Computer-Aided Design of Second and Third-Order Systems With Parameter Variations and Time Response Constraints

BY

John W. Smay

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Department of Electrical Engineering
University of Colorado
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Isaac M. Horowitz
Research Supervisor

Department of Electrical Engineering

University of Colorado

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Abstract--This report presents a systematic design scheme for second and third-order all pole system transfer functions. The system performance specifications are given as inequality constraints on rise time and overshoot of the step response in the time domain. Plant parameters are assumed to vary between known limits. A completed design insures that the time domain constraints are met for all values of the plant. The maximum values of the specifications are assumed at some plant extreme when the structure provides this freedom, resulting in a design which is optimal in the open-loop gain-bandwidth sense. The second-order system is characterized by the usual natural frequency and damping factor. The third-order system is characterized by the coefficients of the denominator polynomial of its transfer function, and these coefficients are related to both the time response and system parameters. The design procedures are reduced to numerical algorithms to permit digital computer solution of the design problem, or give a specific indication when such a solution is not possible. A successful digital computer implementation is given in the appendix.

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INTRODUCTION

1.1 Problem Statement

The purpose of this paper is to present the development of systematic methods for synthesizing second and third-order all pole closed-loop control system transfer functions. The transfer functions contain plant parameters that vary and are required to meet time domain specifications on the unit step response. The methods developed are primarily intended for implementation on a digital computer, but some insight is also provided for conventional design. A digital computer implementation is given in Appendix A.

The system specifications are given as inequality constraints on rise time and overshoot of the unit step response in the time domain. All plant parameters are permitted to vary in an arbitrary manner between known limits. The rate of parameter variation is assumed to be slow when compared to system response time in order that time dependence of the parameters may be neglected.

The final design causes the time response inequalities to be satisfied over the entire range of plant parameters and the specifications are met as equalities at extreme values of the plant. It is shown that the design results in minimum values of feedback loop gain and bandwidth, thus minimizing the possibility of plant saturation due to high frequency noise effects. If it is not possible to satisfy a given set of specifications with the system structure under consideration this information is revealed in a specific way so that alternative specifications may be chosen or the structure abandoned.

1.2 History of the Problem

Large plant parameter variations occur frequently in flight control and chemical process control design problems.

One approach has been to attempt to cancel the effect of parameter variations using an adaptive compensation. Considerable work has been done in the field of adaptive controls and many examples are available in the current literature {1}.

A second approach is to design a non-varying compensation to handle the plant extremes and permit system response to be better than specified for other values of the plant. Rolnik {2} and Olson {3} have investigated this method with specifications given in the s-plane (complex frequency plane) using dominant pole concepts. Barber {4} has applied time response specifications directly to the problem, as they are applied in this paper, by defining a *coefficient space* from the denominator coefficients of the third-order transfer function and transforming the time response specifications into this space. He presents a procedure for solving the third-order all pole problem. However, limitations of the system structure are not investigated and hence the optimality of the design may be questioned. The procedure remains a method of cut and try and is very laborious, thus motivating the application of computer techniques to follow below. The coefficient space is convenient because once the time domain specifications are transformed into it they apply to all system structures resulting in third-order all pole system transfer functions.

1.3 Method of Approach

The second-order system is considered in terms of the familiar natural frequency and damping factor. These variables are quite tractable in the time and frequency domains and can be expressed in terms of plant parameters. The time domain specifications and plant parameter variations are then related through the above variables.

The approach used for the third-order system is through the coefficient space of Barber {4}. A study is made of the transformation of time response specifications into this space. Similarly, plant parameters are viewed in coefficient space. By studying the relations between parameter variations and time response specifications sufficient knowledge is obtained to develop systematic iterative procedures using gradient techniques to accomplish the design.

1.4 Time Response Definitions

Shown in Fig. 1.1 is a typical unit step response of a second or third-order system. Rise time, t_r, and overshoot, OV, are as defined by this figure and the following equations:

$$c(t_r) = 0.9$$

$$OV = c(t_1)-1 \; ; \; c(t_1) > 1$$

These definitions are adhered to throughout the paper.

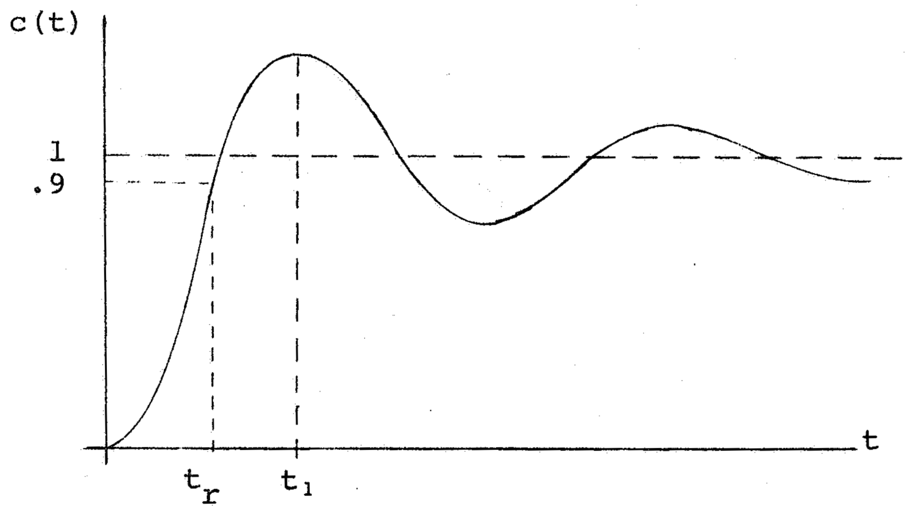


Fig. 1.1 Definition of time response specification.

CHAPTER II

SECOND-ORDER SYSTEM DESIGN

2.1 System Structure

The system structure under consideration is that of Fig. 2.1. From the figure the open-loop transmission, defined in the usual way {5}, is

$$L(s) = P(s)H(s) \tag{2.1}$$

and the resulting systems transfer function becomes

$$T(s) = \frac{C(s)}{R(s)} = \frac{L(s)}{1 + L(s)} = \frac{P(s)H(s)}{1 + P(s)H(s)} \quad . \tag{2.2}$$

The plant is represented by

$$P(s) = \frac{k}{s(s + p)} \tag{2.3}$$

where both k and p are assumed to vary between some known limits. Given a set of time domain specifications on the unit step response of this system, it may be possible to achieve the desired performance with the compensation H(s) consisting of a pure gain. If this is possible, a second-order system transfer function results and may be the most desirable design under the given conditions. Hence a method is sought to determine if a given set of time response specifications are achievable, and if so, a reasonably efficient numercial procedure for determining the minimum value of H(s) = K that will cause the time domain constraints to always be satisfied.

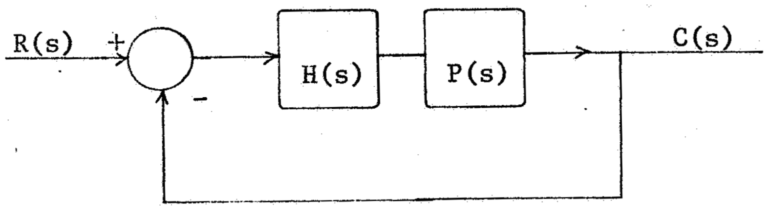


Fig. 2.1 Second-order system structure.

2.2 Relation of Plant Parameters and Time Response

With the compensation assumed to be a pure gain, i.e., $H(s) = K$, substitution into (2.2) gives the system transfer function

$$T(s) = \frac{Kk}{s^2 + ps + Kk} , \tag{2.4}$$

also

$$T(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} . \tag{2.5}$$

In the present notation the complex frequency variable is denoted by $s = \sigma + j\Omega$. The familiar damping factor ζ and natural frequency ω are related to the plant and compensation by equating coefficients in (2.4) and (2.5).

$$\omega^2 = Kk \tag{2.6a}$$

$$2\zeta\omega = p \tag{2.6b}$$

Multiplying (2.5) by 1/s, the Laplace transform of the unit step, and taking the inverse transform gives the time response as

$$c(t) = 1 - \frac{\exp(-\zeta\omega t)}{\sqrt{1-\zeta^2}} \left[\cos\left[\omega\sqrt{1-\zeta^2}t - \mathrm{Tan}^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}}\right]\right] \quad ; \zeta < 1 \tag{2.7a}$$

$$c(t) = 1 + \frac{1}{2}\left[\frac{\exp(-(\zeta-\sqrt{\zeta^2-1})\omega t)}{(\zeta^2-1) + \zeta\sqrt{\zeta^2-1}} + \frac{\exp(-(\zeta+\sqrt{\zeta^2-1})\omega t)}{(\zeta^2-1) - \zeta\sqrt{\zeta^2-1}}\right] \quad ; \zeta > 1 \quad (2.7b)$$

$$= 1 - \exp(-\omega t)(\omega t + 1) \quad ; \zeta = 1 \quad (2.7c)$$

Taking the first time derivative of c(t) for the case $\zeta < 1$ and equating to zero, the smallest positive value of t for which c'(t) $c'(t) = 0$ is obtained. Denoting this value by t_1 and substituting back into (2.7a) results in the second-order system overshoot.

$$OV = c(t_1) - 1 = \exp(-\pi\zeta/\sqrt{1-\zeta^2}) \quad (2.8)$$

Note that overshoot is independent of ω as should be expected since ω does not appear in the amplitude or phase angle of c(t). Several values of overshoot as a function of ζ are shown on Fig. 2.2.

To obtain rise time information for Fig. 2.2, ω is set to unity and rise time computed for several values of ζ. Since ωt always occurs as the product in c(t) the values of t_r obtained may be taken instead as values of ω for which $t_r = 1$ for the corresponding ζ. These values are plotted as the curve of one second rise time. Again due to the occurrence of the ωt product only in c(t), if ω is multiplied and t divided by the same constant $c(\omega t)$ is unchanged for constant ζ. The remaining curves of Fig. 2.2 are plotted in this manner and the figure may be adjusted to any range of ω and t_r desired by this scaling process.

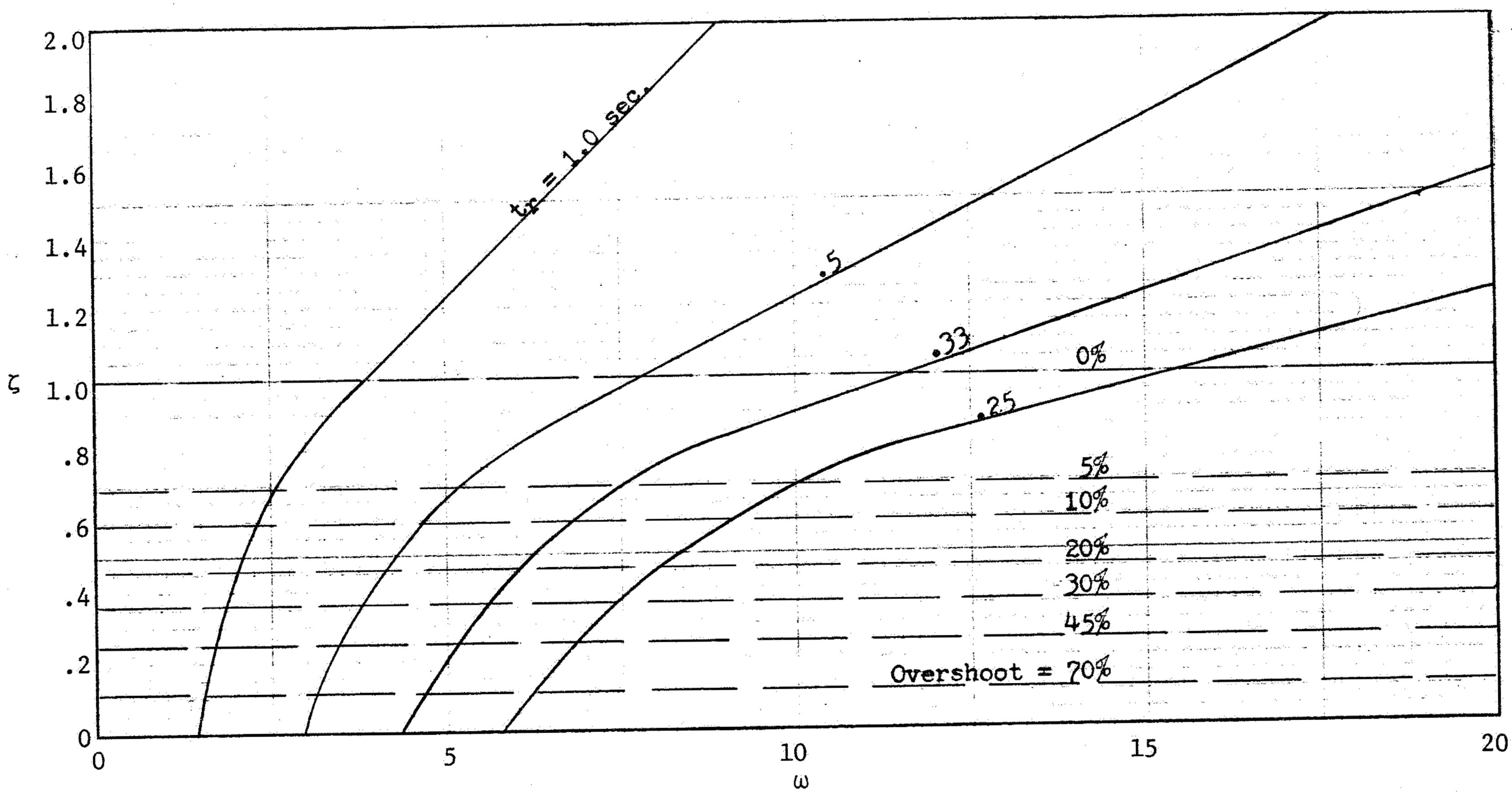


Fig. 2.2 Relation of unit step response rise time and overshoot to parameters of second-order transfer function, $T(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$.

The time response specifications are

$$OV \le OV_s = 10\% \tag{2.9c}$$

and

$$t_r \le t_{rs} = 1 \text{ sec.} \tag{2.9d}$$

The time domain specifications limit the system to the region R_s on the (ω,ζ) plane whose boundaries are the heavy lines of Fig.2.4.

Observe that overshoot is maximized when ζ is at its minimum, i.e., at point A' in Fig. 2.3. From Eqs. (2.6) we get

$$\zeta = \frac{p}{2\sqrt{Kk}} \tag{2.10}$$

which is minimized when $k = k_2$, and $p = p_1$.

Solving (2.8) for ζ,

$$\zeta = \left[\frac{\ln^2 OV}{\pi^2 + \ln^2 OV}\right]^{1/2} . \tag{2.11}$$

Combining (2.10) with (2.11) and using p_1, k_2, and OV_s, the maximum permissible value of K, say K_2, is obtained directly as

$$K_2 = \frac{p_1^2}{4k_2}\left[\frac{\pi^2 + \ln^2 OV_s}{\ln^2 OV_s}\right] . \tag{2.12}$$

The numerical values given result in $K_2 = 20.7$.

Rise time is maximized at the opposite extreme of plant parameters corresponding to point D' of Fig. 2.3. With K_2 now known, ω and ζ are computed at this point as $\omega = \sqrt{K_2 k_1}$ and $\zeta = p_2/2\sqrt{K_2 k_1}$. The numerical example gives $\omega = 4.55$ and $\zeta = 1.1$. Locating this point of Fig. 2.4 there are two possibilities; (1) the corresponding rise time is greater than the specified maximum and the design can not be achieved since K is

Eqs. (2.6) map a rectangle from the (p,k) plane into the region R_p on the (ω,ζ) plane as shown by Fig. 2.3.

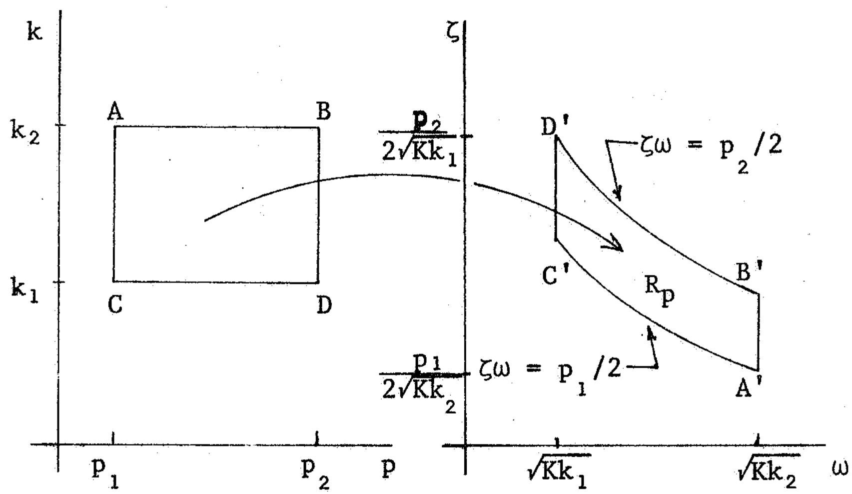


Fig. 2.3 Mapping from plant parameter space to frequency domain parameter space.

The desired relation between plant parameters and time domain specifications is established by mapping both onto the (ω,ζ) plane.

2.3 Design Procedure

Consider a design problem where the plant parameter variations are given as

$$p_1 = 8 < p < 10 = p_2 \tag{2.9a}$$

and

$$k_1 = 1 < k < 2.2 = k_2 \ . \tag{2.9b}$$

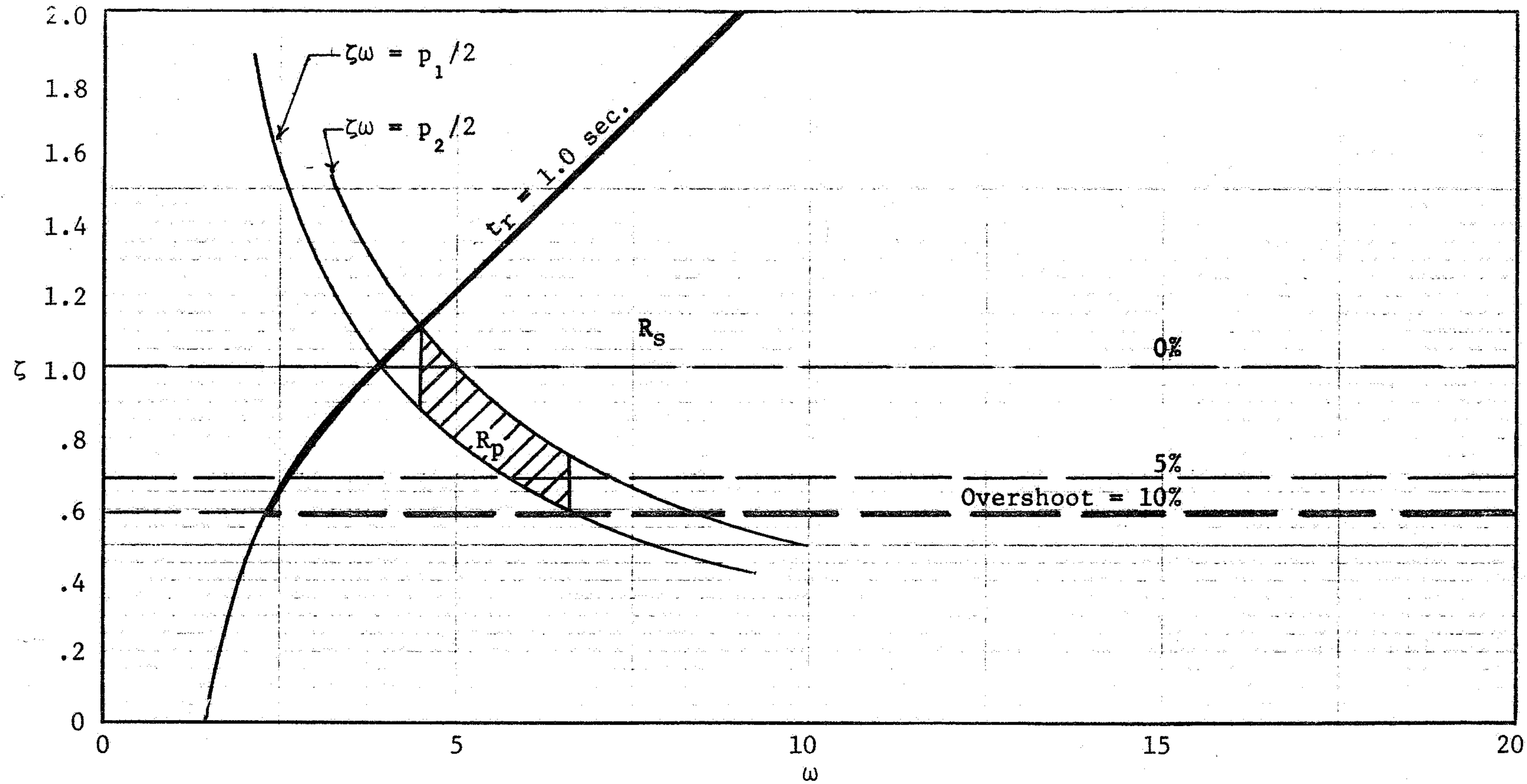


Fig. 2.4 Illustration of second-order design procedure for transfer function,

$$T(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}.$$

already maximum, or (2) the rise time is less than the specified maximum, in which case the design can be accomplished with a smaller gain. To obtain the minimum value of gain, call it K_1, we read from Fig. 2.4 the value of ω_1 at the intersection of the curve $\zeta\omega = p_2/2$ and the curve $t_r = 1$. Using this value and Eq. (2.6a), the minimum value of gain is obtained as $K_1 = \omega_1^2/k_1$. Carrying out the numerical example, $K_1 = (4.5)^2 = 20.2$. The new maximum ω is $\omega = \sqrt{K_1 k_2} = 6.66$.

The design is now complete with a transfer function having ω and ζ which lie in the shaded region R_p of Fig. 2.4. The maximum rise time is 1 second and maximum overshoot is 9.4%. Only one point can be fixed exactly as there is only one design parameter to adjust. It will be shown later that the overdesign on overshoot can be used to good advantage by reducing the open-loop gain-bandwidth in a third-order structure. Note that with the aid of Fig. 2.2 the method lends itself easily to hand calculation.

2.4 Numerical Techniques

To facilitate a computer solution of the above design problem we first require a numerical method of solving for rise time, i.e., a method of solving

$$g(\zeta,\omega,t_r) = c(\zeta,\omega,t_r) - .9 = 0 \tag{2.13}$$

for t_r when given ζ and ω. From Eqs. (2.7) it is seen that (2.13) is a transcendental equation in t_r, hence the need of a numerical method. There are numerous methods for solving equations of this type, most of which require one or more initial guesses of the solution and their convergence is dependent on the initial guesses being close enough in some sense. The method chosen here is known as the method of false position {6}. The only requirements for convergence of the method of false position are that the function be continuous and the initial guesses, t_1 and t_2, be such that $g(\zeta,\omega,t_1)g(\zeta,\omega,t_2) < 0$ with only one root in the interval (t_1,t_2). The initial guesses are obtained by evaluating the function at

$$t = n\delta t \quad ; \quad n = 0,\ 1,\ 2,\cdots,\ m. \tag{2.14}$$

The sequence (2.14) is terminated when the first sign change is observed in g. The initial guesses are $t_1 = m\delta t$, and $t_2 = (m-1)\delta t$. Considering the damped sinusoidal component of c(t), it is reasonable to choose δt proportional to $T = 2\pi/\omega\sqrt{1-\zeta^2}$, the period of the sinusoid. Since $t_r < T$, this will cause m to be relatively small and always less than $T/\delta t + 1$. However, δt must also be small enough to avoid having two roots in (t_1,t_2). For the case when $\zeta \geq 1$ a similar choice of δt is made based on the largest time constant of the function.

The method of solution for the minimum K to satisfy the rise time specification is derived as follows. Although its specific form is not known, rise time has some functional relation to gain K, which can be written,

$$t_r = f(K)\ . \tag{2.15}$$

Let K_0 (K_0 corresponds to K_2 of Sect. 2.3) be an initial guess for K and K_n be the nth computed value. Expand $t_r(K)$ in its Taylor series representation about K_n neglecting all terms except the constant and linear term to get

$$t_r \simeq f(K_n) + f'(K_n)(K - K_n) \ . \tag{2.16}$$

Putting in K_s, the solution we seek, and denoting $t_r(K_s)$ by t_{rs}, the specified rise time is approximated by

$$t_{rs} \simeq f(K_n) + f'(K_n)(K_s - K_n) \ . \tag{2.17}$$

Also noting that $t_{rn} = f(K_n)$, there are two unknowns in (2.17), specifically $f'(K_n)$ and K_s. To get the derivative first note the definition,

$$f'(K_n) = \lim_{\partial K \to 0} \frac{f(K_n + \partial K) - f(K_n)}{\partial K} \ . \tag{2.18}$$

Here we approximate the derivative, using small ∂K, as

$$f'(K_n) \simeq \frac{f(K_n + \partial K) - f(K_n)}{\partial K} = D_n \ , \tag{2.19}$$

also let

$$\Delta K_n = K_s - K_n \ . \tag{2.20}$$

Combining (2.19) and (2.20) in (2.17) gives the result,

$$\Delta K_n = \frac{t_{rs} - t_{rn}}{D_n} \ . \tag{2.21}$$

Now the sequence

$$K_{n+1} = K_n + \Delta K_n \quad ; \quad n = 0, 1, 2, \cdots\cdots \tag{2.22}$$

is computed. This result is just the Newton-Raphson iteration {7}, with a numerical approximation for the derivative. Study of Fig. 2.2 shows that f(K) is monotone decreasing and hence it can be shown that

$$\lim_{n\to\infty} K_n = K_s \ . \tag{2.23}$$

This iteration procedure is carried out until K_n is within the desired accuracy of K_s, or alternatively, until t_{rn} is as close as desired to t_{rs}.

The interpolation error present in Eq. (2.17) sometimes causes a negative K to be computed. The reason for this is evident from Fig. 2.5, which gives a graphical representation of (2.17). Since $\sqrt{K}$ is required in the computation of ω, this result causes computing difficulties and in addition is meaningless in the present problem. The situation is avoided by using $K'_{n+1} = K_n/2$, which decreases K as called for by the interpolation, and will either (1) make K too small but positive, whereas, the normal interpolation will again increase it on the next iteration, or (2) K will become close enough to the solution value K_s that the improved approximation will generate positive K and the iterative process continues normally.

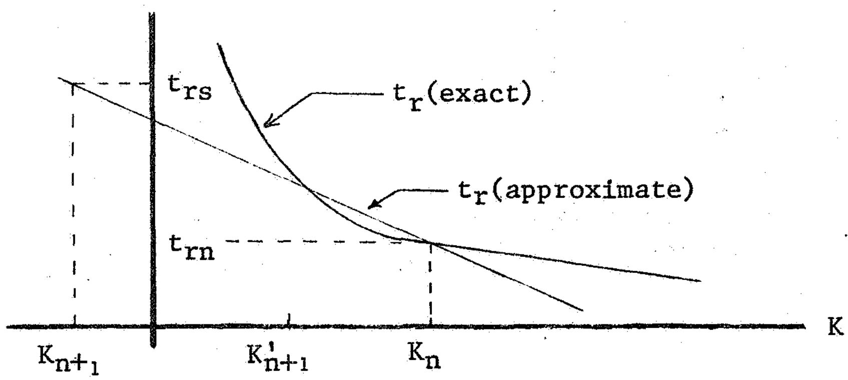


Fig. 2.5 Illustration of rise time interpolation error from Eq. (2.17)

CHAPTER III

TIME DOMAIN TO COEFFICIENT SPACE TRANSFORMATION

3.1 Motivation of Approach

For the third-order system a coefficient space due to Barber {4} is used. The coordinates of this three-space are the coefficients of the denominator polynomial of the third-order system transfer function. Conventional analysis and design gives us considerable feel for time response behavior in terms of frequency domain parameters such as natural frequency and damping factor. Time response specifications will be mapped into the coefficient space and the transformation between frequency domain parameters and coefficients investigated in some detail for two reasons: (1) to develop some feel for the time response in terms of coefficients, and (2) to facilitate computer programming of the design scheme that evolves. The primary advantage of the coefficient space is that its relationship to time response is invariant as the system structure is changed. When a system structure is given, the plant and compensation parameters are related to the coefficients, and only this relation changes if the system structure is changed. The coefficient space has the disadvantage of being limited to systems which have third-order all pole transfer functions.

3.2 Transfer Function and Time Response

The third-order system transfer function to considered is

$$T(s) = \frac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)} \quad . \tag{3.1}$$

Multiplying by 1/s and taking the inverse Laplace transform of this equation gives the following unit step time response.

$$c(t) = 1 - \frac{\exp(-\lambda\zeta\omega t)}{\lambda\zeta^2(\lambda-2) + 1} + \exp(-\zeta\omega t)\left[\frac{\lambda\zeta^2(2-\lambda)\cos(\omega\sqrt{1-\zeta^2}t)}{\lambda\zeta^2(\lambda-2) + 1} + \frac{\lambda\zeta(\zeta^2(2-\lambda)-1)\sin(\omega\sqrt{1-\zeta^2}t)}{\sqrt{1-\zeta^2}[\lambda\zeta^2(\lambda-2) + 1]}\right] \quad ; \begin{matrix}\zeta < 1\\ \lambda \neq 1\end{matrix} \tag{3.2a}$$

$$= 1 - \frac{\exp(-\lambda\omega t)}{(1-\lambda)^2} - \frac{\lambda(\lambda-2)\exp(-\omega t)}{(1-\lambda)^2} - \frac{\lambda\omega t\exp(-\omega t)}{(\lambda-1)} \quad ; \begin{matrix}\zeta = 1\\ \lambda \neq 1\end{matrix} \tag{3.2b}$$

$$= 1 - \left[\frac{\omega^2 t^2}{2} + \omega t + 1\right]\exp(-\omega t) \quad ; \begin{matrix}\zeta = 1\\ \lambda = 1\end{matrix} \tag{3.2c}$$

$$= 1 + \frac{\exp(-\lambda\zeta\omega t)}{\lambda\zeta^2(\lambda-2) - 1} + \frac{\exp(-(\zeta-\sqrt{\zeta^2-1})\omega t)}{2[\zeta(\lambda-2)(\zeta^2-1) + \sqrt{\zeta^2-1}(2\zeta^2-\lambda\zeta^2-1)]} + \frac{\exp(-(\zeta+\sqrt{\zeta^2-1})\omega t)}{2[\zeta(\lambda-2)(\zeta^2-1) - \sqrt{\zeta^2-1}(2\zeta^2-\lambda\zeta^2-1)]} \quad ; \begin{matrix}\zeta > 1\\ \lambda \neq 1\end{matrix} \tag{3.2d}$$

It can be verified that both T(s) and c(t) approach the expressions given in Chapter II for the second-order system as λ becomes infinite.

3.3 Frequency Domain to Coefficient Space Transform and Its Inverse

The coefficient space is defined by rewriting the transfer function as

$$T(s) = \frac{\gamma}{s^3 + \alpha s^2 + \beta s + \gamma} \; , \tag{3.3}$$

and equating to Eq. (3.1). Hence the coefficients (α,β,γ) are given by

$$\alpha = \omega\zeta(\lambda + 2) \tag{3.4a}$$

$$\beta = \omega^2(2\lambda\zeta^2 + 1) \tag{3.4b}$$

$$\gamma = \omega^3\lambda\zeta \,. \tag{3.4c}$$

It is well known that the inverse Laplace transform of a rational function is unique, so given a set of coefficients, and that the input is a unit step, c(t) is well defined and unique. Further, it is clear from (3.4) that for a specific set of frequency domain parameters the coefficients (α,β,γ) are unique. The inverse of this transformation, i.e., the transform from coefficients to frequency domain parameters, is defined implicitly by

$$\lambda = \lambda(\alpha,\beta,\gamma) \tag{3.5a}$$

$$\zeta = \zeta(\alpha,\beta,\gamma) \tag{3.5b}$$

$$\omega = \omega(\alpha,\beta,\gamma) \,. \tag{3.5c}$$

Since it will be necessary to make this transformation on the computer, an investigation of its uniqueness follows and a systematic implementation of the transform is sought. A theorem of the calculus {8}, restated here in the present context, is used to investigate uniqueness.

Theorem

Let (a) $\alpha = \alpha(\lambda,\zeta,\omega)$, $\beta = \beta(\lambda,\zeta,\omega)$, $\gamma = \gamma(\lambda,\zeta,\omega)$ describe a continuously differentiable transformation in a neighborhood S of a point $(\lambda_0,\zeta_0,\omega_0)$ where $\alpha_0 = \alpha_0(\lambda_0,\zeta_0,\omega_0)$ etc., and let

(b) $J\left(\frac{\alpha,\beta,\gamma}{\lambda,\zeta,\omega}\right) \neq 0$ at $(\lambda_0,\zeta_0,\omega_0)$.

Then there exists a neighborhood N of $(\alpha_0,\beta_0,\gamma_0)$ such that

(i) for every (α,β,γ) in N, unique values of (λ,ζ,ω) can be found such that $\alpha = \alpha(\lambda,\zeta,\omega)$, $\beta = \beta(\lambda,\zeta,\omega)$, and $\gamma = \gamma(\lambda,\zeta,\omega)$ and these values are given by a functional relation of the form $\lambda = F(\alpha,\beta,\gamma)$, $\zeta = G(\alpha,\beta,\gamma)$, $\omega = H(\alpha,\beta,\gamma)$.

(ii) the functions F, G, and H are continous and have continuous partial derivatives in S.

The differentiability required by (a) is easily verified from (3.4). Forming the Jacobian of part (b) we get

$$J = \begin{vmatrix} \frac{\partial\alpha}{\partial\lambda} & \frac{\partial\alpha}{\partial\zeta} & \frac{\partial\alpha}{\partial\omega} \\ \frac{\partial\beta}{\partial\lambda} & \frac{\partial\beta}{\partial\zeta} & \frac{\partial\beta}{\partial\omega} \\ \frac{\partial\gamma}{\partial\lambda} & \frac{\partial\gamma}{\partial\zeta} & \frac{\partial\gamma}{\partial\omega} \end{vmatrix} = \begin{vmatrix} \zeta\omega & \omega(\lambda+2) & \zeta(\lambda+2) \\ 2\omega^2\zeta^2 & 4\omega^2\lambda\zeta & 2\omega(2\lambda\zeta^2+1) \\ \omega^3\zeta & \omega^3\lambda & 3\omega^2\lambda\zeta \end{vmatrix}$$

$$= 4\zeta\omega^5(\lambda^2\zeta^2 - 2\lambda\zeta^2 + 1) \quad (3.6)$$

Requiring that ζ and ω be nonzero and equating (3.6) to zero gives the result

$$\lambda = 1 \pm \sqrt{1-1/\zeta^2} \quad . \tag{3.7}$$

Therefore, the transformation is unique except when (3.7) is satisfied for real λ. Thus it is seen that uniqueness holds for all $\zeta < 1$, i.e., when the transfer function has complex conjugate poles.

To perform the actual computation of (λ,ζ,ω), given (α,β,γ), we first eliminate λ and ζ from (3.4) obtaining

$$\omega^6 - \beta\omega^4 + \alpha\gamma\omega^2 - \gamma^2 = 0 \quad . \tag{3.8}$$

Assuming temporarily that this equation can be solved and the proper root ω chosen, the remaining parameters are obtained as follows: solve (3.4a) for ζ, substitute in (3.4c) and simplifying

$$\lambda = \frac{2\gamma}{\alpha\omega^2 - \gamma} \quad . \tag{3.9}$$

Then putting λ from (3.9) into (3.4a) and solving for ζ,

$$\zeta = \frac{\alpha\omega^2 - \gamma}{2\omega^3} \quad . \tag{3.10}$$

Eqs. (3.8) thru (3.10) will be used to implement the desired inverse transformation.

An investigation of the root loci of (3.8) is useful in determining which ω to select from this equation. Letting $x = \omega^2$, and putting into the standard form for root locus techniques, (3.8) is rewritten

$$1 - \frac{\beta x^2}{x^3 + \alpha\gamma x - \gamma^2} = 0 \quad . \tag{3.11}$$

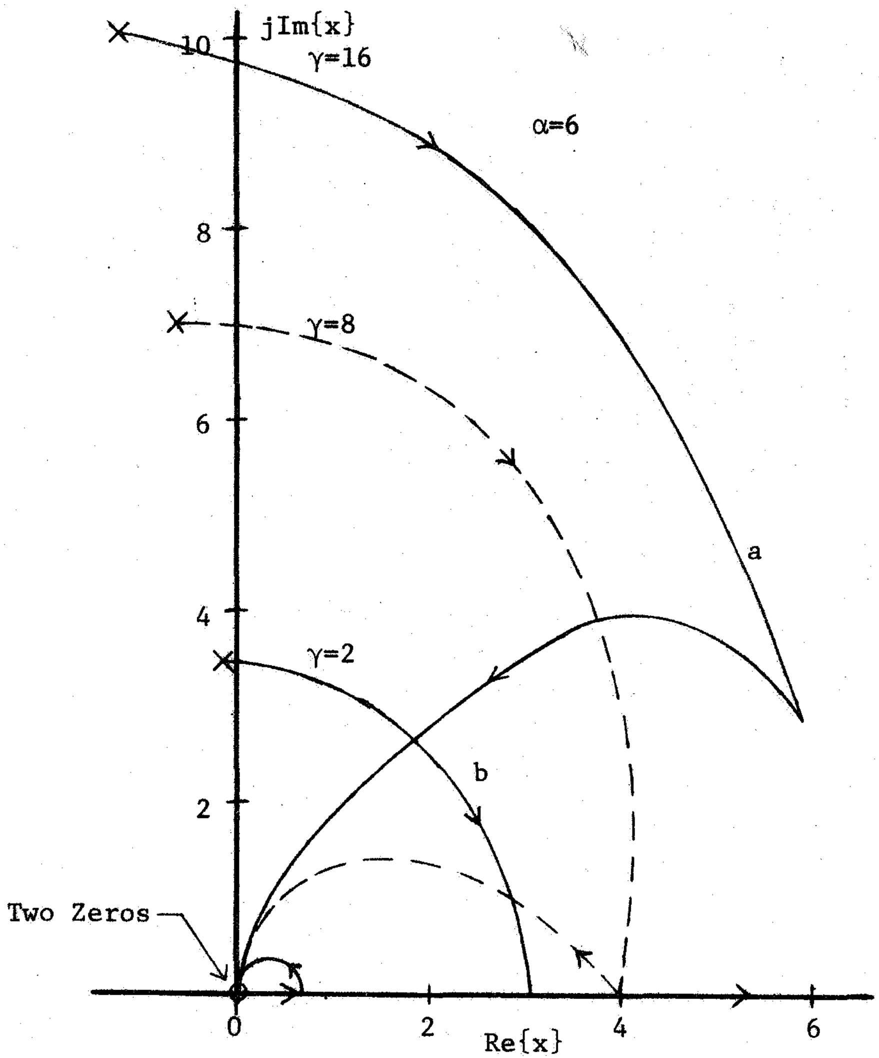


Fig. 3.1 Root loci of x from Eq. (3.11)

Applying the usual rules for constructing root loci {9}, Fig. 3.1 is sketched showing the positive half of the loci of x for several α and γ with β positive. From (3.11) it is easily verified that for $\beta \to 0$ and $\beta \to \infty$ there is one real and positive x and a complex conjugate pair. For sufficiently large γ, locus a, the previous statement is true for all $\beta > 0$. Hence for this case the real x is the only possible selection and its positive square root is the desired ω. For smaller γ, locus b, there are three real roots for some range of β. However, from the previous discussion of uniqueness, this must correspond to the case where $\zeta \geq 1$, and since the time response is known to be unique it is immaterial which x is selected. This correspondes to the case when all three transfer function poles are real and the selection of one x over another causes a corresponding change in λ and ζ thru Eqs. (3.9 & 10) such that the resulting transfer function poles are the same regardless of which x is used. Hence the desired method of accomplishing the transformation is to solve (3.8) for ω^2, taking any real solution, and substitute in (3.9 & 10) to get λ and ζ.

It is instructive to obtain curves generated in coefficient space by individually holding the frequency domain parameters constant. To obtain curves of constant ω we substitute λ and ζ of (3.9 & 10) respectively into (3.4b) and simplify to get

$$\beta = -\frac{\gamma^2}{\omega^4} + \frac{\alpha\gamma}{\omega^2} + \omega^2 \ . \tag{3.12}$$

Setting ω to several values generates a family of parabolas on the (γ,β) plane and straight lines on the (α,β) plane. These are shown in Fig. 3.3 and 3.4 respectively. Eliminating ζ and ω from (3.4) gives

$$\beta = \frac{(\lambda + 2)\gamma}{\alpha\lambda} + \frac{2\lambda\alpha^2}{(\lambda + 2)^2} \quad , \tag{3.13}$$

which permits sketching of the constant λ curves of Fig. 3.2 and 3.4. Eq. (3.13) has a limiting case of interest. As $\lambda \to \pm\infty$ the equation reduces to

$$\beta = \frac{\gamma}{\alpha} \quad . \tag{3.14}$$

This is represented by a straight line in Fig. 3.3. Recalling that this limit corresponds to the second-order system, we can associate this surface in coefficient space with the second-order system. Further, points on the (γ,β) plane below this line correspond to negative λ which results in an unstable transfer function. Thus with α and β constant, a sufficient increase in γ will make the transfer function unstable. The coefficient, γ, is directly proportional to open-loop gain so this is just the manifestation in coefficient space of the well known fact that a third-order system is unstable for sufficiently high gain. However, it is important that this be known and understood in the computational techniques to come later.

It is not convenient to simultaneously eliminate λ and ω from (3.4). Instead, solving (3.4a) for ω and substituting into (3.4b & c) we get, after some manipulation,

$$\lambda^3 + 6\lambda^2 + (12 - \alpha^3/\gamma\zeta^2)\lambda + 8 = 0 \tag{3.15a}$$

and

$$\beta = \frac{\alpha^2(2\lambda\zeta^2 + 1)}{\zeta^2(\lambda + 2)^2} \quad . \tag{3.15b}$$

A root locus investigation of (3.16a) similar to that used for the cubic in ω^2 enables determination of the desired λ. In root locus form (3.15a) is

$$1 - \frac{(\alpha^3/\gamma\zeta^2)\lambda}{(\lambda + 2)^3} = 0 \quad . \tag{3.16}$$

The loci are shown in Fig. 3.2 with α and ζ at some constant value and arrows indicating increasing γ.

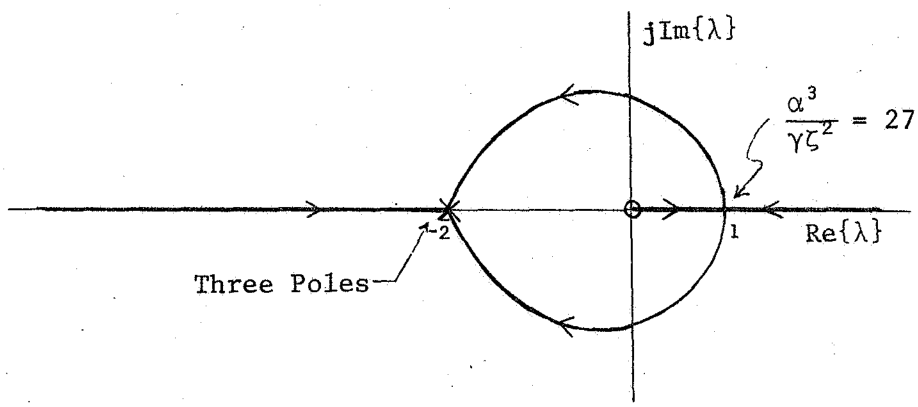


Fig. 3.2 Root loci of Eq. (3.16).

The curves of constants ζ of Fig. 3.3 and 3.4 are obtained by extracting values of λ from (3.15a) and putting these values in (3.15b) to obtain β. Note that curves for different values of constant λ cross in the region where $\zeta > 1$ showing the lack of uniqueness in this region. Similarly the constant ω curves cross in the same region.

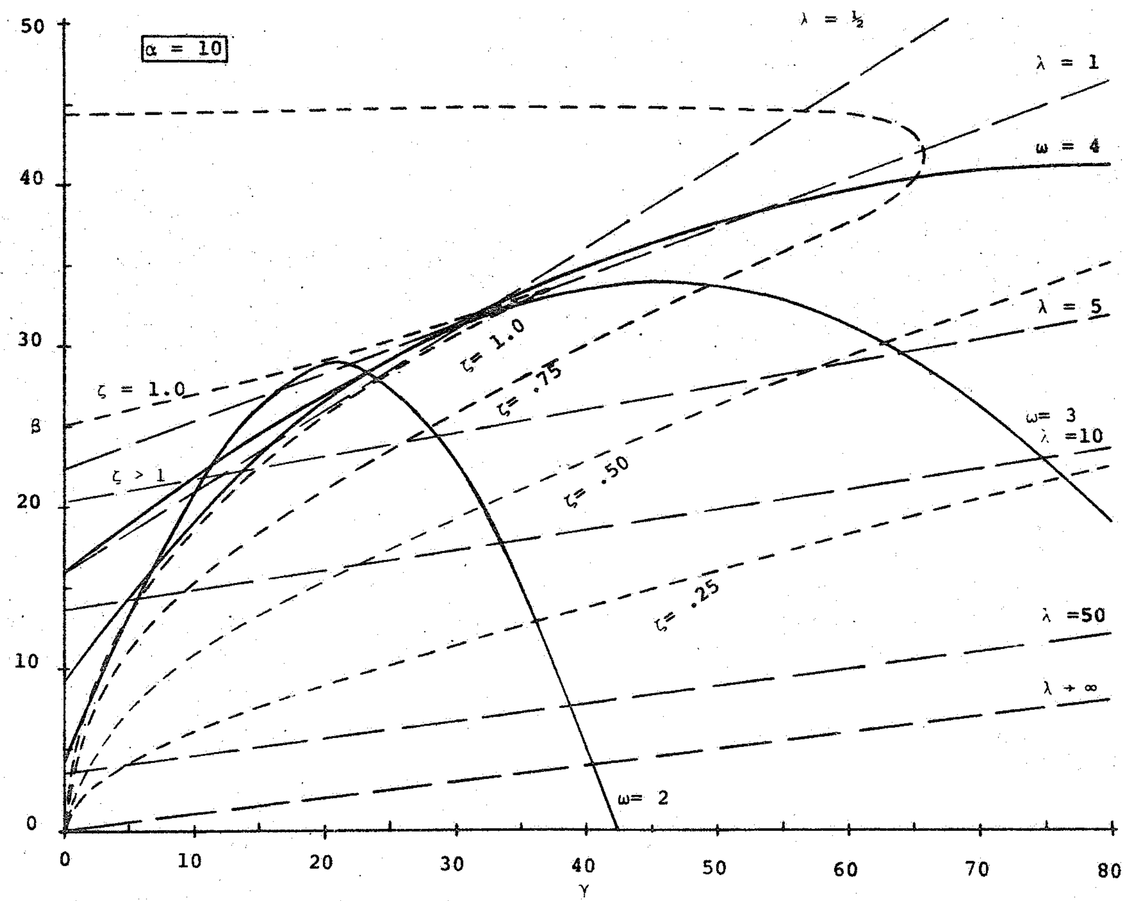


Fig. 3.3 Curves on (γ,β) plane generated in coefficient space by holding frequency domain parameters (λ,ζ,ω) constant.

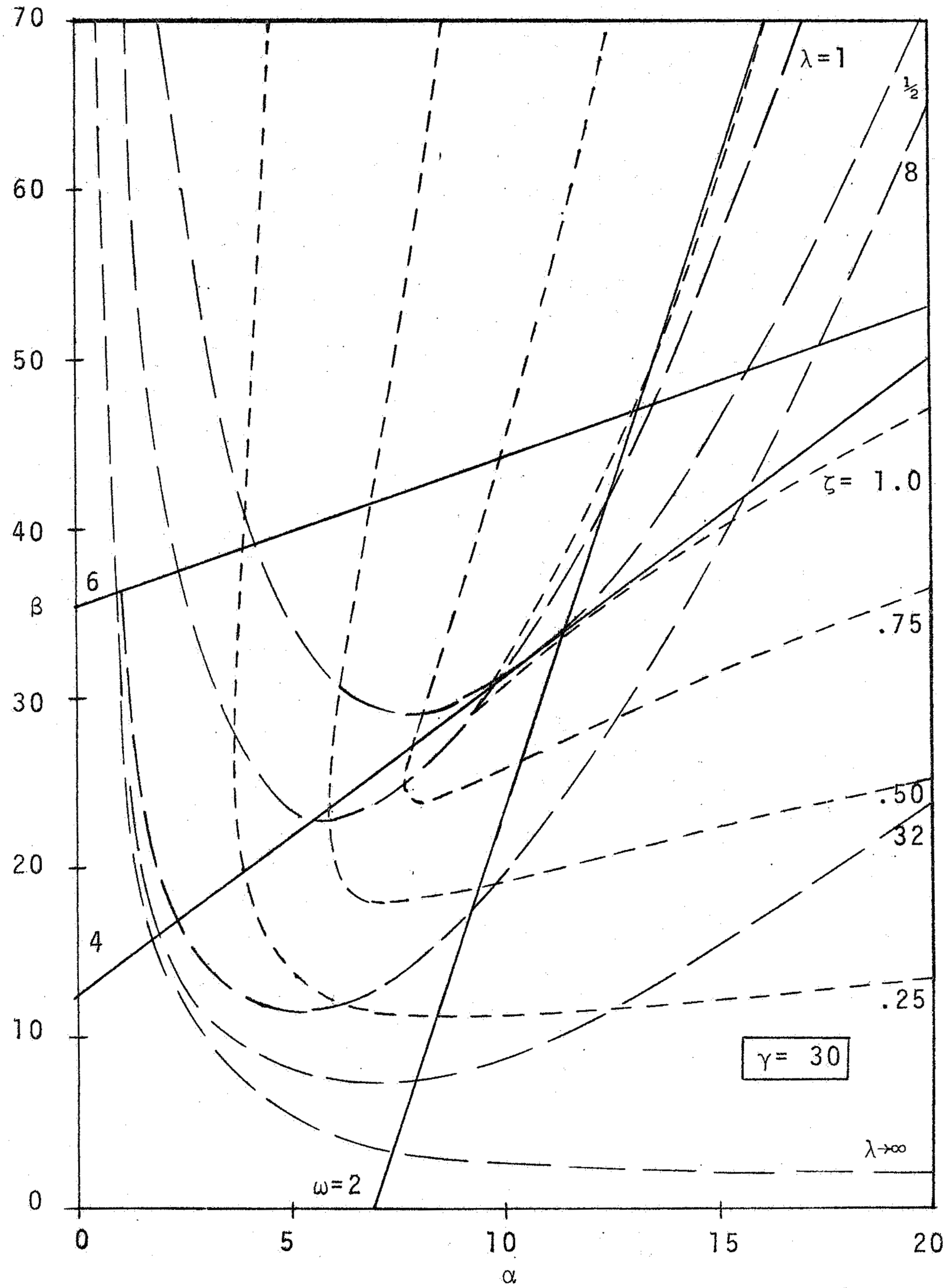


Fig. 3.4 Curves on (α,β) plane generated by holding frequency domain parameters (λ,ζ,ω) constant.

3.4 Time Response Specifications in Coefficient Space.

It is now necessary to obtain surfaces in coefficient space corresponding to given time response specifications. To simplify this task somewhat, two planes of the (α,β,γ) space, the (γ,β) and (α,β) planes, are considered seperately. The problem is then reduced to finding curves in the given planes. In the following chapter the curves will be used to determine the effect of plant parameter variations on time response.

Shown in Figs. 3.5 and 3.6 are rise time and overshoot respectively in the (ζ,λ) plane. Rise time is normalized with respect to ω and overshoot is independent of ω by the same argument used for the second-order transfer function in Chapter II. The following four Figs. 3.7 thru 3.10, show some curves at typical values of rise time and overshoot on the two coefficient planes selected above. Also shown on each figure are three other curves which are useful in evaluating time response in coefficient space. These curves are, (1) the curve representing infinite λ, or the boundary beyond which the transfer function is unstable, (2) the curve corresponding to $\zeta = 1$, indicating the region where the transfer function poles are all real and hence no oscillation is present in the time response, and (3) the curve corresponding to $\lambda\zeta = .56$, which will be discussed in Sect. 3.5.

The curves of all six Figs. 3.5 thru 3.10, are obtained, with the aid of a digital computer, by the following method.

Recall that the time response is completely described by three variables, either (α,β,γ) or (λ,ζ,ω). Let y be the variable on the vertical axis and fix the two corresponding variables at a constant value. Then designating either overshoot or rise time as $h(y)$ and the specified value as h_s, a constant, we get the following equation to be solved.

$$f(y) = h(y) - h_s = 0 \tag{3.17}$$

This equation can be solved by the same method discussed for solving (2.13). In the coefficient plane cases we utilize the transformation of Sect. 3.3 to obtain (λ,ζ,ω) necessary for evaluating the time response $c(t)$. To evaluate overshoot of the third-order response we use the method given in Sect. 2.4 for solving (2.13) to obtain the first zero of the time derivative, i.e., $c'(t_1) = 0$; then $OV = c(t_1) - 1$.

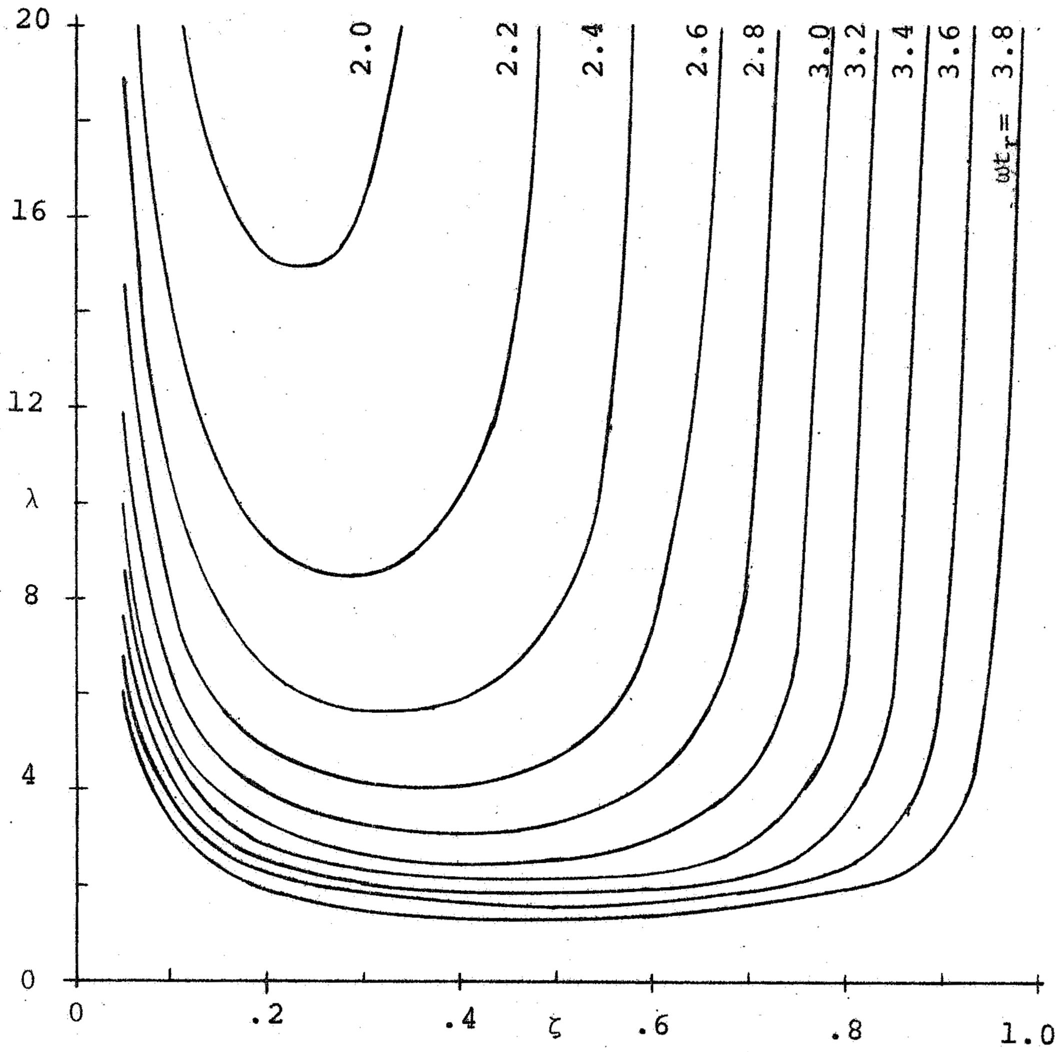


Fig. 3.5 Normalized rise time of step response for third-order transfer function, $T(s) = \dfrac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)}$.

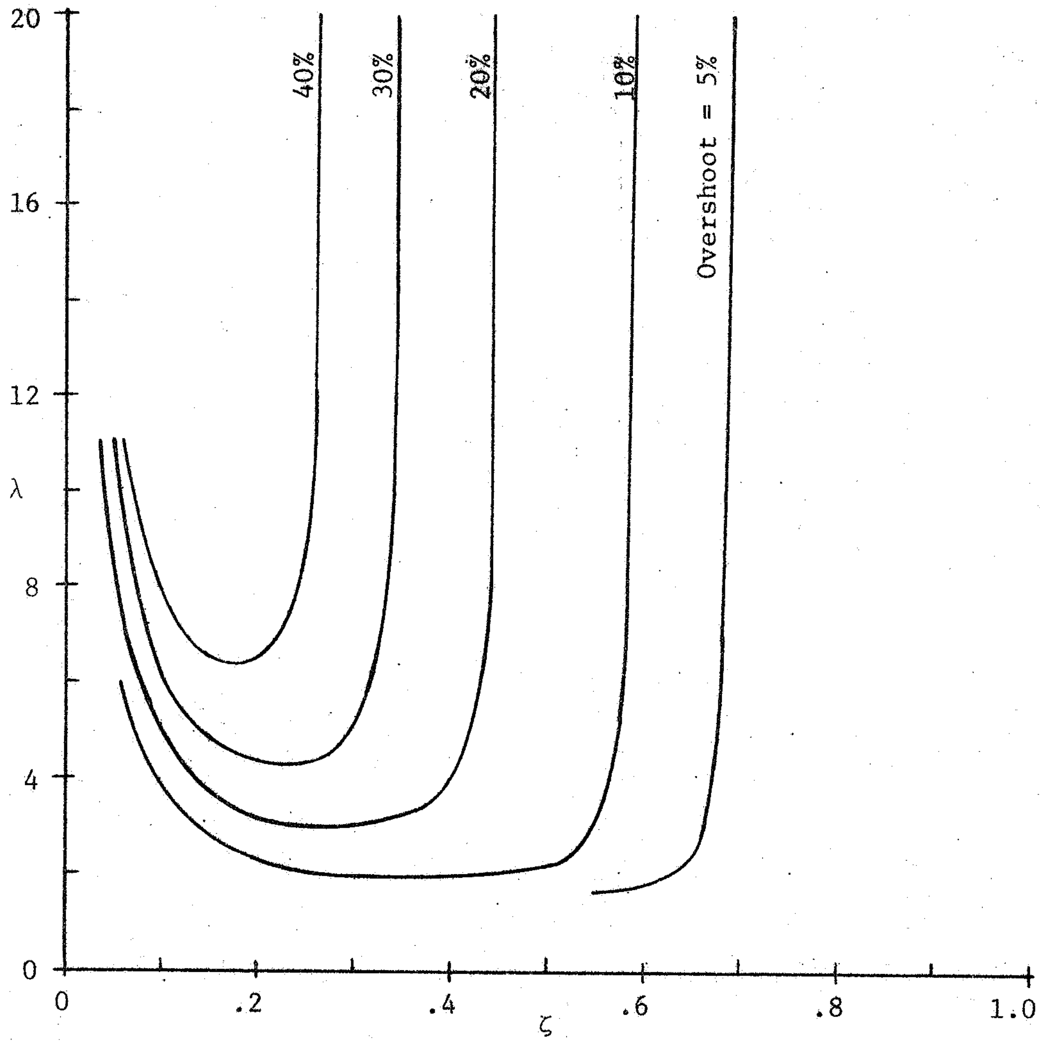


Fig. 3.6 Overshoot of step response for third-order transfer function, $T(s) = \dfrac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)}$.

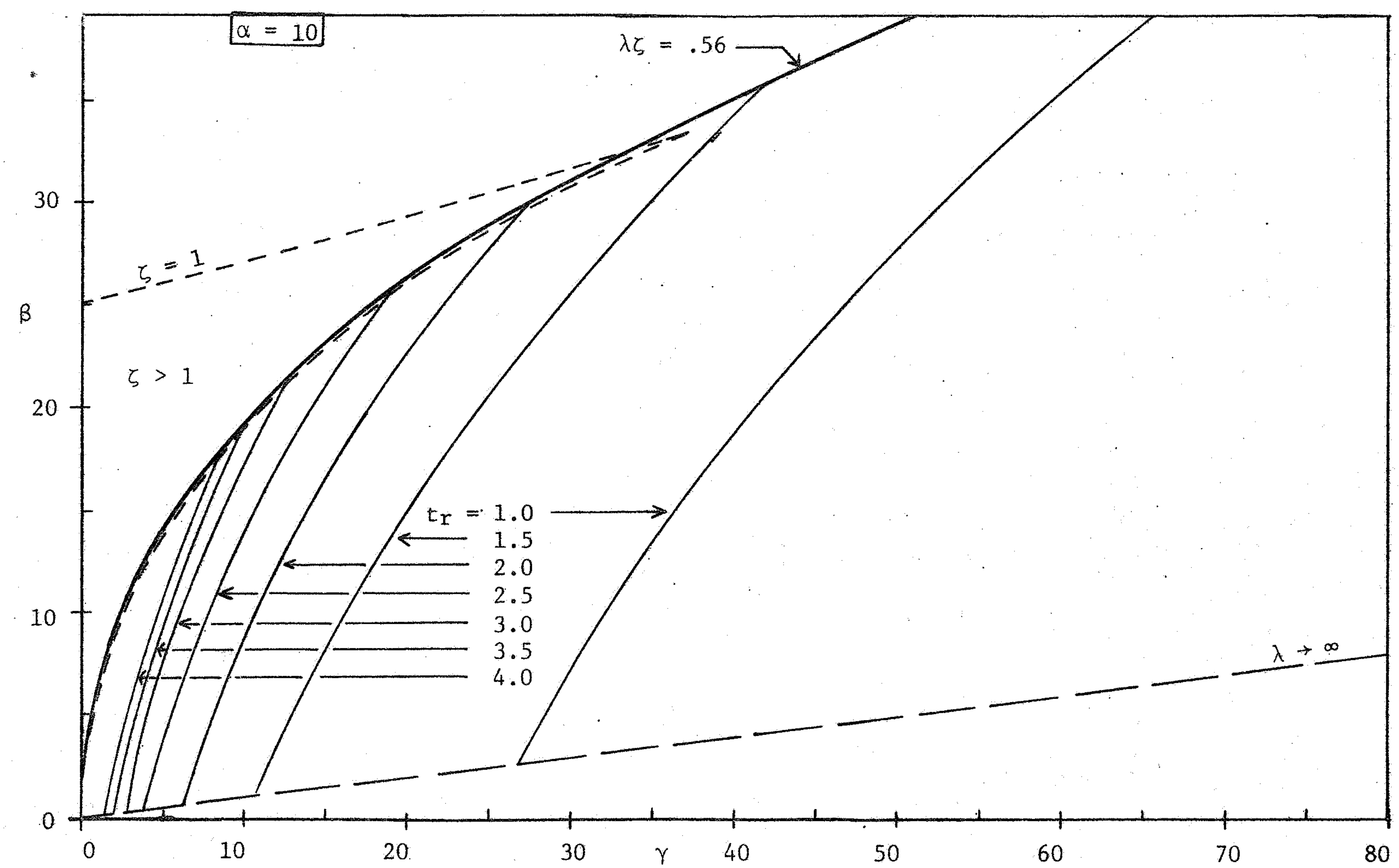


Fig. 3.7 Curves of constant step response rise time for third-order transfer function,

$$T(s) = \frac{\gamma}{s^3 + \alpha s^2 + \beta s + \gamma} = \frac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)} .$$

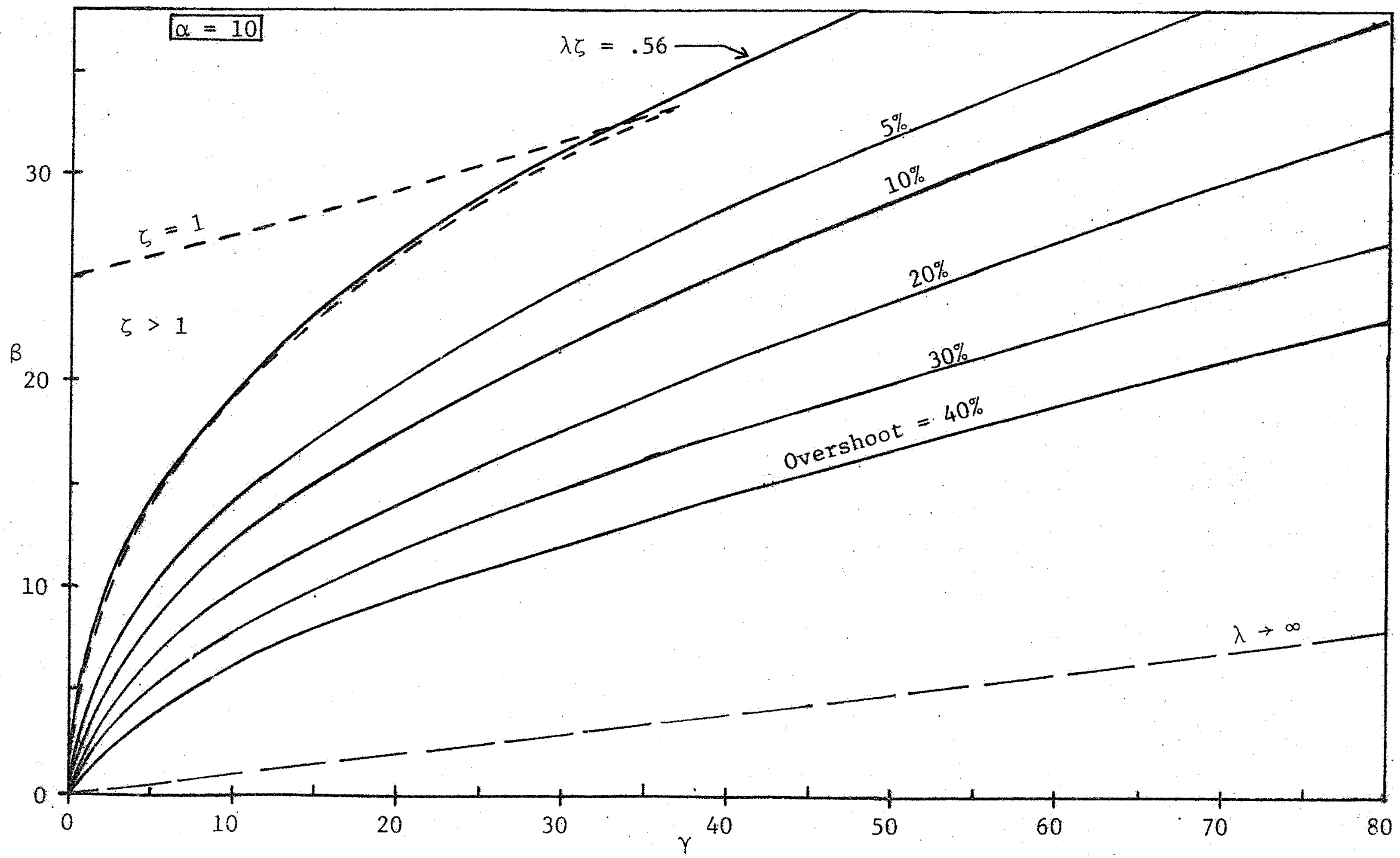


Fig. 3.8 Curves of constant step response overshoot for third-order transfer function,

$$T(s) = \frac{\gamma}{s^3 + \alpha s^2 + \beta s + \gamma} = \frac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)} .$$

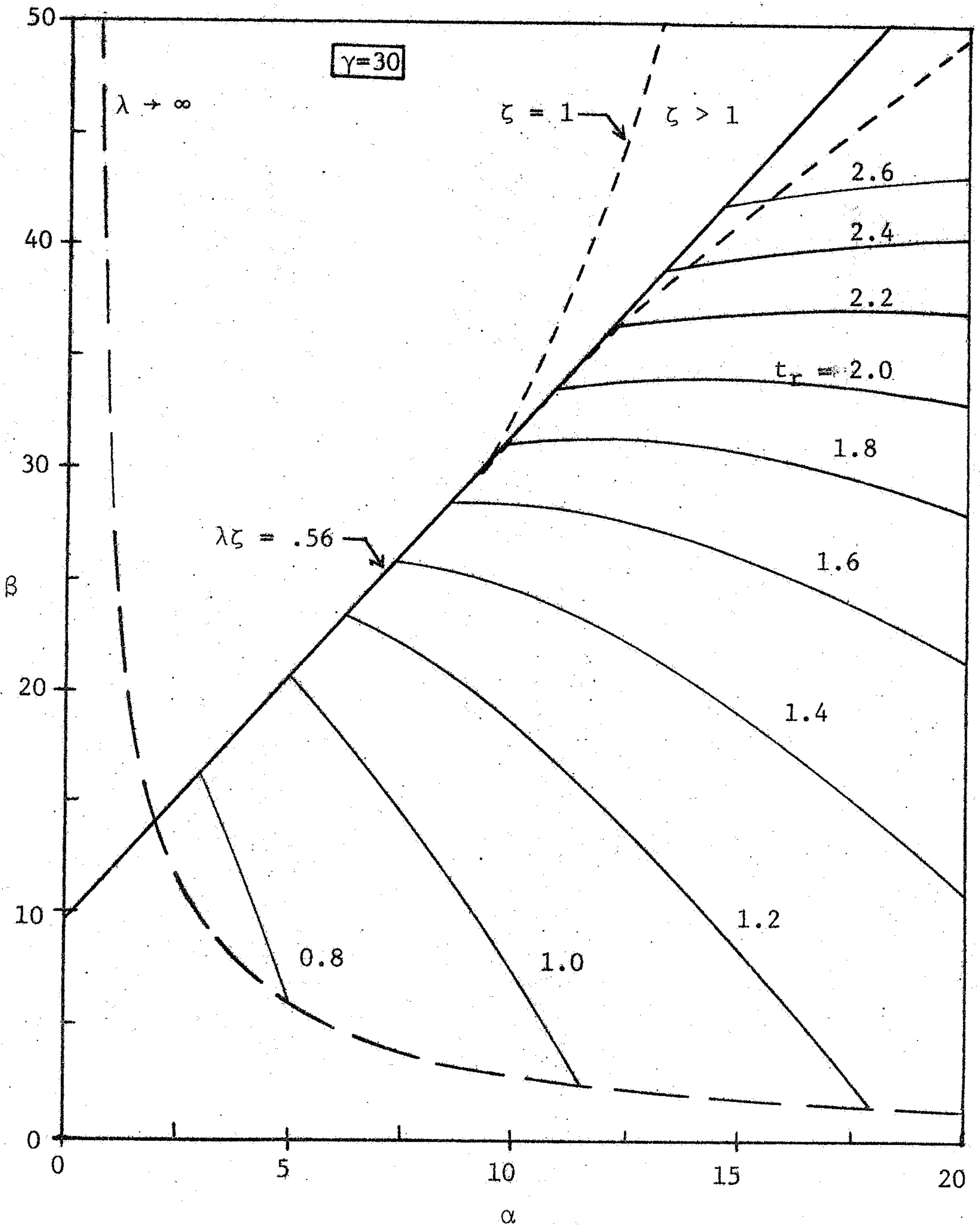


Fig. 3.9 Curves of constant step response rise time for third-order transfer function,

$$T(s) = \frac{\gamma}{s^3 + \alpha s^2 + \beta s + \gamma} = \frac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)} .$$

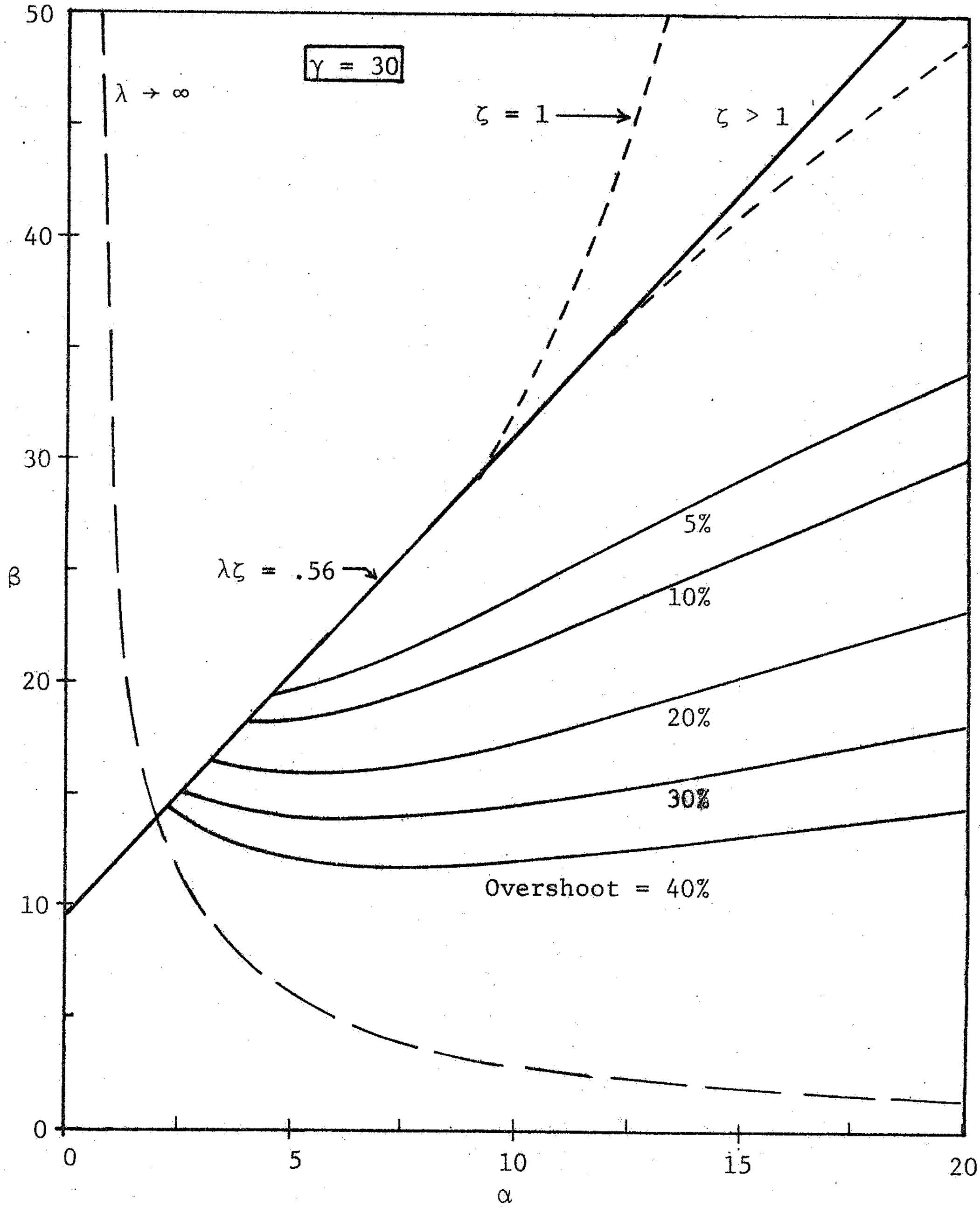


Fig. 3.10 Curves of constant step response overshoot for third-order transfer function,

$$T(s) = \frac{\gamma}{s^3 + \alpha s^2 + \beta s + \gamma} = \frac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)}.$$

3.5 Undesirable Region of Coefficient Space

In the last section a boundary was shown in coefficient space described by setting the $\lambda\zeta$ product equal to a constant. The purpose and method of determining this boundry is now considered. Observe in Fig. 3.9 that the lines of constant rise time appear as though they would meet and cross if extended. Calculation of some additional points indicates that they spirial in toward some focus. Calculation of more points on the rise time curves of Fig. 3.7 reveals that abrupt discontinuities can sometimes occur. The conclusion drawn is that rise time and overshoot as defined in Chapter I are not well-behaved functions in the entire region of coefficient space of interest thus far, i.e., the region where (α,β,γ) are all positive. Hence we want to investigate what this means in terms of time response and attempt to find some criterion for excluding some of the coefficient space from consideration.

Consider Fig. 3.11 which shows the step response for several values of λ with ζ fixed at $\zeta = 0.2$. Observe that the response corresponding to $\lambda = 1.2$ starts to decrease due to its oscillatory component before it reaches the final value of unity the first time. This conflicts with the definition of overshoot given in Chapter I. There it was specified that t_1 be the first nonzero value of time where the derivative of the response went to zero and that $c(t_1) > 1$. The response under consideration in Fig. 3.11 results in a negative overshoot which was no significant meaning in the present problem. Also, a response that oscillates before

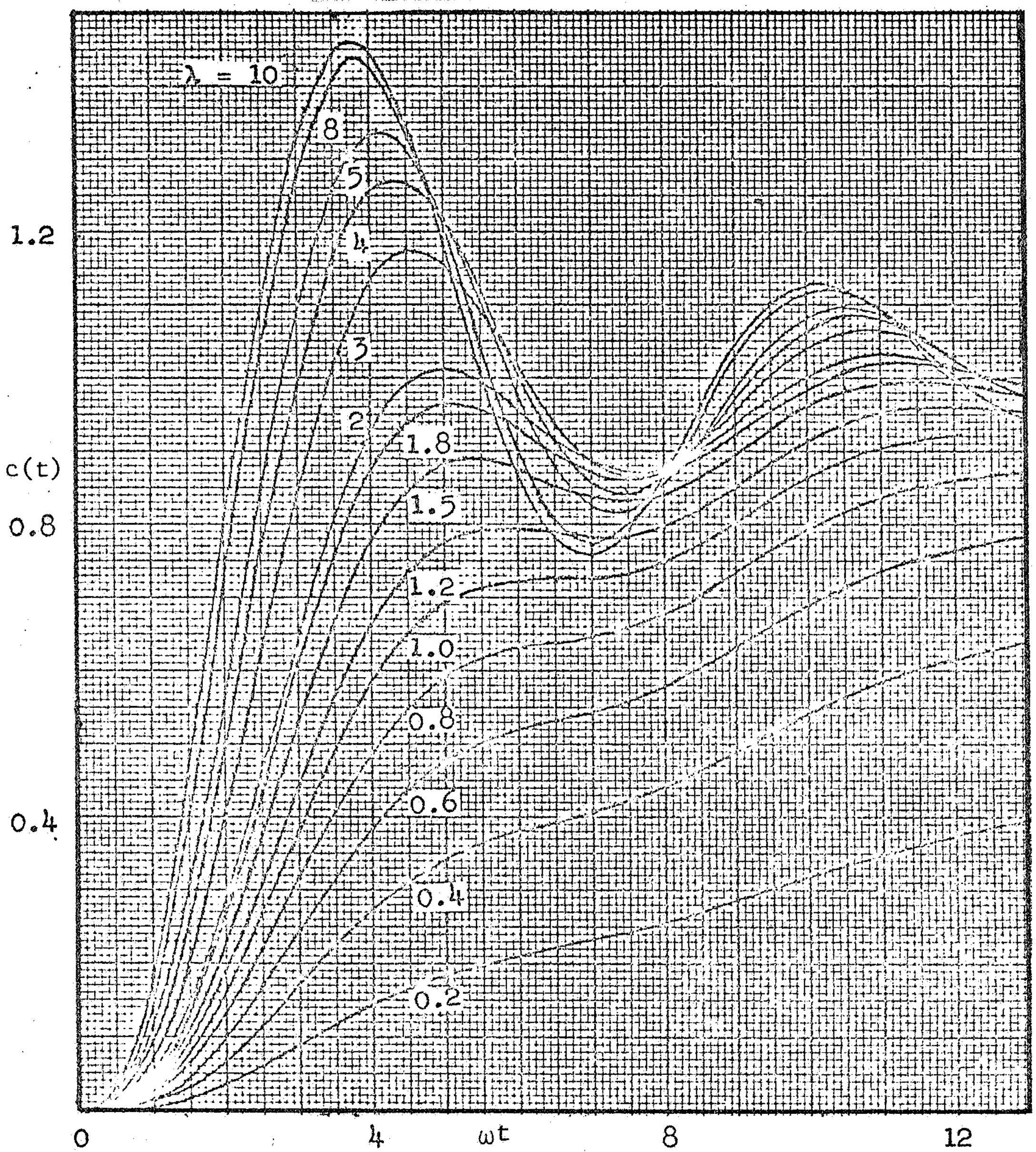


Fig. 3.11 Unit step responses for third-order transfer function,

$$T(s) = \frac{\lambda\zeta\omega^3}{(s + \lambda\zeta\omega)(s^2 + 2\zeta\omega s + \omega^2)} ; \quad \zeta = 0.2.$$

reaching the final value the first time is not very desirable from the practical point of view. Now consider the response for $\lambda = 1.5$. This reaches the value $c(\omega t_0) = .9$, at approximately $\omega t_0 = .9$.However, if λ is increased slightly the hump at $\omega t = 5$ will cross the line of unit magnitude and there results an abrupt change in t_0. Hence the discontinuities of rise time in coefficient space are explained.

To exclude the undesirable possibilities described above the requirement is imposed that the response reach unity magnitude before a zero of the time derivative occurs. From Fig. 3.11 it is noted that with $\zeta = 0.2$ the requirement is met for $\lambda > 2.0$. Additional values are obtained by plotting response curves for other values of ζ. The (λ,ζ) pairs obtained in this way are plotted on the dashed curve of Fig. 3.12.

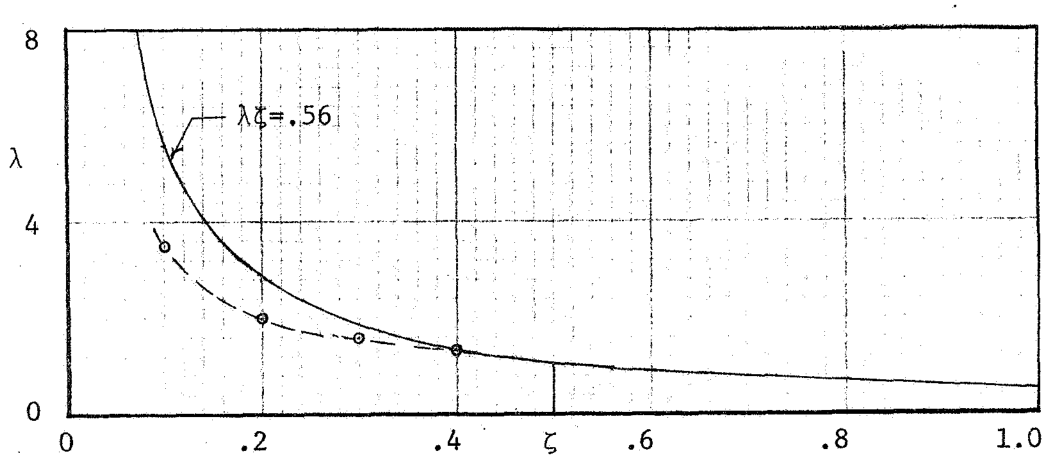


Fig. 3.12 Data for undesirable response boundary.

Since it is highly desirable that this boundary be represented by some known function, the hyperbola $\lambda\zeta = .56$ is selected as a reasonable fit. The hyperbola is also shown in Fig. 3.12. From the investigation of time response it is found that for $\zeta > .5$

and certainly for $\zeta > 1$, the imposed requirement is met for any λ. However due to the desirability of having a simple functional boundary, $\lambda\zeta = .56$ is selected even though it excludes some satisfactory time responses. Transforming the boundary into coefficient space by substituting $\lambda\zeta = b$ in (3.4), the three equations are reduced to one by eliminating ω and the result is

$$\beta = b^{23}\alpha\gamma^{13} + \gamma^{23}(1/b^{23} - b^{43}), \qquad (3.18)$$

or with b = .56,

$$\beta = .6794\alpha\gamma^{13} + 1.010\gamma^{23}. \qquad (3.19)$$

Eq. (3.19) describes a surface in coefficient space whose images in the (β,γ) and (β,α) planes are shown in the figures of the last section as the curve marked $\lambda\zeta = .56$.

3.6 Summary of Results

Sections 3.1 and 3.2 have established the time response and transfer function to be considered in the third-order system study.

In Sections 3.3 and 3.4 the coefficient space is defined and a method of transforming in either direction between coefficients and frequency domain parameters, which describe the time response, is developed. Section 3.5 imposes a restriction on the time response, eliminating some undesirable responses and their deterimental effect on the behavior of the time response specifications in coefficient space.

Combining the results of Sections 3.4 and 3.5, a semi-infinite open region, R, is defined such that

(a) α, β, γ are all positive

and

(b) $\gamma/\alpha < \beta \leq .6794\alpha\gamma^{1/3} + 1.010\gamma^{2/3}$

Further, R is divided into two sub-regions, R_1 and R_2, such that in R_1, $\zeta < 1$; and in R_2, $\zeta \geq 1$. In R_2 overshoot is taken as identically zero by definition. As a result we have that the time response specifications, rise time and overshoot, are well-defined, continuous, and differentiable in R.

CHAPTER IV

THIRD-ORDER SYSTEM DESIGN

4.1 Region of Acceptable Time Response in Coefficient Space

In Chapter III the nature of surfaces corresponding to constant rise time and overshoot were investigated. A region R was defined in coefficient space having time response specifications as well-behaved functions of the coefficients. Selection of a particular set of time response inequality constraints yields a sub-region R_s, contained in R, which is specified by

$$\beta \leq .6794\alpha\gamma^{1/3} + 1.010\gamma^{2/3} \tag{4.1a}$$

$$t_r(\alpha,\beta,\gamma) \leq t_{rs} \tag{4.1b}$$

$$OV(\alpha,\beta,\gamma) \leq OV_s \quad . \tag{4.1c}$$

Such a region is shown in Fig. 4.1 for $t_{rs} = 1$ sec. and $OV_s = 10\%$. For other specifications the region is distorted in shape and/or scaled to a different range of the coefficients. The figure is two dimensional but the image of R_s is shown for several values of γ so that, thinking of γ as an axis positive into the page, a three dimensional region can be visualized. Observe that R_s is convex on the surfaces given by Eqs. (4.1a & b). Returning to Fig. 3.8 it is seen that, although convexity is approached for large γ, the surface given by (4.1c) is not convex. The design problem now becomes that of finding a transfer function whose coefficients remain inside R_s and just graze the inner boundaries at the plant extremes. It will be seen that this results in minimum gain and bandwidth for the open-loop transmission.

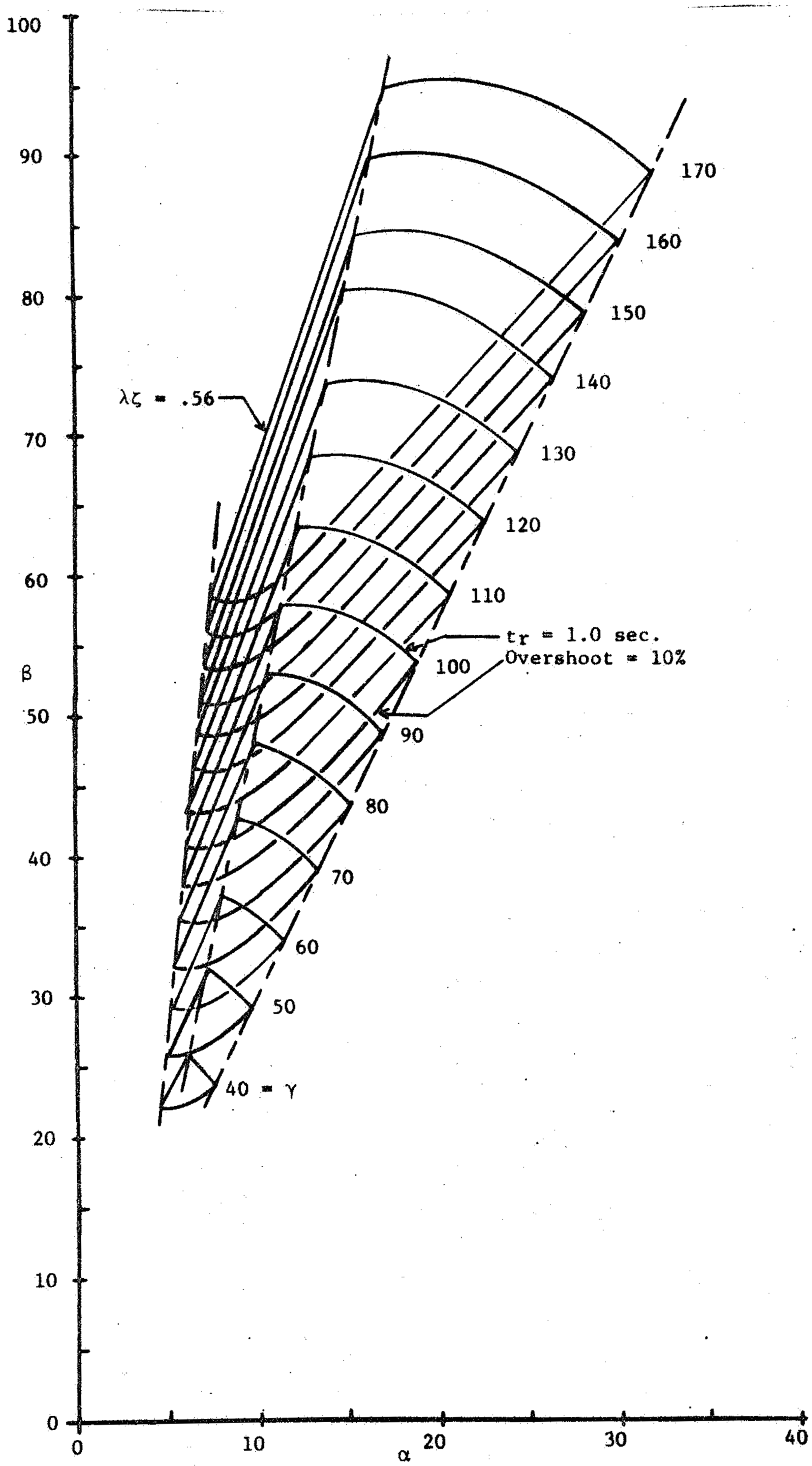


Fig. 4.1 Region of acceptable response in coefficient space for $t_r \leq 1$ sec. and OV $\leq$ 10% for transfer function, $T(s) = \dfrac{\gamma}{s^3 + \alpha s^2 + \beta s + \gamma}$.

4.2 System Structure

The general system structure for which a design scheme is developed in this chapter is shown in Fig. 4.2. This is the same as the second-order structure of Chapter II, however, the plant is now taken to be

$$P(s) = \frac{k}{(s+p_1)(s+p_2)} \tag{4.2}$$

and the compensation is

$$H(s) = \frac{K}{s+a} \quad . \tag{4.3}$$

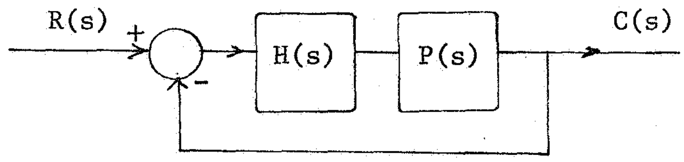


Fig. 4.2 Third-order system structure.

The plant may have a fixed zero which is cancelled by a pole of the compensation before the following design begins and thus does not enter into the calculations. With the assumed forms for plant and compensation the closed-loop transfer function is

$$T(s) = \frac{C(s)}{R(s)}$$

$$= \frac{kK}{s^3 + [a+p_1+p_2]s^2 + [a(p_1+p_2)+p_1p_2]s + [ap_1p_2 + kK]} . \tag{4.4}$$

The transfer function (4.4) has a d-c transmission, obtained by letting $s \to 0$, of $T(0) = kK/(ap_1p_2+kK)$. Usually in a control system it is desired to have $T(0) = 1$ in order that as $t \to \infty$ the output will approach the input. In the given transfer function this can be accomplished in two ways, (1) multiply the transfer by $1/T(0)$, or (2) require that the plant have a pole at the origin of the s-plane, i.e., $p_1 = 0$. The first method requires a prefilter in front of the system having a pure gain of $M(s) = (ap_1p_2 + kK)/kK$. However, if plant parameters are to vary, $M(s)$ must vary accordingly. This infers that the plant parameters can be measured on a continuous basis and thus takes the design problem out of the class being studied here. The reason for considering the more general plant of Eq. (4.2) is that a useful by-product of the design scheme is that it works for plants without parameter variations. When plant parameters do vary it will be assumed that $p_1 = 0$. The resulting coefficients are

$$\alpha = a + p_1 + p_2 \tag{4.5a}$$

$$\beta = a(p_1 + p_2) + p_1p_2 \tag{4.5b}$$

$$\gamma = ap_1p_2 + kK \quad . \tag{4.5c}$$

For any fixed compensation a and K, as the plant parameters are permitted to vary through all possible values, a set of points is generated in coefficient space by Eqs. (4.5). This set is designated as Rp.

4.3 Minimum Point

First consider the design of a system with fixed plant parameters. Taking (4.1b & c) with the equality sign only and using (4.5) gives a system of five equations in five unknowns, the coefficients and the compensation parameters. If this set of

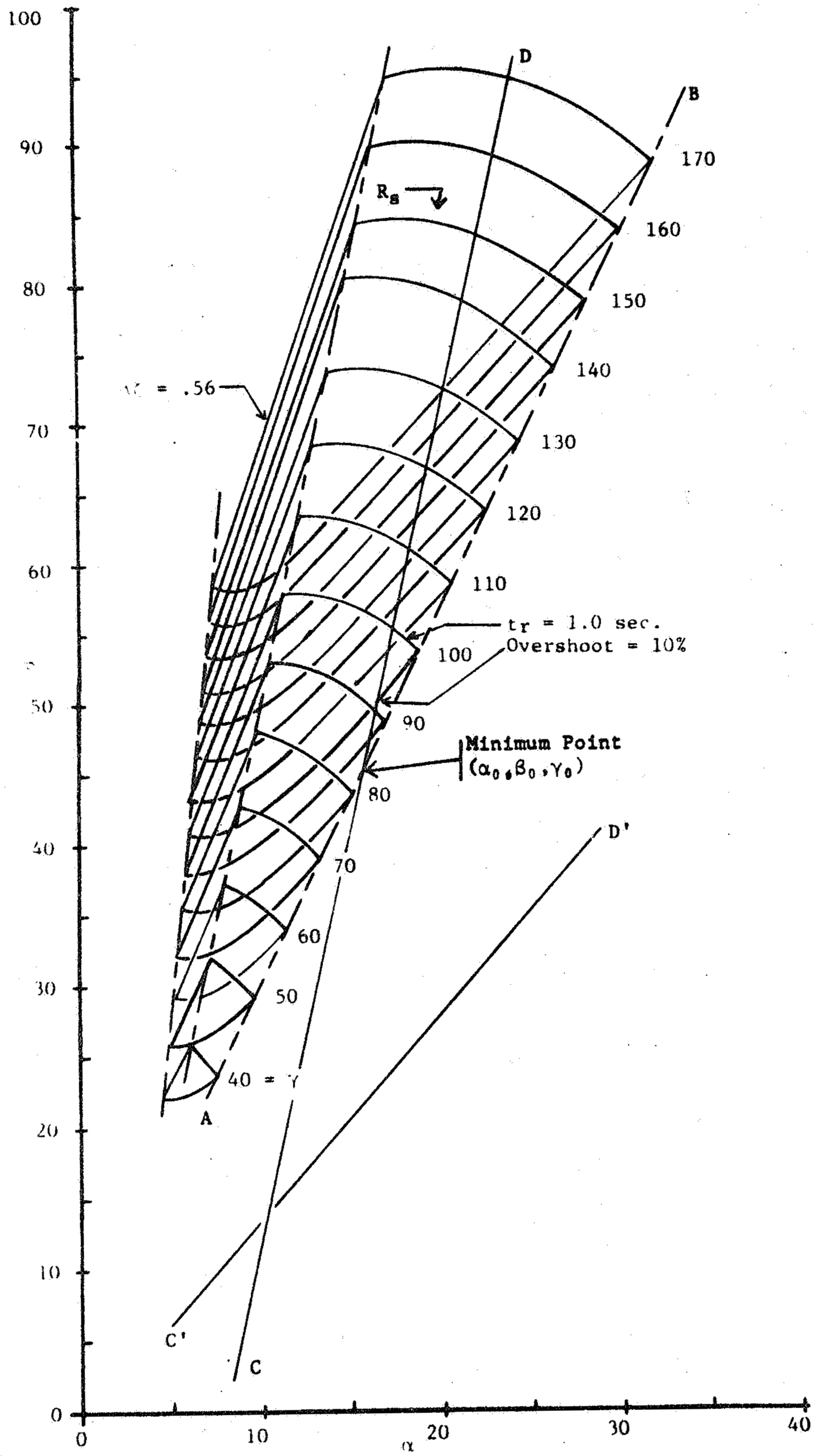


Fig. 4.3 Illustration of fixed plant design.

equations has a solution in R_s then that solution is a candidate for the design. To observe the behavior of the coefficients as the compensation pole a is varied, we eliminate it from (4.5) to get

$$\beta = (p_1 + p_2)\alpha - p_1^2 - p_2^2 - p_1p_2 \tag{4.6a}$$

$$\beta = \frac{(p_1 + p_2)\gamma}{p_1p_2} - \frac{kK(p_1 + p_2)}{p_1p_2} + p_1p_2 \quad . \tag{4.6b}$$

Eq. (4.6a) shows that the plant constrains the system coefficients to a straight line, CD in Fig. 4.3, on the (α,β) plane. Curve AB of Fig. 4.3 shows the intersection of the surface of specified rise time with the surface of specified overshoot. This means that all points in the coefficient space having both rise time and overshoot equal to their specified value lie on curve AB, and hence the fixed plant solution must also lie on AB if such a solution exists. Noting that (4.5a & b) are independent of gain K, values of the coefficients along CD are determined entirely by the compensation pole a. It is seen immediately that the solution sought is (α_0,β_0) where CD and the image of AB cross. At this point K is adjusted to achieve γ_0 and the design is complete for the fixed plant case. It is immediate from Fig. 4.3 that if a solution exists, i.e., there is a real and positive a and K such that Eqs. (4.1b & c) are satisfied as equalities subject to the constraints of Eqs. (4.5), then the solution is both unique and optimal in the gain-bandwidth sense. Also, two cases are readily observed where such a solution does exist. The first is a consequence of the slope of CD, given as $(p_1 + p_2)$ in Eq. (4.6a). If the sum of the plant poles is sufficiently small the result is C'D' of Fig. 4.3 which never crosses AB. A second case corresponds to the underspecified design problem. Taking $a = 0$, α and β have minimum values of $(p_1 + p_2)$ and $p_1 p_2$ respectively. Plant pole values are possible such that this point lies above AB and hence requires a negative a to decrease α and β to the values at the crossing of CD with AB. This problem can not occur if either plant pole is zero because then minimum β is zero. Since a is a system pole, it is required that $a > 0$ for realizability.

Note that the above discussion is equally valid when $p_1 = 0$. The point $(\alpha_0, \beta_0, \gamma_0)$ is called the *minimum point* because it is the first point obtained in the varying parameter case with all plant parameters set to their minimum value.

4.4 Numerical Solution for Minimum Point

A discussion of the numerical solution for the minimum point is given here and the same general technique is used with slight modification for the remaining parts of the third-order problem in this chapter.

The coefficient space is indeed only a useful vehicle to relate plant and compensation parameters to system time response. Once a compensation is found the coefficient values are of only passing interest. With this in mind the minimum point problem can be restated by two equations, instead of the five given above, which are

$$t_r(a,K) = t_{rs} \tag{4.7a}$$

$$OV(a,K) = OV_s \quad . \tag{4.7b}$$

A solution of these equations is equivalent to a solution to the five equations discussed above. With a and K known, everything in (4.5) is known; therefore, the properties of uniqueness and optimality of the solution hold. It follows from the continuity and differentiability of Eq. (4.1b & c) and (4.5) in R, that Eqs. (4.7) are also continuous and differentiable in R. Hence, the solution of two simulataneous equations in two unknowns is desired. It is possible to solve these equations by the Raphson-Newton iteration extended to two equations. However, difficulty is encountered in obtaining initial guesses that cause the iteration to converge.

The method used with good results is the following: first, the specifications are written as functions of a single parameter given by

$$t_r(K) = t_{rs} \tag{4.8a}$$

$$OV(a) = OV_s \quad . \tag{4.8b}$$

The gradient technique derived in Sect. 2.4 for solving Eq. (2.15) is then applied seperately to (4.8a & b). The sequence of steps is

(a) Obtain initial guesses a_0 and K_0.

(b) Approximate derivative of t_r with respect to K and iterate one step ΔK.

(c) Approximate derivative of OV with respect to a and iterate one step Δa.

(d) Repeat b thru d and terminate when t_r and OV are sufficiently close to their specified values.

Initial guesses may be any a_0 and K_0 which give coefficients in R. They are typically taken of order p_2 and $10p_2$ respectively, and a computer routine checks to insure that the coefficients are in R. If the routine finds the initial guesses in violation of the stability boundary of R, K_0 is halved until the violation is removed. Similarly if the initial guesses violate the undesirable response boundary a_0 is halved.

Recall that in R_2, defined in Sect. 3.6, OV is zero by definition, so that if step (c) is attempted in R_2 there is no derivative information available to estimate the increment Δa. When this occurs the present value of a is simply decreased by 10%. This forces the coefficients toward R_1 where overshoot is nonzero and the process depends on this coupled with the successive K iterates to return it to R_1 where it must be to arrive at a specific overshoot solution. A check is also made on the successive iterates to verify that the process is converging to a solution i.e., that $\Delta K_{n+1} < \Delta K_n$ and $\Delta a_{n+1} < \Delta a_n$. The divergent case corresponding to line C'D' of Fig. 4.3 is quickly detected by this check. The discussion of negative K found in the final paragraph of Sect. 2.4 applies to the third-order case as well, and the same corrective action is employed.

It should be understood that the four steps listed as an iteration technique for finding the minimum point rely heavily for their implementation on the background knowledge provided in Chapter III. For example, to evaluate rise time for a given a and K the calculations required are (1) determine the coefficients (α,β,γ), (2) transform coefficients into s-plane parameters (λ,ζ,ω), and (3) calculate rise time of c(t) by the method of Sect. 2.4. A detailed flow chart of the computation procedure is given in Appendix A.

4.5 Plant Gain Variation

The systematic design of a system having a plant gain variation such that

$$k_1 \leq k \leq k_2 \tag{4.9}$$

where k_1 and k_2 are known, is developed in this section. It is assumed that $p_1 = 0$ in Eqs. (4.5) for the reasons stated in that section. The coefficients and system parameters are then related by

$$\alpha = a + p_2 \tag{4.10a}$$

$$\beta = ap_2 \tag{4.10b}$$

$$\gamma = kK \quad . \tag{4.10c}$$

It is also assumed that the minimum point defined in the previous section has been located with $k = k_1$. The gain variation problem is viewed best on the (γ,β) plane shown by Fig. 4.4. On this figure the minimum point is point a and curves of constant rise time and overshoot are shown for several α. Also, the curve AB in Fig. 4.4 is the same as curve AB of Fig. 4.3.

By Eqs. (4.8) it is noted that a gain variation changes the coefficient γ only. This means that when a design is complete α and β are constant with respect to plant parameter variations and thus R_p is a straight line.

Consider the coefficients corresponding to the minimum point a in Fig. 4.4. If the plant gain is increased to k_2, γ increases to some larger value shown at point b. Recall that as the compensation pole is varied the coefficients (α,β) are still confined to CD of Fig. 4.3. We now increase a and K simultaneously in such a manner that the point corresponding to $k = k_1$ remains on the surface of specified rise time. When the point corresponding to $k = k_2$ passes through the surface of R_s given by the overshoot specification the design is complete. The final R_p is shown as the line between points c and d.

Similar to the fixed plant case, there are two possibilities that a solution does not exist in R_s. First, the gain variation may be so large as to cause the coefficients to move up line CD in Fig. 4.3 to the point where the undesirable response boundary marked $\lambda\zeta = .56$ is encountered. If this boundary is crossed

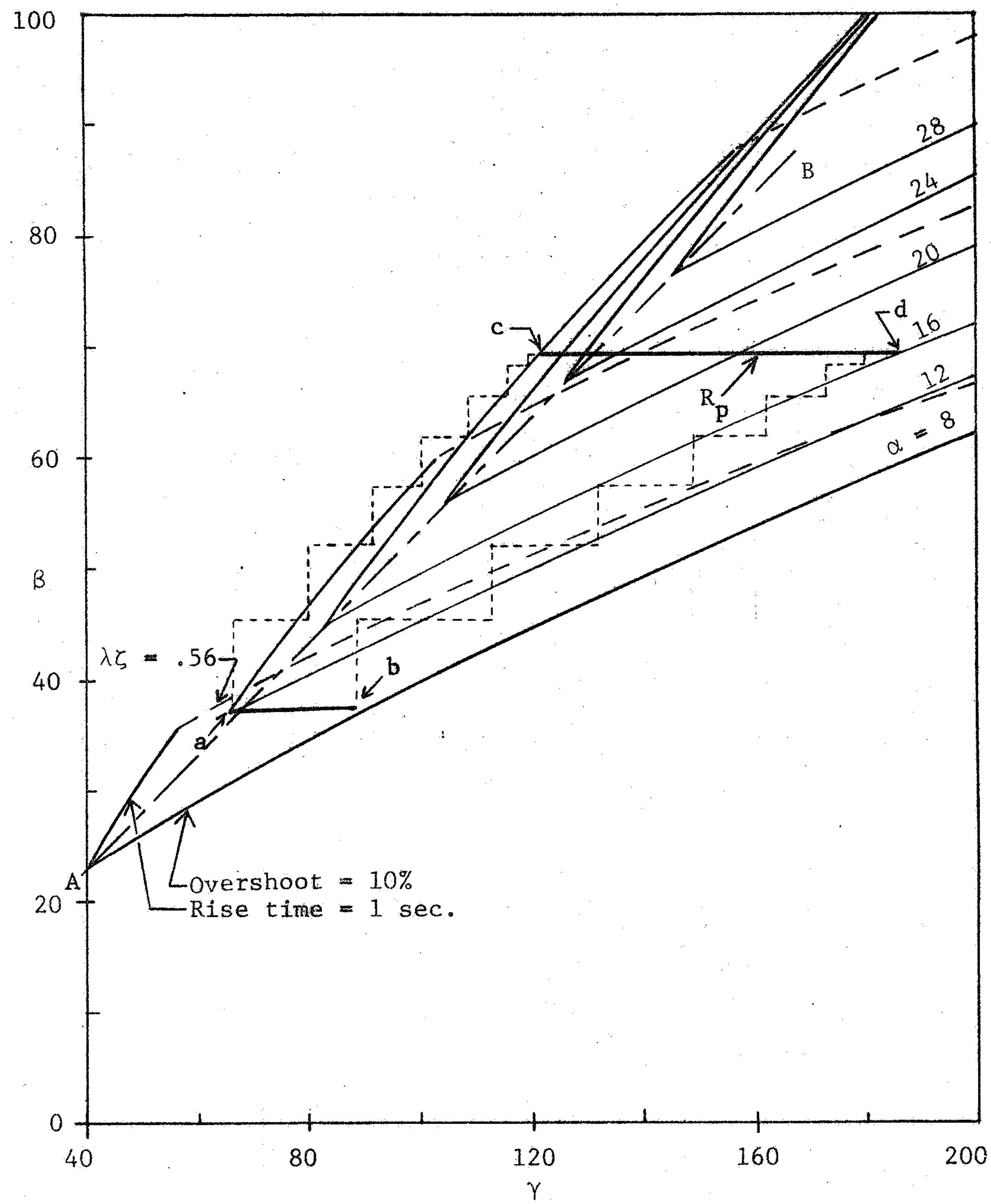


Fig. 4.4 **Illustration of design procedure for plant with gain variation.**

the computation must be terminated due to the poor behavior of the specification functions. The boundary could be crossed and the computation terminated when the iteration procedure is quite close to the final solution. For this reason all current information is retrieved at the termination to permit the designer to evaluate the situation. Possibly a slight relaxation of the specifications will allow a successful design.

The second cause of failure to find a solution is due to the relative expansion rates of R_s and R_p as γ gets large. The γ variation due to plant gain is

$$\Delta\gamma = K(k_2 - k_1) \quad . \tag{4.11}$$

Being proportional to K, the γ variation increases as K is increased in moving up the constant rise time surface. The width of R_s in the γ coordinate also increases, however, if $\Delta\gamma$ increases faster than R_s, then R_p can not be forced to fit in R_s and hence no solution is obtained. This result is detected by divergence of the iterates ΔK and Δa as discussed in Sec. 4.4.

The numerical solution of the variable gain problem is obtained by rewriting Eqs. (4.1b & c) as

$$t_r(\alpha,\beta,\gamma) = t_{rs} \tag{4.12a}$$

$$OV(\alpha,\beta,\gamma+\Delta\gamma) = OV_s \tag{4.12b}$$

where γ is always computed as Kk_1 and $\Delta\gamma$ is given by (4.11). Using (4.12) the technique is identical with that discussed in Sect. 4.4. A typical iteration path is shown by the small dashed line in

Fig. 4.4.

One new problem can arise in the course of the variable gain computation. If k is sufficiently large, point b of Fig. 4.4 may be outside of R in violation of the stability boundary. When this occures overshoot is no longer defined. To prevent this occurence the coefficients at b are tested and if found to be outside of R a temporary value of k_2, say k_2', is used. The problem is solved for k_2', k_2' is increased, and the problem solved again. The sequence is repeated until a solution is found with $k_2' = k_2$. A satisfactory value for k_2' is

$$k_2' = \lambda a p_2 (a+p_2)/(\lambda+2)K - 2\lambda^2 (a+p_2)^3/(\lambda+2)^3 K \quad , \tag{4.13}$$

where λ is taken as a large number. Recalling that the stability boundary is determined by infinite λ, substituting λ and the coefficients in terms of parameters from (4.10) into (3.13) gives the k_2' of (4.13).

4.6 Plant Gain and Pole Variation

A design scheme is now presented for the plant of Eq. (4.4) with a combined gain and pole variation. The parameter variations are stated as

$$k_1 \le k \le k_2 \tag{4.14a}$$

$$p_{21} \le p_2 \le p_{22} \quad . \tag{4.14b}$$

When the plant has a pole variation but does not have a gain variation the problem is handled as a simplified limiting case of the more general situation considered here.

The solution begins by setting p_2 and k to their minimum value and solving for the minimum point as in Sect. 4.3. Note again that $p_1 = 0$. The gain variation is then considered separately with p_2 held at p_{21} and appropriate values of a and K found by the procedure of Sect. 4.5. The line between c and d of Fig. 4.5 shows the region of plant variation in coefficient space R_p at this stage of the design. To observe the shape of R_p for the additional parameter variation p_2 is eliminated from Eqs. (4.5a & b) and (4.5b & c) are rewritten so that

$$\beta = a\alpha - a \tag{4.15a}$$

$$\beta = ap_2 \tag{4.15b}$$

$$\gamma = kK \tag{4.15c}$$

Eqs. (4.15) along with the limits of (4.14) show that R_p is a plane in coefficient space having one edge parallel to the γ axis and lying at slope a on the (α,β) plane. From Fig. 4.5 it is seen that the rise time boundary at R_s is violated immediately as p_2, and hence β, is increased from point c. A little study of how the curves lie on this figure indicates that the rise time violation is aggravated by the simultaneous increase of α with β. The point c' is now moved rightward in Fig. 4.5 to the surface of constant rise time; then subsequent adjustment of a and K move the point c' along the R_s boundary until c' and d are in positions e' and f respectively. The result is that the points e' and f are on

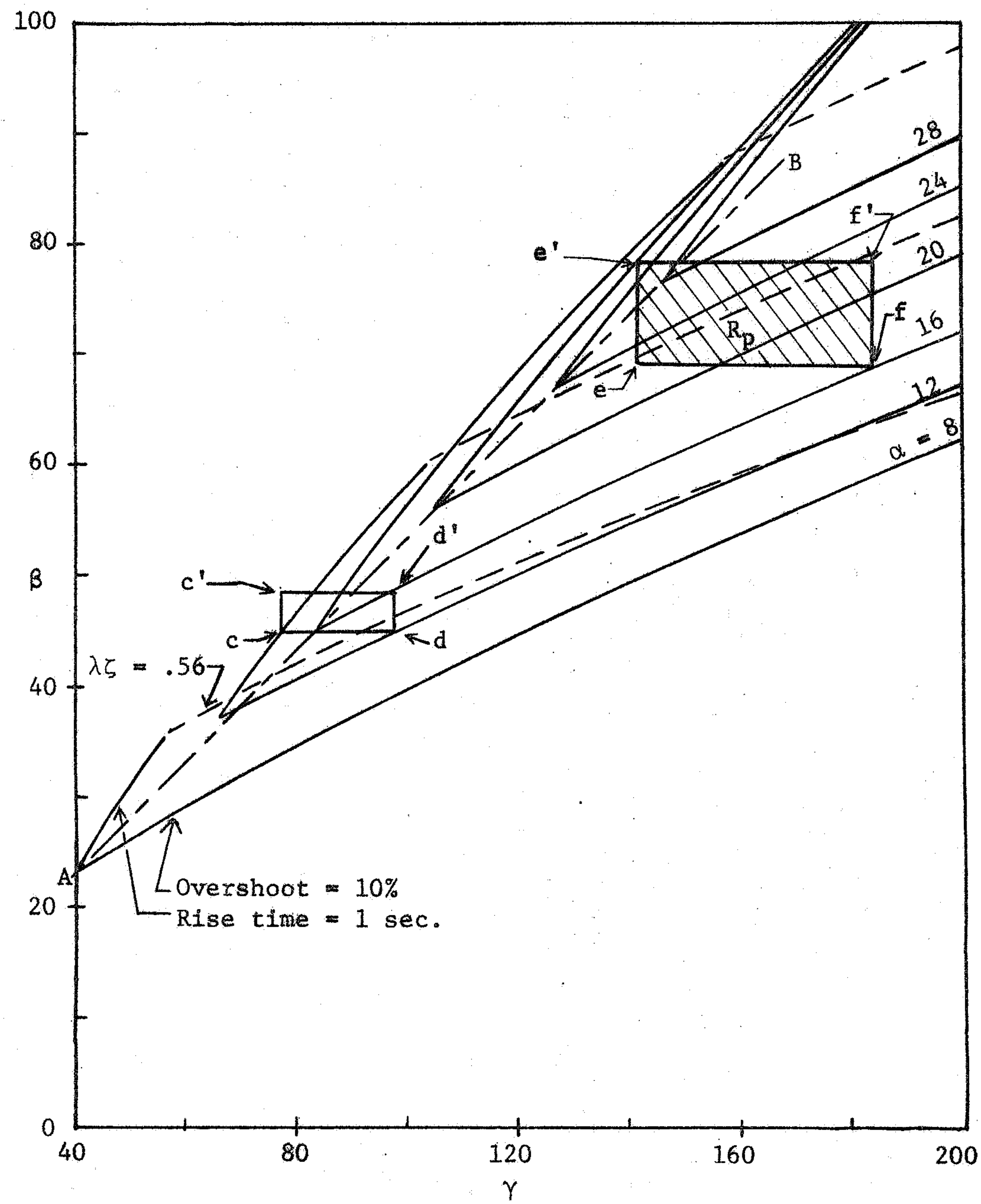


Fig. 4.5 Illustration of design procedure for plant with combined gain and pole variation.

the boundries of R_s and all other points of R_p are in R_s. It is clear from the figure that the solution is unique and optimal.

As in the gain variation case, if the undesirable response boundary is encountered, computation is terminated and any useful information about the point of termination retained. The increase of a and K as point c' moves along the rise time surface causes R_p to expand in two dimensions. The γ variation is given again by Eq. (4.11), but it is already known that this dimension of R_p fits in R_s because the gain variation problem with $p_2 = p_{21}$ would have diverged otherwise. The length of the other edge of the plane R_p, denoted by Δ_0, is calculated using Eqs. (4.15a & b) as

$$\Delta\alpha = (p_{22} - p_{21}) \tag{4.16a}$$

$$\Delta\beta = a(p_{22} - p_{21}) \tag{4.16b}$$

$$\Delta_0 = \sqrt{\Delta\alpha^2 + \Delta\beta^2} - (p_{22} - p_{21})\sqrt{1 + a^2}\ . \tag{4.17}$$

For large a, Δ_0 is approximately proportional to a and the plant and specifications may be such that Δ_0 expands faster than the dimension of R_s of the same orientation. This is determined in the computation by divergence of the Δa and ΔK iterates.

The numerical technique is modified by rewriting (4.1b & c) in the form

$$t_r(\alpha+\Delta\alpha, \beta+\Delta\beta, \gamma) = t_{rs} \tag{4.18a}$$

$$OV(\alpha, \beta, \gamma+\Delta\gamma) = OV_s \tag{4.18b}$$

and applying the procedure of Sect. 4.4. In (4.18) the coefficients (α,β,γ) are those computed for plant parameters at minimum.

4.7 Comparison of Second and Third-Order Designs

In this section the second and third-order designs are compared and some reasons for choosing one over the other are pointed out. It is shown, rather heuristically, that if one design will achieve the time response specifications then the other will also; hence if rise time and overshoot are the only considerations this choice is always available.

On the following page, Fig. 4.6 shows the asympototic Bode plot of open-loop transmission L(s) representing the solution of the following problem. Plant parameters are given by

$$8 \le p_2 \le 10$$

$$1 \le k \le 1.5 \quad ,$$

and time response specifications are

$$t_r \le 1 \text{ sec.}$$

$$OV \le 10\% \quad .$$

The asymptotic Bode plot changes of course as the plant parameters vary, but the plot is shown for the two designs at the time response extremes as labeled. Recall that the second and third order open-loop transmissions are

$$L_2(s) = \frac{K_2 k}{s(s + p)} \tag{4.19a}$$

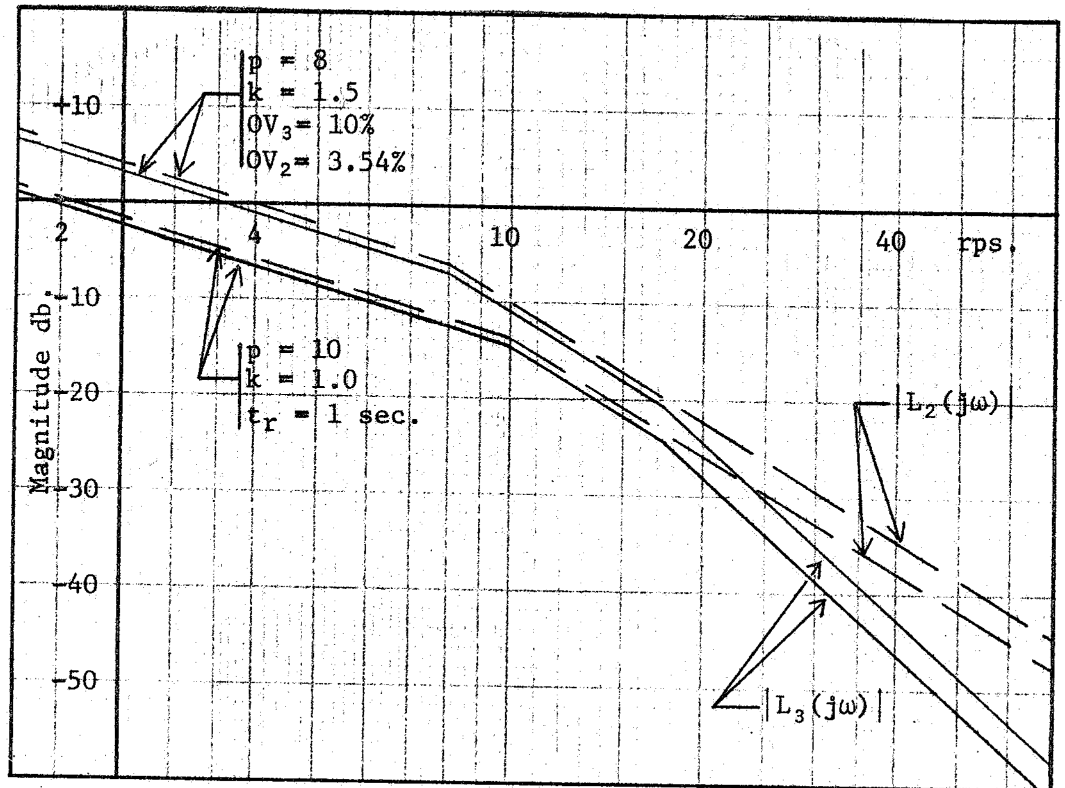


Fig. 4.6 Asymptotic Bode plots of open-loop transmission corresponding to time response extremes.

and

$$L_3(s) = \frac{K_3 k}{s(s + a)(s + p)} \tag{4.19b}$$

respectively. The solutions obtained are for the second-order, $K_2 = 20.1$, and the third-order compensation is $K_3 = 322$ and $a = 17.3$. With these compensations the third-order system for two extremes of plant parameters, touches both the overshoot and rise time boundaries of R_s and thus makes maximum use of the time specifications. The second-order system has a maximum overshoot of 3.54% leaving some freedom in the design specification for improvement.

From conventional frequency domain design techniques we can associate the low frequency portion of the Bode plot with rise time. That is, to achieve some given rise time the system must pass frequencies from zero up to some value with a certain amount of gain. Overshoot in turn, is related to how fast the loop gain is reduced, or equivalently how fast phase lag is added. Consider the lower set of curves in Fig. 4.6 as representing a design for a non-varying plant momentarily. The second-order system has the required gain over the low frequency range to meet the rise time requirement and since more overshoot is allowed the high frequency gain may be lowered faster than is shown by the second order plot. By going to the third-order system we can add pure gain and an additional pole. To again meet the rise time requirement the low frequency gain must be at the same level as it was for the second-order design but with the additional pole the high frequency gain can be lowered at a faster rate as shown by $L_3(s)$. If the second-order system meets both specifications as equalities then the Bode plot of the third-order design must be identical with the second-order. This can only be achieved by letting the compensation pole approach infinity. Indeed, a and K_3 must both go to infinity in such a way that $K_3/a \to K_2$ to give the same charqcteristic as the second-order system. It now becomes clear that if the second-order system cannot be adjusted to meet the response specifications a third-order structure can do no better. Further, at least theoretically, any set or specifications that are achieved with the second-order structure can also be met with the third-order structure.

The third-order structure is more complex and requires more compensation gain thereby increasing the system cost. However, most practical systems require a transducer at the output to generate the feedback signal and such transducers produce high frequency noise; hence it is desirable to reduce the open-loop gain as fast as possible at higher frequencies to attenuate noise in the feedback path. These factors must be weighed by the designer when choosing which structure to use for a particular task.

4.8 Improvements and Simplifications to the Third-Order Design Scheme

With the information of the preceding section the numerical solution can be made somewhat more succinct. First, if a second-order design is not achieved, the third-order need not be attempted. If the second-order try is successful then the solution together with the plant parameter values provide us with reasonable initial guesses to the third-order system compensation. If a successful second-order design yields a gain K_2, then placing the compensation pole for the third-order design far out, say at $20p_2$, and adjusting the initial compensation gain appropriately to $K_3 = 20K_2p_2$, the low frequency Bode characteristic is essentially unchanged and thus a fairly good guess is obtained for the third-order compensation at the start.

Further, instead of iterating to the minimum point and parameter variation solutions seperately, Eqs. (4.18) are used at the outset to iterate directly to the final solution. In the special case of no plant pole variation $\Delta\alpha$ and $\Delta\beta$ of (4.18) simply vanish. Similarly, $\Delta\gamma$ vanishes when the plant has no gain variation. A special case occurs when $p_1 \neq 0$ and the plant is fixed. For this case Eqs. (4.18) are still satisfactory, but initial compensation guesses are chosen as explained in Sect. 4.4 since no second-order design has been attempted.

An attempt to solve a few design problems quickly indicates that the undesirable response boundary adopted in Sect. 3.5 is overly restrictive, preventing the solution of a large class of problems. This results from the mapping of the semi-infinite strip in the (ζ,λ) plane (see Fig. 3.12) bounded by the curves $\zeta = .5$, $\lambda = 0$, and $\lambda\zeta = .56$ into coefficient space. It is found that many solutions lie in the region of coefficient space where $\lambda\zeta < .56$ but $\zeta > .5$ and, as mentioned in Sect. 3.5, the time response specifications are well-behaved functions in this region. Accordingly, a modified region R' is defined as follows:

(a) α,β,γ are all positive,

and

(b) $\gamma/\alpha < \beta < .6794\alpha\gamma^{1/3} + 1.01\gamma^{2/3}$,

or

(c) $\gamma/\alpha < \beta$ and $\zeta > .5$.

This region has all the properties ascribed to R in Sect. 3.5. However, the new region R' and the associated R_s' no longer approach the convexity condition discussed in Sect. 4.1. Fig. 4.7 shows the image in the (γ,β) plane of a general third-order design. In the

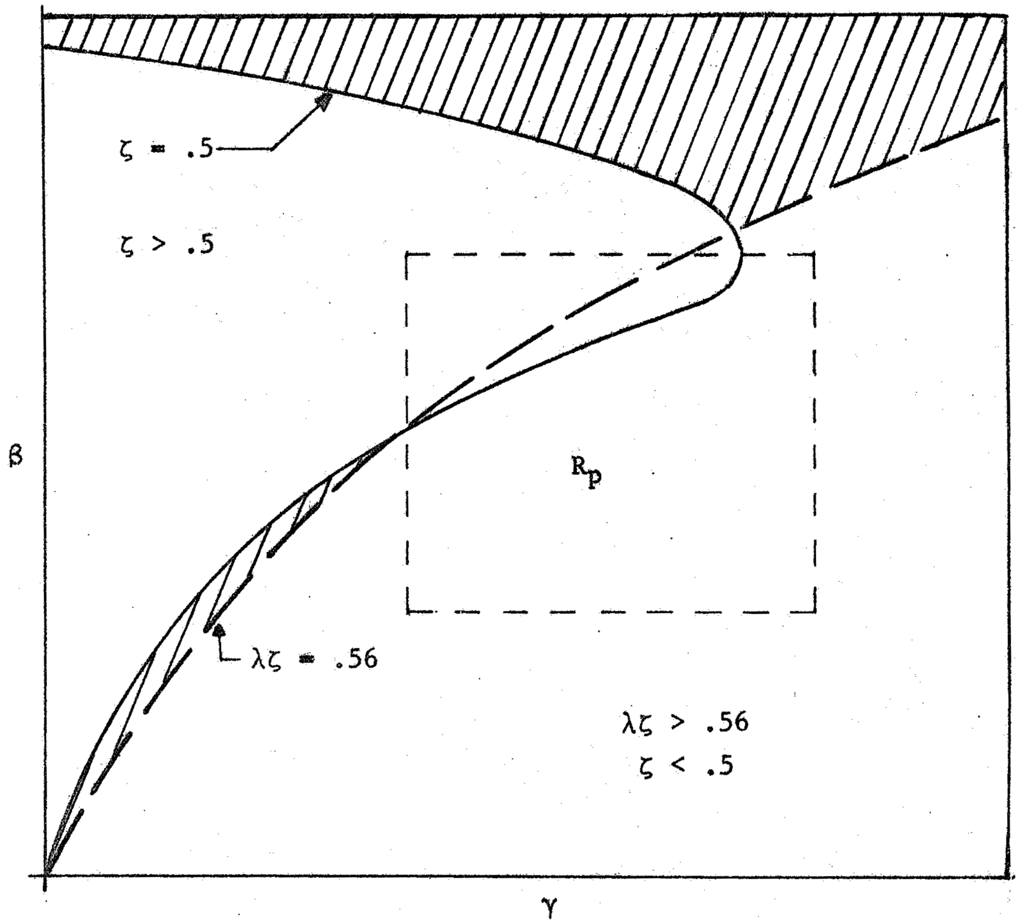


Fig. 4.7 Illustration of general relationship between undesirable response regions and region of plant parameter variation R_p.

shaded region $\zeta < .5$ and $\lambda\zeta < .56$. If the shaded region overlaps with R_p, then undesirable time responses are possible and a special investigation of the system response is required for values of the plant corresponding to the shaded region to ascertain whether or not such responses are acceptable in a particular application. However, a large class of problems result in solutions where R_p and the shaded region are disjoint. The result of defining the new regions R' and R_s' is a significant extension of the usefulness of the design scheme, but with the additional requirement imposed to check the final solution to insure that we never simultaneously have $\zeta < .5$ and $\lambda\zeta < .56$, or if so, that the resulting responses are acceptable. The check is most easily accomplished by plotting the system extreme values on the two dimensional (ζ,λ) plane. Often the check is trivial, e.g., it can be shown using Eqs. (3.4) that ζ takes on its minimum value at one of the corners of R_p in Fig. 4.7. If this minimum is greater than .5, undesirable responses are excluded for all plant values. The possibility remains that the computational algorithm will enter the shaded region before reaching a final solution, in which case computation must be terminated due to the eratic behavior of the time response functions. This has never occurred during any trial problem used in developing the program given in Appendix A.

CHAPTER V
CONCLUSIONS

A design scheme is presented in this paper for achieving inequality constraints on system step response in the time domain while plant parameters vary over some given range. Second and third-order system transfer functions are considered with similar single degree of freedom structures. Criteria are presented as to what conditions permit a design to be achieved with each structure. A numerical algorithm is presented, and implemented in Appendix A, for accomplishing the design with a digital computer.

Possible extentions of the work of this paper are (1) consideration of additional all pole third-order structures, (2) incorporation of zeros in the transfer function, and (3) extention to fourth and higher order system.

A minimal amount of work was done during the course of this research with one two degree of freedom structure {10}. This structure was obtained as follows: in Fig. 4.2, let $H(s) = K$ and put a prefilter with transfer function $b/(s + b)$ between the input and the summing point. It is easily verified that this results in a third-order all pole system transfer function with design parameters K and b. The algorithm given in Sect. 4.4 would not converge for this structure using a test problem with known solution and initial guesses very close to that solution. Specific reasons why convergence was not obtained were not investigated in detail. Convergence was obtained with the Raphson-Newton method using initial guesses quite close to the solution. Obtaining initial guesses that converge for the general case with this structure remains an unsolved problem.

A state-of-the-art summary of computer-aided design techniques is given in {11}. Some pitfalls evident from the present work should be pointed out. Most computer-aided design to date, in the area of transfer function design to achieve some time or frequency response, assume the response specification to be a vector of points through which the response is forced to pass with some error criteria being minimized. Such response specifications result in minimizing a continuous function of system parameters, a property which is not in general true for the specifications used in this treatment. Any extention of the present scheme which results in one or more additional design parameters would require the definition of an additional time response specification for each new design parameter in order that the design problem possess a unique solution. A logical next choice would seem to be settling time, but this function is less well-behaved than those used so far. Thus it is indicated that as the number of design parameters is increased, some other method of specifying the response should be sought. A popular method in conventional design that appears worthy of exploration is the specification of an envelope within which the time response must remain. It is clear that the complexity of the problem increases greatly as more design parameters are allowed.

BIBLIOGRAPHY

{1} I. M. Horowitz, "Plant Adaptive Systems vs. Ordinary Feedback Systems," IRE Trans. On Automatic Control, IEEE, Inc., New York, N. Y., AC-7, Jan. 1962, pp. 48-56.

{2} J. A. Rolnik, "Feedback Control System Synthesis For Plants with Large Parameter Variations," Doctorial Dissertation, Polytechnic Institute of Brooklyn, June 1965, pp. 46-68.

{3} D. Olson, "Design Procedures for Dominant Type Systems with Large Parameter Variations," Master's Thesis, University of Colorado, 1968.

{4} H. Barber, "Third-Order Synthesis for Prescribed Time Domain Tolerances to Parameter Variations," Master's Thesis, University of Colorado, 1968, p. 7, pp. 12-21.

{5} I. M. Horowitz, Synthesis of Feedback Systems, Academic Press, New York, N. Y., 1963, pp. 69-70.

{6} A. Ralston, A First Course in Numerical Analysis, McGraw-Hill Inc., New York, N. Y., 1965, pp. 323-325.

{7} Ibid., pp. 331-332.

{8} W. Fulks, Advanced Calculus, John Wiley & Sons, Inc., New York, N. Y., 1961, p. 237 & 246.

{9} I. M. Horowitz, Synthesis of Feedback Systems, Academic Press, New York, N. Y., 1963, pp. 121-130.

{10} Ibid., p. 246.

{11} G. C. Temes & D. A. Calahan, "Computer-Aided Network Optimization--The State-of-the-Art," Proceedings of the IEEE, IEEE, Inc., New York, N. Y., Nov. 1967, pp. 1832-1863.

APPENDIX A

DIGITAL COMPUTER IMPLEMENTATION OF DESIGN SCHEME

A.1 General Description

The program used to accomplish the second and third-order designs of this paper is listed in A.4. The program is coded in FORTRAN IV source language and has been executed on a Control Data Corporation 6400 computer. Approximately 20,000(octal) units of storage are required on this machine. As stated previously, the basic task of the program is to solve the equation

$$t_r(K) = t_{rs} \tag{A.1a}$$

for the second-order design and the equations

$$t_r(a,K) = t_{rs} \tag{A.1b}$$

$$OV(a,K) = OV_s \tag{A.1c}$$

in the third-order case. The program has been tested over a wide range of problems with the rise time specification being varied by a factor of 10^5. Run times on the machine above have varied from 3 to 8 seconds for a combined second and third-order design of a single system including source program compilation time. A.5 shows some sample runs and includes the numerical example of the text.

A.2 Accuracy

The solutions computed by the program consist of values of a, K, t_r, and OV in Eqs. (A.1). The concept of relative rather than absolute accuracy is employed. The criteria for determining that a solution has been located in the nth iteration is as follows:

$$|t_{rn} - t_{rs}| < .01t_{rs}$$

$$|OV_n - OV_s| < .01OV_s$$

$$|\Delta K_n| < .01K_n$$

$$|\Delta a_n| < .01a_n \quad .$$

All four of these conditions are required to be satisfied in the third-order case, while only the first and third apply to the second-order design. The value t_{rn} is computed by the false position routine discussed in Sect. 2.4 and the convergence criterion here is that

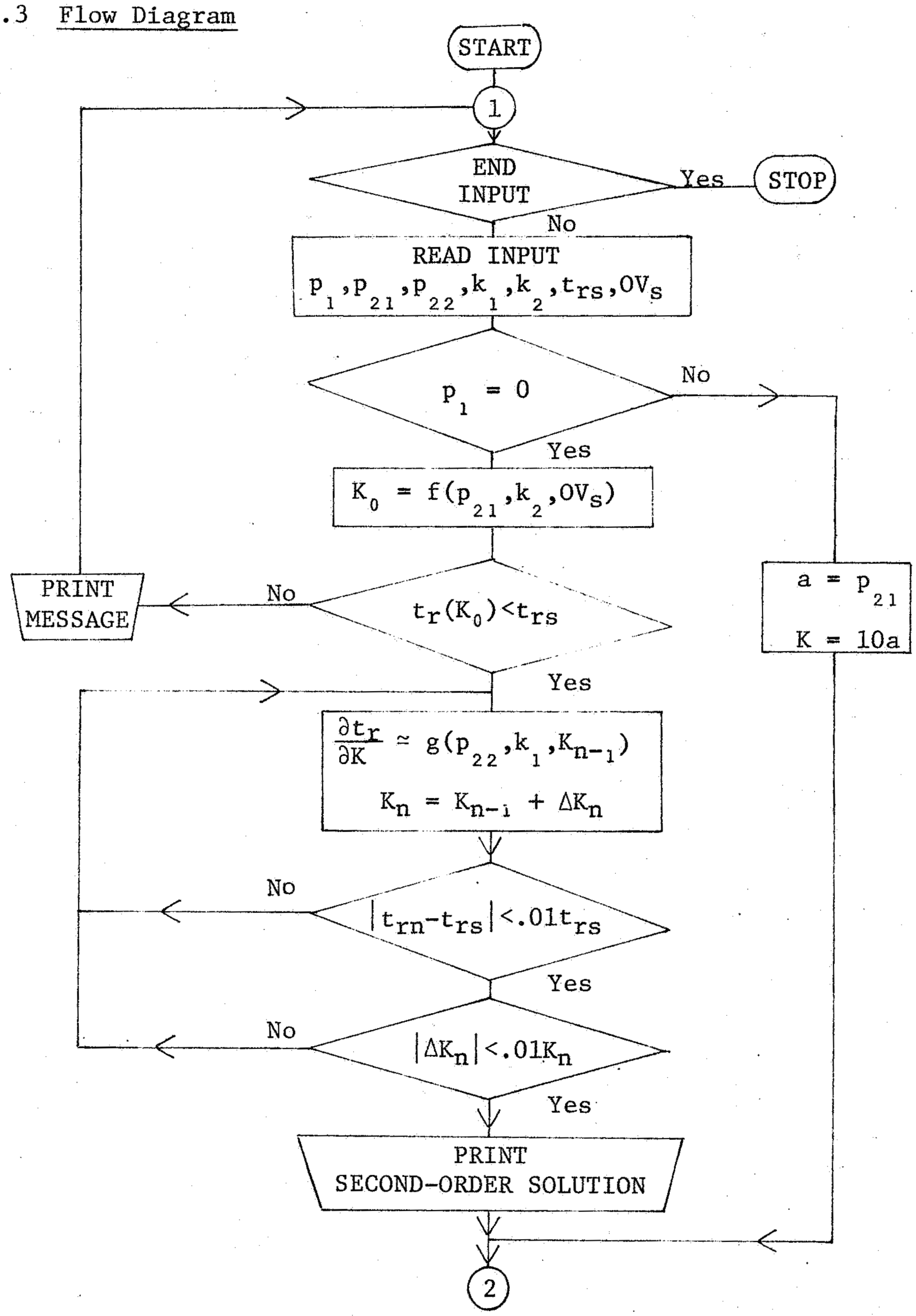
$$|c(t_{rn}) - c(t_{rn-1})| < .001$$

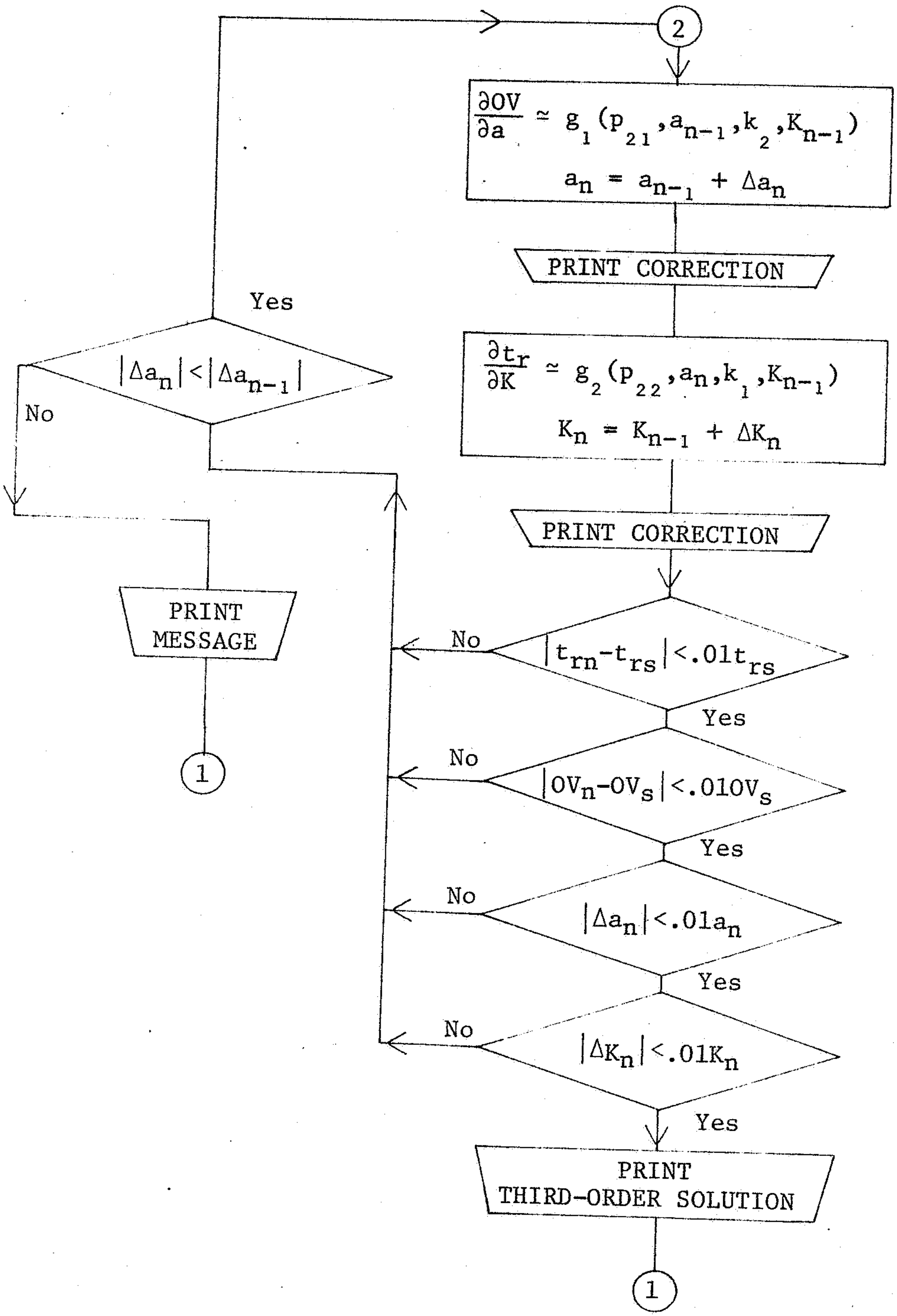
or

$$|t_{rn} - t_{rn-1}| < .001t_{rs} \ ,$$

where the quantities $c(t_{rn}) - .9$ and $c(t_{rn-1}) - .9$ are of opposite sign. In solving for overshoot OV_n the false position routine is used to solve $c'(t_0) = 0$ with the same accuracy criterion as rise time. An explicit bound on the error of OV_n is not available, however, since the derivative is close to zero at t_0, the function is changing slowly with t and the technique is believed to have more than sufficient accuracy for most practical systems designs. Beyond the discussion above, the program accuracy is limited by the capacity of the machine being used and/or the accuracy to which the plant is determined.

A.3 Flow Diagram





2
$\frac{\partial OV}{\partial a} \simeq g_1(p_{21}, a_{n-1}, k_2, K_{n-1})$
$a_n = a_{n-1} + \Delta a_n$
PRINT CORRECTION
$\frac{\partial t_r}{\partial K} \simeq g_2(p_{22}, a_n, k_1, K_{n-1})$
$K_n = K_{n-1} + \Delta K_n$
PRINT CORRECTION
$|t_{rn} - t_{rs}| < .01 t_{rs}$
$|OV_n - OV_s| < .01 OV_s$
$|\Delta a_n| < .01 a_n$
$|\Delta K_n| < .01 K_n$
PRINT THIRD-ORDER SOLUTION
1
$|\Delta a_n| < |\Delta a_{n-1}|$
Yes
No
PRINT MESSAGE
1

The following table relates the FORTRAN variable names to those of the text. Variables not listed but appearing in the program are defined in the program or their relationship to the text is obvious, e.g., λ = LAMBDA.

Table A.3.1

$$\mathrm{P1} = p_1$$

$$\mathrm{P21} = p_{21}$$

$$\mathrm{P22} = p_{22}$$

$$\mathrm{PK1} = k_1$$

$$\mathrm{PK2} = k_2$$

$$\mathrm{CK} = K$$

$$\mathrm{DCK} = \Delta K$$

$$\mathrm{PC} = a$$

$$\mathrm{DPC} = \Delta a$$

$$\mathrm{DOV} = \frac{\partial OV}{\partial a}$$

$$\mathrm{DTR} = \frac{\partial t_r}{\partial K}$$

$$\mathrm{CT2} = c(t) \text{ (second-order)}$$

$$\mathrm{CT3} = c(t) \text{ (third-order)}$$

$$\mathrm{PCT3} = c'(t) \text{ (third-order)}$$

$$\mathrm{TRS} = t_{rs}$$

$$\mathrm{OVS} = OV_s$$

A.4 Computer Program

```
   PROGRAM CFSIGN(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
 1 FORMAT(8F10.4)
 2 FORMAT(1H1,10X,18HSECOND-ORDER PLANT/1H ,10X,21HDESIGN SPECIFICATI
  2ONS/1H0,10X,4HP1 =,E13.5,/1H ,10X,4HP21=,E13.5,5H P22=,E13.5,/1H ,
  310X,4HPK1=,E13.5,5H PK2=,E13.5/1H0,10X,4HTRS=,F10.6,4X,4HOVS=,F8.6
  4/)
 3 FORMAT(1H0,33HUNARLE TO CORRECT INITIAL GUESSES/)
 4 FORMAT(1H0,10X,25HCORRECTED INITIAL GUESSES/1H0,35X,3HPC=,E13.5,1X
  2,3HCK=,E13.5,8X,3(E13.5,2X))
 5 FORMAT(1H0,10X,18HTHIRD-ORDER DESIGN/1H0,45X,2HPC,15X,2HCK,17X,5HA
  2LPHA,10X,4HBETA,11X,5HGAMMA/)
 6 READ(5,1)P1,P21,P22,PK1,PK2,TRS,OVS $ IF(EOF,5)14,7
 7 WRITE(6,2)P1,P21,P22,PK1,PK2,TRS,OVS $ J=0 $ J1=0
   AT=.01*TRS $ AO=.01*OVS $ IF(P1.NE.0.)GO TO 13
   CALL SOLVE2(P21,P22,PK1,PK2,CK,TRS,OVS,AT,J3)
   IF(J3.FQ.1)GO TO 6
   PC=20.*P21 $ CK=PC*CK
 8 WRITE(6,5)
 9 CALL PCTRAN(ALFA,BETA,GAMA,P1,P21,PC,PK1,CK) $ J=J+1
   IF(BETA.GE.GAMA/ALFA+ALFA**2/100.)GO TO 10$CK=CK/2.$IF(J-10)9,9,12
10 IF(BETA.LT..697*ALFA*GAMA**(1./3.)+1.01*GAMA**(2./3.))GO TO 11
   PC=PC/2. $ IF(J-10)9,9,12
11 IF(J.GT.1)WRITE(6,4)PC,CK,ALFA,BETA,GAMA
   CALL SOLVE3(P1,P21,P22,PC,PK1,PK2,CK,TRS,OVS,AT,AO,K,J1) $ GO TO 6
12 WRITE(6,3) $ GO TO 6
13 PC=P21 $ CK=10.*P21 $ GO TO 8
14 STOP $ END
```

```
   SUBROUTINE SOLVE2(PS,PL,PK1,PK2,CK,TRS,OVS,AT,J)
 1 FORMAT(1H0,10X,19HSECOND-ORDER DESIGN/1H0,10X,7HPOINT A,3X,3HTR=,
  2F10.6,3X,3HOV=,F8.6,/1H0,10X,7HPOINT B,3X,3HTR=,F10.6,3X,3HOV=,
  3F8.6,/1H0,20X,3HCK=,E13.5,//)
 2 FORMAT(1H0,39H***SECOND-ORDER DESIGN DO TERMINATED***/)
 3 FORMAT(1H0,10X,34HSECOND-ORDER DESIGN UNSATISFACTORY/)
   IF(OVS.EQ.0.)GO TO 4 $ GO TO 5
 4 CK=PS**2/(4.*PK2) $ GO TO 6
 5 CK=(9.869604+ALOG(OVS)**2)*PS**2/(4.*PK2*ALOG(OVS)**2)
 6 CALL RISFT2(PL/(2.*SQRT(PK1*CK)),SQRT(PK1*CK),TR,AT)
   IF(TRS-TR)7,8,11
 7 WRITF(6,3) $ J=1 $ RETURN
 8 CALL RISET2(PS/(2.*SQRT(PK2*CK)),SQRT(PK2*CK),TR1,AT)
   ZETA=PL/(2.*SQRT(PK2*CK))$IF(ZETA.LT.1.)GO TO 9 $ OV=0. $ GO TO 10
 9 OV=EXP(-3.1415*ZETA/SQRT(1.-ZETA**2))
10 WRITE(6,1) TR,OV,TR1,OVS,CK $ J=0 $ RETURN
11 DO 15 I=1,50 $ CKL=CK $ IF(TRS-TR)12,16,13
12 DC=-.19*CK $ GO TO 14
13 DC=.19*CK
14 CALL RISFT2(PL/(2.*SQRT(PK1*(CK+DC))),SQRT(PK1*(CK+DC)),TR1,AT)
   DCK=(TRS-TR)*DC/(TR1-TR) $ CK=CK+DCK $ IF(CK.LE.0.)CK=CKL/2.
   CALL RISET2(PL/(2.*SQRT(PK1*CK)),SQRT(PK1*CK),TR,AT)
   IF(ABS(TRS-TR).LE.AT.A.ABS(DCK).LE..01*CK)GO TO 16
15 CONTINUE $ WRITE(6,2) $ STOP
16 ZETA=PS/(2.*SQRT(PK2*CK)) $ IF(ZETA.LT.1.)GO TO 17$ OV=0.$GO TO 18
17 OV=EXP(-3.1415*ZETA/SQRT(1.-ZETA**2))
18 CALL RISET2(PS/(2.*SQRT(PK2*CK)),SQRT(PK2*CK),TR1,AT)
   ZETA=PL/(2.*SQRT(PK1*CK))$IF(ZETA.LT.1.)GO TO 19$ OV1=0. $GO TO 20
19 OV1=EXP(-3.1415*ZETA/SQRT(1.-ZETA**2))
20 WRITE(6,1) TR,OV1,TR1,OV,CK $ J=0 $ RETURN $ END
```

```
      SUBROUTINE SOLVE3(P1,P21,P22,PC,PK1,PK2,CK,TRS,OVS,AT,AO,K,J1)
    1 FORMAT(1H0,33H***VARIABLE POLE DO TERMINATED***,2X,3HPC=,E13.5,
     24H CK=,E13.5,/1H0,22X,2HTR,8X,2HOV,9X,6HLAMBDA,8X,4HZETA,
     36X,5HOMEGA,9X,5HALPHA,10X,4HBETA,11X,5HGAMMA)
    2 FORMAT(1H0,10X,54H***UNDESIRABLE RESPONSE BOUNDARY VIOLATED BY SOL
     2VE3***/)
    3 FORMAT(1H0,20HTHIRD-ORDER SOLUTION,15X,3HPC=,E13.5,
     24H CK=,E13.5,/1H0,22X,2HTR,8X,2HOV,9X,6HLAMBDA,8X,4HZETA,
     36X,5HOMEGA,9X,5HALPHA,10X,4HBETA,11X,5HGAMMA)
    4 FORMAT(1H ,7X,5HPOINT,I2,F12.6,2X,F8.6,4X,3(F10.4,2X),3(E13.5,2X))
    5 FORMAT(1H0,27H***THIRD-ORDER DIVERGING***,9X,3HPC=,E13.5,
     24H CK=,E13.5,/1H0,22X,2HTR,8X,2HOV,9X,6HLAMBDA,8X,4HZETA,
     36X,5HOMEGA,9X,5HALPHA,10X,4HBETA,11X,5HGAMMA)
    6 FORMAT(1H1,10X,28HTHIRD-ORDER DESIGN CONTINUED/1H0,45X,2HPC,15X,2H
     2CK,17X,5HALPHA,10X,4HBETA,11X,5HGAMMA/)
      REAL LAMBDA $ K=0 $ DPCL=1000. $ IP=0
      DO 11 I=1,200 $ IP=IP+1
      IF(IP.EQ.11.A.I.EQ.IP)GO TO 7 $IF(IP.EQ.15)GO TO 7 $ GO TO 8
    7 IP=0 $ WRITE(6,6)
    8 CALL OVADJ(P1,P21,PC,PK2,CK,OVS,OV,DPC,AT,J1) $ IF(J1.EQ.1)RETURN
      CALL RTADJ(P1,P22,PC,PK1,CK,TRS,TR,DCK,AT,J1) $ IF(J1.EQ.1)RETURN
      CALL PCTRAN(ALFA,BETA,GAMA,P1,P21,PC,PK2,CK)
      G=GAMA**(1./3.)$IF(BETA.GT..697*ALFA*G+1.01*G**2)GO TO 9$GO TO 10
    9 CALL CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      IF(ZETA.GE..5)GO TO 10 $ J1=1 $ WRITE(6,2) $ RETURN
   10 CALL OVSH3(ALFA,BETA,GAMA,OV,AT)
      IF(ABS(TRS-TR).LE.AT.A.ABS(OVS-OV).LE.AO.A.ABS(DCK).LE..01*CK.A.
     2ABS(DPC).LE..01*PC)GO TO 13
      IF(I.GE.10.A.ABS(DPCL).LT.ABS(DPC))GO TO 12 $ DPCL=DPC
   11 CONTINUE $ WRITE(6,1) PC,CK $ K=1 $ GO TO 14
   12 K=1 $ WRITE(6,5) PC,CK $ GO TO 14
   13 WRITE(6,3) PC,CK
   14 DO 20 I=1,4 $ GO TO(15,16,17,18),I
   15 P2=P21 $ PK=PK1 $ GO TO 19
   16 P2=P22 $ PK=PK1 $ GO TO 19
   17 P2=P21 $ PK=PK2 $ GO TO 19
   18 P2=P22 $ PK=PK2
   19 CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      CALL CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      CALL RISET3(ALFA,BETA,GAMA,TR,AT)
      CALL OVSH3(ALFA,BETA,GAMA,OV,AT)
      WRITE(6,4) I,TR,OV,LAMBDA,ZETA,OMEGA,ALFA,BETA,GAMA
   20 CONTINUE $ RETURN $ END
```

```
      SUBROUTINE RTADJ(P1,P2,PC,PK,CK,TRS,TR,DCK,AT,J) $ REAL LAMBDA
    1 FORMAT(1H ,20HRISE TIME ADJUSTMENT,3X,3HTR=,F10.6,19X,E13.5,8X,3(
     2E13.5,2X))
    2 FORMAT(1H0,10X,72H***UNDESIRABLE RESPONSE BOUNDARY VIOLATED DURING
     2 RISE TIME ADJUSTMENT***/)
    3 FORMAT(1H ,20HRISE TIME STAB LIMIT,3X,3HTR=,F8.4,4H PC=,E13.5,
     24H CK=,E13.5,8X,3(E13.5,2X))
      CALL GRADTRK(P1,P2,PC,PK,CK,TR,TRS,DTR,AT) $ DCK=(TRS-TR)/DTR
      IF(CK+DCK.LE.0.)DCK=-CK/2.
      CK=CK+DCK $ CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      G=GAMA**(1./3.) $ IF(BETA.GT..679*ALFA*G+1.01*G**2)GO TO 5
    4 IF(BETA.LT.GAMA/ALFA+ALFA**2/(1.E+50))GO TO 6
      CALL RISET3(ALFA,BETA,GAMA,TR,AT)
      WRITE(6,1) TR,CK,ALFA,BETA,GAMA $ RETURN
    5 CALL CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      IF(ZETA.GE..5)GO TO 4 $ J=1 $ WRITE(6,2) $ RETURN
    6 CK=(ALFA*(BETA-ALFA**2/(1.E+50))-P1*P2*PC)/PK
      CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      CALL RISET3(ALFA,BETA,GAMA,TR,AT)
      WRITE(6,3) TR,PC,CK,ALFA,BETA,GAMA $ RETURN $ END
```

```
      SUBROUTINE GRADTRK(P1,P2,PC,PK,CK,TR,TRS,DTR,AT) $ K=0
    1 CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      CALL RISET3(ALFA,BETA,GAMA,TR1,AT) $ K=K+1 $ GO TO(2,6),K
    2 IF(TRS-TR1)3,3,4
    3 DK=.1*GAMA/PK  $  GO TO 5
    4 DK=-.1*GAMA/PK
    5 CK=CK+DK  $  TR=TR1  $  GO TO 1
    6 CK=CK-DK  $ DTR=(TR1-TR)/DK $ RETURN $ END
```

```
      SUBROUTINE RISET3(ALFA,BETA,GAMA,T2,AT) $ REAL LAMBDA $ T1=0.
    1 FORMAT(1H0,10X,28H***RISE TIME OUT OF RANGE***,3X,7HLAMBDA=,F12.4,
     23X,5HZETA=,F10.4,3X,6HOMEGA=,F12.4,3X,3HT1=,F10.4/)
    2 FORMAT(1H0,10X,37H***RISE TIME ACCURACY NOT ACHEIVED***,3X,
     27HLAMBDA=,F12.4,3X,5HZETA=,F10.4,3X,6HOMEGA=,F12.4/)
      CALL CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      IF(ZETA-1.)3,4,5
    3 DT=.7/(OMEGA*SQRT(1.-ZETA**2)) $ GO TO 6
    4 DT=1./OMEGA $ IF(LAMBDA.LT.1.)DT=DT/LAMBDA $ GO TO 6
    5 DT=1./(LAMBDA*ZETA*OMEGA) $ DT1=1./(OMEGA*(ZETA-SQRT(ZETA**2-1.)))
      IF(DT1.GT.DT)DT=DT1
    6 DO 7 J=1,50
      C1=CT3(LAMBDA,ZETA,OMEGA,.9,T1) $ IF(C1.GT.0.)GO TO 8
    7 T1=T1+DT $ WRITE(6,1) LAMBDA,ZETA,OMEGA,T1 $ STOP
    8 T2=T1-DT $ C2=CT3(LAMBDA,ZETA,OMEGA,.9,T2) $ DO 11 I=1,40
      T3=(C2*T1-C1*T2)/(C2-C1) $ C3=CT3(LAMBDA,ZETA,OMEGA,.9,T3)
      IF(C3*C1.LT.0.)GO TO 9 $ T1=T2 $ C1=C2
    9 T2=(C3*T1-C1*T3)/(C3-C1) $ C2=CT3(LAMBDA,ZETA,OMEGA,.9,T2)
      IF(C2*C1.LT.0.)GO TO 10 $ T1=T3 $ C1=C3
   10 IF(ABS(T1-T2).LE..1*AT.O.ABS(C1-C2).LE..001)RETURN
   11 CONTINUE $ WRITE(6,2) LAMBDA,ZETA,OMEGA $ RETURN $ END
```

```
      SUBROUTINE OVADJ(P1,P2,PC,PK,CK,OVS,OV,DPC,AT,J) $ REAL LAMBDA
    1 FORMAT(1H ,20HOVERSHOOT ADJUSTMENT,3X,3HOV=,F10.6,2X,E13.5,25X,3(
     2E13.5,2X))
    2 FORMAT(1H0,10X,72H***UNDESIRABLE RESPONSE BOUNDARY VIOLATED DURING
     2 OVERSHOOT ADJUSTMENT***/)
    3 FORMAT(1H0,10X,44H***OVERSHOOT STABILITY BOUNDARY VIOLATION***/)
      CALL GRADOVP(P1,P2,PC,PK,CK,OV,OVS,DOV,AT)
      IF(ABS(DOV).LE.1.E-100)GO TO 8 $ DPC=(OVS-OV)/DOV
      IF(PC+DPC.LE..0)DPC=-PC/2.
    4 PC=PC+DPC $ CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      G=GAMA**(1./3.) $ IF(BETA.GE..679*ALFA*G+1.01*G**2)GO TO 7
    5 IF(BETA.LT.GAMA/ALFA+ALFA**2/(1.E+50))GO TO 9
    6 CALL OVSH3(ALFA,BETA,GAMA,OV,AT)
      WRITE(6,1) OV,PC,ALFA,BETA,GAMA $ RETURN
    7 CALL CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      IF(ZETA.GE..5)GO TO 5 $ J=1 $ WRITE(6,2) $ RETURN
    8 DPC=-PC/10. $ GO TO 4
    9 DO 10 I=1,10 $ PC=PC-DPC/(2.**I)
      CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      IF(BETA.GE.GAMA/ALFA+ALFA**2/(1.E+50))GO TO 6
   10 CONTINUE $ WRITE(6,3) $ RETURN $ END
```

```
      SUBROUTINE GRADOVP(P1,P2,PC,PK,CK,OV,OVS,DOV,AT) $ K=0
    1 CALL PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      CALL OVSH3(ALFA,BETA,GAMA,OV1,AT) $ K=K+1 $ GO TO(2,6),K
    2 IF(OVS-OV1)3,3,4
    3 DP=.05*PC $ GO TO 5
    4 DP=-.05*PC
    5 PC=PC+DP $ OV=OV1 $ GO TO 1
    6 DOV=(OV1-OV)/DP $ PC=PC-DP $ RETURN $ END
```

```
      SUBROUTINE OVSH3(ALFA,BETA,GAMA,OV,AT) $ REAL LAMBDA $ T1=.0
    1 FORMAT(1H0,10X,28H***OVERSHOOT OUT OF RANGE***,3X,7HLAMBDA=,F12.4,
     23X,5HZETA=,F10.4,3X,6HOMEGA=,F12.4,3X,3HT1=,F10.4/)
    2 FORMAT(1H0,10X,37H***OVERSHOOT ACCURACY NOT ACHEIVED***,3X,
     27HLAMBDA=,F12.4,3X,5HZETA=,F10.4,3X,6HOMEGA=,F12.4/)
      CALL CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      IF(ZETA.LT..99)GO TO 3 $ OV=0.0 $ RETURN
    3 DT=.7/(OMEGA*SQRT(1.-ZETA**2))  $ DO 4 I=1,50
      P1=PCT3(LAMBDA,ZETA,OMEGA,T1) $ IF(P1.LT.0.) GO TO 5
    4 T1=T1+DT $ OV=0.0 $ RETURN
    5 T2=T1-DT $ P2=PCT3(LAMBDA,ZETA,OMEGA,T2) $ DO 8 I=1,40
      T3=(P2*T1-P1*T2)/(P2-P1) $ P3=PCT3(LAMBDA,ZETA,OMEGA,T3)
      IF(P3*P1.LT.0.)GO TO 6 $ T1=T2 $ P1=P2
    6 T2=(P3*T1-P1*T3)/(P3-P1) $ P2=PCT3(LAMBDA,ZETA,OMEGA,T2)
      IF(P2*P1.LT.0.)GO TO 7 $ T1=T3 $ P1=P3
    7 IF(ABS(T1-T2).LE..1*AT.O.ABS(C1-C2).LE..001)GO TO 9
    8 CONTINUE $ WRITE(6,2) LAMBDA,ZETA,OMEGA
    9 OV=CT3(LAMBDA,ZETA,OMEGA,1.,T2) $ RETURN $ END
```

```
      SUBROUTINE CSTRAN(ALFA,BETA,GAMA,LAMBDA,ZETA,OMEGA)
      REAL LAMBDA $ A=ALFA*GAMA-BETA**2/3.
      B=-BETA**3/27.+ALFA*BETA*GAMA/6.-GAMA**2/2. $ A1=B**2+A**3/27.
      IF(A1.GE.0.)GO TO 1 $ FE=ACOS(-B/SQRT(-A**3/27.))/3.
      X=2.*SQRT(-A/3.)*COS(FE)+BETA/3. $ GO TO 4
    1 IF(A1.GT.0.)GO TO 3 $ IF(B.GT.0.) GO TO 2
      X=2.*ABS(B)**(1./3.)+BETA/3. $ GO TO 4
    2 X=-2.*B**(1./3.)+BETA/3. $ GO TO 4
    3 XA=-B+SQRT(A1) $ XB=-B-SQRT(A1) $ IF(XA.GT.0.)XA=XA**(1./3.)
      IF(XA.LT.0.)XA=-1.*(ABS(XA)**(1./3.))$IF(XB.GT.0.)XB=XB**(1./3.)
      IF(XB.LT.0.)XB=-1.*(ABS(XB)**(1./3.)) $ X=XA+XB+BETA/3.
    4 OMEGA=SQRT(X) $ LAMBDA=2.*GAMA/(X*ALFA-GAMA)
      ZETA=(X*ALFA-GAMA)/(2.*X*OMEGA) $ RETURN $ END
```

```
      SUBROUTINE PCTRAN(ALFA,BETA,GAMA,P1,P2,PC,PK,CK)
      ALFA=P1+P2+PC $ BETA=P1*P2+PC*(P1+P2)
      GAMA=P1*P2*PC+PK*CK $ RETURN $ END
```

```
      SUBROUTINE RISET2(ZETA,OMEGA,T2,AT) $ T1=0.
    1 FORMAT(1H0,39H**SECOND ORDER RISE TIME OUT OF RANGE**,3X,5HZETA=,
     2F10.4,3X,6HOMEGA=,F10.4,3X,3HT1=,F10.4)
    2 FORMAT(1H0,48H**SECOND ORDER RISE TIME ACCURACY NOT ACHIEVED**,3X,
     25HZETA=,F10.4,3X,6HOMEGA=,F10.4)
      IF(ZETA-1.)3,4,5
    3 DT=.7/(OMEGA*SQRT(1.-ZETA**2)) $ GO TO 6
    4 DT=1./OMEGA $ GO TO 6
    5 DT=1./(OMEGA*(ZETA-SQRT(ZETA**2-1.)))
    6 DO 7 J=1,50 $ C1=CT2(ZETA,OMEGA,.9,T1) $ IF(C1.GT.0.)GO TO 8
    7 T1=T1+DT $ WRITE(6,1) ZETA,OMEGA,T1 $ STOP
    8 T2=T1-DT $ C2=CT2(ZETA,OMEGA,.9,T2) $ DO 11 J=1,20
      T3=(C2*T1-C1*T2)/(C2-C1) $ C3=CT2(ZETA,OMEGA,.9,T3)
      IF(C3*C1.LT.0.)GO TO 9 $ T1=T2 $ C1=C2
    9 T2=(C3*T1-C1*T3)/(C3-C1) $ C2=CT2(ZETA,OMEGA,.9,T2)
      IF(C2*C1.LT.0.)GO TO 10 $ T1=T3 $ C1=C3
   10 IF(ABS(T1-T2).LE..1*AT.O.ABS(C1-C2).LE..001)RETURN
   11 CONTINUE $ WRITE(6,2) ZETA,OMEGA $ RETURN $ END
```

```
      FUNCTION CT3(LAMBDA,ZETA,OMEGA,C,T)
      REAL LAMBDA $ A=LAMBDA*ZETA**2*(LAMBDA-2.)+1.
      IF(ZETA.GE.1.) GO TO 2 $ B=OMEGA*SQRT(1.-ZETA**2)
    1 CT3=1.-EXP(-LAMBDA*ZETA*OMEGA*T)/A+EXP(-ZETA*OMEGA*T)*((1.-A)/A*CO
     2S(B*T)+ZETA*(1.-A-LAMBDA)/(A*B/OMEGA)*SIN(B*T))-C $ RETURN
    2 IF(ZETA.EQ.1.) GO TO 3 $ B=OMEGA*SQRT(ZETA**2-1.)
      D=OMEGA*ZETA*(1.-A-LAMBDA)/B
      CT3=1.+1./A*(-EXP(-LAMBDA*ZETA*OMEGA*T)+(1.-A-D)/2.*EXP((-ZETA*
     2OMEGA-B)*T)+(1.-A+D)/2.*EXP((-ZETA*OMEGA+B)*T))-C $ RETURN
    3 IF(LAMBDA.EQ.1.) GO TO 4
      CT3=1.+(1./(1.-LAMBDA)**2)*(-EXP(-LAMBDA*OMEGA*T)+LAMBDA*(2.-LAMBD
     2A+OMEGA*T-LAMBDA*OMEGA*T)*EXP(-OMEGA*T))-C $ RETURN
    4 D=EXP(-OMEGA*T) $ CT3=1.-D-OMEGA*T*D-OMEGA**2*T**2*D/2.-C $ RETURN
      END
```

```
      FUNCTION PCT3(LAMBDA,ZETA,OMEGA,T)
      REAL LAMBDA $ T1=T*OMEGA $ A=LAMBDA*ZETA**2*(LAMBDA-2.)+1.
      IF(ZETA.GE.1.) GO TO 1 $ B=SQRT(1.-ZETA**2)
      B=SQRT(1.-ZETA**2)
      PCT3=(LAMBDA*ZETA*EXP(-LAMBDA*ZETA*T1)/A+EXP(-ZETA*T1)*(-LAMBDA*ZE
     2TA/A*COS(B*T1)+((ZETA**2+B**2)*(A-1.)+LAMBDA*ZETA**2)/(A*B)*SIN(B*
     3T1)))*OMEGA $ RETURN
    1 PCT3=0.0 $ RETURN $ END
```

```
      FUNCTION CT2(ZETA,OMEGA,C,T1) $ T=OMEGA*T1 $ IF(ZETA-1.)1,2,3
    1 A=SQRT(1.-ZETA**2)
      CT2=1.-EXP(-ZETA*T)/A*COS(A*T-ATAN2(ZETA,A))-C $ RETURN
    2 CT2=1.-(T+1.)*EXP(-T)-C $ RETURN
    3 A=SQRT(ZETA**2-1.) $ CT2=1.+.5*(EXP((-ZETA-A)*T)/(A**2+A*ZETA)+
     2EXP((-ZETA+A)*T)/(A**2-A*ZETA))-C $ RETURN $ END
```

A.5 Numerical Examples

SECOND-ORDER PLANT
DESIGN SPECIFICATIONS

P1 = 0.
P21= 8.00000E+00 P22= 1.00000E+01
PK1= 1.00000E+00 PK2= 2.20000E+00

TRS= 1.000000 OVS= .100000

SECOND-ORDER DESIGN

POINT A TR= 1.002024 OV=0 000000

POINT B TR= .354570 OV= .093584

CK= 2.00630E+01

THIRD-ORDER DESIGN

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.099901	1.65003E+02		1.73003E+02	1.32002E+03	7.06219E+03
RISE TIME ADJUSTMENT	TR=	.997874		3.29441E+03	1.75003E+02	1.65003E+03	3.29441E+03
OVERSHOOT ADJUSTMENT	OV=	.099895	1.69079E+02		1.77079E+02	1.35263E+03	7.24771E+03
RISE TIME ADJUSTMENT	TR=	.997955		3.37626E+03	1.79079E+02	1.69079E+03	3.37626E+03
OVERSHOOT ADJUSTMENT	OV=	.099896	1.73028E+02		1.81028E+02	1.38423E+03	7.42777E+03
RISE TIME ADJUSTMENT	TR=	.998032		3.45554E+03	1.83028E+02	1.73028E+03	3.45554E+03
OVERSHOOT ADJUSTMENT	OV=	.099897	1.76853E+02		1.84853E+02	1.41483E+03	7.60220E+03
RISE TIME ADJUSTMENT	TR=	.998108		3.53232E+03	1.86853E+02	1.76853E+03	3.53232E+03
OVERSHOOT ADJUSTMENT	OV=	.099899	1.80556E+02		1.88556E+02	1.44444E+03	7.77109E+03
RISE TIME ADJUSTMENT	TR=	.998181		3.60662E+03	1.90556E+02	1.80556E+03	3.60662E+03
OVERSHOOT ADJUSTMENT	OV=	.099901	1.84138E+02		1.92138E+02	1.47311E+03	7.93456E+03
RISE TIME ADJUSTMENT	TR=	.998253		3.67851E+03	1.94138E+02	1.84138E+03	3.67851E+03
OVERSHOOT ADJUSTMENT	OV=	.099903	1.87604E+02		1.95604E+02	1.50083E+03	8.09272E+03
RISE TIME ADJUSTMENT	TR=	.998322		3.74804E+03	1.97604E+02	1.87604E+03	3.74804E+03
OVERSHOOT ADJUSTMENT	OV=	.099906	1.90955E+02		1.98955E+02	1.52764E+03	8.24569E+03
RISE TIME ADJUSTMENT	TR=	.998389		3.81527E+03	2.00955E+02	1.90955E+03	3.81527E+03
OVERSHOOT ADJUSTMENT	OV=	.099908	1.94194E+02		2.02194E+02	1.55355E+03	8.39359E+03
RISE TIME ADJUSTMENT	TR=	.998453		3.88025E+03	2.04194E+02	1.94194E+03	3.88025E+03
OVERSHOOT ADJUSTMENT	OV=	.099911	1.97325E+02		2.05325E+02	1.57860E+03	8.53654E+03
RISE TIME ADJUSTMENT	TR=	.998515		3.94304E+03	2.07325E+02	1.97325E+03	3.94304E+03

THIRD-ORDER DESIGN CONTINUED

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.099914	2.00349E+02		2.08349E+02	1.60279E+03	8.67468E+03
RISE TIME ADJUSTMENT	TR=	.998575		4.00370E+03	2.10349E+02	2.00349E+03	4.00370E+03
OVERSHOOT ADJUSTMENT	OV=	.099916	2.03271E+02		2.11271E+02	1.62616E+03	8.80813E+03
RISE TIME ADJUSTMENT	TR=	.998633		4.06228E+03	2.13271E+02	2.03271E+03	4.06228E+03
OVERSHOOT ADJUSTMENT	OV=	.099919	2.06092E+02		2.14092E+02	1.64873E+03	8.93702E+03
RISE TIME ADJUSTMENT	TR=	.998688		4.11885E+03	2.16092E+02	2.06092E+03	4.11885E+03
OVERSHOOT ADJUSTMENT	OV=	.099922	2.08816E+02		2.16816E+02	1.67052E+03	9.04147E+03
RISE TIME ADJUSTMENT	TR=	.998741		4.17347E+03	2.18816E+02	2.08816E+03	4.17347E+03
OVERSHOOT ADJUSTMENT	OV=	.099924	2.11445E+02		2.19445E+02	1.69156E+03	9.18163E+03
RISE TIME ADJUSTMENT	TR=	.998792		4.22618E+03	2.21445E+02	2.11445E+03	4.22618E+03
OVERSHOOT ADJUSTMENT	OV=	.099927	2.13983E+02		2.21983E+02	1.71186E+03	9.29760E+03
RISE TIME ADJUSTMENT	TR=	.998841		4.27706E+03	2.23983E+02	2.13983E+03	4.27706E+03
OVERSHOOT ADJUSTMENT	OV=	.099929	2.16431E+02		2.24431E+02	1.73145E+03	9.40952E+03
RISE TIME ADJUSTMENT	TR=	.998888		4.32614E+03	2.26431E+02	2.16431E+03	4.32614E+03
OVERSHOOT ADJUSTMENT	OV=	.099932	2.18794E+02		2.26794E+02	1.75035E+03	9.51752E+03
RISE TIME ADJUSTMENT	TR=	.998933		4.37350E+03	2.28794E+02	2.18794E+03	4.37350E+03
OVERSHOOT ADJUSTMENT	OV=	.099934	2.21073E+02		2.29073E+02	1.76858E+03	9.62170E+03
RISE TIME ADJUSTMENT	TR=	.998976		4.41918E+03	2.31073E+02	2.21073E+03	4.41918E+03
OVERSHOOT ADJUSTMENT	OV=	.099937	2.23271E+02		2.31271E+02	1.78617E+03	9.72220E+03
RISE TIME ADJUSTMENT	TR=	.999018		4.46324E+03	2.33271E+02	2.23271E+03	4.46324E+03
OVERSHOOT ADJUSTMENT	OV=	.099939	2.25391E+02		2.33391E+02	1.80313E+03	9.81913E+03
RISE TIME ADJUSTMENT	TR=	.999057		4.50573E+03	2.35391E+02	2.25391E+03	4.50573E+03
OVERSHOOT ADJUSTMENT	OV=	.099941	2.27435E+02		2.35435E+02	1.81948E+03	9.91260E+03
RISE TIME ADJUSTMENT	TR=	.999095		4.54669E+03	2.37435E+02	2.27435E+03	4.54669E+03
OVERSHOOT ADJUSTMENT	OV=	.099943	2.29406E+02		2.37406E+02	1.83525E+03	1.00027E+04
RISE TIME ADJUSTMENT	TR=	.999132		4.58619E+03	2.39406E+02	2.29406E+03	4.58619E+03
OVERSHOOT ADJUSTMENT	OV=	.099945	2.31306E+02		2.39306E+02	1.85045E+03	1.00896E+04
RISE TIME ADJUSTMENT	TR=	.999167		4.62427E+03	2.41306E+02	2.31306E+03	4.62427E+03
OVERSHOOT ADJUSTMENT	OV=	.099947	2.33137E+02		2.41137E+02	1.86510E+03	1.01734E+04
RISE TIME ADJUSTMENT	TR=	.999200		4.66097E+03	2.43137E+02	2.33137E+03	4.66097E+03

THIRD-ORDER DESIGN CONTINUED

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.099949	2.34903E+02		2.42903E+02	1.87922E+03	1.02541E+04
RISE TIME ADJUSTMENT	TR=	.999232		4.69634E+03	2.44903E+02	2.34903E+03	4.69634E+03
OVERSHOOT ADJUSTMENT	OV=	.099951	2.36604E+02		2.44604E+02	1.89283E+03	1.03320E+04
RISE TIME ADJUSTMENT	TR=	.999263		4.73044E+03	2.46604E+02	2.36604E+03	4.73044E+03
OVERSHOOT ADJUSTMENT	OV=	.099953	2.38244E+02		2.46244E+02	1.90595E+03	1.04070E+04
RISE TIME ADJUSTMENT	TR=	.999293		4.76329E+03	2.48244E+02	2.38244E+03	4.76329E+03
OVERSHOOT ADJUSTMENT	OV=	.099955	2.39824E+02		2.47824E+02	1.91859E+03	1.04792E+04
RISE TIME ADJUSTMENT	TR=	.999321		4.79494E+03	2.49824E+02	2.39824E+03	4.79494E+03
OVERSHOOT ADJUSTMENT	OV=	.099957	2.41346E+02		2.49346E+02	1.93077E+03	1.05489E+04
RISE TIME ADJUSTMENT	TR=	.999348		4.82544E+03	2.51346E+02	2.41346E+03	4.82544E+03
OVERSHOOT ADJUSTMENT	OV=	.099958	2.42813E+02		2.50813E+02	1.94250E+03	1.06160E+04
RISE TIME ADJUSTMENT	TR=	.999374		4.85483E+03	2.52813E+02	2.42813E+03	4.85483E+03
OVERSHOOT ADJUSTMENT	OV=	.099960	2.44226E+02		2.52226E+02	1.95381E+03	1.06806E+04
RISE TIME ADJUSTMENT	TR=	.999399		4.88314E+03	2.54226E+02	2.44226E+03	4.88314E+03
OVERSHOOT ADJUSTMENT	OV=	.099961	2.45587E+02		2.53587E+02	1.96469E+03	1.07429E+04
RISE TIME ADJUSTMENT	TR=	.999423		4.91041E+03	2.55587E+02	2.45587E+03	4.91041E+03

THIRD-ORDER SOLUTION PC= 2.45587E+02 CK= 4.91041E+03

	TR	OV	LAMBDA	ZETA	OMEGA	ALPHA	BETA	GAMMA
POINT 1	.750893	.002466	62.0702	.8853	4.4708	2.53587E+02	1.96469E+03	4.91041E+03
POINT 2	.999423	0 000000	.0223	3.0262	41.7646	2.55587E+02	2.45587E+03	4.91041E+03
POINT 3	.354683	.100994	62.8963	.5894	6.6299	2.53587E+02	1.96469E+03	1.08029E+04
POINT 4	.420792	.031437	50.0884	.7401	6.6298	2.55587E+02	2.45587E+03	1.08029E+04

SECOND-ORDER PLANT
DESIGN SPECIFICATIONS

P1 = 0.
P21= 8.00000E+00 P22= 1.00000E+01
PK1= 1.00000E+00 PK2= 1.50000E+00

TRS= 1.000000 OVS= .100000

SECOND-ORDER DESIGN

POINT A TR= 1.002133 OV=0 000000

POINT B TR= .496849 OV= .035174

CK= 2.00611E+01

THIRD-ORDER DESIGN

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.132245	1.00106E+02		1.08106E+02	8.00848E+02	4.81467E+03
RISE TIME ADJUSTMENT	TR=	1.614427		1.30125E+03	1.10106E+02	1.00106E+03	1.30125E+03
OVERSHOOT ADJUSTMENT	OV=	.109544	5.00530E+01		5.80530E+01	4.00424E+02	1.95187E+03
RISE TIME ADJUSTMENT	TR=	1.096073		9.04848E+02	6.00530E+01	5.00530E+02	9.04848E+02
OVERSHOOT ADJUSTMENT	OV=	.132403	3.43699E+01		4.23699E+01	2.74960E+02	1.35727E+03
RISE TIME ADJUSTMENT	TR=	1.132755		5.97233E+02	4.43699E+01	3.43699E+02	5.97233E+02
OVERSHOOT ADJUSTMENT	OV=	.123314	2.55933E+01		3.35933E+01	2.04746E+02	8.95849E+02
RISE TIME ADJUSTMENT	TR=	1.033881		4.74034E+02	3.55933E+01	2.55933E+02	4.74034E+02
OVERSHOOT ADJUSTMENT	OV=	.102765	2.28462E+01		3.08462E+01	1.82769E+02	7.11051E+02
RISE TIME ADJUSTMENT	TR=	.998266		4.32971E+02	3.28462E+01	2.28462E+02	4.32971E+02
OVERSHOOT ADJUSTMENT	OV=	.100242	2.14983E+01		2.94983E+01	1.71986E+02	6.49457E+02
RISE TIME ADJUSTMENT	TR=	.996432		4.06592E+02	3.14983E+01	2.14983E+02	4.06592E+02
OVERSHOOT ADJUSTMENT	OV=	.099899	2.05270E+01		2.85270E+01	1.64216E+02	6.09888E+02
RISE TIME ADJUSTMENT	TR=	.996247		3.87183E+02	3.05270E+01	2.05270E+02	3.87183E+02
OVERSHOOT ADJUSTMENT	OV=	.099759	1.97978E+01		2.77978E+01	1.58383E+02	5.80774E+02
RISE TIME ADJUSTMENT	TR=	.996448		3.72516E+02	2.97978E+01	1.97978E+02	3.72516E+02
OVERSHOOT ADJUSTMENT	OV=	.099710	1.92394E+01		2.72394E+01	1.53915E+02	5.58774E+02
RISE TIME ADJUSTMENT	TR=	.996803		3.61233E+02	2.92394E+01	1.92394E+02	3.61233E+02
OVERSHOOT ADJUSTMENT	OV=	.099706	1.88055E+01		2.68055E+01	1.50444E+02	5.41850E+02
RISE TIME ADJUSTMENT	TR=	.997204		3.52439E+02	2.88055E+01	1.88055E+02	3.52439E+02

THIRD-ORDER DESIGN CONTINUED

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.099724	1.84646E+01		2.64646E+01	1.47716E+02	5.28659E+02
RISE TIME ADJUSTMENT	TR=	.997598		3.45515E+02	2.84646E+01	1.84646E+02	3.45515E+02
OVERSHOOT ADJUSTMENT	OV=	.099752	1.81945E+01		2.61945E+01	1.45556E+02	5.18273E+02
RISE TIME ADJUSTMENT	TR=	.997962		3.40022E+02	2.81945E+01	1.81945E+02	3.40022E+02
OVERSHOOT ADJUSTMENT	OV=	.099783	1.79792E+01		2.59792E+01	1.43834E+02	5.10033E+02
RISE TIME ADJUSTMENT	TR=	.998285		3.35638E+02	2.79792E+01	1.79792E+02	3.35638E+02
OVERSHOOT ADJUSTMENT	OV=	.099813	1.78067E+01		2.58067E+01	1.42454E+02	5.03456E+02
RISE TIME ADJUSTMENT	TR=	.998566		3.32121E+02	2.78067E+01	1.78067E+02	3.32121E+02
OVERSHOOT ADJUSTMENT	OV=	.099841	1.76679E+01		2.56679E+01	1.41343E+02	4.98181E+02
RISE TIME ADJUSTMENT	TR=	.998807		3.29289E+02	2.76679E+01	1.76679E+02	3.29289E+02
OVERSHOOT ADJUSTMENT	OV=	.099866	1.75559E+01		2.55559E+01	1.40447E+02	4.93934E+02
RISE TIME ADJUSTMENT	TR=	.999010		3.27003E+02	2.75559E+01	1.75559E+02	3.27003E+02
OVERSHOOT ADJUSTMENT	OV=	.099888	1.74653E+01		2.54653E+01	1.39722E+02	4.90504E+02
RISE TIME ADJUSTMENT	TR=	.999181		3.25152E+02	2.74653E+01	1.74653E+02	3.25152E+02
OVERSHOOT ADJUSTMENT	OV=	.099907	1.73918E+01		2.53918E+01	1.39134E+02	4.87728E+02
RISE TIME ADJUSTMENT	TR=	.999325		3.23650E+02	2.73918E+01	1.73918E+02	3.23650E+02
OVERSHOOT ADJUSTMENT	OV=	.099922	1.73321E+01		2.53321E+01	1.38656E+02	4.85475E+02
RISE TIME ADJUSTMENT	TR=	.999443		3.22430E+02	2.73321E+01	1.73321E+02	3.22430E+02
OVERSHOOT ADJUSTMENT	OV=	.099936	1.72835E+01		2.52835E+01	1.38268E+02	4.83646E+02
RISE TIME ADJUSTMENT	TR=	.999542		3.21438E+02	2.72835E+01	1.72835E+02	3.21438E+02

THIRD-ORDER SOLUTION PC= 1.72835E+01 CK= 3.21438E+02

	TR	OV	LAMBDA	ZETA	OMEGA	ALPHA	BETA	GAMMA
POINT 1	.761055	.019644	5.8651	.7786	4.1290	2.52835E+01	1.38268E+02	3.21438E+02
POINT 2	.999542	0.000000	4.6886	.9950	4.0996	2.72835E+01	1.72835E+02	3.21438E+02
POINT 3	.521777	.099157	6.6667	.5859	4.9791	2.52835E+01	1.38268E+02	4.82157E+02
POINT 4	.630470	.024307	5.2694	.7601	4.9376	2.72835E+01	1.72835E+02	4.82157E+02

SECOND-ORDER PLANT
DESIGN SPECIFICATIONS

P1 = 0.
P21= 5.00000E-02 P22= 7.00000E-02
PK1= 1.00000E+00 PK2= 1.40000E+00

TRS=150.000000 OVS= .180000

SECOND-ORDER DESIGN

POINT A TR=150.245635 OV=0 000000

POINT B TR= 71.437613 OV= .050765

CK= 9.42372E-04

THIRD-ORDER DESIGN

		PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV= .388361	3.16080E-01		3.66080E-01	1.58040E-02	1.31932E-03
RISE TIME ADJUSTMENT	TR= 85.285854		4.71186E-04	3.86080E-01	2.21256E-02	4.71186E-04
OVERSHOOT ADJUSTMENT	OV= .179836	3.21019E-01		3.71019E-01	1.60509E-02	6.59660E-04
RISE TIME ADJUSTMENT	TR=237.058251		1.97459E-04	3.91019E-01	2.24713E-02	1.97459E-04
OVERSHOOT ADJUSTMENT	OV= .186828	1.60509E-01		2.10509E-01	8.02547E-03	2.74442E-04
RISE TIME ADJUSTMENT	TR=173.786829		1.26460E-04	2.30509E-01	1.12357E-02	1.26460E-04
OVERSHOOT ADJUSTMENT	OV= .253267	1.00877E-01		1.50877E-01	5.04384E-03	1.77044E-04
RISE TIME ADJUSTMENT	TR=194.686648		7.02891E-05	1.70877E-01	7.06137E-03	7.02891E-05
OVERSHOOT ADJUSTMENT	OV= .226645	7.09812E-02		1.20981E-01	3.54906E-03	9.84047E-05
RISE TIME ADJUSTMENT	TR=152.664095		5.87307E-05	1.40981E-01	4.96868E-03	5.87307E-05
OVERSHOOT ADJUSTMENT	OV= .179671	6.90044E-02		1.19004E-01	3.45022E-03	8.22229E-05
RISE TIME ADJUSTMENT	TR=149.703001		5.79236E-05	1.39004E-01	4.83031E-03	5.79236E-05
OVERSHOOT ADJUSTMENT	OV= .179795	6.83376E-02		1.18338E-01	3.41688E-03	8.10931E-05
RISE TIME ADJUSTMENT	TR=149.760282		5.73037E-05	1.38338E-01	4.78363E-03	5.73037E-05
OVERSHOOT ADJUSTMENT	OV= .179834	6.78319E-02		1.17832E-01	3.39159E-03	8.02252E-05
RISE TIME ADJUSTMENT	TR=149.810416		5.68321E-05	1.37832E-01	4.74823E-03	5.68321E-05
OVERSHOOT ADJUSTMENT	OV= .179868	6.74459E-02		1.17446E-01	3.37229E-03	7.95650E-05
RISE TIME ADJUSTMENT	TR=149.850877		5.64719E-05	1.37446E-01	4.72121E-03	5.64719E-05
OVERSHOOT ADJUSTMENT	OV= .179895	6.71502E-02		1.17150E-01	3.35751E-03	7.90607E-05
RISE TIME ADJUSTMENT	TR=149.883156		5.61959E-05	1.37150E-01	4.70051E-03	5.61959E-05

THIRD-ORDER SOLUTION PC= 6.71502E-02 CK= 5.61959E-05

	TP	OV	LAMBDA	ZETA	OMEGA	ALPHA	BETA	GAMMA
POINT 1	106.434284	.081138	5.4251	.6158	.0256	1.17150E-01	3.35751E-03	5.61959E-05
POINT 2	149.883156	.001691	4.2376	.8953	.0246	1.37150E-01	4.70051E-03	5.61959E-05
POINT 3	81.719793	.178293	6.4576	.4670	.0297	1.17150E-01	3.35751E-03	7.86743E-05
POINT 4	104.571335	.042034	4.8453	.7038	.0285	1.37150E-01	4.70051E-03	7.86743E-05

SECOND-ORDER PLANT
DESIGN SPECIFICATIONS

P1 = 0.
P21= 3.55000E+03 P22= 3.70000E+03
PK1= 1.00000E+03 PK2= 1.08000E+03

TRS= .001000 OVS= .090000

SECOND-ORDER DESIGN

POINT A TR= .001001 OV= .045211

POINT B TR= .000905 OV= .069015

CK= 6.94541E+03

THIRD-ORDER DESIGN

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.090069	6.70008E+04		7.05508E+04	2.37853E+08	5.32574E+11
RISE TIME ADJUSTMENT	TR=	.000997		4.63222E+08	7.07008E+04	2.47903E+08	4.63222E+11
OVERSHOOT ADJUSTMENT	OV=	.090078	6.31891E+04		6.67391E+04	2.24321E+08	5.00280E+11
RISE TIME ADJUSTMENT	TR=	.000997		4.36711E+08	6.68891E+04	2.33800E+08	4.36711E+11
OVERSHOOT ADJUSTMENT	OV=	.090035	5.98232E+04		6.33732E+04	2.12372E+08	4.71648E+11
RISE TIME ADJUSTMENT	TR=	.000997		4.13293E+08	6.35232E+04	2.21346E+08	4.13293E+11
OVERSHOOT ADJUSTMENT	OV=	.089997	5.68469E+04		6.03969E+04	2.01806E+08	4.46356E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.92575E+08	6.05469E+04	2.10333E+08	3.92575E+11
OVERSHOOT ADJUSTMENT	OV=	.089965	5.42110E+04		5.77610E+04	1.92449E+08	4.23981E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.74218E+08	5.79110E+04	2.00581E+08	3.74218E+11
OVERSHOOT ADJUSTMENT	OV=	.089938	5.18730E+04		5.54230E+04	1.84149E+08	4.04155E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.57928E+08	5.55730E+04	1.91930E+08	3.57928E+11
OVERSHOOT ADJUSTMENT	OV=	.089916	4.97960E+04		5.33460E+04	1.76776E+08	3.86562E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.43451E+08	5.34960E+04	1.84245E+08	3.43451E+11
OVERSHOOT ADJUSTMENT	OV=	.089897	4.79483E+04		5.14983E+04	1.70216E+08	3.70927E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.30566E+08	5.16483E+04	1.77409E+08	3.30566E+11
OVERSHOOT ADJUSTMENT	OV=	.089883	4.63020E+04		4.98520E+04	1.64372E+08	3.57012E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.19082E+08	5.00020E+04	1.71317E+08	3.19082E+11
OVERSHOOT ADJUSTMENT	OV=	.089873	4.48332E+04		4.83832E+04	1.59158E+08	3.44608E+11
RISE TIME ADJUSTMENT	TR=	.000997		3.08832E+08	4.85332E+04	1.65883E+08	3.08832E+11

THIRD-ORDER DESIGN CONTINUED

				PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT	ADJUSTMENT	OV=	.089866	4.35210E+04		4.70710E+04	1.54500E+08	3.33538E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.99671E+08	4.72210E+04	1.61028E+08	2.99671E+11
OVERSHOOT	ADJUSTMENT	OV=	.089861	4.23471E+04		4.58971E+04	1.50332E+08	3.23644E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.91473E+08	4.60471E+04	1.56684E+08	2.91473E+11
OVERSHOOT	ADJUSTMENT	OV=	.089859	4.12957E+04		4.48457E+04	1.46600E+08	3.14791E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.84128E+08	4.49957E+04	1.52794E+08	2.84128E+11
OVERSHOOT	ADJUSTMENT	OV=	.089859	4.03528E+04		4.39028E+04	1.43252E+08	3.06858E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.77540E+08	4.40528E+04	1.49305E+08	2.77540E+11
OVERSHOOT	ADJUSTMENT	OV=	.089860	3.95064E+04		4.30564E+04	1.40248E+08	2.99743E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.71624E+08	4.32064E+04	1.46174E+08	2.71624E+11
OVERSHOOT	ADJUSTMENT	OV=	.089863	3.87457E+04		4.22957E+04	1.37547E+08	2.93353E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.66306E+08	4.24457E+04	1.43359E+08	2.66306E+11
OVERSHOOT	ADJUSTMENT	OV=	.089867	3.80615E+04		4.16115E+04	1.35118E+08	2.87610E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.61521E+08	4.17615E+04	1.40828E+08	2.61521E+11
OVERSHOOT	ADJUSTMENT	OV=	.089872	3.74454E+04		4.09954E+04	1.32931E+08	2.82443E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.57212E+08	4.11454E+04	1.38548E+08	2.57212E+11
OVERSHOOT	ADJUSTMENT	OV=	.089877	3.68902E+04		4.04402E+04	1.30960E+08	2.77789E+11
RISE TIME	ADJUSTMENT	TR=	.000998		2.53328E+08	4.05902E+04	1.36494E+08	2.53328E+11
OVERSHOOT	ADJUSTMENT	OV=	.089883	3.63896E+04		3.99396E+04	1.29183E+08	2.73595E+11
RISE TIME	ADJUSTMENT	TR=	.000999		2.49825E+08	4.00896E+04	1.34641E+08	2.49825E+11
OVERSHOOT	ADJUSTMENT	OV=	.089889	3.59377E+04		3.94877E+04	1.27579E+08	2.69811E+11
RISE TIME	ADJUSTMENT	TR=	.000999		2.46663E+08	3.96377E+04	1.32969E+08	2.46663E+11
OVERSHOOT	ADJUSTMENT	OV=	.089895	3.55295E+04		3.90795E+04	1.26130E+08	2.66396E+11
RISE TIME	ADJUSTMENT	TR=	.000999		2.43807E+08	3.92295E+04	1.31459E+08	2.43807E+11
OVERSHOOT	ADJUSTMENT	OV=	.089901	3.51607E+04		3.87107E+04	1.24821E+08	2.63311E+11
RISE TIME	ADJUSTMENT	TR=	.000999		2.41225E+08	3.88607E+04	1.30095E+08	2.41225E+11
OVERSHOOT	ADJUSTMENT	OV=	.089908	3.48272E+04		3.83772E+04	1.23637E+08	2.60523E+11
RISE TIME	ADJUSTMENT	TR=	.000999		2.38890E+08	3.85272E+04	1.28861E+08	2.38890E+11
OVERSHOOT	ADJUSTMENT	OV=	.089914	3.45254E+04		3.80754E+04	1.22565E+08	2.58002E+11
RISE TIME	ADJUSTMENT	TR=	.000999		2.36778E+08	3.82254E+04	1.27744E+08	2.36778E+11

THIRD-ORDER DESIGN CONTINUED

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.089919	3.42523E+04		3.78023E+04	1.21596E+08	2.55720E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.34865E+08	3.79523E+04	1.26734E+08	2.34865E+11
OVERSHOOT ADJUSTMENT	OV=	.089925	3.40049E+04		3.75549E+04	1.20718E+08	2.53654E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.33133E+08	3.77049E+04	1.25818E+08	2.33133E+11
OVERSHOOT ADJUSTMENT	OV=	.089930	3.37808E+04		3.73308E+04	1.19922E+08	2.51784E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.31563E+08	3.74808E+04	1.24989E+08	2.31563E+11
OVERSHOOT ADJUSTMENT	OV=	.089935	3.35777E+04		3.71277E+04	1.19201E+08	2.50089E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.30141E+08	3.72777E+04	1.24237E+08	2.30141E+11
OVERSHOOT ADJUSTMENT	OV=	.089940	3.33935E+04		3.69435E+04	1.18547E+08	2.48552E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.28851E+08	3.70935E+04	1.23556E+08	2.28851E+11
OVERSHOOT ADJUSTMENT	OV=	.089945	3.32265E+04		3.67765E+04	1.17954E+08	2.47159E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.27681E+08	3.69265E+04	1.22938E+08	2.27681E+11
OVERSHOOT ADJUSTMENT	OV=	.089949	3.30750E+04		3.66250E+04	1.17416E+08	2.45895E+11
RISE TIME ADJUSTMENT	TR=	.000999		2.26619E+08	3.67750E+04	1.22377E+08	2.26619E+11
OVERSHOOT ADJUSTMENT	OV=	.089953	3.29374E+04		3.64874E+04	1.16928E+08	2.44749E+11
RISE TIME ADJUSTMENT	TR=	.001000		2.25656E+08	3.66374E+04	1.21868E+08	2.25656E+11

THIRD-ORDER SOLUTION PC= 3.29374E+04 CK= 2.25656E+08

	TR	OV	LAMBDA	ZETA	OMEGA	ALPHA	BETA	GAMMA
POINT 1	.000969	.074556	19.9785	.6365	2608.3709	3.64874E+04	1.16928E+08	2.25656E+11
POINT 2	.001000	.060723	19.1220	.6650	2608.3256	3.66374E+04	1.21868E+08	2.25656E+11
POINT 3	.000907	.089121	20.0989	.6093	2709.9596	3.64874E+04	1.16928E+08	2.43708E+11
POINT 4	.000934	.074403	19.2332	.6367	2709.9090	3.66374E+04	1.21868E+08	2.43708E+11

SECOND-ORDER PLANT
DESIGN SPECIFICATIONS

P1 = 3.00000E-01
P21= 1.00000E+00 P22= 1.00000E+00
PK1= 1.00000E+00 PK2= 1.00000E+00

TRS= 1.500000 OVS= .150000

THIRD-ORDER DESIGN

CORRECTED INITIAL GUESSES

			PC	CK	ALPHA	BETA	GAMMA
		PC=	1.00000E+00	CK= 2.50000E+00	2.30000E+00	1.40000E+00	2.80000E+00
OVERSHOOT ADJUSTMENT	OV=	.344690	1.97108E+00		3.27108E+00	2.86240E+00	3.09132E+00
RISE TIME ADJUSTMENT	TR=	1.633666		4.00179E+00	3.27108E+00	2.86240E+00	4.59312E+00
OVERSHOOT ADJUSTMENT	OV=	.269270	3.25650E+00		4.55650E+00	4.53346E+00	4.97875E+00
RISE TIME ADJUSTMENT	TR=	1.581788		5.85420E+00	4.55650E+00	4.53346E+00	6.83115E+00
OVERSHOOT ADJUSTMENT	OV=	.221524	4.88789E+00		6.18789E+00	6.65426E+00	7.32057E+00
RISE TIME ADJUSTMENT	TR=	1.562498		8.20029E+00	6.18789E+00	6.65426E+00	9.66666E+00
OVERSHOOT ADJUSTMENT	OV=	.193705	6.93082E+00		8.23082E+00	9.31007E+00	1.02795E+01
RISE TIME ADJUSTMENT	TR=	1.552936		1.12180E+01	8.23082E+00	9.31007E+00	1.32973E+01
OVERSHOOT ADJUSTMENT	OV=	.178068	9.47104E+00		1.07710E+01	1.26123E+01	1.40593E+01
RISE TIME ADJUSTMENT	TR=	1.545650		1.50725E+01	1.07710E+01	1.26123E+01	1.79138E+01
OVERSHOOT ADJUSTMENT	OV=	.169227	1.26103E+01		1.39103E+01	1.66935E+01	1.88556E+01
RISE TIME ADJUSTMENT	TR=	1.539195		1.99255E+01	1.39103E+01	1.66935E+01	2.37086E+01
OVERSHOOT ADJUSTMENT	OV=	.164036	1.64713E+01		1.77713E+01	2.17127E+01	2.48669E+01
RISE TIME ADJUSTMENT	TR=	1.533665		2.59591E+01	1.77713E+01	2.17127E+01	3.09005E+01
OVERSHOOT ADJUSTMENT	OV=	.160826	2.12014E+01		2.25014E+01	2.78618E+01	3.23195E+01
RISE TIME ADJUSTMENT	TR=	1.529161		3.33936E+01	2.25014E+01	2.78618E+01	3.97540E+01
OVERSHOOT ADJUSTMENT	OV=	.158737	2.69777E+01		2.82777E+01	3.53710E+01	4.14869E+01
RISE TIME ADJUSTMENT	TR=	1.525594		4.25002E+01	2.82777E+01	3.53710E+01	5.05935E+01
OVERSHOOT ADJUSTMENT	OV=	.157319	3.40143E+01		3.53143E+01	4.45185E+01	5.27045E+01
RISE TIME ADJUSTMENT	TR=	1.522799		5.36116E+01	3.53143E+01	4.45185E+01	6.38158E+01

THIRD-ORDER DIVERGING PC= 3.40143E+01 CK= 5.36116E+01

	TR	OV	LAMBDA	ZETA	OMEGA	ALPHA	BETA	GAMMA
POINT 1	1.522799	.198589	54.4138	.4573	1.3688	3.53143E+01	4.45185E+01	6.38158E+01
POINT 2	1.522799	.198589	54.4138	.4573	1.3688	3.53143E+01	4.45185E+01	6.38158E+01
POINT 3	1.522799	.198589	54.4138	.4573	1.3688	3.53143E+01	4.45185E+01	6.38158E+01
POINT 4	1.522799	.198589	54.4138	.4573	1.3688	3.53143E+01	4.45185E+01	6.38158E+01

SECOND-ORDER PLANT
DESIGN SPECIFICATIONS

P1 = 3.00000E-01
P21= 3.00000E+00 P22= 3.00000E+00
PK1= 1.00000E+00 PK2= 1.00000E+00

TRS= 1.500000 OVS= .150000

THIRD-ORDER DESIGN

			PC	CK	ALPHA	BETA	GAMMA
OVERSHOOT ADJUSTMENT	OV=	.249386	4.81149E+00		8.11149E+00	1.67779E+01	3.43303E+01
RISE TIME ADJUSTMENT	TR=	2.974179		6.07881E+00	8.11149E+00	1.67779E+01	1.04091E+01
OVERSHOOT ADJUSTMENT	OV=	.092952	2.40574E+00		5.70574E+00	8.83896E+00	8.24398E+00
RISE TIME ADJUSTMENT	TR=	1.567375		8.43682E+00	5.70574E+00	8.83896E+00	1.06020E+01
OVERSHOOT ADJUSTMENT	OV=	.150061	2.53430E+00		5.83430E+00	9.26318E+00	1.07177E+01
RISE TIME ADJUSTMENT	TR=	1.499207		9.42410E+00	5.83430E+00	9.26318E+00	1.17050E+01
OVERSHOOT ADJUSTMENT	OV=	.150691	2.72921E+00		6.02921E+00	9.90639E+00	1.18804E+01
RISE TIME ADJUSTMENT	TR=	1.497292		9.91932E+00	6.02921E+00	9.90639E+00	1.23756E+01
OVERSHOOT ADJUSTMENT	OV=	.149842	2.83310E+00		6.13310E+00	1.02492E+01	1.24691E+01
RISE TIME ADJUSTMENT	TR=	1.498209		1.01660E+01	6.13310E+00	1.02492E+01	1.27158E+01
OVERSHOOT ADJUSTMENT	OV=	.149800	2.88152E+00		6.18152E+00	1.04090E+01	1.27594E+01
RISE TIME ADJUSTMENT	TR=	1.499102		1.02765E+01	6.18152E+00	1.04090E+01	1.28699E+01
OVERSHOOT ADJUSTMENT	OV=	.149889	2.90226E+00		6.20226E+00	1.04775E+01	1.28886E+01
RISE TIME ADJUSTMENT	TR=	1.499608		1.03226E+01	6.20226E+00	1.04775E+01	1.29346E+01

THIRD-ORDER SOLUTION PC= 2.90226E+00 CK= 1.03226E+01

	TR	OV	LAMBDA	ZETA	OMEGA	ALPHA	BETA	GAMMA
POINT 1	1.499608	.151041	5.3586	.4981	1.6923	6.20226E+00	1.04775E+01	1.29346E+01
POINT 2	1.499608	.151041	5.3586	.4981	1.6923	6.20226E+00	1.04775E+01	1.29346E+01
POINT 3	1.499608	.151041	5.3586	.4981	1.6923	6.20226E+00	1.04775E+01	1.29346E+01
POINT 4	1.499608	.151041	5.3586	.4981	1.6923	6.20226E+00	1.04775E+01	1.29346E+01