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# IMPROVED COEFFICIENTS OF THE THIRTEENTH ORDER HARMONICS OF THE GEOPOTENTIAL DERIVED FROM SATELLITE DOPPLER DATA AT THREE DIFFERENT ORBITAL INCLINATIONS

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# ABSTRACT

This paper gives the results of the determination of the coefficients associated with the three odd degree harmonics of the geopotential of order thirteen. The opportunity to refine the values of these coefficients was made possible with the launching of the GEOS-II (1968 02A) satellite.

A brief description of the method used is also included in addition to references where more detailed information can be found.

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## INTRODUCTION

On January 11, 1968 satellite 1968 02A (GEOS-II) was launched into a retrograde orbit. A preliminary analysis of the orbital characteristics of GEOS-II revealed that it was in a near resonant orbit with respect to the geopotential harmonics of order m = 13. From an earlier analysis of the doppler data from satellites 1962 $\beta\mu_1$ and 1965 65A [Yionoulis, 1966] values for the harmonic coefficients of degree and order (n,m) = (13,13) and (15,13) were determined. For processing the 1968 02A doppler data, these values did not adequately account for the effects of the 13th order harmonics of the geopotential. Since the orbital characteristics of these three satellites are significantly different it was possible to use the combined information from all three to more accurately represent the effects of the 13th order harmonics. Coefficients associated with the (13,13), (15,13) and (17,13) harmonics were determined from an analysis of station along track residuals generated by OIP (Orbital Improvement Program).

The geopotential expansion used is given by

$$U = \frac{K}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=o}^{n} \frac{\Pi_n^m(\sin\varphi)}{r^n}\left[C_n^m \cos m(\lambda-\Lambda_G) + S_n^m \sin m(\lambda-\Lambda_G)\right]\right\}$$

where

$r, \varphi, \lambda$ are the geocentric radius, latitude and longitude of the satellite

$\Lambda_G$ = right ascension of Greenwich relative to Aries

K = gravitational constant (398601.5 $Km^3/sec^2$

$$\Pi_n^m(x) = \sqrt{\frac{(n-m)!}{(n+m)!}}\,(1-x^2)^{m/2}\frac{d^m}{dx^m}P_n(x)$$

$x = \sin \varphi$

$C_n^m, S_n^m$ = coefficients of the partially normalized associated Legendre polynomials of degree $n$ and order $m$ ($\Pi_n^m$).

ANALYSIS

In order to generate the error residuals used to determine the resonant coefficients doppler data obtained from the U. S. Navy Satellite Tracking System, TRANET, [Newton, 1963] is first processed by the OIP programs [Guier, 1967]. Given a group of doppler passes from tracking stations covering a specific time span these computer programs determine, via a least squares fit to the experimental doppler data, a "best" orbit for this time span. This orbit is a set of initial conditions for the equations of motion of a satellite and is epoched at the beginning of the time span encompassing the data. A numerical integration scheme is incorporated in the programs to integrate the equations of motion of the satellite. The forces modeled, in addition to those derived from the geopotential model, include the effects of solar radiation pressure, atmospheric drag and the Sun and Moon gravitational forces.

Using this "best" orbit a final analysis is performed on each individual doppler pass within the given time span. The objective here is to express the remaining differences between the theoretical and experimental doppler shifts into a form suitable for further analysis. This is accomplished by determining new tracking station positions via a least squares fit to these doppler residuals. These "navigated" station positions are compared with the current best estimate of the station's true position and the differences are given in a coordinate system defined at the time of closest approach of the satellite to the station. One component of the error (ECR) is given along the slant range vector between the station and satellite. A second component (ECA) is in the direction of the slant range velocity vector as defined at time of closest approach of the satellite to the station.

Guier [1963] showed analytically how these station position errors are related to the satellite position errors. In his analysis the satellite position errors are defined in a coordinate system moving in inertial space with its origin located at the instantaneous satellite position as defined by the theoretical model. The three component directions are specified as follows:

1) H, the altitude error, is defined along the radius vector from the center of mass of the earth to the satellite,
2) C, the cross-track error, is defined in the direction of the negative instantaneous angular momentum vector,
3) L, the along-track error direction, is defined by a vector lying in the orbital plane and perpendicular to the satellite radius vector.

Guier also showed that the station ECA residuals absorbed mainly the satellite along-track errors. This fact is particularly advantageous for studying the near-resonance effects on the satellite's motion as will become clear from the following discussion.

Guier derived a set of differential equations in the H-C-L coordinate system whose solutions yield the time dependence of the satellite position errors due to errors in the geopotential model. A complete discussion of these equations and their solutions is given in Yionoulis [1965]. In this paper it is shown that when a satellite is in a near-resonant orbit with the harmonics of a given order of the geopotential, this results in a long period oscillation in the satellite along-track direction whose amplitude is greatly magnified. This amplification in the along-track component is greater than that which occurs in the altitude component. Also the effects of the geopotential harmonic can be accurately represented by using only those terms whose frequency yields the resonant period. Since this frequency will be the same for all harmonics with the same parity in degree ($n \geq m$) for the given order ($m$), separation of these harmonics must be accomplished through the dependence of their influence coefficients mainly on the orbital inclination. This necessitates the use of data from satellites with significantly different orbital inclinations.

The procedure used for separating the harmonic coefficients was as follows. For each satellite orbit in near-resonance with the same harmonics of a given order, a data span was selected approximately equal to 1 1/2 times the resonant period. The ECA residuals were generated using the OIP computer programs as discussed earlier. From an analysis of these residuals an amplitude and phase was determined for the resonant oscillation contained in each arc. From the analytic expressions for the satellite along-track effects contained in Yionoulis [1965] the desired relationship between the harmonic coefficients were computed for each satellite span.

In Table 1 the orbital characteristics of three satellites used in the

TABLE 1. Orbital Characteristics of Satellites Used in Determination of 13th Order Harmonic Coefficients of the Geopotential

| Satellite No. | Semi-major Axis a | Inclination i (deg.) | Eccentricity e | $T_R^{**}$ (days) |
|---|---|---|---|---|
| 1962 $\beta\mu_1$ | 1.177 $(R_o)^*$ | 50.127 | .0064 | 6 |
| 1965 65A | 1.179 | 90.024 | .0061 | 4 |
| 1968 02A | 1.208 | 105.803 | .0327 | 6 |

$^*R_o$ = 6378.166 Km.

$^{**}T_R$ = Theoretical estimate of resonant period due to 13th order harmonic analysis, are given. The geopotential harmonics of order m = 13 and odd degree n ≥ 13, are the major contributors to the long period resonant oscillations in the ECA residuals. This period, in days, is also tabulated for each satellite.

It should be noted at this point that the even degree harmonics of order 13 also contain terms with periods very close to the resonant period. However, these terms are a factor of eccentricity down from those generated by the odd degree harmonics. Since the eccentricities of the satellite orbits used are quite small, it was felt that their contributions could be ignored. I also arbitrarily chose to absorb the 13th order resonant effects with the coefficients of the three lowest degree harmonics which produced this effect. To solve for the coefficients we generated two sets of three simultaneous equations, one for the $C_n^m$ coefficients and one set for $S_n^m$. They are arranged in order of increasing inclinations. The equations have been normalized by the influence coefficient associated with the (13,13) harmonic.

$$
(1) \quad \begin{cases}
C_{13}^{13} - 4.140\, C_{15}^{13} + 5.722\, C_{17}^{13} = 1.066 \times 10^{-6} \\[2ex]
C_{13}^{13} + 0.394\, C_{15}^{13} + 1.508\, C_{17}^{13} = -0.519 \times 10^{-6} \\[2ex]
C_{13}^{13} - 1.059\, C_{15}^{13} - 1.632\, C_{17}^{13} = -0.248 \times 10^{-6}
\end{cases}
$$

$$
(2) \quad \begin{cases}
S_{13}^{13} - 4.140\, S_{15}^{13} + 5.722\, S_{17}^{13} = 1.562 \times 10^{-6} \\[2ex]
S_{13}^{13} + 0.394\, S_{15}^{13} + 1.508\, S_{17}^{13} = 0.508 \times 10^{-6} \\[2ex]
S_{13}^{13} - 1.059\, S_{15}^{13} - 1.632\, S_{17}^{13} = 0.428 \times 10^{-6}
\end{cases}
$$

Solutions to the above equations yield the following values.

$$C_{13}^{13} = -.4801 \times 10^{-6} \qquad S_{13}^{13} = .4252 \times 10^{-6}$$

$$C_{15}^{13} = -.3007 \times 10^{-6} \qquad S_{15}^{13} = -.1459 \times 10^{-6}$$

$$C_{17}^{13} = .0527 \times 10^{-6} \qquad S_{17}^{13} = .0931 \times 10^{-6}$$

## CONCLUSIONS

The method described in this paper for analyzing residuals obtained from resonant satellite orbits has been successful in accounting for the effects of the resonant geopotential harmonics. It is recognized, however, that this method does not fully utilize all the information that is available for analysis. Therefore, alternate methods are being investigated which hopefully will result in more accurate determinations.

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