ANGULAR AND ENERGY DISTRIBUTION OF PROTONS IN AURORAE

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ABSTRACT: Based on the profiles of the H_α line obtained by observations of aurorae at different zenith distances, calculations were made of the pitch-angle distribution and the energy spectrum of the incident protons. The calculated spectrum corresponded well with the measured spectrum in the solar wind.

A qualitative analysis of the first profiles of the hydrogen lines in the spectra of aurorae [1-3] allowed for drawing the conclusion that the protons entering the atmosphere have a broad spectrum of velocities, with a maximum of about 1000 km/sec [4,5]. Attempts were made to explain the observed profiles of the hydrogen lines by the intrusion of monoenergetic protons with a specially selected angular distribution of the velocities [6, 7]. However, it was shown in [8] that it is impossible to coordinate the profiles at the magnetic zenith and the magnetic horizon by such a method. Using more complete data on effective cross-sections, the authors of other articles [9, 10] drew a conclusion as to the necessity of assuming that some of the incident protons have relatively low velocities. /73

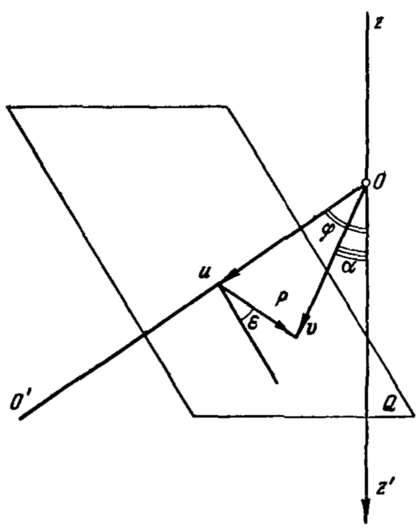

Fig. 1. Geometry of the Observations. *zz'* is the Line of Force; *OO'* is Line of Sight; v is the Total Velocity of the Hydrogen Atom; u is its Projection to the Line of Sight. The Plane is Perpendicular to *OO'*. ε is the Azimuthal Angle in Plane *Q*.

A more detailed analysis of the line profiles showed that, in the range of velocities on the order of several hundreds of kilometers per second (1-5 KeV), the differential energy spectrum of the protons can take the form $E_0^{-1.8}$, where E_0 is the energy of the proton before it enters the atmosphere [11]. If we represent the angular distribution of the velocities for the entering protons by the term $\cos^n \alpha$, where α is the pitch-angle, then a different approach to analysis of the profiles leads to different concepts as to the elongation of this distribution: from $n = 0$ to $n = 6$ [11, 12]. This contradictory result, obvioulsy, cannot be explained in full only by a low accuracy in the data of the observations. One of the possible means of solving this problem is found in denying the assumption that the angular distribution of the protons does not depend on their velocity (this assumption was postulated in an analysis of the hydrogen profiles [9-14]). Even if the protons do not undergo a noticeable scattering during their deceleration in the atmosphere [11, 14], we cannot consider a priori that the angular distribution of the incident protons does not depend on their energy. /74

Let $j(v,\alpha)$ be the flux of the neutralized protons at a unit solid angle which have velocities from v to $v+dv$ and an angle α between the direction of the velocity and the magnetic line of force (Fig. 1). We will consider that this flux is azimuthally symmetrical in relation to the line of force. We will also assume that, regardless of the previous history of the proton, the hydrogen atom has the probability of radiation of a quantum $F(v)$ at a given velocity[1] v. Then the observed intensity of the radiation in the interval of the wavelengths $(\lambda, \lambda+d\lambda)$, or the corresponding interval of the Doppler velocities $(u, u+du)$, for observations at an angle ϕ to the magnetic line of force, is the integral for the plane Q with a parameter u intersecting the distribution of the radiating hydrogen atoms in the velocity space (Fig. 1). In this case, for the sign of the integral, each atom enters with a weight $F(v)$. Calling this intensity $I(u,\phi)du$, we will obtain the following:

$$I(u,\varphi)\,du = 2du \int\limits_{p=0}^{\infty} \int\limits_{\varepsilon=0}^{\pi} j(v,\alpha)\,F(y)\,p\,dp\,d\varepsilon, \qquad (1)$$

where p is the vector difference $v-u$; ε is the angle measured in the plane Q. Substituting p and ε with v and α, we will obtain the following:

$$I(u,\varphi) = 2\int\limits_{v=u}^{\infty} \int\limits_{\alpha_1=\varphi-\arccos\frac{u}{v}}^{\alpha_2=\varphi+\arccos\frac{u}{v}} \frac{j(v,\alpha)\,F(v)\,v\sin\alpha\,dv\cdot d\alpha}{\sqrt{\sin^2\varphi-\cos^2\alpha+2\frac{u}{v}\cos\alpha\cos\varphi-\frac{u^2}{v^2}}}. \qquad (1')$$

This is a strictly valid equation if we assume that $F(v)$ does not depend on the previous history of the emitting hydrogen atom.

Let us now assume that $j(v,\alpha)$ can be represented by the following finite series:

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[1] According to [11], the equilibrium distribution by states is established after the passing of 0.1 of the complete path.

$$j(v, \alpha) = \sum_{n=0}^{N} A_n(v) \cos^n \alpha. \tag{2}$$

Then integration by α is accomplished by the following quadratures:

$$I(u, \varphi) = 2\pi \int_u^\infty \sum_{n=0}^{N} v A_n(v) F(v) K_n\left(\cos\varphi, \frac{u}{v}\right) dv, \tag{3}$$

where $K_n(\cos\phi, u/v)$ are the polynomials of degree n for $\cos\phi$ and u/v. A further solution was made on the assumption that, in the expansion in (2), we can limit ourselves to $N = 6$. The corresponding coefficients of K_n are shown below.

$$K_0 = 1,$$

$$K_1 = \frac{u}{v}\cos\varphi,$$

$$K_2 = \frac{\sin^2\varphi}{2} + \frac{u^2}{v^2}\frac{3\cos^2\varphi - 1}{2},$$

$$K_3 = \frac{u}{v}\frac{3\sin^2\varphi\cos\varphi}{2} + \frac{u^3}{v^3}\frac{\cos\varphi(5\cos^2\varphi - 3)}{2},$$

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$$K_4 = \frac{3\sin^4\varphi}{8} + \frac{u^2}{v^2}\frac{\sin^2\varphi(15\cos^2\varphi - 3)}{4} + \frac{u^4}{v^4} \times \\ \times \frac{35\cos^4\varphi - 30\cos^2\varphi + 3}{8},$$

$$K_5 = \frac{u}{v}\frac{15\sin^4\varphi\cos\varphi}{8} + \frac{u^3}{v^3}\frac{5\sin^2\varphi\cos\varphi(7\cos^2\varphi - 3)}{4} + \\ + \frac{u^5}{v^5}\frac{\cos\varphi(63\cos^4\varphi - 70\cos^2\varphi + 15)}{8},$$

$$K_6 = \frac{5\sin^6\varphi}{16} + \frac{u^2}{v^2}\frac{15\sin^4\varphi(7\cos^2\varphi - 1)}{16} + \frac{u^4}{v^4} \times \\ \times \frac{15\sin^2\varphi(21\cos^4\varphi - 14\cos^2\varphi + 1)}{16} + \frac{u^6}{v^6}\frac{(231\cos^6\varphi - 315\cos^4\varphi + 105\cos^2\varphi - 5)}{16}.$$

The Volterra integral equation of the first type in (3) can be rewritten, in our case, by the following:

$$I(u, \varphi) = 2\pi \int_u^U \sum_{n=0}^{N} B_n(v) K_n\left(\cos\varphi, \frac{u}{v}\right) dv, \tag{3'}$$

where

$$B_n(v) = v A_n(v) F(v), \tag{4}$$

while U is determined by the condition $I(u>U, \phi) = 0$, since the radiation was limited to the finite interval of wavelengths around an unbiased line. Equation (3') can be solved numerically, and, in order to determine the coefficients $B_n(v)$, we must have the values of the function $I(u, \phi)$ for the values of the angle ϕ at $N + 1$. We took the following values of the angles ϕ: 0.30°, 60, 90, 120, 150, and 180°. We used the simplest formula for an approximating

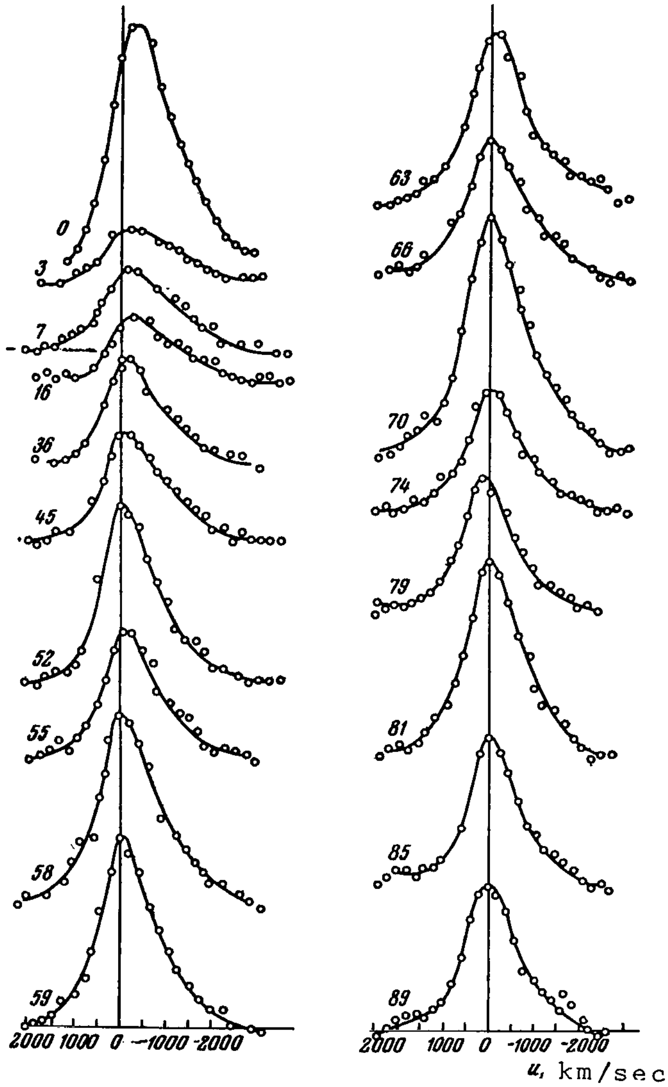

Fig. 2. Profiles of the H α Lines (Not Corrected for the Instrumental Broadening) Obtained by Sighting at Various Angles ϕ (Numbers on the Curves) to the Line of Force. The Width of the Gap in the Spectrometer was Equal to 10 Å (620 km/sec). The Scale of the Ordinates is the Intensity on a Linear Scale.

integration - the trapezoidal rule. The solution was made separately by two grids with various intervals of Δu differing by a factor of 1.5. Since, obviously, $B_n(v>U) = 0$, then as we know, beginning the solution at the point $u = U-\Delta u$, we will obtain very simple systems of equations for determining $B_n(v)$. The problem is also simplified because of the symmetry of the coefficients K_n; instead of a system of equations with seven unknown values of $B_n(v)$, we will obtain two independent systems for $B_n(v)$ with even and odd indices, using $I(u,\phi) \pm I(u,\pi-\phi)$ instead of $I(u,\phi)$.

The values of $I(u,\phi)$ were taken according to the spectra of aurorae obtained at the Loparskaya station during 1959-1960 with the aid of a photoelectric spectrometer [15]. We used the records which were recorded with a gap-width of 10 Å. The profiles of the H_β lines were obtained from 20:30 to 23:00 U.T. on January 25, 1969. During this time, a wide hydrogen field was observed at Loparskaya [15]; there were no clear forms of aurorae. The spectra were obtained at the meridian, at various angles to the line of force; in addition to the H_β line, line 5003 Å N II and the band 4709 N_2^+ were also recorded. Figure 2 shows H_α profiles which were corrected for the instrumental broadening. Each of these profiles was obtained by averaging three to five sequential records. The angle ϕ was shown around each profile. The profile for $\phi = 0$ was obtained on another night, on January 4, 1960 [15][2]. The profiles in Figure 2 were averaged in the following way. After correcting for the /77 instrumental broadening by the graphic method in [16], all the profiles were normalized by area (i.e., we are considering that the hydrogen atoms irradiate isotropically), and, from this, we constructed the relationships between the intensity at the given wavelength (or the given Doppler velocity) and the angle ϕ (Fig. 3). The curves, leveled out for the points, were used for constructing the average profiles (Fig. 4) for the angles $\phi = 0$ (180), 30 (150), 60 (120), and 90°. We can see that the red branch for the zenith profile ($\phi = 0$) and the width of the horizontal profile are greater than for most of the published profiles of hydrogen lines [17]. This can probably be explained in part by the insufficiencies of the method used for corrections of instrumental broadening. Corrections were not made for the effect of scattered light, since the /78 transmittance was good, and since evaluations on the basis of the available data [18] show that the effect of scattered light is within the limits of the accuracy in the observations. The point of view

[2] The correction for instrumental broadening was done graphically in [15] with the coefficient presented in [16]; however, an erroneous value of the coefficient is given in the latter work for a triangular instrumental profile. Therefore, the H_α was not reliably corrected in [16]. The profiles used here were corrected with the coefficient 1/3, instead of 8/3, which was presented in [16] for a triangular operational function.

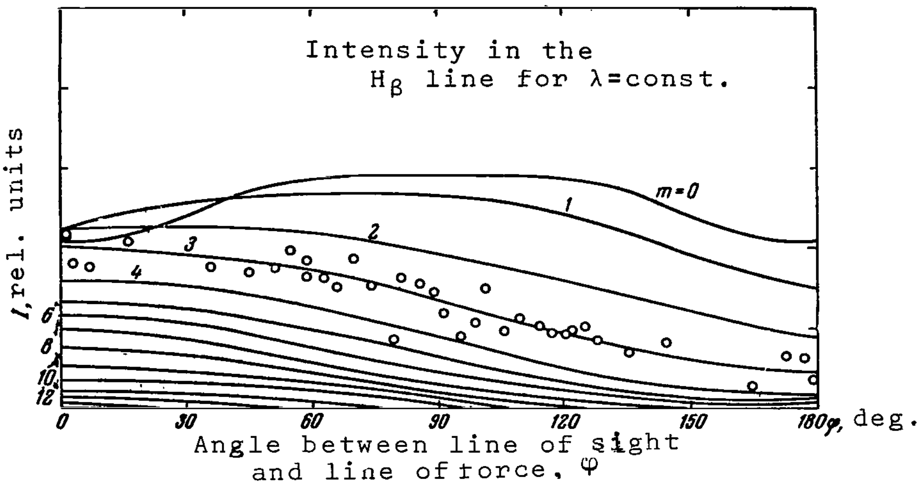

Fig. 3. Relationship Between the Intensity for Various Wavelengths and the Angle ϕ, Constructed from Profiles Normalized for the Area. The Symmetrical Curve is for λm = 4861 Å; the Following are for $\lambda m = 4861 \pm m \cdot 3.88$ Å (Each Profile Gives Two Points: for ϕ and for $180° - \phi$). The Dispersion of Points is Given for m = 3.

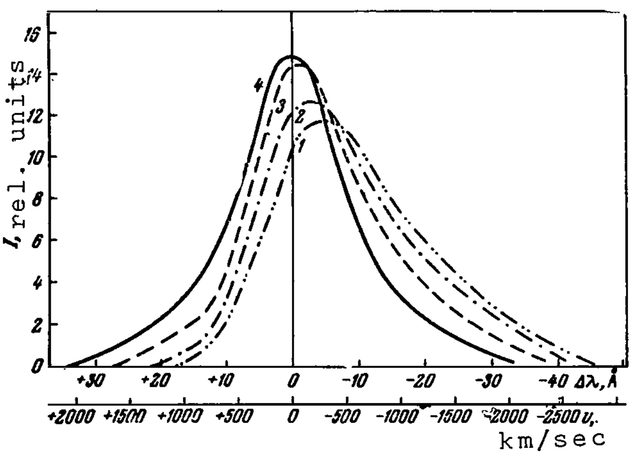

Fig. 4. Profiles of the H_α Lines Used for Calculations. Angle of Sighting: ϕ = 0 (1); 30 (2); 60 (3), and 90° (4).

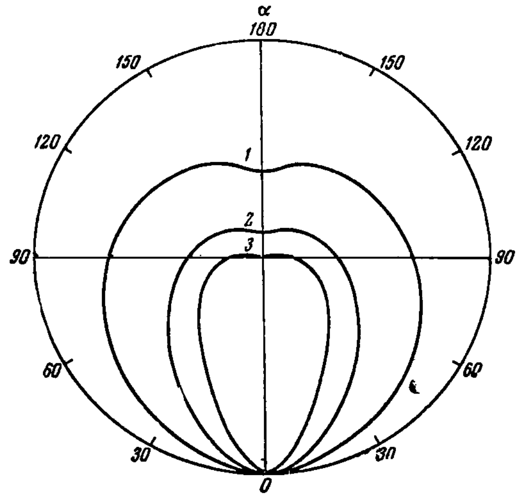

Fig. 5. Angular Distribution of the Directed Flux of Protons in the Atmosphere. (1) $E \leq 1$ KeV; (2) E = 3 KeV; (3) E = 20 KeV.

in [19] was that the red branch of the zenith profile could be explained by the scattering of the light from other regions of the sky. This, obviously, is not completely valid. Other published profiles were not used in this study, since, in this case, there would not be a complete system for the values of $I(u,\alpha)$; later, we will attempt to evaluate the degree of dependence of the results on variation of the original data.

The profiles in Figure 4 were also used for a numerical solution to Equation (3'). The angular characteristics of the distribution,

$$j(v, \alpha) = \frac{1}{vF(v)} \sum_{n=0}^{N} B_n(v) \cos^n \alpha \qquad (2')$$

can be obtained, obviously, without knowing the function $F(v)$. The relative angular distributions for three values of energy are given in Figure 5. We should note that both the angular and the energy characteristics of the beam of protons in the atmosphere were obtained from the Doppler profiles, and relate to the entire strata of the atmosphere. As we can see from Figure 5, the slower the protons are, the closer their angular distribution is to the isotropic. The change in directed flux with the angle, for the examined range of energies, is less abrupt than $\cos^6\alpha$. Therefore, we can consider the representation of $j(v,\alpha)$ in (2) as valid, for $N = 6$.

We took the function presented by Chamberlain [11] for the function $F(v)$:

$$F(v) = K \frac{v^2}{\beta^2} e^{-\frac{v^2}{\beta^2}}, \qquad (5)$$

which is equal to the number of irradiated Balmer quanta per unit interval of a change in velocity. Here $K = 1.84 \cdot 10^{-}$ quanta/km/sec; $\beta = 2000$ km/sec. This function was calculated [11] from the condition of statistical equilibrium for these theoretical and exper- /79
imental values of the effective profiles; the principal contribution in (5) was found in the charge exchange between the protons at the original level and the atoms and molecules of the atmosphere, and in the excitation of the neutralized protons. Having integrated the directed flux by (2') for the sphere, and using (5) in order to consider the effectiveness of de-excitation, we will obtain the function $j(v)$, shown in Figure 6, which is the number of hydrogen atoms with the velocity v passing through a single interval of velocities per second during deceleration (regardless of the altitude in the atmosphere). The scatter in points in Figure 6 characterizes the accuracy of the calculations. The latter was also tested by calculations of $I(0, 0)$ for the calculated values of $j(v,\alpha)$. The values of $I(0, 0)$ calculated by this method differ from those observed by about 10%.

Since the spectrum in Figure 6 related to the entire thickness of the atmosphere, the proton which has an original velocity of v is represented in this spectrum for all the lesser values of

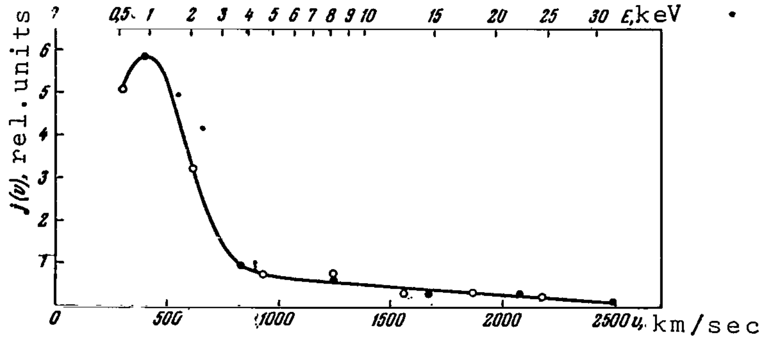

Fig. 6. Number of Protons Passing through a Single Interval of Velocities per Second During Deceleration in the Atmosphere (Relative Units). The Various Indices Show the Values Obtained by Solving Equation (3') for Different Intervals of Δu.

the velocity, if it was not deflected away and did not go beyond that level where it is excited. In this case, the spectrum in Figure 6 should be a non-increasing function of v. However, for energies less than 1 KeV, there is a steep drop in the spectrum; its value exceeds the calculation errors. This probably indicates that the protons with lesser energy undergo a significant scattering in the atmosphere. If we consider that the amount of escaping protons is small (at least for an energy greater than 1 KeV), i.e., that each proton gives an equal contribution to the spectrum in Figure 6 for all velocities less than its original velocity, then the differential energy spectrum of the incident protons can be obtained from the curve in Figure 6 by differentiation according to velocity[3]. This spectrum is shown in Figure 7. The normalization of the spectrum was made on the assumption that the intensity of the H_β line was equal to 100 Rayleighs. Such an intensity was rather normal for the hydrogen fields observed in an auroral zone during the IGY [15]. The dashed line in the same figure shows the energy spectrum calculated by Chamberlain [11] on the assumption that protons of different energies have a similar angular distribution. This spectrum has the form $E^{-1.8}$, while that obtained by a solution of Equation (3) changes as $E_0^{-2.7}$ in the range of energies from 1 to 5 KeV. We can see that denial of the assumption that the angular and energy distributions are not inter-dependent significantly

[3] The energy spectrum of protons in the atmosphere is an integral in relation to the energy spectrum of the entering protons.

changes the appearance of the energy spectrum; some of the divergences, obviously, can be explained by the use of different line profiles in [11] and in this study. /80

A comparison of Figures 5 and 7 provides for making a more reliable determination of the existence of proton scattering during their deceleration in the atmosphere: actually, the broadest angular distribution is observed for those protons ($E \leq 1$ KeV) whose flux is already small in the preliminary beam. After obtaining reliable experimental data on the cross sections of the charge exchange and the excitation for energies less than 5 KeV, it will probably be possible to determine the presence or absence of a relationship between the anisotropy and the energy for the incident beam from the spectral data.

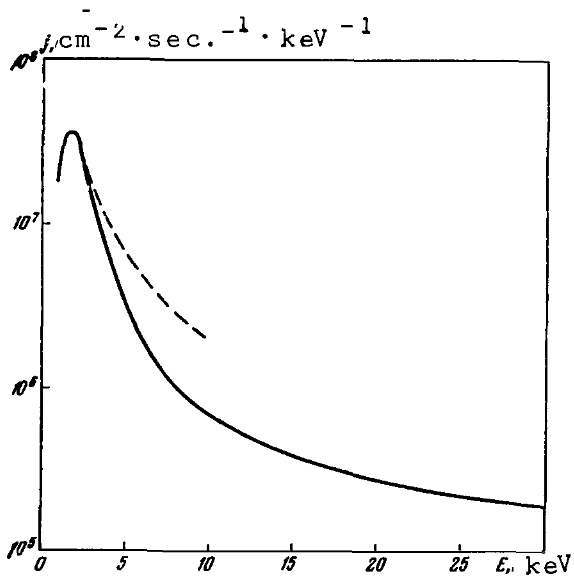

Fig. 7. Calculated Energy Spectrum of the Incident Protons for the Line Intensity H_β = 100 Rayleighs (Solid Line). The Dashes Show the Energy Spectrum Calculated on the Assumption that the Angular and Energy Spectra of the Protons are not Inter-Dependent [11].

Measurements were made recently of the energy spectrum of the particles in the solar wind and in the transitional region where the solar plasma flows around the Earth's magnetosphere [20]. Figure 8 shows the energy spectrum measured in the transitional region on the satellite OGO-I in 1964. The energy spectrum of the protons according to Figure 7 is also shown here. This spectrum was calculated in units of proton concentrations, and not in units of flux. It was displaced along the axis of the ordinates up to the best coincidence with the measurements in the transitional area; this normalization corresponds to the H_β intensity of 70 Rayleighs. We can see that the conformity of the energy spectra of the protons is not substantiated outside the magnetosphere and in the auroral zone, this will indicate that the protons, in contrast to electrons, do not need accelerations in order to penetrate into the auroral zone. The measurements of the flux of protons with energies > 4 KeV in the auroral zone [21] also show that the value of this flux practically coincides with the value of the flux of protons with $E > 4$ KeV in the transitional area.

The divergence between the curve and the points in Figure 8 for E < 1.5 KeV, if it is real, could find a natural explanation in the change in velocity of the solar wind with variation in solar activity [22]. Actually, the curve was obtained from the profiles of the hydrogen lines observed during a year of high solar activity, while the measurements of the transitional area were made during a year of minimum solar activity. Since the change in the spectrum with variation in solar activity, obviously, is greatest in the region with lowest energies, where the profiles of the excitation of Balmer lines are small, it is not surprising that the profiles of the hydrogen lines obtained by different authors and at different times are practically identical. However, it is probable that systematic variations could be found in the Balmer profile observed at the zenith with a change in solar activity. /81

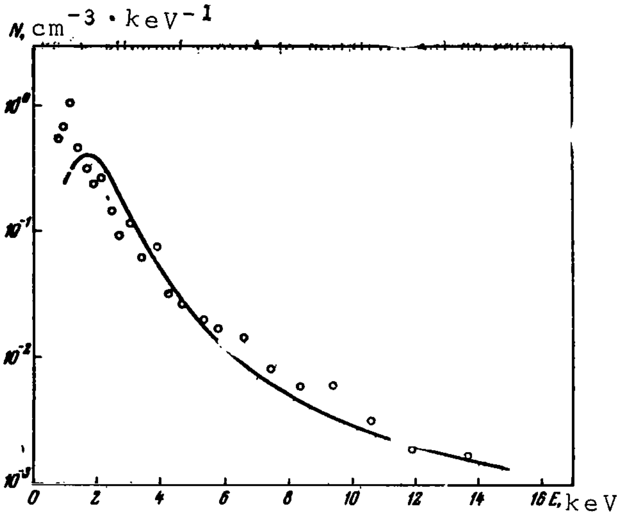

Fig. 8. Comparison Between Proton Density Obtained for Aurorae (Solid Line) and Proton Density in the Transitional Area [20] (Small Circles).

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