NASr-21(05)

MEMORANDUM
RM-5861-NASA
DECEMBER 1968

RELATION OF THE WESTWARD DRIFT OF THE GEOMAGNETIC FIELD TO THE ROTATION OF THE EARTH'S CORE

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PREPARED FOR:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

The RAND Corporation
SANTA MONICA • CALIFORNIA

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This research is supported by the National Aeronautics and Space Administration under Contract No. NASr-21. This report does not necessarily represent the views of the National Aeronautics and Space Administration.

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This study is presented as a competent treatment of the subject, worthy of publication. The Rand Corporation vouches for the quality of the research, without necessarily endorsing the opinions and conclusions of the authors.

Published by The RAND Corporation

PREFACE

At the earth's surface the geomagnetic field drifts slowly westward with time. This drift has been observed for the past three centuries. It suggests that the earth's central core, the seat of the earth's field, rotates more slowly than the solid mantle and crust above.

The present study is one of a series intended to improve predictions of the strength of the geomagnetic field and predictions of the field patterns -- both of which dominate the distribution of the earth's radiation belts. The studies should also assist in the estimates of magnetic fields of other planets. Other recent RAND studies in this series include RM-5191-NASA, Westward Drift of the Geomagnetic Field and its Relation to Motions of the Earth's Core; RM-5192-NASA, Nature of Surface Flow in the Earth's Central Core; and RM-5193-NASA, Comparison of Estimates of Surface Fluid Motions of the Earth's Core for Various Epochs.

This work was supported by the National Aeronautics and Space Administration. The author is a graduate student in the Department of Meteorology at the University of California, Los Angeles, and a consultant to The RAND Corporation.

RELATION OF THE WESTWARD DRIFT OF THE GEOMAGNETIC FIELD TO THE ROTATION OF THE EARTH'S CORE

ABSTRACT

The concept of equating the drift of the geomagnetic field with a similar drifting motion of the earth's core is examined. The drift of the field at the earth's surface and at the core is calculated, and the two values are shown to be considerably different. Analysis of the portion of secular change remaining after westward drift effects have been removed is used to provide an estimate of the error in the drift which results from equating the drift of the field to that of the core. On this basis the best estimate of the westward drift of the core for epoch 1960 is found to be 0.13 degrees per year, with an estimated error of $\pm 0.03^{\circ}$/yr. This drift is considerably smaller than the values usually cited for the core's rotation, such as 0.18 degrees per year obtained by Bullard and others, based mainly on charted differences in surface data.

ACKNOWLEDGMENTS

I am indebted to the late E. H. Vestine for suggesting the present investigation of the westward drift of the geomagnetic field.

CONTENTS

I. INTRODUCTION

As early as 1692, Halley noted that the positions of isogonic lines on magnetic charts moved westward at about 0.5°/yr. Bullard et al. (1950) determined the apparent shift of the field along circles of latitude between the years 1905.5 and 1945, and found a resultant average westward drift rate of 0.18°/yr. Vestine (1952) estimated the westward motion of the eccentric dipole at 0.29°/yr. Yukutake (1962) performed a detailed study of the latitudinal and longitudinal dependence of the westward drift and found that most of the secular change could be accounted for by the drift. Using the Y-component of the magnetic field, he found a mean westward drift of 0.20°/yr. Yukutake (1967) has also recently indicated that an average value of about 0.36°/yr. may apply over the past thousand years or so.

Halley remarked that the earth's core appeared to rotate more slowly than its surface. Later workers have inferred that the drift is directly related to a westward motion of the electrically conducting fluid at the surface of the earth's core.

A knowledge of the rate of westward motion and of how this rate changes with time is of special interest because it may enable us to infer physical features of the core as well as the nature of the magnetic coupling of the core to the mantle, which just above the core has an electrical conductivity estimated to be about 10^{-3} that of copper (Currie, 1967). According to the model of Bullard et al. (1950), the electromagnetic drag of currents induced in the mantle by differential rotation of the core and mantle is offset by a driving torque due to interaction of the main dipole field with toroidal field diffusing out from beneath the core surface. The westward drift should be proportional to the strength of the toroidal field within the core. Changes in the westward drift, if they are indicative of changes in the angular momentum of the entire core, will point to an imbalance of the driving and dragging torques, which may result in a measurable change in the rotation of the mantle (Vestine, 1952).

Lowes (1967) has shown that the calculated drift of the geomagnetic field does not accurately represent the rotation of the core.

There is a significant large-scale component of secular change which cannot be represented by a simple drift, and this circumstance introduces an uncertainty of perhaps $\pm 0.08^{o}$/yr in relating the drift of the field to the rotation of the core. Lowes concluded that small changes in the calculated value of the field drift should not be interpreted as changes in the rate of rotation of the core.

In the present paper, it will be shown that we may considerably reduce the uncertainty in determining the rotation of the core by making more effective use of the higher-order harmonics of the geomagnetic field in the calculation of drift. This drift is calculated at the earth's surface and at the core by the usual method of minimizing the residual secular variation, and is also calculated by a method which treats the field harmonics in a statistical sense and tends to minimize the difference between the calculated field drift and the rotation of the core. It will be seen that the drift of the field at the surface of the earth is likely to be only roughly representative of the rotation of the core.

In order to discuss these matters more fully, we first consider the westward drift of magnetic fields at the surface of the core in relation to motions of the core fluid.

II. CORE MOTIONS AND FIELD DRIFT

Within the core, the time rate of change of the magnetic field is determined by the equation (Elsasser, 1946)

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) + \frac{1}{4\pi\sigma} \nabla^2 \underline{B} \tag{1}$$

where σ is the electrical conductivity of the fluid. In the present paper it is first assumed that σ is large enough so that the diffusion term $1/4\pi\sigma\ \nabla^2\underline{B}$ can be neglected. Secondly, the velocity at the core surface is assumed to be composed of a simple rotation about the earth's geographic axis, plus additional random motions. The shortcomings of these assumptions will be discussed in Section V. It might be noted here, however, that these assumptions cannot account for the facts that the dipole part of the geomagnetic field tends to align itself with the earth's axis of rotation and fails to drift as rapidly as the nondipole field.

The following considerations may thus be best applicable to the non-dipole field. Using spherical coordinates (r,θ,λ), we can write the horizontal components of the velocity as

$$\begin{aligned} v_\lambda &= \omega b \sin\theta + v'_\lambda \\ v_\theta &= v'_\theta \end{aligned} \tag{2}$$

where $\underline{v}'$ is the random component of the velocity, b is the radius of the core, and ω is the rate of rotation of the surface of the core with respect to the solid earth. The radial component of Eq. (1) at the surface of the core (where $v_r = 0$) becomes

$$\frac{\partial B_r}{\partial t} + \omega \frac{\partial B_r}{\partial \lambda} = - \frac{B_r}{b \sin\theta}\left[\frac{\partial(\sin\theta v'_\theta)}{\partial\theta} + \frac{\partial v'_\lambda}{\partial\lambda}\right] - \frac{v'_\theta}{b}\frac{\partial B_r}{\partial\theta} - \frac{v'_\lambda}{b \sin\theta}\frac{\partial B_r}{\partial\lambda}. \tag{3}$$

Multiplying by $\partial B_r/\partial\lambda$ and integrating over the surface of the core yields

$$\int_s \frac{\partial B_r}{\partial t}\frac{\partial B_r}{\partial \lambda}\,da + \int_s \left(\frac{\partial B_r}{\partial \lambda}\right)^2 da$$

$$= \int_s \left\{ -\frac{B_r}{b\sin\theta}\left[\frac{\partial(\sin\theta v'_\theta)}{\partial\theta} + \frac{\partial v'_\lambda}{\partial\lambda}\right] - \frac{v'_\theta}{b}\frac{\partial B_r}{\partial\theta} - \frac{v'_\lambda}{b\sin\theta}\frac{\partial B_r}{\partial\lambda}\right\}\frac{\partial B_r}{\partial\lambda}\,da. \tag{4}$$

The "randomness" of $\underline{v}'$, which has not yet been defined, will be interpreted to mean that $\underline{v}'$ and B_r are independent in a statistical sense, and that the mean values of v'_λ and v'_θ at all points are zero. Because ω may change significantly from epoch to epoch, "mean" cannot be taken as a time average, but rather must be taken as the average over a statistical ensemble of earth cores, all of which have the same $\omega(t)$. Under the assumption of independence, the mean value of the product of a function of B_r and a function of $\underline{v}'$ will be equal to the product of the mean values of the functions. If we take the mean value of Eq. (4), we see that the right-hand side becomes zero, and thus

$$\omega = -\frac{\overline{\int_s \left[\frac{\partial B_r}{\partial t}\frac{\partial B_r}{\partial\lambda}\right] da}}{\overline{\int_s \left[\left(\frac{\partial B_r}{\partial\lambda}\right)^2\right] da}} \tag{5}$$

where the bar over a quantity denotes a mean value.

An exact calculation of ω from Eq. 5 is impossible, as we do not have a full ensemble of earth cores to measure. However, in Section III an estimate of ω is obtained by not taking mean values. An estimate of the error incurred is made in Section IV.

III. CALCULATIONS OF THE WESTWARD DRIFT

Lowes (1967) has shown that the usual methods of determining the westward drift are equivalent to a least-squares fitting of $\partial H/\partial t$ to $\omega\ \partial H/\partial\lambda$ over the earth's surface, where the nondipole parts of the magnetic potential, V, and of the three components of $\underline{B}$ have variously been used in place of H. This method can be outlined as follows.

A quantity $U(\theta,\lambda,t)$ defined on the surface of a sphere will have a time variation $\dot{U}$ which can be divided into drift and nondrift components. Letting W_U be the eastward angular drift rate,

$$\dot{U}_{drift} = - W_U \frac{\partial U}{\partial \lambda} \tag{6}$$

$$\dot{U}_{nondrift} = \frac{\partial U}{\partial t} + W_U \frac{\partial U}{\partial \lambda} \; . \tag{7}$$

W_U can be defined so as to make the integral of $[\dot{U}_{nondrift}]^2$ over the surface of the sphere a minimum. Then

$$\frac{d}{dW_U} \int_s \left(\frac{\partial U}{\partial t} + W_U \frac{\partial U}{\partial \lambda} \right)^2 da = 0$$

which gives

$$W_U = - \frac{\displaystyle\int_s \frac{\partial U}{\partial t} \frac{\partial U}{\partial \lambda}\, da}{\displaystyle\int_s \left(\frac{\partial U}{\partial \lambda}\right)^2 da} \; . \tag{8}$$

If we place $U = B_r$ at the core, then comparison with Eq. (5) shows $W_B(r = b)$ to be an estimate of ω. It will also be of interest to investigate the significance of W when B_r at the surface of the earth is used, and when the magnetic potential V is used instead of B_r.

The spherical harmonic expansion for V is

$$V(r,\theta,\lambda,t) = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \left[g_n^m(t) \cos m\lambda + h_n^m(t) \sin m\lambda\right] P_n^m(\cos\theta) \tag{9}$$

where a is the radius of the earth and P_n^m is Schmidt normalized. If the conductivity of the earth's mantle is small enough, the geomagnetic field can be represented by this potential function down to the surface of the core. Using (8), with U replaced by the series expansion of V in (9), we obtain

$$W_V = - \frac{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{2n+1} \left(\frac{a}{r}\right)^{2n} m\left(\dot{g}_n^m h_n^m - \dot{h}_n^m g_n^m\right)}{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{1}{2n+1} \left(\frac{a}{r}\right)^{2n} m^2\left(g_n^{m2} + h_n^{m2}\right)} . \tag{10}$$

The series expansion for B_r is

$$B_r = - \frac{\partial V}{\partial r} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} (n+1) \left(\frac{a}{r}\right)^{n+2} \left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right) P_n^m \tag{11}$$

and its drift rate is

$$W_B = - \frac{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{(n+1)^2}{2n+1} \left(\frac{a}{r}\right)^{2n} m\left(\dot{g}_n^m h_n^m - \dot{h}_n^m g_n^m\right)}{\sum_{n=1}^{\infty} \sum_{m=1}^{n} \frac{(n+1)^2}{2n+1} \left(\frac{a}{r}\right)^{2n} m^2\left(g_n^{m2} + h_n^{m2}\right)}. \tag{12}$$

It is seen that the expressions (10) and (12) for W_V and W_B are similar in form, but that (12) places more emphasis on higher-degree terms, owing to the factor $(n+1)^2$. It is also seen that the calculated drifts W_V and W_B depend on r. In order to demonstrate the nature of this r-dependence, we shall examine the drifts of different components of the field.

The mean drift of all harmonics of an individual degree n can be found by setting all g_n^m's, h_n^m's, $\dot{g}_n^m$'s and $\dot{h}_n^m$'s equal to zero except those of the degree under consideration. This drift, w(n), is the same whether (10) or (12) is used, and is independent of r:

$$w(n) = -\frac{\sum_{m=1}^{n} m\left(\dot{g}_n^m h_n^m - \dot{h}_n^m g_n^m\right)}{\sum_{m=1}^{n} m^2\left(g_n^{m2} + h_n^{m2}\right)} . \tag{13}$$

In order to obtain quantitative relations showing which degrees of V or B_r are most important in determining the drift of the total field at different values of r, we can define weighting factors $i_U(n)$ so that

$$W_U = \sum_n i_U(n)w(n) \tag{14}$$

where

$$\sum_n i_U(n) = 1. \tag{15}$$

It is seen from (10) and (13) that

$$i_V(n) = \frac{\frac{1}{2n+1}\left(\frac{a}{r}\right)^{2n} \sum_{m=1}^{n} m^2\left(g_n^{m2} + h_n^{m2}\right)}{\sum_k \left[\frac{1}{2k+1}\left(\frac{a}{r}\right)^{2k} \sum_{m=1}^{k} m^2\left(g_k^{m2} + h_k^{m2}\right)\right]} \tag{16}$$

and from (12) and (13) that

$$i_B(n) = \frac{\frac{(n+1)^2}{2n+1}\left(\frac{a}{r}\right)^{2n} \sum_{m=1}^{n} m^2\left(g_n^{m2} + h_n^{m2}\right)}{\sum_k \left[\frac{(k+1)^2}{2k+1}\left(\frac{a}{r}\right)^{2k} \sum_{m=1}^{k} m^2\left(g_k^{m2} + h_k^{m2}\right)\right]} . \tag{17}$$

These weighting factors are not dependent on the secular change coefficients. The calculations below show that the series

$$\sum_{n=1}^{\infty} i_V(n) \quad \text{and} \quad \sum_{n=1}^{\infty} i_B(n)$$

do not converge rapidly at the core, where in fact the values of both $i_V(n)$ and $i_B(n)$ increase with n up to at least n = 6. Extension of the calculations of the drift to higher values of n, however, would involve increased error, due to errors in the measurements of the secular change coefficients, and to a breakdown of the assumption that V and $\dot{V}$ can be extrapolated to the core. Both of these effects become increasingly important with higher n. The assumption is made here that the drift behavior of field components up to n = 6 is representative of the entire field.

Table 1 lists the values of i(n), w(n), and W calculated from the data of Cain et al. (1967) for epoch 1960. The drifts are expressed in degrees/year westward rather than radians/year eastward as in Eqs. (10), (12), and (13). Columns 1, 3, 5, and 7 are for the drift of the total field, and columns 2, 4, 6, and 8 are calculated for the nondipole field by leaving out from all summations the terms in n = 1. The explanations of Δw and of column 9 are given in Section IV; the results contained in Table 1 are discussed in Section V.

Table 1

WEIGHTING FACTORS, VALUES OF THE WESTWARD DRIFT, AND ESTIMATED ERROR FOR EPOCH 1960[a]

n	w(n) in °/yr westward	Δw(n) in °/yr	i(n)								
			1	2	3	4	5	6	7	8	9
			a/r = 1				a/r = 1.8355				
			Using V		Using B_r		Using V		Using B_r		
			Whole Field	Non-Dipole	Whole Field	Non-Dipole	Whole Field	Non-Dipole	Whole Field	Non-Dipole	Minimizing ΔW
1	0.064	0.102	.614		.323		.097		.014		.068
2	0.234	0.153	.222	.576	.264	.390	.118	.130	.040	.040	.030
3	0.096	0.075	.124	.322	.262	.387	.222	.246	.133	.135	.125
4	0.158	0.043	.030	.077	.098	.144	.178	.197	.167	.169	.384
5	0.013	0.088	.006	.016	.029	.042	.122	.135	.165	.167	.091
6	0.157	0.048	.004	.010	.025	.037	.263	.291	.482	.489	.302
W in °/yr westward			0.109	0.180	0.127	0.158	0.126	0.133	0.127	0.128	0.132
ΔW in °/yr			0.072	0.092	0.056	0.067	0.032	0.034	0.031	0.031	0.027

[a] Data of Cain, et al. (1967).

IV. ESTIMATED ERROR

It was mentioned that the drift of the geomagnetic field at a given epoch, as expressed in Eq. (8), is only an estimate of the drift of fluid in the core, as expressed in Eq. (5). Lowes (1967) pointed out that because the features of the geomagnetic field at the earth's surface are large-scale, the drift of the field may differ by a considerable amount from the drift of the core. In this section, this reasoning is extended so as to be applicable to the field at the core.

Consider again the function U, which may represent either V or B_r. Assume that $\dot{U}_{nondrift}$ is random and statistically independent of U. We can write

$$(\Delta W_U)^2 \equiv \overline{\left(W_U - \omega\right)^2} = \overline{\left[\sum_n i_U(n)(w(n) - \omega)\right]^2} \qquad (18)$$

where $(\Delta W_U)^2$ is the mean square deviation of W_U from ω. Writing Eq. (18) in terms of spherical harmonic coefficients gives

$$(\Delta W_U)^2 = \overline{\left[\sum_n i_U(n) \left\{ - \frac{\sum_{m=1}^{n} m \left[(\dot{g}_n^m + m\omega h_n^m)h_n^m - (\dot{h}_n^m - m\omega g_n^m) g_n^m\right]}{\sum_{m=1}^{n} m^2 \left(g_n^{m2} + h_n^{m2}\right)} \right\}\right]^2} \cdot \qquad (19)$$

But $(\dot{g}_n^m + m\omega h_n^m)$ and $(\dot{h}_n^m - m\omega g_n^m)$ are proportional to the spherical harmonic coefficients of $\dot{U}_{nondrift}$. Because of the randomness of $\dot{U}_{nondrift}$,

$$\overline{\left(\dot{g}_n^m + m\omega h_n^m\right)\left(\dot{g}_{n'}^{m'} + m'\omega h_{n'}^{m'}\right)} = 0 \text{ unless } n = n',\ m = m'$$

$$\overline{\left(\dot{h}_n^m - m\omega g_n^m\right)\left(\dot{h}_{n'}^{m'} - m'\omega g_{n'}^{m'}\right)} = 0 \text{ unless } n = n',\ m = m' \qquad (20)$$

$$\overline{\left(\dot{g}_n^m + m\omega h_n^m\right)\left(\dot{h}_{n'}^{m'} - m'\omega g_{n'}^{m'}\right)} = 0 \text{ for all } n, n', m, m'.$$

Remembering also that $\dot{U}_{nondrift}$ is independent of U, we can rewrite Eq. (19) as

$$(\Delta W_U)^2 = \sum_n \sum_{m=1}^{n} \left\{ \overline{\left[\frac{i_U(n) m h_n^m}{\sum_{j=1}^{n} j^2 \left(g_n^{j2} + h_n^{j2}\right)} \right]^2} \, \overline{\left(\dot{g}_n^m + m\omega h_n^m\right)^2} + \overline{\left[\frac{i_U(n) m g_n^m}{\sum_{j=1}^{n} j^2 \left(g_n^{j2} + h_n^{j2}\right)} \right]^2} \, \overline{\left(\dot{h}_n^m - m\omega g_n^m\right)^2} \right\} . \tag{21}$$

It is a consequence of the assumed isotropic randomness of $\dot{U}_{nondrift}$ and of the Schmidt normalization of P_n^m that

$$\overline{\dot{g}_n^{02}} = \overline{\left(\dot{g}_n^1 + \omega h_n^1\right)^2} = \overline{\left(\dot{h}_n^1 - \omega g_n^1\right)^2} = \ldots = \overline{\left(\dot{g}_n^n + n\omega h_n^n\right)^2} = \overline{\left(\dot{h}_n^n - n\omega g_n^n\right)^2}$$

$$= \frac{1}{2n+1} \sum_{m=0}^{n} \left[\overline{\left(\dot{g}_n^m + m\omega h_n^m\right)^2} + \overline{\left(\dot{h}_n^m - m\omega g_n^m\right)^2} \right], \text{ where } h_n^0 \equiv 0. \tag{22}$$

Thus Eq. (21) becomes

$$(\Delta W_U)^2 = \sum_n \left\{ \overline{\left[\frac{i_U^2(n)}{\sum_{m=1}^{n} m^2 \left(g_n^{m2} + h_n^{m2}\right)} \right]} \frac{1}{2n+1} \sum_{m=0}^{n} \left[\overline{\left(\dot{g}_n^m + m\omega h_n^m\right)^2} + \overline{\left(\dot{h}_n^m - m\omega g_n^m\right)^2} \right] \right\} \tag{23}$$

In analogy to the statistical theory of determining variance, it can be shown that we may replace ω by w(n) in Eq. (23) if we also replace the factor $(2n + 1)^{-1}$ by $(2n)^{-1}$, giving

$$(\Delta W_U)^2 = \sum_n \left\{ \overline{\left[\frac{i_U^2(n)}{\sum_{m=1}^{n} m^2 \left(g_n^{m2} + h_n^{m2} \right)} \right]} \frac{1}{2n} \sum_{m=0}^{n} \left[\overline{\left(\dot{g}_n^m + mw(n)h_n^m \right)^2} + \overline{\left(\dot{h}_n^m - mw(n)g_n^m \right)^2} \right] \right\} . \tag{24}$$

An estimate of $(\Delta W_U)^2$ can be obtained by not taking mean values. If we look first at the mean square deviation of $w(n)$ from ω, which is obtained by setting one of the $i(n)$'s equal to one and all of the rest equal to zero, it is seen that

$$[\Delta w(n)]^2 \approx \frac{1}{2n} \frac{\sum_{m=0}^{n} \left[\left(\dot{g}_n^m + mw(n)h_n^m \right)^2 + \left(\dot{h}_n^m - mw(n)g_n^m \right)^2 \right]}{\sum_{m=1}^{n} m^2 \left(g_n^{m2} + h_n^{m2} \right)} . \tag{25}$$

We can then write

$$[\Delta W_U]^2 \approx \sum_n i_U^2(n) [\Delta w(n)]^2 . \tag{26}$$

Equations (25) and (26) were used to calculate the values of $\Delta w(n)$ and ΔW_U listed in Table 1. It is of interest to find a set of $i(n)$'s which will make ΔW a minimum. According to statistical theory, these $i(n)$'s, which will be written $i_o(n)$, are given by

$$i_o(n) = \frac{[\Delta w(n)]^{-2}}{\sum_k [\Delta w(k)]^{-2}} \tag{27}$$

The values of $i_o(n)$, W_o, and ΔW_o are listed in column 9 of Table 1.

V. DISCUSSION AND CONCLUSIONS

The drift values of Table 1 show an encouraging degree of consistency, in light of the simplicity of the assumptions made. The value obtained here for the drift of the nondipole V-field at the earth's surface, 0.180°/yr, agrees well with the drift calculated similarly by other workers, and the error in this case, 0.092°/yr, is similar to the estimate of 0.08°/yr obtained by Lowes. In Sections II and III it was shown that the drift of the B_r-field at the core appears to give an estimate of the drift of the fluid at the surface of the core. Accordingly, we would expect all multipole components of the geomagnetic field to have the same mean westward drift. Combining the drifts of the multipoles in such a way as to minimize error (column 9) is believed to provide the best estimate of ω (0.132 ± 0.027 degrees/yr). This value agrees surprisingly well with the drift of the whole B_r-field at the core ($0.127 \pm 0.031^{\circ}$/yr), an agreement we would expect if the simplified theory used were valid. The drifts $w(n)$ of the individual components of the field show a considerable spread, indicating that nondrift motions in the core are important. However, all of the component drifts except $w(5)$ lie within $\Delta w(n)$ of the value 0.13°/yr.

Inspection of the weighting factors shows that the lower-order multipole fields dominate the drift of the total field at the surface of the earth, whereas the higher-order multipole fields dominate at the core. Because $w(2)$ is considerably larger than any other component drift, and is weighted heavily in determining the drifts of the nondipole V- and B_r-fields at the surface of the earth, the drifts of the nondipole surface fields are likely to provide too large an estimate of ω. The fact that the values of ΔW for these drifts are large gives further reason for the exercise of caution in regarding these drifts as accurate representations of the rotation of the core.

An attempt was also made to determine the time variation of the rotation of the core by calculating W_o and W_B ($r = b$) at different epochs. The results were inconclusive, as the values obtained from the data of different authors for the same epoch sometimes differed by an amount comparable to ΔW. It is believed that these discrepancies arise because the errors in the higher-order secular change coefficients are not random (see item 5 below). For example, when the coefficients are obtained from hand-drawn charts, which are subject to a certain amount of smoothing, higher-order coefficients are likely to be biased towards having small absolute values. Although this may tend to minimize the magnitude of the error between true and given values of the coefficients, the error is not random, and the calculated values of W are likely to be too small. However, two features were noted in the drifts calculated from the data of Vestine et al. (1947) for epochs 1912.5, 1922.5, 1932.5, and 1942.5. Firstly, the component drift w(2) was considerably larger than all other component drifts at all epochs. Because w(2) dominates the nondipole field drift at the earth's surface, the non-dipole surface field has probably drifted faster than the core throughout this century. Secondly, ΔW_o and $\Delta W_B(r=b)$ are comparable to time changes of W_o and $W_B(r=b)$, so that little can be safely said about changes in the rotation of the core on the basis of these drifts.

The following possible sources of error in the determination of ω from the drifting magnetic field might be noted:

1. Diffusion of the magnetic field within the core is probably not entirely negligible, even though the electric conductivity is high. In fact, it is a consequence of the high conductivity of the fluid that velocity shears may produce strong distortions of the magnetic field. These distortions may build up until diffusion tends to dissipate them, in which case diffusion would not be negligible. If the mean rotation of the core varies with depth, the drift of larger-scale features of $\underline{B}$ at the core surface, such as the drift of the dipole field, may tend to reflect the average rotation over a certain depth, rather than the rotation of the core surface alone.

2. The velocities in the core may be poorly represented by a simple rotation with a superimposed random component. Because the $\underline{B}$ configuration at a particular epoch has been determined by convection (and diffusion) of field lines due to $\underline{v}'$, it is not unlikely that $\underline{B}$ and $\underline{v}'$ show a certain degree of correlation. Furthermore, the Lorentz force on the fluid, $\underline{J} \times \underline{B}$, may tend to produce a velocity pattern which is related to the magnetic field. Both of these effects would be present if

hydromagnetic oscillations such as those suggested by Hide (1966) are important in the earth's core. As with diffusion, this source of error may affect larger-scale features more strongly.

3. Finite electrical conductivity of the mantle prevents the magnetic field from having a force-free configuration, so that the use of Eqs. (9) and (11) for V and B_r within the mantle is not strictly valid. The effect of a conducting mantle is to partly mask the more rapidly changing (e.g., more rapidly drifting) components of the field, resulting in a field at the surface of the earth which tends to drift more slowly than the core.

4. Permanent magnetization of portions of the earth's crust will produce a nondrifting component of the observed field in the higher-order terms, also resulting in a calculated net field which tends to drift more slowly than the core.

5. Errors in the given coefficients for the main field and, more importantly, for the secular change field, will give rise to error in the drift calculations. However, it can be seen that the effect of random error in the secular change coefficients will be included in the determination of ΔW and will not produce a mean increase or decrease of the calculated drift. Any errors in the data of Cain et al. (1967) appear to be small enough to make only a minor contribution to ΔW.

6. Neglect of data for $n > 6$ gives rise to error in the calculation of the drift of the total magnetic field, but any resulting loss of accuracy is probably more than offset by the error which would arise from effects 3, 4, and 5 mentioned above. However, the former error may be important in calculating the field drift at the core. For example, a termination of the series at $n = 5$ gives $W_B(r = b) = 0.099^\circ/\text{yr}$, as opposed to the value $0.127^\circ/\text{yr}$ calculated with terms of $n = 6$ included, and a similar discrepancy could occur between the value $0.127^\circ/\text{yr}$ and that for the drift of the entire field.

The influence of the above-named sources of error, except 3 and 4, is probably best minimized by calculating the drift in the method used for W_o (column 9 of Table 1). The weighting factors for W_o are not dependent on the choice of r, but rather are chosen to give most weight to those degrees of n whose multipole drifts show greatest internal consistency, i.e., whose mean-square errors are small, as is the case for

$w(4)$ and $w(6)$ in the present calculations. The value obtained for ΔW_o is probably somewhat underestimated, as it does not fully take into account all sources of error listed above.

In most dynamo theories of the geomagnetic field (for example, Bullard, 1949; Bullard and Gellman, 1954; Elsasser, 1956; Braginskiy, 1964; Malkus, 1968), the westward drift is an important experimental parameter. In the present paper it has been shown that the drift of the nondipole field at the earth's surface may require reinterpretation in terms of the likely drift patterns at the core. It is suggested that the value used for the mean westward rotation of the fluid at the surface of the core should be somewhat less (about 0.13^{o}/yr) than the usual value (about 0.18^{o}/yr), which represents the drift of the field at the earth's surface. Although there remains an uncertainty of at least 0.03^{o}/yr in the determination of the core's rotation, this uncertainty is considerably less than that contained in the usual drift calculations (about 0.09^{o}/yr).

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