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RADIATION FIELDS OF ENERGETIC ELECTRONS IN HELICAL ORBITS WITHIN A MAGNETOACTIVE PLASMA

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ABSTRACT

The radiation fields produced by an energetic electron which is moving in helical orbit in the magnetoactive plasma are derived by solving the Maxwell's equations exactly. Those fields consist of three components which are characterized by different wave frequencies related to the anisotropic nature of the medium.

The result, obtained in this paper, seems to be useful in investigating the suppression of the low frequencies of solar radio type IV bursts and galactic radio emission, and suggests that the explanation for the influence of ambient plasmas on the gyro-synchrotron radiation from energetic electrons must be modified.

1. INTRODUCTION

The influence of the ambient plasmas on the electromagnetic radiation from electrons of arbitrary energy which are helically spiraling in the external magnetic fields have recently been dealt with by many authors (Eidman, 1958; Ginzburg and Syrovatskii, 1964, 1965; McKenzie, 1964; Liemohn, 1965; Ramaty and Lingenfelter, 1967; Mansfield, 1967; Ramaty, 1968; Sakurai and Ogawa, 1968). Since it is, however, very difficult to obtain directly the solution for the electromagnetic fields radiated from an electron moving in the anisotropic medium, various kinds of the devices to solve the Maxwell's equations have been developed.

The suppression of the low frequencies of solar radio type IV bursts suggests that the influence of the ambient solar plasma on the emission characteristics of the bursts must be important as has already been discussed by Ramaty and Lingenfelter (1967, 1968) and Ramaty (1968). However, their treatment has been limited to such frequency ranges as $f > f_p >> f_H$, where f, f_p and f_H are the frequencies of emitted radio wave, the plasma and the electron cyclotron, respectively. In this case, the ambient plasma is, therefore, assumed to be isotropic and the following refractive index, μ applies:

$$\mu = (1- f_p^2/f^2)^{\frac{1}{2}} \tag{1}$$

The conditions which are actually encountered near the sunspot groups, however, are quite different from this simple case (1). In fact, the type IV radio bursts are usually emitted from energetic electrons which are moving within a medium where the strength of magnetic field is very high, say 1000 gauss (for example, Ellison, 1963). The effect of magnetic field, therefore, cannot be neglected in dealing with the emission characteristics of type IV radio bursts (Sakurai, 1964, 1965).

In this paper, we will solve exactly Maxwell's equations applied to an anisotropic medium, and then discuss briefly the properties of the solutions. The calculated emission power flux for some restricted cases is presented in the Appendix.

2. FUNDAMENTAL EQUATIONS AND THE METHOD OF SOLUTION

Maxwell's electromagnetic equations in a magnetoactive plasma are given, by using the anisotropic dielectric tensor $[k]$ (Stix, 1962), as follows:

$$\text{curl}\ \vec{E} = -i\frac{\omega}{c}\vec{H} \tag{2}$$

$$\text{curl}\ \vec{H} = i\frac{\omega}{c}[k]\vec{E} + \frac{4\pi}{c}\vec{I'}, \tag{3}$$

where $\vec{E}$, $\vec{H}$ and $\vec{I'}$ are the electric and magnetic fields which are assumed to change with exp $(i\omega t)$, where ω is the angular frequency, and the electric current due to the electrons moving helically in the medium which are responsible for the emission of electromagnetic waves, and c is the speed of light.

By the use of the vector and scalar potentials $\vec{A}$ and ϕ, the electromagnetic fields, $\vec{E}$ and $\vec{H}$ are usually expressed as

$$\vec{E} = -i\frac{\omega}{c}\vec{A} - \text{grad}\ \phi \tag{4}$$

$$\vec{H} = \text{curl}\ \vec{A} \tag{5}$$

If we here adopt the following subsidiary condition that satisfies the transverse gauge:

$$\text{div}\ \vec{A} = 0, \tag{6}$$

the equation for the vector and scalar potential $\vec{A}$ and ϕ

$$\nabla^2\vec{A} + \frac{\omega^2}{c^2}[k]\vec{A} = -\frac{4\pi}{c}\vec{I'} \tag{7}$$

$$\nabla^2\phi = -\frac{4\pi}{c}e'$$

where e' is electric charge of the moving electron (e' = - e). In this gauge, it is not necessary to take the scalar potential ϕ into consideration. Really, in dealing with the electromagnetic emission from an electron, it is sufficient to take into account only the vector potential $\vec{A}$ in the wave fields. Accordingly, the electromagnetic fields in the wave fields are expressed by

$$\vec{E} = - i \frac{\omega}{c} \vec{A} \qquad (4)'$$

$$\vec{H} = \text{curl } \vec{A}, \qquad (5)'$$

in place of equations (4) and (5).

If we make use of such a Green tensor G ($\vec{r}$, $\vec{r}'$) that satisfies the equation,

$$(\nabla^2 - \frac{\omega^2}{c^2} [k])[G (\vec{r}; \vec{r}')] = \delta (\vec{r}-\vec{r}'), \qquad (8)$$

the vector potential $\vec{A}$ ($\vec{r}$, t) formally solved is mathematically expressed as follows:

$$\vec{A} (\vec{r}, t) = \frac{4\pi}{c} \int [G (\vec{r}, \vec{r}')] \dot{\vec{I}}' (\vec{r}',t') d\vec{r}' \ dt', \qquad (9)$$

where $\vec{r}$ and $\vec{r}'$ are the position vectors from the null point of the coordinate to the point of observation and to the location of the moving charges, respectively. t and t' are, farther, the times with respect to the point of observation and the location of the charges, respectively.

In order to solve equation (8), the Green tensor G($\vec{r}$, $\vec{r}'$) and the δ-function are Fourier-transformed as follows:

$$[G(\vec{r}, \vec{r}')] = \frac{1}{(4\pi)^3} \int [\tilde{G} (\vec{k}, \omega)] e^{i \vec{k} (\vec{r} - \vec{r}')} d\vec{k} \qquad (10)$$

$$\delta(\vec{r} - \vec{r}') = \frac{1}{(4\pi)^3} \int e^{i \vec{k} (\vec{r} - \vec{r}')} d\vec{k} \qquad (11)$$

where $\vec{k}$ and ω are the vector wave number and angular frequency, respectively.

Using the result just obtained, the formal solution of equation (8), $[G(\vec{k}, \omega)]$, is given by

$$[G(\vec{k}, \omega)] = [\chi]^{-1} , \qquad (12)$$

where

$$[\chi] = \begin{bmatrix} k^2 - S\frac{\omega^2}{c^2} & iD\frac{\omega^2}{c^2} & 0 \\ -iD\frac{\omega^2}{c^2} & k^2 - S\frac{\omega^2}{c^2} & 0 \\ 0 & 0 & k^2 - P\frac{\omega^2}{c^2} \end{bmatrix} \qquad (13)$$

The quantities as S , P and D which appeared in the above equation are expressed as follows, by using the notations defined by Stix (1962):

$$S = \frac{1}{2}(R+L) \qquad D = \frac{1}{2}(R-L)$$

and

$$P = 1 - \frac{\omega_p^2}{\omega^2},$$

where

$$R = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega}{\omega - \omega_H}, \quad L = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega}{\omega + \omega_H}$$

$$(\omega = 2\pi f, \ \omega_p = 2\pi f_p \text{ and } \omega_H = 2\pi f_H).$$

Equation (12) is rewritten in the following form:

$$[\tilde{G}(\vec{k},\omega)] = \begin{bmatrix} \frac{k^2 - S\frac{\omega^2}{c^2}}{k_o} & -i\,\frac{D\frac{\omega^2}{c^2}}{k_o} & 0 \\ i\,\frac{D\frac{\omega^2}{c^2}}{k_o} & \frac{k^2 - S\frac{\omega^2}{c^2}}{k_o} & 0 \\ 0 & 0 & \frac{1}{k^2 - P\frac{\omega^2}{c^2}} \end{bmatrix} \quad (12)$$

where

$$k_o = (k^2 - S\frac{\omega^2}{c^2}) - \frac{\omega^4}{c^4} D^2$$

When the above solution $[\tilde{G}(\vec{k},\omega)]$ is substituted into equation (10) and integrated with respect to $|\vec{k}| = k$, the following equation is obtained:

$$[G(\vec{r},\vec{r}')] = \frac{1}{8\pi^2} \int \frac{1}{R'} [G_{ij}] e^{i\omega(t-t')} d\omega, \quad (14)$$

where $R' = |\vec{r} - \vec{r}'|$ and

$$[G_{ij}] = \begin{bmatrix} \frac{1}{2}(e^{\frac{i\omega}{c}R'\sqrt{L}} + e^{\frac{i\omega}{c}R'\sqrt{R}}) & \frac{1}{2}(e^{\frac{i\omega}{c}R'\sqrt{L}} - e^{\frac{i\omega}{c}R'\sqrt{R}}) & 0 \\ -\frac{1}{2}(e^{\frac{i\omega}{c}R'\sqrt{L}} - e^{\frac{i\omega}{c}R'\sqrt{R}}) & \frac{1}{2}(e^{\frac{i\omega}{c}R'\sqrt{L}} + e^{\frac{i\omega}{c}R'\sqrt{R}}) & 0 \\ 0 & 0 & e^{\frac{i\omega}{c}R'\sqrt{P}} \end{bmatrix} \quad (15)$$

By substituting (14) into equation (9), the solution $\vec{A}(\vec{r},t)$ is derived formally as follows:

$$\vec{A}(\vec{r},t) = \frac{4\pi}{c}\int [G(\vec{r},\vec{r}')]\vec{I}(\vec{r}',t')e^{-i\omega(t-t')}\,d\vec{r}'dt'd\omega \quad (16)$$

This result is used to calculate the radiation fields, $\vec{E}$ and $\vec{H}$ which are given by equations (4)' and (5)'.

3. RADIATION FIELDS BY ELECTRONS IN HELICAL ORBITS

The Cartesian coordinate system as shown in Fig. 1, is used here, where the external magnetic field $\vec{H}_0$ is along the Z-axis. In equation (16), the electric current $\vec{I}'$ is expressed as

$$\vec{I}' = e\,\vec{V}(t')\,\delta(\vec{r}'-\vec{r}_e(t')), \quad (17)$$

since the radiating electron produces an electric current, where e, $\vec{V}$ and $\vec{r}_e$ are the electronic charge, the velocity and the position vector of the electron.

As is evident from the figure, $\vec{R}' = \vec{r} - \vec{r}'$ and $|\vec{R}'| = R' = |\vec{r} - \vec{r}'|$. Because of $|\vec{r}'| \ll |\vec{r}|$, the following approximation is possible:

$$R' \simeq r - \vec{n}\cdot\vec{r}',$$

where $\vec{n} = \vec{R}'/R$.

Thus equation (16) can be integrated with respect to $\vec{r}'$, by using the above approximation, as follows:

$$\vec{A}(\vec{r},t) = \frac{e}{2\pi c}\int_{t',\omega} \frac{1}{r}\,[G_{ij}]\,\vec{V}(t')e^{-i\omega(t-t')}dt'd\omega, \quad (18)$$

where R' in $[G_{ij}]$ is expressed as $r - \vec{n}\cdot\vec{r}_e'(t)$. Here $\vec{r}_e'(t')$ is the position vector of a radiating electron, and clearly $|\vec{r}_e| \ll r$, and approximately $\vec{n} = \vec{R}'/R' \sim \vec{r}/|\vec{r}|$.

In the following calculation, the vector $\vec{n}$ is taken within the (y-z) plane, so $\vec{n}$ is expressed as

$$\vec{n} = \vec{j}\sin\theta + \vec{k}\cos\theta,$$

by using the definition given in Fig. 2, where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the unit vectors as for the x-, y- and z-axis, respectively, and θ is the angle between the vector $\vec{r}$ and z-axis.

The position vector $\vec{r}_e{}'(t')$ and the velocity $\vec{V}'(t')(=d\vec{r}_e{}'/dt')$ of the radiating electron are respectively given by

$$\vec{r}_e{}'(t') = \vec{i}\,\frac{v_\perp}{\omega_H'}\cos\omega_H't' + \vec{j}\,\frac{v_\perp}{\omega_H'}\sin\omega'_H t' + \vec{k}\,v_{\parallel}\,t' \tag{19}$$

and

$$\vec{v}'(t') = \vec{i}(v_\perp)\sin\omega_H't' + \vec{j}\,v_\perp\cos\omega_H't' + \vec{k}\,v_{\parallel}; \tag{20}$$

where

$$\omega_H' = \omega_H\sqrt{1-\beta^2},\quad \omega_H = eH_o/m_oc \text{ and } \beta = v/c.$$

In the above two equations, $v_\perp$ and $v_\parallel$ are the components of the electron velocity perpendicular and parallel to the external magnetic field. m_o is the rest mass of an electron.

By taking equation (19) into consideration, R' is expressed as follows:

$$R' = r - \vec{n}.\vec{r}_e{}'(t') = r - \left(\frac{v_\perp}{\omega_H'}\sin\theta\,\sin\omega_H't' + v_\parallel\cos\theta.t'\right)$$

Equation (18) can be easily integrated with respect to time t', using both the above approximation and several mathematical equalities such as $e^{-i\,x\,\sin\psi} = \sum_{s=-\infty}^{\infty} J_s(x)\,e^{-is\psi}$, Thus the result is

$$\vec{A}\,(\vec{r},t) = \sum_{-\infty}^{\infty}\frac{e}{r}\int e^{-i\omega t}\begin{pmatrix}A_{x\omega}\\ A_{y\omega}\\ A_{z\omega}\end{pmatrix}d\omega, \tag{21}$$

where

$$A_{x\omega} = - \frac{i}{2} \frac{v_\perp}{c} \{ [J_s' (X_1) - \frac{s}{X_1} J_s(X_1)] \; a \; \delta \; (\omega_1)$$

$$+ [J_s'(X_2) + \frac{s}{X_2} J_s(X_2)] \; b \; \delta \; (\omega_2) \}$$

$$A_{y\omega} = - \frac{1}{2} \frac{v_\perp}{c} \{ [J_s'(X_1) - \frac{s}{X_1} J_s(X_1)] \; a \; \delta \; (\omega_1)$$

$$- [J_s'(X_2) + \frac{s}{X_2} J_s(X_2)] \; b \; \delta \; (\omega_2) \}$$

$$A_{z\omega} = \frac{v_\parallel}{c} J_s \; (X_3) \; c \; \delta \; (\omega_3)$$

and

$$X_1 = \frac{\omega}{c} \sqrt{L} \frac{v_\perp}{\omega'_H} \sin \theta$$

$$X_2 = \frac{\omega}{c} \sqrt{R} \frac{v_\perp}{\omega'_H} \sin \theta$$

$$X_3 = \frac{\omega}{c} \sqrt{P} \frac{v_\perp}{\omega'_H} \sin \theta$$

$$a = e^{i \frac{\omega}{c} \sqrt{L} r} \qquad b = e^{i \frac{\omega}{c} \sqrt{R} r} \qquad \text{and } c = e^{i \frac{\omega}{c} \sqrt{P} r}.$$

The variables of the three δ-functions are respectively

$$\omega_1 = \omega - s\omega'_H - \omega \beta_\parallel \sqrt{L} \cos \theta$$

$$\omega_2 = \omega - s\omega' - \omega \beta_\parallel \sqrt{R} \cos \theta$$

$$\omega_3 = \omega - s\omega' - \omega \beta_\parallel \sqrt{P} \cos \theta,$$

where $\beta_\parallel = v_\parallel/c$.

The wave frequency ω in equation(21) is defined by the δ-function such as $\delta \; (\omega_1)$, $\delta \; (\omega_2)$ and $\delta \; (\omega_3)$. Accordingly, the equation (21) is easily integrated with respect to the wave frequency and deduced as follows:

$$\vec{A}\ (\vec{r},t) = \sum_{s=-\infty}^{\infty} \frac{e}{r} \begin{pmatrix} A_{sx} \\ A_{sy} \\ A_{sz} \end{pmatrix} [\vec{i},\ \vec{j},\ \vec{k}] \qquad (22)$$

where

$$A_{sx} = \frac{i}{2}\ \frac{v_\perp}{c} \left\{ \frac{\frac{s}{X_1} J_s(X_1) - J_s'(X_1)}{1-\beta_\parallel \cos\theta\ \frac{\partial(\omega\sqrt{L})}{\partial\omega}}\ e^{i(\frac{\omega}{c}\sqrt{L}r - \omega t)}\ \delta(\omega_1) \right.$$

$$\left. - \frac{\frac{s}{X_2} J_s(X_2) + J_s'(X_2)}{1-\beta_\parallel \cos\theta\ \frac{\partial(\omega\sqrt{R})}{\partial\omega}}\ e^{i(\frac{\omega}{c}\sqrt{R}\ r - \omega t)}\ \delta(\omega_2) \right\} \qquad (23a)$$

$$A_{sy} = \frac{1}{2}\ \frac{v_\perp}{c} \left\{ \frac{\frac{s}{X_1} J_s(X_1) - J_s'(X_1)}{1-\beta_\parallel \cos\theta\ \frac{\partial(\omega\sqrt{L})}{\partial\omega}}\ e^{i(\frac{\omega}{c}\sqrt{L}r - \omega t)}\ \delta(\omega_1) \right.$$

$$\left. + \frac{\frac{s}{X_2} J_s(X_2) + J_s'(X_2)}{1-\beta_\parallel \cos\theta\ \frac{\partial(\omega\sqrt{R})}{\partial\omega}}\ e^{i(\frac{\omega}{c}\sqrt{R}r - \omega t)}\ \delta(\omega_2) \right\} \qquad (23b)$$

$$A_{sz} = \frac{v_\parallel}{c}\ \frac{J_3\ (x_3)}{1-\beta_\parallel \cos\theta\ \frac{\partial(\omega\sqrt{p})}{\partial\omega}}\ e^{i(\frac{\omega}{c}\sqrt{P}r - \omega t)}\ \delta\ (\omega_3) \qquad (23c)$$

By substituting the results (22) and (23) into equations (4)' and (5)', the electromagnetic radiation fields produced by an electron in helical orbit can be calculated as follows:

The electric field:

$$E_{sx} = -\frac{1}{2}\frac{v_\perp}{c^2}\frac{e}{r}\left\{\frac{\frac{s}{X_1}J_s(X_1)-J_s'(X_1)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{L})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{L}r-\omega t)}\,\omega\,\delta(\omega_1)\right.$$

$$\left.-\frac{\frac{s}{X_2}J_s(X_2)+J_s'}{1-\beta_{||}\cos\theta\,\partial(\omega\sqrt{R})}\,e^{i(\frac{\omega}{c}\sqrt{R}r-\omega t)}\,\omega\,\delta(\omega_2)\right\} \quad (24a)$$

$$E_{sy} = \frac{i}{2}\frac{v_\perp}{c^2}\frac{e}{r}\left\{\frac{\frac{s}{X_1}J_s(X_1)-J_s(X_1)-J_s'(X_1)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{L})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{L}r-\omega t)}\,\omega\,\delta(\omega_1)\right.$$

$$\left.+\frac{\frac{s}{X_2}J_s(X_2)+J_s'(X_2)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{R})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{R}r-\omega t)}\,\omega\,\delta(\omega_2)\right\} \quad (24b)$$

$$E_{sz} = -i\frac{v_{||}}{c^2}\frac{e}{r}\frac{J_s(X_2)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{p})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{P}r-\omega t)}\,\omega\,\delta(\omega_3) \quad (24c)$$

The magnetic field:

$$H_{sx} = i\frac{e}{r}\left\{\frac{w}{c}\sqrt{p}\sin\theta\,\frac{\beta_{||}\,J_s(X_3)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{p})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{P}r-\omega t)}\,\delta(\omega_3)\right.$$

$$-\frac{1}{2}\frac{v_\perp}{c^2}\sqrt{L}\cos\theta\,\frac{\frac{s}{X_1}J_s(X_1)-J_s'(X_1)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{L})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{L}r-\omega t)}\,\omega\,\delta(\omega_1)$$

$$\left.-\frac{1}{2}\frac{v_\perp}{c^2}\sqrt{R}\cos\theta\,\frac{\frac{s}{X_2}J_s(X_2)+J_s'(X_2)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{R})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{R}r-\omega t)}\,\omega\,\delta(\omega_2)\right\} \quad (25a)$$

$$H_{sy} = -\frac{e}{r}\left\{\frac{1}{2}\frac{v_\perp}{c^2}\cos\theta\left[\sqrt{L}\,\frac{\frac{s}{X_1}J_s(X_1)-J_s'(X_1)}{1-\beta_{||}\cos\theta\,\frac{\partial(\omega\sqrt{L})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{L}r-\omega t)}\,\omega\,\delta(\omega_1)\right.\right.$$

$$- \sqrt{R}\,\frac{\frac{s}{X_2}J_s(X_2)+J_s'(X_2)}{1-\beta_{\parallel}\cos\theta\,\frac{\partial(\omega\sqrt{R})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{R}r-\omega t)}\,\omega\delta(\omega_2)]\} \qquad (25b)$$

$$H_{sz} = \frac{e}{r}\{\frac{1}{2}\frac{v_\perp}{c^2}\sin\theta\,[\sqrt{L}\,\frac{\frac{s}{X_1}J_s(X_1)-J_s'(X_1)}{1-\beta_{\parallel}\cos\theta\,\frac{\partial(\omega\sqrt{L})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{L}r-\omega t)}\,\omega\delta(\omega_1)$$

$$- \sqrt{R}\,\frac{\frac{s}{X_2}J_s(X_2)+J_s'(X)}{1-\beta_{\parallel}\cos\theta\,\frac{\partial(\omega\sqrt{R})}{\partial\omega}}\,e^{i(\frac{\omega}{c}\sqrt{R}r-\omega t)}\,\omega\delta(\omega_2)]\} \qquad (25c)$$

Although the above solutions are quite complicated, the Poynting vector can be calculated by making use of the above electric and magnetic fields. In anisotropic plasmas, it is necessary to calculate both the electric and magnetic fields for the Poynting vector to be derived, because the direction of this vector does not, in general, correspond to that of the wave normal vector $\vec{n}$ (for example, Bekefi, 1966). Thus, the Poynting vector for the S-th harmonic $\vec{I}_s$ is given by using equation (24) and (25) as follows:

$$\vec{I}_s = \frac{c}{8\pi}(\vec{E}_s^* \times \vec{H} + \vec{E}_s \times \vec{H}_s^*), \qquad (26)$$

where $\vec{E}_s = (E_{sx}, E_{sy}, E_{sz})$ and $\vec{H}_s = (H_{sx}, H_{sy}, H_{sz})$. The asterisk denotes the complex conjugate component. In this expression, the three Doppler-shifted emission frequencies are involved in the Poynting flux such as

$$\omega = \frac{s\omega_H'}{1-\beta_{\parallel}\sqrt{L}\cos\theta}, \quad \frac{s\omega_H'}{1-\beta_{\parallel}\sqrt{R}\cos\theta} \quad \text{and} \quad \frac{s\omega_H'}{1-\beta_{\parallel}\sqrt{P}\cos\theta}. \qquad (27)$$

These are obtained from the δ-functions such as $\delta(\omega_1)$, $\delta(\omega_2)$ and $\delta(\omega_3)$.

The frequencies given by equations (27) are identical only when the orbital motion of an emitting electron is circular, i.e., $\beta_{||} = 0$. In this case, all these frequencies are reduced to $s\omega_H'$ ($=s\omega_H \sqrt{1-\beta^2}$), which is the well-known gyro-frequency of relativistic electrons.

When the approximation $\omega > \omega_p >> \omega_H$ is satisfied as has been considered by Ramaty (1968), the medium necessarily becomes isotropic and consequently it follows that $\sqrt{R} = \sqrt{L} = \sqrt{1-\omega_p^2/\omega^2}$. It is, therefore, quite easy to calculate the Poynting vector in this case and one can easily obtain the same result as shown by Ramaty (1968) (See Appendix 1).

If one assumes that $\omega_p^2/\omega^2 << 1$ or that the medium is vacuum, the well-known formulae for the Poynting flux vector is obtained by using equations (24) and (25)(See Appendix 1), which has already been obtained by Schott (1912), Schwinger (1949) and Landau and Lifschitz (1961).

4. CONCLUDING REMARKS

The exact solutions for the radiation fields produced by an electron of arbitrary energy in a magnetoactive plasma have been obtained in this paper. Since the medium is anisotropic, the solutions are very complicated in form as is seen from equations (24) and (25). However, these results make it possible to examine the influence of magnetoactive plasmas on the electromagnetic emission from electrons in helical orbits.

As will be shown in Appendix 1, the solutions obtained here give the same result as that by Ramaty (1968) when both the approximation $f > f_p \gg f_H$ and $\beta_{||} = 0$ are adopted.

In applying these to several problems on solar and galactic radio emissions, the power flux will have to be calculated. These calculations are now under way.

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Appendix 1

When $\omega > \omega_p >> \omega_H$, it follows that $\sqrt{R} = \sqrt{L} = \sqrt{P} = \sqrt{1-\omega_p^2/\omega^2}$, being equal to the isotropic refractive index (1). Since the medium in this case is isotropic, the poynting vector is given as follows:

$$\vec{p}_s = \frac{c}{4\pi} \left| \vec{E}_s^2 \right| \vec{n}$$

$$= \frac{c}{4\pi} \{ \left|E_{sx}\right|^2 + \left|E_{sy}\right|^2 \cos^2\theta + \left|E_{sz}\right|^2 \sin^2\theta \} \vec{n}$$

$$\vec{p}_s = \vec{n} \frac{e^2}{r^2} \frac{c}{4\pi} \left(\frac{\omega}{c}\right)^2 \frac{1}{1-\beta_{\parallel} \cos\theta \frac{\partial(\omega\mu)}{\partial\omega}}^2 \{ [\beta_{\perp} J_s'(X)]^2$$

$$+[\beta_{\perp} \cos\theta \frac{s}{X} J_s(X)]^2 + [\beta_{\parallel} \sin\theta J_s(X)]^2 \} \delta(\omega - s\omega_H' - \omega\beta_{\parallel}\mu\cos\theta) \qquad \text{(A-1)}$$

where
$$X = \frac{s\beta_{\perp} \mu \sin\theta}{1-\beta_{\parallel} \mu \cos\theta} \qquad \text{(A-2)}$$

When the orbit of electron motion is circular, the above equation (A-1) is reduced to

$$\vec{p}_s = \vec{n} \frac{c}{4\pi r^2} \left(\frac{\omega}{c}\right)^2 \{ [\beta J_s'(s\beta\mu\sin\theta)]^2 + [\frac{\cos\theta}{\mu \sin\theta} J_s(s\beta\mu\sin\theta)]^2 \}$$
$$\times \delta(\omega - s\omega_H'), \qquad \text{(A-3)}$$

since $\beta_{\parallel} = 0$. This result is equivalent to that of Ramaty (1968).

In the vacuum, $\mu = 1$. The above equation is further reduced to

$$\vec{p}_s = \vec{n} \frac{c}{4\pi r^2} \left(\frac{s\omega_H}{c}\right)^2 (1-\beta^2) \{ [\beta J_s'(s\beta\sin\theta)]^2$$
$$+ [\cot\theta J_s(s\beta\sin\beta)]^2 \} \qquad \text{(A-4)}$$

which is the well-known formula (for example, Schott, 1912; Landau and Lifshitz, 1961).

CAPTION OF FIGURES

Fig. 1 The coordinate system and the relation among the position vectors.

Fig. 2 The relation between the point of observation and the electron motion.

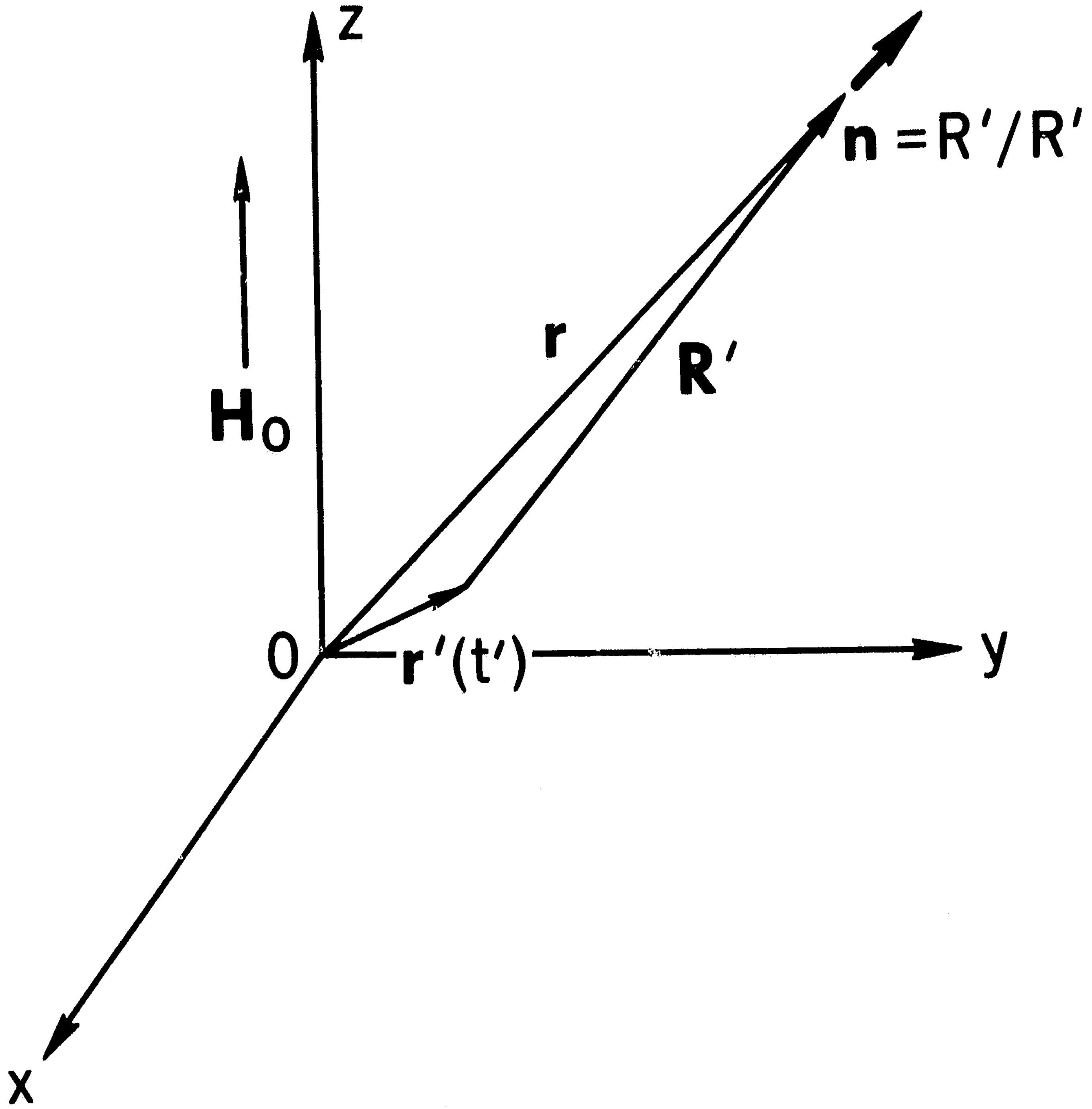

Figure 1

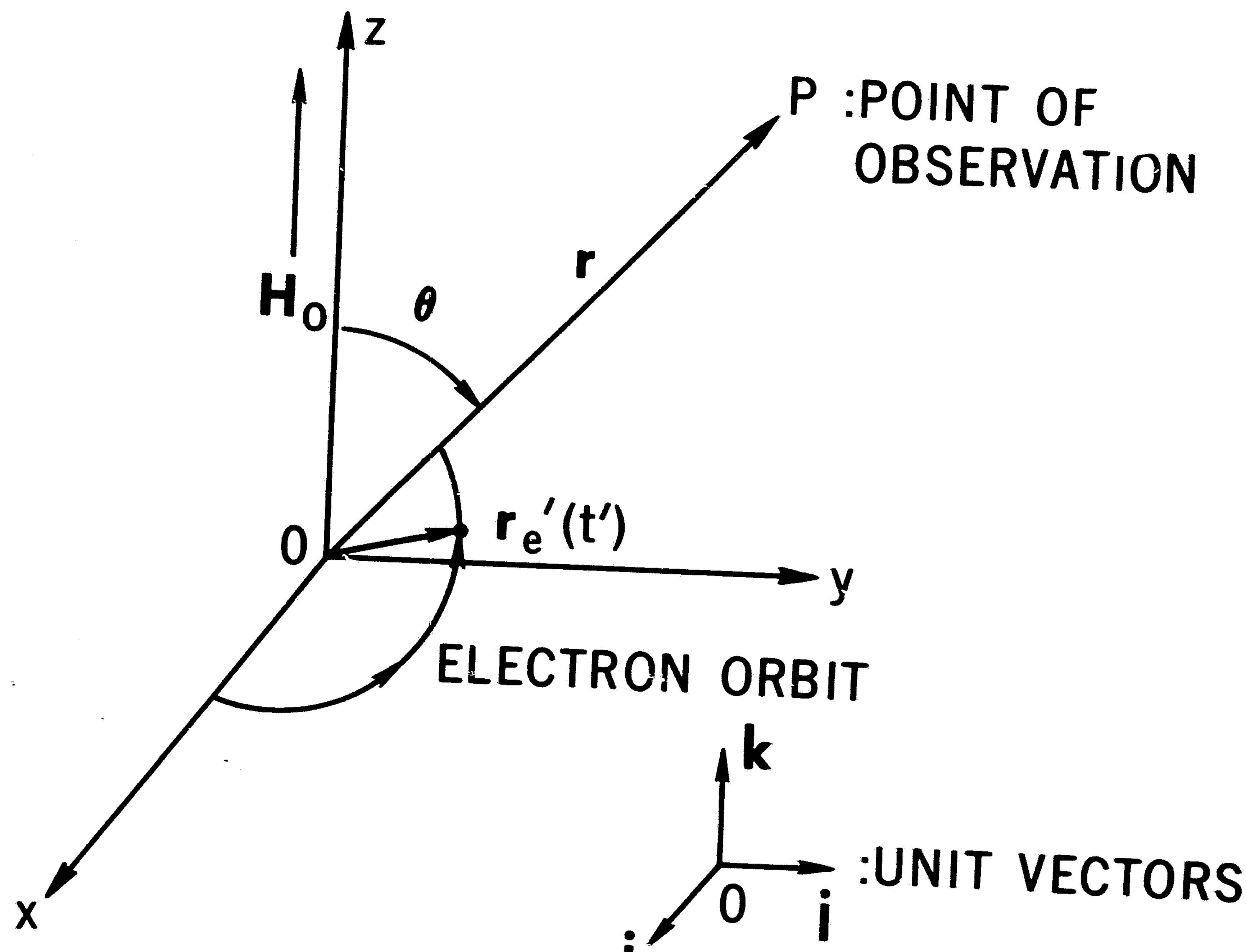
z
P :POINT OF
OBSERVATION
r
H0
θ
0
re′(t′)
y
ELECTRON ORBIT
x
k
:UNIT VECTORS
0
i
i

Figure 2