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Abstract

Power spectra based on Pioneer 6 interplanetary magnetic field data in early 1966 exhibit a frequency dependence of f^{-2} in the range 2.8×10^{-4} to 1.6×10^{-2} cps for periods of both quiet and disturbed field conditions. Both the shape and power levels of these spectra are found to be due to the presence of directional discontinuities in the microstructure (<0.01 AU) of the interplanetary magnetic field. Power spectra at lower frequencies, in the range of 2.3×10^{-6} to 1.4×10^{-4} cps, reflect the field macrostructure (>0.1 AU) and exhibit a frequency dependence roughly between f^{-1} and $f^{-3/2}$. The results are related to theories of galactic cosmic ray modulation and are found to be consistent with recent observations of the modulation.

Introduction

Power spectra or frequency domain studies of fluctuations in the interplanetary magnetic field may be useful in determining the dominant characteristics of the interplanetary magnetic field variations. In addition, recent theoretical work by Jokipii (1966) and Roelof (1966) has related the magnetic field power spectra to "diffusion" coefficients or statistical parameters affecting the propagation of cosmic rays in the interplanetary medium. The results of such theories have been applied to studies of the solar modulation of the galactic cosmic ray intensity (Gloeckler and Jokipii, 1966; Jokipii and Coleman, 1968).

In this paper we report computations of the power spectra of interplanetary magnetic field fluctuations which have been performed with data from the Pioneer 6 satellite. The data used in calculating the power spectra cover the interval of December 1965 to March 1966, and spectral densities were obtained for two frequency ranges: 2.3×10^{-6} to 1.4×10^{-4} cps and 2.8×10^{-4} to 1.6×10^{-2} cps. The shape and levels of the high frequency spectra are found to be directly related to microscale structures in the interplanetary magnetic field with characteristic dimensions of less than 0.01 AU. The spectral results are compared with observations of galactic cosmic ray modulation.

It should be noted that the velocity of convection of the interplanetary medium past the spacecraft by the solar wind, 400-800 km/sec, is much greater than the phase velocity for the propagation of magnetic disturbances, 40-100 km/sec. Thus it is clear that the observed magnetic fluctuations represent principally spatial irregularities rather than explicit temporal variations. If waves are present in the interplanetary medium then it is more appropriate to consider the results as yielding wave number spectra. Previous computations of power spectra (Coleman, 1966; Siscoe, et al., 1968) have employed the frequency convention and it shall be continued here. However, we shall show that the spectral densities and shapes are dominated by the existence of plasma-magnetic field discontinuities which are being convected outward from the sun by the solar wind. We do not support the conclusions of Coleman (1967) that the observed fluctuations of the interplanetary magnetic field, as measured by power spectra, and as correlated with plasma measurements, indicate the existence of waves in the interplanetary medium. Rather we believe that both on the scale in amplitude and frequency which has been studied thus far, the results can be more correctly interpreted in terms of microstructural discontinuities (Burlaga, 1968a).

Spectral Results: High Frequencies

The measurements for this study were made by a fluxgate magnetometer which sampled the interplanetary magnetic field at an average interval of 1.5 seconds with a quantization error of $\pm.25\gamma$, as described in Ness et al. (1966) and Scearce et al. (1968). These measurements were subsequently averaged over 30 second intervals with a resultant precision of approximately $\pm.05\gamma$. The 30 second averages were utilized in obtaining positive frequency power spectra for consecutive twelve hour intervals over the last 14 days in 1965 and the first 14 days in 1966. These spectra cover the frequency range of 2.8×10^{-4} to 1.6×10^{-2} cps, corresponding to periodicities of 60 sec. to 1 hour. Sixty estimates of the spectral density were computed for each twelve hour period, with 40 degrees of freedom per estimate. The noise level, given by the square of the precision divided by the bandwidth of 1/60 cps, is $.013\gamma^2$/cps.

It has been found that the orthogonal component spectra over the frequency interval of 2.8×10^{-4} to 1.6×10^{-2} cps consistently exhibit a frequency dependence with power proportional to f^{-2}. Visual inspection of the magnetic field data when presented in the time domain indicated that this dependence is due to the presence of numerous small-scale directional discontinuities in the interplanetary magnetic field. These have been analyzed by Burlaga (1968b) and are evident generally as distinct and significant changes in the field direction of greater than 30° within a 30 sec. interval. Analysis of discontinuities based on Pioneer 6 data by Burlaga and

Ness (1967) and Burlaga (1968a), and based on Mariner 4 data by Siscoe et al. (1968) has indicated that the directional changes in the field tend to occur by rotation in a plane. This represents a rotational discontinuity formed by a current sheet separating adjacent regions which are in equilibrium. These are one member of a general class of hydromagnetic discontinuities which have recently been discussed by Colburn and Sonett (1966).

Figure 1 shows two 12 hour samples of interplanetary magnetic field data with the discontinuities clearly evident. On this scale the field appears to exhibit a more random rather than a discontinuous behavior. However, if the field is viewed on the smaller scale of an hour, as is discussed in Burlaga (1968b), the fluctuations appear as slow variations in the ambient field between distinct discontinuities, and smaller discontinuities, with angular changes less than 30°, are discernible.

Figures 2 and 3 show representative power spectra of the field components (X, Y, Z) and magnitude F for four twelve hour periods in December 1965. The coordinates used are solar ecliptic, with X and Z referring, respectively, to power per unit bandwidth for component fluctuations in directions out from the sun in the ecliptic plane, and normal to that plane. For visual convenience, the spectra of the components and magnitude are plotted on offset scales rather than superimposed on the same scale. For comparison, each figure contains spectra for periods of quiet and disturbed field conditions as measured by the planetary magnetic activity index Kp. Days 357, hours 12-23 and 358, hours 0-12 are characterized by low values of Kp and few discontinuities, while days 356, hours 0-12 and 361, hours 12-23 are characterized by high Kp and a larger number of discontinuities. The original data for days 356 and 357 have been shown previously in Figure 1.

In general, the power levels of the field components in a given period are comparable, with $P_x(f) \approx P_y(f) \approx P_z(f)$. If the magnetic irregularities are isotropic in the frame of the solar wind, the power spectra should satisfy the relationship (Jokipii, 1967): $P_z(f) = P_y(f) = \frac{1}{2} [P_x(f) - f \frac{d}{df} P_x(f)]$. For power spectra varying as f^{-2} this implies $P_z(f) = P_y(f) = 1.5 P_x(f)$. The results indicate that in this frequency range the magnetic fluctuations are not isotropic. However, since the 95% confidence limits on the spectra fall approximately within the factor of 0.6 - 1.5, we cannot definitely conclude that the fluctuations are consistently non-isotropic.

Statistically significant spectral peaks are observed, and their periodicities in seconds are indicated in Figures 2 and 3. Simultaneous peaks in the components are frequently observed, and significant coherencies between the components, obtained from the cross spectra, are occasionally found at those periodicities. However, over the 24 day period which has been investigated, no constancy has been observed in the frequencies at which the spectral peaks occur.

Except for flattening when the power approaches the noise level, the spectra all exhibit a well-defined f^{-2} dependence, even though the power levels in the components for the disturbed periods vary from 2 to 10 times those in the quiet periods. With the possible exception of Day 356, which is characterized by an abnormally large variance in the field magnitude, (Figure 1), it is noted that the power levels in the field magnitude generally are substantially lower than those of the components, implying that the fluctuations are basically transverse to the average field direction. These results on spectral slope and the relativity small magnitude spectra compared to component spectra are indicative of spectra dominated by discontinuous behavior in the field direction.

Contribution of Discontinuities

The power spectrum of a discontinuity, or step function, varies as f^{-2}, and as discussed by Siscoe et al. (1968) the power spectrum of an ensemble of discontinuities will also vary as f^{-2} at frequencies greater than the frequency corresponding to the average separation between discontinuities. For the Pioneer 6 data in the periods studied, Burlaga (1968b) has determined the average separation to be on the order of 10^{-4} sec.$^{-1}$, which is lower than the frequencies we are sampling. Thus, if the energy of the discontinuities is the dominant contributor, the spectra should vary as f^{-2}. The only change in the spectra for intervals containing a different number of discontinuities, or of discontinuities of differing magnitude, should be in the power levels, and not in the general spectral shape. This indeed has been the result obtained and illustrated in the data of Figures 2 and 3.

In order to test the relative contribution of the discontinuities, power spectra of an artifical time series composed of step functions generated by the spacing and magnitude of the discontinuities, as analysed by Burlaga (1968b), were computed for the four periods shown in Figures 2 and 3. These exhibited an f^{-2} dependence, with power levels during quiet periods generally agreeing within a factor of two of the corresponding spectra of the real data. During disturbed periods the correspondence was somewhat less, approximately within a factor of three. The spectra of the artificial time series also exhibited statistically significant peaks at similar periods to those found in the real spectra. Analysis of the cross spectra indicates the occurrence of significant coherency between the components at the periodicities of simultaneous peaks although, as would be expected, the values of the coherencies were considerably greater than those found at the periodicities of corresponding simultaneous peaks in the spectra of the real data. It seems clear, therefore, that the peaks which are observed in the spectra of the actual data are not, in general, the result of a quasi-stationary wave-like behavior in the field, but rather reflect the magnitude and spacing of the discontinuities in each twelve hour period.

Although the power levels in the spectra of the theoretical time series generally fell within a factor of two of the power levels of the corresponding spectra of the real data, it was felt that better correspondence should be obtained if the more frequently occuring small discontinuities were also used in the simulated time series. Therefore, discontinuities with angular changes less than 30^{o}, discernable from random fluctuations in the ambient field, were also included, and, indeed, a higher degree of correspondence between the theoretical and real spectra was observed, with power levels generally within a factor of 1.5. A scatter plot of the power levels at equivalent frequencies of this result for day 357 is shown in Figure 4. At the lowest frequencies, the theoretical power, $P_t(f)$, is greater than the real power, $P_r(f)$. This is attributed to the fact that the field changes in the components due exclusively to the discontinuities generally have a non-zero average over a twelve hour period. Subsequently, the simulated time series, composed of the superimposed step functions, may exhibit large variances from its mean value for long periods, during which slow changes in the real data have tended to return the field to the average spiral direction and magnitude. At the higher frequencies, however, the values are closely grouped about a one to one correspondence. Thus, it is concluded that the discontinuities are the dominant factor in determining the shape and power spectral density of the spectra for the intervals and frequencies analysed.

Spectral Results: Low Frequency

Hourly averages of 30 sec. data were utilized in obtaining low frequency spectra for fourteen day intervals, beginning at the end of 1965. These were computed with 11 degrees of freedom and cover a frequency range of 2.3×10^{-6} to 1.4×10^{-4} cps, or periods of 2 to 120 hours. Figure 5 shows the results for three such intervals at 28 day separations, approximately the same time in a solar rotation. If peaks are ignored, these spectra exhibit variable frequency dependence, approximately between $f^{-3/2}$ and f^{-1}.

It is not surprising that a regular f^{-2} shape is not observed in this frequency range since the average separation between discontinuities is approximately one hour. Thus, in this range the field macrostructure, rather than the microstructure, will dominate the spectral shape.

Discussion

These results may be compared with power spectra of the interplanetary field previously obtained by Coleman (1966) with data from Mariner 2 in 1962, and by Siscoe et al. (1968) with Mariner 4 data in 1964. Coleman has measured the power in the frequency range of $1.4\text{x}10^{-5}$ to $1.4\text{x}10^{-2}$ cps with a noise level of $9.1\gamma^2$ cps. The results exhibited a general f^{-1} frequency dependence. Spectra obtained by Siscoe covered the frequency range of $3\text{x}10^{-4}$ to $5\text{x}10^{-1}$ cps with a noise level of $.21\gamma^2$/cps, and indicated a frequency dependence of $f^{-3/2}$. The spectra reported in this letter give a slope of 3/2 or less at low frequencies, and a slope of 2 at frequencies between $2.8\text{x}10^{-4}$ and $1.6\text{x}10^{-2}$cps. The situation is summarized in Figure 6.

The discrepancy with prior spectral results is not yet understood. Since in this study, both the spectral slope and magnitude at high frequencies is dominated by the presence of microstructural discontinuities, this may indicate that the microstructure of the interplanetary medium undergoes a significant secular change. In the future we plan to investigate this and obtain spectra over longer time and frequency intervals.

The results may be related to the theoretical determination of the cosmic ray diffusion coefficient by Jokipii (1966) and Roelof (1966) as applied to Parker's (1965) diffusion-convection model of galactic cosmic ray modulation. Parker's model predicts that to a first approximation the outward convection of cosmic rays resulting from scattering by magnetic irregularities "frozen" into the solar wind will, in the steady state, be balanced by the particles' diffusion through the modulating region. This is expressed by

$$n(\vec{r},R)\ \vec{V}_w = D(\vec{r},R,t)\ \vec{\nabla} n(\vec{r},R) \qquad (1)$$

Here n is the particle density; R is the particle's rigidity; D is the diffusion coefficient; and $\vec{V}_w$ is the solar wind velocity. If parallel diffusion along the field lines is much greater than diffusion across the field lines this equation can be reduced to a one-dimensional form and integrated, giving

$$n(r,R) = n(r_o,R)\ \exp\left(\int_r^{r_o} \frac{V_w}{D_{\parallel}}\, dr\right) \qquad (2)$$

where r_o is the boundary of the solar cavity.

The statistical analysis of scattering from magnetic inhomogeneities by Roelof (1966) and Jokipii (1966) has yielded an expression for the parallel diffusion coefficient in terms of the transverse power spectrum of the magnetic field, measured at frequencies corresponding to fluctuations with scale sizes approximately equal to the gyroradii of the cosmic ray in the average field. If the power spectrum throughout the modulating region can be represented by a power law, $f^{-\alpha}$, Jokipii (1966) finds that

$$D_{\parallel} = 2\alpha(\alpha+2)\ c\beta R^2/9V_w P_{zz}(f_o) \qquad (3)$$

Where β is the particle's velocity divided by c, and $P_{zz}(f_o)$ is the power spectrum of a non-radial component of the magnetic field measured at frequencies, $f_o = V_w B\ /\ 2\pi R$. The added factor of 2 in equation (3) arises from the fact that our power spectra are computed for positive frequencies only. From equation (3), we can express (2) as

$$n(r,R) = n(r_0,R) \exp \frac{\eta}{\beta R^{2-\alpha}} \qquad (4)$$

where the spatial dependence and various constants are included in the parameter η. Assuming that the galactic cosmic ray flux is constant, the fractional modulation, or the logarithm of the ratio of the particle densities observed at times t_2 and t_1 will be given by

$$\ln \frac{n(R,t_2)}{n(R,t_1)} \sim \frac{1}{\beta R^{2-\alpha}} \qquad (5)$$

For the period sampled at the end of 1965, the Pioneer 6 spectra indicate that α is between 3/2 and 1 for low frequencies and is 2 for frequencies greater than 2.8×10^{-4} cps. In an average field of $B=6\gamma$, and solar wind velocity, $V_w=400$ km/sec this frequency corresponds to cosmic ray rigidities of $R\sim.4$GV. At rigidities below .4GV, the fractional modulation during this period should vary as $1/\beta$, while at higher rigidities the modulation should vary between $1/\beta R^{\frac{1}{2}}$ to $1/\beta R$. Precisely such a result has been obtained by Ormes and Webber (1968) for the interval 1965-1966 from a direct analysis of cosmic ray data. They find a rigidity independent modulation for rigidities below .5GV and a $1/\beta R^{\frac{1}{2}}$ dependence at greater rigidities. Thus, assuming that the shape of the power spectrum is invariant in the intervals over which the cosmic ray data were obtained the results agree with the observed modulation.

Recent work by Jokipii (1968) has suggested, however, that at low rigidities, $R \lesssim .1$GV, the diffusion coefficient may be independent of rigidity regardless of the spectral shape. He indicates that at low rigidities the particle mean free path, λ, which is related to the diffusion coefficient by

$$\lambda = 3D_{\parallel}/c\beta,$$

may approach the correlation length of the field, L. If this occurs, the assumption that $\lambda \gtrsim L$, leading to the expression for the diffusion coefficient, is violated, and the theory must be modified. He suggests that in a field dominated by discontinuities the mean free path of low rigidity particles should be given by the average distance between discontinuities, which would be approximately the correlation length. The diffusion coefficient would then be given by the rigidity independent form of $Lc\beta/3$. In this regime, therefore, the modulation would also be independent of rigidity.

It should be noted that this will be valid if the discontinuities are basically "kinks" in the magnetic field. However, if as we believe, the discontinuities are basically tangential and there is only weak scattering from small fluctuations in the ambient field between discontinuities, the low rigidity particles will tend to be collimated along the field lines and would encounter discontinuity boundaries less frequently than expected. Thus, the mean free path would in fact be much greater than the distance between discontinuities, and the original form of the diffusion coefficient may still be applicable.

This consideration questions whether the power spectra which are measured by spacecraft and applied directly to modulation theories are the same spectra observed by low rigidity cosmic rays. In future studies we shall determine the contributions to the power spectra from fluctuations between discontinuities.

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FIGURE CAPTIONS

Figure 1 Interplanetary magnetic field data for days 356 and 357 expressed in spherical solar ecliptic coordinates. Dotted lines indicate discontinuities. δF gives the standard deviation for each 30 sec. average.

Figure 2 Power spectra of the interplanetary magnetic field components and magnitude for two 12 hour periods in December 1965. Dotted lines indicate an inverse square frequency dependence. K_p index for each period is included. At this time the satellite was 1×10^6 km from the earth at a Sun-Earth angle of 89^o.

Figure 3 See caption of Figure 2. At this time the satellite was 2×10^6 km from the earth at a Sun-Earth-Probe angle of 90^o.

Figure 4 Scatter plot of the power levels in the field components computed from the real data, $P_r(f)$, versus the theoretical power levels computed from the discontinuities, $P_t(f)$. Scales are offset by two decades. Dotted lines define $P_r(f)=P_t(f)$.

Figure 5 Power spectra computed from hourly averages for three 14 day periods in 1965-1966. Dotted lines indicate $f^{-3/2}$ and f^{-1} frequency dependences.

Figure 6 Comparison of prior spectral results indicating frequency ranges and dependences. Respective noise levels, P_{nl}, are also shown.

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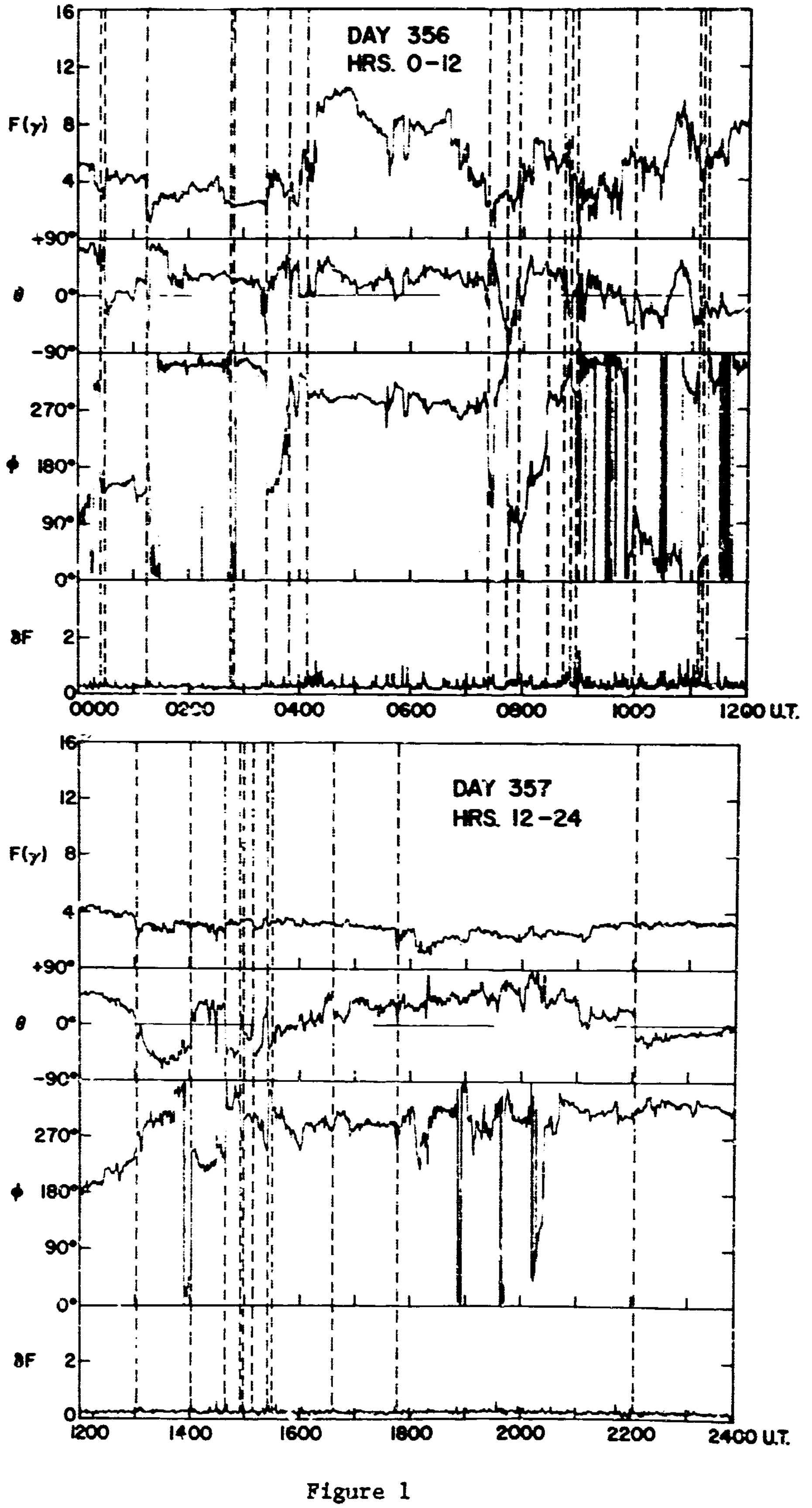


DAY 356
HRS. 0-12
F(γ)
θ
φ
δF
0000 0200 0400 0600 0800 1000 1200 U.T.
DAY 357
HRS. 12-24
F(γ)
θ
φ
δF
1200 1400 1600 1800 2000 2200 2400 U.T.

Figure 1

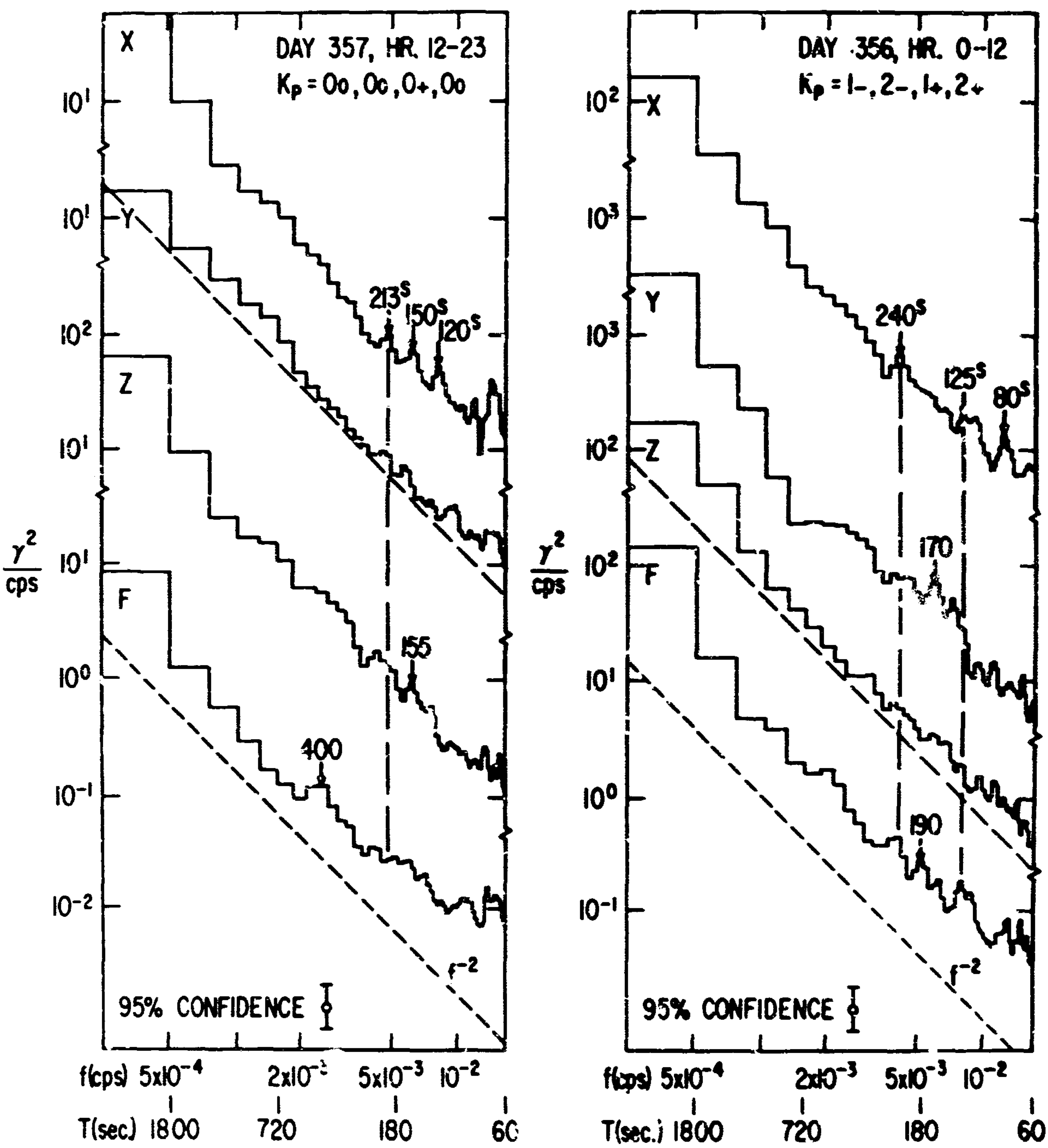

DAY 357, HR. 12-23
Kp = 0o, 0o, 0+, 0o
DAY 356, HR. 0-12
Kp = 1-, 2-, 1+, 2+
X
Y
Z
F
213s
150s
120s
155
400
240s
125s
80s
170
190
f-2
95% CONFIDENCE
γ2/cps
f(cps) 5x10-4 2x10-3 5x10-3 10-2
T(sec) 1800 720 180 60

Figure 2

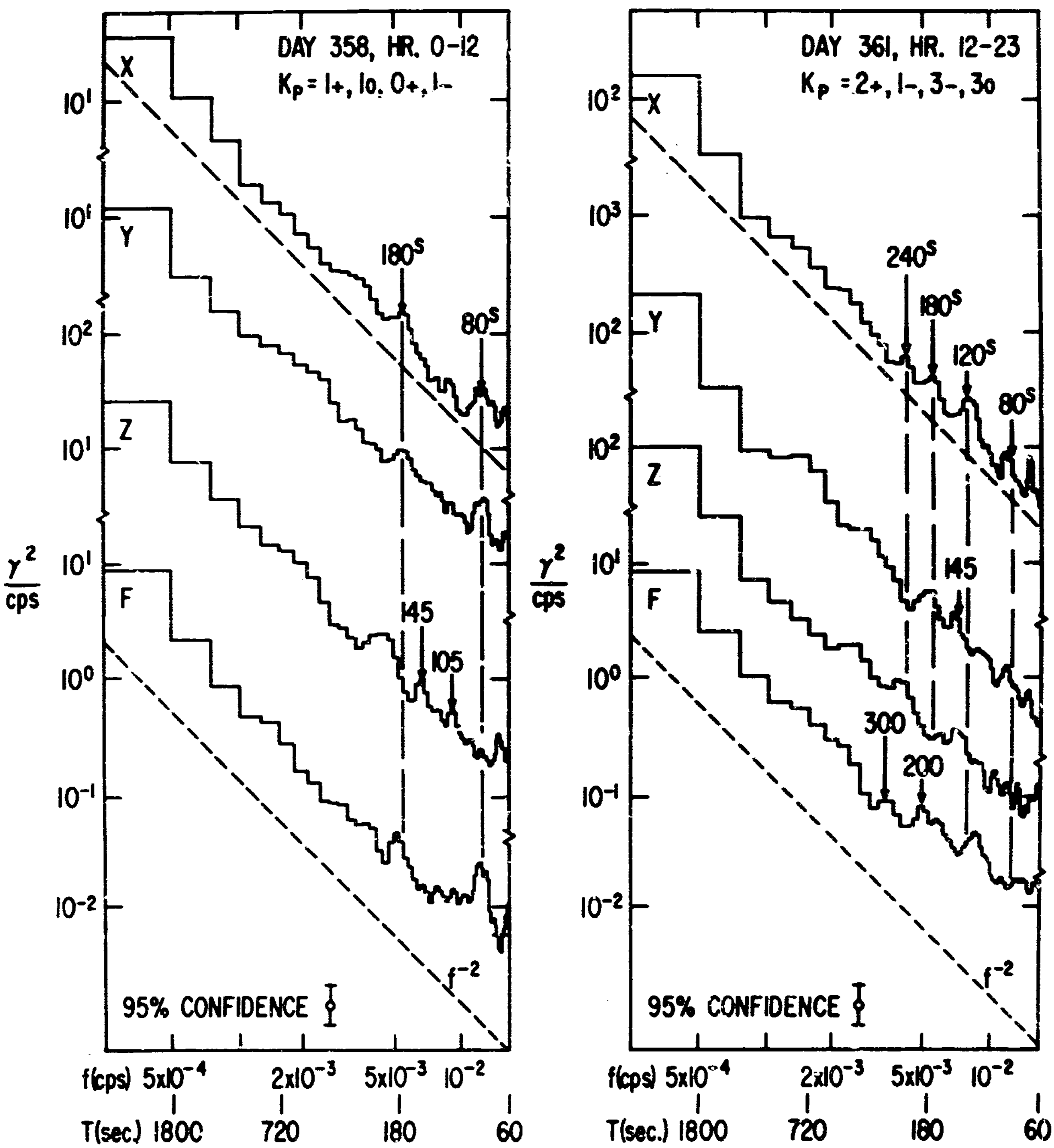

DAY 358, HR. 0-12
Kp = 1+, 1o, 0+, 1-
X
Y
Z
F
180s
80s
145
105
γ²/cps
95% CONFIDENCE
f^-2
f(cps) 5x10^-4 2x10^-3 5x10^-3 10^-2
T(sec.) 1800 720 180 60
DAY 361, HR. 12-23
Kp = 2+, 1-, 3-, 3o
240s
180s
120s
80s
145
300
200

Figure 3

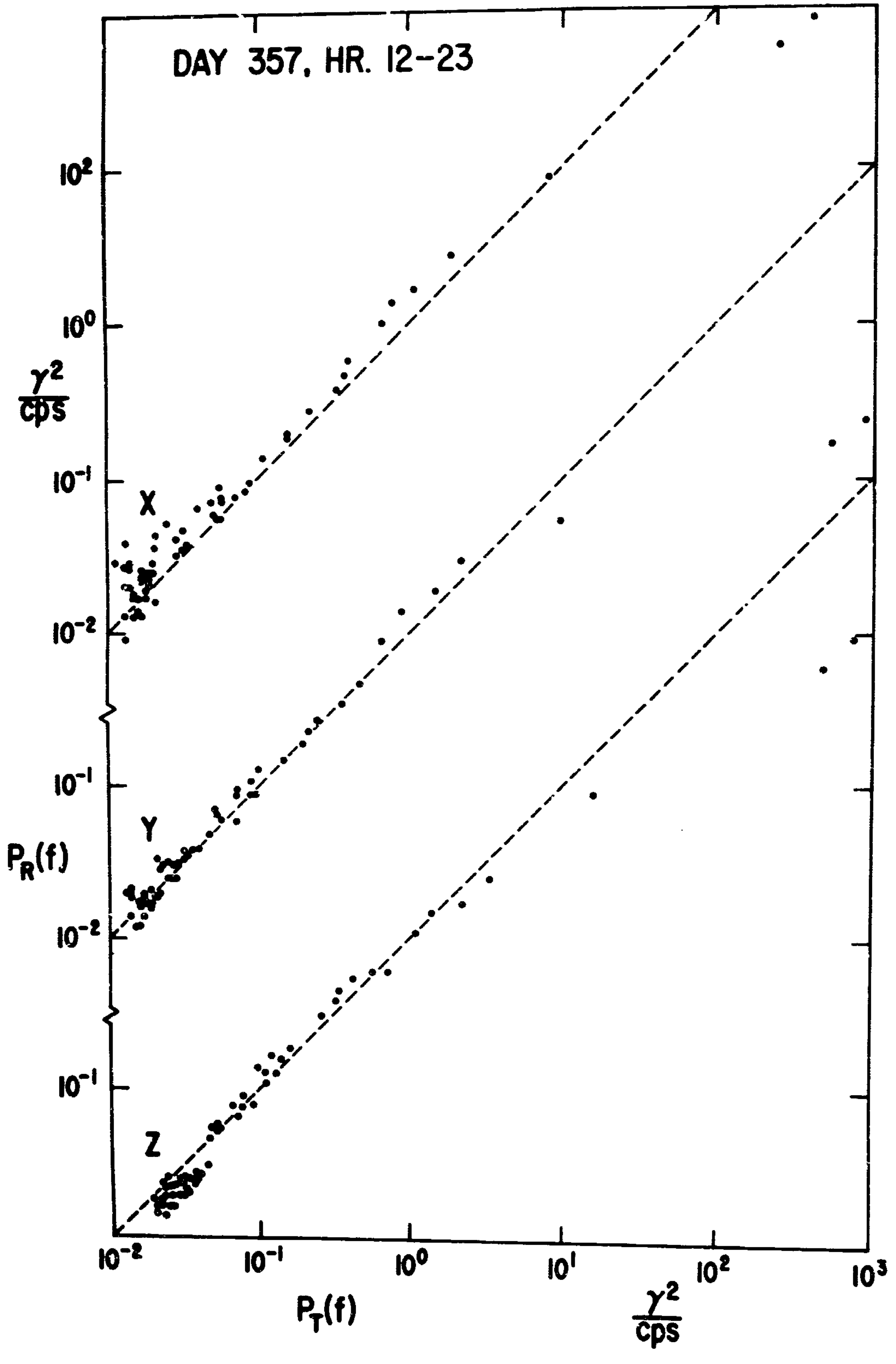
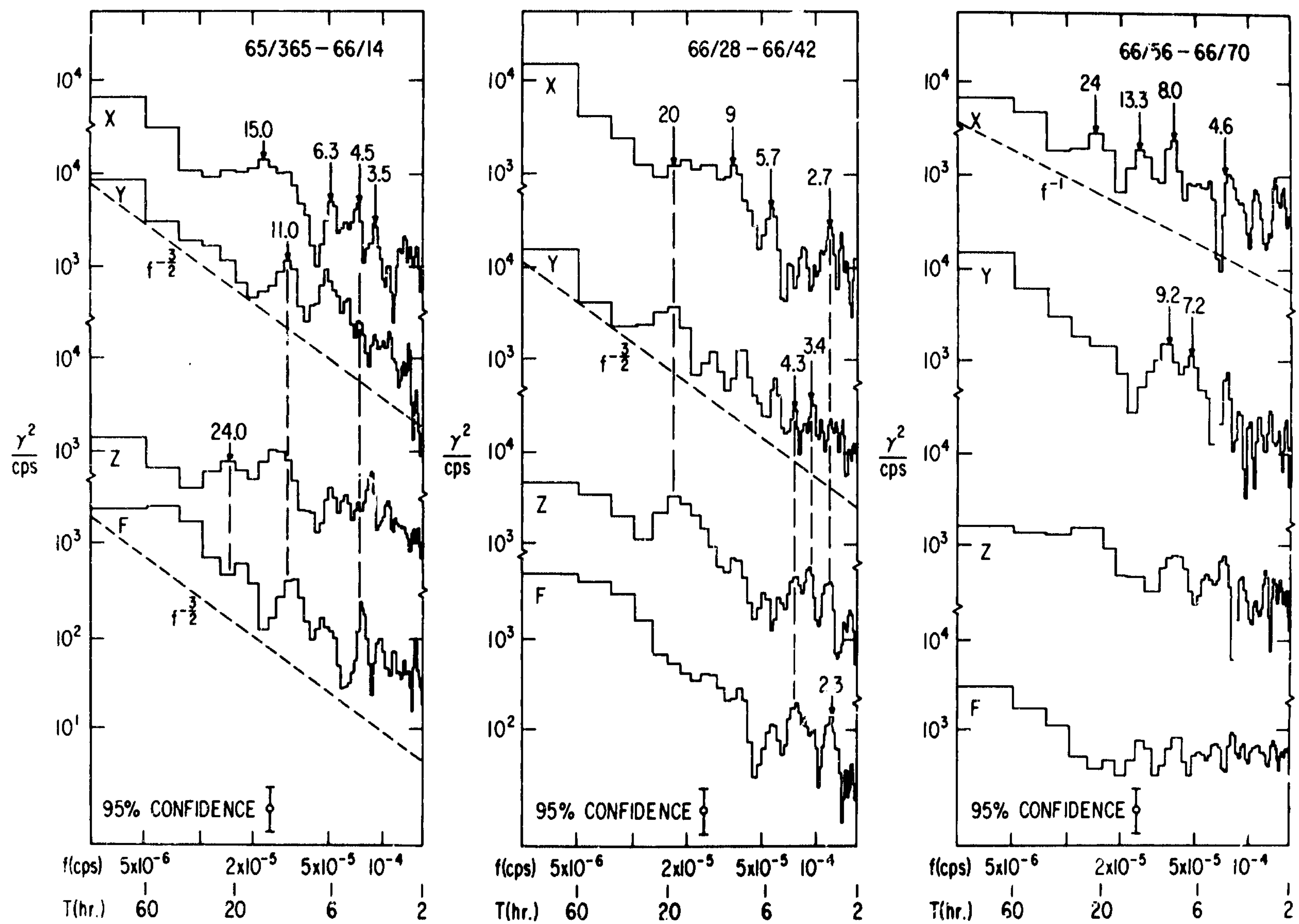

DAY 357, HR. 12-23
X
Y
Z
10^2
10^0
10^{-1}
10^{-2}
10^{-1}
10^{-2}
10^{-1}
$\frac{\gamma^2}{cps}$
$P_R(f)$
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
$P_T(f)$
$\frac{\gamma^2}{cps}$

Figure 4



65/365 – 66/14
66/28 – 66/42
66/56 – 66/70
X
Y
Z
F
15.0
6.3
4.5
3.5
11.0
24.0
20
9
5.7
2.7
4.3
3.4
2.3
24
13.3
8.0
4.6
9.2
7.2
$f^{-\frac{3}{2}}$
f^{-1}
$\frac{\gamma^2}{cps}$
95% CONFIDENCE
f(cps) $5x10^{-6}$ $2x10^{-5}$ $5x10^{-5}$ 10^{-4}
T(hr.) 60 20 6 2

Figure 5

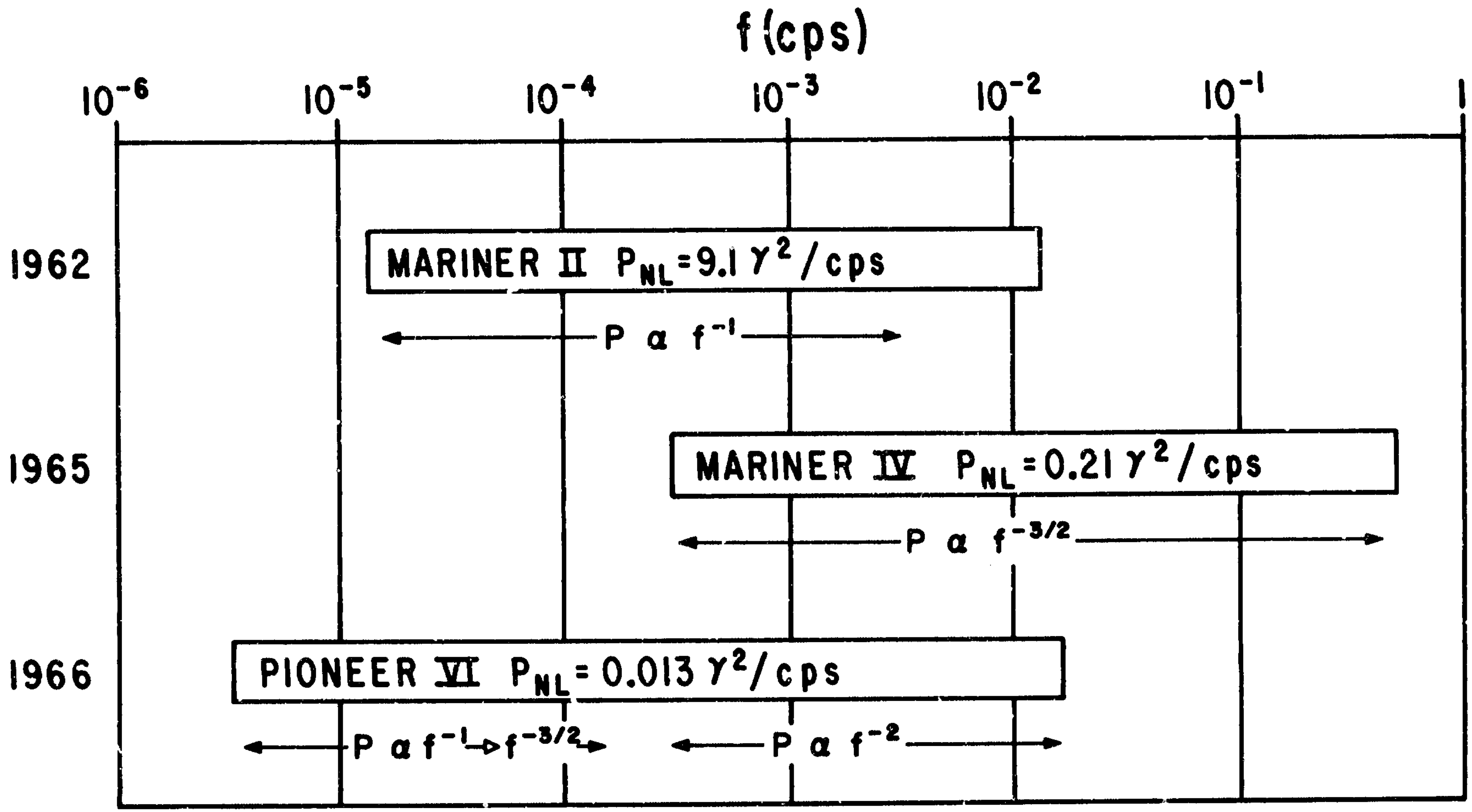

f (cps)
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
1
1962
1965
1966
MARINER II P_NL = 9.1 γ²/cps
P α f^-1
MARINER IV P_NL = 0.21 γ²/cps
P α f^-3/2
PIONEER VI P_NL = 0.013 γ²/cps
P α f^-1 → f^-3/2
P α f^-2

Figure 6