TWO NEW RESULTS IN CYLINDRICAL DIOCOTRON THEORY

R. H. Levy

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AVCO EVERETT RESEARCH LABORATORY
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by

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ABSTRACT

Two new results on the diocotron wave which propagates on crossed-field electron beams at low density are presented. One is a solution of the linearized problem valid for arbitrary unperturbed density profiles; this solution is restricted to the fundamental mode. The other result exhibits a special family of exact non-linear solutions to motion of the diocotron type. Both results contribute to a new diagnostic technique for measuring the total charge contained in cylindrical crossed-field electron beams.

The diocotron wave in a cylindrical geometry was treated by Levy;[1] the assumptions and notation of Ref. 1 are used in what follows, except as noted. The equation for the perturbed potential given in Ref. 1 reduces for the fundamental ($\ell = 1$) mode to:

$$[r\omega + E_0/B] \left\{ \frac{1}{r} \frac{d}{dr} \left(r \frac{d\phi}{dr}\right) - \frac{\phi}{r^2} \right\} = \phi \frac{d}{dr} \left\{ \frac{1}{r} \frac{d}{dr} (r E_0/B) \right\} \tag{1}$$

where $E_0(r)$ is the unperturbed radial electric field. $E_0(r)$ is related through Poisson's equation to the unperturbed density profile $n_0(r)$. In Ref. 1, following earlier work, $n_0(r)$ was taken to be stepwise constant, but it now appears that for the fundamental mode a much more general solution can be obtained. This solution follows from the observation that

$$\phi(r) \propto [r\omega + E_0/B] \tag{2}$$

satisfies Eq. (1), a fact easily shown by direct substitution. Thus. Eq. (2) is one of the two eigenfunctions associated with Eq. (1); the other eigenfunction can quickly be obtained by standard methods, but is singular at $r = 0$. Hence, if the electron cloud is contained inside a single perfectly conducting cylinder of radius d , Eq. (2) is the appropriate eigenfunction. The boundary condition $\phi(d) = 0$ then yields the frequency:

$$\omega = - E_0(d)/dB \tag{3}$$

If Q is the total enclosed charge per unit axial length, Poisson's equation gives:

$$Q = 2\pi\epsilon_0 d E_0(d) \quad . \tag{4}$$

Combining (3) and (4) gives:

$$Q = -2\pi\epsilon_0 d^2 \omega B \quad . \tag{5}$$

This result shows, first, that the fundamental mode is always neutrally stable, and, second, that the frequency of the mode is proportional to the total charge per unit length, regardless of its radial distribution. Identification of the fundamental diocotron mode has been made in experiments reported elsewhere;[2,3] measurement of the frequency ω then permits Q to be evaluated from Eq. (5). This technique has proved very valuable in the study of cylindrical crossed-field electron beams across which the potential cannot be measured by standard techniques.

The non-linear equations governing steady two-dimensional motion under the physical conditions appropriate to Ref. 1 are:

$$-\nabla\phi + \underset{\sim}{v} \times \underset{\approx}{B} = 0 \; ; \quad \nabla^2\phi = n_e e/\epsilon_0 \; ; \quad \underset{\sim}{v} \cdot \nabla n_e = 0 \tag{6}$$

and the general solution to these equations is

$$\nabla^2\phi = f(\phi) \tag{7}$$

where f is an arbitrary function of its argument. Among the many possible solutions of (7) we consider only the special solutions that arise when $f(\phi) = -k^2\phi$ (k arbitrary). Among the allowed solutions corresponding to this choice of $f(\phi)$, we choose

$$\phi = \alpha J_0(kr) + \beta J_\ell(kr) \sin \ell\theta \tag{8}$$

α and β are arbitrary constants, and, to satisfy the boundary condition, k must be a root of the equation

$$J_\ell(kd) = 0 \quad . \tag{9}$$

Let us now suppose that the steady solution just derived actually represents an unsteady flow in the laboratory frame, the flow pattern rotating with uniform angular velocity ω/ℓ about the axis of the cylinder. In this case, the potential in the laboratory frame is

$$\phi(r, \theta, t) = \phi_0 \left[\left(\frac{r}{d}\right)^2 + \alpha' J_0(kr) + \beta' J_\ell(kr) \sin(\ell\theta - \omega t) \right] \tag{10}$$

where the constants ϕ_0 , α' and β' are arbitrary, and ω is given by:

$$\omega = 2\ell\phi_0/Bd^2 \quad . \tag{11}$$

The number density is

$$n_e(r, \theta, t) = \frac{4\epsilon_0\phi_0}{ed^2} \left[1 - \frac{\alpha'}{4} (kd)^2 J_0(kr) - \frac{\beta'}{4} (kd)^2 J_\ell(kr) \sin(\ell\theta - \omega t) \right] \tag{12}$$

The solution (10) or (12) is an exact solution to the unsteady equations of motion. Equation (12) makes physical sense provided only that the constants are so chosen that n_e is non-negative. The solution has the character of a wave propagating with angular phase velocity ω/ℓ on an unperturbed field characterized by the first two terms of (10) or (12), but is, of course, valid for arbitrary amplitude β' of the "perturbation".

There must be a correspondence between the two results quoted in this note since if β' is small, the linear and non-linear solutions to the equations of motion must coincide. Thus, if $\ell = 1$, the "unperturbed" electric field is

$$E_0(r) = -\frac{\phi_0}{d}\left[\frac{2r}{d} - \alpha' kd\, J_1(kr)\right] \tag{13}$$

and it is easily shown that Eqs. (2), (10) and (11) are consistent. It follows that Eq. (5) may be valid for large amplitudes in at least some cases.

The analysis of non-linear motions follows closely the work of Lamb[4] on steady incompressible hydrodynamic flows with vorticity.

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