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A DIGITAL COMPUTER PROGRAM FOR DETERMINING THE ELASTIC-PLASTIC DEFORMATION AND CREEP STRAINS IN CYLINDRICAL RODS, TUBES AND VESSELS

by Richard L. Puthoff

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Cleveland, Ohio

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ABSTRACT

A generalized plane-strain elastic-plastic analysis and a creep analysis of a long hollow (or solid) cylinder are presented. These analyses use the method of successive approximations for solution. The equations are converted into finite difference formulation and programmed on an IBM 7094 Mod II digital computer. The cylinder is subjected to thermal gradients and pressure loads. The resulting stresses and strains calculated are printed as the output. A sample problem is included, along with program operation instructions and listings. Examples of components for which the analysis can be used are reactor pressure shells, fuel elements, and heat-exchanger tubes with symmetrical temperature gradients and constant pressure loads.

A DIGITAL COMPUTER PROGRAM FOR DETERMINING THE ELASTIC-PLASTIC DEFORMATION AND CREEP STRAINS IN CYLINDRICAL RODS, TUBES AND VESSELS

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SUMMARY

Many design applications, such as nuclear reactors, often encounter both high heat fluxes and internal heat generation. As a result, these structures are subjected to large thermal gradients in addition to the pressure loads. These thermal stresses, coupled with high temperatures of operation, necessitate that the material operate near its maximum load-carrying capacity. Therefore, the materials are often operated into the plastic range.

A generalized plane-strain elastic-plastic analysis is presented of a long hollow (or solid) cylinder with thermal stresses and constant pressure loads. The analysis uses the method of successive approximations for solution and applies a one-step loading path called "deformation theory" for the calculations. The equations are converted into finite difference formulation and programmed on an IBM 7094 Mod II digital computer. Stresses and strains due to the thermal gradients and pressure loads are printed as the output. These values represent the elastic-plastic stress state existing on the cylinder when the loads are initially applied (time, 0). Further deformation by creep may then be determined.

The generalized plane-strain creep analysis is similar to the elastic-plastic solution, except that the equivalent creep strain replaces the equivalent plastic strain and plastic flow occurs at all mesh points. Here, however, the loading path is no longer a one-step process, and the "incremental theory" is applied to the calculations. These equations have also been programmed on an IBM 7094 Mod II digital computer.

A sample problem of a hollow steel cylinder (appendix A) has been included to demonstrate the use of the code. Program operation instructions and listings are included in appendixes B and C.

INTRODUCTION

In nuclear reactor applications high temperatures of operation, coupled with internal heat generation, result in unique design problems. Examples of components with these problems are pressure shells, fuel elements, and heat-exchanger tubes. In the case of the pressure shell surrounding the core, not only is heat generated within the shell by gamma radiation emitted from the core, but the usual radiative and conductive heat sources are also present, as in any vessel surrounding a heat source. In the case of the fuel elements, high heat generation occurs because of neutron fissioning. The heat is then carried away by coolant flow through the core. In both cases large temperature gradients occur, resulting in high thermal stresses. These thermal stresses, coupled with high temperatures of operation, necessitate that the material operate near its maximum load-carrying capacity. Therefore, these materials are often operated into the plastic range.

The solution of elastic-plastic flow problems involves the use of equilibrium and compatibility equations combined with nonlinear stress-strain relations instead of the linear Hooke's Law. The solution to the resulting equations requires the use of the method of successive approximations. This method has been used by Millenson and Manson (ref. 1) in connection with the rotating disk and by D. F. Johnson (ref. 2) in analyzing thin-wall cylinders and spheres subjected to internal pressure and nuclear radiation heating.

The method of successive approximations is also applicable to the solution of creep problems. This method can use various assumptions with regard to cumulative creep, such as the strain-hardening, time-hardening, or life-fraction rules (ref. 3). The time-hardening rule was used in this report.

A digital computer program is presented, for use on an IBM 7094 Mod II computer, that calculates stresses and strains in a long hollow (or solid) cylinder undergoing elastic-plastic deformation and creep. FORTRAN IV compiler language has been used. All equations which were used and converted into finite difference formulation are presented. A sample problem and program listings, together with the operating instructions, have been included in the appendixes.

SYMBOLS

A creep law constant

a radius, in.; cm

b radius, in.; cm

E modulus of elasticity, psi; N/cm^2

ΔH	material constant in creep law
h	radial increment in finite difference, $r_n - r_{n-1}$
K	constant
m	creep law exponent
N	creep law exponent
n	creep law exponent
P	internal pressure, psi; N/cm^2
Q/A	surface heat flux, $Btu/(hr)(ft^2)$; W/cm^2
R	universal gas constant
r	radius, in.; cm
T	time, hr
ΔT	time increment, hr
t	temperature, $^{\circ}F$
α	coefficient of thermal expansion, in./in./$^{\circ}F$; cm/cm/K
$\Delta\epsilon$	incremental strain, in./in.; cm/cm
$\dot{\epsilon}$	rate of strain, in./in./hr; cm/cm/hr
ϵ_{ec}	equivalent creep strain, in./in.; cm/cm
$\Delta\epsilon_{ec}$	incremental equivalent creep strain, in./in.; cm/cm
ϵ_{et}	equivalent total strain, in./in.; cm/cm
ϵ_p	equivalent plastic strain, in./in.; cm/cm
ϵ_r	radial strain, in./in.; cm/cm
$\Delta\epsilon_r$	incremental radial strain, in./in.; cm/cm
ϵ_z	longitudinal strain, in./in.; cm/cm
$\Delta\epsilon_z$	incremental longitudinal strain, in./in.; cm/cm
ϵ_θ	tangential strain, in./in.; cm/cm
$\Delta\epsilon_\theta$	incremental tangential strain, in./in.; cm/cm
μ	Poisson's ratio
σ_e	equivalent stress, psi; N/cm^2
σ_r	radial stress, psi; N/cm^2

σ_z longitudinal stress, psi; N/cm^2

σ_θ tangential stress, psi; N/cm^2

Subscript:

n radial mesh points

Superscripts:

c creep

p plastic flow

ANALYSIS

The analysis of a hollow (or solid) cylinder (fig. 1) presented in this section consists of both an elastic-plastic analysis under the initial loads and a creep analysis. They are developed separately and programmed separately. Both analyses use the method of successive approximation for the solution. In the case of elastic-plastic flow, the deformation theory is used (ref. 4), whereas in the creep solution the incremental theory is applied (ref. 4).

To develop the analysis the following assumptions were made:

(1) Axial symmetry exists.

(2) Steady-state heat flow exists.

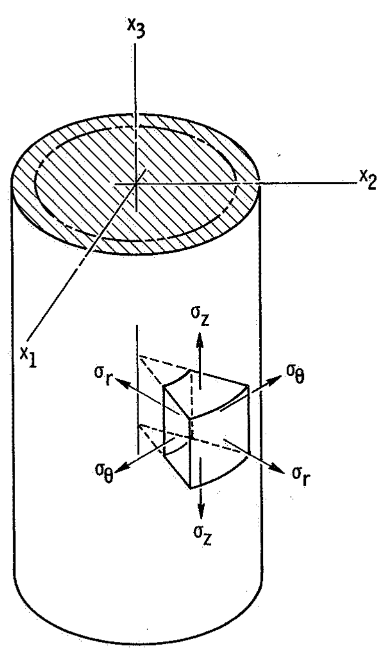


Figure 1. – Stresses of hollow (or solid) cylinder.

(3) Materials are isotropic.
(4) The von Mises' yield criterion is used.[1]
(5) Volume change due to plastic flow is zero.
(6) The cylinder consists of a strain-hardening material.
(7) Generalized plane strain exists.
(8) The cylinder is sufficiently long that Saint Venant's principle applies.

Elastic-Plastic Analysis

The following is a derivation of the equations for calculating the elastic-plastic stresses and strains in a hollow (or solid) cylinder with a symmetrical temperature gradient and internally or externally pressurized. The equilibrium equation is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{1}$$

The compatibility equation is

$$\frac{d\epsilon_\theta}{dr} + \frac{\epsilon_\theta - \epsilon_r}{r} = 0 \tag{2}$$

and the stress-strain relations are

$$\epsilon_r = \frac{1}{E}\left[\sigma_r - \mu(\sigma_\theta + \sigma_z)\right] + \alpha t + \epsilon_r^p \tag{3}$$

$$\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \mu(\sigma_r + \sigma_z)\right] + \alpha t + \epsilon_\theta^p \tag{4}$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_\theta + \sigma_r)\right] + \alpha t - \epsilon_r^p - \epsilon_\theta^p \tag{5}$$

where by assumption (5) the plastic strains are

---

[1]This yield criterion is also referred to as the distortion energy theory. It assumes that yielding begins when the distortion energy equals the distortion energy at yield in simple tension.

$$\epsilon_r^p + \epsilon_\theta^p + \epsilon_z^p = 0 \tag{6}$$

Substituting equations (3) to (5) into the compatibility equation (2) gives

$$\frac{d}{dr}\left[\frac{1}{E}\sigma_\theta - \frac{\mu}{E}\sigma_r - \mu\epsilon_z - \frac{\mu^2}{E}(\sigma_r + \sigma_\theta) + \mu\alpha t + \mu\left(-\epsilon_r^p - \epsilon_\theta^p\right) + \alpha t + \epsilon_\theta^p\right] = \frac{1+\mu}{Er}(\sigma_r - \sigma_\theta) + \frac{\epsilon_r^p - \epsilon_\theta^p}{r} \tag{7}$$

For the generalized plane-strain problem, since ϵ_z is constant,

$$\frac{d(-\mu\epsilon_z)}{dr} = 0$$

For the solid cylindrical configuration,

$$\int_0^a \sigma_z r\, dr = 0$$

and therefore,

$$\epsilon_z = -\frac{\int_0^a \left[\mu(\sigma_r + \sigma_\theta) - E\alpha t + E\left(\epsilon_r^p + \epsilon_\theta^p\right)\right] r\, dr}{\int_0^a Er\, dr} \tag{8}$$

For the case of the hollow cylindrical configuration,

$$\int_a^b \sigma_z r\, dr = \frac{a^2 p}{2}$$

and ϵ_z is changed accordingly.

This equation expresses the axial strain as a function of stresses σ_r and σ_θ, plastic strains ϵ_r^p and ϵ_θ^p, temperature t, and material properties E and α.

The method used for calculating the stress-strain state of the cylinder was the strain-strain method because of its strongly convergent characteristic. This method relates the equivalent total strain with the equivalent plastic strain.

When a material is subjected to the stress state (σ_r, σ_θ, σ_z) the strain components can be determined by application of the following basic assumptions:

(1) The principal plastic shear strains are proportional to the principal shear stresses

$$\frac{\epsilon_r^p - \epsilon_\theta^p}{\sigma_r - \sigma_\theta} = \frac{\epsilon_r^p - \epsilon_z^p}{\sigma_r - \sigma_z} = \frac{\epsilon_\theta^p - \epsilon_z^p}{\sigma_\theta - \sigma_z} = K \tag{9}$$

(2) $\epsilon_r^p + \epsilon_\theta^p + \epsilon_z^p = 0$

(3) A universal relation exists between the equivalent stress σ_e and the equivalent plastic strain ϵ_p where

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_r - \sigma_\theta\right)^2 + \left(\sigma_r - \sigma_z\right)^2 + \left(\sigma_\theta - \sigma_z\right)^2\right]^{1/2} \tag{10}$$

$$\epsilon_p = \frac{\sqrt{2}}{3}\left[\left(\epsilon_r^p - \epsilon_\theta^p\right)^2 + \left(\epsilon_r^p - \epsilon_z^p\right)^2 + \left(\epsilon_\theta^p - \epsilon_z^p\right)^2\right]^{1/2} \tag{11}$$

When the strain-strain method is being used, the elastic and plastic components of strain are separated by using the concept of equivalent total strain defined by

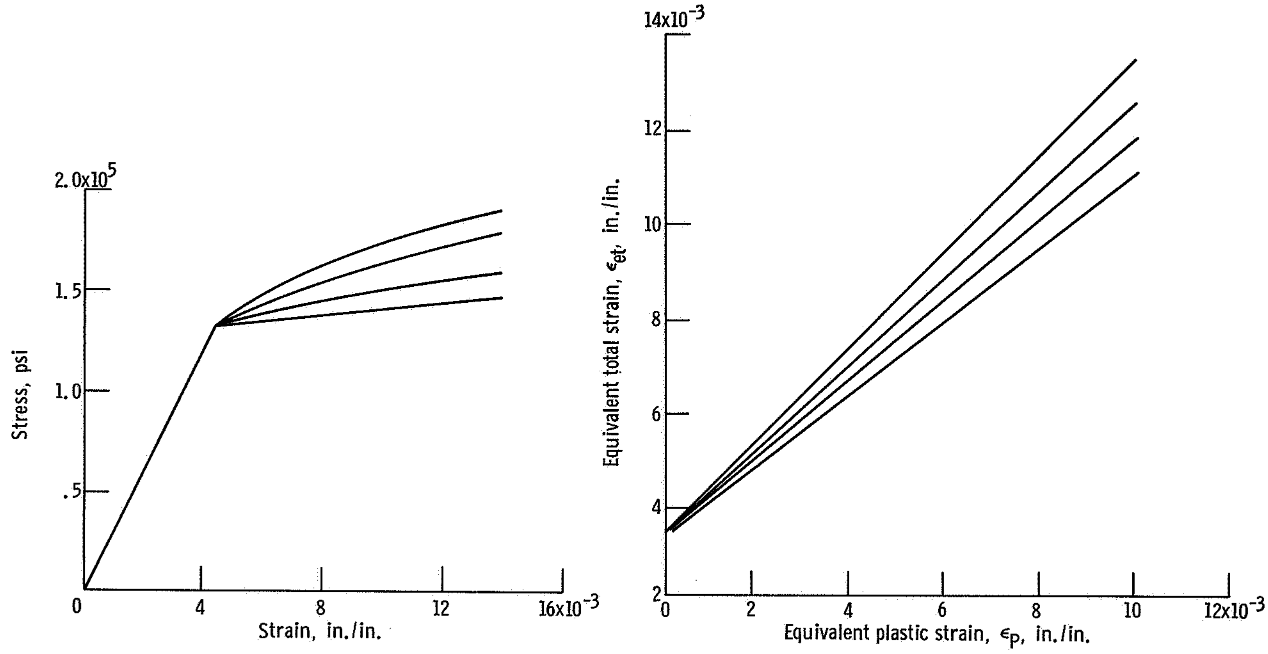
$$\epsilon_{et} = \frac{\sqrt{2}}{3}\left[(\epsilon_r - \epsilon_\theta)^2 + (\epsilon_r - \epsilon_z)^2 + (\epsilon_\theta - \epsilon_z)^2\right]^{1/2} \tag{12}$$

The equivalent plastic strain is related to the equivalent total strain by

$$\epsilon_{et} = \epsilon_p + \frac{2(1 + \mu)}{3}\frac{\sigma_e}{E} \tag{13}$$

From this relation a uniaxial curve (fig. 2(a)) can be replotted as a curve of ϵ_p against ϵ_{et} (fig. 2(b)). From equations (9) to (13), the individual plastic strains for the strain-strain method become

$$\epsilon_r^p = \frac{\epsilon_p}{3\epsilon_{et}}(2\epsilon_r - \epsilon_\theta - \epsilon_z) \tag{14}$$



(a) Uniaxial stress-strain curve.

(b) Strain-strain curve.

Figure 2. - Equivalent plastic strain and equivalent total strain obtained from uniaxial curve.

$$\epsilon_\theta^p = \frac{\epsilon_p}{3\epsilon_{et}}(2\epsilon_\theta - \epsilon_r - \epsilon_z) \tag{15}$$

and ϵ_z^p can be computed from equation (6). These equations express the radial and tangential plastic strains as a function of the equivalent total strain, the equivalent plastic strain, and the total strains.

Finite Difference Formulation

In the finite difference method, a number of discrete mesh points are chosen at intervals along the radius which are not necessarily equal. There are thus $n + 1$ mesh points, the first at the center of the cylinder (or inner radius) and the last at the outer radius. Using the method of middle differences gives the typical equations

$$\left.\begin{aligned} \frac{d\sigma}{dr} &= \frac{\sigma_n - \sigma_{n-1}}{r_n - r_{n-1}} \\ \sigma &= \frac{\sigma_n + \sigma_{n-1}}{2} \end{aligned}\right\} \tag{16}$$

Equation (7) can be restated as

$$\frac{d}{dr}\left[\frac{1-\mu^2}{E}\sigma_\theta - \left(\frac{\mu}{E}+\frac{\mu^2}{E}\right)\sigma_r + (1+\mu)\alpha t + \epsilon_\theta^p\right] = \frac{1+\mu}{Er}\sigma_r - \frac{1+\mu}{Er}\sigma_\theta + \frac{\epsilon_r^p}{r} - \frac{\epsilon_\theta^p}{r} + \mu\left(-\epsilon_r^p - \epsilon_\theta^p\right) \tag{17}$$

Applying equations (16) results in

$$\begin{aligned}&\left\{\frac{1-\mu^2}{Eh_n}(\sigma_{\theta,n} - \sigma_{\theta,n-1}) - \left(\frac{\mu}{E}+\frac{\mu^2}{E}\right)\frac{1}{h_n}(\sigma_{r,n} - \sigma_{r,n-1})\right.\\
&+(1+\mu)\left[(\alpha t)_n - (\alpha t)_{n-1}\right] - \frac{\mu}{h_n}(\epsilon_{z,n} - \epsilon_{z,n-1})\\
&\left.+\frac{\mu}{h_n}\left(-\epsilon_{r,n}^p - \epsilon_{\theta,n}^p + \epsilon_{r,n-1}^p + \epsilon_{\theta,n-1}^p\right) + \frac{1}{h_n}\left(\epsilon_{\theta,n}^p - \epsilon_{\theta,n-1}^p\right)\right\} = \frac{1+\mu}{2Er}(\sigma_{r,n} + \sigma_{r,n-1})\\
&-\frac{1+\mu}{2Er}(\sigma_{\theta,n} + \sigma_{\theta,n-1}) + \frac{1}{2r}\left(\epsilon_{r,n}^p + \epsilon_{r,n-1}^p\right) - \frac{1}{2r}\left(\epsilon_{\theta,n}^p + \epsilon_{\theta,n-1}^p\right)\end{aligned} \tag{18}$$

Also applying equations (16) to equation (1) gives

$$\frac{\sigma_{r,n} - \sigma_{r,n-1}}{r_n - r_{n-1}} + \frac{\sigma_{r,n} + \sigma_{r,n-1}}{2r} - \frac{\sigma_{\theta,n} + \sigma_{\theta,n-1}}{2r} = 0 \tag{19}$$

Now, by collecting all the n and $n - 1$ terms, equations (18) and (19) can be restated as follows:

$$C_n'\sigma_{r,n} + D_n'\sigma_{\theta,n} = F_n'\sigma_{r,n-1} + G_n'\sigma_{\theta,n-1} + H_n' + P_n' \tag{20}$$

$$C_n\sigma_{r,n} - D_n\sigma_{\theta,n} = F_n\sigma_{r,n-1} + G\sigma_{\theta,n-1} \tag{21}$$

where

$$
\left.
\begin{aligned}
C_n &= \frac{1}{h_n} + \frac{1}{2r_n} & C'_n &= \frac{-\mu}{h_nE_n} - \frac{\mu^2}{h_nE_n} - \frac{1+\mu}{2E_nr_n} \\
D_n &= \frac{1}{2r_n} & D'_n &= \frac{1}{h_nE_n} - \frac{\mu^2}{h_nE_n} + \frac{1+\mu}{2E_nr_n} \\
F_n &= \frac{1}{h_n} - \frac{1}{2r_{n-1}} & F'_n &= -\frac{\mu}{h_nE_{n-1}} - \frac{\mu^2}{h_nE_{n-1}} + \frac{1+\mu}{2E_{n-1}r_{n-1}} \\
G_n &= \frac{1}{2r_{n-1}} & G'_n &= \frac{1}{h_nE_{n-1}} - \frac{\mu^2}{h_nE_{n-1}} - \frac{1+\mu}{2E_{n-1}r_{n-1}} \\
& & H'_n &= -\frac{1+\mu}{h_n}\left[(\alpha t)_n - (\alpha t)_{n-1}\right] \\
\end{aligned}
\right\}
\quad (22)
$$

$$
P'_n = \left(-\frac{1}{h_n} - \frac{1}{2r_n} + \frac{\mu}{h_n}\right)\epsilon^p_{\theta,n} + \left(\frac{1}{2r_n} + \frac{\mu}{h_n}\right)\epsilon^p_{r,n} + \left(\frac{1}{h_n} - \frac{1}{2r_{n-1}} - \frac{\mu}{h_n}\right)\epsilon^p_{\theta,n-1} + \left(\frac{1}{2r_{n-1}} - \frac{\mu}{h_n}\right)\epsilon^p_{r,n-1}
$$

Considering the linear nature of equations (20) and (21), it follows that the stresses at any mesh point can ultimately be expressed in linear terms of the stresses at any other mesh point. Solving for the stresses at the n^{th} mesh point in terms of stresses at the $n - 1$ mesh point gives

$$
\sigma_{r,n} = l_{11,n}\sigma_{r,n-1} + l_{12,n}\sigma_{\theta,n-1} + M_{1,n} \tag{23}
$$

$$
\sigma_{\theta,n} = l_{21,n}\sigma_{r,n-1} + l_{22,n}\sigma_{\theta,n-1} + M_{2,n} \tag{24}
$$

where

$$\left.\begin{aligned}
l_{11,n} &= \frac{D'_nF_n + D_nF'_n}{C_nD'_n + C'_nD_n} & l_{12,n} &= \frac{D'_nG_n + D_nG'_n}{C_nD'_n + C'_nD_n} \\
l_{21,n} &= \frac{C_nF'_n - C'_nF_n}{C_nD'_n + C'_nD_n} & l_{22,n} &= \frac{C_nG'_n - C'_nG_n}{C_nD'_n + C'_nD_n} \\
M_{1,n} &= \frac{D_n(H'_n + P'_n)}{C_nD'_n + C'_nD_n} & M_{2,n} &= \frac{C_n(H'_n + P'_n)}{C_nD'_n + C'_nD_n}
\end{aligned}\right\} \tag{25}$$

In matrix notation

$$\sigma_n = L_n\sigma_{n-1} + M_n \tag{26}$$

where σ_n, L_n, σ_{n-1}, and M_n are the indicated matrices. Equation (26) represents a linear recurrence relation between the stresses at the n mesh point and the $n - 1$ mesh point by which they can be linearly related to the stresses at the first mesh point.

$$\sigma_n = A_n\sigma_1 + B_n \tag{27}$$

where A_n and B_n are yet unknown. Substituting equation (27) into equation (26) gives

$$(A_n - A_{n-1}L_n)\sigma_1 = L_nB_{n-1} - B_n + M_n \tag{28}$$

Now σ_1 depends on the boundary conditions and therefore is completely arbitrary; however, equation (28) must be true for all values of σ_1. Therefore, both sides of the equation must vanish identically, as the A's and B's are independent of the boundary conditions and are functions only of the geometry and material properties. Therefore,

$$A_n = A_{n-1}L_n \tag{29}$$

and

$$B_n = L_nB_{n-1} + M_n \tag{30}$$

Now, for the second mesh point, equation (27) gives

$$\sigma_2 = A_2\sigma_1 + B_2 \tag{31}$$

and equation (26) gives

$$\sigma_2 = L_2\sigma_1 + M_2 \tag{32}$$

Therefore, $A_2 = L_2$, $B_2 = M_2$, and by the recurrence relations (eqs. (29) and (30)) all other A_n and B_n can be computed. For the solid cylinder, the following boundary conditions were used:

$$\left.\begin{aligned} \sigma_{r,a} &= -P \qquad \text{at } r = a \\ \sigma_1 = \sigma_{\theta,1} &= \sigma_{r,1} \qquad \text{at } r = 0 \end{aligned}\right\} \tag{33}$$

Substituting equations (33) into equation (27) yields

$$\sigma_{\theta,n} = A_{\theta,n}\sigma_{\theta,1} + B_{\theta,n}$$

$$\sigma_{r,n} = A_{r,n}\sigma_{\theta,1} + B_{r,n}$$

Therefore, $A_{r,1} = A_{\theta,1} = 1$ and $B_{r,1} = B_{\theta,1} = 0$.

$$\sigma_{\theta,1} = \frac{-B_{r,a} + \sigma_{r,a}}{A_{r,a}} \tag{34}$$

The tangential stress at the inner mesh point is a function only of the coefficients $A_{r,a}$, $B_{r,a}$ and the radial stress at $r = a$. After A_n and B_n are derived from equations (29) and (30) and $\sigma_{\theta,1}$ from equation (34), the stresses at every mesh point can be computed from equation (27).

Creep Analysis

The creep analysis is treated in a manner essentially similar to that of the elastic-plastic flow. However, since the loading path is now no longer a one-step process but rather an incremental time-loading process, the equations derived in the previous section must be altered. The following, therefore, is the analysis of a hollow (or solid) cylinder with a symmetrical temperature, internally or externally pressurized and undergoing a creep over a predetermined period of time.

Modification of elastic-plastic equations. - The equilibrium and compatibility equations (eqs. (1) and (2)) remain the same, but the stress-strain relations now reflect the incremental creep strain and the total creep strain. These relations are

$$\epsilon_r = \frac{1}{E}\left[\sigma_r - \mu(\sigma_\theta + \sigma_z)\right] + \alpha t + \epsilon_r^c + \Delta\epsilon_r^c \tag{35}$$

$$\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \mu(\sigma_r + \sigma_z)\right] + \alpha t + \epsilon_\theta^c + \Delta\epsilon_\theta^c \tag{36}$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_\theta + \sigma_r)\right] + \alpha t - \epsilon_r^c - \epsilon_\theta^c - \Delta\epsilon_r^c - \Delta\epsilon_\theta^c \tag{37}$$

where again the incompressibility assumption applies to both the incremental and total creep strains. The compatibility equation in terms of stresses becomes

$$\frac{d}{dr}\left[\frac{1-\mu^2}{E_c}\sigma_\theta - \left(\frac{\mu}{E} + \frac{\mu^2}{E}\right)\sigma_r + (1+\mu)\alpha t + \mu\left(-\epsilon_r^c - \epsilon_\theta^c - \Delta\epsilon_r^c - \Delta\epsilon_\theta^c\right) + \epsilon_\theta^c + \Delta\epsilon_\theta^c\right]$$
$$= \frac{1+\mu}{Er}(\sigma_r - \sigma_\theta) + \frac{\epsilon_r^c + \Delta\epsilon_r^c - \epsilon_\theta^c - \Delta\epsilon_\theta^c}{r} \tag{38}$$

The equivalent stress is calculated by using equation (10). The strain increments due to creep are then calculated by

$$\Delta\epsilon_r^c = \frac{\Delta\epsilon_{ec}}{2\sigma_e}(2\sigma_r - \sigma_\theta - \sigma_z) \tag{39}$$

$$\Delta\epsilon_\theta^c = \frac{\Delta\epsilon_{ec}}{2\sigma_e}(2\sigma_\theta - \sigma_r - \sigma_z) \tag{40}$$

$$\Delta\epsilon_z^c = -\Delta\epsilon_r^c - \Delta\epsilon_\theta^c \tag{41}$$

where $\Delta\epsilon_{ec}$ is obtained from the power law relation between the equivalent stress, equivalent strain, and total time T. This will be discussed further in the next section.

In the finite difference formulation, the middle difference equation (eq. (16)) is again applied to equation (38). The geometric relations are then derived. These equations remain identical to equations (22) with the exception of P_n' which becomes

$$P'_n = \left(-\frac{1}{h_n} - \frac{1}{2r_n} + \frac{\mu}{h_n}\right)\left(\epsilon^c_{\theta,n} + \Delta\epsilon^c_{\theta,n}\right) + \left(\frac{1}{2r_n} + \frac{\mu}{h_n}\right)\left(\epsilon^c_{r,n} + \Delta\epsilon^c_{r,n}\right)$$

$$+ \left(\frac{1}{h_n} - \frac{1}{2r_{n-1}} - \frac{\mu}{h_n}\right)\left(\epsilon^c_{\theta,n-1} + \Delta\epsilon^c_{\theta,n-1}\right) + \left(\frac{1}{2r_{n-1}} - \frac{\mu}{h_n}\right)\left(\epsilon^c_{r,n-1} + \Delta\epsilon^c_{r,n-1}\right) \quad (42)$$

Stress-strain - time relations. - In creep the strain path becomes time dependent. The relation between the strain and time can be expressed by three basic laws (ref. 5), the time-hardening rule, the strain-hardening rule, or the life fraction rule.

There are many stress-strain - time relations in existence (ref. 5); however, the ones used are often dependent upon the data available. A typical example is the constant-temperature stress-strain - time relation. This is one in which the logarithm of the linear creep rate against the logarithm of the stress is linear.

$$\left.\begin{aligned} &\log \dot{\epsilon} = \log A + N \log \sigma \\ \text{or} \quad & \\ &\dot{\epsilon} = A\sigma^N \\ \text{and} \quad & \\ &\Delta\epsilon_{ec} = A\sigma^N \, \Delta T \end{aligned}\right\} \quad (43)$$

where the values of N and A are based upon data available. Since this relation holds only for a constant temperature, similar relations would be necessary for a case where a wide range of temperatures are encountered. Also this relation is valid only in the secondary creep regime.

Other creep functions are a power-law relation of equivalent stress, equivalent strain, and the total time T during which the stress persists

$$\epsilon_{ec} = K\sigma_e^m T^n$$

and a creep function in which the creep curves at a given stress but different temperatures can be brought into coincidence

$$\epsilon_{ec} = K'\sigma_e^m T^n$$

where $K' = Ke^{-n\,\Delta H/RT}$. This relation was first introduced by Dorn (ref. 6).

METHOD OF CALCULATION

The programming of the elastic-plastic and creep analyses was performed separately. The elastic-plastic analysis is used first and represents the cylinder at a time equal to zero. Since the temperature increase is not uniform, the volumetric increase is also not uniform, which results in thermal stresses. Superimposed on these thermal stresses are stresses as a result of pressure loads. The stress state resulting from this initial loading and occuring at a time equal to zero then becomes the input to the creep analysis for calculating the stress state after a period of time.

The following is a brief summary of the program technique of calculating both elastic-plastic deformation and creep. Program listings are given in appendix C.

Summary of Method of Calculation for Elastic-Plastic Deformation

The procedure for computation is as follows:

(1) Set total plastic strains equal to zero, as starting condition.

(2) Or set total plastic strains equal to plastic strains of previous iteration.

(3) Solve for stresses and total strains from the sets of equilibrium equations, compatibility equations, and stress-strain relations, as in any elastic problem. A first approximation is thus obtained for the stresses and total strains. This includes the geometric calculations of equation (22), the matrices L_n and M_n of equation (25) and A_n and B_n of equations (29) and (30), and finally the stresses using equation (27) with the aid of equation (34).

(4) Calculate the equivalent total strain from equation (12).

(5) Calculate the equivalent plastic strain from equation (13).

(6) If ϵ_p is positive, yielding has occurred. Compute plastic strains.

(7) Compute plastic strain components from equations (14) and (15). Return to step (2).

(8) Repeat process until convergence is achieved.

(Convergence is required of the total strains ϵ_r and ϵ_θ at each mesh point.) A sample problem utilizing this method of calculating elastic-plastic deformation is included in appendix A.

Summary of Method of Calculation for Creep Deformation

The final calculations of the elastic-plastic solution (which represent the stress-strain state at time zero) become the input to the creep calculations. The method then proceeds as follows:

(1) Set the initial total strains and P_n' of the creep analysis equal to the total strains and P_n' of the elastic-plastic calculations.

(2) Choose a time increment. (The size of this interval depends on the particular problem.)

(3) Solve for the stresses and total strains from the sets of equilibrium equations, compatibility equations, and stress-strain relations, as in the elastic-plastic calculations.

(4) Calculate the equivalent stress using equation (10).

(5) Calculate the equivalent creep strain from equation (43).

(6) Calculate creep strain increments using equations (39) to (41).

(7) Repeat steps 3 to 6 until two successive calculations of the plastic strain increments $\Delta\epsilon_r^c$, $\Delta\epsilon_\theta^c$ at each mesh point remain within the convergence criteria.

(8) Add incremental strains (eqs. (39) to (41)) to total strain and set incremental strains equal to zero.

(9) Return to step 3 for next time increment.

(10) The problem is completed when total time has been reached.

A sample problem utilizing this method of calculating creep deformation is included in appendix A.

CONCLUDING REMARKS

A computer program for the stress analysis of cylindrical rods, tubes, and vessels is presented. The primary advantage of this analysis is that it allows the material used in a design application to operate in the plastic range. Other advantages are as follows:

(1) It takes into account the material property changes across the cylinder wall.

(2) It calculates stresses and strains in cylinders of composite material.

(3) Both hollow and solid cylindrical geometries can be analyzed.

(4) Computer running times are short (less than 1 minute for the elastic-plastic analysis and 2 minutes for the creep analysis of the example problem).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 1, 1968,
126-15-01-04-22.

APPENDIX A

EXAMPLE SOLUTION

The operating instructions for using the elastic-plastic program and the creep program are presented in appendix B. To further clarify the method of using these programs, an example calculation is presented. The problem involves a typical high-alloy steel tubing 2.50 inches in outside diameter and 2.25 inches in inside diameter with a temperature gradient across the wall. The heat flux Q/A is from the inside wall outward. No pressure loads exist, leaving only thermal stresses. The following steps were taken in the solution of this example problem in the elastic-plastic program:

(1) Obtain uniaxial stress-strain curves for the material at different temperatures (fig. 3).

(2) Convert each stress-strain curve to an equivalent plastic strain - equivalent total strain curve by using equation (13). (See fig. 4 for a typical strain-strain curve.) The stress-strain curve has been approximated by two straight lines. The equations of these straight lines in the plastic region form a part of the subroutine PLSTR.

(3) Divide the 0.125-inch wall into any desired number of mesh points, recording their radial location.

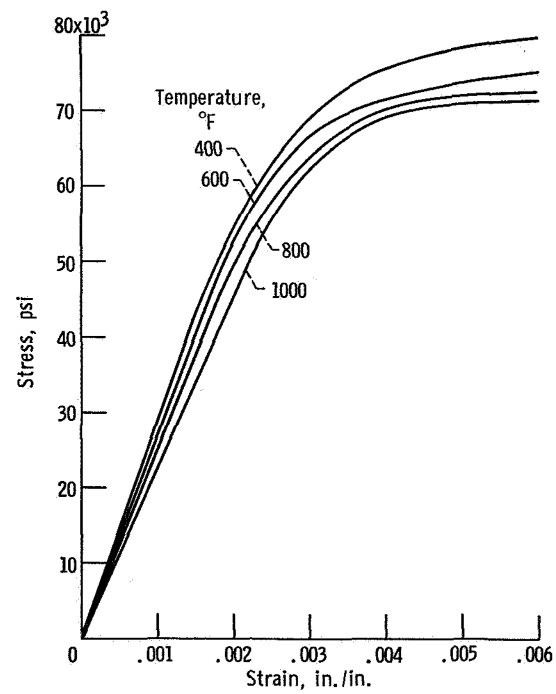


Figure 3. - Stress as function of strain for high-alloy steel tubing.

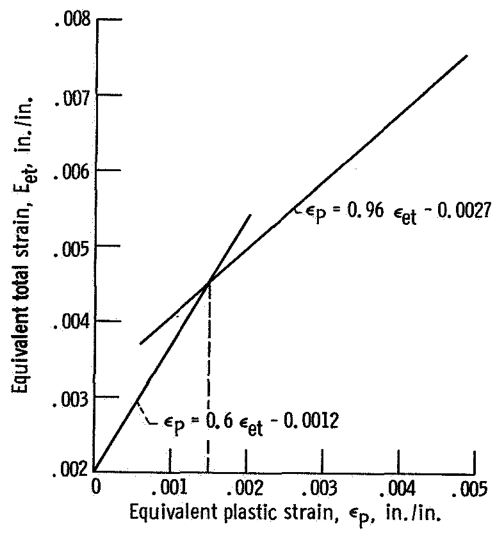


Figure 4. - Equivalent total strain as function of equivalent plastic strain at 1000° F for high-alloy steel tubing.

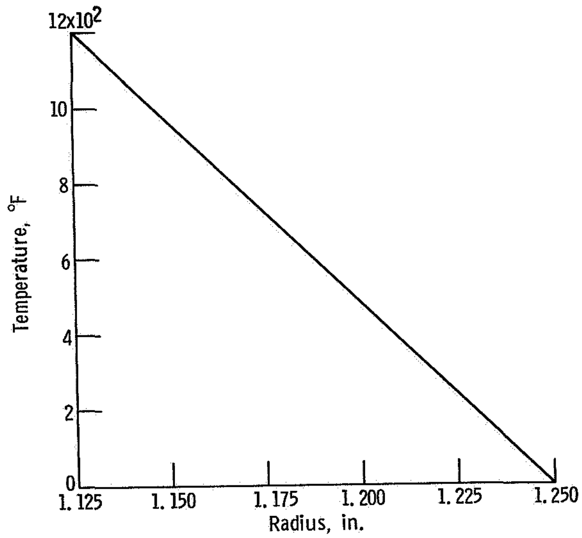


Figure 5. - Temperature profile through wall of tubing.

$$T = \frac{T_i}{\log\left(\frac{b}{a}\right)} \log \frac{b}{r}.$$

(4) Using the thermal temperature gradient of figure 5, record the respective temperatures of each mesh point.

(5) Complete the program input by recording the values of modulus of elasticity and coefficient of thermal expansion at each mesh point for that respective temperature.

The input is complete, and the program output will be as outlined in appendix B. The input-output of the sample problem is as follows:

NO OF ITERATIONS = 1 NO OF STATIONS = 14

STATION NO	CYL RADIUS	TEMPERATURE	ALPHA	E
1	1.125000000	1200.000000	0.102000000E-04	22000000.00
2	1.135000005	1100.000000	0.101000001E-04	22500000.00
3	1.144999996	1000.000000	0.100000000E-04	23000000.00
4	1.155000001	910.0000000	0.990000001E-05	23500000.00
5	1.165000007	812.0000000	0.980000004E-05	24000000.00
6	1.174999997	720.0000000	0.969999996E-05	24500000.00
7	1.185000002	625.0000000	0.960000000E-05	25000000.00
8	1.194999993	530.0000000	0.950000003E-05	25500000.00
9	1.204999998	432.0000000	0.939999995E-05	26000000.00
10	1.215000004	335.0000000	0.929999999E-05	26500000.00
11	1.224999994	240.0000000	0.929999999E-05	27000000.00
12	1.234999999	145.0000000	0.920000002E-05	27500000.00
13	1.245000005	50.00000000	0.920000002E-05	28000000.00
14	1.250000000	0	0.920000002E-05	28500000.00

POISSONS RATIO = 0.30000 RAD R STRESS = 0 RAD I STRESS = 0

STATION NO	TOTAL STRAIN R	TOTAL STRAIN T	TOTAL STRAIN Z
1	0.180260448E-01	0.547730218E-02	0.550192618E-02
2	0.158272595E-01	0.557822443E-02	0.550192595E-02
3	0.136874190E-01	0.565843564E-02	0.550192583E-02
4	0.117882340E-01	0.571976742E-02	0.550192583E-02
5	0.979515305E-02	0.576334196E-02	0.550192571E-02
6	0.796385459E-02	0.578989671E-02	0.550192577E-02
7	0.612923561E-02	0.580053451E-02	0.550192583E-02
8	0.434511906E-02	0.579585147E-02	0.550192565E-02
9	0.255839454E-02	0.577642844E-02	0.550192571E-02
10	0.839047774E-03	0.574289513E-02	0.550192565E-02
11	-0.753082335E-03	0.569639017E-02	0.550192565E-02
12	-0.236056492E-02	0.563768391E-02	0.550192560E-02
13	-0.391244487E-02	0.556723110E-02	0.550192548E-02
14	-0.472697651E-02	0.552768627E-02	0.550192565E-02

STATION NO	PLASTIC STRAIN R	PLASTIC STRAIN T	CYL RADIUS
1	0.549046503E-02	-0.275332076E-02	1.131161943
2	0.402100914E-02	-0.198813717E-02	1.141331270
3	0.259709929E-02	-0.126094700E-02	1.151478887
4	0.172799350E-02	-0.818294313E-03	1.161606327
5	0.491611441E-03	-0.222649933E-03	1.171714291
6	0	-0	1.181803107
7	0	-0	1.191873625
8	-0	0	1.201926023
9	-0	0	1.211960584
10	-0.950788242E-03	0.511317216E-03	1.221977606
11	-0.192332813E-02	0.100582397E-02	1.231978059
12	-0.290233287E-02	0.148842976E-02	1.241962522
13	-0.384348340E-02	0.194166868E-02	1.251931190
14	-0.433673261E-02	0.217654728E-02	1.256909594

STATION NO	RADIAL STRESS	TANGENT STRESS	LONG STRESS	EQUIV STRESS
1	-0	-212363.3438	-211946.6289	212155.8242
2	-1725.279205	-179112.4277	-180432.9785	178051.5410
3	-3127.030182	-145178.2773	-147947.2969	143456.1582
4	-4222.230225	-113921.4355	-117859.3408	111720.5000
5	-5016.575806	-79450.01172	-84276.15820	76960.27734
6	-5510.375732	-46481.12109	-51908.26563	43936.54150
7	-5711.390991	-12032.56641	-17775.04102	10451.45972
8	-5618.997131	22837.67773	17072.20996	26056.86670
9	-5232.874634	59127.80469	53637.74902	61798.97363
10	-4554.446777	95408.59766	90496.52734	97600.00293
11	-3596.340088	130354.2441	126315.3652	131977.8320
12	-2364.607666	166829.1211	163957.3105	167776.6777
13	-859.6215820	203318.0195	201911.4375	203478.5039
14	-0	224813.7637	224249.0117	224532.4824
1.000000000	-1.000000000	-1.000000000	-0.550192577E-02	

STATION NO	ELASTIC STRAIN	PLASTIC STRAIN	P PRIME
1	0.835761137E-02	0.549045909E-02	500000000000
2	0.685824704E-02	0.402108202E-02	-0.913353693E-01
3	0.540556712E-02	0.259745569E-02	-0.892887600E-01
4	0.412016787E-02	0.172879477E-02	-0.542718042E-01
5	0.277910713E-02	0.492336781E-03	-0.773776863E-01
6	0.155420994E-02	0	-0.300273171E-01
7	0.362315437E-03	0	0
8	0.885588393E-03	0	-0
9	0.205995538E-02	0	-0
10	0.319193371E-02	0.951690250E-03	-0.649175057E-01
11	0.423630414E-02	0.192399911E-02	-0.655889912E-01
12	0.528748072E-02	0.290264451E-02	-0.661257198E-01
13	0.629811239E-02	0.384354260E-02	-0.640621968E-01
14	0.682785391E-02	0.433673197E-02	-0.674066823E-01

STATION NO	M1	M2	B1	B2
1	500000000000	500000000000	-0	0
2	158.6089420	36162.81934	158.6089420	36162.81934
3	157.8848305	36313.54590	633.0419922	72956.88867
4	142.7867050	33126.49805	1406.085037	107023.4395
5	153.3958225	35894.60303	2471.819702	144256.0762
6	143.7804546	33932.21973	3829.736908	179939.3086
7	147.0413113	34995.81299	5471.903687	217056.8770
8	145.8653069	35007.70654	7398.780457	254552.2969
9	148.9077873	36035.66455	9610.550659	293425.1719
10	146.0668392	35640.28906	12105.66260	332247.6367
11	137.9669018	33939.89014	14871.37695	369695.1914
12	141.6810799	35136.88916	17901.54907	408633.4414
13	139.2829418	34820.71631	21195.72046	447548.3750
14	37.30787516	18691.26318	22950.60693	472329.9336

NO OF ITERATIONS = 500 NO OF STATIONS = 14

STATION NO	CYL RADIUS	TEMPERATURE	ALPHA	E
1	1.125000000	1200.000000	0.102000000E-04	22000000.00
2	1.135000005	1100.000000	0.101000001E-04	22500000.00
3	1.144999996	1000.000000	0.100000000E-04	23000000.00
4	1.155000001	910.0000000	0.990000001E-05	23500000.00
5	1.165000007	812.0000000	0.980000004E-05	24000000.00
6	1.174999997	720.0000000	0.969999996E-05	24500000.00
7	1.185000002	625.0000000	0.960000000E-05	25000000.00
8	1.194999993	530.0000000	0.950000003E-05	25500000.00
9	1.204999998	432.0000000	0.939999995E-05	26000000.00
10	1.215000004	335.0000000	0.929999999E-05	26500000.00
11	1.224999994	240.0000000	0.929999999E-05	27000000.00
12	1.234999999	145.0000000	0.920000002E-05	27500000.00
13	1.245000005	50.00000000	0.920000002E-05	28000000.00
14	1.250000000	0	0.920000002E-05	28500000.00

POISSONS RATIO = 0.30000 RAD R STRESS = 0 RAD I STRESS = 0

STATION NO	TOTAL STRAIN R	TOTAL STRAIN T	TOTAL STRAIN Z
1	0.232042021E-01	0.530309591E-02	0.550127635E-02
2	0.196806409E-01	0.544536702E-02	0.550127646E-02
3	0.162504725E-01	0.555478351E-02	0.550127629E-02
4	0.134996619E-01	0.563553360E-02	0.550127635E-02
5	0.104154787E-01	0.568985922E-02	0.550127629E-02
6	0.807136204E-02	0.572014606E-02	0.550127629E-02
7	0.623372180E-02	0.573226705E-02	0.550127629E-02
8	0.444671017E-02	0.572901760E-02	0.550127623E-02
9	0.265720961E-02	0.571098103E-02	0.550127617E-02
10	0.163131190E-03	0.567562436E-02	0.550127623E-02
11	-0.230005480E-02	0.562060997E-02	0.550127623E-02
12	-0.478707155E-02	0.554644456E-02	0.550127629E-02
13	-0.718751492E-02	0.545383984E-02	0.550127623E-02
14	-0.844740018E-02	0.540075928E-02	0.550127617E-02

STATION NO	PLASTIC STRAIN R	PLASTIC STRAIN T	CYL RADIUS
1	0.893076870E-02	-0.453995052E-02	1.130965963
2	0.658212387E-02	-0.331048871E-02	1.141180485
3	0.430534745E-02	-0.213656027E-02	1.151360214
4	0.280529633E-02	-0.136703295E-02	1.161509037
5	0.893820659E-03	-0.420681743E-03	1.171628669
6	0	-0	1.181721151
7	0	-0	1.191792712
8	-0	0	1.201846153
9	-0	0	1.211881712
10	-0.134809141E-02	0.706537408E-03	1.221895874
11	-0.285922867E-02	0.146216770E-02	1.231885225
12	-0.437966111E-02	0.220421975E-02	1.241849840
13	-0.584079395E-02	0.290398972E-02	1.251790017
14	-0.660631910E-02	0.326732561E-02	1.256750926

STATION NO	RADIAL STRESS	TANGENT STRESS	LONG STRESS	EQUIV STRESS
1	-0	-74979.28516	-74149.11328	74567.85156
2	-664.3648300	-75829.33203	-75534.03809	75017.94336
3	-1324.145630	-76586.88086	-76963.32520	75451.84961
4	-1941.719864	-68681.23438	-69820.54883	67316.57031
5	-2500.925873	-65478.51660	-67991.67969	64271.19482
6	-2959.774567	-47271.15381	-51396.00781	46511.30518
7	-3189.023010	-12832.38379	-17274.51367	12472.56042
8	-3124.469849	22028.48438	17561.25000	23243.61938
9	-2765.802094	58309.62695	54115.53027	59090.26709
10	-2222.319672	68261.56055	66032.24902	69396.25781
11	-1627.158264	73123.53223	71997.25781	74194.15039
12	-1000.936218	78314.92383	77968.15430	79143.24219
13	-342.7797241	83580.01660	83894.84766	84080.86426
14	-0	87130.21387	87762.55664	87448.31836
24.00000000	0.142497620E-04	0.826648125E-05	-0.550127629E-02	

STATION NO	ELASTIC STRAIN	PLASTIC STRAIN	P PRIME
1	0.118685322E-01	0.893116137E-02	500000000000
2	0.947157713E-02	0.658214552E-02	-0.146176621
3	0.714834378E-02	0.430537685E-02	-0.143307388
4	0.528805936E-02	0.280559069E-02	-0.942491256E-01
5	0.321510996E-02	0.894331373E-03	-0.121218413
6	0.164528955E-02	0	-0.556982309E-01
7	0.432379915E-03	0	0
8	0.789975224E-03	0	-0
9	0.196966564E-02	0	-0
10	0.361827068E-02	0.134860998E-02	-0.907458374E-01
11	0.524110487E-02	0.285946857E-02	-0.100837685
12	0.687398674E-02	0.437968160E-02	-0.101985931
13	0.844340504E-02	0.584081002E-02	-0.989953279E-01
14	0.926577020E-02	0.660643203E-02	-0.104260035

STATION NO	M1	M2	B1	B2
1	500000000000	500000000000	-0	0
2	0.780969054	178.0608482	0.780969054	178.0608482
3	1.086252972	249.8384209	3.425862819	430.2653008
4	38.32816172	8892.128784	45.47364235	9327.938232
5	17.35458302	4060.970306	143.0075321	13505.55957
6	80.51584721	19001.75781	337.9459915	32671.27661
7	147.0413113	34995.81299	759.4712524	68052.39746
8	145.8653069	35007.70654	1471.726669	103840.1387
9	148.9077873	36035.66455	2474.977203	141033.1738
10	38.19695711	9320.052490	3659.877716	151882.4980
11	16.83815813	4142.190857	4893.252502	157627.8203
12	17.90616775	4440.727112	6154.450928	163688.9570
13	17.92784357	4481.958435	7444.316467	169810.6113
14	4.785164595	2397.369751	8103.187988	174520.9336

Both the elastic solution (first iteration) and the plastic solution (convergence) after 24 iterations are shown. The plastic strains indicated in the elastic solution must be ignored, as they represent only the initial plastic calculations and the elastic solution was not yet adjusted. Although the sample problem presented is that of a tube containing a single material, the application can be extended to composite materials (ref. 7). It may be desirable to subscript the Poisson's ratio variable if large differences exist between the composite materials. Also where two dissimilar materials have large differences in properties, it may be necessary to select finer mesh points at these inner faces to improve convergence of the solution.

The following steps were taken in the solution of the example problem in the creep program:

(1) Obtain creep equations similar to equation (43). These equations form a part of the subroutine PLSTR.

(2) Complete the input. The creep input is similar to that of the elastic-plastic program except for the plastic strains and the variable P'_n. These values are obtained directly from the output of the elastic-plastic solution.

With the input complete the output will be as outlined in appendix B. The input-output of the creep portion of the sample problem at the end of 692.5 hours is as follows:

NO OF ITERATIONS = 10 NO OF STATIONS = 14

STATION NO	CYL RADIUS	TEMPERATURE	ALPHA	E
1	1.125000000	1200.000000	0.102000000E-04	22000000.00
2	1.135000005	1100.000000	0.101000001E-04	22500000.00
3	1.144999996	1000.000000	0.100000000E-04	23000000.00
4	1.155000001	910.0000000	0.990000001E-05	23500000.00
5	1.165000007	812.0000000	0.980000004E-05	24000000.00
6	1.174999997	720.0000000	0.969999996E-05	24500000.00
7	1.185000002	625.0000000	0.960000000E-05	25000000.00
8	1.194999993	530.0000000	0.950000003E-05	25500000.00
9	1.204999998	432.0000000	0.939999995E-05	26000000.00
10	1.215000004	335.0000000	0.929999999E-05	26500000.00
11	1.224999994	240.0000000	0.929999999E-05	27000000.00
12	1.234999999	145.0000000	0.920000002E-05	27500000.00
13	1.245000005	50.00000000	0.920000002E-05	28000000.00
14	1.250000000	0	0.920000002E-05	28500000.00

STATION NO	PLASTIC STRAIN R	PLASTIC STRAIN T	P PRIME
1	0.893076998E-02	-0.453989999E-02	0
2	0.658212003E-02	-0.331050000E-02	-0.146176599
3	0.430535001E-02	-0.213650000E-02	-0.143307395
4	0.280528999E-02	-0.136700000E-02	-0.942491004E-01
5	0.893819997E-03	-0.420699998E-03	-0.121218398
6	0	0	-0.556982001E-01
7	0	0	0
8	0	0	0
9	0	0	0
10	-0.134810001E-02	0.706539999E-03	-0.907458002E-01
11	-0.285920000E-02	0.145217000E-02	-0.100837700
12	-0.437970000E-02	0.220422001E-02	-0.101985894
13	-0.584080000E-02	0.290398000E-02	-0.989953000E-01
14	-0.660630001E-02	0.326731999E-02	-0.104259998

POISSONS RATIO = 0.30000 RAD R STRESS = 0 RAD I STRESS = 0

STATION NO	TOTAL STRAIN R	TOTAL STRAIN T	TOTAL STRAIN Z
1	0.269003077E-01	0.451400055E-02	0.492583314E-02
2	0.233316496E-01	0.469933520E-02	0.492591551E-02
3	0.198579582E-01	0.484944438E-02	0.492599356E-02
4	0.151487524E-01	0.496273971E-02	0.491852482E-02
5	0.119721447E-01	0.503708946E-02	0.491899985E-02
6	0.864242378E-02	0.507809949E-02	0.493278564E-02
7	0.678578269E-02	0.510037085E-02	0.493300403E-02
8	0.498755602E-02	0.510698039E-02	0.493302866E-02
9	0.318581908E-02	0.509854540E-02	0.493305258E-02
10	0.679689656E-03	0.507260120E-02	0.493303896E-02
11	-0.179456279E-02	0.502692768E-02	0.493311451E-02
12	-0.429290097E-02	0.496212341E-02	0.493314693E-02
13	-0.670485152E-02	0.487898220E-02	0.493318622E-02
14	-0.797102717E-02	0.483069528E-02	0.493321562E-02

STATION NO	INCR PLAS STR R	INCR PLAS STR T	CYL RADIUS
1	0.158875082E-04	-0.869187045E-05	1.130083382
2	0.154997947E-04	-0.838686697E-05	1.140338764
3	0.151296998E-04	-0.809456474E-05	1.150557488
4	0.293207054E-04	-0.148168149E-04	1.160736635
5	0.277168367E-04	-0.136881382E-04	1.170872688
6	0.458737148E-06	-0.215636042E-06	1.180971146
7	0.434372107E-07	-0.186370823E-07	1.191048294
8	-0.242165470E-11	0.109565168E-10	1.201107159
9	-0.552685973E-07	0.313838449E-07	1.211148039
10	-0.126810606E-06	0.664860282E-07	1.221167475
11	-0.174871866E-06	0.889478242E-07	1.231162205
12	-0.236497234E-06	0.118262479E-06	1.241132423
13	-0.311731835E-06	0.154110356E-06	1.251078501
14	-0.368106019E-06	0.181155308E-06	1.256042525

STATION NO	PLASTIC STRAIN R	PLASTIC STRAIN T
1	0.143595266E-01	-0.736555387E-02
2	0.119319119E-01	-0.606222823E-02
3	0.957866770E-02	-0.481354876E-02
4	0.463992445E-02	-0.228629922E-02
5	0.258208823E-02	-0.124478193E-02
6	0.146360801E-04	-0.680625436E-05
7	0.111598058E-05	-0.464183906E-06
8	-0.214632230E-07	0.158158005E-07
9	-0.358742352E-05	0.201681354E-05
10	-0.135552679E-02	0.710432992E-03
11	-0.286912062E-02	0.146722507E-02
12	-0.439275993E-02	0.221076459E-02
13	-0.585764140E-02	0.291232290E-02
14	-0.662599626E-02	0.327703042E-02

STATION NO	RADIAL STRESS	TANGENT STRESS	LONG STRESS	
1	-0	-10774.02014	-10118.16797	
2	-96.69426823	-10728.58154	-10161.55469	
3	-190.9273396	-10683.74597	-10205.00293	
4	-492.2262650	-58621.31055	-58209.18945	
5	-991.0944977	-57608.57422	-58074.25879	
6	-1520.668396	-68277.66211	-70996.45996	
7	-1942.465012	-34322.64941	-37537.51563	
8	-2069.390594	272.4332275	-3139.496094	
9	-1901.501617	35239.20801	32938.42676	
10	-1549.161514	45880.91797	44369.05469	
11	-1145.579605	50463.48877	49871.96875	
12	-711.3513031	55370.56738	55356.18457	
13	-245.7683792	60350.62012	60807.34375	
14	-0	63535.29199	64206.68066	
4.000000000	0.241283317E-05	0.364772498E-05	-0.493302877E-02	692.4847336

STATION NO	EQUIV. STRESS	PLASTIC STRAIN	P PRIME
1	10461.55042	0.159110161E-04	0
2	10360.04419	0.155172747E-04	-0.142484576
3	10261.85168	0.151421016E-04	-0.140488945
4	57924.26807	0.293213359E-04	-0.309201114
5	56851.89453	0.277176039E-04	-0.128305025
6	68157.24414	0.459012270E-06	-0.162168406
7	34101.54639	0.435828160E-07	-0.859038708E-03
8	3022.467316	0.115109960E-10	-0.692464428E-04
9	36602.20557	0.554380621E-07	-0.252246060E-03
10	46692.62500	0.126860810E-06	-0.908253556E-01
11	51315.99414	0.174881022E-06	-0.101045303
12	56079.86719	0.236497829E-06	-0.102474473
13	60826.18848	0.311739210E-06	-0.997303836E-01
14	63874.28467	0.368122151E-06	-0.105319239

STATION NO	M1	M2	B1	B2
1	500000000000	500000000000	-0	0
2	0.855753057	191.1652775	0.855753057	191.1652775
3	0.821272783	185.7605038	3.378438413	379.4372559
4	-208.6824379	-47801.14648	-201.9848442	-47416.86279
5	7.505106270	1741.419479	-605.7670593	-46264.91553
6	-42.45427990	-9972.145020	-1041.049042	-56796.81592
7	146.4223499	34758.61182	-1369.194733	-22706.71436
8	145.8960609	34988.15723	-1403.083771	12021.40430
9	148.4611835	35963.74561	-1142.771103	48119.23047
10	37.78408575	9248.021851	-698.6522064	57890.10645
11	16.56121421	4096.619507	-203.9648323	62600.05176
12	17.54501176	4385.690857	320.6998253	67632.77246
13	17.50662494	4421.687195	376.0561447	72736.79004
14	4.635173559	2350.302826	1156.672546	76089.63965

The elastic-plastic program is quite short in running time. However, the creep program running time is dependent on the total number of time increments calculated. The example problem ran less than 1 minute for the elastic-plastic analysis and 2 minutes for the creep analysis.

APPENDIX B

OPERATING INSTRUCTIONS

The plastic flow and creep analyses were kept separate, for often only one analysis is used. Control cards were kept at a minimum. Each analysis will require new stress-strain and creep curves. The subroutine PLSTR, therefore, must be rewritten each time. This subroutine occurs by the same name both in the elastic-plastic analysis and the creep analysis. The equations currently in PLSTR of the elastic-plastic solution are for the stress-strain curves of high-alloy steel at 1000°, 800°, and 600° F. The equations currently in PLSTR of the creep analysis are the constant-temperature stress-strain - time relations of high-alloy steel at 1000°, 800°, and 600° F.

Operating Instructions for "Stress" Code (Elastic-Plastic Analysis)

The following is a summary of the input cards for the "Stress" code. They are numbered 1, 2a, 2b, etc. The letter designations are for continuation cards when the number of mesh points exceed a single-card limitation.

Quantity	Format	Card columns	Remarks
Card 1			
NOFS	I5	1-5	Number of mesh points that the cylinder is divided into
NJ	I5	6-10	Control designation: 0 if cylinder contains a hole; 1 if cylinder is solid
Card 2a, 2b ...			
TEMP	8F10.7	1-10 11-20 21-30 etc.	Temperature of each mesh point; additional cards where number of mesh points exceeds 8, 16, etc.
Card 3a, 3b ...			
ALPHA	8F10.7	1-10 11-20 21-30 etc.	Coefficient of thermal expansion corresponding to temperature at each mesh point

Quantity	Format	Card column	Remarks
Card 4a, 4b . . .			
E	8F10.7	1-10 11-20 21-30 etc.	Modulus of elasticity corresponding to temperature of each mesh point
Card 5			
XMU	F10.7	1-10	Poisson's ratio
SIGRR	F10.7	11-20	Pressure load on outside radius of cylinder, psi
SIGRI	F10.7	21-30	Pressure load on inside radius of cylinder, psi; 0 if cylinder is solid
Card 6			
NOFI	I5	1-5	Maximum number of iterations allowed (If NOFI is reached, program continues to print output. If convergence is achieved prior to iteration limitation, program prints final solution.)
Card 7			
RAD	F10.7	1-10 11-20 21-30 etc.	Radial location corresponding to each mesh point, in.

Description of Output for "Stress" Code (Elastic-Plastic Analysis)

The following is a brief summary of the output for the "Stress" code. Each Roman numeral refers to a new section of output.

Section	Quantity	Remarks
I	NO OF ITERATIONS	Iteration limitation (NOFI) placed upon program (input)
	NO OF STATIONS	Number of mesh points (NOFS) the cylinder is divided into (input)
II	STATION NO	Mesh point number
	CYL RADIUS	Radial location (RAD) of mesh point (input)
	TEMPERATURE	Temperature (TEMP) of cylinder at each mesh point (input)
	ALPHA	Coefficient of linear expansion (ALPHA) of material at each mesh point (input)
	E	Modulus of elasticity E of material at each mesh point (input)
III	POISSON'S RATIO	Poisson's ratio of material (input)
	RAD R STRESS	Pressure at cylinder outer radius (input) (SIGRR)
	RAD I STRESS	Pressure at cylinder inner radius (input) (SIGRI)
IV	STATION NO	Mesh point number
	TOTAL STRAIN R	Total radial strain at each mesh point (eq. (3))
	TOTAL STRAIN T	Total tangential strain at each mesh point (eq. (4))
	TOTAL STRAIN Z	Total longitudinal strain at each mesh point (eq. (5))
V	STATION NO	Mesh point number
	PLASTIC STRAIN R	Plastic radial strain at each mesh point (eq. (14))
	PLASTIC STRAIN T	Plastic tangential strain at each mesh point (eq. (15))
	CYL RADIUS	New location of mesh point due to radial strain
VI	STATION NO	Mesh point number
	RADIAL STRESS	Radial stress in cylinder at each mesh point (eq. (27))
	TANGENTIAL STRESS	Tangential stress on cylinder at each mesh point (eq. (27))

Section	Quantity	Remarks
VI	LONG STRESS	Longitudinal stress on cylinder at each mesh point
	EQUIV STRESS	Effective stress at each mesh point (eq. (10))
VII	(No label)	Number of iterations at convergence; last calculated convergence tolerance, DELR, DELT, and constant C3 (See listing)
VIII	STATION NO	Mesh point number
	ELASTIC STRAIN	Equivalent elastic strain at each mesh point (eq. (12))
	PLASTIC STRAIN	Equivalent plastic strain at each mesh point (eq. (11))
	P PRIME	*Value of P'_n of equation (22) at each mesh point
IX	STATION NO	Mesh point number
	M1	*Value of $M_{1,n}$ of equation (25) at each mesh point
	M2	*Value of $M_{2,n}$ of equation (25) at each mesh point
	B1	*Value of equation (30) at each mesh point for radial coefficients
	B2	*Value of equation (30) at each mesh point for circumferential coefficients

(*This information is printed out to aid in evaluating the validity of the solution.)

Operating Instructions for "Creep" Code (Creep Analysis)

The following is a summary of the input cards for the "Creep" code. The cards are numbered similar to the stress code (i.e., 1, 2a, 2b, etc.).

Quantity	Format	Card columns	Remarks
Card 1			
NOFS	I5	1-5	Number of mesh points that the cylinder is divided into
NJ	I5	6-10	Control designation: 0 if cylinder contains a hole; 1 if cylinder is solid
NI	I5	11-15	Control designation; printout of output will occur after NI time increments
Card 2a, 2b ...			
RADX	8F10.7	1-10 11-20 21-30 etc.	Radial location corresponding to each mesh point prior to any creep strain
Card 3a, 3b ...			
TEMP	8F10.7	1-10 11-20 21-30 etc.	Temperature of each mesh point; additional cards where number of mesh points exceeds 8, 16, etc.
Card 4a, 4b ...			
ALPHA	8F10.7	1-10 11-20 21-30 etc.	Coefficient of thermal expansion corresponding to temperature of each mesh point
Card 5a, 5b ...			
E	8F10.7	1-10 11-20 21-30 etc.	Modulus of elasticity corresponding to the temperature of each mesh point

Quantity	Format	Card columns	Remarks
Card 6a, 6b ...			
EPR	8F10.7	1-10 11-20 21-30 etc.	Plastic radial strain at each mesh point, obtained from the output of "Stress" code
Card 7a, 7b ...			
EPT	8F10.7	1-10 11-20 21-30 etc.	Plastic tangential strain at each mesh point, obtained from output of "Stress" code
Card 8a, 8b ...			
PP	8F10.7	1-10 11-20 21-30 etc.	Value of equation (22) of "Stress" code at each mesh point, obtained from output of stress code (P PRIME)
Card 9			
XMU	F10.7	1-10	Poisson's ratio
SIGRR	F10.7	11-20	Pressure on outside radius of cylinder, psi
SIGRI	F10.7	21-30	Pressure on inside radius of cylinder, psi; 0 if cylinder is solid
ACC	F10.7	31-40	Acceleration factor for time increment; initial time increment, 0.01 hr
Card 10			
NOFI	I5	1-5	Maximum number of iterations allowed (If NOFI is reached, program continues to print output. If convergence is achieved prior to iteration limitation, program prints final solution.)
NODT	I5	6-10	Total number of time increments for which creep calculations are performed

Description of Output for "Creep" Code (Creep Analysis)

The following is a brief summary of the output for the "Creep" code. Each Roman numeral refers to a new section of output.

Section	Quantity	Remarks
I	NO OF ITERATIONS	Iteration limitation placed upon program (NOFI in input)
	NO OF STATIONS	Number of mesh points cylinder is divided into (NOFS in input)
II	STATION NO	Mesh point number
	CYL RADIUS	Radial location of mesh points prior to strain (RADX in input)
	TEMPERATURE	Temperature of cylinder at each mesh point (TEMP in input)
	ALPHA	Coefficient of linear expansion of material at each mesh point (ALPHA in input)
	E	Modulus of elasticity of material at each mesh point (E in input)
III	STATION NO	Mesh point number
	PLASTIC STRAIN R	Plastic radial strain at each mesh point (EPR of input)
	PLASTIC STRAIN T	Plastic tangential strain at each mesh point (EPT of input)
	P PRIME	Value of equation (22) of "Stress" code at each mesh point (PP of input)
IV	POISSON'S RATIO	Poisson's ratio of material (XMU of input)
	RAD R STRESS	Pressure at cylinder outer radius (SIGRR of input)
	RAD I STRESS	Pressure at cylinder inner radius (SIGRI of input), 0 if cylinder is solid
V	STATION NO	Mesh point number
	TOTAL STRAIN R	Total radial strain at each mesh point (eq. (35))
	TOTAL STRAIN T	Total tangential strain at each mesh point (eq. (36))

Section	Quantity	Remarks
V	TOTAL STRAIN Z	Total longitudinal strain at each mesh point (eq. (37))
VI	STATION NO	Mesh point number
	INCR PLAS STR R	Plastic radial strain occurring during time increment ΔT at each mesh point (eq. (39))
	INCR PLAS STR T	Plastic tangential strain occurring during time increment ΔT, at each mesh point (eq. (40))
	CYL RADIUS	New radial location of mesh point due to radial strain
VII	STATION NO	Mesh point number
	PLASTIC STRAIN R	Total plastic radial strain
	PLASTIC STRAIN T	Total plastic tangential strain
VIII	STATION NO	Mesh point number
	RADIAL STRESS	Radial stress on cylinder at each mesh point (eq. (27))
	TANGENT STRESS	Tangential stress on cylinder at each mesh point (eq. (27))
	LONG STRESS	Longitudinal stress on cylinder at each mesh point
IX	(No label)	Number of iterations at convergence; last calculated convergence tolerance constant, C3, and total time T (See listing)
X	STATION NO	Mesh point number
	EQUIV STRESS	Equivalent stress at each mesh point (eq. (10))
	PLASTIC STRAIN	Equivalent strain due to creep at each mesh point (eq. (43))
	P PRIME	*Value of equation (42) at each mesh point
XI	STATION NO	Mesh point number
	M1	*Value of $M_{1,n}$ of equation (25) at each mesh point
	M2	*Value of $M_{2,n}$ of equation (25) at each mesh point

Section	Quantity	Remarks
	B1	*Value of equation (30) at each mesh point for radial coefficients
	B2	*Value of equation (30) at each mesh point for circumferential coefficients

(*This information is printed out to aid in evaluating the validity of the solution.)

APPENDIX C

PROGRAM LISTINGS

Elastic-Plastic Program Listing

```
$IBJOB          GO,MAP,SOURCE
$IBFTC STRESS   LIST,DEBUG

      DIMENSION C(30),D(30),F(30),G(30),HT(30),RAD(30),E(30),CP(30),
     1DP(30),FP(30),GP(30),HP(30),PP(30),RL11(30),RL12(30),RL21(30),
     2RL22(30),RM1(30),RM2(30),AR(30),AT(30),B1(30),B2(30),SIGR(30),
     3SIGT(30),SIGZ(30),SIGE(30),TEMP(30),EPR(30),EPT(30),ETR(30),ETT(30
     4),EET(30),EP(30),ALPHA(30),ETZ(30)
      COMMON TEMP,EET,EP,EPR,EPT,RAD,I
  901 FORMAT(8F10.7)
  903 FORMAT (4I5)
      READ (5,903)      NOFS,NJ
      READ (5,901) (TEMP(I),I=1,NOFS)
      READ (5,901) (ALPHA(I),I=1,NOFS)
      READ (5,901) (E(I),I=1,NOFS)
      READ (5,901) XMU,SIGRR,SIGRI
  902 READ (5,903) NOFI
      READ (5,901) (RAD(I),I=1,NOFS)
  904 FORMAT(20H1NO OF ITERATIONS = G13.5,10X,17HNO OF STATIONS = G13.5)
      WRITE (6,904) NOFI,NOFS
  906 FORMAT (11HKSTATION NO,15X,11HCYL  RADIUS,9X,11HTEMPERATURE,9X,
     15HALPHA,15X,1HE)
      WRITE (6,906)
  907 FORMAT(5G20.9)
      DO 908 I=1,NOFS
  908 WRITE (6,907) I,RAD(I),TEMP(I),ALPHA(I),E(I)
  910 FORMAT(18HJPOISSONS RATIO = G15.5,3X,15HRAD R STRESS = G15.5,5X,
     115HRAD I STRESS = G15.5)
      WRITE (6,910) XMU,SIGRR,SIGRI
C
C***************************************
C     BEGIN CALCULATIONS
C***************************************
C
      SUMA = 0.0
      DO 3 I=2,NOFS
    3 HT(I)= RAD(I)-RAD(I-1)
      HT(1) = HT(2)
C
C***************************************
C     CALCULATE THE GEOMETRICAL COEFFICIENTS
C***************************************
C
      DO 1 J=1,NOFS
      IF(J-1) 2,99,2
C     CALCULATE C
    2 C(J) = 1./HT(J)+1./(2.*RAD(J))
C
C     CALCULATE D
      D(J) = 1./(2.*RAD(J))
C
C     CALCULATE F
      F(J) = 1./HT(J)-1./(2.*RAD(J-1))
```

```
C
C     CALCULATE G
      G(J) = 1./(2.*RAD(J-1))
C
C     CALCULATE C PRIME
      CP(J) = -XMU/(HT(J)*E(J))-XMU**2/(HT(J)*E(J))-(1.+XMU)/(2.*E(J)*
     1RAD(J))
C
C     CALCULATE D PRIME
      DP(J) = 1./(HT(J)*E(J))-XMU**2/(HT(J)*E(J))+(1.+XMU)/(2.*E(J)*RAD
     1(J))
C
C     CALCULATE F PRIME
      FP(J) = -XMU/(HT(J)   *E(J-1))-XMU**2/(HT(J)   *E(J-1))+(1.+XMU)/
     1(2.*E(J-1)*RAD(J-1))
C
C     CALCULATE G PRIME
      GP(J) = 1./(HT(J)   *E(J-1))-XMU**2/(HT(J)   *E(J-1))-(1.+XMU)/
     1(2.*E(J-1)*RAD(J-1))
C
C     CALCULATE H PRIME
      HP(J) = -(1.+XMU)/HT(J)*(ALPHA(J)*TEMP(J)-ALPHA(J-1)*TEMP(J-1))
C
C     CALCULATE P PRIME
      PP(J) = 0.0
      DEBUG C(J),D(J),F(J),G(J)
      DEBUG CP(J),DP(J),GP(J),FP(J),HP(J)
C
C     CALCULATE MATRIX COEFFICIENTS L11,L12
      DEN = (C(J)*DP(J)+CP(J)*D(J))
      RL11(J) =(DP(J)*F(J)+D(J)*FP(J))/DEN
      RL12(J) =(DP(J)*G(J)+D(J)*GP(J))/DEN
      RL21(J) =(C(J)*FP(J)-CP(J)*F(J))/DEN
      RL22(J) =(C(J)*GP(J)-CP(J)*G(J))/DEN
      RM1(J) =D(J)*(HP(J)+PP(J))/DEN
      RM2(J) =C(J)*(HP(J)+PP(J))/DEN
      DEBUG RL11(J),RL12(J),RL21(J),RL22(J)
C
C     CALCULATE COEFFICIENTS AR,AT
      AR(J) =RL11(J)*AR(J-1)+RL12(J)*AT(J-1)
      AT(J) =RL21(J)*AR(J-1)+RL22(J)*AT(J-1)
      DEBUG AR(J),AT(J)
C
      GO TO 1
C
   99 IF(NJ) 6,6,7
C
C     SOLID CYLINDER
    7 AR(J) = 1.0
      AT(J) = 1.0
      B1(J) = 0.0
      B2(J) = 0.0
      GO TO 1
C
C     CYLINDER WITH HOLE
    6 AR(J) = 0.0
```

```
      AT(J) = 1.0
      B1(J) = -SIGRI
      B2(J) = 0.0
C
    1 CONTINUE
C
   61 DO 63 IP =1,NOFS
      EP(IP) = 0.0
      EPR(IP) = 0.0
      EPT(IP) = 0.0
      ETR(IP) = 0.0
      ETT(IP) = 0.0
   63 CONTINUE
C
C****************************************
C     ITERATION LOOP
C****************************************
      DO 501 NIT= 1,NOFI
      SUMA = SUMA + 1.0
      NK = 0
      XC4 = 0.0
      XC3 = 0.0
  504 DO 9 K=2,NOFS
C
      DEN= (C(K)*DP(K)+CP(K)*D(K))
      RM1(K)= D(K)*(HP(K)+PP(K))/DEN
      RM2(K)= C(K)*(HP(K)+PP(K))/DEN
C
C
   10 B1(K)= RL11(K)*B1(K-1)+RL12(K)*B2(K-1)+RM1(K)
      B2(K)= RL21(K)*B1(K-1)+RL22(K)*B2(K-1)+RM2(K)
   15 CONTINUE
    9 CONTINUE
C
      DO 39 I= 1,NOFS
C
C     CALCULATE RADIAL AND TANGENTIAL STRESSES
C
      KPT= NOFS
      IF(I-1) 20,25,20
C
   25 IF (NJ) 24,24,23
C
C     CENTER STRESSES FOR A SOLID CYL
C
   23 SIGT(1) = -B1(KPT)/AR(KPT)-SIGRR/AR(KPT)
      SIGR(1) = SIGT(1)
      GO TO 29
C
C     STRESSES FOR HOLED CYL
C
   24 SIGR(1)=-SIGRI
      SIGT(1)= -B1(KPT)/AR(KPT)     - SIGRR/AR(KPT)
      GO TO 29
C
   20 IF(I-KPT) 21,22,21
```

```
   22 SIGR(KPT)=-SIGRR
      GO TO 28
C
   21 SIGR(I)= AR(I)*SIGT(1)+B1(I)
   28 SIGT(I)= AT(I)*SIGT(1)+B2(I)
C
   29 CONTINUE
C
      SIGZ(I) = XMU*(SIGT(I)+SIGR(I)) - E(I)*ALPHA(I)*TEMP(I)+E(I)*EPR(I
     1)+E(I)*EPT(I)
   39 CONTINUE
      DO 35 I = 2,NOFS
      XC2 = (E(I)+E(I-1))/4.*(RAD(I)**2-RAD(I-1)**2)
      XC4 = XC4+XC2
      XC1 = (SIGZ(I) + SIGZ(I-1))/4.*(RAD(I)**2-RAD(I-1)**2)
   35 XC3 = XC3 + XC1
      C3 = (XC3 - (SIGRI * RAD(1) **2)/2.0)/XC4
      DO 37 I=1,NOFS
   37 SIGZ(I) = SIGZ(I) - C3 *E(I)
      DO 30 I=1,NOFS
C     CALCULATE THE TOTAL STRAINS
C
      ETRX= ETR(I)
      ETTX= ETT(I)
C
      ETR(I) = (1./E(I))*(SIGR(I)-XMU*(SIGT(I)+SIGZ(I)))+ALPHA(I)*TEMP(I
     1)+EPR(I)
      ETT(I) = (1./E(I))*(SIGT(I)-XMU*(SIGR(I)+SIGZ(I)))+ALPHA(I)*TEMP(I
     1)+EPT(I)
      ETZ(I)= (1./E(I))*(SIGZ(I)-XMU*(SIGR(I)+SIGT(I)))+ALPHA(I)*TEMP
     1(I)-EPR(I)-EPT(I)
C
C     CALCULATE THE EQUIVALENT STRESS
      PRA= ((SIGR(I)-SIGT(I))**2+(SIGR(I)-SIGZ(I))**2+(SIGT(I)-SIGZ(I))
     1**2)
      SIGE(I)= (1./1.41421)*SQRT(PRA)
C
C****************************************
C     TEST FOR CONVERGENCE
C****************************************
      DELR= (ETRX-ETR(I))/ETR(I)
      DELT= (ETTX-ETT(I))/ETT(I)
C
      IF (ABS(DELR)-.001) 31,31,32
   32 NK= 1
   31 IF (ABS(DELT)-.001) 33,33,34
   34 NK= 1
   33 CONTINUE
C
C     CALCULATE THE EQUIVALENT TOTAL STRAINS
      STA = ((ETR(I)-ETT(I))**2+(ETT(I)-ETZ(I))**2+(ETZ(I)-ETR(I))**2)
      EET(I)= (1.41421/3.0)*SQRT(STA)
C
C
C
      CALL PLSTR
```

```
C
C     CALCULATE PLASTIC STRAINS
C
   66 EPR(I)= EP(I)/(3.*EET(I))*(2.*ETR(I)-ETT(I)-ETZ(I))
      EPT(I)= EP(I)/(3.*EET(I))*(2.*ETT(I)-ETR(I)-ETZ(I))
C
  500 CONTINUE
      IF(I-1) 71,30,71
C
C     CALCULATE NEW VALUE FOR P PRIME
   71 PP(I) = (-1./HT(I)-1./(2.*RAD(I))+XMU/HT(I))*EPT(I)+(1./(2.*RAD(I)
     1)+
     1XMU/HT(I))*EPR(I)+(1./HT(I)-1./(2.*RAD(I-1))-XMU/HT(I))*EPT(I-1)+
     2(1./(2.*RAD(I-1))-XMU/HT(I))*EPR(I-1)
C
   30 CONTINUE
      IF(NK) 501,5C3,501
  501 CONTINUE
  503 CONTINUE
      DO 75 I = 1,NOFS
   75 RAD(I) = RAD(I) *(1. + ETT(I))
C
C*****************************************
C     BEGIN PRINT OUT OF OUTPUT
C*****************************************
  912 FORMAT(11HLSTATION NO,10X,14HTOTAL STRAIN R,6X,14HTOTAL STRAIN T,6
     1X,14HTOTAL STRAIN Z)
      WRITE (6,912)
  914 FORMAT (4G20.9)
      DO 913 I=1,NOFS
  913 WRITE (6,914) I,ETR(I),ETT(I),ETZ(I)
  915 FORMAT (11HKSTATION NO,10X,16HPLASTIC STRAIN R,4X,16HPLASTIC STRAI
     1N T,10X,10HCYL RADIUS)
      WRITE (6,915)
  919 FORMAT (5G20.9)
      DO 918 I=1,NOFS
  918 WRITE (6,919) I,EPR(I),EPT(I),RAD(I)
  920 FORMAT(11HKSTATION NO,10X,13HRADIAL STRESS,7X,14HTANGENT STRESS,6X
     1,11HLONG STRESS,9X,12HEQUIV STRESS)
      WRITE (6,920)
      DO 925 I=1,NOFS
  925 WRITE (6,919) I,SIGR(I),SIGT(I),SIGZ(I),SIGE(I)
      WRITE (6,919) SUMA,DELR,DELT,C3
  929 FORMAT (11HKSTATION NO,10X,14HELASTIC STRAIN,6X,14HPLASTIC STRAIN,
     16X,7HP PRIME )
      WRITE (6,929)
  931 FORMAT (5G20.9)
      DO 930 I=1,NOFS
  930 WRITE (6,931) I,EET(I),EP(I),PP(I)
  928 FORMAT (11HKSTATION NO,10X,2HM1,18X,2HM2,18X,2HB1,18X,2HB2)
      WRITE (6,928)
      DO 936 I=1,NOFS
  936 WRITE (6,931) I,RM1(I),RM2(I),B1(I),B2(I)
      GO TO 902
      END
```

```
$IBFTC PLSTR    LIST,DEBUG

      SUBROUTINE PLSTR
      DIMENSION C(30),D(30),F(30),G(30),HT(30),RAD(30),E(30),CP(30),
     1DP(30),FP(30),GP(30),HP(30),PP(30),RL11(30),RL12(30),RL21(30),
     2RL22(30),RM1(30),RM2(30),AR(30),AT(30),B1(30),B2(30),SIGR(30),
     3SIGT(30),SIGZ(30),SIGE(30),TEMP(30),EPR(30),EPT(30),ETR(30),ETT(30
     4),EET(30),EP(30),ALPHA(30),ETZ(30)
      COMMON TEMP,EET,EP,EPR,EPT,RAD,I
C
C*****************************************
C     MATERIAL IDENTIFICATION
C*****************************************
C
      IF (RAD(I) - RAD(14)) 10,10,11
   11 GO TO 500
C
C*****************************************
C     TEMP IDENTIFICATION MAT 1
C*****************************************
C
   10 IF (TEMP(I) - 1000.) 25,20,20
   25 IF (TEMP(I) - 800.) 35,30,30
   35 IF(TEMP(I) - 600.) 45,40,40
   45 IF (TEMP(I) - 400.) 55,50,50
   55 GO TO 50
C
C     STRAIN CALC TEMP 1000
C
   20 IF (EET(I) - .0045) 26,26,27
   26 EP(I) = .6*EET(I) - .0012
      GO TO 400
C
   27 EP(I) = .98*EET(I) - .0027
      GO TO 400
C
C     STRAIN CALC TEMP 800
C
   30 EP(I) = 0.922 * EET(I) - .00207
      GO TO 400
C
C     STRAIN CALC TEMP 600
C
   40 EP(I) = .898 * EET(I) - .00184
      GO TO 400
C
C     STRAIN CALC TEMP 400
C
   50 IF (EET(I) - .0017) 51,51,52
   51 EP(I) = .81 * EET(I) - .00154
      GO TO 400
C
   52 EP(I) = .931 * EET(I) - .00202
  400 IF (EP(I).LT.0.0) GO TO 80
      GO TO 500
C
C
   80 EP(I) = 0.0
      EPR(I) = 0.0
      EPT(I) = 0.0
  500 RETURN
      END
```

Creep Program Listing

```
$IBJOB        GO,MAP,SOURCE
$IBFTC CREEP  LIST,DEBUG

      DIMENSION C(30),D(30),F(30),G(30),HT(30),RAD(30),E(30),CP(30),
     1DP(30),FP(30),GP(30),HP(30),PP(30),RL11(30),RL12(30),RL21(30),
     2RL22(30),RM1(30),RM2(30),AR(30),AT(30),B1(30),B2(30),SIGR(30),
     3SIGT(30),SIGZ(30),SIGE(30),TEMP(30),EPR(30),EPT(30),ETR(30),ETT(30
     4),ALPHA(30),ETZ(30),DEPR(30),DEPT(30),DEP(30),RADX(30)
      COMMON TEMP,DEP,SIGE,I,DT,RAD
  901 FORMAT(8F10.7)
  903 FORMAT (4I5)
      READ (5,903) NOFS,NJ,NI
      READ (5,901) (RADX(I),I=1,NOFS)
      READ (5,901) (TEMP(I),I=1,NOFS)
      READ (5,901) (ALPHA(I),I=1,NOFS)
      READ (5,901) (E(I),I=1,NOFS)
      READ ( 5,901) (EPR(I), I=1,NOFS)
      READ (5,901) (EPT(I), I=1,NOFS)
      READ (5,901) (PP(I), I=1,NOFS)
      READ (5,901) XMU,SIGRR,SIGRI,      ACC
  902 READ (5,903) NOFI,NODT
  904 FORMAT(20H1NO OF ITERATIONS = G13.5,10X,17HNO OF STATIONS = G13.5)
      WRITE (6,904) NOFI,NOFS
  906 FORMAT (11HKSTATION NO,15X,11HCYL  RADIUS,9X,11HTEMPERATURE,9X,
     15HALPHA,15X,1HE)
      WRITE (6,906)
  907 FORMAT(5G20.9)
      DO 908 I=1,NOFS
  908 WRITE (6,907) I,RADX(I),TEMP(I),ALPHA(I),E(I)
  980 FORMAT (11HKSTATION NO,10X,16HPLASTIC STRAIN R,4X,16HPLASTIC STRAI
     1N T,4X,7HP PRIME)
      WRITE (6,980)
  981 FORMAT (5G20.9)
      DO 982 I = 1,NOFS
  982 WRITE (6,981) I,EPR(I), EPT(I), PP(I)
  910 FORMAT(18HJPOISSONS RATIO = G15.5,3X,15HRAD R STRESS = G15.5,5X,
     115HRAD I STRESS = G15.5)
      WRITE (6,910) XMU,SIGRR,SIGRI
C
C     *********************************
C     BEGIN CALCULATIONS
C     *********************************
      DT= .01
      T= 0.0
      NH = NI-1
   61 DO 63 IP =1,NOFS
      ETR(IP) = 0.0
      ETT(IP) = 0.0
      DEP (IP) = 0.0
      DEPR(IP)= 0.0
      DEPT(IP)= 0.0
   63 CONTINUE
C
C     *********************************
```

```
C      TIME INCREMENTS LOOP
C      **********************************
C
       DO 505 JJ=1,NODT
       SUMA = 0.0
       DT= DT*ACC
       NH = NH+1
    67 DO 68 IJ=1,NOFS
       EPR(IJ)= DEPR(IJ)+ EPR(IJ)
       EPT(IJ)= DEPT(IJ)+EPT(IJ)
C
    68 CONTINUE
       DO 75 I=1,NOFS
    75 RAD(I) = RADX(I)*(1. + ETT(I))
       DO 3 I=2,NOFS
     3 HT(I)= RAD(I)-RAD(I-1)
       HT(1) = HT(2)
C
C      ********************************
C      CALCULATE THE GEOMETRICAL COEFFICIENTS
C      ********************************
C
       DO 1 J=1,NOFS
       IF(J-1) 2,99,2
C      CALCULATE C
     2 C(J) = 1./HT(J)+1./(2.*RAD(J))
C
C      CALCULATE D
       D(J) = 1./(2.*RAD(J))
C
C      CALCULATE F
       F(J) = 1./HT(J)-1./(2.*RAD(J-1))
C
C      CALCULATE G
       G(J) = 1./(2.*RAD(J-1))
C
C      CALCULATE C PRIME
       CP(J) = -XMU/(HT(J)*E(J))-XMU**2/(HT(J)*E(J))-(1.+XMU)/(2.*E(J)*
      1RAD(J))
C
C      CALCULATE D PRIME
       DP(J) = 1./(HT(J)*E(J))-XMU**2/(HT(J)*E(J))+(1.+XMU)/(2.*E(J)*RAD
      1(J))
C
C      CALCULATE F PRIME
       FP(J) = -XMU/(HT(J)   *E(J-1))-XMU**2/(HT(J)   *E(J-1))+(1.+XMU)/
      1(2.*E(J-1)*RAD(J-1))
C
C      CALCULATE G PRIME
       GP(J) = 1./(HT(J)   *E(J-1))-XMU**2/(HT(J)   *E(J-1))-(1.+XMU)/
      1(2.*E(J-1)*RAD(J-1))
C
C      CALCULATE H PRIME
       HP(J) = -(1.+XMU)/HT(J)*(ALPHA(J)*TEMP(J)-ALPHA(J-1)*TEMP(J-1))
C
       DEBUG C(J),D(J),F(J),G(J)
```

```
      DEBUG CP(J),DP(J),GP(J),FP(J),HP(J)
C
C     CALCULATE MATRIX COEFFICIENTS L11,L12
      DEN = (C(J)*DP(J)+CP(J)*D(J))
      RL11(J) =(DP(J)*F(J)+D(J)*FP(J))/DEN
      RL12(J) =(DP(J)*G(J)+D(J)*GP(J))/DEN
      RL21(J) =(C(J)*FP(J)-CP(J)*F(J))/DEN
      RL22(J) =(C(J)*GP(J)-CP(J)*G(J))/DEN
      RM1(J) =D(J)*(HP(J)+PP(J))/DEN
      RM2(J) =C(J)*(HP(J)+PP(J))/DEN
      DEBUG RL11(J),RL12(J),RL21(J),RL22(J)
C
C     CALCULATE COEFFICIENTS AR,AT
      AR(J) =RL11(J)*AR(J-1)+RL12(J)*AT(J-1)
      AT(J) =RL21(J)*AR(J-1)+RL22(J)*AT(J-1)
      DEBUG AR(J),AT(J)
C
      GO TO 1
C
   99 IF(NJ) 6,6,7
C
C     SOLID CYL
C
    7 AR(J) = 1.0
      AT(J) = 1.0
      B1(J) = 0.0
      B2(J) = 0.0
      GO TO 1
C
C     HOLED CYL
C
    6 AR(J) = 0.0
      AT(J) = 1.0
      B1(J) = -SIGRI
      B2(J) = 0.0
C
    1 CONTINUE
C
C     ********************************
C     ITERATION LOOP
C     ********************************
C
   66 DO 501 NIT= 1,NOFI
      SUMA = SUMA + 1.0
      NK = 0
      XC4 = 0.0
      XC3 = 0.0
  504 DO 9 K=2,NOFS
C
      DEN= (C(K)*DP(K)+CP(K)*D(K))
      RM1(K)= D(K)*(HP(K)+PP(K))/DEN
      RM2(K)= C(K)*(HP(K)+PP(K))/DEN
C
C
   10 B1(K)= RL11(K)*B1(K-1)+RL12(K)*B2(K-1)+RM1(K)
      B2(K)= RL21(K)*B1(K-1)+RL22(K)*B2(K-1)+RM2(K)
```

```
   15 CONTINUE
    9 CONTINUE
C
      DO 39 I= 1,NOFS
C
C     CALCULATE RADIAL AND TANGENTIAL STRESSES
C
      KPT= NOFS
      IF(I-1) 20,25,20
C
   25 IF (NJ) 24,24,23
C
C     CENTER STRESSES FOR A SOLID CYL
C
   23 SIGT(1) = -B1(KPT)/AR(KPT)-SIGRR/AR(KPT)
      SIGR(1) = SIGT(1)
      GO TO 29
C
C     STRESSES FOR HOLED CYL
C
   24 SIGR(1)=-SIGRI
      SIGT(1)= -B1(KPT)/AR(KPT)     - SIGRR/AR(KPT)
      GO TO 29
C
   20 IF(I-KPT) 21,22,21
   22 SIGR(KPT)=-SIGRR
      GO TO 28
C
   21 SIGR(I)= AR(I)*SIGT(1)+B1(I)
   28 SIGT(I)= AT(I)*SIGT(1)+B2(I)
C
   29 CONTINUE
C
      SIGZ(I) = XMU*(SIGT(I)+SIGR(I)) - E(I)*ALPHA(I)*TEMP(I)+E(I)*EPR(I
     1)+E(I)*EPT(I)
   39 CONTINUE
      DO 35 I = 2,NOFS
      XC2 = (E(I)+E(I-1))/4.*(RAD(I)**2-RAD(I-1)**2)
      XC4 = XC4+XC2
      XC1 = (SIGZ(I) + SIGZ(I-1))/4.*(RAD(I)**2-RAD(I-1)**2)
   35 XC3 = XC3 + XC1
      C3 = (XC3 - (SIGRI * RAD(1) **2)/2.0)/XC4
      DO 37 I=1,NOFS
   37 SIGZ(I) = SIGZ(I) - C3 *E(I)
      DO 30 I=1,NOFS
C     CALCULATE THE TOTAL STRAINS
C
      ETR(I) = (1./E(I))*(SIGR(I)-XMU*(SIGT(I)+SIGZ(I)))+ALPHA(I)*TEMP(I
     1)+EPR(I)+ DEPR(I)
      ETT(I) = (1./E(I))*(SIGT(I)-XMU*(SIGR(I)+SIGZ(I)))+ALPHA(I)*TEMP(I
     1)+EPT(I) +DEPT(I)
      ETZ(I)= (1./E(I))*(SIGZ(I)-XMU*(SIGR(I)+SIGT(I)))+ALPHA(I)*TEMP
     1(I)-EPR(I)-EPT(I)    -DEPT(I) - DEPR(I)
C
C     CALCULATE THE EQUIVALENT STRESS
      PRA= ((SIGR(I)-SIGT(I))**2+(SIGR(I)-SIGZ(I))**2+(SIGT(I)-SIGZ(I))
```

```
     1**2)
      SIGE(I)= (1./1.41421)*SQRT(PRA)
C
      CALL PLSTR
C
C
C     CALCULATE PLASTIC STRAIN INCREMENTS
      EPRX = DEPR(I)
      EPTX = DEPT(I)
C
      DEPR(I)= DEP(I)/(2.*SIGE(I))*(2.*SIGR(I)-SIGT(I)-SIGZ(I))
      DEPT(I)= DEP(I)/(2.*SIGE(I))*(2.*SIGT(I)-SIGR(I)-SIGZ(I))
C
C     *********************************
C     TEST FOR CONVERGENCE
C     *********************************
C
      DELR = (EPRX-DEPR(I))/DEPR(I)
      DELT = (EPTX-DEPT(I))/DEPT(I)
C
  500 CONTINUE
C
      IF (ABS(DELR)-.001) 31,31,32
   32 NK= 1
   31 IF (ABS(DELT)-.001) 33,33,34
   34 NK= 1
   33 CONTINUE
C
C
C
      IF(I-1) 71,30,71
C
C     CALCULATE NEW VALUE FOR P PRIME
   71 PP(I)= (-1./HT(I)-1./(2.*RAD(I))+XMU/HT(I))*(EPT(I)+DEPT(I))+
     1(1./(2.*RAD(I))+XMU/HT(I))*(EPR(I)+DEPR(I))+(1./HT(I)-1./(2.*RAD
     2(I-1))-XMU/HT(I))*(EPT(I-1)+DEPT(I-1))+(1./(2.*RAD(I-1))-XMU/
     3HT(I))*(EPR(I-1)+DEPR(I-1))
C
   30 CONTINUE
      IF(NK) 501,503,501
  501 CONTINUE
  503 CONTINUE
      T= T+DT
      IF(NI.NE.NH) GO TO 505
      NH = 0
C     *********************************
C     BEGIN PRINT OUT OF OUTPUT
C     *********************************
  912 FORMAT(11HLSTATION NO,10X,14HTOTAL STRAIN R,6X,14HTOTAL STRAIN T,6
     1X,14HTOTAL STRAIN Z)
      WRITE (6,912)
  914 FORMAT (4G20.9)
      DO 913 I=1,NOFS
  913 WRITE (6,914) I,ETR(I),ETT(I),ETZ(I)
  915 FORMAT (11HKSTATION NO,10X,16HINCR PLAS STR  R,4X,16HINCR PLAS STR
     1  T,4X,10HCYL RADIUS)
```

```
      WRITE (6,915)
  919 FORMAT (5G20.9)
      DO 918 I=1,NOFS
  918 WRITE (6,919) I,DEPR(I),DEPT(I),RAD(I)
  960 FORMAT (11HKSTATION NO,10X,16HPLASTIC STRAIN R,4X,16HPLASTIC STRAI
     1N T)
      WRITE (6,960)
  961 FORMAT (5G20.9)
      DO 962 I= 1,NOFS
  962 WRITE (6,961) I,EPR(I),EPT(I)
  920 FORMAT(11HKSTATION NO,10X,13HRADIAL STRESS,7X,14HTANGENT STRESS,6X
     1,11HLONG STRESS)
      WRITE (6,920)
      DO 925 I=1,NOFS
  925 WRITE (6,919) I,SIGR(I),SIGT(I),SIGZ(I)
      WRITE (6,919) SUMA,DELR,DELT,C3,T
  929 FORMAT (11HKSTATION NO,10X,14HEQUIV.  STRESS,6X,14HPLASTIC STRAIN,
     16X,7HP PRIME )
      WRITE (6,929)
  931 FORMAT (5G20.9)
      DO 930 I=1,NOFS
  930 WRITE (6,931) I,SIGE(I),DEP(I),PP(I)
  928 FORMAT (11HKSTATION NO,10X,2HM1,18X,2HM2,18X,2HB1,18X,2HB2)
      WRITE (6,928)
      DO 936 I=1,NOFS
  936 WRITE (6,931) I,RM1(I),RM2(I),B1(I),B2(I)
  505 CONTINUE
      GO TO 502
      END
```

```
$IBFTC PLSTR    LIST,DEBUG

      SUBROUTINE PLSTR
      DIMENSION C(30),D(30),F(30),G(30),HT(30),RAD(30),E(30),CP(30),
     1DP(30),FP(30),GP(30),HP(30),PP(30),RL11(30),RL12(30),RL21(30),
     2RL22(30),RM1(30),RM2(30),AR(30),AT(30),B1(30),B2(30),SIGR(30),
     3SIGT(30),SIGZ(30),SIGE(30),TEMP(30),EPR(30),EPT(30),ETR(30),ETT(30
     4),ALPHA(30),ETZ(30),DEPR(30),DEPT(30),DEP(30),RADX(30)
      COMMON TEMP,DEP,SIGE,I,DT,RAD
C
C     ********************************
C     MATERIAL IDENTIFICATION
C     ********************************
C
      IF (RAD(I) - RAD(14)) 10,10,11
   11 GO TO 500
C
C     ********************************
C     TEMP IDENTIFICATION MAT 1
C     ********************************
C
   10 IF (TEMP(I) - 1000.) 25,20,20
   25 IF (TEMP(I) - 800.) 35,30,30
   35 IF (TEMP(I) - 600.) 45,40,40
   45 GO TO 40
C
C     STRAIN CALC TEMP 1000
C
   20 DEP(I) = 4.40E-17*SIGE(I)**2.57 *DT
      GO TO 400
C
C     STRAIN CALC TEMP 800
   30 DEP(I) = 8.0E-21*SIGE(I)**3.01    *DT
      GO TO 400
C
C     STRAIN CALC TEMP 600
   40 DEP(I) = 1.0E-24*SIGE(I)**3.4 *DT
C
  400 CONTINUE
  500 RETURN
      END
```

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