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Technical Report 32-1312

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Preface

The work described in this report was performed by the Systems Division of the Jet Propulsion Laboratory.

Contents

Abstract

A systematic procedure is described for the determination of coefficients in linear smoothing formulas and related derivative estimation formulas, and coefficients are tabulated for a selection of commonly useful formulas of this type.

Polynomial Smoothing Formulas and Derivative Formulas for One or Two Independent Variables

I. Introduction

The first part of this report discusses the mathematical basis of smoothing formulas and related derivative estimation formulas, with special emphasis on polynomial fitting in one or two independent variables at equally spaced abscissas. The task of constructing smoothing formulas is shown to reduce to that of solving certain linear least squares problems. A method for solving linear least squares problems by computer can then be employed to obtain the desired formulas.

The appendixes provide tables of smoothing formulas and derivative formulas for least squares polynomial fitting at equally spaced abscissas. In the case of one independent variable, smoothing formulas are given for each odd number m of points up to 11 and for each degree n of polynomial approximation such that $1 \leq n < m$. First and second derivative formulas for unit spacing are included. In the case of two independent variables, smoothing formulas are given for 3×3, 3×5, and 5×5 planar arrays of points with linear and quadratic approximations. Derivative formulas are provided for all non-zero partial derivatives.

The calculations for this report were made on an IBM 7094 machine with the help of a linear least squares program LSQSOL[1] developed at the Jet Propulsion Laboratory (JPL) by R. J. Hanson. A brief concluding section explains how this program can be used to develop smoothing formulas for more than two variables and smoothing formulas based on other sets of functions.

II. Formulas for One Independent Variable

A. Smoothing Formulas

Let $x_1, \cdots, x_m$ be equally spaced values of one independent variable at which data $y_1, \cdots, y_m$ are given. Let n satisfy $0 \leq n < m$. Consider the problem of computing approximating values $P(x_1), \cdots, P(x_m)$ assumed by a polynomial P of degree n at most which minimizes the expression

$$\sigma(P) = \sum_{i=1}^{m} [P(x_i) - y_i]^2 \tag{1}$$

This problem can be solved by means of smoothing formulas without constructing the actual polynomial.

[1]Write-ups for LSQSOL, COVLSQ, and SEQLSQ/SEQLQ2, April 24, 1968.

A polynomial smoothing formula of degree n for m points is a matrix **C** such that the product of **C** with a column vector whose entries are $y_1, \cdots, y_m$ is the column vector whose entries are the values $P(x_1), \cdots, P(x_m)$ of the least squares fitted polynomial of degree $\leq n$. The matrix **C** must be independent of the data $y_1, \cdots, y_m$. The existence of such a smoothing formula for each m and n, $(n < m)$, will be shown to follow from known facts about the solution of linear least squares problems. (For another treatment see Hildebrand, Ref. 1.) Also it will be shown that **C** is independent of the abscissas $x_1, \cdots, x_m$, as long as these are evenly spaced.

We write $P(x) = b_0 + b_1 x + \cdots + b_n x^n$, where the coefficients are to be determined so that $\sigma(P)$ is minimized. Let **b** and **y** be two column vectors with entries $b_0, \cdots, b_n$ and $y_1, \cdots, y_m$ respectively, and let $\mathbf{A} = (a_{ij})$ be the $m \times (n+1)$ matrix such that, for each $i = 1, \cdots, m$,

$$a_{ij} = \begin{cases} 1 & \text{if } j = 1 \\ (x_i)^{j-1} & \text{if } 1 < j \leq (n+1) \end{cases} \tag{2}$$

The product **Ab** is the column vector $\hat{\mathbf{y}}$ with entries $P(x_1), \cdots, P(x_n)$. Evidently, the problem of minimizing Eq. (1) by a proper choice of P is equivalent to that of minimizing the Euclidean length of the vector

$$\mathbf{Ab} - \mathbf{y}$$

by a proper choice of **b**. The latter problem is known to have the solution

$$\mathbf{b} = \mathbf{A}^+\mathbf{y}$$

where $\mathbf{A}^+$ is the pseudoinverse of the matrix **A**. (See Ref. 2.) The values of the minimizing polynomial are then given by

$$\hat{\mathbf{y}} = \mathbf{Ab} = \mathbf{AA}^+\mathbf{y}$$

Thus the smoothing formula **C** is the matrix $\mathbf{AA}^+$. It is independent of the data **y**, since **A** and $\mathbf{A}^+$ are defined only in terms of $x_1, \cdots, x_m$.

It should be noted that if $n < m$, there is only one polynomial of degree $\leq n$ which minimizes σ. For suppose P and Q are two such polynomials. Define a new polynomial R_t for each $0 \leq t \leq 1$ by

$$R_t(x) = tP(x) + (1-t)Q(x)$$

Now

$$\sigma(R_t) = \sum_{i=1}^{m} [tP(x_i) + (1-t)Q(x_i) - y_i]^2$$

so that

$$\frac{d^2}{dt^2}\sigma(R_t) = 2\sum_{i=1}^{m} [P(x_i) - Q(x_i)]^2$$

If $P(x_i) \neq Q(x_i)$ for some i, then $(d^2/dt^2)\,\sigma(Rt) > 0$ and the graph of R_t for $0 \leq t \leq 1$ is concave upward. This means that $\sigma(R_{1/2}) < \sigma(P)$, contradicting the fact that P minimizes σ. Hence, it must be the case that $P(x_i) = Q(x_i)$ for $i = 1, \cdots, m$. The two polynomials P and Q of degree $n < m$ agree at m points; therefore, they are identical.

The results of the previous paragraph can be used to show that the smoothing formula **C** is independent of the points $x_1, \cdots, x_m$ as long as there is even spacing.

Let $\bar{x}_1, \cdots, \bar{x}_m$ be any m equally spaced points (not necessarily with the same spacing as $x_1, \cdots, x_m$) and let $\bar{\mathbf{C}}$ be a smoothing formula of degree $(n < m)$ for use with data given at $\bar{x}_1, \cdots, \bar{x}_m$. It suffices to show that $\mathbf{Cy} = \bar{\mathbf{C}}\mathbf{y}$ for an arbitrary data vector **y**, since this will imply that $\mathbf{C} = \bar{\mathbf{C}}$. For an arbitrary **y**, let σ be defined as in Eq. (1) and define $\bar{\sigma}$ by

$$\bar{\sigma}(Q) = \sum_{i=1}^{m} [Q(\bar{x}_i) - y_i]^2$$

for any polynomial Q. It is possible to define a one-to-one mapping from the set of all polynomials of degree $\leq n$ onto itself, the image of each P being denoted by $\bar{P}$, such that

$$P(x_i) = \bar{P}(\bar{x}_i) \tag{3}$$

for $i = 1, \cdots, m$ and all P. Specifically, $\bar{P}$ is obtained from P by the formula

$$\bar{P}(\bar{x}) = P\left(x_1 + \frac{x_2 - x_1}{\bar{x}_2 - \bar{x}_1}\,[\bar{x} - \bar{x}_1]\right)$$

Equation (3) immediately implies

$$\sigma(P) = \bar{\sigma}(\bar{P})$$

for all P. From this it follows that $\bar{\sigma}$ has the same minimal value as σ over polynomials of degree $\leq n$. Let P_0 be the unique polynomial which minimizes σ (uniqueness is assured by the previous discussion). Then $\bar{P}_0$ is the unique polynomial minimizing $\bar{\sigma}$. The entries of $\mathbf{Cy}$ and $\bar{\mathbf{C}}\mathbf{y}$ are given by $P_0(x_1), \cdots, P_0(x_m)$ and $\bar{P}_0(\bar{x}_1), \cdots, \bar{P}_0(\bar{x}_m)$, respectively. Equality holds entry by entry, according to Eq. (3). Therefore $\mathbf{Cy} = \bar{\mathbf{C}}\mathbf{y}$.[2]

B. Derivative Formulas

A polynomial derivative formula of degree n for m points is a matrix whose product with an arbitrary data vector having m entries lists the values of the derivative of the least squares fitted polynomial having degree $\leq n$. Derivative formulas can be constructed for all orders of derivatives. Since the coefficients of the approximating polynomial are given by $\mathbf{b} = \mathbf{A}^+\mathbf{y}$, the values of its first derivative at $x_1, \cdots, x_m$ are given by $\mathbf{A}'\mathbf{b} = \mathbf{A}'\mathbf{A}^+\mathbf{y}$, where $\mathbf{A}' = (a'_{ij})$ is the $m \times (n + 1)$ matrix such that

$$a'_{ij} = \begin{cases} 0 & \text{if } j = 1 \\ 1 & \text{if } j = 2 \\ (j - 1)\,(x_i)^{j-2} & \text{if } j > 2 \end{cases} \qquad (4)$$

Thus the first derivative formula is $\mathbf{A}'\mathbf{A}^+$. Similarly, the second derivative formula is $\mathbf{A}''\mathbf{A}^+$, where $\mathbf{A}'' = (a''_{ij})$ is the $m \times (n + 1)$ matrix such that

$$a''_{ij} = \begin{cases} 0 & \text{if } j \leq 2 \\ 2 & \text{if } j = 3 \\ (j - 2)\,(j - 1)\,(x_i)^{j-3} & \text{if } j > 3 \end{cases} \qquad (5)$$

Derivative formulas are not independent of the spacing of the array, although they are independent of translations. Let $\mathbf{D}$ and $\mathbf{E}$ be the first and second derivative formulas when unit spacing is employed, and let $\mathbf{D}_h$ and $\mathbf{E}_h$ be the corresponding formulas when the uniform spacing is h ($h = x_2 - x_1$). Then it is easy to see that

$$\mathbf{D}_h = \frac{1}{h}\,\mathbf{D} \text{ and } \mathbf{E}_h = \frac{1}{h^2}\,\mathbf{E}$$

The derivative formulas $\mathbf{D}$ and $\mathbf{E}$ are given in the tables, with the formulas for non-unit spacing obtainable by dividing by h and h^2, respectively.

C. Computation

The smoothing formulas and derivative formulas listed in Appendix A were obtained by carrying out the following steps for each odd integer m with $3 \leq m \leq 11$ and each integer n with $1 \leq n < m$:

(1) Construct $m \times (n + 1)$ matrices $\mathbf{A}$, $\mathbf{A}'$, and $\mathbf{A}''$ as described in Eqs. (2), (4), and (5) using $x_i = i - (m + 1)/2$. The actual multiplications need only be done once to form $\mathbf{A}$, $\mathbf{A}'$, $\mathbf{A}''$ with $m = 11$ and $n = 10$. These results can be stored in new locations $\mathbf{B}$, $\mathbf{B}'$, $\mathbf{B}''$. Then matrices $\mathbf{A}$, $\mathbf{A}'$, $\mathbf{A}''$ for arbitrary m and n are formed by intersecting the middle m rows and the first $n + 1$ columns of each of the matrices $\mathbf{B}$, $\mathbf{B}'$, $\mathbf{B}''$. The particular choice of the x_i's makes the matrix entries as small as possible in absolute value, subject to the restriction of unit spacing needed to construct $\mathbf{D}$ and $\mathbf{E}$.

(2) Compute the pseudoinverse $\mathbf{A}^+$ of the matrix $\mathbf{A}^+$ using the JPL Fortran IV subroutine **LSQSOL**. Although this subroutine actually computes $\mathbf{A}^+\mathbf{y}$ for any vector $\mathbf{y}$, the matrix $\mathbf{A}^+$ can be reconstructed by virtue of the fact that if $\mathbf{e}_i$ is the ith column of the identity matrix, then $\mathbf{A}^+\mathbf{e}_i$ is the ith column of $\mathbf{A}^+$.

(3) Form the three matrix products $\mathbf{A}\mathbf{A}^+$, $\mathbf{A}'\mathbf{A}^+$, $\mathbf{A}''\mathbf{A}^+$, and print them out together with the numbers m and n. These matrices are just the smoothing formula $\mathbf{C}$ and the first two derivative formulas $\mathbf{D}$ and $\mathbf{E}$, respectively.

The smoothing formulas generated in this way can be checked against the several smoothing formulas given in Hildebrand (Ref. 1), which have exact fractional entries.

[2]Smoothing formulas could also be developed for abscissas with non-uniform spacing. The construction would be identical with that given above. Once a smoothing formula is obtained for one set of points, the same formula can be used for any geometrically similar set of points. This follows from an argument similar to the one of the previous paragraph.

In the case $m = 7$ and $n = 5$, for instance, the decimal expansions agree to 7 significant figures, with a possible error of ± 1 in the eighth digit.

In each case where $n = m - 1$, the smoothing formula is simply the identity matrix, since m points can be fitted exactly by an interpolating polynomial of degree $m - 1$. In such a case, the first and second derivative formulas can be used to compute the values of the first two derivatives of the interpolating polynomial through m points.

D. Use of Appendix A

As previously indicated, each smoothing formula in Appendix A is a matrix whose product with a data vector with entries $y_1, \cdots, y_m$ is the vector of smoothed values $P(x_1), \cdots, P(x_m)$ assumed by the least squares fitted polynomial of degree n. Thus, the value $P(x_j)$ is obtained by multiplying each entry in the row labeled $P(J)$ by the component of the data vector indicated in the column heading, and then summing the results. Similarly, by carrying out the same procedure for the row of the derivative formula labeled $H*P'(J)$, the value $h \cdot P'(x_j)$ is obtained, where h is the uniform spacing of the abscissas. The row labeled $H*H*P''(J)$ is used to compute $h^2 \cdot P''(x_j)$. The center row of each formula is offset by extra space, since this row is used most often in applications.

To illustrate the use of the tables, let data take the values 3, 5, 4, 2, 6 at the points -4, -2, 0, 2, 4, respectively. Suppose we wish to compute the values of the third-degree least squares fitted polynomial and its first two derivatives evaluated at 0. These are obtained by using the third row of each of the formulas for $m = 5$ and $n = 3$ so that

$$P(x_3) = 3(-0.0857143) + 5(0.3428571) + 4(0.4857143) + 2(0.3428571) + 6(-0.0857143) = 3.571$$

$$P'(x_3) = \tfrac{1}{2}[3(0.0833333) + 5(0.6666667) + 4(0.0000000) + 2(0.6666667) + 6(-0.0833333)] = -1.125$$

$$P''(x_3) = (\tfrac{1}{2})^2[3(0.2857143) + 5(-0.1428571) + 4(-0.2857143) + 2(-0.1428571) + 6(0.2857143)] = 0.107$$

III. Formulas for Two Independent Variables

A. Smoothing Formulas

Let $(x_1,y_1), \cdots, (x_m,y_m)$ be the points of an $r \times s$ array of points in the plane, listed row by row (thus $m = rs$). Suppose that any two horizontally adjacent points are separated by a distance h and that any two vertically adjacent points are separated by a distance k. Given data $z_1, \cdots, z_m$ at these points, consider the problem of computing the approximating values $P(x_1,y_1), \cdots, P(x_m,y_m)$ assumed by the function P of the form $P(x,y) = a + bx + cy$ which minimizes the expression

$$\sigma(P) = \sum_{i=1}^{m} [P(x_i,y_i) - z_i]^2 \tag{6}$$

This problem can be solved easily with a *linear smoothing formula for an $r \times s$ array*, by which we mean a matrix C whose product with the column vector z having entries $z_1, \cdots, z_m$ is the column vector $\hat{\mathbf{z}}$ having the entries $P(x_1,y_1), \cdots, P(x_m,y_m)$, for the linear function P minimizing σ. Similarly, the problem of computing approximating values assumed by a function of the form $P(x,y) = a + bx + cy + dx^2 + exy + fy^2$ which minimizes σ can be solved by means of a *quadratic smoothing formula for an $r \times s$ array*, defined in an analogous way. As is so with smoothing formulas for one independent variable, each of these problems is equivalent to one of minimizing the Euclidean length of the vector

$$\mathbf{Ab} - \mathbf{z}$$

by a proper choice of **b**, where for the linear problem

$$\mathbf{b} = \begin{vmatrix} a \\ b \\ c \end{vmatrix}, \qquad \mathbf{A} = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & x_m & y_m \end{vmatrix} \tag{7}$$

and for the quadratic problem

$$\mathbf{b} = \begin{vmatrix} a \\ b \\ c \\ d \\ e \\ f \end{vmatrix}, \qquad \mathbf{A} = \begin{vmatrix} 1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 \\ 1 & x_2 & y_2 & x_2^2 & x_2y_2 & y_2^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & x_m & y_m & x_m^2 & x_my_m & y_m^2 \end{vmatrix} \tag{8}$$

The minimum Euclidean length of $\mathbf{Ab} - \mathbf{z}$ is attained by putting $\mathbf{b} = \mathbf{A}^+\mathbf{z}$, where $\mathbf{A}^+$ is the pseudoinverse of A. The vector $\hat{\mathbf{z}}$ with entries $P(x_1,y_1), \cdots, P(x_m,y_m)$ is given by

$$\hat{\mathbf{z}} = \mathbf{Ab} = \mathbf{AA}^+\mathbf{z}$$

so that the smoothing formula C is the matrix $\mathbf{AA}^+$.

It can be shown that C is independent of the points $(x_1,y_1), \cdots, (x_m,y_m)$ by the same type of argument as that given in Section II.A.

B. Derivative Formulas

A linear (or quadratic) derivative formula for an $r \times s$ array of points is a matrix whose product with an arbitrary data vector having $m = rs$ entries lists the values of a derivative of the least squares fitted linear (or quadratic, respectively) function. A derivative formula can be constructed for a partial derivative of any given order by multiplying the appropriate formal partial derivative of the matrix A with the matrix $\mathbf{A}^+$. Here A is given by Eq. (7) for the linear case and by Eq. (8) for the quadratic case. *A formal partial derivative of the matrix* A is obtained by replacing each entry of A, regarded as a function $f(x,y)$ evaluated at (x_i,y_i), by the specified partial derivative of f evaluated at (x_i,y_i). We shall denote the formal derivatives of first order by $\mathbf{A}_x$ and $\mathbf{A}_y$ and those of second order by $\mathbf{A}_{xx}$, $\mathbf{A}_{xy}$, and $\mathbf{A}_{yy}$. The corresponding derivative formulas will be denoted by $\mathbf{D}_x$, $\mathbf{D}_y$, $\mathbf{D}_{xx}$, $\mathbf{D}_{xy}$, and $\mathbf{D}_{yy}$. They are given by $\mathbf{D}_x = \mathbf{A}_x\mathbf{A}^+, \cdots, \mathbf{D}_{yy} = \mathbf{A}_{yy}\mathbf{A}^+$.

Derivative formulas depend on the horizontal spacing h and the vertical spacing k of the array of points. If we use $\mathbf{D}_x$, $\mathbf{D}_y$, $\mathbf{D}_{xx}$, $\mathbf{D}_{xy}$, and $\mathbf{D}_{yy}$ to denote derivative formulas for spacing h and k, and use $\mathbf{E}_x$, $\mathbf{E}_y$, $\mathbf{E}_{xx}$, $\mathbf{E}_{xy}$, and $\mathbf{E}_{yy}$ to denote derivative formulas for unit horizontal and vertical spacing, then it is easy to see that these are related by the equations

$$\mathbf{D}_x = \frac{1}{h}\mathbf{E}_x, \qquad \mathbf{D}_y = \frac{1}{k}\mathbf{E}_y$$

$$\mathbf{D}_{xx} = \frac{1}{h^2}\mathbf{E}_{xx}, \quad \mathbf{D}_{xy} = \frac{1}{hk}\mathbf{E}_{xy}, \quad \mathbf{D}_{yy} = \frac{1}{k^2}\mathbf{E}_{yy}$$

C. Computation

The procedure by which the formulas of Appendix B were obtained is as follows. A matrix A is constructed as in Eq. (8) for quadratic approximation with a 5×5 array of points whose corners are (1,0), (5,0), (1,−5), (5,−5). The points are taken row by row. The formal first partials $\mathbf{A}_x$ and $\mathbf{A}_y$ are formed as shown in Section III.B. By putting together submatrices of these three matrices, the corresponding matrices for quadratic approximation with 3×5 and 3×3 arrays are obtained, as well as those for linear approximation with each of the three arrays.

For quadratic approximation with each of the arrays, $\mathbf{A}^+$ is computed using **LSQSOL** (cf. *step* 2 in Section II.C), and the products $\mathbf{AA}^+$, $\mathbf{A}_x\mathbf{A}^+$, and $\mathbf{A}_y\mathbf{A}^+$ are formed. Then the following are printed: $\mathbf{AA}^+$, $\mathbf{A}_x\mathbf{A}^+$, $\mathbf{A}_y\mathbf{A}^+$, the fourth row of $2\mathbf{A}^+$, the fifth row of $\mathbf{A}^+$, and the sixth row of $2\mathbf{A}^+$. The first three items are, of course, the smoothing formula C and the derivative formulas $\mathbf{E}_x$ and $\mathbf{E}_y$. As is easily established, the last three items are just the first rows of the second order partial derivative formulas $\mathbf{E}_{xx}$, $\mathbf{E}_{xy}$, and $\mathbf{E}_{yy}$. Only the first row of each of these is printed, since the other rows are the same as the first.

For linear approximation with each of the arrays, the results which are printed out are the smoothing formula $\mathbf{AA}^+$, the second row of $\mathbf{A}^+$ and the third row of $\mathbf{A}^+$. The last two items are the first rows of $\mathbf{E}_x$ and $\mathbf{E}_y$. Each of these matrices has all other rows identical with the first row. The formulas $\mathbf{E}_{xx}$, $\mathbf{E}_{xy}$, and $\mathbf{E}_{yy}$ are not listed, since they are identically zero.

D. Use of Appendix B

To apply the formulas of Appendix B to an $r \times s$ array, the points of the array should be numbered from 1 to $m = rs$ by letting the successive points of the top row from left to right take the numbers 1 through s, the successive points of the second row take the numbers $s + 1$ to $2s$, and so forth, until all r rows are counted. Given a data vector with components $y_1, \cdots, y_m$, where the subscript on each entry indicates the number of the point at which the data value is given, the values of the least squares fitted linear (or quadratic) function P approximating the data are the components of the matrix product of the appropriate smoothing formula and the data vector. Thus, the value of P at the jth point is obtained by multiplying each entry in the row of the smoothing formula labeled $P(J)$ by the component of the data vector indicated in the column heading and then summing the results. A similar procedure applied to a row of a derivative matrix gives the value of the indicated derivative at a point, multiplied by a constant depending on the uniform horizontal spacing h and the uniform vertical spacing k.

When all rows of a derivative formula are the same, only the first row is given. The center row of each formula is offset by extra space, since it is used for computations at the important center point of the array.

IV. Generalizations

The procedures of the previous sections can be applied to similar problems. With the help of the subroutine **LSQSOL**, it is possible to construct smoothing formulas and derivative formulas for k-dimensional arrays with least squares fitting by linear combinations of arbitrary functions $Q_1, \cdots, Q_n$ of a k-dimensional variable $\mathbf{x}$. Also, formulas can be developed for arrays of nonuniform spacing, such formulas applying to classes of geometrically similar arrays. The next paragraph indicates briefly how these generalizations can be made.

Let $\mathbf{x}_1, \cdots, \mathbf{x}_m$ be points of a k-dimensional array, listed in some convenient order. Suppose we wish to approximate data $y_1, \cdots, y_m$ by values $P(\mathbf{x}_1), \cdots, P(\mathbf{x}_m)$ assumed by the function of the form

$$P(\mathbf{x}) = \sum_{j=1}^{n} b_j Q_j(\mathbf{x})$$

which minimizes

$$\sigma(P) = \sum_{i=1}^{m} [P(\mathbf{x}_i) - y_i]^2$$

As in the previous sections, the vector $\mathbf{b}$ of coefficients for the appropriate P is given by

$$\mathbf{b} = \mathbf{A}^+\mathbf{y}$$

where $\mathbf{A}^+$ is the pseudoinverse of the matrix $\mathbf{A} = (a_{ij})$ such that $a_{ij} = Q_j(\mathbf{x}_i)$ for $i = 1, \cdots, m$ and $j = 1, \cdots, n$. The matrix $\mathbf{A}^+$ can be found as in *step* (2) of Section II. C by using **LSQSOL**. The smoothing formula which computes $P(\mathbf{x}_1), \cdots, P(\mathbf{x}_m)$ by matrix multiplication with a data vector $\mathbf{y}$ is just $\mathbf{AA}^+$. Partial derivative formulas are obtained by multiplying the appropriate formal derivative of $\mathbf{A}$ with $\mathbf{A}^+$. The smoothing formula will be the same for any other array of points that is similar to the original one, whereas derivative formulas will be changed by multiplicative constants depending on the spacing.

References

1. Hildebrand, F. B., *Introduction to Numerical Analysis*, McGraw-Hill Book Company, Inc., New York, 1956.
2. Hanson, R. J., and Lawson, C. L., "Extensions and Applications of the Householder Algorithm for Solving Linear Least Squares Problems. Part I: Extensions," *Mathematics of Computation* (in press).

Appendix A

Smoothing and Derivative Formulas for One Independent Variable

The single variable smoothing formulas given in this appendix are listed below with their page numbers. The use of these formulas is explained in Section II.D. on page 4.

M = 3 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)
P(1)	0.8333333	0.3333333	-0.1666667
P(2)	0.3333333	0.3333333	0.3333333
P(3)	-0.1666667	0.3333333	0.8333333

M = 3 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)
H*P'(1)	-0.5000000	0.0000000	0.5000000
H*P'(2)	-0.5000000	0.0000000	0.5000000
H*P'(3)	-0.5000000	0.0000000	0.5000000

M = 3 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)
H*H*P''(1)	-0.0000000	0.0000000	0.0000000
H*H*P''(2)	-0.0000000	0.0000000	0.0000000
H*H*P''(3)	-0.0000000	0.0000000	0.0000000

M = 3 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)
P(1)	1.0000000	-0.0000000	0.0000000
P(2)	0.0000000	1.0000000	0.0000000
P(3)	0.0000000	-0.0000000	1.0000000

M = 3 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)
H*P'(1)	-1.5000000	2.0000000	-0.5000000
H*P'(2)	-0.5000000	0.0000000	0.5000000
H*P'(3)	0.5000000	-2.0000000	1.5000000

M = 3 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)
H*H*P''(1)	1.0000000	-2.0000000	1.0000000
H*H*P''(2)	1.0000000	-2.0000000	1.0000000
H*H*P''(3)	1.0000000	-2.0000000	1.0000000

M = 5 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
P(1)	0.6000000	0.4000000	0.2000000	-0.0000000	-0.2000000
P(2)	0.4000000	0.3000000	0.2000000	0.1000000	-0.0000000
P(3)	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000
P(4)	-0.0000000	0.1000000	0.2000000	0.3000000	0.4000000
P(5)	-0.2000000	-0.0000000	0.2000000	0.4000000	0.6000000

M = 5 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*P'(1)	-0.2000000	-0.1000000	-0.0000000	0.1000000	0.2000000
H*P'(2)	-0.2000000	-0.1000000	-0.0000000	0.1000000	0.2000000
H*P'(3)	-0.2000000	-0.1000000	-0.0000000	0.1000000	0.2000000
H*P'(4)	-0.2000000	-0.1000000	-0.0000000	0.1000000	0.2000000
H*P'(5)	-0.2000000	-0.1000000	-0.0000000	0.1000000	0.2000000

M = 5 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*H*P''(1)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(2)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(3)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(4)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(5)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000

M = 5 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
P(1)	0.8857143	0.2571429	-0.0857143	-0.1428571	0.0857143
P(2)	0.2571429	0.3714286	0.3428571	0.1714286	-0.1428571
P(3)	-0.0857143	0.3428571	0.4857143	0.3428571	-0.0857143
P(4)	-0.1428571	0.1714286	0.3428571	0.3714286	0.2571429
P(5)	0.0857143	-0.1428571	-0.0857143	0.2571429	0.8857143

M = 5 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*P'(1)	-0.7714286	0.1857143	0.5714286	0.3857143	-0.3714286
H*P'(2)	-0.4857143	0.0428571	0.2857143	0.2428571	-0.0857143
H*P'(3)	-0.2000000	-0.1000000	0.0000000	0.1000000	0.2000000
H*P'(4)	0.0857143	-0.2428571	-0.2857143	-0.0428571	0.4857143
H*P'(5)	0.3714286	-0.3857143	-0.5714286	-0.1857143	0.7714286

M = 5 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*H*P''(1)	0.2857143	-0.1428571	-0.2857143	-0.1428571	0.2857143
H*H*P''(2)	0.2857143	-0.1428571	-0.2857143	-0.1428571	0.2857143
H*H*P''(3)	0.2857143	-0.1428571	-0.2857143	-0.1428571	0.2857143
H*H*P''(4)	0.2857143	-0.1428571	-0.2857143	-0.1428571	0.2857143
H*H*P''(5)	0.2857143	-0.1428571	-0.2857143	-0.1428571	0.2857143

M = 5 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
P(1)	0.9857143	0.0571429	-0.0857143	0.0571429	-0.0142857
P(2)	0.0571429	0.7714286	0.3428571	-0.2285714	0.0571429
P(3)	-0.0857143	0.3428571	0.4857143	0.3428571	-0.0857143
P(4)	0.0571429	-0.2285714	0.3428571	0.7714286	0.0571429
P(5)	-0.0142857	0.0571429	-0.0857143	0.0571429	0.9857143

M = 5 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*P'(1)	-1.4880952	1.6190476	0.5714286	-1.0476190	0.3452381
H*P'(2)	-0.4523809	-0.0238095	0.2857143	0.3095238	-0.1190476
H*P'(3)	0.0833333	-0.6666667	0.0000000	0.6666667	-0.0833333
H*P'(4)	0.1190476	-0.3095238	-0.2857143	0.0238095	0.4523809
H*P'(5)	-0.3452381	1.0476190	-0.5714286	-1.6190476	1.4880952

M = 5 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*H*P''(1)	1.2857143	-2.1428571	-0.2857143	1.8571429	-0.7142857
H*H*P''(2)	0.7857143	-1.1428571	-0.2857143	0.8571429	-0.2142857
H*H*P''(3)	0.2857143	-0.1428571	-0.2857143	-0.1428571	0.2857143
H*H*P''(4)	-0.2142857	0.8571429	-0.2857143	-1.1428571	0.7857143
H*H*P''(5)	-0.7142857	1.8571429	-0.2857143	-2.1428571	1.2857143

M = 5 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
P(1)	1.0000000	-0.0000000	0.0000000	-0.0000000	0.0000000
P(2)	0.0000000	1.0000000	0.0000000	-0.0000000	0.0000000
P(3)	-0.0000000	-0.0000000	1.0000000	0.0000000	-0.0000000
P(4)	0.0000000	-0.0000000	0.0000000	1.0000000	0.0000000
P(5)	0.0000000	-0.0000000	0.0000000	-0.0000000	1.0000000

M = 5 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*P'(1)	-2.0833333	4.0000000	-3.0000000	1.3333333	-0.2500000
H*P'(2)	-0.2500000	-0.8333333	1.5000000	-0.5000000	0.0833333
H*P'(3)	0.0833333	-0.6666667	-0.0000000	0.6666667	-0.0833333
H*P'(4)	-0.0833333	0.5000000	-1.5000000	0.8333333	0.2500000
H*P'(5)	0.2500000	-1.3333333	3.0000000	-4.0000000	2.0833333

M = 5 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
H*H*P''(1)	2.9166666	-8.6666666	9.4999999	-4.6666666	0.9166667
H*H*P''(2)	0.9166667	-1.6666667	0.5000000	0.3333333	-0.0833333
H*H*P''(3)	-0.0833333	1.3333333	-2.5000000	1.3333333	-0.0833333
H*H*P''(4)	-0.0833333	0.3333333	0.5000000	-1.6666667	0.9166667
H*H*P''(5)	0.9166667	-4.6666666	9.4999999	-8.6666666	2.9166666

M = 7 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.4642857	0.3571429	0.2500000	0.1428571	0.0357143	-0.0714286	-0.1785714
P(2)	0.3571429	0.2857143	0.2142857	0.1428571	0.0714286	-0.0000000	-0.0714286
P(3)	0.2500000	0.2142857	0.1785714	0.1428571	0.1071429	0.0714286	0.0357143
P(4)	0.1428571	0.1428571	0.1428571	0.1428571	0.1428571	0.1428571	0.1428571
P(5)	0.0357143	0.0714286	0.1071429	0.1428571	0.1785714	0.2142857	0.2500000
P(6)	-0.0714286	-0.0000000	0.0714286	0.1428571	0.2142857	0.2857143	0.3571429
P(7)	-0.1785714	-0.0714286	0.0357143	0.1428571	0.2500000	0.3571429	0.4642857

M = 7 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429
H*P'(2)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429
H*P'(3)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429
H*P'(4)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429
H*P'(5)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429
H*P'(6)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429
H*P'(7)	-0.1071429	-0.0714286	-0.0357143	0.0000000	0.0357143	0.0714286	0.1071429

M = 7 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(2)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(3)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(4)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(5)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(6)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(7)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

M = 7 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.7619048	0.3571429	0.0714286	-0.0952381	-0.1428571	-0.0714286	0.1190476
P(2)	0.3571429	0.2857143	0.2142857	0.1428571	0.0714286	0.0000000	-0.0714286
P(3)	0.0714286	0.2142857	0.2857143	0.2857143	0.2142857	0.0714286	-0.1428571
P(4)	-0.0952381	0.1428571	0.2857143	0.3333333	0.2857143	0.1428571	-0.0952381
P(5)	-0.1428571	0.0714286	0.2142857	0.2857143	0.2857143	0.2142857	0.0714286
P(6)	-0.0714286	0.0000000	0.0714286	0.1428571	0.2142857	0.2857143	0.3571429
P(7)	0.1190476	-0.0714286	-0.1428571	-0.0952381	0.0714286	0.3571429	0.7619048

M = 7 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.4642857	-0.0714286	0.1785714	0.2857143	0.2500000	0.0714286	-0.2500000
H*P'(2)	-0.3452381	-0.0714286	0.1071429	0.1904762	0.1785714	0.0714286	-0.1309524
H*P'(3)	-0.2261905	-0.0714286	0.0357143	0.0952381	0.1071429	0.0714286	-0.0119048
H*P'(4)	-0.1071429	-0.0714286	-0.0357143	-0.0000000	0.0357143	0.0714286	0.1071429
H*P'(5)	0.0119048	-0.0714286	-0.1071429	-0.0952381	-0.0357143	0.0714286	0.2261905
H*P'(6)	0.1309524	-0.0714286	-0.1785714	-0.1904762	-0.1071429	0.0714286	0.3452381
H*P'(7)	0.2500000	-0.0714286	-0.2500000	-0.2857143	-0.1785714	0.0714286	0.4642857

M = 7 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(2)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(3)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(4)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(5)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(6)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(7)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476

M = 7 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9285714	0.1904762	-0.0952381	-0.0952381	0.0238095	0.0952381	-0.0476190
P(2)	0.1904762	0.4523809	0.3809524	0.1428571	-0.0952381	-0.1666667	0.0952381
P(3)	-0.0952381	0.3809524	0.4523809	0.2857143	0.0476190	-0.0952381	0.0238095
P(4)	-0.0952381	0.1428571	0.2857143	0.3333333	0.2857143	0.1428571	-0.0952381
P(5)	0.0238095	-0.0952381	0.0476190	0.2857143	0.4523809	0.3809524	-0.0952381
P(6)	0.0952381	-0.1666667	-0.0952381	0.1428571	0.3809524	0.4523809	0.1904762
P(7)	-0.0476190	0.0952381	0.0238095	-0.0952381	-0.0952381	0.1904762	0.9285714

M = 7 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-1.0198413	0.4841270	0.7341270	0.2857143	-0.3055556	-0.4841270	0.3055556
H*P'(2)	-0.4841270	0.0674603	0.2460317	0.1904762	0.0396825	-0.0674603	0.0079365
H*P'(3)	-0.1150794	-0.1825397	-0.0753968	0.0952381	0.2182540	0.1825397	-0.1230159
H*P'(4)	0.0873016	-0.2658730	-0.2301587	-0.0000000	0.2301587	0.2658730	-0.0873016
H*P'(5)	0.1230159	-0.1825397	-0.2182540	-0.0952381	0.0753968	0.1825397	0.1150794
H*P'(6)	-0.0079365	0.0674603	-0.0396825	-0.1904762	-0.2460317	-0.0674603	0.4841270
H*P'(7)	-0.3055555	0.4841270	0.3055556	-0.2857143	-0.7341270	-0.4841270	1.0198413

M = 7 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.6190476	-0.5000000	-0.5714286	-0.0952381	0.4285714	0.5000000	-0.3809524
H*H*P''(2)	0.4523809	-0.3333333	-0.4047619	-0.0952381	0.2619048	0.3333333	-0.2142857
H*H*P''(3)	0.2857143	-0.1666667	-0.2380952	-0.0952381	0.0952381	0.1666667	-0.0476190
H*H*P''(4)	0.1190476	0.0000000	-0.0714286	-0.0952381	-0.0714286	0.0000000	0.1190476
H*H*P''(5)	-0.0476190	0.1666667	0.0952381	-0.0952381	-0.2380952	-0.1666667	0.2857143
H*H*P''(6)	-0.2142857	0.3333333	0.2619048	-0.0952381	-0.4047619	-0.3333333	0.4523810
H*H*P''(7)	-0.3809524	0.5000000	0.4285714	-0.0952381	-0.5714286	-0.5000000	0.6190476

M = 7 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9870130	0.0541125	-0.0757576	0.0216450	0.0432900	-0.0411255	0.0108225
P(2)	0.0541126	0.7705628	0.3354978	-0.1298701	-0.1406926	0.1515152	-0.0411255
P(3)	-0.0757576	0.3354978	0.4588745	0.3246753	0.0541126	-0.1406926	0.0432900
P(4)	0.0216450	-0.1298701	0.3246753	0.5670996	0.3246753	-0.1298701	0.0216450
P(5)	0.0432900	-0.1406926	0.0541126	0.3246753	0.4588745	0.3354978	-0.0757576
P(6)	-0.0411255	0.1515151	-0.1406926	-0.1298701	0.3354978	0.7705628	0.0541126
P(7)	0.0108225	-0.0411256	0.0432900	0.0216450	-0.0757576	0.0541125	0.9870130

M = 7 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-1.5945166	1.8250361	0.5425685	-0.8636364	-0.4971140	0.8567821	-0.2691198
H*P'(2)	-0.4126984	-0.0992063	0.2698413	0.3333333	0.0634921	-0.2341270	0.0793651
H*P'(3)	0.0569986	-0.5840548	-0.0180375	0.4393939	0.2756133	-0.2189755	0.0490620
H*P'(4)	0.0873016	-0.2658730	-0.2301587	0.0000000	0.2301587	0.2658730	-0.0873016
H*P'(5)	-0.0490620	0.2189755	-0.2756133	-0.4393939	0.0180375	0.5840548	-0.0569986
H*P'(6)	-0.0793651	0.2341270	-0.0634921	-0.3333333	-0.2698413	0.0992063	0.4126984
H*P'(7)	0.2691198	-0.8567821	0.4971140	0.8636364	-0.5425685	-1.8250360	1.5945166

M = 7 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	1.6287879	-2.8560606	-0.2348485	1.9242424	0.7651515	-1.8560606	0.6287879
H*H*P''(2)	0.7803030	-1.0984848	-0.2954545	0.5606061	0.3712121	-0.4318182	0.1136364
H*H*P''(3)	0.2045455	0.0227273	-0.2651515	-0.2575758	0.0681818	0.3560606	-0.1287879
H*H*P''(4)	-0.0984848	0.5075758	-0.1439394	-0.5303030	-0.1439394	0.5075758	-0.0984848
H*H*P''(5)	-0.1287879	0.3560606	0.0681818	-0.2575758	-0.2651515	0.0227273	0.2045455
H*H*P''(6)	0.1136364	-0.4318182	0.3712121	0.5606061	-0.2954545	-1.0984848	0.7803030
H*H*P''(7)	0.6287879	-1.8560606	0.7651515	1.9242424	-0.2348485	-2.8560606	1.6287879

M = 7 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9989177	0.0064935	-0.0162337	0.0216450	-0.0162338	0.0064935	-0.0010823
P(2)	0.0064935	0.9610389	0.0974026	-0.1298701	0.0974026	-0.0389611	0.0064935
P(3)	-0.0162338	0.0974026	0.7564935	0.3246753	-0.2435065	0.0974026	-0.0162338
P(4)	0.0216450	-0.1298701	0.3246753	0.5670996	0.3246753	-0.1298701	0.0216450
P(5)	-0.0162338	0.0974026	-0.2435065	0.3246753	0.7564935	0.0974026	-0.0162338
P(6)	0.0064935	-0.0389611	0.0974026	-0.1298701	0.0974026	0.9610389	0.0064935
P(7)	-0.0010822	0.0064935	-0.0162338	0.0216450	-0.0162337	0.0064935	0.9989177

M = 7 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.0734848	3.7409091	-1.8522727	-0.8636364	1.8977273	-1.0590909	0.2098485
H*P'(2)	-0.2666667	-0.6833333	1.0000000	0.3333333	-0.6666667	0.3500000	-0.0666667
H*P'(3)	0.0780303	-0.6681818	0.0871212	0.4393939	0.1704545	-0.1348485	0.0280303
H*P'(4)	-0.0166667	0.1500000	-0.7500000	0.0000000	0.7500000	-0.1500000	0.0166667
H*P'(5)	-0.0280303	0.1348485	-0.1704545	-0.4393939	-0.0871212	0.6681818	-0.0780303
H*P'(6)	0.0666667	-0.3500000	0.6666667	-0.3333333	-1.0000000	0.6833333	0.2666667
H*P'(7)	-0.2098485	1.0590909	-1.8977273	0.8636364	1.8522727	-3.7409091	2.0734848

M = 7 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	3.0037878	-8.3560605	6.6401514	1.9242424	-6.1098484	3.6439394	-0.7462121
H*H*P''(2)	0.8636364	-1.4318182	0.1212121	0.5606061	-0.0454545	-0.0984849	0.0303030
H*H*P''(3)	-0.0037879	0.8560606	-1.3068182	-0.2575758	1.1098485	-0.4772727	0.0795455
H*H*P''(4)	-0.0984848	0.5075758	-0.1439394	-0.5303030	-0.1439394	0.5075758	-0.0984848
H*H*P''(5)	0.0795455	-0.4772727	1.1098485	-0.2575758	-1.3068182	0.8560606	-0.0037879
H*H*P''(6)	0.0303030	-0.0984849	-0.0454545	0.5606061	0.1212121	-1.4318182	0.8636364
H*H*P''(7)	-0.7462121	3.6439394	-6.1098484	1.9242424	6.6401514	-8.3560605	3.0037878

M = 7 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	1.0000000	0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	-0.0000000
P(2)	-0.0000000	1.0000000	-0.0000000	0.0000000	0.0000000	-0.0000000	-0.0000000
P(3)	-0.0000000	0.0000000	1.0000000	0.0000000	0.0000000	-0.0000000	-0.0000000
P(4)	0.0000000	-0.0000000	0.0000000	1.0000000	-0.0000000	-0.0000000	0.0000000
P(5)	-0.0000000	-0.0000000	0.0000000	0.0000000	1.0000000	0.0000000	-0.0000000
P(6)	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	1.0000000	-0.0000000
P(7)	-0.0000000	0.0000000	-0.0000000	0.0000000	0.0000001	0.0000000	1.0000000

M = 7 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.4500000	5.9999999	-7.4999999	6.6666666	-3.7500000	1.2000000	-0.1666667
H*P'(2)	-0.1666667	-1.2833333	2.5000000	-1.6666667	0.8333333	-0.2500000	0.0333333
H*P'(3)	0.0333333	-0.4000000	-0.5833333	1.3333333	-0.5000000	0.1333333	-0.0166667
H*P'(4)	-0.0166667	0.1500000	-0.7500000	0.0000000	0.7500000	-0.1500000	0.0166667
H*P'(5)	0.0166667	-0.1333333	0.5000000	-1.3333333	0.5833333	0.4000000	-0.0333333
H*P'(6)	-0.0333333	0.2500000	-0.8333333	1.6666667	-2.4999999	1.2833333	0.1666667
H*P'(7)	0.1666667	-1.2000000	3.7500000	-6.6666666	7.5000000	-5.9999999	2.4500000

M = 7 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	4.5111110	-17.3999999	29.2499998	-28.2222221	16.4999998	-5.4000000	0.7611111
H*H*P''(2)	0.7611111	-0.8166667	-1.4166667	2.6111111	-1.5833333	0.5166667	-0.0722222
H*H*P''(3)	-0.0722222	1.2666667	-2.3333333	1.1111111	0.0833333	-0.0666667	0.0111111
H*H*P''(4)	0.0111111	-0.1500000	1.5000000	-2.7222222	1.5000000	-0.1500000	0.0111111
H*H*P''(5)	0.0111111	-0.0666667	0.0833333	1.1111111	-2.3333333	1.2666667	-0.0722222
H*H*P''(6)	-0.0722222	0.5166667	-1.5833333	2.6111111	-1.4166666	-0.8166667	0.7611111
H*H*P''(7)	0.7611111	-5.4000000	16.4999998	-28.2222221	29.2499998	-17.3999999	4.5111110

M = 9 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.3777778	0.3111111	0.2444444	0.1777778	0.1111111	0.0444444	-0.0222222
P(2)	0.3111111	0.2611111	0.2111111	0.1611111	0.1111111	0.0611111	0.0111111
P(3)	0.2444444	0.2111111	0.1777778	0.1444444	0.1111111	0.0777778	0.0444444
P(4)	0.1777778	0.1611111	0.1444444	0.1277778	0.1111111	0.0944444	0.0777778
P(5)	0.1111111	0.1111111	0.1111111	0.1111111	0.1111111	0.1111111	0.1111111
P(6)	0.0444444	0.0611111	0.0777778	0.0944444	0.1111111	0.1277778	0.1444444
P(7)	-0.0222222	0.0111111	0.0444444	0.0777778	0.1111111	0.1444444	0.1777778
P(8)	-0.0888889	-0.0388889	0.0111111	0.0611111	0.1111111	0.1611111	0.2111111
P(9)	-0.1555556	-0.0888889	-0.0222222	0.0444444	0.1111111	0.1777778	0.2444444

	Y(8)	Y(9)
P(1)	-0.0888889	-0.1555556
P(2)	-0.0388889	-0.0888889
P(3)	0.0111111	-0.0222222
P(4)	0.0611111	0.0444444
P(5)	0.1111111	0.1111111
P(6)	0.1611111	0.1777778
P(7)	0.2111111	0.2444444
P(8)	0.2611111	0.3111111
P(9)	0.3111111	0.3777778

M = 9 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(2)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(3)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(4)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(5)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(6)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(7)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(8)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333
H*P'(9)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	0.0000000	0.0166667	0.0333333

	Y(8)	Y(9)
H*P'(1)	0.0500000	0.0666667
H*P'(2)	0.0500000	0.0666667
H*P'(3)	0.0500000	0.0666667
H*P'(4)	0.0500000	0.0666667
H*P'(5)	0.0500000	0.0666667
H*P'(6)	0.0500000	0.0666667
H*P'(7)	0.0500000	0.0666667
H*P'(8)	0.0500000	0.0666667
H*P'(9)	0.0500000	0.0666667

M = 9 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(2)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(3)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(4)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(5)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(6)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(7)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(8)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(9)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000

	Y(8)	Y(9)
H*H*P''(1)	0.0000000	0.0000000
H*H*P''(2)	0.0000000	0.0000000
H*H*P''(3)	0.0000000	0.0000000
H*H*P''(4)	0.0000000	0.0000000
H*H*P''(5)	0.0000000	0.0000000
H*H*P''(6)	0.0000000	0.0000000
H*H*P''(7)	0.0000000	0.0000000
H*H*P''(8)	0.0000000	0.0000000
H*H*P''(9)	0.0000000	0.0000000

M = 9 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.6606061	0.3818182	0.1636364	0.0060606	-0.0909091	-0.1272727	-0.1030303
P(2)	0.3818182	0.2787879	0.1909091	0.1181818	0.0606061	0.0181818	-0.0090909
P(3)	0.1636364	0.1909091	0.2008658	0.1935065	0.1688312	0.1268398	0.0675325
P(4)	0.0060606	0.1181818	0.1935065	0.2320346	0.2337662	0.1987013	0.1268398
P(5)	-0.0909091	0.0606061	0.1688312	0.2337662	0.2554113	0.2337662	0.1688312
P(6)	-0.1272727	0.0181818	0.1268398	0.1987013	0.2337662	0.2320346	0.1935065
P(7)	-0.1030303	-0.0090909	0.0675325	0.1268398	0.1688312	0.1935065	0.2008658
P(8)	-0.0181818	-0.0212121	-0.0090909	0.0181818	0.0606061	0.1181818	0.1909091
P(9)	0.1272727	-0.0181818	-0.1030303	-0.1272727	-0.0909091	0.0060606	0.1636364

	Y(8)	Y(9)
P(1)	-0.0181818	0.1272727
P(2)	-0.0212121	-0.0181818
P(3)	-0.0090909	-0.1030303
P(4)	0.0181818	-0.1272727
P(5)	0.0606061	-0.0909091
P(6)	0.1181818	0.0060606
P(7)	0.1909091	0.1636364
P(8)	0.2787879	0.3818182
P(9)	0.3818182	0.6606061

M = 9 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.3090909	-0.1106061	0.0359307	0.1305195	0.1731602	0.1638528	0.1025974
H*P'(2)	-0.2484848	-0.0954545	0.0186147	0.0937229	0.1298701	0.1270563	0.0852814
H*P'(3)	-0.1878788	-0.0803030	0.0012987	0.0569264	0.0865801	0.0902597	0.0679654
H*P'(4)	-0.1272727	-0.0651515	-0.0160173	0.0201299	0.0432900	0.0534632	0.0506494
H*P'(5)	-0.0666667	-0.0500000	-0.0333333	-0.0166667	-0.0000000	0.0166667	0.0333333
H*P'(6)	-0.0060606	-0.0348485	-0.0506494	-0.0534632	-0.0432900	-0.0201299	0.0160173
H*P'(7)	0.0545455	-0.0196970	-0.0679654	-0.0902597	-0.0865801	-0.0569264	-0.0012987
H*P'(8)	0.1151515	-0.0045455	-0.0852814	-0.1270563	-0.1298701	-0.0937229	-0.0186147
H*P'(9)	0.1757576	0.0106061	-0.1025974	-0.1638528	-0.1731602	-0.1305195	-0.0359307

	Y(8)	Y(9)
H*P'(1)	-0.0106061	-0.1757576
H*P'(2)	0.0045455	-0.1151515
H*P'(3)	0.0196970	-0.0545455
H*P'(4)	0.0348485	0.0060606
H*P'(5)	0.0500000	0.0666667
H*P'(6)	0.0651515	0.1272727
H*P'(7)	0.0803030	0.1878788
H*P'(8)	0.0954545	0.2484848
H*P'(9)	0.1106061	0.3090909

M = 9 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(2)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(3)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(4)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(5)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(6)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(7)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(8)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(9)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160

	Y(8)	Y(9)
H*H*P''(1)	0.0151515	0.0606061
H*H*P''(2)	0.0151515	0.0606061
H*H*P''(3)	0.0151515	0.0606061
H*H*P''(4)	0.0151515	0.0606061
H*H*P''(5)	0.0151515	0.0606061
H*H*P''(6)	0.0151515	0.0606061
H*H*P''(7)	0.0151515	0.0606061
H*H*P''(8)	0.0151515	0.0606061
H*H*P''(9)	0.0151515	0.0606061

M = 9 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.8585858	0.2828283	-0.0202020	-0.1212121	-0.0909091	0.0000000	0.0808081
P(2)	0.2828283	0.3282828	0.2828283	0.1818182	0.0606061	-0.0454545	-0.1010101
P(3)	-0.0202020	0.2828283	0.3715729	0.3116883	0.1688312	0.0086580	-0.1031746
P(4)	-0.1212121	0.1818182	0.3116883	0.3138528	0.2337662	0.1168831	0.0086580
P(5)	-0.0909091	0.0606061	0.1688312	0.2337662	0.2554113	0.2337662	0.1688312
P(6)	-0.0000000	-0.0454545	0.0086580	0.1168831	0.2337662	0.3138528	0.3116883
P(7)	0.0808081	-0.1010101	-0.1031746	0.0086580	0.1688312	0.3116883	0.3715729
P(8)	0.0808081	-0.0707071	-0.1010101	-0.0454545	0.0606061	0.1818182	0.2828283
P(9)	-0.0707071	0.0808081	0.0808081	0.0000000	-0.0909091	-0.1212121	-0.0202020

	Y(8)	Y(9)
P(1)	0.0808081	-0.0707071
P(2)	-0.0707071	0.0808081
P(3)	-0.1010101	0.0808081
P(4)	-0.0454545	0.0000000
P(5)	0.0606061	-0.0909091
P(6)	0.1818182	-0.1212121
P(7)	0.2828283	-0.0202020
P(8)	0.3282828	0.2828283
P(9)	0.2828283	0.8585858

M = 9 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.7356902	0.1026936	0.4320587	0.4047619	0.1731602	-0.1103896	-0.2935305
H*P'(2)	-0.4276094	-0.0058923	0.1849447	0.2088745	0.1298701	0.0119048	-0.0810486
H*P'(3)	-0.1902357	-0.0791246	0.0034873	0.0584416	0.0865801	0.0887446	0.0657768
H*P'(4)	-0.0235690	-0.1170034	-0.1123136	-0.0465368	0.0432900	0.1201299	0.1469456
H*P'(5)	0.0723906	-0.1195286	-0.1624579	-0.1060606	0.0000000	0.1060606	0.1624579
H*P'(6)	0.0976431	-0.0867003	-0.1469456	-0.1201299	-0.0432900	0.0465368	0.1123136
H*P'(7)	0.0521886	-0.0185185	-0.0657768	-0.0887446	-0.0865801	-0.0584416	-0.0034873
H*P'(8)	-0.0639731	0.0850168	0.0810486	-0.0119048	-0.1298701	-0.2088745	-0.1849447
H*P'(9)	-0.2508417	0.2239057	0.2935305	0.1103896	-0.1731602	-0.4047619	-0.4320587

	Y(8)	Y(9)
H*P'(1)	-0.2239057	0.2508417
H*P'(2)	-0.0850168	0.0639731
H*P'(3)	0.0185185	-0.0521886
H*P'(4)	0.0867003	-0.0976431
H*P'(5)	0.1195286	-0.0723906
H*P'(6)	0.1170034	0.0235690
H*P'(7)	0.0791246	0.1902357
H*P'(8)	0.0058923	0.4276094
H*P'(9)	-0.1026936	0.7356902

M = 9 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.3434343	-0.1262626	-0.2799423	-0.2186147	-0.0432900	0.1450216	0.2453102
H*H*P''(2)	0.2727273	-0.0909091	-0.2142857	-0.1731602	-0.0432900	0.0995671	0.1796537
H*H*P''(3)	0.2020202	-0.0555556	-0.1486291	-0.1277056	-0.0432900	0.0541126	0.1139971
H*H*P''(4)	0.1313131	-0.0202020	-0.0829726	-0.0822511	-0.0432900	0.0086580	0.0483405
H*H*P''(5)	0.0606061	0.0151515	-0.0173160	-0.0367965	-0.0432900	-0.0367965	-0.0173160
H*H*P''(6)	-0.0101010	0.0505050	0.0483405	0.0086580	-0.0432900	-0.0822511	-0.0829726
H*H*P''(7)	-0.0808081	0.0858586	0.1139971	0.0541126	-0.0432900	-0.1277056	-0.1486291
H*H*P''(8)	-0.1515152	0.1212121	0.1796537	0.0995671	-0.0432900	-0.1731602	-0.2142857
H*H*P''(9)	-0.2222222	0.1565657	0.2453102	0.1450216	-0.0432900	-0.2186147	-0.2799423

	Y(8)	Y(9)
H*H*P''(1)	0.1565657	-0.2222222
H*H*P''(2)	0.1212121	-0.1515152
H*H*P''(3)	0.0858586	-0.0808081
H*H*P''(4)	0.0505050	-0.0101010
H*H*P''(5)	0.0151515	0.0606061
H*H*P''(6)	-0.0202020	0.1313131
H*H*P''(7)	-0.0555556	0.2020202
H*H*P''(8)	-0.0909091	0.2727273
H*H*P''(9)	-0.1262626	0.3434343

M = 9 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9564879	0.1359751	-0.0971251	-0.0582751	0.0349650	0.0629371	0.0038850
P(2)	0.1359751	0.5485625	0.3982129	0.0874126	-0.1282051	-0.1398601	0.0143745
P(3)	-0.0971251	0.3982129	0.4320124	0.2622378	0.0699301	-0.0407925	-0.0427350
P(4)	-0.0582751	0.0874126	0.2622378	0.3543123	0.3146853	0.1573427	-0.0407925
P(5)	0.0349650	-0.1282051	0.0699301	0.3146853	0.4172494	0.3146853	0.0699301
P(6)	0.0629371	-0.1398601	-0.0407925	0.1573427	0.3146853	0.3543123	0.2622378
P(7)	0.0038850	0.0143745	-0.0427350	-0.0407925	0.0699301	0.2622378	0.4320124
P(8)	-0.0660451	0.1495726	0.0143745	-0.1398601	-0.1282051	0.0874126	0.3982129
P(9)	0.0271950	-0.0660451	0.0038850	0.0629371	0.0349650	-0.0582751	-0.0971251

	Y(8)	Y(9)
P(1)	-0.0660451	0.0271950
P(2)	0.1495726	-0.0660451
P(3)	0.0143745	0.0038850
P(4)	-0.1398601	0.0629371
P(5)	-0.1282051	0.0349650
P(6)	0.0874126	-0.0582751
P(7)	0.3982129	-0.0971251
P(8)	0.5485625	0.1359751
P(9)	0.1359751	0.9564880

M = 9 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-1.2438487	0.8649314	0.8313261	0.0780886	-0.4801865	-0.4370629	0.1057369
H*P'(2)	-0.4660710	0.0518001	0.2151645	0.1841492	0.0804196	-0.0128205	-0.0508288
H*P'(3)	-0.0527066	-0.2854183	-0.1045714	0.1468531	0.2634033	0.1771562	-0.0422818
H*P'(4)	0.0941466	-0.2935768	-0.2048045	0.0291375	0.1946387	0.1958042	0.0544548
H*P'(5)	0.0723906	-0.1195286	-0.1624579	-0.1060606	0.0000000	0.1060606	0.1624579
H*P'(6)	-0.0200725	0.0898731	-0.0544548	-0.1958042	-0.1946387	-0.0291375	0.2048045
H*P'(7)	-0.0853406	0.1877752	0.0422818	-0.1771562	-0.2634033	-0.1468531	0.1045714
H*P'(8)	-0.0255115	0.0273245	0.0508288	0.0128205	-0.0804196	-0.1841492	-0.2151645
H*P'(9)	0.2573167	-0.5383320	-0.1057369	0.4370629	0.4801865	-0.0780886	-0.8313261

	Y(8)	Y(9)
H*P'(1)	0.5383320	-0.2573168
H*P'(2)	-0.0273245	0.0255115
H*P'(3)	-0.1877752	0.0853406
H*P'(4)	-0.0898731	0.0200725
H*P'(5)	0.1195286	-0.0723906
H*P'(6)	0.2935768	-0.0941466
H*P'(7)	0.2854183	0.0527066
H*P'(8)	-0.0518001	0.4660710
H*P'(9)	-0.8649314	1.2438487

M = 9 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.9926185	-1.1000388	-0.7900155	0.1987179	0.7913753	0.5623543	-0.2647630
H*H*P''(2)	0.5792541	-0.5506993	-0.4551282	0.0238928	0.3508158	0.2966200	-0.0611888
H*H*P''(3)	0.2637918	-0.1482129	-0.1971639	-0.0879953	0.0361305	0.0938228	0.0654623
H*H*P''(4)	0.0462315	0.1074204	-0.0161228	-0.1369464	-0.1526807	-0.0460373	0.1151904
H*H*P''(5)	-0.0734266	0.2162005	0.0879953	-0.1229604	-0.2156177	-0.1229604	0.0879953
H*H*P''(6)	-0.0951826	0.1781274	0.1151904	-0.0460373	-0.1526807	-0.1369464	-0.0161228
H*H*P''(7)	-0.0190365	-0.0067988	0.0654623	0.0938228	0.0361305	-0.0879953	-0.1971639
H*H*P''(8)	0.1550117	-0.3385781	-0.0611888	0.2966200	0.3508158	0.0238928	-0.4551282
H*H*P''(9)	0.4269619	-0.8172106	-0.2647630	0.5623543	0.7913753	0.1987179	-0.7900155

	Y(8)	Y(9)
H*H*P''(1)	-0.8172106	0.4269619
H*H*P''(2)	-0.3385781	0.1550117
H*H*P''(3)	-0.0067988	-0.0190365
H*H*P''(4)	0.1781274	-0.0951826
H*H*P''(5)	0.2162005	-0.0734266
H*H*P''(6)	0.1074204	0.0462315
H*H*P''(7)	-0.1482129	0.2637918
H*H*P''(8)	-0.5506993	0.5792541
H*H*P''(9)	-1.1000388	0.9926185

M = 9 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9906760	0.0419580	-0.0629371	0.0186480	0.0349650	-0.0139860	-0.0303030
P(2)	0.0419580	0.8071095	0.3041958	-0.1241259	-0.1282051	0.0716783	0.1083916
P(3)	-0.0629371	0.3041958	0.4662005	0.3391608	0.0699301	-0.1177156	-0.0769231
P(4)	0.0186480	-0.1241259	0.3391608	0.5273893	0.3146853	-0.0157343	-0.1177156
P(5)	0.0349650	-0.1282051	0.0699301	0.3146853	0.4172494	0.3146853	0.0699301
P(6)	-0.0139860	0.0716783	-0.1177156	-0.0157343	0.3146853	0.5273893	0.3391608
P(7)	-0.0303030	0.1083916	-0.0769231	-0.1177156	0.0699301	0.3391608	0.4662005
P(8)	0.0279720	-0.1089743	0.1083916	0.0716783	-0.1282051	-0.1241259	0.3041958
P(9)	-0.0069930	0.0279721	-0.0303030	-0.0139860	0.0349650	0.0186480	-0.0629371

	Y(8)	Y(9)
P(1)	0.0279721	-0.0069930
P(2)	-0.1089743	0.0279720
P(3)	0.1083916	-0.0303030
P(4)	0.0716783	-0.0139860
P(5)	-0.1282051	0.0349650
P(6)	-0.1241259	0.0186480
P(7)	0.3041958	-0.0629371
P(8)	0.8071095	0.0419580
P(9)	0.0419580	0.9906760

M = 9 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-1.7219114	2.1796037	0.3532634	-0.9975524	-0.4801865	0.6385781	0.5837995
H*P'(2)	-0.3757576	-0.1965618	0.3054779	0.3873543	0.0804196	-0.2160256	-0.1411422
H*P'(3)	0.0589744	-0.5925408	0.0071096	0.3981352	0.2634033	-0.0741259	-0.1539627
H*P'(4)	0.0648019	-0.2128788	-0.2341492	-0.0368881	0.1946387	0.2618298	0.0837995
H*P'(5)	-0.0296037	0.1609557	-0.2644522	-0.3355478	0.0000000	0.3355478	0.2644522
H*P'(6)	-0.0494172	0.1705711	-0.0837995	-0.2618298	-0.1946387	0.0368881	0.2341492
H*P'(7)	0.0263403	-0.1193473	0.1539627	0.0741259	-0.2634033	-0.3981352	-0.0071095
H*P'(8)	0.0648019	-0.2210373	0.1411422	0.2160256	-0.0804196	-0.3873543	-0.3054778
H*P'(9)	-0.2207459	0.7763403	-0.5837995	-0.6385781	0.4801865	0.9975524	-0.3532634

	Y(8)	Y(9)
H*P'(1)	-0.7763403	0.2207459
H*P'(2)	0.2210373	-0.0648019
H*P'(3)	0.1193473	-0.0263403
H*P'(4)	-0.1705711	0.0494172
H*P'(5)	-0.1609557	0.0296037
H*P'(6)	0.2128788	-0.0648019
H*P'(7)	0.5925408	-0.0589744
H*P'(8)	0.1965618	0.3757576
H*P'(9)	-2.1796037	1.7219114

M = 9 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	2.0011655	-3.8735431	0.2185315	2.4679487	0.7913753	-1.7068764	-1.2733100
H*H*P''(2)	0.7972028	-1.1500583	-0.2371795	0.5142774	0.3508159	-0.1937646	-0.2791375
H*H*P''(3)	0.1526807	0.1573426	-0.3082751	-0.3379953	0.0361305	0.3438228	0.1765734
H*H*P''(4)	-0.0862471	0.4717366	-0.1486014	-0.4350233	-0.1526807	0.2520396	0.2476690
H*H*P''(5)	-0.0734266	0.2162005	0.0879953	-0.1229604	-0.2156177	-0.1229604	0.0879953
H*H*P''(6)	0.0372960	-0.1861888	0.2476690	0.2520396	-0.1526807	-0.4350233	-0.1486014
H*H*P''(7)	0.0920746	-0.3123543	0.1765734	0.3438228	0.0361305	-0.3379953	-0.3082750
H*H*P''(8)	-0.0629371	0.2607809	-0.2791375	-0.1937646	0.3508159	0.5142774	-0.2371795
H*H*P''(9)	-0.5815851	1.9562937	-1.2733100	-1.7068764	0.7913753	2.4679487	0.2185314

	Y(8)	Y(9)
H*H*P''(1)	1.9562937	-0.5815851
H*H*P''(2)	0.2607809	-0.0629371
H*H*P''(3)	-0.3123543	0.0920746
H*H*P''(4)	-0.1861888	0.0372960
H*H*P''(5)	0.2162005	-0.0734266
H*H*P''(6)	0.4717366	-0.0862471
H*H*P''(7)	0.1573426	0.1526807
H*H*P''(8)	-1.1500583	0.7972028
H*H*P''(9)	-3.8735431	2.0011655

M = 9 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9987568	0.0076145	-0.0184927	0.0206682	-0.0054390	-0.0119658	0.0141414
P(2)	0.0076146	0.9530691	0.1153069	-0.1327117	0.0435121	0.0630925	-0.0804973
P(3)	-0.0184926	0.1153069	0.7106449	0.3502719	-0.1522921	-0.1066045	0.1675214
P(4)	0.0206682	-0.1327117	0.3502719	0.5278943	0.3045843	-0.0152292	-0.1066045
P(5)	-0.0054390	0.0435120	-0.1522922	0.3045843	0.6192696	0.3045843	-0.1522922
P(6)	-0.0119658	0.0630925	-0.1066045	-0.0152292	0.3045843	0.5278943	0.3502719
P(7)	0.0141414	-0.0804973	0.1675214	-0.1066045	-0.1522921	0.3502719	0.7106449
P(8)	-0.0063714	0.0369852	-0.0804973	0.0630925	0.0435121	-0.1327117	0.1153069
P(9)	0.0010878	-0.0063714	0.0141413	-0.0119658	-0.0054390	0.0206682	-0.0184926

	Y(8)	Y(9)
P(1)	-0.0063714	0.0010878
P(2)	0.0369852	-0.0063714
P(3)	-0.0804973	0.0141414
P(4)	0.0630925	-0.0119658
P(5)	0.0435120	-0.0054390
P(6)	-0.1327117	0.0206682
P(7)	0.1153069	-0.0184926
P(8)	0.9530691	0.0076146
P(9)	0.0076145	0.9987568

M = 9 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.1211033	3.8761694	-1.8422920	-1.0973504	1.5157731	0.5387801	-1.6117560
H*P'(2)	-0.2579798	-0.6971173	0.9532557	0.4167988	-0.5084693	-0.1865812	0.5066356
H*P'(3)	0.0694794	-0.6371872	0.0648873	0.4007615	0.2108780	-0.0714996	-0.0961849
H*P'(4)	-0.0036830	0.0781818	-0.6108158	-0.0540093	0.5370629	0.2447086	-0.2928671
H*P'(5)	-0.0296037	0.1609557	-0.2644522	-0.3355478	-0.0000000	0.3355478	0.2644522
H*P'(6)	0.0190676	-0.1204895	0.2928671	-0.2447086	-0.5370629	0.0540093	0.6108158
H*P'(7)	0.0158353	-0.0747009	0.0961849	-0.0714996	-0.2108780	-0.4007615	-0.0648873
H*P'(8)	-0.0529759	0.2795183	-0.5066356	0.1865812	0.5084693	-0.4167988	-0.9532556
H*P'(9)	0.1784460	-0.9202253	1.6117559	-0.5387801	-1.5157731	1.0973504	1.8422921

	Y(8)	Y(9)
H*P'(1)	0.9202253	-0.1784460
H*P'(2)	-0.2795183	0.0529759
H*P'(3)	0.0747009	-0.0158353
H*P'(4)	0.1204895	-0.0190676
H*P'(5)	-0.1609557	0.0296037
H*P'(6)	-0.0781818	0.0036830
H*P'(7)	0.6371872	-0.0694794
H*P'(8)	0.6971173	0.2579798
H*P'(9)	-3.8761694	2.1211033

M = 9 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	3.2344315	-9.1149235	7.0014942	2.7762652	-5.3749546	-1.3985599	5.5096529
H*H*P''(2)	0.8355193	-1.3129034	-0.0264388	0.5238565	0.1592334	-0.1841854	-0.0683968
H*H*P''(3)	0.0041285	0.7886894	-1.1253121	-0.3751334	0.7788915	0.3066848	-0.6404636
H*H*P''(4)	-0.0802538	0.4462652	-0.1156384	-0.4335250	-0.1826470	0.2535379	0.2806319
H*H*P''(5)	0.0285263	-0.2170992	0.6487361	-0.0974722	-0.7253820	-0.0974722	0.6487361
H*H*P''(6)	0.0432893	-0.2116602	0.2806320	0.2535379	-0.1826470	-0.4335250	-0.1156384
H*H*P''(7)	-0.0564776	0.3189925	-0.6404636	0.3066848	0.7788915	-0.3751334	1.1253121
H*H*P''(8)	-0.0246206	0.0979358	-0.0683968	-0.1841854	0.1592334	0.5238565	0.0264388
H*H*P''(9)	0.6516809	-3.2850867	5.5096528	-1.3985599	-5.3749546	2.7762652	7.0014943

	Y(8)	Y(9)
H*H*P''(1)	-3.2850867	0.6516809
H*H*P''(2)	0.0979358	-0.0246206
H*H*P''(3)	0.3189925	-0.0564776
H*H*P''(4)	-0.2116602	0.0432893
H*H*P''(5)	-0.2170992	0.0285263
H*H*P''(6)	0.4462652	-0.0802538
H*H*P''(7)	0.7886894	0.0041285
H*H*P''(8)	-1.3129034	0.8355193
H*H*P''(9)	-9.1149235	3.2344315

M = 9 = NUMBER OF POINTS
N = 7 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9999223	0.0006216	-0.0021756	0.0043514	-0.0054390	0.0043510	-0.0021758
P(2)	0.0006216	0.9950272	0.0174048	-0.0348096	0.0435121	-0.0348097	0.0174048
P(3)	-0.0021756	0.0174048	0.9390831	0.1218337	-0.1522921	0.1218337	-0.0609169
P(4)	0.0043512	-0.0348096	0.1218337	0.7563325	0.3045843	-0.2436674	0.1218337
P(5)	-0.0054390	0.0435120	-0.1522922	0.3045843	0.6192696	0.3045843	-0.1522922
P(6)	0.0043512	-0.0348096	0.1218337	-0.2436674	0.3045843	0.7563325	0.1218337
P(7)	-0.0021756	0.0174048	-0.0609169	0.1218337	-0.1522921	0.1218337	0.9390831
P(8)	0.0006216	-0.0049728	0.0174048	-0.0348097	0.0435121	-0.0348096	0.0174048
P(9)	-0.0000777	0.0006215	-0.0021758	0.0043510	-0.0054390	0.0043514	-0.0021756

	Y(8)	Y(9)
P(1)	0.0006215	-0.0000777
P(2)	-0.0049728	0.0006216
P(3)	0.0174048	-0.0021756
P(4)	-0.0348096	0.0043512
P(5)	0.0435120	-0.0054390
P(6)	-0.0348096	0.0043512
P(7)	0.0174048	-0.0021756
P(8)	0.9950272	0.0006216
P(9)	0.0006216	0.9999223

M = 9 = NUMBER OF POINTS
N = 7 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.4462032	5.8267686	-6.3936906	3.4540479	1.5157731	-4.0126182	2.9396427
H*P'(2)	-0.1739305	-1.2014130	2.1299456	-0.7598913	-0.5084693	0.9901088	-0.6700543
H*P'(3)	0.0387268	-0.4526718	-0.3656488	0.8312976	0.2108780	-0.5020357	0.3343512
H*P'(4)	-0.0161372	0.1529071	-0.7851748	0.1203496	0.5370629	0.0703497	-0.1185082
H*P'(5)	0.0035714	-0.0380952	0.2000000	-0.8000000	-0.0000000	0.8000000	-0.2000000
H*P'(6)	0.0066134	-0.0457642	0.1185082	-0.0703497	-0.5370629	-0.1203496	0.7851748
H*P'(7)	-0.0149173	0.1098146	-0.3343512	0.5020357	-0.2108780	-0.8312976	0.3656488
H*P'(8)	0.0310734	-0.2247775	0.6700543	-0.9901088	0.5084693	0.7598913	-2.1299456
H*P'(9)	-0.1466539	1.0303739	-2.9396427	4.0126182	-1.5157731	-3.4540479	6.3936906

	Y(8)	Y(9)
H*P'(1)	-1.0303739	0.1466539
H*P'(2)	0.2247775	-0.0310734
H*P'(3)	-0.1098146	0.0149173
H*P'(4)	0.0457642	-0.0066134
H*P'(5)	0.0380952	-0.0035714
H*P'(6)	-0.1529071	0.0161372
H*P'(7)	0.4526718	-0.0387268
H*P'(8)	1.2014130	0.1739305
H*P'(9)	-5.8267686	2.4462032

M = 9 = NUMBER OF POINTS
N = 7 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	4.5486117	-17.0000048	25.4000177	-15.6222581	-5.3749546	16.9999633	-12.8888707
H*H*P''(2)	0.7554494	-0.8324838	-1.1474178	1.6448356	0.1592334	-1.3051645	1.0525821
H*H*P''(3)	-0.0567301	1.1538410	-1.9773323	0.4768868	0.7788915	-0.5453354	0.2115566
H*H*P''(4)	-0.0129267	0.0423025	0.8269412	-1.3761046	-0.1826470	1.1961176	-0.6619477
H*H*P''(5)	0.0285263	-0.2170992	0.6487361	-0.0974722	-0.7253820	-0.0974722	0.6487361
H*H*P''(6)	-0.0240378	0.1923025	-0.6619477	1.1961176	-0.1826470	-1.3761046	0.8269412
H*H*P''(7)	0.0043810	-0.0461590	0.2115566	-0.5453354	0.7788915	0.4768868	-1.9773323
H*H*P''(8)	0.0554494	-0.3824839	1.0525821	-1.3051645	0.1592334	1.6448356	-1.1474178
H*H*P''(9)	-0.6624993	4.5999946	-12.8888707	16.9999633	-5.3749546	-15.6222581	25.4000177

	Y(8)	Y(9)
H*H*P''(1)	4.5999946	-0.6624993
H*H*P''(2)	-0.3824839	0.0554494
H*H*P''(3)	-0.0461590	0.0043810
H*H*P''(4)	0.1923025	-0.0240378
H*H*P''(5)	-0.2170992	0.0285263
H*H*P''(6)	0.0423025	-0.0129267
H*H*P''(7)	1.1538410	-0.0567301
H*H*P''(8)	-0.8324838	0.7554494
H*H*P''(9)	-17.0000048	4.5486117

M = 9 = NUMBER OF POINTS
N = 8 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	1.0000000	0.0000001	0.0000002	0.0000009	0.0000010	0.0000005	0.0000001
P(2)	-0.0000000	1.0000000	0.0000001	0.0000002	0.0000002	0.0000001	0.0000000
P(3)	-0.0000000	-0.0000000	1.0000000	0.0000000	0.0000000	0.0000000	0.0000000
P(4)	-0.0000000	-0.0000000	0.0000000	1.0000000	0.0000000	-0.0000000	-0.0000000
P(5)	0.0000000	-0.0000000	0.0000000	-0.0000000	1.0000000	-0.0000000	0.0000000
P(6)	0.0000000	0.0000000	-0.0000000	-0.0000000	0.0000000	1.0000000	0.0000000
P(7)	0.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	1.0000000
P(8)	0.0000000	-0.0000000	0.0000000	0.0000001	0.0000002	0.0000002	0.0000001
P(9)	0.0000000	-0.0000001	0.0000001	0.0000005	0.0000010	0.0000009	0.0000002

	Y(8)	Y(9)
P(1)	-0.0000001	0.0000000
P(2)	-0.0000000	0.0000000
P(3)	-0.0000000	0.0000000
P(4)	0.0000000	0.0000000
P(5)	-0.0000000	0.0000000
P(6)	-0.0000000	-0.0000000
P(7)	-0.0000000	-0.0000000
P(8)	1.0000000	-0.0000000
P(9)	0.0000001	1.0000000

M = 9 = NUMBER OF POINTS
N = 8 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.7178571	7.9999998	-14.0000001	18.6666651	-17.5000014	11.1999990	-4.6666667
H*P'(2)	-0.1250000	-1.5928572	3.4999999	-3.5000003	2.9166663	-1.7500001	0.7000000
H*P'(3)	0.0178571	-0.2857143	-0.9500000	1.9999999	-1.2500000	0.6666666	-0.2500000
H*P'(4)	-0.0059524	0.0714286	-0.5000000	-0.4500000	1.2500000	-0.5000000	0.1666667
H*P'(5)	0.0035714	-0.0380952	0.2000000	-0.8000000	0.0000000	0.8000000	-0.2000000
H*P'(6)	-0.0035714	0.0357143	-0.1666667	0.5000000	-1.2500000	0.4500000	0.5000000
H*P'(7)	0.0059524	-0.0571429	0.2500000	-0.6666666	1.2500000	-1.9999999	0.9500000
H*P'(8)	-0.0178571	0.1666666	-0.7000000	1.7500001	-2.9166663	3.5000003	-3.4999999
H*P'(9)	0.1250000	-1.1428572	4.6666667	-11.1999990	17.5000012	-18.6666651	14.0000001

	Y(8)	Y(9)
H*P'(1)	1.1428572	-0.1250000
H*P'(2)	-0.1666666	0.0178571
H*P'(3)	0.0571429	-0.0059524
H*P'(4)	-0.0357143	0.0035714
H*P'(5)	0.0380952	-0.0035714
H*P'(6)	-0.0714286	0.0059524
H*P'(7)	0.2857143	-0.0178571
H*P'(8)	1.5928572	0.1250000
H*P'(9)	-7.9999998	2.7178571

M = 9 = NUMBER OF POINTS
N = 8 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	5.8593253	-27.4857140	62.0999994	-89.0222197	86.3750010	-56.3999982	23.8111110
H*H*P''(2)	0.6482143	0.0253969	-4.1499999	7.6500005	-7.3472216	4.7000003	-1.9499999
H*H*P''(3)	-0.0517857	1.1142857	-1.8388888	0.2000001	1.1250001	-0.8222222	0.3500000
H*H*P''(4)	0.0093254	-0.1357143	1.4500000	-2.6222222	1.3750000	-0.0500000	-0.0388889
H*H*P''(5)	-0.0017857	0.0253968	-0.2000000	1.6000000	-2.8472222	1.6000000	-0.2000000
H*H*P''(6)	-0.0017857	0.0142857	-0.0388889	-0.0500000	1.3750000	-2.6222222	1.4500000
H*H*P''(7)	0.0093254	-0.0857143	0.3500000	-0.8222222	1.1250001	0.2000001	-1.8388888
H*H*P''(8)	-0.0517857	0.4753968	-1.9499999	4.7000003	-7.3472216	7.6500005	-4.1499999
H*H*P''(9)	0.6482143	-5.8857144	23.8111110	-56.3999982	86.3750010	-89.0222197	62.0999994

	Y(8)	Y(9)
H*H*P''(1)	-5.8857144	0.6482143
H*H*P''(2)	0.4753968	-0.0517857
H*H*P''(3)	-0.0857143	0.0093254
H*H*P''(4)	0.0142857	-0.0017857
H*H*P''(5)	0.0253968	-0.0017857
H*H*P''(6)	-0.1357143	0.0093254
H*H*P''(7)	1.1142857	-0.0517857
H*H*P''(8)	0.0253969	0.6482143
H*H*P''(9)	-27.4857140	5.8593253

M = 11 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.3181818	0.2727273	0.2272727	0.1818182	0.1363636	0.0909091	0.0454545
P(2)	0.2727273	0.2363636	0.2000000	0.1636364	0.1272727	0.0909091	0.0545455
P(3)	0.2272727	0.2000000	0.1727273	0.1454545	0.1181818	0.0909091	0.0636364
P(4)	0.1818182	0.1636364	0.1454545	0.1272727	0.1090909	0.0909091	0.0727273
P(5)	0.1363636	0.1272727	0.1181818	0.1090909	0.1000000	0.0909091	0.0818182
P(6)	0.0909091	0.0909091	0.0909091	0.0909091	0.0909091	0.0909091	0.0909091
P(7)	0.0454545	0.0545455	0.0636364	0.0727273	0.0818182	0.0909091	0.1000000
P(8)	-0.0000000	0.0181818	0.0363636	0.0545455	0.0727273	0.0909091	0.1090909
P(9)	-0.0454545	-0.0181818	0.0090909	0.0363636	0.0636364	0.0909091	0.1181818
P(10)	-0.0909091	-0.0545455	-0.0181818	0.0181818	0.0545455	0.0909091	0.1272727
P(11)	-0.1363636	-0.0909091	-0.0454545	0.0000000	0.0454545	0.0909091	0.1363636

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	-0.0000000	-0.0454545	-0.0909091	-0.1363636
P(2)	0.0181818	-0.0181818	-0.0545455	-0.0909091
P(3)	0.0363636	0.0090909	-0.0181818	-0.0454545
P(4)	0.0545455	0.0363636	0.0181818	-0.0000000
P(5)	0.0727273	0.0636364	0.0545455	0.0454545
P(6)	0.0909091	0.0909091	0.0909091	0.0909091
P(7)	0.1090909	0.1181818	0.1272727	0.1363636
P(8)	0.1272727	0.1454545	0.1636364	0.1818182
P(9)	0.1454545	0.1727273	0.2000000	0.2272727
P(10)	0.1636364	0.2000000	0.2363636	0.2727273
P(11)	0.1818182	0.2272727	0.2727273	0.3181818

M = 11 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(2)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(3)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(4)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(5)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(6)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(7)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(8)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(9)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(10)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909
H*P'(11)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	0.0000000	0.0090909

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(2)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(3)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(4)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(5)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(6)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(7)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(8)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(9)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(10)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(11)	0.0181818	0.0272727	0.0363636	0.0454545

M = 11 = NUMBER OF POINTS
N = 1 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(2)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(3)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(4)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(5)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(6)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(7)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(8)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(9)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(10)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000
H*H*P''(11)	-0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0000000

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(2)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(3)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(4)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(5)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(6)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(7)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(8)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(9)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(10)	0.0000000	0.0000000	0.0000000	0.0000000
H*H*P''(11)	0.0000000	0.0000000	0.0000000	0.0000000

M = 11 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.5804196	0.3776224	0.2097902	0.0769231	-0.0209790	-0.0839161	-0.1118881
P(2)	0.3776224	0.2783217	0.1930070	0.1216783	0.0643357	0.0209790	-0.0083916
P(3)	0.2097902	0.1930070	0.1738928	0.1524475	0.1286713	0.1025641	0.0741259
P(4)	0.0769231	0.1216783	0.1524476	0.1692308	0.1720280	0.1608392	0.1356643
P(5)	-0.0209790	0.0643357	0.1286713	0.1720280	0.1944056	0.1958042	0.1762238
P(6)	-0.0839161	0.0209790	0.1025641	0.1608392	0.1958042	0.2074592	0.1958042
P(7)	-0.1118881	-0.0083916	0.0741259	0.1356643	0.1762238	0.1958042	0.1944056
P(8)	-0.1048951	-0.0237762	0.0433566	0.0965035	0.1356643	0.1608392	0.1720280
P(9)	-0.0629371	-0.0251748	0.0102564	0.0433566	0.0741259	0.1025641	0.1286713
P(10)	0.0139860	-0.0125874	-0.0251748	-0.0237762	-0.0083916	0.0209790	0.0643357
P(11)	0.1258741	0.0139860	-0.0629371	-0.1048951	-0.1118881	-0.0839161	-0.0209790

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	-0.1048951	-0.0629371	0.0139860	0.1258741
P(2)	-0.0237762	-0.0251748	-0.0125874	0.0139860
P(3)	0.0433566	0.0102564	-0.0251748	-0.0629371
P(4)	0.0965035	0.0433566	-0.0237762	-0.1048951
P(5)	0.1356643	0.0741259	-0.0083916	-0.1118881
P(6)	0.1608392	0.1025641	0.0209790	-0.0839161
P(7)	0.1720280	0.1286713	0.0643357	-0.0209790
P(8)	0.1692308	0.1524476	0.1216783	0.0769231
P(9)	0.1524475	0.1738928	0.1930070	0.2097902
P(10)	0.1216783	0.1930070	0.2783217	0.3776224
P(11)	0.0769231	0.2097902	0.3776224	0.5804196

M = 11 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.2202797	-0.1062937	-0.0156177	0.0517483	0.0958042	0.1165501	0.1139860
H*P'(2)	-0.1853147	-0.0923077	-0.0179487	0.0377622	0.0748252	0.0932401	0.0930070
H*P'(3)	-0.1503496	-0.0783217	-0.0202797	0.0237762	0.0538462	0.0699301	0.0720280
H*P'(4)	-0.1153846	-0.0643357	-0.0226107	0.0097902	0.0328671	0.0466200	0.0510490
H*P'(5)	-0.0804196	-0.0503496	-0.0249417	-0.0041958	0.0118881	0.0233100	0.0300699
H*P'(6)	-0.0454545	-0.0363636	-0.0272727	-0.0181818	-0.0090909	-0.0000000	0.0090909
H*P'(7)	-0.0104895	-0.0223776	-0.0296037	-0.0321678	-0.0300699	-0.0233100	-0.0118881
H*P'(8)	0.0244755	-0.0083916	-0.0319347	-0.0461538	-0.0510490	-0.0466200	-0.0328671
H*P'(9)	0.0594406	0.0055944	-0.0342657	-0.0601399	-0.0720280	-0.0699301	-0.0538462
H*P'(10)	0.0944056	0.0195804	-0.0365967	-0.0741259	-0.0930070	-0.0932401	-0.0748252
H*P'(11)	0.1293706	0.0335664	-0.0389277	-0.0881119	-0.1139860	-0.1165501	-0.0958042

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	0.0881119	0.0389277	-0.0335664	-0.1293706
H*P'(2)	0.0741259	0.0365967	-0.0195804	-0.0944056
H*P'(3)	0.0601399	0.0342657	-0.0055944	-0.0594406
H*P'(4)	0.0461538	0.0319347	0.0083916	-0.0244755
H*P'(5)	0.0321678	0.0296037	0.0223776	0.0104895
H*P'(6)	0.0181818	0.0272727	0.0363636	0.0454545
H*P'(7)	0.0041958	0.0249417	0.0503496	0.0804196
H*P'(8)	-0.0097902	0.0226107	0.0643357	0.1153846
H*P'(9)	-0.0237762	0.0202797	0.0783217	0.1503496
H*P'(10)	-0.0377622	0.0179487	0.0923077	0.1853147
H*P'(11)	-0.0517483	0.0156177	0.1062937	0.2202797

M = 11 = NUMBER OF POINTS
N = 2 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(2)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(3)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(4)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(5)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(6)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(7)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(8)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(9)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(10)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(11)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(2)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(3)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(4)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(5)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(6)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(7)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(8)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(9)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(10)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(11)	-0.0139860	-0.0023310	0.0139860	0.0349650

M = 11 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.7902098	0.3356643	0.0559441	-0.0839161	-0.1188811	-0.0839161	-0.0139860
P(2)	0.3356643	0.2867133	0.2237762	0.1538462	0.0839161	0.0209790	-0.0279720
P(3)	0.0559441	0.2237762	0.2867133	0.2703963	0.2004662	0.1025641	0.0023310
P(4)	-0.0839161	0.1538462	0.2703963	0.2925408	0.2470862	0.1608392	0.0606061
P(5)	-0.1188811	0.0839161	0.2004662	0.2470862	0.2400932	0.1958042	0.1305361
P(6)	-0.0839161	0.0209790	0.1025641	0.1608392	0.1958042	0.2074592	0.1958042
P(7)	-0.0139860	-0.0279720	0.0023310	0.0606061	0.1305361	0.1958042	0.2400932
P(8)	0.0559441	-0.0559441	-0.0745921	-0.0268065	0.0606061	0.1608392	0.2470862
P(9)	0.0909091	-0.0559441	-0.1025641	-0.0745921	0.0023310	0.1025641	0.2004662
P(10)	0.0559441	-0.0209790	-0.0559441	-0.0559441	-0.0279720	0.0209790	0.0839161
P(11)	-0.0839161	0.0559441	0.0909091	0.0559441	-0.0139860	-0.0839161	-0.1188811

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	0.0559441	0.0909091	0.0559441	-0.0839161
P(2)	-0.0559441	-0.0559441	-0.0209790	0.0559441
P(3)	-0.0745921	-0.1025641	-0.0559441	0.0909091
P(4)	-0.0268065	-0.0745921	-0.0559441	0.0559441
P(5)	0.0606061	0.0023310	-0.0279720	-0.0139860
P(6)	0.1608392	0.1025641	0.0209790	-0.0839161
P(7)	0.2470862	0.2004662	0.0839161	-0.1188811
P(8)	0.2925408	0.2703963	0.1538462	-0.0839161
P(9)	0.2703963	0.2867133	0.2237762	0.0559441
P(10)	0.1538462	0.2237762	0.2867133	0.3356643
P(11)	-0.0839161	0.0559441	0.3356643	0.7902098

M = 11 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.5536130	-0.0396270	0.2288267	0.3073038	0.2513597	0.1165501	-0.0415695
H*P'(2)	-0.3613054	-0.0571096	0.1111111	0.1726884	0.1569542	0.0932401	0.0108780
H*P'(3)	-0.2039627	-0.0675991	0.0190365	0.0648796	0.0788656	0.0699301	0.0470085
H*P'(4)	-0.0815851	-0.0710956	-0.0473970	-0.0161228	0.0170940	0.0466200	0.0668221
H*P'(5)	0.0058275	-0.0675991	-0.0881896	-0.0703186	-0.0283605	0.0233100	0.0703186
H*P'(6)	0.0582751	-0.0571096	-0.1033411	-0.0977078	-0.0574981	-0.0000000	0.0574981
H*P'(7)	0.0757576	-0.0396270	-0.0928516	-0.0982906	-0.0703186	-0.0233100	0.0283605
H*P'(8)	0.0582751	-0.0151515	-0.0567211	-0.0720668	-0.0668221	-0.0466200	-0.0170940
H*P'(9)	0.0058275	0.0163170	0.0050505	-0.0190365	-0.0470085	-0.0699301	-0.0788656
H*P'(10)	-0.0815851	0.0547786	0.0924631	0.0608003	-0.0108780	-0.0932401	-0.1569542
H*P'(11)	-0.2039627	0.1002331	0.2055167	0.1674437	0.0415695	-0.1165501	-0.2513597

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	-0.1674437	-0.2055167	-0.1002331	0.2039627
H*P'(2)	-0.0608003	-0.0924631	-0.0547786	0.0815851
H*P'(3)	0.0190365	-0.0050505	-0.0163170	-0.0058275
H*P'(4)	0.0720668	0.0567211	0.0151515	-0.0582751
H*P'(5)	0.0982906	0.0928516	0.0396270	-0.0757576
H*P'(6)	0.0977078	0.1033411	0.0571096	-0.0582751
H*P'(7)	0.0703186	0.0881896	0.0675991	-0.0058275
H*P'(8)	0.0161228	0.0473970	0.0710956	0.0815851
H*P'(9)	-0.0648796	-0.0190365	0.0675991	0.2039627
H*P'(10)	-0.1726884	-0.1111111	0.0571096	0.3613054
H*P'(11)	-0.3073038	-0.2288267	0.0396270	0.5536130

M = 11 = NUMBER OF POINTS
N = 3 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.2097902	-0.0209790	-0.1305361	-0.1480186	-0.1025641	-0.0233100	0.0606061
H*H*P''(2)	0.1748252	-0.0139860	-0.1048951	-0.1212121	-0.0862471	-0.0233100	0.0442890
H*H*P''(3)	0.1398601	-0.0069930	-0.0792541	-0.0944056	-0.0699301	-0.0233100	0.0279720
H*H*P''(4)	0.1048951	-0.0000000	-0.0536131	-0.0675991	-0.0536131	-0.0233100	0.0116550
H*H*P''(5)	0.0699301	0.0069930	-0.0279720	-0.0407925	-0.0372960	-0.0233100	-0.0046620
H*H*P''(6)	0.0349650	0.0139860	-0.0023310	-0.0139860	-0.0209790	-0.0233100	-0.0209790
H*H*P''(7)	-0.0000000	0.0209790	0.0233100	0.0128205	-0.0046620	-0.0233100	-0.0372960
H*H*P''(8)	-0.0349650	0.0279720	0.0489510	0.0396270	0.0116550	-0.0233100	-0.0536131
H*H*P''(9)	-0.0699301	0.0349650	0.0745921	0.0664336	0.0279720	-0.0233100	-0.0699301
H*H*P''(10)	-0.1048951	0.0419580	0.1002331	0.0932401	0.0442890	-0.0233100	-0.0862471
H*H*P''(11)	-0.1398601	0.0489510	0.1258741	0.1200466	0.0606061	-0.0233100	-0.1025641

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	0.1200466	0.1258741	0.0489510	-0.1398601
H*H*P''(2)	0.0932401	0.1002331	0.0419580	-0.1048951
H*H*P''(3)	0.0664336	0.0745921	0.0349650	-0.0699301
H*H*P''(4)	0.0396270	0.0489510	0.0279720	-0.0349650
H*H*P''(5)	0.0128205	0.0233100	0.0209790	-0.0000000
H*H*P''(6)	-0.0139860	-0.0023310	0.0139860	0.0349650
H*H*P''(7)	-0.0407925	-0.0279720	0.0069930	0.0699301
H*H*P''(8)	-0.0675991	-0.0536131	0.0000000	0.1048951
H*H*P''(9)	-0.0944056	-0.0792541	-0.0069930	0.1398601
H*H*P''(10)	-0.1212121	-0.1048951	-0.0139860	0.1748252
H*H*P''(11)	-0.1480186	-0.1305361	-0.0209790	0.2097902

M = 11 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9160839	0.2097902	-0.0699301	-0.1048951	-0.0349650	0.0419580	0.0699301
P(2)	0.2097902	0.4125874	0.3496503	0.1748252	0.0000000	-0.1048951	-0.1118881
P(3)	-0.0699301	0.3496503	0.4125874	0.2913753	0.1165501	-0.0233100	-0.0815851
P(4)	-0.1048951	0.1748252	0.2913753	0.2960373	0.2331002	0.1398601	0.0466200
P(5)	-0.0349650	0.0000000	0.1165501	0.2331002	0.2960373	0.2797203	0.1864802
P(6)	0.0419580	-0.1048951	-0.0233100	0.1398601	0.2797203	0.3333333	0.2797203
P(7)	0.0699301	-0.1118881	-0.0815851	0.0466200	0.1864802	0.2797203	0.2960373
P(8)	0.0349650	-0.0349650	-0.0536131	-0.0233100	0.0466200	0.1398601	0.2331002
P(9)	-0.0349650	0.0699301	0.0233100	-0.0536131	-0.0815851	-0.0233100	0.1165501
P(10)	-0.0699301	0.1048951	0.0699301	-0.0349650	-0.1118881	-0.1048951	0.0000000
P(11)	0.0419580	-0.0699301	-0.0349650	0.0349650	0.0699301	0.0419580	-0.0349650

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	0.0349650	-0.0349650	-0.0699301	0.0419580
P(2)	-0.0349650	0.0699301	0.1048951	-0.0699301
P(3)	-0.0536131	0.0233100	0.0699301	-0.0349650
P(4)	-0.0233100	-0.0536131	-0.0349650	0.0349650
P(5)	0.0466200	-0.0815851	-0.1118881	0.0699301
P(6)	0.1398601	-0.0233100	-0.1048951	0.0419580
P(7)	0.2331002	0.1165501	0.0000000	-0.0349650
P(8)	0.2960373	0.2913753	0.1748252	-0.1048951
P(9)	0.2913753	0.4125874	0.3496503	-0.0699301
P(10)	0.1748252	0.3496503	0.4125874	0.2097902
P(11)	-0.1048951	-0.0699301	0.2097902	0.9160839

M = 11 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-0.9906760	0.3974359	0.6658897	0.3801476	-0.0400155	-0.3205128	-0.3329448
H*P'(2)	-0.4592075	0.0407925	0.2090132	0.1890054	0.0916861	-0.0046620	-0.0543901
H*P'(3)	-0.1305361	-0.1410256	-0.0543901	0.0526418	0.1278166	0.1433566	0.0959596
H*P'(4)	0.0372960	-0.1899767	-0.1662782	-0.0359363	0.0963481	0.1655012	0.1460761
H*P'(5)	0.0862471	-0.1480186	-0.1686092	-0.0837218	0.0252525	0.1037296	0.1239316
H*P'(6)	0.0582751	-0.0571096	-0.1033411	-0.0977078	-0.0574981	-0.0000000	0.0574981
H*P'(7)	-0.0046620	0.0407925	-0.0124320	-0.0848873	-0.1239316	-0.1037296	-0.0252525
H*P'(8)	-0.0606061	0.1037296	0.0621601	-0.0522533	-0.1460761	-0.1655012	-0.0963481
H*P'(9)	-0.0675991	0.0897436	0.0784771	-0.0067988	-0.0959596	-0.1433566	-0.1278166
H*P'(10)	0.0163170	-0.0431235	-0.0054390	0.0444833	0.0543901	0.0046620	-0.0916861
H*P'(11)	0.2331002	-0.3368298	-0.2315462	0.0945998	0.3329448	0.3205128	0.0400155

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	-0.0945998	0.2315462	0.3368298	-0.2331002
H*P'(2)	-0.0444833	0.0054390	0.0431235	-0.0163170
H*P'(3)	0.0067988	-0.0784771	-0.0897436	0.0675991
H*P'(4)	0.0522533	-0.0621601	-0.1037296	0.0606061
H*P'(5)	0.0848873	0.0124320	-0.0407925	0.0046620
H*P'(6)	0.0977078	0.1033411	0.0571096	-0.0582751
H*P'(7)	0.0837218	0.1686092	0.1480186	-0.0862471
H*P'(8)	0.0359363	0.1662782	0.1899767	-0.0372960
H*P'(9)	-0.0526418	0.0543901	0.1410256	0.1305361
H*P'(10)	-0.1890054	-0.2090132	-0.0407925	0.4592075
H*P'(11)	-0.3801476	-0.6658897	-0.3974359	0.9906760

M = 11 = NUMBER OF POINTS
N = 4 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	0.6468531	-0.4580420	-0.5675991	-0.2208625	0.1888112	0.4137529	0.3519813
H*H*P''(2)	0.4230769	-0.2622378	-0.3531469	-0.1625874	0.0792541	0.2249417	0.2097902
H*H*P''(3)	0.2412587	-0.1083916	-0.1806527	-0.1113054	-0.0023310	0.0780886	0.0955711
H*H*P''(4)	0.1013986	0.0034965	-0.0501165	-0.0670163	-0.0559441	-0.0268065	0.0093240
H*H*P''(5)	0.0034965	0.0734266	0.0384615	-0.0297203	-0.0815851	-0.0897436	-0.0489510
H*H*P''(6)	-0.0524476	0.1013986	0.0850816	0.0005828	-0.0792541	-0.1107226	-0.0792541
H*H*P''(7)	-0.0664336	0.0874126	0.0897436	0.0238928	-0.0489510	-0.0897436	-0.0815851
H*H*P''(8)	-0.0384615	0.0314685	0.0524476	0.0402098	0.0093240	-0.0268065	-0.0559441
H*H*P''(9)	0.0314685	-0.0664336	-0.0268065	0.0495338	0.0955711	0.0780886	-0.0023310
H*H*P''(10)	0.1433566	-0.2062937	-0.1480186	0.0518648	0.2097902	0.2249417	0.0792541
H*H*P''(11)	0.2972028	-0.3881119	-0.3111888	0.0472028	0.3519813	0.4137529	0.1888112

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	0.0472028	-0.3111888	-0.3881119	0.2972028
H*H*P''(2)	0.0518648	-0.1480186	-0.2062937	0.1433566
H*H*P''(3)	0.0495338	-0.0268065	-0.0664336	0.0314685
H*H*P''(4)	0.0402098	0.0524476	0.0314685	-0.0384615
H*H*P''(5)	0.0238928	0.0897436	0.0874126	-0.0664336
H*H*P''(6)	0.0005828	0.0850816	0.1013986	-0.0524476
H*H*P''(7)	-0.0297203	0.0384615	0.0734266	0.0034965
H*H*P''(8)	-0.0670163	-0.0501165	0.0034965	0.1013986
H*H*P''(9)	-0.1113054	-0.1806527	-0.1083916	0.2412587
H*H*P''(10)	-0.1625874	-0.3531469	-0.2622378	0.4230769
H*H*P''(11)	-0.2208625	-0.5675991	-0.4580420	0.6468531

M = 11 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9737762	0.0944056	-0.0891608	-0.0279720	0.0419580	0.0419580	-0.0069930
P(2)	0.0944056	0.6433566	0.3881119	0.0209790	-0.1538462	-0.1048951	0.0419580
P(3)	-0.0891608	0.3881119	0.4189977	0.2657343	0.0909091	-0.0233100	-0.0559441
P(4)	-0.0279720	0.0209790	0.2657343	0.3986014	0.3356643	0.1398601	-0.0559441
P(5)	0.0419580	-0.1538462	0.0909091	0.3356643	0.3986014	0.2797203	0.0839161
P(6)	0.0419580	-0.1048951	-0.0233100	0.1398601	0.2797203	0.3333333	0.2797203
P(7)	-0.0069930	0.0419580	-0.0559441	-0.0559441	0.0839161	0.2797203	0.3986014
P(8)	-0.0419580	0.1188811	-0.0279720	-0.1258741	-0.0559441	0.1398601	0.3356643
P(9)	-0.0157343	0.0314685	0.0168998	-0.0279720	-0.0559441	-0.0233100	0.0909091
P(10)	0.0454545	-0.1258741	0.0314685	0.1188811	0.0419580	-0.1048951	-0.1538462
P(11)	-0.0157343	0.0454545	-0.0157343	-0.0419581	-0.0069930	0.0419580	0.0419580

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	-0.0419581	-0.0157343	0.0454545	-0.0157343
P(2)	0.1188811	0.0314685	-0.1258741	0.0454545
P(3)	-0.0279720	0.0168998	0.0314685	-0.0157343
P(4)	-0.1258741	-0.0279720	0.1188811	-0.0419580
P(5)	-0.0559441	-0.0559441	0.0419580	-0.0069930
P(6)	0.1398601	-0.0233100	-0.1048951	0.0419580
P(7)	0.3356643	0.0909091	-0.1538462	0.0419580
P(8)	0.3986014	0.2657343	0.0209790	-0.0279720
P(9)	0.2657343	0.4189977	0.3881119	-0.0891608
P(10)	0.0209790	0.3881119	0.6433566	0.0944056
P(11)	-0.0279720	-0.0891608	0.0944056	0.9737762

M = 11 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-1.4429196	1.3019231	0.8166375	-0.2228438	-0.6430070	-0.3205128	0.2700466
H*P'(2)	-0.4354895	-0.0066434	0.2011072	0.2206294	0.1233100	-0.0046620	-0.0860140
H*P'(3)	-0.0058566	-0.3903846	-0.0959499	0.2188811	0.2940559	0.1433566	-0.0702797
H*P'(4)	0.0898601	-0.2951049	-0.1837995	0.0341492	0.1664336	0.1655012	0.0759907
H*P'(5)	0.0378496	-0.0512238	-0.1524767	-0.1482517	-0.0392774	0.1037296	0.1884615
H*P'(6)	-0.0333916	0.1262238	-0.0727855	-0.2199301	-0.1797203	-0.0000000	0.1797203
H*P'(7)	-0.0530594	0.1375874	0.0037005	-0.1494172	-0.1884615	-0.1037296	0.0392774
H*P'(8)	-0.0080420	-0.0013986	0.0446387	0.0178322	-0.0759907	-0.1655012	-0.1664336
H*P'(9)	0.0570804	-0.1596154	0.0369172	0.1594406	0.0702797	-0.1433566	-0.2940559
H*P'(10)	0.0400350	-0.0905594	-0.0133450	0.0761072	0.0860140	0.0046620	-0.1233100
H*P'(11)	-0.2191434	0.5676573	-0.0807984	-0.5083916	-0.2700466	0.3205128	0.6430070

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	0.5083916	0.0807984	-0.5676573	0.2191434
H*P'(2)	-0.0761072	0.0133450	0.0905594	-0.0400350
H*P'(3)	-0.1594406	-0.0369172	0.1596154	-0.0570804
H*P'(4)	-0.0178322	-0.0446387	0.0013986	0.0080420
H*P'(5)	0.1494172	-0.0037005	-0.1375874	0.0530594
H*P'(6)	0.2199301	0.0727855	-0.1262238	0.0333916
H*P'(7)	0.1482517	0.1524767	0.0512238	-0.0378496
H*P'(8)	-0.0341492	0.1837995	0.2951049	-0.0898601
H*P'(9)	-0.2188811	0.0959499	0.3903846	0.0058566
H*P'(10)	-0.2206294	-0.2011072	0.0066434	0.4354895
H*P'(11)	0.2228438	-0.8166375	-1.3019231	1.4429196

M = 11 = NUMBER OF POINTS
N = 5 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	1.3920455	-1.9484265	-0.8159965	0.7727273	1.1824009	0.4137529	-0.6416084
H*H*P''(2)	0.6730769	-0.7622377	-0.4364802	0.1707459	0.4125874	0.2249417	-0.1235431
H*H*P''(3)	0.2268357	-0.0795455	-0.1758450	-0.1305361	-0.0215618	0.0780886	0.1148019
H*H*P''(4)	-0.0043706	0.2150350	-0.0148601	-0.2080420	-0.1969697	-0.0268065	0.1503496
H*H*P''(5)	-0.0782343	0.2368881	0.0657051	-0.1386946	-0.1905594	-0.0897436	0.0600233
H*H*P''(6)	-0.0524476	0.1013986	0.0850816	0.0005828	-0.0792541	-0.1107226	-0.0792541
H*H*P''(7)	0.0152972	-0.0760490	0.0625000	0.1328671	0.0600233	-0.0897436	-0.1905594
H*H*P''(8)	0.0673077	-0.1800699	0.0171911	0.1812354	0.1503496	-0.0268065	-0.1969697
H*H*P''(9)	0.0458916	-0.0952797	-0.0316142	0.0687646	0.1148019	0.0780886	-0.0215618
H*H*P''(10)	-0.1066434	0.2937063	-0.0646853	-0.2814685	-0.1235431	0.2249417	0.4125874
H*H*P''(11)	-0.4479895	1.1022727	-0.0627914	-0.9463869	-0.6416084	0.4137529	1.1824009

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	-0.9463869	-0.0627914	1.1022727	-0.4479895
H*H*P''(2)	-0.2814685	-0.0646853	0.2937063	-0.1066434
H*H*P''(3)	0.0687646	-0.0316142	-0.0952797	0.0458916
H*H*P''(4)	0.1812354	0.0171911	-0.1800699	0.0673077
H*H*P''(5)	0.1328671	0.0625000	-0.0760490	0.0152972
H*H*P''(6)	0.0005828	0.0850816	0.1013986	-0.0524476
H*H*P''(7)	-0.1386946	0.0657051	0.2368881	-0.0782343
H*H*P''(8)	-0.2080420	-0.0148601	0.2150350	-0.0043706
H*H*P''(9)	-0.1305361	-0.1758450	-0.0795455	0.2268357
H*H*P''(10)	0.1707459	-0.4364802	-0.7622377	0.6730769
H*H*P''(11)	0.7727273	-0.8159965	-1.9484265	1.3920454

M = 11 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9938297	0.0302345	-0.0503908	0.0201564	0.0259152	-0.0115179	-0.0230358
P(2)	0.0302345	0.8487042	0.2640477	-0.1330317	-0.1025093	0.0662279	0.0932949
P(3)	-0.0503908	0.2640477	0.4939531	0.3587824	0.0598930	-0.1266968	-0.0869601
P(4)	0.0201563	-0.1330317	0.3587824	0.5141094	0.2971617	0.0115179	-0.0944467
P(5)	0.0259153	-0.1025093	0.0598930	0.2971617	0.4114356	0.3225010	0.0967503
P(6)	-0.0115179	0.0662279	-0.1266968	0.0115179	0.3225010	0.4759358	0.3225010
P(7)	-0.0230358	0.0932949	-0.0869601	-0.0944467	0.0967503	0.3225010	0.4114356
P(8)	0.0061703	-0.0351296	0.0650761	-0.0103661	-0.0944467	0.0115179	0.2971617
P(9)	0.0230358	-0.0925956	0.0918552	0.0650761	-0.0869601	-0.1266968	0.0598930
P(10)	-0.0187166	0.0794735	-0.0925956	-0.0351296	0.0932949	0.0662279	-0.1025093
P(11)	0.0043192	-0.0187166	0.0230358	0.0061703	-0.0230358	-0.0115179	0.0259152

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	0.0061703	0.0230358	-0.0187166	0.0043192
P(2)	-0.0351296	-0.0925956	0.0794735	-0.0187166
P(3)	0.0650761	0.0918552	-0.0925956	0.0230358
P(4)	-0.0103661	0.0650761	-0.0351296	0.0061703
P(5)	-0.0944467	-0.0869601	0.0932949	-0.0230358
P(6)	0.0115179	-0.1266968	0.0662279	-0.0115179
P(7)	0.2971617	0.0598930	-0.1025093	0.0259153
P(8)	0.5141094	0.3587824	-0.1330317	0.0201563
P(9)	0.3587824	0.4939531	0.2640477	-0.0503908
P(10)	-0.1330317	0.2640477	0.8487042	0.0302345
P(11)	0.0201564	-0.0503908	0.0302345	0.9938297

M = 11 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-1.8483340	2.5992492	0.0328363	-1.1958385	-0.3186754	0.7605923	0.5943782
H*P'(2)	-0.3403023	-0.3112423	0.3851357	0.4490785	0.0471603	-0.2584945	-0.1621637
H*P'(3)	0.0631942	-0.6113472	0.0375483	0.3846030	0.2388153	-0.0407788	-0.1255204
H*P'(4)	0.0465446	-0.1564953	-0.2675428	-0.0698080	0.2010860	0.2810092	0.1106431
H*P'(5)	-0.0282600	0.1603270	-0.2802886	-0.3069148	0.0136103	0.2800219	0.2413492
H*P'(6)	-0.0333916	0.1262238	-0.0727855	-0.2199301	-0.1797203	-0.0000000	0.1797203
H*P'(7)	0.0130502	-0.0739634	0.1315124	0.0092459	-0.2413492	-0.2800219	-0.0136103
H*P'(8)	0.0352735	-0.1400082	0.1283820	0.1217894	-0.1106431	-0.2810092	-0.2010860
H*P'(9)	-0.0119704	0.0613472	-0.0965810	-0.0062814	0.1255204	0.0407788	-0.2388153
H*P'(10)	-0.0551522	0.2140395	-0.1973735	-0.1523420	0.1621637	0.2584945	-0.0471603
H*P'(11)	0.1862711	-0.7296689	0.7030029	0.4646031	-0.5943782	-0.7605923	0.3186754

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	-0.4646031	-0.7030029	0.7296689	-0.1862711
H*P'(2)	0.1523420	0.1973735	-0.2140395	0.0551522
H*P'(3)	0.0062814	0.0965810	-0.0613472	0.0119704
H*P'(4)	-0.1217894	-0.1283820	0.1400082	-0.0352735
H*P'(5)	-0.0092459	-0.1315124	0.0739634	-0.0130502
H*P'(6)	0.2199301	0.0727855	-0.1262238	0.0333916
H*P'(7)	0.3069148	0.2802886	-0.1603270	0.0282600
H*P'(8)	0.0698080	0.2675428	0.1564953	-0.0465446
H*P'(9)	-0.3846030	-0.0375483	0.6113472	-0.0631942
H*P'(10)	-0.4490785	-0.3851357	0.3112423	0.3403023
H*P'(11)	1.1958385	-0.0328363	-2.5992492	1.8483340

M = 11 = NUMBER OF POINTS
N = 6 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	2.3866756	-5.1312429	1.1069551	3.1598396	0.3866968	-2.2385941	-1.4373125
H*H*P''(2)	0.8104210	-1.2017386	-0.1709484	0.5003716	0.3027122	-0.1413090	-0.2334184
H*H*P''(3)	0.1091663	0.2969964	-0.4033390	-0.4129425	0.0725737	0.3918735	0.2089373
H*H*P''(4)	-0.0836041	0.4685822	-0.1680449	-0.3982024	-0.1335829	0.1844828	0.2137365
H*H*P''(5)	-0.0461710	0.1342856	0.1276941	-0.0617428	-0.2162101	-0.1752457	0.0343727
H*H*P''(6)	0.0314205	-0.1669793	0.2472266	0.2018662	-0.1463486	-0.3343709	-0.1463486
H*H*P''(7)	0.0473605	-0.1786514	0.1244890	0.2098190	0.0343727	-0.1752457	-0.2162101
H*H*P''(8)	-0.0119258	0.0734773	-0.1359936	-0.0089250	0.2137365	0.1844828	-0.1335829
H*H*P''(9)	-0.0717777	0.2812622	-0.2591083	-0.2136418	0.2089373	0.3918735	0.0725737
H*H*P''(10)	0.0307007	-0.1457946	0.2008465	0.0481571	-0.2334184	-0.1413090	0.3027122
H*H*P''(11)	0.5466406	-2.0805436	1.8601602	1.4407254	-1.4373125	-2.2385941	0.3866968

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	1.4407254	1.8601602	-2.0805436	0.5466406
H*H*P''(2)	0.0481571	0.2008465	-0.1457946	0.0307007
H*H*P''(3)	-0.2136418	-0.2591083	0.2812622	-0.0717777
H*H*P''(4)	-0.0089250	-0.1359936	0.0734773	-0.0119258
H*H*P''(5)	0.2098190	0.1244890	-0.1786514	0.0473605
H*H*P''(6)	0.2018662	0.2472266	-0.1669793	0.0314205
H*H*P''(7)	-0.0617428	0.1276941	0.1342856	-0.0461710
H*H*P''(8)	-0.3982024	-0.1680449	0.4685822	-0.0836041
H*H*P''(9)	-0.4129425	-0.4033390	0.2969964	0.1091663
H*H*P''(10)	0.5003716	-0.1709484	-1.2017386	0.8104210
H*H*P''(11)	3.1598396	1.1069551	-5.1312429	2.3866756

M = 11 = NUMBER OF POINTS
N = 7 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9989716	0.0065815	-0.0164541	0.0180996	-0.0028795	-0.0115179	0.0057590
P(2)	0.0065817	0.9575071	0.1079391	-0.1235705	0.0299465	0.0662279	-0.0391608
P(3)	-0.0164541	0.1079391	0.7179350	0.3452077	-0.1301522	-0.1266968	0.1030851
P(4)	0.0180995	-0.1235705	0.3452077	0.5149321	0.3086796	0.0115179	-0.1059646
P(5)	-0.0028795	0.0299465	-0.1301522	0.3086796	0.5726861	0.3225010	-0.0645002
P(6)	-0.0115179	0.0662279	-0.1266968	0.0115179	0.3225010	0.4759358	0.3225010
P(7)	0.0057589	-0.0391608	0.1030851	-0.1059646	-0.0645002	0.3225010	0.5726861
P(8)	0.0082271	-0.0445907	0.0786508	-0.0111888	-0.1059646	0.0115179	0.3086796
P(9)	-0.0109009	0.0635130	-0.1321267	0.0786508	0.1030851	-0.1266968	-0.1301522
P(10)	0.0049362	-0.0293295	0.0635130	-0.0445907	-0.0391608	0.0662279	0.0299465
P(11)	-0.0008227	0.0049364	-0.0109009	0.0082271	0.0057590	-0.0115179	-0.0028795

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	0.0082271	-0.0109009	0.0049364	-0.0008227
P(2)	-0.0445907	0.0635130	-0.0293295	0.0049362
P(3)	0.0786508	-0.1321267	0.0635130	-0.0109009
P(4)	-0.0111888	0.0786508	-0.0445907	0.0082271
P(5)	-0.1059646	0.1030851	-0.0391608	0.0057589
P(6)	0.0115179	-0.1266968	0.0662279	-0.0115179
P(7)	0.3086796	-0.1301522	0.0299465	-0.0028795
P(8)	0.5149321	0.3452077	-0.1235705	0.0180995
P(9)	0.3452077	0.7179350	0.1079391	-0.0164541
P(10)	-0.1235705	0.1079391	0.9575071	0.0065817
P(11)	0.0180996	-0.0164541	0.0065815	0.9989716

M = 11 = NUMBER OF POINTS
N = 7 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.1889125	4.1659106	-2.2149821	-1.0596071	1.5885644	0.7605923	-1.3128617
H*P'(2)	-0.2420697	-0.7631124	1.0334712	0.4097855	-0.5029425	-0.2584945	0.3879391
H*P'(3)	0.0628048	-0.6095563	0.0349788	0.3847588	0.2409955	-0.0407788	-0.1277005
H*P'(4)	-0.0026072	0.0696031	-0.5919449	-0.0501473	0.4763362	0.2810092	-0.1646072
H*P'(5)	-0.0244672	0.1428803	-0.2552563	-0.3084320	-0.0076292	0.2800219	0.2625888
H*P'(6)	0.0091027	-0.0692499	0.2076766	-0.2369278	-0.4176882	0.0000000	0.4176882
H*P'(7)	0.0168430	-0.0914102	0.1565447	0.0077287	-0.2625888	-0.2800219	0.0076292
H*P'(8)	-0.0138783	0.0860902	-0.1960201	0.1414501	0.1646072	-0.2810092	-0.4763362
H*P'(9)	-0.0123597	0.0631381	-0.0991505	-0.0061256	0.1277005	0.0407788	-0.2409955
H*P'(10)	0.0430805	-0.2378307	0.4509620	-0.1916350	-0.3879391	0.2584945	0.5029425
H*P'(11)	-0.1543074	0.8369925	-1.5448155	0.6008345	1.3128617	-0.7605923	-1.5885644

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	-0.6008345	1.5448155	-0.8369925	0.1543074
H*P'(2)	0.1916350	-0.4509620	0.2378307	-0.0430805
H*P'(3)	0.0061256	0.0991505	-0.0631381	0.0123597
H*P'(4)	-0.1414501	0.1960201	-0.0860902	0.0138783
H*P'(5)	-0.0077287	-0.1565447	0.0914102	-0.0168430
H*P'(6)	0.2369278	-0.2076766	0.0692499	-0.0091027
H*P'(7)	0.3084320	0.2552563	-0.1428803	0.0244672
H*P'(8)	0.0501473	0.5919449	-0.0696031	0.0026072
H*P'(9)	-0.3847588	-0.0349788	0.6095563	-0.0628048
H*P'(10)	-0.4097855	-1.0334712	0.7631124	0.2420697
H*P'(11)	1.0596071	2.2149821	-4.1659106	2.1889125

M = 11 = NUMBER OF POINTS
N = 7 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	3.5218680	-10.3531281	8.5992250	2.7057626	-5.9703807	-2.2385941	4.9197651
H*H*P''(2)	0.8147287	-1.2215545	-0.1425170	0.4986485	0.2785886	-0.1413090	-0.2092947
H*H*P''(3)	-0.0030817	0.8133374	-1.1441761	-0.3680433	0.7011627	0.3918735	-0.4196517
H*H*P''(4)	-0.0663673	0.3892927	-0.0542817	-0.4050971	-0.2301092	0.1844828	0.3102628
H*H*P''(5)	0.0194799	-0.1677083	0.5609898	-0.0880031	-0.5838548	-0.1752457	0.4020175
H*H*P''(6)	0.0314205	-0.1669793	0.2472266	0.2018662	-0.1463486	-0.3343709	-0.1463486
H*H*P''(7)	-0.0182904	0.1233425	-0.3088066	0.2360793	0.4020175	-0.1752457	-0.5838548
H*H*P''(8)	-0.0291627	0.1527668	-0.2497568	-0.0020303	0.3102628	0.1844828	-0.2301092
H*H*P''(9)	0.0404703	-0.2350788	0.4817288	-0.2585411	-0.4196517	0.3918735	0.7011627
H*H*P''(10)	0.0263929	-0.1259788	0.1724151	0.0498803	-0.2092947	-0.1413090	0.2785886
H*H*P''(11)	-0.5885518	3.1413415	-5.6321098	1.8948023	4.9197651	-2.2385941	-5.9703807

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	1.8948023	-5.6321098	3.1413415	-0.5885518
H*H*P''(2)	0.0498803	0.1724151	-0.1259788	0.0263929
H*H*P''(3)	-0.2585411	0.4817288	-0.2350788	0.0404703
H*H*P''(4)	-0.0020303	-0.2497568	0.1527668	-0.0291627
H*H*P''(5)	0.2360793	-0.3088066	0.1233425	-0.0182904
H*H*P''(6)	0.2018662	0.2472266	-0.1669793	0.0314205
H*H*P''(7)	-0.0880031	0.5609898	-0.1677083	0.0194799
H*H*P''(8)	-0.4050971	-0.0542817	0.3892927	-0.0663673
H*H*P''(9)	-0.3680433	-1.1441761	0.8133374	-0.0030817
H*H*P''(10)	0.4986485	-0.1425170	-1.2215545	0.8147287
H*H*P''(11)	2.7057626	8.5992250	-10.3531281	3.5218680

M = 11 = NUMBER OF POINTS
N = 8 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9998917	0.0008765	-0.0030203	0.0055857	-0.0054559	0.0013641	0.0031826
P(2)	0.0008768	0.9928770	0.0246487	-0.0459849	0.0459200	-0.0136396	-0.0231873
P(3)	-0.0030202	0.0246487	0.9140704	0.1625062	-0.1677672	0.0613782	0.0654701
P(4)	0.0055857	-0.0459850	0.1625062	0.6851198	0.3437182	-0.1636753	-0.0709260
P(5)	-0.0054558	0.0459200	-0.1677672	0.3437182	0.5799000	0.2864318	-0.0572864
P(6)	0.0013640	-0.0136396	0.0613782	-0.1636753	0.2864318	0.6562818	0.2864318
P(7)	0.0031826	-0.0231873	0.0654701	-0.0709260	-0.0572864	0.2864318	0.5799000
P(8)	-0.0042867	0.0329949	-0.1040508	0.1589989	-0.0709260	-0.1636753	0.3437182
P(9)	0.0025331	-0.0197774	0.0640087	-0.1040507	0.0654701	0.0613782	-0.1677672
P(10)	-0.0007686	0.0060404	-0.0197775	0.0329949	-0.0231873	-0.0136396	0.0459200
P(11)	0.0000974	-0.0007686	0.0025328	-0.0042868	0.0031826	0.0013641	-0.0054559

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	-0.0042868	0.0025328	-0.0007686	0.0000974
P(2)	0.0329949	-0.0197775	0.0060404	-0.0007686
P(3)	-0.1040507	0.0640087	-0.0197774	0.0025331
P(4)	0.1589989	-0.1040508	0.0329949	-0.0042867
P(5)	-0.0709260	0.0654701	-0.0231873	0.0031826
P(6)	-0.1636753	0.0613782	-0.0136396	0.0013640
P(7)	0.3437182	-0.1677672	0.0459200	-0.0054558
P(8)	0.6851198	0.1625062	-0.0459850	0.0055857
P(9)	0.1625062	0.9140704	0.0246487	-0.0030202
P(10)	-0.0459849	0.0246487	0.9928770	0.0008768
P(11)	0.0055857	-0.0030203	0.0008765	0.9998917

M = 11 = NUMBER OF POINTS
N = 8 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.4733123	5.9291896	-6.3672190	2.8082305	2.3848839	-3.2210053	-0.5165422
H*P'(2)	-0.1718163	-1.1986836	2.0591713	-0.5456611	-0.6996521	0.7250535	0.1912295
H*P'(3)	0.0383188	-0.4577428	-0.3225173	0.7177689	0.3095564	-0.3835834	-0.0591396
H*P'(4)	-0.0137018	0.1383897	-0.7539262	0.1007393	0.5074011	0.1256847	-0.1335423
H*P'(5)	-0.0012527	-0.0010497	0.0836755	-0.6241492	-0.0726298	0.6050250	0.1975882
H*P'(6)	0.0091027	-0.0692499	0.2076766	-0.2369278	-0.4176882	-0.0000000	0.4176882
H*P'(7)	-0.0063715	0.0525198	-0.1823870	0.3234460	-0.1975882	-0.6050250	0.0726298
H*P'(8)	-0.0027837	0.0173036	-0.0340388	-0.0094365	0.1335423	-0.1256847	-0.5074011
H*P'(9)	0.0121263	-0.0886754	0.2583457	-0.3391358	0.0591396	0.3835834	-0.3095564
H*P'(10)	-0.0271730	0.1977406	-0.5747382	0.7638116	-0.1912295	-0.7250535	0.6996521
H*P'(11)	0.1300924	-0.9262865	2.6074214	-3.2670031	0.5165422	3.2210053	-2.3848839

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	3.2670031	-2.6074214	0.9262865	-0.1300924
H*P'(2)	-0.7638116	0.5747382	-0.1977406	0.0271730
H*P'(3)	0.3391358	-0.2583457	0.0886754	-0.0121263
H*P'(4)	0.0094365	0.0340388	-0.0173036	0.0027837
H*P'(5)	-0.3234460	0.1823870	-0.0525198	0.0063715
H*P'(6)	0.2369278	-0.2076766	0.0692499	-0.0091027
H*P'(7)	0.6241492	-0.0836755	0.0010497	0.0012527
H*P'(8)	-0.1007393	0.7539262	-0.1383897	0.0137018
H*P'(9)	-0.7177689	0.3225173	0.4577428	-0.0383188
H*P'(10)	0.5456611	-2.0591713	1.1986836	0.1718163
H*P'(11)	-2.8082305	6.3672190	-5.9291896	2.4733123

M = 11 = NUMBER OF POINTS
N = 8 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	4.7042580	-17.6839466	25.8621192	-13.3747422	-9.2810729	14.3148669	1.6090729
H*H*P''(2)	0.7415520	-0.7678587	-1.2108977	1.4938522	0.4834835	-1.1657834	-0.0043999
H*H*P''(3)	-0.0498865	1.1035267	-1.8275252	0.2685010	0.8322160	-0.2633927	-0.2885985
H*H*P''(4)	-0.0175805	0.0868144	0.6580057	-1.0685977	-0.3667123	0.8674982	0.1736597
H*H*P''(5)	0.0247444	-0.2003485	0.6378521	-0.1596009	-0.5985956	-0.1015420	0.3872767
H*H*P''(6)	-0.0077931	0.0761453	-0.3252925	0.7351716	-0.0365504	-0.8833618	-0.0365504
H*H*P''(7)	-0.0130258	0.0907023	-0.2319443	0.1644815	0.3872767	-0.1015420	-0.5985956
H*H*P''(8)	0.0196241	-0.1497114	0.4625306	-0.6655309	0.1736597	0.8674982	-0.3667123
H*H*P''(9)	-0.0063344	0.0551105	-0.2016203	0.3780033	-0.2885985	-0.2633927	0.8322160
H*H*P''(10)	-0.0467839	0.3277170	-0.8959656	1.0450840	-0.0043999	-1.1657834	0.4834835
H*H*P''(11)	0.5938383	-4.1894770	11.6307844	-14.1857024	1.6090729	14.3148669	-9.2810729

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	-14.1857024	11.6307844	-4.1894770	0.5938383
H*H*P''(2)	1.0450840	-0.8959656	0.3277170	-0.0467839
H*H*P''(3)	0.3780033	-0.2016203	0.0551105	-0.0063344
H*H*P''(4)	-0.6655309	0.4625306	-0.1497114	0.0196241
H*H*P''(5)	0.1644815	-0.2319443	0.0907023	-0.0130258
H*H*P''(6)	0.7351716	-0.3252925	0.0761453	-0.0077931
H*H*P''(7)	-0.1596009	0.6378521	-0.2003485	0.0247444
H*H*P''(8)	-1.0685977	0.6580057	0.0868144	-0.0175805
H*H*P''(9)	0.2685010	-1.8275252	1.1035267	-0.0498865
H*H*P''(10)	1.4938522	-1.2108977	-0.7678587	0.7415520
H*H*P''(11)	-13.3747422	25.8621192	-17.6839466	4.7042580

M = 11 = NUMBER OF POINTS
N = 9 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.9999946	0.0000541	-0.0002451	0.0006486	-0.0011363	0.0013641	-0.0011371
P(2)	0.0000541	0.9994587	0.0024353	-0.0064951	0.0113664	-0.0136396	0.0113663
P(3)	-0.0002436	0.0024356	0.9890395	0.0292278	-0.0511486	0.0613782	-0.0511485
P(4)	0.0006495	-0.0064951	0.0292277	0.9220594	0.1363961	-0.1636753	0.1363961
P(5)	-0.0011366	0.0113663	-0.0511485	0.1363961	0.7613068	0.2864318	-0.2386932
P(6)	0.0013640	-0.0136396	0.0613782	-0.1636753	0.2864318	0.6562818	0.2864318
P(7)	-0.0011366	0.0113663	-0.0511485	0.1363961	-0.2386932	0.2864318	0.7613068
P(8)	0.0006495	-0.0064951	0.0292277	-0.0779406	0.1363961	-0.1636753	0.1363961
P(9)	-0.0002436	0.0024356	-0.0109604	0.0292277	-0.0511485	0.0613782	-0.0511486
P(10)	0.0000541	-0.0005413	0.0024359	-0.0064950	0.0113663	-0.0136396	0.0113664
P(11)	-0.0000054	0.0000538	-0.0002424	0.0006503	-0.0011371	0.0013641	-0.0011363

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	0.0006503	-0.0002424	0.0000538	-0.0000054
P(2)	-0.0064950	0.0024359	-0.0005413	0.0000541
P(3)	0.0292277	-0.0109604	0.0024356	-0.0002436
P(4)	-0.0779406	0.0292277	-0.0064951	0.0006495
P(5)	0.1363961	-0.0511485	0.0113663	-0.0011366
P(6)	-0.1636753	0.0613782	-0.0136396	0.0013640
P(7)	0.1363961	-0.0511485	0.0113663	-0.0011366
P(8)	0.9220594	0.0292277	-0.0064951	0.0006495
P(9)	0.0292278	0.9890395	0.0024356	-0.0002436
P(10)	-0.0064951	0.0024353	0.9994587	0.0000541
P(11)	0.0006486	-0.0002451	0.0000541	0.9999946

M = 11 = NUMBER OF POINTS
N = 9 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.7161865	7.8721824	-12.9248183	14.4661897	-7.8158297	-3.2210053	9.6841714
H*P'(2)	-0.1278772	-1.5501963	3.2455266	-2.6547361	1.1457885	0.7250535	-1.6542111
H*P'(3)	0.0200407	-0.3115179	-0.8160267	1.5951190	-0.4581250	-0.3835834	0.7085417
H*P'(4)	-0.0074432	0.0883208	-0.5849437	-0.1996740	0.7702628	0.1256847	-0.3964039
H*P'(5)	0.0035515	-0.0394831	0.2133884	-0.8547500	0.1291458	0.6050250	-0.0041875
H*P'(6)	-0.0007937	0.0099206	-0.0595238	0.2380952	-0.8333333	-0.0000000	0.8333333
H*P'(7)	-0.0015674	0.0140863	-0.0526741	0.0928452	0.0041875	-0.6050250	-0.1291458
H*P'(8)	0.0034749	-0.0327653	0.1349437	-0.3098498	0.3964039	-0.1256847	-0.7702628
H*P'(9)	-0.0061518	0.0575496	-0.2351636	0.5382143	-0.7085417	0.3835834	0.4581250
H*P'(10)	0.0167661	-0.1537721	0.6116171	-1.3452634	1.6542111	-0.7250535	-1.1457885
H*P'(11)	-0.1127818	1.0167062	-3.9501778	8.3909562	-9.6841714	3.2210053	7.8158297

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	-8.3909562	3.9501778	-1.0167062	0.1127818
H*P'(2)	1.3452634	-0.6116171	0.1537721	-0.0167661
H*P'(3)	-0.5382143	0.2351636	-0.0575496	0.0061518
H*P'(4)	0.3098498	-0.1349437	0.0327653	-0.0034749
H*P'(5)	-0.0928452	0.0526741	-0.0140863	0.0015674
H*P'(6)	-0.2380952	0.0595238	-0.0099206	0.0007937
H*P'(7)	0.8547500	-0.2133884	0.0394831	-0.0035515
H*P'(8)	0.1996740	0.5849437	-0.0883208	0.0074432
H*P'(9)	-1.5951190	0.8160267	0.3115179	-0.0200407
H*P'(10)	2.6547361	-3.2455266	1.5501963	0.1278772
H*P'(11)	-14.4661897	12.9248183	-7.8721824	2.7161865

M = 11 = NUMBER OF POINTS
N = 9 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	5.8806950	-27.0954416	57.6259131	-69.8437166	40.1292782	14.3148669	-47.8012781
H*H*P''(2)	0.6493682	-0.0303883	-3.6998609	5.9186738	-3.3882357	-1.1657834	3.8673193
H*H*P''(3)	-0.0502334	1.1063027	-1.8368943	0.2851568	0.8176421	-0.2633927	-0.2740246
H*H*P''(4)	0.0047917	-0.0921628	1.2620538	-2.1424611	0.5729182	0.8674982	-0.7659707
H*H*P''(5)	0.0049664	-0.0421247	0.1038468	0.7897419	-1.4292705	-0.1015420	1.2179517
H*H*P''(6)	-0.0077931	0.0761453	-0.3252925	0.7351716	-0.0365504	-0.8833618	-0.0365504
H*H*P''(7)	0.0067522	-0.0675215	0.3020611	-0.7848613	1.2179517	-0.1015420	-1.4292705
H*H*P''(8)	-0.0027480	0.0292658	-0.1415175	0.4083325	-0.7659707	0.8674982	0.5729182
H*H*P''(9)	-0.0059874	0.0523345	-0.1922512	0.3613474	-0.2740246	-0.2633927	0.8176421
H*H*P''(10)	0.0453999	-0.4097534	1.5929977	-3.3797376	3.8673193	-1.1657834	-3.3882357
H*H*P''(11)	-0.5825987	5.2220182	-20.1330094	42.2832718	-47.8012781	14.3148669	40.1292782

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	42.2832718	-20.1330094	5.2220182	-0.5825987
H*H*P''(2)	-3.3797376	1.5929977	-0.4097534	0.0453999
H*H*P''(3)	0.3613474	-0.1922512	0.0523345	-0.0059874
H*H*P''(4)	0.4083325	-0.1415175	0.0292658	-0.0027480
H*H*P''(5)	-0.7848613	0.3020611	-0.0675215	0.0067522
H*H*P''(6)	0.7351716	-0.3252925	0.0761453	-0.0077931
H*H*P''(7)	0.7897419	0.1038468	-0.0421247	0.0049664
H*H*P''(8)	-2.1424611	1.2620538	-0.0921628	0.0047917
H*H*P''(9)	0.2851568	-1.8368943	1.1063027	-0.0502334
H*H*P''(10)	5.9186738	-3.6998609	-0.0303883	0.6493682
H*H*P''(11)	-69.8437166	57.6259131	-27.0954416	5.8806950

M = 11 = NUMBER OF POINTS
N = 10 = DEGREE OF POLY.

THE SMOOTHING FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	1.0000000	0.0000003	-0.0000005	-0.0000029	0.0000025	-0.0000000	0.0000016
P(2)	-0.0000000	1.0000001	-0.0000001	-0.0000002	0.0000003	0.0000002	0.0000002
P(3)	-0.0000000	0.0000000	1.0000000	0.0000001	-0.0000000	-0.0000000	-0.0000000
P(4)	0.0000000	0.0000000	-0.0000000	1.0000000	-0.0000000	-0.0000000	-0.0000000
P(5)	0.0000000	0.0000000	-0.0000000	0.0000000	1.0000000	-0.0000000	0.0000000
P(6)	0.0000000	-0.0000000	0.0000000	-0.0000000	-0.0000000	1.0000000	-0.0000000
P(7)	-0.0000000	-0.0000000	0.0000000	-0.0000000	0.0000000	-0.0000000	1.0000000
P(8)	-0.0000000	-0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000
P(9)	-0.0000000	-0.0000000	0.0000001	0.0000000	-0.0000000	-0.0000000	-0.0000000
P(10)	-0.0000000	-0.0000000	0.0000004	-0.0000001	0.0000002	0.0000002	0.0000003
P(11)	-0.0000000	0.0000000	0.0000021	-0.0000013	0.0000016	-0.0000000	0.0000025

	Y(8)	Y(9)	Y(10)	Y(11)
P(1)	-0.0000013	0.0000021	0.0000000	-0.0000000
P(2)	-0.0000001	0.0000004	-0.0000000	-0.0000000
P(3)	0.0000000	0.0000001	-0.0000000	-0.0000000
P(4)	-0.0000000	0.0000000	-0.0000000	-0.0000000
P(5)	-0.0000000	0.0000000	-0.0000000	-0.0000000
P(6)	0.0000000	0.0000000	-0.0000000	0.0000000
P(7)	0.0000000	-0.0000000	0.0000000	0.0000000
P(8)	1.0000000	-0.0000000	0.0000000	0.0000000
P(9)	0.0000001	1.0000000	0.0000000	-0.0000000
P(10)	-0.0000002	-0.0000001	1.0000001	-0.0000000
P(11)	-0.0000029	-0.0000005	0.0000003	1.0000000

M = 11 = NUMBER OF POINTS
N = 10 = DEGREE OF POLY.

THE FIRST DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*P'(1)	-2.9289682	9.9999996	-22.4999993	40.0000057	-52.5000038	50.4000015	-35.0000024
H*P'(2)	-0.1000000	-1.8289683	4.5000002	-5.9999992	6.9999991	-6.3000003	4.1999995
H*P'(3)	0.0111111	-0.2222222	-1.2178571	2.6666666	-2.3333334	1.8666665	-1.1666667
H*P'(4)	-0.0027778	0.0416667	-0.3750000	-0.7595238	1.7500000	-1.0500000	0.5833333
H*P'(5)	0.0011905	-0.0158730	0.1071429	-0.5714286	-0.3666666	1.2000000	-0.5000000
H*P'(6)	-0.0007937	0.0099206	-0.0595238	0.2380952	-0.8333333	-0.0000000	0.8333333
H*P'(7)	0.0007937	-0.0095238	0.0535714	-0.1904762	0.5000000	-1.2000000	0.3666666
H*P'(8)	-0.0011905	0.0138889	-0.0750000	0.2500000	-0.5833333	1.0500000	-1.7500000
H*P'(9)	0.0027778	-0.0317460	0.1666668	-0.5333333	1.1666667	-1.8666665	2.3333334
H*P'(10)	-0.0111111	0.1250000	-0.6428564	1.9999997	-4.1999995	6.3000003	-6.9999991
H*P'(11)	0.0999999	-1.1111110	5.6250032	-17.1428599	35.0000024	-50.4000015	52.5000038

	Y(8)	Y(9)	Y(10)	Y(11)
H*P'(1)	17.1428599	-5.6250032	1.1111110	-0.0999999
H*P'(2)	-1.9999997	0.6428564	-0.1250000	0.0111111
H*P'(3)	0.5333333	-0.1666668	0.0317460	-0.0027778
H*P'(4)	-0.2500000	0.0750000	-0.0138889	0.0011905
H*P'(5)	0.1904762	-0.0535714	0.0095238	-0.0007937
H*P'(6)	-0.2380952	0.0595238	-0.0099206	0.0007937
H*P'(7)	0.5714286	-0.1071429	0.0158730	-0.0011905
H*P'(8)	0.7595238	0.3750000	-0.0416667	0.0027778
H*P'(9)	-2.6666666	1.2178571	0.2222222	-0.0111111
H*P'(10)	5.9999992	-4.5000002	1.8289683	0.1000000
H*P'(11)	-40.0000057	22.4999993	-9.9999996	2.9289682

M = 11 = NUMBER OF POINTS
N = 10 = DEGREE OF POLY.

THE SECOND DERIVATIVE FORMULA FOR THE ABOVE M AND N IS GIVEN BY

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*P''(1)	7.0290872	-38.5793643	109.3035698	-207.6508026	281.2916718	-275.0800018	193.3611145
H*H*P''(2)	0.5657936	0.8053573	-7.4607145	15.9476169	-20.9388871	19.8949997	-13.6833322
H*H*P''(3)	-0.0381746	0.9857143	-1.2942461	-1.1619049	3.3500002	-3.3022219	2.2583335
H*H*P''(4)	0.0060714	-0.1049603	1.3196428	-2.2960317	0.8416667	0.5450000	-0.4972222
H*H*P''(5)	-0.0014683	0.0222222	-0.1857143	1.5619048	-2.7805555	1.5200000	-0.1333333
H*H*P''(6)	0.0003175	-0.0049603	0.0396825	-0.2380952	1.6666667	-2.9272222	1.6666667
H*H*P''(7)	0.0003175	-0.0031746	0.0125000	-0.0126984	-0.1333333	1.5200000	-2.7805555
H*H*P''(8)	-0.0014683	0.0164683	-0.0839285	0.2547619	-0.4972222	0.5450000	0.8416667
H*H*P''(9)	0.0060714	-0.0682540	0.3503971	-1.0857143	2.2583335	-3.3022219	3.3500002
H*H*P''(10)	-0.0381746	0.4259921	-2.1678559	6.6492054	-13.6833322	19.8949997	-20.9388871
H*H*P''(11)	0.5657935	-6.2619046	31.5446470	-95.5238142	193.3611145	-275.0800018	281.2916718

	Y(8)	Y(9)	Y(10)	Y(11)
H*H*P''(1)	-95.5238142	31.5446470	-6.2619046	0.5657935
H*H*P''(2)	6.6492054	-2.1678559	0.4259921	-0.0381746
H*H*P''(3)	-1.0857143	0.3503971	-0.0682540	0.0060714
H*H*P''(4)	0.2547619	-0.0839285	0.0164683	-0.0014683
H*H*P''(5)	-0.0126984	0.0125000	-0.0031746	0.0003175
H*H*P''(6)	-0.2380952	0.0396825	-0.0049603	0.0003175
H*H*P''(7)	1.5619048	-0.1857143	0.0222222	-0.0014683
H*H*P''(8)	-2.2960317	1.3196428	-0.1049603	0.0060714
H*H*P''(9)	-1.1619049	-1.2942461	0.9857143	-0.0381746
H*H*P''(10)	15.9476169	-7.4607145	0.8053573	0.5657936
H*H*P''(11)	-207.6508026	109.3035698	-38.5793643	7.0290872

Appendix B

Smoothing and Derivative Formulas for Two Independent Variables

See Section III.D. on the use of this appendix.

[a]Linear function.

[b]Quadratic function.

3 X 3 ARRAY
LINEAR FITTING

SMOOTHING FORMULA

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.4444444	0.2777778	0.1111111	0.2777778	0.1111111	-0.0555556	0.1111111
P(2)	0.2777778	0.2777778	0.2777778	0.1111111	0.1111111	0.1111111	-0.0555556
P(3)	0.1111111	0.2777778	0.4444444	-0.0555556	0.1111111	0.2777778	-0.2222222
P(4)	0.2777778	0.1111111	-0.0555556	0.2777778	0.1111111	-0.0555556	0.2777778
P(5)	0.1111111	0.1111111	0.1111111	0.1111111	0.1111111	0.1111111	0.1111111
P(6)	-0.0555556	0.1111111	0.2777778	-0.0555556	0.1111111	0.2777778	-0.0555556
P(7)	0.1111111	-0.0555556	-0.2222222	0.2777778	0.1111111	-0.0555556	0.4444444
P(8)	-0.0555556	-0.0555556	-0.0555556	0.1111111	0.1111111	0.1111111	0.2777778
P(9)	-0.2222222	-0.0555556	0.1111111	-0.0555556	0.1111111	0.2777778	0.1111111

	Y(8)	Y(9)
P(1)	-0.0555556	-0.2222222
P(2)	-0.0555556	-0.0555556
P(3)	-0.0555556	0.1111111
P(4)	0.1111111	-0.0555556
P(5)	0.1111111	0.1111111
P(6)	0.1111111	0.2777778
P(7)	0.2777778	0.1111111
P(8)	0.2777778	0.2777778
P(9)	0.2777778	0.4444444

3 X 3 ARRAY
LINEAR FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T.* X). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*DX(1)	-0.1666667	-0.0000000	0.1666667	-0.1666667	-0.0000000	0.1666667	-0.1666667

	Y(8)	Y(9)
H*DX(1)	-0.0000000	0.1666667

3 X 3 ARRAY
LINEAR FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*DY(1)	0.1666667	0.1666667	0.1666667	-0.0000000	0.0000000	-0.0000000	-0.1666667

	Y(8)	Y(9)
K*DY(1)	-0.1666667	-0.1666667

*W.R.T.: with respect to

3 X 3 ARRAY
QUADRATIC FITTING

SMOOTHING FORMULA

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.8055555	0.2222222	-0.0277778	0.2222222	-0.1111111	-0.1111111	-0.0277778
P(2)	0.2222222	0.5555556	0.2222222	-0.1111111	0.2222222	-0.1111111	-0.1111111
P(3)	-0.0277778	0.2222222	0.8055555	-0.1111111	-0.1111111	0.2222222	0.1388889
P(4)	0.2222222	-0.1111111	-0.1111111	0.5555556	0.2222222	0.2222222	0.2222222
P(5)	-0.1111111	0.2222222	-0.1111111	0.2222222	0.5555555	0.2222222	-0.1111111
P(6)	-0.1111111	-0.1111111	0.2222222	0.2222222	0.2222222	0.5555556	-0.1111111
P(7)	-0.0277778	-0.1111111	0.1388889	0.2222222	-0.1111111	-0.1111111	0.8055555
P(8)	-0.1111111	0.2222222	-0.1111111	-0.1111111	0.2222222	-0.1111111	0.2222222
P(9)	0.1388889	-0.1111111	-0.0277778	-0.1111111	-0.1111111	0.2222222	-0.0277778

	Y(8)	Y(9)
P(1)	-0.1111111	0.1388889
P(2)	0.2222222	-0.1111111
P(3)	-0.1111111	-0.0277778
P(4)	-0.1111111	-0.1111111
P(5)	0.2222222	-0.1111111
P(6)	-0.1111111	0.2222222
P(7)	0.2222222	-0.0277778
P(8)	0.5555556	0.2222222
P(9)	0.2222222	0.8055555

3 X 3 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. X)

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*DX(1)	-0.7500000	0.6666667	0.0833333	-0.5000000	0.6666667	-0.1666667	-0.2500000
H*DX(2)	-0.4166667	0.0000000	0.4166667	-0.1666667	-0.0000000	0.1666667	0.0833333
H*DX(3)	-0.0833333	-0.6666667	0.7500000	0.1666667	-0.6666667	0.5000000	0.4166667
H*DX(4)	-0.5000000	0.6666667	-0.1666667	-0.5000000	0.6666667	-0.1666667	-0.5000000
H*DX(5)	-0.1666667	0.0000000	0.1666667	-0.1666667	-0.0000000	0.1666667	-0.1666667
H*DX(6)	0.1666667	-0.6666667	0.5000000	0.1666667	-0.6666667	0.5000000	0.1666667
H*DX(7)	-0.2500000	0.6666667	-0.4166667	-0.5000000	0.6666667	-0.1666667	-0.7500000
H*DX(8)	0.0833333	0.0000000	-0.0833333	-0.1666667	-0.0000000	0.1666667	-0.4166667
H*DX(9)	0.4166667	-0.6666667	0.2500000	0.1666667	-0.6666667	0.5000000	-0.0833333

	Y(8)	Y(9)
H*DX(1)	0.6666667	-0.4166667
H*DX(2)	0.0000000	-0.0833333
H*DX(3)	-0.6666667	0.2500000
H*DX(4)	0.6666667	-0.1666667
H*DX(5)	0.0000000	0.1666667
H*DX(6)	-0.6666667	0.5000000
H*DX(7)	0.6666667	0.0833333
H*DX(8)	0.0000000	0.4166667
H*DX(9)	-0.6666667	0.7500000

3 X 3 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. Y)

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*DY(1)	0.7500000	0.5000000	0.2500000	-0.6666667	-0.6666667	-0.6666667	-0.0833333
K*DY(2)	0.5000000	0.5000000	0.5000000	-0.6666667	-0.6666667	-0.6666667	0.1666667
K*DY(3)	0.2500000	0.5000000	0.7500000	-0.6666667	-0.6666667	-0.6666667	0.4166667
K*DY(4)	0.4166667	0.1666667	-0.0833333	0.0000000	0.0000000	0.0000000	-0.4166667
K*DY(5)	0.1666667	0.1666667	0.1666667	0.0000000	0.0000000	0.0000000	-0.1666667
K*DY(6)	-0.0833333	0.1666667	0.4166667	0.0000000	0.0000000	-0.0000000	0.0833333
K*DY(7)	0.0833333	-0.1666667	-0.4166667	0.6666667	0.6666667	0.6666667	-0.7500000
K*DY(8)	-0.1666667	-0.1666667	-0.1666667	0.6666667	0.6666667	0.6666667	-0.5000000
K*DY(9)	-0.4166667	-0.1666667	0.0833333	0.6666667	0.6666667	0.6666667	-0.2500000

	Y(8)	Y(9)
K*DY(1)	0.1666667	0.4166667
K*DY(2)	0.1666667	0.1666667
K*DY(3)	0.1666667	-0.0833333
K*DY(4)	-0.1666667	0.0833333
K*DY(5)	-0.1666667	-0.1666667
K*DY(6)	-0.1666667	-0.4166667
K*DY(7)	-0.5000000	-0.2500000
K*DY(8)	-0.5000000	-0.5000000
K*DY(9)	-0.5000000	-0.7500000

3 X 3 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. X AND X). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*DXX(1)	0.3333333	-0.6666667	0.3333333	0.3333333	-0.6666667	0.3333333	0.3333333

	Y(8)	Y(9)
H*H*DXX(1)	-0.6666667	0.3333333

3 X 3 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. X AND Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*K*DXY(1)	-0.2500000	-0.0000000	0.2500000	-0.0000000	0.0000000	-0.0000000	0.2500000

	Y(8)	Y(9)
H*K*DXY(1)	-0.0000000	-0.2500000

3 X 3 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. Y AND Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*K*DYY(1)	0.3333333	0.3333333	0.3333333	-0.6666667	-0.6666667	-0.6666667	0.3333333

	Y(8)	Y(9)
K*K*DYY(1)	0.3333333	0.3333333

3 X 5 ARRAY
LINEAR FITTING

SMOOTHING FORMULA

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.3000000	0.2333333	0.1666667	0.1000000	0.0333333	0.2000000	0.1333333
P(2)	0.2333333	0.2000000	0.1666667	0.1333333	0.1000000	0.1333333	0.1000000
P(3)	0.1666667	0.1666667	0.1666667	0.1666667	0.1666667	0.0666667	0.0666667
P(4)	0.1000000	0.1333333	0.1666667	0.2000000	0.2333333	0.0000000	0.0333333
P(5)	0.0333333	0.1000000	0.1666667	0.2333333	0.3000000	-0.0666667	-0.0000000
P(6)	0.2000000	0.1333333	0.0666667	-0.0000000	-0.0666667	0.2000000	0.1333333
P(7)	0.1333333	0.1000000	0.0666667	0.0333333	-0.0000000	0.1333333	0.1000000
P(8)	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
P(9)	-0.0000000	0.0333333	0.0666667	0.1000000	0.1333333	0.0000000	0.0333333
P(10)	-0.0666667	-0.0000000	0.0666667	0.1333333	0.2000000	-0.0666667	-0.0000000
P(11)	0.1000000	0.0333333	-0.0333333	-0.1000000	-0.1666667	0.2000000	0.1333333
P(12)	0.0333333	0.0000000	-0.0333333	-0.0666667	-0.1000000	0.1333333	0.1000000
P(13)	-0.0333333	-0.0333333	-0.0333333	-0.0333333	-0.0333333	0.0666667	0.0666667
P(14)	-0.1000000	-0.0666667	-0.0333333	-0.0000000	0.0333333	0.0000000	0.0333333
P(15)	-0.1666667	-0.1000000	-0.0333333	0.0333333	0.1000000	-0.0666667	-0.0000000

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
P(1)	0.0666667	-0.0000000	-0.0666667	0.1000000	0.0333333	-0.0333333	-0.1000000
P(2)	0.0666667	0.0333333	-0.0000000	0.0333333	0.0000000	-0.0333333	-0.0666667
P(3)	0.0666667	0.0666667	0.0666667	-0.0333333	-0.0333333	-0.0333333	-0.0333333
P(4)	0.0666667	0.1000000	0.1333333	-0.1000000	-0.0666667	-0.0333333	0.0000000
P(5)	0.0666667	0.1333333	0.2000000	-0.1666667	-0.1000000	-0.0333333	0.0333333
P(6)	0.0666667	-0.0000000	-0.0666667	0.2000000	0.1333333	0.0666667	0.0000000
P(7)	0.0666667	0.0333333	-0.0000000	0.1333333	0.1000000	0.0666667	0.0333333
P(8)	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
P(9)	0.0666667	0.1000000	0.1333333	0.0000000	0.0333333	0.0666667	0.1000000
P(10)	0.0666667	0.1333333	0.2000000	-0.0666667	0.0000000	0.0666667	0.1333333
P(11)	0.0666667	-0.0000000	-0.0666667	0.3000000	0.2333333	0.1666667	0.1000000
P(12)	0.0666667	0.0333333	-0.0000000	0.2333333	0.2000000	0.1666667	0.1333333
P(13)	0.0666667	0.0666667	0.0666667	0.1666667	0.1666667	0.1666667	0.1666667
P(14)	0.0666667	0.1000000	0.1333333	0.1000000	0.1333333	0.1666667	0.2000000
P(15)	.0666667	0.1333333	0.2000000	0.0333333	0.1000000	0.1666667	0.2333333

	Y(15)
P(1)	-0.1666667
P(2)	-0.1000000
P(3)	-0.0333333
P(4)	0.0333333
P(5)	0.1000000
P(6)	-0.0666667
P(7)	0.0000000
P(8)	0.0666667
P(9)	0.1333333
P(10)	0.2000000
P(11)	0.0333333
P(12)	0.1000000
P(13)	0.1666667
P(14)	0.2333333
P(15)	0.3000000

3 X 5 ARRAY
LINEAR FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. X). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*DX(1)	-0.0666667	-0.0333333	-0.0000000	0.0333333	0.0666667	-0.0666667	-0.0333333
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*DX(1)	-0.0000000	0.0333333	0.0666667	-0.0666667	-0.0333333	-0.0000000	0.0333333
	Y(15)						
H*DX(1)	0.0666667						

3 X 5 ARRAY
LINEAR FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*DY(1)	0.1000000	0.1000000	0.1000000	0.1000000	0.1000000	-0.0000000	0.0000000
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
K*DY(1)	0.0000000	0.0000000	0.0000000	-0.1000000	-0.1000000	-0.1000000	-0.1000000
	Y(15)						
K*DY(1)	-0.1000000						

3 X 5 ARRAY
QUADRATIC FITTING

SMOOTHING FORMULA

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.6285714	0.3190476	0.1047619	-0.0142857	-0.0380952	0.2285714	0.0190476
P(2)	0.3190476	0.3071429	0.2476190	0.1404762	-0.0142857	0.0190476	0.0571429
P(3)	0.1047619	0.2476190	0.2952381	0.2476190	0.1047619	-0.0952381	0.0476190
P(4)	-0.0142857	0.1404762	0.2476190	0.3071429	0.3190476	-0.1142857	-0.0095238
P(5)	-0.0380952	-0.0142857	0.1047619	0.3190476	0.6285714	-0.0380952	-0.1142857
P(6)	0.2285714	0.0190476	-0.0952381	-0.1142857	-0.0380952	0.4285714	0.2190476
P(7)	0.0190476	0.0571429	0.0476190	-0.0095238	-0.1142857	0.2190476	0.2571429
P(8)	-0.0952381	0.0476190	0.0952381	0.0476190	-0.0952381	0.1047619	0.2476190
P(9)	-0.1142857	-0.0095238	0.0476190	0.0571429	0.0190476	0.0857143	0.1904762
P(10)	-0.0380952	-0.1142857	-0.0952381	0.0190476	0.2285714	0.1619048	0.0857143
P(11)	0.0285714	-0.0809524	-0.0952381	-0.0142857	0.1619048	0.2285714	0.0190476
P(12)	-0.0809524	0.0071429	0.0476190	0.0404762	-0.0142857	0.0190476	0.0571429
P(13)	-0.0952381	0.0476190	0.0952381	0.0476190	-0.0952381	-0.0952381	0.0476190
P(14)	-0.0142857	0.0404762	0.0476190	0.0071429	-0.0809524	-0.1142857	-0.0095238
P(15)	0.1619048	-0.0142857	-0.0952381	-0.0809524	0.0285714	-0.0380952	-0.1142857

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
P(1)	-0.0952381	-0.1142857	-0.0380952	0.0285714	-0.0809524	-0.0952381	-0.0142857
P(2)	0.0476190	-0.0095238	-0.1142857	-0.0809524	0.0071429	0.0476190	0.0404762
P(3)	0.0952381	0.0476190	-0.0952381	-0.0952381	0.0476190	0.0952381	0.0476190
P(4)	0.0476190	0.0571429	0.0190476	-0.0142857	0.0404762	0.0476190	0.0071429
P(5)	-0.0952381	0.0190476	0.2285714	0.1619048	-0.0142857	-0.0952381	-0.0809524
P(6)	0.1047619	0.0857143	0.1619048	0.2285714	0.0190476	-0.0952381	-0.1142857
P(7)	0.2476190	0.1904762	0.0857143	0.0190476	0.0571429	0.0476190	-0.0095238
P(8)	0.2952381	0.2476190	0.1047619	-0.0952381	0.0476190	0.0952381	0.0476190
P(9)	0.2476190	0.2571429	0.2190476	-0.1142857	-0.0095238	0.0476190	0.0571429
P(10)	0.1047619	0.2190476	0.4285714	-0.0380952	-0.1142857	-0.0952381	0.0190476
P(11)	-0.0952381	-0.1142857	-0.0380952	0.6285714	0.3190476	0.1047619	-0.0142857
P(12)	0.0476190	-0.0095238	-0.1142857	0.3190476	0.3071429	0.2476190	0.1404762
P(13)	0.0952381	0.0476190	-0.0952381	0.1047619	0.2476190	0.2952381	0.2476190
P(14)	0.0476190	0.0571429	0.0190476	-0.0142857	0.1404762	0.2476190	0.3071429
P(15)	-0.0952381	0.0190476	0.2285714	-0.0380952	-0.0142857	0.1047619	0.3190476

	Y(15)
P(1)	0.1619048
P(2)	-0.0142857
P(3)	-0.0952381
P(4)	-0.0809524
P(5)	0.0285714
P(6)	-0.0380952
P(7)	-0.1142857
P(8)	-0.0952381
P(9)	0.0190476
P(10)	0.2285714
P(11)	-0.0380952
P(12)	-0.0142857
P(13)	0.1047619
P(14)	0.3190476
P(15)	0.6285714

3 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. X)

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*DX(1)	-0.3571429	0.0119048	0.1904762	0.1785714	-0.0238095	-0.2571429	0.0619048
H*DX(2)	-0.2619048	-0.0357143	0.0952381	0.1309524	0.0714286	-0.1619048	0.0142857
H*DX(3)	-0.1666667	-0.0833333	-0.0000000	0.0833333	0.1666667	-0.0666667	-0.0333333
H*DX(4)	-0.0714286	-0.1309524	-0.0952381	0.0357143	0.2619048	0.0285714	-0.0809524
H*DX(5)	0.0238095	-0.1785714	-0.1904762	-0.0119048	0.3571429	0.1238095	-0.1285714
H*DX(6)	-0.2571429	0.0619048	0.1904762	0.1285714	-0.1238095	-0.2571429	0.0619048
H*DX(7)	-0.1619048	0.0142857	0.0952381	0.0809524	-0.0285714	-0.1619048	0.0142857
H*DX(8)	-0.0666667	-0.0333333	-0.0000000	0.0333333	0.0666667	-0.0666667	-0.0333333
H*DX(9)	0.0285714	-0.0809524	-0.0952381	-0.0142857	0.1619048	0.0285714	-0.0809524
H*DX(10)	0.1238095	-0.1285714	-0.1904762	-0.0619048	0.2571429	0.1238095	-0.1285714
H*DX(11)	-0.1571429	0.1119048	0.1904762	0.0785714	-0.2238095	-0.2571429	0.0619048
H*DX(12)	-0.0619048	0.0642857	0.0952381	0.0309524	-0.1285714	-0.1619048	0.0142857
H*DX(13)	0.0333333	0.0166667	-0.0000000	-0.0166667	-0.0333333	-0.0666667	-0.0333333
H*DX(14)	0.1285714	-0.0309524	-0.0952381	-0.0642857	0.0619048	0.0285714	-0.0809524
H*DX(15)	0.2238095	-0.0785714	-0.1904762	-0.1119048	0.1571429	0.1238095	-0.1285714

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*DX(1)	0.1904762	0.1285714	-0.1238095	-0.1571429	0.1119048	0.1904762	0.0785714
H*DX(2)	0.0952381	0.0809524	-0.0285714	-0.0619048	0.0642857	0.0952381	0.0309524
H*DX(3)	-0.0000000	0.0333333	0.0666667	0.0333333	0.0166667	-0.0000000	-0.0166667
H*DX(4)	-0.0952381	-0.0142857	0.1619048	0.1285714	-0.0309524	-0.0952381	-0.0642857
H*DX(5)	-0.1904762	-0.0619048	0.2571429	0.2238095	-0.0785714	-0.1904762	-0.1119048
H*DX(6)	0.1904762	0.1285714	-0.1238095	-0.2571429	0.0619048	0.1904762	0.1285714
H*DX(7)	0.0952381	0.0809524	-0.0285714	-0.1619048	0.0142857	0.0952381	0.0809524
H*DX(8)	-0.0000000	0.0333333	0.0666667	-0.0666667	-0.0333333	-0.0000000	0.0333333
H*DX(9)	-0.0952381	-0.0142857	0.1619048	0.0285714	-0.0809524	-0.0952381	-0.0142857
H*DX(10)	-0.1904762	-0.0619048	0.2571429	0.1238095	-0.1285714	-0.1904762	-0.0619048
H*DX(11)	0.1904762	0.1285714	-0.1238095	-0.3571429	0.0119048	0.1904762	0.1785714
H*DX(12)	0.0952381	0.0809524	-0.0285714	-0.2619048	-0.0357143	0.0952381	0.1309524
H*DX(13)	-0.0000000	0.0333333	0.0666667	-0.1666667	-0.0833333	-0.0000000	0.0833333
H*DX(14)	-0.0952381	-0.0142857	0.1619048	-0.0714286	-0.1309524	-0.0952381	0.0357143
H*DX(15)	-0.1904762	-0.0619048	0.2571429	0.0238095	-0.1785714	-0.1904762	-0.0119048

	Y(15)
H*DX(1)	-0.2238095
H*DX(2)	-0.1285714
H*DX(3)	-0.0333333
H*DX(4)	0.0619048
H*DX(5)	0.1571429
H*DX(6)	-0.1238095
H*DX(7)	-0.0285714
H*DX(8)	0.0666667
H*DX(9)	0.1619048
H*DX(10)	0.2571429
H*DX(11)	-0.0238095
H*DX(12)	0.0714286
H*DX(13)	0.1666667
H*DX(14)	0.2619048
H*DX(15)	0.3571429

3 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. Y)

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*DY(1)	0.5000000	0.4000000	0.3000000	0.2000000	0.1000000	-0.4000000	-0.4000000
K*DY(2)	0.4000000	0.3500000	0.3000000	0.2500000	0.2000000	-0.4000000	-0.4000000
K*DY(3)	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	-0.4000000	-0.4000000
K*DY(4)	0.2000000	0.2500000	0.3000000	0.3500000	0.4000000	-0.4000000	-0.4000000
K*DY(5)	0.1000000	0.2000000	0.3000000	0.4000000	0.5000000	-0.4000000	-0.4000000
K*DY(6)	0.3000000	0.2000000	0.1000000	-0.0000000	-0.1000000	0.0000000	0.0000000
K*DY(7)	0.2000000	0.1500000	0.1000000	0.0500000	-0.0000000	0.0000000	0.0000000
K*DY(8)	0.1000000	0.1000000	0.1000000	0.1000000	0.1000000	0.0000000	0.0000000
K*DY(9)	-0.0000000	0.0500000	0.1000000	0.1500000	0.2000000	0.0000000	0.0000000
K*DY(10)	-0.1000000	0.0000000	0.1000000	0.2000000	0.3000000	-0.0000000	0.0000000
K*DY(11)	0.1000000	0.0000000	-0.1000000	-0.2000000	-0.3000000	0.4000000	0.4000000
K*DY(12)	-0.0000000	-0.0500000	-0.1000000	-0.1500000	-0.2000000	0.4000000	0.4000000
K*DY(13)	-0.1000000	-0.1000000	-0.1000000	-0.1000000	-0.1000000	0.4000000	0.4000000
K*DY(14)	-0.2000000	-0.1500000	-0.1000000	-0.0500000	-0.0000000	0.4000000	0.4000000
K*DY(15)	-0.3000000	-0.2000000	-0.1000000	-0.0000000	0.1000000	0.4000000	0.4000000

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
K*DY(1)	-0.4000000	-0.4000000	-0.4000000	-0.1000000	-0.0000000	0.1000000	0.2000000
K*DY(2)	-0.4000000	-0.4000000	-0.4000000	0.0000000	0.0500000	0.1000000	0.1500000
K*DY(3)	-0.4000000	-0.4000000	-0.4000000	0.1000000	0.1000000	0.1000000	0.1000000
K*DY(4)	-0.4000000	-0.4000000	-0.4000000	0.2000000	0.1500000	0.1000000	0.0500000
K*DY(5)	-0.4000000	-0.4000000	-0.4000000	0.3000000	0.2000000	0.1000000	0.0000000
K*DY(6)	0.0000000	0.0000000	0.0000000	-0.3000000	-0.2000000	-0.1000000	0.0000000
K*DY(7)	-0.0000000	0.0000000	0.0000000	-0.2000000	-0.1500000	-0.1000000	-0.0500000
K*DY(8)	-0.0000000	0.0000000	0.0000000	-0.1000000	-0.1000000	-0.1000000	-0.1000000
K*DY(9)	-0.0000000	0.0000000	0.0000000	0.0000000	-0.0500000	-0.1000000	-0.1500000
K*DY(10)	-0.0000000	0.0000000	0.0000000	0.1000000	-0.0000000	-0.1000000	-0.2000000
K*DY(11)	0.4000000	0.4000000	0.4000000	-0.5000000	-0.4000000	-0.3000000	-0.2000000
K*DY(12)	0.4000000	0.4000000	0.4000000	-0.4000000	-0.3500000	-0.3000000	-0.2500000
K*DY(13)	0.4000000	0.4000000	0.4000000	-0.3000000	-0.3000000	-0.3000000	-0.3000000
K*DY(14)	0.4000000	0.4000000	0.4000000	-0.2000000	-0.2500000	-0.3000000	-0.3500000
K*DY(15)	0.4000000	0.4000000	0.4000000	-0.1000000	-0.2000000	-0.3000000	-0.4000000

	Y(15)
K*DY(1)	0.3000000
K*DY(2)	0.2000000
K*DY(3)	0.1000000
K*DY(4)	-0.0000000
K*DY(5)	-0.1000000
K*DY(6)	0.1000000
K*DY(7)	-0.0000000
K*DY(8)	-0.1000000
K*DY(9)	-0.2000000
K*DY(10)	-0.3000000
K*DY(11)	-0.1000000
K*DY(12)	-0.2000000
K*DY(13)	-0.3000000
K*DY(14)	-0.4000000
K*DY(15)	-0.5000000

3 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. X AND X). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*DXX(1)	0.0952381	-0.0476190	-0.0952381	-0.0476190	0.0952381	0.0952381	-0.0476190
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*H*DXX(1)	-0.0952381	-0.0476190	0.0952381	0.0952381	-0.0476190	-0.0952381	-0.0476190
	Y(15)						
H*H*DXX(1)	0.0952381						

3 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. X AND Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*K*DXY(1)	-0.1000000	-0.0500000	-0.0000000	.0500000	0.1000000	-0.0000000	0.0000000
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*K*DXY(1)	-0.0000000	-0.0000000	0.0000000	0.1000000	0.0500000	0.0000000	-0.0500000
	Y(15)						
H*K*DXY(1)	-0.1000000						

3 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. Y AND Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*K*DYY(1)	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000	-0.4000000	-0.4000000
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
K*K*DYY(1)	-0.4000000	-0.4000000	-0.4000000	0.2000000	0.2000000	0.2000000	0.2000000
	Y(15)						
K*K*DYY(1)	0.2000000						

5 X 5 ARRAY
LINEAR FITTING

SMOOTHING FORMULA

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.2000000	0.1600000	0.1200000	0.0800000	0.0400000	0.1600000	0.1200000
P(2)	0.1600000	0.1400000	0.1200000	0.1000000	0.0800000	0.1200000	0.1000000
P(3)	0.1200000	0.1200000	0.1200000	0.1200000	0.1200000	0.0800000	0.0800000
P(4)	0.0800000	0.1000000	0.1200000	0.1400000	0.1600000	0.0400000	0.0600000
P(5)	0.0400000	0.0800000	0.1200000	0.1600000	0.2000000	0.0000000	0.0400000
P(6)	0.1600000	0.1200000	0.0800000	0.0400000	0.0000000	0.1400000	0.1000000
P(7)	0.1200000	0.1000000	0.0800000	0.0600000	0.0400000	0.1000000	0.0800000
P(8)	0.0800000	0.0800000	0.0800000	0.0800000	0.0800000	0.0600000	0.0600000
P(9)	0.0400000	0.0600000	0.0800000	0.1000000	0.1200000	0.0200000	0.0400000
P(10)	-0.0000000	0.0400000	0.0800000	0.1200000	0.1600000	-0.0200000	0.0200000
P(11)	0.1200000	0.0800000	0.0400000	0.0000000	-0.0400000	0.1200000	0.0800000
P(12)	0.0800000	0.0600000	0.0400000	0.0200000	0.0000000	0.0800000	0.0600000
P(13)	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000
P(14)	-0.0000000	0.0200000	0.0400000	0.0600000	0.0800000	0.0000000	0.0200000
P(15)	-0.0400000	0.0000000	0.0400000	0.0800000	0.1200000	-0.0400000	-0.0000000
P(16)	0.0800000	0.0400000	0.0000000	-0.0400000	-0.0800000	0.1000000	0.0600000
P(17)	0.0400000	0.0200000	0.0000000	-0.0200000	-0.0400000	0.0600000	0.0400000
P(18)	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0200000	0.0200000
P(19)	-0.0400000	-0.0200000	0.0000000	0.0200000	0.0400000	-0.0200000	-0.0000000
P(20)	-0.0800000	-0.0400000	0.0000000	0.0400000	0.0800000	-0.0600000	-0.0200000
P(21)	0.0400000	0.0000000	-0.0400000	-0.0800000	-0.1200000	0.0800000	0.0400000
P(22)	-0.0000000	-0.0200000	-0.0400000	-0.0600000	-0.0800000	0.0400000	0.0200000
P(23)	-0.0400000	-0.0400000	-0.0400000	-0.0400000	-0.0400000	0.0000000	-0.0000000
P(24)	-0.0800000	-0.0600000	-0.0400000	-0.0200000	0.0000000	-0.0400000	-0.0200000
P(25)	-0.1200000	-0.0800000	-0.0400000	0.0000000	0.0400000	-0.0800000	-0.0400000

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
P(1)	0.0800000	0.0400000	-0.0000000	0.1200000	0.0800000	0.0400000	-0.0000000
P(2)	0.0800000	0.0600000	0.0400000	0.0800000	0.0600000	0.0400000	0.0200000
P(3)	0.0800000	0.0800000	0.0800000	0.0400000	0.0400000	0.0400000	0.0400000
P(4)	0.0800000	0.1000000	0.1200000	0.0000000	0.0200000	0.0400000	0.0600000
P(5)	0.0800000	0.1200000	0.1600000	-0.0400000	0.0000000	0.0400000	0.0800000
P(6)	0.0600000	0.0200000	-0.0200000	0.1200000	0.0800000	0.0400000	-0.0000000
P(7)	0.0600000	0.0400000	0.0200000	0.0800000	0.0600000	0.0400000	0.0200000
P(8)	0.0600000	0.0600000	0.0600000	0.0400000	0.0400000	0.0400000	0.0400000
P(9)	0.0600000	0.0800000	0.1000000	0.0000000	0.0200000	0.0400000	0.0600000
P(10)	0.0600000	0.1000000	0.1400000	-0.0400000	0.0000000	0.0400000	0.0800000
P(11)	0.0400000	-0.0000000	-0.0400000	0.1200000	0.0800000	0.0400000	-0.0000000
P(12)	0.0400000	0.0200000	-0.0000000	0.0800000	0.0600000	0.0400000	0.0200000
P(13)	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000
P(14)	0.0400000	0.0600000	0.0800000	0.0000000	0.0200000	0.0400000	0.0600000
P(15)	0.0400000	0.0800000	0.1200000	-0.0400000	0.0000000	0.0400000	0.0800000
P(16)	0.0200000	-0.0200000	-0.0600000	0.1200000	0.0800000	0.0400000	-0.0000000
P(17)	0.0200000	-0.0000000	-0.0200000	0.0800000	0.0600000	0.0400000	0.0200000
P(18)	0.0200000	0.0200000	0.0200000	0.0400000	0.0400000	0.0400000	0.0400000
P(19)	0.0200000	0.0400000	0.0600000	0.0000000	0.0200000	0.0400000	0.0600000
P(20)	0.0200000	0.0600000	0.1000000	-0.0400000	0.0000000	0.0400000	0.0800000
P(21)	-0.0000000	-0.0400000	-0.0800000	0.1200000	0.0800000	0.0400000	-0.0000000
P(22)	-0.0000000	-0.0200000	-0.0400000	0.0800000	0.0600000	0.0400000	0.0200000
P(23)	-0.0000000	-0.0000000	-0.0000000	0.0400000	0.0400000	0.0400000	0.0400000
P(24)	-0.0000000	0.0200000	0.0400000	0.0000000	0.0200000	0.0400000	0.0600000
P(25)	-0.0000000	0.0400000	0.0800000	-0.0400000	0.0000000	0.0400000	0.0800000

	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
P(1)	-0.0400000	0.0800000	0.0400000	0.0000000	-0.0400000	-0.0800000	0.0400000
P(2)	-0.0000000	0.0400000	0.0200000	-0.0000000	-0.0200000	-0.0400000	0.0000000
P(3)	0.0400000	0.0000000	0.0000000	-0.0000000	0.0000000	-0.0000000	-0.0400000
P(4)	0.0800000	-0.0400000	-0.0200000	-0.0000000	0.0200000	0.0400000	-0.0800000
P(5)	0.1200000	-0.0800000	-0.0400000	-0.0000000	0.0400000	0.0800000	-0.1200000
P(6)	-0.0400000	0.1000000	0.0600000	0.0200000	-0.0200000	-0.0600000	0.0800000
P(7)	-0.0000000	0.0600000	0.0400000	0.0200000	0.0000000	-0.0200000	0.0400000
P(8)	0.0400000	0.0200000	0.0200000	0.0200000	0.0200000	0.0200000	0.0000000
P(9)	0.0800000	-0.0200000	0.0000000	0.0200000	0.0400000	0.0600000	-0.0400000
P(10)	0.1200000	-0.0600000	-0.0200000	0.0200000	0.0600000	0.1000000	-0.0800000
P(11)	-0.0400000	0.1200000	0.0800000	0.0400000	0.0000000	-0.0400000	0.1200000
P(12)	-0.0000000	0.0800000	0.0600000	0.0400000	0.0200000	-0.0000000	0.0800000
P(13)	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000
P(14)	0.0800000	0.0000000	0.0200000	0.0400000	0.0600000	0.0800000	0.0000000
P(15)	0.1200000	-0.0400000	0.0000000	0.0400000	0.0800000	0.1200000	-0.0400000
P(16)	-0.0400000	0.1400000	0.1000000	0.0600000	0.0200000	-0.0200000	0.1600000
P(17)	-0.0000000	0.1000000	0.0800000	0.0600000	0.0400000	0.0200000	0.1200000
P(18)	0.0400000	0.0600000	0.0600000	0.0600000	0.0600000	0.0600000	0.0800000
P(19)	0.0800000	0.0200000	0.0400000	0.0600000	0.0800000	0.1000000	0.0400000
P(20)	0.1200000	-0.0200000	0.0200000	0.0600000	0.1000000	0.1400000	0.0000000
P(21)	-0.0400000	0.1600000	0.1200000	0.0800000	0.0400000	-0.0000000	0.2000000
P(22)	-0.0000000	0.1200000	0.1000000	0.0800000	0.0600000	0.0400000	0.1600000
P(23)	0.0400000	0.0800000	0.0800000	0.0800000	0.0800000	0.0800000	0.1200000
P(24)	0.0800000	0.0400000	0.0600000	0.0800000	0.1000000	0.1200000	0.0800000
P(25)	0.1200000	0.0000000	0.0400000	0.0800000	0.1200000	0.1600000	0.0400000

	Y(22)	Y(23)	Y(24)	Y(25)
P(1)	0.0000000	-0.0400000	-0.0800000	-0.1200000
P(2)	-0.0200000	-0.0400000	-0.0600000	-0.0800000
P(3)	-0.0400000	-0.0400000	-0.0400000	-0.0400000
P(4)	-0.0600000	-0.0400000	-0.0200000	-0.0000000
P(5)	-0.0800000	-0.0400000	-0.0000000	0.0400000
P(6)	0.0400000	0.0000000	-0.0400000	-0.0800000
P(7)	0.0200000	0.0000000	-0.0200000	-0.0400000
P(8)	0.0000000	-0.0000000	-0.0000000	-0.0000000
P(9)	-0.0200000	-0.0000000	0.0200000	0.0400000
P(10)	-0.0400000	-0.0000000	0.0400000	0.0800000
P(11)	0.0800000	0.0400000	-0.0000000	-0.0400000
P(12)	0.0600000	0.0400000	0.0200000	-0.0000000
P(13)	0.0400000	0.0400000	0.0400000	0.0400000
P(14)	0.0200000	0.0400000	0.0600000	0.0800000
P(15)	0.0000000	0.0400000	0.0800000	0.1200000
P(16)	0.1200000	0.0800000	0.0400000	-0.0000000
P(17)	0.1000000	0.0800000	0.0600000	0.0400000
P(18)	0.0800000	0.0800000	0.0800000	0.0800000
P(19)	0.0600000	0.0800000	0.1000000	0.1200000
P(20)	0.0400000	0.0800000	0.1200000	0.1600000
P(21)	0.1600000	0.1200000	0.0800000	0.0400000
P(22)	0.1400000	0.1200000	0.1000000	0.0800000
P(23)	0.1200000	0.1200000	0.1200000	0.1200000
P(24)	0.1000000	0.1200000	0.1400000	0.1600000
P(25)	0.0800000	0.1200000	0.1600000	0.2000000

5 X 5 ARRAY
LINEAR FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. X). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*DX(1)	-0.0400000	-0.0200000	-0.0000000	0.0200000	0.0400000	-0.0400000	-0.0200000
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*DX(1)	-0.0000000	0.0200000	0.0400000	-0.0400000	-0.0200000	-0.0000000	0.0200000
	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
H*DX(1)	0.0400000	-0.0400000	-0.0200000	-0.0000000	0.0200000	0.0400000	-0.0400000
	Y(22)	Y(23)	Y(24)	Y(25)			
H*DX(1)	-0.0200000	-0.0000000	0.0200000	0.0400000			

5 X 5 ARRAY
LINEAR FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*DY(1)	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0200000	0.0200000
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
K*DY(1)	0.0200000	0.0200000	0.0200000	-0.0000000	-0.0000000	-0.0200000	0.0000000
	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
K*DY(1)	0.0000000	-0.0200000	-0.0200000	-0.0200000	-0.0200000	-0.0200000	-0.0400000
	Y(22)	Y(23)	Y(24)	Y(25)			
K*DY(1)	-0.0400000	-0.0400000	-0.0400000	-0.0400000			

5 X 5 ARRAY
QUADRATIC FITTING

SMOOTHING FORMULA

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
P(1)	0.4742857	0.2685714	0.1200000	0.0285714	-0.0057143	0.2685714	0.1028571
P(2)	0.2685714	0.2514286	0.2057143	0.1314286	0.0285714	0.1028571	0.1057143
P(3)	0.1200000	0.2057143	0.2342857	0.2057143	0.1200000	-0.0057143	0.0800000
P(4)	0.0285714	0.1314286	0.2057143	0.2514286	0.2685714	-0.0571429	0.0257143
P(5)	-0.0057143	0.0285714	0.1200000	0.2685714	0.4742857	-0.0514286	-0.0571429
P(6)	0.2685714	0.1028571	-0.0057143	-0.0571429	-0.0514286	0.2514286	0.1057143
P(7)	0.1028571	0.1057143	0.0800000	0.0257143	-0.0571429	0.1057143	0.1185714
P(8)	-0.0057143	0.0800000	0.1085714	0.0800000	-0.0057143	0.0171429	0.1028571
P(9)	-0.0571429	0.0257143	0.0800000	0.1057143	0.1028571	-0.0142857	0.0585714
P(10)	-0.0514286	-0.0571429	-0.0057143	0.1028571	0.2685714	0.0114286	-0.0142857
P(11)	0.1200000	-0.0057143	-0.0742857	-0.0857143	-0.0400000	0.2057143	0.0800000
P(12)	-0.0057143	0.0171429	0.0114286	-0.0228571	-0.0857143	0.0800000	0.1028571
P(13)	-0.0742857	0.0114286	0.0400000	0.0114286	-0.0742857	0.0114286	0.0971429
P(14)	-0.0857143	-0.0228571	0.0114286	0.0171429	-0.0057143	0.0000000	0.0628571
P(15)	-0.0400000	-0.0857143	-0.0742857	-0.0057143	0.1200000	0.0457143	-0.0000000
P(16)	0.0285714	-0.0571429	-0.0857143	-0.0571429	0.0285714	0.1314286	0.0257143
P(17)	-0.0571429	-0.0142857	0.0000000	-0.0142857	-0.0571429	0.0257143	0.0585714
P(18)	-0.0857143	0.0000000	0.0285714	-0.0000000	-0.0857143	-0.0228571	0.0628571
P(19)	-0.0571429	-0.0142857	0.0000000	-0.0142857	-0.0571429	-0.0142857	0.0385714
P(20)	0.0285714	-0.0571429	-0.0857143	-0.0571429	0.0285714	0.0514286	-0.0142857
P(21)	-0.0057143	-0.0514286	-0.0400000	0.0285714	0.1542857	0.0285714	-0.0571429
P(22)	-0.0514286	0.0114286	0.0457143	0.0514286	0.0285714	-0.0571429	-0.0142857
P(23)	-0.0400000	0.0457143	0.0742857	0.0457143	-0.0400000	-0.0857143	-0.0000000
P(24)	0.0285714	0.0514286	0.0457143	0.0114286	-0.0514286	-0.0571429	-0.0142857
P(25)	0.1542857	0.0285714	-0.0400000	-0.0514286	-0.0057143	0.0285714	-0.0571429

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
P(1)	-0.0057143	-0.0571429	-0.0514286	0.1200000	-0.0057143	-0.0742857	-0.0857143
P(2)	0.0800000	0.0257143	-0.0571429	-0.0057143	0.0171429	0.0114286	-0.0228571
P(3)	0.1085714	0.0800000	-0.0057143	-0.0742857	0.0114286	0.0400000	0.0114286
P(4)	0.0800000	0.1057143	0.1028571	-0.0857143	-0.0228571	0.0114286	0.0171429
P(5)	-0.0057143	0.1028571	0.2685714	-0.0400000	-0.0857143	-0.0742857	-0.0057143
P(6)	0.0171429	-0.0142857	0.0114286	0.2057143	0.0800000	0.0114286	0.0000000
P(7)	0.1028571	0.0585714	-0.0142857	0.0800000	0.1028571	0.0971429	0.0628571
P(8)	0.1314286	0.1028571	0.0171429	0.0114286	0.0971429	0.1257143	0.0971429
P(9)	0.1028571	0.1185714	0.1057143	0.0000000	0.0628571	0.0971429	0.1028571
P(10)	0.0171429	0.1057143	0.2514286	0.0457143	0.0000000	0.0114286	0.0800000
P(11)	0.0114286	-0.0000000	0.0457143	0.2342857	0.1085714	0.0400000	0.0285714
P(12)	0.0971429	0.0628571	0.0000000	0.1085714	0.1314286	0.1257143	0.0914286
P(13)	0.1257143	0.0971429	0.0114286	0.0400000	0.1257143	0.1542857	0.1257143
P(14)	0.0971429	0.1028571	0.0800000	0.0285714	0.0914286	0.1257143	0.1314286
P(15)	0.0114286	0.0800000	0.2057143	0.0742857	0.0285714	0.0400000	0.1085714
P(16)	-0.0228571	-0.0142857	0.0514286	0.2057143	0.0800000	0.0114286	0.0000000
P(17)	0.0628571	0.0385714	-0.0142857	0.0800000	0.1028571	0.0971429	0.0628571
P(18)	0.0914286	0.0628571	-0.0228571	0.0114286	0.0971429	0.1257143	0.0971429
P(19)	0.0628571	0.0585714	0.0257143	0.0000000	0.0628571	0.0971429	0.1028571
P(20)	-0.0228571	0.0257143	0.1314286	0.0457143	-0.0000000	0.0114286	0.0800000
P(21)	-0.0857143	-0.0571429	0.0285714	0.1200000	-0.0057143	-0.0742857	-0.0857143
P(22)	0.0000000	-0.0142857	-0.0571429	-0.0057143	0.0171429	0.0114286	0.0228571
P(23)	0.0285714	-0.0000000	-0.0857143	-0.0742857	0.0114286	0.0400000	0.0114286
P(24)	-0.0000000	-0.0142857	-0.0571429	-0.0857143	-0.0228571	0.0114286	0.0171429
P(25)	-0.0857143	-0.0571429	0.0285714	-0.0400000	-0.0857143	-0.0742857	-0.0057143

	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
P(1)	-0.0400000	0.0285714	-0.0571429	-0.0857143	-0.0571429	0.0285714	-0.0057143
P(2)	-0.0857143	-0.0571429	-0.0142857	0.0000000	-0.0142857	-0.0571429	-0.0514286
P(3)	-0.0742857	-0.0857143	-0.0000000	0.0285714	-0.0000000	-0.0857143	-0.0400000
P(4)	-0.0057143	-0.0571429	-0.0142857	0.0000000	-0.0142857	-0.0571429	0.0285714
P(5)	0.1200000	0.0285714	-0.0571429	-0.0857143	-0.0571429	0.0285714	0.1542857
P(6)	0.0457143	0.1314286	0.0257143	-0.0228571	-0.0142857	0.0514286	0.0285714
P(7)	-0.0000000	0.0257143	0.0585714	0.0628571	0.0385714	-0.0142857	-0.0571429
P(8)	0.0114286	-0.0228571	0.0628571	0.0914286	0.0628571	-0.0228571	-0.0857143
P(9)	0.0800000	-0.0142857	0.0385714	0.0628571	0.0585714	0.0257143	-0.0571429
P(10)	0.2057143	0.0514286	-0.0142857	-0.0228571	0.0257143	0.1314286	0.0285714
P(11)	0.0742857	0.2057143	0.0800000	0.0114286	-0.0000000	0.0457143	0.1200000
P(12)	0.0285714	0.0800000	0.1028571	0.0971429	0.0628571	0.0000000	-0.0057143
P(13)	0.0400000	0.0114286	0.0971429	0.1257143	0.0971429	0.0114286	-0.0742857
P(14)	0.1085714	-0.0000000	0.0628571	0.0971429	0.1028571	0.0800000	-0.0857143
P(15)	0.2342857	0.0457143	-0.0000000	0.0114286	0.0800000	0.2057143	-0.0400000
P(16)	0.0457143	0.2514286	0.1057143	0.0171429	-0.0142857	0.0114286	0.2685714
P(17)	-0.0000000	0.1057143	0.1185714	0.1028571	0.0585714	-0.0142857	0.1028571
P(18)	0.0114286	0.0171429	0.1028571	0.1314286	0.1028571	0.0171429	-0.0057143
P(19)	0.0800000	-0.0142857	0.0585714	0.1028571	0.1185714	0.1057143	-0.0571429
P(20)	0.2057143	0.0114286	-0.0142857	0.0171429	0.1057143	0.2514286	-0.0514286
P(21)	-0.0400000	0.2685714	0.1028571	-0.0057143	-0.0571429	-0.0514286	0.4742857
P(22)	-0.0857143	0.1028571	0.1057143	0.0800000	0.0257143	-0.0571429	0.2685714
P(23)	-0.0742857	-0.0057143	0.0800000	0.1085714	0.0800000	-0.0057143	0.1200000
P(24)	-0.0057143	-0.0571429	0.0257143	0.0800000	0.1057143	0.1028571	0.0285714
P(25)	0.1200000	-0.0514286	-0.0571429	-0.0057143	0.1028571	0.2685714	-0.0057143

	Y(22)	Y(23)	Y(24)	Y(25)
P(1)	-0.0514286	-0.0400000	0.0285714	0.1542857
P(2)	0.0114286	0.0457143	0.0514286	0.0285714
P(3)	0.0457143	0.0742857	0.0457143	-0.0400000
P(4)	0.0514286	0.0457143	0.0114286	-0.0514286
P(5)	0.0285714	-0.0400000	-0.0514286	-0.0057143
P(6)	-0.0571429	-0.0857143	-0.0571429	0.0285714
P(7)	-0.0142857	-0.0000000	-0.0142857	-0.0571429
P(8)	0.0000000	0.0285714	0.0000000	-0.0857143
P(9)	-0.0142857	-0.0000000	-0.0142857	-0.0571429
P(10)	-0.0571429	-0.0857143	-0.0571429	0.0285714
P(11)	-0.0057143	-0.0742857	-0.0857143	-0.0400000
P(12)	0.0171429	0.0114286	-0.0228571	-0.0857143
P(13)	0.0114286	0.0400000	0.0114286	-0.0742857
P(14)	-0.0228571	0.0114286	0.0171429	-0.0057143
P(15)	-0.0857143	-0.0742857	-0.0057143	0.1200000
P(16)	0.1028571	-0.0057143	-0.0571429	-0.0514286
P(17)	0.1057143	0.0800000	0.0257143	-0.0571429
P(18)	0.0800000	0.1085714	0.0800000	-0.0057143
P(19)	0.0257143	0.0800000	0.1057143	0.1028571
P(20)	-0.0571429	-0.0057143	0.1028571	0.2685714
P(21)	0.2685714	0.1200000	0.0285714	-0.0057143
P(22)	0.2514286	0.2057143	0.1314286	0.0285714
P(23)	0.2057143	0.2342857	0.2057143	0.1200000
P(24)	0.1314286	0.2057143	0.2514286	0.2685714
P(25)	0.0285714	0.1200000	0.2685714	0.4742857

5 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. X)

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*DX(1)	-0.2342857	-0.0028571	0.1142857	0.1171429	0.0057143	-0.1942857	0.0171429
H*DX(2)	-0.1771429	-0.0314286	0.0571429	0.0885714	0.0628571	-0.1371429	-0.0114286
H*DX(3)	-0.1200000	-0.0600000	-0.0000000	0.0600000	0.1200000	-0.0800000	-0.0400000
H*DX(4)	-0.0628571	-0.0885714	-0.0571429	0.0314286	0.1771429	-0.0228571	-0.0685714
H*DX(5)	-0.0057143	-0.1171429	-0.1142857	0.0028571	0.2342857	0.0342857	-0.0971429
H*DX(6)	-0.1942857	0.0171429	0.1142857	0.0971429	-0.0342857	-0.1742857	0.0271429
H*DX(7)	-0.1371429	-0.0114286	0.0571429	0.0685714	0.0228571	-0.1171429	-0.0014286
H*DX(8)	-0.0800000	-0.0400000	-0.0000000	0.0400000	0.0800000	-0.0600000	-0.0300000
H*DX(9)	-0.0228571	-0.0685714	-0.0571429	0.0114286	0.1371429	-0.0028571	-0.0585714
H*DX(10)	0.0342857	-0.0971429	-0.1142857	-0.0171429	0.1942857	0.0542857	-0.0871429
H*DX(11)	-0.1542857	0.0371429	0.1142857	0.0771429	-0.0742857	-0.1542857	0.0371429
H*DX(12)	-0.0971429	0.0085714	0.0571429	0.0485714	-0.0171429	-0.0971429	0.0085714
H*DX(13)	-0.0400000	-0.0200000	-0.0000000	0.0200000	0.0400000	-0.0400000	-0.0200000
H*DX(14)	0.0171429	-0.0485714	-0.0571429	-0.0085714	0.0971429	0.0171429	-0.0485714
H*DX(15)	0.0742857	-0.0771429	-0.1142857	-0.0371429	0.1542857	0.0742857	-0.0771429
H*DX(16)	-0.1142857	0.0571429	0.1142857	0.0571429	-0.1142857	-0.1342857	0.0471429
H*DX(17)	-0.0571429	0.0285714	0.0571429	0.0285714	-0.0571429	-0.0771429	0.0185714
H*DX(18)	0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	-0.0200000	-0.0100000
H*DX(19)	0.0571429	-0.0285714	-0.0571429	-0.0285714	0.0571429	0.0371429	-0.0385714
H*DX(20)	0.1142857	-0.0571429	-0.1142857	-0.0571429	0.1142857	0.0942857	-0.0671429
H*DX(21)	-0.0742857	0.0771429	0.1142857	0.0371429	-0.1542857	-0.1142857	0.0571429
H*DX(22)	-0.0171429	0.0485714	0.0571429	0.0085714	-0.0971429	-0.0571429	0.0285714
H*DX(23)	0.0400000	0.0200000	-0.0000000	-0.0200000	-0.0400000	-0.0000000	-0.0000000
H*DX(24)	0.0971429	-0.0085714	-0.0571429	-0.0485714	0.0171429	0.0571429	-0.0285714
H*DX(25)	0.1542857	-0.0371429	-0.1142857	-0.0771429	0.0742857	0.1142857	-0.0571429

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*DX(1)	0.1142857	0.0971429	-0.0342857	-0.1542857	0.0371429	0.1142857	0.0771429
H*DX(2)	0.0571429	0.0685714	0.0228571	-0.0971429	0.0085714	0.0571429	0.0485714
H*DX(3)	-0.0000000	0.0400000	0.0800000	-0.0400000	-0.0200000	-0.0000000	0.0200000
H*DX(4)	-0.0571429	0.0114286	0.1371429	0.0171429	-0.0485714	-0.0571429	-0.0085714
H*DX(5)	-0.1142857	-0.0171429	0.1942857	0.0742857	-0.0771429	-0.1142857	-0.0371429
H*DX(6)	0.1142857	0.0871429	-0.0542857	-0.1542857	0.0371429	0.1142857	0.0771429
H*DX(7)	0.0571429	0.0585714	0.0028571	-0.0971429	0.0085714	0.0571429	0.0485714
H*DX(8)	-0.0000000	0.0300000	0.0600000	-0.0400000	-0.0200000	-0.0000000	0.0200000
H*DX(9)	-0.0571429	0.0014286	0.1171429	0.0171429	-0.0485714	-0.0571429	-0.0085714
H*DX(10)	-0.1142857	-0.0271429	0.1742857	0.0742857	-0.0771429	-0.1142857	-0.0371429
H*DX(11)	0.1142857	0.0771429	-0.0742857	-0.1542857	0.0371429	0.1142857	0.0771429
H*DX(12)	0.0571429	0.0485714	-0.0171429	-0.0971429	0.0085714	0.0571429	0.0485714
H*DX(13)	-0.0000000	0.0200000	0.0400000	-0.0400000	-0.0200000	-0.0000000	0.0200000
H*DX(14)	-0.0571429	-0.0085714	0.0971429	0.0171429	-0.0485714	-0.0571429	-0.0085714
H*DX(15)	-0.1142857	-0.0371429	0.1542857	0.0742857	-0.0771429	-0.1142857	-0.0371429
H*DX(16)	0.1142857	0.0671429	-0.0942857	-0.1542857	0.0371429	0.1142857	0.0771429
H*DX(17)	0.0571429	0.0385714	-0.0371429	-0.0971429	0.0085714	0.0571429	0.0485714
H*DX(18)	-0.0000000	0.0100000	0.0200000	-0.0400000	-0.0200000	-0.0000000	0.0200000
H*DX(19)	-0.0571429	-0.0185714	0.0771429	0.0171429	-0.0485714	-0.0571429	-0.0085714
H*DX(20)	-0.1142857	-0.0471429	0.1342857	0.0742857	-0.0771429	-0.1142857	-0.0371429
H*DX(21)	0.1142857	0.0571429	-0.1142857	-0.1542857	0.0371429	0.1142857	0.0771429
H*DX(22)	0.0571429	0.0285714	-0.0571429	-0.0971429	0.0085714	0.0571429	0.0485714
H*DX(23)	-0.0000000	0.0000000	0.0000000	-0.0400000	-0.0200000	-0.0000000	0.0200000
H*DX(24)	-0.0571429	-0.0285714	0.0571429	0.0171429	-0.0485714	-0.0571429	-0.0085714
H*DX(25)	-0.1142857	-0.0571429	0.1142857	0.0742857	-0.0771429	-0.1142857	-0.0371429

	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
H*DX(1)	-0.0742857	-0.1142857	0.0571429	0.1142857	0.0571429	-0.1142857	-0.0742857
H*DX(2)	-0.0171429	-0.0571429	0.0285714	0.0571429	0.0285714	-0.0571429	-0.0171429
H*DX(3)	0.0400000	0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	0.0400000
H*DX(4)	0.0971429	0.0571429	-0.0285714	-0.0571429	-0.0285714	0.0571429	0.0971429
H*DX(5)	0.1542857	0.1142857	-0.0571429	-0.1142857	-0.0571429	0.1142857	0.1542857
H*DX(6)	-0.0742857	-0.1342857	0.0471429	0.1142857	0.0671429	-0.0942857	-0.1142857
H*DX(7)	-0.0171429	-0.0771429	0.0185714	0.0571429	0.0385714	-0.0371429	-0.0571429
H*DX(8)	0.0400000	-0.0200000	-0.0100000	-0.0000000	0.0100000	0.0200000	0.0000000
H*DX(9)	0.0971429	0.0371429	-0.0385714	-0.0571429	-0.0185714	0.0771429	0.0571429
H*DX(10)	0.1542857	0.0942857	-0.0671429	-0.1142857	-0.0471429	0.1342857	0.1142857
H*DX(11)	-0.0742857	-0.1542857	0.0371429	0.1142857	0.0771429	-0.0742857	-0.1542857
H*DX(12)	-0.0171429	-0.0971429	0.0085714	0.0571429	0.0485714	-0.0171429	-0.0971429
H*DX(13)	0.0400000	-0.0400000	-0.0200000	-0.0000000	0.0200000	0.0400000	-0.0400000
H*DX(14)	0.0971429	0.0171429	-0.0485714	-0.0571429	-0.0085714	0.0971429	0.0171429
H*DX(15)	0.1542857	0.0742857	-0.0771429	-0.1142857	-0.0371429	0.1542857	0.0742857
H*DX(16)	-0.0742857	-0.1742857	0.0271429	0.1142857	0.0871429	-0.0542857	-0.1942857
H*DX(17)	-0.0171429	-0.1171429	-0.0014286	0.0571429	0.0585714	0.0028571	-0.1371429
H*DX(18)	0.0400000	-0.0600000	-0.0300000	-0.0000000	0.0300000	0.0600000	-0.0800000
H*DX(19)	0.0971429	-0.0028571	-0.0585714	-0.0571429	0.0014286	0.1171429	-0.0228571
H*DX(20)	0.1542857	0.0542857	-0.0871429	-0.1142857	-0.0271429	0.1742857	0.0342857
H*DX(21)	-0.0742857	-0.1942857	0.0171429	0.1142857	0.0971429	-0.0342857	-0.2342857
H*DX(22)	-0.0171429	-0.1371429	-0.0114286	0.0571429	0.0685714	0.0228571	-0.1771429
H*DX(23)	0.0400000	-0.0800000	-0.0400000	-0.0000000	0.0400000	0.0800000	-0.1200000
H*DX(24)	0.0971429	-0.0228571	-0.0685714	-0.0571429	0.0114286	0.1371429	-0.0628571
H*DX(25)	0.1542857	0.0342857	-0.0971429	-0.1142857	-0.0171429	0.1942857	-0.0057143

	Y(22)	Y(23)	Y(24)	Y(25)
H*DX(1)	0.0771429	0.1142857	0.0371429	-0.1542857
H*DX(2)	0.0485714	0.0571429	0.0085714	-0.0971429
H*DX(3)	0.0200000	-0.0000000	-0.0200000	-0.0400000
H*DX(4)	-0.0085714	-0.0571429	-0.0485714	0.0171429
H*DX(5)	-0.0371429	-0.1142857	-0.0771429	0.0742857
H*DX(6)	0.0571429	0.1142857	0.0571429	-0.1142857
H*DX(7)	0.0285714	0.0571429	0.0285714	-0.0571429
H*DX(8)	0.0000000	-0.0000000	-0.0000000	-0.0000000
H*DX(9)	-0.0285714	-0.0571429	-0.0285714	0.0571429
H*DX(10)	-0.0571429	-0.1142857	-0.0571429	0.1142857
H*DX(11)	0.0371429	0.1142857	0.0771429	-0.0742857
H*DX(12)	0.0085714	0.0571429	0.0485714	-0.0171429
H*DX(13)	-0.0200000	-0.0000000	0.0200000	0.0400000
H*DX(14)	-0.0485714	-0.0571429	-0.0085714	0.0971429
H*DX(15)	-0.0771429	-0.1142857	-0.0371429	0.1542857
H*DX(16)	0.0171429	0.1142857	0.0971429	-0.0342857
H*DX(17)	-0.0114286	0.0571429	0.0685714	0.0228571
H*DX(18)	-0.0400000	-0.0000000	0.0400000	0.0800000
H*DX(19)	-0.0685714	-0.0571429	0.0114286	0.1371429
H*DX(20)	-0.0971429	-0.1142857	-0.0171429	0.1942857
H*DX(21)	-0.0028571	0.1142857	0.1171429	0.0057143
H*DX(22)	-0.0314286	0.0571429	0.0885714	0.0628571
H*DX(23)	-0.0600000	-0.0000000	0.0600000	0.1200000
H*DX(24)	-0.0885714	-0.0571429	0.0314286	0.1771429
H*DX(25)	-0.1171429	-0.1142857	0.0028571	0.2342857

5 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (FIRST PARTIAL W.R.T. Y)

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*DY(1)	0.2342857	0.1942857	0.1542857	0.1142857	0.0742857	0.0028571	-0.0171429
K*DY(2)	0.1942857	0.1742857	0.1542857	0.1342857	0.1142857	-0.0171429	-0.0271429
K*DY(3)	0.1542857	0.1542857	0.1542857	0.1542857	0.1542857	-0.0371429	-0.0371429
K*DY(4)	0.1142857	0.1342857	0.1542857	0.1742857	0.1942857	-0.0571429	-0.0471429
K*DY(5)	0.0742857	0.1142857	0.1542857	0.1942857	0.2342857	-0.0771429	-0.0571429
K*DY(6)	0.1771429	0.1371429	0.0971429	0.0571429	0.0171429	0.0314286	0.0114286
K*DY(7)	0.1371429	0.1171429	0.0971429	0.0771429	0.0571429	0.0114286	0.0014286
K*DY(8)	0.0971429	0.0971429	0.0971429	0.0971429	0.0971429	-0.0085714	-0.0085714
K*DY(9)	0.0571429	0.0771429	0.0971429	0.1171429	0.1371429	-0.0285714	-0.0185714
K*DY(10)	0.0171429	0.0571429	0.0971429	0.1371429	0.1771429	-0.0485714	-0.0285714
K*DY(11)	0.1200000	0.0800000	0.0400000	-0.0000000	-0.0400000	0.0600000	0.0400000
K*DY(12)	0.0800000	0.0600000	0.0400000	0.0200000	-0.0000000	0.0400000	0.0300000
K*DY(13)	0.0400000	0.0400000	0.0400000	0.0400000	0.0400000	0.0200000	0.0200000
K*DY(14)	-0.0000000	0.0200000	0.0400000	0.0600000	0.0800000	0.0000000	0.0100000
K*DY(15)	-0.0400000	0.0000000	0.0400000	0.0800000	0.1200000	-0.0200000	0.0000000
K*DY(16)	0.0628571	0.0228571	-0.0171429	-0.0571429	-0.0971429	0.0885714	0.0685714
K*DY(17)	0.0228571	0.0028571	-0.0171429	-0.0371429	-0.0571429	0.0685714	0.0585714
K*DY(18)	-0.0171429	-0.0171429	-0.0171429	-0.0171429	-0.0171429	0.0485714	0.0485714
K*DY(19)	-0.0571429	-0.0371429	-0.0171429	0.0028571	0.0228571	0.0285714	0.0385714
K*DY(20)	-0.0971429	-0.0571429	-0.0171429	0.0228571	0.0628571	0.0085714	0.0285714
K*DY(21)	0.0057143	-0.0342857	-0.0742857	-0.1142857	-0.1542857	0.1171429	0.0971429
K*DY(22)	-0.0342857	-0.0542857	-0.0742857	-0.0942857	-0.1142857	0.0971429	0.0871429
K*DY(23)	-0.0742857	-0.0742857	-0.0742857	-0.0742857	-0.0742857	0.0771429	0.0771429
K*DY(24)	-0.1142857	-0.0942857	-0.0742857	-0.0542857	-0.0342857	0.0571429	0.0671429
K*DY(25)	-0.1542857	-0.1142857	-0.0742857	-0.0342857	0.0057143	0.0371429	0.0571429

	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
K*DY(1)	-0.0371429	-0.0571429	-0.0771429	-0.1142857	-0.1142857	-0.1142857	-0.1142857
K*DY(2)	-0.0371429	-0.0471429	-0.0571429	-0.1142857	-0.1142857	-0.1142857	-0.1142857
K*DY(3)	-0.0371429	-0.0371429	-0.0371429	-0.1142857	-0.1142857	-0.1142857	-0.1142857
K*DY(4)	-0.0371429	-0.0271429	-0.0171429	-0.1142857	-0.1142857	-0.1142857	-0.1142857
K*DY(5)	-0.0371429	-0.0171429	0.0028571	-0.1142857	-0.1142857	-0.1142857	-0.1142857
K*DY(6)	-0.0085714	-0.0285714	-0.0485714	-0.0571429	-0.0571429	-0.0571429	-0.0571429
K*DY(7)	-0.0085714	-0.0185714	-0.0285714	-0.0571429	-0.0571429	-0.0571429	-0.0571429
K*DY(8)	-0.0085714	-0.0085714	-0.0085714	-0.0571429	-0.0571429	-0.0571429	-0.0571429
K*DY(9)	-0.0085714	0.0014286	0.0114286	-0.0571429	-0.0571429	-0.0571429	-0.0571429
K*DY(10)	-0.0085714	0.0114286	0.0314286	-0.0571429	-0.0571429	-0.0571429	-0.0571429
K*DY(11)	0.0200000	0.0000000	-0.0200000	0.0000000	0.0000000	0.0000000	0.0000000
K*DY(12)	0.0200000	0.0100000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
K*DY(13)	0.0200000	0.0200000	0.0200000	0.0000000	0.0000000	0.0000000	0.0000000
K*DY(14)	0.0200000	0.0300000	0.0400000	0.0000000	0.0000000	0.0000000	0.0000000
K*DY(15)	0.0200000	0.0400000	0.0600000	0.0000000	0.0000000	0.0000000	0.0000000
K*DY(16)	0.0485714	0.0285714	0.0085714	0.0571429	0.0571429	0.0571429	0.0571429
K*DY(17)	0.0485714	0.0385714	0.0285714	0.0571429	0.0571429	0.0571429	0.0571429
K*DY(18)	0.0485714	0.0485714	0.0485714	0.0571429	0.0571429	0.0571429	0.0571429
K*DY(19)	0.0485714	0.0585714	0.0685714	0.0571429	0.0571429	0.0571429	0.0571429
K*DY(20)	0.0485714	0.0685714	0.0885714	0.0571429	0.0571429	0.0571429	0.0571429
K*DY(21)	0.0771429	0.0571429	0.0371429	0.1142857	0.1142857	0.1142857	0.1142857
K*DY(22)	0.0771429	0.0671429	0.0571429	0.1142857	0.1142857	0.1142857	0.1142857
K*DY(23)	0.0771429	0.0771429	0.0771429	0.1142857	0.1142857	0.1142857	0.1142857
K*DY(24)	0.0771429	0.0871429	0.0971429	0.1142857	0.1142857	0.1142857	0.1142857
K*DY(25)	0.0771429	0.0971429	0.1171429	0.1142857	0.1142857	0.1142857	0.1142857

	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
K*DY(1)	-0.1142857	-0.1171429	-0.0971429	-0.0771429	-0.0571429	-0.0371429	-0.0057143
K*DY(2)	-0.1142857	-0.0971429	-0.0871429	-0.0771429	-0.0671429	-0.0571429	0.0342857
K*DY(3)	-0.1142857	-0.0771429	-0.0771429	-0.0771429	-0.0771429	-0.0771429	0.0742857
K*DY(4)	-0.1142857	-0.0571429	-0.0671429	-0.0771429	-0.0871429	-0.0971429	0.1142857
K*DY(5)	-0.1142857	-0.0371429	-0.0571429	-0.0771429	-0.0971429	-0.1171429	0.1542857
K*DY(6)	-0.0571429	-0.0885714	-0.0685714	-0.0485714	-0.0285714	-0.0085714	-0.0628571
K*DY(7)	-0.0571429	-0.0685714	-0.0585714	-0.0485714	-0.0385714	-0.0285714	-0.0228571
K*DY(8)	-0.0571429	-0.0485714	-0.0485714	-0.0485714	-0.0485714	-0.0485714	0.0171429
K*DY(9)	-0.0571429	-0.0285714	-0.0385714	-0.0485714	-0.0585714	-0.0685714	0.0571429
K*DY(10)	-0.0571429	-0.0085714	-0.0285714	-0.0485714	-0.0685714	-0.0885714	0.0971429
K*DY(11)	0.0000000	-0.0600000	-0.0400000	-0.0200000	0.0000000	0.0200000	-0.1200000
K*DY(12)	0.0000000	-0.0400000	-0.0300000	-0.0200000	-0.0100000	0.0000000	-0.0800000
K*DY(13)	0.0000000	-0.0200000	-0.0200000	-0.0200000	-0.0200000	-0.0200000	-0.0400000
K*DY(14)	0.0000000	0.0000000	-0.0100000	-0.0200000	-0.0300000	-0.0400000	-0.0000000
K*DY(15)	0.0000000	0.0200000	-0.0000000	-0.0200000	-0.0400000	-0.0600000	0.0400000
K*DY(16)	0.0571429	-0.0314286	-0.0114286	0.0085714	0.0285714	0.0485714	-0.1771429
K*DY(17)	0.0571429	-0.0114286	-0.0014286	0.0085714	0.0185714	0.0285714	-0.1371429
K*DY(18)	0.0571429	0.0085714	0.0085714	0.0085714	0.0085714	0.0085714	-0.0971429
K*DY(19)	0.0571429	0.0285714	0.0185714	0.0085714	-0.0014286	-0.0114286	-0.0571429
K*DY(20)	0.0571429	0.0485714	0.0285714	0.0085714	-0.0114286	-0.0314286	-0.0171429
K*DY(21)	0.1142857	-0.0028571	0.0171429	0.0371429	0.0571429	0.0771429	-0.2342857
K*DY(22)	0.1142857	0.0171429	0.0271429	0.0371429	0.0471429	0.0571429	-0.1942857
K*DY(23)	0.1142857	0.0371429	0.0371429	0.0371429	0.0371429	0.0371429	-0.1542857
K*DY(24)	0.1142857	0.0571429	0.0471429	0.0371429	0.0271429	0.0171429	-0.1142857
K*DY(25)	0.1142857	0.0771429	0.0571429	0.0371429	0.0171429	-0.0028571	-0.0742857

	Y(22)	Y(23)	Y(24)	Y(25)
K*DY(1)	0.0342857	0.0742857	0.1142857	0.1542857
K*DY(2)	0.0542857	0.0742857	0.0942857	0.1142857
K*DY(3)	0.0742857	0.0742857	0.0742857	0.0742857
K*DY(4)	0.0942857	0.0742857	0.0542857	0.0342857
K*DY(5)	0.1142857	0.0742857	0.0342857	-0.0057143
K*DY(6)	-0.0228571	0.0171429	0.0571429	0.0971429
K*DY(7)	-0.0028571	0.0171429	0.0371429	0.0571429
K*DY(8)	0.0171429	0.0171429	0.0171429	0.0171429
K*DY(9)	0.0371429	0.0171429	-0.0028571	-0.0228571
K*DY(10)	0.0571429	0.0171429	-0.0228571	-0.0628571
K*DY(11)	-0.0800000	-0.0400000	-0.0000000	0.0400000
K*DY(12)	-0.0600000	-0.0400000	-0.0200000	0.0000000
K*DY(13)	-0.0400000	-0.0400000	-0.0400000	-0.0400000
K*DY(14)	-0.0200000	-0.0400000	-0.0600000	-0.0800000
K*DY(15)	-0.0000000	-0.0400000	-0.0800000	-0.1200000
K*DY(16)	-0.1371429	-0.0971429	-0.0571429	-0.0171429
K*DY(17)	-0.1171429	-0.0971429	-0.0771429	-0.0571429
K*DY(18)	-0.0971429	-0.0971429	-0.0971429	-0.0971429
K*DY(19)	-0.0771429	-0.0971429	-0.1171429	-0.1371429
K*DY(20)	-0.0571429	-0.0971429	-0.1371429	-0.1771429
K*DY(21)	-0.1942857	-0.1542857	-0.1142857	-0.0742857
K*DY(22)	-0.1742857	-0.1542857	-0.1342857	-0.1142857
K*DY(23)	-0.1542857	-0.1542857	-0.1542857	-0.1542857
K*DY(24)	-0.1342857	-0.1542857	-0.1742857	-0.1942857
K*DY(25)	-0.1142857	-0.1542857	-0.1942857	-0.2342857

5 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. X AND X). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*H*DXX(1)	0.0571429	-0.0285714	-0.0571429	-0.0285714	0.0571429	0.0571429	-0.0285714
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*H*DXX(1)	-0.0571429	-0.0285714	0.0571429	0.0571429	-0.0285714	-0.0571429	-0.0285714
	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
H*H*DXX(1)	0.0571429	0.0571429	-0.0285714	-0.0571429	-0.0285714	0.0571429	0.0571429
	Y(22)	Y(23)	Y(24)	Y(25)			
H*H*DXX(1)	-0.0285714	-0.0571429	-0.0285714	0.0571429			

5 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. X AND Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
H*K*DXY(1)	-0.0400000	-0.0200000	-0.0000000	0.0200000	0.0400000	-0.0200000	-0.0100000
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
H*K*DXY(1)	0.0000000	0.0100000	0.0200000	0.0000000	-0.0000000	0.0000000	-0.0000000
	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
H*K*DXY(1)	-0.0000000	0.0200000	0.0100000	-0.0000000	-0.0100000	-0.0200000	0.0400000
	Y(22)	Y(23)	Y(24)	Y(25)			
H*K*DXY(1)	0.0200000	0.0000000	-0.0200000	-0.0400000			

5 X 5 ARRAY
QUADRATIC FITTING

DERIV. FORMULA (SECOND PARTIAL W.R.T. Y AND Y). ONLY ONE ROW IS GIVEN.

	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)	Y(7)
K*K*DYY(1)	0.0571429	0.0571429	0.0571429	0.0571429	0.0571429	-0.0285714	-0.0285714
	Y(8)	Y(9)	Y(10)	Y(11)	Y(12)	Y(13)	Y(14)
K*K*DYY(1)	-0.0285714	-0.0285714	-0.0285714	-0.0571429	-0.0571429	-0.0571429	-0.0571429
	Y(15)	Y(16)	Y(17)	Y(18)	Y(19)	Y(20)	Y(21)
K*K*DYY(1)	-0.0571429	-0.0285714	-0.0285714	-0.0285714	-0.0285714	-0.0285714	0.0571429
	Y(22)	Y(23)	Y(24)	Y(25)			
K*K*DYY(1)	0.0571429	0.0571429	0.0571429	0.0571429			