V. Spacecraft Control

GUIDANCE AND CONTROL DIVISION

A. Partial Inertial System Integration Test, *G. Paine*

1. Introduction

The LAB 1 Alert program was exercised during integration of the strapdown electrostatic aerospace navigator (SEAN) system from August 19 through 28, 1968. The system included the Alert computer,[1] the computer adapter and display (CAD), and the inertial measurement unit (IMU) with digital velocity meters (DVMs) but without the electrostatic gyros (ESGs). Tests were conducted to determine that: (1) the basic inertial integrations had been correctly mechanized, (2) data were being correctly received from the CAD, (3) the schemes for using altimeter data to damp the position and velocity errors were correct, and (4) the proper computations were being performed to convert from inertial to local coordinates. These tests were successfully completed after several minor errors were discovered and corrected. As the LAB 1 program[2] constitutes about one-fourth of the final flight navigation program, the successful completion of the tests signifies a major milestone in the development of a flight program.

[1]Manufactured by Honeywell, Inc., Minneapolis, Minn.

[2]Paine, G., *Preliminary SEAN Navigation Program (LAB 1)—Request for Programming*, Mar. 25, 1968; Markiewicz, B. R., *SEAN Navigation Equations for the Alert Computer*, Apr. 30, 1968 (JPL internal documents).

The short length of time required for the integration proves the value of the complete system simulations performed in generating the equations for LAB 1 and the value of program checkout employing a computer simulator. These simulation efforts have been described previously in SPS 37-52, Vol. III, pp. 52–55.

2. Test Setup

No attempt was made to align the IMU with the local geodetic vertical. Instead, the DVM biases were set to zero in the computer program and the IMU was tilted until the level accelerometers were indicating less than 50-μg output (less than 1 pulse/45 s), thus aligning the IMU to astronomic vertical, offset by the real DVM biases and the residual DVM outputs.

To replace the gyros, LAB 1 computes a transformation matrix based on time and geodetic latitude to convert between geodetic and inertial coordinates. Consequently, the outputs from the three DVMs are processed as though the IMU were aligned to geodetic vertical and there were no local gravitational anomalies.

3. Test Results

The results of two of the test runs, a 52.266-min test without altimeter damping and a 26.133-min test with altimeter damping, are presented in Table 1. A 14-min

Table 1. LAB 1 test results of observed position, computed equivalent bias, and measured equivalent bias errors

Time, min	Cross-range error			Down-range error			Altitude error	
	Observed position, ft	Computed equivalent bias, μg	Measured equivalent bias, μg	Observed position, ft	Computed equivalent bias, μg	Measured equivalent bias, μg	Observed position, ft	Computed equivalent bias, μg
52.266-min run (without altitude control)								
11.2	−249	−36	−22	236	34	44	−403	−49
46.67	−1800	−43	−22	2875	48	44	−29000	−40
52.27	−1750	−48	−22	3689	24	44	−50000	−39
26.133-min run (with altitude control)[a]								
26.13	−1459	−51	−22	1344	47	44	−60	—
26.13[b]	−1078	−38	−22	1307	44	44	−174	—

[a]CV = 1/32, CR = 1/16, corrections every 14 s, dominant time constant 15.3 min.
[b]Based on 52.266-min run with corrections for addition of altitude control.

test was also run with altimeter damping and a large vertical accelerometer bias. The residual errors (the difference between expected and observed) were small. The data in each case were analyzed to provide equivalent DVM bias errors. Gain constants *CR* and *CV* were used to control the position and velocity feedback during altitude control.

These biases are actually the combination of IMU misalignments with DVM biases, and do not reflect the accuracy of DVM calibration. The misalignments could have been reduced, but this was not done in order to provide more data on system performance. In addition, the measured equivalent values of the down-range error (DRE) and cross-range error (CRE) were obtained by examining the level DVM outputs directly. No such estimate was obtained from the vertical (ALT) DVM. The computed equivalent bias shifts of ±10 μg seen in the data can be attributed to a wide variety of sources (computation errors, real bias shifts, etc.). Since the magnitude is so small, this shift is considered secondary.

The 52.266-min run data show good agreement between the measured values of the DVM bias and the values needed to produce the position errors. The difference between the value of level acceleration observed on the DVM outputs directly and that needed to produce the CRE is probably caused by an attitude misalignment in the body-to-inertial transformation matrix. The offset of about 22 μg is equivalent to an attitude error of only 5 arc sec. The value of CRE decreases after the 48-min value because it is almost a pure Schuler error. The vertical DVM bias remained relatively constant throughout the test period. The large altitude error (ALTE) of 50,000 ft is to be expected since the altitude channel is unstable and no altimeter damping was employed.

The 26.133-min run with altimeter damping followed the conclusion of the previous run. The observed CRE did not match, as well as desired, the measured value based on the 52.266-min run when corrected for the addition of altitude control. However, there was excellent agreement between the measured and computed values of CRE, DRE, and ALTE. At the end of the test period, the ALTE was drifting upwards so that, if data had been taken over an extended period, still better agreement could have been expected.

The 14-min run with altimeter damping (CV = 1/16, CR = 1/8, corrections every 14 s) was made with a large vertical DVM bias introduced (1140 μg) to make the comparison easier between the observed results and those predicted from theory. The IMU was not carefully leveled at this point, so no comparisons of DRE or CRE could be made. There was an excellent comparison between the altitude error observed (805 ft) and those predicted (848 ft). In addition, the predicted dominant time constant (304 s) was in good agreement with the observed value (270 s).

B. Automatic Lens Design Program, *L. F. Schmidt*

The objective of this study is to further develop an automatic lens design program into a practical design tool, which will be flexible enough to handle most optical designs and yet not be cumbersome for designing simple optical systems.

The lens design program was rewritten into Fortran IV language and submitted to the COSMIC Computer Center[3] in March 1968. (A detailed explanation of the program is provided in a three-volume document.) Prior to its submission to COSMIC, 10 organizations were supplied with copies of the program. This increased use of the program is possible since several different computer systems can handle the Fortran version. A new computer system is being planned that is not compatible with the machine language version of the program; however, no problems are anticipated in converting the Fortran version to the new computer.

The Fortran version contains a newly developed option that provides a graphical plot of the optical system configuration. This eliminates the necessity for a manually drawn optical system to determine if a design is progressing in a satisfactory manner or needs new input data to direct its progress.

Several improvements to the code have been undertaken since it was submitted to COSMIC.

An improved method of handling a curved image plane has been devised. The general technique used is to shift the position of the flat image plane for each object and corresponding image height. This position is calculated to cause the image plane to intersect the desired curve at the expected image height. In the old method, the image plane shift was calculated manually and entered. As the image height changed during design operations, the image plane shifts were updated in a trial-and-error fashion throughout the design phase.

In the new method, the image plane shift is automatically recalculated by the code after each design iteration. In this way the time necessary for manual calculation is saved as well as the extra computer time that was expended during the trial-and-error process of adjusting the image plane.

An option has been developed that will generate a punched card output for use on another program (PAGOS), which examines optical systems in terms of modulation transfer function (MTF). This provides a fast method of MTF analysis for image-forming optical systems.

A method of controlling the contrast of the image has been developed. Weights can be entered that place more importance on the central image rays, compared to the outer ones. The program assigns weights for each ray in a gaussian fashion when this option is exercised.

These program modifications are currently being incorporated into the three-volume document.

An attempt was made to develop a method of controlling the ratio of spot sizes relative to each object height. This was discontinued when the initial efforts were unsuccessful and it was concluded that such an option was not worth the effort required to get it operating properly.

[3]Computer Software Management and Information Center, Computer Center, University of Georgia, Athens, Georgia.

C. Vibration and Shock Analysis: Strapdown Electrostatic Aerospace Navigator, *G. T. Starks*

The purpose of the vibration and shock analysis was to (1) determine the *g* level that could be transmitted through various combinations of natural frequency and damping ratios (ζ) of a mechanical isolation system, which would be detrimental to the survival of the electrostatic gyroscope (ESG), and (2) determine the minimum parameters that would isolate the destructive *g* input.

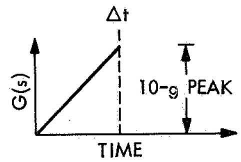
Part of the strapdown electrostatic aerospace navigator (SEAN) system consists of an inertial measuring unit (IMU) housing the two ESGs, three accelerometers, and their associated electronic circuits in a rigid body structure weighing approximately 88 lb.

The *g* capability of the ESG to be used in the first developmental navigational test is 4 *g*, of which 1 *g* is allotted to the static *g* environment. Exceeding the 3-*g* capability above the 1-*g* static environment of the ESG will result in destruction of the gyroscope.

The environment specified by MIL STD 810A that the ESG is expected to survive is:

Shock, 10-*g* terminal sawtooth	0.011 s
Vibration:	
Military aircraft	
0.10-in. double amplitude (DA)	5–14 Hz
1 *g*	14–23 Hz
0.036-in. DA	23–74 Hz
10 *g*	74–500 Hz
Van (truck), smooth roads	
1.00-in. DA	0–5 Hz
1.3 *g*	5–500 Hz

The transfer function for a shock input of a 10-g, 0.011-s terminal sawtooth driving function



is

$$G(s) = \frac{10g}{\Delta t}\left(\frac{1}{s^2} - \frac{e^{-\Delta t s}}{s^2} - \frac{\Delta t\, e^{-\Delta t s}}{s}\right) \tag{1}$$

where the duration of input force $\Delta t = 0.011$ s.

The response of the IMU (Fig. 1) is

$$M\ddot{X} + K(\dot{X} - \dot{Y}) + D(\dot{X} - \dot{Y}) = \text{forcing function } F(s) \tag{2}$$

where

D = damping of isolator

K = equivalent spring constant of isolator

M = W/g of IMU

with

M = mass of IMU

W = weight of IMU

The transfer function describing the response of the IMU in g to a shock input is

$$G(s)_{IMU} = \frac{10g}{\Delta t}\left(\frac{1 + \frac{2\zeta}{\omega_n}s}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}\right)\left(\frac{1}{s^2} - \frac{e^{-\Delta t s}}{s^2} - \frac{\Delta t\, e^{-\Delta t s}}{s}\right) \tag{3}$$

The inverse of $G(s)$ is

$$\begin{aligned} G(t)_{IMU} = \frac{10g}{\Delta t}\Bigg(&\left[\Delta t + \frac{\sin}{e^{\alpha \Delta t}\beta}\left(\beta t + 2\tan^{-1}\frac{\beta}{\alpha} + \tan^{-1}\frac{\beta}{a_0 - \alpha}\right)\right] U(t) \\ &- \left\{t - \Delta t + \frac{\sin}{\beta\, e^{\alpha(t-\Delta t)}}\left[\beta(t - \Delta t) + 2\tan^{-1}\frac{\beta}{\alpha} + \tan^{-1}\frac{\beta}{a_0 - \alpha}\right]\right\} U(t - \Delta t) \\ &- \Delta t\left\{1 - \frac{\omega_n \sin}{\beta\, e^{\alpha(t-\Delta t)}}\left[\beta(t - \Delta t) + \tan^{-1}\frac{\beta}{\alpha} + \tan^{-1}\frac{\beta}{a_0 - \alpha}\right]\right\} U(t - \Delta t)\Bigg) \end{aligned} \tag{4}$$

where

$U(t)$ = unit step function equal to unity to the right of $t = 0$ and is zero to the left of $t = 0$

$U(t - \Delta t)$ = unit step function equal to zero to the left of $t = \Delta t$ and is unity to the right of $t = \Delta t$

Δt = 0.011 s

ω_n = natural frequency of isolator design

$\beta = (\omega_n^2 - \zeta^2\omega_n^2)^{1/2}$

$a_0 = \frac{\omega_n}{2\zeta}$

$\alpha = \zeta\,\omega_n$

The values for various natural frequency isolators given in Table 2 were obtained using Eq. (4) at damping ratios 0.3 and 0.2.

The gain or transmissibility of a second-order system to a sinusoidal input forcing function (configuration shown in Fig. 1) is

$$\text{Transmissibility} = \frac{\left[1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2}} \tag{5}$$

Equation (5) was used to obtain the transmissibility factors presented in Table 3 for damping ratios equal to 0.2 and 0.3.

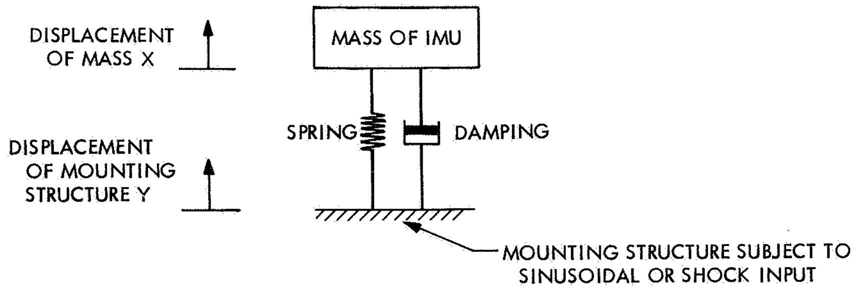


Fig. 1. Shock mount spring mass configuration

The terminal or peak acceleration in g for a sinusoidal input can be computed from

$$\frac{a_n}{g} = \frac{r\omega^2}{12g} \tag{6}$$

where

a_n = peak acceleration, ft/s^2

r = ½ double amplitude of driving function, ft

ω = angular frequency, rad

Table 4 lists the values of acceleration using Eq. (6). The computations were based on the environment specified for the military aircraft portion of MIL STD 810A. Using data from Tables 3 and 4, the values of acceleration transmitted to the IMU are shown in Table 5 for some representative frequencies.

Table 2. Acceleration response of IMU to 10-g input

Frequency, Hz	Damping ratio	Acceleration level at 0.011 s, g	Peak acceleration, g
18	0.3	4.64	4.95
12	0.3	3.1	3.4
10	0.3	2.47	2.8
	0.2	1.94	2.8
8	0.3	1.93	2.24
	0.2	1.47	2.24
6	0.3	1.44	1.7
	0.2	0.95	1.7

Table 3. Frequency transmissibility values for damping ratios 0.3 and 0.2

Frequency, Hz	Transmissibility factor	
	Damping ratio 0.3	Damping ratio 0.2
8	1.995 (peak)	2.734 (peak)
10	1.333	1.486
20	0.330	0.265
30	0.186	0.137
40	0.131	0.093
50	0.1015	0.071
60	0.0832	0.0572
74	0.0666	0.0453

Assuming a simultaneous input from both shock (Table 2) and sinusoidal frequency acceleration (Table 5), the total possible peak accelerations that are transmitted to the IMU are given in Table 6 for both 6- and 8-Hz shock mounts.

Table 4. Acceleration values versus frequency input

Frequency, Hz	Peak acceleration, g
6	0.184
8	0.304
14	1.000
30	1.6535
40	2.9395
50	4.5930
60	6.6139
70	9.000
74	10.000

Table 5. Acceleration levels transmitted to IMU[a]

Frequency, Hz	Acceleration response of IMU, g
8	0.832
30	0.23
40	0.273
50	0.324
60	0.378
74	0.453

[a]Shock mount of $\zeta = 0.2$, natural frequency = 8 Hz.

Table 6. Possible total peak acceleration transmitted through 6- and 8-Hz shock mounts

Frequency, Hz	Peak acceleration, g			
	6-Hz shock mount		8-Hz shock mount	
	$\zeta = 0.2$	$\zeta = 0.3$	$\zeta = 0.2$	$\zeta = 0.3$
6	2.2	2.1	—	—
8	—	—	3.07	2.85
74	2.14	2.4	2.69	2.71

The 6-Hz shock mount would provide about 0.6- to 0.8-g isolation to the IMU for the expected aircraft environment and combined 10-g shock. This is shown graphically in Fig. 2, where transmissibility is decreasing prior to the 14-Hz, 1-g point.

However, in reference to the expected van environment as shown in Fig. 2, the 1.3-g environment extends back to 5 Hz inside of the peak transmissibility point. At resonance the 6-Hz ($\zeta = 0.3$) shock mount could transmit approximately 2.6 g independent of the shock spectrum. An approximately 4-Hz isolator would be required to reduce the expected van environment to a level compatible with a shock environment such that the total expected input does not exceed 3 g.

Typical response curves of the IMU using an 8-Hz isolator to a 10-g shock input are shown in Fig. 3.

A van to be used in performing navigational tests has been procured for the SEAN system. The van has been instrumented and operated over terrain that is expected to be encountered during the navigational tests.

Data are being reduced to determine the criteria for the best design to isolate the IMU.

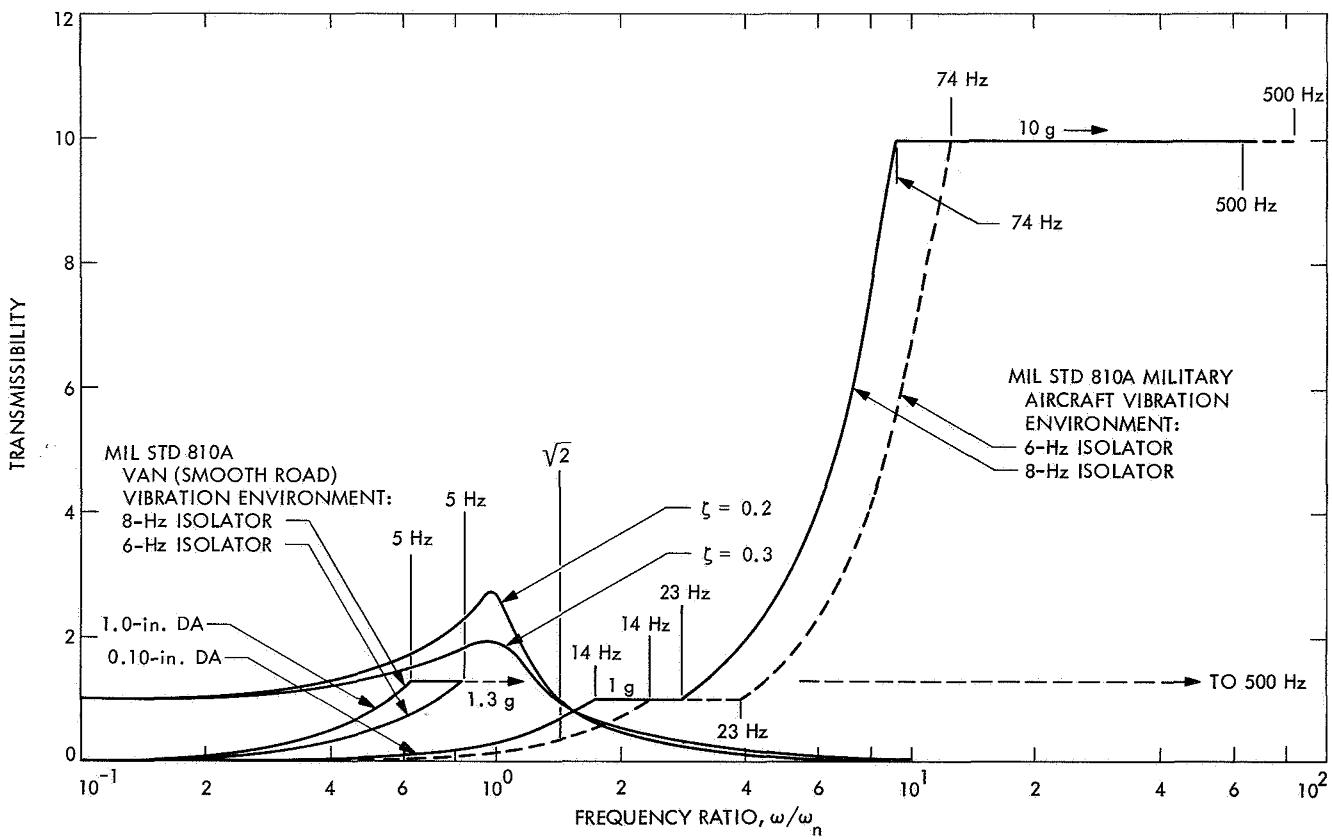


Fig. 2. 6- and 8-Hz transmissibility plots related to expected environments

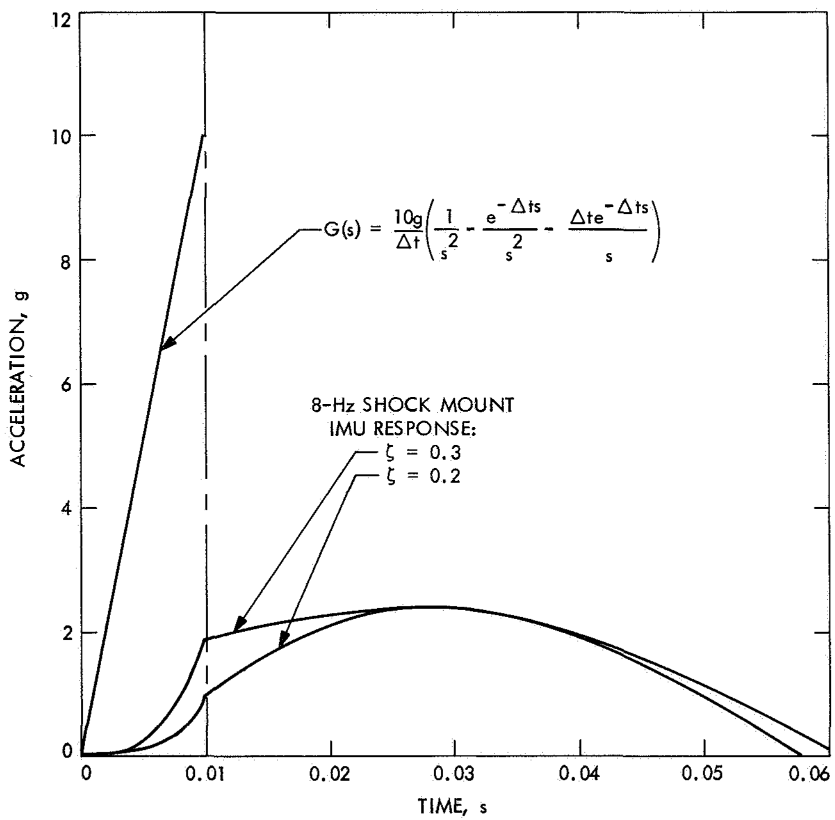


Fig. 3. Typical response curves of IMU using an 8-Hz isolator to a shock input