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ABSTRACT

The atmospheric tide within the thermosphere driven by the solar EUV heat input has a predominant diurnal component, contrary to the tides within the lower atmosphere where the semidiurnal component prevails. Based on this observatorial fact, a theory of the thermospheric tides is given in which the spherical harmonics of lowest degree are considered as the eigenfunctions of the problem. Perturbation theory is used which leads to a complete separation between a zonal and a diurnal system. A general solution is given and the zonal and the diurnal horizontal windsystem at 300 km height is calculated. This windsystem has an equatorward directed component at low latitudes during daytime which allows to explain the equatorial F2 anomaly as resulting from an enhancement of electrons at $\pm 10°$ latitude due to such winds.

ON THE DIURNAL TIDE WITHIN THE THERMOSPHERE

1. Introduction

While the knowledge as well as an adequate basic theory of the tidal motion of the lower atmosphere are more than 100 years old [see e.g. the review articles of Wilkes (1951), Kertz (1957) and Siebert (1961)], the diurnal tidal variation of the thermospheric density has been discovered only 10 years ago from satellite drag measurements (see e.g. the review article by Priester, et al. 1967). The most important difference between the atmospheric tides within lower and upper atmosphere is the predominance of the semidiurnal component in the lower atmosphere and the predominance of the diurnal component in the upper atmosphere.

There exists general agreement that the driving force within the whole atmosphere is predominately solar radiation — absorbed by water vapor in the troposphere, by ozon in the stratosphere and mesosphere and by oxygen within the thermosphere (Lindzen, 1967). Thus, the diurnal component of the driving force is much larger than the higher harmonics. The predominance of the semidiurnal tide within the lower atmosphere in spite of its smaller excitement has originally been considered as a resonance effect. One eigenvalue of the atmosphere comes very close to 12 hours, the period of the semidiurnal solar tide. Recently, however, suggested by Siebert (1961) and quantitatively worked out by Kato (1966) and Lindzen (1967), it has been shown that the diurnal component really is the extraordinary one being an evanescent mode with negative equivalent depth so that its propagation is suppressed. It will be shown in section two of this paper that the diurnal solar tidal wave becomes a propagation mode within the thermosphere, thus predominating the tidal motion there.

Tidal theory of the thermosphere started with the one dimensional model of Harris and Priester (1962) in which heating within a vertical column and its resultant dynamic behavior have been studied. The main shortcomings of this theory were that the calculated diurnal density variation was not in agreement with the observations. Harris and Priester therefore introduced an ad hoc second heat source in order to shift amplitude and phase of the calculated values toward the measured data. It has been shown from a two dimensional equatorial model (Volland et al., 1969) that the natural response time of the thermosphere with respect to maximum solar heating is just 2 hours in agreement with the observations and that a diurnal tidal gravity wave propagating from the lower atmosphere into the thermosphere is superposed to the diurnal waves generated within the thermosphere by the EUV heat source.

A complete theory of the diurnal tide within the thermosphere is more complicated than the classical tidal theory because heat conduction has to be taken into account. Therefore, the basic tidal differential equation becomes a fourth order equation contrary to the second order Laplace equation in the classical case. Moreover, the number of eigenvalues doubles, and there does not more exist a clear distinction between propagation modes and evanescent modes because the eigenvalues are complex. On the other hand, the theory of the thermospheric tides in some ways becomes easier to handle than the classical theory because of the predominance of the diurnal component and because the boundaries of a given thermospheric model behave like free internal boundaries.

Subject of this paper is to give the basic equations and general solutions of the thermospheric diurnal tide. Furthermore, we shall discuss the implications of these solutions on the thermospheric wind system and calculate the horizontal wind field at 300 km altitude. An exact numerical integration of the tidal motions will be given in a subsequent paper.

2. Propagation Modes and Evanescent Modes

In this section we will give a very crude description of the propagation modes and the evanescent modes in tidal theory based on a plane model of the earth and neglecting Coriolis force and heat conductivity. We do this merely to achieve a basic physical picture of the nature of the wave modes and its change within the different atmospheric levels.

We idealize the earth's atmosphere during equinox by a rectangular trough of length $\lambda_y = 2\pi R$ and of width λ_x. R is the earth's radius. x represents the meridional component, positive in south direction. y represents the longitudinal component, positive in east direction. The length of λ_x will be defined below. The lower boundary of the trough is the earth's surface. The trough is open in the upward direction. The atmosphere within this trough shall be isothermal and shall behave adiabatically. In such atmosphere the diurnal solar heating can generate gravity waves of the following type:

$$c_{nm} = A_{nm} \cdot \cos(k_{nm} y) \cdot \exp(j\,\omega_m \tau \pm j\,\Lambda_{nm} z) \tag{1}$$

with

$$k_{nm} = \frac{\pi}{\lambda_x} = \frac{\sqrt{n(n+1) - m^2}}{R} \qquad \text{x-component of horizontal wave number}$$

$$k_m = \frac{2\pi m}{\lambda_y} = \frac{m}{R} \qquad \text{y-component of horizontal wave number}$$

$$k_n = \frac{\sqrt{n(n+1)}}{a} = \sqrt{k_m^2 + k_{nm}^2} \qquad \text{horizontal wave number}$$

$\omega_m = m\,\Omega$ angular frequency of the wave mode

Ω angular frequency of the rotating earth

$\tau = t + \frac{k_m}{\omega_m}\, y$ local time

t universal time

x, y, z coordinates in south-, east- and upward direction

(m, n) integers determining the domain of wave mode.

$$\Lambda_{nm} = \frac{1}{C}\sqrt{\omega_m^2 - \omega_a^2 - \frac{C^2 k_n^2}{\omega_m^2}\left(\omega_m^2 - \omega_b^2\right)} \sim \frac{\omega_a}{C}\sqrt{S_{nm}^2 - 1} \tag{2}$$

eigenvalue of the gravity wave of domain (n, m) [see Hines (1960), Equation (14); and Siebert (1961), Equation (4.12)]

C velocity of sound

$$\omega_a = \frac{\gamma g}{2C}$$

$$\omega_b = \frac{2\sqrt{\gamma - 1}}{\gamma}\,\omega_a \qquad \text{Brunt-Väisälä frequency}$$

g gravitational acceleration force

γ ratio between the specific heats at constant volume and at constant pressure.

$$S_{nm} = \frac{2C\sqrt{n(n+1)(\gamma-1)}}{mR\gamma\Omega} \tag{3}$$

The last approximation in Equation (2) is valid because within the ranges of frequency and height, which we consider, it is $\omega_a >> \omega_m$.

The plus sign of the exponential factor in Equation (1) is related to an upward propagating gravity wave, the minus sign is related to a downward propagating wave. We define as propagation modes waves with real eigenvalues Λ_{nm} [$S_{nm} > 1$ in Equation (2)] and as evanescent modes waves with imaginary eigenvalues $\Lambda_{nm} = j|\Lambda_{nm}|$ [$S_{nm} < 1$ in Equation (2)]. Evanescent waves are suppressed very rapidly during their propagation. They do not participate in an effective wave energy flux. A wave changes its character from a propagation mode into an evanescent mode at $S_{nm} = 1$. The boundary condition of zero wind velocity at the ground ($z = 0$) determines the remaining amplitude factors A_{nm} of a combination of an upgoing and an downgoing wave of the same domain (n, m).

The specific choice of the x-component of the horizontal wavenumber in Equation (1) has been made in order to allow a direct comparison with the eigenvalues on a sphere. These are for a nonrotating spherical earth the spherical functions $P_{nm}(\theta)$, where P_{nm} are the associated Legendre's polynoms and θ is the colatitude. This comparison can be made by replacing the term $\cos(k_{nm}y)$ in Equation (1) by P_{nm}. In the classical theory of tides the horizontal wavenumber k_n is equal to

$$k_n = \frac{\omega_m}{\sqrt{gh_n}} = \frac{\omega_m}{V_n}, \qquad (4)$$

where h_n is the so-called equivalent denth, because water waves in shallow oceans of depth h_n have in fact the horizontal phase velocity

$$V_n = \sqrt{gh_n} .$$

According to our definition, k_n is the horizontal wavenumber which determines the horizontal phase velocity V_n of the characteristic waves of frequency ω_m.

Table I contains the numbers S_{n1} of the first three symmetric functions P_{n1} from Equation (3) for the diurnal modes ($m = 1$). Row 1 in Table I has been calculated for an isothermal lower atmosphere with temperature $T_0 = 250°$ ($C =$ 320 m/sec). We notice that the eigenfunction P_{11} has a value $S_{11} < 1$ and thus belongs to the evanescent modes, while the higher eigenfunctions P_{31} and P_{51} are propagation modes. In an exact calculation taking into account Coriolis force the eigenfunctions P_{nm} must be replaced by the so-called Hough functions

$$\theta_{nm} = \sum_{\ell=m}^{\infty} C_{\ell m}^{(n)} P_{\ell m}$$

which are sums of the associated Legendre's polynoms centered around the predominant function P_{nm}. The determination of the eigenvalues of θ_{nm} is much more complicated than in the case of the nonrotating earth. Here, imaginary horizontal wavenumbers k_n and therefore negative equivalent depths h_n appear in Equation (2) for θ_1, and θ_{31} (Lindzen, 1967). Thus, the condition of an evanescent mode for P_{11} is even stronger valid than in the case of a nonrotating earth. It is this fact, the evanescent mode type of the function

$$\theta_{11} \sim C_{11}^{(1)} P_{11} ,$$

which causes its weak generation within the lower atmosphere in spite of the predominant exitation force which goes into this wave domain by the solar heating.

Table II, row 2, contains the numbers S_{n1} calculated for a temperature of $T_0 = 1000°K$ ($C = 750$ m/sec) which is a typical thermospheric temperature. Now we notice that even the first eigenfunction P_{11} belongs to the propagation modes. Heat conduction and ion drag reduce the influence of the Coriolis force within the thermosphere (Volland, 1969). On the other hand, the difference between propagation modes and evanescent modes becomes vague because the eigenvalues are complex giving rise to wave energy dissipation even of propagation modes. Full wave calculations of the propagation of free internal gravity waves within the thermosphere show that maximum transmission of gravity waves exists near $S_{nm} = 1$ and that with increasing $S_{nm} > 1$ the transmission coefficient of gravity waves drops like

$$|T_{nm}| \sim (S_{nm})^{-b\,z} \qquad (S_{nm} > 1)$$

where b is a positive number depending on frequency and the model adopted (Volland, 1968b).

We expect therefore that the high temperature, the specific geometric and rotational data of the earth and the influence of heat conduction and ion drag cause the preference of the function P_{11} in the diurnal tidal wave propagation within the thermosphere. This is exac‸ what we notice from the observed data of density and temperature (see e.g. Jacchia and Slowey, 1967; Priester et al., 1967, Taeusch et al., 1968). The predominance of the term P_{11} begins at about 110 km altitude as we can see from the analysis of the geomagnetic S_q current (Kato, 1956).

3. The Basic Equations

The complete basic equations of conservation of mass, momentum and energy and the equation of state written in spherical coordinates are given elsewhere (Priester et al., 1967). The following analysis makes use of some assumptions:

a. <u>Application of the perturbation theory and use of a given mean atmospheric model</u>

Perturbation theory implies that all physical parameters, depending on time or latitude, are small compared with the mean values averaged over time and sphere. Thus, higher order terms of these parameters shall be neglected. This is a sufficient approximation as long as the relative magnitudes of the coefficients in the series of spherical harmonics are smaller than 0.3:

$$\frac{|a_{nm}|}{a_{00}} < 0.3 \qquad (n > 0) \; ,$$

This condition holds in fact for all parameters of the thermosphere below about 400 km altitude (see e.g. CIRA, 1965; Volland et al., 1969). Numerical calculations, which will be presented elsewhere, therefore, are limited to this height range.

An important result of perturbation theory is, that the variations can be completely separated into domains of numbers m, because any coupling between the different domains m only occurs via nonlinear terms of the perturbation variables. The validity of perturbation theory, on the other hand, justifies the static diffusion model of Jacchia (1964). Therefore, we can use mean values of temperature, density and molecular weight from the Jacchia model. Molecular weight as well as the coefficient of heat conductivity show small time and latitudinal variations below 400 km altitude in the Jacchia model. These values therefore are considered to be only height dependent. The collision number between ions and neutrals is proportional to the ion density. This value strongly depends on height, latitude and time. In order to be consistent in our model we can only take into account its height and latitude dependence.

b. Use of a given independent EUV heat source

The EUV heating of the neutral atmosphere via inelastic collisions with the ions depends itself on the temporal state of density, temperature and composition of the neutral air. For convenience we shall use a given independent EUV heat source which can e.g. be derived from the calculations of Harris and Priester (1962) and extrapolated into the whole sphere. In view of the large uncertainties in the determination of the efficiency factor of the heating this does not add very much to the errors already involved in the problem.

c. Negligence of molecular viscosity

Molecular viscosity mainly influences the horizontal wind field via vertical windshear. It can be shown that below 400 km its influence on the dynamics of the diurnal tides within the thermosphere is small compared with the acceleration force or the ion drag force in the equation of conservation of horizontal momentum (Geisler, 1967; Kohl and King, 1967; Volland et al., 1969). The negligence of viscosity greatly simplifies the handling of the differential equations.

d. Wind components are perturbation variables

We shall treat all wind components including a mean longitudinal wind v_{00} as perturbation variables, thus neglecting squares and higher order terms. This is justified because generally the condition

$$\frac{|a_{nm}|}{C} < 0.3$$

holds below 400 km altitude. a_{nm} is the coefficient of one of the wind components within the series of spherical harmonics, and C is the velocity of sound in the particular height. Furthermore, we shall assume that the ions are completely bound by the earth's magnetic field, but can move freely with the neutral wind along the geomagnetic lines of force. Then the relation between ion velocity $\vec{v}_i$ and neutral air velocity $\vec{v}$ is given by

$$\vec{v}_i = \frac{(\vec{v} \cdot \vec{B}_0)\, \vec{B}_0}{B_0^2} \tag{5}$$

where $\bar{B}_0$ is the magnetic field of a dipole which approximates the geomagnetic field.

e. Predominance of lowest domain numbers n

Our model is valid for equinox conditions. Therefore, the asymmetric spherical harmonics of density and temperature variation disappear. Moreover, we confine ourselves to the diurnal component of the tides. We treat, therefore, only the domains $m = 0$ and $m = 1$ of number m. As outlined in section 2 we have reason to believe from a theoretical point of view that the term P_{11} is predominant throughout the thermosphere. This is confirmed by the measurements (Priester et al., 1967; Jacchia and Slowey, 1967; Taeusch et al., 1968). Likewise, the density observations show a predominance of the term P_{20} in the meridional variation (Newton, 1968). We, therefore, assume that within the domains $m = 0$ and $m = 1$ the higher order n- terms of the density variations are small compared with the perturbation coefficients of P_{20} and P_{11} and can be neglected. Equivalent assumptions are made for the other physical parameters. This classification of the coefficients of the spherical harmonics into three classes of successive importance:

$$\begin{aligned} a_{00} &>> |a_{20}| >> |a_{n0}| \qquad (n \geqq 4) \\ a_{00} &>> |a_{11}| >> |a_{n1}| \qquad (n \geqq 3) \end{aligned} \tag{6}$$

greatly simplifies the calculations. It means that we can in fact treat the spherical harmonics P_{20} and P_{11} as the eigenfunctions of systems $m = 0$ and 1, respectively.

With the assumptions outlined above the system of equations governing the dynamic behavior of the thermosphere is given by

$$\frac{\partial \rho}{\partial t} + \rho_{00} \operatorname{div} \vec{v} + w \frac{\partial \rho_{00}}{\partial r} = 0 \tag{7}$$

$$\rho_{00} \left\{ \frac{\partial u}{\partial t} + \nu \sin^2 I \, u - 2\Omega \cos\theta \, v - \nu \sin I \cos I \, w \right\} + \frac{\partial p}{r \, \partial \theta} = 0 \tag{8}$$

$$\rho_{00} \left\{ \frac{\partial v}{\partial t} + 2\Omega \cos\theta \, u + \nu \, v + 2\Omega \sin\theta \, w \right\} + \frac{1}{r \sin\theta} \frac{\partial p}{\partial \lambda} = 0 \tag{9}$$

$$\rho_{00} \left\{ \frac{\partial w}{\partial t} - \nu \sin I \cos I \, u - 2\Omega \sin\theta \, v + \nu \cos^2 I \, w \right\} + g\rho + \frac{\partial p}{\partial r} = 0 \tag{10}$$

$$\rho_{00} c_v \frac{\partial T}{\partial t} + \rho_{00} c_v w \frac{\partial T_{00}}{\partial r} - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \kappa_{00} \frac{\partial T}{\partial r} \right) - \kappa_{00} \Delta T + p_{00} \operatorname{div} \vec{v} = Q \tag{11}$$

$$p = \frac{R}{M_{00}} \rho T \tag{12}$$

Equations (7) to (12) are the equations of conservation of mass, momentum and energy, respectively. Equation (12) is the equation of state. It is

(θ, λ, r) spherical coordinates

t time

$\vec{v} = (u, v, w)$ vector of wind velocity with i's components in south-, east- and upward direction.

p pressure (p_{00} mean pressure)

ρ density (ρ_{00} mean density)

T temperature (T_{00} mean temperature)

Ω angular frequency of the earth's rotation

ν number of collisions between the ions and one neutral

I Inclination angle of the earth's magnetic dipole field

g magnitude of gravitational force

c_v specific heat at constant volume

κ_{00} mean coefficient of heat conductivity

R gas constant

M_{00} mean molecular weight

$$\operatorname{div} \vec{v} = \frac{1}{r \sin\theta} \frac{\partial}{\partial\theta} (\sin\theta \, u) + \frac{1}{r \sin\theta} \frac{\partial v}{\partial\lambda} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 w)$$

$$\Delta = \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial\lambda^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta}\right)$$

4. Development of the Physical Parameters in Series of Spherical Harmonics

Our model shall be valid during equinox conditions. Therefore all asymmetric spherical functions in the development of density, temperature and pressure disappear. Because of our assumption in Equation (6) higher order terms of the coefficients in the series of spherical harmonics are neglected. We use the unnormalized associated Legendre polynoms, which are

$$P_{00} = 1$$

$$P_{10} = \cos\theta \quad ; \quad P_{11} = \sin\theta$$

$$P_{20} = \frac{1}{2}(3\cos^2\theta - 1) \; ; \quad P_{21} = 3 \sin\theta \cos\theta$$

We start with the expressions for temperature, density, pressure and vertical wind. They have the general form

$$a(r, \theta, \lambda) = a_{00}(r)P_{00} + a_{20}(r)P_{20}(\theta) + a_{11}(r)P_{11}(\theta)\, e^{j\Omega\tau} \qquad (13)$$

Here, of course, the mean vertical wind disappears ($w_{00} = 0$). Because a_{11} (r) is complex, we consider the real part

$$2 \text{ Real} \left\{ a_{11}(r)\, e^{j\Omega\tau} \right\} = |A_{11}| \cos\left\{ \Omega(\tau - \tau_{11}) \right\}$$

as the physical solution.

$$|A_{11}| = 2|a_{11}|$$

is the magnitude and

$$\tau_{11} = -\frac{\arg(a_{11})}{\Omega}$$

is the time lag.

It is

$$\tau = t + \frac{\lambda}{\Omega}$$

the local time, while t is the universal time.

In the development of horizontal winds into series of spherical harmonics it is convenient to add a factor $(\sin\theta)^{\pm 1}$ to the series. This allows a direct transformation of the spherical harmonics of equal domain m within the system of Equations (7) to (12), merely by using the well known recurrence formulae of spherical functions.

We have to introduce a mean longitudinal wind v_{00} in order to obtain a unique solution of the problem. Meridional winds as well as longitudinal winds are symmetrical with respect to the equator during equinox. Longitudinal winds and the zonal-component of the meridional wind disappear at the poles. But the time dependent component of the meridional wind is different from zero there, because a finite pressure gradient can be maintained at the poles.

In order to fulfill the conditions outlined above we introduce the expressions

$$u(r, \theta, \lambda) = \sin\theta\, u_{10}(r) P_{10}(\theta) + \frac{u_{21}(r)}{\sin\theta} P_{21}(\theta)\, e^{j\Omega\tau} \tag{14}$$

$$v(r, \theta, \lambda) = \sin\theta \left\{ v_{00}(r) P_{00} + v_{20}(r) P_{20}(\theta) + v_{11}(r) P_{11}(\theta)\, e^{j\Omega\tau} \right\} \tag{15}$$

We have reasons to expect that this representation of the horizontal winds possesses the fastest convergence rate, thus allowing as a first order approximation the negligence of higher order terms in the series.

We furthermore assume known functions of altitude of the coefficient of heat conductivity $\kappa_{00} = A(r)\sqrt{T_{00}(r)}$ and of molecular height $M_{00}(r)$. Likewise, the EUV heat source Q is assumed to be developed in a general form of Equation (13). As one can easily see from a harmonic analysis e.g. of a Chapman-function of the ion production the three coefficients in Equation (11) constitute the predominant heating terms.

Finally, we take a known function of the collision number

$$\nu = \nu_{00}(r) P_{00} + \nu_{20}(r) P_{20}(\theta) \tag{16}$$

because the coefficients ν_{00} and ν_{20} have comparable magnitudes. The angle of incidence I of the geomagnetic dipole is related to the colatitude in a well known manner. We obtain

$$\left.\begin{array}{l}\cos^2 I \\ \\ \sin^2 I \\ \\ \sin I \cos I\end{array}\right\} = \left\{\begin{array}{l}\sin^2\theta \\ \\ 4\cos^2\theta \\ \\ 2\sin\theta\cos\theta\end{array}\right\} \frac{1}{(1 + 3\cos^2\theta)}$$

and develop

$$\frac{1}{1 + 3\cos^2\theta} = \alpha_{00}P_{00} + \alpha_{20}P_{20}(\theta) + \ldots$$

with

$$\alpha_{00} = 0.604$$
$$\alpha_{20} = -0.520,$$

neglecting higher order terms.

With these assumptions we enter Equations (7) to (12), use the recu:rence formulae of the spherical harmonics (see e.g. Jahr.ke and Emde, 1945), collect all coefficients belonging to the domain (n, m) and ob· ·in for each Equation (i), numbered according to the Equation-number (7) to (1. ; expressions of the general form

$$\sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} F_{nm}^{(i)}(r)\, P_{nm}(\vartheta)\, e^{i m \lambda} = 0 \tag{17}$$

Because of the orthogonality of the spherical harmonics each coefficient $F_{nm}^{(i)}$ must be zero. Thus, we find the set of equations (for abbreviation we write: $\frac{\partial}{\partial r} = {}'$)

$$F_{00}^{(10)} \equiv \frac{1}{\rho_{00}} p'_{00} + g = 0 \tag{18a}$$

$$F_{00}^{(11)} \equiv \frac{1}{r^2} (r^2 \kappa_{00} T'_{00})' + Q_{00} = 0 \tag{18b}$$

$$F_{00}^{(12)} \equiv p_{00} - \frac{R}{M_{00}} \rho_{00} T_{00} = 0 \tag{18c}$$

$$F_{20}^{(10)} \equiv \frac{1}{\rho_{00}} p'_{20} + g \frac{\rho_{20}}{\rho_{00}} + \frac{8}{15} \gamma_{10} u_{10} + \frac{4}{3} \Omega v_{00} - \frac{20}{21} \Omega v_{20} - \frac{2}{3}\left(\gamma_{00} - \frac{5}{7}\gamma_{20}\right) w_{20} = 0 \tag{19a}$$

$$F_{20}^{(11)} \equiv -\frac{1}{r^2}\left(r^2 \kappa_{00} T'_{20}\right)' + \frac{6}{r^2} \kappa_{00} T_{20} + \left(c_v \rho_{00} T'_{00} - \frac{p_{00}}{\rho_{00}} \rho'_{00}\right) w_{20} - Q_{20} = 0 \tag{19b}$$

$$F_{20}^{(7)} \equiv \frac{1}{r^2}\left(r^2 w_{20}\right)' + \frac{\rho'_{00}}{\rho_{00}} w_{20} + \frac{2}{r} u_{10} = 0 \tag{19c}$$

$$F_{20}^{(12)} \equiv p_{20} - \frac{R}{M_{00}} (\rho_{00} T_{20} + \rho_{20} T_{00}) = 0 \tag{19d}$$

$$F_{10}^{(8)} \equiv \frac{6}{7} \gamma_{10} u_{10} - \Omega v_{00} - \frac{\Omega}{7} v_{20} - \left(\gamma_{00} + \frac{\gamma_{20}}{7}\right) w_{20} - \frac{3}{2} \frac{p_{20}}{r \rho_{00}} = 0 \tag{19e}$$

$$F_{00}^{(9)} \equiv 2\Omega u_{10} + 5\left(\nu_{00} - \frac{\nu_{20}}{5}\right) v_{00} - \left(\nu_{00} - \frac{5}{7}\nu_{20}\right) v_{20} - 2\Omega w_{20} = 0 \tag{19f}$$

$$F_{20}^{(9)} \equiv 2\Omega u_{10} - (7\nu_{00} - 5\nu_{20}) v_{00} + \left(5\nu_{00} + \frac{\nu_{20}}{35}\right) v_{20} + 10\Omega w_{20} = 0 \tag{19g}$$

$$F_{11}^{(10)} \equiv \frac{1}{\rho_{00}} p_{11}' + g\frac{\rho_{11}}{\rho_{00}} - \frac{6}{5}\gamma_{21}u_{21} - \frac{8}{5}\Omega v_{11} + \left(j\Omega - \frac{4}{5}\gamma_{11}\right) w_{11} = 0 \qquad (20a)$$

$$F_{11}^{(11)} \equiv -\frac{1}{r^2}\left(r^2 \kappa_{00} T_{11}'\right)' + \left(j\Omega\rho_{00}c_v + \frac{2}{r^2}\kappa_{00}\right) T_{11}$$

$$- \frac{p_{00}}{\rho_{00}} j\Omega\rho_{11} \left(c_v \rho_{00} T_{00}' - \frac{p_{00}}{\rho_{00}}\rho_{00}'\right) w_{11} - Q_{11} = 0 \qquad (20b)$$

$$F_{11}^{(7)} \equiv \frac{1}{r^2}\left(r^2 w_{11}\right)' + \frac{\rho_{00}'}{\rho_{00}} w_{11} - \frac{9}{4}\frac{u_{21}}{r} + \frac{j\,v_{11}}{r} + \frac{j\Omega\rho_{11}}{\rho_{00}} = 0 \qquad (20c)$$

$$F_{11}^{(12)} \equiv p_{11} - \frac{R}{M_{00}}\left(\rho_{00}T_{11} + \rho_{11}T_{00}\right) = 0 \qquad (20d)$$

$$F_{21}^{(8)} \equiv \left(j\Omega + \frac{12}{7}\gamma_{21}\right) u_{21} - \frac{8}{21}\Omega v_{11} - \frac{8}{21}\gamma_{11}w_{11} + \frac{p_{11}}{3r\rho_{00}} = 0 \qquad (20e)$$

$$F_{11}^{(9)} \equiv \frac{3}{2}\Omega u_{21} + \left(j\Omega + \nu_{00} - \frac{\nu_{20}}{5}\right) v_{11} + 2\Omega w_{11} + j\,\frac{5}{4}\frac{p_{11}}{r\rho_{00}} = 0 \qquad (20f)$$

with

$$\beta_{00} = \nu_{00}\alpha_{00} + \frac{\nu_{20}\alpha_{20}}{5}$$

$$\beta_{20} = \nu_{00}\alpha_{20} + \nu_{20}\alpha_{00} + \frac{2}{7}\nu_{20}\alpha_{20}$$

with

$$\gamma_{00} = \frac{\beta_{20}}{5}; \; \gamma_{11} = \beta_{00} - \frac{\beta_{20}}{5}$$

$$\gamma_{10} = \beta_{00} + \frac{2}{5}\beta_{20}\,; \quad \gamma_{21} = \beta_{00} + \frac{\beta_{20}}{7}$$

$$\gamma_{20} = \beta_{00} + \frac{2}{7}\beta_{20}$$

The right hand side of Equation (18a) is not exactly zero but consists of terms of the same order of magnitude as the terms in Equation (19f). As one can easily see, these terms are two orders of magnitudes smaller than the terms on the left hand side in Equation (18a). If they are neglected, Equation (18a) is the barometric height formula. Now the system of Equations (18) is completely decoupled from the system of Equations (19) and gives the mean values averaged over time and sphere.

Equations (19) belonging to the domain $m = 0$ describe the time independent latitudinal variations of the thermospheric parameters. This system will be called the zonal system. Equations (20) belonging to the domain $m = 1$ give the diurnal variations. We call this domain the diurnal system.

5. General Solution

The approximate validity of the barometric height formula [Equation (18a)] allows the determination of mean temperature, pressure and molecular weight from the observed mean density by a static diffusion model (Jacchia, 1964). In our treatment of the diurnal tide within the thermosphere we therefore start with these known parameters of the thermosphere and use them for the normalization of the set of Equations (19) and (20). We write for abbreviation

$$
\begin{aligned}
w^{(0)} &= w_{20}/C & w^{(1)} &= w_{11}/C \\
p^{(0)} &= p_{20}/p_{00} & p^{(1)} &= p_{11}/p_{00} \\
T^{(0)} &= T_{20}/T_{00} & T^{(1)} &= T_{11}/T_{00} \\
y^{(0)} &= \kappa_{00} T'_{20}/(Cp_{00}) & y^{(1)} &= \kappa_{00} T'_{11}/(Cp_{00}) \\
V^{(0)} &= v_{00}/C & \rho^{(1)} &= \rho_{11}/\rho_{00} \\
v^{(0)} &= v_{20}/C & v^{(1)} &= v_{11}/C \\
u^{(0)} &= u_{10}/C & u^{(1)} &= u_{21}/C \\
\rho^{(0)} &= \rho_{20}/\rho_{00} \\
Q^{(0)} &= Q_{20}/(\Omega p_{00}) & Q^{(1)} &= Q_{11}/(\Omega p_{00})
\end{aligned}
\tag{21}
$$

We eliminate the parameters $\rho^{(i)}$, $v^{(i)}$, $u^{(i)}$ and $V^{(0)}$ from Equations (19d) to (19g) and (20d) to (20f), respectively, and obtain two independent systems of first order linear ordinary differential equations, which we write in concise matrix form

$$
\frac{d\mathbf{e}^{(i)}}{d\xi} = \mathbf{K}^{(i)}(\xi)\,\mathbf{e}^{(i)} + \mathbf{h}^{(i)}(\xi) \tag{22}
$$

with

$$\mathbf{e}^{(i)'} = \begin{pmatrix} w^{(i)} \\ p^{(i)} \\ T^{(i)} \\ y^{(i)} \end{pmatrix}; \quad \mathbf{h}^{(i)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -Q^{(i)} \end{pmatrix}; \quad (i = 0, 1)$$

and

$$\xi = \frac{\Omega}{C} r$$

C = velocity of sound

$\mathbf{K}^{(0)}(\xi)$ is a 4×4-matrix with real elements, $\mathbf{K}^{(1)}(\xi)$ is a 4×4-matrix with complex elements. The elements of $\mathbf{K}^{(i)}$ can be found from Equations (19) and (20), respectively. The elements of $\mathbf{K}^{(i)}$ as well as of $\mathbf{e}_i$ are dimensionless and are of comparable magnitudes.

Equation (22) can be solved by standard methods. If we approximate the real atmosphere by a number of isothermal homogeneous slabs of thickness $\Delta\xi_r$ in which the elements of $\mathbf{K}_r^{(i)}$ are constant, then the solution of Equation (22) is

$$\mathbf{e}(\xi_n) = \mathbf{P}_0^n \, \mathbf{e}(\xi_0) + \boldsymbol{\Phi}_0^n \tag{23}$$

with

$$\mathbf{P}_i^j = \prod_{r=0}^{j-i-1} e^{K_{j-r}\,\Delta\xi_{j-r}}\, \Gamma_{j-r}\,; \quad \mathbf{P}_0^0 = \mathbf{E}$$

$$\Phi_i^j = \sum_{r=i+1}^{j} \mathbf{P}_r^j\, \Lambda_r\, h_r\, \Delta\xi_r$$

$$e^{K_i\,\Delta\xi_i} = \sum_{\ell=0}^{\infty} \frac{(K_i\,\Delta\xi_i)^\ell}{\ell\,!}$$

$$\Lambda_i = \sum_{\ell=0}^{\infty} \frac{(K_i\,\Delta\xi_i)^\ell}{(\ell+1)\,!}$$

$\mathbf{E}$ unit matrix,

$$\Gamma_i = \begin{pmatrix} C_{i-1}/C_i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & T_{i-1}/T_i & 0 \\ 0 & 0 & 0 & C_{i-1}/C_i \end{pmatrix}$$

The matrix Γ_i matches the boundary conditions of continuous wave parameters w, p, T and κ T' at the boundary between two adjacent slabs of temperature T_{i-1} and T_i.

6. The Horizontal Wind System

From Equations (19e) to (19g) and from (20e) and (20f), respectively, we notice that the horizontal winds are connected with the vertical wind and the pressure by linear equations. Since the vertical wind is at least one order of magnitude smaller than the horizontal wind components, we can neglect the vertical wind. In the following discussion we shall neglect furthermore the higher order terms ν_{20} and α_{20} of the collision number and of the magnetic dip angle, because they influence the numerical values of the winds but not their general behavior. In the numerical calculations which will follow in the next section we shall however take into account these higher order terms.

Negligence of vertical winds and of latitudinal dependence of collision number and dip angle leads then to very simple relations between the horizontal wind components and the pressure field. These are for the zonal system (m = 0)

$$\begin{aligned} \frac{v_{00}}{C} &= -\frac{7}{6} \frac{1}{\left(1 + \frac{\nu_{00}^2}{\Omega^2} \alpha_{00}\right) \gamma \xi} \frac{p_{20}}{p_{00}} \\ v_{20} &= 2\, v_{00} \\ u_{10} &= -\frac{3}{2} \frac{\nu_{00}}{\Omega} v_{00} \, ; \end{aligned} \tag{24}$$

For the diurnal system (m = 1) we obtain

$$\frac{u_{21}}{C} = \frac{17}{9}\,\frac{j\left(1 - \frac{7}{17}\,j\,\frac{\nu_{00}}{\Omega}\right)}{\gamma\xi\Delta}\,\frac{p_{11}}{p_{00}}$$

$$\frac{v_{11}}{C} = -\frac{49}{12}\left(1 - \frac{60}{49}\,j\,\frac{\nu_{00}}{\Omega}\,\alpha_{00}\right)\frac{1}{\gamma\xi\Delta}\,\frac{p_{11}}{p_{00}} \tag{25}$$

with

$$\Delta = 1 - \frac{7}{3}\,j\,\frac{\nu_{00}}{\Omega}\left(1 + \frac{12}{7}\,\alpha_{00}\right) - 4\frac{\nu_{00}^2}{\Omega^2}$$

$$\xi = \frac{\Omega}{C}\,r$$

$$\gamma = \frac{c_p}{c_v} \sim 1.5$$

For large collision numbers ($\nu_{00}/\Omega >> 1$) it is

$$\left.\begin{matrix} v_{00} \\ v_{20} \end{matrix}\right\} \propto \frac{1}{\nu_{00}^2} \quad \text{and} \quad \left.\begin{matrix} u_{10} \\ |u_{21}| \\ |v_{11}| \end{matrix}\right\} \propto \frac{1}{\nu_{00}}$$

For small collision numbers ($\nu_{00}/\Omega << 1$) only the meridional zonal velocity $u_{10} \propto \nu_{00}$ depends on ν_{00}. The magnitude of the horizontal winds therefore is very sensitive with respect to the collisions between neutrals and ions.

The longitudinal wind of system $m = 0$ is

$$v_{zonal} = \sin\theta\ (v_{00}P_{00} + v_{20}P_{20}) = 3\sin\theta\ \cos^2\theta\ v_{00} \tag{26}$$

This wind is the geostrophic wind due to the meridional pressure gradient p_{20}. Collisions between neutral and ions cause a mean meridional wind u_{10} to flow in the direction of the pressure gradient. Collisions moreover reduce the magnitude of the geostrophic wind v_{zonal}.

A mean longitudinal wind [Equation (26)] means that the whole atmosphere rotates with respect to the earth surface. It has been pointed out by King-Hele and Allan (1966) that such wind exists within thermospheric heights. King-Hele's winds, derived from the observation of the changes in the orbit inclinations of satellites, blow from west to east and are of the order of 100 m/sec. One of King-Hele's explanations for the origin of these winds is in fact the geostrophic wind hypothesis. The pressure gradient built up to generate this wind must be directed toward the poles. Observation of Jacchia and Slowey (1967) as well as density measurements on board of Explorer 32 (Newton, 1968) however show that during low solar activity ($F < 150$) the zonal pressure gradient is directed toward the equator above 300 km altitude ($p_{20} > 0$). According to Equation (24) this pressure gradient causes a meridional wind u_{10} blowing toward the equator which is in agreement with a hypothesis to explain the equatorial F2 anomaly (Mayr and Volland, 1969). At moderate solar activity ($F = 150$) the zonal pressure p_{20} disappears and it seems likely that it becomes negative at high solar activity. According to Equations (24) and (26) we therefore expect a zonal wind blowing toward the west at low solar activity and blowing toward the east at high solar activity. Geisler (1967) too finds this westward zonal wind at solar minimum conditions from his numerical study of the horizontal wind-system in the thermosphere. King-Hele however states from his observations that the mean wind always blows toward the east regardless of the solar activity with even a greater magnitude at solar maximum than at solar minimum.

There exist several possibilities to explain this discrepancy between King-Hele's observations and the theoretical results. First, it will be shown in Figure 2 that due to the higher order terms of the collision number and the geo-magnetic dip angle the calculated zonal wind blows toward the east within ±15° from the equator at 300 km altitude. Second, we neglected in our calculations the time dependence of the collision number. A diurnal term ν_{11} would couple the zonal system with the diurnal system. It has been pointed out by Volland (1966), that such coupling would in fact give rise to an additional eastward component of the mean wind. Third, electric fields, entirely neglected in our treatment, could cause ion drift such that the observed mean wind results (Hines, 1965).

We should bear in mind however that the zonal pressure gradient not only depends on solar activity but also on height. For a better understanding of this problem we need numerical calculations of the three dimensional wind and pressure fields, and furthermore we have to wait for more detailed observational data of the zonal wind depending on height, latitude and solar activity before a final answer can be given.

7. Numerical Calculations of the Horizontal Wind Field

For a numerical study of the horizontal wind field at 300 km altitude we used the following data valid at low solar activity (F = 100):

Mean temperature and molecular weight from CIRA model 3. From (Volland et al., 1969) the relative diurnal pressure

$$
\begin{aligned}
(A_{11})_p &= \frac{2|p_{11}|}{p_{00}} = 0.48 \\
(\tau_{11})_p &= 14^{30} \text{ local time}
\end{aligned}
\tag{27}
$$

From Newton (1968) the zonal pressure, extrapolated from his density and temperature data

$$
\frac{p_{20}}{p_{00}} = 0.25. \tag{28}
$$

The collision number from the maps of the F2 critical frequency (Martyn, 1955)

$$
\begin{aligned}
\nu_{00} &= 2.4 \times 10^{-4} \text{ sec}^{-1} \\
\nu_{20} &= -1.9 \times 10^{-4} \text{ sec}^{-1}.
\end{aligned}
\tag{29}
$$

We calculated the horizontal winds from Equations (19e) to (19g) and (20e) and (20f), respectively, and neglected the vertical wind components. Figures 1 and 2 give the calculated zonal winds

$$
\begin{aligned}
u_{zonal} &= \sin\theta \; u_{10} P_{10}(\theta) \\
v_{zonal} &= \sin\theta \left\{ v_{00} P_{00} + v_{20} P_{20}(\theta) \right\},
\end{aligned}
\tag{30}
$$

the mean longitudinal wind

$$v_{mean} = \sin\theta \, v_{00} P_{00} \tag{31}$$

and the magnitudes of the diurnal winds

$$\begin{aligned} u_{diurnal} &= 2|u_{21}| \frac{P_{21}(\theta)}{\sin\theta} \\ v_{diurnal} &= 2|v_{11}| \sin\theta \, P_{11}(\theta) \end{aligned} \tag{32}$$

versus latitude. Moreover, the total horizontal winds

$$\begin{aligned} u &= u_{zonal} + u_{diurnal} \cos\left[\Omega(\tau - \tau_{21})\right] \\ v &= v_{zonal} + v_{diurnal} \cos\left[\Omega(\tau - \tau_{11})\right] , \end{aligned} \tag{33}$$

where the time lags have been calculated to

$$\tau_{11} = 21^{00} \text{ local time}$$

$$\tau_{21} = 2^{20} \text{ local time},$$

are plotted for the time of maximum and minimum winds respectively.

We notice that due to the higher order terms ν_{20} and α_{20} of the collision number and of the geomagnetic dip angle the zonal longitudinal wind v_{zonal} has a small eastward component near the equator (Figure 2).

Furthermore, the meridional wind u blows at low latitudes toward the equator even at day time. This unexpected result follows from the relatively large zonal u_{10}-component which predominates the meridional velocity at low latitudes. Such equatorial wind at noon can in fact create the equatorial F2 anomaly: the relative maximum of the F2 electron density at ±10° latitude and the trough at the equator. The electrons drift upwards by the equatorial wind and cause an enhancement of the electron density near the equator (Mayr and Volland, 1969).

The wind u_{10} depends on the zonal pressure gradient p_{20}. This pressure gradient disappears at heights above 400 km at moderate solar activity and probably becomes negative with further increasing solar activity (Newton, 1968). As a consequence the equatorward directed wind at noon decreases or even reverses with increasing solar activity, giving rise to a weakening or disappearance of the F2 anomaly. This picture is in qualitative agreement with the observations.

An equatorward wind implies of course a total pressure gradient directed toward the equator. The combination of the pressure terms p_{20} and p_{11} chosen in Equations (27) and (28) therefore gives rise to a small relative pressure minimum at the equator at noon. The elongation of the density bulge at noon and at low solar activity observed by Jacchia and Slowey (1967) points in fact into this direction. It is possible that due to the low degree of space and time resolution in the satellite drag measurements this minimum has been overlooked.

Whether the p_{20} term is mainly created by solar EUV or by another heating mechanism (e.g.: the regular component of the precipitation of high energy electrons into the auroral zones related to the geomagnetic S_D-current) is an open question. The different components of the pressure gradient are certainly functions of height. Since most of the heat is deposited within a height range below 200 km we expect a pressure gradient built up there and directed away from the heating zone. In the case of solar EUV heating this would be a gradient toward the poles. Due to this pressure gradient a global wind system is set up, in which the vertical wind plays a decisive role. There must be a height range — probably the region above the main heating zone — where a return flow achieves flow continuity. This return flow however must be connected with a pressure gradient directed opposite to the pressure gradient within the main heating zone. Because of the important role of the vertical wind a quantitative treatment of such problem is only possible by a numerical integration of the dynamic equations of the spherical thermosphere [Equations (23)]. An attempt to determine the vertical winds from the pressure field of the Jacchia model and from the horizontal winds of Geisler (1967) via the equation of continuity (Dickinson and Geisler, 1968) suffers from the unaccuracy of the density observations below 200 km altitude.

Figure 3 finally presents the horizontal wind field in the northern hemisphere at 300 km height in a form used by Kohl and King (1967). This allows a direct comparison between Kohl and King's winds (their Figure 5) and our wind. The striking difference between both wind fields is the presence of the equatorward wind component at low latitudes and of daytime in our wind field. This component is due to the relatively large zonal wind. Its generating zonal pressure gradient p_{20} can be observed already in Jacchia and Slowey's (1967) data at low solar activity, though Jacchia's number value is smaller than the value observed by

Newton (1968). Kohl and King (1967) entirely neglected the zonal winds. They moreover used a constant collision number for the whole hemisphere.

Therefore we can only compare our diurnal windsystem (u_{21}, v_{11}) with their data, which is in reasonable agreement. It shows that slight differences in the numerical data of the pressure gradient p_{11} and of the collision number ν are relatively unimportant compared with the influence of the zonal winds.

Geisler's (1967) winds are very similar to Kohl and King's (1967) winds because they are based on the same calculation method. Geisler, however, found already the mean component v_{00} from Jacchia's data. Since Jacchia's zonal pressure component is supposed to be too small, Geisler could not detect the equatorial winds at daytime, though he observes "a convergence in low latitudes" in his horizontal wind field, which is the direct consequence of the superposition of the diurnal system with the zonal system.

8. Conclusion

The solar diurnal tides of the spherical thermosphere have been treated using spherical harmonics of lowest degree as eigenfunctions of the problem. Perturbation theory is a sufficient approximation within the thermosphere below 400 km altitude. It allows to determine the mean physical parameters like pressure, temperature and molecular weight from the observed density data by a static diffusion model (Jacchia, 1964). Thus, these data were the basis for a perturbation treatment of the hydrodynamic equations leading to a complete separation between a zonal component, depending only on latitude and height, and a diurnal component, dependent on latitude, height and local time. A general solution for both the zonal and the diurnal system is given.

The horizontal wind field turns out to depend mainly on the horizontal pressure gradients. Based on observed data for the zonal and the diurnal components of the pressure gradient, the horizontal wind at 300 km altitude and for low solar activity has been calculated. Its results show an equatorward directed wind flow at low latitudes even at day time which is due to the relatively large zonal pressure gradient. This wind could possibly cause the equatorial F2 anomaly: the trough in the electron density at the geomagnetic equator during low solar activity.

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Table I

Number S_{nm} of the .igenfunctions P_{nm} of the diurnal tidal component ($m = 1$) for two different isothermal atmospheres of temperature $T_0 = 250°K$ and $T_0 = 1000°K$.

	P_{11}	P_{31}	P_{51}
$T_0 = 250°K$	0.88	2.16	3.41
$T_0 = 1000°K$	2.16	5.29	8.36

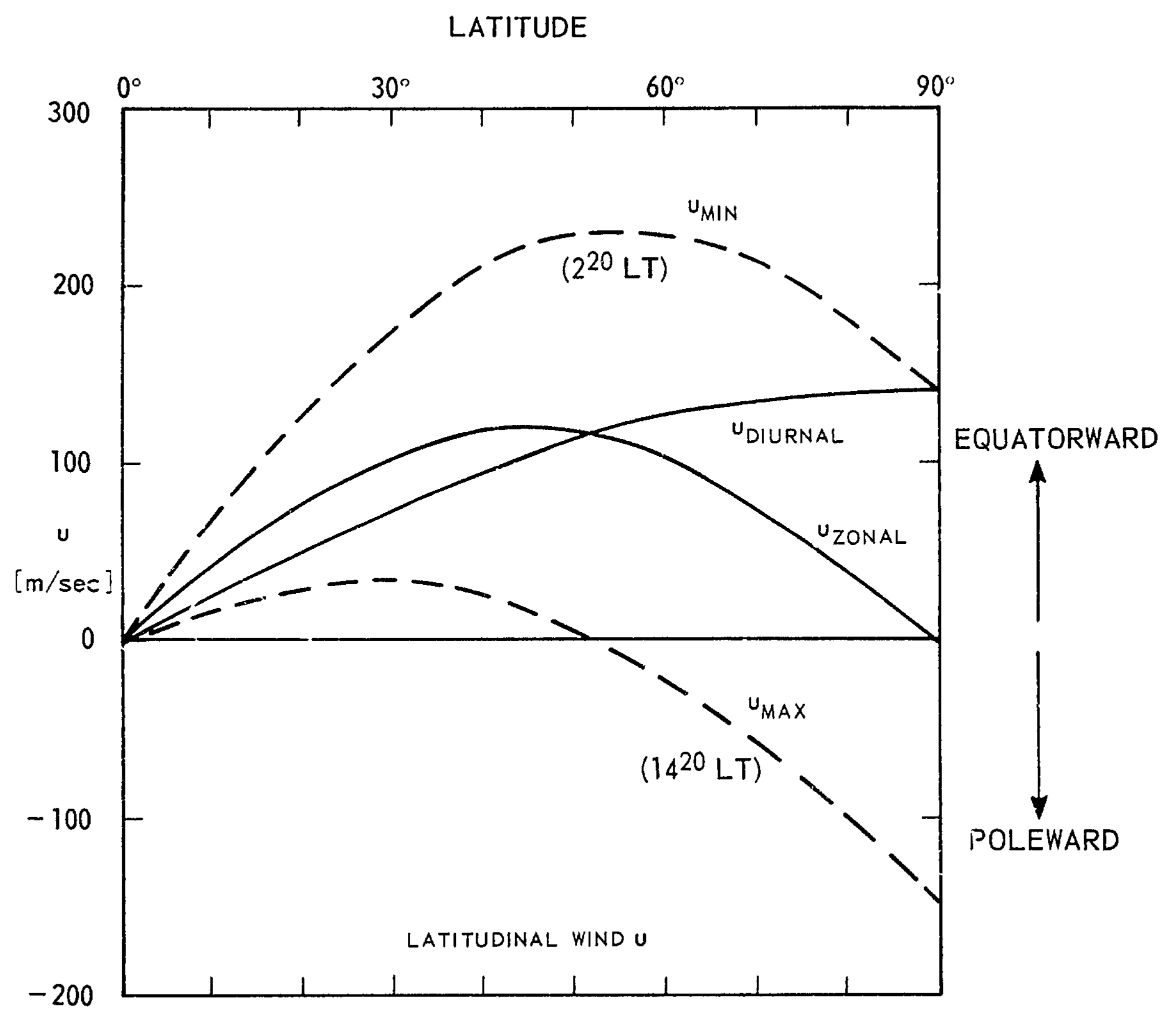

Figure 1. Components of calculated latitudinal wind u

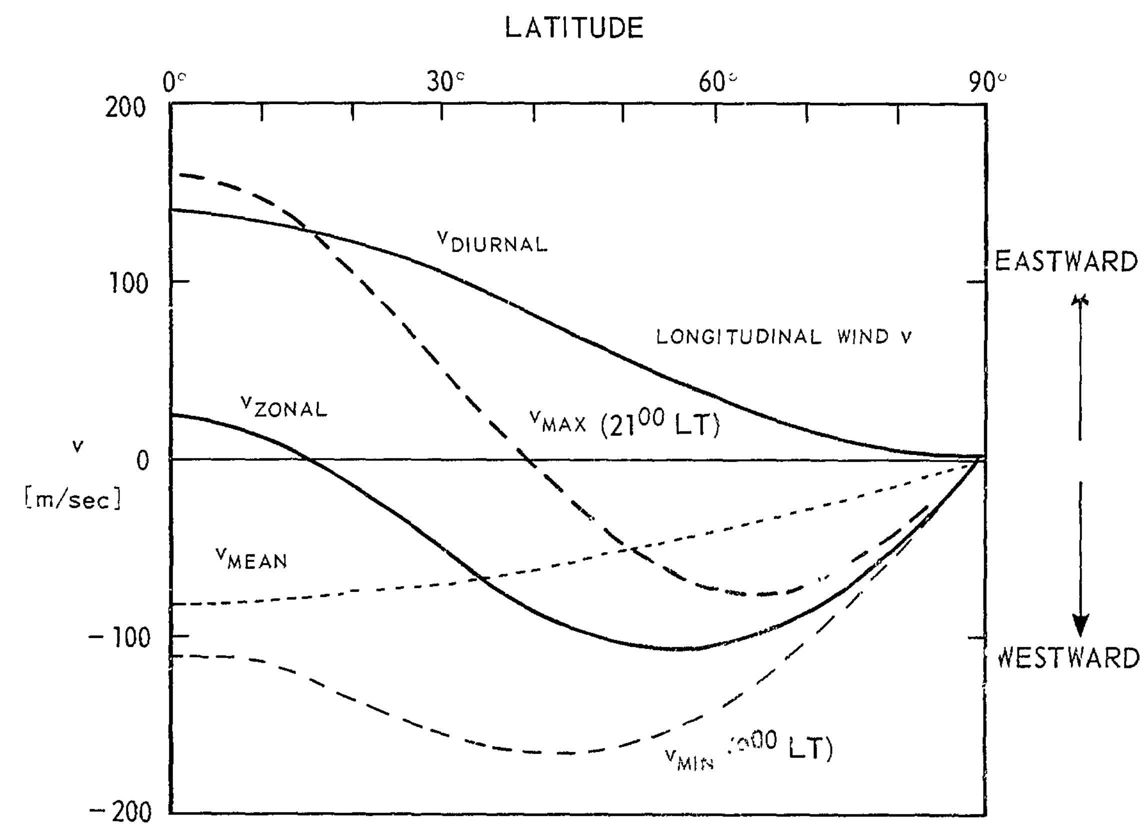

Figure 2. Components of calculated longitudinal wind v

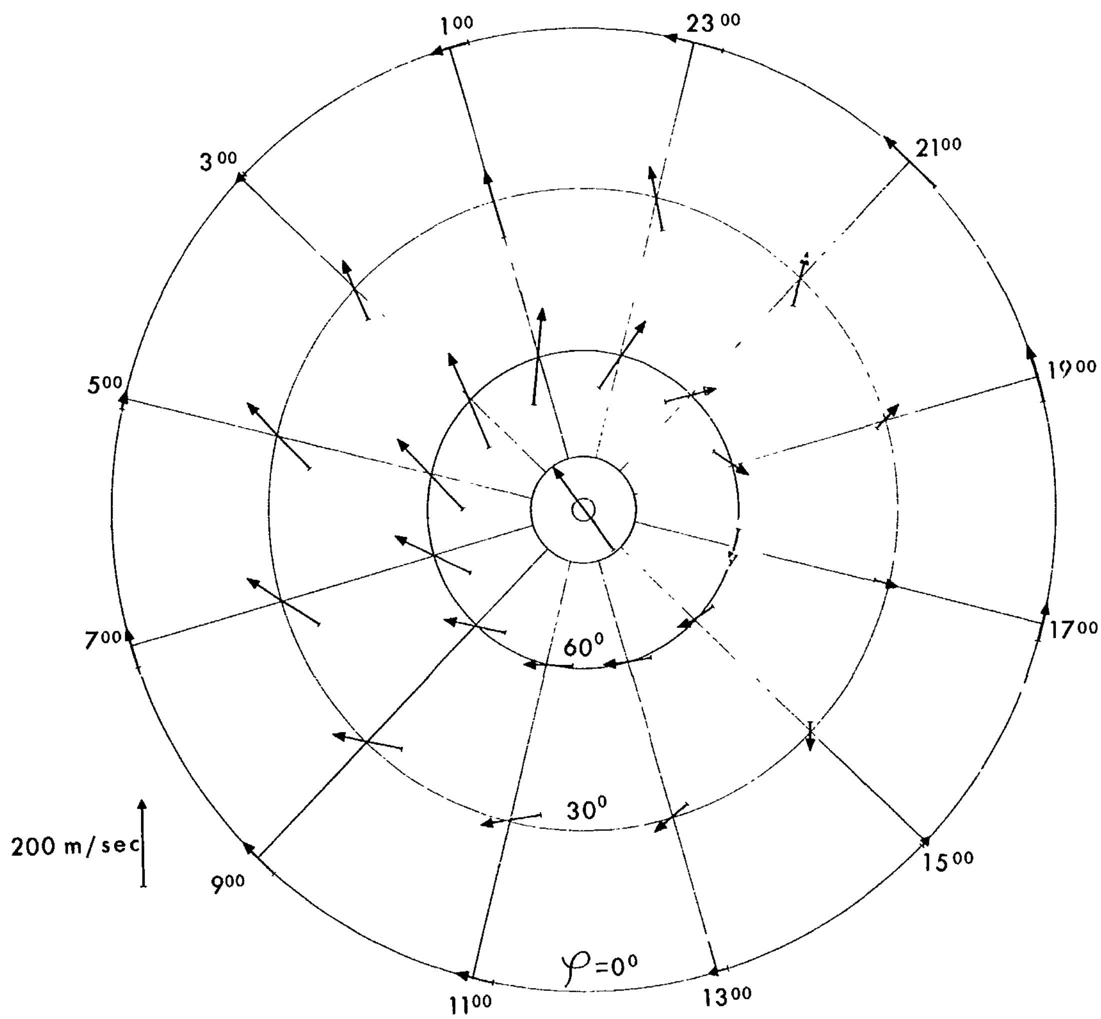

Figure 3. Calculated horizontal wind system at 300 km altitude in the northern hemisphere versus local time