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CURTAILED ATTRIBUTE SAMPLING

Prepared under Contract No. NAS 8-11175 by
A. Clifford Cohen

THE UNIVERSITY OF GEORGIA

For

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama

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The University of Georgia

SUMMARY

This paper is concerned with maximum likelihood estimation of a process (or lot) average proportion of defectives based on attribute samples that have been curtailed either with rejection of a lot on finding the kth defective or with acceptance on find the Kth non-defective. The MLE, $\hat{p}$, based on inspections from a sequence of m lots is shown to be simply the ratio of the number of defectives found to the total number of items inspected, and the asymptotic variance of this estimator is shown to be approximately $\hat{p}(1-\hat{p})/\Sigma_1^m y_i$, where $\Sigma_1^m y_i$ is the total of all items inspected.

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1. INTRODUCTION

Maximum likelihood estimation of the fraction defective and of the ASN (average sample number) in single stage curtailed sampling has been considered by Phatak and Bhatt [5]. In two recent papers, Craig [1,2] derived simplified expressions for the ASN in both single-stage and in two-stage curtailed sampling plans. In the present paper, arguments previously advanced in obtaining estimators are simplified by choosing the number of items inspected as the basic random variable rather than the artificial variable employed in [5]. The end results obtained here are the same as corresponding results obtained by Phatak and Bhatt.

Our concern here is with attribute acceptance plans in which randomly selected individuals units from a lot are inspected in sequence until either

1, an accumulated total of k defectives are found, in which case the lot is rejected, or until

2, an accumulated total of K non-defectives are found, in which case the lot is accepted.

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The number of items Y, inspected in order to reach a decision with respect to acceptance or rejection of a given lot is thereby a discrete random variable which may assume the values k, k+1, . . ., n, where

$$n = k + K - 1. \tag{1}$$

Sampling may thus be terminated after inspecting as few as k items from a lot, and in no case will more than n items be inspected from a given lot.

2. THE RANDOM VARIABLE Y, NUMBER ITEMS INSPECTED

Let p designate the probability of selecting a defective in a single trial, and let this probability remain constant from trial to trial. Furthermore, let the trials be stochastically independent. Thus, p may be interpreted as the process average proportion of defectives. For sufficiently large lots, however, we may justify considering p as the proportion of defectives in the lot. The probability function of Y may be expressed as

$$f(y;p) = f(y \cap R;p) + f(y \cap A;p), \tag{2}$$

where $f(y \cap R;p)$ is the joint probability that $Y = y$, and that the lot will be rejected, whereas $f(y \cap A;p)$ is the joint probability that $Y = y$, and that the lot will be accepted. It follows that

$$f(y \cap R;p) = \binom{y-1}{k-1} p^k q^{y-k}; \quad y = k,\ k+1,\ \ldots,\ n, \tag{3}$$

and

$$f(y \cap A;p) = \binom{y-1}{K-1} q^K p^{y-K}; \quad y = K,\ K+1,\ \ldots,\ n, \tag{4}$$

where $q = 1 - p$.

Functions (3) and (4) are recognized as probability functions of one-parameter negative binomial distributions. The probability function (2) may be expressed as

$$f(y;p) = \begin{cases} f(y \cap R;p); & y = k,\ k+1,\ \ldots,\ K-1. \\ \\ f(y \cap R;p) + f(y \cap A;p); & y = K,\ K+1,\ \ldots,\ n. \end{cases} \tag{5}$$

On substituting (3) and (4) into (5), we have

$$
f(y;p) = \begin{cases} \binom{y-1}{k-1} p^k q^{y-k}; & y = k,\ k+1,\ \ldots,\ K-1, \\ \binom{y-1}{k-1} p^k q^{y-k} + \binom{y-1}{K-1} q^K p^{y-K}; & y = K,\ K+1,\ \ldots,\ n. \\ 0, \text{ elsewhere.} & \end{cases} \tag{6}
$$

The probability of rejecting a lot follows from (3) when we sum on y from k to n. Thus

$$
P(R) = \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k} . \tag{7}
$$

It has been shown by Patil [3,4] by Morris [6], and perhaps by others that the following identity holds between the binomial and the negative binomial distributions

$$
\sum_{z=k}^{n} \binom{n}{z} p^z q^{n-z} = \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k} . \tag{8}
$$

Therefore, the probability of rejecting a lot can be expressed in the alternate form

$$P(R) = \sum_{z=k}^{n} \binom{n}{z} p^z q^{n-z} , \tag{9}$$

where Z is the number of defectives present in a sample of size n, and where n is given by (1).

It consequently follows that the operating characteristic curve of the curtailed sampling plan described above as specified by k and K is identical with the operating characteristic curve of an ordinary single sampling plan specified by n and k where $n = k + K-1$. Curtailment of course permits a decision to be reached with fewer inspections than when fixed size samples are employed.

3. ESTIMATION USING CURTAILED SAMPLES

Suppose that m lots have been subjected to inspection in accordance with the curtailed plan described in Section 2. Let a be the number of lots that were accepted and let r be the number rejected so that

$$m = a + r. \tag{10}$$

Let the number of defectives found and the number of items inspected be recorded for each lot. The sample data then consists of the paired values (z_1, y_1), (z_2, y_2), . . ., (z_a, y_a), (k, y_{a+1}), (k, y_{a+2}), . . ., (k, y_{a+r}) where z_i $(i = 1, \ldots, a)$ is the number of defectives found in the ith accepted lot $(z_i < k)$ and k, of course, is the number of defectives found in each rejected lot. We could describe our sample more concisely as consisting of the paired values (z_i, y_i); $i = 1, 2, \ldots, m$, with $z_i < k$ for accepted lots and $z_i = k$ for rejected lots. Using (6), the likelihood function for such a sample follows as

$$L[\,(z_1,y_1),\ (z_2,y_2),\ \ldots,\ (z_m,y_m)\,] = \tag{11}$$

$$\prod_{i=1}^{r} \binom{y_i - 1}{k-1} p^k q^{y_i - k} \prod_{j=1}^{a} \binom{y_j - 1}{K-1} q^K p^{y_j - K} .$$

On taking logarithms of (11), differentiating with respect to p, equating to zero, and solving for p we obtain

$$\hat{p} = \frac{\left[\sum_{j=1}^{a} y_j - aK\right] + rk}{\sum_{i=1}^{m} y_i} . \qquad (12)$$

We note that in (12),

rk = number of defectives in r rejected lots.

aK = number of non-defectives in a accepted lots,

$\sum_{1}^{a} y_j$ = total number inspections from a accepted lots,

$\sum_{1}^{m} y_i$ = total number inspections from m = a + r lots.

Accordingly, (12) can be written as

$$\hat{p} = \frac{\text{Total number of defectives found}}{\text{Total number of units inspected}} . \qquad (13)$$

The asymptotic variance of p in this case can be obtained as $V(\hat{p}) \sim -1/E(\partial^2 \ln L/\partial p^2)$. which after certain algebraic reduction becomes

$$V(\hat{p}) \sim \hat{p}\hat{q}/mE(Y). \qquad (14)$$

The expected value of Y is given in Section 4 which follows. However, for a sufficiently large number of lots, the mean of the observed values of Y should provide a reasonable approximation to E(Y). Accordingly, the variance of (14) might be approximated as

$$V(\hat{p}) \doteq \hat{p}\hat{q}/\Sigma_1^m y_i \quad . \tag{15}$$

4. AVERAGE SAMPLE SIZE IN CURTAILED SAMPLES

In the notation of this paper, the average sample size (or average sample number, ASN) is merely the expected value of Y.

$$ASN = E(Y). \tag{16}$$

It follows from (6) that

$$E(Y) = \sum_{y=k}^{n} y \binom{y-1}{k-1} p^k q^{y-k} + \sum_{y=K}^{n} y \binom{y-1}{K-1} q^K p^{y-K} \, . \tag{17}$$

which, using the identity of (8), becomes

$$E(Y) = \frac{k}{p} \left\{ 1 - B[p, n+1, k] \right\} + \frac{K}{q} B[p, n+1, k-1]. \tag{18}$$

where

$$B[p,n,k] = \sum_{x=0}^{k} \binom{n}{x} p^x q^{n-x}, \text{ and } n = k + K - 1. \tag{19}$$

The expression given above in (18) was given by Phatak and Bhatt [5], and an equivalent form which enjoys certain advantages for computational purposes was given by Craig [1.2].

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