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IDENTIFICATION OF LINEAR SYSTEMS, FINAL REPORT ON SIMULATION STUDIES

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OBJECTIVE

The development of a practical computational technique for the identification of constant parameter linear systems based upon the system response to random or sinusoidal excitation.

INTRODUCTION

This study of duration February 1967 - April 1968 has been devoted to the analytical development and digital simulation experiences of a parameter estimation technique that appears to be of superior practical significance for the identification of real systems.

The technique is applicable to linear systems and is simply based upon the properties of statistical expectations and time averages. It also has a potential application to non-linear system identification. The physical situation that one often encounters is that a dynamical model of the system, via differential equations is given, but the various physical parameters, in particular, the mass, spring, and damping factors are unknown. In order to obtain a complete useful model of the system, it is necessary that these parameters are known. The technique presented in this report requires knowledge of the dynamics of the system; that is, the displacements, velocities, and accelerations, as well as the input data. From these data all unknown parameters can be determined by forming various moments, or time averages, of the input and the dynamical output variables of the system. The resulting linear equations in the unknown parameters are then solved to yield the desired estimates. A theoretical study of the technique was accomplished for linear systems in both the random and deterministic input case.

These results appear in Chapter II. This theoretical study greatly clarifies the role of such parameters as the length of time over which the system is to be observed, the nature of the spectrum of the excitation, as well as the role of the steady-state dynamics of the system in effecting a usable identification scheme.

When the mass is known, only the displacements and velocities are required in order to determine the estimates of the spring and damping constants.

The technique is applied to study digitally simulated models of one-dimensional linear systems with five degrees of freedom. The parameters for simulation are taken as those of a NASA-Goddard 5-mass experimental model. The simulated model is subjected to various random as well as sinusoidal excitations. The estimated parameters are found to agree with the actual parameters up to four and five place accuracy! Even more of a significant feature is that the actual system simulated parameters have a spread of five or six orders of magnitude between the mass and the spring constant. A major problem in parameter search techniques is to determine the range of parameter values. For the present technique this presents no problem as can be seen by the extremely accurate estimated parameter values.

A completely detailed program for simulation, as well as estimation of parameters has been developed for linear chain-like mass-spring-dashpot systems with arbitrary degrees of freedom and an arbitrary number of force inputs to the system. This appears in Chapter II.

The technique was also applied to a simulated two-dimensional system of masses, springs, and dashpots supplied to us by NASA-Goddard. Again, parameter estimation was truly outstanding as can be seen in Chapter III of this report.

It can safely be stated at this point that when displacement, velocity and acceleration data is available, system identification can be accomplished quite satisfactorily by this method.

It was hoped that actual data taken from vibration tests on the NASA-Goddard five mass system could have been analyzed to obtain an estimate of the real system parameters. However, the tests only yielded acceleration data. Digital integration of this data was attempted in order to yield an estimate of the velocities and displacements of the five masses. A least squares trend was removed to account for the fact that the initial conditions of the velocities and displacements are unknown at the point at which the acceleration record commences. Due to the numerical inaccuracies present when integrating and removing trends twice, satisfactory estimates were not obtainable from the real system data during the duration of the contract period. (This is, in part, due to the time required to put the vibration data on tape and then digitize it in a form suitable for computation. This was all accomplished by NASA-Goddard.) However, we do not hesitate to add that this is merely a numerical problem of simulation, which can certainly be resolved with future investigations. We present a first step in this direction in the present report, by integrating the acceleration once and identifying two parameters of a damped oscillator.

Thus, we can say in summary that:

A. A method has been proposed for identification of linear and non-linear constant coefficient systems, by random or sinusoidal excitations as discussed in Chapter I, Parameter Estimation.

B. The method is studied here for linear systems, subjected to random or sinusoidal excitations.

C. The theoretical studies have generated a rather broad understanding of the method, as presented in Chapter II, Theoretical Development.

D. The method yeilds extremely accurate parameter identification for rather complex systems, as presented in Chapter III, Identification of Simulated Linear Dynamical Systems.

E. A complete discussion of the simulation techniques as well as the program details are presented in Chapter IV, Computer Simulation and Identification.

F. Suggestions for future investigations are presented in Chapter V, Summary and Conclusions.

It is to be noted that the identification scheme proposed here in general places no restriction on what combination of variables are to be multiplied together and subsequently time averaged in order to create the necessary algebraic equations. On the other hand, it is to be recognized that if the well known method of least squares curve fitting technique is applied to the identification problem, there will result an identification of the type proposed here with a particular form of variable products. For the hypothetical situation where one has available both the exact form of the system equations and error-free response and forcing function data, a trivial case exists that can be solved without resorting to time averages of variable products. By trivial it is meant that all one has to do is select data at a sufficient number of distinct times to form the algebraic equations. In such a situation, the least squares curve fitting criterion has no real meaning or significance since there is no error to minimize. But when one considers "real life" situations, where a system with an infinite number of degrees of freedom is approximated by one with a finite number, non-linearities are either ignored or guessed at in form, measured data contains errors, or the coupling is incorrectly assumed; the question remains to be answered as to what "product form" to use to produce the "best" estimate of the parameters. In fact, the "best product form" to use will probably be a function of the particular assumed form of system equations.

CHAPTER I

PARAMETER IDENTIFICATION

The problem of identification of a system or of a process is now recognized as a basic part of modern engineering technique. It is clear that we must identify in order to design and in order to control in any optimal fashion. Thus, the subject of identification has been actively studied in the past decade, and will continue to develop both theoretically and practically as engineers continue to expand our technology.

Identification problems in engineering have been most actively pursued by electrical engineers in the past 10-15 years primarily motivated by the desire for adaptive and optimal control of systems and processes. Thus, the ideas of cross-correlations and cross-spectral densities for estimating the impulse response function or the frequency response function have been generated by them. Furthermore, electrical engineers and optimal control engineers have been forward in their efforts to apply parameter estimation schemes for identification purposes.

Vibrations engineers have, to a large extent, remained with the classical technique of driving a structural system by sinusoidal excitations at various frquencies to determine the frequency response characteristics of structural systems. Parameter estimation ideas have not as yet permeated the bag of tricks that structural vibrations engineers can use freely in determining models of structural systems. Although new techniques based upon second order statistics, mean square approximations, or energy techniques are beginning to change that picture somewhat.

The purpose of this report is to present a parameter identification technique. As we indicated above, there is certainly no lack of parameter identification techniques in the literature. However, the technique that is presented in the present report possesses noteworthy features.

In the first place, the technique is simple to comprehend and to apply. Second, the same theoretical concept applies to both random and sinusoidal excitations; indeed, even sweep sinusoidal excitations. Third, it appears that the technique can be extended to non-linear systems with unknown parameters since the basic theory would remain unchanged. Fourth, the technique does not appear to be affected by wide ranges of the parameter values that often plague optimum parameter search techniques. Finally, in simulation studies the technique has produced highly accurate parameter estimates for reasonably complex systems.

Thus, it appears that the identification technique proposed in this report holds promise of being of practical significance for identification of arbitrary systems with unknown constant parameters.

This report is limited to the study of linear systems. It presents a theoretical development of the ideas, and estimates of parameters of simulated 5-mass chain-like systems as well as a two-mass two-dimensional system, among others. These systems, as well as the actual parameters, were supplied by NASA-Goddard. In each case, the mass, spring, and damping constants are estimated from the digitally simulated system subjected to random as well as sinusoidal inputs. It is noteworthy to point out that the mass and spring constants are six orders of magnitude apart in their values and yet each is estimated with very high accuracy. It is also noteworthy to add that a single sinusoidal excitation at what appears to be any arbitrary frequency will yield the identification of the system parameters. Thus, one does not have to excite the system at a multitude of frequencies, or in some frequency bandwidth as has been implied by many previous investigations. Finally, the description of the digital simulation techniques as well as the program for simulation and identification are presented.

One point must be made concerning the identification of systems via parameter estimation techniques. That is, the methods that have been developed as well as the method we describe in this report will identify the analytical or simulated model of the actual physical system. Hence, if the analytical model is not a satisfactory equivalent or approximation to the system, then clearly, one is not identifying the real system. Thus, any analytical model identification scheme (such as parameter estimation) is only as good as the model that will be used to describe the physical system. (We note that identification schemes can sometimes be used to help provide a better system model. However, we will not dwell on that point here.)

With this understanding of the proper role of identification by parameter estimation, we can now proceed to describe our approach.

CHAPTER II

THEORETICAL DEVELOPMENT

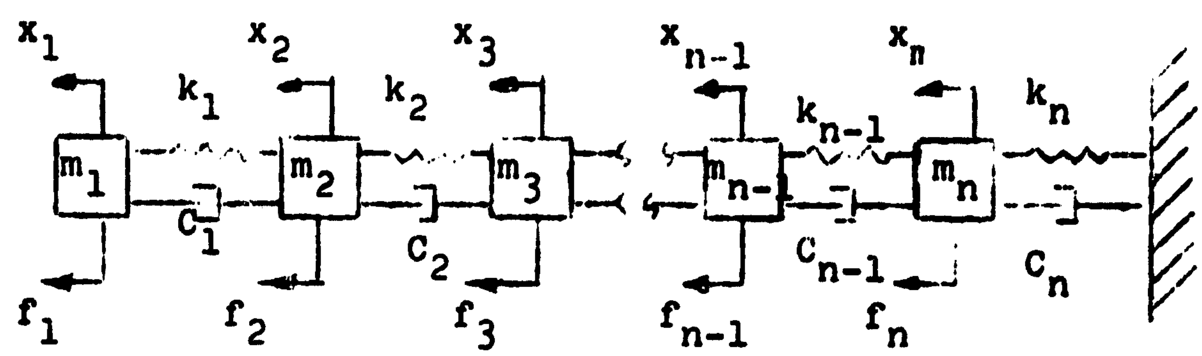
2.1 Introduction

The basic assumptions that we make in this chapter are that a linear time invariant system is being driven by some excitation, either random or deterministic. It will serve our purposes to think of our system as being composed of a number of masses connected to one another by linear springs and linear dashpots. We further assume that each mass may be driven by a separate excitation and further that the accelerations, velocities and displacements of each mass as well as the various excitations may be noiselessly observed, or at least, obtainable by suitable means. We point out that this rules out pure white noise as an input for reasons that shall be discussed below.

We assume that the various displacements, velocities and accelerations of the masses are related to the excitations via a system of linear differential equations of known form and order, but with unknown mass, spring, and damping parameters.

In general, our systems have the character of those given in Figure I. Figure I (a) shows a one-dimensional

(a) Chain-Like, One-Dimensional System



(b) Two-Dimensional System

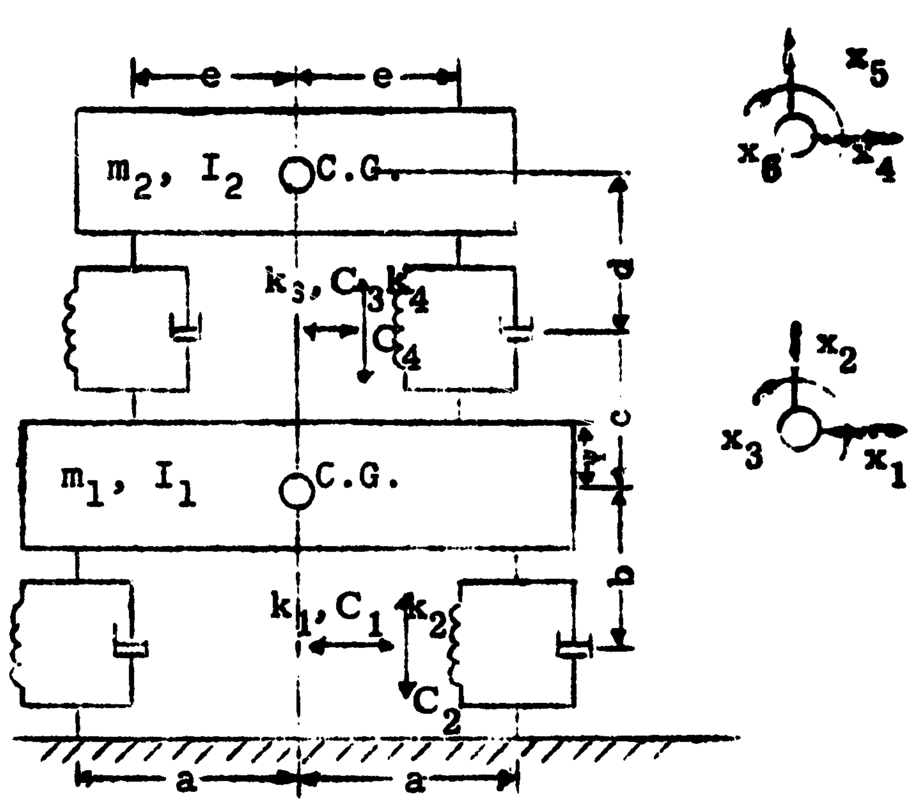


Figure I

chain-like system; Figure I(b) shows a two-dimensional system.

In either case the form of the linear differential equation that governs the dynamics of these systems is

$$\dot{\bar{y}}(t) = A\,\bar{y}(t) + \bar{f}(t) \tag{2.1.1}$$

where the $\bar{y}$ vector is the vector of all the states of system. That is, its components are the displacements and velocities of the masses, the A matrix is a constant matrix made up of the various spring and damping constants, the $\bar{f}$ vector is the excitation vector,

We have assumed all mass and inertia constants to be unity in the general equation (2.1.1). These constants will enter explicitly in the specific classes of systems we study below.

Thus, we write

$$\left.\bar{y}_{(t)} = \begin{pmatrix} y_1(t) \\ . \\ . \\ y_n(t) \end{pmatrix} \qquad \bar{f} = \begin{pmatrix} f_1(t) \\ . \\ . \\ f_n(t) \end{pmatrix} \qquad A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ . & & . \\ . & & . \\ a_{n1} & & a_{nn} \end{pmatrix} \right\} \tag{2.1.2}$$

for the vectors and matrix defining the system.

In our development below we shall be concerned with both steady state oscillations as well as transient oscillations. Naturally, we assume the system to be stable in order that the steady state oscillation exists. Having stated these few introductory remarks, we can now present our detailed analytical development.

2.2 Random Excitations

Let us assume that the components $f_i(t)$, $i = 1, \ldots, n$ of the $\overline{f}(t)$ excitation vector are stationary random processes possessing as many moments as may be required by our analysis (in general, for linear systems only second moment properties will be required). We will further assume that the excitation processes are smooth enough to guarantee that all the derivatives $\dot{y}_i(t)$ exist. (The reader may recall that this is not the case if the excitation is a Gaussian White noise with Dirac "Delta" function for its covariance. See appendix)

We can immediately write a general solution to the A matrix for the linear system 2.1.1 as follows. We multiply equation 2.1.1 by the transpose vector $\overline{y}'(t)$ and take expectations of the resulting equation to yield

$$E\{\dot{\bar{y}}(t)\ \bar{y}'(t)\} = A\ E\{\bar{y}(t)\ \bar{y}'(t)\} + E\{\bar{f}(t)\ \bar{y}'(t)\} \quad (2.2.1)$$

The equation 2.2.1 can be solved for the matrix A as,

$$[E\{\dot{\bar{y}}(t)\ \bar{y}'(t)\} - E\{\bar{f}(t)\ \bar{y}'(t)\}][E\{\bar{y}(t)\ \bar{y}'(t)\}]^{-1} = A \quad (2.2.2)$$

assuming that the inverse matrix

$$\left[E\{\bar{y}(t)\ \bar{y}'(t)\}\right]^{-1} \quad (2.2.3)$$

exists.

The relation 2.2.2 presents a general solution of the identification of A for linear systems of the form 2.1.1, if 2.2.3 exists. The existence of this inverse is guaranteed if there is no linear relationship among the components of the $\bar{y}(t)$ vector, since the covariance matrix $E\{\bar{y}(t)\ \bar{y}'(t)\}$ is symmetric and non-negative definite. The non-negative definiteness follows from the fact that

$$E\{(\sum_{i=1}^{n} \alpha_i\, y_i(t))^2\} \geq 0 \qquad (2.2.4)$$

for any constants $(\alpha_1, \ldots, \alpha_n)$. Furthermore, the equality sign in 2.2.4 can only hold if there exists a linear relationship among the components of $\bar{y}(t)$. Thus, 2.2.3 exists and the constant matrix A is solvable as given by equation 2.2.2 on the basis of observations of the $\bar{y}$ and $\bar{f}$ vectors.

This is somewhat more general than we wish to consider. Since the estimation of the various moments in 2.2.2 as well as the inverse matrix, especially in the transient situation where the moments are functions of t, are difficult to estimate. Therefore, to proceed with our development let us assume that the transients have, for all practical purposes, died out and the system is operating in the steady state. It is known that the $\bar{y}$-process is a statistically stationary process in that case and all the moments present in equation 2.2.2 are constant.

Furthermore, they can be estimated simply by taking time averages over discrete or continuous values of the time parameter.

Therefore, it follows, in the stationary case, that

$$E\{y_i^r(t)\}\ ,\ E\{y_i^r(t)\ y_j^s(t)\}\ ,\quad \begin{matrix} i,j = 1,\ \ldots,\ n \\ r,s = 0,1,2,\ \ldots \end{matrix} \tag{2.2.4}$$

exist and are constant in time. Hence,

$$\left.\begin{aligned} \frac{d}{dt}\,E\{y_i^r(t)\} &= 0 \\ \frac{d}{dt}\,E\{y_i^r(t)\ y_j^s(t)\} &= 0 \end{aligned}\right\} \tag{2.2.5}$$

for i,j and r,s as above.

We now specify that the $\bar{y}$-process is a stationary, mean square differentiable process. Such processes are generated, for example, by passing a stationary mean square continuous process (i.e., a process with continuous covariance function) through a time invariant linear filter. Thus, if the excitation process $\bar{f}$ is mean square continuous, we are assured that the stationary $\bar{y}$-process is mean square continuous. It is because of the desired differentiability properties of the $\bar{y}$-process that we are ruling out the white noise type excitations in the random case. We explain this in full detail in the Appendix.

Now as a result of the mean square differentiability of the $\bar{y}$-process the derivative operator in 2.2.5 can be taken into the expectation operator to give

$$E\{y_i^{r-1}(t)\,\dot{y}_i(t)\} = 0 \qquad \text{(a)}$$

$$(2.2.6)$$

$$rE\{y_i^{r-1}(t)\,\dot{y}_i(t)\,y_j^{s}(t)\} + sE\{y_i^{r}(t)\,y_j^{s-1}(t)\,\dot{y}_j(t)\} = 0 \qquad \text{(b)}$$

In particular, it follows that for $r = 2$ in 2.2.6 a and $r = s = 1$ in 2.2.6 b we obtain

$$E\{y_i(t)\,\dot{y}_i(t)\} = 0 \qquad \text{(a)}$$

$$(2.2.7)$$

$$E\{y_i(t)\,\dot{y}_j(t)\} + E\{\dot{y}_i(t)\,y_j(t)\} = 0 \qquad \text{(b)}$$

The first equality in equation 2.2.7 states the well-known fact that a stationary process and its derivative are uncorrelated at any given time. We repeat that the equations 2.2.7 <u>do not hold</u> if the excitation process is a white noise as will be seen in the Appendix.

Thus, on the basis of equations 2.2.6 and 2.2.7, the identification of the parameter matrix A as given by formula 2.2.2 reduces greatly in the stationary case. We illustrate these ideas by a few very simple analytical examples.

Example I.

Consider the system

$$\dot{y}(t) + ay(t) = f(t) \tag{2.2.8}$$

Upon multiplying 2.2.8 by $y(t)$ and taking expectations, we find

$$E\{y(t)\,\dot{y}(t)\} + aE\{y^2(t)\} = E\{f(t)\,y(t)\} \tag{2.2.9}$$

However, from 2.2.7 a it follows that

$$a = \frac{E\{f(t)\,y(t)\}}{E\{y^2(t)\}} \tag{2.2.10}$$

For our estimate of a, therefore, one merely estimates the moments that appear in 2.2.10.

Example II.

We consider here the somewhat more complex system of coupled oscillators as shown in Figure II.

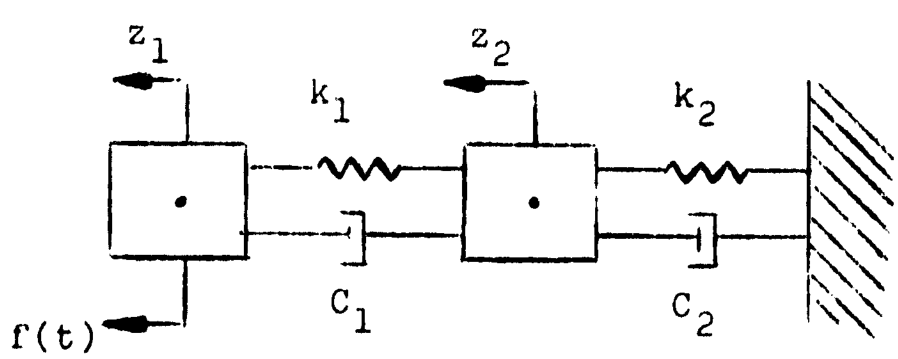


Figure II

Assuming the masses to be unity, we may write the equations of the system as,

$$\left.\begin{aligned} &\ddot{z}_1(t) + C_1[\dot{z}_1(t) - \dot{z}_2(t)] + k_1[z_1(t) - z_2(t)] = f(t) \\ &\ddot{z}_2(t) + C_2\,\dot{z}_2(t) + k_2\,z_2(t) - C_1[\dot{z}_1(t) - \dot{z}_2(t)] \\ &\qquad\qquad - k_1[z_1(t) - z_2(t)] = 0 \end{aligned}\right\} \quad (2.2.11)$$

Upon setting

$$\begin{pmatrix} z_1 \\ \dot{z}_1 \\ z_2 \\ \dot{z}_2 \end{pmatrix} \equiv \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \tag{2.2.12}$$

we may rewrite 2.2.11 as the system

$$\left.\begin{aligned}
\dot{y}_1(t) &= y_2(t) \\
\dot{y}_2(t) &= -\ C_1[y_2(t) - y_4(t)] - k_1[y_1(t) - y_3(t)] + f(t) \\
\dot{y}_3(t) &= y_4(t) \\
\dot{y}_4(t) &= -\ C_2 y_4(t) - k_2 y_3(t) + C_1[y_2(t) - y_4(t)] \\
&\qquad + k_1[y_1(t) - y_3(t)]
\end{aligned}\right\} \tag{2.2.13}$$

The second equation in 2.2.13 is multiplied by y_1, y_2 and then averaged. The fourth equation in 2.2.13 is multiplied by y_3, y_4 and averaged. On the basis of equations 2.2.7 it will follow that,

$$
\left.
\begin{aligned}
E\{y_2^2\} + E\{fy_1\} &= -C_1\ E\{y_1y_4\} + k_1(E\{y_1^2\} - E\{y_1y_3\}) \\
E\{fy_2\} &= C_1\ (E\{y_2^2\} - E\{y_2y_4\}) - k_1\ E\{y_2y_3\} \\
-E\{y_4^2\} &= -\ k_2\ E\{y_3^2\} + C_1\ E\{y_2y_3\} + k_1(E\{y_1y_3\} - E\{y_3^2\}) \\
0 &= -\ C_2\ E\{y_4^2\} + C_1\ (E\{y_2y_4\} - E\{y_4^2\}) + k_1\ E\{y_1y_4\}
\end{aligned}
\right. \qquad (2.2.14)
$$

The set of four linear algebraic equations, 2.2.14, in the four unknowns are easily solved to determine the parameters C_1, k_1, C_2, and k_2.

Thus, for example,

$$
\left.
\begin{aligned}
C_1 &= \frac{\begin{vmatrix} E\{y_2^2\} + E\{fy_1\} & E\{y_1^2\} - E\{y_1y_3\} \\ E\{fy_2\} & -\ E\{y_2y_3\} \end{vmatrix}}{B} \\
k_1 &= \frac{\begin{vmatrix} -\ E\{y_1y_4\} & E\{y_2^2\} + E\{fy_1\} \\ E\{y_2^2\} - E\{y_2y_4\} & E\{fy_2\} \end{vmatrix}}{B}
\end{aligned}
\right\} \qquad (2.2.15)
$$

where

$$B = \begin{vmatrix} -E\{y_1y_4\} & E\{y_1^2\} - E\{y_1y_3\} \\ E\{y_2^2\} - E\{y_2y_4\} & -E\{y_2y_3\} \end{vmatrix}$$

Similar equations yield C_2, k_2 as well. We shall leave the discussion of numerical results of this system for the next chapter.

For the case of unknown mass, spring and damping constants one must obtain one more set of moment equations in order to obtain a solvable set of linear simultaneous equations. We illustrate the problems that may occur in the proper choice of the third moment equation by the following simple oscillator.

Let us consider the case of the system given by

$$m\,\ddot{y}(t) + C\,\dot{y}(t) + k\,y(t) = f(t) \quad (2.2.16)$$

where m, C, k are all unknown.

Upon multiplying 2.2.16 by y, $\dot{y}$ and taking expectations, we obtain equations analogous to those in our previous examples, as,

$$m\,E\{y\ddot{y}\} + C\,E\{y\dot{y}\} + k\,E\{y^2\} = E\{fy\}$$

$$m\,E\{\dot{y}\ddot{y}\} + C\,E\{\dot{y}^2\} + k\,E\{\dot{y}y\} = E\{f\dot{y}\} \tag{2.2.17}$$

By equation 2.2.7 a we can reduce these equations to yield

$$\left.\begin{aligned} -\,m\,E\{\dot{y}^2\} \qquad + k\,E\{y^2\} &= E\{fy\} && \text{(a)} \\ C\,E\{\dot{y}^2\} \qquad &= E\{f\dot{y}\} && \text{(b)} \end{aligned}\right\} \tag{2.2.18}$$

However, we require one more equation. One might consider multiplying 2.2.16 by y^2, for example, and then take expectations to yield

$$m\,E\{y^2\ddot{y}\} + C\,E\{y^2\dot{y}\} + k\,E\{y^3\} = E\{fy^2\} \tag{2.2.19}$$

where, by 2.2.6 a it follows that $E\{y^2\dot{y}\}$ is identically zero.

But, in general, in practice the excitation function will be a zero mean Gaussian random process. Hence, y, $\dot{y}$, $\ddot{y}$ are all Gaussian processes; each is a linear operator of the input process f.

Thus, specifically one has,

$$\left.\begin{aligned} y(t) &= \int_{-\infty}^{t} H(t-\tau)\, f(\tau)\, d\tau \\ \dot{y}(t) &= \int_{-\infty}^{t} \dot{H}(t-\tau)\, f(\tau)\, d\tau \\ \ddot{y}(t) &= -\frac{1}{m} \int_{-\infty}^{t} [C\dot{H}(t-\tau) + kH(t-\tau)]\, f(\tau)\, d\tau + \frac{1}{m} f(t) \end{aligned}\right\} \quad (2.2.20)$$

Thus, any third order moment in y, $\dot{y}$, $\ddot{y}$ will involve moments of the form

$$E\{f(t_1)\, f(t_2)\, f(t_3)\} \quad (2.2.21)$$

However, it is a well-known fact that all third order moments of a Gaussian process are identically zero. Indeed, all odd order moments of a Gaussian process are zero so that equation 2.2.19 cannot yield any new information as all its terms are zero.

Hence, if we cannot apply odd order moments to yield our third equation, the next question is: "What about even moments?". Here, we get into trouble of a different nature, as we now demonstrate. Let us, for example choose

$$m\,E\{y^3\ddot{y}\} + C\,E\{y^3\dot{y}\} + kE\{y^4\} = E\{fy^3\} \tag{2.2.22}$$

as our third equation, again the term $E\{y^3\dot{y}\}$ is zero. It is well known that the even product moments of zero mean jointly distributed Gaussian random variables can be evaluated in terms of the second moments. In particular for X_1, X_2, X_3, X_4 jointly Gaussian, one has

$$\begin{aligned} E\{X_1X_2X_3X_4\} &= E\{X_1X_2\}\,E\{X_3X_4\} \\ &\quad + E\{X_1X_3\}\,E\{X_2X_4\} \\ &\quad + E\{X_1X_4\}\,E\{X_2X_3\} \end{aligned} \tag{2.2.23}$$

If we apply the identity 2.2.23 to the terms of equation 2.2.22 we obtain

$$
\begin{aligned}
E\{y^3\ddot{y}\} &= 3\,E\{y^2\}\;E\{y\ddot{y}\} \\
E\{y^4\} &= 3\,(E\{y^2\})^2 \qquad\qquad (2.2.24) \\
E\{fy^3\} &= 3\,E\{y^2\}\;E\{yf\}
\end{aligned}
$$

Thus, equation 2.2.22 may be written as,

$$3\,m\,E\{y^2\}\,E\{\dot{y}^2\} + 3\,k\,(E\{y^2\})^2 = 3\,E\{y^2\}\,E\{yf\} \qquad (2.2.25)$$

We immediately recognize that equation 2.2.25 is equation 2.2.18 a multiplied by the factor $3\,E\{y^2\}$.

Hence, equation 2.2.22 yields no new information. Again, if one chooses any even moment equation, it will always reduce to a linear combination of the equations 2.2.18 for the zero mean Gaussian case.

Since, in practice, noise generators yield Gaussian or near Gaussian processes, it will not be possible to identify the three unknowns on the basis of moments obtained from equation 2.2.16 by multiplying by powers of y, $\dot{y}$ and averaging. However, if the noise process used for excitation is definitely non-Gaussian, then one can establish moments of the nature of those we have described. We shall present data that displays this phenomenon in the next chapter. How then are we to obtain a third condition for evaluating the unknown parameters? The most obvious choice is to use the acceleration variable. This is quite practical since, in general, it is the acceleration data that is actually obtained from experimental tests.

Therefore, we can multiply equation 2.2.16 by $\ddot{y}$ and take expectations giving us our third, and independent, equation

$$m\,E\{\ddot{y}^2\} + C\,E\{\dot{y}\ddot{y}\} + k\,E\{y\ddot{y}\} = E\{f\ddot{y}\} \tag{2.2.26}$$

Finally, the system of linear equations available for parameter identification are given as,

$$\left.\begin{aligned} -m\,E\{\dot{y}^2\} \qquad\qquad + k\,E\{y^2\} &= E\{fy\} \\ C\,E\{\dot{y}^2\} \qquad\qquad &= E\{f\dot{y}\} \\ m\,E\{\ddot{y}^2\} \qquad\qquad - k\,E\{\dot{y}^2\} &= E\{f\ddot{y}\} \end{aligned}\right\} \tag{2.2.27}$$

Thus, in order to identify linear systems by random excitations, we shall construct moment equations by multiplying the coupled equations by the displacement, the velocity and then the acceleration. We illustrate this for the general one-dimensional chain-like system as shown in Figure I (a).

Denoting the displacement, velocity, and acceleration of the i^{th} mass by x_i, $\dot{x}_i$, $\ddot{x}_i$, the equations of motion for N masses in the chain are,

$$\left.\begin{aligned}
m_1\ddot{x}_1 &= f_1 - C_1(\dot{x}_1-\dot{x}_2) - k_1(x_1-x_2) \\
m_2\ddot{x}_2 &= f_2 - C_2(\dot{x}_2-\dot{x}_3) - k_2(x_2-x_3) + C_1(\dot{x}_1-\dot{x}_2) \\
&\qquad + k_1(x_1-x_2) \\
&\vdots \\
m_N\ddot{x}_N &= f_N - C_N\dot{x}_N - k_Nx_N + C_{N-1}(\dot{x}_{N-1}-\dot{x}_N) \\
&\qquad + k_{N-1}(x_{N-1}-x_N)
\end{aligned}\right\} \quad (2.2.28)$$

Upon multiplying the i^{th} equation in 2.2.28 by x_i, $\dot{x}_i$, $\ddot{x}_i$ respectively, we obtain the following system of parameter identification equations.

$$\left.\begin{aligned}
m_1E\{x_1\ddot{x}_1\} &= E\{x_1f_1\} - C_1E\{x_1(\dot{x}_1-\dot{x}_2)\} - k_1E\{x_1(x_1-x_2)\}\\
m_1E\{\dot{x}_1\ddot{x}_1\} &= E\{\dot{x}_1f_1\} - C_1E\{\dot{x}_1(\dot{x}_1-\dot{x}_2)\} - k_1E\{\dot{x}_1(x_1-x_2)\}\\
m_1E\{\ddot{x}_1{}^2\} &= E\{\ddot{x}_1f_1\} - C_1E\{\ddot{x}_1(\dot{x}_1-\dot{x}_2)\} - k_1E\{\ddot{x}_1(x_1-x_2)\}\\
m_2E\{x_2\ddot{x}_2\} &= E\{x_2f_2\} - C_2E\{x_2(\dot{x}_2-\dot{x}_3)\} - k_2E\{x_2(x_2-x_3)\}\\
&\quad + C_1E\{x_2(\dot{x}_1-\dot{x}_2)\} + k_1E\{x_2(x_1-x_2)\}\\
m_2E\{\dot{x}_2\ddot{x}_2\} &= E\{\dot{x}_2f_2\} - C_2E\{\dot{x}_2(\dot{x}_2-\dot{x}_3)\} - k_2E\{\dot{x}_2(x_2-x_3)\}\\
&\quad + C_1E\{\dot{x}_2(\dot{x}_1-\dot{x}_2)\} + k_1E\{\dot{x}_2(x_1-x_2)\}\\
m_2E\{\ddot{x}_2{}^2\} &= E\{\ddot{x}_2f_2\} - C_2E\{\ddot{x}_2(\dot{x}_2-\dot{x}_3)\} - k_2E\{\ddot{x}_2(x_2-x_3)\}\\
&\quad + C_1E\{\ddot{x}_2(\dot{x}_1-\dot{x}_2)\} + k_1E\{\ddot{x}_2(x_1-x_2)\}\\
&\vdots\\
&\text{etc.}
\end{aligned}\right\}\quad(2.2.29)$$

Naturally, in the steady state case we can apply the identities 2.2.7 in order to simplify further the system of equations 2.2.29. One point, in passing, is that the chain-like nature of the system 2.2.28 yields a set of parameter estimation equations that can be solved sequentially. Thus, in 2.2.29 we can solve for m_1, C_1, and k_1 from the first three equations, then substitute these estimates in the next set of three equations to yield estimates of m_2, C_2, and k_2. This procedure may be continued along the chain. We shall amplify this in Chapter IV on the details of computer simulation of the various systems.

One obvious question that one must consider concerns the errors that are made when the system is not yet in the stationary state. This is likely to occur when the damping factor, C, is small relative to the spring constant k. We can easily illustrate the effects of an error that is made in the estimated value $E\{y\dot{y}\}$, say, for the simple oscillator.

Let us assume the mass, m, is unity in the equation 2.2.16 for the simple one degree of freedom oscillator. We assume C, k are unknown.

Then equations 2.2.17 yield for estimates

$$\left.\begin{aligned}\hat{C} &= \frac{\widehat{E\{f\dot{y}\}} - k\,\widehat{E\{y\dot{y}\}} - \widehat{E\{\dot{y}\ddot{y}\}}}{\widehat{E\{\dot{y}^2\}}} \\[2ex] \hat{k} &= \frac{\widehat{E\{fy\}} - C\,\widehat{E\{y\dot{y}\}} - \widehat{E\{y\ddot{y}\}}}{\widehat{E\{y^2\}}}\end{aligned}\right\}\qquad (2.2.30)$$

where "^" denotes estimate.

If k is very large relative to C, then a small error in the estimate of $E\{y\dot{y}\}$ shall create a large error in $\hat{C}$. Thus, upon placing $E\{y\dot{y}\}$ equal to zero when it is not quite zero can yield large errors in $\hat{C}$. Indeed, the $k\,\widehat{E\{y\dot{y}\}}$ greatly dominates the $E\{\dot{y}\ddot{y}\}$ term. Hence a small error in $E\{\dot{y}\ddot{y}\}$ will not affect the $\hat{C}$ as much.

The story is quite different for the estimate of k, that is, $\hat{k}$. The error in $E\{y\dot{y}\}$ by setting it equal to zero has very little effect on $\hat{k}$ if $\hat{C} << k$. Thus, one would conclude that $\hat{k}$ could still be estimated reasonably well by setting various of the second moments equal to zero, even when this is in error due to the fact that the system has not reached steady state yet. One would also conclude that the damping coefficient estimate would suffer greatly. This is exactly what was observed in our simulation experiments as discussed in Chapter III.

When the system is in the steady state, the estimates obtained from simulations were quite acceptable.

2.3 Deterministic Excitations

We shall now concentrate upon the problem of identification of linear systems by means of deterministic excitations of the type that are readily available in laboratory test situations. The most common types of excitations that can be generated in the laboratory are the pure sinusoidal and the sweep sinusoidal oscillations.

In order to illustrate the approach one takes for such a deterministic input, let us consider the simple oscillator with a sinusoidal excitation.

Thus, consider

$$m\ddot{y} + C\dot{y} + ky = \sin \omega t \tag{2.3.1}$$

We assume the system to be asymptotically stable in order that the transients will die out. In this case, it is equivalent to specify that bounded inputs yield bounded outputs.

For zero initial conditions the solution may be written as,

$$y_{(t)} = \int_0^t d\tau \, H(t-\tau) \sin \omega\tau \tag{2.3.2}$$

where

$$H(t) = \frac{1}{m\sqrt{\frac{k}{m} - \frac{C^2}{4m^2}}} e^{-\frac{C}{2m}t} \sin\sqrt{\frac{k}{m} - \frac{C^2}{4m^2}}\ t \qquad (2.3.3)$$

We define an average operator of the form

$$\langle u(t) \rangle \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T u(t)\, dt, \qquad (2.3.4)$$

for functions $u(t)$ for which this operator exists.

Now on the basis of this operator, let us attempt to identify the constants m, C, and k. We shall multiply equation 2.3.1 by y, $\dot{y}$, $\ddot{y}$, respectively, and take averages as defined by equation 2.3.4 to yield

$$\begin{aligned} m\langle y\ddot{y}\rangle + C\langle y\dot{y}\rangle + k\langle y^2\rangle &= \langle fy\rangle \\ m\langle \dot{y}\ddot{y}\rangle + C\langle \dot{y}^2\rangle + k\langle y\dot{y}\rangle &= \langle f\dot{y}\rangle \qquad (2.3.5) \\ m\langle \ddot{y}^2\rangle + C\langle \dot{y}\ddot{y}\rangle + k\langle y\ddot{y}\rangle &= \langle f\ddot{y}\rangle \end{aligned}$$

We now wish to investigate these terms in somewhat more detail.

One simply obtains

$$\langle y\dot{y}\rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T y(t)\,\dot{y}(t)\,dt = \lim_{T\to\infty} \frac{1}{2T}\,[y^2(t)] = 0 \quad (2.3.6)$$

where we have used the fact that the initial conditions are zero and $y(t)$ is bounded on the interval $(0,\infty)$ since the excitation is bounded.

Therefore, we can in the same fashion determine that

$$\left.\begin{aligned} \langle \dot{y}\ddot{y}\rangle &= 0 \\ \langle y\ddot{y}\rangle &= -\langle \dot{y}^2\rangle \end{aligned}\right\} \quad (2.3.7)$$

It will follow that 2.3.5 can be written as,

$$\left.\begin{aligned} -m\langle \dot{y}^2\rangle \qquad + k\langle y^2\rangle &= \langle fy\rangle \\ c\langle \dot{y}^2\rangle \qquad\qquad &= \langle f\dot{y}\rangle \\ m\langle \ddot{y}^2\rangle \qquad - k\langle \dot{y}^2\rangle &= \langle f\ddot{y}\rangle \end{aligned}\right\} \quad (2.3.9)$$

The reader will recognize the equations 2.3.9 to be identical to the parameter identification equations 2.2.27 except that the expectation operator in 2.2.27 is replaced by the time average operator as defined in 2.3.4.

However, there is a very significant distinction to be made in the derivation of 2.3.9 as opposed to the derivation of 2.2.27. In the derivation of 2.2.27 it was assumed that the processes over which the expectations are applied are stationary processes (at least up to the second moments). This is guaranteed by our assumptions of a stationary process input, into the system 2.1.1, where the constant matrix A is a stability matrix. In that case, the steady state solution is a stationary random process and equation 2.2.27 applies. On the other hand, the equation 2.3.9 did not make use of the assumption of stationarity, or steady state, for its derivation. This is a very significant point. In fact, for the sinusoidal input case, if $y_s(t)$ is the steady state solution, we cannot identify all three constants m, C, k. This can be illustrated very simply in the following fashion. The steady state solution is given.

$$y_s(t) = \int_{-\infty}^{t} H(t-\tau) \sin \omega\tau d\tau = \int_{0}^{\infty} H(\tau) \sin \omega(t-\tau) d\tau$$

$$= Q \sin (\omega t - \phi) \qquad (2.\ 3.10)$$

where Q, ϕ are easily determined in terms of the integrals

$$\int_0^\infty H(\tau) \sin \omega\tau \, d\tau. \quad \int_0^\infty H(\tau) \cos \omega\tau \, d\tau.$$

Therefore, we find,

$$\left.\begin{aligned} \dot{y}_s(t) &= \omega Q \cos(\omega t - \phi) \\ \ddot{y}_s(t) &= -\omega^2 Q \sin(\omega t - \phi) \end{aligned}\right\} \qquad (2.3.11)$$

Upon applying 2.3.11 into the equations 2.3.9 we find the equations

$$\begin{aligned} -m\langle \dot{y}_s^{\,2}\rangle \qquad\qquad + k\langle y_s^{\,2}\rangle &= \langle f y_s\rangle \\ C\langle \dot{y}_s^{\,2}\rangle \qquad\qquad &= \langle f\dot{y}_s\rangle \qquad (2.3.12) \\ m\langle \ddot{y}_s^{\,2}\rangle \qquad\qquad - k\langle \dot{y}_s^{\,2}\rangle &= \langle f\ddot{y}_s\rangle \end{aligned}$$

We easily evaluate

$$\langle y_s^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T Q^2 \sin^2 (\omega t-\phi)dt = \frac{Q^2}{2}$$

$$\langle \dot{y}_s^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T \omega^2 Q^2 \cos^2 (\omega t-\phi)dt = \frac{\omega^2 Q^2}{2} \qquad (2.3.13)$$

$$\langle \ddot{y}_s^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T \omega^4 Q^2 \sin^2 (\omega t-\phi)dt = \frac{\omega^4 Q^2}{2}$$

The determinant of the linear system of equations 2.3.12 in the unknowns m, C, k is

$$\begin{vmatrix} -\frac{\omega^2Q^2}{2} & 0 & \frac{Q^2}{2} \\ 0 & \frac{\omega^2Q^2}{2} & 0 \\ \frac{\omega^4Q^2}{2} & 0 & -\frac{\omega^2Q^2}{2} \end{vmatrix} = 0 \qquad (2.3.14)$$

since the third row is $-\omega^2$ times the first row.

Hence, we <u>cannot</u> identify all three unknowns by these equations in the steady-state case with sinusoidal excitations. The reason is quite clear: y_s and its two derivatives are not linearly independent of one another. However, any two unknown parameters can be identified. This will be illustrated in simulated examples in the next chapter. Of course, this problem does not enter into the random excitation case simply because the sample functions and their derivatives are not linearly dependent. Therefore, one would not expect to obtain linearly dependent moments from the stationary random solution processes.

The equation 2.3.9 was obtained for a simple oscillator. We can easily establish that similar equations are possible in higher degree of freedom systems as well. Thus, let us consider the general solution to the system

$$\frac{d\bar{y}(t)}{dt} = A\bar{y}(t) + \bar{f}(t) \tag{2.3.15}$$

where A, $\bar{y}$, $\bar{f}$ are defined in Section 2.1.

For zero initial conditions, the solution process can be written as,

$$\bar{y}(t) = \int_0^t e^{A(t-\tau)} \bar{f}(\tau)\, d\tau \tag{2.3.16}$$

Any component $y_i(t)$ of the state vector $\bar{y}$ can be given by the integral

$$y_i(t) = \sum_{j=1}^{n} \int_0^t H_{ij}(t-\tau)\, f_j(\tau)\, d\tau \qquad (2.3.17)$$

where $f_j(\tau)$ is the excitation at the j^{th} driving point and $H_{ij}(t-\tau)$ is the influence function or impulsc response between the j^{th} driving point and the i^{th} state. From our assumed stability we have,

$$\int_0^\infty | H_{ij}(\tau) |\, d\tau < \infty \qquad \text{for } i, j = 1, \ldots, n \quad (2.3.18)$$

We are assuming the $f_j(t)$ to be of the form

$$f_j(t) = F_j \sin \omega_j t \qquad (2.3.19)$$

Thus, it follows that y_i and its derivatives are bounded.

In order to achieve equations of the form analogous to 2.3.9 we must establish

$$\left\langle y_i^n(t) \; \frac{dy_i(t)}{dt} \right\rangle = 0 \qquad \text{(a)}$$

$$\left\langle \frac{dy_i(t)}{dt} \; y_j(t) \right\rangle = - \left\langle y_i(t) \; \frac{dy_j(t)}{dt} \right\rangle \qquad \text{(b)} \tag{2.3.20}$$

But this is immediate for by definition

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T y_i^n(t) \; \frac{dy_i(t)}{dt} \; dt = \lim_{T\to\infty} \frac{1}{T} \; \frac{1}{n+1} \quad y_i^{n+1}(T) = 0 \text{ (a)}$$

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \frac{dy_i(t)}{dt} \quad y_j(t) \; dt \tag{2.3.21}$$

$$= \lim_{T\to\infty} \frac{1}{T} \, y_i(t) \; y_j(T) - \lim_{T\to\infty} \frac{1}{T} \int_0^T y_i(t) \quad \frac{dy_j(t)}{dt} \quad dt$$

$$= - \lim_{T\to\infty} \frac{1}{T} \int_0^T y_i(t) \quad \frac{dy_j(t)}{dt} \quad dt \qquad \text{(b)}$$

In the derivation of 2.3.21 we have used only the fact that the components $y_i(t)$ are bounded on $(0,\infty)$ and that the initial conditions are zero. (This last condition is clearly not a requirement for establishing 2.3.21.)

It was pointed out above that the steady state solution is not required for identification. We should now notice another remarkable fact. That is, the requirement of sinusoidal excitations is not basic in the derivations above. All that is basic is that the system is stable in the sense of bounded inputs yielding bounded outputs. Therefore, all that we require is that a bounded function, <u>any</u> function for that matter, is used for excitation purposes. Thus, for example, a sweep sinusoidal or even a damped sinusoidal function can lead, theoretically, to the identification of the parameters of the system Even more interesting, a sample excitation from a random process such as those discussed in Section 2.2 will yield identification on this basis. We must stop a moment and reflect upon this last point.

Our whole approach to the identification of the unknown parameters was originally motivated by the statistical reasoning as put forth in Section 2.2. Now we see that there is perhaps a more fundamental point that underlies the procedure that we are proposing. We are led to think this way, simply because we can establish the same technique for identification with <u>any</u> test excitation. Furthermore, it appears that steady state properties are not required.

There is, in fact, a more basic mechanism going on here that subsumes all of the previous analyses of Section 2.2 and this section as special cases.

We illustrate this mechanism, again, with the simple linear damped oscillator.

We consider the oscillator driven by a bounded excitation $f(t)$,

$$m\ddot{y}(t) + C\dot{y}(t) + ky(t) = f(t) \tag{2.3.22}$$

Suppose we are able to observe the excitation, the displacement, velocity and acceleration <u>exactly</u> at three different instants t_1, t_2, t_3. It would then follow that we would have three equations,

$$\left.\begin{aligned} m\ddot{y}_1 + C\dot{y}_1 + ky_1 &= f_1 \\ m\ddot{y}_2 + C\dot{y}_2 + ky_2 &= f_2 \\ m\ddot{y}_3 + C\dot{y}_3 + ky_3 &= f_3 \end{aligned}\right\} \tag{2.3.23}$$

at the observation times.

Hence, with only three observations, these equations, if not singular, would lead to identification of the parameters. However, even if we can obtain simultaneous records of the excitation, displacement, etc., we cannot expect to achieve exact observations of the four quantities at any given time.

Ineed, the observations would yield some error that is of an independent random error type with zero mean. That is, the observations would be of the form,

$$\left.\begin{aligned}
m(\ddot{y}_1 + n_{11}) + C(\dot{y}_1 + n_{12}) + k(y_1 + n_{13}) &= f_1 + n_{14} \\
m(\ddot{y}_2 + n_{21}) + C(\dot{y}_2 + n_{22}) + k(y_2 + n_{23}) &= f_2 + n_{24} \\
m(\ddot{y}_3 + n_{31}) + C(\dot{y}_3 + n_{32}) + k(y_3 + n_{33}) &= f_3 + n_{34}
\end{aligned}\right\} \quad (2.3.24)$$

where the n's are independent, identically distributed and $E\{n_{ij}\} = 0$.

But because the noise in the observations has mean zero, it follows that

$$\frac{1}{T} \int_0^T n(t)dt \to 0 \qquad \text{as} \qquad T \to \infty$$

If we write the observed excitation and dynamical variables as,

$$\tilde{f}(t), \quad \tilde{y}(t), \quad \tilde{\dot{y}}(t), \quad \tilde{\ddot{y}}(t)$$

then it will follow that

$$\left.\begin{aligned} \langle \tilde{f}(t) \rangle &= \langle f(t) \rangle \\ \langle \tilde{y}(t) \rangle &= \langle y(t) \rangle \\ \langle \tilde{\dot{y}}(t) \rangle &= \langle \dot{y}(t) \rangle \\ \langle \tilde{\ddot{y}}(t) \rangle &= \langle \ddot{y}(t) \rangle \end{aligned}\right\} \quad (2.3.25)$$

that is, the errors in observation will, so to speak, average out to zero.

If $\langle f(t) \rangle$ is identically zero, then all terms in 2.3.25 will be zero. Hence, directly taking a time average of 2.3.22 would not yield a usable equation for identification purposes. For this reason we will use, as before, time averages of second powers for identification of linear systems.

Hence, the actual identification equations that we apply in the case of the linear system 2.3.22 are given as,

$$\left.\begin{aligned}
&m\,\frac{1}{T}\int_{0}^{T} y(t)\;\ddot{y}(t)\;dt + C\,\frac{1}{T}\int_{0}^{T} y(t)\;\dot{y}(t)\;dt \\
&\qquad + k\,\frac{1}{T}\int_{0}^{T} y^{2}(t)\;dt \;=\; \frac{1}{T}\int_{0}^{T} f(t)\;y(t)\;dt \\
&m\,\frac{1}{T}\int_{0}^{T} \dot{y}(t)\;\ddot{y}(t)\;dt + C\,\frac{1}{T}\int_{0}^{T} \dot{y}^{2}(t)\;dt \\
&\qquad + k\,\frac{1}{T}\int_{0}^{T} y(t)\;\dot{y}(t)\;dt \;=\; \frac{1}{T}\int_{0}^{T} f(t)\;\dot{y}(t)\;dt \\
&m\,\frac{1}{T}\int_{0}^{T} \ddot{y}^{2}(t)dt + C\,\frac{1}{T}\int_{0}^{T} \dot{y}(t)\ddot{y}(t)dt \\
&\qquad + k\,\frac{1}{T}\int_{0}^{T} y(t)\ddot{y}(t)dt \;=\; \frac{1}{T}\int_{0}^{T} f(t)\ddot{y}(t)dt
\end{aligned}\right\}\quad(2.3.26)$$

where the variables are all considered to be the observed variables.

We wish to make it quite clear that equation 2.3.26 holds in all cases, random or deterministic. If $f(t)$ is a sample of a random process and T is large enough, then the averages can be replaced by expectation operators, allowing a number of the terms to go to zero as we have discussed before and follow the analysis of Section 2.2. But, whether T is large or not, equations 2.3.26 will always hold. As we shall see, they yield extremely accurate parameter estimates.

The equations analogous to 2.2.29 for an N mass chain are,

$$m_1 \frac{1}{T}\int_0^T y_1(t)\ddot{y}_1(t)dt = \frac{1}{T}\int_0^T y_1(t)f_1(t)dt$$

$$- C_1 \frac{1}{T}\int_0^T y_1(t)(\dot{y}_1(t)-\dot{y}_2(t))dt - k_1 \frac{1}{T}\int_0^T y_1(t)(y_1(t)-y_2(t))dt$$

$$m_1 \frac{1}{T}\int_0^T \dot{y}_1(t)\ddot{y}_1(t)dt = \frac{1}{T}\int_0^T \dot{y}_1(t)f_1(t)dt$$

$$- C_1 \frac{1}{T}\int_0^T \dot{y}_1(t)(\dot{y}_1(t)-\dot{y}_2(t))dt - k_1 \frac{1}{T}\int_0^T \dot{y}_1(t)(y_1(t)-y_2(t))dt$$

$$m_1 \frac{1}{T}\int_0^T \ddot{y}_1^2(t)dt = \frac{1}{T}\int_0^T \ddot{y}_1(t)f_1(t)dt$$

$$- C_1 \frac{1}{T}\int_0^T \ddot{y}_1(t)(\dot{y}_1(t)-\dot{y}_2(t))dt - k_1 \frac{1}{T}\int_0^T \ddot{y}_1(t)(y_1(t)-y_2(t))dt$$

$$m_2 \frac{1}{T}\int_0^T y_2(t)\ddot{y}_2(t)dt = \frac{1}{T}\int_0^T y_2(t)f_2(t)dt$$

$$- C_2 \frac{1}{T}\int_0^T y_2(t)(\dot{y}_2(t)-\dot{y}_3(t))dt - k_2 \frac{1}{T}\int_0^T y_2(t)(y_2(t)-y_3(t))dt$$

$$+ C_1 \frac{1}{T}\int_0^T y_2(t)(\dot{y}_1(t)-\dot{y}_2(t))dt + k_1 \frac{1}{T}\int_0^T y_2(t)(y_1(t)-y_2(t))dt$$

etc. (2.3.27)

We repeat that these equations hold for any T. In somes cases many of the terms will be close to zero relative to the other terms. This is especially so when the excitation is a sample from a stationary process and the system is operating in steady state. In that case, the analysis of Section 2.2 will apply.

We also wish to repeat that equations 2.3.27 hold for any excitation function as long as there is an appreciable magnitude of the output vector. This is quite distinct from the analysis of Section 2.2 where the stationary properties were significant.

In the next chapter we present the results of our experiments on simulated systems using both the statistical as well as the deterministic approaches to the identification of the unknown parameters for a variety of systems.

CHAPTER III
IDENTIFICATION OF SIMULATED LINEAR DYNAMICAL SYSTEMS

3.1 Introduction

Simulation studies were carried out for a number of linear dynamical systems. Coupled oscillators of the one-dimensional type as shown in Figure I (a) as well as a two-dimensional system shown in Figure I (b) were simulated and studied on the digital computer. For the coupled oscillator case studies were made on two and five mass systems. The most significant five mass system was defined by mass, damping and spring constants provided by NASA-Goddard. Inputs to the simulated systems were random as well as sinusoidal. The random excitations used were of two types. One was generated on the computer by passing white noise samples through a filter with selected band pass properties. The other excitation process was that taken from di ital tape records of noise generator sources as provided by NASA-Goddard. In the case of sinusoidal excitations, both fixed frequency as well as sweep frequency excitations were applied. In all cases, identification was accomplished. In many cases, as will be seen below, the estimated parameters are remarkably close to the actual parameters. In the following sections we present these results as well as various significantly chosen cases to shed as much light as possible on the present approach to the problem of identification. In Section 3.2 we shall present and discuss results based upon random excitations. In Section 3.3 we present and discuss results based upon sinusoidal excitation. In Section 3.4 we present preliminary results related to the estimation of parameters when only the excitation and acceleration data are known. In such a case one must integrate the acceleration data to yield the velocity up to an unknown initial constant. The initial condition is then determined by a least squares linear fit of the integrated acceleration data.

3.2 Identification of Simulated Systems One-Dimensional by Random Excitations

Among the random excitation simulation experiments, we have considered cases in which we achieve, for all practical purposes, a steady state condition so that equations 2.2.7 hold and can be applied to simplify the parameter identification equations 2.2.29.

We have also considered the situation in which the damping factors are small so that the stationary solution process has not developed. In that situation we merely revert to the time average and apply all terms in the equation 2.2.29. In all cases identification of the parameters is quite successful. We shall describe our results in the experiments that follow.

Experiment I. The object of this experiment is to study the effect of the transient period before observations are taken.

For this case we consider a two mass chainlike system whose dynamical equations are given by 2.2.28 with $N = 2$. The parameters for the simulated system are

$$m_1 = m_2 = 1, \quad k_1 = 16, \quad k_2 = 9, \quad C_1 = 4, \quad C_2 = 3 \tag{3.2.1}$$

Only one mass, m_1, is driven. The simulation as well as the sampling intervals are 0.05 sec. The number of samples used for estimation is 2000, which is equivalent to an identification period of 100 secs. The random excitation function is of the form

$$f_1(t_j) = \sum_{i=1}^{10} a_i \tilde{W}_{j-i} \tag{3.2.2}$$

where the $\{\bar{W}_j\}$ is a sequence of independent zero mean Gaussian random variables (white noise) and

$$a_1 = .1, \quad a_2 = .2, \quad a_3 = .3, \quad a_4 = .4$$

$$a_5 = \cdots = a_{10} = .5$$

The standard deviation of the white noise was chosen to be 30.

The simulation was initiated and observations were taken for a period of 100 seconds commencing at 50 secs, 150 secs, etc. for five successive observation periods. The parameters were estimated on the basis of the observations in the periods 50-150, 150-250, ..., 450-550. The estimates are given in the following table.

Transient Interval Observation Starting At

Estimate	50 sec	150 sec	250 sec	350 sec	450 sec	True
$\hat{m}_1$	1.0034	1.0176	.9865	1.002	1.003	1.0
$\hat{k}_1$	16.037	16.3286	15.80	16.08	15.945	16.0
$\hat{c}_1$	4.015	4.1219	3.89	4.039	3.954	4.0
$\hat{m}_2$	1.0038	1.069	.9338	1.009	0.985	1.0
$\hat{k}_2$	9.022	9.3616	8.75	9.067	8.906	9.0
$\hat{c}_2$	3.0079	3.1093	2.924	3.034	2.945	3.0

Table I

Clearly, the estimated parameters are excellent. The variations are merely random variations and are not a function of the transient interval. This, of course, is due to the fact that after a period of 50 seconds, the system is already in the steady state because of relatively high ratio of critical damping.

It would appear that after the transients have died out, observation can be started at an arbitrary time to yield satisfactory estimates.

It is interesting to note that the average of these five runs gives

$$\overline{m_1} = 1.0025 \quad \overline{k_1} = 16.0381 \quad \overline{c_1} = 4.0039$$

$$\overline{m_2} = 1.0001 \quad \overline{k_2} = 9.0213 \quad \overline{c_2} = 3.041$$

which are very close to the true parameters.

Experiment II. The object of this experiment is to demonstrate the variation in the parameter estimates when passing from transient state into steady state conditions. For this experiment a two mass chainlike oscillato was used. Hence, the system equations are identical to those of experiment I. The system parameters we hosen as,

$$m = m_1 = m_2 = .707, \quad C = C_1 = C_2 = 50, \quad k = k_1 = k_2 = 1.5 \times 10^5$$

For this system we have calculated all of its characteristic numbers which are given by the roots of the polynomial,

$$(m\, s^2 + \tfrac{3}{2} C\, s + \tfrac{3}{2} k)^2 - \tfrac{5}{4} (C\, s + k)^2 = 0 \qquad (3.2.3)$$

One finds the solutions to this characteristic equation to be

$$\frac{-\left(\frac{3 - \sqrt{5}}{2}\right)C \pm \sqrt{\left(\frac{3 - \sqrt{5}}{2}\right)^2 C^2 - 2\,(3 - \sqrt{5})k}}{2m}$$

$$(3.2.4)$$

$$\frac{-\left(\frac{3 + \sqrt{5}}{2}\right)C \pm \sqrt{\left(\frac{3 + \sqrt{5}}{2}\right)^2 C^2 - 2\,(3 + \sqrt{5})k}}{2m}$$

The mode frequencies are approximately 53 c.p.s. and 140 c.p.s. with corresponding damping factors approximately 26.8 and 185.

The system was excited by a random force generated by passing white noise samples through a band pass filter with center frequency 60 c.p.s. and effective band-width (defined by location of 1/2 power frequency) 40 c.p.s. Thus, the band has a range of 40 c.p.s. to 80 c.p.s.

This is generated by a system of the form

$$\ddot{f} + B\dot{f} + \omega^2 f = B\dot{x} \tag{3.2.5}$$

A description of the generation of these excitations is given in Chapter IV. The sampling rate is 1600 samples/sec. corresponding to approximately 11 samples per cycle of highest mode frequency. The estimates of parameters were taken from observations on the intervals 0 - 0.4, 0.4 - 0.8, 0.8 - 1.2, and 0 - 0.8, 0.8 - 1.6, 1.6 - 2.4. The 0 - 0.4 and 0 - 0.8 contain the transient intervals which are relatively short with the present damping constants. The estimates are given in the following table. Run number one and run number two are excited by two different samples from the excitation process.

The major features of these estimates are that they vary in accordance with the variations of the estimates of the moments that are used in the identification formulas. Thus, the errors that are present are due, in part, to setting those moments to zero, such as, $E\{y_1\dot{y}_1\}$, $E\{y_2\dot{y}_2\}$ etc., when they may not actually be small. The greatest errors

Observation Interval

Estimated Parameters	0 - 0.4	0.4 - 0.8	0.8 - 1.2	0 - 0.8	0.8 - 1.6	1.6 - 2.4	True Parameter Values
$\hat{m}_1$	.7095	.7142	.6999	.7079	.7103	.7062	.707
$\hat{k}_1$	1.5173×10^5	1.5124×10^5	1.4825×10^5	1.5032×10^5	1.5096×10^5	1.4962×10^5	1.5×10^5
$\hat{C}_1$	72.701	44.271	46.633	52.293	55.963	44.603	50
$\hat{m}_2$	.7533	.7146	.6742	.7125	.7255	.6945	.707
$\hat{k}_2$	1.5532×10^5	1.5127×10^5	1.4598×10^5	1.5072×10^5	1.5220×10^5	1.4860×10^5	1.5×10^5
$\hat{C}_2$	76.349	41.902	45.941	52.607	56.599	43.704	50
	Run No. 1			Run No. 2			

Table II

occur in the interval o - 0.4 since in this interval the transient period is roughly 1/3 of the interval length. Thus, the errors are greatest for estimating "stationary" moments. As we have stated in Section 2.2, these errors will be reflected greatly in the damping constant estimates, and to a much lesser extent in the spring constant estimates. This is exactly what is shown in the first column of the table above. Indeed, the spring constant estimates are uniformly good as well as the mass estimates. The greatest variation occurs in $\hat{C}$. In run number one, we see an obvious change in the accuracy of the estimates after the interval o - 0.4 since then we are effectively in the steady state case. Due to the increased length of intervals of observation, there is a more definite change in the error of estimation from 0 - 0.4 to 0.4 - 0.8 than from 0 - 0.8 to 0.8 - 1.6 because the transient period is a much smaller part of the observation period in 0 - 0.8. Thus, the variations in the estimates of run number two can be considered as truly random variations.

<u>Experiment III</u>. In this experiment, which was performed at an early stage of the study, a five mass chainlike system was simulated with only the first mass being excited. The input excitation is identical to the excitation used in experiment I. The interval of simulation was .05 secs with

5000 samples used for identification purposes. The transient interval was chosen as 50 secs, 300 secs, 550 secs. Thus, after the first 50 second transient interval successive runs of 250 secs were used for identification. The results of the three runs are shown in Table III (a). We denote the estimates for the three successive runs as $\hat{m}_1(1)$, $\hat{m}_1(2)$, $\hat{m}_1(3)$, $\hat{k}_1(1)$, etc.

The first and third runs are quite acceptable as estimates of the true parameters. However, the second set does possess large estimate errors, especially for $\hat{C}_1$. Of course, as we have already seen in Chapter II, we expect errors to be greater in the damping constant estimates. Part of the source of error here was traced to the way in which the excitation was being simulated. A modification in the simulation of the excitation, which was used in all future simulations, brought the new estimations given here, in Table III (b), only for the first run.

All estimates in Table III (b) are quite acceptable. Of course, one should expect good estimates since the damping is relatively high so that after a transient interval of 50 seconds the system is in steady state. Furthermore, the observation period is quite long. Hence, stationary moment estimates should be close to time estimates.

i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	$\hat{m}_i(3)$	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	$\hat{k}_i(3)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	$\hat{C}_i(3)$
1	1.0	0.879	.655	.940	49.0	49.604	46.555	51.594	7.0	9.126	12.433	9.035
2	1.0	1.163	1.054	1.048	36.0	36.886	33.623	37.971	6.0	5.318	7.033	7.647
3	1.0	.946	.924	.928	25.0	25.201	23.084	25.528	5.0	4.891	5.344	6.498
4	1.0	1.027	.955	.904	16.0	16.198	14.804	15.831	4.0	3.884	4.036	4.870
5	1.0	1.020	1.000	.885	9.0	9.136	8.390	8.702	3.0	2.940	2.947	3.420

Table III (a)

	m_i	$\hat{m}_i$	k_i	$\hat{k}_i$	C_i	$\hat{C}_i$
1	1.0	1.096	49.0	50.32	7.0	5.78
2	1.0	0.710	36.0	33.80	6.0	7.40
3	1.0	1.210	25.0	24.78	5.0	4.36
4	1.0	0.917	16.0	15.65	4.0	3.72
5	1.0	0.939	9.0	8.76	3.0	2.82

Table III (b)

Of course, this system with high damping factors does not truly reflect actual structural systems in which the damping ratio is very low. We, thus, considered a system with parameters that are comparable to those found in an actual structural system.

<u>Experiment IV</u>. In this experiment, we simulated a five mass chainlike system as shown in Figure I(a) of Chapter II. The system equations, again, are given by 2.2.28 with $N = 5$. We based our estimations upon the assumption of steady state with a statistically stationary solution process so that the various moments were set to zero as given by equation 2.2.7. The spring and damping constants were supplied by NASA-Goddard as reflecting the true parameter values of a NASA-Goddard five mass experimental model. The simulated system was excited by the same function as in experiment III, as well as by randomly generated excitations as provided on a tape recording supplied by NASA-Goddard. The spectrum of the taped excitation is given by the following figure.

The sampling frequency was 1400 c.p.s., a total of 42,000 samples were used, representing 30 secs of observation time. A typical set of estimates is given in the table below to three places beyond the decimal point.

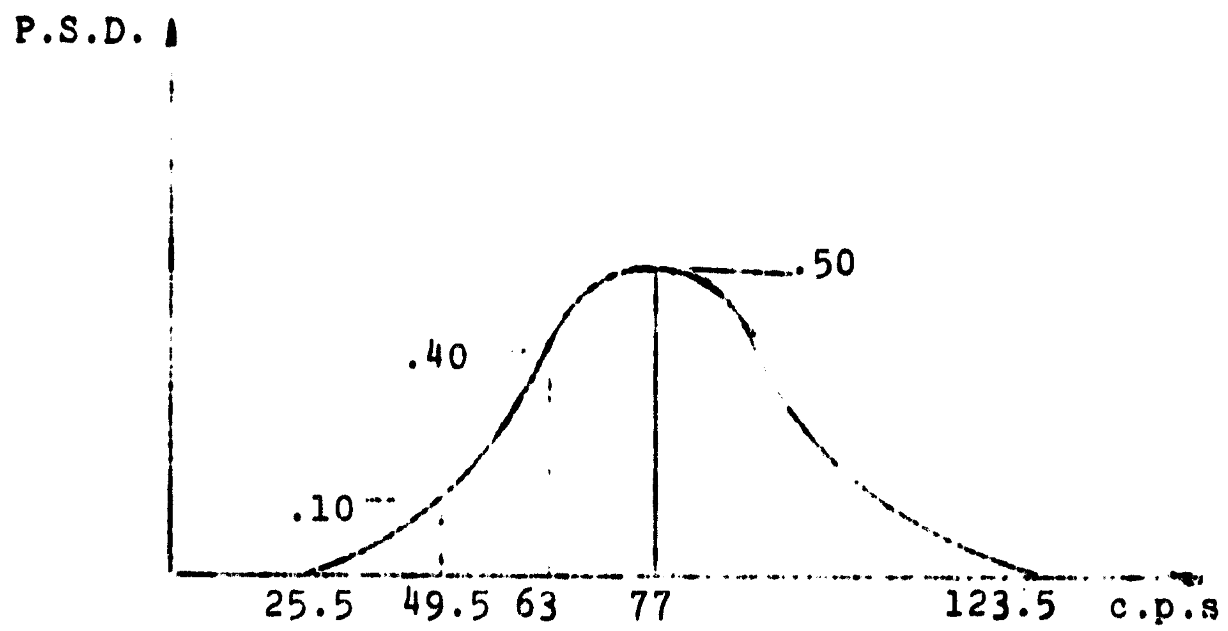


Figure III

i	m_i	$\hat{m}_i$	k_i	$\hat{k}_i$	C_i	$\hat{C}_i$
1	.667	.666	$.930 \times 10^6$	$.929 \times 10^6$	.900	18.519
2	.667	.667	$.830 \times 10^5$	$.829 \times 10^5$	1.900	3.513
3	.667	.666	$.930 \times 10^6$	$.929 \times 10^6$	.900	44.960
4	.667	.667	$.830 \times 10^5$	$.829 \times 10^5$	1.900	-4.916
5	.667	.666	$.530 \times 10^3$	$.529 \times 10^3$	.100	6.023

Table IV

The very striking feature of this set of estimates is that the spring constants and mass constants are extremely accurate, whereas the damping constant estimates are not even close to the true values. Of course, we recognize this to be a case where the damping constant is extremely small and for all practical purposes the damping ratio is zero in view of the high spring rate. As we have indicated in the analysis of Chapter II, we will expect large errors in $\hat{C}_1$ and small errors in $\hat{k}_1$ in this situation. As we see, the fact that $C_1 << k_1$ gives us almost no effect on $\hat{k}_1$ of an error in $\hat{C}_1$. It can only effect $\hat{k}_1$ in the fifth and sixth digits which are outside any practical significance.

The question is, why do we have such a great error in $\hat{C}_1$ in the first place? The reason is clear; $\hat{C}_1$ is so small that it simply takes too long for the steady state to develop. Hence, we did not observe the steady state over the interval used for identification purposes. Furthermore, applying longer transient intervals before observations were performed on the simulated system did not appreciably help matters. Although $\hat{m}_1$, $\hat{k}_1$ were always quite good, the $\hat{C}_1$ were very poor.

It was at this point that we applied the time average ideas as described by equations 2.3.27 of Chapter II in order to attempt the identification of the parameters.

<u>Experiment V</u>. We present in this experiment, the results of identification of simulated five-mass chainlike system as given by equations 2.3.27 where the system is not in the steady state due to the extremely small damping ratio. Simulated, as well as tape force, inputs were used. Various sampling rates and periods of observations were applied; these are presented in the following tables: V(a) and V(b). <u>No</u> transient interval was used. That is, observations for identification purposes were started at $t = 0$.

For this table, the force input was the taped excitation provided by NASA-Goddard with spectrum as shown in Experiment IV. The sampling frequency was 1400 c.p.s. with 10 secs, 20 secs, and 30 secs used for observation intervals all starting at $t = 0$. $\hat{m}_i(1)$, $\hat{k}_i(1)$, $\hat{C}_i(1)$ corresponds to 10 second observation interval, similarly for 20 secs and 30 secs. The most obvious feature of Table V (a) is the remarkable accuracy of the estimates. This appears to be independent of the length of the observation interval in this case. But, we do notice, that even at 10 secs, we are taking 14,000 points for identification purposes which is a large number of observations.

Table V (b) shows the results of identification for exactly the same system and same excitation as in Table V (a) except that the taped excitation was sampled as

	True Value	Estimated Interval 10 secs	20 secs	30 secs	True Value	Estimation Interval 10 secs	20 secs	30 secs	True Value	Estimation Interval 10 secs	20 secs	30 secs
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	$\hat{m}_i(3)$	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	$\hat{k}_i(3)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	$\hat{C}_i(3)$
1	.667	.667	.667	.667	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	.900	.900	.895	.894
2	.667	.667	.667	.667	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.900	1.900	1.900
3	.667	.667	.667	.667	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	.900	.904	.885	.878
4	.667	.667	.667	.667	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.900	1.905	1.904
5	.667	.667	.667	.667	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	.100	.100	.096	.096

Table V (a)

i	m_i	$\hat{m}_i$	k_i	$\hat{k}_i$	C_i	$\hat{C}_i$
1	.667	.666	$.93 \times 10^6$	$.93 \times 10^6$	.900	.914
2	.667	.666	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.901
3	.667	.666	$.93 \times 10^6$	$.93 \times 10^6$	.900	.973
4	.667	.666	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.897
5	.667	.666	$.53 \times 10^3$	$.53 \times 10^3$	.100	.106

Table V (b)

700 c.p.s. rather than the previous 1400 c.p.s. to determine the significance of the sampling rates. Thus, the system was simulated at an interval of .00143 secs. Table V (b) shows these results for 10 second observation time. Again, the identification observations commence at t = 0.

We notice a slight change in the accuracy of the $\hat{C}_1$; however, $\hat{m}_1$ and $\hat{k}_1$ do not appear to be affected by this change in sampling rate.

The following table, V (c), shows the parameter identification of the linear system, all of whose parameters are taken from the five-mass NASA-Goddard system. The spring and damping constants are the same as in Tables V (a) and V (b). The only change is the mass parameter, which is .052 instead of .667. Thus, the natural frequencies of this system are somewhat higher than in the previous cases. Upon simulation at 1400 samples per second, quite a bit of error in the estimates were present. It was felt that the simulation rate was not large enough to account for the frequency range of this new system which had gone up about 3.5 times. Thus, it was decided to raise the simulation rate to 5000 c.p.s. and make a time scale change on the taped input excitation to account for this new sampling rate. Therefore, effectively, the excitation was at a higher frequency range than in real time.

The two sets of estimates are accomplished by the taped input force with the time scale change for 1 second and 2 seconds observation intervals starting at $t = 0$.

We see in Table V (c) that the 2 second estimates of C_1 are better than the 1 second estimates. Longer observation times will undoubtedly yield extremely accurate $\hat{C}_1$. Again, $\hat{m}_1$ and $\hat{k}_1$ are uniformly excellent.

We can summarize all of the results in this section by stating that if it is reasonably established that the system is in steady state operation, then Equation 2.2.7 may be applied to reduce the number of constants in the identification equations. However, if there is any doubt or merely in order to be somewhat more confident of the estimates, it appears that the time average equations 2.3.27 will always give good estimates without waiting for the transients to die out. Of course, if transients have effectively disappeared, the moments that should reduce to zero will be effectively zero upon estimation. The frequency range of the excitation does not appear to be a factor. However, the time of observation is certainly a factor as well as the sampling rate. A rough figure that can probably be adhered to is 5 - 10 samples per cycle of the highest observed frequency of the output of the system.

	Estimation Interval			Estimation Interval			Estimation Interval		
i	True Value m_i	1 sec $\hat{m}_i(1)$	2 sec $\hat{m}_i(2)$	True Value k_i	1 sec $\hat{k}_i(1)$	2 sec $\hat{k}_i(2)$	True Value C_i	1 sec $\hat{C}_i(1)$	2 sec $\hat{C}_i(2)$
1	.052	.052	.052	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	0.900	1.041	1.082
2	.052	.052	.052	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.900	1.903
3	.052	.052	.052	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	0.900	1.179	1.037
4	.052	.052	.052	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.865	1.907
5	.052	.052	.052	$.43 \times 10^2$	$.43 \times 10^2$	$.43 \times 10^2$	0.100	0.135	0.088

Table V (c)

The lowest frequency should be observed for 5 - 10 cycles as well. These could be approximated by observation on the oscilloscope during the period that identification tests are being conducted. This was made quite clear during Experiment V where the sampling interval had to be changed due to the higher frequency range of the system with smaller mass.

In the next section, we shall see how these points are reflected in identification by sinusoidal excitations.

3.3 Identification of Simulated One Dimensional Systems by Sinusoidal Excitations

Experiment I. In this first experiment, we present the results of identification studies of the same system described in Experiment II of Section 3.2. In this case, the simulated system was excited at these distinct frequencies: 20 c.p.s., 60 c.p.s., and 160 c.p.s. with amplitude 20. The sampling rate was 1600 c.p.s. which is approximately 11 samples per cycle of the highest mode frequency of the system which is 140 c.p.s. The low mode is approximately 53 c.p.s. Hence, we see that the three excitations lie below, in between and above the system's natural frequencies. For each excitation the observation intervals were .05 secs,

.1 sec, .2 sec, .4 sec, .6 sec, .8 sec, 1.0 sec, all starting at t = 0. For the lowest mode, this represents 2.5 cycles, 5.3 cycles, 10.6 cycles, etc., up to 53 cycles, or 80 samples, 160 samples, etc.

The results of these estimates are given in tables VI (a), (b) and (c). The estimates are given by equation 2.3.27 for N = 2.

Needless to say, the estimations presented in Tables VI (a), (b), and (c) are extremely monotonous. They are, in fact, monotonous to four and five place accuracy! Only in the sixth and seventh places does one detect significant differences.

It is obvious that the transient period is still in effect during the entire period over which we are performing the parameter identifications since the estimates are excellent for every estimation interval up to 1.0 sec. This is reflected in the estimates for the random case of Experiment II, Section 3.2, where it was clear that the extimates were better in the periods .8 sec on to 2.4 secs (except for random fluctuations which one must expect when setting various of the moments to zero).

In the next section, we illustrate an example of how the transient periods and steady state periods affect estimation with sinusoidal excitations.

	True Value	Estimation Interval .05 sec	.1 sec	.2 sec	.4 sec	.6 sec	.8 sec	1.0 sec
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	$\hat{m}_i(3)$	$\hat{m}_i(4)$	$\hat{m}_i(5)$	$\hat{m}_i(6)$	$\hat{m}_i(7)$
1	.707	.707	.707	.707	.707	.707	.707	.707
2	.707	.707	.707	.707	.707	.707	.707	.707
i	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	$\hat{k}_i(3)$	$\hat{k}_i(4)$	$\hat{k}_i(5)$	$\hat{k}_i(6)$	$\hat{k}_i(7)$
1	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$
2	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$
i	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	$\hat{C}_i(3)$	$\hat{C}_i(4)$	$\hat{C}_i(5)$	$\hat{C}_i(6)$	$\hat{C}_i(7)$
1	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
2	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0

Table VI (a)

Excitation Frequency 20 c.p.s.

	True Value	Estimation Interval .05 sec	.1 sec	.2 sec	.4 sec	.6 sec	.8 sec	1.0 sec
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	$\hat{m}_i(3)$	$\hat{m}_i(4)$	$\hat{m}_i(5)$	$\hat{m}_i(6)$	$\hat{m}_i(7)$
1	.707	.707	.707	.707	.707	.707	.707	.707
2	.707	.707	.707	.707	.707	.707	.707	.707
i	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	$\hat{k}_i(3)$	$\hat{k}_i(4)$	$\hat{k}_i(5)$	$\hat{k}_i(6)$	$\hat{k}_i(7)$
1	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$
2	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$
i	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	$\hat{C}_i(3)$	$\hat{C}_i(4)$	$\hat{C}_i(5)$	$\hat{C}_i(6)$	$\hat{C}_i(7)$
1	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
2	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0

Table VI (b)

Excitation Frequency 60 c.p.s.

Estimation Interval

	True Value	.05 sec	.1 sec	.2 sec	.4 sec	.6 sec	.8 sec	1.0 sec
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	$\hat{m}_i(3)$	$\hat{m}_i(4)$	$\hat{m}_i(5)$	$\hat{m}_i(6)$	$\hat{m}_i(7)$
1	.707	.707	.707	.707	.707	.707	.707	.707
2	.707	.707	.707	.707	.707	.707	.707	.707
i	k_i	$k_i(1)$	$\hat{k}_i(2)$	$\hat{k}_i(3)$	$\hat{k}_i(4)$	$\hat{k}_i(5)$	$\hat{k}_i(6)$	$\hat{k}_i(7)$
1	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$
2	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$	$.15 \times 10^6$
i	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	$\hat{C}_i(3)$	$\hat{C}_i(4)$	$\hat{C}_i(5)$	$\hat{C}_i(6)$	$\hat{C}_i(7)$
1	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
2	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0

Table VI (c)

Excitation Frequency 160 c.p.s.

Experiment II. In Section 2.3 of Chapter II we presented an analysis of the identification technique for sinusoidal excitations when the system is in steady state. It was shown there that one cannot expect to identify all three unknown parameters m, k, and C in the steady state since the steady state solution under sinusoidal excitation will not yield sufficient linearly independent equations for identification purposes. We illustrate this with the present experiment. We take a system of five masses in the usual one-dimensional chain. The damping was chosen high in order to put the system into the steady state condition relatively rapidly. Both 10 c.p.s. and 100 c.p.s. sinusoidal excitations were applied. Estimations were taken over an interval of 1 sec. The first observation period was 0.0 - 1.0 sec, the next 1.0 - 2.0 secs, then 2.0 - 3.0 secs, etc. Thus, the transient period before observation was taken was 0.0 sec, 1.0 sec., 2.0 secs, etc. The sampling interval for simulation and estimation purposes was .001 second for the 10 c.p.s. excitation and .0005 second for 100 c.p.s. excitation. We shall show the variations in estimates of all three unknowns over 0.0 - 1.0 sec and 1.0 - 2.0 secs for 10 c.p.s. and 100 c.p.s. excitations as well as the estimates of only spring and damping constants for given mass.

i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$
	True	Transient Period 0.0 sec	Transient Period 1.0 sec	True	Transient Period 0.0 sec	Transient Period 1.0 sec	True	Transient Period 0.0 sec	Transient Period 1.0 sec
1	.667	.666	4.428	$.93 \times 10^6$	$.93 \times 10^6$	$.83 \times 10^5$	200	200.687	$-.12 \times 10^5$
2	.667	.667	-1.590	$.83 \times 10^5$	$.83 \times 10^5$	$.55 \times 10^5$	100	100.072	$-.51 \times 10^3$
3	.667	.668	3.129	$.93 \times 10^6$	$.93 \times 10^6$	$.45 \times 10^5$	200	191.842	$-.24 \times 10^5$
4	.6[illegible]	.670	-118.337	$.83 \times 10^5$	$.83 \times 10^5$	$.10 \times 10^7$	100	96.653	$.88 \times 10^5$
5	.667	.662	105.593	$.53 \times 10^3$	$.53 \times 10^3$	$-.31 \times 10^5$	100	100.020	$-.45 \times 10^3$

Table VII (a)

Excitation Frequency 10 cps

Mass, Spring, Damping Constants Unknown

i	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$
	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec
1	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	200	199.905	199.969
2	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	100	99.978	99.999
3	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	200	199.917	199.998
4	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	100	100.001	100.001
5	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	100	100.000	100.000

Table VII (b)

Excitation Frequency 10 cps

Mass Known, Spring, Damping Unknown

	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$
1	.667	.667	1.679	$.93 \times 10^6$	$.93 \times 10^6$	$-.23 \times 10^6$	200	199.981	$-.14 \times 10^4$
2	.667	.667	.494	$.83 \times 10^5$	$.83 \times 10^5$	$.32 \times 10^6$	100	99.998	$.93 \times 10^2$
3	.667	.667	25.470	$.93 \times 10^6$	$.93 \times 10^6$	$-.18 \times 10^8$	200	199.896	$-.40 \times 10^5$
4	.667	.667	-4.047	$.83 \times 10^5$	$.83 \times 10^5$	$.48 \times 10^7$	100	100.030	$.87 \times 10^2$
5	.667	.667	20.508	$.53 \times 10^3$	$.53 \times 10^3$	$.78 \times 10^5$	100	99.991	$.15 \times 10^6$

Table VII (c)

Excitation Frequency 100 c.p.s.

Mass, Spring, Damping Constants Unknown

	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec	True Value	Transient Period 0.0 sec	Transient Period 1.0 sec
i	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$
1	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	200	199.999	200.000
2	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	100	100.000	100.000
3	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	200	199.989	200.001
4	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	100	100.002	99.999
5	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	100	100.000	100.002

Table VII (d)

Excitation Frequency 100 c.p.s.

Mass Known, Spring, Damping Constants Unknown

Tables VII (a), (b), (c) and (d) show a very dramatic difference in going from transient to steady state intervals in attempting to identify the unknown parameters. In Tables VII (a) and (c), the first transient interval produces excellent estimates, whereas in the very next interval the estimates completely fall apart. They are in no way related to the actual parameter values. Yet, with the mass known in Tables VII (b) & (d) the $\hat{C}_1$ and $\hat{k}_1$ are excellent during both estimation periods. The reason, as we have developed in Chapter II, is that the equations for estimation become singular in the steady state case for three unknowns. But, they are not singular for two unknowns. Thus, the estimates will break down as shown, for three unknowns, in passing from primarily transient to primarily steady state conditions.

This experiment shows quite clearly that in order to identify all unknowns by sinusoidal excitations, it is important that the system be in the transient state.

Experiment III. In this experiment we simulate the NASA-Goddard five-mass system as in Experiment III of Section 3.2. The excitation is sinusoidal with frequency 70 c.p.s.

The system is sampled at 5000 c.p.s. Each run starts the observations at $t = 0$. The observation period is 0.0 - 2.0 secs, 0.0 - 4.0 secs. The results are shown in Table VIII. These estimates are very accurate, which clearly establishes the importance of the sinusoidal excitation for identification purposes.

<u>Experiment IV</u>. Since it was stated in Section 2.3 of Chapter II that it appears possible to identify with almost any "suitable" excitation function, it was decided that we should generate such an excitation for purposes of identification. It seemed reasonable that an excitation that could easily be achieved in the laboratory was the logical choice for identification. Thus, we generated a sweep sinusoidal excitation of the form

$$f_1(t) = A \sin \alpha t^2$$

where $A = 25$, $\alpha = 100$.

		Estimation Interval			Estimation Interval			Estimation Interval	
	True Value	0.0-2.0	0.0-4.0	True Value	0.0-2.0	0.0-4.0	True Value	0.0-2.0	0.0-4.0
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$
1	.052	.052	.052	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	1.900	1.895	1.883
2	.052	.052	.052	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	.900	.896	.895
3	.052	.052	.052	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	1.900	1.850	1.843
4	.052	.052	.052	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	.900	.898	.898
5	.052	.052	.052	$.43 \times 10^2$	$.43 \times 10^2$	$.43 \times 10^2$	.900	.900	.900

Table VIII

The sampling interval for simulation and estimation was 0.0014 secs. All observations commenced at t = 0.70 secs in order for the frequency to become reasonably large. We shall show the results of four runs of duration 3.92 secs, 9.80 secs, 13.72 secs, and 19.60 secs corresponding to 2800 samples, 7000 samples, 9800 samples, and 14,000 samples, respectively. The results are shown in Table IX.

The results of this experiment show that identification can be suitably accomplished by functions other than pure sinusoids or random excitations.

3.4 Identification of Multi-Dimensional Systems

It was found desirable to demonstrate that the proposed identification technique is not dependent upon the fact that the system to be identified was a one-dimensional system. The system shown in Figure I (b) of Chapter II was considered. The equations of motion are given in NASA Technical Note TN D-3865 "Mechanical Impedance Analysis for Lumped Parameter Multi-Degree of Freedom/Multi-Dimensional Systems" by F. J. On, May 1967.

	Estimation Interval						Estimation Interval					Estimation Interval			
	True Value	3.92 secs	9.80 secs	13.72 secs		True Value	3.92 secs	9.80 secs	13.72 secs		True Value	3.92 secs	9.80 secs	13.72 secs	
i	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	$\hat{m}_i(3)$	$\hat{m}_i(4)$	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	$\hat{k}_i(3)$	$\hat{k}_i(4)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	$\hat{C}_i(3)$	$\hat{C}_i(4)$
1	.667	.668	.666	.666	.666	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	.900	.737	.927	.963	1.083
2	.667	.668	.665	.666	.665	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.895	1.903	1.902	1.903
3	.667	.668	.666	.666	.666	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	$.93 \times 10^6$	.900	.741	.984	.946	1.030
4	.667	.668	.666	.666	.666	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	$.83 \times 10^5$	1.900	1.876	1.912	1.908	1.941
5	.667	.668	.666	.666	.666	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	$.53 \times 10^3$	.100	.108	.097	.099	.095

Table IX

These equations are as follows.

$$
\begin{aligned}
&m_1\ddot{x}_1 + 2(C_1+C_3)\dot{x}_1 - 2\,C_3\dot{x}_4 + 2(k_1+k_3)x_1 - 2k_3x_4 = f_1 \\
&m_1\ddot{x}_2 + 2(C_2+C_4)\dot{x}_2 - 2C_4\dot{x}_5 + 2(k_2+k_4)x_2 - 2k_4x_5 = f_2 \\
&I_1\ddot{x}_3 + 2(C_1b^2+C_2a^2+C_3c^2+C_4e^2)\dot{x}_3 - 2(C_3cd+C_4e^2)\dot{x}_6 \\
&\quad + 2(k_1b^2+k_2a^2+k_3c^2+k_4e^2)x_3 - 2(k_3cd+k_4e^2)x_6 = f_3 \\
&m_2\ddot{x}_4 - 2C_3\dot{x}_1 + 2C_3\dot{x}_4 - 2\,k_3x_1 + 2k_3x_4 = f_4 \\
&m_2\ddot{x}_5 - 2C_4\dot{x}_2 + 2C_4\dot{x}_5 - 2k_4x_2 + 2k_4x_5 = f_5 \\
&I_2\ddot{x}_6 - 2(C_3c\,d+C_4e^2)\dot{x}_3 + 2(C_3d^2+C_4e^2)\dot{x}_6 \\
&\quad - 2(k_3c\,d+k_4e^2)x_3 + 2(k_3d^2+k_4e^2)x_6 = f_6
\end{aligned}
\tag{3.4.1}
$$

The constants a, b, c, and d represent the various vertical and horizontal distances between the two-mass centers and the spring-damper components.

The system parameters supplied by Mr. F. On of NASA-Goddard are as follows.

$m_1 = m_2 = .26, \quad I_1 = I_2 = 70, \quad k_1 = k_3 = 10^5,$

$k_2 = k_4 = 4 \times 10^5, \quad C_1 = C_3 = 10.0, \; C_2 = C_4 = 20.0,$

$a = 10$ ins, $b = 20$ ins, $c = 15$ ins, $d = 6$ ins, $e = 12$ ins

This system was simulated and driven by random excitations as well as sinusoidal excitations. The system was simulated and sampled at intervals of .0005 secs. Observations commenced at $t = 0$ and were made at intervals of 0.0 - 1.0 secs and 0.0 - 2.0 secs. In the random case f_4, f_5, and f_6 were independent random excitations generated as previously by passing white noise through a filter to yield a process with spectrum having center frequency 70 c.p.s. and bandwidth 20 c.p.s. Furthermore, f_1, f_2, and f_3 were identically zero. For the sinusoidal case f_1, f_2, and f_3 again were zero, f_4, f_5, and f_6 were given as,

$$f_4 = 25 \sin 100 \pi t, \qquad f_5 = 25 \sin (100 \pi t + \tfrac{\pi}{2})$$

$$f_6 = 25 \sin (100 \pi t + \tfrac{3}{4} \pi).$$

The results are presented in tables X (a) and (b) first for random then for sinusoidal excitations.

Estimation Intervals

	True Value	0.0-1.0 sec	1.0-2.0 secs	True Value	0.0-1.0 sec	1.0-2.0 secs	True Value	0.0-1.0 sec	1.0-2.0 sec	True Value	0.0-1.0 sec	1.0-2.0 secs
i	k_i	$\hat{k}_i(1)$	$\hat{k}_i(2)$	C_i	$\hat{C}_i(1)$	$\hat{C}_i(2)$	m_i	$\hat{m}_i(1)$	$\hat{m}_i(2)$	I_i	$\hat{I}_i(1)$	$\hat{I}_i(2)$
1	$.99 \times 10^5$	$.99 \times 10^5$	$.10 \times 10^6$	10.0	10.0	10.0	.26	.26	.26	70.0	70.0	70.0
2	$.40 \times 10^6$	$.40 \times 10^6$	$.40 \times 10^6$	20.0	20.0	20.0	.26	.26	.26	70.0	70.0	70.0
3	$.99 \times 10^5$	$.10 \times 10^6$	$.10 \times 10^6$	10.0	10.0	10.0						
4	$.40 \times 10^6$	$.40 \times 10^6$	$.40 \times 10^6$	20.0	20.0	20.0						

Table X (a)

Random Excitation

Estimation Intervals

i	k_i True Value	$\hat{k}_i(1)$ 0.0-1.0 sec	$\hat{k}_i(2)$ 1.0-2.0 secs	C_i True Value	$\hat{C}_i(1)$ 0.0-1.0 sec	$\hat{C}_i(2)$ 1.0-2.0 secs	m_i True Value	$\hat{m}_i(1)$ 0.0-1.0 sec	$\hat{m}_i(2)$ 1.0-2.0 secs	I_i True Value	$\hat{I}_i(1)$ 0.0-1.0 sec	$\hat{I}_i(2)$ 1.0-2.0 secs
1	$.99 \times 10^5$	$.10 \times 10^6$	$.10 \times 10^6$	10.0	10.0	10.0	.26	.26	.26	70.0	70.0	70.0
2	$.40 \times 10^6$	$.40 \times 10^6$	$.40 \times 10^6$	20.0	20.0	20.0	.26	.26	.26	70.0	70.0	70.0
3	$.99 \times 10^5$	$.99 \times 10^5$	$.99 \times 10^5$	10.0	10.0	10.0						
4	$.40 \times 10^6$	$.40 \times 10^6$	$.40 \times 10^6$	20.0	20.0	20.0						

Table X (b)

Sinusoidal Excitation

Again, to the first few places, the estimated parameters are identical with the actual system parameters. It is only at the fifth and sixth places where deviation occurs.

Clearly, multidimensional systems give no problems for identification as long as it is merely a problem of parameter identification with known structure.

3.5 Identification Using Only Acceleration Data

It was hoped that we could present a comprehensive study of the problem of parameter identification by the proposed method when only acceleration data is available. This, in a sense, is very important to establish since, in general, acceleration data is the most commonly recorded. Velocity and displacement data are not usually present in vibrations recordings. For the proposed scheme acceleration data alone is not sufficient for identification purposes. Thus, it was thought desirable to determine how well we can identify by integrating the acceleration data in order to obtain velocity as well as displacement data and accomplish identification on the basis of these calculated data. This numerical problem was not resolved during the course of the present research project. Part of the problem that exists here is that the initial values of the velocities and displacements are unknown so that one has to remove some type of trend in order to satisfy, for example, that the mean values should be zero. Upon attempting two integrations and two trend removals, the estimated parameters obtained possessed large errors. As there was insufficient time remaining in order to resolve these problems as well as the many other problems that came up during the course of this research, we decided to present a simple example in which only one integration of the acceleration is required and the initial value of the velocity is known so that no trend removal is required. This example does give credance to the approach and opens doors for future investigations.

The system chosen for simulation is

$$\ddot{x}(t) + 30\,\dot{x}(t) + 900\,x(t) = f(t) \qquad (3.5.1)$$

For this system the damping ratio is 1/2, so that we can expect the transient to last but a few cycles. We are especially interested in the steady state here for it will allow us to identify k and C on the basis of acceleration and velocity data alone. The excitation was a random function with spectrum as shown in Experiment IV of Section 3.2.

Based upon the assumption of stationarity we can apply the following equations for identification

$$
\begin{aligned}
E\{\ddot{x}^2\} + 30\,E\{\dot{x}\ddot{x}\} + 900\,E\{x\ddot{x}\} &= E\{f\ddot{x}\} \\
E\{\dot{x}\ddot{x}\} + 30\,E\{\dot{x}^2\} + 900\,E\{x\dot{x}\} &= E\{f\dot{x}\}
\end{aligned}
\qquad (3.5.2)
$$

Now, assuming stationarity, we can reduce these equations to

$$
\begin{aligned}
E\{\ddot{x}^2\} - 900\,E\{\dot{x}^2\} &= E\{f\ddot{x}\} \\
30\,E\{\dot{x}^2\} &= E\{f\dot{x}\}
\end{aligned}
\qquad (3.5.3)
$$

Therefore, identification is accomplished on the basis of velocity and acceleration data alone.

The sampling frequency was 140 c.p.s. The periods of observation were taken on successive intervals of 5.96 secs duration. This corresponds to 835 samples used for parameter estimation purposes.

The results of this parameter estimate are shown in the following table, XI.

Naturally, we feel quite encouraged by the success of this simple simulation experiment. It is clear, however, that much remains to be accomplished before we can apply our approach to parameter estimation with acceleration data alone.

	True Value		Estimation Interval 5.96-11.92	11.92-17.88	17.88-23.84	23.84-29.80
k	900.0	$\hat{k}$	860.0	860.0	886.0	900.0
C	30.0	$\hat{C}$	29.1	29.1	29.8	30.2

Table XI

CHAPTER IV

COMPUTER SIMULATION AND IDENTIFICATION

4.1 Computer Techniques

In order to verify numerically the proposed identification technique, the chain-like system shown in Figure I-a and described by equation 2.2.28 is simulated, necessary moments are computed and the system parameters are estimated on an IBM 7094 computer system. The operations performed by the computer can clearly be divided into three distinct functions:

a. Simulation of the system
b. Estimation of the moments
c. Estimation of the system parameters

A description of the computing method used in each case follows.

Simulation of the System

When a computer is used to simulate a system, it is actually programmed to integrate, with respect to time, the set of differential equations describing that system. In this case, equation 2.2.28 is rewritten into a set of first-order differential equations by change of variables. That is, let

$$\left.\begin{aligned}
x_1 &= y_1 \quad ; \qquad \dot{x}_1 = y_2 \\
x_2 &= y_3 \quad ; \qquad \dot{x}_2 = y_4 \\
&\vdots \\
x_n &= y_{2n-1} \quad ; \qquad \dot{x}_n = y_{2n}
\end{aligned}\right\} \qquad (4.1.1)$$

$$\left.\begin{aligned}
\frac{dy_1}{dt} &= \dot{y}_1 = y_2 \\
\frac{dy_2}{dt} &= \dot{y}_2 = -\frac{C_1}{m_1}(y_2-y_4) - \frac{k_1}{m_1}(y_1-y_3) + \frac{f_1}{m_1} \\
&\vdots \\
\frac{dy_{2i-1}}{dt} &= \dot{y}_{2i-1} = y_{2i} \\
\frac{dy_{2i}}{dt} &= \dot{y}_{2i} = -\frac{C_i}{m_i}(y_{2i}-y_{2i+2}) - \frac{k_i}{m_i}(y_{2i-1}-y_{2i+1}) \\
&\quad + \frac{C_{i-1}}{m_i}(y_{2i-2}-y_{2i}) + \frac{k_{i-1}}{m_i}(y_{2i-3}-y_{2i-1}) + \frac{f_i}{m_i} \\
&\vdots
\end{aligned}\right\} \qquad (4.1.2)$$

$$\left.\begin{aligned}
\frac{dy_{2n-1}}{dy} &= \dot{y}_{2n-1} = y_{2n} \\
\frac{dy_{2n}}{dt} &= \dot{y}_{2n} = -\frac{C_n}{m_n}\, y_{2n} - \frac{k_n}{m_n}\, y_{2n-1} + \frac{C_{n-1}}{m_n}\,(y_{2n-2}-y_{2n}) \\
&\quad + \frac{k_{n-1}}{m_n}\,(y_{2n-3}-y_{2n-1}) + \frac{f_n}{m_n}
\end{aligned}\right\}$$

or in a more compact notation

$$\frac{d\bar{y}}{dt} = g(\bar{y}, \bar{F}) \tag{4.1.3}$$

where g in this case is a linear function of $\bar{y}$ and F.

The operation performed by the computer in the integration of the set of differential equations consists of two parts, the generation of the derivative functions, namely $g(\bar{y}, F)$ and the integration with respect to time. The flow of information is shown in Figure IV.

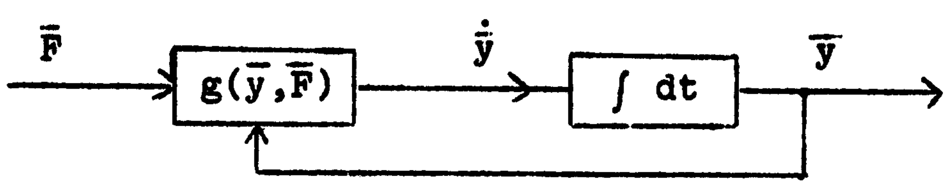


Figure IV

A fourth-order Runge-Kutta technique of integration is used to produce tabulated values of the computer integral at equal time interval H. This is a single-step method in which the value of $\bar{y}_n$ and F_n at $t_n = nH$ is used to compute $\bar{y}_{n+1}$ at $t_{n+1} = (n+1)H$. The relevant formulas are:

$$\bar{y}_{n+1} = \bar{y}_n + \frac{1}{6}(S_0 + 2S_1 + 2S_2 + S_3) \qquad (4.1.4)$$

where

$$S_0 = H\, g(\bar{y}_n, \bar{F}_n, t_n)$$

$$S_1 = H\, g(\bar{y}_n + \frac{S_0}{2}, \frac{\bar{F}_n + \bar{F}_{n+1}}{2}, t_n + \frac{H}{2})$$

$$S_2 = H\, g(\bar{y}_n + \frac{S_1}{2}, \frac{F_n + F_{n+1}}{2}, t_n + \frac{H}{2})$$

$$S_3 = H\, g(\bar{y}_n + S_2, F_{n+1}, t_{n+1})$$

Estimation of the Moments

The moments as coefficients of the simultaneous equations in equation 2.2.29 for solving the system parameters m_i, C_i, k_i, $i=1$ to 5 are estimated from equally-spaced samples of the system response functions generated in the process of simulating the system.

The least square estimate of a moment such as $\mu_{xz} = E[xz]$ from N duplets $\{x_i, z_i\}$, $i=1,2,\ldots N$ is simply

$$\hat{\mu}_{xz} = \frac{1}{N} \sum_{i=1}^{N} x_i z_i \qquad (4.1.5)$$

Estimation of the System Parameters

The set of simultaneous equations for estimating the system parameters is given by equation 2.2.29. They can be expressed in matrix notation sequentially for each ith mass, spring, and dashpot section from i=1 to 5 as follows

$$A_i \quad P_i = B_i \qquad (4.1.6)$$

where P_i is the unknown system parameter vector of the ith mass, spring, and dashpot section

$$P_i = \begin{bmatrix} m_i \\ C_i \\ k_i \end{bmatrix} \qquad (4.1.7)$$

The coefficient matrix A_i for i=1 to 4 is

$$A_i = \begin{bmatrix} \overline{y_{2i-1}\dot{y}_{2i}} , & \overline{(y_{2i-1}y_{2i}-y_{2i-1}y_{2i+2})} , & \overline{(y_{2i-1}^{2}-y_{2i-1}y_{2i+1})} \\ \overline{y_{2i}\dot{y}_{2i}} , & \overline{(y_{2i}^{2}-y_{2i}y_{2i+2})} , & \overline{(y_{2i-1}y_{2i}-y_{2i}y_{2i+1})} \\ \overline{\dot{y}_{2i}^{2}} , & \overline{(y_{2i}\dot{y}_{2i}-\dot{y}_{2i}y_{2i+2})} , & \overline{(y_{2i-1}\dot{y}_{2i}-\dot{y}_{2i}y_{2i+1})} \end{bmatrix} \qquad (4.1.8)$$

and for i=5

$$A_5 = \begin{bmatrix} \overline{y_9\dot{y}_{10}} , & \overline{y_9 y_{10}} , & \overline{y_9^{\,2}} \\ \overline{y_{10}\dot{y}_{10}} , & \overline{y_{10}^{\,2}} , & \overline{y_9 y_{10}} \\ \overline{\dot{y}_{10}^{\,2}} , & \overline{y_{10}\dot{y}_{10}} , & \overline{\dot{y}_{10}y_9} \end{bmatrix} \qquad (4.1.9)$$

The constant vector B_i for i=1 is

$$B_1 = \begin{bmatrix} \overline{y_1 f_1} \\ \overline{y_2 f_1} \\ \overline{\dot{y}_2 f_1} \end{bmatrix} \qquad (4.1.10)$$

and for $i=2$ to 5 is (4.1.11)

$$B_i = \begin{bmatrix} (\overline{y_{2i-2}y_{2i-1}} - \overline{y_{2i-1}y_{2i}})\, c_{i-1} + (\overline{y_{2i-3}y_{2i-1}} - \overline{y^2_{2i-1}})\, k_{i-1} + \overline{y_{2i-1}f_i} \\ \\ (\overline{y_{2i-2}y_{2i}} - \overline{y^2_{2i}})\, c_{i-1} + (\overline{y_{2i-3}y_{2i}} - \overline{y_{2i-1}y_{2i}})\, k_{i-1} + \overline{y_{2i}f_i} \\ \\ (\overline{y_{2i-2}\dot{y}_{2i}} - \overline{y_{2i}\dot{y}_{2i}})\, c_{i-1} + (\overline{y_{2i-3}\dot{y}_{2i}} - \overline{y_{2i-1}\dot{y}_{2i}})\, k_{i-1} + \dot{y}_{2i}f_i \end{bmatrix}$$

Notice that the vectors B_i for $i=2$ to 5 are functions only of the previous (i-1)th system parameters. Instead of solving 15 equations simultaneously, each $A_i \quad P_i = B_i$ is used to solve for the unknown vector P_i sequentially from $i=1$ to $i=5$.

4.2 Computer Program Abstract I

a) This program simulates the one-dimensional chainlike lumped parameter spring-mass-dashpot system whose equations of motion are given by equation 4.1.2. From the simulated equally-spaced samples of the system dynamical outputs y_j,

$\dot{y}_j$, $j=1$ to $2N$ and the system inputs F_i, $i=1$ to NF, the second moments or time averages of $\overline{y_j y_k}$, $\overline{\dot{y}_j y_k}$, $\overline{\dot{y}_j^{\,2}}$, $\overline{F_i y_{2i-1}}$, $\overline{F_i y_{2i}}$, and $\overline{F_i \dot{y}_{2i}}$ are computed and the system parameter triplets (m_i, k_i, C_i), $i=1$ to N are estimated.

b) Outputs from this program include:

1) Sample moments or time averages of $\overline{y_j y_k}$; $j=1$ to $2N$, $k=j, \ldots (j+3)$ or $2N$; $\overline{\dot{y}_j y_k}$; $j=1$ to $2N$, $k=(j-3)$ or $1, \ldots (j+3)$ or $2N$; $\overline{\dot{y}_j^{\,2}}$; $j=1$ to $2N$; $\overline{F_i y_{2i-1}}$, $\overline{F_i y_{2i}}$ and $\overline{F_i \dot{y}_{2i}}$; $i=1, \ldots NF$.

2) Sets of simultaneous equations for solving each system parameter triplet $(\hat{m}_i, \hat{k}_i, \hat{C}_i)$ and simultaneous equations for solving each duplet (k_i^*, C_i^*) with m_i given for $i=1$ to N.

3) Tabulation of the true parameters (m_i, k_i, C_i) against the corresponding estimated parameters $(\hat{m}_i, \hat{k}_i, \hat{C}_i)$ and (k_i^*, C_i^*) for $i=1$ to N.

c) Limitation of this program*

1) N, the number of mass, is limited to 10 ($N \leq 10$).

2) NF, the number of input forcing functions, is limited to N ($NF \leq N$).

*Maximum number of masses is arbitrary and is specified by dimension statement.

d) Required supporting subprograms

1) RKD
2) DERSUB
3) FØRSUB
4) PAMSQT
5) XSQ
6) XMØNT3
7) LØC2
8) LØC3
9) CØEF
10) RLMTX
11) GAURN (if random input is desired)

The descriptions for these subroutines are given in section 3.

Input Cards

a) Degree of Freedom Card

Col. 1-2 N - Number of mass for the chainlike system ($N \leq 10$)

b) System Parameter Card(s)

A card is used to specify each system parameter triplet (m_i, k_i, C_i). N cards are then needed, and they should be arranged consecutively from i=1 to N.

Col. 1-10 CM(I) - Floating point constant for the ith mass coefficient

Col. 11-20 CK(I) - Floating point constant for the ith spring coefficient

Col. 21-30 CC(I) - Floating point constant for the ith damper coefficient

c) Number of Input Card

Col. 1 - 2 NF - Number of input forcing functions

d) Input Forcing Characteristics Card(s)

A card is used to specify each input forcing function characteristics. NF cards are needed and they should be arranged consecutively from I=1 to NF.

Col. 1 - 5 FOST(I) - Amplitude of the sinusoid input to ith mass, or standard deviation of the white noise input to a band-pass filter whose output is the input to ith mass of the system.

Col. 6 - 10 FW(I) - Frequency (cps) of the sinusoid input to ith mass, or center frequency (cps) of the band-pass filter for the ith input

Col. 11-15 FB(I) - Phase shift (in degrees) with respect to t = 0 of the sinusoid input to ith mass, or bandwidth of the band-pass filter.

e) Simulation Specification Card

Col. 1 - 5 FREQ - Sampling frequency for simulation of the system. The simulation interval is then 1/FREQ.

Col. 6 - 10 NI - Number of samples to be simulated before samples are taken for estimation of the moments and the system parameters.

Col. 11-15 NO - Number of samples (after initial NI samples) to be used for estimation of the moments and the system parameters.

Col. 16-20 K - Only every kth sample (after initial NI samples) of the equally-spaced samples are to be used for estimation of the moments and the system parameters.

Col. 21-25 NORUN - Number of successive times the moments and the system parameters are to be estimated.

Col. 26-30 INIT - Control index for how the samples are taken for each successive estimation of the moments and parameters.

If INIT < 0, successive NO samples (after the initial NI samples) taken at every kth sample are to be used for estimation of the moments and the parameters.

If INIT = 0, system is reinitialized each time, NO samples (after initial NI samples) taken at every kth sample are used for estimation of the moments and parameters.

If INIT > 0, (after initial NI samples) NO samples are used commulatively each time; that is, NO, then 2 x NO, ..., then NORVN x NO samples are successively used for estimation of the moments and parameters

f) Repeat a to e for a different choice of system parameters as many times as desired. A blank card after e will cause a stop.

Description of Supporting Subprograms

a) RKD (DERSUB, FORSUB, M, NF, H, TI, YI, FOS, K, N, F, VAL, DVAL, Y)

This Fortran subroutine generates the solution to a set of M simultaneous first-order, ordinary differential equations by the classical fourth-order Runge-Kutta method of integration. Where

DERSUB - Name of the external subroutine used to compute the derivatives.

FORSUB - Name of the external subroutine used to generate the input forcing functions.

M - Number of equations for expressing the system.

NF - Number of input forcing functions

H - Step size for integration

TI - Initial value of T

YI - Initial values of Y, an array of M

FOS - Initial values of F, an array of NF. Destroyed in the process and replaced by the final value of F.

K - The desired number of steps of size H between values of the integrals to be stored in VAL, values of the derivatives to be stored in DVAL and the values of the input forcings to be stored in F.

N - The number of values to be stored in VAL, DVAL and F. The final value of T will be TI + (N*K*H).

F - A matrix of NF by N containing the values of input forcing functions generated by the external subroutine FORSUB.

VAL - A matrix of M by N containing the integrated values of the M derivatives generated by DERSUB.

DVAL - A matrix of M by N containing the derivatives generated by the external subroutine DERSUB.

Y - The final integrated values of the M derivatives, an array of M.

b) DERSUB (T, VAR, FS, M, NF, DER)

This Fortran subroutine computes the derivatives given by equation 4.1.2 for the integration subroutine RKD, where DER(I); I=1,M are derivatives of VAR(I), I=1,M with respect to T, and are functions of T, VAR and FS (input forcing vector of length NF). This subroutine describes the dynamics of the system; therefore, a different system can be simulated by simply using a corresponding subroutine DERSUB that describes the system dynamics by a set of first-order ordinary differential equations.

c) FORSUB (T, FOS, NF)

This Fortran subroutine generates the input forcing functions to the system for the integration subroutine RKD, where FOS(I), I=1,NF are the values of the input forcing functions at time T. A subprogram for sinusoidal input and another subprogram for random input are included. The user can use either one as desired, and only one is to be used at a time.

The random forcing function f_i is generated by passing a white noise sequence (simulated by subprogram RANDPK) through a bandpass digital filter with center frequency w_i and bandwidth B_i. The corresponding continuous filter can be described by the differential equation

$$\ddot{f}_i + B_i\dot{f}_i + w_i^2 f_i = B_i\dot{x} \qquad (4.2.1)$$

which will yield the following recurrence formula for the digital simulation

$$F_i(N+2) = A_1F_i(N+1) + A_2F_i(N) + B_iH(x(N+1)-x(N)) \qquad (4.2.2)$$

where

$$A_1 = 2\, e^{-\frac{B_iH}{2}} \cos\ w_dH$$

$$A_2 = -\, e^{-B_iH}$$

$$w_d = \sqrt{w_i^{\,2} - B_i^{\,2}/4}$$

and H is the simulation interval

d) PAMSQT (A, NV, NO, M, SQ, SUM, ID)

This Fortran subroutine computes either sums and sums of products of two variables or averages and averages of products of two variables from a set of sample vectors. Where

A - Sample matrix NV by NO of a set of vectors.

NV - Number of variables or length of vector

NO - Number of samples

M - Number of adjacent variables ($M \leq NV - 1$) for computing products of two variables. $Y(J) * Y(L)$ where $|L - J| \leq M$.

SQ - Sums or averages of products of two variables $Y(J) * Y(L)$, $J=1, NV; L=J, (J+M)$ or NV, a matrix of $(M+1) * NV - (M * (M+1))/2$.

SUM - Sums or averages, a vector of length NV

ID - Control index

ID = 0 computing sums and sums of products

ID ≠ 0 computing averages and averages of products

e) Subroutine XSQ (A, B, NV, NO, M, ASQ, AB, IFLAG)

This Fortran subroutine computes either sums or averages of products of variables between two sets of vectors and squares of one set of vector. Where

A - Sample matrix of NV by NO of one set of vectors

B - Sample matrix of NV by NO of another set of vectors

NV - Number of variables or length of vector

NO - Number of samples

M - Number of adjacent variables ($M \leq NV - 1$) for computing products of variables from the two sets of vectors $Y(J) * Z(L)$, 1 or $(J - M) \leq L \leq (J + M)$ or NV.

ASQ - Sums or averages of squares of variable of the vectors stored in sample matrix A, a vector of length NV.

AB - Sums or averages of products of one variable of vectors stored in A and another variable of vectors stored in B. Y(J) * Z(L), J=1,NV. A matrix of (2 * M + 1) * NV - M * (M + 1).

IFLAG - Control index IFLAG = 0, compute sums
IFLAG ≠ 0, compute averages

f) Subroutine XMONT3 (A, NA, B, C, NBC, NO, XM, ID)

This Fortran subroutine computes sums or averages of products between variable of one vector and variable of two other vectors. Where

A - Sample matrix of NA by NO of one set of vectors

NA - Number of variable or length of vector stored in A

B - Sample matrix of NBC by NO of another set of vectors

C - Sample matrix of NBC by NO of the third set of vectors

NBC - Number of variables or length of vector stored in B and C

NO - Number of samples

XM - Sums or averages of products of variables
A(J) * B(2J-1); A(J) * B(2J) and A(J) * C(2J),
J=1, NA

ID - Control index
ID = 0 computing sums
ID ≠ 0 computing averages

g) Subroutine LOC2 (L, J, LJ, NV, M)

This Fortran subroutine computes a vector subscript for an element in a symmetric matrix where only M elements from the diagonal elements of the upper (or lower) triangular matrix are stored in a vector. To illustrate the storage mode, Figure V shows the vectors subscripts for elements in a 6 x 6 matrix where only 2 elements from the diagonal elements are stored.

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 2 & 4 & 5 & 6 & 0 & 0 \\ 3 & 5 & 7 & 8 & 9 & 0 \\ 0 & 6 & 8 & 10 & 11 & 12 \\ 0 & 0 & 9 & 11 & 13 & 14 \\ 0 & 0 & 0 & 12 & 14 & 15 \end{bmatrix}$$

Figure V

Storage Mode

L - Row number of element or (J-M) $\leq$ L $\leq$ (J+M) or NV

J - Column number of element

LJ - Resultant vector subscript

NV - Number of columns in matrix

M - Number of adjacent elements from the diagonal elements

M $\leq$ (NV-1)

h) Subroutine LOC3 (L, J, LJ, NV, M)

This Fortran subroutine computes a vector subscript for an element in a matrix where only M adjacent elements from the diagonal elements are stored in a vector. To illustrate the storage mode, Figure VI shows the vector subscripts for elements in a 6 x 6 matrix where only two adjacent elements from the diagonal elements are stored.

$$\begin{bmatrix} 1 & 4 & 8 & 0 & 0 & 0 \\ 2 & 5 & 9 & 13 & 0 & 0 \\ 3 & 6 & 10 & 14 & 18 & 0 \\ 0 & 7 & 11 & 15 & 19 & 22 \\ 0 & 0 & 12 & 16 & 20 & 23 \\ 0 & 0 & 0 & 17 & 21 & 24 \end{bmatrix}$$

Figure VI

L - Row number of element 1 or $(J-M) \leq L \leq (J+M)$ or NV
J - Column number of element
LJ - Resultant vector subscript
NV - Number of column
M - Number of adjacent elements from the diagonal elements that are stored

i) COEF (J, N, NF, SQ, DYSQ, YDY, XM, EC, EK, A, B)

This Fortran subroutine computes the coefficients for the three simultaneous equations in solving the jth system parameter triplet (m_j, k_j, C_j). Where

J - Number of the system parameter triplet to be solved

N - Number of mass (or degree of freedom) of the one-dimensional chainlike system

NF - Number of input forcing functions

SQ - Averages of products of the system dynamical outputs

DYSQ - Averages of squares of the derivatives

YDY - Averages of products of the system dynamical cutputs and the derivatives

XM - Averages of products of the system input and the dynamic outputs

EC - Previously estimated damping coefficient of the (J-1)th system parameter triplet

EK - Previously estimated spring coefficient of the (J-1)th system parameter triplet

A - A 3 x 3 matrix containing the left-hand side coefficients of the equations.

B - A vector of length 3 containing right-hand sides constants

j) RLMTX (A, NR, NSYS, MARK, DET, INOPT)

This Fortran subprogram evalutes the determinant of a matrix A with real elements and at the user's option finds the inverse or solves one or more simultaneous systems. Where

NR - Order of matrix A

NSYS - Number of simultaneous systems to be solved if the system solving option is chosen. Otherwise, NSYS is irrelevant.

MARK - Singularity indicator. If Mark = 1 on return to calling program, the matrix A is singular.

DET - Determinant of A

INOPT Option flag

= 0 for determinant evaluation only

= -1 for system solving option

= +1 for inverse option

A - Array name of the augmented matrix C/B. The subroutine is compiled with the dimension A(3,4). This dimension must be changed if it is inconsistent with the dimension of A in the calling program. A must be at least dimension

1) (N by (N+NSYS)) for the system solving option
2) (N by 2N) for the inverse option

If the system option is chosen, the known vectors b of (Cx = b) must be stored in the (N+1)st through

(N+NSYS) column of A (i.e., must constitute the (N by NSYS) matrix B). The solution vectors will be returned in these same columns.

If the inverse option is chosen C^{-1} will be returned in B. The original matrix C is destroyed (on return C will contain the triangularized matrix).

k) RANDPK (Entry names GAURN, EXPRN, and FLRAN)

This MAP function subprogram generates pseudo-random numbers and includes three entry points for three different distributors, Gaussian (normal), Exponential or Rectangular (uniform). The function names are GAURN, EXPRN, and FLRAN corresponding to Gaussian, exponential, and rectangular, respectively. An example of the generation of a Gaussian-distribution pseudo-random number which is to be assigned to the variable Y is as follows:

Y = GAURN(X)

where X is a dummy variable and has no effect on the random number generation.

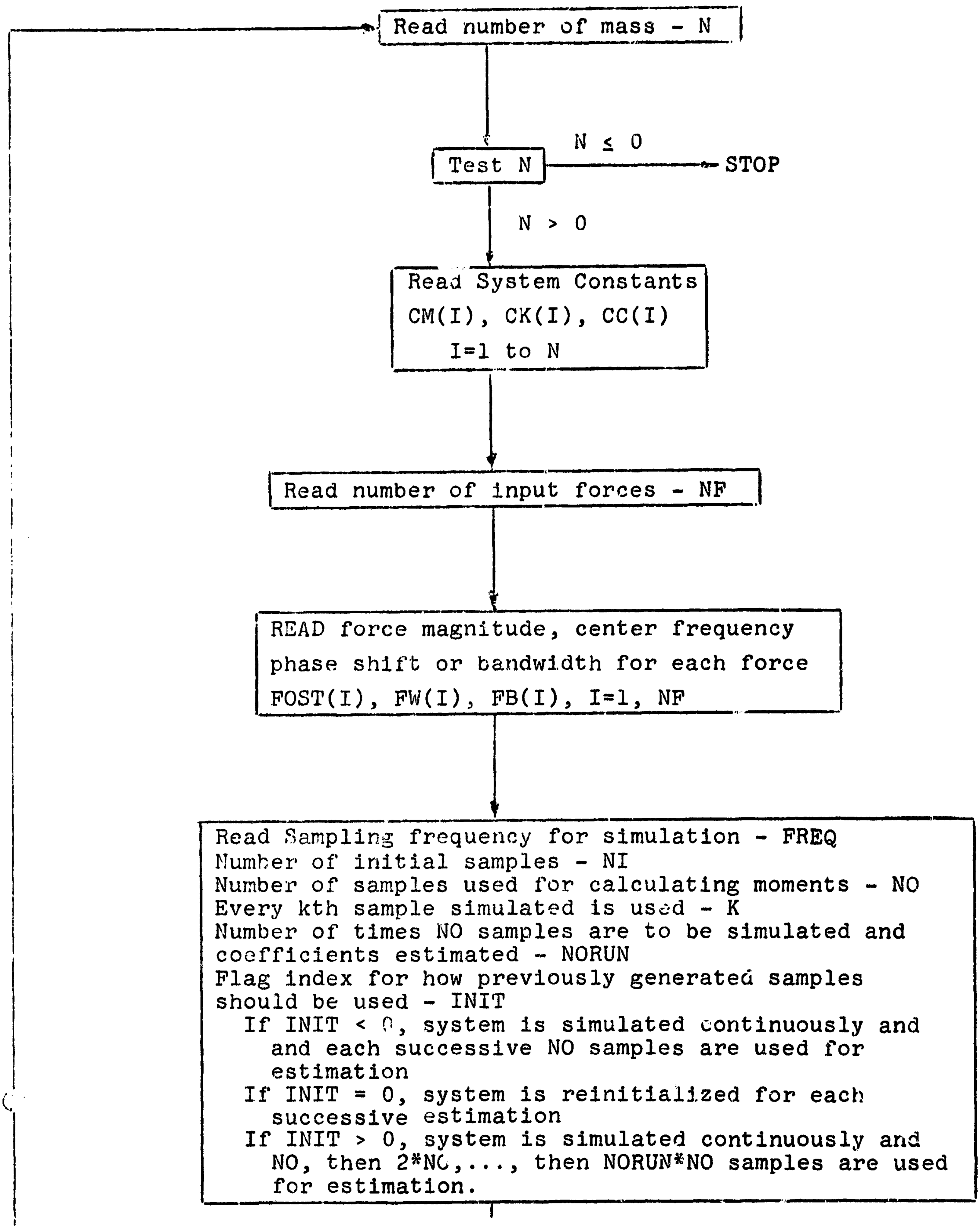
This package of routines has the characteristics that each separate run of the program produces the same sequence of numbers. If a different set of numbers is desired, it is possible to preset the routine at a different value by presetting the initializing regulatory variable inside the routine. One possible way to generate a different sequence each run is to get the value of this special variable after all the random numbers for that particular run have been obtained. Then this value could be stored in the routine in the next program to continue the old sequence. To implement this capability, included in the package are two subroutines. The one to get the number from the routine may be executed by

```
CALL  GETNM (NUM)
```

where NUM is the integer variable into which the special number is stored. The other . . to store an integer (NUM) into this special location may be executed as

```
CALL  STORNM (NUM).
```

Block Diagram



Set NON = 0

Determine number of samples can be simulated at one time (limited by the allocated storage space) - NIS

Set NOS = NO

Generate NIS samples of each state variables

Y_i, i=1,2N and DY_i, i=1,2N

where $DY_i = \dot{Y}_i$ and $Y_{2i} = \dot{Y}_{2i-1}$

by calling

Subroutine RKD - fourth-order Runge-Kutta integration process which calls for two auxiliary subroutines

Subroutine DERSUB - generates derivatives DY as functions of Y and T and

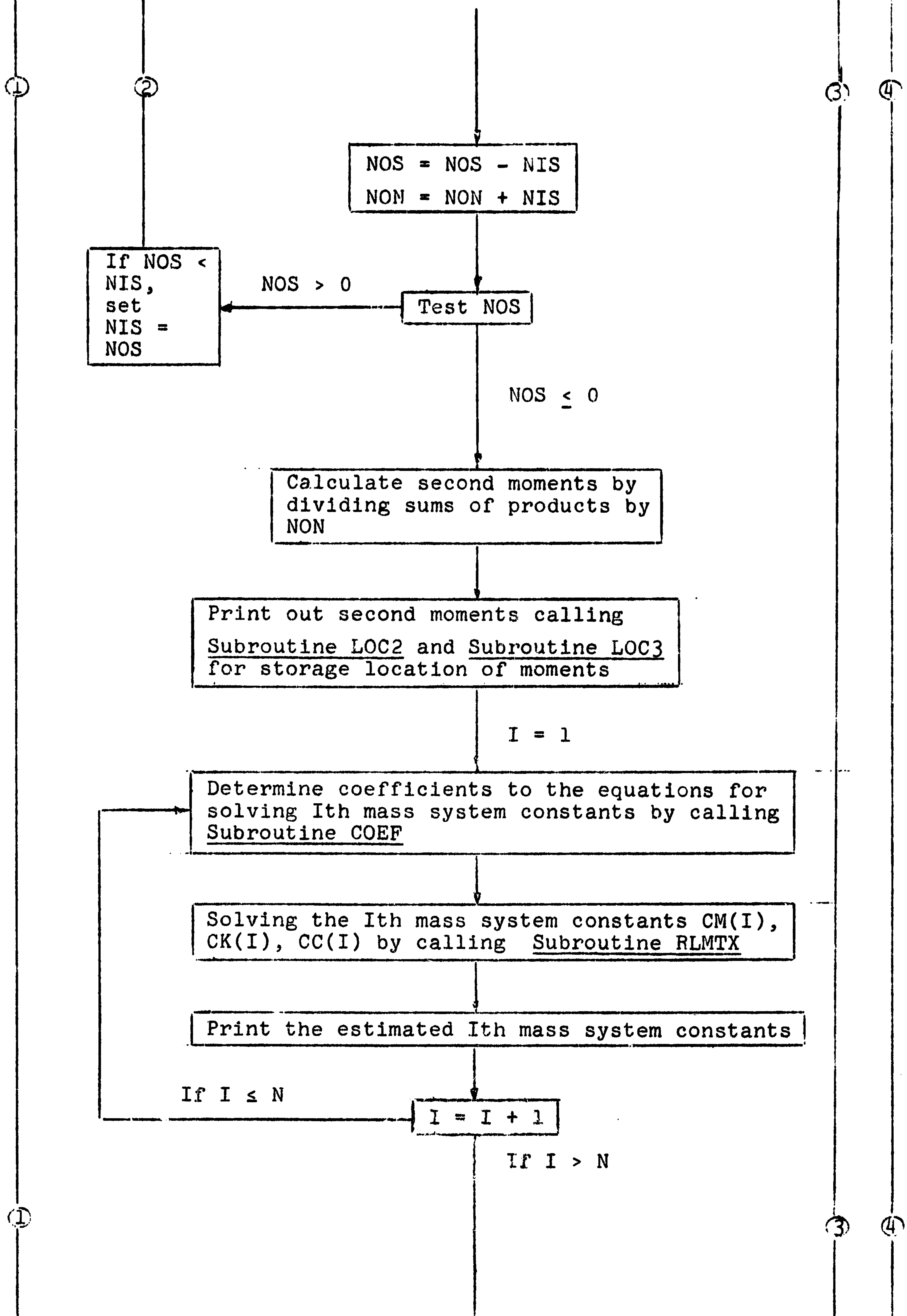
Subroutine FORSUB - generates forcing functions

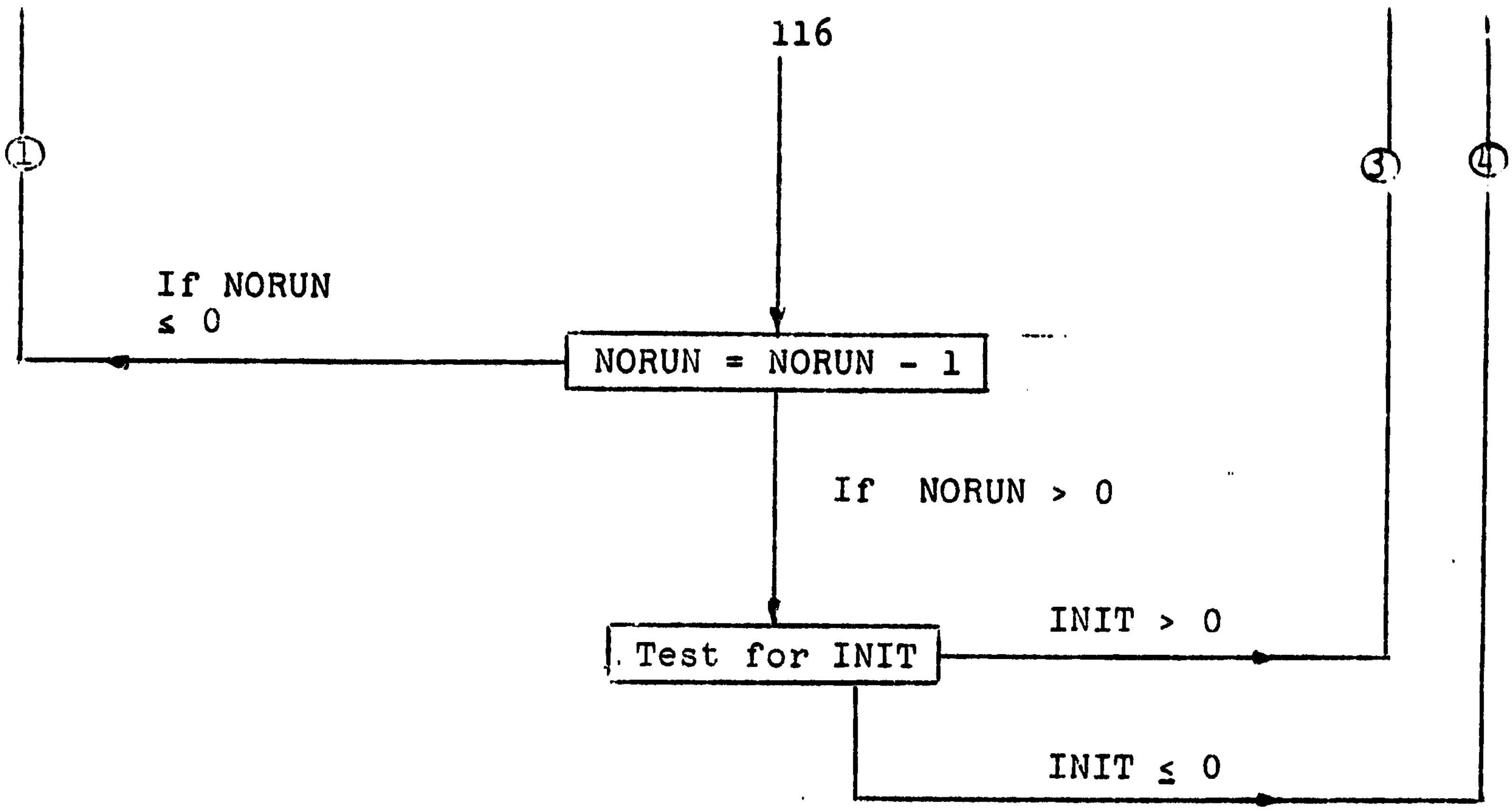
Calculate sum of squares by

Subroutine PAMSQT - calculate sum of squares of Y and products of Y_i Y_j

Subroutine XSQ - calculate sum of products DY with Y

Subroutine AMONT3 - calculate sum of products of Force F with DY or Y


① ② ③ ④
NOS = NOS - NIS
NON = NON + NIS
Test NOS
NOS > 0
If NOS < NIS, set NIS = NOS
NOS ≤ 0
Calculate second moments by dividing sums of products by NON
Print out second moments calling Subroutine LOC2 and Subroutine LOC3 for storage location of moments
I = 1
Determine coefficients to the equations for solving Ith mass system constants by calling Subroutine COEF
Solving the Ith mass system constants CM(I), CK(I), CC(I) by calling Subroutine RLMTX
Print the estimated Ith mass system constants
If I ≤ N
I = I + 1
If I > N
① ③ ④


116
①
③
④
If NORUN ≤ 0
NORUN = NORUN - 1
If NORUN > 0
Test for INIT
INIT > 0
INIT ≤ 0

Listing of the Main Program and Associated Subprograms

```
C     PROGRAM TO SIMULATE N DEGREE FREEDOM,ONE DIMENSIONAL, SPRING MASS
C       AND DAMPER SYSTEM AND ESTIMATE THE COEFFICIENTS
C
      DIMENSION CC(10),CK(10),CM(10),FI(10),Y(5000),DY(5000),YF(20),
     1F(2500),EC(10),EK(10),EM(10),FW(10),FB(10),DIS(3,10),YI(20)   ,
     2FOST(10),          SQ(80), SUM(20),XM(30),A(3,3),B(3),R(3,4)
     3,DYSQ(20),YDY(140),SQT(80),SUMT(20),XMT(30),DYSQT(20),YDYT(140)
     4,EFK(10),EFC(10),SUMS(20),XMS(30),SQS(80),DYSQS(20),YDYS(140)
      DOUBLE PRECISION SQS,SUMS,XMS,DYSQS,YDYS
      EQUIVALENCE (R(1,1),A(1,1)),(R(1,4),B(1))
      COMMON CC,CK,CM,FOST,FW,FB,DIS
      EXTERNAL DERSUB,FORSUB
C
    5 READ(5,500) N
      IF(N) 400,400,8
    8 READ(5,501)   (CM(I),CK(I),CC(I),I=1,N)
      READ(5,502) NF,(FOST(I),FW(I),FB(I),I=1,NF)
      READ(5,506) FREQ,NI,NO,K ,NORUN ,INIT
C
C     INIT .GT. 0 ALL SAMPLES USED FOR SUCCESIVE ESTIMATION OF MOMENTS
C     INIT .EQ. 0  EACH SUCCESIVE RUN IS REINITIALIZED
C     INIT .LT. 0 NO SAMPLES USED FOR SUCCESIVE ESTIMATION OF MOMENTS
C
C     INITIALIZATION
C
      H=1.0/FREQ
      N2=2*N
      MX=3
      IF(N2.LE.3) MX=N2-1
      NSQ=(MX+1)*N2-(MX*(MX+1))/2
      NXM=3*NF
      NXC=(2*MX+1)*N2-(MX+1)*MX
      DO 10 I=1,NF
      BH=6.283185*FB(I)*H
      WD=FW(I)*FW(I)-0.25*FB(I)*FB(I)
      IF(WD.LT.0.0) WD=-WD
      WD=6.283185*H*SQRT(WD)
      DIS(1,I)=2.0*COS(WD)*EXP(-0.5*BH)
      DIS(2,I)=-1.0*EXP(-1.0*BH)
      DIS(3,I)=BH
   10 CONTINUE
   15 DO 30 I=1,N2
   30 YI(I)=0.0
      TI=0.0
```

```
      CALL FORSUB(TI,FI,NF)
      IF(NI.LE.0) GO TO 41
      CALL RKD(DERSUB,FORSUB,N2,NF,H,0.0,YI,FI,NI,1,F,Y,DY,YE)
      DO 40 I=1,N2
   40 YI(I)=YE(I)
      TI=     FLOAT(NI)*H
   41 DO 42 J=1,NXM
   42 XMS(J)=0.0
      DO 43 J=1,NSQ
   43 SQS(J)=0.0
      DO 44 J=1,N2
      DYSQS(J)=0.0
   44 SUMS(J)=0.0
      DO 45 J=1,NXQ
   45 YDYS(J)=0.0
      NON=0
      TIS=TI
  455 IF((NO*NF).LE.2500) GO TO 49
      MNF=2500/NF
      IF((NO*N2).LE.5000) GO TO 46
      MN2= 5000/N2
      IF(MNF-MN2) 46,46,48
   46 NIS=MNF
      GO TO 50
   48 NIS=MN2
      GO TO 50
   49 IF((NO*N2).LE. 5000) GO TO 140
      NIS= 5000/N2
      GO TO 50
  140 NIS=NO
   50 NOS=NO
   51 CALL RKD(DERSUB,FORSUB,N2,NF,H,TI,YI,FI,K,NIS,F,Y,DY,YE)
      CALL PAMSQT(Y,N2,NIS,MX,SQT,SUMT,0)
      CALL XSQ(DY,Y,N2,NIS,MX,DYSQT,YDYT,0)
      CALL XMONT3(F,NF,Y,DY,N2,NIS,XMT,0)
      NOS=NOS-NIS
      DO 52 J=1,NSQ
   52 SQS(J)=SQS(J)+SQT(J)
      DO 54 J=1,N2
      DYSQS(J)=DYSQS(J)+DYSQT(J)
   54 SUMS(J)=SUMS(J)+SUMT(J)
      DO 55 J=1,NXQ
   55 YDYS(J)=YDYS(J)+YDYT(J)
      DO 56 J=1,NXM
   56 XMS(J)=XMS(J)+XMT(J)
      DO 58 J=1,N2
   58 YI(J)=YE(J)
      NON=NON+NIS
      TN=NIS*K
   60 TI=TI+TN*H
      IF(NOS) 100,100,65
   65 IF(NOS-NIS) 70,51,51
   70 NIS=NOS
      GO TO 51
  100 DO 110 J=1,NSQ
  110 SQ(J)=SQS(J)/FLOAT(NON)
```

```
      DO 112 J=1,N2
      DYSQ(J)=DYSQS(J)/FLOAT(NON)
  112 SUM(J)=SUMS(J)/FLOAT(NON)
      DO 114 J=1,NXM
  114 XM(J)=XMS(J)/FLOAT(NON)
      DO 116 J=1,NXQ
  116 YDY(J)=YDYS(J)/FLOAT(NON)
      WRITE(6,606) N
      WRITE(6,622)  (SUM(J),J=1,N2)
      WRITE(6,620)
      DO 155 J=1,N2
      CALL LOC2(J,J,K1,N2,MX)
      J3=J+MX
      IF(J3.GT.N2) J3=N2
      CALL LOC2(J3,J,K2,N2,MX    )
  155 WRITE(6,621) J,(SQ(JJ),JJ=K1,K2)

      WRITE(6,623) (DYSQ(J),J=1,N2)
      WRITE(6,628)
      DO 160 J=1,N2
      J3=J+MX
      IF(J3.GT.N2) J3=N2
      CALL LOC3(J3,J,K2,N2,MX)
      J3=J-3
      IF(J3.LT.1) J3=1
      CALL LOC3(J3,J,K1,N2,MX)
  160 WRITE(6,629) J,J3,(YDY(JJ),JJ=K1,K2)

      WRITE(6,626)
      DO 165 J=1,NF
  165 WRITE(6,627) J,XM(3*J-2),XM(3*J-1),XM(3*J)
      WRITE(6,607)
      PC=0.0
      PK=0.0
      DO 200 I=1,N
      CALL COEF(I,N,NF,SQ,DYSQ,YDY,XM,PC,PK,A,B)
      WRITE(6,624) I
      DO 170 J=1,3
  170 WRITE(6,625) (A(J,L),L=1,3),B(J)

      CALL RLMTX(R,3,1,MARK,DET,-1)
      EM(I)=R(1,4)
      EK(I)=R(2,4)
      EC(I)=R(3,4)
      WRITE(6,631) DET
C
      IF(I.EQ.1) GO TO 175
      PK=EEK(I-1)
      PC=EEC(I-1)
  175 CALL COEF(I,N,NF,SQ,DYSQ,YDY,XM,PC,PK,A,B)
      B(1)=B(1)-CM(I)*A(1,1)
      R(1,1)=A(1,2)
      R(1,2) = A(1,3)
      R(1,3)=B(1)
      B(2)=B(2)-CM(I)*A(2,1)
      R(2,1)=A(2,2)
```

```
      R(2,2)=A(2,3)
      R(2,3)=B(2)
      WRITE(6,632)
      DO 180 J=1,2
  180 WRITE(6,633) (R(J,L),L=1,3)

      CALL RLMTX(R,2,1,MARK,DET,-1)
      WRITE(6,631) DET
      EEK(I)=R(1,3)
      EEC(I)=R(2,3)
      PK=EK(I)
      PC=EC(I)
  200 CONTINUE
C
      WRITE(6,600) N
      WRITE(6,601) NF,(I,FOST(I),FW(I),FB(I),I=1,NF)

      HK=FLOAT(K)*H
      TIE=H*FLOAT(NON)
      WRITE(6,603) FREQ,H,TIS,HK,NON,TIE
      WRITE(6,604)
      DO 330 I=1,N
  330 WRITE(6,605)  CM(I),EM(I),CK(I),EK(I),CC(I),EC(I)
      WRITE(6,634)
      DO 335 I=1,N
  335 WRITE(6,635) CK(I),EEK(I),CC(I),EEC(I)
      NORUN=NORUN-1
      IF(NORUN) 5,5,340
  340 IF(INIT) 41,15,455
  500 FORMAT(I2)
  501 FORMAT(3F10.1)
  502 FORMAT(I2/( 3F5.1) )
  504 FORMAT(8F10.4)
  506 FORMAT(F5.3,5I5)
  600 FORMAT(41H1 ESTIMATED COEFFICIENTS OF THE SIMULATED,I2,
 6001 11HMASS SYSTEM )
  601 FORMAT(1H0,18HNO OF EXCITATION =, I3/( 1H0,10X,14HFOR EXCITATION,
 6011 I3,5X,11HAMPLITUDE =,F10.2,5X,18HCENTER FREQUENCY =,F10.1,5X,
 601226HPHASE SHIFT OR BANDWIDTH =,F10.2) )
  603 FORMAT(21HOSAMPLING FREQUENCY =,F10.1, 3HCPS/
 6031        1H0,30HSAMPLING INT. FOR SIMULATION =,F8.5/
 60311H0,16HTRANSIENT INT. =    ,F10.2/
 60321H0,39HSAMPLING INT. FOR CALCULATING MOMENTS =,F8.5/
 60331H0,42HTOTAL NO. OF SAMPLES USED FOR ESTIMATION =,I6/
 603431HOTIME INTERVAL FOR ESTIMATION =,F10.2,3HSEC)
  604 FORMAT(1H0,16X,4HM(I),34X,4HK(I),34X,4HC(I)/3(14X,4HTRUE,8X,
 60419HESTIMATED,3X))
  605 FORMAT(1H0,3(4X,2E17.8))
  606 FORMAT(48H1ESTIMATION OF MOMENTS FOR THE IDENTIFICATION OF,
 6061 I2,11HMASS SYSTEM)
  607 FORMAT(52H1SETS OF EQUATIONS FOR SOLVING THE SYSTEM PARAMETERS)
  620 FORMAT(1H0,40H2ND MOMENTS Y(J)Y(L), L=J TO J+3 (OR 2N))
  621 FORMAT(1H0,8X,2HY(,I2,1H),5E14.4)
  622 FORMAT(29HOFIRST MOMENTS OF Y(J),J=1,2N//(5E20.8))
  623 FORMAT(43HOSECOND MOMENTS OF DERIVATIVES DY(J),J=1,2N//(5E20.8))
  624 FORMAT(1H0,20HEQUATION FOR SOLVING,I3,27HTH MASS SYSTEM COEFFICIE
```

```
6241TS)
625 FORMAT(1H0,3X,E15.8,6H*M + (,E15.8,7H)*K + (,E15.8,5H)*C =,E15.8)
626 FORMAT(1H0,53HCROSS MOMENTS OF INPUT FORCE AND Y(2J-1),Y(2J),DY(2
6261))
627 FORMAT(1H0,3X,I2,8HTH FORCE,3X,3F14.4)
628 FORMAT(1H0,51H2ND MOMENTS OF DY(J)*Y(L),L=J-3,OR(1) TO J+3(OR 2N))
629 FORMAT(4H0DY(,I2,4H)*Y(,I2,1H),7F14.4)
630 FORMAT(1H1,I20)
631 FORMAT(14H0DETERMINNANT=,E20.8)
632 FORMAT(31H0     EQUATIONS WHEN CM IS KNOWN)
633 FORMAT(1H0,24X,E15.8,6H*K + (,E15.8,5H)*C =,E20.8)
634 FORMAT(26H0ESTIMATES WITH MASS GIVEN)
635 FORMAT(1H0,38X,2(4X,2E17.8))
400 STOP
    END
```

```
      SUBROUTINE RKD(DERSUB,FORSUB,M,NF,H,TI,YI,FOS,K,N,F,VAL,DVAL,Y)
      DIMENSION Y(20),S1(20),S2(20),S3(20),S4(20),DY(20),FOS( 1),YI( 1),
     1VAL(1),F(1) ,A(20),DVAL(1) ,FOSP(20),FOSA(20)
      H2=H/2.
      T=TI
      L=0
      LF=0
      DO 10 I=1,M
   10 Y(I)=YI(I)
      CALL DERSUB(T,Y,FOS,M,NF,DY)
      DO 80 LL=1,N
      DO 65 JJ=1,K
      TP=T+H
      CALL FORSUB(TP,FOSP,NF)
      DO 12 I=1,NF
   12 FOSA(I)=(FOS(I)+FOSP(I))/2.0
C
C     COMPUTE K SUB 0
C
   25 DO 30 I=1,M
      S1(I)=H*DY(I)
   30 A(I)=Y(I)+S1(I)/2.
      TA=T+H2
C
C     COMPUTE K SUB 1
C
      CALL DERSUB(TA,A,FOSA,M,NF,DY)
      DO 40 I=1,M
      S2(I)=H*DY(I)
   40 A(I)=Y(I)+S2(I)/2.
C
C     COMPUTE K SUB 2
C
      CALL DERSUB(TA,A,FOSA,M,NF,DY)
      DO 50 I=1,M
      S3(I)=H*DY(I)
   50 A(I)=Y(I)+S3(I)
      TA=T+H
C
C     COMPUTE K SUB 3
      CALL DERSUB(TA,A,FOSP,M,NF,DY)
      DO 60 I=1,M
   60 S4(I)=H*DY(I)
      T=T+H
      DO 62 I=1,NF
   62 FOS(I)=FOSP(I)
C
C     COMPUTE NEW VALUES OF INTEGRALS
C
      DO 63 I=1,M
   63 Y(I)=Y(I)+(S1(I)+2.*S2(I)+2.*S3(I)+S4(I))/6.0
   65 CALL DERSUB(T,Y,FOS,M,NF,DY)
      DO 70 I=1,M
      L=L+1
      DVAL(L)=DY(I)
```

```
 70 VAL(I) =Y(I)
    DO 75 I=1,NF
    LF=LF+1
 75 F(LF)=FCS(I)
 80 CONTINUE
    RETURN
    END
```

```
C      -----------------------------------------------------------------
C       SUBROUTINE DERSUB(T,VAR,FS,M,NF,DER)
C      PURPOSE
C        AUXILIARY SUBROUTINE WHICH COMPUTES THE DERIVATIVES FOR THE
C          INTEGRATION SUBROUTINE RKD
C      -----------------------------------------------------------------
      SUBROUTINE DERSUB(T,VAR,FS,M,NF,DER)
      DIMENSION VAR(20),DER(20),CC(10),CK(10),CM(10),FS(10)
      COMMON CC,CK,CM
      N=M/2
      DO 10 I=1,N
   10 DER(2*I-1)=VAR(2*I)
      IF(N-2) 50,15,15
   15 DER(2)=(CK(1)/CM(1))*(VAR(3)-VAR(1))+(CC(1)/CM(1))*(VAR(4)-VAR(2))
     1       +FS(1)/CM(1)
      IF(N.EQ.2) GO TO 30
      N1=N-1
      DO 25 I=2,N1
      DER(2*I)   =(CK(I)/CM(I))*(VAR(2*I+1)-VAR(2*I-1))
     1            +(CC(I)/CM(I))*(VAR(2*I+2)-VAR(2*I))
     1            -(CK(I-1)/CM(I))*(VAR(2*I-1)-VAR(2*I-3))
     1            -(CC(I-1)/CM(I))*(VAR(2*I)  -VAR(2*I-2))
      IF(NF.LT.I) GO TO 25
      DER(2*I)   =DER(2*I)   +FS(I)/CM(I)
   25 CONTINUE
   30 DER(2*N) =-(CK(N)/CM(N))*VAR(2*N-1)-(CC(N)/CM(N))*VAR(2*N)
     1             -(CK(N-1)/CM(N))*(VAR(2*N-1)-VAR(2*N-3))
     1             -(CC(N-1)/CM(N))*(VAR(2*N)  -VAR(2*N-2))
      IF(NF.LT.N) GO TO 100
      DER(2*N)   =DER(2*N)   +FS(N)/CM(N)
      GO TO 100
   50 DER(2)=-(CK(1)/CM(1))*VAR(1)-(CC(1)/CM(1))*VAR(2)+FS(1)/CM(1)
  100 RETURN
      END
```

```
      SUBROUTINE FORSUB(T,F,NF)
      DIMENSION CC(10),CK(10),CM(10),FOST(10),FW(10),FB(10),F(10)
      COMMON CC,CK,CM,FOST,FW,FB
      DO 50 I=1,NF
      THETA=6.283185*(FW(I)*T+FB(I)/360.)
      F(I)=FOST(I)*SIN(THETA)
   50 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FORSUB(T,FOS,NF)
      DIMENSION FOS(10),FOST(10),FW(10),FB(10),DIS(3,10),XF(2,10),
     1CC(10),CK(10),CM(10)  ,XR(10)
      COMMON CC,CK,CM,FOST,FW,FB,DIS
      DO 50 I=1,NF
      TEMP=FOST(I)*GAURN(F)
      FOS(I)=DIS(1,I)*XF(1,I)+DIS(2,I)*XF(2,I)+DIS(3,I)*(TEMP-XR(I))
      XF(2,I)=XF(1,I)
      XF(1,I)=FOS(I)
      XR(I)=TEMP
   50 CONTINUE
      RETURN
      END
```

```
C     ----------------------------------------------------------------
C     SUBROUTINE  PAMSQT(A,NV,NO,M,SQ,SUM,ID)
C     PURPOSE
C       TO CALCULATE EITHER PARTIAL FIRST AND SECOND MOMENTS
C       OR SUM OF SQUARES AND SUM
C         A  - DATA MATRIX, NV BY NO
C         NV - NO OF VARIABLES
C         NO - NO OF OBSERVATIONS
C         M  - NO OF ADJACENT VARIABLES FOR CROSS MOEMNTS,M LESS THAN N
C         SQ - SUM OF SQUARES WHEN ID=0
C              SECOND MOMENTS WHEN ID OTHER THAN 0
C              MATRIX OF (M+1)*NV-(M*(M+1))/2
C         SUM - ARRAY OF NV
C     ----------------------------------------------------------------
      SUBROUTINE PAMSQT(A,NV,NO,M,SQ,SUM,ID)
      DIMENSION A(1),SQ(1), SUM(1)
      LJ=0
      KJ=0
      DO 20 J=1,NV
      IF(NV-J-M) 4,4,6
    4 JM=NV
      GO TO 8
    6 JM=J+M
    8 DO 10 L=J,JM
      LJ=LJ+1
   10 SQ(LJ)=0.0
   20 SUM(J)=0.0
C
C     CALCULATE SUM OF SQUARES AND SUM
C
      DO 40 I=1,NO
      LJ=0
      DO 40 J=1,NV
      IF(NV-J-M) 24,24,26
   24 JM=NV
      GO TO 28
   26 JM=J+M
   28 IJ=(I-1)*NV+J
      DO 30 L=J,JM
      IL=(I-1)*NV+L
      LJ=LJ+1
   30 SQ(LJ)=SQ(LJ)+A(IJ)*A(IL)
   40 SUM(J)= SUM(J)+A(IJ)
C
C     CALCULATE MOMENTS IF ID NOT 0
C
      IF(ID) 60,100,60
   60 JN=(M+1)*NV-(M*(M+1))/2
      CNO=NO
      DO 70 J=1,JN
   70 SQ(J)=SQ(J)/CNO
      DO 80 K=1,NV
   80 SUM(K)=SUM(K)/CNO
  100 RETURN
      END
```

```
      SUBROUTINE XSQ(A,B,NV,NO,M,ASQ,AB,IFLAG)
C     A - DATA MATRIX OF NV BY NO
C     B - DATA MATRIX OF NV BY NO
C     NV -  NO OF VARIABLES
C     NO - NO OF OBSERVATIONS
C     M - NO OF VARIABLES FOR THE CROSS MOMENTS
C      ASQ - IFLAG EQ 0  SUM OF SQUARES OF A
C            IFLAG NE 0 SECOND MOMENTS OF A
C     AB - IFLAG EQ 0 SUM OF SQUARES OF A(I)*B(J),J=J1,JM
C          IFLAG NE 0 SECOND MOMENTS OF A(I)*B(J),J=J1,JM
      DIMENSION A(1),B(1),ASQ(1),AB(1)
      DO 10 I=1,NV
   10 ASQ(I)=0.0
      NM=(2*M+1)*NV-M*(M+1)
      DO 20 I=1,NM
   20 AB(I)=0.0
      IN=0
      DO 80 N=1,NO
      IJ=0
      DO 80 I=1,NV
      IN=IN+1
      ASQ(I)=ASQ(I)+A(IN)*A(IN)
      IF(I-M) 25,25,30
   25 I1=1
      GO TO 35
   30 I1=I-M
   35 IF(NV-I-M) 40,40,50
   40 IM=NV
      GO TO 55
   50 IM=I+M
   55 DO 60 J=I1,IM
      JN=(N-1)*NV+J
      IJ=IJ+1
   60 AB(IJ)=AB(IJ)+A(IN)*B(JN)
   80 CONTINUE
      IF(IFLAG) 90,100,90
   90 CNO=NO
      DO 94 I=1,NV
   94 ASQ(I)=ASQ(I)/CNO
      DO 96 I=1,NM
   96 AB(I)=AB(I)/CNO
  100 RETURN
      END
```

```
C     --- ----------------------------------------------------------------
C        SUBROUTINE XMONT3(A,NA,B,C,NBC,NO,XM,ID)
C        PURPOSE
C          TO COMPUTE CROSS MOMENT OF A(J)*B(2*J-1),A(J)*B(2*J)
C          AND A(J)*C(2*J)
C        A  -BASE DATA MATRIX,NA BY NO
C        NA -NO.OF VARIABLES OF A
C        B  -CROSS VARIABLE DATA MATRIX
C        C    CROSS VARIABLE DATA MATRIX
C        NBC-NO.OF VARIABLES OF B AND C, NB .GE. (2*NA)
C        NO -NO.OF OBSERVATIONS
C        XM -CROSS MOMENTS,MATRIX OF 3*NA
C        ID -ID NOT ZERO,COMPUTE THE MOMENTS
C            ID ZERO, COMPUTE THE SUMS
C     --------------------------------------------------------------------
      SUBROUTINE XMONT3(A,NA,B,C,NBC,NO,XM,ID)
      DIMENSION A(1),B(1),XM(3) ,C(1)
      N3=3*NA
      DO 10 J=1,N3
   10 XM(J)=0.0
      IJA=0
      DO 30 I=1,NO
      DO 30 J=1,NA
      IJA=IJA+1
      IJ2B=(I-1)*NBC+2*J
      IJ1B=IJ2B-1
      XM(3*J-2)=XM(3*J-2)+A(IJA)*B(IJ1B)
      XM(3*J-1)=XM(3*J-1)+A(IJA)*B(IJ2B)
      XM(3*J)=XM(3*J)+A(IJA)*C(IJ2B)
   30 CONTINUE
C
C     COMPUTE MOMENTS IF ID NOT ZERO
C
      IF(ID) 40,50,40
   40 CNO=NO
      DO 45 J=1,N3
   45 XM(J)=XM(J)/CNO
   50 RETURN
      END
```

```
      ------------------------------------------------------------------
      SUBROUTINE LOC2(L,J,LJ,NV,M)
      M  -THE NO.OF ROWS FROM THE DIAGONAL
C     PURPOSE
C       TO CALCULATE THE STORAGE LOCATION OF A PARTIAL LOWER HALF MATRI
C     L   -ROW NO.
C     J   -COLUMN NO.
C     LJ -STORAGE LOCATION
C     NV -RANGE OF THE MATRIX
C     ------------------------------------------------------------------
      SUBROUTINE LOC2(L,J,LJ,NV,M)
      IF(L.LT.(J-M))  GO TO 50
      IF(L.GT.(J+M))  GO TO 50
      IF(L.GE.J) GO TO 10
      LT=J
      JT=L
      GO TO 15
   10 LT=L
      JT=J
   15 IF((JT+M-1).LE.NV)  GO TO 20
      N=(JT+M-1)-NV
      N=(N*(N+1))/2
      LJ=(M+1)*(JT-1)+LT-JT-N+1
      GO TO 60
   20 LJ=(M+1)*(JT-1)+LT-JT+1
      GO TO 60
   50 WRITE(6,601)
  601 FORMAT(1H0,49HL AND J ARE NOT IN THE RANGE OF THE STORED MATRIX)
      LJ=0
   60 RETURN
      END
```

```
C     ---------------------------------------------------------------
C     SUBROUTINE LOC3(L,J,LJ,NV,M)
C     PURPOSE
C       TO CALCULATE THE STORAGE LOCATION OF A PARTIAL MATRIX
C     L  - NO. OF ROW OF THE MATRIX
C     J  - NO  OF COLUMN OF THE MATRIX
C     LJ - STORAGE LOCATION
C     NV - RANGE OF THE SQUARE MATRIX
C     M  - ONLY M ROW FROM THE DIAGONAL  ARE STORED,LESS THAN NV
C     ---------------------------------------------------------------
      SUBROUTINE LOC3(L,J,LJ,NV,M)
      IF(L.LT.(J-M)) GO TO 100
      IF(L.GT.(J+M)) GO TO 100
      IF(J-1-M) 10,10,50
   10 IF((J-1+M).LE.NV) GO TO 20
      NT=M+2-J
      NL=J-1+M-NV
      LJ=(2*M+1)*(J-1)-((M+NT)*(M-NT+1))/2-(NL*(NL+1))/2+L
      GO TO 110
   20 NT=M+2-J
      LJ=(2*M+1)*(J-1)-((M+NT)*(M-NT+1))/2+L
      GO TO 110
   50 IF((J-1+M).LE.NV) GO TO 70
      NL=J-1+M-NV
      LM=L-J+M+1
      LJ=(2*M+1)*(J-1)-(M*(M+1))/2-(NL*(NL+1))/2+LM
      GO TO 110
   70 LM=L-J+M+1
      LJ=(2*M+1)*(J-1)-(M*(M+1))/2+LM
      GO TO 110
  100 WRITE(6,601)
  601 FORMAT(1H0,49HL AND J ARE OUT OF THE RANGE OF THE STORED MATRIX)
  110 RETURN
      END
```

```
C     ------------------------------------------------------------------
C     SUBROUTINE COFF(J,N,NF,SQ,DYSQ,YDY,XM,EC,EK,A,B)
C     PURPOSE
C       TO OBTAIN THE COEFFICIENTS OF THE 3 BY 3 EQUATIONS FOR
C       SOLVING THE JTH SET OF SYSTEM CONSTANTS OF THE ONE DIMENSIONAL
C       N DEGREE FREEDOM,SPRING,MASS AND DAMPER SYSTEM EXCITED BY
C       NF SEQUENTIAL FORCES
C       SQ -SECOND MOMENTS OF THE SYSTEM OUTPUTS
C     DYSQ - 2ND MOMENTS OF DY
C     YDY - 2ND MOMENTS OF DY*Y
C       XM -CROSS MOMENTS OF THE SYSTEM INPUTS AND OUTPUTS
C     ------------------------------------------------------------------
      SUBROUTINE COEF(J,N,NF,SQ,DYSQ,YDY,XM,EC,EK,A,B)
      DIMENSION A(3,3),B(3),SQ(1),DYSQ(1),YDY(1),XM(3)
      NV=2*N
      MX=3
      IF(NV.LE.3) MX=NV-1
      A(3,1)=-DYSQ(2*J)
      JR=2*J
      JC=JR-1
      CALL LOC3(JR,JR,K1,NV,MX)
      A(2,1)=-YDY(K1)
      CALL LOC3(JC,JR,K1,NV,MX)
      A(1,1)=-YDY(K1)
      CALL LOC2(JR,JC,K3,NV,MX)
      CALL LOC2(JC,JC,K2,NV,MX)
      JR=JR+1
      IF(J.EQ.N) GO TO 20
C
      CALL LOC2(JR,JC,K1,NV,MX)
      A(1,2)=SQ(K1)-SQ(K2)
      JR=JR+1
      CALL LOC2(JR,JC,K1,NV,MX)
      A(1,3)=SQ(K1)-SQ(K3)
      JR=2*J+1
      JC=2*J
      CALL LOC2(JR,JC,K1,NV,MX)
      A(2,2)=SQ(K1)-SQ(K3)
      CALL LOC3(JR,JC,K1,NV,MX)
      JJ=JC-1
      CALL LOC3(JJ,JC,K2,NV,MX)
      A(3,2)=YDY(K1)-YDY(K2)
      JR=JR+1
      CALL LOC2(JR,JC,K1,NV,MX)
      CALL LOC2(JC,JC,K2,NV,MX)
      A(2,3)=SQ(K1)-SQ(K2)
      CALL LOC3(JR,JC,K1,NV,MX)
      A(3,3)=YDY(K1)+A(2,1)
      GO TO 30
C
C      COMPUTE COEFFICIENTS WHEN J=N
C
   20 A(1,2)=-SQ(K2)
      A(1,3)=-SQ(K3)
      A(2,2)=-SQ(K3)
```

```
      JC=2*J
      JR=JC-1
      CALL LOC2(JC,JC,K2,NV,MX)
      A(2,3)=-SQ(K2)
      CALL LOC3(JR,JC,K2,NV,MX)
      A(3,2)=-YDY(K2)
      A(3,3)=A(2,1)
C
C     COMPUTE B(1),B(2) B(3)
C     EC -ESTIMATED (J-1)TH DAMPER CONSTANT
C     EK -ESTIMATED (J-1)TH SPRING CONSTANT
C
   30 IF(J.EQ.1) GO TO 50
      JR=2*J-1
      JC=2*J-3
      CALL LOC2(JR,JR,K1,NV,MX)
      CALL LOC2(JR,JC,K2,NV,MX)
      JC=JC+1
      CALL LOC2(JR,JC,K3,NV,MX)
      JJ=2*J
      CALL LOC2(JJ,JR,K4,NV,MX)
      B(1)=(SQ(K1)-SQ(K2))*EK + (SQ(K4)-SQ(K3))*EC
      JR=JR+1
      CALL LOC2(JR,JC,K3,NV,MX)
      CALL LOC2(JR,JR,K2,NV,MX)
      JC=JC-1
      CALL LOC2(JR,JC,K1,NV,MX)
      B(2)=(SQ(K4)-SQ(K1))*EK + (SQ(K2)-SQ(K3))*EC
      JC=2*J
      JR=JC-1
      CALL LOC3(JR,JC,K1,NV,MX)
      JR=JR-2
      CALL LOC3(JR,JC,K2,NV,MX)
      CALL LOC3(JC,JC,K3,NV,MX)
      JR=JR+1
      CALL LOC3(JR,JC,K4,NV,MX)
      B(3)=(YDY(K1)-YDY(K2))*EK+(YDY(K3)-YDY(K4))*EC
   45 IF(J.GT.NF) GO TO 100
      B(1)=B(1)-XM(3*J-2)
      B(2)=B(2)-XM(3*J-1)
      B(3)=B(3)-XM(3*J)
      GO TO 100
   50 B(1)=-XM(1)
      B(2)=-XM(2)
      B(3)=-XM(3)
  100 RETURN
      END
```

```
      SUBROUTINE [illegible] (A,NR,NSYS,MARK,DET,INOPT)
C     NSYS = NO. OF SYSTEMS TO BE SOLVED
C     NR = ORDER OF A
C     A = INPUT MATRIX
C     MARK = SINGULARITY INDICATOR (MARK=1 FOR SINGULAR A)
C     DET = DET(A)
C     INOPT = -1 FOR SYSTEM SOLN. AND DET
C              0 FOR DET ONLY
C             +1 FOR INVERSE AND DET
C     DIMENSION STMNT. MUST AGREE WITH DIM. STMNT. IN MAIN PROGRAM
C
      DIMENSION A(3,4),X(4)
C
C     PRESET PARAMETERS
      SIGN = 1.
      MARK = 0
      IFLAG = INOPT
      N = NR
      NPL = N+1
      NMI = N-1
      NN = N+N
      NPLSY = N+NSYS
      IF (IFLAG) 40,40,10
C
C     INVERSE OPTION - PRESET AUGMENTED PART TO I
 10   DO 20 I=1,N
      DO 20 J=NPL,NN
 20   A(I,J) = 0.
      DO 30 I=1,N
      J = I+N
 30   A(I,J) = 1.
      NPLSY = NN
C
C     TRIANGULARIZE A
 40   DO 120 I=1,NMI
      IPL = I+1
C     DETERMINE PIVOT ELEMENT
      MAX = I
      AMAX = ABS(A(I,I))
      DO 60 K=IPL,N
      IF (AMAX-ABS(A(K,I))) 50,60,60
 50   MAX = K
      AMAX = ABS(A(K,I))
 60   CONTINUE
      IF (MAX-I) 70,90,70
C     PIVOTING NECESSARY - INTERCHANGE ROWS
 70   DO 80 L=I,NPLSY
      TEMP = A(I,L)
      A(I,L) = A(MAX,L)
 80   A(MAX,L) = TEMP
      SIGN = -SIGN
C     ELIMINATE A(I+1,I)---A(N,I)
 90   DO 120 J=IPL,N
      TEMP = A(J,I)
      IF (TEMP) 100,120,100
```

```
  100 CONST = -TEMP/A(I,I)
      DO 110 L=I,NPLSY
  110 A(J,L) = A(J,L)+A(I,L)*CONST
  120 CONTINUE
C
C     COMPUTE VALUE OF DETERMINANT
      TEMP = 1.
      DO 130 I=1,N
      AGG = A(I,I)
      IF (AGG) 130,125,130
C     MATRIX SINGULAR
  125 WRITE (6,900)
      MARK = 1
      GO TO 135
  130 TEMP = TEMP*AGG
      DET = SIGN*TEMP
C
C     EXIT IF DET ONLY OPTION
  135 IF (IFLAG) 140,250,140
C     CHECK FOR INVERSE OPTION OR SYSTEMS OPTION
  140 IF (IFLAG-1) 160,150,160
C     INVERSE OPTION - ABORT IF A IS SINGULAR
  150 IF (MARK-1) 160,250,160
C
C     BACK SUBSTITUTE TO OBTAIN INVERSE OR SYSTEM SOLUTION(S)
  160 DO 240 I=NPL,NPLSY
      K = N
  170 X(K) = A(K,I)
      IF (K-N) 180,200,180
  180 DO 190 J=KPL,N
  190 X(K) = X(K)-A(K,J)*X(J)
  200 X(K) = X(K)/A(K,K)
      IF (K-1) 210,220,210
  210 KPL = K
      K = K-1
      GO TO 170
C     PUT SOLN. VECT. INTO APPROPRIATE COLUMN OF A
  220 DO 230 L=1,N
  230 A(L,I) = X(L)
  240 CONTINUE
C
  250 RETURN
  900 FORMAT (//1X15HSINGULAR MATRIX//)
      END
```

```
C       SUBROUTINE RANDKP for random number generation           RAND0001
        ENTRY     EXPRN
        ENTRY     GAURN
        ENTRY     FLRAN
        ENTRY     GETNM
        ENTRY     STORNM


EXPRN   LDQ       RANDOM
C       PXD       952,0
H       STA       A
        MPY       GENERA
        STQ       COMMON+1

        STQ       COMMON
F       MPY       GENERA
        STQ       RANDOM
        CLA       COMMON
        TLQ       B
        LDQ       COMMON+1

        RQL       12
        CAL       C
        LGL       24
        STO       COMMON
        CLA       A
        LLS       12


E       FAD       COMMON
G       TNZ       1,4
        TRA       E
B       MPY       GENERA
        STQ       COMMON
        CLA       RANDOM
        TLQ       F
        CLA       A
        ADM       G
        TRA       H
GAURN   SXD       COMMON+3,4
CC      TSX       EXPRN,4
        ADD       AA
        STO       COMMON+4

        TSX       EXPRN,4
        STO       COMMON
        FSB       BB


        STO       COMMON+1

        LDQ       COMMON+1

        FMP       COMMON+1
```

```
         SUB     COMMON+4

         TPL     CC
         LXD     COMMON+3,4

         CLA     COMMON
S        LDQ     RANDOM
         RQL     20
         LLS     0
         TRA     1,4
 FLRAN   LDQ     RANDOM
         MPY     GENERA
         STQ     RANDOM


         CLA     AAA
         LGL     28
         FAD     AAA
         TRA     S
GETNM    CLA     RANDOM
         STO*    3,4
         TRA     1,4
STORNM   CLA*    3,4
         STO     RANDOM
         TRA     1,4
GENERA   OCT     343277244615
RANDOM   DEC     30517578125
 AA      OCT     001000000000
 BB      DEC     1.
 AAA     OCT     172000000100
 A       OCT     000217000000
COMMON   BSS     5
         END
```

Input Cards and Output Tabulations for a Sample Problem

The sample problem is to simulate a five-mass system whose parameters are

$$m_1 = m_2 = \ldots = m_5 = 0.052$$

$$k_1 = k_3 = 9.3 \times 10^5 \quad , \quad k_2 = k_4 = 8.3 \times 10^4 \quad , \quad k_5 = 43$$

$$C_1 = C_3 = 1.9 \quad , \quad C_2 = C_4 = C_5 = 0.9$$

The system is to be driven at mass No. 1 by a random force which is generated by a white noise of standard deviation 30 being passed through a bandpass filter with center frequency 70 cps and a bandwidth of 20 cps. The sampling frequency for simulation is chosen to be 5000 samples per sec. 10,000 samples are to be simulated and used for estimation of the parameters.

a) Input Cards

Col. 1	6	11	16	21	26
5					
0.052		930000.0		.9	
0.052		83000.0		0.9	
0.052		930000.0		1.9	
0.052		83000.0		0.9	
0.052		43.0		0.9	
1					
30.0	70.0	20.0			
5000.		10000	1	1	1

b) Sample Output

```
ESTIMATION OF MOMENTS FOR THE IDENTIFICATION OF 5MASS SYSTEM

FIRST MOMENTS OF Y(J),J=1,2N

   -0.56105125E-05    -0.11423636E-03    -0.56138458E-05    -0.11669857E-03    -0.56358206E-05
   -0.12976357E-03    -0.56367663E-05    -0.12998723E-03    -0.56357353E-05    -0.12637553E-03

2ND MOMENTS Y(J)Y(L), L=J TO J+3 (OR 2N)

        Y( 1)    0.7758E-07    0.1305E-07    0.7759E-07    0.2477E-07

        Y( 2)    0.2882E-02    0.1851E-08    0.2748E-02   -0.8469E-07

        Y( 3)    0.7761E-07    0.1367E-07    0.7621E-07    0.1127E-06

        Y( 4)    0.2662E-02   -0.8239E-07   -0.3467E-04   -0.8743E-07

        Y( 5)    0.7955E-07    0.1684E-07    0.7976E-07    0.2412E-07

        Y( 6)    0.1524E-02    0.9619E-08    0.1621E-02   -0.7096E-07

        Y( 7)    0.8000E-07    0.1690E-07    0.8155E-07    0.1025E-06

        Y( 8)    0.1745E-02   -0.6962E-07    0.2554E-02

        Y( 9)    0.8481E-07    0.1598E-07

        Y(10)    0.5264E-02

SECOND MOMENTS OF DERIVATIVES DY(J),J=1,2N

     0.28823444E-02     0.30359671E 04     0.26616850E-02     0.27681042E 04     0.15237063E-02
     0.74332625E 03     0.17446624E-02     0.91664041E 03     0.52641622E-02     0.43313226E 04

2ND MOMENTS OF DY(J)*Y(L),L=J-3,OR(1) TO J+3(OR 2N)

DY( 1)*Y( 1)    0.1305E-07    0.2882E-02    0.1851E-08    0.2748E-02

DY( 2)*Y( 1)   -0.2890E-02    0.2775E-02   -0.2755E-02   -0.1791E-01    0.1266E-03

DY( 3)*Y( 1)    0.2477E-07    0.2748E-02    0.1367E-07    0.2662E-02   -0.8239E-07   -0.3467E-04

DY( 4)*Y( 1)   -0.2753E-02    0.1713E-01   -0.2667E-02    0.1557E-02    0.2817E-04   -0.1016E-00    0.2306E-03

DY( 5)*Y( 2)   -0.1359E-03    0.1127E-06   -0.3467E-04    0.1684E-07    0.1524E-02    0.9619E-08    0.1621E-02

DY( 6)*Y( 3)    0.3046E-04    0.1025E-30   -0.1528E-02    0.4126E-03   -0.1626E-02   -0.7178E-02   -0.2244E-02

DY( 7)*Y( 4)   -0.2370E-03    0.2412E-07    0.1621E-02    0.1690E-07    0.1745E-02   -0.6962E-07    0.2554E-02

DY( 8)*Y( 5)   -0.1626E-02    0.7717E-02   -0.1750E-02    0.4346E-03   -0.2559E-02   -0.7977E-01

DY( 9)*Y( 6)    0.2240E-02    0.1025E-06    0.2554E-02    0.1598E-07    0.5264E-02

DY(10)*Y( 7)   -0.2555E-02    0.7595E-01   -0.5265E-02    0.4139E-04

CROSS MOMENTS OF INPUT FORCE AND Y(2J-1),Y(2J),DY(2J)

    1TH FORCE      -0.1545E-03    0.1081E-01    0.3282E 02
```

SETS OF EQUATIONS FOR SOLVING THE SYSTEM PARAMETERS

EQUATION FOR SOLVING 1TH MASS SYSTEM COEFFICIENTS

0.28856820E-02*M + (0.49485301E-11)*K + (0.11721747E-07)*C = 0.15488926E-03
-0.27754712E-02*M + (-0.11196143E-07)*K + (-0.13477858E-03)*C =-0.10812799E-01
-0.30359671E 04*M + (0.13450605E-03)*K + (-0.20680859E-01)*C =-0.32818981E 02

DETERMINANT= 0.54670725E-10

EQUATIONS WHEN CM IS KNOWN

0.49485301E-11*K + (0.11721747E-07)*C = 0.46258064E-05
-0.11196143E-07*K + (-0.13477858E-03)*C = -0.10668474E-01

DETERMINANT= -0.53577143E-15

EQUATION FOR SOLVING 2TH MASS SYSTEM COEFFICIENTS

0.26674548E-02*M + (-0.13982903E-08)*K + (0.99036615E-07)*C = 0.22740278E-04
-0.15571713E-02*M + (-0.96061180E-07)*K + (-0.26963624E-02)*C =-0.10480815E-01
-0.27681342E 04*M + (0.26956235E-02)*K + (-0.10311664E-00)*C = 0.79703196E 02

DETERMINANT= 0.89513931E-08

EQUATIONS WHEN CM IS KNOWN

-0.13982903E-08*K + (0.99036615E-07)*C = -0.11596145E-03
-0.96061180E-07*K + (-0.26963624E-02)*C = -0.10400841E-01

DETERMINANT= 0.37798112E-11

EQUATION FOR SOLVING 3TH MASS SYSTEM COEFFICIENTS

0.15284548E-02*M + (0.21236567E-09)*K + (0.72742409E-08)*C = 0.27710023E-03
-0.41259130E-03*M + (-0.72242052E-08)*K + (0.97638374E-04)*C =-0.65547096E-02
-0.74332625E 03*M + (-0.97650044E-04)*K + (-0.75510815E-02)*C =-0.12948482E 03

DETERMINANT= -0.79591747E-12

EQUATIONS WHEN CM IS KNOWN

0.21236567E-09*K + (0.72742409E-08)*C = 0.19749673E-03
-0.72242052E-08*K + (0.97638374E-04)*C = -0.65315912E-02

DETERMINANT= 0.20787980E-13

EQUATION FOR SOLVING 4TH MASS SYSTEM COEFFICIENTS

0.17497303E-02*M + (0.15567152E-08)*K + (0.85582849E-07)*C = 0.22020365E-03
-0.43463121E-03*M + (-0.86526524E-07)*K + (0.80525805E-03)*C =-0.64767434E-02
-0.91664341E 03*M + (-0.80912324E-03)*K + (-0.80206463E-01)*C =-0.11486047E 03

```
DETERMINANT=      -0.37310325E-11
     EQUATIONS WHEN CM IS KNOWN
                           0.15567192E-08*K + ( 0.85582849E-07)*C =      0.12925986E-03
                          -0.86526524E-07*K + ( 0.80925605E-03)*C =     -0.64517181E-02
DETERMINANT=       0.12671927E-11
EQUATION FOR SOLVING  5TH MASS SYSTEM COEFFICIENTS
     0.52654621E-02*M + (-0.84806616E-07)*K + (-0.15983417E-07)*C = 0.26901576E-03
    -0.41387929E-04*M + (-0.15983417E-07)*K + (-0.52641622E-02)*C =-0.48129715E-02
    -0.43313226E 04*M + ( 0.52654621E-02)*K + (-0.41387929E-04)*C =-0.22406352E 03
DETERMINANT=      -0.17877524E-05
     EQUATIONS WHEN CM IS KNOWN
                          -0.84806616E-07*K + (-0.15983417E-07)*C =     -0.37133540E-05
                          -0.15983417E-07*K + (-0.52641622E-02)*C =     -0.47361505E-02
DETERMINANT=       0.44644605E-09
```

```
ESTIMATED COEFFICIENTS OF THE SIMULATED 5MASS SYSTEM
NO OF EXCITATION =  1
        FOR EXCITATION  1     AMPLITUDE =     30.00     CENTER FREQUENCY =      70.0     PHASE SHIFT OR BANDWIDTH =     20.00
SAMPLING FREQUENCY =    5000.0CPS
SAMPLING INT. FOR SIMULATION = 0.00020
TRANSIENT INT. =       0.
SAMPLING INT. FOR CALCULATING MOMENTS = 0.00020
TOTAL NO. OF SAMPLES USED FOR ESTIMATION = 10000
TIME INTERVAL FOR ESTIMATION =      2.00SEC
```

M(I) TRUE	M(I) ESTIMATED	K(I) TRUE	K(I) ESTIMATED	C(I) TRUE	C(I) ESTIMATED
0.52000000E-01	0.52030345E-01	0.92999999E 06	0.93000745E 06	0.19000000E 01	0.18992361E 01
0.52000000E-01	0.52001563E-01	0.82999999E 05	0.83010836E 05	0.90000000E 00	0.89994701E 00
0.52000000E-01	0.52046265E-01	0.92999999E 06	0.92972419E 06	0.19000000E 01	0.18771925E 01
0.52000000E-01	0.52270773E-01	0.82999999E 05	0.82654660E 05	0.90000000E 00	0.86226630E 00
0.52000000E-01	0.51783010E-01	0.43000000E 02	0.42815404E 02	0.90000000E 00	0.91375299E 00

ESTIMATES WITH MASS GIVEN

K(I) TRUE	K(I) ESTIMATED	C(I) TRUE	C(I) ESTIMATED
0.92999999E 06	0.93025953E 06	0.19000000E 01	0.18783012E 01
0.82999999E 05	0.82996670E 05	0.90000000E 00	0.90057462E 00
0.92999999E 06	0.92990141E 06	0.19000000E 01	0.19071131E 01
0.82999999E 05	0.82984015E 05	0.90000000E 00	0.90033124E 00
0.43000000E 02	0.43615568E 02	0.90000000E 00	0.89956447E 00

4.3 Computer Program Abstract II

General Description

a) This program is basically the same as I except for changes made to enable the input force to be read from a tape unit instead of being simulated. The tape with the desired input force samples is to be mounted on the tape unit with logical address 21 and consists of binary records. The first record should have 21 words with the 5th word being the number of input force samples per second. All succeeding records should have 1003 words with the first word being the record number and followed by 167 blocks of six words. The first words of each six word block should be the input force samples.

b) Output from this program includes the same output as I.

c) Limitation of this program:

1) N, the number of mass, is limited to 10

2) NF, the number of input forcing function, is limited to 1 and is the driving force at the first mass.

d) Required supporting subprogram

1) RKFOR

2) DERF

3) PAMSQT

4) XSQ

5) XMONT3

6) LOC2

7) LOC3

8) COEF

9) RLMTX

Input Cards

a) Degree of Freedom Card

Col. 1 - 2 N - Number of mass for the chainlike system

b) System Parameter Card(s)

A card is used to specify each system parameter triplet (m_i, k_i, C_i). N cards are then needed and should be arranged consecutively from i=1 to N.

Col. 1 - 10 CM(I) - Floating point constant for the ith mass coefficient

Col. 11-20 CK(I) - Floating point constant for the ith spring coefficient

Col. 21-30 CC(I) - Floating point constant for the ith damper coefficient

c) Simulation Specification Card

Col. 1-10 NI - Number of samples to be simulated before samples are taken for estimation of the moments and the system parameters

Col. 11-20 NO - Number of samples to be used for estimation of moments and system parameters

Col. 21-30 K - Only every kth samples of the equally-spaced samples are to be used for estimation of the moments and system parameters.

Col. 31-40 NORUN - Number of successive times the moments and the system parameters are to be estimated

Col. 41-50 INIT - Control index for how the samples are taken for each successive estimation of the moments and parameters

If INIT < 0, successive NO samples (after the initial NI samples) taken at every kth sample are to be used for the estimation process.

If INIT = 0, system is reinitialized each time. NO samples (after initial NI samples) taken at every kth sample are used for the estimation process.

If INIT > 0, (after initial NI samples) NO samples are used cummulatively each time; that is, NO, then 2 x NO, ... then NORUN x NO are successively used for the estimation process.

Col. 51-60 IH - Index to change sampling rate of the random force recorded on digital tape by changing the time scale which will also change the bandwidth of frequency spectrum.

IH = 0, the sampling rate of the input force recorded on tape is to be used.

IH $\neq$ 0, the sampling interval is given by HTEP

Col. 61-70 HTEP - Floating point constant for sampling interval, ignored when IH = 0.

d) Repeat a to c for a different choice of system as many times as desired. A blank card after c will cause a stop.

Description of Supporting Subprograms

a) RKFOR (DERF, M, H, TI, YI, FI, K, N, F, VAL, DVAL, Y, FOSP)

This Fortran subprogram generates the solution to a set of M simultaneous first-order, ordinary differential equations by the classical fourth-order Runge-Kutta method of integration. Where

DERF - Name of the external subroutine used to compute the derivatives

M - Number of equations for expressing the system

H - Step size for integration

TI - Initial value of T

YI - Initial value of Y

FI - Initial value of F

K - The desired number of steps of size H between values of the integrals to be stored in VAL and the derivatives to be stored in DVAL

N - The number of values to be stored in VAL and DVAL

F - Input forcing function values read from tape. On the return, only every kth value will remain in the array

VAL - A matrix of M by N containing the integrated value of the M derivatives generated by DERF

DVAL - A matrix of M by N containing the derivatives generated by the external subroutine DERF

Y - The final integrated value of M derivatives. An array of M

FOSP - The last value of F

b) DERF (T, VAR, FS, M, DER)

This Fortran subroutine computes the derivatives for the integration subroutine RKFOR where DER(I), I=1, M are the derivatives of VAR(I), I=1, M with respect to T, and are functions of T, VAR and FS (input forcing function value).

c) The descriptions for subprograms PAMSQT, XSQ, XMONT3, LOC2, LOC3, COEF and RLMTX are given in Section 4.2.

Listing of the Main Program and the Subprograms

The main program and only the subprograms RKFOR and DERF will be listed as follows since the other subprograms have already been listed in Section 4.2.

```
C       PROGRAM TO SIMULATE N DEGREE FREEDOM,ONE DIMENSIONAL, SPRING MASS
C         AND DAMPER SYSTEM AND ESTIMATE THE COEFFICIENTS
C
C       READ FORCE FROM TAPE UNIT 21
C
        DIMENSION CC(10),CK(10),CM(10),         Y(5000),DY(5000),YE(20),
       1F(2500),EC(10),EK(10),EM(10),YI(20),
       2REC(6,167),SQ(40),SUM(10),XM(15),A(3,3),B(3),R(3,4) ,ID(20)
       3,DYSQ(10),YDY( 70),SQT(40),SUMT(10),XMT(15),DYSQT(10),YDYT( 70)
       4,EEK(10),EEC(10) ,SQS(40),SUMS(10),XMS(15),DYSQS(10),YDYS(70)
        DOUBLE PRECISION SQS,SUMS,XMS,DYSQS,YDYS
        EQUIVALENCE (R(1,1),A(1,1)),(R(1,4),B(1))

        COMMON CC,CK,CM
        EXTERNAL DERF
C
      5 REWIND 21
        READ(5,500) N
        IF(N) 400,400,8
      8 READ(5,501)    (CM(I),CK(I),CC(I),I=1,N)
        READ(5,506) NI,NO,K,NORUN,INIT,IH,HTEP

        READ(21) ID
C
C       INITIALIZATION
C
        IF(IH) 10,12,10
     10 H=HTEP
        GO TO 14
     12 H=1.0/FLOAT(ID(5))
     14 N2=2*N
        NF=1
        MX=3
        IF(N2.LE.3) MX=N2-1
        NSQ=(MX+1)*N2-(MX*(MX+1))/2
        NXM=3*NF
        NXQ=(2*MX+1)*N2-(MX+1)*MX
        JTEST=168
     15 DO 30 I=1,N2
     30 YI(I)=0.0
        TI=0.0
        READ(21) COUNT,((REC(L,J),L=1,6),J=1,167)

        FI=REC(1,167)
        IF(NI) 41,41,31
     31 LF=0
     32 READ(21) COUNT,((REC(L,J),L=1,6),J=1,167)
```

```
      DO 34 J=1,167
      LF=LF+1
      F(LF)=REC(1,J)
      IF(LF.EQ.NI) GO TO 35
   34 CONTINUE
      GO TO 32
   35 JTEST=J+1
      CALL RKFOR(DERF,N2,H,TI,YI,FI,NI,1,F,Y,DY,YE,FE)
      DO 40 I=1,N2
   40 YI(I)=YE(I)
      TI=0.0+FLOAT(NI)*H
      FI=FE
   41 DO 42 J=1,NXM
   42 XMS(J)=0.0
      DO 43 J=1,NSQ
   43 SQS(J)=0.0
      DO 44 J=1,N2
      DYSQS(J)=0.0
   44 SUMS(J)=0.0
      DO 45 J=1,NXQ
   45 YDYS(J)=0.0
      NON=0
      TIS=TI
  455 IF((NO* K).LE.2500) GO TO 49
      MNF=2500/K
      IF((NO*N2).LE.5000) GO TO 46
      MN2= 5000/N2
      IF(MNF-MN2) 46,46,48
   46 NIS=MNF
      GO TO 50
   48 NIS=MN2
      GO TO 50
   49 IF((NO*N2).LE. 5000) GO TO 140
      NIS= 5000/N2
      GO TO 50
  140 NIS=NO
   50 NCS=NO
   51 LF=0
      IF(JTEST.GE.168) GO TO 1055
      DO 1052 J=JTEST,167
      LF=LF+1
 1052 F(LF)=REC(1,J)
 1055 READ(21) COUNT,((REC(L,J),L=1,6),J=1,167)

      DO 1057 J=1,167
      LF=LF+1
      F(LF)=REC(1,J)
      IF(LF.EQ.(NIS*K)) GO TO 1059
 1057 CONTINUE
      GO TO 1055
 1059 JTEST=J+1
C
C     SIMULATE SAMPLES AND CALCULATE MOMENTS
C
      CALL RKFOR(DERF,N2,H,TI,YI,FI,K,NIS,F,Y,DY,YE,FE)
```

```
      CALL PAYSQT(Y,N2,NIS,MX,SQT,SUMT,C)
      CALL XSQ(DY,Y,N2,NIS,MX,DYSQT,YDYT,C)
      CALL XMCNT3(F,NF,Y,DY,N2,NIS,XMT,C)
      NOS=NOS-NIS
      DO 52 J=1,NSQ
   52 SQS(J)=SQS(J)+SQT(J)
      DO 54 J=1,N2
      DYSQS(J)=DYSQS(J)+DYSQT(J)
   54 SUMS(J)=SUMS(J)+SUMT(J)
      DO 55 J=1,NXQ
   55 YDYS(J)=YDYS(J)+YDYT(J)
      DO 56 J=1,NXM
   56 XMS(J)=XMS(J)+XMT(J)
      DO 58 J=1,N2
   58 YI(J)=YE(J)
      FI=FF
      TN=NIS*K
      NON=NON+NIS
   60 TI=TI+TN*H
      IF(NOS) 100,100,65
   65 IF(NOS-NIS) 70,51,51
   70 NIS=NOS
      GO TO 51
  100 DO 110 J=1,NSQ
  110 SQ(J)=SQS(J)/FLOAT(NON)
      DO 112 J=1,N2
      DYSQ(J)=DYSQS(J)/FLOAT(NON)
  112 SUM(J)=SUMS(J)/FLOAT(NON)
      DO 114 J=1,NXM
  114 XM(J)=XMS(J)/FLOAT(NON)
      DO 116 J=1,NXQ
  116 YDY(J)=YDYS(J)/FLOAT(NON)
C
C     WRITE THE MOMENTS
C
  153 WRITE(6,620) N
      DO 155 J=1,N2
      CALL LOC2(J,J,K1,N2,MX)
      J3=J+MX
      IF(J3.GT.N2) J3=N2
      CALL LOC2(J3,J,K2,N2,MX     )
  155 WRITE(6,621) J,(SQ(JJ),JJ=K1,K2)

      WRITE(6,622)  (SUM(J),J=1,N2)
      WRITE(6,623) (DYSQ(J),J=1,N2)
      WRITE(6,628)
      DO 160 J=1,N2
      J3=J+MX
      IF(J3.GT.N2) J3=N2
      CALL LOC3(J3,J,K2,N2,MX)
      J3=J-3
      IF(J3.LT.1) J3=1
      CALL LOC3(J3,J,K1,N2,MX)
  160 WRITE(6,629) J,J3,(YDY(JJ),JJ=K1,K2)

      WRITE(6,626)
```

```
      DO 165 J=1,NF
  165 WRITE(6,627) J,XM(3*J-2),XM(3*J-1),XM(3*J)
      PC=0.0
      PK=0.0
C
C     ESTIMATE THE COEFFICIENTS
C
      DO 200 I=1,N
      CALL COEF(I,N,NF,SQ,DYSQ,YDY,XM,PC,PK,A,B)
      WRITE(6,624) I
      DO 170 J=1,3
  170 WRITE(6,625) (A(J,L),L=1,3),B(J)

      CALL RLMTX(R,3,1,MARK,DET,-1)
      EM(I)=R(1,4)
      EK(I)=R(2,4)
      EC(I)=R(3,4)
      WRITE(6,631) DET
C
      IF(I.EQ.1) GO TO 175
      PK=EEK(I-1)
      PC=EEC(I-1)
  175 CALL COEF(I,N,NF,SQ,DYSQ,YDY,XM,PC,PK,A,B)
      B(1)=B(1)-CM(I)*A(1,1)
      R(1,1)=A(1,2)
      R(1,2) = A(1,3)
      R(1,3)=B(1)
      B(2)=B(2)-CM(I)*A(2,1)
      R(2,1)=A(2,2)
      R(2,2)=A(2,3)
      R(2,3)=B(2)
      WRITE(6,632)
      DO 180 J=1,2
  180 WRITE(6,633) (R(J,L),L=1,3)

      CALL RLMTX(R,2,1,MARK,DET,-1)
      WRITE(6,631) DET
      EEK(I)=R(1,3)
      EEC(I)=R(2,3)
      PK=EK(I)
      PC=EC(I)
  200 CONTINUE
C
      WRI.      )1) (ID(I),I=1,8)
      HK=FLOAT(K)*H
      TIE=H*FLOAT(NON)
      WRITE(6,603) H,TIS,HK,NON ,TIE
      WRITE(6,604)
      DO 330 I=1,N
  330 WRITE(6,605)  CM(I),EM(I),CK(I),EK(I),CC(I),EC(I)
      WRITE(6,634)
      DO 335 I=1,N
  335 WRITE(6,635) CK(I),EEK(I),CC(I),EEC(I)
      NORUN=NORUN-1
      IF(NORUN) 5,5,340
  340 IF(INIT) 41,15,455
```

```
500 FORMAT(I2)
501 FORMAT(8F10.1)
504 FORMAT(8F10.4)
506 FORMAT(6I10,F10.8)
601 FORMAT(16H1EXPERIMENT NO =,I10,5X,31HINPUT SPECTRUM CLASSIFICATIO
6011 =,I3,5X,4HNV =,I3,5X,4HNC =,I3//21H SAMPLING FREQUENCY =,I8,3HCP
6012,5X,19HDATE OF EXPERIMENT -,I2,1H/,I2,1H/,I2)
603 FORMAT(1H0,30HSAMPLING INT. FOR SIMULATION =,F8.5/
60311H0,16HTRANSIENT INT. =     ,F10.2/
60321H0,39HSAMPLING INT. FOR CALCULATING MOMENTS =,F8.5/
60331H0,42HTOTAL NO. OF SAMPLES USED FOR ESTIMATION =,I6/
603431H0TIME INTERVAL FOR ESTIMATION =,F10.2,3HSEC)
604 FORMAT(1H0,16X,4HM(I),34X,4HK(I),34X,4HC(I)/3(14X,4HTRUE,8X,
60419HESTIMATED,3X))
605 FORMAT(1H0,3(4X,2E17.8))
620 FORMAT(1H1,43H2ND MOMENTS Y(J)Y(L), L=J TO J+3 (OR 2N) OF,I3,
620112H MASS SYSTEM)
621 FORMAT(1H0,8X,2HY(,I2,1H),5E14.4)
622 FORMAT(1H0,13HFIRST MOMENTS//(5F20.8))
623 FORMAT(34H0SECOND MOMENTS OF 2ND DERIVATIVES//(5E20.8))
624 FORMAT(1H0,20HEQUATION FOR SOLVING,I3,27HTH MASS SYSTEM COEFFICIE
6241TS)
625 FORMAT(1H0,3X,E15.8,6H*M + (,E15.8,7H)*K + (,E15.8,5H)*C =,E15.8)
626 FORMAT(1H0,53HCROSS MOMENTS OF INPUT FORCE AND Y(2J-1),Y(2J),DY(2
6261))
627 FORMAT(1H0,3X,I2,8HTH FORCE,3X,3E14.4)
628 FORMAT(1H0,51H2ND MOMENTS OF DY(J)*Y(L),L=J-3,OR 1) TO J+3(OR 2N))
629 FORMAT(4H0DY(,I2,4H)*Y(,I2,1H),7E14.4)
631 FORMAT(14H0DETERMINNANT=,E20.8)
632 FORMAT(31H0     EQUATIONS WHEN CM IS KNOWN)
633 FORMAT(1H0,24X,E15.8,6H*K + (,E15.8,5H)*C =,E20.8)
634 FORMAT(26H0ESTIMATES WITH MASS GIVEN)
635 FORMAT(1H0,38X,2(4X,2E17.8))
400 STOP
    END
```

```
      SUBROUTINE RKFOR(DERF,M,H,TI,YI,FI,K,N,F,VAL,DVAL,Y,FOSP)
      DIMENSION Y(20),S1(20),S2(20),S3(20),S4(20),DY(20),YI(20),
     1VAL(1),F(1) ,A(20),DVAL(1)
      NK=0
      H2=0.5*H
      T=TI
      L=0
      DO 10 I=1,M
   10 Y(I)=YI(I)
      FOS=FI
      CALL DERF(T,Y,FOS,M,DY)
      DO 20 I=1,M
   20 S1(I)=H*DY(I)
      DO 80 LL=1,N
      DO 65 JJ=1,K
      NK=NK+1
      FOSP=F(NK)
      FOSA=(FOS+FOSP)/2.0
C
C     COMPUTE K SUB 0
C
   25 DO 30 I=1,M
   30 A(I)=Y(I)+S1(I)/2.
      TA=T+H2
C
C     COMPUTE K SUB 1
C
      CALL DERF(TA,A,FOSA,M,DY)
      DO 40 I=1,M
      S2(I)=H*DY(I)
   40 A(I)=Y(I)+S2(I)/2.
C
C     COMPUTE K SUB 2
C
      CALL DERF(TA,A,FOSA,M,DY)
      DO 50 I=1,M
      S3(I)=H*DY(I)
   50 A(I)=Y(I)+S3(I)
      TA=T+H
C
C     COMPUTE K SUB 3
      CALL DERF(TA,A,FOSP,M,DY)
      DO 60 I=1,M
   60 S4(I)=H*DY(I)
      T=T+H
      FOS=FOSP
C
C     COMPUTE NEW VALUES OF INTEGRALS
C
      DO 63 I=1,M
   63 Y(I)=Y(I)+(S1(I)+2.*S2(I)+2.*S3(I)+S4(I))/6.0
      CALL DERF(T,Y,FOS, M,DY)
      DO 65 I=1,M
   65 S1(I)=H*DY(I)
      DO 70 I=1,M
```

```
      L=L+1
      DVAL(L)=DY(I)
   70 VAL(L) =Y(I)
      F(LL)=FCSP
   80 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DERF(T,VAR,FS,M,DER)
C     ------------------------------------------------------------------
C     PURPOSE
C       AUXILIARY SUBROUTINE WHICH COMPUTES THE DERIVATIVES FOR THE
C         INTEGRATION SUBROUTINE    RKFOR
C         OF ONE DIMENSION,N DEGREE FREEDOM,SPRING,MASS,DAMPER SYSTEM,
C         EXCITED BY NF FORCES FROM MASS 1 TO MASS NF WHERE NF .LE.N
C     ------------------------------------------------------------------
      DIMENSION VAR(20),DER(20),CC(10),CK(10),CM(10)
      COMMON CC,CK,CM
      N=M/2
      DO 10 I=1,N
   10 DER(2*I-1)=VAR(2*I)
      IF(N-2) 50,15,15
   15 DER(2)=(CK(1)/CM(1))*(VAR(3)-VAR(1))+(CC(1)/CM(1))*(VAR(4)-VAR(2))
     1         +FS/CM(1)
      IF(N.EQ.2) GO TO 30
      N1=N-1
      DO 25 I=2,N1
      DER(2*I)   =(CK(I)/CM(I))*(VAR(2*I+1)-VAR(2*I-1))
     1            +(CC(I)/CM(I))*(VAR(2*I+2)-VAR(2*I))
     1            -(CK(I-1)/CM(I))*(VAR(2*I-1)-VAR(2*I-3))
     1            -(CC(I-1)/CM(I))*(VAR(2*I)   -VAR(2*I-2))
   25 CONTINUE
   30 DER(2*N) =-(CK(N)/CM(N))*VAR(2*N-1)-(CC(N)/CM(N))*VAR(2*N)
     1            -(CK(N-1)/CM(N))*(VAR(2*N-1)-VAR(2*N-3))
     1            -(CC(N-1)/CM(N))*(VAR(2*N)   -VAR(2*N-2))
      GO TO 100
   50 DER(2)=-(CK(1)/CM(1))*VAR(1)-(CC(1)/CM(1))*VAR(2)+FS/CM(1)
  100 RETURN
      END
```

Input Cards and Output Tabulations for a Sample Problem

The sample problem is to simulate a five-mass system whose parameters are

$$m_1 = m_2 = \ldots = m_5 = 0.66667$$

$$k_1 = k_3 = 9.3 \times 10^5, \; k_2 = k_4 = 8.3 \times 10^4, \; k_5 = 530$$

$$C_1 = C_3 = 0.9 \;, \quad C_2 = C_4 = 1.9 \;, \quad C_5 = 0.1$$

The system is to be driven at mass No. 1 by a random force that has been digitally recorded on a magnetic digital tape. 3500 samples are to be simulated and used for estimation of the moments and the system parameters.

a) Input Cards

```
Col. 1   6       11       16       21       26       31       36       41       46
. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
5
0.666666       930000.0       0.9
0.666666        83000.0       1.9
0.666666       930000.0       0.9
0.666666        83000.0       1.9
0.666666          530.0       0.1
                        3500              1               1               1
```

b) Sample Output

```
2ND MOMENTS Y(J)Y(L), L=J TO J+3 (OR 2N) OF  5 MASS SYSTEM

        Y( 1)     0.2773E-06    0.1293E-09    0.2763E-06    0.1461E-07

        Y( 2)     0.1439E-02   -0.1330E-07    0.1354E-02   -0.1020E-06

        Y( 3)     0.2753E-06    0.1239E-08    0.2545E-06    0.1086E-06

        Y( 4)     0.1276E-02   -0.1083E-06   -0.5076E-03   -0.1079E-06

        Y( 5)     0.2629E-06    0.1739E-08    0.2632E-06    0.8791E-08

        Y( 6)     0.6111E-03   -0.5421E-08    0.6327E-03   -0.5900E-07

        Y( 7)     0.2636E-06    0.1638E-08    0.2633E-06    0.6403E-07

        Y( 8)     0.6667E-03   -0.6528E-07    0.1728E-03

        Y( 9)     0.2821E-06    0.1804E-08

        Y(10)     0.2567E-02

FIRST MOMENTS

    -0.17794580E-03     0.11294794E-04     -0.17784372E-03     0.11732835E-04     -0.17678886E-03
     0.18132111E-04    -0.17669534E-03      0.17810012E-04    -0.17564707E-03      0.42849763E-05

SECOND MOMENTS OF     DERIVATIVES

     0.14387309E-02     0.13035915E 03      0.12760448E-02     0.11301515E 03      0.61112563E-03
     0.10925123E 03     0.66668891E-03      0.10890906E 03     0.25668876E-02      0.30021939E 03

2ND MOMENTS OF DV(J)*Y(L),L=J-3,OR 1) TO J+3(OR 2N)

DV( 1)*Y( 1)    0.1293E-09    0.1439E-02   -0.1330E-07    0.1354E-02

DV( 2)*Y( 1)   -0.1442E-02    0.1254E-02   -0.1358E-02   -0.1214E-02    0.5596E-03

DV( 3)*Y( 1)    0.1461E-07    0.1354E-02    0.1239E-08    0.1276E-02   -0.1083E-06   -0.5076E-03

DV( 4)*Y( 1)   -0.1354E-02    0.2338E-02   -0.1276E-02    0.2967E-04    0.5066E-03   -0.1880E-01    0.6184E-03

DV( 5)*Y( 2)   -0.5606E-03    0.1086E-06   -0.5076E-03    0.1739E-08    0.6111E-03   -0.5421E-08    0.6327E-03

DV( 6)*Y( 3)    0.5077E-03    0.1924E-01   -0.6112E-03    0.1562E-03   -0.6328E-03   -0.1147E-02   -0.2928E-05

DV( 7)*Y( 4)   -0.6193E-03    0.8791E-08    0.6327E-03    0.1638E-08    0.6667E-03   -0.6528E-07    0.1728E-03

DV( 8)*Y( 5)   -0.6327E-03    0.1494E-02   -0.6667E-03    0.1930E-03   -0.1728E-03   -0.1019E-01

DV( 9)*Y( 6)    0.2962E-05    0.6403E-07    0.1728E-03    0.1804E-08    0.2567E-02

DV(10)*Y( 7)   -0.1719E-03    0.9765E-02   -0.2567E-02    0.5371E-03

CROSS MOMENTS OF INPUT FORCE AND Y(2J-1),Y(2J),DY(2J)

    1TH FORCE       -0.2719E-04    0.1340E-01    0.8641E 01

EQUATION FOR SOLVING  1TH MASS SYSTEM COEFFICIENTS

    0.14421741E-02*M + (-0.10046115E-08)*K + ( 0.14478014E-07)*C = 0.27191465E-04

   -0.12544415E-02*M + (-0.13426846E-07)*K + (-0.84582993E-04)*C =-0.13399392E-01
```

```
   -0.13035915E 03*M + ( 0.84158862E-04)*K + (-0.24687601E-02)*C =-0.86406012E 01
DETERMINNANT=     -0.78698362E-12
    EQUATIONS WHEN CM IS KNOWN
                         -0.10046115E-08*K + ( 0.14478014E-07)*C =     -0.93425793E-03
                         -0.13426846E-07*K + (-0.84582993E-04)*C =     -0.12563098E-01
DETERMINNANT=      0.85167440E-13
EQUATION FOR SOLVING  2TH MASS SYSTEM COEFFICIENTS
    0.12758565E-02*M + (-0.20809391E-07)*K + ( 0.10735772E-06)*C =-0.87616081E-03
   -0.29672290E-04*M + (-0.10956997E-06)*K + (-0.17836130E-02)*C =-0.12502900E-01
   -0.11301515E 03*M + ( 0.17824998E-02)*K + (-0.18831640E-01)*C = 0.72549009E 02
DETERMINNANT=     -0.13702607E-09
    EQUATIONS WHEN CM IS KNOWN
                         -0.20809391E-07*K + ( 0.10735772E-06)*C =     -0.17269481E-02
                         -0.10956997E-06*K + (-0.17836130E-02)*C =     -0.12483154E-01
DETERMINNANT=      0.37127664E-10
EQUATION FOR SOLVING  3TH MASS SYSTEM COEFFICIENTS
    0.61122148E-03*M + ( 0.30598812E-09)*K + ( 0.70517970E-08)*C = 0.69176064E-03
   -0.15618455E-03*M + (-0.71601526E-08)*K + ( 0.21573403E-04)*C =-0.67372260E-02
   -0.10925123E 03*M + (-0.21584848E-04)*K + (-0.13034393E-02)*C =-0.92868278E 02
DETERMINNANT=     -0.43641966E-12
    EQUATIONS WHEN CM IS KNOWN
                          0.30598812E-09*K + ( 0.70517970E-08)*C =      0.28457380E-03
                         -0.71601526E-08*K + ( 0.21573403E-04)*C =     -0.66392884E-02
DETERMINNANT=      0.66516971E-14
EQUATION FOR SOLVING  4TH MASS SYSTEM COEFFICIENTS
    0.66668987E-03*M + (-0.30219737E-09)*K + ( 0.62389190E-07)*C = 0.41934446E-03
   -0.19300176E-03*M + (-0.66918778E-07)*K + (-0.49393757E-03)*C =-0.66128585E-02
   -0.10890906E 03*M + ( 0.49390024E-03)*K + (-0.10385732E-01)*C =-0.31620084E 02
DETERMINNANT=      0.14638974E-09
    EQUATIONS WHEN CM IS KNOWN
                         -0.30219737E-09*K + ( 0.62389190E-07)*C =     -0.24957123E-04
                         -0.66918778E-07*K + (-0.49393757E-03)*C =     -0.64924011E-02
DETERMINNANT=      0.15344165E-12
```

```
EQUATION FOR SOLVING  5TH MASS SYSTEM COEFFICIENTS
     0.25667022E-02*M + (-0.28213149E-06)*K + (-0.18038338E-08)*C = 0.15610611E-02
    -0.53710299E-03*M + (-0.18038338E-08)*K + (-0.25668876E-02)*C =-0.64580160E-03
    -0.30021939E 03*M + ( 0.25667022E-02)*K + (-0.53710299E-03)*C =-0.19871689E 03
DETERMINNANT=     -0.20050818E-06
     EQUATIONS WHEN CM IS KNOWN
                          -0.28213149E-06*K + (-0.18038338E-08)*C =     -0.14994060E-03
                          -0.18038338E-08*K + (-0.25668876E-02)*C =     -0.25087469E-03
DETERMINNANT=      0.72419984E-05
```

EXPERIMENT NO = 2 INPUT SPECTRUM CLASSIFICATION = 2 NV = 6 NO =167

SAMPLING FREQUENCY = 700CPS DATE OF EXPERIENT - 67/11/13

SAMPLING INT. FOR SIMULATION = 0.00143

TRANSIENT INT. = C.

SAMPLING INT. FOR CALCULATING MOMENTS = 0.00143

TOTAL NO. OF SAMPLES USED FOR ESTIMATION = 3500

TIME INTERVAL FOR ESTIMATION = 5.00SEC

M(I)		K(I)		C(I)	
TRUE	ESTIMATED	TRUE	ESTIMATED	TRUE	ESTIMATED
0.66666666E 00	C.66650777E 00	0.92999999E 06	0.92975486E 06	0.90000000E 00	0.94135029E 00
0.66666666E 00	0.66625895E 00	0.82999999E 05	0.82963368E 05	0.19000000E 01	0.19022267E 01
0.66666666E 00	0.66636050E 00	0.92999999E 06	0.92964128E 06	0.90000000E 00	0.10766544E 01
0.66666666E 00	0.66642746E 00	0.82999999E 05	0.82971213E 05	0.19000000E 01	0.18866852E 01
0.66666666E 00	C.66643504E 00	0.53000000E 03	0.52982040E 03	0.09999999E-00	0.11177024E-00

ESTIMATES WITH MASS GIVEN

K(I) TRUE	K(I) ESTIMATED	C(I) TRUE	C(I) ESTIMATED
0.92999999E 06	0.92998238E 06	0.90000000E 00	0.90287752E 00
0.82999999E 05	0.82998683E 05	0.19000000E 01	0.19000705E 01
0.92999999E 06	0.92999488E 06	0.90000000E 00	0.90931174E 00
0.82999999E 05	0.82978232E 05	0.19000000E 01	0.19022629E 01
0.53000000E 03	0.53145582E 03	0.09999999E-00	0.97361500E-01

4.4 Computer Program Abstract III

General Description

This Fortran program simulates the two-dimensional, six degrees of freedom system shown in Figure I b and whose equation of motion, after change of variable, is given as

$$
\left.
\begin{aligned}
\dot{y}_{2i-1} &= y_{2i} \\
\dot{y}_2 &= -\frac{2C_1}{m_1} y_2 - \frac{2k_1}{m_1} y_1 + \frac{2C_3}{m_1}(y_8-y_2) + \frac{2k_3}{m_1}(y_7-y_1) + \frac{f_1}{m_1} \\
\dot{y}_4 &= -\frac{2C_2}{m_1} y_4 - \frac{2k_2}{m_1} y_3 + \frac{2C_4}{m_1}(y_{10}-y_4) + \frac{2k_4}{m_1}(y_9-y_3) + \frac{f_2}{m_1} \\
\dot{y}_6 &= -\frac{2(C_1b^2+C_2a^2+C_3c^2+C_4e^2)}{I_1} y_6 - \frac{2(k_1b^2+k_2a^2+k_3c^2+k_4e^2)}{I_1} y_5 \\
&\quad + \frac{2(C_3ed + C_4e2)}{I_1} Y_{12} + \frac{2(K_3ed + K_4e2)}{I_1} Y_{11} + \frac{f_3}{I_1} \\
\dot{y}_8 &= -\frac{2C_3}{m_2}(y_8-y_2) - \frac{2k_3}{m_2}(y_7-y_1) + \frac{f_4}{m_2} \\
\dot{y}_{10} &= -\frac{2C_4}{m_2}(y_{10}-y_4) - \frac{2k_4}{m_2}(y_9-y_3) + \frac{f_5}{m_2} \\
\dot{y}_{12} &= -\frac{2(C_3d^2+C_4e^2)}{I_2} y_{12} - \frac{2(k_3d^2+k_4e^2)}{I_2} y_{11} \\
&\quad + \frac{2(C_3cd+C_4e^2)}{I_2} y_6 + \frac{2(k_3cd+k_4e^2)}{I_2} y_5 + \frac{f_6}{I_2}
\end{aligned}
\right\} \quad (4.4.1)
$$

From the simulated equally-spaced samples of the system dynamical outputs y_j, $\dot{y}_j$, j=1 to 12 and the system inputs f_i, i=1 to 6, the second moments or time averages of $\overline{y_j y_k}$, $\overline{\dot{y}_j y_k}$, $\overline{\dot{y}_j^{\,2}}$, $\overline{f_i y_{2i-1}}$, $\overline{f_i y_{2i}}$, and $\overline{f_i \dot{y}_{2i}}$ are computed and the system parameters m_i, I_i, k_{2i-1}, k_{2i}, C_{2i-1}, C_{2i} for i=1 and 2 are estimated.

b) Outputs from this program include:

1) Sample moments or time averages of

$\overline{y_j y_k}$; j=1 to 6N ; k=j, ..., (j+7) or 12

$\overline{\dot{y}_j y_k}$; j=1 to 6N ; k=(j-3) or 1, ..., (j+7) or 12

$\overline{\dot{y}_j^{\,2}}$; j=1 to 12

$\overline{f_i y_{2i-1}}$, $\overline{f_i y_{2i}}$ and $\overline{f_i \dot{y}_{2i}}$; i=1 to 6

2) Sets of simultaneous equations for solving each set of system parameters.

3) Tabulation of true parameters against the estimated parameters.

c) Required Supporting Subprograms

1) RKD
2) MULDEV
3) MULFOS
4) PAMSQT
5) XSQ
6) XFOT
7) LOC2
8) LOC3
9) RLMTX

Input Cards

a) Number of Mass Card

Col. 1-2 N - Number of mass for the two-dimensional system (N=2).

b) System Parameter Card(s)

A card is used to specify each set of system parameter (m_i, I_i, k_{2i-1}, k_{2i}, C_{2i-1}, C_{2i}). Therefore, N(=2) cards are needed.

Col. 1-10 CM(I) - Floating point constant for the ith mass coefficient

Col. 11-20 CI(I) - Floating point constant for the ith moment of inertia

Col. 21-30 CK(2I-1) - Floating point constant for the spring coefficient in one direction

Col. 31-40 CK(2I) - Floating point constant for the spring coefficient in the other direction

Col. 41-50 CC(2I-1) - Floating point constant for the damper coefficient in one direction

Col. 51-60 CC(2I) - Floating point constant for the damper coefficient in the other direction

c) System Dimension Card

Col. 1-10 A - Floating point constant for the distance between C. G. of Mass 1 and the spring-dashpot unit (k_2, C_2).

Col. 11-20 B - Floating point constant for the distance between C. G. of Mass 1 and the spring-dashpot unit (k_1, C_1).

Col. 21-30 C - Floating point constant for the distance between C. G. of Mass 1 and the spring-dashpot unit (k_3, C_3).

Col. 31-40 D - Floating point constant for the distance between C. G. of Mass 2 and the spring-dashpot unit (k_3, C_3).

Col. 41-50 E - Floating point constant for the distance between C. G. of Mass 2 and the spring-dashpot unit (k_4, C_4).

d) Number of Input Card

Col. 1-2 NF - Number of input forcing functions (NF=6)

e) Input Forcing Characteristics Card(s)

A card is used to specify each input forcing function characteristics. NF (=6) cards are needed and they should be arranged consecutively from I=1 to NF (6).

Col. 1-5 NVEC(I) - Index for application of forcing function f_i

If NVEC(I) $\leq$ 0, f_i is not applied

If NVEC(I) > 0, f_i is applied to mass i

Col. 6-10 FOST(I) - Amplitude of sinusoid input of the ith force, or the standard deviation of the white noise input to a bandpass filter whose output is the ith input force.

Col. 11-15 FW(I) - Frequency (cps) of the sinusoid function or center frequency of the bandpass filter for the ith input

Col. 16-20 FB(I) - Phase shift (in degrees) with respect to t = 0 of the sinusoid function or bandwidth of the bandpass filter for the ith input.

f) Simulation Specification Card

Col. 1-5	FREQ	- Sampling frequency for simulation of the system. The simulation interval is then 1/FREQ.
Col. 6-10	NI	- Number of initial samples to be simulated before samples are taken for estimation of the moments and the system parameters.
Col. 11-15	NO	- Number of samples (after initial NI samples) to be used for estimation.
Col. 16-20	K	- Only every kth sample (after the initial NI samples) of the equally-spaced (of interval 1/FREQ) samples are to be used for estimation.
Col. 21-25	NORUN	- Number of successive times the moments and the system parameters are to be estimated.
Col. 26-30	INIT	- Control index for how the samples are taken for each successive estimation. If INIT < 0, successive NO samples (after the initial NI samples) taken at every kth sample are to be used for estimation. If INIT = 0, system is reinitialized each time, NO samples (after initial NI samples) are used for estimation.

If INIT > 0, (after initial NI samples) NO samples are used cummulatively each time; that is, NO, then 2 x NO, ..., then NORUN x NO samples are successively used for estimation.

g) Repeat a to f for a different choice of system parameters as many times as desired. A blank card after f will cause a stop.

Description of Supporting Subprograms

a) MULDEV (T, Y, F, N, NF, DY)

This Fortran subroutine computes the derivatives as given by equation 4.4.1 for the integration subroutine RKD where DY(I), I=1, N are the derivatives of Y(I), I=1, N with respect to T, and are functions of T, Y, and F where F(I), I=1, NF are the input forcing values at T.

b) MULFOS (T, FOS, NF)

This Fortran subroutine generates the input forcing functions to the system for the integration subroutine RKD, where FOS(I), I=1, NF are the values of the input forcing values at time T. A subprogram for sinusoid input and another for random input are included. The user can use either one as desired, and only one is to be used at a time.

d) XFOT (A, NA, NVEC, B, C, NBC, NO, XM, ID)

This Fortran subroutine computes sums or averages of product between variables of one vector and variables of two other vectors, where

A - Sample matrix of NA by NO of one set of vectors

NA - Number of variables or length of vectors stored in A

NVEC - If NVEC(I) > 0, then the ith variable of vector stored in A is greater than 0.

B - Sample matrix of NBC by NO of the second set of vectors

C - Sample matrix of NBC by NO of the third set of vectors

NBC - Number of variables or length of vectors stored in B and C

NO - Number of vectors

XM - Sums or averages of products of variables A(J)*B(2J-1); A(J)*B(2J); and A(J)*C(2J); J=1, NA

ID - Control index

ID = 0, computing sums

ID ≠ 0, computing averages

e) The descriptions for subprograms RKD, PAMSQT, XSQ, LOC2, LOC3, and RLMTX are given in Section 4.2.

Listing of the Main Program and the Subprogram

The main program and only the subprograms MULDEV, MULFOS, and XFOT are listed since the other subprograms have already been listed in Section 4.2.

```
C     PROGRAM TO SIMULATE MULTIDIMENSIONAL 2 MASS,SPRING AND DAMPER
C     SYSTEM AND ESTIMATE THE PARAMETERS
C
      DIMENSION CC(4),CK(4),CM(2),CI(2),FS(6),Y(4800),DY(4800),YE(20),
     1F(2400),EC( 4),EK( 4),EM(2),YI(20),EI(2),NVEC(6),FW(6),FB(6),
     2FOST( 6),DD(6),    SQ(80), SUM(20),XM(30),FI(6),       R(3,4)
     3,DYSQ(20),YDY(140),SQT(80),SUMT(20),XMT(30),DYSQT(20),YDYT(140)
     4,DIS(3,6),ED(6), SUMS(20),XMS(30),SQS(80),DYSQS(20),YDYS(140)
      DOUBLE PRECISION SQS,SUMS,XMS,DYSQS,YDYS
      COMMON CM,CI,CC,CK,DD,NVEC,FOST,FW,FB,DIS
      EXTERNAL MULDEV,MULFCS
C
    5 READ(5,500) N
      IF(N) 400,400,8
    8 READ(5,501) (CM(I),CI(I),CK(2*I-1),CK(2*I),CC(2*I-1),CC(2*I),
     1 I=1,N)
      READ(5,501) A,B,C,D,E
      READ(5,503) NF,(NVEC(I),FOST(I),FW(I),FB(I),I=1,NF)

      READ(5,506) FREQ,NI,NO,K ,NCRUN ,INIT
C
C     INIT .GT. 0 ALL SAMPLES USED FOR SUCCESIVE ESTIMATION OF MOMENTS
C     INIT .EQ. 0  EACH SUCCESIVE RUN IS REINITIALIZED
C     INIT .LT. 0 NO SAMPLES USED FOR SUCCESIVE ESTIMATION OF MOMENTS
C
C     INITIALIZATION
C

      DD(1)=(B*B)*CC(1)+(A*A)*CC(2)+(C*C)*CC(3)+(E*E)*CC(4)
      DD(2)=(B*B)*CK(1)+(A*A)*CK(2)+(C*C)*CK(3)+(E*E)*CK(4)
      DD(3)=(C*D)*CC(3)+(E*E)*CC(4)
      DD(4)=(C*D)*CK(3)+(E*E)*CK(4)
      DD(5)=(D*D)*CC(3)+(E*E)*CC(4)
      DD(6)=(D*D)*CK(3)+(E*E)*CK(4)
      H=1.0/FREQ
      N2=6*N
      MX=7
      IF(N2.LE.6) MX=N2-1
      DO 10 I=1,NF
      IF(NVEC(I).LE.0) GO TO 10
      BH=6.283185*FB(I)*H
      WD=FW(I)*FW(I)-0.25*FB(I)*FB(I)
      IF(WD.LT.0.0) WD=-WD
      WD=6.283185*H*SQRT(WD)
      DIS(1,I)=2.0*COS(WD)*EXP(-0.5*BH)
      DIS(2,I)=-1.0*EXP(-1.0*BH)
      DIS(3,I)=BH
   10 CONTINUE
      NSQ=(MX+1)*N2-(MX*(MX+1))/2
      NXM=3*NF
      NXQ=(2*MX+1)*N2-(MX+1)*MX
      IF(NSQ.GT.80) GO TO 350
      IF (NXM.GT.30) GO TO 350
      IF(NXQ.GT. 140) GO TO 350
   15 DO 30 I=1,N2
```

```
30 YI(I)=0.0
   TI=0.0
   CALL MULFOS(TI,FI,NF)
   IF(NI.LE.0) GO TO 41
   CALL RKD(MULDEV,MULFOS,N2,NF,H,0.0,YI,FI,NI,1,F,Y,DY,YE)
   DO 40 I=1,N2
40 YI(I)=YE(I)
   TI=     FLOAT(NI)*H
41 DO 42 J=1,NXM
42 XMS(J)=0.0
   DO 43 J=1,NSC
43 SCS(J)=0.0
   DO 44 J=1,N2
   DYSCS(J)=0.0
44 SUMS(J)=0.0
   DO 45 J=1,NXC
45 YDYS(J)=0.0
   NON=0
   TIS=TI
455 IF((NO*NF).LE.2400) GO TO 49
   MNF=2400/NF
   IF((NO*N2).LE.4800) GO TO 46
   MN2= 4800/N2
   IF(MNF-MN2) 46,46,48
46 NIS=MNF
   GO TO 50
48 NIS=MN2
   GO TO 50
49 IF((NO*N2).LE. 4800) GO TO 140
   NIS= 4800/N2
   GO TO 50
140 NIS=NO
50 NOS=NO
51 CALL RKD(MULDEV,MULFOS,N2,NF,H,TI,YI,FI,K,NIS,F,Y,DY,YE)
   CALL PAMSQT(Y,N2,NIS,MX,SQT,SUMT,0)
   CALL XSC(DY,Y,N2,NIS,MX,DYSQT,YDYT,0)
   CALL XFOT(F,NF,NVEC,Y,DY,N2,NIS,XMT,0)
   NOS=NOS-NIS
   DO 52 J=1,NSC
52 SCS(J)=SCS(J)+SCT(J)
   DO 54 J=1,N2
   DYSCS(J)=DYSCS(J)+DYSQT(J)
54 SUMS(J)=SUMS(J)+SUMT(J)
   DO 55 J=1,NXC
55 YDYS(J)=YDYS(J)+YDYT(J)
   DO 56 J=1,NXM
56 XMS(J)=XMS(J)+XMT(J)
   DO 58 J=1,[illegible]
58 YI(J)=YE(J)
   NON=NON+NIS
   TN=NIS*K
60 TI=TI+TN*H
   IF(NOS) 100,100,65
65 IF(NOS-NIS) 70,51,51
70 NIS=NOS
   GO TO 51
```

```
C
C       WRITE THE MOMENTS
C
  100 DO 110 J=1,NSQ
  110 SQ(J)=SQS(J)/FLOAT(NON)
      DO 112 J=1,N2
      DYSQ(J)=DYSQS(J)/FLOAT(NON)
  112 SUM(J)=SUMS(J)/FLOAT(NON)
      DO 114 J=1,NXM
  114 XM(J)=XMS(J)/FLOAT(NON)
      DO 116 J=1,NXQ
  116 YDY(J)=YDYS(J)/FLOAT(NON)
      WRITE(6,606)  N
      WRITE(6,622)  (SUM(J),J=1,N2)
      WRITE(6,620)
      DO 155 J=1,N2
      CALL LOC2(J,J,K1,N2,MX)
      J3=J+MX
      IF(J3.GT.N2) J3=N2
      CALL LOC2(J3,J,K2,N2,MX    )
  155 WRITE(6,621) J,(SQ(JJ),JJ=K1,K2)

      WRITE(6,623) (DYSQ(J),J=1,N2)
      WRITE(6,628)
      DO 160 J=1,N2
      J3=J+MX
      IF(J3.GT.N2) J3=N2
      CALL LOC3(J3,J,K2,N2,MX)
      J3=J-MX
      IF(J3.LT.1) J3=1
      CALL LOC3(J3,J,K1,N2,MX)
  160 WRITE(6,625) J,J3,(YDY(JJ),JJ=K1,K2)

      WRITE(6,626)
      DO 165 J=1,NF
  165 WRITE(6,627) NVEC(J), XM(3*J-2),XM(3*J-1),XM(3*J)
C
C       ESTIMATE THE COEFFICIENTS
C
      IF(NVEC(4)) 173,173,175
  173 IF(NVEC(5)) 175,175,185
C
C      FOR F(4).GT. 0 OR BOTH F(4) AND F(5) .LE.0
C
  175 CALL LOC2(8,7,K1,N2,MX)
      CALL LOC2(7,2,K2,N2,MX)
      R(1,1) =SQ(K1)-SQ(K2)
      CALL LOC2(8,1,K2,N2,MX)
      R(2,2) =SQ(K1)-SQ(K2)
      CALL LOC2(7,7,K1,N2,MX)
      CALL LOC2(7,1,K2,N2,MX)
      R(1,2) =SQ(K1)-SQ(K2)
      CALL LOC2(8,8,K1,N2,MX)
      CALL LOC2(8,2,K2,N2,MX)
      R(2,1) =SQ(K1)-SQ(K2)
      CALL LOC3(7,8,K1,N2,MX)
```

```
      CALL LCC3(8,8,K2,N2,MX)
      IF(NVEC(4).LE.0) GO TO 180
C
C     FOR F(4) .GT. 0, ESTIMATE C(3),K(3) AND M(2) FIRST
C
      R(1,3)=0.5*YDY(K1)
      R(2,3)=0.5*YDY(K2)
      CALL LCC3(1,8,K3,N2,MX)
      R(3,2)=YDY(K1)-YDY(K3)
      CALL LCC3(2,8,K3,N2,MX)
      R(3,1)=YDY(K2)-YDY(K3)
      R(3,3)=0.5*DYSQ(8)
      R(1,4)=0.5*XM(10)
      R(2,4)=0.5*XM(11)
      R(3,4)=0.5*XM(12)
      J=3
      I=2
      WRITE(6,624) J,J,I
      WRITE(6,625) ((R(L,J),J=1,4),L=1,3)

      CALL RLMTX(R,3,1,MARK,DET,-1)
      WRITE(6,631) DET
      EC(3)=R(1,4)
      EK(3)=R(2,4)
      EM(2)=R(3,4)
      GO TO 185
C
C     F(4) AND F(5) ARE BOTH 0, ESTIMATE ONLY C(3) AND K(3) WITH M(2)
C     GIVEN
C
  180 R(1,3)=-0.5*CM(2)*YDY(K1)
      R(2,3)=-0.5*CM(2)*YDY(K2)
      J=3
      WRITE(6,610) J,J
      WRITE(6,633) ((R(L,J),J=1,3),L=1,2)

      CALL RLMTX(R,2,1,MARK,DET,-1)
      WRITE(6,631) DET
      EC(3)=R(1,3)
      EK(3)=R(2,3)
C
  185 CALL LCC2(10,9,K1,N2,MX)
      CALL LCC2( 9,4,K2,N2,MX)
      R(1,1)=SQ(K1)-SQ(K2)
      CALL LCC2(10,3,K2,N2,MX)
      R(2,2)=SQ(K1)-SQ(K2)
      CALL LCC2( 9,9,K1,N2,MX)
      CALL LCC2( 9,3,K2,N2,MX)
      R(1,2)=SQ(K1)-SQ(K2)
      CALL LCC2(10,10,K1,N2,MX)
      CALL LCC2(10, 4,K2,N2,MX)
      R(2,1)=SQ(K1)-SQ(K2)
      CALL LCC3(9,10,K1,N2,MX)
      CALL LCC3(10,10,K2,N2,MX)
      IF(NVEC(4).LE.0) GO TO 190
C
```

```
C      F(4) .GT. 0 ESTIMATE C(4) AND K(4)
C
       R(1,3)=0.5*(XM(13)-YDY(K1)*EM(2))
       R(2,3)=0.5*(XM(14)-YDY(K2)*EM(2))
       J=4
       WRITE(6,610) J,J
       WRITE(6,633) ((R(L,J),J=1,3),L=1,2)

       CALL RLMTX(R,2,1,MARK,DET,-1)
       WRITE(6,631) DET
       EC(4)=R(1,3)
       EK(4)=R(2,3)
       GO TO 250
  190  IF(NVEC(5).GT.0 ) GO TO 210
C
C      BOTH F(4) AND F(5) ARE 0, ESTIMATE C(4) AND K(4) WITH M(2) GIVEN
C
       R(1,3)=-0.5*CM(2)*YDY(K1)
       R(2,3)=-0.5*CM(2)*YDY(K2)
       J=4
       WRITE(6,610) J,J
       WRITE(6,633) ((R(L,J),J=1,3),L=1,2)

       CALL RLMTX(R,2,1,MARK,DET,-1)
       WRITE(6,631) DET
       EC(4)=R(1,3)
       EK(4)=R(2,3)
       GO TO 250
C
C       F(5) .GT. 0 AND F(4) .LE. 0, ESTIMATE C(4) ,K(4) AND M(2) FIRST
C
  210  R(1,3) = 0.5*YDY(K1)
       R(2,3) = 0.5*YDY(K2)
       CALL LCC3(4,10,K3,N2,MX)
       R(3,1)=YDY(K2)-YDY(K3)
       CALL LCC3(3,10,K3,N2,MX)
       R(3,2)=YDY(K1)-YDY(K3)
       R(3,3)=0.5*DYSC(10)
       R(1,4)=0.5*XM(13)
       R(2,4)=0.5*XM(14)
       R(3,4)=0.5*XM(15)
       I=2
       J=4
       WRITE(6,624) J,J,I
       WRITE(6,625) ((R(L,J),J=1,4),L=1,3)

       CALL RLMTX(R,3,1,MARK,DET,-1)
       WRITE(6,631) DET
       EC(4)=R(1,4)
       EK(4)=R(2,4)
       EM(2)=R(3,4)
C
C       ESTIMATE C(3) AND K(3)
C
       CALL LCC2(8,7,K1,N2,MX)
       CALL LCC2(7,2,K2,N2,MX)
```

```
      R(1,1) =SQ(K1)-SQ(K2)
      CALL LCC2(8,1,K2,N2,MX)
      R(2,2) =SQ(K1)-SQ(K2)
      CALL LCC2(7,7,K1,N2,MX)
      CALL LCC2(7,1,K2,N2,MX)
      R(1,2) =SQ(K1)-SQ(K2)
      CALL LCC2(8,8,K1,N2,MX)
      CALL LCC2(8,2,K2,N2,MX)
      R(2,1) =SQ(K1)-SQ(K2)
      CALL LCC3(7,8,K1,N2,MX)
      CALL LCC3(8,8,K2,N2,MX)
      R(1,3)=-0.5*EM(2)*YDY(K1)
      R(2,3)=-0.5*EM(2)*YDY(K2)
      J=3
      WRITE(6,610) J,J
      WRITE(6,633) ((R(L,J),J=1,3),L=1,2)

      CALL RLMTX(R,2,1,MARK,DET,-1)
      WRITE(6,631) DET
      EC(3)=R(1,3)
      EK(3)=R(2,3)
C
C     ESTIMATE I(2)
C
  250 ED(3)=(C*D)*EC(3)+(E*E)*EC(4)
      ED(4)=(C*D)*EK(3)+(E*E)*EK(4)
      ED(5)=(D*D)*EC(3)+(E*E)*EC(4)
      ED(6)=(D*D)*EK(3)+(E*E)*EK(4)
      CALL LCC2(12,11,K1,N2,MX)
      CALL LCC2(11, 6,K2,N2,MX)
      CALL LCC2(11,11,K3,N2,MX)
      CALL LCC2(11, 5,K4,N2,MX)
      CALL LCC3(11,12,K5,N2,MX)
      EI(2)=(2.0*(ED(3)*SQ(K2)+ED(4)*SQ(K4)-ED(5)*SQ(K1)-ED(6)*SQ(K3))+
     1 XM(16))/YDY(K5)
C
C     ESTIMATE C(1),K(1) AND M(1)
C
      CALL LCC2(2,1,K1,N2,MX)
      R(1,1)=SQ(K1)
      R(3,2)=SQ(K1)
      CALL LCC2(1,1,K1,N2,MX)
      R(1,2)=SQ(K1)
      CALL LCC2(2,2,K1,N2,MX)
      R(2,1)=SQ(K1)
      CALL LCC3(1,2,K1,N2,MX)
      R(1,3)=0.5*YDY(K1)
      R(3,2)=YDY(K1)
      CALL LCC3(2,2,K1,N2,MX)
      R(2,3)=0.5*YDY(K1)
      R(3,1)=YDY(K1)
      R(3,3)=0.5*DYSQ(2)
      CALL LCC2(8,1,K1,N2,MX)
      CALL LCC2(2,1,K2,N2,MX)
      CALL LCC2(7,1,K3,N2,MX)
      CALL LCC2(1,1,K4,N2,MX)
```

```
     ,4)=EC(3)*(SQ(K1)-SQ(K2))+EK(3)*(SQ(K3)-SQ(K4))+0.5*XM(1)
      CALL LCC2(8,2,K1,N2,MX)
      CALL LCC2(2,2,K4,N2,MX)
      CALL LCC2(7,2,K3,N2,MX)
      R(2,4)=EC(3)*(SQ(K1)-SQ(K4))+EK(3)*(SQ(K3)-SQ(K2))+0.5*XM(2)
      CALL LCC3(8,2,K1,N2,MX)
      CALL LCC3(2,2,K2,N2,MX)
      CALL LCC3(7,2,K3,N2,MX)
      CALL LCC3(1,2,K4,N2,MX)
      R(3,4)=EC(3)*(YDY(K1)-YDY(K2))+EK(3)*(YDY(K3)-YDY(K4))+0.5*XM(3)
      I=1
      WRITE(6,624) I,I,I
      WRITE(6,625) ((R(L,J),J=1,4),L=1,3)

      CALL RLMTX(R,3,1,MARK,DET,-1)
      WRITE(6,631) DET
      EC(1)=R(1,4)
      EK(1)=R(2,4)
      EM(1)=R(3,4)
C
C     ESTIMATE C(2),K(2)
C
      CALL LCC2(4,3,K1,N2,MX)
      R(1,1)=SQ(K1)
      R(2,2)=SQ(K1)
      CALL LCC2(3,3,K1,N2,MX)
      R(1,2)=SQ(K1)
      CALL LCC2(4,4,K1,N2,MX)
      R(2,1)=SQ(K1)
      CALL LCC3(3,4,K5,N2,MX)
      CALL LCC2(10,3,K1,N2,MX)
      CALL LCC2( 4,3,K2,N2,MX)
      CALL LCC2( 9,3,K3,N2,MX)
      CALL LCC2( 3,3,K4,N2,MX)
      R(1,3)=EC(4)*(SQ(K1)-SQ(K2))+EK(4)*(SQ(K3)-SQ(K4))+0.5*XM(4)
     1-0.5*EM(1)*YDY(K5)
      CALL LCC2(10,4,K1,N2,MX)
      CALL LCC2( 4,4,K4,N2,MX)
      CALL LCC2( 9,4,K3,N2,MX)
      CALL LCC3( 4,4,K5,N2,MX)
      R(2,3)=EC(4)*(SQ(K1)-SQ(K4))+EK(4)*(SQ(K3)-SQ(K2))+0.5*XM(4)
     1-0.5*EM(1)*YDY(K5)
      J=2
      WRITE(6,610) J,J
      WRITE(6,633) ((R(L,J),J=1,3),L=1,2)

      CALL RLMTX(R,2,1,MARK,DET,-1)
      WRITE(6,631) DET
      EC(2)=R(1,3)
      EK(2)=R(2,3)
C
C     ESTIMATE I(2)
C
      ED(1)=(B*B)*EC(1)+(A*A)*EC(2)+(C*C)*EC(3)+(E*E)*EC(4)
      ED(2)=(B*B)*EK(1)+(A*A)*EK(2)+(C*C)*EK(3)+(E*E)*EK(4)
      CALL LCC2( 6,5,K1,N2,MX)
```

```
      CALL LCC2(12,5,K2,N2,MX)
      CALL LCC2( 5,5,K3,N2,MX)
      CALL LCC2(11,5,K4,N2,MX)
      CALL LCC3( 5,6,K5,N2,MX)
      EI(1)=(2.0*(EC(3)*SQ(K2)+EC(4)*SQ(K4)-EC(1)*SQ(K1)-EC(2)*SQ(K3))
     1 +XM(7))/YCY(K5)
      WRITE(6,600)
      WRITE(6,601) NF,(I,FOST(I),FW(I),FB(I),I=1,NF)

      HK=FLCAT(K)*H
      TIE=H*FLCAT(NON)
      WRITE(6,603) H,TIS,HK,NON,TIE
      WRITE(6,604)
      WRITE(6,635)  (I,CC(I),EC(I),CK(I),EK(I),I=1,4)
      WRITE(6,637)   (I,CM(I),EM(I),CI(I),EI(I),I=1,2)
      NCRUN=NCRUN-1
      IF(NCRUN) 5,5,340
  340 IF(INIT) 41,15,455
  350 WRITE(6,636) NSC,NXM,NXQ
  500 FORMAT(I2)
  501 FORMAT(6F10.1)
  503 FORMAT(I2/(I5,3F5.1))
  506 FORMAT(F5.3,5I5)
  600 FORMAT(53H1ESTIMATION OF THE PARAMETERS OF THE SIMULATED SYSTEM)
  601 FORMAT(1H0, 18HNO OF EXCITATION =,I3/(17H0FOR THE FORCE NO,I2,
  60114X,11HAMPLITUDE =,F10.2,4X,18HCENTER FREQUENCY =,F10.2,4X,
  6012 27HPHASE SHIFT OR BAND WIDTH =,F10.2) )
  603 FORMAT(1H0,30HSAMPLING INT. FOR SIMULATION =,F8.5/
  60311H0,16HTRANSIENT INT. =     ,F10.2/
  60321H0,39HSAMPLING INT. FOR CALCULATING MOMENTS =,F8.5/
  60331H0,42HTOTAL NO. OF SAMPLES USED FOR ESTIMATION =,I6/
  603431H0TIME INTERVAL FOR ESTIMATION =,F10.2,3HSEC)
  604 FORMAT(  1H0,2(20X,4HTRUE,18X,9HESTIMATED) )
  606 FORMAT(29H1ESTIMATION OF THE MOMENTS OF,I2,12H MASS SYSTEM)
  610 FORMAT(24H0EQUATION FOR SOLVING C(,I1,4H),K(,I1,1H) )
  620 FORMAT(1H0,40H2ND MOMENTS Y(J)Y(L), L=J TO J+7 (OR 2N))
  621 FORMAT(1H0,2X,2HY(,I2,1H),8E14.4)
  622 FORMAT(29H0FIRST MOMENTS OF Y(J),J=1,2N//(5E20.8))
  623 FORMAT(43H0SECOND MOMENTS OF DERIVATIVES DY(J),J=1,2N//(5E20.8))
  624 FORMAT(24H0EQUATION FOR SOLVING C(,I1,4H),K(,I1,8H) AND M(,I1,1H))
  625 FORMAT(1H0,3X,E15.8,6H*C + (,E15.8,7H)*K + (,E15.8,5H)*M =,E15.8)
  626 FORMAT(1H0,53HCROSS MOMENTS OF INPUT FORCE AND Y(2J-1),Y(2J),DY(2
  6261))
  627 FORMAT(1H0,3X,I2,8HTH FORCE,3X,3E14.4)
  628 FORMAT(1H0,51H2ND MOMENTS OF DY(J)*Y(L),L=J-7,OR 1) TO J+7(OR 2N))
  629 FORMAT(4H0DY(,I2,4H)*Y(,I2,1H)/(12X,7E14.4))
  631 FORMAT(14H0DETERMINNANT=,E20.8)
  632 FORMAT(31H0      EQUATIONS WHEN CM IS KNOWN)
  633 FORMAT(1H0,24X,E15.8,6H*C + (,E15.8,5H)*K =,E20.8)
  635 FORMAT(1H0,31X,4HC(I),49X,4HK(I),/(3HCI=,I1,2(10X,2E20.8)))
  636 FORMAT(40H1AT LEAST ONE OF THE FOLLOWING NOS. NSQ=,I2,6H, NXM=,I2
  6361 ,6H, NXC=,I3,44H IS GREATER THAN THE CORRESPONDING 80,30,140 )
  637 FORMAT(1H0,31X,4HM(I),49X,4HI(I),/(3HCI=,I1,2(10X,2E20.8)))
  400 STOP
      END
```

```
      SUBROUTINE MLLDEV(T,Y,F,N,NF,DY)
      DIMENSION CM(2),CI(2),CK(4),CC(4),DD(6),Y(12),F(6),DY(12)
      COMMON CM,CI,CC,CK,DD
C
C     DD(1)=(B*B)*CC(1)+(A*A)*CC(2)+(C*C)*CC(3)+(E*E)*CC(4)
C     DD(2)=(B*B)*CK(1)+(A*A)*CK(2)+(C*C)*CK(3)+(E*E)*CK(4)
C     DD(3)=(C*D)*CC(3)+(E*E)*CC(4)
C     DD(4)=(C*D)*CK(3)+(E*E)*CK(4)
C     DD(5)=(D*D)*CC(3)+(E*E)*CC(4)
C     DD(6)=(D*D)*CK(3)+(E*E)*CK(4)
C
      N2=N/2
      DO 10 I=1,N2
   10 DY(2*I-1)=Y(2*I)
      DY(2) = -(2.0/CM(1)) * (CC(1)*Y(2) + CC(3)*(Y(2)-Y(8))
     1 + CK(1)*Y(1) + CK(3)*(Y(1)-Y(7))) + F(1)/CM(1)
      DY(4) = -(2.0/CM(1)) * (CC(2)*Y(4) + CC(4)*(Y(4)-Y(10))
     1 + CK(2)*Y(3) + CK(4)*(Y(3)-Y(9))) + F(2)/CM(1)
      DY(6) = -(2.0/CI(1)) * (DD(1)*Y(6) +DD(2)*Y(5) - DD(3)*Y(12)
     1 - DD(4)*Y(11)) + F(3)/CI(1)
      DY(8) = (2.0/CM(2)) * (CC(3)*(Y(2)-Y(8)) + CK(3)*(Y(1)-Y(7)))
     1 + F(4)/CM(2)
      DY(10) = (2.0/CM(2)) * (CC(4)*(Y(4)-Y(10)) + CK(4)*(Y(3)-Y(9)))
     1 + F(5)/CM(2)
      DY(12) = (2.0/CI(2)) * (DD(3)*Y(6)+DD(4)*Y(5) - DD(5)*Y(12)
     1 -DD(6)*Y(11)) + F(6)/CI(2)
      RETURN
      END
```

```
      SUBROUTINE MLLFCS(T,FOS,NF)
      DIMENSION CM(2),CI(2),CC(4),CK(4),DD(6),NVEC(6),FW(6),FB(6),
     1FOST(6),FOS(6)
      COMMON CM,CI,CC,CK,DD,NVEC,FOST,FW,FB
      DO 20 I=1,NF
      IF(NVEC(I).LE.0) GO TO 10
      THETA=6.283185*(T*FW(I)+FB(I)/360.)
      FOS(I)=FOST(I)*SIN (THETA)
      GO TO 20
   10 FOS(I)=0.0
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FULFOS(T,FOS,NF)
      DIMENSION CM(2),CI(3),CC(4),CK(4),DD(6),NVEC(6),FW(6),FB(6),
     1FCST(6),FOS(6),DIS(3,6),XF(2,6),XR(6)
      COMMON CM,CI,CC,CK,DD,NVEC,FCST,FW,FB,DIS
      DO 20 I=1,NF
      IF(NVEC(I).LE.0) GO TO 10
      TEMP=FCST(I)*GAURN(L)
      FOS(I)=DIS(1,I)*XF(1,I)+DIS(2,I)*XF(2,I)+DIS(3,I)*(TEMP-XR(I))
      XF(2,I)=XF(1,I)
      XF(1,I)=FOS(I)
      XR(I)=TEMP
      GO TO 20
   10 FOS(I)=0.0
   20 CONTINUE
      RETURN
      END


C     ----------------------------------------------------------------
C        PURPCSE
C          TC CCMPUTE CRCSS MCMENT CF A(J)*B(2*J-1),A(J)*B(2*J)
C          ANC A(J)*C(2*J)
C        A   -BASE CATA MATRIX,NA BY NC
C        NA  -NC.CF VARIABLES OF A
C        NVEC - IF NVEC(I) .GT. 0,THEN A(I) .GT. 0
C        B   -CRCSS VARIABLE DATA MATRIX
C        C -  CRCSS VARIABLE CATA MATRIX
C        NBC-NC.CF VARIABLES CF B ANC C, NB .GE. (2*NA)
C        NC  -NC.CF CBSERVATIONS
C        XM  -CRCSS MCMENTS,MATRIX CF 3*NA
C        IC  -ID NCT ZERC,COMPUTE THE MCMENTS
C            IC ZERC, CCMPUTE THE SUMS
C     ----------------------------------------------------------------
      SUBROUTINE XFOT(A,NA,NVEC,B,C,NBC,NC,XM,ID)
      CIMENSICN A(1),B(1),XM(3) ,C(1),NVEC(1)
      N3=3*NA
      CO 10 J=1,N3
   10 XM(J)=0.0
      IJA=0
      DO 30 I=1,NO
      CO 30 J=1,NA
      IJA=IJA+1
      IF(NVEC(J).LE.0) GO TO 30
      IJ2B=(I-1)*NBC+2*J
      IJ1B=IJ2B-1
      XM(3*J-2)=XM(3*J-2)+A(IJA)*B(IJ1B)
      XM(3*J-1)=XM(3*J-1)+A(IJA)*B(IJ2B)
      XM(3*J)=XM(3*J)+A(IJA)*C(IJ2B)
   30 CONTINUE
C
C     COMPUTE MOMENTS IF ID NOT ZERC
C
      IF(IC) 40,5C,4C
   40 CNC=NC
      CO 45 J=1,N3
   45 XM(J)=XM(J)/CNC
   50 RETURN
      END
```

Input Cards and Output Tabulations for a Sample Problem

The sample problem is to simulate a two-dimensional, six degrees of freedom system shown in Figure I b whose parameters are

$$m_1 = m_2 = 0.26 \quad , \quad I_1 = I_2 = 70$$

$$k_1 = k_3 = 10^5 \quad , \quad k_2 = k_4 = 4 \times 10^5$$

$$C_1 = C_3 = 10 \quad , \quad C_2 = C_4 = 20$$

$$a = 10 \ , \ b = 20, \ C = 15 \ , \ d = 6 \ , \ e = 12$$

The system is to be driven by three independent random forces f_4, f_5, and f_6 applied onto mass 2. Each random force is generated by passing a white noise of standard deviation 30 through a bandpass filter with center frequency 70 cps and bandwidth 20 cps. The sampling frequency is to be 2000 cps and 2000 samples are to be generated and used for estimation of the moments and system parameters.

a) Input Cards

```
Col. 1  6    11    16    21    26    31    36    41    46    51
    ..............................................................

    2

    0.26      70.0      100000.0    400000.0      10.0        20.0

    0.26      70.0      100000.0    400000.0      10.0        20.0

    10.0      20.0      15.0        6.0           12.0

    6

    -1

    -2

    -3

    4     30. 70.  20.

    5     30. 70.  20.

    6     30. 70.  20.

    2000.     2000  1      1      1
```

b) Sample Output

ESTIMATION OF THE MOMENT OF A MASS SYSTEM

FIRST MOMENTS OF V(J),J=1,2N

```
   -0.2141[illegible]E-07     [illegible]E-04      [illegible]           0.178[illegible]E-06   -0.11[illegible]583E-09
   -0.2562[illegible]E-06     [illegible]E-05      0.67[illegible]E-06   -0.1888[illegible]E-07  0.26[illegible]E-06
   [illegible]                [illegible]
```

2ND MOMENTS V(J)V(L), L=J TO J+7 (OR 6N)

```
V( 1)   [illegible]E-07  0.1153E-08     [illegible]E-11  -0.5592E-04     0.3329E-12     -0.1130E-08    0.6520E-07     -0.4460E-06
V( 2)   0.1155E-01       [illegible]E-04 0.1488E-06     [illegible]E-08  0.1408E-06     0.6695E-06     0.1864E-01     0.1534E-07
V( 3)   0.2531E-09       [illegible]E-05 -0.5662E-13    0.62[illegible]E-10 -0.3232E-10  0.2102E-07     0.4309E-09     -0.4949E-08
V( 4)   0.2351E-03       -0.4750E-10    0.3760E-07     -0.15[illegible]E-07 0.1796E-04  0.5116E-08     0.3792E-03     -0.1030E-09
V( 5)   [illegible]E-14  [illegible]E-12 [illegible]E-12 0.1772E-04    -0.9764E-13    -0.8300E-10    0.1645E-13     -0.2466E-12
V( 6)   0.54[illegible]E-08 -0.179[illegible]E-08 0.2037E-06 0.7673E-10   -0.6238E-07    0.3773E-12     0.1069E-07
V( 7)   [illegible]E-06  0.2555E-08     -0.4566E-10    -0.4711E-07     0.1369E-11     -0.4019E-08
V( 8)   0.1026E-01       0.4478E-07     0.3264E-04     0.1984E-08      0.4291E-06
V( 9)   0.7773E-09       0.4975E-05     -0.1988E-12    0.1678E-09
V(10)   [illegible]E-03  -0.1862E-05    -0.1164E-06
V(11)   [illegible]E-13  0.1300E-12
V(12)   0.2157E-07
```

SECOND MOMENTS OF DERIVATIVES DV(J),J=1,6N

```
   0.115[illegible]E-01    [illegible]E 04       0.235103[illegible]E-03   0.28149673E 03      0.56049322E-08
   0.44C742[illegible]E-02 [illegible]E-01       0.889[illegible]E 04      0.62431593E-03      0.70303627E 03
   0.2156[illegible]E-17   [illegible]E-01
```

2ND MOMENTS OF DV(J)*V(L),L=J-7,OR 1) TO J+7(OR 6N)

```
DV( 1)*V( 1)
             0.1153E-08    0.1155E-01    0.6376E-08    0.1488E-06    0.1119E-08    0.1408E-06    0.4495E-06
             0.1864E-01

DV( 2)*V( 1)
             -0.1155E-01   0.6626E-07    -0.1433E-04   -0.7548E-02   -0.1488E-06   0.3561E-03    -0.1863E-01
             0.1468E-03    -0.2603E-04

DV( 3)*V( 1)
             -0.5592E-08   0.1488E-04    0.2330E-09    0.2351E-03    -0.4750E-10   -0.3760E-07   -0.1982E-07
             0.1796E-04    0.5116E-08    0.3792E-03

DV( 4)*V( 1)
             -0.1481E-06   0.7950E-02    -0.2351E-03   0.3077E-03    0.3744E-07    -0.2590E-06   -0.1786E-04
             0.7511E-02    -0.3751E-03   0.8747E-02    0.6770E-07

DV( 5)*V( 1)
             -0.1130E-03   0.1458E-08    0.4292E-10    -0.3760E-07   0.3318E-13    0.5605E-08    -0.1790E-08
             0.2037E-05    0.7679E-10    -0.6238E-07   0.3773E-12    0.1069E-07

DV( 6)*V( 1)
             -0.1410E-06   -0.3556E-03   0.3755E-07    0.2597E-04    -0.5454E-08   0.2288E-08    -0.2938E-06
             -0.5441E-03   0.6232E-07    0.4411E-04    -0.1069E-07   0.3049E-06
```

```
DY( 7)*Y( 1)
     [illegible]      0.1[illegible]   0.2102E-07   0.1706E-04   0.1772E-08   0.2037E-06   0.2559E-08
     [illegible]      [illegible]      0.3264E-0[illegible]   0.1984E-08   0.6291E-06
DY( 8)*Y( 1)
     -0.1[illegible]  -0.14[illegible] -0.1722E-04  -0.8018E-02  -0.2174E-06   0.5450E-03  -0.7726E-01
     [illegible]      -0.3174E-04      0.135[illegible]  -0.46[illegible]   0.1223E-02
DY( 9)*Y( 2)
     [illegible]      -0.4[illegible]  0.3792E-0[illegible]   [illegible]E-10  -0.6218E-07  -0.4311E-07   0.3264E-04
     [illegible]      0.62[illegible]  -0.10[illegible]   -0.1164E-06
DY(10)*Y( 3)
     [illegible]      -0.8[illegible]  0.6129E-07   -0.4346E-04  -0.3185E-04   0.1380E-01  -0.6232E-03
     0.1774E-[illegible]   0.1135E-0[illegible]   -0.1[illegible]
DY(11)*Y( 4)
     [illegible]      0.24[illegible]E-12   0.1265E-07  -0.6019E-08   0.4251E-06   0.1679E-09  -0.1164E-06
     [illegible]      0.21[illegible]
DY(12)*Y( 5)
     -0.1[illegible]  -0.3[illegible]  -0.4236E-0[illegible]  -0.1206E-02   0.1162E-06   0.1011E-03  -0.2157E-07
     [illegible]

CROSS MOMENTS OF [illegible] FORCE AND Y(2J-1),Y(2J),DY(2J)
  -1TH FORCE        .           0.           0.
  -2TH FORCE        .           .            0.
  -3TH FORCE        .           0.           0.
   4TH FORCE   [illegible]E-03   0.[illegible]   -0.20[illegible] 02
   5TH FORCE   [illegible]E-04   0.1[illegible]E-01   -0.1237E 02
   6TH FORCE   [illegible]E-04   0.1[illegible]E-03   -0.6788E-01
EQUATION FOR SOLVING C(2),K(2) AND M(2)
  -0.446[illegible]*C + ( 0.610[illegible]-07)*K + (-0.15127840E-01)*M = 0.1684[illegible]E-03
  0.11[illegible]*C + ( [illegible]E-02)*K + ( 0.4529[illegible]-02)*M = 0.1622[illegible]E-00
  0.15[illegible]*C + (-0.1162224[illegible])*K + ( [illegible]446[illegible] 04)*M =-0.1019[illegible]E 02
DETERMINANT        = [illegible]17[illegible]E-07
EQUATION FOR SOLVING C(4),K(4)
        -[illegible]*C + ( 0.707[illegible]E-[illegible])*K =       0.12261789E-03
        [illegible]0.2451126[illegible]*C + ( [illegible]E-[illegible])*K =       [illegible]994E-02
DETERMINANT        = [illegible]E-1[illegible]
EQUATION FOR SOLVING C(1),K(1) AND M(1)
  0.11[illegible]*C + ( [illegible]E-[illegible])*K + (-0.57738022E-[illegible]2)*M = 0.2506543[illegible]E-02
  [illegible]*C + ( [illegible]0.119[illegible]E-[illegible])*K + ( 0.3312[illegible]E-03)*M = 0.115614[illegible]E-00
  [illegible]*C + ( [illegible]0.11[illegible]E-[illegible]1)*K + ( [illegible]0.1721[illegible] 04)*M =-0.7071[illegible]E 03
DETERMINANT=        [illegible]
```

```
EQUATION FOR SOLVING C(2),K(2)

     ( 0.23205565[illegible])*C + ( 0.25206311E-0[illegible])*K =    0.10122985E-03
     ( [illegible])*C + ( 0.2320556[illegible]E-0[illegible])*K =    0.4795285[illegible]1E-02

DETERMINANT=    -0.[illegible]71E-1[illegible]
```

ESTIMATION OF THE PARAMETERS OF THE SIMULATED SYSTEM

NO OF EXCITATION = 6

FOR THE FORCE NO 1	AMPLITUDE =	-0.	CENTER FREQUENCY =	-0.	PHASE SHIFT OR BAND WIDTH =	-0.
FOR THE FORCE NO 2	AMPLITUDE =	-0.	CENTER FREQUENCY =	-0.	PHASE SHIFT OR BAND WIDTH =	-0.
FOR THE FORCE NO 3	AMPLITUDE =	-0.	CENTER FREQUENCY =	-0.	PHASE SHIFT OR BAND WIDTH =	-0.
FOR THE FORCE NO 4	AMPLITUDE =	3[illegible].00	CENTER FREQUENCY =	70.00	PHASE SHIFT OR BAND WIDTH =	20.00
FOR THE FORCE NO 5	AMPLITUDE =	3[illegible].00	CENTER FREQUENCY =	70.00	PHASE SHIFT OR BAND WIDTH =	20.00
FOR THE FORCE NO 6	AMPLITUDE =	3[illegible].00	CENTER FREQUENCY =	70.00	PHASE SHIFT OR BAND WIDTH =	20.00

SAMPLING INT. FOR SIMULATION = 0.00050

TRANSIENT INT. = 0.

SAMPLING INT. FOR CALCULATING MOMENTS = 0.00050

TOTAL NO. OF SAMPLES USED FOR ESTIMATION = 2000

TIME INTERVAL FOR ESTIMATION = 1.00SEC

	TRUE	ESTIMATED	TRUE	ESTIMATED
	C(I)		K(I)	
I=1	0.[illegible]9999999E 02	0.99999256E 01	0.[illegible]9999999E 06	0.99999972E 05
I=2	0.2[illegible]E 02	0.20000004E 02	0.40[illegible]E 06	0.39999977E 06
I=3	0.[illegible]9999[illegible]9E 02	0.59999513E 01	0.09999999E 06	0.10000011E 06
I=4	0.2[illegible]E 02	0.19999994E 02	0.40000000E 06	0.400000[illegible] 06
	M(I)		I(I)	
I=1	0.26[illegible]E-00	0.25999923E-00	0.70[illegible]E 02	0.69999851E 02
I=2	0.26[illegible]E-00	0.26000026E-00	0.70000000E 02	0.70000035E 02

CHAPTER V

SUMMARY AND CONCLUSIONS

We have developed analytically and presented numerical examples of a parameter identification approach when the dynamical structure of the system is known. We have also constructed the program for system simulation and identification of chainlike systems in Chapter IV. Equivalently, we have studied systems of known differential equations with unknown parameters. We have seen in Chapter II that these parameters can be theoretically identified by random as well as sinusoidal excitations. The identification procedure makes use of displacement and velocity as well as acceleration information. The method is very straightforward, it does not make use of subtle theoretical points. Even more, on the basis of the simulation experiments of Chapter III, we see that the technique works and works well. The technique does not appear to be sensitive to the type of excitation used. Random excitations of various spectral properties as well as sinusoidal excitations with frequency in a wide range all yield very good parameter estimates.

The procedure that is to be applied when identifying a real system that can be described by linear differential equations is reasonably simple. If the identification is to be done digitally, then the displacement, velocity, and acceleration data as well as the excitation data should be roughly digitized at a rate 5 - 10 times greater than the highest frequency present. This is rough, but it is sufficient to obtain some estimate of the highest frequency through oscilloscope observations. A record of sufficient length to cover 5 - 10 cycles at the lowest frequency present will then suffice for identification purposes. If steady-state conditions have been achieved for a random excitation, then various moments will be zero making the estimation equations simpler. However, if all moments are kept regarding them merely as time averages, one does not need to reflect upon whether the system is in steady state. In the event the system is being excited by sinusoidal oscillations and is in the steady state, then not all parameters can be estimated. Identification by sinusoidal excitations is best achieved during the transient stage of the dynamics of the system.

One very significant point that must be re-iterated is that this is a direct method, it is not a search technique. Hence, the relative magnitudes of the parameters do not present the problem common to all search techniques. The problem involved is the size of the step that must be taken for searching parameters. When parameters are large, a small step will get one to the correct value; when a parameter is small, one can easily exceed the parameter value in one step.

This is evident in many of the examples we presented in Chapter III where the parameter values are many orders of magnitude apart. Yet, as we have seen the parameter estimates are amazingly accurate., by the proposed technique.

Although we have not developed the details during the period of the present contract, it is clear that the same approach will be applicable to non-linear systems whose dynamics are describable by differential equations with polynomial non-linearities and unknown constant coefficients. It could be of significant interest and applicability to study the present parameter identification approach for non-linear systems.

The major point to be settled relative to the present study is the practicability of the present approach with acceleration and excitation data only. If this can be affirmatively resolved then parameter estimation by the present technique should become a useful and commonly used procedure.

We shall close this report by stating emphatically that our motivation was to bring forth what appears to be a useful idea, not a deep idea nor an idea for which one can only feel a desire to study theoretically. Our analysis of the idea as well as the great number of simulation experiments reflects the attitude with which we have performed this study. Thus, we did not look at this idea in all generality or from the purely fundamental theoretical point of view. We were interested most of all in how well the idea works.

We sincerely hope that the present study can be looked upon as having practical significance as it relates especially to present-day engineering problems.

APPENDIX

THE CONCEPT OF THE GAUSSIAN WHITE NOISE PROCESS

It has been stated throughout the development and study of the present parameter estimation technique that the Gaussian white noise is an unsuitable excitation for a system that is undergoing investigation by the proposed identification technique. This point has been brought out in Chapter II of this final report.

The detailed reasons behind this statement shall be developed and discussed in this appendix.

The Wiener process is a Gaussian process with stationary independent increments. It satisfies,

$$\begin{aligned}
&\text{Prob } \{B(o) = 0\} = 1 && \text{(a)} \\
&E\{B(t)\} = 0 && \text{(b)} \\
&E\{(B(t) - B(s))^2\} = \sigma^2 |t-s| && \text{(c)} \\
&E\{(B(t_4)-B(t_3))(B(t_2)-B(t))\} = 0 && \text{(d)}
\end{aligned} \qquad \text{(I.1)}$$

for any $t_4 > t_3 \geq t_2 > t_1$

Condition I.1 (c) yields the statistical stationarity of the increment $[B(t)-B(s)]$, and condition I.1 (d) yields the independence of the increments. The joint density function for the B-process at times $0 < t_1 < \dots < t_n$ is given by the Gaussian density function.

$$f(x_1,t_1;\ x_2,t_2;\ \dots\ x_n,t_n) = \frac{1}{(2\pi)^{n/2}\ \sigma^n\ [t_1(t_2-t_1)\dots(t_n-t_{n-1})]^{1/2}}$$

$$\exp\left[-\frac{1}{2\sigma^2}\left(\frac{x_1^2}{t_1^2} + \sum_{i=1}^{n-1}\frac{(x_{i+1}-x_i)^2}{t_{i+1}-t_i}\right)\right] \qquad \text{(I.2)}$$

It is known that the sample functions of the B-process are continuous and nowhere differentiable. These classical results were obtained by Norbert Wiener and are the primary reason that the process bears his name. We note that the non-differentiability of the sample functions make this process somewhat unacceptable as a model of displacements of actual physical particles in which velocities and accelerations are present because the process does not possess velocities and accelerations.

The pathological properties of the Gaussian white noise are primarily due to the non-differentiability of the Wiener Process as we shall see.

A white noise refers to any noise for which the spectral density function

$$f(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(\tau) e^{-i\lambda\tau} d\tau \qquad (I.3)$$

is supposed to exist and be constant for all $\lambda \epsilon (-\infty,\infty)$. Thus, the term "white" means that the noise contains all frequencies of the same average power.

The Gaussian white noise is a white noise for which the distribution functions are Gaussian.

We recall that the power spectral density and the covariance are Fourier transform pairs. This is referred to commonly as the Wiener-Khintchin relation. Thus, from I.3 we have

$$\Gamma(\tau) = \int_{-\infty}^{\infty} f(\lambda)\ e^{i\lambda\tau}\ d\lambda \qquad (I.4)$$

Strictly speaking, a white noise process can never occur in nature since its second moment is given by

$$\sigma^2 = \Gamma(0) = \int_{-\infty}^{\infty} f(\lambda)\, d\lambda \tag{I.5}$$

which is not a convergent integral for $f(\lambda)$ constant.

It follows that the covariance of the W-process (white noise) is given as,

$$E\{W(s)\, W(t)\} = \sigma^2\, \delta(t-s) \tag{I.6}$$

where δ represents the impulse function. That is,

$$\delta(\tau) = 0, \quad \tau \neq 0 \qquad \text{(a)}$$

$$\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1 \qquad \text{(b)} \tag{I.7}$$

Hence, no matter how near t_1, t_2 are, if $t_1 \neq t_2$, then $W(t_1)$, $W(t_2)$ are uncorrelated. But even more, since the process is Gaussian, then $W(t_1)$, $W(t_2)$ are not only uncorrelated, but they are actually independent of one another.

If one first stops a moment and reflects upon what the preceding statements imply for the sample functions, the pathological nature of Gaussian White Noise becomes apparent. Indeed if we may quote Doob on page 78 of his treatise, "These processes (independent or purely random) are discussed only in the discrete parameter case, since the sample functions in the continuous parameter case are too irregular to arise in practice."

We recall that all of the regularity properties of analysis (such as continuity, differentiability, etc.) depend upon the relative values of a function for argument values that are close to one another.

However, as we have seen above, the values of the Gaussian White Noise samples are completely independent no matter how close the arguments t_1, t_2 are to one another. We must expect therefore, that the sample functions for the Gaussian White Noise to be quite pathological and unnatural. In fact, as we have seen, the Gaussian White Noise is only a mathematical abstraction that cannot be represented in nature. We shall continue our discussion with an approach to the White Noise process that brings these points clearly to the surface.

One rather useful way in which one can think of a White Noise is to consider initially a process with a covariance of the form,

$$C\, e^{-\alpha|\tau|}, \quad C>0, \ \alpha>0. \tag{I.8}$$

The spectral density corresponding to the covariance I.8 is

$$f(\lambda) = \frac{1}{2\pi}\int_{-\infty}^{\infty} C\, e^{-\alpha|\tau|}\, e^{-i\lambda\tau} d\tau = \frac{C}{\pi}\,\frac{\alpha}{\alpha^2+\lambda^2} \tag{I.9}$$

$$= \frac{A}{\alpha^2+\lambda^2}\ , \quad A = \frac{C\alpha}{\pi}\ .$$

If we let $C, \alpha\to\infty$ so that $\frac{A}{\alpha^2}\ (= \frac{C}{\pi\alpha})$ is constant, then one obtains the constant spectral density.

We notice that for $C, \alpha\to\infty$ such that $\frac{C}{\pi\alpha}$ is constant we must have

$$\lim_{C,\alpha\to\infty} Ce^{-\alpha|\tau|} \to 0 \quad \text{for all} \quad \tau \neq 0. \tag{I.10}$$

The limit I.10 comes from the fact that the exponential beats any power and C,α approach infinity at the same rate. We have, furthermore,

$$C\int_{-\infty}^{\infty} e^{-\alpha|\tau|} d\tau = C\left(\frac{2}{\alpha}\right) = 2\pi\left(\frac{C}{\pi\alpha}\right) \to \text{constant}. \tag{I.11}$$

All of the results above may be summarized and put into order as follows.

We consider the Gaussian White Noise $W(t)$, $t\varepsilon(-\infty,\infty)$. We have re-established by limiting operations

a) the covariance function must be an impulse function. This follows from I.10,, I.11. Thus, $W(t)$ is uncorrelated at any two distinct times. That is, $t_1 \neq t_2$

implies

$$E\{W(t_1)W(t_2)\} = 0$$

b) Since $W(t)$ is Gaussian, then a) implies it is completely random. Thus, $t_1 \neq t_2$ implies $W(t_1)$ is statistically independent of $W(t_2)$.

Now let us consider $\{\xi_{C,\alpha}(t), t\epsilon[o,\infty]\}$, a Gaussian process for which $E\{\xi_{C,\alpha}(t)\} = 0$ and

$$E\{\xi_{C,\alpha}(t)\ \xi_{C,\alpha}(t+\tau)\} = Ce^{-\alpha|\tau|} \tag{I.12}$$

From the mean square calculus theorem we know that

$$y_{C,\alpha}(t) = \int_o^t \xi_{C,\alpha}(\tau)d\tau \tag{I.13}$$

exists as an integral in the mean square sense. Furthermore, $\{y_{C,\alpha}(t), t\epsilon[o,\infty]\}$ is a Gaussian process for which $P\{y_{C,\alpha}(0) = 0\} = 1$ and $E\{y_{C,\alpha}(t)\} = 0$.

Now, we are interested in what happens to $y_{C,\alpha}(t)$ as the covariance of $\xi_{C,\alpha}(t)$ goes to the impulse function limit that we described above. The random variables $\xi_{C,\alpha}(t)$ will approach $W(t)$.

What can we say concerning the second moments of the $y_{C,\alpha}$-process? Clearly,

$$E\{y^2_{C,\alpha}(t)\} = E\{\int_0^t d\tau_1 \int_0^t d\tau_2\ \xi_{C,\alpha}(\tau_1)\ \xi_{C,\alpha}(\tau_2)\}$$

$$= \int_0^t d\tau_1 \int_0^t d\tau_2\ \ E\{\xi_{C,\alpha}(\tau_1)\ \xi_{C,\alpha}(\tau_2)\} \qquad (I.14)$$

(because $\xi_{C,\alpha}(\tau_1)\ \xi_{C,\alpha}(\tau_2)$ are absolutely integrable and their absolute average exists by the Schwarz inequality.)

$$E\{y^2_{C,\alpha}(t)\} = \int_0^t d\tau_1 \int_0^t d\tau_2\ \ Ce^{-\alpha|\tau_1-\tau_2|}$$

$$= \int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2\ Ce^{-\alpha(\tau_1-\tau_2)} + \int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1\ Ce^{-\alpha(\tau_2-\tau_1)}$$

$$= 2\int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2\ \ Ce^{-\alpha(\tau_1-\tau_2)}$$

$$= \frac{2C}{\alpha}\left[\tau_1 + \frac{1}{\alpha}e^{-\alpha\tau_1}\right]\Bigg|_0^t$$

$\rightarrow \sigma^2 t$ as $C,\alpha\rightarrow\infty$ in such a way that $\frac{C}{\pi\alpha}$ is constant.

We now have that $y_{C,\alpha}(t) \rightarrow B(t)$ as $\xi_{C,\alpha}(t) \rightarrow W(t)$ through covariances, where

$$E\{B^2(t)\} = \sigma^2 t. \qquad (I.15)$$

Furthermore, for any (t_1, t_2), we obtain in the same fashion as above that

$$E\{|B(t_2) - B(t_1)|^2\} = \sigma^2|t_2-t_1|. \tag{I.16}$$

Now suppose that $t_1<t_2<t_3$ then

$$\begin{aligned} E\{[B(t_3)-B(t_1)]^2\} &= E\{[B(t_3)-B(t_2)+B(t_2)-B(t_1)]^2\} \\ &= E\{[B(t_3) - B(t_2)]^2\} \\ &\quad + 2\, E\{[B(t_3) - B(t_2)][B(t_2) - B(t_1)]\} \\ &\quad + E\{[B(t_2) - B(t_1)]^2\} \end{aligned} \tag{I.17}$$

or

$$\sigma^2(t_3-t_1) = \sigma^2(t_3-t_2) + 2E\{[B(t_3)-B(t_2)][B(t_2)-B(t_1)]\} + \sigma^2(t_2-t_1). \tag{I.18}$$

Therefore, we must have

$$E\{[B(t_3) - B(t_2)][B(t_2) - B(t_1)]\} = 0. \tag{I.19}$$

The last expression, (I.19) says that the B-process possesses independent increments. Furthermore, the B-process is Gaussian, since the $y_{C,\alpha}$-process is Gaussian for every C,α. The covariances of the $y_{C,\alpha}$-process converge to the covariance of the B-process. Hence, the process $\{B(t), t\epsilon[o,\infty]\}$ is a Gaussian process with stationary independent increments for which we have

$$E\{B(t)\} = 0$$
$$E\{B^2(t)\} = \sigma^2 t \qquad \text{(I.20)}$$
$$\text{Prob } \{B(0) = 0\} = 1.$$

But, this is the definition of the Wiener Process! Hence, it would follow that

$$B(t) = \int_0^t W(\tau)d\tau. \qquad \text{(I.21)}$$

Therefore, one would obtain from 1.21)

$$\frac{dB(t)}{dt} = W(t). \qquad \text{(I.22)}$$

That is, the Gaussian White Noise is formally the derivative of the Wiener process. However, as we recall from our earlier discussions, the Wiener process does not possess a derivative. Hence, the pathological nature of the Gaussian White Noise is explicitly brought out by the formal relation I.22 . The relation I.22 simply states that the Gaussian White Noise is a mathematical fiction.

Hence, when we write an equation such as

$$\frac{dx(t)}{dt} + \beta x(t) = W(t) \quad \left(\equiv \frac{dB(t)}{dt}\right) \qquad \text{(I.23)}$$

we must understand that it does not exist as an ordinary differential equation of elementary calculus since the White Noise inp is a fiction.

The question we must ask ourselves is can one make analytical sense of the Equation I.23 . The answer is yes, and this was first accomplished by K. Ito (*1).

The idea is simply that, instead of treating I.23 as a differential equation, one should instead study an equation in differentials

$$dx(t) + \beta x(t)dt = dB(t). \tag{I.24}$$

The equation I.24 is given content and meaning by the stochastic integral which is a well defined operation introduced by Ito. Hence, the meaning of I.24 is

$$x(t)-x(t_o) + \beta\int_{t_o}^{t} x(\tau)d\tau = \int_{t_o}^{t} dB(\tau) = B(t)-B(t_o). \tag{I.25}$$

For the most general first order equation,

$$\frac{dx(t)}{dt} \equiv m(x(t),t) + \sigma(x(t),t)\frac{dB(t)}{dt} , \tag{I.26}$$

the meaning of this is given by the stochastic integral equation,

$$x(t)-x(t_o) = \int_{t_o}^{t} m(x(\tau),\tau)d\tau + \int_{t_o}^{t} \sigma(x(\tau),\tau)dB(\tau), \tag{I.27}$$

where the integrals are well defined.

(*1) K. Ito. Memoirs of American Mathematical Society No. 4, 1951.

On the basis of this definition, the solution processes of I.26 , or I.27 are well defined and have been the subject of a great deal of research in the past ten years. [For a description of the properties of these processes, see the treatise by Dynkin (2) on Markov processes.]

We are now in a position to indicate the reason why the Gaussian White Noise is unsuitable as an input process for the purposes of identification by the technique we have proposed.

Since the derivative $\frac{dB(t)}{dt}$ in equation I.23 does not exist, it follows that the derivative $\frac{dx(t)}{dt}$ in I.23 , also, does not exist. Therefore, we do not have the equality

$$\frac{d}{dt} E\{x^2(t)\} = E\{x(t) \frac{dx(t)}{dt}\}, \tag{I.28}$$

that is required in our identification procedure.

Indeed, we can say even more. In particular,

$$E\{x(t) \frac{dx(t)}{dt}\} \neq 0 \tag{I.29}$$

for the process defined by equation I.23 , or more correctly, by equation I.24 .

We recall that the B-process possesses independent increments, thus

$$dB(t) \equiv B(t + dt) - B(t) \tag{I.30}$$

is independent of all prior increments of $B(s)$ for $s \leq t$.

(*2)Dynkin. Markov Processes, Springer-Verlag. Berlin, 1965.

Finally, in the procedure of identification, one's initial reaction is to ask for a wide band-width excitation in order to assure that enough frequencies are sufficiently excited so that the parameters of the system can be correctly estimated. But, as we have already seen in Chapters II and III, not only is this not required but a pure sinusoidal excitation with a single frequency is enough to allow identification. Thus, the fact that we do not want "white noise" is no weakness in the present approach. Indeed, it is even to our advantage. The reader may recall that many of the easy identification techniques made use of the Gaussian white noise for their theoretical development. But such noise is impossible to achieve in the laboratory. On the other hand, the excitations we use in our theoretical development are exactly those that can and are commonly used in laboratory testing.