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APPLICATION OF HANSEN'S METHOD TO THE Xth SATELLITE OF JUPITER

by

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Goddard Space Flight Center

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ABSTRACT

This article contains the new orbital elements and the table of the general perturbations of the X^{th} satellite of Jupiter.

A modification of Hansen's theory suggested by Musen was the theoretical foundation for the programming and for the expansion of Hansen's coordinates of the satellite into trigonometric series in four basic arguments. The complete collection of formulas is given in the exposition. Also described are the program and the operations for handling the expansions and for integrating the differential equations of the theory by means of iteration. This program was developed and the actual numerical computations were performed by Charnow and Maury. The new set of elements represents the observations in the interval 1938-1967 better than the previous sets. The residuals are now of the order of only a few seconds of arc. Several second order effects—for example, the planetary perturbations and the effect of the variability of the orbital elements of Jupiter—will be treated by the subsequent work.

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INTRODUCTION

We have accumulated the observations of the X^{th} satellite of Jupiter published over an interval of 29 years. With this observational material, an attempt can now be made to develop a better theory and a better system of mean orbital elements.

The X^{th} satellite of Jupiter was discovered in 1938 by Seth Nicholson at the Mount Wilson Observatory. It belongs to the group of outer satellites of Jupiter whose motion is affected predominantly by solar perturbations but also, to some extent, by the disturbing action of Saturn. The satellite is very weak visually: of 19^{th} stellar magnitude. Its orbital inclination toward the orbital plane of Jupiter is large, about 27°. Thus the development of an analytical theory for this satellite is not an easy problem. Soon after the discovery, two preliminary orbits were computed: one by Wilson (1939) without taking into account the solar perturbations, and one by Herget (UAI) Circ. 727) using a modified Laplacian method. Herget included the solar effect in the computations from the very start, and thus his preliminary orbit is a better one. Herget (1947) also suggested a modification of Encke's device to compute the perturbed rectangular coordinates of the satellite. Using this method Herget, up to 1943, and later Musen produced the geocentric ephemerides of the X^{th} satellite.

A preliminary orbit correction obtained by Musen at Cincinnati was used by Herget as a foundation for the numerical integration of the differential equations of motion of this satellite. Within two years after the completion of the numerical integration, Musen found that his corrected elements were not accurate enough. This might be explained by the mutual influence between the correction of elements and the perturbative effects. That influence could not be investigated then, because of lack of technical means. The possibility of such investigation exists now and enables us to obtain better elements as well as better general perturbations.

Lemekhova (1961) has applied the theory of Delaunay (1860, 1867) to the motion of Jupiter X and obtained a corrected set of elements using observations from 1938 to 1942. She reduced these observations to 14 normal places. The largest residuals after her orbit correction still remain 36" in right ascension and 24" in declination.

The theory of Delaunay was used by Lemekhova because it was the only analytical lunar theory extensively developed. One must bear in mind, however, that Delaunay had in view the application of his theory to the moon and not to the satellites of Jupiter. For this reason, he did not include the higher powers of the inclination in his expansions. It is questionable whether his theory can be applied uncritically to a satellite with the high orbital inclination of Jupiter X. In order for the theory of Delaunay to be applied to Jupiter X too, it must be extended several orders higher. This is a formidable task that has been undertaken by several groups, but not yet completed. So far no extension of the theory of Delaunay exists that would enable us to obtain an accurate representation of observations of such satellites as Jupiter X. The authors, however, have no doubt that in the future such an extension will become a reality.

The work of Lemekhova still retains much of its importance because it permits us to form an idea of the relative importance of terms with different arguments. During Musen's stay in Cincinnati, Herget called his attention to the fact that the lunar theory of Hansen (1862) can be applied to obtain the expansion of the solar perturbations of the outer satellites of Jupiter. Since then several possible modifications of Hansen's theory have been developed by Musen (1963, 1965) at Goddard Space Flight Center in a form adaptable to the use of electronic machines. One of those modifications (1963) is being utilized in this work. The programming and the numerical computations were performed by M. Charnow (1966) and Jesse Maury.

Hansen's lunar theory can be applied to the cases of high inclination. It is also usable for obtaining the expansions of perturbations up to any degree of accuracy compatible with the accuracy of observations. These expansions are obtained in the form of trigonometric series with *purely numerical* coefficients, and this makes the programming easier. So far this is the only theory which permits relatively rapid calculations. Some numerical inaccuracies do exist in Hansen's theory of the moon. However, his basic theoretical idea is correct and can be applied to satellites on which solar effects strongly dominate and for which several small non-gravitational effects, important in the lunar theory, can be neglected in the computation of yearly ephemerides. The intermediary orbit of Hansen's theory is a rotating ellipse of fixed shape. The use of the variational solution (Hill, 1878) is essential only when a literal development in powers of the constants of the eccentricities and of the inclination is being pursued (Brown, 1897-1908). The variational part of the theory for the satellites is not necessarily the most significant part numerically. The largest amplitude normally is associated with the evection and contains the eccentricity of the satellite orbit as a factor. Thus, from the numerical point of view, the use of the mean rotating ellipse as an intermediary represents a natural start.

The appearance of squares of small divisors in the process of double integration presents considerable difficulty in every lunar theory. In Hansen's theory, the squares of small divisors are associated with the long period terms in the perturbations of the mean anomaly and with the element Ξ. The amplitudes of these terms are small. In the process of iteration they are obtained as a ratio of two small numbers. Thus a loss of accuracy is inevitable. Every lunar theory has a special procedure for handling these terms. Here we made use of Hansen's transformation (1862) of the element Ξ. The expression for this element is written by Hansen in such a manner that the terms to be integrated twice contain small factors. The presence of these factors diminishes the influence of division by the mean motion of long period arguments in the double integration.

After the programming had been completed, we applied several checks to test its correctness. The expansion of the disturbing function pertaining to the first cycle of integration must coincide with that obtained by Delaunay when the numerical values of the eccentricities and of the inclination (both constants) are substituted. We found that the agreement was very good and was always in accord with the accepted numerical accuracy. The process of iteration was checked by the partial reproductions of Hansen's lunar theory. The final output of the theory comprises the Fourier expansions of the Hansen coordinates and of the Euler parameters of the satellite. Their accuracy as given here is quite sufficient to produce a good ephemeris of the X^{th} satellite.

Using Lemekhova's elements and our expansions of perturbations, M. Charnow and J. Maury computed an ephemeris for 1967. The satellite was found by E. Roemer in very close proximity to the predicted position. Lemekhova in her work expressed the opinion that some observations of Jupiter X still remain unpublished. The authors also hold this not impossible. It would be highly desirable that such unpublished observations be made available to the scientific community.

The orbit correction and the perturbations have now been recomputed using the new observations. The present work contains the final results of these computations, the complete set of formulas used, and also the comparison with the observations. One can see that the set of new elements represents the observations much better than the set of elements with which we started.

We prefer the general perturbations because they are superior to numerical integration in the description of physical characteristics of an oscillatory system. All the theoretical and technical difficulties associated with the work on the general theories are compensated for by the gratifying results. We obtain an accurate representation of the observations, an accurate prediction, and also an insight into the behavior of the orbit over a long interval of time.

Work on the general theories of motion of bodies in the solar system must proceed because flights to the planets and to their satellites will become a reality in the future. Some day we will have artificial planetary satellites too. The numerical and theoretical techniques must be ready to meet these new challenges.

COLLECTION OF FORMULAS

The notations used in this work are the same as in Musen's previous theory (1963).

A Bessel-function routine is employed to generate the numerical coefficients of the input series $(\rho/a_0)\cos\phi$ through a'/r'. These series have literal arguments. The major iterative loop begins with the calculation of the s_i series. The disturbing function is generated from the s and the p series. When the series representing the $d\Psi/dt$, $d\lambda_2/dt$, and $d\lambda_3/dt$ functions have been formed, the values of $n_0 y$, $n_0 \alpha$, and $n_0 \eta$ are determined by imposing the condition that the series representing those functions must contain no constant terms.

The values $n_0 y$, $n_0 \alpha$, $n_0 \eta$, and $n_0 y'$ relate to the motion of the lunar perigee, lunar node, and solar perigee. The series

$$\left[\frac{h_0}{h}\right], \quad [\Upsilon], \quad [\Psi], \quad [\lambda_i] \quad . \quad (i = 1, 2, 3, 4)$$

are obtained by formal integration. The $n_0 \delta z$ series is computed from the $dn_0 \delta z/dt$ series after c_1 and c_2 have been calculated, so that $n_0 \delta z$ contains neither constant nor $\sin g$ terms.

The $n_0 \delta z$ series has been computed; now the

$$\frac{h_0}{h}, \quad \Xi, \quad \frac{h}{h_0}, \quad \Upsilon, \quad \Psi, \quad \overline{W} \quad \text{and} \quad \nu$$

are constructed for final output or for use in the next iteration. They are complete when the values of $n_0 y$, $n_0 \alpha$, and $n_0 \eta$ have converged.

Input Information:

$$g_0, \ \omega_0, \ \omega_0', \ I_0, \ a_0, \ e_0, \ n_0 \ ,$$

$$g_0', \ a', \ e', \ n', \ i', \ \Omega' \ .$$

Basic Arguments:

$$g = g_0 + n_0 t \ , \qquad g' = g_0' + n_0' t \ ,$$

$$\omega = \omega_0 + n_0 (y + \alpha - \eta) t \ ,$$

$$\omega' = \omega_0' + n_0 (\alpha + \eta + y') t \ .$$

The following standard formulas of the elliptic motion are used; however, if preferred, Cayley's Tables or the harmonic analysis can be used instead.

$$\frac{\rho}{a_0} \cos\phi = -\frac{3}{2} e_0 + 2 \sum_{p=1}^{+\infty} \frac{J_p'(pe_0)}{p} \cos p\gamma \ ,$$

$$\frac{\rho}{a_0} \sin\phi = \frac{2\sqrt{1-e_0^2}}{e_0} \sum_{p=1}^{+\infty} \frac{J_p(pe_0)}{p} \sin p\gamma \ ,$$

$$\frac{\rho}{a_0} = 1 + \frac{1}{2} e_0^2 - 2e_0 \sum_{p=1}^{+\infty} \frac{J_p'\left(pe_0\right)}{p} \cos p\gamma \ ,$$

$$\frac{\rho^2}{a_0^2} = 1 + \frac{3}{2} e_0^2 - 4 \sum_{p=1}^{+\infty} \frac{J_p\left(pe_0\right)}{p^2} \cos p\gamma \ ,$$

$$\frac{a_0}{\rho} = 1 + 2 \sum_{p=1}^{+\infty} J_p\left(pe_0\right) \cos p\gamma \ ,$$

$$\frac{a'}{r'} = 1 + 2 \sum_{p=1}^{+\infty} J_p\left(pe'\right) \cos pg' \ ;$$

$$s_1 = (1+\nu) \cdot \frac{\rho}{a_0} \cdot \frac{a'}{r'} \cos\left(\phi + \overline{f}' + \omega + \omega'\right) \ ,$$

$$s_2 = (1+\nu) \cdot \frac{\rho}{a_0} \cdot \frac{a'}{r'} \sin\left(\phi + \overline{f}' + \omega + \omega'\right) \ ,$$

$$s_3 = (1+\nu) \cdot \frac{\rho}{a_0} \cdot \frac{a'}{r'} \cdot \cos\left(\phi - \overline{f}' + \omega - \omega'\right) \ ,$$

$$s_4 = (1+\nu) \cdot \frac{\rho}{a_0} \cdot \frac{a'}{r'} \cdot \sin\left(\phi - \overline{f}' + \omega - \omega'\right) \ ,$$

$$s = +\left(\lambda_1^2 - \lambda_2^2\right) s_1 - 2\lambda_1 \lambda_2 s_2 + \left(\lambda_4^2 - \lambda_3^2\right) s_3 - 2\lambda_3 \lambda_4 s_4 \ ;$$

$$p = (1+\nu) \frac{\rho}{a_0} \cdot \frac{a'}{r'} \ ;$$

$$\frac{1}{2} \frac{\partial s}{\partial \lambda_1} = +\lambda_1 s_1 - \lambda_2 s_2 = \sigma_1 \ ,$$

$$\frac{1}{2} \frac{\partial s}{\partial \lambda_2} = -\lambda_2 s_1 - \lambda_1 s_2 = \sigma_2 \ ,$$

$$\frac{1}{2} \frac{\partial s}{\partial \lambda_3} = -\lambda_3 s_3 - \lambda_4 s_4 = \sigma_3 \ ,$$

$$\frac{1}{2} \frac{\partial s}{\partial \lambda_4} = +\lambda_4 s_3 - \lambda_3 s_4 = \sigma_4 \ ;$$

$$M_1 = \frac{2a_0 n_0}{1 - e_0^2} \left[\frac{1}{e_0} \left(1 - e_0^2 - \frac{\rho^2}{a_0^2}\right) - \frac{\nu}{1+\nu} \frac{1}{e_0} \left(1 - e_0^2 - \frac{\rho}{a_0}\right) + \left(\frac{h^2}{h_0^2} - 1\right) \frac{1}{e_0} \frac{\rho}{a_0} \left(1 - \frac{\rho}{a_0}\right) \right] ,$$

$$N_1 = \frac{2a_0 n_0}{1 - e_0^2} \cdot \frac{\rho}{a_0} \cdot \frac{\sin\phi}{\sqrt{1 - e_0^2}} \left[1 - \frac{a_0}{\rho} \cdot \frac{\nu}{1+\nu} - \left(\frac{h^2}{h_0^2} - 1\right)\left(\frac{a_0}{\rho} - 1\right) \right] ,$$

$$M_2 = \frac{2a_0 n_0}{1 - e_0^2} \left[\frac{1}{\sqrt{1 - e_0^2}} \int \left(2 \frac{\rho}{a_0} \cos\phi + 3e_0\right) d\gamma - \frac{\nu}{1+\nu} \frac{\rho}{a_0} \sin\phi + \left(\frac{h^2}{h_0^2} - 1\right) \frac{\rho^2}{a_0^2} \cdot \frac{\sin\phi}{1 - e_0^2} \right] ,$$

$$N_2 = \frac{2a_0 n_0}{\left(1 - e_0^2\right)^{3/2}} \left[-\left(\frac{\rho}{a_0} \cos\phi + 2e_0\right) + \sqrt{1 - e_0^2} \frac{\nu}{1+\nu} \frac{d}{d\gamma} \frac{\rho}{a_0} \sin\phi \right.$$

$$\left. + \left(\frac{h^2}{h_0^2} - 1\right) e_0 \frac{\rho}{a_0} \frac{\sin\phi}{\sqrt{1 - e_0^2}} \frac{d}{d\gamma} \frac{\rho}{a_0} \cos\phi \right] ,$$

$$M_3 = + \frac{n_0 a_0}{1 - e_0^2} \frac{\rho^2}{a_0^2} ,$$

$$N_3 = - \frac{n_0 a_0}{1 - e_0^2} \cdot \frac{\rho}{a_0} \cdot \frac{e_0 \sin\phi}{\sqrt{1 - e_0^2}} ;$$

$$\Omega_1 = \frac{m' a^2}{a'^3} \cdot \frac{a'}{r'} \cdot \left(\frac{3}{2} s^2 - \frac{1}{2} p^2\right) ,$$

$$\Omega_2 = \frac{m' a^3}{a'^4} \cdot \frac{a'}{r'} \cdot \left(\frac{5}{2} s^3 - \frac{3}{2} sp^2\right) ,$$

$$\Omega_3 = \frac{m' a^4}{a'^5} \cdot \frac{a'}{r'} \cdot \left(\frac{35}{8} s^4 - \frac{15}{4} s^2 p^2 + \frac{3}{8} p^4\right) ,$$

. . . .

$$\Omega = \Omega_1 + \Omega_2 + \Omega_3 + \cdots ,$$

$$\frac{\partial\Omega}{\partial\gamma} = \frac{\partial\Omega_1}{\partial\gamma} + \frac{\partial\Omega_2}{\partial\gamma} + \frac{\partial\Omega_3}{\partial\gamma} + \cdots ,$$

$$\rho \frac{\partial\Omega}{\partial\rho} = 2\Omega_1 + 3\Omega_2 + 4\Omega_3 + \cdots ,$$

$$\frac{1}{1+\nu} \cdot \frac{a}{\rho} \cdot \frac{\bar{r}'}{a'} \cdot \frac{\partial\Omega}{\partial S} = + \frac{m' a^2}{a'^3} \cdot \frac{a'}{r'} \cdot 3s + \frac{m' a^3}{a'^4} \cdot \frac{a'}{r'} \cdot \left(\frac{15}{2} s^2 - \frac{3}{2} p^2\right) + \frac{m' a^4}{a'^5} \cdot \frac{a'}{r'} \cdot \left(\frac{35}{2} s^3 - \frac{15}{2} sp^2\right) + \cdots ,$$

$$T_i = M_i \frac{\partial \Omega}{\partial \gamma} + N_i \cdot \rho \frac{\partial \Omega}{\partial \rho} \qquad (i = 1, 2, 3) ;$$

$$F_i = \sum_n \frac{1}{n!} \frac{\overline{\partial^n T_i}}{\partial \gamma^n} \left(n_0 \, \delta z\right)^n \qquad (n = 0, 1, 2, 3, \cdots) ;$$

$$\frac{d\Upsilon}{dt} = + n_0 \, y \Psi + F_1 ,$$

$$\frac{d\Psi}{dt} = - n_0 \, y \left(\Upsilon + 2 \frac{h}{h_0} \cdot \frac{e_0}{1 - e_0^2}\right) + F_2 ,$$

$$\frac{d}{dt} \frac{h_0}{h} = F_3 ;$$

$$G_1 = \frac{1}{2} \frac{h}{h_0} \cdot \frac{a_0 \, n_0}{\sqrt{1 - e_0^2}} \cdot \left(\frac{a}{\rho} \cdot \frac{\overline{r}'}{a'} \cdot \frac{1}{1 + \nu} \frac{\partial \Omega}{\partial S}\right) \cdot \left[+ \left(\lambda_3^2 + \lambda_4^2\right) \sigma_2 - \left(\lambda_1 \lambda_4 + \lambda_2 \lambda_3\right) \sigma_3 - \left(\lambda_2 \lambda_4 - \lambda_1 \lambda_3\right) \sigma_4\right] ,$$

$$G_2 = \frac{1}{2} \frac{h}{h_0} \cdot \frac{a_0 \, n_0}{\sqrt{1 - e_0^2}} \cdot \left(\frac{a}{\rho} \cdot \frac{\overline{r}'}{a'} \cdot \frac{1}{1 + \nu} \frac{\partial \Omega}{\partial S}\right) \left[- \left(\lambda_4^2 + \lambda_3^2\right) \sigma_1 - \left(\lambda_2 \lambda_4 - \lambda_1 \lambda_3\right) \sigma_3 + \left(\lambda_1 \lambda_4 + \lambda_2 \lambda_3\right) \sigma_4\right] ,$$

$$G_3 = \frac{1}{2} \frac{h}{h_0} \cdot \frac{a_0 \, n_0}{\sqrt{1 - e_0^2}} \cdot \left(\frac{a}{\rho} \cdot \frac{\overline{r}'}{a'} \cdot \frac{1}{1 + \nu} \frac{\partial \Omega}{\partial S}\right) \left[- \left(\lambda_1^2 + \lambda_2^2\right) \sigma_4 + \left(\lambda_1 \lambda_4 + \lambda_2 \lambda_3\right) \sigma_1 + \left(\lambda_2 \lambda_4 - \lambda_1 \lambda_3\right) \sigma_2\right] ,$$

$$G_4 = \frac{1}{2} \frac{h}{h_0} \cdot \frac{a_0 \, n_0}{\sqrt{1 - e_0^2}} \cdot \left(\frac{a}{\rho} \cdot \frac{\overline{r}'}{a'} \cdot \frac{1}{1 + \nu} \frac{\partial \Omega}{\partial S}\right) \cdot \left[+ \left(\lambda_1^2 + \lambda_2^2\right) \sigma_3 + \left(\lambda_2 \lambda_4 - \lambda_1 \lambda_3\right) \sigma_1 - \left(\lambda_1 \lambda_4 + \lambda_2 \lambda_3\right) \sigma_2\right] ;$$

$$H_i = \sum_n \frac{1}{n!} \left(n_0 \, \delta z\right)^n \frac{\overline{\partial^n G_i}}{\partial \gamma^n} \qquad (i = 1, 2, 3, 4) ;$$

$$\frac{d\lambda_1}{dt} = + n_0 \, \alpha \lambda_2 + H_1 ,$$

$$\frac{d\lambda_2}{dt} = - n_0 \, \alpha \lambda_1 + H_2 ,$$

$$\frac{d\lambda_3}{dt} = + n_0 \, \eta \lambda_4 + H_3 ,$$

$$\frac{d\lambda_4}{dt} = - n_0 \, \eta \lambda_3 + H_4 .$$

Designating by

$$\left[\frac{h_0}{h}\right], \quad [\Upsilon], \quad [\Psi], \quad [\lambda_i] \qquad (i = 1, 2, 3, 4)$$

the series obtained by formal integration, we have

$$\Psi = [\Psi],$$

$$[\Xi] = -3\left[\frac{h_0}{h}\right] - \frac{3}{2} e_0 [\Upsilon] + 2\left(\Delta^2 - \Delta^3 + \cdots\right);$$

$$\left(\frac{\bar{r}}{a_0} \cos \bar{f}\right) - \left(\frac{\bar{\rho}}{a_0} \cos \bar{\phi}\right) = \sum_n \frac{(n_0\, \delta z)^n}{n!} \frac{d^n}{dg^n} \frac{\bar{\rho}}{a_0} \cos \bar{\phi},$$

$$\left(\frac{\bar{r}}{a_0} \sin \bar{f}\right) - \left(\frac{\bar{\rho}}{a_0} \sin \bar{\phi}\right) = \sum_n \frac{(n_0\, \delta z)^n}{n!} \frac{d^n}{dg^n} \frac{\bar{\rho}}{a_0} \sin \phi,$$

$$\left(\frac{\bar{r}}{a_0}\right)^2 - \left(\frac{\bar{\rho}}{a_0}\right)^2 = \sum_n \frac{(n_0\, \delta z)^n}{n!} \frac{d^n}{dg^n} \frac{\bar{\rho}^2}{a_0^2};$$

$$\left[\bar{W}_0\right] = [\Xi] + [\Upsilon]\left(\frac{\bar{\rho}}{a_0} \cos \bar{\phi} + \frac{3}{2} e_0\right) + [\Psi] \frac{\bar{\rho}}{a_0} \sin \bar{\phi},$$

$$B = n_0 [\Upsilon]\left(\frac{\bar{r}}{a_0} \cos \bar{f} - \frac{\bar{\rho}}{a_0} \cos \bar{\phi}\right) + n_0 [\Psi]\left(\frac{\bar{r}}{a_0} \sin \bar{f} - \frac{\bar{\rho}}{a_0} \sin \bar{\phi}\right)$$

$$- \frac{n_0\, y}{\sqrt{1 - e_0^2}}\left(\frac{\bar{r}^2}{a_0^2} - \frac{\bar{\rho}^2}{a_0^2}\right) + \frac{n_0\, \nu^2 (1 + \bar{W})}{1 - \nu^2} + n_0\, c_2\left(\frac{\bar{r}}{a} \cos \bar{f} - \frac{\bar{\rho}}{a} \cos \bar{\phi}\right),$$

$$n_0\left[\bar{W}_0\right] - \frac{n_0\, y}{\sqrt{1 - e_0^2}} \cdot \frac{\bar{\rho}^2}{a_0^2} + B = A_1 + A_2 \cos g + \cdots,$$

$$\frac{\bar{\rho}}{a_0} \cos \bar{\phi} + \frac{3}{2} e_0 = \beta \cos g + \cdots;$$

$$c_1 = \frac{A_1}{3n_0} + \frac{A_2\, e_0}{2\beta n_0},$$

$$c_2 = -\frac{A_2}{\beta n_0};$$

$$\frac{dn_0\,\delta z}{dt} = n_0\left(-3c_1 - \frac{3}{2}e_0 c_2\right) + n_0 c_2\left(\frac{\bar{\rho}}{a_0}\cos\bar{\phi} + \frac{3}{2}e_0\right) - \frac{n_0 y}{\sqrt{1-e_0^2}}\frac{\bar{\rho}^2}{a_0^2} + n_0\left[\bar{W}_0\right] + B ,$$

$$n_0\,\delta z = \int \frac{dn_0\,\delta z}{dt} ,$$

$$\frac{h_0}{h} = 1 + c_1 + \left[\frac{h_0}{h}\right] = 1 + \Delta ,$$

$$\Upsilon = c_2 + [\Upsilon] ,$$

$$\Xi = -3\Delta - \frac{3}{2}e_0\Upsilon + 2\left(\Delta^2 - \Delta^3 + \cdots\right) ;$$

$$\frac{h}{h_0} = 1 - \Delta + \Delta^2 - \Delta^3 + \Delta^4 - \Delta^5 + \cdots ,$$

$$\bar{W} = \Xi + \Upsilon\left(\frac{\bar{r}}{a_0}\cos\bar{f} + \frac{3}{2}e_0\right) + \Psi\frac{\bar{r}}{a_0}\sin\bar{f} ,$$

$$\nu = \frac{1}{2}(\Delta - \bar{W}) - \frac{1}{2}(\Delta + \bar{W})\nu ;$$

$$A^2 + 2A\left(\cos\frac{1}{2}I_0 + \sin\frac{1}{2}I_0\right) + (11) = 0 ,$$

$$B^2 - 2B\left(\cos\frac{1}{2}I_0 - \sin\frac{1}{2}I_0\right) + (12) = 0 ,$$

where

$$(11) = \text{constant term in}\left\{\left([\lambda_1] + [\lambda_4]\right)^2 + \left([\lambda_2] - [\lambda_3]\right)^2\right\} ,$$

$$(12) = \text{constant term in}\left\{\left([\lambda_1] - [\lambda_4]\right)^2 + \left([\lambda_2] + [\lambda_3]\right)^2\right\} ;$$

$$\lambda_1 = \sin\frac{1}{2}I_0 + \frac{1}{2}(A+B) + [\lambda_1] ,$$

$$\lambda_2 = [\lambda_2] ,$$

$$\lambda_3 = [\lambda_3] ,$$

$$\lambda_4 = \cos\frac{1}{2}I_0 + \frac{1}{2}(A-B) + [\lambda_4] .$$

DETERMINATION OF THE LONG-PERIOD EFFECTS

The terms containing the short period arguments g or g' can be handled by the process of iteration easily. The difficulty arises, for reasons explained in the Introduction, when we want to determine the long period terms in the perturbations of the mean anomaly, among which the term with the argument 2ω is the most important. For the computation of these terms, a special procedure is necessary. The most significant long period terms in the perturbations of the mean anomaly are transferred into $n_0\, \delta z$ through the element Ξ. An additional integration is performed. Consequently, we need a form of Ξ split into two parts. The first part will be affected by a single integration only. The second part, which will be affected by a double integration, will contain small factors which will diminish the effect of division by the mean motion of the long period arguments. We are here using, without modification, Hansen's formulas given on page 374 of the Darlegung (Hansen, 1862):

$$\Xi = -3a_0\,\Omega - 3\frac{n'}{n_0}\sqrt{1-e_0^2}\cdot\sqrt{1-e'^2}\left(\frac{h_0}{h}\cos I + k\right)\left(\frac{a'}{r'}\right)^2$$

$$+\frac{1}{2}\left[\left(\frac{h}{h_0}-1\right)-\Xi\right]\left(\frac{h}{h_0}-1\right)+\frac{3}{8}\left(1-e_0^2\right)\left(\Upsilon^2+\Psi^2\right)+Z\ ,$$

$$\cos I = 1 - 2\lambda_1^2 - 2\lambda_2^2\ ,$$

$$\frac{dZ/n_0}{dt} = \frac{3e'}{\sqrt{1-e'^2}}\frac{n'}{n_0}\left(r'\frac{\partial a_0\,\Omega}{\partial r'}\right)\left(\frac{a'}{r'}\sin f'\right)$$

$$-6\left(\frac{n'}{n_0}\right)^2 e'\sqrt{1-e_0^2}\left(\frac{h_0}{h}\cos I + k\right)\left(\frac{a'}{r'}\right)^3\sin f'\ ,$$

$$r'\frac{\partial a_0\,\Omega}{\partial r'} = -3\left(a_0\,\Omega_1\right) - 4\left(a_0\,\Omega_2\right) - 5\left(a_0\,\Omega_3\right) - \cdots\ .$$

The series

$$\frac{h}{h_0} - 1 = -\Delta + \Delta^2 - \Delta^3 + \cdots\ ,\quad \left(\frac{h}{h_0}-1\right)\Xi\ ,\quad \left(\frac{h}{h_0}-1\right)^2,\quad \Upsilon^2\ ,\quad \Psi^2$$

can be taken from the previous iteration.

At each iteration step we shall use these formulas only for the purpose of determining the long period terms. In this case we can disregard the constant of integration k.

COMPUTATION OF THE COORDINATES AND VELOCITIES

After the process of iteration has been completed, we determine the components of the satellite position and velocity vectors from the following system of formulas:

$$A_1(\alpha) = \begin{bmatrix} +1 & 0 & 0 \\ 0 & +\cos\alpha & -\sin\alpha \\ 0 & +\sin\alpha & +\cos\alpha \end{bmatrix}$$

$$A_3(\alpha) = \begin{bmatrix} +\cos\alpha & -\sin\alpha & 0 \\ +\sin\alpha & +\cos\alpha & 0 \\ 0 & 0 & +1 \end{bmatrix}$$

Let the Gibbsian vectors of Jupiter's orbit be

$$\vec{P}', \quad \vec{Q}', \quad \vec{R}' \; .$$

The components of these vectors and of the satellite position and velocity vectors are referred to the mean equator and equinox 1950.0.

We have

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \Gamma \begin{bmatrix} \xi \\ \eta \\ 0 \end{bmatrix} , \qquad \frac{\vec{v}}{n_0} = \Gamma \begin{bmatrix} u \\ w \\ 0 \end{bmatrix}$$

where

$$\xi = (1+\nu)\, a_0 \left(\cos E - e_0\right) ,$$

$$\eta = (1+\nu)\, a_0 \sqrt{1-e_0^2}\, \sin E ,$$

$$u = -\, a_0 \left(\frac{h}{h_0}\, \frac{a_0}{r}\, \sin E + \frac{1}{2} \sqrt{1-e_0^2}\, \Psi \right) ,$$

$$w = +\, a_0 \left(\frac{h}{h_0} \cdot \frac{a_0}{r} \sqrt{1-e_0^2}\, \cos E + \frac{1}{2} \sqrt{1-e_0^2}\, \Upsilon \right) ,$$

$$\Gamma = A_1(\epsilon) \cdot \left[-\vec{P}', -\vec{Q}', +\vec{R}' \right] \cdot A_3(-\omega') \cdot \Lambda \cdot A_3(+\omega) ,$$

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix}$$

$$\lambda_{11} = +\lambda_1^2 - \lambda_2^2 - \lambda_3^2 + \lambda_4^2 ,$$

$$\lambda_{12} = -2(\lambda_3 \lambda_4 + \lambda_1 \lambda_2) ,$$

$$\lambda_{13} = +2(\lambda_1 \lambda_3 - \lambda_2 \lambda_4) ,$$

$$\lambda_{21} = +2(\lambda_3 \lambda_4 - \lambda_1 \lambda_2) ,$$

$$\lambda_{22} = -\lambda_1^2 + \lambda_2^2 - \lambda_3^2 + \lambda_4^2 ,$$

$$\lambda_{23} = -2(\lambda_1 \lambda_4 + \lambda_2 \lambda_3) ,$$

$$\lambda_{31} = +2(\lambda_1 \lambda_3 + \lambda_2 \lambda_4) ,$$

$$\lambda_{32} = +2(\lambda_1 \lambda_4 - \lambda_2 \lambda_3) ,$$

$$\lambda_{33} = -\lambda_1^2 - \lambda_2^2 + \lambda_3^2 + \lambda_4^2 .$$

The geocentric right ascension and declination and the distance from the earth are computed from the standard formulas

$$\rho \cos\delta \cos\alpha = x + x' + X ,$$

$$\rho \cos\delta \sin\alpha = y + y' + Y ,$$

$$\rho \sin\delta = z + z' + Z ,$$

where x', y', z' are the heliocentric coordinates of Jupiter and X, Y, Z the geocentric coordinates of the sun.

ORBIT CORRECTION

The formulas used for the orbit correction represent a transformation of the Eckert-Brouwer method (1937). In this transformation we make use of our expressions for the position and velocity vectors in the inertial system.

$$\vec{M} = \begin{bmatrix} -\sin\alpha/\rho \\ +\cos\alpha/\rho \\ 0 \end{bmatrix} , \qquad \vec{N} = \begin{bmatrix} -\sin\delta\cos\alpha/\rho \\ -\sin\delta\sin\alpha/\rho \\ +\cos\delta/\rho \end{bmatrix} ,$$

$$\alpha_1 = \vec{R} \cdot \vec{M}, \quad \alpha_2 = \vec{Q} \cdot \vec{M}, \quad \alpha_3 = \vec{P} \cdot \vec{M}, \quad \alpha_4 = \vec{v} \cdot \vec{M}/n_0, \quad \alpha_5 = \vec{r} \cdot \vec{M},$$

$$\beta_1 = \vec{R} \cdot \vec{N}, \quad \beta_2 = \vec{Q} \cdot \vec{N}, \quad \beta_3 = \vec{P} \cdot \vec{N}, \quad \beta_4 = \vec{v} \cdot \vec{N}/n_0, \quad \beta_5 = \vec{r} \cdot \vec{N},$$

$$H = -\frac{\cos E + e_0}{1 - e_0^2}, \qquad K = \left(\frac{2}{1 - e_0^2} + e_0 H\right) \sin E,$$

$$A_1 = \alpha_1 (\xi \sin \omega + \eta \cos \omega), \qquad B_1 = \beta_1 (\xi \sin \omega + \eta \cos \omega),$$

$$A_2 = \alpha_1 (\xi \cos \omega - \eta \sin \omega), \qquad B_2 = \beta_1 (\xi \cos \omega - \eta \sin \omega),$$

$$A_3 = \alpha_2 \xi - \alpha_3 \eta, \qquad B_3 = \beta_2 \xi - \beta_3 \eta,$$

$$A_4 = \alpha_4, \qquad B_4 = \beta_4,$$

$$A_5 = \alpha_5 + (m) \alpha_4, \qquad B_5 = \beta_5 + (m) \beta_4,$$

$$A_6 = H \alpha_5 + K \alpha_4, \qquad B_6 = H \beta_5 + K \beta_4,$$

$$(m) = -0.02617994 \left(g - g_0\right)^\circ,$$

$$A_1 \Delta I + A_2 \sin I \Delta\omega' + A_3 \Delta\chi + A_4 \Delta g_0 + A_5 \frac{\Delta a_0}{a_0} + A_6 \Delta e_0 = \cos \delta \, \Delta\alpha,$$

$$B_1 \Delta I + B_2 \sin I \Delta\omega' + B_3 \Delta\chi + B_4 \Delta g_0 + B_5 \frac{\Delta a_0}{a_0} + B_6 \Delta e_0 = \Delta\delta,$$

$$\Delta\omega = \Delta\chi + \cos I_0 \Delta\omega'.$$

PROGRAMMING

The adaptation of Hansen's lunar theory as a model has made possible the effective application of digital computing equipment to this problem. Hansen required a qualitative personal judgment at every step of the development of his long Fourier series. We replace this with a feedback controlled iteration which now can be automated. A computer program can be constructed of a sequence of relatively short steps: this sequence is repeated until we reach the accepted numerical accuracy. These short steps are comparatively straightforward to program and easily checked.

The basic tools of the Hansen lunar theory are Fourier series with five periodic arguments, g, g', ω, ω', and γ. Since all needed operations on these series (series multiplication, series addition, differentiation, integration, etc.) are closed, a set of computer subroutines to execute these operations can readily be defined. In the technique adopted by the authors, each term of a series is composed of two words. The first is the numerical coefficient in the usual floating-point form. The second is a "logic" word containing the periodic arguments. This logic word relies on positional representation and bias (normalization) around 50. It consists of a base logic word having the value +005050505050505050 with the integral multipliers of the periodic arguments introduced into it in the order g, g', ω, ω', γ. The coefficient of the argument g can assume any integral value between 0 and 99. The coefficients of the four remaining arguments can assume any integral value between -49 and +49 normalized by 50. By maintaining all logic words so that the first non-zero argument is positive (i.e., $A \sin(-\theta) = -A \sin(\theta)$ and $B \cos(-\theta) = B \cos(\theta)$), it was possible to use the sign of the logic word to denote sine or cosine. This effects a saving in computation time and storage space. Cosine is denoted by a positive logic word; sine by a negative word. An example is +029329805 $\sin(1g - 2g' + 2\omega - 2\omega')$, the evection term in $n_0 \, \delta z$. In the computer, this term is represented by the numeric, floating-point coefficient +029329805 followed by the logic word -014852485050505050.

The constant term in any series is carried as a coefficient of $\cos(0)$: that is, a floating-point coefficient followed by the base logic word +005050505050505050. This representation is unique. To each series is prefixed a single word denoting the number of terms in the series. The terms of a series are arranged in descending order of the absolute value of the numeric coefficients.

This representation of series was used to program a complete set of computer subroutines for the algebraic manipulation of Fourier series. This package of subroutines can multiply, add, subtract, scalar multiply, differentiate, integrate, and evaluate trigonometric series, as well as perform several specialized functions such as extracting a specific term with a given argument. Series multiplication employs the standard half-angle formulas: addition and subtraction of the arguments are realized through logical addition and subtraction of the logic words (Charnow, 1961): all five arguments are handled simultaneously without recourse to breaking apart the logic word. In executing any of these operations, all duplication of terms is eliminated (maintaining a positive non-zero first argument is an aid in this). An operation on a series or a binary operation on two series is terminated by the exhaustion of terms; or, since the terms of a series appear in descending order, it may be terminated on the basis of a numerical criterion applied to the coefficients of the resultant series. At the conclusion of any operation which may produce a disordered series, the resultant series is ordered by a separate subroutine, the arranger. By the use of these tools, algebraic series operations are programmed almost as easily as numerical calculations.

The original version of this program was written in a pseudo-machine language (Maury, 1964), (Gorman, 1964) on the IBM 7090 and 7094. This version was capable of operating on series with fifty or fewer terms. Subsequently, versions allowing longer and longer series were developed. The goal was to ensure that the computation would be controlled only by numerical criteria and not by storage restrictions. To realize this, the program was shifted to the Univac 1107 to take advantage of the high-speed, random-access drums. These drums were used as temporary storage for series, while the bulk of the computer storage was reserved for two operand registers of 10,000 words each and a result register of 20,000 words.

As the length of the series increased, the time to perform the series operations also increased (exponentially in the case of the *series multiply* operation). To ease the time burden, a number of the subroutines were rewritten in machine language and the program was again shifted, this time to the Univac 1108. Currently, all the main subroutines including multiplication, addition, subtraction, and the arranger are in assembly language, while the statements reflecting the lunar theory are in symbolic language. This version of the program can handle series having up to 2,500 terms, and 750,000 words of high-speed (4.25 ms access time) drum storage is reserved for storing series. The computer storage contains the subroutines, the lunar theory, a small resident I/O monitor, and the 40,000 words comprising the three series registers.

The application of this program to the X^{th} satellite of Jupiter required two hours. The first iteration took only 40 seconds, since the perturbations are zero during this iteration. The second iteration required 16 minutes. The third iteration, in which all the perturbations were completely represented and the series had just begun to converge, required 26 minutes. Thereafter, the time to complete one iteration dropped off markedly, to approximately 13 minutes. Convergence was achieved in eight iterations.

CORRECTED ELEMENTS OF THE X^{th} SATELLITE

The initial elements of the motion of Jupiter X are derived from those developed by E. N. Lemekhova:

Epoch: 1938 July 27.3128 = J.D. 2429106.8128

X Satellite	Jupiter
The elements are referred to the orbital plane of Jupiter	Mean ecliptic and equinox 1950.0
$g_0 = 216^{\circ}.6928$	$\lambda' = 329^{\circ}.20254$
$n = 1^{\circ}.384557$ per day	$n' = 0^{\circ}.083091$ per day
$a = 0.078345$ a.u.	$a' = 5.202561$ a.u.
$e = 0.10739$	$e' = 0.048398$
$\omega_0 = 244^{\circ}.7381$	$\omega' = 273^{\circ}.57223$
$\omega_0' = 113.5835$	$\Omega' = 99.92939$
$I = 27.5748$	$i' = 1.30614$

The system was adjusted for use with a modified Hansen instead of Delaunay theory.

Using the initial elements, the Hansen parameters $n_0 y$, $n_0 \alpha$, and $n_0 \eta$ were obtained from the general perturbation theory:

$n_0 y = 1^\circ\!.64886$ per year

$n_0 \alpha = 1.15605$ per year

$n_0 \eta = 0.06700$ per year

The mean motions of the perigee and node are:

mean motion of perigee $= n_0 (y - 2\eta) = +1^\circ\!.51486$ per year

mean motion of node $= - n_0 (\alpha + \eta) = -1^\circ\!.22305$ per year

We obtained the following corrections:

$\Delta g_0 = +3^\circ\!.6449 \pm 0^\circ\!.0310$

$\Delta \chi = -3.4020 \pm 0.0308$

$\sin I \Delta\omega' = -0.1320 \pm 0.0090$

$\Delta I = +0.0256 \pm 0.0072$

$\Delta e = +0.00851 \pm 0.00009$

$\Delta a/a = -0.000128 \pm 0.000001$

The system of the corrected elements is

Epoch J.D. 2429106.8128

$g_0 = 220^\circ\!.3377$

$n = 1^\circ\!.384687$ per day

$a = 0.078335$

$e = 0.11590$

$\omega_0 = 241^\circ\!.0835$

$\omega_0' = 113^\circ\!.2985$

$I = 27^\circ\!.6004$

$n_0 y = 1^\circ\!.65492$ per year

$n_0 \alpha = 1^\circ\!.16939$ per year

$n_0 \eta = 0^\circ\!.06831$ per year

This set of elements was obtained through the repeated use of a differential correction procedure. The new elements resulted in better agreement between the observed and predicted positions of the satellite.

The Representation of Observations Used.

	UT		Observed α (1950.0)	Observed δ (1950.0)	O - C Δα cos δ	O - C Δδ	
1	1938 July	6^d.3743	22^{h}17^{m}15^s.21	-11°13'32".3	-1".3	+2".6	Mt. Wilson
2		6.4667	22 17 13.69	-11 13 41.6	-0.9	+1.5	Mt. Wilson
3		9.4229	22 16 22.89	-11 18 23.4	-1.5	+1.4	Mt. Wilson
4		9.4521	22 16 22.38	-11 18 26.0	-0.9	+1.8	Mt. Wilson
5		27.3361	22 09 13.67	-12 00 14.9	-0.2	+0.0	Mt. Wilson
7		28.3174	22 08 45.05	-12 03 06.3	-0.3	+0.3	Mt. Wilson
8		29.4667	22 08 10.91	-12 06 30.2	+0.1	+1.1	Mt. Wilson
10	Aug.	24.4167	21 54 07.40	-13 33 16.2	+2.4	+0.2	Mt. Wilson
11		25.2129	21 53 41.88	-13 36 00.3	+3.2	-0.8	Mt. Wilson
12	Oct.	18.1285	21 39 13.15	-15 25 53.6	+0.7	+0.1	Mt. Wilson
13		20.1257	21 39 25.45	-15 25 54.0	+0.7	-0.5	Mt. Wilson
14		23.1319	21 39 50.17	-15 25 15.1	+0.2	-0.0	Mt. Wilson
15	Nov.	21.1188	21 49 30.06	-14 41 12.0	+1.0	+0.2	Mt. Wilson
16	1939 July	15.4205	0 31 26.11	+ 1 36 35.3	+3.0	-0.6	Mt. Wilson
17		15.4528	0 31 26.49	+ 1 36 36.4	+2.2	-0.7	Mt. Wilson
18	Aug.	16.4365	0 33 47.03	+ 1 33 14.1	+1.3	+2.6	Mt. Wilson
19		16.4806	0 33 46.72	+ 1 33 11.8	+1.0	+2.4	Mt. Wilson
20	Oct.	8.3998	0 15 53.39	+ 0 00 48.7	-0.3	+3.3	Mt. Wilson
21	Dec.	15.0885	0 01 26.93	- 0 48 17.8	-2.8	-0.6	Mt. Wilson
24	1940 Sept.	8.3736	2 57 26.16	+16 02 05.9	-0.2	-0.1	Mt. Wilson
25		8.4438	2 57 25.74	+16 02 03.0	+0.1	-0.6	Mt. Wilson
26	Oct.	25.3488	2 38 37.74	+14 14 20.0	+0.9	-0.7	Mt. Wilson
27		25.3731	2 38 36.88	+14 14 15.2	+1.6	-0.3	Mt. Wilson
28	Nov.	1.2160	2 34 23.98	+13 49 17.5	+0.0	-1.0	Mt. Wilson
29		1.4181	2 34 16.35	+13 48 33.3	+0.1	-0.6	Mt. Wilson
30	1941 Dec.	23.2569	4 56 06.78	+22 24 12.2	-1.0	+0.2	Mt. Wilson
31	1942 Feb.	17.2342	4 44 28.41	+22 23 09.5	+0.7	-1.1	Mt. Wilson
32		18.2618	4 44 36.92	+22 23 19.4	+2.0	-1.5	Mt. Wilson
33	Nov.	8.4951	7 53 12.49	+21 30 26.6	-0.1	-0.4	Mt. Wilson
34		9.5090	7 53 17.86	+21 30 12.4	+0.2	-0.6	Mt. Wilson
35	1943 Jan.	6.2400	7 32 51.54	+21 57 36.7	+6.1	+1.1	Mt. Wilson
36		6.3230	7 32 48.03	+21 57 40.5	+5.6	+0.5	Mt. Wilson
37	1951 Sept.	30.29101	0 39 41.51	+ 2 17 48.3	-1.1	-3.8	Mt. Wilson
38		30.30663	0 39 41.08	+ 2 17 47.1	-1.6	-3.3	Mt. Wilson
50	1958 Apr.	25.30100	13 34 27.90	- 7 41 04.4	-2.2	-2.0	Flagstaff
51		25.36887	13 34 26.09	- 7 40 53.6	-1.3	-2.3	Flagstaff
52	1967 Jan.	7.31247	8 17 49.28	+20 23 30.2	-0.1	+4.2	Tucson
53		7.38122	8 17 47.19	+20 23 34.4	+1.4	+3.3	Tucson
54	Feb.	11.17166	7 57 31.87	+21 06 02.6	-6.3	+2.7	Tucson
55		11.25846	7 57 28.83	+21 06 08.1	-7.3	+2.9	Tucson

Argument				$n_0\,\delta z$	ν
g	g′	ω	ω′	sin	cos
0	+0	+2	+0	+0°113	+0.00094
0	+1	-3	+1	+0.007	
0	+1	-2	+0	+0.043	-0.00002
0	+1	-2	+2	-0.078	+0.00008
0	+1	-1	+1	+0.028	
0	+1	+0	+0	-0.310	+0.00017
0	+1	+0	+2	+0.002	
0	+1	+1	+1	-0.028	+0.00002
0	+1	+2	+0	-0.011	
0	+1	+2	+2	+0.002	
0	+2	-4	+2	+0.010	-0.00001
0	+2	-2	+0	-0.002	
0	+2	-2	+2	+0.040	-0.00074
0	+2	-1	+1	+0.008	
0	+2	+0	+0	-0.006	
0	+2	+0	+2	-0.476	+0.00037
0	+2	+1	+1	-0.002	
0	+2	+2	+0	+0.001	
0	+2	+2	+2	+0.007	
0	+3	-2	+2	+0.009	-0.00009
0	+3	-1	+3	-0.004	+0.00001
0	+3	+0	+2	-0.054	+0.00006
0	+3	+1	+3	-0.001	
0	+3	+2	+2	+0.001	
0	+4	-2	+2	+0.003	-0.00001
0	+4	-2	+4	+0.002	-0.00001
0	+4	-1	+3	+0.001	
0	+4	+0	+2	-0.004	
0	+4	+0	+4	-0.001	
0	+5	-2	+4	+0.001	
1	-5	+2	-2	+0.001	-0.00001
1	-4	+0	-2	-0.002	+0.00001
1	-4	+1	-3	-0.002	+0.00001
1	-4	+2	-4	+0.001	-0.00002
1	-4	+2	-2	+0.021	-0.00016
1	-3	+0	-2	-0.024	+0.00017

Argument				$n_0\,\delta z$	ν
g	g′	ω	ω′	sin	cos
1	-3	+1	-3	-0°007	+0.00005
1	-3	+2	-2	+0.213	-0.00170
1	-3	+3	-3	-0.005	+0.00004
1	-3	+4	-4	+0.001	
1	-2	+0	-2	-0.172	+0.00131
1	-2	+0	+0	+0.002	-0.00002
1	-2	+1	-1	-0.006	+0.00005
1	-2	+2	-2	+1.684	-0.01392
1	-2	+4	-2	-0.006	+0.00004
1	-1	+0	-2	+0.010	-0.00008
1	-1	+0	+0	+0.079	-0.00064
1	-1	+1	-1	-0.070	+0.00058
1	-1	+2	-2	-0.060	+0.00051
1	-1	+2	+0	+0.011	-0.00009
1	-1	+3	-1	-0.001	+0.00001
1	-1	+4	-2	-0.001	+0.00001
1	+0	-2	+0	-0.022	+0.00019
1	+0	+0	+0		+0.00039
1	+0	+2	-2	-0.004	+0.00003
1	+0	+2	+0	-1.861	+0.01628
1	+0	+4	+0	+0.004	-0.00004
1	+1	-1	+1	+0.001	-0.00001
1	+1	+0	+0	-0.058	+0.00054
1	+1	+0	+2	+0.002	-0.00002
1	+1	+1	+1	+0.024	-0.00022
1	+1	+2	+0	-0.012	+0.00011
1	+2	-2	+2	+0.001	-0.00001
1	+2	+0	+0	-0.002	+0.00002
1	+2	+0	+2	-0.052	+0.00048
1	+2	+1	+1	+0.001	-0.00001
1	+2	+2	+2	+0.010	-0.00009
1	+3	+0	+2	-0.005	+0.00005
2	-5	+2	-2	+0.001	-0.00001
2	-5	+4	-4	+0.003	-0.00002
2	-4	+2	-4	-0.001	
2	-4	+2	-2	+0.009	-0.00011

Argument				$n_0\,\delta z$	ν
g	g′	ω	ω′	sin	cos
2	-4	+3	-3	-0°001	
2	-4	+4	-4	+0.015	-0.00010
2	-3	+2	-2	+0.068	-0.00085
2	-3	+3	-3	-0.004	+0.00004
2	-3	+4	-2	-0.004	+0.00003
2	-2	+0	-2	-0.003	+0.00006
2	-2	+2	-2	+0.357	-0.00461
2	-2	+4	-2	-0.034	+0.00025
2	-1	+0	+0	+0.001	-0.00002
2	-1	+1	-1	-0.001	+0.00003
2	-1	+2	-2	-0.011	+0.00014
2	-1	+2	+0		-0.00001
2	-1	+3	-1	+0.001	
2	+0	+0	+0		+0.00003
2	+0	+2	+0	-0.027	+0.00060
2	+0	+4	+0	+0.019	-0.00013
2	+1	+0	+0	-0.001	+0.00003
2	+1	+1	+1		-0.00001
2	+1	+2	+0	+0.002	-0.00002
2	+2	+0	+2	-0.001	+0.00002
2	+2	+2	+2	+0.002	-0.00002
3	-5	+4	-4	+0.002	-0.00002
3	-4	+4	-4	+0.008	-0.00008
3	-3	+2	-2	+0.004	-0.00008
3	-3	+3	-3		-0.00001
3	-3	+4	-2	-0.001	+0.00001
3	-2	+2	-2	+0.021	-0.00044
3	-2	+4	-2	-0.010	+0.00009
3	-1	+2	-2	-0.001	+0.00001
3	+0	+2	+0	-0.001	+0.00005
3	+0	+4	+0	+0.001	-0.00001
4	-5	+4	-4		-0.00001
4	-4	+4	-4	+0.001	-0.00001
4	-2	+2	-2	+0.001	-0.00004
4	-2	+4	-2	-0.001	+0.00000

Argument				λ_1	λ_2	λ_3	λ_4
g	g′	ω	ω′	cos	sin	sin	cos
0	+0	+0	+0	+0.23855			+0.97111
0	+0	+2	+0	+0.00163	-0.00264	+0.00028	-0.00039
0	+1	-2	+0	-0.00002	-0.00002		
0	+1	-1	+1		-0.00010	+0.00001	-0.00001
0	+1	+0	+0	+0.00001	+0.00129	-0.00029	
0	+1	+0	+2	-0.00026	+0.00024	-0.00006	+0.00006
0	+1	+1	+1	-0.00004	+0.00004		
0	+1	+2	+0	+0.00001	-0.00001		
0	+2	-2	+2	+0.00017	+0.00019	-0.00002	-0.00006
0	+2	+0	+0		+0.00004	-0.00001	
0	+2	+0	+2	+0.00490	-0.00447	+0.00104	-0.00120
0	+2	+2	+2	-0.00002	+0.00003		
0	+3	-2	+2	+0.00002	+0.00001		
0	+3	-1	+3	-0.00001	+0.00001		
0	+3	+0	+2	+0.00056	-0.00050	+0.00011	-0.00014
0	+4	+0	+2	+0.00004	-0.00004	+0.00001	-0.00001
1	-3	+0	-2	+0.00001	+0.00001		
1	-3	+1	-3	-0.00001	-0.00001		
1	-3	+2	-2	+0.00003	-0.00004		
1	-2	+0	-2	+0.00004	+0.00010	-0.00002	-0.00001

Argument				λ_1	λ_2	λ_3	λ_4
g	g′	ω	ω′	cos	sin	sin	cos
1	-2	+2	-2	+0.00021	-0.00024	+0.00005	-0.00005
1	-1	+0	+0		-0.00001		
1	-1	+1	-1		+0.00002		
1	-1	+2	+0	-0.00001	+0.00001		
1	+0	+0	+0	+0.00005	-0.00017	+0.00004	-0.00001
1	+0	+2	+0	-0.00019	+0.00021	-0.00005	+0.00004
1	+1	+0	+0		-0.00001		
1	+1	+1	+1	+0.00001			
1	+1	+2	+0	-0.00001	+0.00001		
1	+2	+0	+2	-0.00008	+0.00007	-0.00001	+0.00002
1	+3	+0	+2	-0.00001	+0.00001		
2	-3	+2	-2	-0.00005	+0.00005	-0.00001	+0.00001
2	-2	+2	-2	-0.00028	+0.00028	-0.00006	+0.00006
2	-1	+2	+0	+0.00001	-0.00001		
2	+0	+2	+0	+0.00025	-0.00025	+0.00006	-0.00006
2	+1	+2	+0	+0.00001	-0.00001		
2	+2	+2	+2	+0.00001	-0.00001		
3	-2	+2	-2	-0.00002	+0.00002		
3	+0	+2	+0	+0.00002	-0.00002		

Argument				Ψ	ϒ	h_0/h
g	g′	ω	ω′	sin	cos	cos
0	+0	+0	+0		-0.00119	+1.00207
0	+0	+2	-2	+0.00007	-0.00007	
0	+0	+2	+0	+0.03280	-0.03347	-0.00192
0	+0	+4	+0	-0.00007	+0.00007	
0	+1	-3	+1	-0.00003	-0.00003	
0	+1	-2	+0	+0.00027	+0.00026	+0.00003
0	+1	-2	+2	-0.00089	-0.00089	-0.00006
0	+1	-1	+1	-0.00115	-0.00108	-0.00006
0	+1	+0	+0	+0.00239	+0.00017	+0.00003
0	+1	+0	+2	+0.00012	+0.00018	
0	+1	+1	+1	-0.00048	+0.00048	+0.00002
0	+1	+2	+0	+0.00033	-0.00033	-0.00003
0	+2	-4	+2	-0.00009	-0.00009	
0	+2	-2	+0	+0.00001	+0.00001	
0	+2	-2	+2	+0.02435	+0.02431	+0.00141
0	+2	-1	+1	-0.00008	-0.00008	
0	+2	-1	+3	+0.00001	+0.00001	
0	+2	+0	+0	+0.00009		
0	+2	+0	+2	-0.00127	-0.00350	-0.00038
0	+2	+1	+1	-0.00003	+0.00003	
0	+2	+2	+0	+0.00001	-0.00001	
0	+2	+2	+2	-0.00005	+0.00007	+0.00002
0	+3	-3	+3	-0.00005	-0.00005	
0	+3	-2	+2	+0.00267	+0.00267	+0.00015
0	+3	-1	+3	-0.00009	-0.00008	
0	+3	+0	+2	-0.00015	-0.00042	-0.00004
0	+4	-2	+2	+0.00023	+0.00023	+0.00001
0	+4	-2	+4	-0.00005	-0.00006	
0	+4	-1	+3	-0.00002	-0.00002	
0	+4	+0	+2	-0.00001	-0.00003	
0	+4	+0	+4	+0.00001		
0	+5	-2	+2	+0.00001	+0.00001	
0	+5	-2	+4	-0.00001	-0.00001	
1	-5	+2	-2	-0.00004	+0.00004	
1	-4	+0	-2	-0.00001	-0.00001	
1	-4	+2	-4		-0.00001	
1	-4	+2	-2	-0.00038	+0.00039	+0.00004

Argument				Ψ	ϒ	h_0/h
g	g′	ω	ω′	sin	cos	cos
1	-4	+3	-3	+0.00003	-0.00003	
1	-4	+4	-4	-0.00002	+0.00002	
1	-3	+0	-2	-0.00014	-0.00012	-0.00005
1	-3	+1	-3	-0.00001	+0.00002	
1	-3	+2	-2	-0.00299	+0.00304	+0.00034
1	-3	+3	-3	+0.00011	-0.00012	-0.00001
1	-3	+4	-4		+0.00001	
1	-3	+4	-2	+0.00001	-0.00001	
1	-2	+0	-2	-0.00074	-0.00069	-0.00026
1	-2	+0	+0	-0.00001	-0.00001	
1	-2	+2	-2	-0.01606	+0.01639	+0.00188
1	-2	+4	-2	+0.00004	-0.00004	
1	-1	+0	-2	+0.00001	+0.00002	
1	-1	+0	+0	-0.00017	-0.00020	+0.00002
1	-1	+1	-1	+0.00001		-0.00002
1	-1	+2	-2	+0.00041	-0.00042	-0.00003
1	-1	+2	+0	-0.00011	+0.00011	+0.00001
1	-1	+3	-1	+0.00001	-0.00001	
1	+0	-2	+0	-0.00001	-0.00001	
1	+0	+0	+0	-0.00242	-0.00231	+0.00022
1	+0	+2	+0	-0.00164	+0.00169	+0.00017
1	+0	+4	+0	+0.00001	-0.00001	
1	+1	+0	+0	-0.00016	-0.00016	+0.00001
1	+1	+0	+2	+0.00001	+0.00001	
1	+1	+2	+0	-0.00011	+0.00011	+0.00001
1	+2	-2	+2	-0.00005	-0.00005	
1	+2	+0	+0	-0.00001	-0.00001	
1	+2	+0	+2	-0.00046	-0.00045	+0.00002
1	+2	+2	+2	-0.00004	+0.00004	
1	+3	+0	+2	-0.00007	-0.00007	
2	-5	+4	-4	-0.00001	+0.00001	
2	-4	+2	-4			-0.00001
2	-4	+2	-2	-0.00003		-0.00005
2	-4	+3	-3	-0.00003	+0.00003	
2	-4	+4	-4	-0.00005	+0.00005	
2	-3	+0	-2	+0.00002	+0.00002	
2	-3	+2	-2	-0.00026	-0.00004	-0.00044

Argument				Ψ	ϒ	h_0/h
g	g′	ω	ω′	sin	cos	cos
2	-3	+3	-3	-0.00012	+0.00013	+0.00002
2	-2	+0	-2	+0.00013	+0.00014	-0.00001
2	-2	+2	-2	-0.00147	-0.00029	-0.00247
2	-2	+4	-2	+0.00003	-0.00003	
2	-1	+0	+0	-0.00002	-0.00002	
2	-1	+2	-2	+0.00003		+0.00007
2	-1	+2	+0			-0.00001
2	-1	+3	-1	-0.00002	+0.00002	
2	+0	+0	+0	-0.00019	-0.00020	
2	+0	+2	+0	-0.00013		-0.00026
2	+1	+0	+0	-0.00001	-0.00001	
2	+1	+2	+0			-0.00001
2	+2	+0	+2	-0.00002	-0.00002	
3	-4	+2	-4	+0.00001	+0.00001	
3	-4	+2	-2	+0.00003	+0.00003	
3	-4	+4	-4	-0.00001		-0.00002
3	-3	+2	-2	+0.00027	+0.00028	-0.00003
3	-3	+3	-3	-0.00002		-0.00002
3	-2	+0	-2	+0.00002	+0.00002	
3	-2	+2	-2	+0.00151	+0.00158	-0.00018
3	-2	+4	-2	+0.00001		+0.00002
3	-1	+2	-2	-0.00005	-0.00005	
3	-1	+2	+0	+0.00001	+0.00001	
3	+0	+0	+0	-0.00002	-0.00002	
3	+0	+2	+0	+0.00016	+0.00017	-0.00001
3	+1	+2	+0	+0.00001	+0.00001	
4	-5	+4	-4	+0.00001	+0.00001	
4	-4	+4	-4	+0.00003	+0.00003	
4	-3	+2	-2	+0.00005	+0.00005	
4	-3	+3	-3	+0.00001	+0.00001	
4	-2	+2	-2	+0.00026	+0.00027	-0.00001
4	-2	+4	-2	-0.00003	-0.00003	
4	-1	+2	-2	-0.00001	-0.00001	
4	+0	+2	+0	+0.00002	+0.00002	
5	-4	+4	-4	+0.00001	+0.00001	
5	-2	+2	-2	+0.00003	+0.00003	

CONCLUSION

It has been shown that electronic data processing machines can be used to automatize the production of semi-analytical theories of the "natural" satellites. The theory of motion of the Xth satellite of Jupiter as given in this article represents the solution of the so-called "main problem." The direct solar effects were expanded under the assumption that the heliocentric motion of Jupiter is Keplerian. We are justified in such an approach because the solar perturbations are undoubtedly strongly dominant in the motion of Jupiter X. This can be seen from the fact that the new set of elements leads to a considerably better representation than the old one. The difference between the observed and the computed positions is now reduced to only a few seconds of arc. However, to make the theory more complete, several other secondary small effects must also be considered. We have still to include in the theory the direct planetary influence as well as the influence of the variability of the elements of Jupiter on the motion of the satellite. We expect these effects to change the difference between the observed and the computed position slightly, about 1" - 2".

The comparison between the theory of Hansen and that of Delaunay also is a topic of considerable theoretical and numerical importance. The authors plan to expand and to discuss these problems in their next paper.

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