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QUARTERLY

PROGRESS REPORT

CONVOLUTIONAL CODING TECHNIQUES FOR DATA PROTECTION

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1. Research in Sequential Decoding

(a) Theoretical Investigations

A new way of viewing the Fano sequential decoding algorithm, which was mentioned in the previous progress report, has now been fully developed. The approach is to view the Fano algorithm as a technique for searching an arbitrary value tree, i.e. a tree in which the nodes are assigned a real number value. Precise expressions have been developed for the number of "forward looks" and "forward moves" made at any node in the value tree, and a precise characterization has been obtained for the nature of the final path through the tree found by the algorithm. The final path is determined as follows: At each node beginning with the origin, the final next node chosen by the Fano algorithm is that on the path through the full tree stemming from the former node which has the greatest minimum node value along its length. This research is documented in:

J. L. Massey and M. K. Sain, "Trunk and Tree Searching Properties of the Fano Sequential Decoding Algorithm," Elec. Engr. Memo. No. EE-687, U. of Notre Dame, Notre Dame, Ind., Oct. 1, 1968 (Preprint of article to appear in the Proceeding of the Sixth Annual Allerton Conference on Circuit and System Theory, Univ. of Illinois, Oct. 2-4, 1968.)

It has subsequently been realized that the concepts in this new approach to the Fano algorithm can be significantly generalized. In this generalization, the Fano algorithm can be extended to become an algorithm for searching an arbitrary branch-weighted graph. Defining the value of a path as the sum of the weights on its component branches, the Fano algorithm becomes an efficient procedure for finding the path from a specified starting node to any one of a set of specified stopping nodes for which the minimum of the path values from the starting node to nodes on the path is greatest. It is believed that this will make the Fano algorithm a useful sub-optimal solution for dynamic programming problems and other graph searches. Work is continuing on this subject.

Research has also been completed on the task of calculating the probability distribution for the minimum path value along the correct or transmitted path in a value tree resulting from communication over a memoryless symmetric channel such as the additive white Gaussian noise channel. An abstract of this research is given in:

J. L. Massey and M. K. Sain, "Distribution of the Minimum Cumulative Metric for Sequential Decoding." (Abstract of Short Paper to be presented at the 1969 IEEE International Symposium on Information Theory, Nevelle, N. Y., Jan. 28-31, 1969.)

A later technical report will fully document this work on the distribution of the cumulative metric.

(b) Experimental Investigations

During this period, programming was completed for a sequential decoding facility for a simulated white gaussian noise channel. The facility was used to evaluate relative performance of 11 different rate one-half convolutional codes with constraint lengths of 37 branches (74 bits). A subsequent technical report will give complete details of the tests made, but table I gives a sampling of the results.

From table I, it is clear that codes 5 and 8 perform about equally well on the Gaussian channel at a signal-to-noise level of 0 db. It is also clear that code 8 is significantly better than code 5 at high noise levels on the binary symmetric channel, and it is expected that this superiority will also obtain at higher noise levels on the Gaussian channel--this simulation will be made in the near future. Code 5 is the first 36 branches of the 48 branch code presently used at Goddard, while code 8 is a new non-systematic code developed on this project. This code has the property that its two generators differ only in the second bit position so that the information bits can be recovered almost as easily as with a systematic code when the receiver site does not have a decoder available.

It is planned to extend the capability of the simulation facility to 73 branches so that the full Goddard code can be tested against this non-systematic code to determine if the superiority of the latter is maintained.

2. Other Investigations.

The investigation of error propagation in convolutional codes via a stochastic sequential machines approach has been completed. This work is described in:

T. N. Morrissey, Jr., "A Unified Markovian Analysis of Decoders for Convolutional Codes," Tech. Rpt. No. EE-687, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., Oct. 24, 1968.

This report formed the Ph.D. dissertation of its author. The results of this work make it possible for the first time to calculate steady-state error probabilities for fixed-span decoders for convolutional codes. The analysis was also extended to the variable-span Viterbi decoder which is a maximum-likelihood decoder for convolutional codes. It is believed that this work will form the basis for many additional investigations of convolutional coding.

Code	Code Generators (octal)	Type Channel	Decoding Errors	Decoding Erasures	Number of Frames with Computation exceeding 700	1000	1500	10,000
5	400,000,000,000 715,473,701,317	Gaussian with E_b/N_o of 0 db	0	4	523	404	188	19
5'	400,000,000,000,0 715,473,601,317,4	Binary symmetric Prob. Error .033	0	0	189	77	27	2
5'		Binary symmetric Prob. Error .045	0	4	510	324	197	26
5'		Binary symmetric Prob. Error .057	82	127	825	667	535	262
8	533,533,676,737 733,533,676,737	Gaussian with E_b/N_o of 0 db	0	5	567	358	225	17
8		Binary Symmetric Prob. Error .033	0	0	223	92	47	2
8		Binary symmetric Prob. Error .045	0	8	581	382	240	36
8		Binary symmetric Prob. Error .057	0	245	--data not yet available, see-- note 2 below			

Notes: (1) Code 5' is trivially different from 5 and it is expected that the performance of code 5 on the binary symmetric channel will be equivalent--this run has not yet been made.
(2) Only the first 993 of the 1000 frames have been decoded for code 8 on the binary symmetric channel with Prob. Error of .057--The conclusion is already clear however that decoding errors have been eliminated.
(3) Each frame contains 256 information bits. A frame was declared erased when the number of decoding computations reached 50,000

TABLE I. Results of Decoding 1000 frames for various codes and various channels