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EMPIRICAL CHARACTERIZATION OF SOME PRESSURE WAVE SHAPES IN STRONG TRAVELING TRANSVERSE ACOUSTIC MODES

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ABSTRACT

Pressure wave shapes measured at the wall of a short cylindrical cavity containing a relatively "pure" traveling transverse acoustic mode were evaluated for harmonic content. The empirical relation $p' = \epsilon \cos t + \epsilon^2 \cos 2t + \epsilon^3 \cos 3t + \ldots$, was found to adequately represent wave shapes for peak-to-peak pressure amplitudes approaching the mean cavity pressure. The relation is relatively simple and provides a convenient method of characterizing strong wave shapes for many analytical and experimental purposes.

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SUMMARY

Pressure wave shapes measured at the wall of a short cylindrical cavity containing a relatively "pure" traveling transverse acoustic mode were evaluated for harmonic content. The empirical relation $p' = \epsilon \cos t + \epsilon^2 \cos 2t + \epsilon^2 \cos 3t + \ldots$, was found to adequately represent wave shapes for peak-to-peak pressure amplitudes approaching the mean cavity pressure. The relation is relatively simple and provides a convenient method of characterizing strong wave shapes for many analytical and experimental purposes.

INTRODUCTION

In realistic systems, the concept of sinusoidal variations of pressure with time for any acoustic oscillation is valid only for relatively minute pressure amplitudes. Harmonic distortion increases with amplitude and substantially changes the pressure wave shape. A description of such wave shape distortion as a function of amplitude is frequently required for experimental and analytical purposes.

Presented herein is the harmonic content observed in some wave shapes measured at the wall of a short cylindrical cavity containing strong but relatively pure traveling transverse acoustic modes. The evaluation of harmonic content was required for a nonlinear analysis of a process which responds to such pressure oscillations and only a few wave shapes were examined. A simple mathematical expression for the coefficients of harmonic components was sought which could be conveniently applied for such waves. The results, although limited in scope, may have utility in other analytical or experimental studies. The wave characterization is restricted to a particular resonant condition, however, this transverse mode is frequently observed in the combustors of large scale rocket and turbojet engines and in the compressor and fan ducting of turbojet engines.

DISCUSSION

Traveling Transverse Acoustic Modes

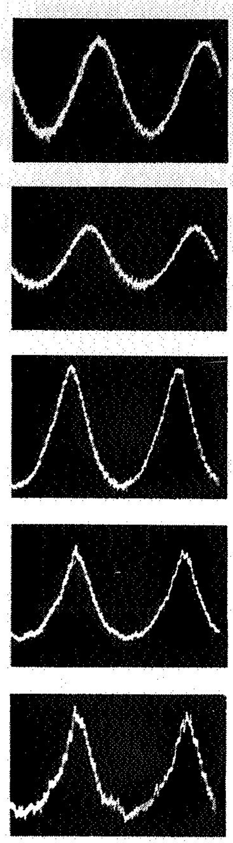
The traveling transverse acoustic mode is one of the fundamental modes of resonance in a cylindrical cavity. In this mode the pressure wave is alined along a circumferential path with the pressure crests and valleys having a radial orientation. This wave form rotates about the cylindrical axis with a velocity at the circumference of the cylinder equal to 1.84 times the normal acoustic velocity. Theoretical properties of high amplitude wave shapes are presented in reference 1. One of the distinguishing features of the high amplitude theoretical wave shapes described in reference 1 is that the wave shape remains symmetrical with time about the maximum pressure. The wave does not steepen or become shock fronted as is the usual case for high amplitude pressure waves. The distinguishing wave shape is restricted to pure modes. However, relatively pure modes can and frequently are excited by combustion and flow processes.

Pressure wave shapes having the distinguishing shape of a pure mode were obtained in a study of the generation of strong traveling transverse acoustic modes with a rotating gas jet (ref. 2). The wave shapes obtained from wall pressure measurements are shown in figure 1. The main cavity pressure was at or near atmospheric conditions. Wave shapes for amplitudes up to and above the mean cavity pressure are shown. Some nonsymmetrical wave distortion is obvious for the higher amplitude waves. The smaller but still substantial wave amplitudes appear to be ideal, that is, symmetrical in time about the peak pressure.

In reference 1, where the characteristics of strong or high amplitude waves for the transverse modes are analytically derived, the wave shape is expressed as a power series in an amplitude parameter $\mathscr{E}$. The pressure at a fixed wall position is of the form

$$\frac{p}{p_o} = 1 + \mathscr{E}(\gamma P_{10} \cos t) + \mathscr{E}^2\left[2\gamma(P_{20} \cos 2t + P_{23})\right] + \mathscr{E}^3(4\gamma P_{30} \cos 3t + \gamma P_{34} \cos t) + \mathscr{E}^4 \ . \ . \ . \qquad (1)$$

where the symbols are defined in the appendix and the P_{ij} values are constants related to the radius and given in appendix C of reference 1.



Peak to peak pressure amplitude, $(\Delta P)_{pp}$, psi	(N/m^2)	Frequency, f_n, Hz
2.8	(0.19×10^5)	647
5.0	(0.35×10^5)	641
10.0	(0.69×10^5)	644
15.5	(1.07×10^5)	634
24.0	(1.65×10^5)	629

Figure 1. - Pressure - time wave shapes, ref. 1.

In many applications a harmonic series rather than a power series provides a more useful expression of the wave form. This is true for the analysis of experimental data where methods for harmonic analysis are available. A harmonic series is also useful in the nonlinear analysis of processes which respond to pressure changes.

Rearranging equation (1) as a harmonic series gives

$$p' \simeq \frac{p - p_o\left(1 + 2\gamma P_{23}\mathscr{E}^2 + \ldots \mathscr{E}^4\right)}{p_o} = \gamma\left(\mathscr{E}^2 P_{10} + \mathscr{E}^3 P_{34} + \mathscr{E}^5 \ldots\right)\cos t$$

$$+ \gamma\left(2\mathscr{E}^2 P_{20} + \mathscr{E}^4 \ldots\right)\cos 2t + \gamma\left(4\mathscr{E}^3 P_{30} + \mathscr{E}^5 \ldots\right)\cos 3t + \ldots \qquad (2)$$

As a harmonic series each of the coefficients takes the form of a separate power series in $\mathscr{E}$. Coefficients of this type are not convenient to use either in the evaluation of experimental data or in nonlinear analysis. Also, within the scope of reference 1, the coefficients are not adequately defined for high amplitude waves. Additional P_{ij} values are required and these require considerable analysis and numerical computations to evaluate. It is concluded, therefore, that the amplitude parameter $\mathcal{E}$ does not provide a convenient way of identifying a wave of a given amplitude for many applications.

Actually, the amplitude parameter $\mathcal{E}$ in reference 1 was arbitrarily chosen to facilitate the analytical procedures used in that study. It does not, in the general sense, represent a fundamental wave property, and a redefinition of an amplitude parameter may provide a more acceptable method of characterizing strong waves as a harmonic series.

Analysis of Experimental Wave Shapes

The pressure wave shapes shown in figure 1 were numerically evaluated for harmonic content and found to be adequately represented by the Fourier series.

$$p = C_0 + C_1 \cos t + C_2 \cos 2t + C_3 \cos 3t + \dots \qquad (3)$$

The values of the coefficients are shown by the symbols plotted in figure 2. In calculating these coefficients it was necessary to digitize the wave shapes, a procedure which limits the accuracy of the coefficients. A general pattern is established, however, where for each amplitude the logarithm of the coefficient decreases nearly linearly with an increase in harmonic order. This establishes the relation $(C_n/C_0) = \epsilon^n$.

The value of C_0 is the mean pressure according to equation (3) and is indicated by the extrapolation shown in figure 2. A value of about 15 psi is shown for the 2.8, 5.0, and 10 psi amplitude conditions. For these amplitudes, the wave shape is very nearly given by

$$\frac{p - C_0}{C_0} = p' = \epsilon \cos t + \epsilon^2 \cos 2t + \epsilon^3 \cos 3t + \epsilon^4 \dots \qquad (4)$$

where ϵ is a parameter sufficient to describe a wave of a given amplitude.

The mean cavity pressure indicator for the 15.5 and 24 psi amplitude conditions was severely affected by the large pressure perturbations during the experimental tests and reliable values could not be established. Extrapolated mean pressure values of 17 and 21 psi are indicated in figure 2 for the 15.5 and 24 psi amplitudes, respectively. Elevated mean pressures for these amplitudes are expected for several reasons.

(1) Pressure amplitude was experimentally increased by increasing the mean gas

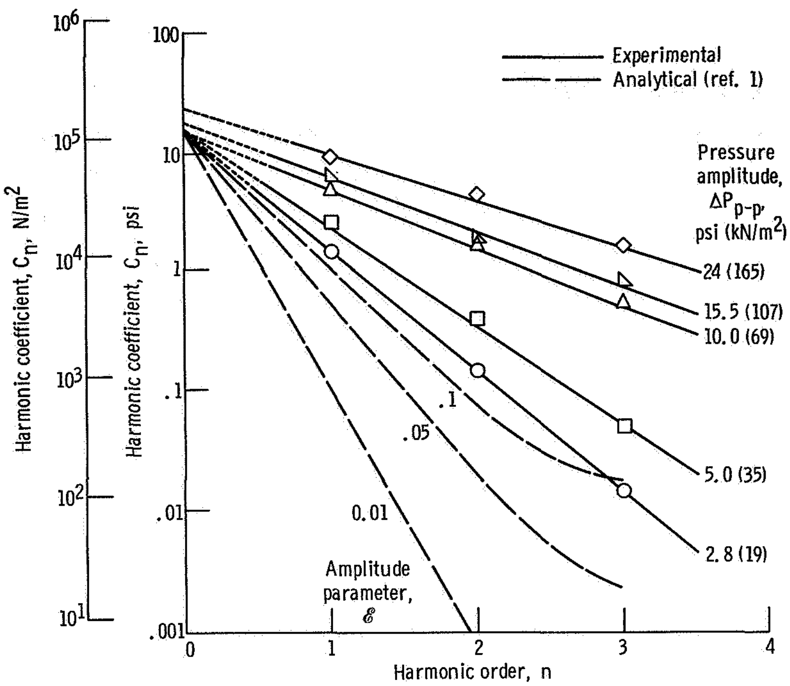


Figure 2. - Harmonic content of some pressure-wave shapes where $p = C_0 + C_1 \cos t + C_2 \cos 2t + C_3 \cos 3t + \ldots$ and comparison with reference analysis.

flow rate through the cavity and this would elevate the mean cavity pressure.

(2) An effect on mean cavity pressure with increasing amplitude is expected according to reference 1 and as shown by equation (2) where the initial pressure p_0 is modified by the amplitude factor to give the mean pressure $\overline{p}$.

(3) As indicated previously, these higher amplitude waves appear to deviate from ideal waves and the extrapolation for mean values may be less reliable.

For these reasons, it is concluded that ideal waves with amplitudes greater than 15.5 psi are also approximated by equation (4).

Figure 2 also shows, for several values of $\mathcal{E}$, the decrease in the harmonic coefficients with increasing harmonic order predicted by equation (3). For the limited number of P_{ij} values given in reference 1, the accuracy of the analytical coefficients decreases for an $\mathcal{E}$ larger than about 0.1 or a pressure amplitude above 2.8 psi. Up to this amplitude, a linear decrease in coefficients with increasing order appears adequate to characterize the theoretical wave shape and a continuation of this trend is implied for higher amplitude waves. The complex task of obtaining higher order solutions by the method of reference 1 must be performed, however, to analytically justify a linear variation for higher amplitude waves.

Properties of the Wave Shape Function

Assuming equation (4) adequately represents the pressure wave form, certain generalization can be established from such a series expression.

The maximum pressure p_{max} occurs when t is equal to zero. The periodic terms are then all equal to unity and the maximum pressure is related to the coefficients by

$$\frac{p_{max} - \overline{p}}{\overline{p}} = \epsilon + \epsilon^2 + \epsilon^3 + \ldots \tag{5}$$

This series expression in ϵ can be reduced to an analytical form such that

$$\frac{p_{max} - \overline{p}}{\overline{p}} = \frac{\epsilon}{1 - \epsilon} \tag{6}$$

Similarly, the minimum pressure p_{min} occurs when t is equal to π and

$$\frac{p_{min} - \overline{p}}{\overline{p}} = -\epsilon + \epsilon^2 - \epsilon^3 + \ldots \tag{7}$$

or

$$\frac{p_{min} - \overline{p}}{\overline{p}} = - \frac{\epsilon}{1 + \epsilon} \tag{8}$$

Using equations (6) and (8), the peak-to-peak pressure amplitude ΔP_{p-p} becomes

$$\frac{\Delta P_{p-p}}{\overline{p}} = \frac{p_{max} - p_{min}}{\overline{p}} = \frac{2\epsilon}{1 - \epsilon^2} \tag{9}$$

and the pressure ratio takes the form

$$\frac{p_{max}}{p_{min}} = \frac{1 + \epsilon}{1 - \epsilon} \tag{10}$$

Equations (6) and (8) to (10) are readily evaluated expressions which relate the amplitude parameter ϵ to some of the properties frequently used to describe strong waves.

An upper limit value of ϵ equal to unity allows for waves of infinite amplitude. Equation (8), however, shows that when ϵ approaches unity the minimum pressure p_{min} does not decrease below one-half the mean pressure.

CONCLUDING REMARKS

The characterization of some pressure wave shapes in strong traveling transverse acoustic modes by

$$p' = \epsilon \cos t + \epsilon^2 \cos 2t + \epsilon^3 \cos 3t + . . .$$

was adequate for the intended purpose of examining a process which responds to such pressure oscillations. Although the analysis of these wave shapes is very limited in scope, it has been presented in this paper because the functional form for the wave shape appears to have utility in other analytical and experimental studies. Additional experimental and theoretical studies are required, however, to rigorously establish the functional form or its limitations.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 1, 1968,
128-31-06-02-22.

APPENDIX - SYMBOLS

C_n pressure coefficient for n harmonic order, psi

$\mathcal{E}, \epsilon$ amplitude parameters, dimensionless

P_{ij} constants related to radius, ref. 1, dimensionless

ΔP_{p-p} peak-to-peak pressure amplitude, $p_{max} - p_{min}$, psi

p instantaneous pressure, psi

p_o initial pressure, psi

p_{max} maximum pressure during oscillation, psi

p_{min} minimum pressure during oscillation, psi

$\overline{p}$ mean pressure, psi

p' pressure perturbation, $(p - \overline{p})/\overline{p}$, dimensionless

t cycle time, rad

α dimensionless radial coordinate

γ ratio of specific heats, dimensionless

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