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TITLE- Stability of Attitude Motion of an Orbiting Vehicle Containing a Gyrostat

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ABSTRACT

A study is made on the stability of equilibrium attitude of a space vehicle containing a gyrostat with respect to the earth-pointing rotating frame in a circular orbit. Based on the first-approximation stability analysis incorporated with damping, criteria for asymptotic stability are obtained for a few equilibrium attitudes in which the gyrostat is oriented normal to the orbital plane. One interesting result is that the attitude configuration with the axis of minimum moment of inertia being the axis of rotation, which is asymptotically unstable in the classical case, can be made asymptotically stable by the gyrostat.

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From: E. Y. Yu

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TECHNICAL MEMORANDUM

STABILITY OF ATTITUDE MOTION OF AN ORBITING VEHICLE CONTAINING A GYROSTAT

I. INTRODUCTION

It is of interest to find the stable attitude motion of an orbiting vehicle containing a gyrostat (a symmetric, constant-speed rotor with its axis fixed to the vehicle). The attitude equilibrium and the stability of motion of such a gyrostat-mounted orbiting rigid body is a simple generalization of the classical problem (without the gyrostat). Recently, Roberson and Hooker[1] have studied the existence of the gravitational equilibria in the case of circular orbit. For a special gyrostat orientation their result is explicit, whereas in the general orientation they reduce their result to a tedious numerical computational algorithm. We wish to state that the problem of finding the equilibrium for a given arbitrary orientation of the gyrostat (the so-called most general case in Reference [1]) will not be considered here, since an arbitrary equilibrium is of little usefulness. What particularly interests us, in as far as astronautical application is concerned, is the equilibrium in which the principal axes of inertia of the body are in line with the earth-pointing rotating coordinates. Such an attitude equilibrium can be achieved by aligning the angular momentum of the gyrostat with the normal of the orbital plane as has been proved mathematically in Reference [1] and can also be realized from the physical point of view.

The stability of motion is studied here by including damping in the linearized equations of motion as damping will actually be provided in the vehicle. (The practical damping scheme will not be discussed here.) The criteria for asymptotic stability obtained here in the first-approximation stability theory also imply[2] nonlinear asymptotic stability. It will be shown that a useful orientation in the attitude motion, which would have been asymptotically unstable in the classical case, now becomes asymptotically stable as a result of the stabilizing effect of the gyrostat.

II. EQUILIBRIUM

Following the notations of Reference [1], the Euler equation of motion, in matrix form, of a rigid body, containing a symmetric rotor, orbiting in a circular orbit around the earth without taking any external disturbing torques into account is

$$I\dot{\omega} + \tilde{\omega}(I\omega + h) = 3\Omega^2 \tilde{\xi}_3 I \xi_3 \qquad (1)$$

Here, $I = [I_{\alpha\beta}]$ is the 3x3 inertia matrix of the body (including rotors), $\omega = [\omega_\alpha]$ is the 3x1 matrix of body components of inertia angular velocity, $\tilde{\omega} = [\epsilon_{\alpha\lambda\beta}\ \omega_\lambda]$ the 3x3 matrix, ($\epsilon_{\alpha\lambda\beta}$ the "Epsilon symbols" of tensor analysis), $h = [h_\alpha]$ the 3x1 matrix of body components of internal angular momentum from the rotor, and Ω the constant orbital angular speed. The unit vectors in the body coordinates, $\hat{x}_\alpha$, and in the earth-pointing rotating coordinates, $\hat{\xi}_\alpha$, are related by the Euler angles transformation matrix $[\Theta_{\alpha\lambda}]$, i.e.,

$$\hat{x}_\alpha = \Theta_{\alpha\lambda}\ \hat{\xi}_\lambda \quad , \quad \alpha=1,2,3 \tag{2}$$

where $\hat{\xi}_3$ is along the outward local vertical, $\hat{\xi}$,parallel to the orbital velocity, and $\hat{\xi}_2$ normal to the orbital plane. The body components of $\hat{\xi}_\lambda$ are given as 3x1 matrices, i.e., $\xi_\lambda = [\Theta_{\alpha\lambda}]$ so that $\tilde{\xi}_\lambda = [\epsilon_{\alpha\gamma\beta}\ \Theta_{\gamma\lambda}]$ are 3x3 matrices.

According to Likens and Roberson,[3] a necessary and sufficient condition for equilibrium is that $\omega = \Omega\ \xi_2$ or from (1)

$$\tilde{\xi}_2(I\ \xi_2 + H) = 3\tilde{\xi}_3\ I\ \xi_3 \tag{3}$$

where $H = h/\Omega$. Roberson and Hooker's formulation for the solution of (3) is to express $I\xi_\lambda$ and H as a linear combination of the bases ξ_λ, $\lambda=1,2,3$, in the physical vector space, e.g., $H = \sum J_\alpha \xi_\alpha$, etc. They then reduce the problem to the determination of the eigenvalues and the corresponding eigenvectors $(\xi_{1\alpha} : \xi_{2\alpha} : \xi_{3\alpha})$, from which one can determine the three directions (or the three Euler angles) in the physical space. We will not reproduce their work here, though we will present the explicit results of their two special cases with refinements. These explicit results will be obtained here otherwise by a brief, simple-minded analysis of Equation (3), since it enables us to gain insight into the equilibrium, stated in Reference [1] in the form of mathematical theorems.

First, we consider the case when either side of (3) is zero. From $3\tilde{\xi}_3 I \xi_3 = 0$, we have $\hat{\xi}_3 // I \xi_3$, indicating that $\hat{\xi}_3$ is in line with a principal axis, labelled as x_3, and that $I\xi_1$ and $I\xi_2$ are on the $\xi_1\xi_2$-plane. From $\tilde{\xi}_2(I\xi_2+H) = 0$, we find that either (i) $I\xi_2 = -H$ or (ii) $(I\xi_2 + H) // \hat{\xi}_2$ so that H must lie on the $\xi_1\xi_2$- or the local horizontal plane, with the extreme case being $I\xi_2 // H // \hat{\xi}_2$. In Reference [1], this is the case $J_1 \neq 0$, $J_3 = 0$ (so $H = J_1\hat{\xi}_1 + J_2\hat{\xi}_2$) and is covered by Theorem 2 there. This result is plotted in Figure 1 for the case of $I_1 < I_2$ (I_i = moments of inertia about the x_i- axes, $i = 1,2,3$, respectively), where the indicated angles are $\theta = \cos^{-1}(\hat{x}_1 \cdot \hat{\xi}_1)$ and $\phi = \cos^{-1}(\hat{x}_1 \cdot \hat{h})$. For given magnitude and direction of H, the angle θ (or ϕ) can be determined either by graphical method or analytically by the equilibrium condition, $\tilde{\xi}_2(I\xi_2 + H) = 0$, written out as

$$(I_2-I_1)\sin\theta\cos\theta - H\cos(\phi+\theta) = 0 \qquad (4)$$

or in the notation of [1] as

$$\cos(\theta+\phi) = \kappa_{21}\sin 2\theta \qquad (5)$$

where $\kappa_{21} = (I_2-I_1)/2H$. Relation (5) is plotted in Figures 2a,b, for various values of κ_{21} (>0 for $I_2>I_1$) and κ_{12} ($=-\kappa_{21} > 0$ for $I_1 > I_2$) between 0 and ∞. The extreme case $\kappa = 0$, i.e., $I_1 = I_2$ indicates $\theta+\phi = \pi/2$ so that H is always pointing normal to the orbital plane. The case $\kappa>1$, which is not noted in [1], signifies the small magnitude of H, with the limiting case being the classical situation since as $H\to 0$ or $\kappa_{21}\to\infty$ the curve shrinks down to a single point ($\theta = 0$, $\phi = \pi/2$) or ($\theta=\pi/2$, $\phi=0$).

The only other case is when neither side of (3) is zero. Since in (3) the R.H.S. $\perp \hat{\xi}_3$ and L.H.S. $\perp \hat{\xi}_2$, their equality signifies that they are both parallel to $\hat{\xi}_1$ so that both $I\xi_3$ and $I\xi_2 + H$ lie on the $\xi_2\xi_3$-plane. But H may or may not lie on the $\xi_2\xi_3$-plane. When H does not lie on that plane we have the most general case as it is called in Reference [1], which, as indicated earlier, is of no practical interest to us and is therefore not considered here. When H lies on the $\xi_2\xi_3$-plane (i.e., $H = J_2\hat{\xi}_2 + J_3\hat{\xi}_3$), so does $I\xi_2$; hence, it can be shown that $\hat{\xi}_1 \parallel \hat{x}_1$. This is the other special case, $J_1 = 0$, $J_2 \neq 0$, considered in Reference [1]. As shown in Figure 3 for $I_3>I_2$, where $\theta' = \cos^{-1}(\hat{x}_2\cdot\hat{\xi}_2)$ and $\phi' = \cos^{-1}(\hat{x}_2\cdot\hat{h})$, the equilibrium condition (3) is written out as

$$(I_2-I_3)\sin\theta'\cos\theta' + H\sin(\theta'+\phi') = 3(I_3-I_2)\sin\theta'\cos\theta'$$

or in the notation of [1]

$$\sin(\theta'+\phi') = \kappa_{32}\sin 2\theta' \qquad (6)$$

where $\kappa_{32} = (I_3-I_2)/(H/2)$, ($\kappa_{32} \gtrless 0$ as $I_3 \gtrless I_2$). The plots of (6) are the same as those of (5) in Figures 2a,b if the relation $\theta' = \theta-\frac{\pi}{2}$ is used (or if the origin is translated to the left by $\frac{\pi}{2}$ and at the same time the curves are moved down by π).

III. STABILITY

For stability, let us first consider the equilibrium for the case with H in the $\xi_1\xi_2$-plane. We denote the coordinates situated at the equilibrium position by (X_1,X_2,X_3), where $\hat{X}_3 \parallel \hat{\xi}_3$ and $(\hat{X}_1,\hat{X}_2)$ are obtained by rotating about the ξ_3-axis with a constant angle θ given in (5). The angular position of the body coordinates (x_1,x_2,x_3) from the equilibrium is resulted from rotations by angles $\theta_3, \theta_1, \theta_2$ in the 3-1-2 order with the X_3-axis being the first and the x_2-axis being the last axis of rotation. For the linear stability analysis treated here, we assume that these three angles are small. Then the transformation matrix in $\hat{x}_\alpha = \Theta_{\alpha\lambda}\hat{\xi}_\lambda$ is given by

$$\Theta = \begin{pmatrix} \cos\theta-\theta_3\sin\theta & \sin\theta+\theta_3\cos\theta & -\theta_2 \\ -\theta_3\cos\theta-\sin\theta & -\theta_3\sin\theta+\cos\theta & \theta_1 \\ \theta_2\cos\theta+\theta_1\sin\theta & \theta_2\sin\theta-\theta_1\cos\theta & 1 \end{pmatrix} \tag{7}$$

and the components in the body coordinates of the absolute angular velocity are

$$\begin{aligned} \vec{\omega} = {} & [\dot{\theta}_1 + \Omega(\sin\theta + \theta_3\cos\theta)]\hat{x}_1 + \\ & [\dot{\theta}_2 + \Omega(\cos\theta - \theta_3\sin\theta)]\hat{x}_2 + \\ & [\dot{\theta}_3 + \Omega(\theta_2\sin\theta - \theta_1\cos\theta)]\hat{x}_3 \end{aligned} \tag{8}$$

where $\dot{\theta}_i$, $i = 1,2,3$ are assumed to be of the same order of smallness as θ_i. Furthermore, viscous damping is provided in the vehicle to dissipate the angular oscillations with respect to the earth-pointing rotating frame, with the damping coefficients c_i, $i = 1,2,3$ being all positive definite. Then the linearized equations of motion incorporated with viscous damping can be shown to be, in matrix form,

$$I v'' + D v' + G^{(12)} v' + K^{(12)} v = 0 \tag{9}$$

Here, the primes denote derivatives with respect to $\tau = \Omega t$, and the matrices are

$$v = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}, \quad I = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}, \quad D = \begin{pmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{pmatrix}$$

$$G^{(12)} = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ -g_{13} & -g_{23} & 0 \end{pmatrix} \quad K^{(12)} = \begin{pmatrix} k_{11} & k_{12} & 0 \\ k_{21} & k_{22} & \\ 0 & 0 & k_{33} \end{pmatrix}, \tag{10}$$

where $g_{13} = (I_1 + I_3 - I_2)\cos\theta - H\sin\phi$, $g_{23} = -(I_2+I_3-I_1)\sin\theta + H\cos\phi$;
$k_{11} = (I_2-I_3)(3 + \cos^2\theta) + H\sin\phi\cos\theta$, $k_{12} = -(I_2-I_3)\sin\theta\cos\theta - H\sin\phi\sin\theta$,

$k_{21} = -(I_1-I_3)\sin\theta\cos\theta - H\cos\phi\cos\theta$, $k_{22} = (I_1-I_3)(3 + \sin^2\theta) + H\sin\theta\cos\phi$,
and $k_{33} = (I_2-I_1)\cos 2\theta + H\sin(\phi+\theta)$. (11)

First of all, it is noted that when the gyrostat is not provided then, with H=0 and θ=0, equation (9) reduces, in the absence of damping, to the classical set of equations with the equation in θ_2 decoupled from the equations in θ_1 and θ_3. The stability criteria in the classical situation are well known, namely, the Lagrange[4] configuration as that of the moon being

$I_2 > I_1 > I_3$, where I_2 of the axis of rotation is the largest, and the other configuration, sometimes referred to as the Delp[5,6] configuration in the modern attitude control literature, being $I_1 > I_3 > I_2$, where I_2 of the axis of rotation is the least.

Criteria of asymptotic stability for the solution of Equation (9) in the case when $K^{(12)}$ is not symmetric can be obtained from the characteristic equation, $|Is^2+(D+G^{(12)})s + K^{(12)}| =$ or

$$a_0 s^5 + a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s + a_6 = 0 \qquad (12)$$

where

$$a_0 = I_1 I_2 I_3, \; a_1 = I_1 I_2 c_3 + I_2 I_3 c_1 + I_3 I_1 c_2,$$

$$\begin{aligned}
a_2 &= I_1 I_2 k_{33} + I_2 I_3 k_{11} + I_3 I_1 k_{22} + I_1 c_2 c_3 + I_2 c_3 c_1 + \\
&\quad I_3 c_1 c_2 + I_1 g_{23}^2 + I_2 g_{13}^2 \\
a_3 &= I_1(k_{22}c_3+k_{33}c_2) + I_2(k_{11}c_3+k_{33}c_1) + I_3(k_{11}c_2+k_{22}c_1) + \\
&\quad c_1 g_{23}^2 + c_2 g_{13}^2 \\
a_4 &= I_1 k_{22} k_{33} + I_2 k_{11} k_{33} + I_3(k_{11}k_{22}-k_{12}k_{21}) + k_{11}c_2c_3 + \\
&\quad k_{22}c_3c_1 + k_{33}c_1c_2 + k_{11}g_{23}^2 + k_{22}g_{13}^2 - \\
&\quad (k_{12} + k_{21})g_{13}g_{23} \\
a_5 &= (k_{11}k_{22}-k_{12}k_{21})c_3 + k_{22}k_{33}c_1 + k_{33}k_{11}c_2, \\
a_6 &= k_{33}(k_{11}k_{22}-k_{12}k_{21}). \qquad (12a)
\end{aligned}$$

The necessary conditions for asymptotic stability are that all the a_i's are positive definite, or simply that

$$k_{11}, k_{22}, k_{33} > 0 \qquad (13a)$$

and

$$\begin{vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{vmatrix} > 0 \qquad (13b)$$

To obtain the sufficient conditions we have to construct either the Routh array or the Hurwitz determinants from the coefficients, a_i, from which four additional inequalities are obtained. These conditions are lengthy and complex and are rather difficult to interpret; so they are not presented here.

In the case when h lies on the $\xi_2\xi_3$-plane, the linearized equation of motion is the same as Equation (9) except that the matrices G and K are defined as

$$G^{(23)} = \begin{pmatrix} 0 & g_{12} & g_{13} \\ -g_{12} & 0 & 0 \\ -g_{13} & 0 & 0 \end{pmatrix}, \quad K^{(23)} = \begin{pmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & k_{23} \\ 0 & k_{23} & k_{23} \end{pmatrix} \qquad (14)$$

where $g_{12} = (I_1 + I_2 - I_3)\sin\theta' + H\sin\phi'$,

$g_{13} = (I_1 + I_3 - I_2)\cos\theta' - H\cos\phi'$,

$k_{11} = 4(I_2 - I_3)\cos 2\theta' + H\cos(\phi'+\theta')$,

$k_{22} = (I_1 - I_3)(3\cos^2\theta' - \sin^2\theta') - H\sin\theta'\sin\phi'$,

$k_{23} = 2(I_3 - I_1)\sin 2\theta' - H\sin\phi'\cos\theta'$,

$$k_{32} = 2(I_2-I_1)\sin 2\theta' + H\cos\phi'\sin\theta',$$

and $$k_{33} = (I_2-I_1)(\cos^2\theta'-3\sin^2\theta') + H\cos\phi'\cos\theta'.$$

The necessary conditions for asymptotic stability are the same as those in (13a) and (13b) for the preceding case except (13b) be replaced by

$$\begin{vmatrix} k_{22} & k_{23} \\ k_{32} & k_{33} \end{vmatrix} > 0 \tag{15}$$

We may now reduce either of the two foregoing cases to an aligned equilibrium attitude in which the x_i-axes are in line with the ξ_i-axes, $i=1,2,3$, and h points in the x_2- or ξ_2- direction. This may be effected by turning h either from $\xi_1\xi_2$-plane until $\theta=0$ and $\phi=\pi/2$ or from the $\xi_2\xi_3$-plane until $\theta'=0$ and $\phi'=0$. Then Equation (9) (either with $G^{(12)}$, $K^{(12)}$ or with $G^{(23)}$ and $K^{(23)}$) reduce to

$$Iv'' + Dv' + G^{(2)}v' + K^{(2)}v = 0 \tag{16}$$

where

$$G^{(2)} = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & 0 \\ -g_{13} & 0 & 0 \end{pmatrix} \text{ and } K^{(2)} = \begin{pmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{pmatrix} \tag{17}$$

with $$g_{13} = (I_1 + I_3 - I_2) - H,$$

$$k_{11} = 4(I_2-I_3) + H,\ k_{22} = 3(I_1-I_3),\ k_{33} = (I_2-I_1) + H.$$

Equation (16) consists of an equation in θ_2 alone and a set of two equations in θ_1 and θ_3. This indicates that to the first order, the gyrostat along the x_2-axis affects not the pitch motion (about the same axis) but the yaw and roll motions. Thus the gyrostat in the pitch direction will either stiffen or soften the frequency of the roll and yaw librational modes, depending on the magnitude of H and the moments of inertia. Now since the $K^{(2)}$ matrix is symmetric, we may apply the Kelvin-Tait-Chetayev Theorem[7] for asymptotic stability of the solution of Equation (16), where I and D are symmetric positive definite and G is skew symmetric. The criteria of asymptotic stability are simply that the three eigenvalues of $K^{(2)}$ in (17) be positive definite, namely,

$$\begin{aligned} &4(I_2-I_3) + H > 0 \\ &(I_2-I_1) + H > 0 \\ &I_1 - I_3 > 0 \end{aligned} \tag{18}$$

This result adds nothing new to the Lagrange configuration, $I_2>I_1>I_3$, since the Lagrange configuration is already asymptotically stable without the gryostat. (Note: the gyrostat, though not needed to achieve asymptotic stability for the Lagrange configuration, would probably have a stabilizing effect so far as to reduce the error angles produced by an external disturbance is concerned.) Nevertheless, for the attitude configuration $I_1 > I_3 > I_2$, the inequalities in (18) mean that this attitude equilibrium which was stable[5,6] but not asymptotically stable[7] can now be made asymptotically stable by the addition of a gyrostat (see Figure 4). The role of the gyrostat is simply to convert two of the eigenvalues of $K^{(2)}$ from negative to positive. Such a result may be stated as follows: The attitude configuration, $I_1 > I_3 > I_2$, can be made asymptotically stable by adding along the axis of rotation with the minimum of inertia a gyrostat of an angular momentum,

$$H(=h/\Omega) > \text{larger}\ [4(I_3-I_2),(I_1-I_2)] \tag{19}$$

For an axisymmetric body with $I_1 = I_2$, the criteria for asymptotic stability reduce from (18) (or directly from $K^{(12)}$ in (10) with $\theta = 0$ and $\phi = \pi/2$) to

$$\begin{aligned} 4(I_2-I_3) + H &> 0 \\ H &> 0 \\ I_2-I_3 &> 0 \end{aligned} \tag{20}$$

Hence, as long as I_3 is the minimum moment of inertia, the earth-pointing attitude motion of such an elongated body (see Figure 5) is asymptotically stable. Here, the gyrostat is used to control the yaw motion as may be seen from the equations in θ_1 and θ_3.

If the body is symmetric with respect to the x_1-axis, i.e., $I_2 = I_3$, then the criteria for asymptotic stability reduce from (18) (or directly from $K^{(23)}$ in (14) with $\theta'=0$ and $\phi' = 0$) to

$$\begin{aligned} H &> 0 \\ (I_2-I_1) + H &> 0 \\ I_1-I_2 &> 0 \end{aligned} \tag{21}$$

Hence, a flat drum-shaped body, $I_1>I_2$, equipped along the x_2-axis of rotation with a gyrostat of angular momentum $H>I_1-I_2$,(see Figure 6), will have an asymptotically stable attitude motion.

It is worth remarking here that for an axisymmetric body with $I_1=I_3$ Equations (16) reduce in the absence of damping to the ones investigated in References [8], [9] for the case of spin stabilization if the spin angular momentum there is identified as our gyrostat angular momentum here. But when the dissipative effect is taken into consideration, the results in these two cases are divergently different. This is because of the fact that in the spin case energy dissipation tends to increase the angle of precession when the spin axis has a minimum moment of inertia whereas here with the gyrostat energy dissipation results in asymptotic stability. For this reason it is of no significance to study the oscillation stability (with D=0) as done in [8] and [9].

Suppose that the gyrostat is not perfectly in line with the axis of rotation, let us now calculate the error angles induced by the misalignment. Denote ϕ_2 $(= \pi/2 - \phi)$ as the angle that h is deviated from the x_2-axis in the x_1x_2-plane and ϕ_1 as the out-of-plane misaligned angle (ϕ_1 is positive if h has a positive x_3-component). Within the linearized framework we assume that ϕ_1 and ϕ_2 are of the same order of smallness as θ_1, θ_2, θ_3. It can be easily shown that there result a cocked angle in θ_1

$$\theta_1{}^* = -H\phi_1/[4(I_2-I_3) + H]$$

due to the out-of-plane misalignment and a cocked angle in θ_3

$$\theta_3{}^* = H\phi_2/[I_2-I_1 + H]$$

due to the in-plane misalignment. For the attitude configuration, $I_2>I_1>I_3$, we see that the gyrostat makes the cocked angles smaller than the misalignment angles. For the attitude configuration, $I_1>I_3>I_2$, we find that the magnitude of H should not be chosen to be nearly equal to the minimum value specified in (19) if small cocked angles are desired.

IV. CONCLUDING REMARKS

It has been shown that stable attitude motions can be achieved from a vehicle containing a gyrostat. For the equilibrium attitude of the $I_1>I_3>I_2$ configuration where I_2 is the moment of inertia of the axis of rotation, which is asymptotically unstable in the classical case, we found that the gyrostat, oriented normal to the orbital plane, will make it become asymptotically stable. For the Lagrange configuration, $I_2 > I_1 > I_3$, which is already asymptotically stable in the classical case, the gyrostat would probably render a stabilizing effect in the presence of external disturbances, such as aerodynamic and solar torques. Such a stability problem remains to be investigated though the result depends on the particular nature of the disturbance.

The gyrostat employed for stability purpose could already have been available in the vehicle. An example of this would be that in the Apollo Applications Program mission for solar or stellar astronomy observations the vehicle is oriented toward a fixed direction in the inertial frame by means of gimbal-mounted control-moment gyros. Whenever desired, the gimbals are turned and locked so that the attitude motion of the vehicle is converted to an earth-pointing one. Such a stable motion (either in the $I_1 > I_3 > I_2$ or the $I_2 > I_1 > I_3$ attitude configuration) may then be used for earth observation or, by use of a counter-rotating device at an angular rate of $-\Omega$, for star observation.

V. ACKNOWLEDGMENTS

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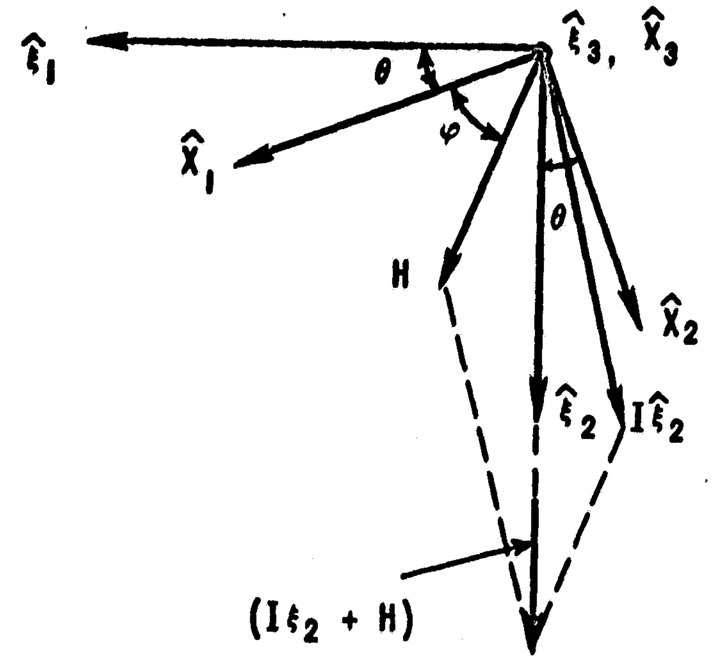
E. Y. Yu

1022-EYY   E. Y. Yu

Attachments
Figures 1-6
References



$\hat{\xi}_1$
$\hat{\xi}_3, \hat{X}_3$
θ
$\hat{X}_1$
φ
θ
H
$\hat{X}_2$
$\hat{\xi}_2$
$I\hat{\xi}_2$
$(I\xi_2 + H)$

FIGURE I

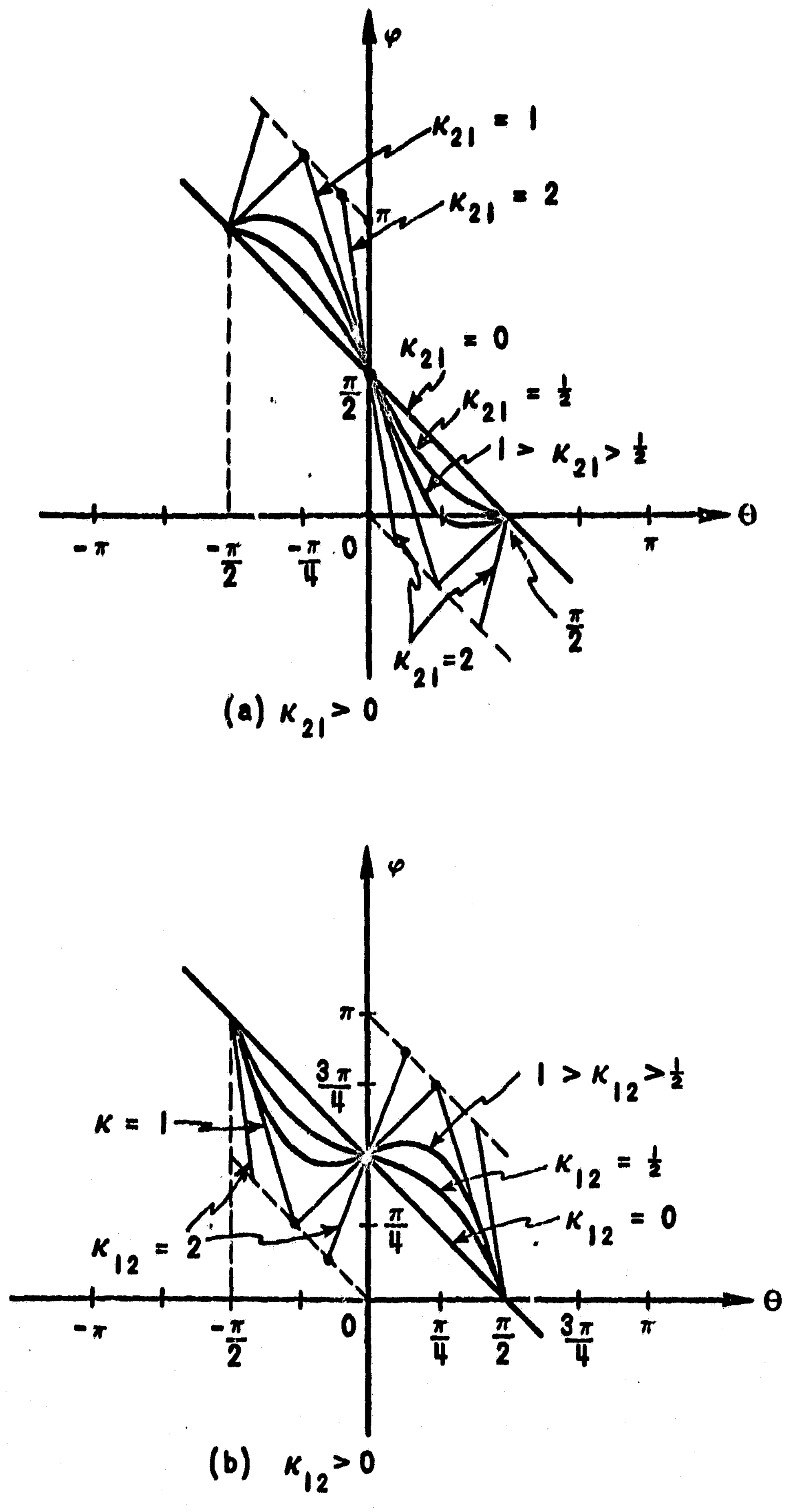


FIGURE 2 - PLOT OF COS $(\theta + \varphi) = \kappa_{21}$ SIN 2Θ

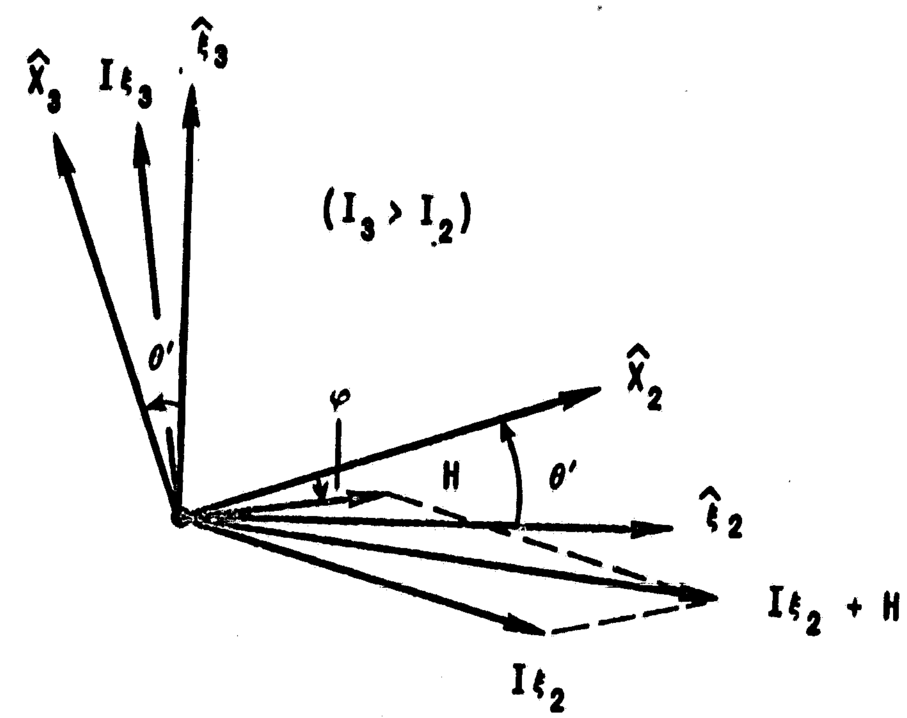

$\hat{X}_3$
$I\xi_3$
$\hat{\xi}_3$
$(I_3 > I_2)$
θ'
$\hat{X}_2$
φ
H
θ'
$\hat{\xi}_2$
$I\xi_2 + H$
$I\xi_2$

FIGURE 3

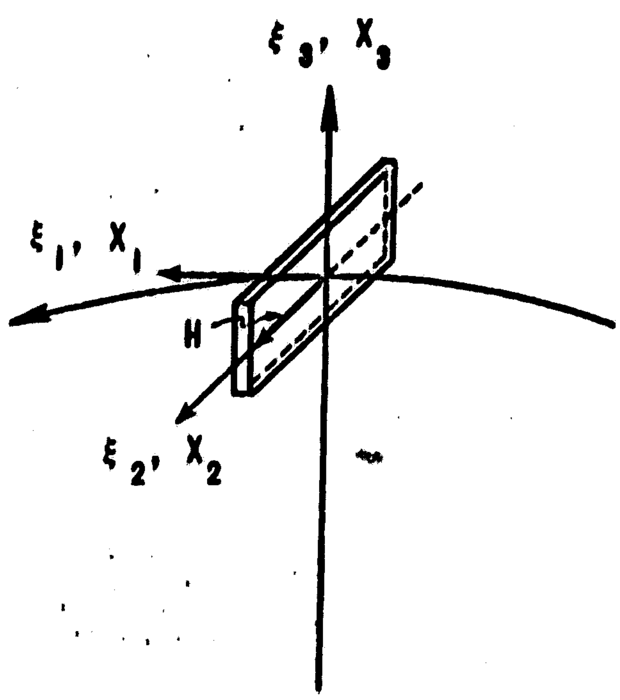


FIGURE 4

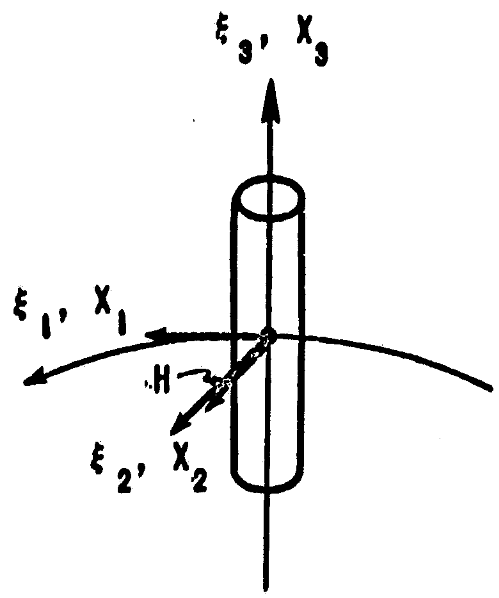


FIGURE 5

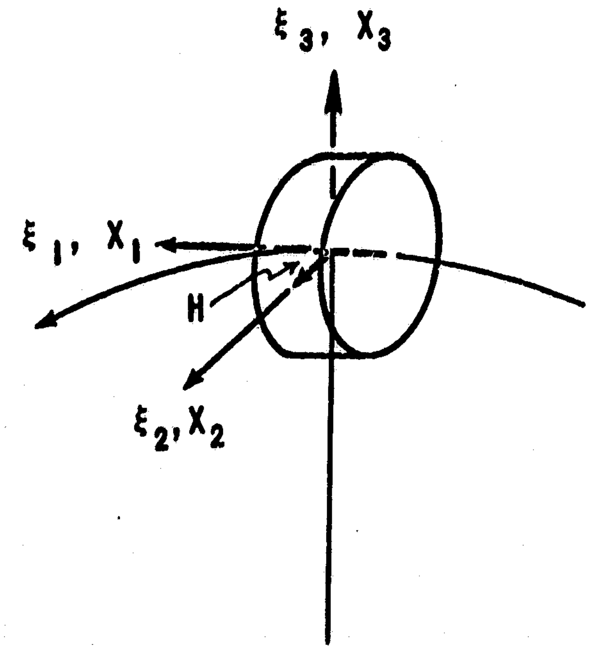


FIGURE 6

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