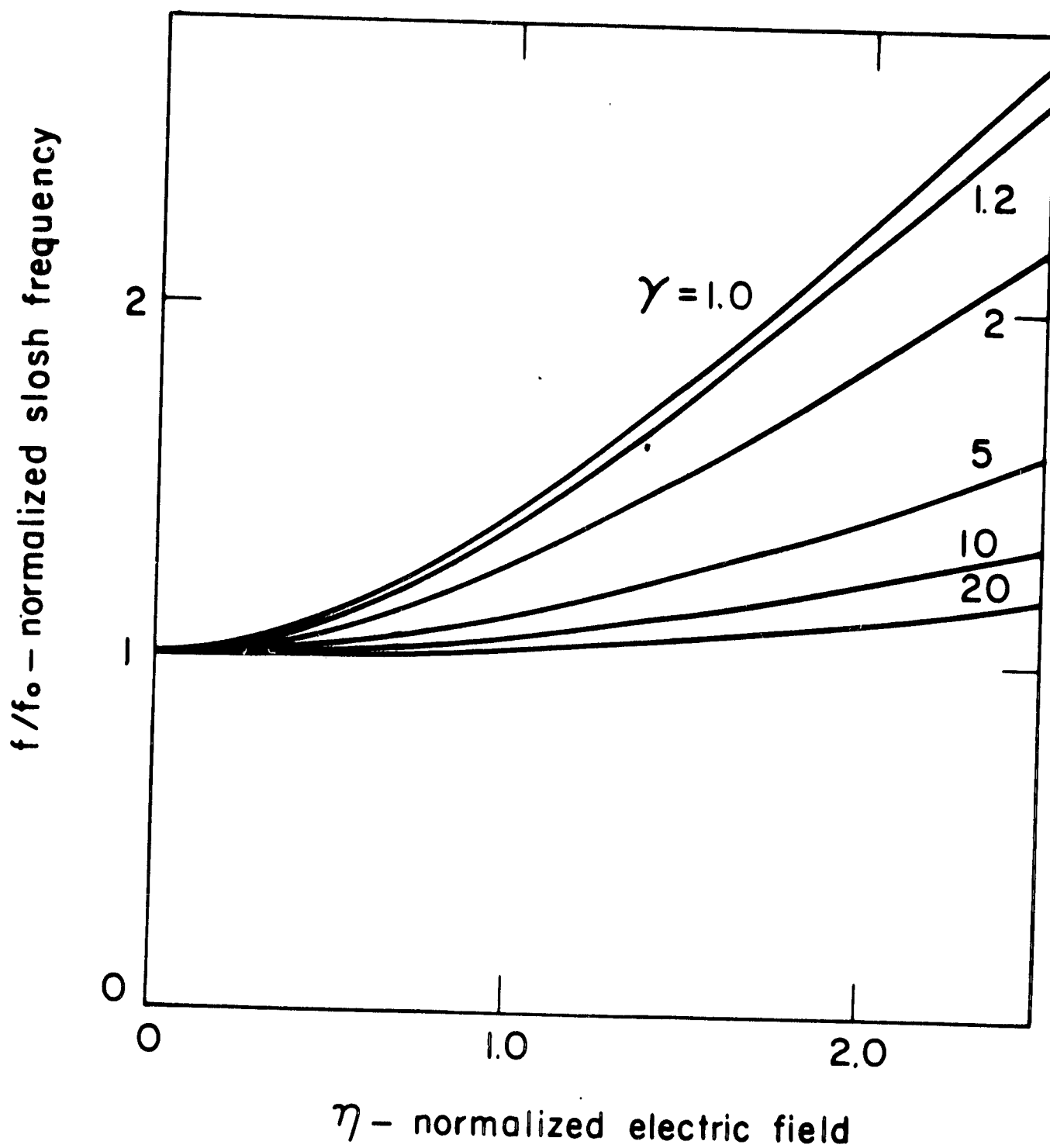


Fig. 5 Slosh frequency f normalized to frequency f_0 in the absence of electric field as a function of the normalized electric field intensity. γ is the slosh amplitude normalized to half of the interelectrode spacing.

2.4 Stability

Stability is of major interest when the bang-bang concept is applied to orientation systems assuming the configuration illustrated in Fig. 2. Here, the fluid is in an unstable equilibrium in the absence of the electric field, a situation equivalent to subjecting the pendulum model of Fig. 3 to an adverse acceleration g , as shown. Thus, g in Eqs. (11) and (12) is replaced by $-g$, and the potentials have the dependence on ξ sketched in Fig. 4c.

For a peak excursion ξ_m that does not exceed $s/2$, the point of incipient instability follows from Eq. (11) ($g \rightarrow -g$) as:

$$\eta = \eta^* = 1 ; \quad \gamma < 1 \quad (21)$$

where again $\eta = E_0 \sqrt{(\epsilon - \epsilon_0)/2\rho g s}$ and η^* is the critical normalized E_0 . That is, if the normalized electric field intensity does not exceed unity, even infinitesimal oscillations are unstable.

If, for the adverse g case, the amplitude of the excursion ξ_m exceeds $s/2$ ($\gamma > 1$), the equilibrium is unstable for amplitudes ξ_m that exceed the deflection ξ_p as shown in Fig. 4c. Note, ξ_p is the deflection where the potential reaches its maximum value. Thus, in the bang-bang range of peak slosh amplitudes, the equilibrium is stable if

$$\eta < \eta^* = \sqrt{\gamma} \quad (22)$$

The stability regimes, as a function of the normalized slosh amplitude, are summarized in Fig. 6. As might be expected on physical grounds, a greater electric field is required to stabilize the larger slosh amplitudes. Of course, the piecewise continuous marginal stability curve is a consequence of the piecewise continuous potential model. In actuality, it would be expected that the curve would be smooth in the region of transition near $\gamma = 1$.

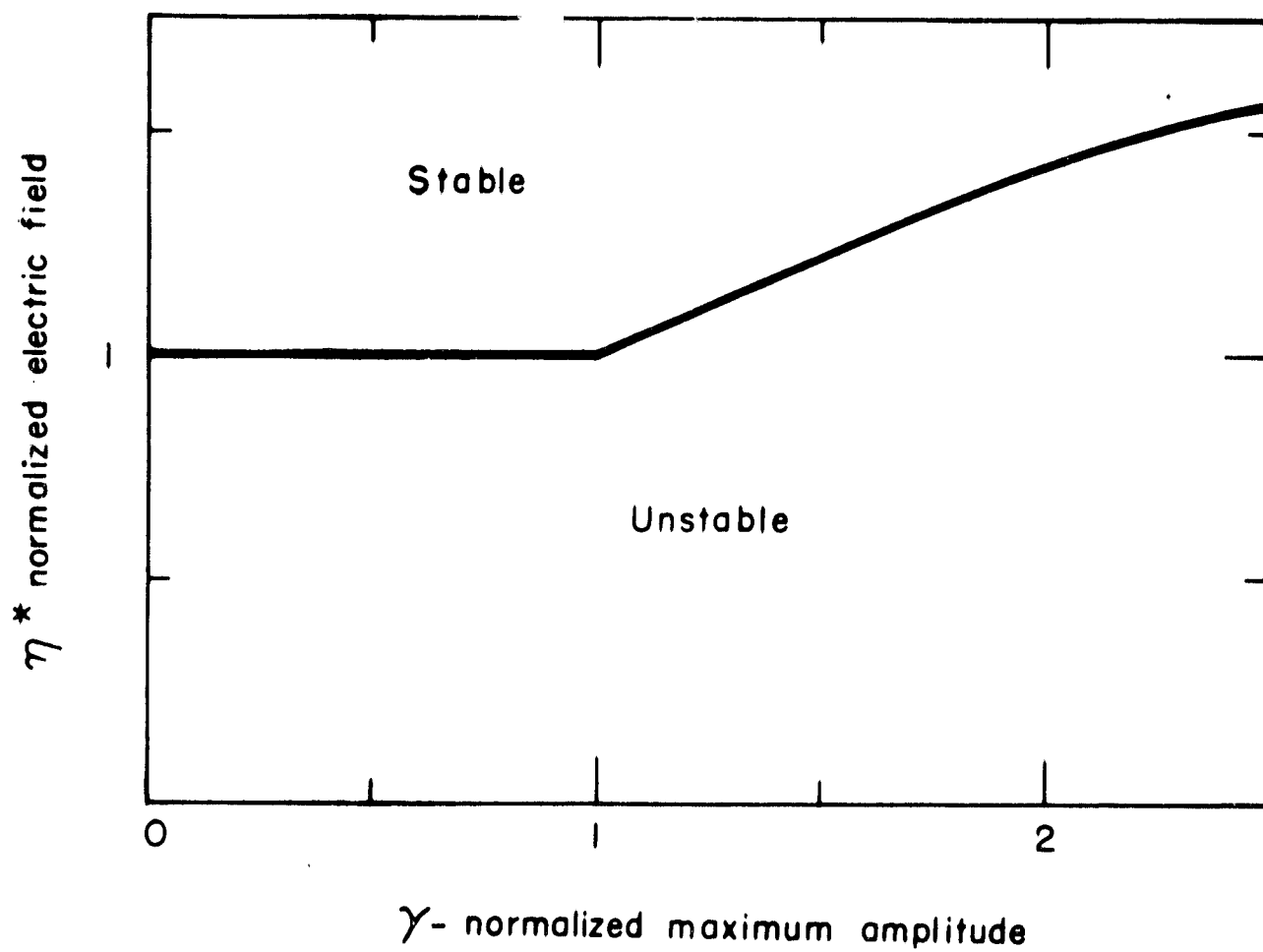


Fig. 6 Critical normalized electric field intensity for stable slosh, as a function of the maximum amplitude normalized to the half interplate spacing

2.5 Experiments: AC Fields

It is useful to obtain experimental conditions wherein the polarization force density of Eq. (2) is not dominated by free charge effects. Fields of alternating polarity, having a frequency much greater than the reciprocal relaxation time, can be used to avoid effects from free charges that accrue in the fluid through relaxation (conduction) phenomena. In the experiments to be described in this section, the electric field had a frequency of 400 Hz., a value considerably in excess of that required to avoid relaxation effects in the fluids used. This relatively high frequency was used because with 60 Hz. fields, the hydrodynamic response of the liquid-gas interface to the 120 Hz. component of the force was significant, as evidenced by short wavelength vibrations of the interface. This dynamic response, characterized by a frequency twice that of the imposed ac fields, is not of interest here. Hence, the 400 Hz. fields were used both to avoid charge relaxation effects and to eliminate fluid interactions involving the second harmonic of the imposed fields. In computing the electrical force, the ac field is considered as a dc field having the same rms value.

Two classes of experiment are of interest: the first to study slosh modes in a positive g , and the second to study stability in an adverse g , and the second to study stability in an adverse g , as exemplified by the situations of Figs. 1 and 2. Measurements of the slosh dynamics were obtained from a Plexiglas tank of rectangular cross-section (7.5 x 12.5 cm) having the electrode geometry of Fig. 1. The electrodes were 1/8 inch thick brass plates with a 2.2 cm vertical height and 0.38 cm. spacing.

The objective of the slosh experiments was to see if the simple predictions summarized in Fig. 5 are meaningful in an engineering context. Thus, measurements were made of the lowest slosh mode resonance frequency (the natural frequency predicted by the potential well mode) as a function of applied electric field and slosh amplitude.

In the dynamic experiments it was essential to retain a relatively high Q for the resonance in spite of the damping effect of the electrodes. Hence, a low viscosity

fluid, Freon 113, was used and horizontal vibrations of the tank in the direction of the electrodes were used to excite the mode shown in Fig. 7a. In this picture, the fluid is seen at its peak amplitude, essentially as sketched in Fig. 1, with no applied electric field. The effect of applying 14 KV between the plates is evident in Fig. 7b, where all other conditions of Fig. 7a are retained and the picture was taken with the amplitude peaking to the left. The original slosh amplitude is almost completely attenuated.

If the frequency of excitation is raised, keeping the applied field and excitation the same, the slosh amplitude is returned to its original value, although somewhat revised in shape, as shown in Fig. 7c. Thus, the electric field had a stiffening effect on the interface that was on the same order as that produced by a normal (one g) gravitational acceleration. It is important to recognize that the frequency shift, rather than the slosh attenuation at a given frequency, is the meaningful measure of the degree to which the field prevents slosh amplification in a space application as g is suddenly reduced.

The discrete pendulum model, of course, can describe only the average of the interface motions. Even so, the simple model gives a remarkably quantitative prediction of the dynamics. It is reasonable to define peak deflection of the equivalent pendulum as $1/\sqrt{2}$ that of the slosh amplitude. The measured frequency shifts for small and large amplitude slosh are compared to the predictions in Fig. 8. Note that $\gamma = 3.5$ corresponds to a peak amplitude that is still less than the vertical height of the electrodes.

The pendulum model ignores the continuum nature of the bang-bang interaction. Thus, it does not account for changes in the slosh mode shape, even as observed over the limited range of variables in a one-g experiment (compare Figs. 7a and 7c). Although, in the experiments discussed here, the effects of gravity and the electric field were clearly demonstrated to be on the same order, there were no situations in which the effect of gravity was ignorable compared to that of the electric field. It must be recognized that in the low-gravity condition, the predictions of the simple pendulum model might not be in as close agreement with experiment as is indicated by Fig. 8. This is true because the continuum aspects of the bang-bang dynamics

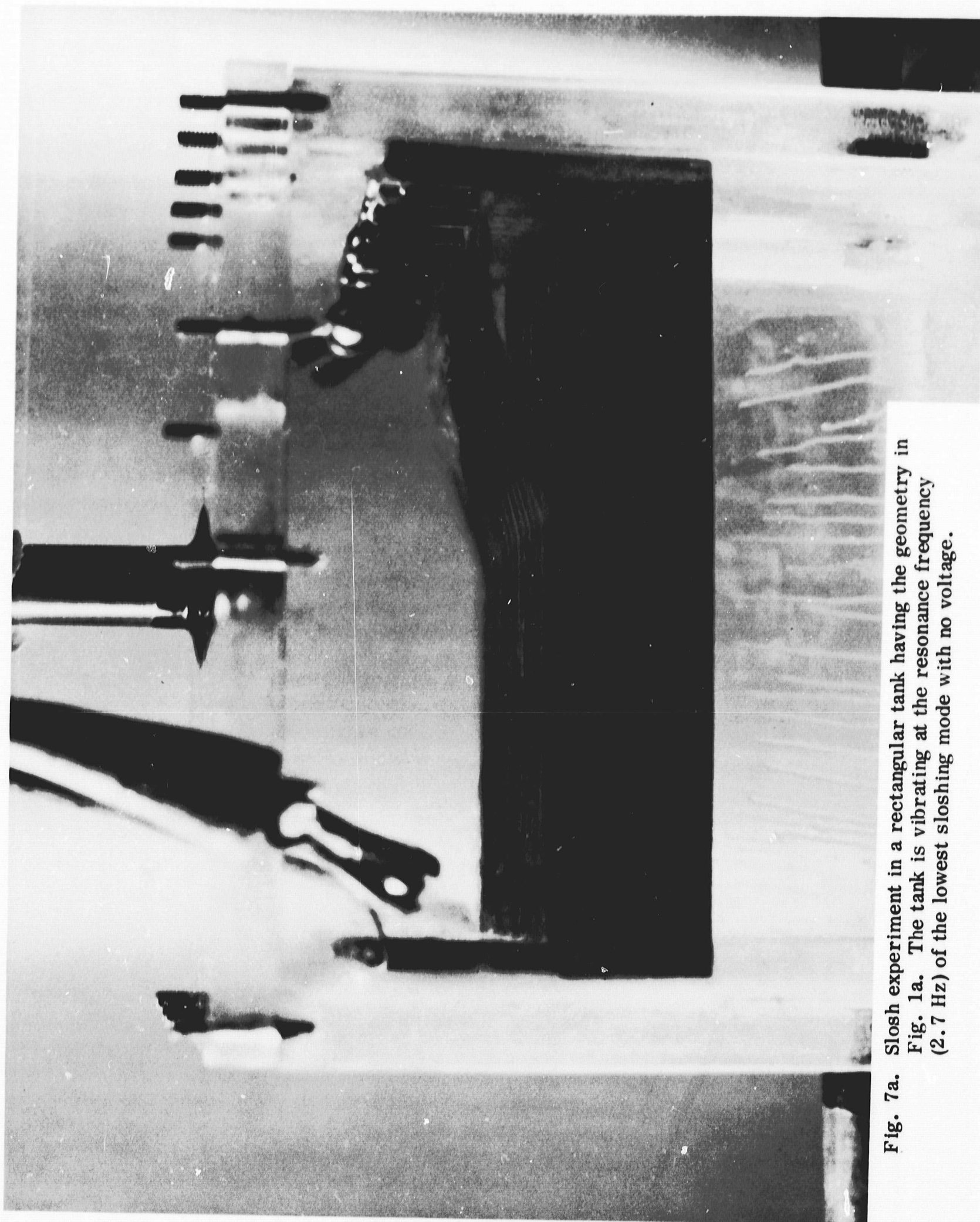


Fig. 7a. Slosh experiment in a rectangular tank having the geometry in Fig. 1a. The tank is vibrating at the resonance frequency (2.7 Hz) of the lowest sloshing mode with no voltage.

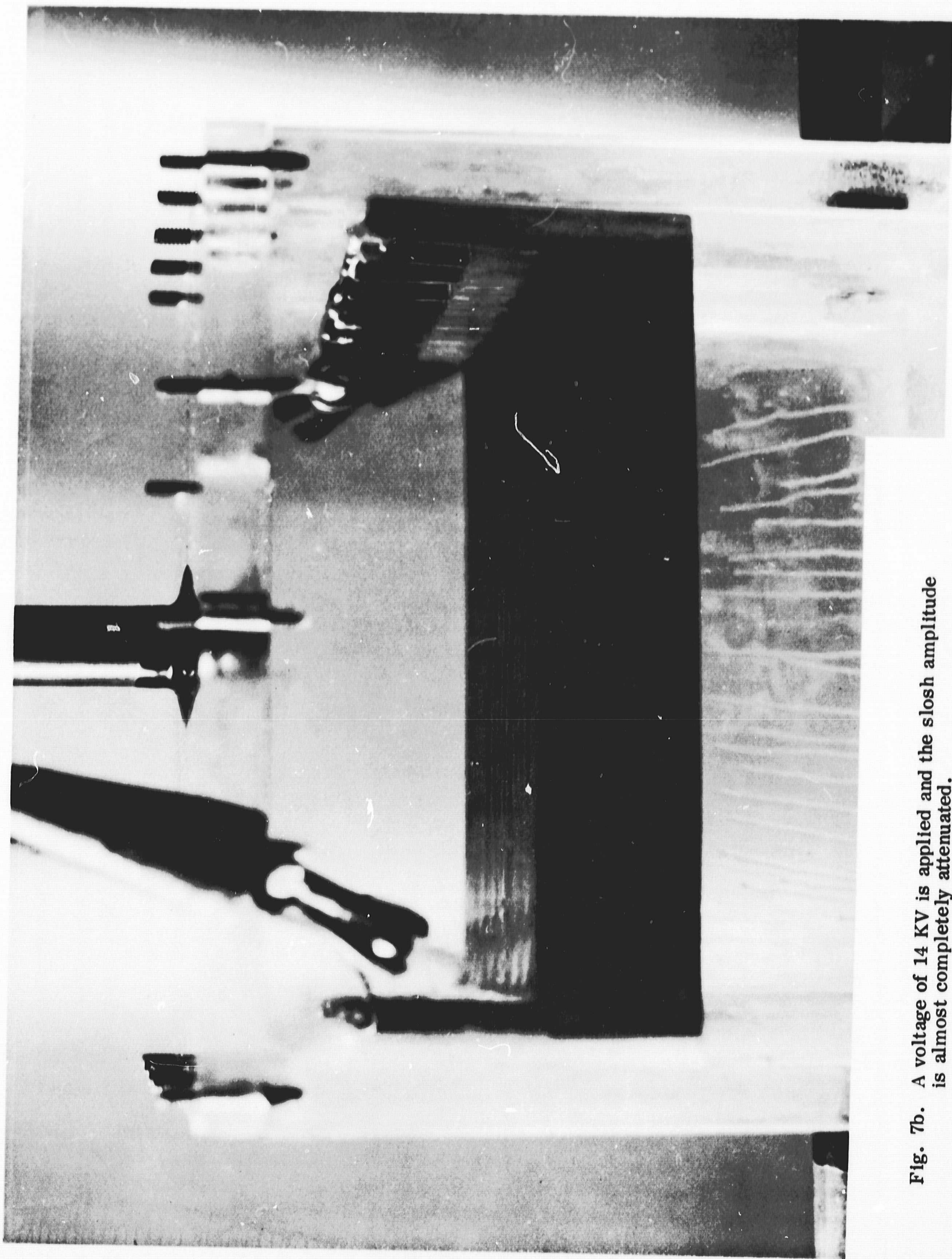


Fig. 7b. A voltage of 14 KV is applied and the slosh amplitude is almost completely attenuated.

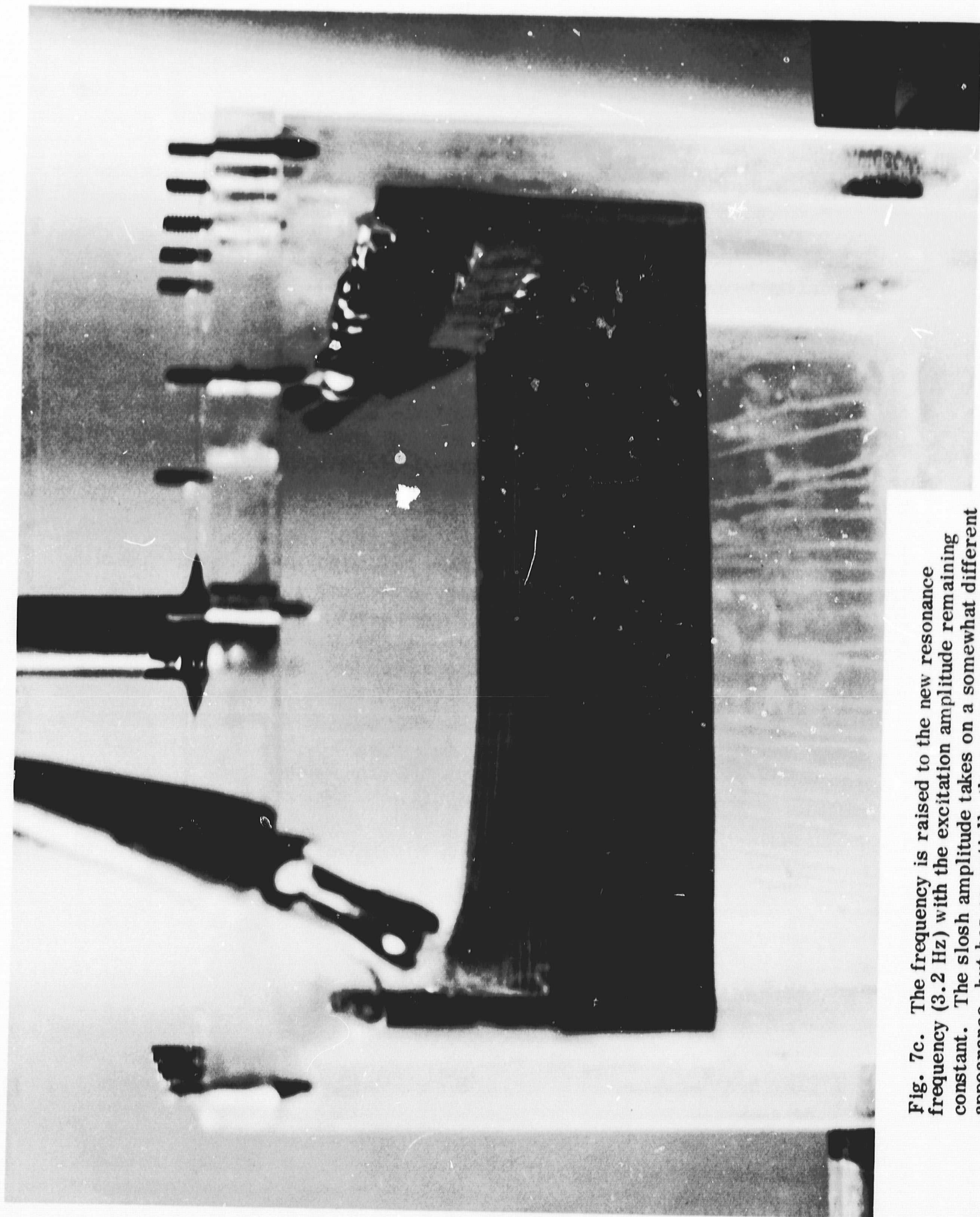


Fig. 7c. The frequency is raised to the new resonance frequency (3.2 Hz) with the excitation amplitude remaining constant. The slosh amplitude takes on a somewhat different appearance, but has essentially the same amplitude as in Fig. 7a.

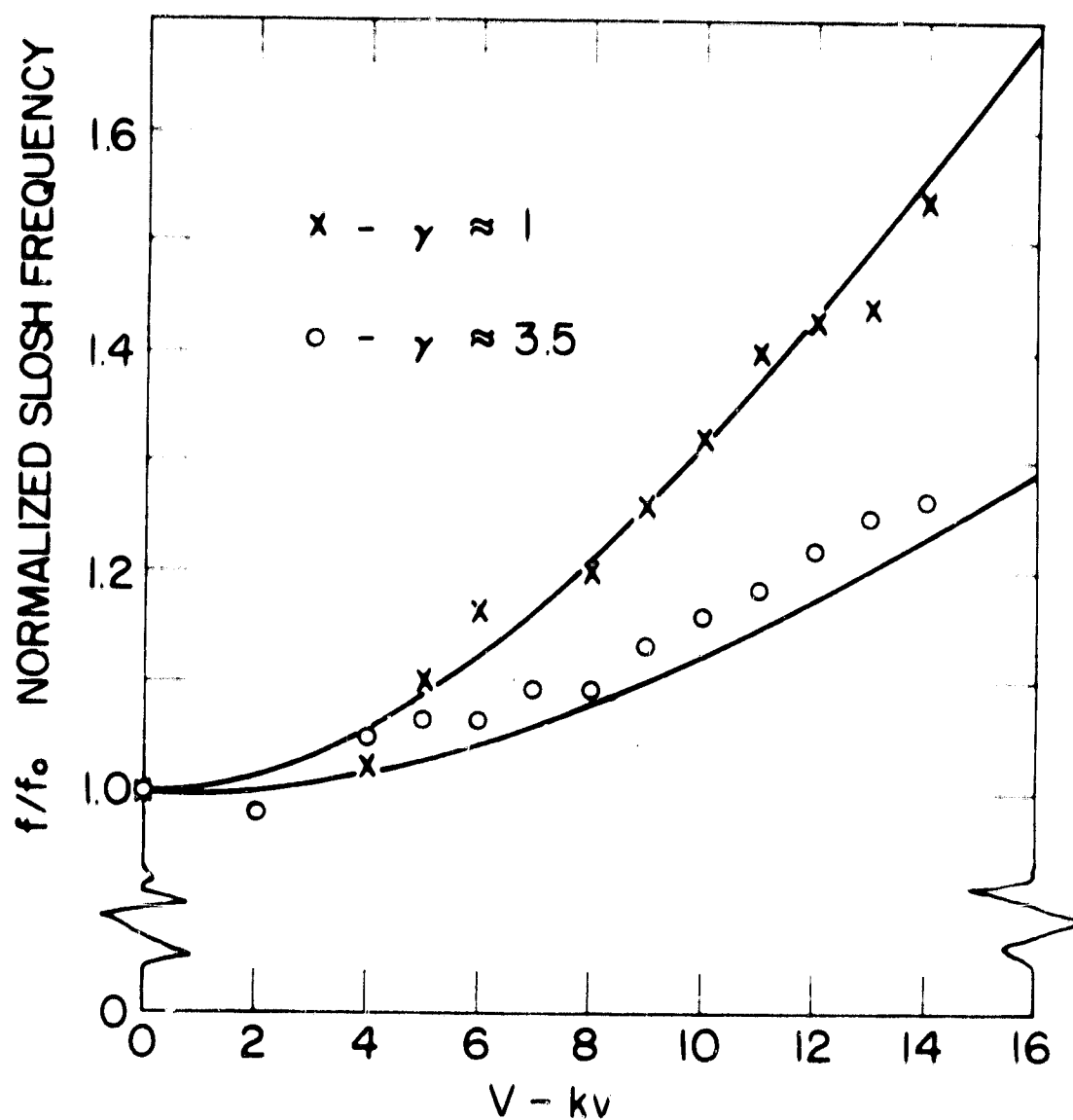


Fig. 8 Experimentally observed frequency shift as a function of voltage for the experiment of Fig. 7. The fluid is Freon 113. The solid curves are given by the theoretical model and the correlation clearly shows the effect of the amplitude.

would then be dominant in determining the spatial distribution of the interface deflections.

A second class of ac experiments had the objective of investigating the stability predictions of Eq. (21) in an adverse acceleration. The configuration sketched in Fig. 2 is shown experimentally in Fig. 9. A glass cylinder of circular cross-section, 6 cm. inside diameter, was separated into two sections by coaxial electrodes (inter-electrode spacing $s = 0.32$ cm.). The top of the cylinder was closed off by a rubber stopper and the bottom placed in a pan of dielectric liquid (in the picture, corn oil). Thus, with the voltage off the liquid could be sucked into the cylinder so that both the regions above and below the electrodes were filled. Then the voltage was applied to the electrodes and the liquid allowed to drain from the lower section of the cylinder by bubbling Freon gas into the lower section, to avoid breakdown. Because of the bang-bang stabilization of the interface at the lower edges of the electrodes, the bubbles were stopped at the electrodes when the field was applied, and the liquid remained in the upper part of the cylinder as shown in Fig. 9 (where the corn oil was dyed for photographic purposes). A closeup view of the stabilized interface is shown in Fig. 10a, where, incidentally, it can be seen that the electrodes were wetted by the fluid. When the voltage was reduced below the critical value predicted by Eq. (21), the configuration was unstable, and the liquid fell from the upper section of the cylinder. The view of Fig. 10b was taken shortly after incipience of instability. If the field was reapplied, after the fluid began to fall through the electrodes, the motion could be stopped and the interface returned to stable equilibrium.

It is important to distinguish between stabilizing the interface and supporting a column of liquid. The idea that the electric field supports the column of liquid deserves careful consideration in any case, since the force density of Eq. (2) acts downward in the apparatus of Fig. 9. The hydrostatic pressure in a liquid must always be taken into account in determining the static equilibrium. Even if the electric field did act in a direction to support the liquid column, the maximum force would be only sufficient to support a column 0.65 cm. high under the experimental conditions. Yet, a column 10 cm. high was supported indefinitely. Clearly, what supports the liquid column is the pressure of the gas below the interface, not the electric pressure. The electric field simply stabilizes the interface.



Figure 9. Experiment to Demonstrate the Bang-Bang Stabilization of an Interface in an Adverse Acceleration. The dark liquid is suspended in the upper half of a glass container with electrodes having essentially the geometry of Figure 2. The liquid (corn-oil) has been removed from the bottom half of the cylinder, the liquid/vapor interface is stabilized by the electric field.



Fig. 10 Closeup view of lower edges of electrodes for the experiment of Fig. 9.

Fig. 10a Voltage (10 KV) exceeds that required by Fig. 9



Fig. 10b Instability resulting from a reduction in voltage below ~ 6 KV required for stability. Note that the interface to the right has risen, while that to the left is falling—a form of Rayleigh-Taylor instability, as predicted.

A quantitative comparison between experimental conditions for incipient instability with infinitesimal slosh amplitude and the prediction of the pendulum model is made in Fig. 11. The solid line, which is predicted by Eq. (21), gives a reasonable estimate of the conditions for instability. Experimental errors arise because of variations in the way in which the liquid wets the electrodes and the glass walls of the container. Most of the liquids tended to set both. Pertinent fluid parameters are summarized in Table 1.

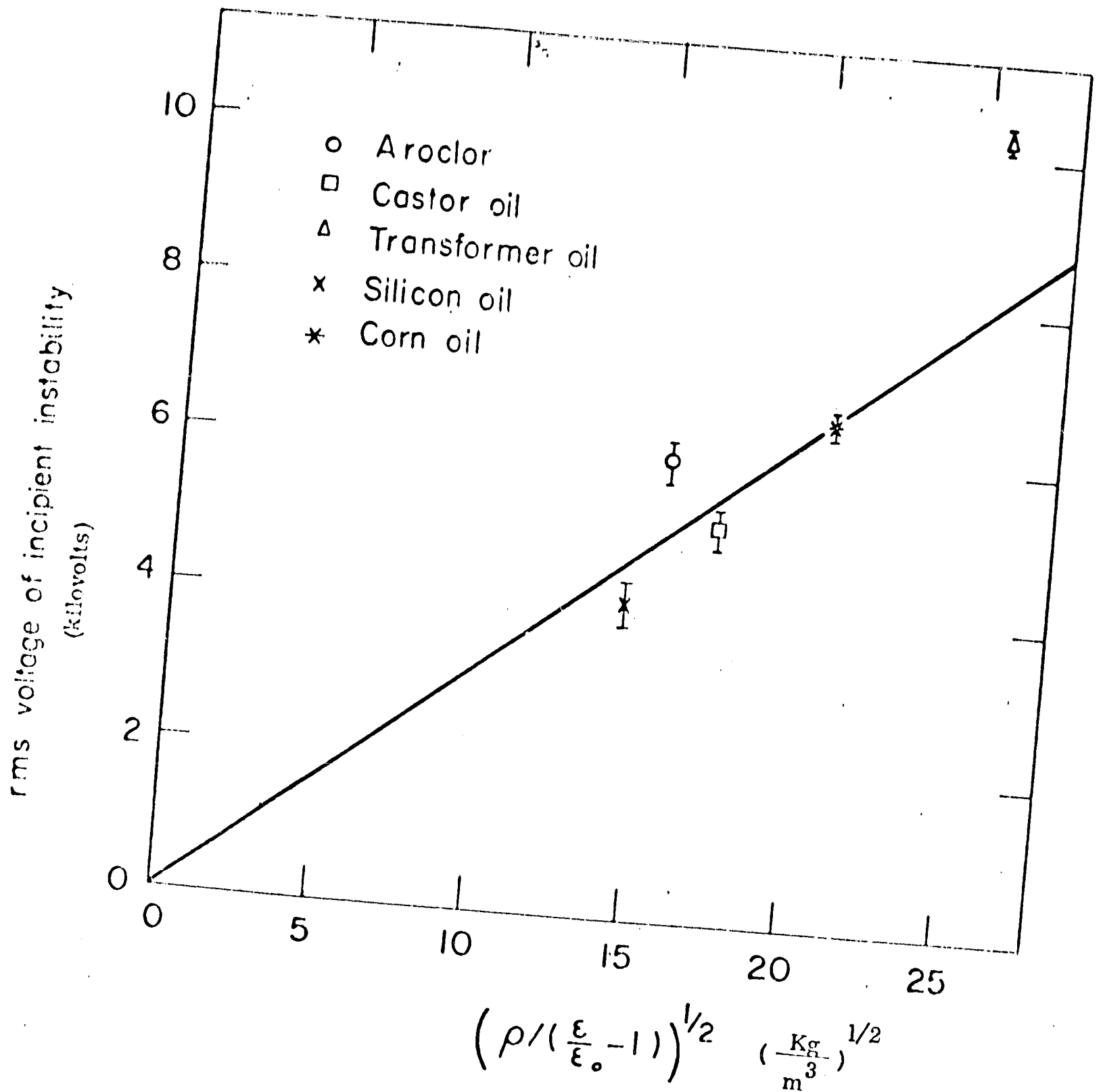


Fig. 11 AC Voltage for incipient instability in experiments of Figs. 9 and 10 as function of pertinent fluid parameters. The solid line is the theory, Eq. (21).

Table 1. Data for Fluids Used in Incipient Instability Experiments

<u>Fluid</u>	<u>Relative Permittivity ϵ/ϵ_0</u>	<u>Density (Units of 10^3 kg/m^3)</u>
Aroclor 1232 (Monsanto)	5.7	1.14
Castor Oil (Howe & French)	4.7	1.1
Transformer Oil (GE 10-C)	2.2	0.8
Silicon Oil (Dow Corning 1265)	6.95	1.25
Corn Oil (Mazola)	3.1	0.91
Freon 113	2.41	1.56
Liquid Nitrogen	1.43	0.8

2.6 Experiments: dc Fields

There are three effects that generally account for differences between results, such as those just described, where ac fields are used and where dc fields are used. In the dc fields, the fluids are prone to forms of instability related to the relaxation of free charge, while the electrical breakdown strength of the dielectric can tend to be degraded by the accumulation of space charge due to emission effects at the electrodes.¹⁰ On the other hand, interfacial vibration and instability induced by the pulsating nature of the ac field-induced forces is not present with the dc fields. In the ac experiments this difficulty was avoided by using a sufficiently high-frequency electric field.

Experiments were conducted in essentially the same configuration as previously described (see Fig. 9), but with dc fields. For the cases tested (fluids with relaxation times exceeding 10^{-2} seconds) it was found that the voltage for incipient instability with dc fields agreed well with the theory, Eq. (21). The dc stability data is plotted in Fig. 12.

Of greatest significance were stability experiments conducted using liquid nitrogen with dc fields. The experimental configuration for liquid nitrogen was essentially as described in the previous section, with provision made for maintaining the cryogenic environment as shown in Fig. 13. The apparatus consisted of a glass tube with a set of concentric cylindrical electrodes carefully fitted to prevent leakage between the outer electrode and the tube. The glass tube with its electrode array was positioned in an unsilvered glass dewar and suspended from a fiberglass cover as shown in Fig. 12. The experiment began by drawing liquid nitrogen into the glass tube using a vacuum pump. When the tube was nearly full, the tube vent was constricted to permit continuous removal of the nitrogen boil-off. The electric field was established by connecting the electrode array to a filtered source of dc high voltage. Following this, nitrogen gas was bubbled beneath the electrodes where it displaced the liquid. Thus the upper portion of the tube was filled with liquid nitrogen supported by the pressure of the gas beneath the electrodes, and stabilized by the electrohydrodynamic, bang-bang interaction at the liquid-vapor interface. The voltage was then decreased gradually, until the liquid-vapor interface became unstable and the liquid in the glass column fell.

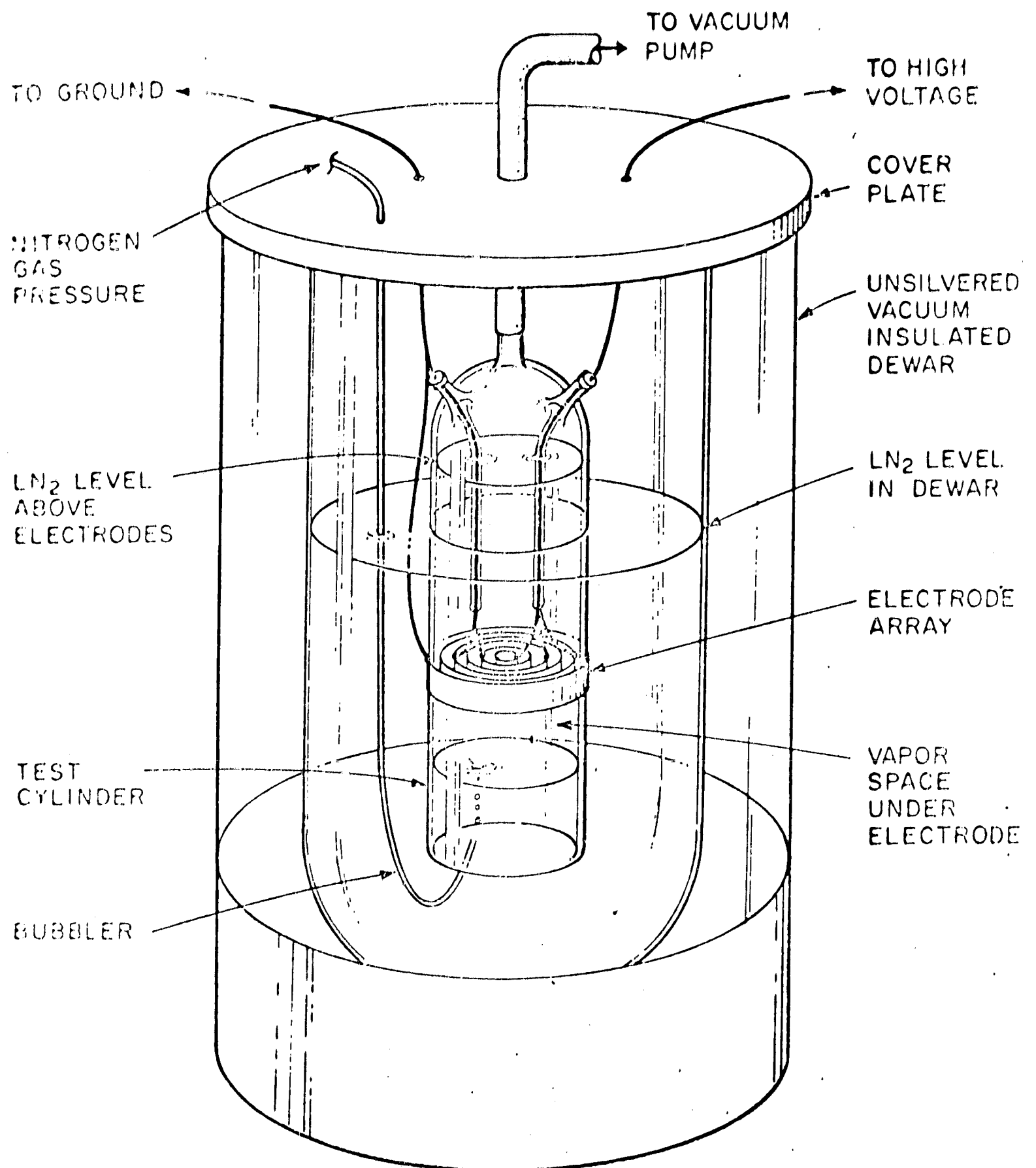


Fig. 13. Experiment to demonstrate bang-bang stabilization of cryogenic liquids. The electrodes are concentric cylinders, as in the apparatus of Fig. 9. The fluid is liquid nitrogen.

The point of instability with liquid nitrogen was found to be 21.5 kv. Eq. (21) predicts the incipience of instability at

It has been found that, with dc fields, there is an upper limit on the potential required for stable orientation of liquid Freon as well as a lower limit. As the voltage is raised, the interface is observed to become overstable, the interfacial oscillations often reaching amplitudes such that the liquid orientation is lost. There are two possible mechanisms for this overstability, both under investigation. One concerns the effects of charge relaxation on interactions at the interface¹⁰ and the second concerns bulk instability of the fluid due to space charge accumulation from a non-ohmic conduction process connected with the generation of carriers at the electrodes.

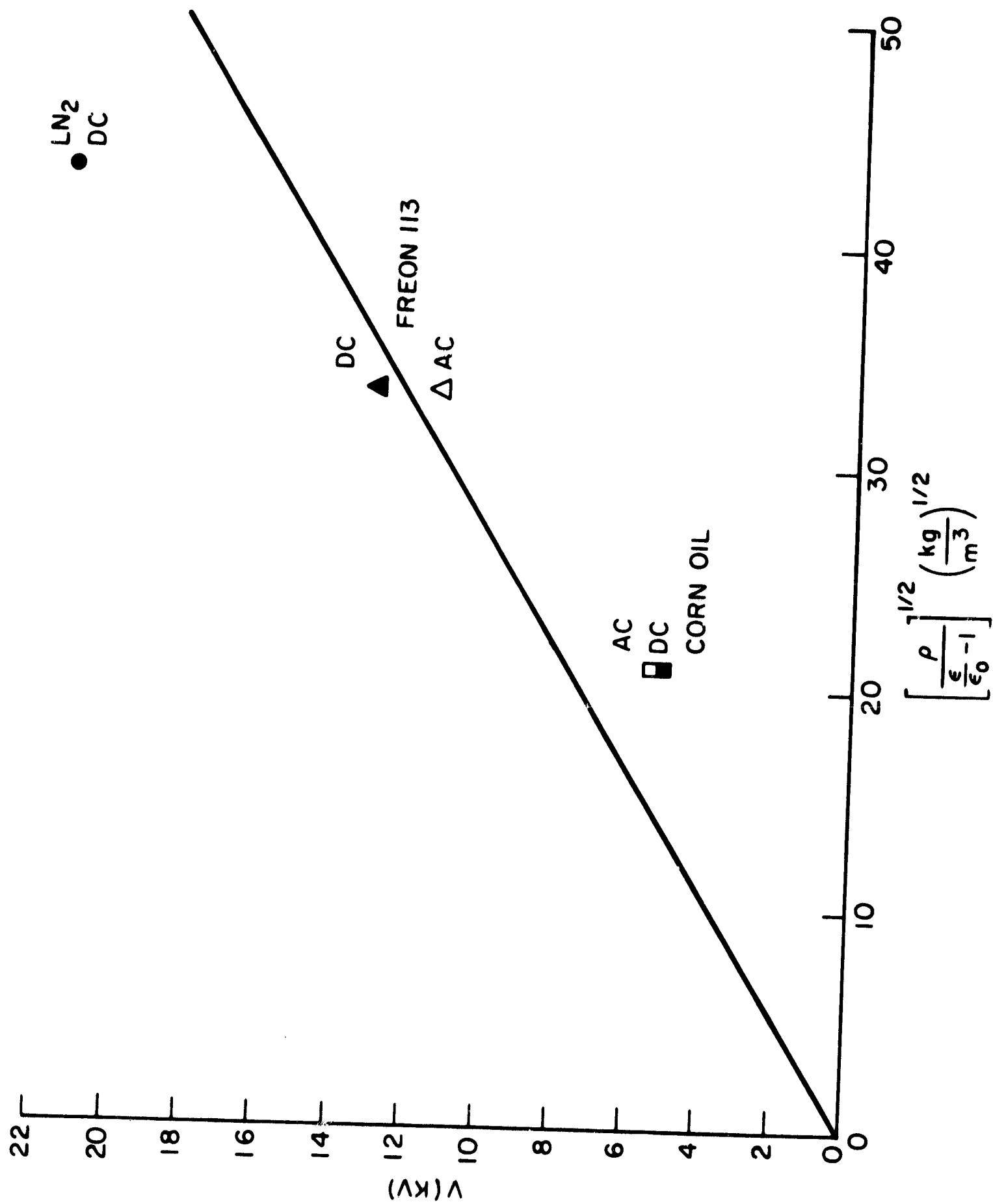


Fig. 12. DC Voltage for Incipient Instability in Experiment of Figs. 9 and 12. The solid line is the theory, Eq. (21).

Section 3

DESIGN OF AN ELECTROHYDRODYNAMIC SLOSH BAFFLE

In what follows, we assume at the outset that the height of the electrode array is sufficiently large that the sloshing liquid/vapor interface does not reach the lower edges of the electrodes. This will in fact be a design condition that determines the height b (Fig. 14). The maximum amplitude of the oscillations after cut-off ($g = g^*$) is determined by recognizing that the peak slosh amplitude occurs when $d\xi/dt = 0$ and the potential energy is just equal to the kinetic energy established prior to engine cut-off. That is

$$\epsilon_o - V(\xi_m) = 0 = \epsilon_o - g^* (\rho_\ell - \rho_v) \xi_m^2 - \frac{1}{2} (\epsilon_\ell - \epsilon_v) E_o^2 \xi_m \quad (23)$$

Remember that ϵ_o is known. Hence, this expression can be solved for the amplitude ξ_m (only one root is physical).

$$\frac{\xi_m}{\xi_a} = -P + \sqrt{P^2 + 1} \quad (24)$$

where we have defined the parameters:

$$\xi_a = \xi_o \sqrt{g_o/g^*} \quad (25)$$

$$P = \frac{(\epsilon_\ell - \epsilon_v) E_o^2}{4 \xi_o g^* (\rho_\ell - \rho_v)} \sqrt{g^*/g_o} \quad (26)$$

Note that ξ_a is the amplitude of the peak oscillations after engine cut-off if the electric field were not present. The effect of the electric field in attenuating the amplification of the slosh amplitude is reflected by the magnitude of P . A plot of the maximum slosh amplitude ξ_m , normalized to ξ_a , as a function of P , is shown in Fig. 15.

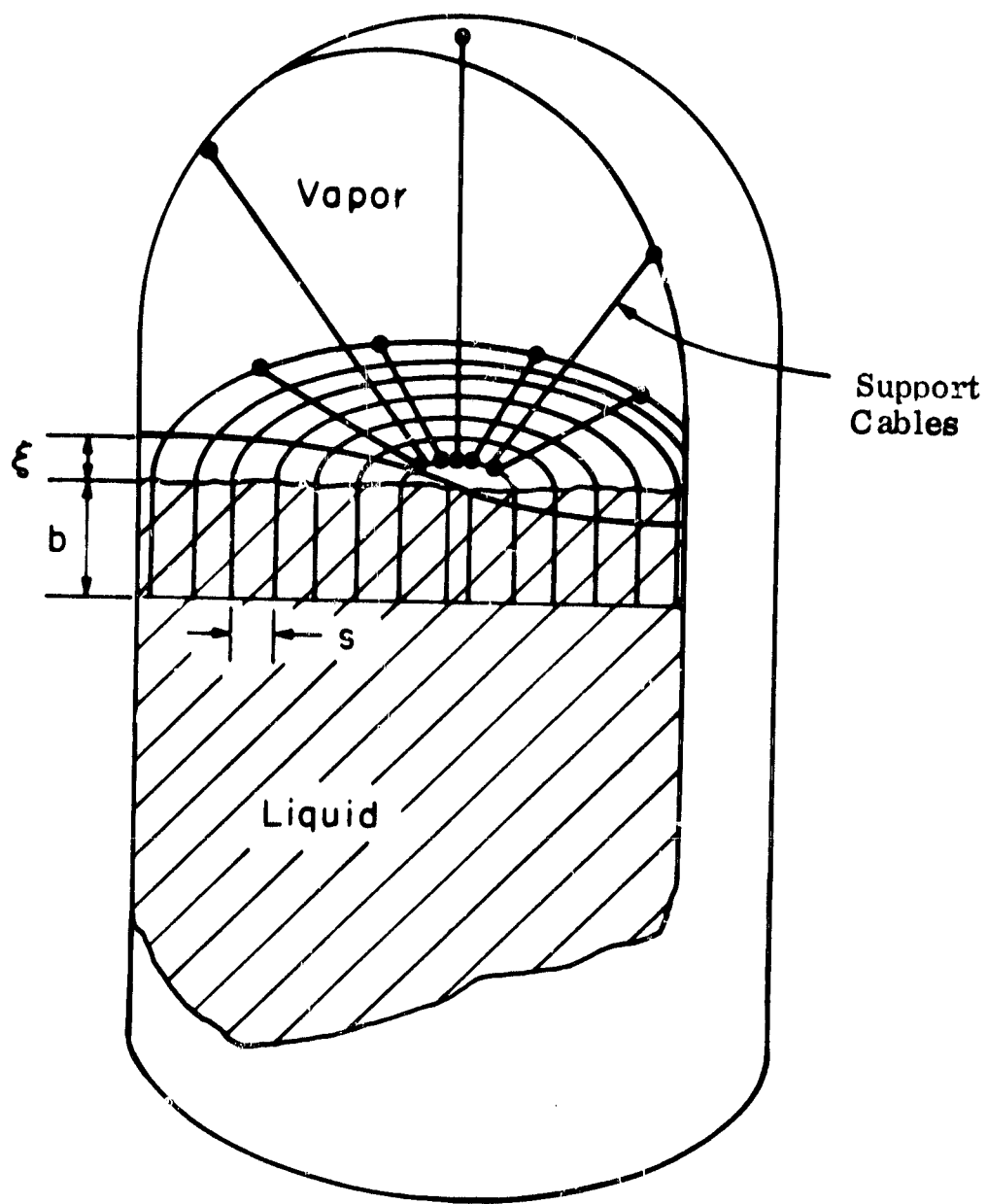


Fig. 14. Electrohydrodynamic Slosh Baffle for SIV-B LH_2 Tank

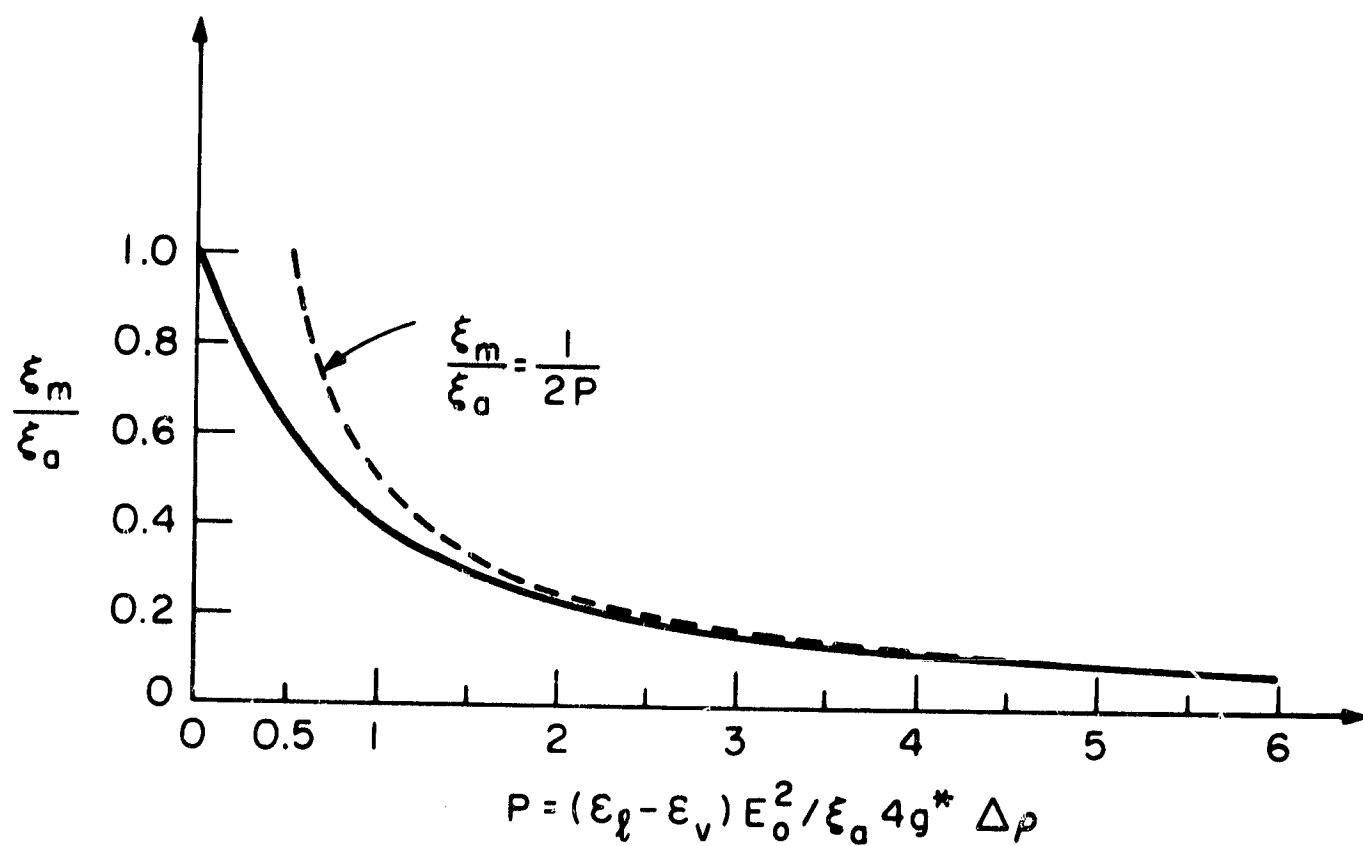


Fig. 15. Normalized Slosh Amplitude as a Function of the Normalized Electric Pressure. The dotted line indicates the approximation used in much of the design optimization.

Practical interest is primarily confined to cases where the attenuation of the slosh amplitude after engine cut-off exceeds 0.5, and for this reason, it is possible to write Eq. (24) in a simple form which takes advantage of the fact that $P \gg 1$ for most cases of interest.

$$\frac{\xi}{\xi_a} = \frac{1}{2P} ; P > 1 \quad (27)$$

The validity of this approximation is indicated by the dotted curve in Fig. 14. As long as P is greater than unity, a negligible error is incurred in using Eq. (27) rather than Eq. (24).

3.1 Electrode Weight

The following parameters are summarized and defined for purposes of using the results of the previous sections to optimize a slosh baffle design.

R	= tank radius
b	= vertical height of electrodes
s	= spacing of electrodes in baffle
V _o	= rms voltage across electrodes
W _b	= weight of baffle structure
W _{sup}	= weight of power supply
W _{ft}	= weight of feedthrough
g _o	= acceleration before engine cut-off
ξ _{out}	= amplitude of slosh at outer wall when g = g _o (E _o = 0)
ξ _a	= amplitude of slosh represented by equivalent pendulum when g = g*
ξ _m	= maximum amplitude of equivalent pendulum when g = g*, E = E _o

For design purposes, the electrode weight, together with the weight of the support hardware, is of primary importance. The weight of the baffle alone has the functional dependence

$$W_b = CR^2 b/s \quad (28)$$

where the constant C can be determined from a point design derived in detail in Appendix B. Equation (28) becomes

$$W_b = 1.86 \frac{R^2 b}{s} \text{ lbs} \quad (29)$$

As a design condition, we require that the slosh amplitude at the outside radius be equal to b. Within the spirit of approximations implicit to this design, we relate the amplitudes at the outer wall to those of the pendulum by a factor of 2. Hence,

the kinetic energy of the equivalent pendulum is determined from a maximum amplitude found by using half of the amplitude evaluated at the wall

$$\xi_o = \xi_{out}/2$$

Similarly, the plates must have a sufficient vertical height that the slosh amplitude at the wall not exceed b . In terms of the equivalent pendulum maximum amplitude this requires that

$$b = 2 \xi_m \quad (30)$$

Then, from Eq. (27), this last condition requires that

$$b = \xi_{out}^2 g_o \frac{(\rho_l - \rho_v)}{(\epsilon_l - \epsilon_v)} \frac{S^2}{V_o^2} \quad (31)$$

We are given the following information for the SIV-B hydrogen tank:

$$R = 3.3 \text{ meter}$$

$$\xi_{out} = 7.61 \times 10^{-2} \text{ meter}$$

$$\rho_l - \rho_v = 70 \text{ kg/m}^3$$

$$\epsilon_l - \epsilon_v = (0.25) (8.85 \times 10^{-12}) \text{ farad/meter}$$

From Eq. (31) it follows that

$$b = 1.84 \times 10^{12} \frac{S^2}{V_o^2} \quad (32)$$

From this, it follows that the weight of the baffle, as given by Eq. (29), can be expressed as

$$W_b = 3.42 \frac{R^2 S}{V_o^2} \times 10^{12} \text{ (lbs.)} \quad (33)$$

It seems surprising that the weight is decreased by making s small (since this increases the number of electrodes). Certainly the weight is proportional to $1/s$ (Eq. (29)), but by making s small, it is possible to make b small and this is reflected in the numerator of Eq. (29). Since b is proportional to the square of s (Eq. (31)), it is advantageous to make s small for a given voltage. There is, however, a fundamental limitation on how small s can be made. Remember that s must always be much less than b . As a design condition, we require that

$$\frac{b}{s} > 5 \quad (34)$$

It follows from Eq. (32) that

$$(1.84 \times 10^{12}) \frac{s}{V_o^2} > 5 \quad (35)$$

and this in turn puts a lower bound on the weight given by Eq. (33),

$$W_b > 8.46 R^2 \quad (36)$$

A second limitation arises from the fact that the electric field V_o/s has some maximum value (limited by the electrical breakdown strength of the gas), and this imposes an upper bound on b ,

$$b > (1.84 \times 10^{12}) (E^*)^2 \quad (37)$$

where E^* is the electrical breakdown field between the electrodes.

The design curves given by Eqs. (33), (36), and (37) are shown in Fig. 16. The weight W_b in pounds is plotted as a function of power supply voltage V_o , with the plate spacing s as a parameter. Points below the horizontal dotted line fail to satisfy the condition that b be at least five times as large as s . The broken line labeled $E_o > E^*$ represents the breakdown condition based on an assumed value of

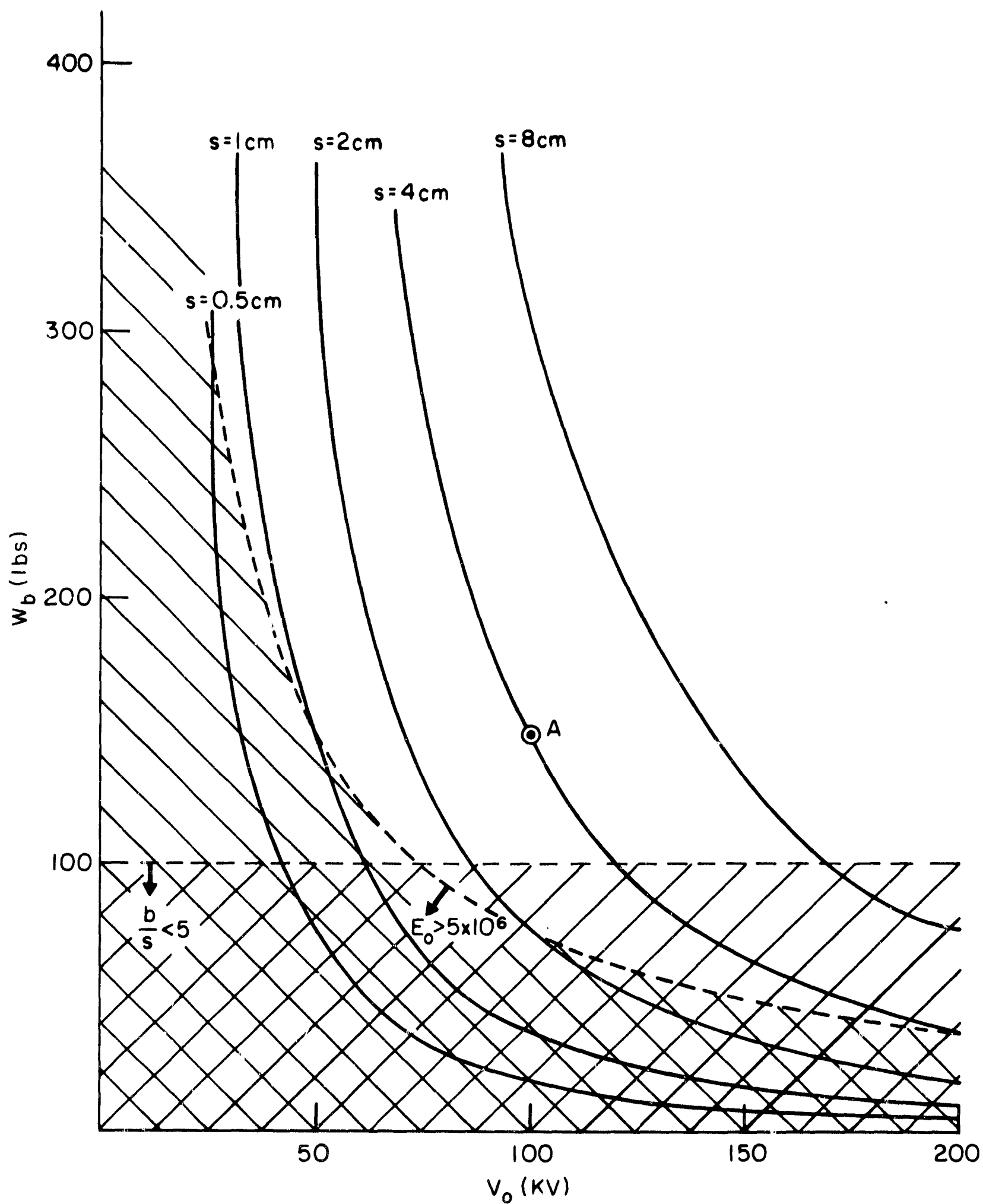


Fig. 16. Electrode Weight vs. Voltage for a Tank having a 260" Diameter. Point Design, A, is described on page 44 .

$E^* = 5 \times 10^6$ v/m.* Design points to the left of this line are not valid because the system will not operate at the design voltage without electrical breakdown.

As an example, consider the design point (A) shown in the figure. We use the graph, together with Eq. 32, to conclude that the parameters of the system are

$$W_b = 150 \text{ lbs.}$$

$$V_o = 100 \text{ KV}$$

$$s = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$$

$$b = 29.5 \text{ cm} = 2.95 \times 10^{-1} \text{ m}$$

*A conservative estimate for the breakdown voltage under normal operating conditions for the SIV-B hydrogen tank. Refer to the Phase III report of this contract (NAS 8-20553) for a discussion of the electrical breakdown properties of hydrogen.

3.2 Total System Weight

The electrohydrodynamic slosh baffle system consists of the electrode array (as designed in Section 3.1), a high-voltage power supply, and a high-voltage feedthrough to carry the high voltage from the power supply, through the tank wall, and to the electrode array.

The weight of the high-voltage power supply increases with output voltage as shown in Fig. 17. The design and weight analysis for the power supply is presented in Appendix C. This design utilizes state-of-the-art components.

The high-voltage feedthrough must be of a special design to minimize both weight and heat leakage. Appendix C describes the design of a new type of high-voltage feedthrough. A feasibility model of this component has been constructed and successfully tested to voltages exceeding 100 kv under Phase III of this contract. Figure 18 summarizes the weight of the feedthrough as a function of voltage.

The weight of the complete slosh baffle system (electrode array, power supply, and feedthrough) is shown in Fig. 19.

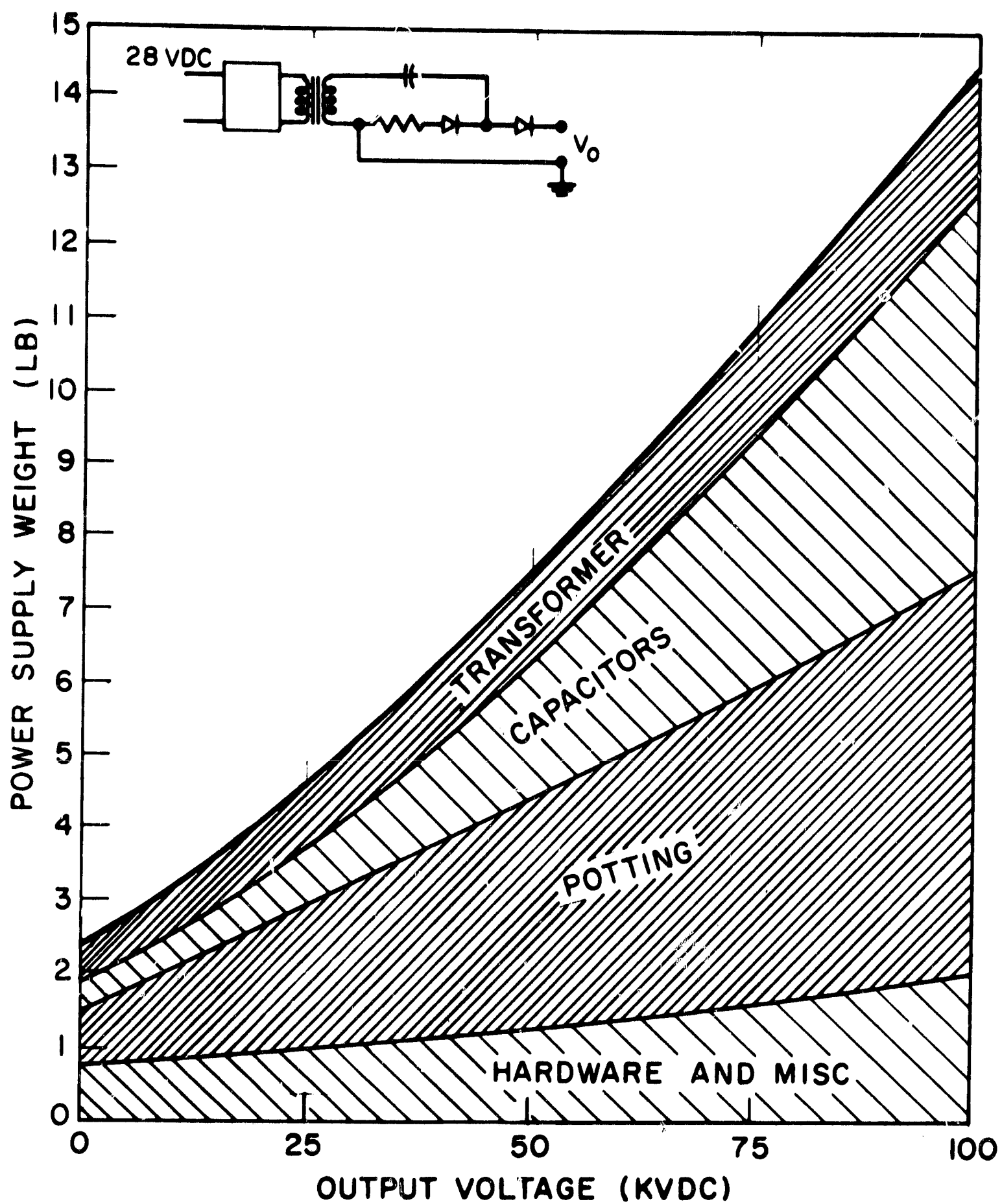


Fig. 17. High-Voltage Power Supply Weight

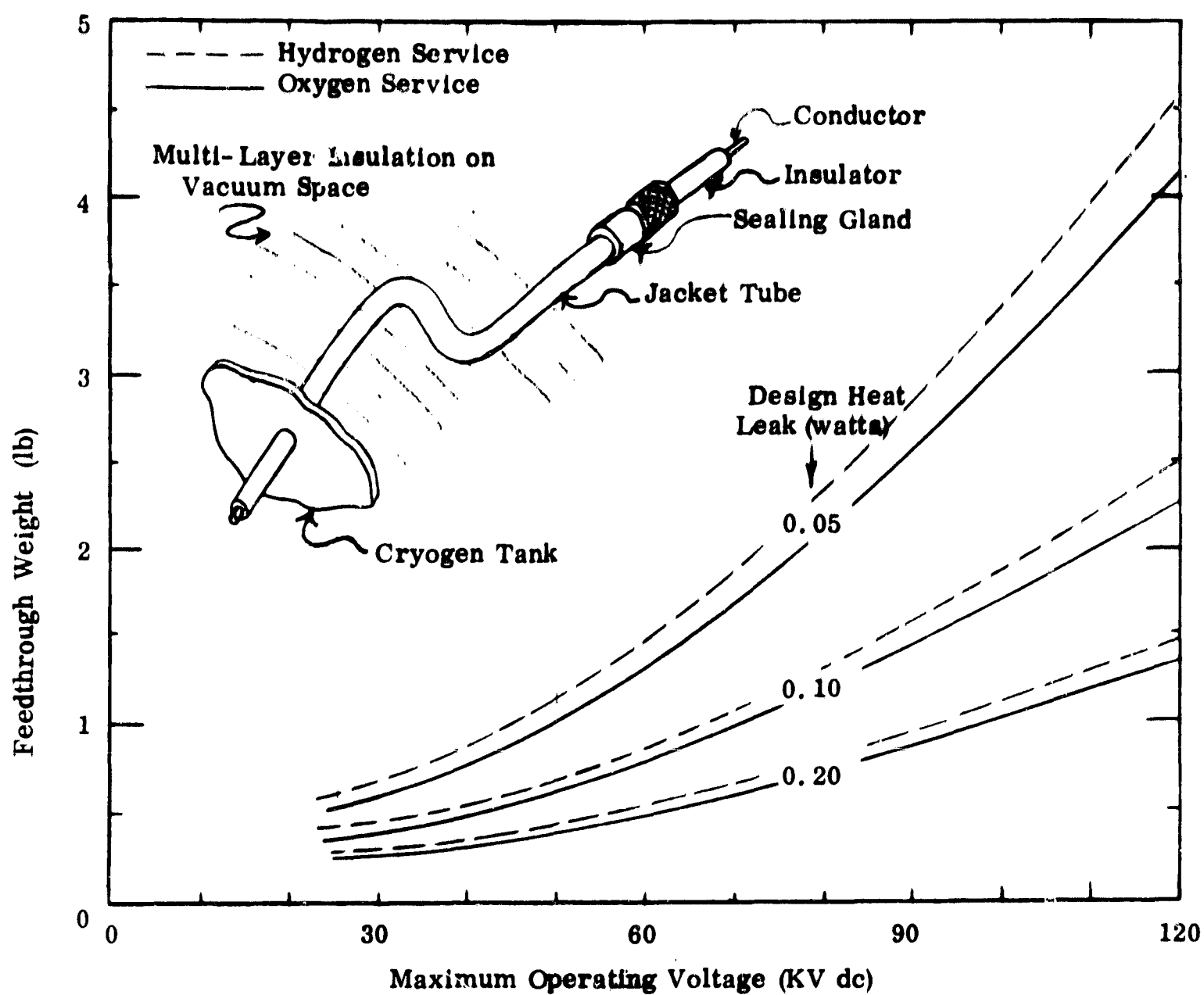


Figure 18. Weights of Coaxial High Voltage Feedthrough for Dielectrophoretic Liquid Expulsion Devices for Cryogenic Tankage

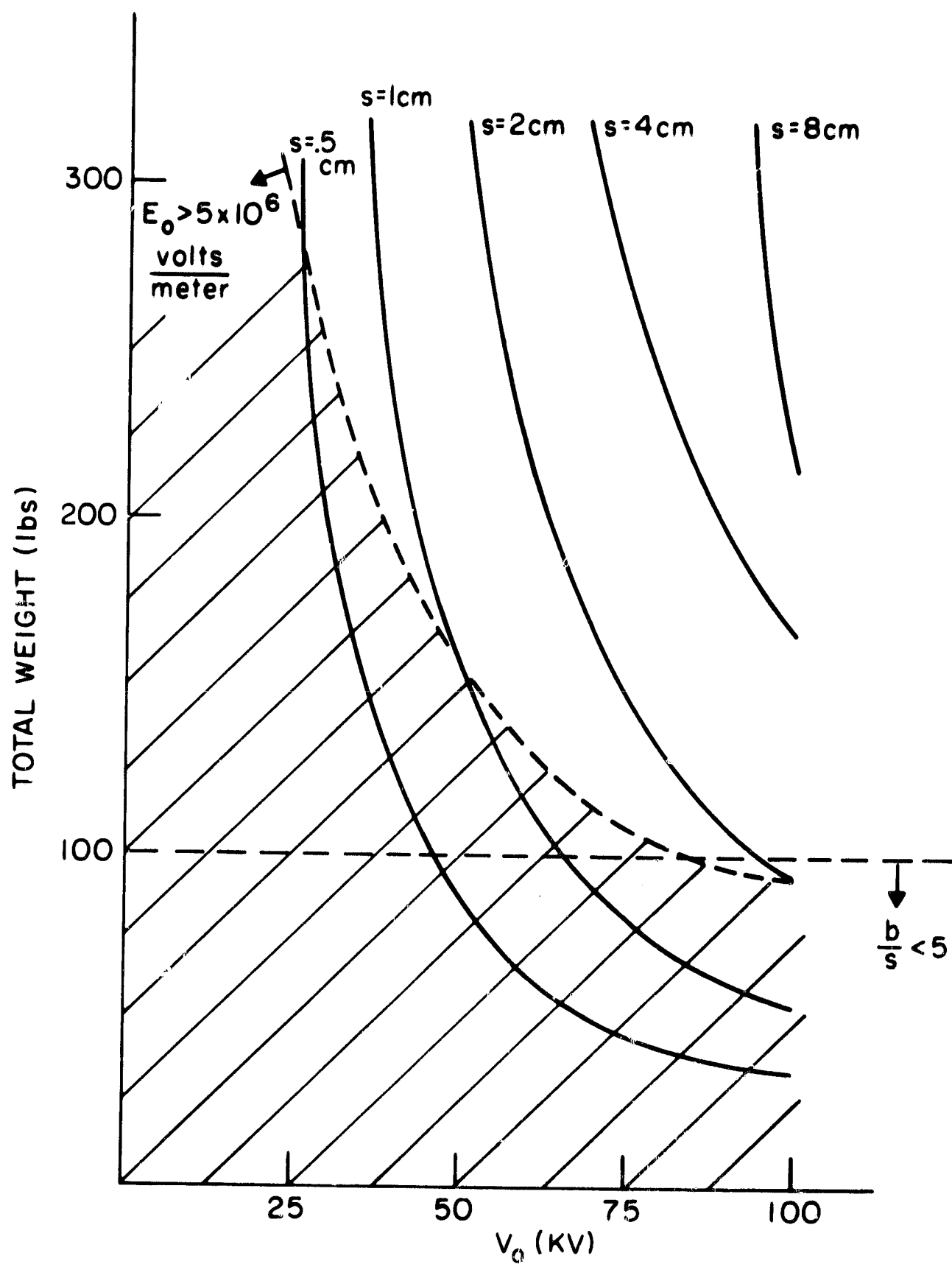


Fig. 19. Total Weight of Electrodes, Supports, Feedthrough and Power Supply

Section 4

CONCLUSIONS AND RECOMMENDATIONS

The analyses, experiments, and designs developed under this program have shown that electrohydrodynamic forces can exert a gross stabilizing effect to control sloshing of cryogenic propellants in low gravity.

The electrohydrodynamic bang-bang stabilization mechanism acts to stiffen a liquid/vapor surface to such a great extent that electrohydrodynamic orientation of a liquid/vapor system can now be demonstrated and studied in 1-g. Tests under this program have shown that stable electrohydrodynamic orientation with dc fields is potentially feasible. Orientation of a cryogenic liquid (LN_2) has been demonstrated.

The design of an electrohydrodynamic slosh baffle to prevent slosh wave growth following engine cut-off has been developed. The weight of such a baffle system for the SIV-B hydrogen tank has been calculated.

The electrohydrodynamic bang-bang stabilization mechanism is useful not only for preventing slosh growth following abrupt changes in vehicle acceleration, but also for providing liquid/vapor stability in the design of EHD partial orientation devices and liquid expulsion systems.

It is recommended that further experimental work be undertaken to demonstrate the operation of EHD partial orientation devices using bang-bang stabilization.

The bang-bang stabilization phenomenon is of critical importance to the stability of the EHD liquid expulsion system discussed under Phase II of this contract. We recommend that experiments be conducted to test the dynamic stability theory for this ribbon electrode liquid expulsion device.

The implication of the dc experiments that dc fields are potentially feasible for electrohydrodynamic cryogenic orientation systems is an important breakthrough (since dc power supplies are simple, lighter, and more reliable than ac supplies). Further experiments should be undertaken to investigate the limitations, if any, on the use of dc fields for EHD propellant orientation.

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Appendix A

To establish that the dielectrophoretic force should be approximated as shown in Fig. 4a, an analog experiment was performed. To see the connection between this experiment and Fig. 2, consider the two regions of fluid between the symmetry ground planes of Fig. A-1a. Although the symmetry ground planes are imaginary surfaces in the actual bang-bang apparatus, they can be simulated by real metal electrodes, as shown in Fig. A-1b. Two channels are so used that fluid is conserved as it is in an actual slosh.

We have already argued that for such a mode, the average dielectrophoretic force per unit area on the slosh pendulum has extreme values of zero (for $\xi = 0$) and $\pm \frac{1}{2}(\epsilon - \epsilon_0)E_0^2$ (for $|\xi| \gg s$; see Fig. 4). It remains only to determine the shape of the curve between these two values. If we assume that the plates have a radius of curvature large enough for the problem to be modeled as two-dimensional, and if the plate height h is sufficiently large, with respect to the plate thickness $d = s/2$, the shape of the $\tau_e(\xi)$ curve can only depend on the two-dimensionless parameters ξ/s and ϵ/ϵ_0 .

We recall that, for a constant potential electric-field system having a capacitance C , dependent on a geometrical displacement ξ , the force is proportional to ¹² $(\partial C / \partial \xi)$. Since we already know the asymptotic value of the average force, it is necessary only to determine the dimensionless curve:

$$\tau_e / \tau_{\max} = \frac{\frac{\partial C}{\partial \xi}}{\left(\frac{\partial C}{\partial \xi} \right)_{\max}}$$

experimentally for the apparatus of Fig. A-1b scaled so as to preserve the dimensionless parameters ξ/s and ϵ/ϵ_0 .

The results of such an experiment are shown in Fig.A-2 for two fluids, aroclor ($\epsilon/\epsilon_0 = 5.7$) and corn oil ($\epsilon/\epsilon_0 = 3.1$). It is seen that there is only a weak dependence on the factor ϵ/ϵ_0 . The piecewise continuous solid curve is the one used for the pendulum force (see Fig. 4).

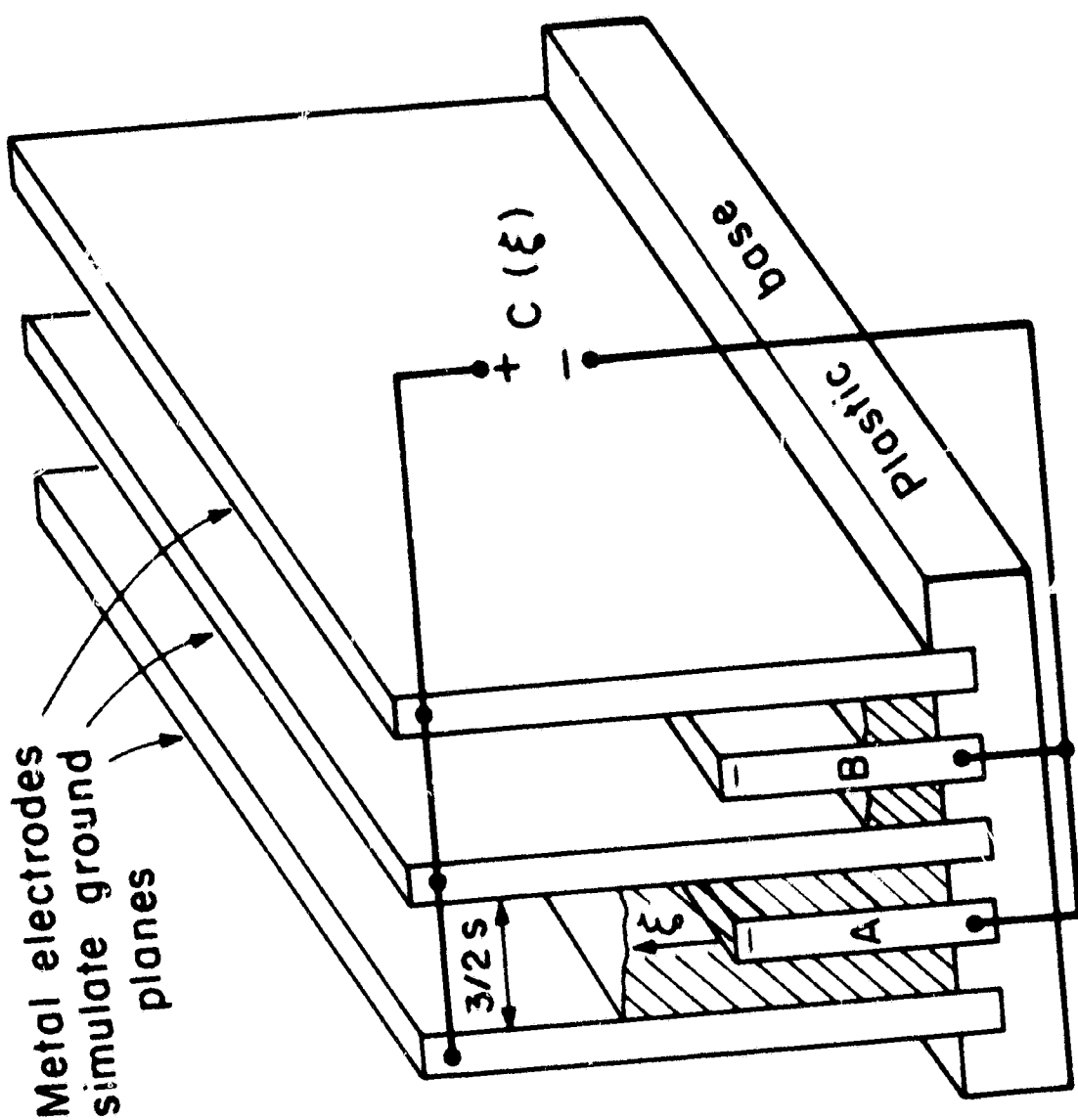


Fig. A-lb. Analog Apparatus for Measuring Capacitance as a Function of Interface Deflections

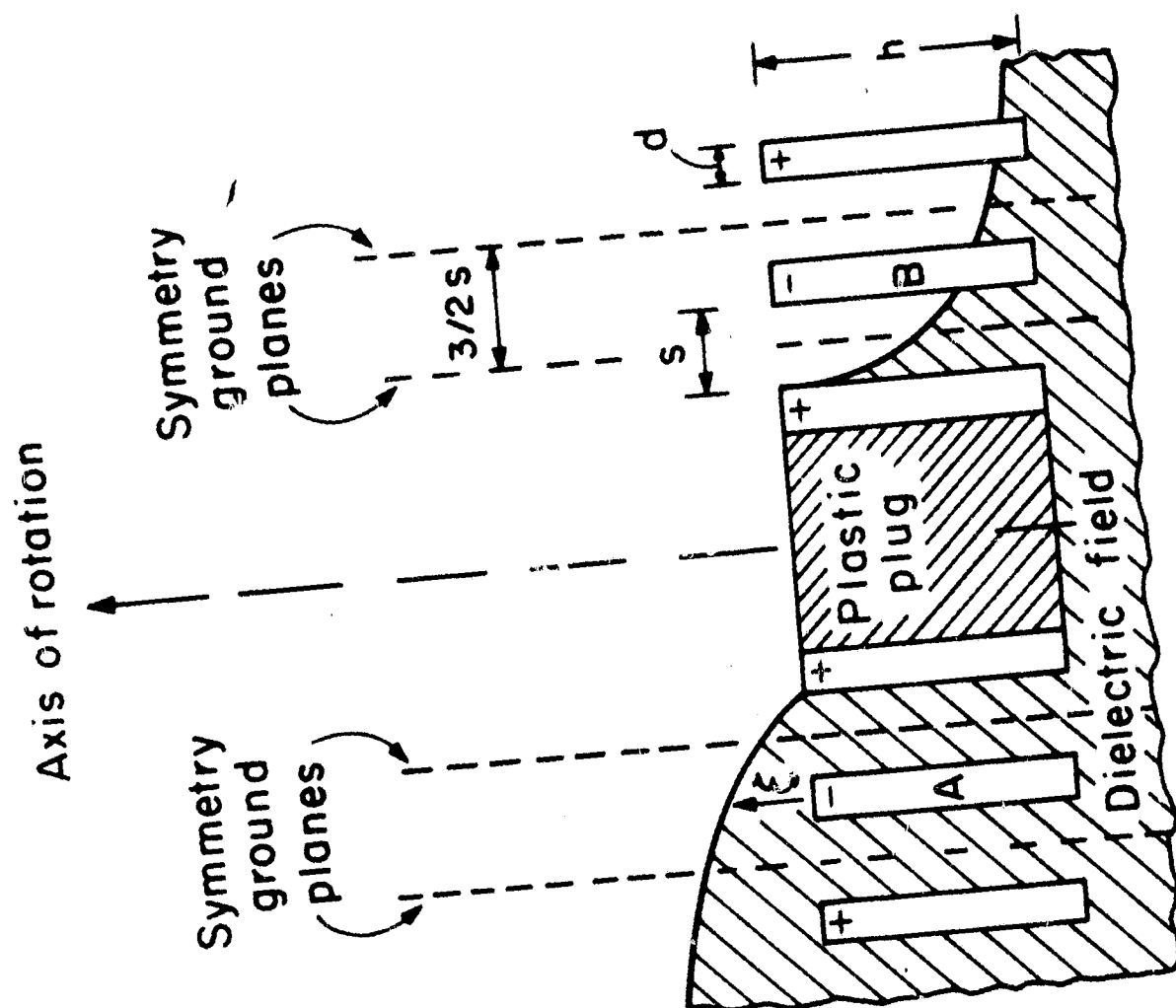


Fig. A-la. Cross-Sectional View of Electrodes and Fluid Interface for the Geometry of Fig. 2, but with the Relative Positions of Air and Liquid Reversed

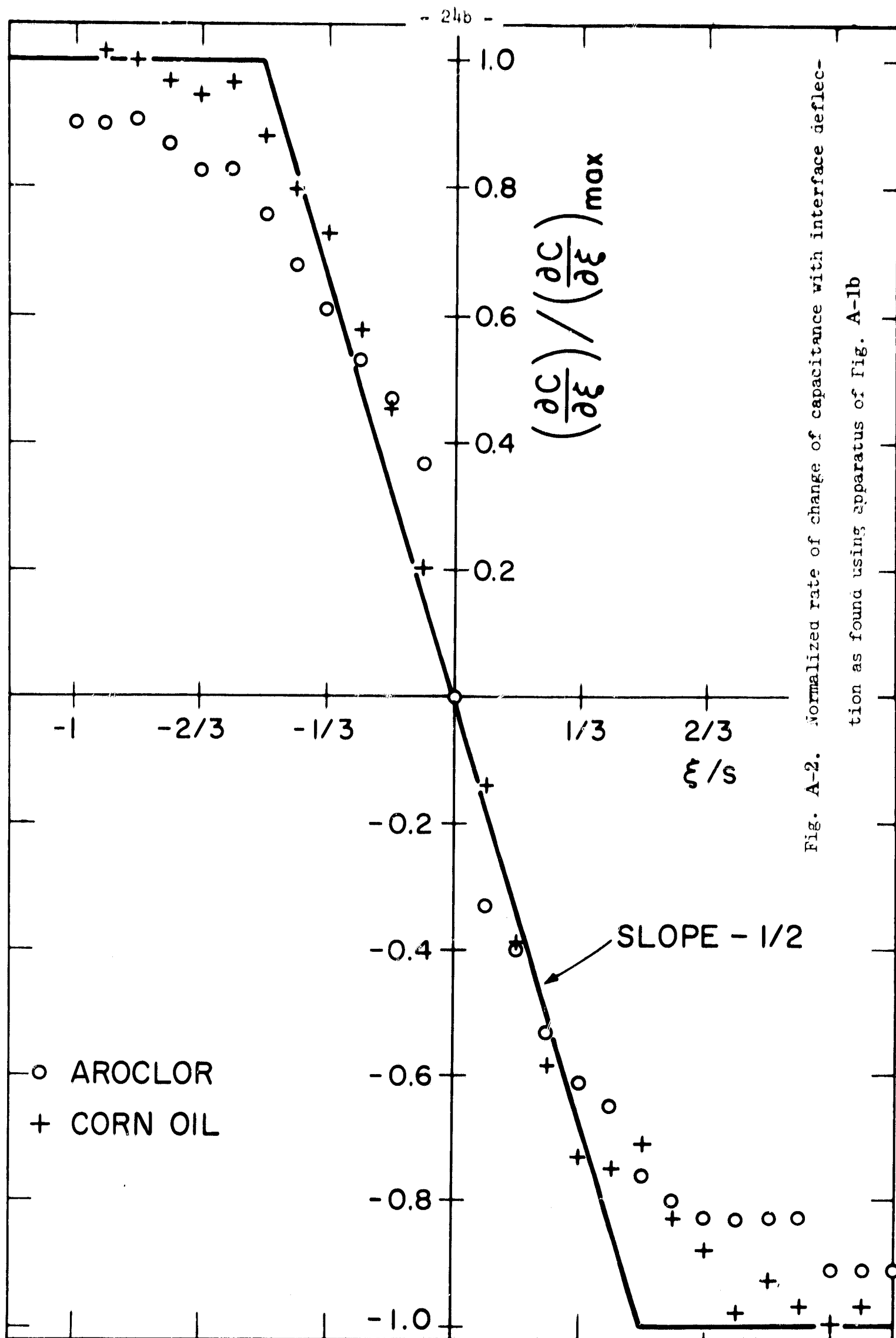


Fig. A-2. Normalized rate of change of capacitance with interface deflection as found using apparatus of Fig. A-1b

Appendix B

ELECTRODE WEIGHT: POINT DESIGNS

A detailed weight analysis was conducted by Lockheed Missiles and Space Company* for the EHD slosh baffle electrode and support hardware. Three candidate electrode configurations were evaluated:

- 1) concentric cylindrical screens
- 2) horizontal parallel screens
- 3) vertical parallel screens

Slosh baffle point designs were conducted for two tank diameters: 260-inch, and 396-inch. For each point design the electrodes were assumed to be fabricated of steel screen mesh having an area weight density of 0.01 lbs/ft^2 . The screens were edged with steel tubing having a diameter of $3/8$ inch. This is necessary in the design of dielectrophoretic screen electrodes to prevent electrical breakdown from the sharp edges of the screens and to reinforce the screen edges as well. For these point designs the electrode spacing was held at 2 inches and the height of each array was 12 inches. The point design philosophy is a time saver: weights of electrode arrays having different spacings and heights can be extrapolated straightforwardly from these point designs using Eq. (28).

Each of the three electrode configurations was designed to the same constraints:

Environmental Criteria

1. Maximum acceleration during ascent - 6 g.
2. Maximum pressure loading from sloshing propellant - 0.1 psi.

Pressure loading of dielectrophoretic screens assumed that the area for pressure loading is 10 per cent of the total area.

*Lockheed conducted these designs and weight analyses under subcontract

Design Criteria

1. Vertical deflection of the entire array is not to exceed six inches.
2. Plate separation must be maintained at 2 ± 0.25 inches or better.
3. Electrodes must not be permanently deformed during launch.

● Concentric Cylindrical Screens

The principle design approach is shown in Fig. B-1. A central support cylinder is used to distribute the loads, via cable assemblies, to the tank dome. The outer ends of the support struts are attached to the tank wall, employing a slotted connection to allow tank expansion and contraction.

An alternate approach to a concentric cylinder is shown in Fig. B-2. In this system the support struts have been replaced by a network of pretensioned thin straps, and the central support and cable system has been eliminated. An outer ring is required for end attachment of the tension straps.

The weight of the electrodes carried by each strut and the distribution of this load was determined. The struts were designed to be angles. The required angle weights were determined to support this electrode mass and the self mass of the supports. The cylindrical electrodes are stabilized by the tubular edging.

The weight analysis indicated the following for principle design:

	<u>20-Inch Tank</u>	<u>396-Inch Tank</u>
Electrode	22.2	49
Edge Doubler	66	153
Support Structure	<u>35</u>	<u>97</u>
Total	123.2 lbs.	299 lbs.

The maximum deflection of this array was 1.13 inches for the 260-inch tank and 4.6 inches for the 396-inch tank.

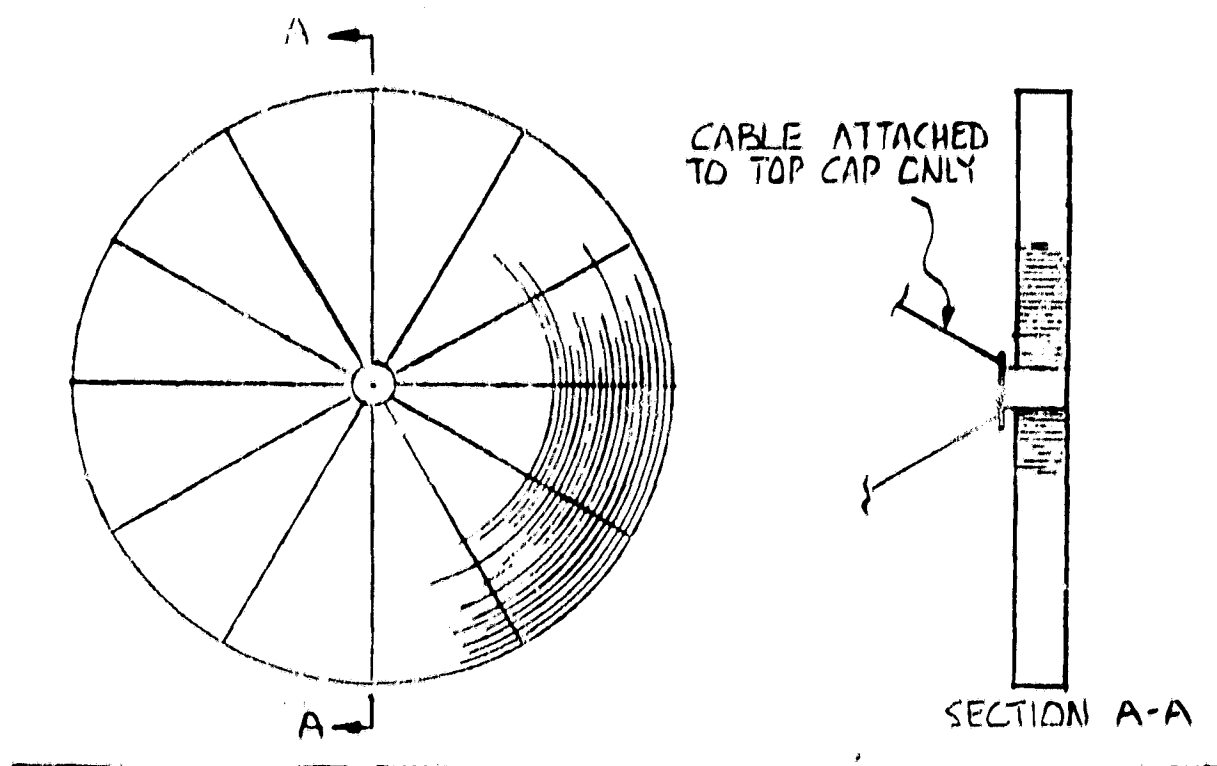


Fig. B-1. Concentric Cylinder Slosh Baffle Design with Cable Support

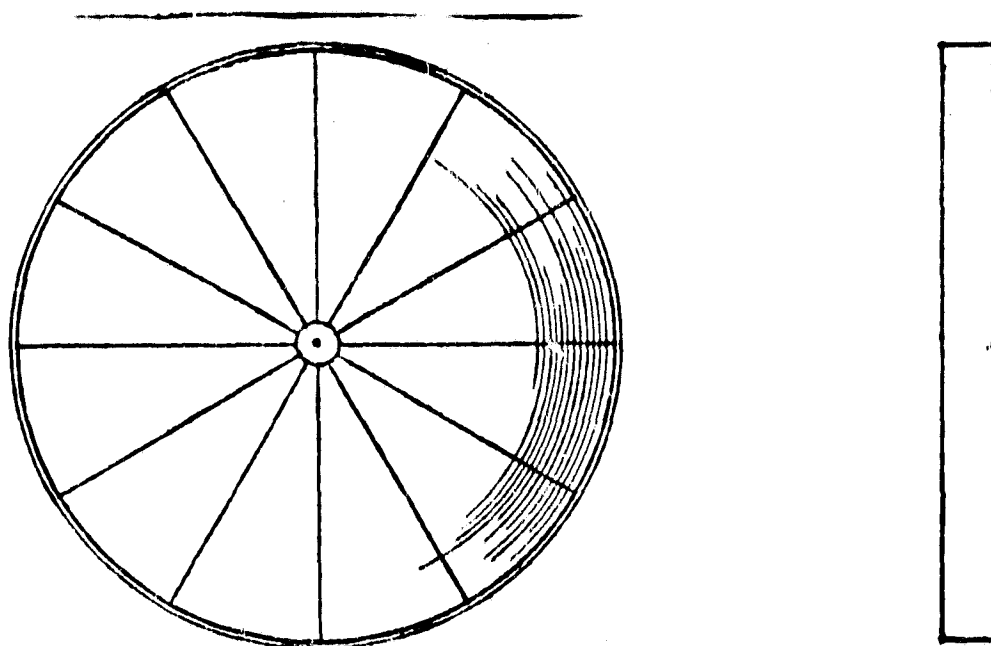


Fig. B-2. Alternate Concentric Cylinder Slosh Baffle Design without Cable Support

The weight analysis of the alternate design was as follows:

	<u>260-Inch Tank</u>	<u>396-Inch Tank</u>
Electrode	22.2	49
Edge Doubler	66	153
Support Structure	<u>60</u>	<u>180</u>
Total	148.2 lbs.	382 lbs.

● Horizontal Parallel Screens

The stacked plate array consists of horizontal grids placed two inches apart with rings at the top and bottom of the stack. The pretensioning of the grid is necessary to assure the grids are self-supporting and rigid enough to preclude excessive deflections under maximum loading conditions (Fig. B-4 and Fig. B-6).

The analysis was conducted with particular attention to prevention of buckling. The deflection during ascent was held to 0.25 inches.

The weight of the system was:

	<u>260-Inch Tank</u>	<u>396-Inch Tank</u>
Electrode Screens	26.2	60.5
Edge Doublers	6.7	10.2
Support Structure	<u>130.0</u>	<u>420.0</u>
Total	162.9 lbs.	490.7 lbs.

● Vertical Parallel Screens

The design concept is sketched in Fig. B-3. An outer ring is used for attachment of top and bottom edge doublers (or caps). The attachment to the tank wall is shown in Fig. B-5. The baffle system is self-supporting. Rods or wires, spaced at intervals, are required to prevent lateral movement.

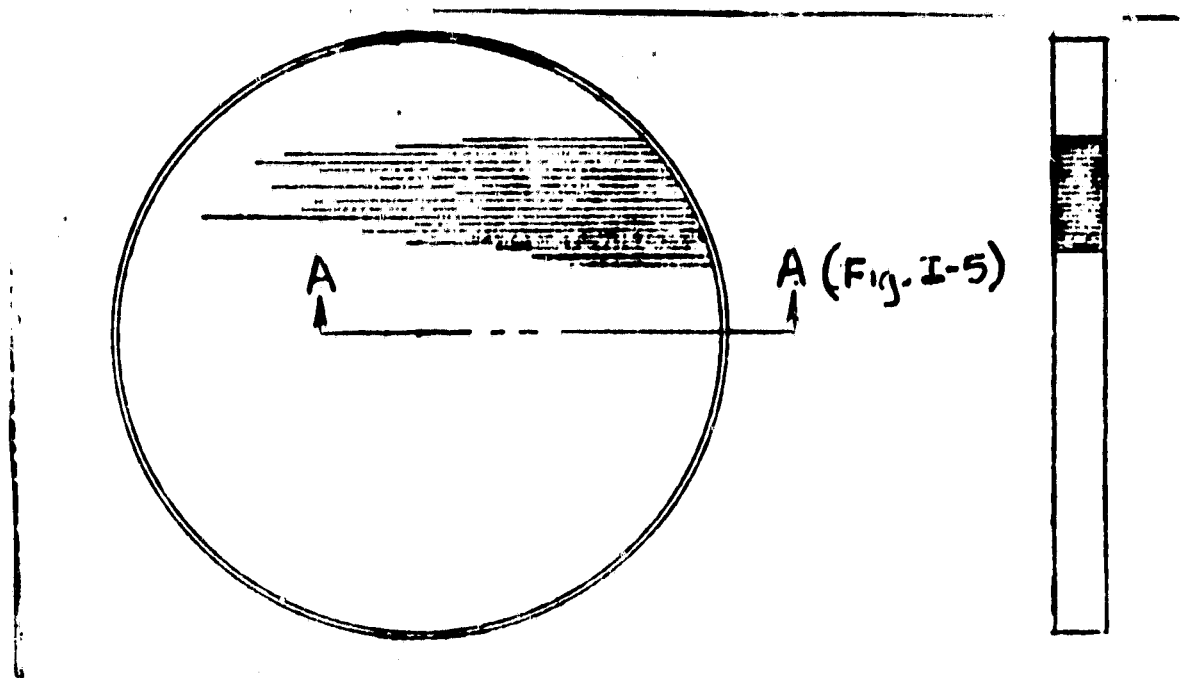


Fig. B-3. Parallel Plate Slosh Baffle Design

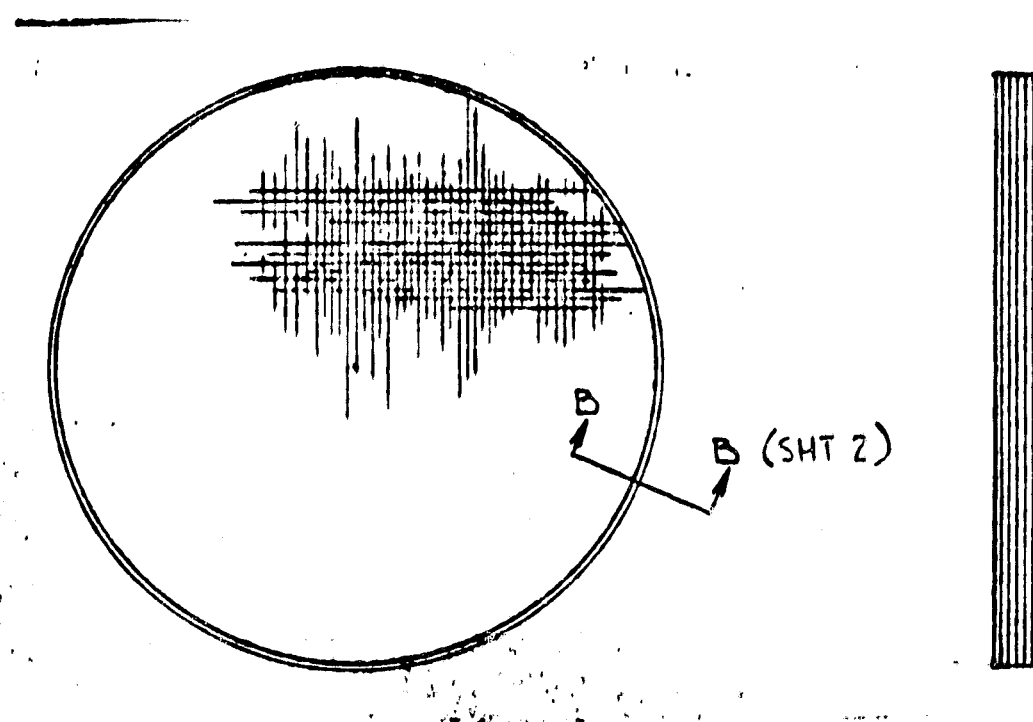


Fig. B-4. Stacked Plate Slosh Baffle Design

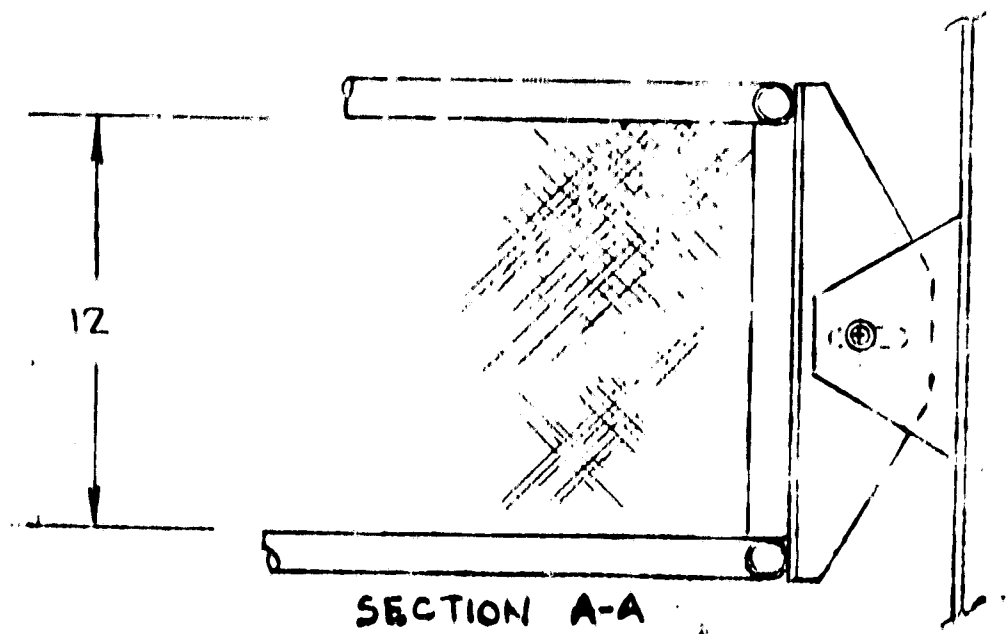


Fig. B-5. Tank Attachment for Parallel Plate Electrodes

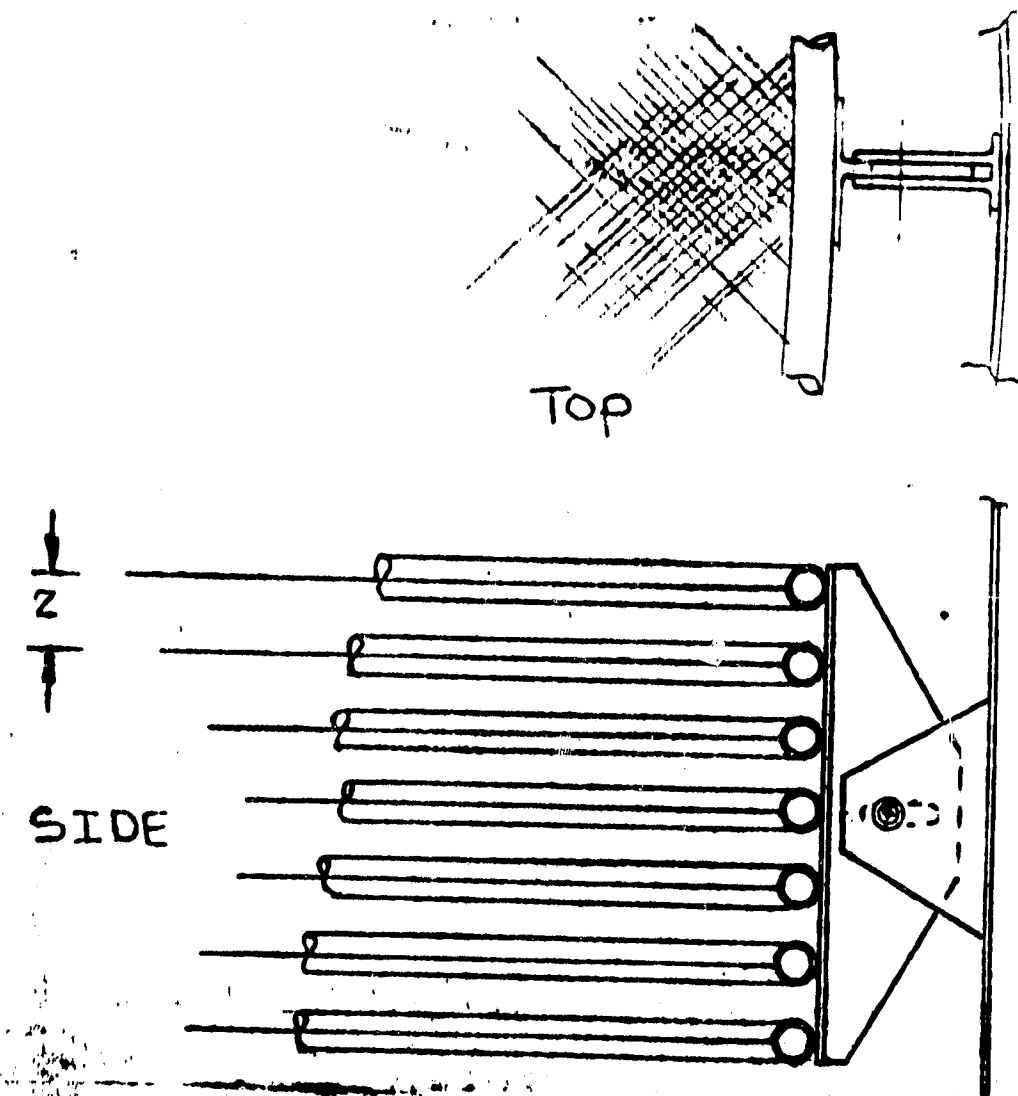


Fig. B-6. Tank Attachment for Stacked Plate Electrodes

Two approaches were used in the analyses:

a) The electrode screens were not considered to act as webs between the edge doublers.

b) The electrode screens were considered to act as webs between the edge doublers (baffles acting as "I" beams).

The analyses produced the following for the assumption a):

	<u>260-Inch Tank</u>	<u>396-Inch Tank</u>
Electrode Screen	20	46
Edge Doubler	60	139
Support Structure	<u>170</u>	<u>1710</u>
Total	250 (lbs.)	1895 (lbs.)

The weights corresponding to assumption b) are:

	<u>260-Inch Tank</u>	<u>396-Inch Tank</u>
Electrode Screen	20	46
Edge Doublers	60	139
Support Structure	<u>55</u>	<u>530</u>
Total	135 (lbs.)	715 (lbs.)

● Selection of Most Promising Configuration

The concentric cylindrical electrode with cable supports was selected as the most promising configuration because it was the lightest weight candidate by a significant margin. The point design weight for the 260-inch diameter case was used to determine the constant in Eq. (28).

$$W_b = CR^2 b/s$$

where W_b is the total baffle electrode and support weight (lbs.)
 R is the tank radius (meters)
 b is the height of the baffle (meters)
 s is the electrode spacing. (meters)

For our point design

$$W_b = 123 \text{ lbs.}$$

$$R = 130 \text{ inch} = 3.3 \text{ m}$$

$$b = 12 \text{ inch} = 0.304 \text{ m}$$

$$s = 2 \text{ inch} = 0.0508 \text{ m}$$

Thus $C = 1.86 \text{ lbs/m}^2$

Appendix C

HIGH-VOLTAGE POWER SUPPLY AND FEEDTHROUGH DESIGNS

High-Voltage Power Supply

The power supply design is a straightforward, state-of-the-art circuit consisting of a step-up transformer with the output rectified by a half-wave voltage doubler circuit. A schematic of the high-voltage DC power supply circuit is shown in Fig. C-1. A solid state inverter converts the DC input to 400 cps, 120 volt sinusoidal power. The step-up transformer boosts the inverter output to a peak voltage of one-half the required DC voltage (RMS output voltage of the step-up transformer equals $1/2\sqrt{2}$ times the required DC voltage). A voltage doubler circuit and current-limiting resistor complete the power supply circuit. The load capacitance of the dielectrophoretic device is used as a component of the doubler circuit.

The high-voltage step-up transformer is of conventional, tape wound core construction. Conventional design procedures as found in standard design manuals* were followed in the design of the transformer. Three simplifying assumptions followed in the transformer design were: (1) a 2 mil, tape wound core has a maximum magnetic field strength ($B_{\max}$) of 1 weber/m², (2) the secondary coil is wound with #44 AWG copper wire, the finest wire that can be handled practically on coil winding machines, and (3) the secondary coil has a space factor of 1/4. The 2 mil tape core has a maximum hysteresis loss of 1/2 watt/lb. of core weight when operating at 400 cps. In the 100 KV power supply design this amounts to a total core loss of less than 0.4 watt.

The capacitors are conventional molded high-voltage units with a peak rating of 30 KV. The power supply design assumes derating by a factor of 1.5. The voltage doubler circuit operates satisfactorily as long as the two capacitors are the same size within a factor of about 3. The power supply design assumes a load capacitance of 1000 pico-farads.

The high-voltage diodes are standard units with a rating of 20 KV P.I.V. These were derated by a factor of 2 for this design.

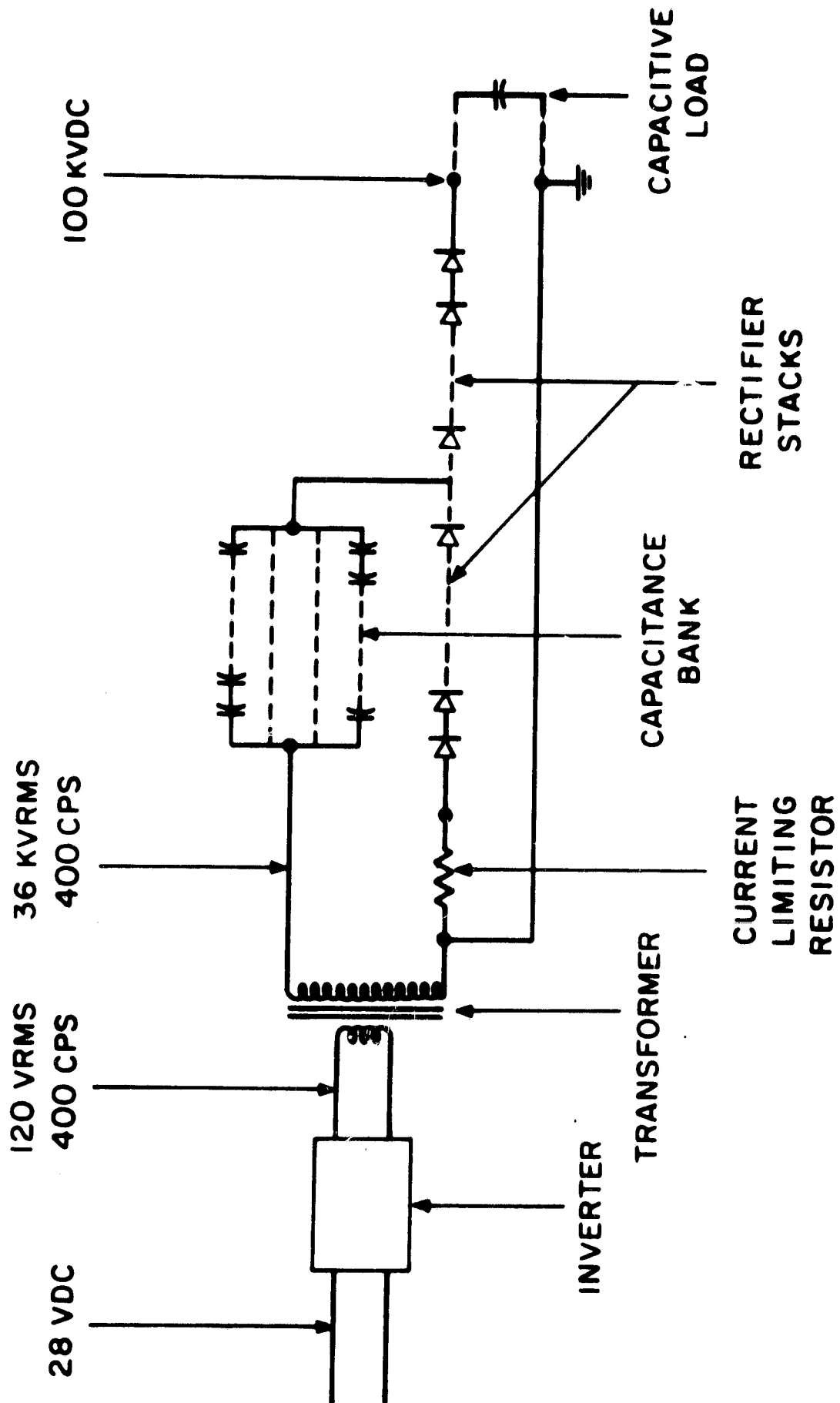


Figure C-1. High Voltage Power Supply Circuit Schematic

The resistor on the secondary output of the transformer limits the short circuit power to 25 watts. All the power supply components are rated to withstand this loading.

The entire power supply is encapsulated as a single unit in polyurethane resin. The dielectric strength of the polyurethane is 550 volts/mil. A lightweight aluminum container protects the flexible potting material from mechanical abuse. The arrangement of the 100 KV power supply is shown in detail in Fig. C-2.

The results of a component weight analysis for the power supply are summarized in Fig. C-3.

The general specifications of a 100-KV power supply are:

1. Output voltage	100 KVDC into a 1000 pf capacitive load with not more than 1.0 mw of ohmic dissipation.
2. Input voltage	28 V DC.
3. Transformer frequency	400 cps.
4. Core loss	less than 1 watt.
5. Weight	not more than 16 lb.

High-Voltage Feedthrough

The high-voltage feedthrough design consists of a Teflon insulated, high-voltage lead passing through a thin walled stainless steel tube. This tubular jacket is welded to the propellant tank on one end and to a gland-type pressure seal on the other. The length of the feedthrough is brought through the multi-layer tank insulation in a spiral to minimize radiational heat leaks. The length of the feedthrough is determined to minimize the conductive heat leak. The gland seal is maintained at ambient temperature and provides a very effective pressure sealing of the feedthrough assembly.

To analyze the heat leakage of the coaxial feedthrough design, the feedthrough can be modeled as in Fig. C-4.

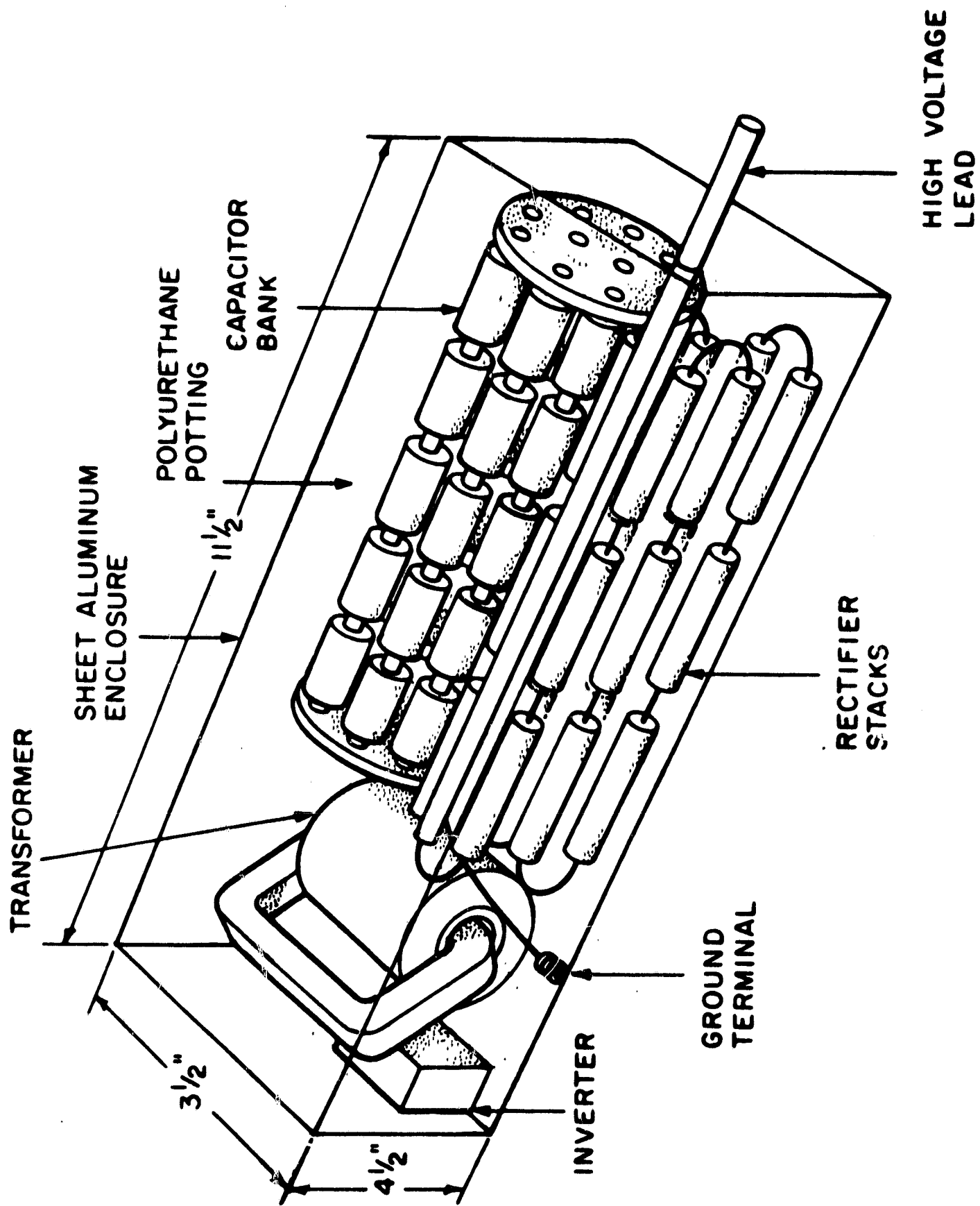


Figure C-2. General Arrangement of 100 KV DC High Voltage Power Supply

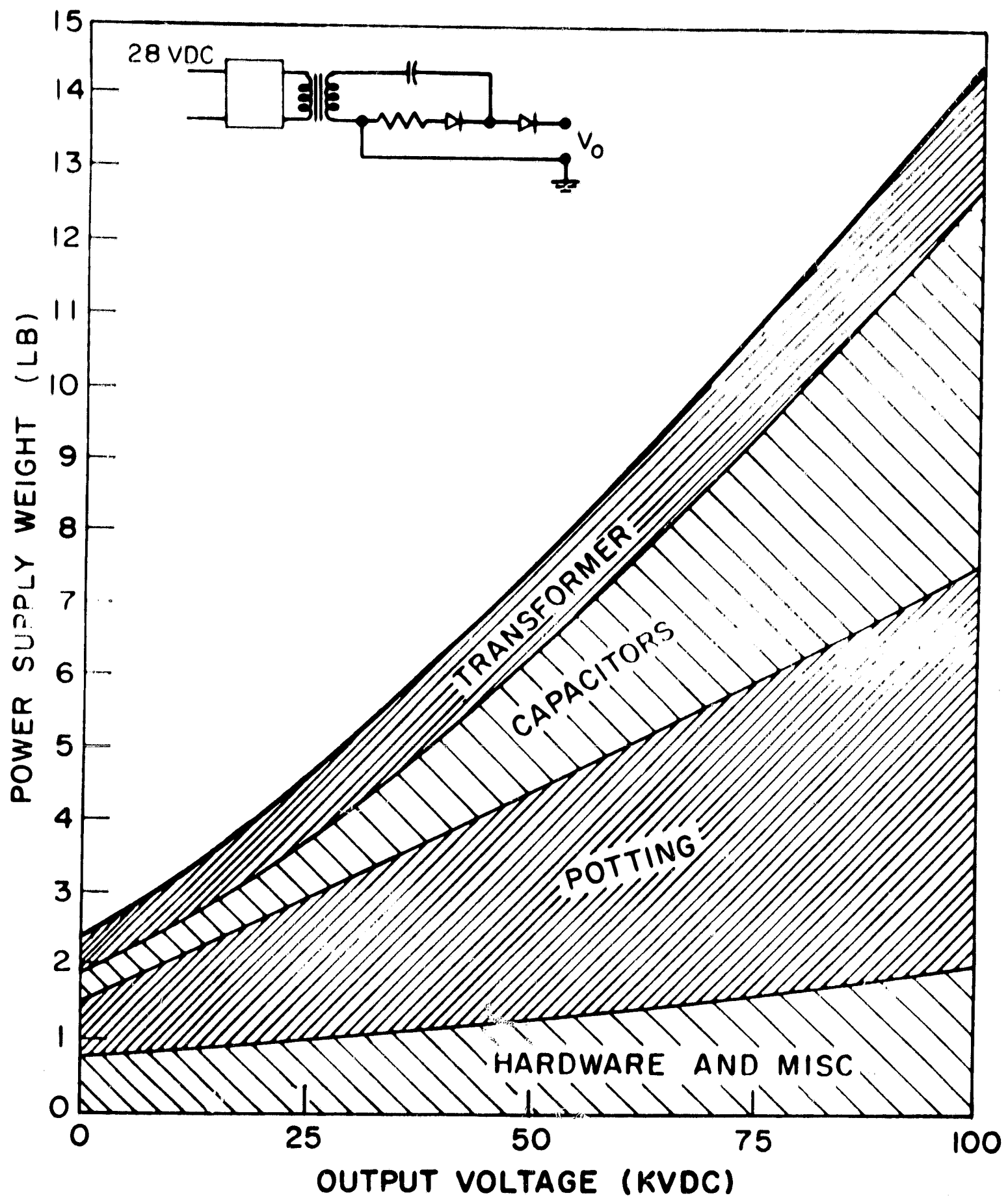


Fig. C-3. High-Voltage Power Supply Weight

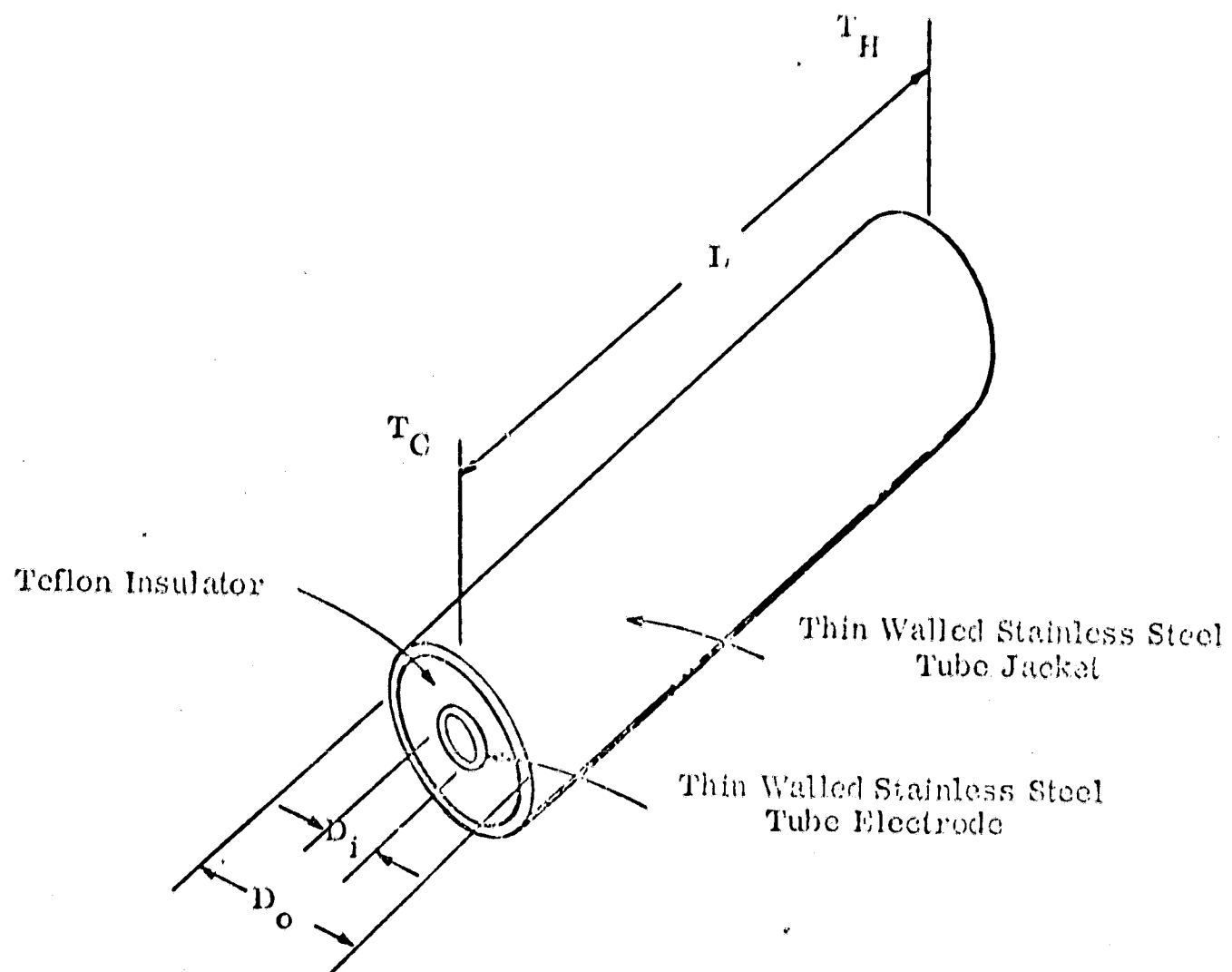


Fig. C-4. Model for Heat Leak Analysis of the Coaxial High-Voltage Feedthrough

The total heat leak through the feedthrough is given by:

$$Q = \left(\frac{A_s}{L} \int_{T_c}^{T_H} \lambda dT \right)_{\text{Steel}} + \left(\frac{A_T}{L} \int_{T_c}^{T_H} \lambda dT \right)_{\text{Teflon}} \quad (23)$$

where

- Q = heat leak
- A = cross-sectional area
- L = length
- λ = thermal conductivity
- T_c = cold-end temperature
- T_H = warm-end temperature

The conductivity integral* over the range from saturated liquid oxygen and hydrogen temperature to ambient temperature for stainless steel and Teflon is:

	$\int_{T_c}^{T_H} \lambda dT$ Watts cm °K	
Material	LH_2 (20° - 300° K)	LO_2 (90° - 300° K)
Stainless	30.437	26.24
Teflon	0.6856	0.539

*Stewart, R. B., and Johnson, V. J., "A Compendium of the Properties of Materials at Low Temperatures (Phase II), "WADD-TR-60-56 Part IV (December, 1961).

Substituting the appropriate relationships for the areas and the conductivity integrals, the heat leak is given by:

$$Q = \frac{(D_o + D_i)}{L} [210 t + 1.08 (D_o - D_i)] \quad \text{for oxygen}$$

$$Q = \frac{(D_o + D_i)}{L} [243 t + 1.37 (D_o - D_i)] \quad \text{for hydrogen}$$

where the dimensions are in inches.

Electrical tests of two coaxial feedthrough geometries have been conducted. Both test models used a 1/4" diameter center electrode. One unit had 1/8" thickness of Teflon insulation while the second model had 1/4" insulation thickness. The first unit performed satisfactorily at up to 60 KV with a factor of safety, while the second unit was conservatively rated at 120 KV (the maximum voltage limit of the laboratory power supply).

An analysis of the weight of the coaxial high-voltage feedthrough was conducted. The weights are presented as a function of the operating voltage and the design heat leak rate in Fig. C-5.

It should be noted that even at a design heat leak of 0.05 watt (less than 10 per cent of the total heat leak in a superinsulated 40" hydrogen tank) the feedthrough is only a small fraction of the total weight of the dielectrophoretic liquid expulsion system.

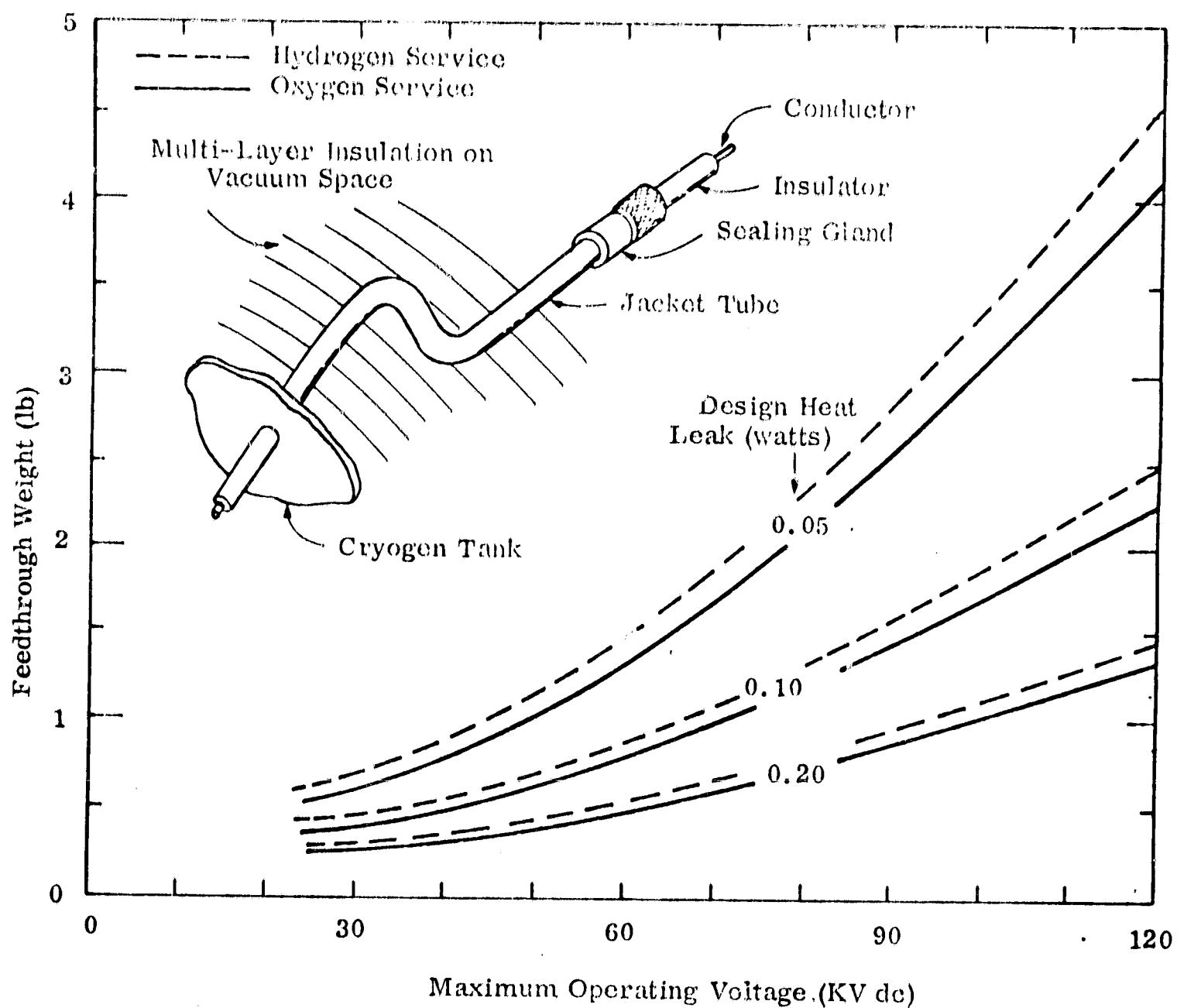


Fig. C-5. Weights of Coaxial High-Voltage Feedthrough for Dielectrophoretic Liquid Expulsion Devices for Cryogenic Tankage